Computer Algebra Independent Integration Tests

Summer 2024

2-Exponentials/2.5/164-2.5.4

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**919**]. This is test number [164].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	98.91 (909)	1.09 (10)
Mathematica	97.61 (897)	2.39 (22)
Fricas	88.36 (812)	11.64 (107)
Maple	82.26 (756)	17.74 (163)
Reduce	78.45 (721)	21.55 (198)
Giac	57.56 (529)	42.44 (390)
Mupad	52.45 (482)	47.55 (437)
Maxima	51.47 (473)	48.53 (446)
Sympy	21.11 (194)	78.89 (725)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

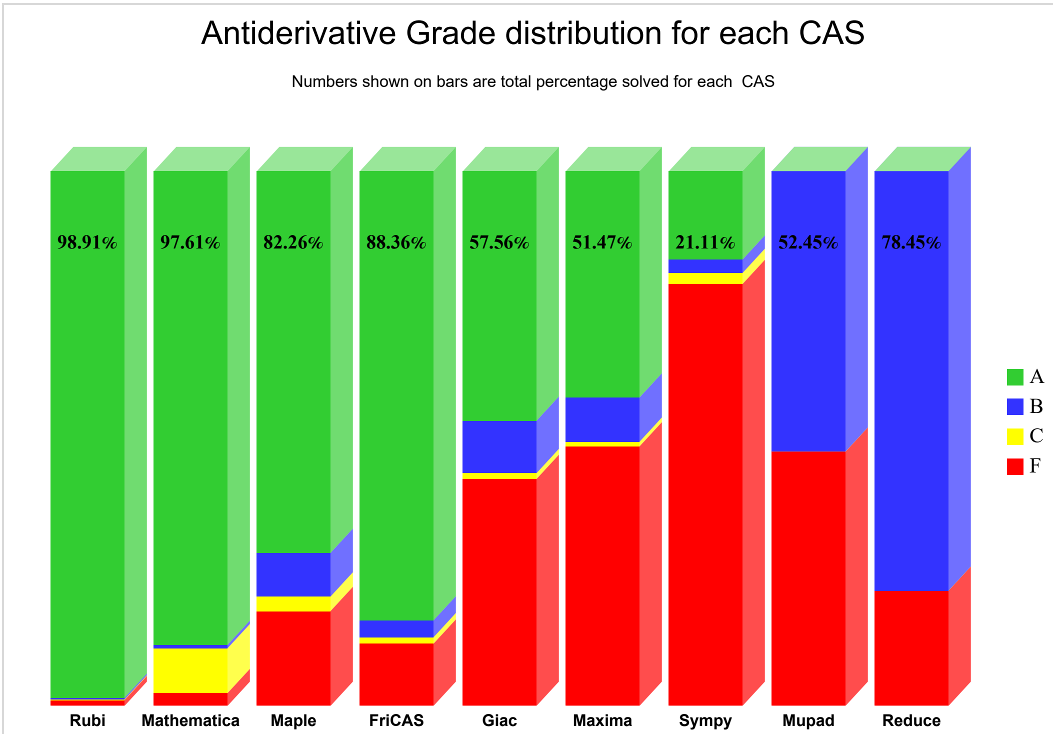
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

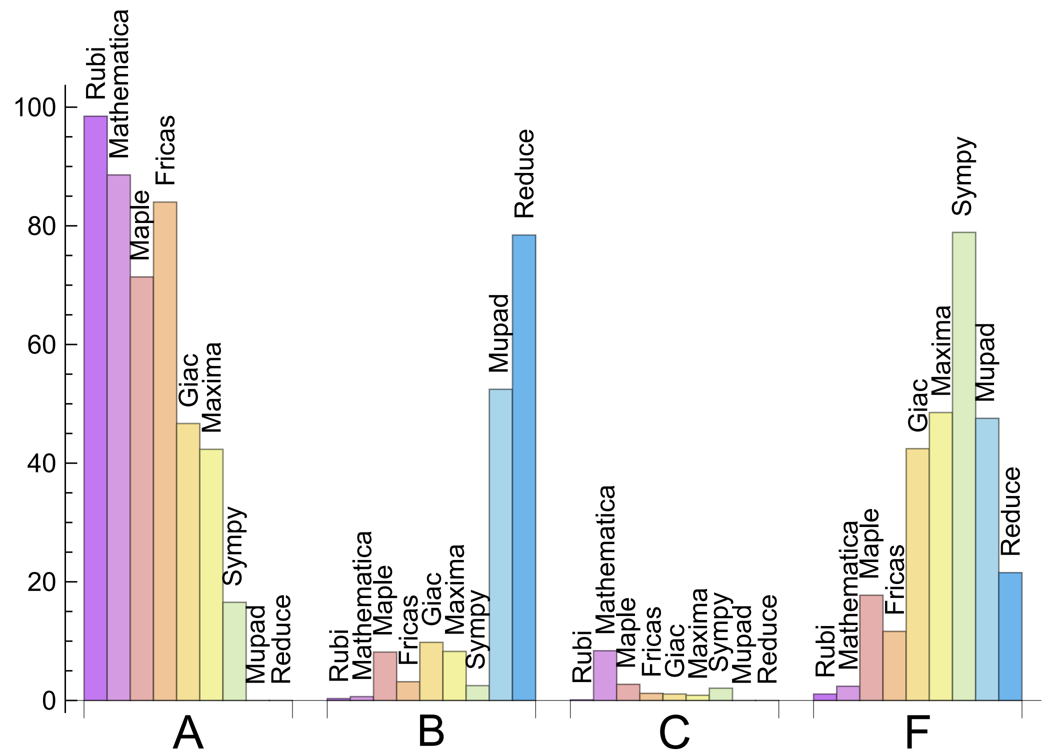
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.477	0.326	0.109	1.088
Mathematica	88.575	0.653	8.379	2.394
Fricas	84.004	3.156	1.197	11.643
Maple	71.382	8.161	2.720	17.737
Giac	46.681	9.793	1.088	42.437
Maxima	42.329	8.270	0.871	48.531
Sympy	16.540	2.503	2.067	78.890
Mupad	0.000	52.448	0.000	47.552
Reduce	0.000	78.455	0.000	21.545

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	10	100.00	0.00	0.00
Mathematica	22	81.82	18.18	0.00
Fricas	107	100.00	0.00	0.00
Maple	163	100.00	0.00	0.00
Reduce	198	100.00	0.00	0.00
Giac	390	50.51	0.26	49.23
Mupad	437	0.00	100.00	0.00
Maxima	446	100.00	0.00	0.00
Sympy	725	69.24	30.48	0.28

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.06
Fricas	0.10
Giac	0.14
Reduce	0.16
Mathematica	0.29
Maple	0.29
Rubi	0.68
Sympy	3.25
Mupad	8.55

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	79.57	1.22	70.00	0.71
Mupad	92.07	1.22	78.00	0.91
Reduce	94.28	0.92	67.00	0.85
Maxima	110.57	1.51	93.00	1.03
Rubi	111.43	0.97	95.00	1.00
Giac	114.51	1.44	82.00	0.98
Maple	118.62	2.02	85.50	0.90
Sympy	120.88	1.60	58.00	1.03
Fricas	140.01	1.85	98.50	1.12

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

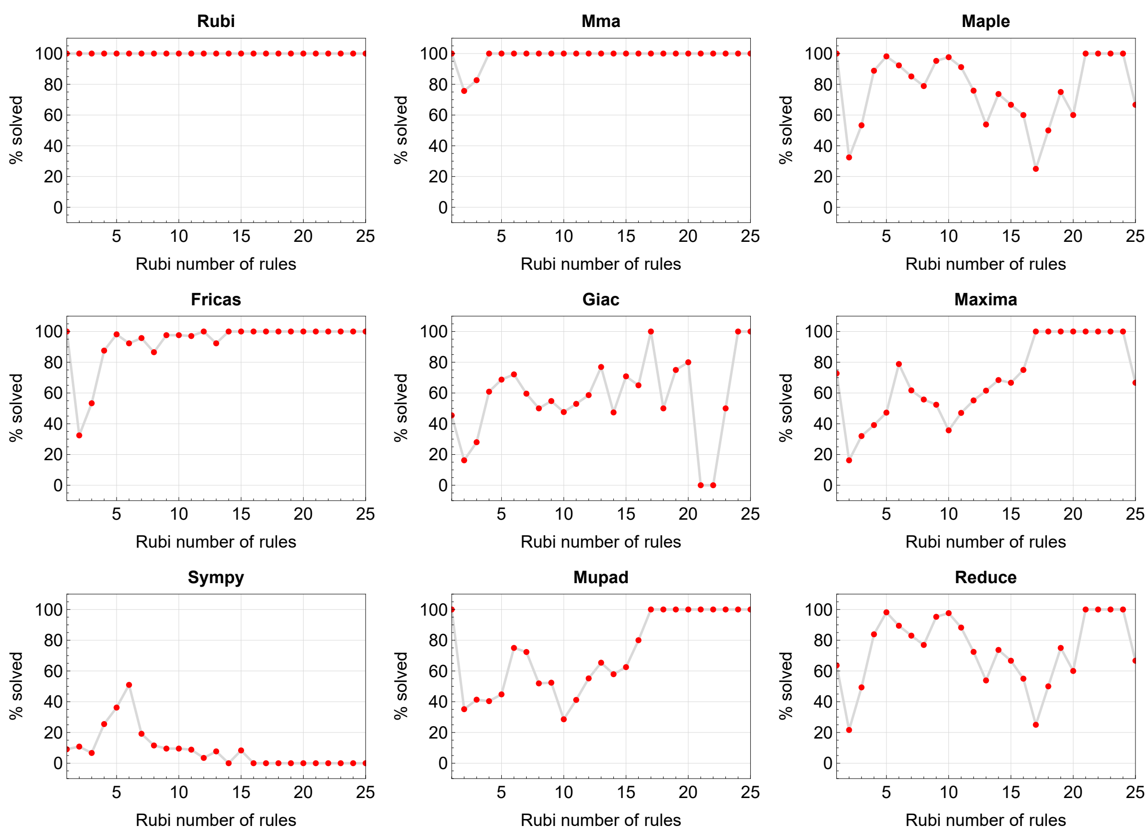


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

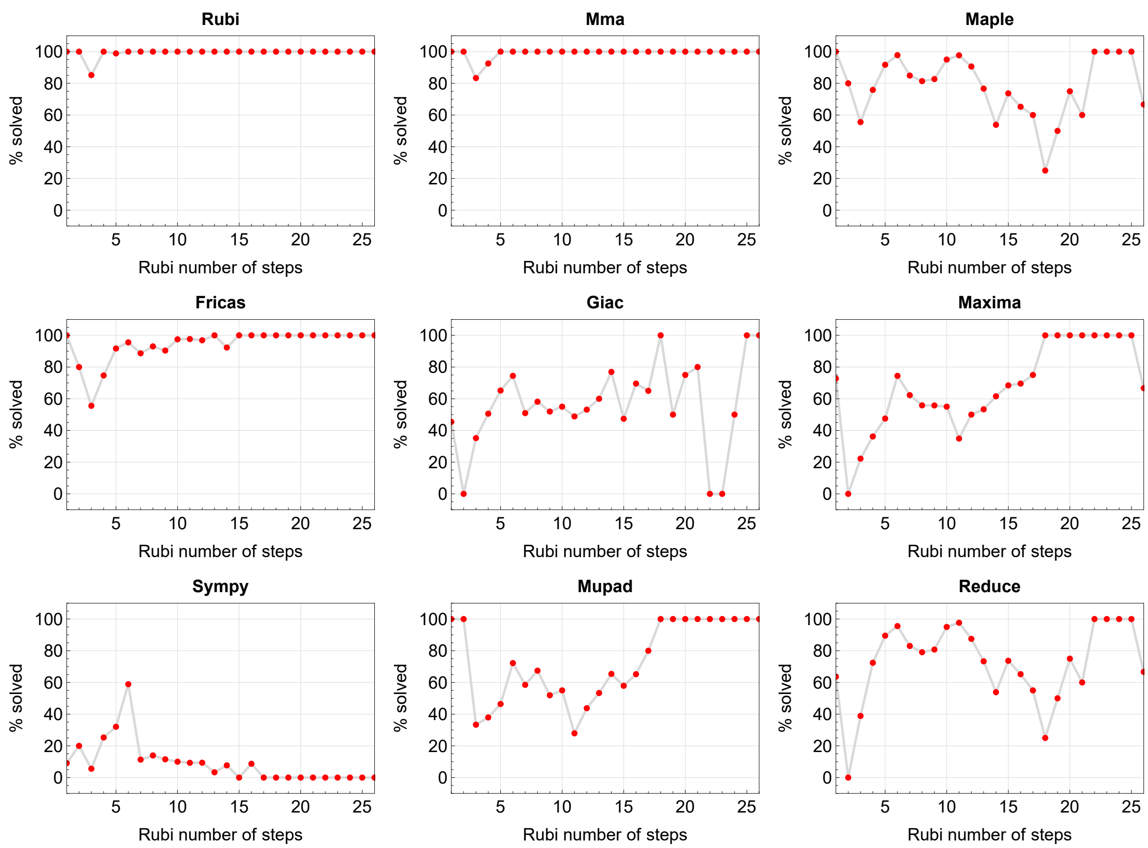


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

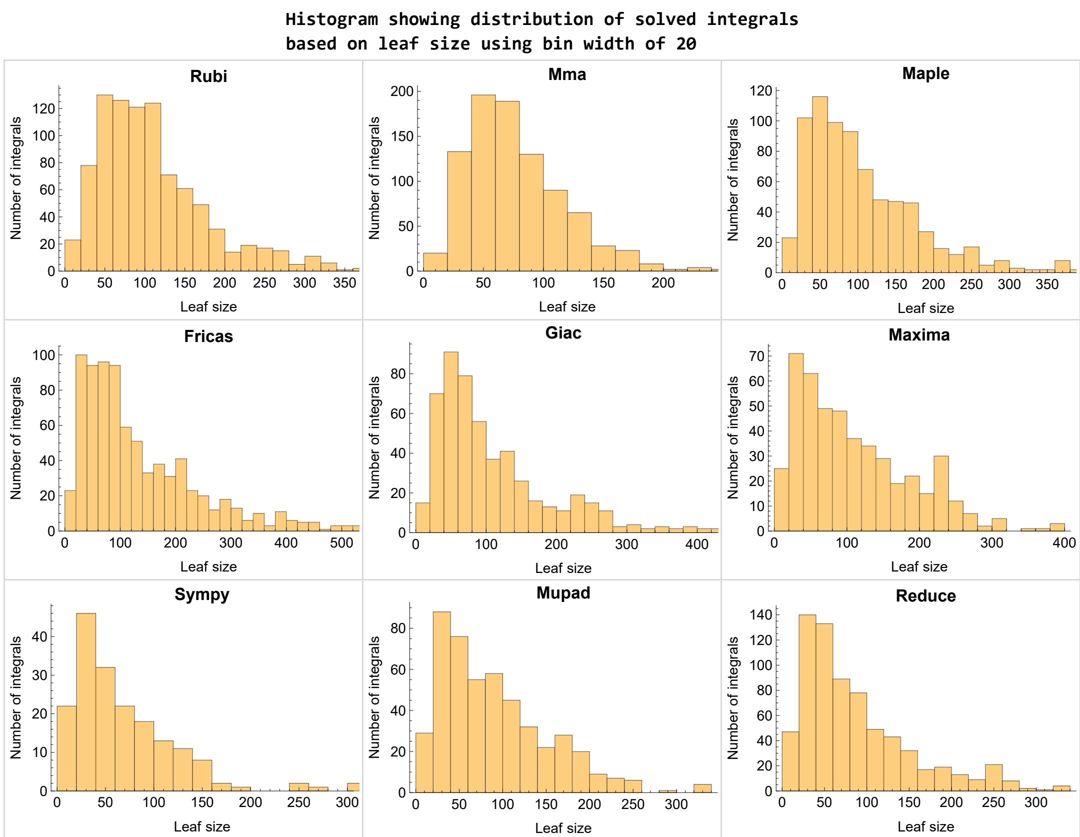


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

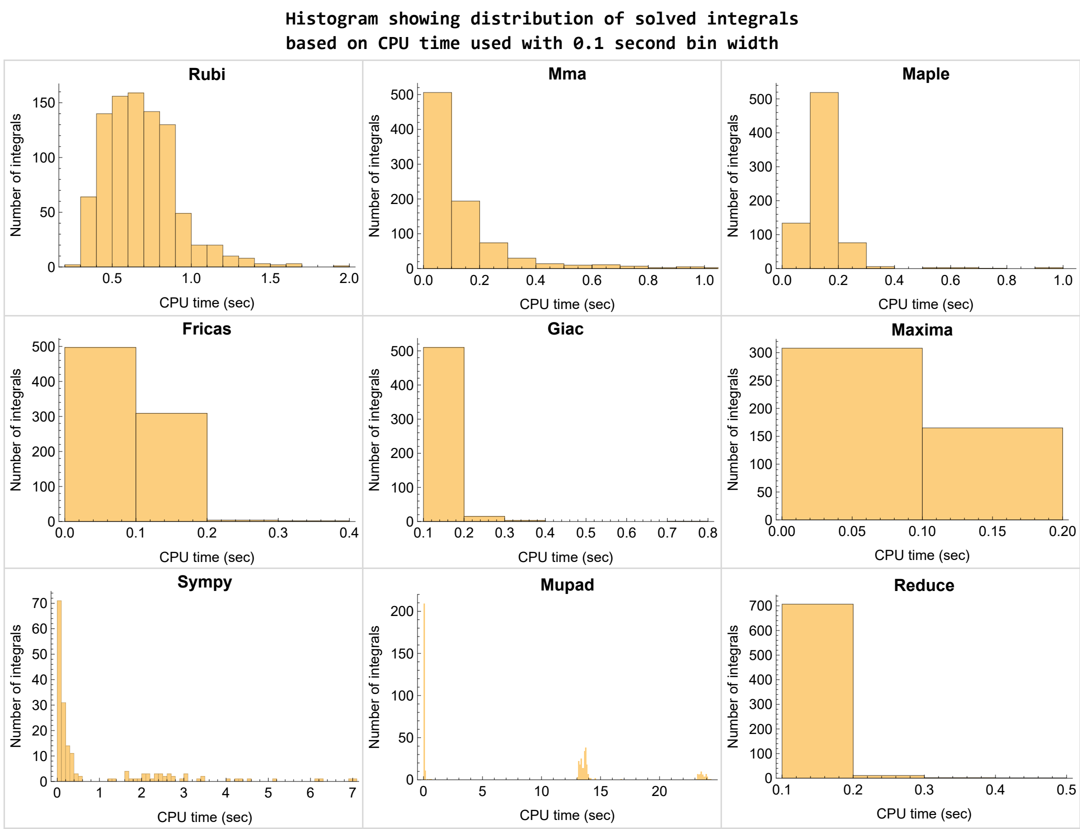


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

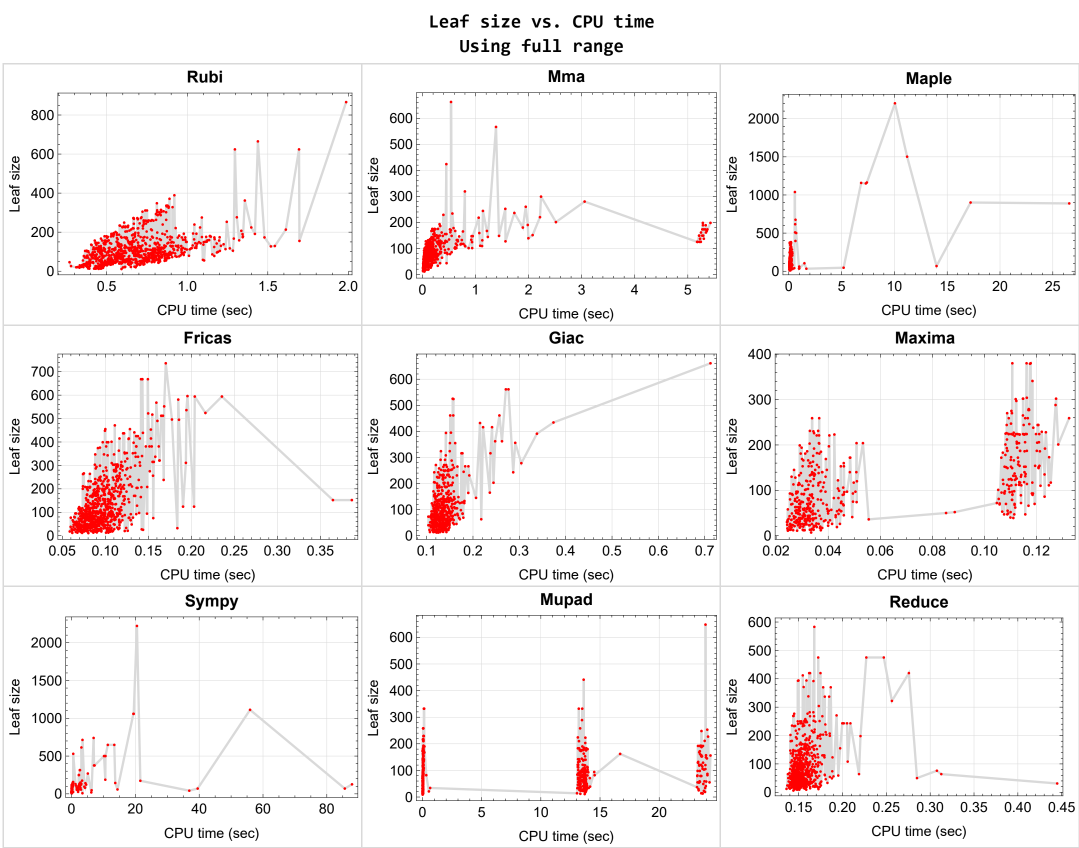


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {72, 73, 74, 75, 81, 82, 83, 84, 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 111, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136, 137, 138, 139, 165, 166, 167, 168, 183, 184, 185, 306, 309, 310, 559, 560, 561, 562, 563, 565, 917}

Mathematica {5, 8, 144, 145, 146, 147, 279, 355, 385, 386, 387, 444, 445, 446, 447, 713, 716, 718, 722, 724, 730, 731, 732, 733, 734, 735, 736, 744, 745, 747, 755, 771, 772, 789, 805}

Maple {125, 127}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

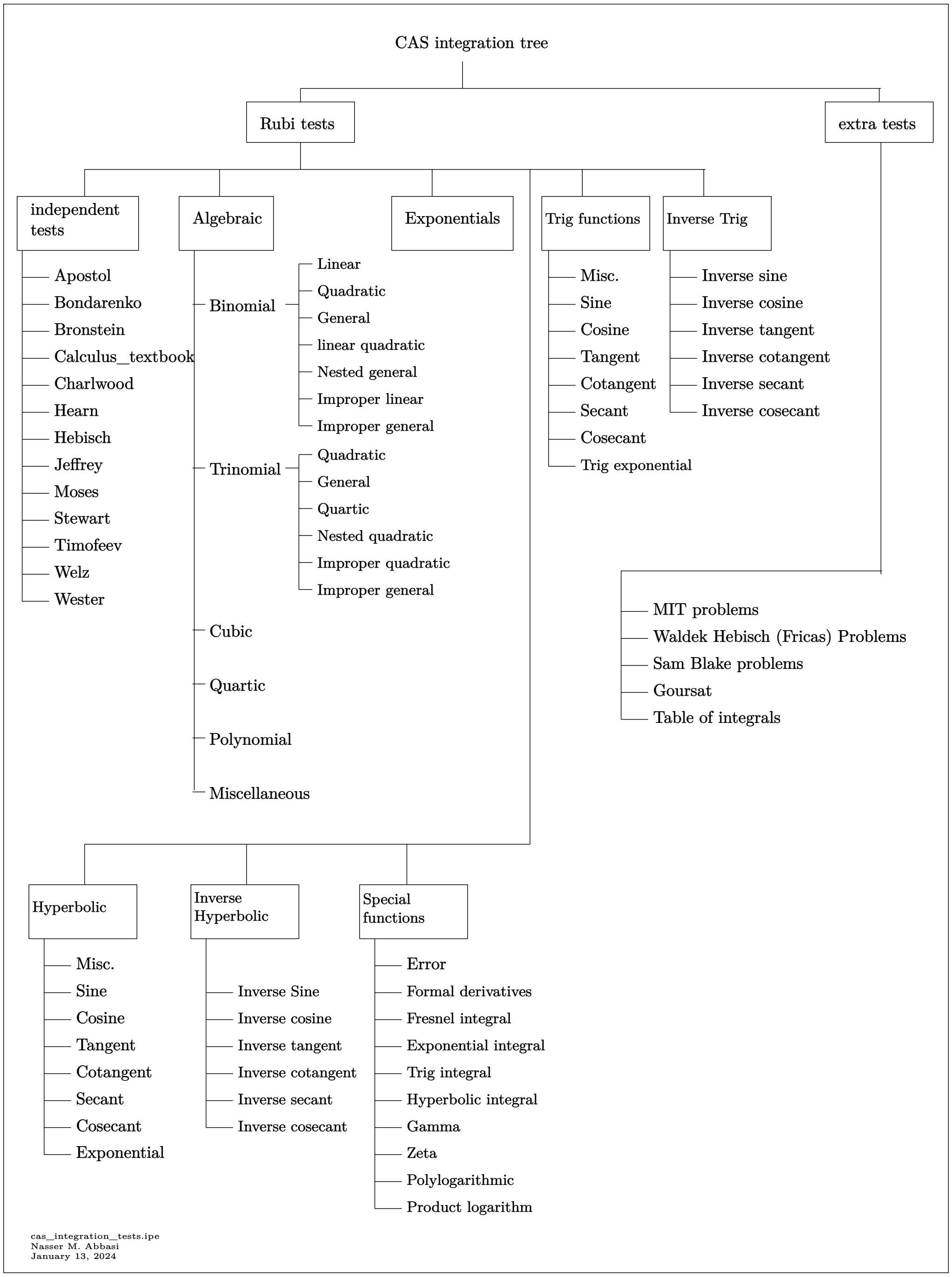
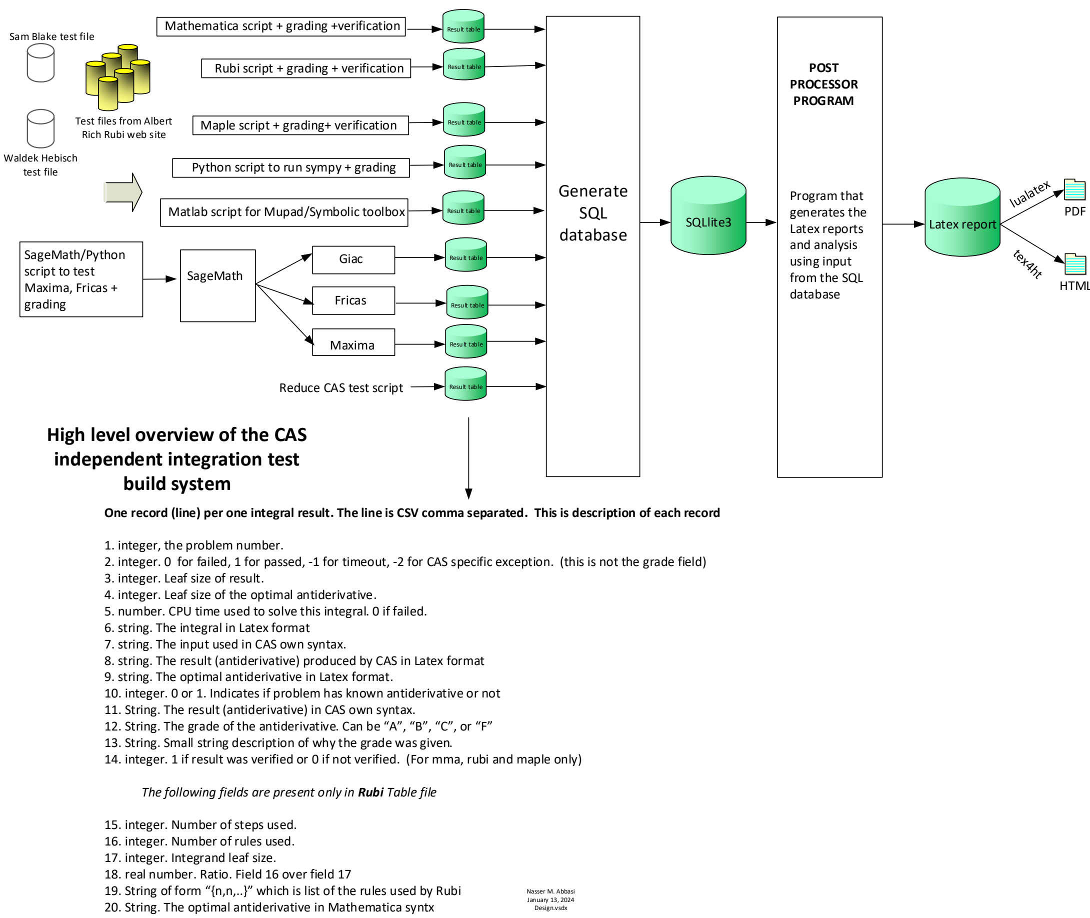


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919 }

B grade { 279, 297, 745 }

C grade { 178 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 554, 555 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 76, 77, 78, 79, 85, 86, 87, 88, 89, 91, 92, 94, 95, 96, 97, 103, 104, 105, 106, 107, 109, 110, 112, 113, 114, 115, 121, 122, 128, 129, 130, 132, 135, 140, 141, 142, 143, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 388, 389, 390, 393, 394, 395, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 470, 471, 472, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 498, 503, 504, 505, 506, 507, 508, 509, 510, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 564, 566, 569, 570, 571, 572, 573, 574, 576, 577, 578, 579, 580, 581, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 714, 715, 717, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916 }

B grade { 284, 396, 413, 582, 718, 745 }

C grade { 1, 2, 3, 4, 5, 6, 7, 8, 72, 75, 80, 81, 82, 83, 84, 90, 93, 98, 99, 100, 101, 102, 108, 111, 116, 117, 118, 119, 120, 123, 124, 125, 126, 127, 131, 133, 134, 136, 137, 138, 139, 144, 145, 146, 147, 178, 265, 266, 267, 385, 386, 387, 444, 445, 446, 447, 466, 467, 468, 469, 473, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 511, 512, 575, 592, 713, 716 }

F normal fail { 148, 149, 150, 151, 152, 153, 154, 155, 156, 391, 392, 563, 565, 567, 568, 917, 918, 919 }

F(-1) timedout fail { 559, 560, 561, 562 }

F(-2) exception fail { }

Maple

A grade { 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 165, 166, 167, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 288, 289, 290, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 394, 395, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 432, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 450, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 490, 491, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 714, 715, 721, 722, 723, 724, 727, 728, 731, 732, 733, 734, 737, 738, 748, 749, 750, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 769, 770, 771, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 844, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 908, 909 }

B grade { 12, 13, 14, 28, 29, 30, 44, 45, 46, 61, 62, 63, 168, 169, 187, 204, 220, 312, 396, 397, 398, 413, 414, 431, 433, 447, 448, 449, 451, 452, 465, 466, 467, 468, 469, 488, 489, 492, 493, 494, 514, 515, 539, 540, 541, 659, 751, 752, 753, 754, 755, 768, 772, 785, 786, 787, 788, 789, 801, 817, 818, 819, 820, 821, 841, 842, 843, 845, 846, 865, 866, 867, 889, 890, 891 }

C grade { 1, 2, 3, 4, 5, 6, 7, 8, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 140, 141, 142, 143 }

F normal fail { 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 134, 135, 136, 137, 138, 139, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 291, 292, 293, 294, 295, 296, 297, 301, 302, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 712, 713, 716, 717, 718, 719, 720, 725, 726, 729, 730, 735, 736, 739, 740, 741, 742, 743, 744, 745, 746, 747, 906, 907, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 288, 289, 290, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 394, 395, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 475, 476, 477, 478, 479, 480, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 525, 526, 527, 528, 529, 530, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 570, 571, 572, 573, 574, 575, 576, 577, 579, 580, 581, 583, 584, 587, 588, 589, 590, 591, 592, 593, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 714, 715, 721, 722, 723, 724, 727, 728, 731, 732, 733, 734, 737, 738, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 908, 909 }

B grade { 13, 14, 29, 45, 46, 188, 198, 204, 242, 312, 396, 457, 458, 474, 481, 482, 483, 505, 506, 523, 524, 531, 532, 578, 582, 585, 586, 595, 785 }

C grade { 1, 2, 3, 4, 5, 6, 7, 8, 137, 138, 139 }

F normal fail { 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 291, 292, 293, 294, 295, 296, 297, 301, 302, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 712, 713, 716, 717, 718, 719, 720, 725, 726, 729, 730, 735, 736, 739, 740, 741, 742, 743, 744, 745, 746, 747, 906, 907, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 205, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 252, 253, 254, 255, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 277, 298, 299, 300, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 342, 343, 346, 347, 350, 351, 358, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 430, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 451, 452, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 583, 584, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 655, 656, 657, 658, 659, 694, 695, 696, 697, 698, 714, 715, 737, 738, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 801, 802, 803, 804, 805, 904, 905, 908, 909 }

B grade { 9, 10, 11, 12, 13, 14, 15, 16, 17, 29, 41, 42, 43, 44, 45, 46, 47, 48, 49, 62, 165, 166, 167, 168, 183, 184, 185, 186, 198, 201, 202, 203, 204, 214, 222, 276, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 394, 395, 396, 397, 411, 412, 413, 414, 429, 431, 432, 450, 582, 585, 618, 644, 748, 749, 750, 751, 765, 766, 767, 782, 783, 784, 785, 798, 799, 800 }

C grade { 1, 2, 3, 4, 344, 345, 348, 349 }

F normal fail { 5, 6, 7, 8, 124, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 231, 232, 233, 234, 247, 248, 249, 250, 251, 256, 257, 258, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 301, 302, 322, 323, 333, 334, 335, 336, 337, 338, 339, 340, 341, 352, 353, 354, 355, 356, 357, 361, 362, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 596, 597, 598, 599, 600, 601, 602, 603, 604, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 739, 740, 741, 742, 743, 744, 745, 746, 747, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 906, 907, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 9, 11, 12, 18, 19, 20, 21, 22, 23, 24, 25, 33, 34, 35, 36, 38, 39, 40, 42, 43, 44, 50, 51, 52, 53, 54, 55, 56, 57, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 165, 166, 167, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 190, 191, 192, 193, 194, 196, 197, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 222, 226, 228, 230, 231, 232, 233, 234, 238, 239, 240, 241, 242, 243, 245, 248, 249, 250, 251, 254, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 272, 274, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 320, 321, 322, 323, 326, 327, 328, 329, 330, 331, 332, 334, 344, 345, 348, 349, 352, 353, 356, 357, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 397, 400, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 415, 416, 418, 424, 425, 426, 427, 430, 431, 432, 435, 436, 437, 438, 439, 440, 441, 442, 443, 450, 511, 512, 513, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 600, 605, 606, 607, 608, 609, 619, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 641, 642, 643, 644, 645, 647, 648, 649, 652, 653, 654, 656, 657, 658, 659, 660, 665, 666, 667, 668, 669, 670, 671, 672, 689, 690, 691, 692, 693, 695, 696, 697, 698, 699, 703, 704, 705, 706, 707, 708, 709, 710, 711, 756, 757, 758, 759, 760, 761, 762, 763, 764, 770, 773, 774, 777, 778, 779, 780, 781, 790, 791, 792, 793, 794, 795, 796, 797, 801, 802, 803, 806, 807, 808, 809, 822, 823, 824, 825, 826, 831, 832, 833, 834, 847, 848, 849, 850, 851, 858, 859, 860, 861, 862, 863, 864, 866, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 890, 895, 896, 897, 898, 899, 900, 901, 902, 903 }

B grade { 13, 14, 15, 16, 17, 37, 45, 47, 49, 180, 188, 189, 195, 198, 214, 235, 236, 237, 312, 318, 324, 325, 394, 395, 396, 411, 413, 414, 419, 420, 421, 422, 423, 428, 429, 465, 466, 467, 468, 469, 514, 516, 517, 518, 519, 542, 543, 544, 581, 582, 584, 611, 637, 661, 662, 663, 700, 701, 702, 748, 749, 750, 751, 765, 766, 767, 768, 775, 776, 782, 783, 784, 785, 798, 799, 800, 814, 815, 816, 839, 840, 841, 867, 868, 869, 870, 891, 892, 893, 894 }

C grade { 1, 2, 3, 4, 342, 343, 346, 347, 350, 351 }

F normal fail { 5, 6, 7, 8, 26, 27, 28, 29, 30, 31, 32, 58, 59, 60, 61, 62, 63, 64, 65, 66, 136, 140, 141, 142, 143, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 169, 205, 218, 219, 220, 221, 223, 224, 225, 276, 277, 278, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 381, 383, 385, 386, 387, 388, 390, 391, 392, 393, 398, 433, 444, 445, 446, 447, 448, 449, 451, 453, 455, 457, 458, 459, 460, 470, 472, 477, 481, 482, 483, 485, 499, 500, 502, 503, 504, 505, 506, 507, 508, 509, 510, 520, 530, 531, 532, 533, 534, 535, 554, 555, 556, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 612, 613, 614, 638, 639, 640, 715, 717, 720, 721, 722, 723, 724, 726, 727, 728, 729, 730, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 752, 753, 754, 755, 771, 772, 786, 787, 788, 789, 804, 805, 904, 906, 908, 910, 912, 916, 917, 918, 919 }

F(-1) timedout fail { 541 }

F(-2) exception fail { 10, 41, 46, 48, 144, 147, 227, 229, 244, 246, 247, 252, 253, 255, 257, 268, 269, 270, 273, 275, 279, 282, 319, 333, 335, 336, 337, 338, 339, 340, 341, 354, 355, 358, 372, 373, 374, 375, 376, 377, 378, 379, 380, 382, 384, 389, 399, 401, 417, 434, 452, 454, 456, 461, 462, 463, 464, 471, 473, 474, 475, 476, 478, 479, 480, 484, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 501, 515, 521, 522, 523, 524, 525, 526, 527, 528, 529, 536, 537, 538, 539, 540, 545, 546, 547, 548, 549, 550, 551, 552, 553, 557, 558, 596, 597, 598, 599, 601, 602, 603, 604, 610, 615, 616, 617, 618, 620, 621, 622, 623, 636, 646, 650, 651, 655, 664, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 694, 712, 713, 714, 716, 718, 719, 725, 731, 769, 810, 811, 812, 813, 817, 818, 819, 820, 821, 827, 828, 829, 830, 835, 836, 837, 838, 842, 843, 844, 845, 846, 852, 853, 854, 855, 856, 857, 865, 889, 905, 907, 909, 911, 913, 914, 915 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 252, 253, 254, 255, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 276, 277, 288, 289, 290, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 358, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 507, 508, 509, 510, 516, 517, 518, 519, 533, 534, 535, 550, 551, 552, 553, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 611, 612, 613, 614, 621, 637, 638, 639, 640, 647, 714, 715, 721, 722, 723, 724, 727, 728, 731, 732, 733, 734, 737, 738, 741, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 856, 857, 860, 880, 881, 884 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 7, 8, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 231, 232, 233, 234, 247, 248, 249, 250, 251, 256, 257, 258, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 291, 292, 293, 294, 295, 296, 297, 301, 302, 322, 323, 333, 334, 335, 336, 337, 338, 339, 340, 341, 352, 353, 354, 355, 356, 357, 361, 362, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 511, 512, 513, 514, 515, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 615, 616, 617, 618, 619, 620, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 641, 642, 643, 644, 645, 646, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 716, 717, 718, 719, 720, 725, 726, 729, 730, 735, 736, 739, 740, 742, 743, 744, 745, 746, 747, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 858, 859, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 882, 883, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919 }

F(-2) exception fail { }

Sympy

A grade { 18, 19, 20, 21, 22, 23, 24, 25, 33, 34, 35, 36, 37, 38, 39, 40, 50, 51, 52, 53, 54, 55, 56, 57, 175, 176, 177, 178, 179, 180, 181, 192, 193, 194, 195, 196, 197, 199, 200, 209, 210, 211, 212, 213, 214, 215, 216, 217, 235, 236, 237, 238, 239, 240, 241, 242, 259, 260, 261, 262, 263, 264, 265, 266, 267, 316, 324, 325, 326, 327, 343, 347, 353, 355, 356, 357, 363, 364, 365, 366, 367, 402, 403, 404, 405, 406, 407, 408, 409, 410, 419, 420, 421, 422, 423, 424, 425, 426, 427, 436, 437, 438, 439, 440, 442, 443, 570, 571, 572, 573, 574, 575, 576, 577, 578, 583, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 756, 757, 758, 759, 760, 761, 762, 763, 764, 773, 774, 775, 776, 777, 778, 779, 780, 781, 790, 791, 792, 793, 794, 795, 796, 797 }

B grade { 141, 174, 182, 198, 276, 277, 312, 328, 441, 515, 540, 579, 580, 581, 582, 584, 585, 605, 606, 607, 608, 633, 634 }

C grade { 142, 288, 289, 350, 351, 461, 462, 463, 464, 566, 740, 741, 742, 814, 815, 816, 839, 840, 841 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 26, 27, 28, 29, 30, 31, 32, 41, 42, 43, 44, 45, 46, 47, 48, 49, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 143, 144, 145, 146, 149, 150, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 183, 184, 185, 186, 187, 188, 189, 190, 191, 201, 202, 203, 204, 205, 206, 207, 208, 218, 219, 220, 221, 222, 223, 224, 225, 229, 230, 231, 232, 247, 248, 254, 255, 256, 278, 279, 280, 281, 283, 284, 285, 286, 287, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 329, 330, 331, 332, 333, 334, 335, 336, 337, 342, 345, 346, 348, 349, 352, 354, 358, 359, 360, 361, 362, 368, 369, 370, 371, 381, 382, 383, 384, 385, 386, 390, 394, 395, 396, 397, 398, 399, 400, 401, 411, 412, 413, 414, 415, 416, 417, 418, 428, 429, 430, 431, 432, 433, 434, 435, 444, 445, 446, 447, 448, 449, 450, 451, 457, 458, 465, 466, 467, 468, 469, 474, 481, 482, 486, 487, 488, 489, 490, 491, 492, 493, 494, 503, 504, 505, 506, 507, 511, 512, 513, 514, 516, 517, 518, 519, 521, 522, 523, 530, 531, 532, 533, 536, 537, 538, 539, 541, 542, 543, 544, 556, 557, 558, 560, 561, 562, 563, 564, 565, 567, 568, 569, 599, 600, 601, 602, 609, 610, 611, 612, 613, 614, 618, 619, 620, 628, 629, 630, 635, 636, 637, 638, 639, 640, 646, 647, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 673, 674, 675, 676, 677, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 707, 713, 714, 716, 718, 719, 720, 721, 722, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 743, 744, 745, 746, 748, 749, 750, 751, 752, 753, 754, 755, 765, 766, 767, 768, 769, 770, 771, 772, 782, 783, 784, 785, 786, 787, 788, 789, 799, 800, 801, 802, 803, 809, 817, 818, 819, 820, 821, 834, 835, 842, 843, 844, 845, 846, 857, 858, 859, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 882, 883, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 912, 913, 914, 915, 916, 917, 918, 919 }

F(-1) timedout fail { 112, 147, 148, 151, 152, 155, 226, 227, 228, 233, 234, 243, 244, 245, 246, 249, 250, 251, 252, 253, 257, 258, 268, 269, 270, 271, 272, 273, 274, 275, 282, 290, 338, 339, 340, 341, 344, 372, 373, 374, 375, 376, 377, 378, 379, 380, 387, 388, 389, 391, 392, 393, 452, 453, 454, 455, 456, 459, 460, 470, 471, 472, 473, 475, 476, 477, 478, 479, 480, 483, 484, 485, 495, 496, 497, 498, 499, 500, 501, 502, 508, 509, 510, 520, 524, 525, 526, 527, 528, 529, 534, 535, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 559, 596, 597, 598, 603, 604, 615, 616, 617, 621, 622, 623, 624, 625, 626, 627, 631, 632, 641, 642, 643, 644, 645, 648, 649, 670, 671, 672, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 703, 704, 705, 706, 708, 709, 710, 711, 712, 715, 717, 723, 724, 747, 798, 804, 805, 806, 807, 808, 810, 811, 812, 813, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 836, 837, 838, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 860, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 884, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911 }

F(-2) exception fail { 291, 297 }

Reduce

A grade { }

B grade { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 124, 125, 126, 127, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 288, 289, 290, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 710, 711, 714, 715, 737, 738, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 908, 909 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 709, 712, 713, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 739, 740, 741, 742, 743, 744, 745, 746, 747, 906, 907, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F	C	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	34	35	57	50	0	53	16	34
N.S.	1	0.00	34.00	35.00	57.00	50.00	0.00	53.00	16.00	34.00
time (sec)	N/A	0.000	0.039	0.178	0.041	0.079	0.000	0.116	0.155	0.646

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F	C	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	24	20	19	21	0	20	16	19
N.S.	1	0.00	24.00	20.00	19.00	21.00	0.00	20.00	16.00	19.00
time (sec)	N/A	0.000	0.027	0.171	0.043	0.074	0.000	0.133	0.151	23.873

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F	C	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	21	20	19	21	0	20	18	21
N.S.	1	0.00	21.00	20.00	19.00	21.00	0.00	20.00	18.00	21.00
time (sec)	N/A	0.000	0.028	0.174	0.034	0.071	0.000	0.129	0.157	0.538

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	C	C	F	C	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	31	35	54	52	0	53	18	34
N.S.	1	0.00	31.00	35.00	54.00	52.00	0.00	53.00	18.00	34.00
time (sec)	N/A	0.000	0.040	0.172	0.040	0.066	0.000	0.110	0.152	23.882

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	C	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	78	168	0	79	0	0	16	0
N.S.	1	0.00	78.00	168.00	0.00	79.00	0.00	0.00	16.00	0.00
time (sec)	N/A	0.000	0.135	0.191	0.000	0.080	0.000	0.000	0.153	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	C	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	67	119	0	78	0	0	14	0
N.S.	1	0.00	67.00	119.00	0.00	78.00	0.00	0.00	14.00	0.00
time (sec)	N/A	0.000	0.118	0.174	0.000	0.085	0.000	0.000	0.161	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	C	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	69	120	0	78	0	0	16	0
N.S.	1	0.00	69.00	120.00	0.00	78.00	0.00	0.00	16.00	0.00
time (sec)	N/A	0.000	0.120	0.182	0.000	0.095	0.000	0.000	0.158	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	C	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	78	168	0	119	0	0	18	0
N.S.	1	0.00	78.00	168.00	0.00	119.00	0.00	0.00	18.00	0.00
time (sec)	N/A	0.000	0.129	0.197	0.000	0.108	0.000	0.000	0.153	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	125	68	117	203	92	0	111	97	171
N.S.	1	1.10	0.60	1.03	1.78	0.81	0.00	0.97	0.85	1.50
time (sec)	N/A	0.587	0.090	0.121	0.037	0.076	0.000	0.146	0.162	0.106

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	95	60	109	166	84	0	0	77	133
N.S.	1	1.06	0.67	1.21	1.84	0.93	0.00	0.00	0.86	1.48
time (sec)	N/A	0.509	0.058	0.094	0.030	0.077	0.000	0.000	0.158	0.063

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	65	49	100	128	73	0	77	56	98
N.S.	1	1.03	0.78	1.59	2.03	1.16	0.00	1.22	0.89	1.56
time (sec)	N/A	0.443	0.043	0.078	0.027	0.091	0.000	0.131	0.155	23.245

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	41	90	90	64	0	57	39	58
N.S.	1	1.00	1.14	2.50	2.50	1.78	0.00	1.58	1.08	1.61
time (sec)	N/A	0.367	0.036	0.070	0.033	0.107	0.000	0.150	0.167	0.048

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	26	36	131	69	57	0	63	56	37
N.S.	1	1.18	1.64	5.95	3.14	2.59	0.00	2.86	2.55	1.68
time (sec)	N/A	0.392	0.019	0.078	0.106	0.074	0.000	0.139	0.170	23.195

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	28	27	76	53	46	0	66	59	55
N.S.	1	1.17	1.12	3.17	2.21	1.92	0.00	2.75	2.46	2.29
time (sec)	N/A	0.348	0.024	0.085	0.107	0.077	0.000	0.138	0.163	23.240

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	60	42	85	91	60	0	143	81	81
N.S.	1	1.15	0.81	1.63	1.75	1.15	0.00	2.75	1.56	1.56
time (sec)	N/A	0.394	0.054	0.084	0.111	0.092	0.000	0.135	0.154	23.229

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	90	51	93	136	68	0	148	110	105
N.S.	1	1.14	0.65	1.18	1.72	0.86	0.00	1.87	1.39	1.33
time (sec)	N/A	0.461	0.096	0.092	0.116	0.084	0.000	0.119	0.169	0.067

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	120	59	101	172	76	0	226	130	129
N.S.	1	1.17	0.57	0.98	1.67	0.74	0.00	2.19	1.26	1.25
time (sec)	N/A	0.518	0.111	0.098	0.106	0.077	0.000	0.150	0.163	23.268

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	43	42	37	47	42	38
N.S.	1	1.00	1.00	0.91	1.00	0.98	0.86	1.09	0.98	0.88
time (sec)	N/A	0.530	0.025	0.114	0.025	0.079	0.053	0.131	0.158	0.037

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	34	33	27	38	33	30
N.S.	1	1.00	1.00	0.94	1.03	1.00	0.82	1.15	1.00	0.91
time (sec)	N/A	0.460	0.021	0.117	0.028	0.069	0.047	0.111	0.150	0.037

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	26	25	20	30	25	23
N.S.	1	1.00	1.00	0.92	1.00	0.96	0.77	1.15	0.96	0.88
time (sec)	N/A	0.431	0.018	0.109	0.030	0.076	0.042	0.136	0.158	0.038

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	13	16	10	14	16	13
N.S.	1	1.00	1.00	1.00	0.93	1.14	0.71	1.00	1.14	0.93
time (sec)	N/A	0.385	0.020	0.111	0.033	0.071	0.037	0.107	0.152	0.028

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	13	13	10	15	13	14
N.S.	1	1.00	1.00	1.00	0.93	0.93	0.71	1.07	0.93	1.00
time (sec)	N/A	0.428	0.014	0.119	0.026	0.068	0.062	0.115	0.157	23.634

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	18	22	15	20	22	14
N.S.	1	1.00	1.00	1.00	0.95	1.16	0.79	1.05	1.16	0.74
time (sec)	N/A	0.430	0.017	0.127	0.029	0.069	0.078	0.130	0.157	23.557

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	30	30	35	26	32	35	23
N.S.	1	1.00	1.00	0.91	0.91	1.06	0.79	0.97	1.06	0.70
time (sec)	N/A	0.463	0.019	0.135	0.024	0.078	0.092	0.121	0.157	0.044

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	37	38	43	34	40	43	30
N.S.	1	1.00	1.00	0.92	0.95	1.08	0.85	1.00	1.08	0.75
time (sec)	N/A	0.474	0.023	0.138	0.034	0.099	0.102	0.114	0.152	23.315

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	75	151	182	112	0	0	95	154
N.S.	1	1.00	0.64	1.28	1.54	0.95	0.00	0.00	0.81	1.31
time (sec)	N/A	1.028	0.102	0.111	0.036	0.103	0.000	0.000	0.160	23.552

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	66	142	145	103	0	0	80	117
N.S.	1	1.00	0.72	1.54	1.58	1.12	0.00	0.00	0.87	1.27
time (sec)	N/A	0.975	0.080	0.101	0.027	0.082	0.000	0.000	0.153	0.066

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	133	110	92	0	0	67	59
N.S.	1	1.00	0.87	2.15	1.77	1.48	0.00	0.00	1.08	0.95
time (sec)	N/A	0.866	0.065	0.089	0.034	0.077	0.000	0.000	0.159	23.414

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	83	53	372	90	104	0	0	98	54
N.S.	1	1.80	1.15	8.09	1.96	2.26	0.00	0.00	2.13	1.17
time (sec)	N/A	0.978	0.075	0.089	0.111	0.072	0.000	0.000	0.162	23.364

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	57	41	116	72	74	0	0	89	57
N.S.	1	1.08	0.77	2.19	1.36	1.40	0.00	0.00	1.68	1.08
time (sec)	N/A	0.457	0.103	0.104	0.114	0.074	0.000	0.000	0.157	23.441

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	99	56	125	110	88	0	0	116	83
N.S.	1	1.22	0.69	1.54	1.36	1.09	0.00	0.00	1.43	1.02
time (sec)	N/A	1.191	0.109	0.115	0.122	0.075	0.000	0.000	0.155	0.081

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	129	66	135	154	96	0	0	132	152
N.S.	1	1.24	0.63	1.30	1.48	0.92	0.00	0.00	1.27	1.46
time (sec)	N/A	1.544	0.128	0.112	0.119	0.104	0.000	0.000	0.168	23.588

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	58	66	49	78	67	57
N.S.	1	1.00	1.00	0.91	1.02	1.16	0.86	1.37	1.18	1.00
time (sec)	N/A	0.517	0.078	0.139	0.024	0.062	0.092	0.115	0.153	0.042

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	49	57	39	69	58	49
N.S.	1	1.00	1.00	0.94	1.04	1.21	0.83	1.47	1.23	1.04
time (sec)	N/A	0.497	0.060	0.131	0.034	0.085	0.088	0.120	0.150	0.033

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	41	49	31	64	50	38
N.S.	1	1.00	1.00	0.92	1.05	1.26	0.79	1.64	1.28	0.97
time (sec)	N/A	0.447	0.046	0.129	0.027	0.069	0.082	0.134	0.152	0.040

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	26	38	19	46	41	25
N.S.	1	1.00	0.96	0.96	0.96	1.41	0.70	1.70	1.52	0.93
time (sec)	N/A	0.396	0.034	0.125	0.025	0.060	0.080	0.128	0.148	23.336

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	12	18	8	57	22	12
N.S.	1	1.00	1.00	1.00	0.92	1.38	0.62	4.38	1.69	0.92
time (sec)	N/A	0.433	0.016	0.140	0.030	0.075	0.092	0.127	0.147	0.039

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	34	55	26	40	61	28
N.S.	1	1.00	1.00	0.97	1.06	1.72	0.81	1.25	1.91	0.88
time (sec)	N/A	0.458	0.037	0.145	0.024	0.076	0.126	0.118	0.160	0.056

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	43	48	73	41	62	74	41
N.S.	1	1.00	1.00	0.93	1.04	1.59	0.89	1.35	1.61	0.89
time (sec)	N/A	0.489	0.050	0.156	0.034	0.072	0.155	0.138	0.160	23.940

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	56	81	49	74	82	49
N.S.	1	1.00	1.00	0.94	1.04	1.50	0.91	1.37	1.52	0.91
time (sec)	N/A	0.489	0.067	0.161	0.031	0.074	0.166	0.134	0.150	23.726

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	125	68	117	203	91	0	0	97	172
N.S.	1	1.10	0.60	1.03	1.78	0.80	0.00	0.00	0.85	1.51
time (sec)	N/A	0.577	0.114	0.085	0.033	0.100	0.000	0.000	0.156	23.431

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	95	60	109	166	83	0	86	77	134
N.S.	1	1.06	0.67	1.21	1.84	0.92	0.00	0.96	0.86	1.49
time (sec)	N/A	0.500	0.063	0.077	0.032	0.095	0.000	0.131	0.147	0.056

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	63	49	100	130	73	0	71	56	97
N.S.	1	0.98	0.77	1.56	2.03	1.14	0.00	1.11	0.88	1.52
time (sec)	N/A	0.468	0.048	0.076	0.026	0.077	0.000	0.135	0.151	0.059

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	42	91	90	64	0	52	39	58
N.S.	1	1.00	1.14	2.46	2.43	1.73	0.00	1.41	1.05	1.57
time (sec)	N/A	0.392	0.034	0.063	0.026	0.094	0.000	0.143	0.148	23.978

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	24	34	133	70	57	0	59	56	37
N.S.	1	1.20	1.70	6.65	3.50	2.85	0.00	2.95	2.80	1.85
time (sec)	N/A	0.392	0.020	0.071	0.112	0.073	0.000	0.132	0.144	0.032

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	29	26	77	55	47	0	0	61	55
N.S.	1	1.16	1.04	3.08	2.20	1.88	0.00	0.00	2.44	2.20
time (sec)	N/A	0.350	0.039	0.082	0.109	0.097	0.000	0.000	0.148	24.252

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	59	41	85	93	60	0	157	82	82
N.S.	1	1.13	0.79	1.63	1.79	1.15	0.00	3.02	1.58	1.58
time (sec)	N/A	0.400	0.060	0.092	0.108	0.067	0.000	0.148	0.152	24.075

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	90	52	93	137	68	0	0	110	105
N.S.	1	1.14	0.66	1.18	1.73	0.86	0.00	0.00	1.39	1.33
time (sec)	N/A	0.471	0.095	0.092	0.112	0.082	0.000	0.000	0.143	23.905

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	120	59	101	173	77	0	258	130	129
N.S.	1	1.17	0.57	0.98	1.68	0.75	0.00	2.50	1.26	1.25
time (sec)	N/A	0.554	0.109	0.093	0.110	0.094	0.000	0.127	0.156	23.707

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	39	43	42	37	47	42	38
N.S.	1	1.00	1.00	0.93	1.02	1.00	0.88	1.12	1.00	0.90
time (sec)	N/A	0.483	0.026	0.118	0.027	0.076	0.056	0.148	0.143	0.033

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	32	34	33	27	38	33	31
N.S.	1	1.00	1.00	0.97	1.03	1.00	0.82	1.15	1.00	0.94
time (sec)	N/A	0.477	0.019	0.120	0.031	0.075	0.049	0.133	0.143	0.037

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	26	25	20	30	25	23
N.S.	1	1.00	1.00	0.96	1.04	1.00	0.80	1.20	1.00	0.92
time (sec)	N/A	0.439	0.016	0.113	0.029	0.064	0.049	0.133	0.147	23.777

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	16	10	14	16	13
N.S.	1	1.00	1.00	1.08	1.00	1.23	0.77	1.08	1.23	1.00
time (sec)	N/A	0.392	0.017	0.108	0.025	0.066	0.046	0.119	0.148	23.674

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	10	15	13	14
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.77	1.15	1.00	1.08
time (sec)	N/A	0.422	0.011	0.121	0.030	0.101	0.069	0.115	0.145	0.047

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	23	15	20	22	14
N.S.	1	1.00	1.00	1.06	1.00	1.28	0.83	1.11	1.22	0.78
time (sec)	N/A	0.434	0.014	0.130	0.029	0.070	0.073	0.125	0.152	0.040

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	30	30	35	26	32	35	24
N.S.	1	1.00	1.00	0.94	0.94	1.09	0.81	1.00	1.09	0.75
time (sec)	N/A	0.461	0.016	0.138	0.028	0.114	0.084	0.115	0.147	0.047

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	38	38	43	34	40	43	31
N.S.	1	1.00	1.00	0.95	0.95	1.08	0.85	1.00	1.08	0.78
time (sec)	N/A	0.472	0.019	0.138	0.030	0.069	0.094	0.107	0.154	0.038

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	83	153	223	92	0	0	152	192
N.S.	1	1.00	0.61	1.12	1.64	0.68	0.00	0.00	1.12	1.41
time (sec)	N/A	1.100	0.119	0.105	0.042	0.082	0.000	0.000	0.157	23.419

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	75	145	186	84	0	0	132	156
N.S.	1	1.00	0.65	1.25	1.60	0.72	0.00	0.00	1.14	1.34
time (sec)	N/A	1.023	0.101	0.093	0.032	0.073	0.000	0.000	0.153	24.333

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	66	136	151	75	0	0	111	120
N.S.	1	1.00	0.73	1.51	1.68	0.83	0.00	0.00	1.23	1.33
time (sec)	N/A	0.968	0.085	0.099	0.036	0.080	0.000	0.000	0.158	23.903

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	60	54	127	111	66	0	0	92	78
N.S.	1	0.95	0.86	2.02	1.76	1.05	0.00	0.00	1.46	1.24
time (sec)	N/A	0.868	0.066	0.086	0.036	0.101	0.000	0.000	0.142	0.042

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	81	55	369	89	74	0	0	139	54
N.S.	1	1.69	1.15	7.69	1.85	1.54	0.00	0.00	2.90	1.12
time (sec)	N/A	0.959	0.075	0.089	0.114	0.097	0.000	0.000	0.165	0.034

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	54	41	109	72	49	0	0	137	59
N.S.	1	1.15	0.87	2.32	1.53	1.04	0.00	0.00	2.91	1.26
time (sec)	N/A	0.453	0.116	0.102	0.105	0.088	0.000	0.000	0.157	23.823

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	96	56	119	112	61	0	0	170	118
N.S.	1	1.19	0.69	1.47	1.38	0.75	0.00	0.00	2.10	1.46
time (sec)	N/A	1.184	0.155	0.108	0.113	0.072	0.000	0.000	0.155	0.062

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	127	66	129	157	69	0	0	191	153
N.S.	1	1.25	0.65	1.26	1.54	0.68	0.00	0.00	1.87	1.50
time (sec)	N/A	1.519	0.136	0.110	0.113	0.081	0.000	0.000	0.162	23.616

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	155	75	137	193	77	0	0	211	190
N.S.	1	1.28	0.62	1.13	1.60	0.64	0.00	0.00	1.74	1.57
time (sec)	N/A	1.696	0.064	0.109	0.109	0.132	0.000	0.000	0.167	0.081

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	253	272	173	0	259	119	0	234	20	229
N.S.	1	1.08	0.68	0.00	1.02	0.47	0.00	0.92	0.08	0.91
time (sec)	N/A	0.690	5.302	0.000	0.116	0.089	0.000	0.136	0.213	0.110

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	229	149	0	224	111	0	203	20	192
N.S.	1	1.06	0.69	0.00	1.04	0.51	0.00	0.94	0.09	0.89
time (sec)	N/A	0.605	5.213	0.000	0.110	0.082	0.000	0.139	0.230	23.698

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	186	125	0	187	103	0	172	20	157
N.S.	1	1.04	0.70	0.00	1.04	0.58	0.00	0.96	0.11	0.88
time (sec)	N/A	0.531	5.175	0.000	0.109	0.087	0.000	0.147	0.212	23.955

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	145	66	0	149	93	0	139	18	120
N.S.	1	1.02	0.46	0.00	1.05	0.65	0.00	0.98	0.13	0.85
time (sec)	N/A	0.437	0.201	0.000	0.109	0.092	0.000	0.139	0.204	0.075

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	102	51	0	111	84	0	108	17	78
N.S.	1	1.06	0.53	0.00	1.16	0.88	0.00	1.12	0.18	0.81
time (sec)	N/A	0.376	0.105	0.000	0.110	0.087	0.000	0.132	0.170	0.065

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	222	258	30	0	224	194	0	232	20	101
N.S.	1	1.16	0.14	0.00	1.01	0.87	0.00	1.05	0.09	0.45
time (sec)	N/A	0.752	0.057	0.000	0.121	0.086	0.000	0.147	0.185	0.092

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	219	148	0	186	174	0	186	20	87
N.S.	1	1.14	0.77	0.00	0.97	0.91	0.00	0.97	0.10	0.45
time (sec)	N/A	0.693	0.295	0.000	0.115	0.079	0.000	0.132	0.189	24.073

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	244	260	173	0	226	199	0	223	20	132
N.S.	1	1.07	0.71	0.00	0.93	0.82	0.00	0.91	0.08	0.54
time (sec)	N/A	0.761	0.231	0.000	0.109	0.099	0.000	0.141	0.205	0.080

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	303	93	0	270	208	0	271	20	168
N.S.	1	1.08	0.33	0.00	0.96	0.74	0.00	0.96	0.07	0.60
time (sec)	N/A	0.839	0.136	0.000	0.109	0.077	0.000	0.120	0.196	0.083

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	253	272	173	0	259	119	0	234	20	229
N.S.	1	1.08	0.68	0.00	1.02	0.47	0.00	0.92	0.08	0.91
time (sec)	N/A	0.667	5.273	0.000	0.133	0.084	0.000	0.145	0.214	0.102

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	227	149	0	224	111	0	203	20	192
N.S.	1	1.05	0.69	0.00	1.04	0.51	0.00	0.94	0.09	0.89
time (sec)	N/A	0.599	5.214	0.000	0.113	0.099	0.000	0.136	0.191	23.280

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	186	125	0	187	103	0	172	20	157
N.S.	1	1.04	0.70	0.00	1.04	0.58	0.00	0.96	0.11	0.88
time (sec)	N/A	0.563	5.182	0.000	0.109	0.088	0.000	0.124	0.190	0.087

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	145	70	0	152	95	0	141	18	120
N.S.	1	1.02	0.49	0.00	1.07	0.67	0.00	0.99	0.13	0.85
time (sec)	N/A	0.437	0.197	0.000	0.110	0.082	0.000	0.148	0.184	0.083

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	102	56	0	112	86	0	109	17	79
N.S.	1	1.04	0.57	0.00	1.14	0.88	0.00	1.11	0.17	0.81
time (sec)	N/A	0.385	0.092	0.000	0.109	0.106	0.000	0.140	0.167	23.220

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	258	30	0	224	194	0	232	20	101
N.S.	1	1.17	0.14	0.00	1.01	0.88	0.00	1.05	0.09	0.46
time (sec)	N/A	0.758	0.058	0.000	0.109	0.080	0.000	0.137	0.194	0.055

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	219	46	0	187	175	0	187	20	88
N.S.	1	1.13	0.24	0.00	0.97	0.91	0.00	0.97	0.10	0.46
time (sec)	N/A	0.667	0.086	0.000	0.114	0.087	0.000	0.143	0.180	0.080

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	244	260	76	0	229	200	0	225	20	132
N.S.	1	1.07	0.31	0.00	0.94	0.82	0.00	0.92	0.08	0.54
time (sec)	N/A	0.729	0.102	0.000	0.122	0.074	0.000	0.123	0.178	23.296

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	303	93	0	277	208	0	271	20	168
N.S.	1	1.08	0.33	0.00	0.99	0.74	0.00	0.96	0.07	0.60
time (sec)	N/A	0.812	0.152	0.000	0.117	0.082	0.000	0.138	0.185	23.381

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	285	311	198	0	275	152	0	254	73	248
N.S.	1	1.09	0.69	0.00	0.96	0.53	0.00	0.89	0.26	0.87
time (sec)	N/A	0.739	5.283	0.000	0.110	0.080	0.000	0.147	0.249	23.572

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	248	268	161	0	238	144	0	223	73	211
N.S.	1	1.08	0.65	0.00	0.96	0.58	0.00	0.90	0.29	0.85
time (sec)	N/A	0.669	5.248	0.000	0.119	0.080	0.000	0.157	0.255	23.842

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	225	137	0	203	136	0	192	73	176
N.S.	1	1.07	0.65	0.00	0.96	0.64	0.00	0.91	0.35	0.83
time (sec)	N/A	0.597	5.229	0.000	0.112	0.085	0.000	0.127	0.213	0.096

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	179	80	0	166	128	0	161	71	139
N.S.	1	1.02	0.45	0.00	0.94	0.73	0.00	0.91	0.40	0.79
time (sec)	N/A	0.487	0.244	0.000	0.111	0.080	0.000	0.131	0.217	0.086

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	136	67	0	131	117	0	141	68	98
N.S.	1	1.05	0.52	0.00	1.01	0.90	0.00	1.08	0.52	0.75
time (sec)	N/A	0.435	0.186	0.000	0.112	0.078	0.000	0.146	0.195	23.430

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	250	287	30	0	244	263	0	252	70	118
N.S.	1	1.15	0.12	0.00	0.98	1.05	0.00	1.01	0.28	0.47
time (sec)	N/A	0.817	0.107	0.000	0.121	0.074	0.000	0.131	0.249	0.043

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	224	253	173	0	204	232	0	217	75	107
N.S.	1	1.13	0.77	0.00	0.91	1.04	0.00	0.97	0.33	0.48
time (sec)	N/A	0.755	0.517	0.000	0.113	0.083	0.000	0.135	0.220	0.082

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	276	293	186	0	244	258	0	243	77	152
N.S.	1	1.06	0.67	0.00	0.88	0.93	0.00	0.88	0.28	0.55
time (sec)	N/A	0.806	0.335	0.000	0.120	0.099	0.000	0.125	0.246	0.083

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	332	104	0	288	266	0	291	77	188
N.S.	1	1.07	0.34	0.00	0.93	0.86	0.00	0.94	0.25	0.61
time (sec)	N/A	0.872	0.198	0.000	0.127	0.075	0.000	0.140	0.236	23.563

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	253	272	173	0	259	119	0	234	20	229
N.S.	1	1.08	0.68	0.00	1.02	0.47	0.00	0.92	0.08	0.91
time (sec)	N/A	0.665	5.329	0.000	0.117	0.087	0.000	0.128	0.214	0.084

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	229	149	0	224	111	0	203	20	193
N.S.	1	1.06	0.69	0.00	1.04	0.51	0.00	0.94	0.09	0.89
time (sec)	N/A	0.618	5.271	0.000	0.109	0.082	0.000	0.146	0.220	23.523

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	186	125	0	187	102	0	172	20	157
N.S.	1	1.04	0.70	0.00	1.04	0.57	0.00	0.96	0.11	0.88
time (sec)	N/A	0.532	5.221	0.000	0.114	0.117	0.000	0.126	0.189	23.556

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	145	66	0	151	93	0	140	18	121
N.S.	1	1.02	0.46	0.00	1.06	0.65	0.00	0.99	0.13	0.85
time (sec)	N/A	0.430	0.191	0.000	0.121	0.104	0.000	0.143	0.177	23.545

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	102	33	0	111	84	0	108	17	79
N.S.	1	1.05	0.34	0.00	1.14	0.87	0.00	1.11	0.18	0.81
time (sec)	N/A	0.385	0.063	0.000	0.125	0.082	0.000	0.129	0.181	23.559

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	222	258	30	0	224	196	0	232	20	101
N.S.	1	1.16	0.14	0.00	1.01	0.88	0.00	1.05	0.09	0.45
time (sec)	N/A	0.751	0.057	0.000	0.120	0.097	0.000	0.145	0.172	0.056

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	220	33	0	186	174	0	186	20	88
N.S.	1	1.15	0.17	0.00	0.97	0.91	0.00	0.97	0.10	0.46
time (sec)	N/A	0.662	0.064	0.000	0.121	0.085	0.000	0.141	0.181	0.058

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	244	259	56	0	227	198	0	223	20	132
N.S.	1	1.06	0.23	0.00	0.93	0.81	0.00	0.91	0.08	0.54
time (sec)	N/A	0.739	0.094	0.000	0.111	0.088	0.000	0.124	0.208	0.065

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	302	93	0	277	207	0	271	20	169
N.S.	1	1.07	0.33	0.00	0.99	0.74	0.00	0.96	0.07	0.60
time (sec)	N/A	0.869	0.150	0.000	0.117	0.079	0.000	0.156	0.203	0.086

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	253	272	173	0	259	119	0	234	20	229
N.S.	1	1.08	0.68	0.00	1.02	0.47	0.00	0.92	0.08	0.91
time (sec)	N/A	0.713	5.369	0.000	0.109	0.079	0.000	0.147	0.219	0.093

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	227	149	0	224	111	0	203	20	193
N.S.	1	1.05	0.69	0.00	1.04	0.51	0.00	0.94	0.09	0.89
time (sec)	N/A	0.603	5.289	0.000	0.112	0.116	0.000	0.156	0.201	0.082

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	186	125	0	187	103	0	172	20	157
N.S.	1	1.04	0.70	0.00	1.04	0.58	0.00	0.96	0.11	0.88
time (sec)	N/A	0.543	5.245	0.000	0.120	0.080	0.000	0.134	0.204	0.060

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	145	70	0	152	95	0	141	18	121
N.S.	1	1.02	0.49	0.00	1.07	0.67	0.00	0.99	0.13	0.85
time (sec)	N/A	0.449	0.227	0.000	0.116	0.085	0.000	0.160	0.184	23.381

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	102	55	0	112	86	0	109	17	79
N.S.	1	1.04	0.56	0.00	1.14	0.88	0.00	1.11	0.17	0.81
time (sec)	N/A	0.385	0.140	0.000	0.112	0.098	0.000	0.160	0.172	23.483

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	258	28	0	224	196	0	232	20	101
N.S.	1	1.17	0.13	0.00	1.01	0.89	0.00	1.05	0.09	0.46
time (sec)	N/A	0.748	0.086	0.000	0.116	0.105	0.000	0.130	0.195	0.047

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	220	149	0	187	175	0	187	20	88
N.S.	1	1.13	0.77	0.00	0.96	0.90	0.00	0.96	0.10	0.45
time (sec)	N/A	0.695	0.326	0.000	0.116	0.098	0.000	0.143	0.190	0.054

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	244	259	174	0	228	200	0	225	20	132
N.S.	1	1.06	0.71	0.00	0.93	0.82	0.00	0.92	0.08	0.54
time (sec)	N/A	0.790	0.242	0.000	0.124	0.079	0.000	0.135	0.192	0.056

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	302	93	0	270	208	0	271	20	169
N.S.	1	1.07	0.33	0.00	0.96	0.74	0.00	0.96	0.07	0.60
time (sec)	N/A	0.825	0.163	0.000	0.111	0.082	0.000	0.142	0.201	0.059

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	287	311	198	0	279	119	0	254	71	253
N.S.	1	1.08	0.69	0.00	0.97	0.41	0.00	0.89	0.25	0.88
time (sec)	N/A	0.750	5.427	0.000	0.117	0.081	0.000	0.155	0.252	24.064

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	250	268	161	0	244	111	0	223	71	217
N.S.	1	1.07	0.64	0.00	0.98	0.44	0.00	0.89	0.28	0.87
time (sec)	N/A	0.694	5.342	0.000	0.115	0.082	0.000	0.127	0.261	0.080

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	213	225	137	0	207	103	0	192	71	181
N.S.	1	1.06	0.64	0.00	0.97	0.48	0.00	0.90	0.33	0.85
time (sec)	N/A	0.615	5.302	0.000	0.112	0.092	0.000	0.148	0.228	0.078

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	179	121	0	172	95	0	161	69	145
N.S.	1	1.02	0.69	0.00	0.98	0.54	0.00	0.91	0.39	0.82
time (sec)	N/A	0.499	0.282	0.000	0.113	0.083	0.000	0.152	0.214	24.030

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	136	31	0	132	86	0	129	66	103
N.S.	1	1.05	0.24	0.00	1.02	0.66	0.00	0.99	0.51	0.79
time (sec)	N/A	0.436	0.092	0.000	0.111	0.078	0.000	0.148	0.191	23.783

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	250	287	28	0	244	211	0	252	69	118
N.S.	1	1.15	0.11	0.00	0.98	0.84	0.00	1.01	0.28	0.47
time (sec)	N/A	0.835	0.113	0.000	0.109	0.074	0.000	0.152	0.223	0.031

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	224	252	31	0	204	176	0	204	75	106
N.S.	1	1.12	0.14	0.00	0.91	0.79	0.00	0.91	0.33	0.47
time (sec)	N/A	0.730	0.094	0.000	0.113	0.079	0.000	0.118	0.226	23.664

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	276	292	101	0	247	200	0	243	77	153
N.S.	1	1.06	0.37	0.00	0.89	0.72	0.00	0.88	0.28	0.55
time (sec)	N/A	0.829	0.183	0.000	0.110	0.097	0.000	0.116	0.237	0.072

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	331	104	0	297	208	0	291	77	188
N.S.	1	1.07	0.34	0.00	0.96	0.67	0.00	0.94	0.25	0.61
time (sec)	N/A	0.884	0.199	0.000	0.112	0.101	0.000	0.130	0.259	23.586

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	266	189	1163	220	173	0	215	16	168
N.S.	1	1.12	0.80	4.91	0.93	0.73	0.00	0.91	0.07	0.71
time (sec)	N/A	0.693	5.285	7.394	0.112	0.107	0.000	0.136	0.295	0.148

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	233	167	1158	194	168	0	191	14	142
N.S.	1	1.11	0.80	5.51	0.92	0.80	0.00	0.91	0.07	0.68
time (sec)	N/A	0.633	0.616	6.851	0.113	0.082	0.000	0.123	0.247	0.117

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	201	35	1151	167	160	0	168	13	115
N.S.	1	1.15	0.20	6.58	0.95	0.91	0.00	0.96	0.07	0.66
time (sec)	N/A	0.555	0.061	7.297	0.110	0.078	0.000	0.147	0.197	23.641

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	371	26	2202	0	264	0	261	90	167
N.S.	1	1.28	0.09	7.59	0.00	0.91	0.00	0.90	0.31	0.58
time (sec)	N/A	0.890	0.049	10.052	0.000	0.082	0.000	0.159	0.161	0.151

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	F	A	B	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	173	214	39	889	152	151	0	152	50	109
N.S.	1	1.24	0.23	5.14	0.88	0.87	0.00	0.88	0.29	0.63
time (sec)	N/A	0.652	0.067	26.557	0.115	0.080	0.000	0.118	0.285	0.110

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	243	124	1502	178	168	0	175	64	136
N.S.	1	1.20	0.61	7.44	0.88	0.83	0.00	0.87	0.32	0.67
time (sec)	N/A	0.668	0.934	11.204	0.108	0.084	0.000	0.134	0.313	0.113

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	F	A	B	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	229	276	133	901	205	174	0	199	76	161
N.S.	1	1.21	0.58	3.93	0.90	0.76	0.00	0.87	0.33	0.70
time (sec)	N/A	0.716	0.237	17.214	0.120	0.100	0.000	0.137	0.308	23.453

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	167	189	616	149	100	0	144	16	171
N.S.	1	1.06	1.20	3.92	0.95	0.64	0.00	0.92	0.10	1.09
time (sec)	N/A	0.464	5.337	0.604	0.111	0.105	0.000	0.135	0.223	0.062

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	136	165	509	123	95	0	120	14	145
N.S.	1	1.05	1.27	3.92	0.95	0.73	0.00	0.92	0.11	1.12
time (sec)	N/A	0.404	0.762	0.596	0.110	0.083	0.000	0.122	0.226	0.048

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	104	85	397	96	87	0	97	13	118
N.S.	1	1.08	0.89	4.14	1.00	0.91	0.00	1.01	0.14	1.23
time (sec)	N/A	0.366	0.257	0.629	0.110	0.082	0.000	0.134	0.205	23.387

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	157	26	1039	140	86	0	79	16	82
N.S.	1	1.01	0.17	6.70	0.90	0.55	0.00	0.51	0.10	0.53
time (sec)	N/A	0.426	0.053	0.580	0.115	0.078	0.000	0.124	0.301	0.321

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	105	87	501	98	97	0	99	16	118
N.S.	1	1.06	0.88	5.06	0.99	0.98	0.00	1.00	0.16	1.19
time (sec)	N/A	0.376	0.184	0.705	0.111	0.077	0.000	0.118	0.311	0.026

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	134	134	675	124	111	0	122	16	145
N.S.	1	1.03	1.03	5.19	0.95	0.85	0.00	0.94	0.12	1.12
time (sec)	N/A	0.407	0.501	0.636	0.109	0.094	0.000	0.134	0.348	0.028

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	354	389	167	0	341	242	0	308	20	227
N.S.	1	1.10	0.47	0.00	0.96	0.68	0.00	0.87	0.06	0.64
time (sec)	N/A	0.920	5.357	0.000	0.118	0.088	0.000	0.141	0.259	24.129

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	317	348	319	0	304	233	0	288	18	190
N.S.	1	1.10	1.01	0.00	0.96	0.74	0.00	0.91	0.06	0.60
time (sec)	N/A	0.811	0.799	0.000	0.116	0.114	0.000	0.154	0.215	0.163

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	277	305	56	0	265	224	0	0	17	149
N.S.	1	1.10	0.20	0.00	0.96	0.81	0.00	0.00	0.06	0.54
time (sec)	N/A	0.722	0.069	0.000	0.115	0.090	0.000	0.000	0.193	23.949

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	C	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	673	866	30	0	0	441	0	661	20	648
N.S.	1	1.29	0.04	0.00	0.00	0.66	0.00	0.98	0.03	0.96
time (sec)	N/A	1.986	0.053	0.000	0.000	0.134	0.000	0.712	0.224	23.913

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	C	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	502	624	46	0	0	374	0	432	316	162
N.S.	1	1.24	0.09	0.00	0.00	0.75	0.00	0.86	0.63	0.32
time (sec)	N/A	1.296	0.075	0.000	0.000	0.091	0.000	0.215	3.024	16.700

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	C	F	A	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	553	665	72	0	0	399	0	461	346	210
N.S.	1	1.20	0.13	0.00	0.00	0.72	0.00	0.83	0.63	0.38
time (sec)	N/A	1.438	0.101	0.000	0.000	0.097	0.000	0.257	3.149	0.075

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	50	231	0	0	0	0	494	0
N.S.	1	1.00	0.83	3.85	0.00	0.00	0.00	0.00	8.23	0.00
time (sec)	N/A	0.516	0.034	0.343	0.000	0.000	0.000	0.000	0.206	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	28	126	0	0	110	0	76	0
N.S.	1	1.00	0.62	2.80	0.00	0.00	2.44	0.00	1.69	0.00
time (sec)	N/A	0.470	0.013	0.191	0.000	0.000	1.316	0.000	0.200	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	29	113	0	0	129	0	72	0
N.S.	1	1.00	0.63	2.46	0.00	0.00	2.80	0.00	1.57	0.00
time (sec)	N/A	0.464	0.015	0.188	0.000	0.000	1.251	0.000	0.200	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	206	0	0	0	0	493	0
N.S.	1	1.00	0.85	3.43	0.00	0.00	0.00	0.00	8.22	0.00
time (sec)	N/A	0.506	0.026	0.328	0.000	0.000	0.000	0.000	0.201	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	180	229	0	0	0	0	0	72	0
N.S.	1	1.54	1.96	0.00	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	1.141	0.394	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	97	129	0	0	0	0	0	24	0
N.S.	1	1.26	1.68	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.467	0.433	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	96	116	0	0	0	0	0	24	0
N.S.	1	1.23	1.49	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.457	0.296	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	180	193	0	0	0	0	0	70	0
N.S.	1	1.50	1.61	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	1.140	0.322	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	0	0	0	0	0	78	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.86	0.00
time (sec)	N/A	0.399	0.000	0.000	0.000	0.000	0.000	0.000	0.354	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	0	0	0	0	0	24	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.394	0.000	0.000	0.000	0.000	0.000	0.000	0.295	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	0	0	0	0	0	24	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.386	0.000	0.000	0.000	0.000	0.000	0.000	0.317	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	0	0	0	0	0	24	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.389	0.000	0.000	0.000	0.000	0.000	0.000	0.311	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	0	0	0	0	0	24	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.386	0.000	0.000	0.000	0.000	0.000	0.000	0.305	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	0	0	0	0	0	0	20	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.357	0.000	0.000	0.000	0.000	0.000	0.000	0.390	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	0	0	0	0	0	0	20	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.363	0.000	0.000	0.000	0.000	0.000	0.000	0.399	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	0	0	0	0	0	24	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.382	0.000	0.000	0.000	0.000	0.000	0.000	0.351	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	0	0	0	0	0	0	18	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.393	0.000	0.000	0.000	0.000	0.000	0.000	0.143	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	178	118	0	0	0	0	0	14	0
N.S.	1	1.02	0.68	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.526	0.630	0.000	0.000	0.000	0.000	0.000	0.143	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	98	0	0	0	0	0	12	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.419	0.348	0.000	0.000	0.000	0.000	0.000	0.145	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	82	0	0	0	0	0	10	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.358	0.207	0.000	0.000	0.000	0.000	0.000	0.152	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	142	0	0	0	0	0	14	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.451	0.198	0.000	0.000	0.000	0.000	0.000	0.144	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	44	0	0	0	0	0	43	0
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.384	0.046	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	107	0	0	0	0	0	48	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.429	0.399	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	171	132	0	0	0	0	0	121	0
N.S.	1	1.02	0.79	0.00	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.502	0.662	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	187	148	0	0	0	0	0	121	0
N.S.	1	0.85	0.67	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.531	0.549	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	128	80	128	259	125	0	138	120	214
N.S.	1	0.97	0.61	0.97	1.96	0.95	0.00	1.05	0.91	1.62
time (sec)	N/A	0.704	0.234	0.118	0.037	0.079	0.000	0.127	0.151	0.069

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	98	73	120	221	115	0	118	100	177
N.S.	1	0.93	0.70	1.14	2.10	1.10	0.00	1.12	0.95	1.69
time (sec)	N/A	0.531	0.230	0.123	0.040	0.080	0.000	0.134	0.146	13.760

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	70	64	112	181	103	0	98	80	138
N.S.	1	0.90	0.82	1.44	2.32	1.32	0.00	1.26	1.03	1.77
time (sec)	N/A	0.474	0.167	0.121	0.029	0.071	0.000	0.136	0.150	0.041

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	43	51	92	132	77	0	58	44	94
N.S.	1	0.91	1.09	1.96	2.81	1.64	0.00	1.23	0.94	2.00
time (sec)	N/A	0.390	0.110	0.064	0.035	0.092	0.000	0.128	0.146	13.417

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	54	60	249	78	87	0	0	56	48
N.S.	1	1.04	1.15	4.79	1.50	1.67	0.00	0.00	1.08	0.92
time (sec)	N/A	0.469	0.135	0.126	0.032	0.077	0.000	0.000	0.150	0.045

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	34	36	23	57	0	49	58	23
N.S.	1	1.00	1.03	1.09	0.70	1.73	0.00	1.48	1.76	0.70
time (sec)	N/A	0.377	0.174	0.127	0.026	0.095	0.000	0.157	0.152	0.023

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	64	42	41	39	77	0	85	93	39
N.S.	1	0.96	0.63	0.61	0.58	1.15	0.00	1.27	1.39	0.58
time (sec)	N/A	0.429	0.183	0.129	0.035	0.076	0.000	0.143	0.155	13.294

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	128	51	50	55	96	0	105	132	56
N.S.	1	1.28	0.51	0.50	0.55	0.96	0.00	1.05	1.32	0.56
time (sec)	N/A	0.575	0.179	0.129	0.033	0.100	0.000	0.166	0.151	13.318

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	192	59	58	71	116	0	125	168	72
N.S.	1	1.44	0.44	0.44	0.53	0.87	0.00	0.94	1.26	0.54
time (sec)	N/A	0.846	0.207	0.141	0.033	0.072	0.000	0.172	0.160	0.026

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	36	23	45	60	60	66	60	44	60
N.S.	1	0.97	0.62	1.22	1.62	1.62	1.78	1.62	1.19	1.62
time (sec)	N/A	0.538	0.033	0.132	0.033	0.063	0.038	0.118	0.146	13.324

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	36	30	29	37	37	36	37	28	37
N.S.	1	0.97	0.81	0.78	1.00	1.00	0.97	1.00	0.76	1.00
time (sec)	N/A	0.512	0.025	0.129	0.029	0.069	0.033	0.111	0.146	0.026

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	36	30	29	38	38	37	38	28	38
N.S.	1	0.97	0.81	0.78	1.03	1.03	1.00	1.03	0.76	1.03
time (sec)	N/A	0.499	0.024	0.127	0.027	0.063	0.031	0.110	0.150	0.027

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	17	17	16	18	18	15	18	15	15
N.S.	1	0.85	0.85	0.80	0.90	0.90	0.75	0.90	0.75	0.75
time (sec)	N/A	0.463	0.015	0.125	0.026	0.059	0.024	0.118	0.156	0.015

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	26	26	10	12	12	12	12	10	9
N.S.	1	1.86	1.86	0.71	0.86	0.86	0.86	0.86	0.71	0.64
time (sec)	N/A	0.278	0.014	0.104	0.031	0.062	0.021	0.122	0.145	0.013

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	30	30	29	30	29	20	31	35	29
N.S.	1	0.94	0.94	0.91	0.94	0.91	0.62	0.97	1.09	0.91
time (sec)	N/A	0.502	0.023	0.131	0.027	0.065	0.084	0.132	0.152	0.029

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	25	13	26	26	24	34	33	13
N.S.	1	1.00	1.79	0.93	1.86	1.86	1.71	2.43	2.36	0.93
time (sec)	N/A	0.433	0.017	0.128	0.031	0.063	0.092	0.114	0.142	0.029

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	36	23	21	47	47	49	21	37	46
N.S.	1	0.97	0.62	0.57	1.27	1.27	1.32	0.57	1.00	1.24
time (sec)	N/A	0.489	0.027	0.131	0.034	0.065	0.146	0.123	0.142	13.344

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	36	23	21	57	57	60	21	45	56
N.S.	1	0.97	0.62	0.57	1.54	1.54	1.62	0.57	1.22	1.51
time (sec)	N/A	0.491	0.026	0.135	0.029	0.065	0.175	0.112	0.149	0.049

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	101	80	128	259	126	0	138	120	214
N.S.	1	0.96	0.76	1.22	2.47	1.20	0.00	1.31	1.14	2.04
time (sec)	N/A	0.490	0.322	0.120	0.034	0.095	0.000	0.136	0.145	0.055

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	76	64	106	221	109	0	84	66	176
N.S.	1	0.97	0.82	1.36	2.83	1.40	0.00	1.08	0.85	2.26
time (sec)	N/A	0.434	0.208	0.119	0.036	0.075	0.000	0.178	0.146	0.042

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	77	64	112	181	103	0	98	80	139
N.S.	1	0.99	0.82	1.44	2.32	1.32	0.00	1.26	1.03	1.78
time (sec)	N/A	0.517	0.193	0.118	0.037	0.078	0.000	0.126	0.147	13.606

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	70	53	99	135	81	0	74	58	97
N.S.	1	1.08	0.82	1.52	2.08	1.25	0.00	1.14	0.89	1.49
time (sec)	N/A	0.551	0.124	0.086	0.034	0.088	0.000	0.138	0.152	13.373

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	63	345	95	120	0	35	98	63
N.S.	1	1.00	0.78	4.26	1.17	1.48	0.00	0.43	1.21	0.78
time (sec)	N/A	0.739	0.194	0.129	0.028	0.081	0.000	0.133	0.149	13.408

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	36	36	23	77	0	69	96	23
N.S.	1	1.00	1.09	1.09	0.70	2.33	0.00	2.09	2.91	0.70
time (sec)	N/A	0.395	0.163	0.126	0.028	0.078	0.000	0.179	0.153	0.018

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	66	41	41	39	95	0	125	131	39
N.S.	1	0.99	0.61	0.61	0.58	1.42	0.00	1.87	1.96	0.58
time (sec)	N/A	0.444	0.177	0.132	0.029	0.071	0.000	0.181	0.150	14.162

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	101	50	50	55	116	0	145	168	56
N.S.	1	1.01	0.50	0.50	0.55	1.16	0.00	1.45	1.68	0.56
time (sec)	N/A	0.570	0.181	0.129	0.029	0.069	0.000	0.207	0.153	0.022

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	134	58	58	71	134	0	165	204	72
N.S.	1	1.01	0.44	0.44	0.53	1.01	0.00	1.24	1.53	0.54
time (sec)	N/A	0.827	0.183	0.156	0.029	0.078	0.000	0.236	0.157	13.340

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	48	31	45	59	59	63	42	44	59
N.S.	1	0.91	0.58	0.85	1.11	1.11	1.19	0.79	0.83	1.11
time (sec)	N/A	0.531	0.030	0.151	0.030	0.063	0.053	0.123	0.144	0.018

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	26	26	23	28	28	29	42	24	24
N.S.	1	0.81	0.81	0.72	0.88	0.88	0.91	1.31	0.75	0.75
time (sec)	N/A	0.482	0.023	0.145	0.028	0.061	0.040	0.141	0.142	0.023

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	33	30	29	37	37	37	42	28	37
N.S.	1	0.94	0.86	0.83	1.06	1.06	1.06	1.20	0.80	1.06
time (sec)	N/A	0.487	0.023	0.359	0.033	0.071	0.039	0.114	0.145	0.026

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	21	16	25	25	24	40	19	19
N.S.	1	1.00	1.24	0.94	1.47	1.47	1.41	2.35	1.12	1.12
time (sec)	N/A	0.436	0.021	0.131	0.032	0.067	0.030	0.104	0.159	0.018

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	26	24	24	28	26	50	27	26
N.S.	1	0.96	0.96	0.89	0.89	1.04	0.96	1.85	1.00	0.96
time (sec)	N/A	0.469	0.018	0.128	0.031	0.065	0.069	0.113	0.140	13.263

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	43	36	32	44	49	37	57	63	42
N.S.	1	0.90	0.75	0.67	0.92	1.02	0.77	1.19	1.31	0.88
time (sec)	N/A	0.554	0.028	0.139	0.037	0.082	0.129	0.134	0.144	0.033

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	51	51	51	50	39	25
N.S.	1	1.00	1.00	0.96	2.04	2.04	2.04	2.00	1.56	1.00
time (sec)	N/A	0.449	0.013	0.141	0.029	0.070	0.153	0.113	0.147	13.374

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	47	31	27	65	65	70	42	53	29
N.S.	1	0.90	0.60	0.52	1.25	1.25	1.35	0.81	1.02	0.56
time (sec)	N/A	0.519	0.024	0.144	0.032	0.077	0.192	0.134	0.153	0.051

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	48	31	27	77	77	80	42	61	29
N.S.	1	0.91	0.58	0.51	1.45	1.45	1.51	0.79	1.15	0.55
time (sec)	N/A	0.512	0.024	0.148	0.027	0.060	0.230	0.126	0.142	0.064

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	130	72	120	221	114	0	109	100	176
N.S.	1	1.02	0.57	0.94	1.74	0.90	0.00	0.86	0.79	1.39
time (sec)	N/A	0.896	0.261	0.128	0.036	0.072	0.000	0.137	0.151	13.557

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	102	64	112	181	104	0	90	80	140
N.S.	1	1.02	0.64	1.12	1.81	1.04	0.00	0.90	0.80	1.40
time (sec)	N/A	0.681	0.184	0.121	0.031	0.077	0.000	0.144	0.143	13.727

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	70	53	99	135	81	0	68	58	96
N.S.	1	1.08	0.82	1.52	2.08	1.25	0.00	1.05	0.89	1.48
time (sec)	N/A	0.479	0.122	0.080	0.034	0.077	0.000	0.118	0.145	0.034

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	34	76	55	47	0	33	27	24
N.S.	1	1.00	1.48	3.30	2.39	2.04	0.00	1.43	1.17	1.04
time (sec)	N/A	0.377	0.134	0.121	0.034	0.071	0.000	0.127	0.149	13.350

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	36	23	39	0	0	32	23
N.S.	1	1.00	0.96	1.29	0.82	1.39	0.00	0.00	1.14	0.82
time (sec)	N/A	0.375	0.145	0.125	0.033	0.067	0.000	0.000	0.142	0.021

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	66	34	41	39	57	0	45	59	38
N.S.	1	1.06	0.55	0.66	0.63	0.92	0.00	0.73	0.95	0.61
time (sec)	N/A	0.415	0.149	0.128	0.029	0.084	0.000	0.164	0.153	13.294

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	130	43	50	55	77	0	65	96	55
N.S.	1	1.37	0.45	0.53	0.58	0.81	0.00	0.68	1.01	0.58
time (sec)	N/A	0.569	0.170	0.125	0.036	0.075	0.000	0.155	0.145	0.020

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	167	51	58	71	95	0	85	132	71
N.S.	1	1.30	0.40	0.45	0.55	0.74	0.00	0.66	1.03	0.55
time (sec)	N/A	0.621	0.165	0.139	0.038	0.096	0.000	0.178	0.147	0.022

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	78	56	50	63	68	68	75	53	63
N.S.	1	0.86	0.62	0.55	0.69	0.75	0.75	0.82	0.58	0.69
time (sec)	N/A	0.552	0.025	0.144	0.028	0.070	0.078	0.113	0.150	0.023

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	63	48	42	52	57	56	64	45	52
N.S.	1	0.86	0.66	0.58	0.71	0.78	0.77	0.88	0.62	0.71
time (sec)	N/A	0.529	0.023	0.154	0.033	0.083	0.071	0.111	0.152	0.020

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	47	39	34	41	45	41	52	36	41
N.S.	1	0.85	0.71	0.62	0.75	0.82	0.75	0.95	0.65	0.75
time (sec)	N/A	0.512	0.019	0.133	0.033	0.085	0.070	0.128	0.144	0.025

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	24	28	24	35	27	26
N.S.	1	1.00	1.00	0.92	0.92	1.08	0.92	1.35	1.04	1.00
time (sec)	N/A	0.457	0.013	0.121	0.026	0.092	0.061	0.135	0.151	13.310

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	12	15	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	0.86	1.07	1.00	1.00
time (sec)	N/A	0.447	0.012	0.129	0.025	0.090	0.025	0.136	0.147	0.021

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	24	29	23	20	25	29	12
N.S.	1	1.00	1.00	2.00	2.42	1.92	1.67	2.08	2.42	1.00
time (sec)	N/A	0.451	0.014	0.129	0.028	0.083	0.076	0.134	0.141	13.339

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	32	32	40	48	46	39	46	53	31
N.S.	1	0.97	0.97	1.21	1.45	1.39	1.18	1.39	1.61	0.94
time (sec)	N/A	0.495	0.029	0.141	0.035	0.068	0.124	0.114	0.144	0.036

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	47	35	51	63	76	54	51	93	46
N.S.	1	0.92	0.69	1.00	1.24	1.49	1.06	1.00	1.82	0.90
time (sec)	N/A	0.514	0.035	0.144	0.027	0.084	0.155	0.137	0.146	13.308

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	62	44	57	84	113	78	89	137	65
N.S.	1	0.90	0.64	0.83	1.22	1.64	1.13	1.29	1.99	0.94
time (sec)	N/A	0.531	0.041	0.145	0.028	0.111	0.216	0.134	0.141	0.045

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	131	78	148	204	104	0	0	135	163
N.S.	1	1.02	0.60	1.15	1.58	0.81	0.00	0.00	1.05	1.26
time (sec)	N/A	0.944	0.245	0.129	0.036	0.077	0.000	0.000	0.144	0.042

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	99	68	136	156	81	0	0	113	117
N.S.	1	1.08	0.74	1.48	1.70	0.88	0.00	0.00	1.23	1.27
time (sec)	N/A	0.694	0.193	0.099	0.036	0.084	0.000	0.000	0.142	0.037

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	52	54	248	78	63	0	0	77	48
N.S.	1	0.98	1.02	4.68	1.47	1.19	0.00	0.00	1.45	0.91
time (sec)	N/A	0.458	0.186	0.122	0.029	0.085	0.000	0.000	0.145	13.325

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	25	22	22	0	0	32	22
N.S.	1	1.00	0.93	0.89	0.79	0.79	0.00	0.00	1.14	0.79
time (sec)	N/A	0.370	0.157	0.125	0.035	0.105	0.000	0.000	0.147	0.017

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	33	30	48	31	0	22	47	38
N.S.	1	1.00	1.57	1.43	2.29	1.48	0.00	1.05	2.24	1.81
time (sec)	N/A	0.355	0.163	0.123	0.029	0.090	0.000	0.124	0.150	13.372

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	45	65	58	0	0	82	50
N.S.	1	1.00	0.82	0.74	1.07	0.95	0.00	0.00	1.34	0.82
time (sec)	N/A	0.439	0.176	0.122	0.034	0.086	0.000	0.000	0.144	13.292

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	94	57	54	82	77	0	0	131	51
N.S.	1	0.85	0.52	0.49	0.75	0.70	0.00	0.00	1.19	0.46
time (sec)	N/A	0.662	0.171	0.129	0.033	0.091	0.000	0.000	0.148	13.356

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	129	66	63	97	96	0	0	146	60
N.S.	1	0.90	0.46	0.44	0.68	0.67	0.00	0.00	1.02	0.42
time (sec)	N/A	0.963	0.192	0.141	0.039	0.078	0.000	0.000	0.150	0.039

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	195	86	69	99	105	0	147	55	76
N.S.	1	0.77	0.34	0.27	0.39	0.41	0.00	0.58	0.22	0.30
time (sec)	N/A	0.543	0.087	0.141	0.044	0.078	0.000	0.146	0.149	13.468

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	157	78	61	83	94	0	0	47	102
N.S.	1	0.80	0.40	0.31	0.42	0.48	0.00	0.00	0.24	0.52
time (sec)	N/A	0.493	0.064	0.135	0.042	0.069	0.000	0.000	0.142	13.463

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	119	70	53	67	83	0	97	39	60
N.S.	1	1.03	0.61	0.46	0.58	0.72	0.00	0.84	0.34	0.52
time (sec)	N/A	0.450	0.056	0.134	0.046	0.105	0.000	0.133	0.152	13.480

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	87	57	43	45	64	0	0	29	50
N.S.	1	1.13	0.74	0.56	0.58	0.83	0.00	0.00	0.38	0.65
time (sec)	N/A	0.414	0.036	0.132	0.043	0.091	0.000	0.000	0.146	13.879

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	43	35	26	50	0	49	19	43
N.S.	1	1.00	1.48	1.21	0.90	1.72	0.00	1.69	0.66	1.48
time (sec)	N/A	0.345	0.025	0.129	0.041	0.078	0.000	0.129	0.147	13.902

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	105	99	82	0	245	0	94	42	0
N.S.	1	0.88	0.83	0.69	0.00	2.06	0.00	0.79	0.35	0.00
time (sec)	N/A	0.456	0.080	0.151	0.000	0.116	0.000	0.149	0.158	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	114	116	118	0	287	0	60	70	0
N.S.	1	0.88	0.90	0.91	0.00	2.22	0.00	0.47	0.54	0.00
time (sec)	N/A	0.464	0.082	0.145	0.000	0.112	0.000	0.158	0.145	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	154	123	165	0	343	0	78	88	0
N.S.	1	0.79	0.63	0.85	0.00	1.77	0.00	0.40	0.45	0.00
time (sec)	N/A	0.504	0.175	0.154	0.000	0.103	0.000	0.152	0.148	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	190	139	219	0	399	0	105	165	0
N.S.	1	0.76	0.56	0.88	0.00	1.60	0.00	0.42	0.66	0.00
time (sec)	N/A	0.544	0.196	0.148	0.000	0.102	0.000	0.161	0.160	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	46	34	21	32	60	58	205	47	32
N.S.	1	1.15	0.85	0.52	0.80	1.50	1.45	5.12	1.18	0.80
time (sec)	N/A	0.595	0.052	0.170	0.038	0.098	2.342	0.116	0.141	13.710

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	46	34	21	32	49	58	141	39	32
N.S.	1	1.15	0.85	0.52	0.80	1.22	1.45	3.52	0.98	0.80
time (sec)	N/A	0.540	0.047	0.165	0.029	0.069	2.157	0.137	0.143	0.020

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	46	30	21	32	32	58	71	29	32
N.S.	1	1.15	0.75	0.52	0.80	0.80	1.45	1.78	0.72	0.80
time (sec)	N/A	0.535	0.038	0.167	0.033	0.071	2.085	0.115	0.147	0.020

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	44	23	20	30	19	56	44	19	32
N.S.	1	1.16	0.61	0.53	0.79	0.50	1.47	1.16	0.50	0.84
time (sec)	N/A	0.534	0.031	0.149	0.031	0.085	1.968	0.116	0.142	0.018

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	42	21	20	30	29	54	32	24	19
N.S.	1	1.17	0.58	0.56	0.83	0.81	1.50	0.89	0.67	0.53
time (sec)	N/A	0.531	0.029	0.164	0.028	0.091	1.616	0.110	0.137	13.710

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	44	34	21	26	44	56	36	32	20
N.S.	1	1.16	0.89	0.55	0.68	1.16	1.47	0.95	0.84	0.53
time (sec)	N/A	0.545	0.075	0.157	0.036	0.097	1.783	0.119	0.144	0.018

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	46	34	21	24	56	58	34	40	20
N.S.	1	1.15	0.85	0.52	0.60	1.40	1.45	0.85	1.00	0.50
time (sec)	N/A	0.527	0.080	0.167	0.031	0.095	1.693	0.113	0.149	13.721

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	46	34	21	26	66	58	36	48	20
N.S.	1	1.15	0.85	0.52	0.65	1.65	1.45	0.90	1.20	0.50
time (sec)	N/A	0.525	0.075	0.158	0.029	0.085	1.860	0.136	0.143	0.018

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	157	77	64	106	105	0	130	55	110
N.S.	1	0.80	0.39	0.32	0.54	0.53	0.00	0.66	0.28	0.56
time (sec)	N/A	0.489	0.060	0.137	0.046	0.095	0.000	0.138	0.154	13.948

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	119	69	56	90	94	0	0	47	102
N.S.	1	0.87	0.50	0.41	0.66	0.69	0.00	0.00	0.34	0.74
time (sec)	N/A	0.450	0.050	0.136	0.046	0.089	0.000	0.000	0.146	13.956

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	87	59	48	74	83	0	80	39	93
N.S.	1	0.98	0.66	0.54	0.83	0.93	0.00	0.90	0.44	1.04
time (sec)	N/A	0.409	0.050	0.138	0.051	0.089	0.000	0.125	0.143	14.009

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	43	35	41	61	0	0	28	81
N.S.	1	1.00	1.39	1.13	1.32	1.97	0.00	0.00	0.90	2.61
time (sec)	N/A	0.356	0.044	0.137	0.049	0.101	0.000	0.000	0.148	13.938

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	138	105	107	0	256	0	0	52	0
N.S.	1	0.84	0.64	0.65	0.00	1.56	0.00	0.00	0.32	0.00
time (sec)	N/A	0.492	0.079	0.150	0.000	0.114	0.000	0.000	0.151	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	148	116	135	0	294	0	75	91	0
N.S.	1	0.87	0.68	0.79	0.00	1.73	0.00	0.44	0.54	0.00
time (sec)	N/A	0.495	0.160	0.163	0.000	0.113	0.000	0.145	0.150	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	158	125	174	0	347	0	78	116	0
N.S.	1	0.84	0.66	0.93	0.00	1.85	0.00	0.41	0.62	0.00
time (sec)	N/A	0.512	0.188	0.141	0.000	0.098	0.000	0.167	0.152	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	196	142	226	0	399	0	105	165	0
N.S.	1	0.78	0.57	0.90	0.00	1.59	0.00	0.42	0.66	0.00
time (sec)	N/A	0.562	0.170	0.152	0.000	0.103	0.000	0.164	0.148	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	234	147	278	0	455	0	129	130	0
N.S.	1	0.76	0.48	0.90	0.00	1.48	0.00	0.42	0.42	0.00
time (sec)	N/A	0.580	0.234	0.144	0.000	0.100	0.000	0.179	0.150	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	157	70	61	96	83	0	0	39	94
N.S.	1	1.23	0.55	0.48	0.75	0.65	0.00	0.00	0.30	0.73
time (sec)	N/A	0.472	0.059	0.134	0.046	0.071	0.000	0.000	0.147	14.511

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	119	60	53	72	64	0	0	29	82
N.S.	1	1.25	0.63	0.56	0.76	0.67	0.00	0.00	0.31	0.86
time (sec)	N/A	0.464	0.050	0.136	0.045	0.097	0.000	0.000	0.158	14.546

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	87	50	42	54	50	0	43	20	71
N.S.	1	1.40	0.81	0.68	0.87	0.81	0.00	0.69	0.32	1.15
time (sec)	N/A	0.413	0.038	0.128	0.043	0.093	0.000	0.119	0.145	14.138

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	35	29	44	0	0	17	34
N.S.	1	1.00	0.97	1.21	1.00	1.52	0.00	0.00	0.59	1.17
time (sec)	N/A	0.348	0.032	0.130	0.040	0.100	0.000	0.000	0.146	14.041

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	76	76	78	0	147	0	64	30	0
N.S.	1	0.99	0.99	1.01	0.00	1.91	0.00	0.83	0.39	0.00
time (sec)	N/A	0.424	0.105	0.143	0.000	0.100	0.000	0.124	0.142	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	117	116	123	0	287	0	0	70	0
N.S.	1	0.85	0.85	0.90	0.00	2.09	0.00	0.00	0.51	0.00
time (sec)	N/A	0.462	0.107	0.144	0.000	0.089	0.000	0.000	0.144	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	155	125	172	0	347	0	88	116	0
N.S.	1	0.80	0.64	0.89	0.00	1.79	0.00	0.45	0.60	0.00
time (sec)	N/A	0.506	0.142	0.152	0.000	0.110	0.000	0.152	0.146	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	148	88	79	123	211	150	161	108	112
N.S.	1	1.08	0.64	0.58	0.90	1.54	1.09	1.18	0.79	0.82
time (sec)	N/A	0.644	0.093	0.188	0.107	0.091	2.768	0.118	0.145	0.049

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	126	80	71	109	187	133	134	93	95
N.S.	1	1.09	0.69	0.61	0.94	1.61	1.15	1.16	0.80	0.82
time (sec)	N/A	0.618	0.074	0.191	0.110	0.089	2.620	0.121	0.145	0.037

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	104	71	61	95	153	116	107	76	78
N.S.	1	1.09	0.75	0.64	1.00	1.61	1.22	1.13	0.80	0.82
time (sec)	N/A	0.587	0.054	0.182	0.108	0.090	2.563	0.118	0.145	13.742

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	82	61	57	79	124	99	77	59	61
N.S.	1	1.08	0.80	0.75	1.04	1.63	1.30	1.01	0.78	0.80
time (sec)	N/A	0.563	0.042	0.165	0.111	0.105	2.469	0.125	0.142	13.704

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	60	58	45	68	110	82	51	50	47
N.S.	1	1.03	1.00	0.78	1.17	1.90	1.41	0.88	0.86	0.81
time (sec)	N/A	0.530	0.033	0.165	0.112	0.116	1.622	0.115	0.145	13.740

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	29	52	83	66	36	39	28
N.S.	1	1.00	1.00	0.78	1.41	2.24	1.78	0.97	1.05	0.76
time (sec)	N/A	0.502	0.024	0.148	0.110	0.099	1.660	0.113	0.143	0.051

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	62	37	50	71	152	83	54	66	47
N.S.	1	1.09	0.65	0.88	1.25	2.67	1.46	0.95	1.16	0.82
time (sec)	N/A	0.555	0.031	0.161	0.106	0.078	2.022	0.113	0.150	0.054

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	91	39	64	81	202	104	73	128	65
N.S.	1	1.10	0.47	0.77	0.98	2.43	1.25	0.88	1.54	0.78
time (sec)	N/A	0.571	0.034	0.178	0.106	0.091	2.147	0.104	0.143	13.821

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	120	39	78	101	258	121	93	202	79
N.S.	1	1.15	0.38	0.75	0.97	2.48	1.16	0.89	1.94	0.76
time (sec)	N/A	0.610	0.039	0.181	0.105	0.123	2.397	0.110	0.141	0.055

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	191	68	72	120	83	0	0	49	94
N.S.	1	0.76	0.27	0.29	0.48	0.33	0.00	0.00	0.20	0.38
time (sec)	N/A	0.529	0.054	0.149	0.051	0.088	0.000	0.000	0.144	13.885

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	153	57	63	93	63	0	0	38	81
N.S.	1	0.81	0.30	0.33	0.49	0.33	0.00	0.00	0.20	0.43
time (sec)	N/A	0.489	0.047	0.150	0.049	0.077	0.000	0.000	0.143	13.831

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	115	48	55	75	50	0	0	30	71
N.S.	1	0.84	0.35	0.40	0.55	0.36	0.00	0.00	0.22	0.52
time (sec)	N/A	0.440	0.034	0.144	0.045	0.076	0.000	0.000	0.142	13.803

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	48	47	48	44	0	42	24	34
N.S.	1	0.98	0.56	0.55	0.56	0.52	0.00	0.49	0.28	0.40
time (sec)	N/A	0.410	0.038	0.144	0.044	0.079	0.000	0.121	0.150	13.909

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	41	35	45	43	0	41	19	32
N.S.	1	1.00	1.41	1.21	1.55	1.48	0.00	1.41	0.66	1.10
time (sec)	N/A	0.353	0.033	0.130	0.046	0.078	0.000	0.120	0.152	13.815

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	101	122	85	0	241	0	0	57	0
N.S.	1	0.83	1.01	0.70	0.00	1.99	0.00	0.00	0.47	0.00
time (sec)	N/A	0.447	0.091	0.141	0.000	0.099	0.000	0.000	0.145	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	141	130	129	0	291	0	90	110	0
N.S.	1	0.76	0.70	0.70	0.00	1.57	0.00	0.49	0.59	0.00
time (sec)	N/A	0.528	0.332	0.141	0.000	0.101	0.000	0.153	0.152	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	179	140	180	0	347	0	0	207	0
N.S.	1	0.74	0.58	0.74	0.00	1.43	0.00	0.00	0.86	0.00
time (sec)	N/A	0.541	0.277	0.140	0.000	0.088	0.000	0.000	0.149	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	75	50	74	153	81	530	0	78	57
N.S.	1	1.14	0.76	1.12	2.32	1.23	8.03	0.00	1.18	0.86
time (sec)	N/A	0.562	0.124	0.186	0.046	0.079	0.535	0.000	0.148	13.809

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	47	28	29	49	28	124	0	28	28
N.S.	1	1.12	0.67	0.69	1.17	0.67	2.95	0.00	0.67	0.67
time (sec)	N/A	0.519	0.020	0.148	0.040	0.110	0.335	0.000	0.146	13.862

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0	110	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.50	0.00
time (sec)	N/A	0.473	0.023	0.000	0.000	0.000	0.000	0.000	0.145	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	233	138	0	0	0	0	0	80	0
N.S.	1	2.48	1.47	0.00	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.583	0.159	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	176	131	0	0	0	0	0	26	0
N.S.	1	1.87	1.39	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.507	0.104	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	76	0	0	0	0	0	26	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.459	0.058	0.000	0.000	0.000	0.000	0.000	0.145	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	96	0	0	0	0	0	78	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.459	0.072	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	0	0	20	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.467	0.059	0.000	0.000	0.000	0.000	0.000	0.146	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	190	0	0	0	0	0	68	0
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.423	1.984	0.000	0.000	0.000	0.000	0.000	0.143	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	144	0	0	0	0	0	48	0
N.S.	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.429	1.231	0.000	0.000	0.000	0.000	0.000	0.146	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0	28	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.405	0.614	0.000	0.000	0.000	0.000	0.000	0.141	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	87	0	0	0	0	0	23	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.408	0.333	0.000	0.000	0.000	0.000	0.000	0.141	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	33	33	0	58	187	0	35	32
N.S.	1	1.00	0.69	0.69	0.00	1.21	3.90	0.00	0.73	0.67
time (sec)	N/A	0.389	0.288	1.664	0.000	0.084	10.584	0.000	0.140	13.829

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	105	64	46	0	128	1112	0	90	113
N.S.	1	1.01	0.62	0.44	0.00	1.23	10.69	0.00	0.87	1.09
time (sec)	N/A	0.464	0.368	5.191	0.000	0.087	55.945	0.000	0.143	13.929

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	208	83	68	0	228	0	0	180	180
N.S.	1	0.93	0.37	0.30	0.00	1.02	0.00	0.00	0.80	0.80
time (sec)	N/A	0.641	0.424	13.991	0.000	0.088	0.000	0.000	0.141	13.949

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	106	103	0	0	0	0	0	72	0
N.S.	1	1.08	1.05	0.00	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.478	0.123	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	106	101	0	0	0	0	0	45	0
N.S.	1	1.08	1.03	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.474	0.087	0.000	0.000	0.000	0.000	0.000	0.148	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	106	98	0	0	0	0	0	21	0
N.S.	1	1.08	1.00	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.464	0.081	0.000	0.000	0.000	0.000	0.000	0.139	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	104	96	0	0	0	0	0	25	0
N.S.	1	1.08	1.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.460	0.072	0.000	0.000	0.000	0.000	0.000	0.141	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	104	94	0	0	0	0	0	42	0
N.S.	1	1.08	0.98	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.463	0.053	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	187	117	0	0	0	0	0	54	0
N.S.	1	1.91	1.19	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.532	0.135	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	249	138	0	0	0	0	0	72	0
N.S.	1	2.54	1.41	0.00	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.608	0.204	0.000	0.000	0.000	0.000	0.000	0.145	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	226	116	104	122	185	0	0	82	223
N.S.	1	0.81	0.42	0.37	0.44	0.67	0.00	0.00	0.29	0.80
time (sec)	N/A	0.631	0.086	1.479	0.044	0.087	0.000	0.000	0.158	13.762

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	129	78	61	68	93	0	0	51	140
N.S.	1	1.02	0.61	0.48	0.54	0.73	0.00	0.00	0.40	1.10
time (sec)	N/A	0.463	0.061	0.989	0.039	0.118	0.000	0.000	0.152	13.779

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	58	34	37	44	0	0	22	55
N.S.	1	1.00	1.61	0.94	1.03	1.22	0.00	0.00	0.61	1.53
time (sec)	N/A	0.358	0.025	0.971	0.037	0.099	0.000	0.000	0.146	13.653

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	96	78	0	0	0	0	0	34	0
N.S.	1	1.20	0.98	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.459	0.035	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	105	89	0	0	0	0	0	41	0
N.S.	1	1.19	1.01	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.464	0.048	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	112	41	62	110	57	0	59	51	94
N.S.	1	1.81	0.66	1.00	1.77	0.92	0.00	0.95	0.82	1.52
time (sec)	N/A	0.424	0.047	0.125	0.030	0.118	0.000	0.120	0.151	0.047

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	80	40	57	87	54	0	48	39	68
N.S.	1	1.63	0.82	1.16	1.78	1.10	0.00	0.98	0.80	1.39
time (sec)	N/A	0.380	0.042	0.083	0.034	0.087	0.000	0.139	0.153	13.661

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	21	22	50	24	0	15	17	18
N.S.	1	1.00	1.17	1.22	2.78	1.33	0.00	0.83	0.94	1.00
time (sec)	N/A	0.331	0.035	0.048	0.028	0.092	0.000	0.136	0.157	0.033

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	31	39	48	83	51	0	38	27	63
N.S.	1	0.89	1.11	1.37	2.37	1.46	0.00	1.09	0.77	1.80
time (sec)	N/A	0.348	0.027	0.078	0.034	0.084	0.000	0.138	0.152	13.546

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	144	52	70	138	66	0	71	64	118
N.S.	1	1.76	0.63	0.85	1.68	0.80	0.00	0.87	0.78	1.44
time (sec)	N/A	0.481	0.075	0.124	0.036	0.089	0.000	0.149	0.152	0.027

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	112	47	65	112	61	0	60	52	94
N.S.	1	1.62	0.68	0.94	1.62	0.88	0.00	0.87	0.75	1.36
time (sec)	N/A	0.416	0.041	0.122	0.028	0.070	0.000	0.120	0.154	13.599

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	74	52	70	138	66	0	72	65	118
N.S.	1	1.04	0.73	0.99	1.94	0.93	0.00	1.01	0.92	1.66
time (sec)	N/A	0.455	0.068	0.137	0.030	0.072	0.000	0.147	0.142	13.710

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	50	47	60	112	61	0	60	50	90
N.S.	1	0.94	0.89	1.13	2.11	1.15	0.00	1.13	0.94	1.70
time (sec)	N/A	0.393	0.047	0.129	0.032	0.076	0.000	0.141	0.148	0.023

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	22	15	16	26	15	0	14	9	26
N.S.	1	1.69	1.15	1.23	2.00	1.15	0.00	1.08	0.69	2.00
time (sec)	N/A	0.343	0.030	0.132	0.034	0.073	0.000	0.143	0.159	0.031

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	22	18	35	31	31	26	21	17	14
N.S.	1	1.83	1.50	2.92	2.58	2.58	2.17	1.75	1.42	1.17
time (sec)	N/A	0.333	0.017	0.124	0.027	0.098	2.058	0.118	0.152	0.017

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	65	74	66	0	65	49	43
N.S.	1	1.00	0.80	1.27	1.45	1.29	0.00	1.27	0.96	0.84
time (sec)	N/A	0.436	0.088	0.139	0.030	0.129	0.000	0.144	0.149	13.641

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	38	52	44	61	0	49	38	28
N.S.	1	1.00	1.03	1.41	1.19	1.65	0.00	1.32	1.03	0.76
time (sec)	N/A	0.392	0.059	0.135	0.030	0.075	0.000	0.147	0.148	13.546

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	45	36	56	44	44	0	44	56	28
N.S.	1	1.36	1.09	1.70	1.33	1.33	0.00	1.33	1.70	0.85
time (sec)	N/A	0.376	0.051	0.126	0.031	0.072	0.000	0.149	0.142	0.017

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	21	18	12	11	11	8	22	18	11
N.S.	1	1.11	0.95	0.63	0.58	0.58	0.42	1.16	0.95	0.58
time (sec)	N/A	0.318	0.029	0.121	0.028	0.080	3.417	0.143	0.147	13.705

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	60	43	69	56	84	0	79	82	40
N.S.	1	0.98	0.70	1.13	0.92	1.38	0.00	1.30	1.34	0.66
time (sec)	N/A	0.466	0.092	0.138	0.028	0.073	0.000	0.139	0.140	0.024

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	22	13	31	0	46	38	13
N.S.	1	1.00	1.00	0.92	0.54	1.29	0.00	1.92	1.58	0.54
time (sec)	N/A	0.332	0.031	0.132	0.033	0.077	0.000	0.133	0.145	0.013

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	118	67	49	55	69	0	0	36	57
N.S.	1	0.84	0.48	0.35	0.39	0.49	0.00	0.00	0.26	0.41
time (sec)	N/A	0.513	0.058	0.140	0.040	0.080	0.000	0.000	0.151	13.721

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	86	56	41	41	61	0	81	28	49
N.S.	1	0.93	0.61	0.45	0.45	0.66	0.00	0.88	0.30	0.53
time (sec)	N/A	0.438	0.039	0.135	0.043	0.100	0.000	0.125	0.152	13.708

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	43	35	26	50	0	49	19	43
N.S.	1	1.00	1.48	1.21	0.90	1.72	0.00	1.69	0.66	1.48
time (sec)	N/A	0.348	0.015	0.130	0.045	0.088	0.000	0.113	0.147	0.001

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	78	75	69	0	212	0	88	47	0
N.S.	1	0.86	0.82	0.76	0.00	2.33	0.00	0.97	0.52	0.00
time (sec)	N/A	0.488	0.057	0.141	0.000	0.098	0.000	0.134	0.151	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	76	76	78	0	234	0	101	56	0
N.S.	1	0.80	0.80	0.82	0.00	2.46	0.00	1.06	0.59	0.00
time (sec)	N/A	0.475	0.046	0.147	0.000	0.113	0.000	0.148	0.150	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	107	48	45	74	44	134	189	44	83
N.S.	1	1.06	0.48	0.45	0.73	0.44	1.33	1.87	0.44	0.82
time (sec)	N/A	0.786	0.081	0.204	0.030	0.074	2.476	0.139	0.144	0.024

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	40	37	60	36	109	142	36	66
N.S.	1	1.08	0.50	0.46	0.75	0.45	1.36	1.78	0.45	0.82
time (sec)	N/A	0.770	0.063	0.194	0.033	0.074	2.420	0.119	0.148	13.631

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	63	31	28	44	27	80	92	27	46
N.S.	1	1.11	0.54	0.49	0.77	0.47	1.40	1.61	0.47	0.81
time (sec)	N/A	0.652	0.058	0.198	0.032	0.105	2.793	0.116	0.153	0.028

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	44	23	20	30	19	56	44	19	32
N.S.	1	1.16	0.61	0.53	0.79	0.50	1.47	1.16	0.50	0.84
time (sec)	N/A	0.529	0.010	0.154	0.027	0.079	2.170	0.129	0.142	0.001

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	45	39	32	49	89	80	40	35	31
N.S.	1	1.15	1.00	0.82	1.26	2.28	2.05	1.03	0.90	0.79
time (sec)	N/A	0.682	0.047	0.173	0.109	0.103	4.230	0.136	0.153	13.702

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	49	42	43	62	101	0	48	45	34
N.S.	1	1.17	1.00	1.02	1.48	2.40	0.00	1.14	1.07	0.81
time (sec)	N/A	0.685	0.040	0.181	0.112	0.108	0.000	0.111	0.147	0.035

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	76	55	52	103	123	0	76	64	54
N.S.	1	1.12	0.81	0.76	1.51	1.81	0.00	1.12	0.94	0.79
time (sec)	N/A	0.696	0.061	0.194	0.109	0.104	0.000	0.109	0.144	13.680

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	103	63	62	134	139	0	104	79	74
N.S.	1	1.16	0.71	0.70	1.51	1.56	0.00	1.17	0.89	0.83
time (sec)	N/A	0.724	0.071	0.204	0.114	0.082	0.000	0.124	0.152	13.574

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	130	71	70	163	155	0	131	94	91
N.S.	1	1.18	0.65	0.64	1.48	1.41	0.00	1.19	0.85	0.83
time (sec)	N/A	0.767	0.080	0.224	0.113	0.103	0.000	0.124	0.155	0.039

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	233	130	146	0	310	0	0	94	0
N.S.	1	0.75	0.42	0.47	0.00	1.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.725	0.143	0.154	0.000	0.107	0.000	0.000	0.156	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	197	122	138	0	294	0	145	80	0
N.S.	1	0.75	0.47	0.53	0.00	1.12	0.00	0.55	0.31	0.00
time (sec)	N/A	0.678	0.109	0.158	0.000	0.090	0.000	0.151	0.153	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	170	114	125	0	278	0	0	66	0
N.S.	1	0.79	0.53	0.58	0.00	1.30	0.00	0.00	0.31	0.00
time (sec)	N/A	0.564	0.127	0.161	0.000	0.101	0.000	0.000	0.157	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	138	105	107	0	256	0	0	52	0
N.S.	1	0.84	0.64	0.65	0.00	1.56	0.00	0.00	0.32	0.00
time (sec)	N/A	0.485	0.061	0.148	0.000	0.116	0.000	0.000	0.146	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	131	120	110	0	363	0	0	130	0
N.S.	1	0.79	0.73	0.67	0.00	2.20	0.00	0.00	0.79	0.00
time (sec)	N/A	0.592	0.098	0.148	0.000	0.116	0.000	0.000	0.152	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	134	120	119	0	401	0	0	190	0
N.S.	1	0.79	0.71	0.70	0.00	2.37	0.00	0.00	1.12	0.00
time (sec)	N/A	0.574	0.089	0.172	0.000	0.107	0.000	0.000	0.159	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	157	132	144	0	439	0	0	184	0
N.S.	1	0.70	0.59	0.64	0.00	1.95	0.00	0.00	0.82	0.00
time (sec)	N/A	0.631	0.236	0.175	0.000	0.144	0.000	0.000	0.162	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	188	140	165	0	455	0	0	198	0
N.S.	1	0.68	0.51	0.60	0.00	1.65	0.00	0.00	0.72	0.00
time (sec)	N/A	0.665	0.300	0.175	0.000	0.129	0.000	0.000	0.172	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	219	148	181	0	471	0	0	212	0
N.S.	1	0.68	0.46	0.56	0.00	1.46	0.00	0.00	0.66	0.00
time (sec)	N/A	0.725	0.382	0.168	0.000	0.111	0.000	0.000	0.181	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	120	46	37	27	33	0	47	21	48
N.S.	1	0.91	0.35	0.28	0.20	0.25	0.00	0.36	0.16	0.36
time (sec)	N/A	0.436	0.046	0.139	0.035	0.072	0.000	0.115	0.153	13.752

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	97	41	32	22	28	70	40	16	38
N.S.	1	0.99	0.42	0.33	0.22	0.29	0.71	0.41	0.16	0.39
time (sec)	N/A	0.387	0.034	0.136	0.034	0.068	39.527	0.137	0.172	13.621

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	95	48	34	22	45	0	46	22	35
N.S.	1	0.91	0.46	0.33	0.21	0.43	0.00	0.44	0.21	0.34
time (sec)	N/A	0.412	0.039	0.147	0.040	0.086	0.000	0.116	0.156	13.804

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	70	43	29	17	40	0	32	17	30
N.S.	1	1.03	0.63	0.43	0.25	0.59	0.00	0.47	0.25	0.44
time (sec)	N/A	0.364	0.033	0.143	0.038	0.074	0.000	0.114	0.151	13.848

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	97	39	30	20	26	0	38	14	38
N.S.	1	0.99	0.40	0.31	0.20	0.27	0.00	0.39	0.14	0.39
time (sec)	N/A	0.415	0.028	0.141	0.037	0.094	0.000	0.121	0.149	13.880

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	70	34	25	15	21	56	31	9	21
N.S.	1	1.09	0.53	0.39	0.23	0.33	0.88	0.48	0.14	0.33
time (sec)	N/A	0.382	0.024	0.138	0.034	0.108	2.950	0.115	0.151	13.807

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	70	41	29	17	40	0	32	17	30
N.S.	1	0.99	0.58	0.41	0.24	0.56	0.00	0.45	0.24	0.42
time (sec)	N/A	0.373	0.025	0.145	0.037	0.084	0.000	0.138	0.150	13.710

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	34	24	12	33	0	15	10	25
N.S.	1	1.00	1.70	1.20	0.60	1.65	0.00	0.75	0.50	1.25
time (sec)	N/A	0.311	0.019	0.139	0.037	0.084	0.000	0.126	0.149	13.817

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	70	26	25	13	21	48	31	9	21
N.S.	1	1.04	0.39	0.37	0.19	0.31	0.72	0.46	0.13	0.31
time (sec)	N/A	0.376	0.026	0.136	0.034	0.109	6.290	0.133	0.148	13.793

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	21	22	7	18	19	23	6	18
N.S.	1	1.00	0.70	0.73	0.23	0.60	0.63	0.77	0.20	0.60
time (sec)	N/A	0.333	0.019	0.131	0.033	0.095	6.195	0.115	0.144	13.773

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	98	69	68	0	76	0	50	40	0
N.S.	1	0.89	0.63	0.62	0.00	0.69	0.00	0.45	0.36	0.00
time (sec)	N/A	0.426	0.077	0.160	0.000	0.102	0.000	0.138	0.146	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	78	63	58	0	70	58	35	31	0
N.S.	1	0.87	0.70	0.64	0.00	0.78	0.64	0.39	0.34	0.00
time (sec)	N/A	0.376	0.045	0.151	0.000	0.093	14.399	0.121	0.143	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	78	65	47	0	46	0	0	21	0
N.S.	1	0.90	0.75	0.54	0.00	0.53	0.00	0.00	0.24	0.00
time (sec)	N/A	0.406	0.057	0.142	0.000	0.090	0.000	0.000	0.146	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	41	37	0	26	42	0	13	0
N.S.	1	1.00	0.75	0.67	0.00	0.47	0.76	0.00	0.24	0.00
time (sec)	N/A	0.353	0.041	0.132	0.000	0.079	36.898	0.000	0.151	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	112	75	88	0	88	126	50	73	0
N.S.	1	0.89	0.60	0.70	0.00	0.70	1.00	0.40	0.58	0.00
time (sec)	N/A	0.442	0.123	0.160	0.000	0.109	87.894	0.142	0.145	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	85	69	77	0	80	70	40	56	0
N.S.	1	0.94	0.77	0.86	0.00	0.89	0.78	0.44	0.62	0.00
time (sec)	N/A	0.396	0.079	0.155	0.000	0.111	85.631	0.124	0.149	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	151	67	59	83	69	0	0	36	88
N.S.	1	1.06	0.47	0.42	0.58	0.49	0.00	0.00	0.25	0.62
time (sec)	N/A	0.561	0.081	0.148	0.044	0.071	0.000	0.000	0.165	13.707

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	119	58	50	69	60	0	69	28	57
N.S.	1	1.14	0.56	0.48	0.66	0.58	0.00	0.66	0.27	0.55
time (sec)	N/A	0.482	0.057	0.147	0.046	0.093	0.000	0.147	0.148	13.680

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	87	50	42	54	50	0	43	20	71
N.S.	1	1.40	0.81	0.68	0.87	0.81	0.00	0.69	0.32	1.15
time (sec)	N/A	0.413	0.031	0.135	0.045	0.096	0.000	0.148	0.150	0.001

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	82	74	80	0	211	0	40	47	0
N.S.	1	0.90	0.81	0.88	0.00	2.32	0.00	0.44	0.52	0.00
time (sec)	N/A	0.500	0.074	0.145	0.000	0.116	0.000	0.121	0.163	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	82	76	90	0	237	0	48	57	0
N.S.	1	0.87	0.81	0.96	0.00	2.52	0.00	0.51	0.61	0.00
time (sec)	N/A	0.515	0.064	0.158	0.000	0.104	0.000	0.134	0.150	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	142	85	75	123	175	175	159	105	114
N.S.	1	1.02	0.61	0.54	0.88	1.26	1.26	1.14	0.76	0.82
time (sec)	N/A	0.877	0.195	0.233	0.112	0.108	3.311	0.126	0.152	0.055

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	100	78	68	97	158	134	105	90	80
N.S.	1	1.03	0.80	0.70	1.00	1.63	1.38	1.08	0.93	0.82
time (sec)	N/A	0.801	0.127	0.211	0.111	0.111	3.051	0.119	0.153	13.624

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	109	70	59	95	143	122	105	75	80
N.S.	1	1.12	0.72	0.61	0.98	1.47	1.26	1.08	0.77	0.82
time (sec)	N/A	0.671	0.140	0.202	0.107	0.100	3.000	0.115	0.162	13.659

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	82	61	57	79	124	99	77	59	61
N.S.	1	1.08	0.80	0.75	1.04	1.63	1.30	1.01	0.78	0.80
time (sec)	N/A	0.547	0.032	0.190	0.111	0.102	2.604	0.117	0.165	0.001

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	85	74	58	98	170	122	67	67	57
N.S.	1	1.15	1.00	0.78	1.32	2.30	1.65	0.91	0.91	0.77
time (sec)	N/A	0.741	0.056	0.190	0.108	0.108	4.590	0.138	0.169	0.053

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	92	78	70	111	186	0	71	81	61
N.S.	1	1.18	1.00	0.90	1.42	2.38	0.00	0.91	1.04	0.78
time (sec)	N/A	0.757	0.064	0.210	0.114	0.118	0.000	0.119	0.165	0.063

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	119	93	80	152	216	0	106	108	88
N.S.	1	1.12	0.88	0.75	1.43	2.04	0.00	1.00	1.02	0.83
time (sec)	N/A	0.839	0.105	0.242	0.108	0.127	0.000	0.135	0.206	13.683

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	146	101	89	183	232	0	133	123	105
N.S.	1	1.15	0.80	0.70	1.44	1.83	0.00	1.05	0.97	0.83
time (sec)	N/A	0.883	0.125	0.232	0.107	0.098	0.000	0.122	0.189	13.700

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	173	109	97	212	248	0	160	138	122
N.S.	1	1.17	0.74	0.66	1.43	1.68	0.00	1.08	0.93	0.82
time (sec)	N/A	0.904	0.133	0.260	0.109	0.109	0.000	0.143	0.161	0.077

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	204	73	80	117	77	0	0	54	74
N.S.	1	0.73	0.26	0.28	0.42	0.27	0.00	0.00	0.19	0.26
time (sec)	N/A	0.626	0.052	0.168	0.050	0.111	0.000	0.000	0.163	13.725

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	176	65	72	104	69	0	0	46	88
N.S.	1	0.76	0.28	0.31	0.45	0.30	0.00	0.00	0.20	0.38
time (sec)	N/A	0.589	0.048	0.177	0.044	0.112	0.000	0.000	0.154	13.711

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	144	57	64	91	61	0	0	38	58
N.S.	1	0.79	0.31	0.35	0.50	0.34	0.00	0.00	0.21	0.32
time (sec)	N/A	0.520	0.039	0.175	0.046	0.092	0.000	0.000	0.163	13.637

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	115	48	55	75	50	0	0	30	71
N.S.	1	0.84	0.35	0.40	0.55	0.36	0.00	0.00	0.22	0.52
time (sec)	N/A	0.450	0.020	0.158	0.050	0.092	0.000	0.000	0.157	0.001

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	107	78	80	0	212	0	0	65	0
N.S.	1	0.78	0.57	0.58	0.00	1.55	0.00	0.00	0.47	0.00
time (sec)	N/A	0.547	0.133	0.167	0.000	0.091	0.000	0.000	0.156	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	111	79	86	0	238	0	0	74	0
N.S.	1	0.80	0.57	0.62	0.00	1.72	0.00	0.00	0.54	0.00
time (sec)	N/A	0.554	0.112	0.178	0.000	0.092	0.000	0.000	0.162	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	141	90	103	0	267	0	0	90	0
N.S.	1	0.74	0.47	0.54	0.00	1.41	0.00	0.00	0.47	0.00
time (sec)	N/A	0.577	0.103	0.191	0.000	0.117	0.000	0.000	0.165	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	172	98	114	0	283	0	0	98	0
N.S.	1	0.72	0.41	0.48	0.00	1.19	0.00	0.00	0.41	0.00
time (sec)	N/A	0.593	0.122	0.188	0.000	0.125	0.000	0.000	0.160	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	203	106	125	0	299	0	0	106	0
N.S.	1	0.71	0.37	0.44	0.00	1.05	0.00	0.00	0.37	0.00
time (sec)	N/A	0.616	0.118	0.252	0.000	0.094	0.000	0.000	0.160	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	203	161	0	0	0	0	0	135	0
N.S.	1	1.07	0.85	0.00	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.748	0.361	0.000	0.000	0.000	0.000	0.000	1.295	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	155	120	0	0	0	0	0	108	0
N.S.	1	1.07	0.83	0.00	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.662	0.297	0.000	0.000	0.000	0.000	0.000	0.421	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	107	87	0	0	0	0	0	79	0
N.S.	1	1.14	0.93	0.00	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.517	0.236	0.000	0.000	0.000	0.000	0.000	0.255	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	40	0	0	0	0	0	50	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	1.19	0.00
time (sec)	N/A	0.417	0.164	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	152	121	0	0	0	0	0	40	0
N.S.	1	1.14	0.91	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.671	0.303	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	214	121	0	0	0	0	0	51	0
N.S.	1	1.37	0.78	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.775	0.313	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	201	263	121	0	0	0	0	0	68	0
N.S.	1	1.31	0.60	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.869	0.349	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	196	133	0	0	0	0	0	102	0
N.S.	1	0.92	0.62	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.696	0.238	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	137	106	0	0	0	0	0	66	0
N.S.	1	1.02	0.79	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.590	0.121	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	68	0	0	0	0	0	33	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.512	0.039	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	0	0	0	0	0	0	37	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.550	0.000	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	0	0	0	0	0	0	58	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.567	0.000	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	137	104	0	0	0	0	0	33	0
N.S.	1	1.04	0.79	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.570	0.088	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	115	175	157	224	156	0	248	163	183
N.S.	1	1.01	1.54	1.38	1.96	1.37	0.00	2.18	1.43	1.61
time (sec)	N/A	0.800	0.226	0.105	0.116	0.100	0.000	0.138	0.167	13.848

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	167	140	201	146	0	221	135	163
N.S.	1	1.00	1.90	1.59	2.28	1.66	0.00	2.51	1.53	1.85
time (sec)	N/A	0.582	0.205	0.098	0.108	0.103	0.000	0.136	0.159	13.624

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	56	158	131	125	119	0	137	103	90
N.S.	1	0.90	2.55	2.11	2.02	1.92	0.00	2.21	1.66	1.45
time (sec)	N/A	0.508	0.226	0.095	0.112	0.106	0.000	0.158	0.151	0.051

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	34	31	63	66	48	0	42	53	60
N.S.	1	1.26	1.15	2.33	2.44	1.78	0.00	1.56	1.96	2.22
time (sec)	N/A	0.378	0.054	0.069	0.117	0.131	0.000	0.140	0.150	13.702

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	77	63	143	116	94	0	0	70	62
N.S.	1	1.08	0.89	2.01	1.63	1.32	0.00	0.00	0.99	0.87
time (sec)	N/A	0.553	0.133	0.114	0.036	0.148	0.000	0.000	0.165	0.049

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	104	94	185	137	134	0	0	130	104
N.S.	1	0.87	0.78	1.54	1.14	1.12	0.00	0.00	1.08	0.87
time (sec)	N/A	0.790	0.072	0.125	0.034	0.092	0.000	0.000	0.177	13.699

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	143	104	225	153	170	0	63	199	121
N.S.	1	0.93	0.68	1.46	0.99	1.10	0.00	0.41	1.29	0.79
time (sec)	N/A	1.050	0.093	0.142	0.032	0.133	0.000	0.142	0.174	0.051

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	178	112	265	169	204	0	0	268	137
N.S.	1	0.95	0.60	1.41	0.90	1.09	0.00	0.00	1.43	0.73
time (sec)	N/A	1.346	0.117	0.144	0.030	0.137	0.000	0.000	0.168	13.582

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	48	57	47	57	67	63	58	51	51
N.S.	1	0.79	0.93	0.77	0.93	1.10	1.03	0.95	0.84	0.84
time (sec)	N/A	0.733	0.100	0.180	0.030	0.111	0.141	0.116	0.171	0.042

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	34	41	32	37	43	39	38	35	35
N.S.	1	0.85	1.02	0.80	0.92	1.08	0.98	0.95	0.88	0.88
time (sec)	N/A	0.710	0.067	0.177	0.025	0.082	0.095	0.113	0.162	13.662

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	32	38	31	37	45	37	38	35	35
N.S.	1	0.82	0.97	0.79	0.95	1.15	0.95	0.97	0.90	0.90
time (sec)	N/A	0.705	0.059	0.156	0.031	0.116	0.084	0.134	0.159	13.833

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	20	16	17	16	21	15	16	19	19
N.S.	1	1.25	1.00	1.06	1.00	1.31	0.94	1.00	1.19	1.19
time (sec)	N/A	0.680	0.049	0.152	0.024	0.126	0.042	0.127	0.163	0.021

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	13	10	12	11	11
N.S.	1	1.00	1.00	1.09	1.00	1.18	0.91	1.09	1.00	1.00
time (sec)	N/A	0.592	0.036	0.132	0.025	0.104	0.039	0.125	0.173	0.018

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	36	30	35	35	40	26	36	44	34
N.S.	1	0.97	0.81	0.95	0.95	1.08	0.70	0.97	1.19	0.92
time (sec)	N/A	0.696	0.039	0.128	0.026	0.080	0.092	0.119	0.169	13.719

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	52	52	47	55	70	49	42	71	54
N.S.	1	0.98	0.98	0.89	1.04	1.32	0.92	0.79	1.34	1.02
time (sec)	N/A	0.760	0.078	0.138	0.027	0.086	0.136	0.119	0.171	0.040

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	68	63	56	75	100	73	50	99	71
N.S.	1	0.93	0.86	0.77	1.03	1.37	1.00	0.68	1.36	0.97
time (sec)	N/A	0.776	0.082	0.132	0.028	0.094	0.190	0.120	0.176	13.669

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	81	71	67	93	126	94	58	129	90
N.S.	1	0.93	0.82	0.77	1.07	1.45	1.08	0.67	1.48	1.03
time (sec)	N/A	0.800	0.099	0.133	0.025	0.091	0.256	0.131	0.179	13.650

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	102	175	157	223	156	0	248	155	183
N.S.	1	0.99	1.70	1.52	2.17	1.51	0.00	2.41	1.50	1.78
time (sec)	N/A	0.602	0.292	0.092	0.115	0.123	0.000	0.147	0.197	0.070

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	63	51	105	151	85	0	69	91	119
N.S.	1	1.03	0.84	1.72	2.48	1.39	0.00	1.13	1.49	1.95
time (sec)	N/A	0.433	0.094	0.080	0.108	0.114	0.000	0.131	0.172	13.740

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	67	154	131	125	114	0	138	105	90
N.S.	1	1.06	2.44	2.08	1.98	1.81	0.00	2.19	1.67	1.43
time (sec)	N/A	0.586	0.204	0.080	0.110	0.116	0.000	0.148	0.172	13.732

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	55	73	145	114	88	0	91	74	82
N.S.	1	1.12	1.49	2.96	2.33	1.80	0.00	1.86	1.51	1.67
time (sec)	N/A	0.566	0.143	0.087	0.116	0.098	0.000	0.149	0.166	13.550

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	109	70	183	133	128	0	63	130	100
N.S.	1	0.94	0.60	1.58	1.15	1.10	0.00	0.54	1.12	0.86
time (sec)	N/A	0.834	0.206	0.116	0.033	0.100	0.000	0.151	0.179	0.056

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	143	104	225	153	170	0	63	199	120
N.S.	1	0.96	0.70	1.51	1.03	1.14	0.00	0.42	1.34	0.81
time (sec)	N/A	1.083	0.100	0.128	0.030	0.086	0.000	0.156	0.186	13.653

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	158	112	265	169	204	0	0	268	137
N.S.	1	0.87	0.62	1.46	0.93	1.12	0.00	0.00	1.47	0.75
time (sec)	N/A	1.322	0.102	0.136	0.039	0.115	0.000	0.000	0.177	0.058

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	213	120	305	185	240	0	63	337	153
N.S.	1	0.99	0.56	1.42	0.86	1.12	0.00	0.29	1.57	0.71
time (sec)	N/A	1.612	0.119	0.149	0.037	0.118	0.000	0.218	0.180	13.738

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	51	51	48	59	67	63	123	51	51
N.S.	1	0.80	0.80	0.75	0.92	1.05	0.98	1.92	0.80	0.80
time (sec)	N/A	0.704	0.092	0.234	0.032	0.129	0.150	0.127	0.154	0.036

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	28	28	27	31	36	31	59	29	27
N.S.	1	0.93	0.93	0.90	1.03	1.20	1.03	1.97	0.97	0.90
time (sec)	N/A	0.686	0.064	0.199	0.024	0.089	0.087	0.117	0.163	13.844

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	35	38	30	34	43	37	98	35	31
N.S.	1	0.92	1.00	0.79	0.89	1.13	0.97	2.58	0.92	0.82
time (sec)	N/A	0.689	0.064	0.191	0.041	0.106	0.088	0.137	0.175	0.026

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	23	23	24	27	32	26	94	25	25
N.S.	1	0.85	0.85	0.89	1.00	1.19	0.96	3.48	0.93	0.93
time (sec)	N/A	0.672	0.050	0.174	0.029	0.184	0.058	0.115	0.169	0.024

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	22	22	22	24	23	17	55	21	24
N.S.	1	0.88	0.88	0.88	0.96	0.92	0.68	2.20	0.84	0.96
time (sec)	N/A	0.615	0.043	0.165	0.024	0.107	0.116	0.112	0.171	13.466

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	51	51	43	49	64	41	74	73	48
N.S.	1	0.96	0.96	0.81	0.92	1.21	0.77	1.40	1.38	0.91
time (sec)	N/A	0.725	0.040	0.150	0.031	0.081	0.136	0.127	0.169	0.035

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	66	63	56	75	100	73	94	99	71
N.S.	1	0.93	0.89	0.79	1.06	1.41	1.03	1.32	1.39	1.00
time (sec)	N/A	0.794	0.075	0.154	0.033	0.089	0.197	0.135	0.166	13.780

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	82	71	67	93	126	94	109	129	90
N.S.	1	0.92	0.80	0.75	1.04	1.42	1.06	1.22	1.45	1.01
time (sec)	N/A	0.789	0.099	0.163	0.028	0.077	0.307	0.123	0.169	0.047

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	94	79	78	113	154	114	124	159	109
N.S.	1	0.90	0.75	0.74	1.08	1.47	1.09	1.18	1.51	1.04
time (sec)	N/A	0.811	0.112	0.155	0.027	0.088	0.373	0.111	0.165	13.670

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	142	175	157	223	156	0	265	163	185
N.S.	1	1.09	1.35	1.21	1.72	1.20	0.00	2.04	1.25	1.42
time (sec)	N/A	1.183	0.185	0.121	0.115	0.127	0.000	0.166	0.174	13.657

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	107	167	149	201	143	0	232	134	163
N.S.	1	1.01	1.58	1.41	1.90	1.35	0.00	2.19	1.26	1.54
time (sec)	N/A	0.965	0.168	0.116	0.128	0.128	0.000	0.133	0.172	0.064

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	80	55	134	126	113	0	130	105	90
N.S.	1	1.04	0.71	1.74	1.64	1.47	0.00	1.69	1.36	1.17
time (sec)	N/A	0.780	0.229	0.115	0.112	0.115	0.000	0.136	0.173	0.045

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	55	73	137	114	88	0	85	74	82
N.S.	1	1.12	1.49	2.80	2.33	1.80	0.00	1.73	1.51	1.67
time (sec)	N/A	0.525	0.157	0.096	0.107	0.098	0.000	0.151	0.168	13.479

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	28	44	27	0	24	19	39
N.S.	1	1.00	1.00	1.47	2.32	1.42	0.00	1.26	1.00	2.05
time (sec)	N/A	0.384	0.113	0.096	0.033	0.091	0.000	0.114	0.171	0.030

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	74	69	144	120	97	0	0	70	62
N.S.	1	1.06	0.99	2.06	1.71	1.39	0.00	0.00	1.00	0.89
time (sec)	N/A	0.564	0.051	0.129	0.032	0.139	0.000	0.000	0.160	13.484

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	100	94	185	137	134	0	0	130	105
N.S.	1	0.91	0.85	1.68	1.25	1.22	0.00	0.00	1.18	0.95
time (sec)	N/A	0.802	0.082	0.145	0.035	0.108	0.000	0.000	0.178	0.056

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	129	104	225	153	170	0	59	199	121
N.S.	1	0.90	0.73	1.57	1.07	1.19	0.00	0.41	1.39	0.85
time (sec)	N/A	1.057	0.091	0.168	0.045	0.089	0.000	0.152	0.174	13.528

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	54	54	51	60	71	56	62	57	61
N.S.	1	0.83	0.83	0.78	0.92	1.09	0.86	0.95	0.88	0.94
time (sec)	N/A	0.748	0.085	0.201	0.029	0.100	0.257	0.117	0.170	0.053

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	44	46	43	51	62	42	53	49	51
N.S.	1	0.81	0.85	0.80	0.94	1.15	0.78	0.98	0.91	0.94
time (sec)	N/A	0.719	0.076	0.181	0.028	0.107	0.200	0.144	0.188	0.046

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	34	34	31	40	43	31	42	35	40
N.S.	1	0.85	0.85	0.78	1.00	1.08	0.78	1.05	0.88	1.00
time (sec)	N/A	0.689	0.061	0.165	0.032	0.079	0.158	0.113	0.164	0.043

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	19	23	20	23	22	17	25	19	23
N.S.	1	0.83	1.00	0.87	1.00	0.96	0.74	1.09	0.83	1.00
time (sec)	N/A	0.601	0.042	0.148	0.033	0.086	0.122	0.126	0.168	13.526

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	22	22	20	20	19	17	21	19	19
N.S.	1	1.10	1.10	1.00	1.00	0.95	0.85	1.05	0.95	0.95
time (sec)	N/A	0.650	0.025	0.124	0.030	0.089	0.052	0.116	0.174	0.023

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	23	18	28	34	27	34	36	33	17
N.S.	1	1.28	1.00	1.56	1.89	1.50	1.89	2.00	1.83	0.94
time (sec)	N/A	0.650	0.047	0.144	0.034	0.094	0.091	0.143	0.183	0.037

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	55	57	48	53	59	56	51	62	52
N.S.	1	0.96	1.00	0.84	0.93	1.04	0.98	0.89	1.09	0.91
time (sec)	N/A	0.733	0.076	0.141	0.032	0.106	0.173	0.113	0.180	13.781

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	72	72	60	69	93	73	57	105	68
N.S.	1	0.96	0.96	0.80	0.92	1.24	0.97	0.76	1.40	0.91
time (sec)	N/A	0.765	0.098	0.145	0.026	0.115	0.227	0.117	0.175	0.052

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	173	567	191	246	157	0	0	271	211
N.S.	1	1.09	3.57	1.20	1.55	0.99	0.00	0.00	1.70	1.33
time (sec)	N/A	1.479	1.385	0.136	0.117	0.097	0.000	0.000	0.193	0.081

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	138	663	181	225	146	0	0	251	190
N.S.	1	1.02	4.91	1.34	1.67	1.08	0.00	0.00	1.86	1.41
time (sec)	N/A	1.226	0.537	0.137	0.108	0.100	0.000	0.000	0.174	13.509

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	111	424	169	149	120	0	0	212	117
N.S.	1	1.06	4.04	1.61	1.42	1.14	0.00	0.00	2.02	1.11
time (sec)	N/A	0.964	0.450	0.133	0.121	0.101	0.000	0.000	0.168	0.056

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	87	234	376	135	92	0	0	165	107
N.S.	1	1.16	3.12	5.01	1.80	1.23	0.00	0.00	2.20	1.43
time (sec)	N/A	0.732	0.563	0.096	0.123	0.136	0.000	0.000	0.169	13.555

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	70	61	137	120	69	0	0	95	87
N.S.	1	0.97	0.85	1.90	1.67	0.96	0.00	0.00	1.32	1.21
time (sec)	N/A	0.552	0.172	0.108	0.033	0.110	0.000	0.000	0.172	0.032

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	68	69	138	125	67	0	0	95	90
N.S.	1	0.96	0.97	1.94	1.76	0.94	0.00	0.00	1.34	1.27
time (sec)	N/A	0.529	0.050	0.129	0.034	0.095	0.000	0.000	0.150	13.643

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	45	33	41	92	42	0	42	37	41
N.S.	1	1.10	0.80	1.00	2.24	1.02	0.00	1.02	0.90	1.00
time (sec)	N/A	0.402	0.029	0.121	0.031	0.090	0.000	0.138	0.167	13.574

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	105	94	218	160	134	0	0	156	128
N.S.	1	0.95	0.85	1.98	1.45	1.22	0.00	0.00	1.42	1.16
time (sec)	N/A	0.614	0.080	0.155	0.046	0.084	0.000	0.000	0.162	0.046

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	132	104	259	176	170	0	0	268	144
N.S.	1	0.92	0.73	1.81	1.23	1.19	0.00	0.00	1.87	1.01
time (sec)	N/A	1.034	0.106	0.170	0.034	0.083	0.000	0.000	0.163	13.459

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	183	109	166	0	437	0	0	123	0
N.S.	1	0.78	0.46	0.71	0.00	1.86	0.00	0.00	0.52	0.00
time (sec)	N/A	0.697	0.142	0.104	0.000	0.122	0.000	0.000	0.179	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	150	101	149	0	415	0	0	105	0
N.S.	1	0.77	0.52	0.76	0.00	2.12	0.00	0.00	0.54	0.00
time (sec)	N/A	0.635	0.090	0.098	0.000	0.127	0.000	0.000	0.163	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	115	89	132	0	381	0	0	87	0
N.S.	1	0.73	0.57	0.84	0.00	2.43	0.00	0.00	0.55	0.00
time (sec)	N/A	0.589	0.082	0.088	0.000	0.154	0.000	0.000	0.154	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	122	70	106	0	313	0	0	59	0
N.S.	1	1.04	0.60	0.91	0.00	2.68	0.00	0.00	0.50	0.00
time (sec)	N/A	0.667	0.061	0.105	0.000	0.140	0.000	0.000	0.160	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	82	66	87	0	295	0	0	31	0
N.S.	1	1.05	0.85	1.12	0.00	3.78	0.00	0.00	0.40	0.00
time (sec)	N/A	0.560	0.048	0.083	0.000	0.120	0.000	0.000	0.153	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	103	95	151	0	517	0	0	118	0
N.S.	1	0.77	0.71	1.13	0.00	3.86	0.00	0.00	0.88	0.00
time (sec)	N/A	0.601	0.090	0.181	0.000	0.155	0.000	0.000	0.166	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	137	122	259	0	594	0	0	252	0
N.S.	1	0.81	0.72	1.52	0.00	3.49	0.00	0.00	1.48	0.00
time (sec)	N/A	0.644	0.108	0.193	0.000	0.204	0.000	0.000	0.167	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	175	135	328	0	668	0	0	420	0
N.S.	1	0.82	0.63	1.54	0.00	3.14	0.00	0.00	1.97	0.00
time (sec)	N/A	0.711	0.142	0.197	0.000	0.149	0.000	0.000	0.175	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	146	91	155	0	243	2222	0	123	0
N.S.	1	1.02	0.64	1.08	0.00	1.70	15.54	0.00	0.86	0.00
time (sec)	N/A	0.816	0.167	0.174	0.000	0.126	20.493	0.000	0.172	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	124	83	144	0	221	740	0	105	0
N.S.	1	1.05	0.70	1.22	0.00	1.87	6.27	0.00	0.89	0.00
time (sec)	N/A	0.815	0.130	0.170	0.000	0.086	6.914	0.000	0.159	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	103	75	108	0	191	144	0	87	0
N.S.	1	1.08	0.79	1.14	0.00	2.01	1.52	0.00	0.92	0.00
time (sec)	N/A	0.741	0.092	0.168	0.000	0.112	13.704	0.000	0.148	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	80	55	103	0	146	173	0	59	0
N.S.	1	1.14	0.79	1.47	0.00	2.09	2.47	0.00	0.84	0.00
time (sec)	N/A	0.715	0.072	0.154	0.000	0.091	21.563	0.000	0.147	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	58	50	98	0	133	0	96	33	0
N.S.	1	1.16	1.00	1.96	0.00	2.66	0.00	1.92	0.66	0.00
time (sec)	N/A	0.679	0.038	0.150	0.000	0.086	0.000	0.138	0.147	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	83	43	150	0	185	0	170	62	0
N.S.	1	1.19	0.61	2.14	0.00	2.64	0.00	2.43	0.89	0.00
time (sec)	N/A	0.701	0.039	0.181	0.000	0.105	0.000	0.159	0.148	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	110	55	201	0	247	0	245	114	0
N.S.	1	1.16	0.58	2.12	0.00	2.60	0.00	2.58	1.20	0.00
time (sec)	N/A	0.769	0.040	0.169	0.000	0.101	0.000	0.180	0.149	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	137	58	247	0	303	0	316	175	0
N.S.	1	1.16	0.49	2.09	0.00	2.57	0.00	2.68	1.48	0.00
time (sec)	N/A	0.806	0.039	0.177	0.000	0.088	0.000	0.238	0.152	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	164	46	291	0	355	0	391	236	0
N.S.	1	1.13	0.32	2.01	0.00	2.45	0.00	2.70	1.63	0.00
time (sec)	N/A	0.829	0.049	0.184	0.000	0.122	0.000	0.338	0.162	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	168	109	178	0	437	0	0	123	0
N.S.	1	0.71	0.46	0.76	0.00	1.86	0.00	0.00	0.52	0.00
time (sec)	N/A	0.665	0.121	0.094	0.000	0.121	0.000	0.000	0.169	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	132	101	161	0	415	0	0	105	0
N.S.	1	0.68	0.52	0.83	0.00	2.13	0.00	0.00	0.54	0.00
time (sec)	N/A	0.612	0.094	0.089	0.000	0.140	0.000	0.000	0.160	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	162	89	144	0	381	0	0	87	0
N.S.	1	1.04	0.57	0.92	0.00	2.44	0.00	0.00	0.56	0.00
time (sec)	N/A	0.772	0.074	0.092	0.000	0.151	0.000	0.000	0.156	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	123	66	118	0	315	0	0	59	0
N.S.	1	1.04	0.56	1.00	0.00	2.67	0.00	0.00	0.50	0.00
time (sec)	N/A	0.660	0.052	0.074	0.000	0.155	0.000	0.000	0.195	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	104	95	160	0	512	0	0	117	0
N.S.	1	0.77	0.70	1.19	0.00	3.79	0.00	0.00	0.87	0.00
time (sec)	N/A	0.588	0.062	0.184	0.000	0.167	0.000	0.000	0.156	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	135	115	259	0	581	0	0	252	0
N.S.	1	0.79	0.68	1.52	0.00	3.42	0.00	0.00	1.48	0.00
time (sec)	N/A	0.657	0.115	0.191	0.000	0.185	0.000	0.000	0.159	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	172	135	326	0	668	0	0	420	0
N.S.	1	0.83	0.65	1.57	0.00	3.21	0.00	0.00	2.02	0.00
time (sec)	N/A	0.727	0.144	0.194	0.000	0.142	0.000	0.000	0.163	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	210	143	376	0	736	0	0	583	0
N.S.	1	0.84	0.57	1.51	0.00	2.96	0.00	0.00	2.34	0.00
time (sec)	N/A	0.781	0.176	0.198	0.000	0.170	0.000	0.000	0.168	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	148	101	161	0	415	0	0	105	0
N.S.	1	0.76	0.52	0.82	0.00	2.12	0.00	0.00	0.54	0.00
time (sec)	N/A	0.654	0.112	0.115	0.000	0.116	0.000	0.000	0.159	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	113	89	144	0	381	0	0	87	0
N.S.	1	0.72	0.57	0.92	0.00	2.43	0.00	0.00	0.55	0.00
time (sec)	N/A	0.591	0.076	0.114	0.000	0.162	0.000	0.000	0.151	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	93	70	118	0	315	0	0	59	0
N.S.	1	0.80	0.60	1.02	0.00	2.72	0.00	0.00	0.51	0.00
time (sec)	N/A	0.574	0.057	0.112	0.000	0.156	0.000	0.000	0.151	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	87	65	101	0	297	0	0	33	0
N.S.	1	1.10	0.82	1.28	0.00	3.76	0.00	0.00	0.42	0.00
time (sec)	N/A	0.575	0.046	0.102	0.000	0.118	0.000	0.000	0.154	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	84	65	102	0	299	0	0	36	0
N.S.	1	1.08	0.83	1.31	0.00	3.83	0.00	0.00	0.46	0.00
time (sec)	N/A	0.568	0.053	0.097	0.000	0.135	0.000	0.000	0.145	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	112	91	162	0	522	0	0	120	0
N.S.	1	0.82	0.67	1.19	0.00	3.84	0.00	0.00	0.88	0.00
time (sec)	N/A	0.588	0.075	0.188	0.000	0.150	0.000	0.000	0.149	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	140	123	264	0	596	0	0	252	0
N.S.	1	0.78	0.69	1.47	0.00	3.33	0.00	0.00	1.41	0.00
time (sec)	N/A	0.656	0.112	0.223	0.000	0.195	0.000	0.000	0.157	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	175	135	316	0	668	0	0	420	0
N.S.	1	0.81	0.62	1.46	0.00	3.09	0.00	0.00	1.94	0.00
time (sec)	N/A	0.735	0.160	0.224	0.000	0.143	0.000	0.000	0.162	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	172	125	223	0	340	0	0	197	0
N.S.	1	1.06	0.77	1.37	0.00	2.09	0.00	0.00	1.21	0.00
time (sec)	N/A	0.941	0.271	0.225	0.000	0.143	0.000	0.000	0.163	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	150	116	213	0	302	0	0	179	0
N.S.	1	1.09	0.84	1.54	0.00	2.19	0.00	0.00	1.30	0.00
time (sec)	N/A	0.905	0.124	0.214	0.000	0.118	0.000	0.000	0.165	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	128	95	193	0	252	0	0	139	0
N.S.	1	1.13	0.84	1.71	0.00	2.23	0.00	0.00	1.23	0.00
time (sec)	N/A	0.866	0.093	0.194	0.000	0.117	0.000	0.000	0.150	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	108	92	168	0	236	0	0	107	0
N.S.	1	1.17	1.00	1.83	0.00	2.57	0.00	0.00	1.16	0.00
time (sec)	N/A	0.777	0.056	0.183	0.000	0.092	0.000	0.000	0.158	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	105	95	136	0	235	0	0	108	0
N.S.	1	1.11	1.00	1.43	0.00	2.47	0.00	0.00	1.14	0.00
time (sec)	N/A	0.794	0.057	0.183	0.000	0.095	0.000	0.000	0.163	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	116	94	136	0	232	0	0	110	0
N.S.	1	1.23	1.00	1.45	0.00	2.47	0.00	0.00	1.17	0.00
time (sec)	N/A	0.852	0.070	0.203	0.000	0.103	0.000	0.000	0.153	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	132	70	227	0	304	0	0	152	0
N.S.	1	1.14	0.60	1.96	0.00	2.62	0.00	0.00	1.31	0.00
time (sec)	N/A	0.858	0.069	0.226	0.000	0.112	0.000	0.000	0.171	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	162	79	271	0	376	0	0	304	0
N.S.	1	1.10	0.54	1.84	0.00	2.56	0.00	0.00	2.07	0.00
time (sec)	N/A	0.894	0.072	0.227	0.000	0.094	0.000	0.000	0.173	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	192	82	315	0	448	0	0	475	0
N.S.	1	1.12	0.48	1.83	0.00	2.60	0.00	0.00	2.76	0.00
time (sec)	N/A	0.978	0.075	0.237	0.000	0.100	0.000	0.000	0.172	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	177	132	212	0	415	0	0	113	0
N.S.	1	0.76	0.57	0.91	0.00	1.79	0.00	0.00	0.49	0.00
time (sec)	N/A	0.675	0.133	0.144	0.000	0.153	0.000	0.000	0.158	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	144	124	195	0	381	0	0	101	0
N.S.	1	0.75	0.64	1.01	0.00	1.97	0.00	0.00	0.52	0.00
time (sec)	N/A	0.637	0.123	0.124	0.000	0.136	0.000	0.000	0.150	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	109	71	169	0	315	0	0	77	0
N.S.	1	0.72	0.47	1.11	0.00	2.07	0.00	0.00	0.51	0.00
time (sec)	N/A	0.577	0.088	0.136	0.000	0.119	0.000	0.000	0.149	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	92	67	146	0	299	0	0	59	0
N.S.	1	0.80	0.58	1.27	0.00	2.60	0.00	0.00	0.51	0.00
time (sec)	N/A	0.568	0.067	0.113	0.000	0.140	0.000	0.000	0.150	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	125	69	149	0	303	0	0	62	0
N.S.	1	1.06	0.58	1.26	0.00	2.57	0.00	0.00	0.53	0.00
time (sec)	N/A	0.677	0.068	0.114	0.000	0.110	0.000	0.000	0.158	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	121	64	149	0	311	0	0	62	0
N.S.	1	1.03	0.55	1.27	0.00	2.66	0.00	0.00	0.53	0.00
time (sec)	N/A	0.654	0.059	0.112	0.000	0.194	0.000	0.000	0.147	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	121	90	258	0	524	0	0	168	0
N.S.	1	0.70	0.52	1.48	0.00	3.01	0.00	0.00	0.97	0.00
time (sec)	N/A	0.619	0.072	0.195	0.000	0.216	0.000	0.000	0.157	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	160	121	290	0	594	0	0	338	0
N.S.	1	0.74	0.56	1.35	0.00	2.76	0.00	0.00	1.57	0.00
time (sec)	N/A	0.683	0.096	0.202	0.000	0.235	0.000	0.000	0.161	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	174	147	121	0	337	0	0	67	0
N.S.	1	1.06	0.90	0.74	0.00	2.05	0.00	0.00	0.41	0.00
time (sec)	N/A	0.877	0.925	0.069	0.000	0.138	0.000	0.000	0.145	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	129	148	104	0	317	0	0	48	0
N.S.	1	1.04	1.19	0.84	0.00	2.56	0.00	0.00	0.39	0.00
time (sec)	N/A	0.701	1.442	0.068	0.000	0.123	0.000	0.000	0.158	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	82	66	87	0	295	0	0	31	0
N.S.	1	1.05	0.85	1.12	0.00	3.78	0.00	0.00	0.40	0.00
time (sec)	N/A	0.530	0.046	0.056	0.000	0.114	0.000	0.000	0.145	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	78	132	88	0	275	0	0	43	0
N.S.	1	1.03	1.74	1.16	0.00	3.62	0.00	0.00	0.57	0.00
time (sec)	N/A	0.640	0.646	0.075	0.000	0.110	0.000	0.000	0.145	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	45	39	0	58	0	0	42	47
N.S.	1	1.00	1.22	1.05	0.00	1.57	0.00	0.00	1.14	1.27
time (sec)	N/A	0.528	0.215	0.062	0.000	0.103	0.000	0.000	0.151	13.674

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	58	45	0	68	0	0	63	53
N.S.	1	1.00	0.75	0.58	0.00	0.88	0.00	0.00	0.82	0.69
time (sec)	N/A	0.585	0.259	0.070	0.000	0.079	0.000	0.000	0.151	13.747

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	122	66	53	0	77	0	0	81	100
N.S.	1	1.08	0.58	0.47	0.00	0.68	0.00	0.00	0.72	0.88
time (sec)	N/A	0.711	0.249	0.063	0.000	0.094	0.000	0.000	0.151	13.685

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	175	74	61	0	84	0	0	99	108
N.S.	1	1.15	0.49	0.40	0.00	0.55	0.00	0.00	0.65	0.71
time (sec)	N/A	1.091	0.297	0.066	0.000	0.079	0.000	0.000	0.154	13.710

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	157	50	127	0	188	0	142	85	0
N.S.	1	1.21	0.38	0.98	0.00	1.45	0.00	1.09	0.65	0.00
time (sec)	N/A	1.198	0.050	0.139	0.000	0.114	0.000	0.137	0.148	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	124	50	119	0	172	0	127	67	0
N.S.	1	1.18	0.48	1.13	0.00	1.64	0.00	1.21	0.64	0.00
time (sec)	N/A	1.167	0.048	0.137	0.000	0.109	0.000	0.138	0.143	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	91	77	111	0	156	0	112	49	0
N.S.	1	1.14	0.96	1.39	0.00	1.95	0.00	1.40	0.61	0.00
time (sec)	N/A	0.925	0.092	0.139	0.000	0.089	0.000	0.153	0.151	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	58	50	98	0	133	0	96	33	0
N.S.	1	1.16	1.00	1.96	0.00	2.66	0.00	1.92	0.66	0.00
time (sec)	N/A	0.683	0.027	0.129	0.000	0.091	0.000	0.141	0.154	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	B	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	58	47	98	0	120	95	0	41	0
N.S.	1	1.23	1.00	2.09	0.00	2.55	2.02	0.00	0.87	0.00
time (sec)	N/A	1.097	0.038	0.141	0.000	0.104	4.009	0.000	0.151	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	55	28	25	0	28	0	121	44	24
N.S.	1	1.31	0.67	0.60	0.00	0.67	0.00	2.88	1.05	0.57
time (sec)	N/A	1.105	0.046	0.133	0.000	0.078	0.000	0.156	0.157	13.649

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	78	36	33	0	36	0	164	62	32
N.S.	1	1.13	0.52	0.48	0.00	0.52	0.00	2.38	0.90	0.46
time (sec)	N/A	1.168	0.052	0.138	0.000	0.084	0.000	0.190	0.152	13.649

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	105	44	41	0	44	0	203	81	77
N.S.	1	1.09	0.46	0.43	0.00	0.46	0.00	2.11	0.84	0.80
time (sec)	N/A	1.283	0.066	0.139	0.000	0.103	0.000	0.244	0.155	13.586

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	130	52	49	0	52	0	243	99	98
N.S.	1	1.07	0.43	0.40	0.00	0.43	0.00	2.01	0.82	0.81
time (sec)	N/A	1.209	0.079	0.136	0.000	0.083	0.000	0.287	0.156	13.705

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	188	252	224	0	568	0	0	169	0
N.S.	1	0.72	0.97	0.86	0.00	2.18	0.00	0.00	0.65	0.00
time (sec)	N/A	0.861	1.561	0.171	0.000	0.159	0.000	0.000	0.151	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	160	244	202	0	552	0	0	151	0
N.S.	1	0.73	1.11	0.92	0.00	2.51	0.00	0.00	0.69	0.00
time (sec)	N/A	0.836	1.139	0.161	0.000	0.168	0.000	0.000	0.151	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	132	236	180	0	536	0	0	133	0
N.S.	1	0.73	1.31	1.00	0.00	2.98	0.00	0.00	0.74	0.00
time (sec)	N/A	0.696	1.731	0.159	0.000	0.194	0.000	0.000	0.160	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	104	95	160	0	512	0	0	117	0
N.S.	1	0.77	0.70	1.19	0.00	3.79	0.00	0.00	0.87	0.00
time (sec)	N/A	0.592	0.089	0.151	0.000	0.165	0.000	0.000	0.146	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	100	218	161	0	490	0	0	131	0
N.S.	1	0.78	1.69	1.25	0.00	3.80	0.00	0.00	1.02	0.00
time (sec)	N/A	0.708	1.059	0.163	0.000	0.161	0.000	0.000	0.149	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	138	155	140	0	353	0	0	152	0
N.S.	1	1.10	1.24	1.12	0.00	2.82	0.00	0.00	1.22	0.00
time (sec)	N/A	0.744	0.425	0.166	0.000	0.125	0.000	0.000	0.157	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	177	162	165	0	381	0	0	171	0
N.S.	1	1.04	0.95	0.97	0.00	2.24	0.00	0.00	1.01	0.00
time (sec)	N/A	0.781	0.525	0.163	0.000	0.147	0.000	0.000	0.158	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	221	170	187	0	397	0	0	189	0
N.S.	1	1.06	0.81	0.89	0.00	1.90	0.00	0.00	0.90	0.00
time (sec)	N/A	0.928	0.535	0.167	0.000	0.133	0.000	0.000	0.175	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	276	178	206	0	413	0	0	207	0
N.S.	1	1.12	0.72	0.84	0.00	1.68	0.00	0.00	0.84	0.00
time (sec)	N/A	1.308	0.585	0.171	0.000	0.127	0.000	0.000	0.185	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	177	147	133	0	337	0	0	67	0
N.S.	1	1.08	0.90	0.81	0.00	2.05	0.00	0.00	0.41	0.00
time (sec)	N/A	0.895	0.947	0.102	0.000	0.158	0.000	0.000	0.152	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	132	139	116	0	321	0	0	49	0
N.S.	1	1.06	1.12	0.94	0.00	2.59	0.00	0.00	0.40	0.00
time (sec)	N/A	0.722	1.996	0.099	0.000	0.163	0.000	0.000	0.156	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	87	65	101	0	297	0	0	33	0
N.S.	1	1.10	0.82	1.28	0.00	3.76	0.00	0.00	0.42	0.00
time (sec)	N/A	0.542	0.047	0.073	0.000	0.120	0.000	0.000	0.142	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	83	132	100	0	275	0	0	41	0
N.S.	1	1.09	1.74	1.32	0.00	3.62	0.00	0.00	0.54	0.00
time (sec)	N/A	0.671	0.741	0.086	0.000	0.152	0.000	0.000	0.154	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	78	46	52	0	59	0	0	44	54
N.S.	1	1.11	0.66	0.74	0.00	0.84	0.00	0.00	0.63	0.77
time (sec)	N/A	0.609	0.251	0.085	0.000	0.117	0.000	0.000	0.156	13.606

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	120	58	60	0	69	0	0	62	62
N.S.	1	1.06	0.51	0.53	0.00	0.61	0.00	0.00	0.55	0.55
time (sec)	N/A	0.655	0.288	0.084	0.000	0.107	0.000	0.000	0.153	13.688

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	169	66	68	0	77	0	0	81	100
N.S.	1	1.11	0.43	0.45	0.00	0.51	0.00	0.00	0.53	0.66
time (sec)	N/A	0.811	0.268	0.092	0.000	0.128	0.000	0.000	0.149	13.687

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	207	116	197	0	288	0	0	159	0
N.S.	1	1.20	0.67	1.15	0.00	1.67	0.00	0.00	0.92	0.00
time (sec)	N/A	1.336	0.168	0.168	0.000	0.126	0.000	0.000	0.157	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	174	108	189	0	276	0	0	141	0
N.S.	1	1.18	0.73	1.29	0.00	1.88	0.00	0.00	0.96	0.00
time (sec)	N/A	1.344	0.129	0.165	0.000	0.112	0.000	0.000	0.153	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	141	100	181	0	256	0	0	123	0
N.S.	1	1.16	0.82	1.48	0.00	2.10	0.00	0.00	1.01	0.00
time (sec)	N/A	1.062	0.111	0.168	0.000	0.155	0.000	0.000	0.147	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	108	92	168	0	236	0	0	107	0
N.S.	1	1.17	1.00	1.83	0.00	2.57	0.00	0.00	1.16	0.00
time (sec)	N/A	0.799	0.030	0.150	0.000	0.115	0.000	0.000	0.148	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	B	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	104	86	165	0	220	141	0	120	0
N.S.	1	1.21	1.00	1.92	0.00	2.56	1.64	0.00	1.40	0.00
time (sec)	N/A	1.202	0.050	0.164	0.000	0.096	4.350	0.000	0.154	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	86	69	142	0	169	0	0	135	0
N.S.	1	1.05	0.84	1.73	0.00	2.06	0.00	0.00	1.65	0.00
time (sec)	N/A	1.156	0.073	0.168	0.000	0.116	0.000	0.000	0.152	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	79	152	0	189	0	278	155	0
N.S.	1	1.00	0.70	1.35	0.00	1.67	0.00	2.46	1.37	0.00
time (sec)	N/A	1.237	0.102	0.165	0.000	0.092	0.000	0.304	0.158	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	117	87	160	0	209	0	356	173	0
N.S.	1	1.04	0.77	1.42	0.00	1.85	0.00	3.15	1.53	0.00
time (sec)	N/A	1.264	0.167	0.172	0.000	0.102	0.000	0.291	0.163	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	167	95	168	0	221	0	434	191	0
N.S.	1	1.02	0.58	1.03	0.00	1.36	0.00	2.66	1.17	0.00
time (sec)	N/A	1.289	0.163	0.175	0.000	0.088	0.000	0.374	0.172	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	172	167	197	0	353	0	0	95	0
N.S.	1	0.70	0.68	0.81	0.00	1.45	0.00	0.00	0.39	0.00
time (sec)	N/A	0.787	1.214	0.105	0.000	0.135	0.000	0.000	0.149	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	144	159	180	0	337	0	0	83	0
N.S.	1	0.71	0.79	0.89	0.00	1.67	0.00	0.00	0.41	0.00
time (sec)	N/A	0.740	1.071	0.105	0.000	0.135	0.000	0.000	0.150	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	116	151	163	0	321	0	0	71	0
N.S.	1	0.72	0.94	1.01	0.00	1.99	0.00	0.00	0.44	0.00
time (sec)	N/A	0.644	2.075	0.098	0.000	0.163	0.000	0.000	0.151	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	92	67	146	0	299	0	0	59	0
N.S.	1	0.80	0.58	1.27	0.00	2.60	0.00	0.00	0.51	0.00
time (sec)	N/A	0.560	0.064	0.086	0.000	0.114	0.000	0.000	0.145	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	90	131	151	0	277	0	0	65	0
N.S.	1	0.83	1.20	1.39	0.00	2.54	0.00	0.00	0.60	0.00
time (sec)	N/A	0.695	1.127	0.101	0.000	0.113	0.000	0.000	0.150	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	115	58	60	0	59	0	0	59	54
N.S.	1	1.06	0.53	0.55	0.00	0.54	0.00	0.00	0.54	0.50
time (sec)	N/A	0.689	0.266	0.105	0.000	0.091	0.000	0.000	0.144	13.629

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	157	70	68	0	69	0	0	71	62
N.S.	1	1.05	0.47	0.45	0.00	0.46	0.00	0.00	0.47	0.41
time (sec)	N/A	0.722	0.280	0.105	0.000	0.088	0.000	0.000	0.149	13.653

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	205	78	76	0	77	0	0	83	100
N.S.	1	1.09	0.41	0.40	0.00	0.41	0.00	0.00	0.44	0.53
time (sec)	N/A	0.873	0.279	0.107	0.000	0.094	0.000	0.000	0.151	13.787

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	253	86	84	0	85	0	0	95	108
N.S.	1	1.11	0.38	0.37	0.00	0.37	0.00	0.00	0.42	0.47
time (sec)	N/A	1.245	0.350	0.110	0.000	0.095	0.000	0.000	0.149	13.727

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	0	61	0	0	0	0	0	26	0
N.S.	1	0.00	0.98	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.000	0.041	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	0	95	0	0	0	0	0	52	0
N.S.	1	0.00	0.80	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.000	0.069	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	185	187	155	0	0	0	0	0	34	0
N.S.	1	1.01	0.84	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.577	0.452	0.000	0.000	0.000	0.000	0.000	0.145	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	97	0	0	0	0	0	24	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.489	0.491	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	163	113	0	0	0	0	0	36	0
N.S.	1	0.98	0.68	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.568	0.086	0.000	0.000	0.000	0.000	0.000	0.147	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F(-1)	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	111	0	0	0	0	0	0	61	0
N.S.	1	1.07	0.00	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.715	0.000	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F(-1)	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	111	0	0	0	0	0	0	28	0
N.S.	1	1.07	0.00	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.681	0.000	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F(-1)	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	111	0	0	0	0	0	0	28	0
N.S.	1	1.07	0.00	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.690	0.000	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F(-1)	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	111	0	0	0	0	0	0	45	0
N.S.	1	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.749	0.000	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	110	0	0	0	0	0	0	32	0
N.S.	1	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.638	0.000	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	0	0	0	33	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.568	0.034	0.000	0.000	0.000	0.000	0.000	0.143	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	93	0	0	0	0	0	0	35	0
N.S.	1	1.16	0.00	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.619	0.000	0.000	0.000	0.000	0.000	0.000	0.145	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	69	46	0	0	0	269	0	89	0
N.S.	1	1.21	0.81	0.00	0.00	0.00	4.72	0.00	1.56	0.00
time (sec)	N/A	0.597	0.031	0.000	0.000	0.000	5.101	0.000	0.145	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	90	0	0	0	0	0	0	38	0
N.S.	1	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.580	0.000	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	88	0	0	0	0	0	0	38	0
N.S.	1	1.07	0.00	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.594	0.000	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	124	87	0	0	0	0	0	218	0
N.S.	1	1.09	0.76	0.00	0.00	0.00	0.00	0.00	1.91	0.00
time (sec)	N/A	0.698	0.060	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	74	47	85	113	113	119	113	84	113
N.S.	1	0.88	0.56	1.01	1.35	1.35	1.42	1.35	1.00	1.35
time (sec)	N/A	0.637	0.047	0.188	0.029	0.078	0.043	0.114	0.144	13.123

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	62	39	61	82	82	87	82	60	82
N.S.	1	0.90	0.57	0.88	1.19	1.19	1.26	1.19	0.87	1.19
time (sec)	N/A	0.587	0.036	0.167	0.032	0.078	0.041	0.113	0.143	0.023

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	48	31	53	70	70	70	70	52	70
N.S.	1	0.92	0.60	1.02	1.35	1.35	1.35	1.35	1.00	1.35
time (sec)	N/A	0.553	0.029	0.163	0.029	0.088	0.033	0.109	0.142	0.021

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	34	23	29	38	38	36	38	28	38
N.S.	1	0.97	0.66	0.83	1.09	1.09	1.03	1.09	0.80	1.09
time (sec)	N/A	0.523	0.022	0.152	0.035	0.079	0.031	0.137	0.157	0.027

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	20	14	21	21	20	21	18	17
N.S.	1	1.00	1.33	0.93	1.40	1.40	1.33	1.40	1.20	1.13
time (sec)	N/A	0.429	0.018	0.119	0.027	0.061	0.022	0.135	0.146	0.019

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	13	13	13	10	14	12	14
N.S.	1	1.00	1.12	0.81	0.81	0.81	0.62	0.88	0.75	0.88
time (sec)	N/A	0.467	0.064	0.136	0.024	0.086	0.060	0.114	0.151	13.057

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	47	35	51	63	76	54	51	93	46
N.S.	1	0.92	0.69	1.00	1.24	1.49	1.06	1.00	1.82	0.90
time (sec)	N/A	0.553	0.036	0.202	0.029	0.083	0.149	0.108	0.142	0.043

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	76	63	76	91	121	85	74	145	73
N.S.	1	0.88	0.73	0.88	1.06	1.41	0.99	0.86	1.69	0.85
time (sec)	N/A	0.602	0.048	0.152	0.034	0.089	0.222	0.120	0.152	13.149

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	105	82	92	140	217	141	91	269	121
N.S.	1	0.87	0.68	0.76	1.16	1.79	1.17	0.75	2.22	1.00
time (sec)	N/A	0.656	0.077	0.159	0.038	0.102	0.344	0.136	0.144	13.138

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	58	39	75	101	101	109	102	76	101
N.S.	1	0.88	0.59	1.14	1.53	1.53	1.65	1.55	1.15	1.53
time (sec)	N/A	0.598	0.037	0.200	0.031	0.093	0.052	0.115	0.145	13.158

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	47	31	69	92	92	100	90	68	92
N.S.	1	0.90	0.60	1.33	1.77	1.77	1.92	1.73	1.31	1.77
time (sec)	N/A	0.560	0.033	0.182	0.030	0.073	0.045	0.122	0.143	0.026

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	33	23	45	59	59	63	78	44	59
N.S.	1	0.94	0.66	1.29	1.69	1.69	1.80	2.23	1.26	1.69
time (sec)	N/A	0.525	0.026	0.174	0.032	0.088	0.046	0.120	0.151	0.019

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	16	47	47	48	64	35	47
N.S.	1	1.00	2.18	0.94	2.76	2.76	2.82	3.76	2.06	2.76
time (sec)	N/A	0.477	0.027	0.148	0.029	0.063	0.051	0.138	0.141	0.017

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	44	38	32	33	37	36	60	35	33
N.S.	1	0.96	0.83	0.70	0.72	0.80	0.78	1.30	0.76	0.72
time (sec)	N/A	0.502	0.023	0.141	0.031	0.076	0.066	0.115	0.146	13.085

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	13	19	19	17	27	32	12
N.S.	1	1.00	1.92	1.00	1.46	1.46	1.31	2.08	2.46	0.92
time (sec)	N/A	0.463	0.016	0.152	0.036	0.079	0.089	0.115	0.146	0.028

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	41	41	42	15	31	40
N.S.	1	1.00	1.00	0.89	2.28	2.28	2.33	0.83	1.72	2.22
time (sec)	N/A	0.476	0.103	0.152	0.030	0.071	0.118	0.117	0.143	0.035

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	76	52	65	102	147	99	91	181	83
N.S.	1	0.87	0.60	0.75	1.17	1.69	1.14	1.05	2.08	0.95
time (sec)	N/A	0.591	0.048	0.168	0.032	0.086	0.241	0.134	0.142	0.053

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	105	80	92	130	191	129	127	233	111
N.S.	1	0.86	0.66	0.75	1.07	1.57	1.06	1.04	1.91	0.91
time (sec)	N/A	0.642	0.063	0.171	0.039	0.099	0.316	0.117	0.145	13.156

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	66	39	61	80	80	87	80	60	80
N.S.	1	0.90	0.53	0.84	1.10	1.10	1.19	1.10	0.82	1.10
time (sec)	N/A	0.587	0.036	0.168	0.025	0.064	0.040	0.137	0.150	0.023

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	51	31	53	70	70	70	70	52	70
N.S.	1	0.93	0.56	0.96	1.27	1.27	1.27	1.27	0.95	1.27
time (sec)	N/A	0.567	0.031	0.159	0.033	0.087	0.036	0.129	0.140	0.020

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	36	30	29	37	37	36	37	28	37
N.S.	1	0.97	0.81	0.78	1.00	1.00	0.97	1.00	0.76	1.00
time (sec)	N/A	0.518	0.022	0.154	0.029	0.068	0.034	0.128	0.143	0.028

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	14	20	20	19	20	18	17
N.S.	1	1.00	1.31	0.88	1.25	1.25	1.19	1.25	1.12	1.06
time (sec)	N/A	0.427	0.018	0.122	0.025	0.069	0.023	0.128	0.139	0.020

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	14	12	12	10	14	13	12
N.S.	1	1.00	1.29	1.00	0.86	0.86	0.71	1.00	0.93	0.86
time (sec)	N/A	0.467	0.060	0.135	0.027	0.073	0.055	0.129	0.140	0.029

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	45	33	51	63	76	54	51	94	46
N.S.	1	0.92	0.67	1.04	1.29	1.55	1.10	1.04	1.92	0.94
time (sec)	N/A	0.546	0.033	0.149	0.028	0.077	0.143	0.142	0.142	13.124

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	74	61	76	91	121	85	74	145	73
N.S.	1	0.88	0.73	0.90	1.08	1.44	1.01	0.88	1.73	0.87
time (sec)	N/A	0.599	0.051	0.152	0.031	0.086	0.222	0.133	0.147	0.042

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	103	80	92	140	217	141	91	269	121
N.S.	1	0.87	0.67	0.77	1.18	1.82	1.18	0.76	2.26	1.02
time (sec)	N/A	0.670	0.068	0.157	0.038	0.104	0.328	0.143	0.144	13.114

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	105	79	113	0	117	0	0	89	0
N.S.	1	0.46	0.34	0.49	0.00	0.51	0.00	0.00	0.39	0.00
time (sec)	N/A	0.823	0.078	0.142	0.000	0.085	0.000	0.000	0.148	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	92	71	97	0	95	0	0	73	0
N.S.	1	0.50	0.39	0.53	0.00	0.52	0.00	0.00	0.40	0.00
time (sec)	N/A	0.782	0.060	0.145	0.000	0.088	0.000	0.000	0.143	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	76	63	81	0	73	0	0	57	0
N.S.	1	0.56	0.46	0.60	0.00	0.54	0.00	0.00	0.42	0.00
time (sec)	N/A	0.762	0.050	0.137	0.000	0.102	0.000	0.000	0.147	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	66	53	63	0	43	0	0	39	0
N.S.	1	0.71	0.57	0.68	0.00	0.46	0.00	0.00	0.42	0.00
time (sec)	N/A	0.736	0.038	0.138	0.000	0.080	0.000	0.000	0.149	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	41	44	0	22	0	36	22	0
N.S.	1	1.00	0.89	0.96	0.00	0.48	0.00	0.78	0.48	0.00
time (sec)	N/A	0.614	0.024	0.135	0.000	0.094	0.000	0.109	0.154	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	51	0	22	0	0	19	0
N.S.	1	1.00	1.00	1.34	0.00	0.58	0.00	0.00	0.50	0.00
time (sec)	N/A	0.651	0.026	0.135	0.000	0.079	0.000	0.000	0.147	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	63	56	84	0	86	0	0	98	0
N.S.	1	0.69	0.62	0.92	0.00	0.95	0.00	0.00	1.08	0.00
time (sec)	N/A	0.756	0.059	0.139	0.000	0.109	0.000	0.000	0.148	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	93	83	169	0	136	0	0	250	0
N.S.	1	0.51	0.45	0.92	0.00	0.74	0.00	0.00	1.36	0.00
time (sec)	N/A	0.824	0.071	0.154	0.000	0.087	0.000	0.000	0.152	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	121	101	241	0	191	0	0	393	0
N.S.	1	0.44	0.36	0.87	0.00	0.69	0.00	0.00	1.42	0.00
time (sec)	N/A	0.858	0.104	0.142	0.000	0.090	0.000	0.000	0.149	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	198	167	138	192	329	714	164	198	0
N.S.	1	1.17	0.99	0.82	1.14	1.95	4.22	0.97	1.17	0.00
time (sec)	N/A	0.814	0.216	0.181	0.123	0.126	3.430	0.143	0.221	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	172	151	122	173	286	614	141	160	0
N.S.	1	1.18	1.03	0.84	1.18	1.96	4.21	0.97	1.10	0.00
time (sec)	N/A	0.751	0.227	0.185	0.125	0.118	3.094	0.136	0.162	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	146	135	106	154	241	314	116	122	0
N.S.	1	1.19	1.10	0.86	1.25	1.96	2.55	0.94	0.99	0.00
time (sec)	N/A	0.694	0.161	0.182	0.114	0.094	2.592	0.139	0.155	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	120	117	90	131	180	250	85	82	0
N.S.	1	1.20	1.17	0.90	1.31	1.80	2.50	0.85	0.82	0.00
time (sec)	N/A	0.643	0.134	0.178	0.116	0.099	2.309	0.145	0.164	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	95	76	69	47	134	0	62	42	0
N.S.	1	1.23	0.99	0.90	0.61	1.74	0.00	0.81	0.55	0.00
time (sec)	N/A	0.607	0.075	0.177	0.111	0.109	0.000	0.127	0.155	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	66	82	79	40	153	0	0	48	0
N.S.	1	1.12	1.39	1.34	0.68	2.59	0.00	0.00	0.81	0.00
time (sec)	N/A	0.575	0.055	0.177	0.110	0.106	0.000	0.000	0.156	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	57	64	31	61	47	0	148	54	33
N.S.	1	1.12	1.25	0.61	1.20	0.92	0.00	2.90	1.06	0.65
time (sec)	N/A	0.563	0.065	0.178	0.032	0.111	0.000	0.159	0.161	13.358

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	88	53	47	80	75	0	0	104	56
N.S.	1	1.19	0.72	0.64	1.08	1.01	0.00	0.00	1.41	0.76
time (sec)	N/A	0.597	0.051	0.183	0.036	0.111	0.000	0.000	0.152	13.232

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	119	96	64	99	124	0	0	150	134
N.S.	1	1.23	0.99	0.66	1.02	1.28	0.00	0.00	1.55	1.38
time (sec)	N/A	0.645	0.064	0.184	0.037	0.203	0.000	0.000	0.168	13.265

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	150	112	80	118	152	0	0	247	177
N.S.	1	1.25	0.93	0.67	0.98	1.27	0.00	0.00	2.06	1.48
time (sec)	N/A	0.665	0.080	0.188	0.037	0.386	0.000	0.000	0.158	13.293

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	94	71	100	204	95	0	0	73	0
N.S.	1	0.51	0.38	0.54	1.10	0.51	0.00	0.00	0.39	0.00
time (sec)	N/A	0.818	0.074	0.146	0.053	0.074	0.000	0.000	0.160	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	79	63	100	172	95	0	0	73	0
N.S.	1	0.57	0.45	0.72	1.24	0.68	0.00	0.00	0.53	0.00
time (sec)	N/A	0.801	0.062	0.148	0.048	0.077	0.000	0.000	0.154	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	66	55	84	140	73	0	0	57	0
N.S.	1	0.71	0.59	0.90	1.51	0.78	0.00	0.00	0.61	0.00
time (sec)	N/A	0.744	0.050	0.142	0.051	0.084	0.000	0.000	0.161	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	58	48	97	42	0	0	38	0
N.S.	1	1.00	1.26	1.04	2.11	0.91	0.00	0.00	0.83	0.00
time (sec)	N/A	0.687	0.039	0.145	0.050	0.077	0.000	0.000	0.150	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	61	59	67	0	33	0	76	32	0
N.S.	1	0.54	0.52	0.59	0.00	0.29	0.00	0.67	0.28	0.00
time (sec)	N/A	0.677	0.038	0.144	0.000	0.100	0.000	0.138	0.165	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	57	49	64	0	39	0	0	41	0
N.S.	1	0.72	0.62	0.81	0.00	0.49	0.00	0.00	0.52	0.00
time (sec)	N/A	0.739	0.045	0.145	0.000	0.075	0.000	0.000	0.168	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	51	39	0	39	0	0	26	90
N.S.	1	1.00	1.09	0.83	0.00	0.83	0.00	0.00	0.55	1.91
time (sec)	N/A	0.690	0.096	0.145	0.000	0.087	0.000	0.000	0.149	13.364

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	93	71	169	0	139	0	0	242	0
N.S.	1	0.50	0.38	0.91	0.00	0.75	0.00	0.00	1.31	0.00
time (sec)	N/A	0.792	0.074	0.152	0.000	0.088	0.000	0.000	0.155	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	123	99	241	0	190	0	0	394	0
N.S.	1	0.44	0.36	0.87	0.00	0.68	0.00	0.00	1.42	0.00
time (sec)	N/A	0.859	0.104	0.148	0.000	0.092	0.000	0.000	0.159	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	111	79	113	0	117	0	185	89	0
N.S.	1	0.47	0.34	0.48	0.00	0.50	0.00	0.79	0.38	0.00
time (sec)	N/A	0.837	0.084	0.143	0.000	0.073	0.000	0.128	0.154	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	95	71	97	0	95	0	151	73	0
N.S.	1	0.51	0.38	0.52	0.00	0.51	0.00	0.81	0.39	0.00
time (sec)	N/A	0.805	0.064	0.148	0.000	0.074	0.000	0.122	0.159	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	81	63	81	0	73	0	117	57	0
N.S.	1	0.58	0.45	0.58	0.00	0.53	0.00	0.84	0.41	0.00
time (sec)	N/A	0.759	0.051	0.142	0.000	0.069	0.000	0.120	0.151	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	67	53	63	0	43	0	73	39	0
N.S.	1	0.71	0.56	0.66	0.00	0.45	0.00	0.77	0.41	0.00
time (sec)	N/A	0.728	0.041	0.154	0.000	0.069	0.000	0.113	0.149	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	44	0	22	0	39	22	0
N.S.	1	1.00	0.87	0.94	0.00	0.47	0.00	0.83	0.47	0.00
time (sec)	N/A	0.616	0.026	0.135	0.000	0.086	0.000	0.149	0.167	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	51	0	22	0	22	35	0
N.S.	1	1.00	1.00	1.38	0.00	0.59	0.00	0.59	0.95	0.00
time (sec)	N/A	0.633	0.033	0.137	0.000	0.069	0.000	0.147	0.159	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	63	54	84	0	83	0	58	99	0
N.S.	1	0.70	0.60	0.93	0.00	0.92	0.00	0.64	1.10	0.00
time (sec)	N/A	0.728	0.057	0.140	0.000	0.120	0.000	0.143	0.170	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	91	81	169	0	137	0	80	249	0
N.S.	1	0.50	0.44	0.92	0.00	0.75	0.00	0.44	1.36	0.00
time (sec)	N/A	0.769	0.084	0.142	0.000	0.100	0.000	0.161	0.169	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	121	99	241	0	186	0	96	392	0
N.S.	1	0.44	0.36	0.87	0.00	0.67	0.00	0.35	1.42	0.00
time (sec)	N/A	0.841	0.114	0.142	0.000	0.105	0.000	0.156	0.167	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	147	136	106	154	241	314	117	122	0
N.S.	1	1.20	1.11	0.86	1.25	1.96	2.55	0.95	0.99	0.00
time (sec)	N/A	0.694	0.194	0.188	0.121	0.114	2.644	0.136	0.152	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	121	117	90	130	180	248	85	82	0
N.S.	1	1.21	1.17	0.90	1.30	1.80	2.48	0.85	0.82	0.00
time (sec)	N/A	0.652	0.123	0.179	0.114	0.104	2.292	0.129	0.159	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	95	100	69	47	134	0	62	42	0
N.S.	1	1.23	1.30	0.90	0.61	1.74	0.00	0.81	0.55	0.00
time (sec)	N/A	0.620	0.073	0.185	0.109	0.103	0.000	0.123	0.148	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	67	100	73	39	151	0	0	46	0
N.S.	1	1.05	1.56	1.14	0.61	2.36	0.00	0.00	0.72	0.00
time (sec)	N/A	0.599	0.118	0.178	0.111	0.098	0.000	0.000	0.164	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	58	63	31	60	47	0	148	56	33
N.S.	1	0.89	0.97	0.48	0.92	0.72	0.00	2.28	0.86	0.51
time (sec)	N/A	0.609	0.053	0.177	0.031	0.084	0.000	0.169	0.150	13.157

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	89	79	47	79	75	0	0	107	56
N.S.	1	1.01	0.90	0.53	0.90	0.85	0.00	0.00	1.22	0.64
time (sec)	N/A	0.590	0.085	0.178	0.037	0.156	0.000	0.000	0.152	13.232

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	120	96	64	98	124	0	0	150	134
N.S.	1	1.08	0.86	0.58	0.88	1.12	0.00	0.00	1.35	1.21
time (sec)	N/A	0.631	0.105	0.181	0.037	0.190	0.000	0.000	0.158	13.272

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	151	112	80	117	152	0	0	247	177
N.S.	1	1.13	0.84	0.60	0.87	1.13	0.00	0.00	1.84	1.32
time (sec)	N/A	0.653	0.125	0.181	0.038	0.365	0.000	0.000	0.159	13.308

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	97	71	100	204	95	0	151	73	0
N.S.	1	0.51	0.38	0.53	1.08	0.50	0.00	0.80	0.39	0.00
time (sec)	N/A	0.835	0.088	0.145	0.051	0.066	0.000	0.137	0.175	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	83	63	100	172	95	0	151	73	0
N.S.	1	0.58	0.44	0.70	1.21	0.67	0.00	1.06	0.51	0.00
time (sec)	N/A	0.792	0.081	0.142	0.048	0.065	0.000	0.116	0.168	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	67	55	84	140	73	0	117	57	0
N.S.	1	0.71	0.58	0.88	1.47	0.77	0.00	1.23	0.60	0.00
time (sec)	N/A	0.758	0.063	0.138	0.048	0.079	0.000	0.113	0.160	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	58	48	97	42	0	72	38	0
N.S.	1	1.00	1.23	1.02	2.06	0.89	0.00	1.53	0.81	0.00
time (sec)	N/A	0.680	0.051	0.138	0.047	0.077	0.000	0.121	0.160	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	59	58	67	0	33	0	56	30	0
N.S.	1	0.53	0.52	0.60	0.00	0.29	0.00	0.50	0.27	0.00
time (sec)	N/A	0.664	0.046	0.139	0.000	0.103	0.000	0.118	0.174	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	57	47	62	0	38	0	0	77	0
N.S.	1	0.74	0.61	0.81	0.00	0.49	0.00	0.00	1.00	0.00
time (sec)	N/A	0.739	0.055	0.136	0.000	0.075	0.000	0.000	0.149	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	51	39	0	39	0	26	26	58
N.S.	1	1.00	1.11	0.85	0.00	0.85	0.00	0.57	0.57	1.26
time (sec)	N/A	0.685	0.129	0.132	0.000	0.088	0.000	0.147	0.147	13.225

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	91	71	169	0	136	0	98	250	0
N.S.	1	0.50	0.39	0.93	0.00	0.75	0.00	0.54	1.37	0.00
time (sec)	N/A	0.822	0.107	0.139	0.000	0.091	0.000	0.174	0.153	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	119	99	241	0	193	0	133	394	0
N.S.	1	0.43	0.36	0.88	0.00	0.70	0.00	0.48	1.43	0.00
time (sec)	N/A	0.849	0.145	0.139	0.000	0.097	0.000	0.182	0.150	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	52	45	47	0	25	0	0	26	0
N.S.	1	0.68	0.59	0.62	0.00	0.33	0.00	0.00	0.34	0.00
time (sec)	N/A	0.785	0.034	0.136	0.000	0.078	0.000	0.000	0.145	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	52	43	47	0	25	0	0	26	0
N.S.	1	0.70	0.58	0.64	0.00	0.34	0.00	0.00	0.35	0.00
time (sec)	N/A	0.717	0.034	0.134	0.000	0.072	0.000	0.000	0.150	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	41	44	0	22	0	36	22	0
N.S.	1	1.00	0.89	0.96	0.00	0.48	0.00	0.78	0.48	0.00
time (sec)	N/A	0.582	0.014	0.132	0.000	0.077	0.000	0.116	0.139	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	42	41	44	0	18	0	31	34	0
N.S.	1	0.61	0.59	0.64	0.00	0.26	0.00	0.45	0.49	0.00
time (sec)	N/A	0.635	0.031	0.135	0.000	0.108	0.000	0.149	0.149	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	46	44	48	0	22	0	25	41	0
N.S.	1	0.63	0.60	0.66	0.00	0.30	0.00	0.34	0.56	0.00
time (sec)	N/A	0.751	0.033	0.135	0.000	0.095	0.000	0.132	0.146	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	192	96	97	117	184	0	0	100	0
N.S.	1	1.25	0.62	0.63	0.76	1.19	0.00	0.00	0.65	0.00
time (sec)	N/A	1.057	0.222	0.211	0.126	0.104	0.000	0.000	0.151	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	161	88	89	93	168	0	84	81	0
N.S.	1	1.25	0.68	0.69	0.72	1.30	0.00	0.65	0.63	0.00
time (sec)	N/A	0.974	0.160	0.195	0.119	0.122	0.000	0.166	0.149	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	120	79	80	70	150	0	72	61	0
N.S.	1	1.21	0.80	0.81	0.71	1.52	0.00	0.73	0.62	0.00
time (sec)	N/A	0.821	0.148	0.190	0.119	0.094	0.000	0.126	0.157	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	95	76	69	47	134	0	62	42	0
N.S.	1	1.23	0.99	0.90	0.61	1.74	0.00	0.81	0.55	0.00
time (sec)	N/A	0.613	0.049	0.171	0.108	0.090	0.000	0.136	0.148	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	84	97	129	90	181	0	95	32	0
N.S.	1	1.12	1.29	1.72	1.20	2.41	0.00	1.27	0.43	0.00
time (sec)	N/A	0.900	0.217	0.178	0.118	0.088	0.000	0.132	0.152	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	89	104	102	0	199	0	134	38	0
N.S.	1	1.09	1.27	1.24	0.00	2.43	0.00	1.63	0.46	0.00
time (sec)	N/A	0.894	0.188	0.201	0.000	0.108	0.000	0.155	0.155	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	88	76	78	0	138	0	200	50	0
N.S.	1	1.13	0.97	1.00	0.00	1.77	0.00	2.56	0.64	0.00
time (sec)	N/A	0.860	0.232	0.217	0.000	0.107	0.000	0.123	0.151	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	114	82	86	0	154	0	250	69	0
N.S.	1	1.15	0.83	0.87	0.00	1.56	0.00	2.53	0.70	0.00
time (sec)	N/A	0.915	0.244	0.225	0.000	0.104	0.000	0.137	0.154	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	150	95	95	0	170	0	324	90	0
N.S.	1	1.15	0.73	0.73	0.00	1.31	0.00	2.49	0.69	0.00
time (sec)	N/A	0.997	0.278	0.266	0.000	0.135	0.000	0.150	0.156	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	89	87	92	0	58	0	0	56	0
N.S.	1	0.39	0.38	0.40	0.00	0.25	0.00	0.00	0.25	0.00
time (sec)	N/A	0.872	0.084	0.140	0.000	0.090	0.000	0.000	0.147	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	77	74	83	0	49	0	123	48	0
N.S.	1	0.41	0.40	0.45	0.00	0.26	0.00	0.66	0.26	0.00
time (sec)	N/A	0.926	0.069	0.138	0.000	0.085	0.000	0.161	0.150	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	71	66	76	0	42	0	93	40	0
N.S.	1	0.47	0.43	0.50	0.00	0.28	0.00	0.61	0.26	0.00
time (sec)	N/A	0.830	0.078	0.142	0.000	0.084	0.000	0.137	0.145	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	61	59	67	0	33	0	76	32	0
N.S.	1	0.54	0.52	0.59	0.00	0.29	0.00	0.67	0.28	0.00
time (sec)	N/A	0.682	0.018	0.139	0.000	0.124	0.000	0.134	0.149	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	53	54	59	0	28	0	49	40	0
N.S.	1	0.46	0.47	0.52	0.00	0.25	0.00	0.43	0.35	0.00
time (sec)	N/A	0.715	0.046	0.141	0.000	0.115	0.000	0.149	0.153	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	58	54	64	0	33	0	34	52	0
N.S.	1	0.51	0.47	0.56	0.00	0.29	0.00	0.30	0.46	0.00
time (sec)	N/A	0.823	0.041	0.149	0.000	0.107	0.000	0.141	0.153	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	70	64	77	0	90	0	46	74	0
N.S.	1	0.46	0.42	0.50	0.00	0.59	0.00	0.30	0.48	0.00
time (sec)	N/A	0.841	0.054	0.145	0.000	0.106	0.000	0.136	0.159	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	80	76	85	0	98	0	54	82	0
N.S.	1	0.41	0.39	0.44	0.00	0.51	0.00	0.28	0.42	0.00
time (sec)	N/A	0.842	0.077	0.145	0.000	0.097	0.000	0.144	0.154	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	86	80	93	0	106	0	62	90	0
N.S.	1	0.38	0.35	0.41	0.00	0.46	0.00	0.27	0.39	0.00
time (sec)	N/A	0.839	0.075	0.145	0.000	0.078	0.000	0.153	0.153	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	94	83	106	0	76	0	0	118	0
N.S.	1	0.45	0.39	0.50	0.00	0.36	0.00	0.00	0.56	0.00
time (sec)	N/A	0.827	0.111	0.144	0.000	0.127	0.000	0.000	0.159	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	84	74	98	0	69	0	0	112	0
N.S.	1	0.49	0.43	0.57	0.00	0.40	0.00	0.00	0.65	0.00
time (sec)	N/A	0.776	0.082	0.144	0.000	0.094	0.000	0.000	0.151	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	79	62	86	0	56	0	0	98	0
N.S.	1	0.61	0.48	0.66	0.00	0.43	0.00	0.00	0.75	0.00
time (sec)	N/A	0.875	0.064	0.146	0.000	0.116	0.000	0.000	0.154	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	63	52	84	0	86	0	0	100	0
N.S.	1	0.72	0.60	0.97	0.00	0.99	0.00	0.00	1.15	0.00
time (sec)	N/A	0.791	0.055	0.146	0.000	0.114	0.000	0.000	0.144	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	63	56	84	0	86	0	0	98	0
N.S.	1	0.69	0.62	0.92	0.00	0.95	0.00	0.00	1.08	0.00
time (sec)	N/A	0.736	0.030	0.139	0.000	0.083	0.000	0.000	0.147	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	72	68	93	0	63	0	0	149	0
N.S.	1	0.41	0.38	0.53	0.00	0.36	0.00	0.00	0.84	0.00
time (sec)	N/A	0.828	0.070	0.145	0.000	0.116	0.000	0.000	0.144	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	82	79	118	0	92	0	0	194	0
N.S.	1	0.38	0.37	0.55	0.00	0.43	0.00	0.00	0.91	0.00
time (sec)	N/A	0.864	0.077	0.148	0.000	0.130	0.000	0.000	0.147	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	99	94	138	0	113	0	0	222	0
N.S.	1	0.39	0.37	0.55	0.00	0.45	0.00	0.00	0.88	0.00
time (sec)	N/A	0.893	0.104	0.145	0.000	0.102	0.000	0.000	0.151	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	113	100	185	0	138	0	0	258	0
N.S.	1	0.43	0.38	0.71	0.00	0.53	0.00	0.00	0.98	0.00
time (sec)	N/A	0.827	0.133	0.150	0.000	0.106	0.000	0.000	0.158	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	109	84	169	0	122	0	0	250	0
N.S.	1	0.50	0.39	0.78	0.00	0.56	0.00	0.00	1.15	0.00
time (sec)	N/A	0.918	0.112	0.149	0.000	0.118	0.000	0.000	0.157	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	93	72	169	0	136	0	0	250	0
N.S.	1	0.53	0.41	0.96	0.00	0.77	0.00	0.00	1.42	0.00
time (sec)	N/A	0.901	0.117	0.152	0.000	0.104	0.000	0.000	0.157	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	93	85	164	0	134	0	0	243	0
N.S.	1	0.51	0.46	0.89	0.00	0.73	0.00	0.00	1.32	0.00
time (sec)	N/A	0.895	0.071	0.148	0.000	0.096	0.000	0.000	0.152	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	78	62	164	0	134	0	0	242	0
N.S.	1	0.57	0.45	1.20	0.00	0.98	0.00	0.00	1.77	0.00
time (sec)	N/A	0.809	0.071	0.150	0.000	0.122	0.000	0.000	0.154	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	93	83	169	0	136	0	0	250	0
N.S.	1	0.51	0.45	0.92	0.00	0.74	0.00	0.00	1.36	0.00
time (sec)	N/A	0.777	0.032	0.142	0.000	0.093	0.000	0.000	0.149	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	96	88	196	0	145	0	0	371	0
N.S.	1	0.35	0.32	0.72	0.00	0.54	0.00	0.00	1.37	0.00
time (sec)	N/A	0.891	0.127	0.151	0.000	0.106	0.000	0.000	0.156	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	108	99	225	0	174	0	0	412	0
N.S.	1	0.35	0.32	0.73	0.00	0.57	0.00	0.00	1.34	0.00
time (sec)	N/A	0.940	0.103	0.148	0.000	0.092	0.000	0.000	0.155	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	53	45	47	0	25	0	42	26	0
N.S.	1	0.70	0.59	0.62	0.00	0.33	0.00	0.55	0.34	0.00
time (sec)	N/A	0.792	0.034	0.136	0.000	0.075	0.000	0.115	0.161	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	53	43	47	0	25	0	42	26	0
N.S.	1	0.72	0.58	0.64	0.00	0.34	0.00	0.57	0.35	0.00
time (sec)	N/A	0.740	0.028	0.138	0.000	0.076	0.000	0.144	0.153	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	44	0	22	0	39	22	0
N.S.	1	1.00	0.87	0.94	0.00	0.47	0.00	0.83	0.47	0.00
time (sec)	N/A	0.602	0.015	0.135	0.000	0.090	0.000	0.107	0.152	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	44	42	46	0	20	0	27	14	0
N.S.	1	0.63	0.60	0.66	0.00	0.29	0.00	0.39	0.20	0.00
time (sec)	N/A	0.658	0.033	0.135	0.000	0.123	0.000	0.141	0.163	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	48	43	48	0	22	0	28	20	0
N.S.	1	0.67	0.60	0.67	0.00	0.31	0.00	0.39	0.28	0.00
time (sec)	N/A	0.822	0.030	0.135	0.000	0.086	0.000	0.121	0.166	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	193	96	97	117	184	0	0	100	0
N.S.	1	1.38	0.69	0.69	0.84	1.31	0.00	0.00	0.71	0.00
time (sec)	N/A	1.036	0.210	0.223	0.118	0.088	0.000	0.000	0.155	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	161	88	89	93	168	0	84	81	0
N.S.	1	1.25	0.68	0.69	0.72	1.30	0.00	0.65	0.63	0.00
time (sec)	N/A	0.987	0.174	0.198	0.118	0.121	0.000	0.120	0.154	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	121	79	79	70	150	0	73	61	0
N.S.	1	1.22	0.80	0.80	0.71	1.52	0.00	0.74	0.62	0.00
time (sec)	N/A	0.829	0.140	0.194	0.108	0.121	0.000	0.156	0.157	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	95	100	69	47	134	0	62	42	0
N.S.	1	1.23	1.30	0.90	0.61	1.74	0.00	0.81	0.55	0.00
time (sec)	N/A	0.606	0.059	0.178	0.116	0.112	0.000	0.132	0.149	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	84	97	121	86	181	0	95	32	0
N.S.	1	1.12	1.29	1.61	1.15	2.41	0.00	1.27	0.43	0.00
time (sec)	N/A	0.883	0.173	0.191	0.123	0.086	0.000	0.149	0.157	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	91	104	102	0	198	0	134	38	0
N.S.	1	1.11	1.27	1.24	0.00	2.41	0.00	1.63	0.46	0.00
time (sec)	N/A	0.888	0.157	0.220	0.000	0.119	0.000	0.128	0.153	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	88	76	79	0	137	0	200	50	0
N.S.	1	1.13	0.97	1.01	0.00	1.76	0.00	2.56	0.64	0.00
time (sec)	N/A	0.865	0.202	0.226	0.000	0.092	0.000	0.146	0.156	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	115	82	87	0	153	0	250	69	0
N.S.	1	1.14	0.81	0.86	0.00	1.51	0.00	2.48	0.68	0.00
time (sec)	N/A	0.914	0.178	0.233	0.000	0.106	0.000	0.140	0.159	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	150	95	95	0	169	0	324	90	0
N.S.	1	1.15	0.73	0.73	0.00	1.30	0.00	2.49	0.69	0.00
time (sec)	N/A	0.992	0.222	0.279	0.000	0.115	0.000	0.131	0.168	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	87	86	92	0	58	0	100	54	0
N.S.	1	0.38	0.38	0.41	0.00	0.26	0.00	0.44	0.24	0.00
time (sec)	N/A	0.873	0.071	0.148	0.000	0.097	0.000	0.121	0.186	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	75	74	83	0	49	0	84	46	0
N.S.	1	0.40	0.40	0.45	0.00	0.26	0.00	0.45	0.25	0.00
time (sec)	N/A	0.842	0.053	0.148	0.000	0.086	0.000	0.117	0.181	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	69	65	76	0	42	0	72	38	0
N.S.	1	0.46	0.43	0.50	0.00	0.28	0.00	0.48	0.25	0.00
time (sec)	N/A	0.776	0.048	0.152	0.000	0.090	0.000	0.130	0.176	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	59	58	67	0	33	0	56	30	0
N.S.	1	0.53	0.52	0.60	0.00	0.29	0.00	0.50	0.27	0.00
time (sec)	N/A	0.647	0.019	0.145	0.000	0.080	0.000	0.115	0.169	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	50	50	61	0	26	0	41	24	0
N.S.	1	0.45	0.45	0.54	0.00	0.23	0.00	0.37	0.21	0.00
time (sec)	N/A	0.686	0.034	0.151	0.000	0.087	0.000	0.122	0.175	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	56	54	64	0	33	0	46	31	0
N.S.	1	0.49	0.47	0.56	0.00	0.29	0.00	0.40	0.27	0.00
time (sec)	N/A	0.804	0.046	0.147	0.000	0.089	0.000	0.136	0.445	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	68	63	77	0	88	0	64	34	0
N.S.	1	0.45	0.41	0.51	0.00	0.58	0.00	0.42	0.22	0.00
time (sec)	N/A	0.828	0.054	0.151	0.000	0.123	0.000	0.141	200.027	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	78	75	85	0	98	0	78	56	0
N.S.	1	0.40	0.39	0.44	0.00	0.51	0.00	0.40	0.29	0.00
time (sec)	N/A	0.825	0.066	0.152	0.000	0.092	0.000	0.117	0.190	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	83	79	93	0	104	0	92	64	0
N.S.	1	0.37	0.35	0.41	0.00	0.46	0.00	0.41	0.28	0.00
time (sec)	N/A	0.820	0.059	0.190	0.000	0.086	0.000	0.130	0.219	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	86	74	0	0	0	0	0	437	0
N.S.	1	0.63	0.54	0.00	0.00	0.00	0.00	0.00	3.21	0.00
time (sec)	N/A	0.854	0.067	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	173	129	0	0	0	0	0	55	0
N.S.	1	1.01	0.75	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.988	0.307	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	60	56	61	54	74	0	0	32	93
N.S.	1	0.73	0.68	0.74	0.66	0.90	0.00	0.00	0.39	1.13
time (sec)	N/A	0.811	0.065	0.204	0.045	0.082	0.000	0.000	0.150	13.546

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	62	60	63	57	75	0	0	30	94
N.S.	1	0.75	0.72	0.76	0.69	0.90	0.00	0.00	0.36	1.13
time (sec)	N/A	0.820	0.052	0.155	0.044	0.091	0.000	0.000	0.151	13.287

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	173	110	0	0	0	0	0	57	0
N.S.	1	1.01	0.64	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.966	0.331	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	85	75	0	0	0	0	0	91	0
N.S.	1	0.62	0.55	0.00	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.841	0.073	0.000	0.000	0.000	0.000	0.000	0.264	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	280	0	0	0	0	0	57	0
N.S.	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.799	3.055	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	101	0	0	0	0	0	25	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.788	0.869	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	81	0	0	0	0	0	29	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.756	0.339	0.000	0.000	0.000	0.000	0.000	0.143	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	80	0	0	54	78
N.S.	1	1.00	0.93	1.07	0.00	1.74	0.00	0.00	1.17	1.70
time (sec)	N/A	0.413	0.332	0.149	0.000	0.084	0.000	0.000	0.153	13.352

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	110	84	0	165	0	0	70	173
N.S.	1	1.00	1.08	0.82	0.00	1.62	0.00	0.00	0.69	1.70
time (sec)	N/A	0.710	0.800	0.152	0.000	0.141	0.000	0.000	0.154	13.387

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	163	299	140	0	291	0	0	92	289
N.S.	1	0.98	1.80	0.84	0.00	1.75	0.00	0.00	0.55	1.74
time (sec)	N/A	1.043	2.234	0.152	0.000	0.130	0.000	0.000	0.167	13.476

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	224	260	218	0	453	0	0	108	441
N.S.	1	0.94	1.09	0.91	0.00	1.90	0.00	0.00	0.45	1.85
time (sec)	N/A	1.397	1.946	0.161	0.000	0.153	0.000	0.000	0.161	13.616

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	356	275	133	0	0	0	0	0	57	0
N.S.	1	0.77	0.37	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.090	0.919	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	127	0	0	0	0	0	57	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	1.170	0.666	0.000	0.000	0.000	0.000	0.000	0.152	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	82	0	0	55	81
N.S.	1	1.00	0.93	1.07	0.00	1.78	0.00	0.00	1.20	1.76
time (sec)	N/A	0.475	0.454	0.153	0.000	0.084	0.000	0.000	0.155	13.316

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	80	0	0	54	78
N.S.	1	1.00	0.93	1.07	0.00	1.74	0.00	0.00	1.17	1.70
time (sec)	N/A	0.415	0.044	0.266	0.000	0.113	0.000	0.000	0.147	0.001

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	277	224	127	0	0	0	0	0	55	0
N.S.	1	0.81	0.46	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.080	1.564	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	460	362	201	0	0	0	0	0	73	0
N.S.	1	0.79	0.44	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.357	2.514	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	239	110	93	0	175	0	0	73	175
N.S.	1	0.72	0.33	0.28	0.00	0.53	0.00	0.00	0.22	0.53
time (sec)	N/A	1.044	1.088	0.160	0.000	0.085	0.000	0.000	0.165	13.298

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	108	109	96	0	180	0	0	73	175
N.S.	1	1.06	1.07	0.94	0.00	1.76	0.00	0.00	0.72	1.72
time (sec)	N/A	0.817	1.122	0.162	0.000	0.087	0.000	0.000	0.156	13.822

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	103	108	86	0	171	0	0	71	176
N.S.	1	1.06	1.11	0.89	0.00	1.76	0.00	0.00	0.73	1.81
time (sec)	N/A	0.741	0.804	0.161	0.000	0.089	0.000	0.000	0.155	13.446

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	110	84	0	165	0	0	70	173
N.S.	1	1.00	1.08	0.82	0.00	1.62	0.00	0.00	0.69	1.70
time (sec)	N/A	0.665	0.194	0.145	0.000	0.085	0.000	0.000	0.154	0.001

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	944	624	220	0	0	0	0	0	72	0
N.S.	1	0.66	0.23	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.694	2.214	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	83	0	0	0	0	0	24	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.713	0.215	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	36	38	34	42	0	0	38	59
N.S.	1	1.00	0.71	0.75	0.67	0.82	0.00	0.00	0.75	1.16
time (sec)	N/A	0.631	0.098	0.994	0.039	0.093	0.000	0.000	0.139	13.383

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	36	40	36	44	0	0	40	59
N.S.	1	1.00	0.69	0.77	0.69	0.85	0.00	0.00	0.77	1.13
time (sec)	N/A	0.631	0.076	0.522	0.056	0.080	0.000	0.000	0.150	13.198

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	72	0	0	0	0	0	720	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	11.43	0.00
time (sec)	N/A	0.553	0.029	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	67	0	0	0	648	0	194	0
N.S.	1	1.00	1.24	0.00	0.00	0.00	12.00	0.00	3.59	0.00
time (sec)	N/A	0.537	0.022	0.000	0.000	0.000	13.403	0.000	0.157	0.000

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	24	0	110	43
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.52	0.00	2.39	0.93
time (sec)	N/A	0.269	0.105	0.000	0.000	0.000	0.493	0.000	0.156	13.727

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	55	73	0	0	0	648	0	196	0
N.S.	1	0.85	1.12	0.00	0.00	0.00	9.97	0.00	3.02	0.00
time (sec)	N/A	0.540	0.034	0.000	0.000	0.000	11.362	0.000	0.164	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	64	74	0	0	0	0	0	712	0
N.S.	1	0.97	1.12	0.00	0.00	0.00	0.00	0.00	10.79	0.00
time (sec)	N/A	0.559	0.036	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	118	122	0	0	0	0	0	88	0
N.S.	1	1.39	1.44	0.00	0.00	0.00	0.00	0.00	1.04	0.00
time (sec)	N/A	0.690	0.305	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	118	122	0	0	0	0	0	30	0
N.S.	1	2.07	2.14	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.674	0.327	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	118	118	0	0	0	0	0	30	0
N.S.	1	1.39	1.39	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.713	0.319	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	118	119	0	0	0	0	0	86	0
N.S.	1	1.39	1.40	0.00	0.00	0.00	0.00	0.00	1.01	0.00
time (sec)	N/A	0.707	0.352	0.000	0.000	0.000	0.000	0.000	0.289	0.000

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	336	120	188	380	201	0	461	243	332
N.S.	1	1.98	0.71	1.11	2.24	1.18	0.00	2.71	1.43	1.95
time (sec)	N/A	0.870	0.401	0.119	0.116	0.093	0.000	0.151	0.209	13.497

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	262	104	172	302	179	0	355	203	258
N.S.	1	1.91	0.76	1.26	2.20	1.31	0.00	2.59	1.48	1.88
time (sec)	N/A	0.781	0.336	0.106	0.115	0.087	0.000	0.162	0.178	13.318

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	188	94	156	223	157	0	249	155	183
N.S.	1	1.81	0.90	1.50	2.14	1.51	0.00	2.39	1.49	1.76
time (sec)	N/A	0.671	0.229	0.105	0.114	0.110	0.000	0.145	0.160	13.623

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	113	53	127	117	104	0	130	103	84
N.S.	1	1.98	0.93	2.23	2.05	1.82	0.00	2.28	1.81	1.47
time (sec)	N/A	0.543	0.160	0.088	0.114	0.090	0.000	0.167	0.154	0.036

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	101	56	144	116	93	0	0	70	62
N.S.	1	1.46	0.81	2.09	1.68	1.35	0.00	0.00	1.01	0.90
time (sec)	N/A	0.516	0.191	0.132	0.029	0.107	0.000	0.000	0.161	0.056

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	170	83	218	160	134	0	0	156	128
N.S.	1	1.63	0.80	2.10	1.54	1.29	0.00	0.00	1.50	1.23
time (sec)	N/A	0.626	0.458	0.161	0.033	0.092	0.000	0.000	0.153	0.041

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	232	99	292	194	178	0	0	239	171
N.S.	1	1.69	0.72	2.13	1.42	1.30	0.00	0.00	1.74	1.25
time (sec)	N/A	0.725	0.614	0.174	0.030	0.086	0.000	0.000	0.180	0.053

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	307	115	366	230	274	0	0	322	210
N.S.	1	1.81	0.68	2.15	1.35	1.61	0.00	0.00	1.89	1.24
time (sec)	N/A	0.853	0.717	0.198	0.034	0.112	0.000	0.000	0.256	13.219

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	99	127	88	114	122	124	115	91	89
N.S.	1	0.78	1.00	0.69	0.90	0.96	0.98	0.91	0.72	0.70
time (sec)	N/A	0.829	0.063	0.299	0.029	0.070	0.364	0.121	0.152	13.157

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	73	90	64	81	89	88	82	67	65
N.S.	1	0.81	1.00	0.71	0.90	0.99	0.98	0.91	0.74	0.72
time (sec)	N/A	0.783	0.037	0.257	0.027	0.110	0.237	0.134	0.158	0.040

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	60	76	55	70	78	76	71	59	56
N.S.	1	0.79	1.00	0.72	0.92	1.03	1.00	0.93	0.78	0.74
time (sec)	N/A	0.762	0.033	0.217	0.032	0.064	0.184	0.124	0.145	0.035

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	35	39	31	35	43	39	36	35	32
N.S.	1	0.90	1.00	0.79	0.90	1.10	1.00	0.92	0.90	0.82
time (sec)	N/A	0.709	0.024	0.204	0.026	0.069	0.088	0.120	0.154	0.029

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	21	26	20	22	23	23
N.S.	1	1.00	1.00	1.05	1.00	1.24	0.95	1.05	1.10	1.10
time (sec)	N/A	0.611	0.019	0.168	0.029	0.071	0.049	0.140	0.150	0.025

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	37	28	36	35	40	36	36	44	33
N.S.	1	1.03	0.78	1.00	0.97	1.11	1.00	1.00	1.22	0.92
time (sec)	N/A	0.734	0.038	0.145	0.032	0.077	0.071	0.135	0.154	13.161

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	72	75	60	69	93	73	57	105	68
N.S.	1	0.96	1.00	0.80	0.92	1.24	0.97	0.76	1.40	0.91
time (sec)	N/A	0.799	0.059	0.162	0.029	0.082	0.200	0.114	0.152	0.056

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	99	82	84	97	137	102	80	153	94
N.S.	1	0.90	0.75	0.76	0.88	1.25	0.93	0.73	1.39	0.85
time (sec)	N/A	0.848	0.093	0.169	0.028	0.098	0.344	0.118	0.167	0.064

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	130	98	108	145	233	156	96	277	142
N.S.	1	0.90	0.68	0.74	1.00	1.61	1.08	0.66	1.91	0.98
time (sec)	N/A	0.929	0.130	0.173	0.039	0.080	0.476	0.117	0.160	13.140

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	336	126	180	380	201	0	461	243	332
N.S.	1	1.87	0.70	1.00	2.11	1.12	0.00	2.56	1.35	1.84
time (sec)	N/A	0.878	0.359	0.122	0.118	0.122	0.000	0.158	0.205	0.140

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	263	110	173	302	179	0	355	203	258
N.S.	1	1.79	0.75	1.18	2.05	1.22	0.00	2.41	1.38	1.76
time (sec)	N/A	0.806	0.328	0.110	0.115	0.105	0.000	0.150	0.182	0.074

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	187	94	157	223	156	0	248	163	183
N.S.	1	1.67	0.84	1.40	1.99	1.39	0.00	2.21	1.46	1.63
time (sec)	N/A	0.675	0.259	0.093	0.114	0.093	0.000	0.146	0.158	13.231

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	79	57	129	118	106	0	130	101	84
N.S.	1	1.16	0.84	1.90	1.74	1.56	0.00	1.91	1.49	1.24
time (sec)	N/A	0.480	0.173	0.108	0.113	0.086	0.000	0.143	0.162	13.193

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	140	69	185	133	128	0	0	130	100
N.S.	1	1.33	0.66	1.76	1.27	1.22	0.00	0.00	1.24	0.95
time (sec)	N/A	0.568	0.217	0.128	0.029	0.137	0.000	0.000	0.155	13.492

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	173	78	225	153	170	0	63	199	121
N.S.	1	1.25	0.57	1.63	1.11	1.23	0.00	0.46	1.44	0.88
time (sec)	N/A	0.609	0.520	0.154	0.034	0.095	0.000	0.172	0.155	13.156

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	244	101	299	192	204	0	0	282	160
N.S.	1	1.43	0.59	1.75	1.12	1.19	0.00	0.00	1.65	0.94
time (sec)	N/A	0.744	0.936	0.171	0.035	0.108	0.000	0.000	0.167	0.054

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	313	117	373	226	248	0	0	475	203
N.S.	1	1.53	0.57	1.83	1.11	1.22	0.00	0.00	2.33	1.00
time (sec)	N/A	0.838	0.772	0.190	0.035	0.093	0.000	0.000	0.247	13.754

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	93	116	80	103	111	112	184	83	81
N.S.	1	0.80	1.00	0.69	0.89	0.96	0.97	1.59	0.72	0.70
time (sec)	N/A	0.814	0.061	0.340	0.032	0.070	0.359	0.123	0.160	0.049

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	78	100	71	92	100	100	160	75	72
N.S.	1	0.78	1.00	0.71	0.92	1.00	1.00	1.60	0.75	0.72
time (sec)	N/A	0.788	0.038	0.276	0.026	0.085	0.258	0.131	0.145	13.343

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	53	63	47	59	67	65	136	51	48
N.S.	1	0.84	1.00	0.75	0.94	1.06	1.03	2.16	0.81	0.76
time (sec)	N/A	0.760	0.032	0.223	0.040	0.079	0.164	0.123	0.146	13.324

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	41	51	40	46	56	53	112	43	43
N.S.	1	0.80	1.00	0.78	0.90	1.10	1.04	2.20	0.84	0.84
time (sec)	N/A	0.710	0.025	0.204	0.031	0.067	0.112	0.125	0.148	13.306

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	32	35	26	66	33	32
N.S.	1	1.00	1.00	0.88	0.97	1.06	0.79	2.00	1.00	0.97
time (sec)	N/A	0.639	0.025	0.184	0.032	0.085	0.155	0.124	0.144	0.039

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	52	53	43	49	64	41	74	71	48
N.S.	1	0.98	1.00	0.81	0.92	1.21	0.77	1.40	1.34	0.91
time (sec)	N/A	0.764	0.040	0.159	0.026	0.076	0.133	0.114	0.147	0.039

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	66	63	56	75	100	83	93	99	71
N.S.	1	0.93	0.89	0.79	1.06	1.41	1.17	1.31	1.39	1.00
time (sec)	N/A	0.787	0.044	0.154	0.030	0.093	0.179	0.112	0.159	0.042

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	102	89	82	107	163	114	130	189	104
N.S.	1	0.92	0.80	0.74	0.96	1.47	1.03	1.17	1.70	0.94
time (sec)	N/A	0.861	0.081	0.177	0.038	0.092	0.357	0.118	0.148	0.061

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	129	98	108	135	207	144	170	241	131
N.S.	1	0.88	0.67	0.74	0.92	1.42	0.99	1.16	1.65	0.90
time (sec)	N/A	0.910	0.125	0.175	0.035	0.081	0.510	0.115	0.149	13.159

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	328	120	189	380	201	0	524	243	332
N.S.	1	1.94	0.71	1.12	2.25	1.19	0.00	3.10	1.44	1.96
time (sec)	N/A	0.909	0.415	0.118	0.111	0.100	0.000	0.158	0.201	0.118

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	258	110	173	302	179	0	394	203	258
N.S.	1	1.90	0.81	1.27	2.22	1.32	0.00	2.90	1.49	1.90
time (sec)	N/A	0.774	0.323	0.108	0.128	0.117	0.000	0.142	0.175	0.056

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	184	94	157	223	156	0	264	155	183
N.S.	1	1.79	0.91	1.52	2.17	1.51	0.00	2.56	1.50	1.78
time (sec)	N/A	0.658	0.275	0.101	0.111	0.103	0.000	0.136	0.158	0.049

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	112	55	127	117	107	0	121	101	84
N.S.	1	2.00	0.98	2.27	2.09	1.91	0.00	2.16	1.80	1.50
time (sec)	N/A	0.534	0.171	0.079	0.113	0.100	0.000	0.134	0.155	13.085

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	102	57	138	121	67	0	0	95	86
N.S.	1	1.44	0.80	1.94	1.70	0.94	0.00	0.00	1.34	1.21
time (sec)	N/A	0.510	0.197	0.128	0.032	0.097	0.000	0.000	0.145	13.146

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	171	85	213	163	119	0	0	199	137
N.S.	1	1.60	0.79	1.99	1.52	1.11	0.00	0.00	1.86	1.28
time (sec)	N/A	0.623	0.549	0.157	0.034	0.103	0.000	0.000	0.155	0.039

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	241	101	287	197	161	0	0	282	178
N.S.	1	1.72	0.72	2.05	1.41	1.15	0.00	0.00	2.01	1.27
time (sec)	N/A	0.730	0.606	0.156	0.040	0.094	0.000	0.000	0.163	0.032

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	312	117	361	231	205	0	0	475	217
N.S.	1	1.80	0.68	2.09	1.34	1.18	0.00	0.00	2.75	1.25
time (sec)	N/A	0.813	0.726	0.175	0.033	0.082	0.000	0.000	0.227	0.035

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	73	90	64	81	89	88	82	67	67
N.S.	1	0.81	1.00	0.71	0.90	0.99	0.98	0.91	0.74	0.74
time (sec)	N/A	0.824	0.057	0.243	0.032	0.078	0.238	0.111	0.154	13.151

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	60	76	55	70	78	76	71	59	56
N.S.	1	0.79	1.00	0.72	0.92	1.03	1.00	0.93	0.78	0.74
time (sec)	N/A	0.789	0.041	0.225	0.026	0.092	0.171	0.122	0.156	13.156

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	34	40	32	37	43	39	38	35	35
N.S.	1	0.85	1.00	0.80	0.92	1.08	0.98	0.95	0.88	0.88
time (sec)	N/A	0.728	0.029	0.193	0.032	0.071	0.090	0.120	0.154	13.127

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	21	26	20	22	23	25
N.S.	1	1.00	1.00	1.05	1.00	1.24	0.95	1.05	1.10	1.19
time (sec)	N/A	0.613	0.024	0.158	0.025	0.093	0.049	0.106	0.155	0.023

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	36	28	36	34	38	36	36	44	33
N.S.	1	1.03	0.80	1.03	0.97	1.09	1.03	1.03	1.26	0.94
time (sec)	N/A	0.733	0.052	0.142	0.028	0.087	0.071	0.125	0.168	0.030

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	70	70	60	69	92	75	57	102	68
N.S.	1	0.96	0.96	0.82	0.95	1.26	1.03	0.78	1.40	0.93
time (sec)	N/A	0.806	0.081	0.150	0.034	0.080	0.207	0.121	0.148	13.151

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	97	104	84	97	137	102	80	153	94
N.S.	1	0.90	0.96	0.78	0.90	1.27	0.94	0.74	1.42	0.87
time (sec)	N/A	0.857	0.103	0.159	0.028	0.085	0.337	0.116	0.155	13.416

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	128	124	108	145	233	156	96	277	142
N.S.	1	0.90	0.87	0.76	1.01	1.63	1.09	0.67	1.94	0.99
time (sec)	N/A	0.903	0.133	0.171	0.035	0.103	0.490	0.138	0.145	13.229

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	325	126	180	379	201	0	525	243	332
N.S.	1	1.81	0.70	1.00	2.11	1.12	0.00	2.92	1.35	1.84
time (sec)	N/A	0.884	0.462	0.131	0.118	0.091	0.000	0.156	0.200	13.206

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	257	110	173	301	179	0	395	203	258
N.S.	1	1.80	0.77	1.21	2.10	1.25	0.00	2.76	1.42	1.80
time (sec)	N/A	0.751	0.333	0.103	0.112	0.107	0.000	0.152	0.175	0.056

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	189	94	157	224	156	0	264	163	183
N.S.	1	1.66	0.82	1.38	1.96	1.37	0.00	2.32	1.43	1.61
time (sec)	N/A	0.673	0.274	0.095	0.115	0.098	0.000	0.144	0.157	0.048

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	81	57	130	118	103	0	122	103	84
N.S.	1	1.17	0.83	1.88	1.71	1.49	0.00	1.77	1.49	1.22
time (sec)	N/A	0.469	0.175	0.102	0.115	0.088	0.000	0.155	0.151	0.032

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	141	69	173	140	96	0	59	158	114
N.S.	1	1.32	0.64	1.62	1.31	0.90	0.00	0.55	1.48	1.07
time (sec)	N/A	0.575	0.210	0.122	0.034	0.098	0.000	0.142	0.157	0.039

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	173	78	207	161	135	0	59	221	141
N.S.	1	1.24	0.56	1.48	1.15	0.96	0.00	0.42	1.58	1.01
time (sec)	N/A	0.621	0.471	0.155	0.033	0.089	0.000	0.148	0.168	0.025

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	245	101	281	199	179	0	0	337	183
N.S.	1	1.42	0.58	1.62	1.15	1.03	0.00	0.00	1.95	1.06
time (sec)	N/A	0.704	0.639	0.174	0.029	0.091	0.000	0.000	0.185	13.128

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	314	117	355	231	275	0	0	420	224
N.S.	1	1.52	0.57	1.72	1.12	1.33	0.00	0.00	2.04	1.09
time (sec)	N/A	0.836	0.738	0.192	0.036	0.098	0.000	0.000	0.276	13.127

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	101	94	112	0	96	0	132	69	0
N.S.	1	0.31	0.29	0.35	0.00	0.30	0.00	0.41	0.21	0.00
time (sec)	N/A	0.700	0.103	0.079	0.000	0.082	0.000	0.141	0.169	0.000

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	78	73	96	0	74	0	106	53	0
N.S.	1	0.33	0.31	0.41	0.00	0.31	0.00	0.45	0.22	0.00
time (sec)	N/A	0.671	0.074	0.071	0.000	0.076	0.000	0.139	0.152	0.000

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	62	62	80	0	42	0	70	35	0
N.S.	1	0.42	0.42	0.55	0.00	0.29	0.00	0.48	0.24	0.00
time (sec)	N/A	0.642	0.052	0.069	0.000	0.091	0.000	0.134	0.153	0.000

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	39	38	50	0	17	0	40	12	0
N.S.	1	0.58	0.57	0.75	0.00	0.25	0.00	0.60	0.18	0.00
time (sec)	N/A	0.578	0.035	0.063	0.000	0.089	0.000	0.132	0.158	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	51	43	57	0	24	0	0	19	0
N.S.	1	0.71	0.60	0.79	0.00	0.33	0.00	0.00	0.26	0.00
time (sec)	N/A	0.620	0.033	0.067	0.000	0.087	0.000	0.000	0.149	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	84	71	102	0	68	0	0	64	0
N.S.	1	0.49	0.41	0.59	0.00	0.39	0.00	0.00	0.37	0.00
time (sec)	N/A	0.672	0.081	0.076	0.000	0.116	0.000	0.000	0.148	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	113	97	175	0	137	0	0	147	0
N.S.	1	0.43	0.37	0.67	0.00	0.52	0.00	0.00	0.56	0.00
time (sec)	N/A	0.741	0.136	0.082	0.000	0.120	0.000	0.000	0.149	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	142	119	247	0	205	0	0	234	0
N.S.	1	0.40	0.33	0.69	0.00	0.57	0.00	0.00	0.65	0.00
time (sec)	N/A	0.798	0.201	0.091	0.000	0.102	0.000	0.000	0.146	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	182	150	250	0	425	1059	561	184	0
N.S.	1	0.96	0.79	1.32	0.00	2.24	5.57	2.95	0.97	0.00
time (sec)	N/A	0.950	0.265	0.315	0.000	0.138	19.383	0.277	0.158	0.000

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	147	134	234	0	381	500	416	148	0
N.S.	1	0.94	0.86	1.50	0.00	2.44	3.21	2.67	0.95	0.00
time (sec)	N/A	0.900	0.207	0.289	0.000	0.136	10.636	0.222	0.161	0.000

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	112	115	202	0	304	376	266	102	0
N.S.	1	0.92	0.94	1.66	0.00	2.49	3.08	2.18	0.84	0.00
time (sec)	N/A	0.804	0.186	0.250	0.000	0.114	7.214	0.190	0.160	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	72	80	197	0	252	0	0	51	0
N.S.	1	0.82	0.91	2.24	0.00	2.86	0.00	0.00	0.58	0.00
time (sec)	N/A	0.705	0.141	0.211	0.000	0.129	0.000	0.000	0.152	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	93	68	160	0	216	0	0	85	0
N.S.	1	1.19	0.87	2.05	0.00	2.77	0.00	0.00	1.09	0.00
time (sec)	N/A	0.664	0.168	0.233	0.000	0.122	0.000	0.000	0.155	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	126	95	214	0	280	0	0	142	0
N.S.	1	1.07	0.81	1.81	0.00	2.37	0.00	0.00	1.20	0.00
time (sec)	N/A	0.902	0.154	0.178	0.000	0.110	0.000	0.000	0.159	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	158	105	298	0	352	0	0	217	0
N.S.	1	1.04	0.69	1.96	0.00	2.32	0.00	0.00	1.43	0.00
time (sec)	N/A	1.125	0.169	0.188	0.000	0.132	0.000	0.000	0.165	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	190	133	382	0	496	0	0	370	0
N.S.	1	1.02	0.71	2.04	0.00	2.65	0.00	0.00	1.98	0.00
time (sec)	N/A	1.343	0.218	0.207	0.000	0.177	0.000	0.000	0.187	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	102	95	112	0	96	0	132	69	0
N.S.	1	0.32	0.30	0.35	0.00	0.30	0.00	0.41	0.21	0.00
time (sec)	N/A	0.711	0.090	0.082	0.000	0.095	0.000	0.147	0.153	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	102	97	112	0	96	0	133	69	0
N.S.	1	0.31	0.30	0.35	0.00	0.30	0.00	0.41	0.21	0.00
time (sec)	N/A	0.691	0.079	0.079	0.000	0.108	0.000	0.150	0.170	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	80	78	96	0	72	0	107	53	0
N.S.	1	0.34	0.33	0.41	0.00	0.31	0.00	0.46	0.23	0.00
time (sec)	N/A	0.683	0.071	0.076	0.000	0.087	0.000	0.147	0.155	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	60	57	69	0	44	0	71	35	0
N.S.	1	0.41	0.39	0.47	0.00	0.30	0.00	0.48	0.24	0.00
time (sec)	N/A	0.635	0.054	0.075	0.000	0.091	0.000	0.134	0.146	0.000

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	50	51	65	0	27	0	63	22	0
N.S.	1	0.46	0.47	0.60	0.00	0.25	0.00	0.58	0.20	0.00
time (sec)	N/A	0.620	0.042	0.069	0.000	0.142	0.000	0.135	0.149	0.000

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	62	57	85	0	49	0	0	46	0
N.S.	1	0.54	0.50	0.74	0.00	0.43	0.00	0.00	0.40	0.00
time (sec)	N/A	0.638	0.071	0.082	0.000	0.110	0.000	0.000	0.147	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	84	64	102	0	81	0	0	75	0
N.S.	1	0.49	0.37	0.60	0.00	0.47	0.00	0.00	0.44	0.00
time (sec)	N/A	0.701	0.097	0.072	0.000	0.107	0.000	0.000	0.142	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	114	96	175	0	138	0	0	147	0
N.S.	1	0.43	0.36	0.66	0.00	0.52	0.00	0.00	0.55	0.00
time (sec)	N/A	0.744	0.129	0.092	0.000	0.108	0.000	0.000	0.150	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	145	116	247	0	207	0	0	235	0
N.S.	1	0.40	0.32	0.69	0.00	0.58	0.00	0.00	0.65	0.00
time (sec)	N/A	0.799	0.204	0.089	0.000	0.142	0.000	0.000	0.156	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	100	95	112	0	96	0	156	79	0
N.S.	1	0.31	0.30	0.35	0.00	0.30	0.00	0.48	0.25	0.00
time (sec)	N/A	0.702	0.080	0.076	0.000	0.113	0.000	0.116	0.153	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	80	75	96	0	74	0	118	63	0
N.S.	1	0.34	0.32	0.40	0.00	0.31	0.00	0.50	0.26	0.00
time (sec)	N/A	0.683	0.068	0.075	0.000	0.097	0.000	0.123	0.162	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	59	63	80	0	44	0	72	45	0
N.S.	1	0.40	0.43	0.54	0.00	0.30	0.00	0.49	0.31	0.00
time (sec)	N/A	0.618	0.050	0.069	0.000	0.100	0.000	0.133	0.159	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	41	39	52	0	19	0	36	17	0
N.S.	1	0.60	0.57	0.76	0.00	0.28	0.00	0.53	0.25	0.00
time (sec)	N/A	0.594	0.038	0.057	0.000	0.085	0.000	0.116	0.152	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	48	43	59	0	26	0	0	21	0
N.S.	1	0.67	0.60	0.82	0.00	0.36	0.00	0.00	0.29	0.00
time (sec)	N/A	0.619	0.035	0.057	0.000	0.092	0.000	0.000	0.152	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	85	70	103	0	66	0	0	63	0
N.S.	1	0.49	0.41	0.60	0.00	0.38	0.00	0.00	0.37	0.00
time (sec)	N/A	0.684	0.080	0.072	0.000	0.120	0.000	0.000	0.149	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	110	95	175	0	135	0	0	146	0
N.S.	1	0.42	0.36	0.67	0.00	0.51	0.00	0.00	0.56	0.00
time (sec)	N/A	0.743	0.175	0.085	0.000	0.112	0.000	0.000	0.146	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	143	119	247	0	201	0	0	233	0
N.S.	1	0.40	0.33	0.69	0.00	0.56	0.00	0.00	0.65	0.00
time (sec)	N/A	0.818	0.198	0.080	0.000	0.118	0.000	0.000	0.155	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	182	150	250	0	425	1059	561	184	0
N.S.	1	0.96	0.79	1.32	0.00	2.24	5.57	2.95	0.97	0.00
time (sec)	N/A	0.950	0.265	0.326	0.000	0.136	19.478	0.271	0.159	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	147	134	234	0	381	500	416	148	0
N.S.	1	0.94	0.86	1.50	0.00	2.44	3.21	2.67	0.95	0.00
time (sec)	N/A	0.915	0.205	0.286	0.000	0.101	10.179	0.241	0.151	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	110	115	202	0	303	376	266	102	0
N.S.	1	0.92	0.96	1.68	0.00	2.52	3.13	2.22	0.85	0.00
time (sec)	N/A	0.817	0.182	0.244	0.000	0.109	7.048	0.183	0.153	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	72	80	196	0	252	0	0	51	0
N.S.	1	0.82	0.91	2.23	0.00	2.86	0.00	0.00	0.58	0.00
time (sec)	N/A	0.745	0.131	0.213	0.000	0.113	0.000	0.000	0.146	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	93	68	154	0	212	0	0	85	0
N.S.	1	1.16	0.85	1.92	0.00	2.65	0.00	0.00	1.06	0.00
time (sec)	N/A	0.679	0.184	0.239	0.000	0.112	0.000	0.000	0.149	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	127	95	202	0	279	0	0	142	0
N.S.	1	1.06	0.79	1.68	0.00	2.32	0.00	0.00	1.18	0.00
time (sec)	N/A	0.937	0.145	0.180	0.000	0.120	0.000	0.000	0.144	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	159	105	286	0	351	0	0	217	0
N.S.	1	1.03	0.68	1.86	0.00	2.28	0.00	0.00	1.41	0.00
time (sec)	N/A	1.136	0.164	0.296	0.000	0.123	0.000	0.000	0.147	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	191	131	370	0	495	0	0	370	0
N.S.	1	1.01	0.69	1.96	0.00	2.62	0.00	0.00	1.96	0.00
time (sec)	N/A	1.420	0.209	0.207	0.000	0.186	0.000	0.000	0.178	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	100	95	112	0	96	0	157	79	0
N.S.	1	0.31	0.30	0.35	0.00	0.30	0.00	0.49	0.25	0.00
time (sec)	N/A	0.787	0.092	0.085	0.000	0.106	0.000	0.125	0.167	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	103	97	112	0	96	0	157	79	0
N.S.	1	0.32	0.30	0.35	0.00	0.30	0.00	0.48	0.24	0.00
time (sec)	N/A	0.713	0.083	0.082	0.000	0.089	0.000	0.126	0.160	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	77	79	96	0	74	0	118	63	0
N.S.	1	0.33	0.34	0.41	0.00	0.31	0.00	0.50	0.27	0.00
time (sec)	N/A	0.680	0.079	0.082	0.000	0.099	0.000	0.116	0.159	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	62	57	69	0	42	0	73	37	0
N.S.	1	0.42	0.39	0.47	0.00	0.28	0.00	0.49	0.25	0.00
time (sec)	N/A	0.627	0.056	0.066	0.000	0.089	0.000	0.118	0.157	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	47	47	67	0	25	0	55	23	0
N.S.	1	0.44	0.44	0.63	0.00	0.23	0.00	0.51	0.21	0.00
time (sec)	N/A	0.612	0.038	0.069	0.000	0.095	0.000	0.135	0.154	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	62	55	87	0	47	0	0	46	0
N.S.	1	0.55	0.49	0.77	0.00	0.42	0.00	0.00	0.41	0.00
time (sec)	N/A	0.637	0.068	0.072	0.000	0.099	0.000	0.000	0.147	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	79	63	102	0	81	0	0	75	0
N.S.	1	0.47	0.38	0.61	0.00	0.48	0.00	0.00	0.45	0.00
time (sec)	N/A	0.693	0.100	0.073	0.000	0.102	0.000	0.000	0.148	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	113	95	175	0	137	0	0	147	0
N.S.	1	0.43	0.36	0.66	0.00	0.52	0.00	0.00	0.56	0.00
time (sec)	N/A	0.748	0.137	0.081	0.000	0.106	0.000	0.000	0.140	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	140	108	247	0	208	0	0	235	0
N.S.	1	0.39	0.30	0.69	0.00	0.58	0.00	0.00	0.66	0.00
time (sec)	N/A	0.804	0.251	0.094	0.000	0.149	0.000	0.000	0.142	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	49	45	47	0	24	0	0	16	46
N.S.	1	0.64	0.59	0.62	0.00	0.32	0.00	0.00	0.21	0.61
time (sec)	N/A	0.826	0.034	0.063	0.000	0.104	0.000	0.000	0.139	13.338

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	46	0	21	0	0	13	45
N.S.	1	1.00	1.00	1.07	0.00	0.49	0.00	0.00	0.30	1.05
time (sec)	N/A	0.683	0.034	0.062	0.000	0.102	0.000	0.000	0.140	13.253

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	39	38	50	0	17	0	40	12	0
N.S.	1	0.58	0.57	0.75	0.00	0.25	0.00	0.60	0.18	0.00
time (sec)	N/A	0.590	0.020	0.052	0.000	0.106	0.000	0.133	0.136	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	43	41	50	0	21	0	32	16	0
N.S.	1	0.61	0.59	0.71	0.00	0.30	0.00	0.46	0.23	0.00
time (sec)	N/A	0.821	0.030	0.079	0.000	0.088	0.000	0.130	0.144	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	47	0	21	0	30	16	63
N.S.	1	1.00	1.00	1.02	0.00	0.46	0.00	0.65	0.35	1.37
time (sec)	N/A	0.780	0.031	0.072	0.000	0.090	0.000	0.139	0.142	13.247

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	176	101	142	0	238	0	139	105	0
N.S.	1	1.14	0.65	0.92	0.00	1.54	0.00	0.90	0.68	0.00
time (sec)	N/A	1.117	0.169	0.237	0.000	0.168	0.000	0.133	0.146	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	149	93	134	0	222	0	128	87	0
N.S.	1	1.15	0.72	1.03	0.00	1.71	0.00	0.98	0.67	0.00
time (sec)	N/A	1.073	0.207	0.240	0.000	0.112	0.000	0.132	0.145	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	112	84	124	0	204	0	116	68	0
N.S.	1	1.13	0.85	1.25	0.00	2.06	0.00	1.17	0.69	0.00
time (sec)	N/A	1.016	0.174	0.228	0.000	0.136	0.000	0.129	0.140	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	91	77	114	0	188	0	106	50	0
N.S.	1	1.17	0.99	1.46	0.00	2.41	0.00	1.36	0.64	0.00
time (sec)	N/A	0.822	0.081	0.217	0.000	0.110	0.000	0.132	0.138	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	72	80	196	0	252	0	0	51	0
N.S.	1	0.82	0.91	2.23	0.00	2.86	0.00	0.00	0.58	0.00
time (sec)	N/A	0.717	0.045	0.217	0.000	0.101	0.000	0.000	0.143	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	76	82	145	0	238	0	127	60	0
N.S.	1	0.96	1.04	1.84	0.00	3.01	0.00	1.61	0.76	0.00
time (sec)	N/A	0.950	0.220	0.239	0.000	0.110	0.000	0.176	0.151	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	97	78	142	0	162	0	194	58	0
N.S.	1	1.23	0.99	1.80	0.00	2.05	0.00	2.46	0.73	0.00
time (sec)	N/A	0.965	0.143	0.248	0.000	0.111	0.000	0.154	0.140	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	119	86	151	0	186	0	231	84	0
N.S.	1	1.16	0.83	1.47	0.00	1.81	0.00	2.24	0.82	0.00
time (sec)	N/A	1.056	0.156	0.264	0.000	0.117	0.000	0.165	0.149	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	151	94	159	0	202	0	316	152	0
N.S.	1	1.14	0.71	1.20	0.00	1.52	0.00	2.38	1.14	0.00
time (sec)	N/A	1.112	0.152	0.247	0.000	0.111	0.000	0.177	0.153	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	173	102	167	0	218	0	362	122	0
N.S.	1	1.09	0.65	1.06	0.00	1.38	0.00	2.29	0.77	0.00
time (sec)	N/A	1.168	0.172	0.247	0.000	0.118	0.000	0.248	0.148	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	74	71	89	0	48	0	141	43	0
N.S.	1	0.40	0.38	0.48	0.00	0.26	0.00	0.76	0.23	0.00
time (sec)	N/A	0.934	0.065	0.097	0.000	0.103	0.000	0.131	0.144	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	68	63	82	0	41	0	107	36	0
N.S.	1	0.45	0.41	0.54	0.00	0.27	0.00	0.70	0.24	0.00
time (sec)	N/A	0.888	0.049	0.076	0.000	0.099	0.000	0.126	0.145	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	58	56	73	0	32	0	89	27	0
N.S.	1	0.51	0.50	0.65	0.00	0.28	0.00	0.79	0.24	0.00
time (sec)	N/A	0.786	0.037	0.092	0.000	0.082	0.000	0.154	0.140	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	50	51	65	0	27	0	63	22	0
N.S.	1	0.46	0.47	0.60	0.00	0.25	0.00	0.58	0.20	0.00
time (sec)	N/A	0.644	0.021	0.069	0.000	0.102	0.000	0.136	0.142	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	55	51	66	0	32	0	44	27	0
N.S.	1	0.51	0.47	0.61	0.00	0.30	0.00	0.41	0.25	0.00
time (sec)	N/A	0.869	0.048	0.081	0.000	0.116	0.000	0.159	0.143	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	67	61	82	0	85	0	56	40	0
N.S.	1	0.46	0.41	0.56	0.00	0.58	0.00	0.38	0.27	0.00
time (sec)	N/A	0.881	0.067	0.076	0.000	0.085	0.000	0.146	0.149	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	77	73	90	0	93	0	68	48	0
N.S.	1	0.41	0.39	0.48	0.00	0.49	0.00	0.36	0.26	0.00
time (sec)	N/A	0.878	0.072	0.090	0.000	0.098	0.000	0.170	0.141	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	83	77	98	0	101	0	76	56	0
N.S.	1	0.37	0.35	0.44	0.00	0.45	0.00	0.34	0.25	0.00
time (sec)	N/A	0.888	0.076	0.071	0.000	0.091	0.000	0.156	0.147	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	95	91	106	0	109	0	88	64	0
N.S.	1	0.36	0.34	0.40	0.00	0.41	0.00	0.33	0.24	0.00
time (sec)	N/A	0.912	0.075	0.098	0.000	0.119	0.000	0.149	0.138	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	50	45	47	0	24	0	53	25	46
N.S.	1	0.66	0.59	0.62	0.00	0.32	0.00	0.70	0.33	0.61
time (sec)	N/A	0.831	0.040	0.084	0.000	0.102	0.000	0.120	0.144	13.745

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	46	0	21	0	50	20	45
N.S.	1	1.00	1.00	1.05	0.00	0.48	0.00	1.14	0.45	1.02
time (sec)	N/A	0.679	0.031	0.064	0.000	0.080	0.000	0.117	0.155	13.743

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	41	39	52	0	19	0	36	17	0
N.S.	1	0.60	0.57	0.76	0.00	0.28	0.00	0.53	0.25	0.00
time (sec)	N/A	0.593	0.023	0.051	0.000	0.084	0.000	0.119	0.144	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	45	40	50	0	21	0	37	22	0
N.S.	1	0.65	0.58	0.72	0.00	0.30	0.00	0.54	0.32	0.00
time (sec)	N/A	0.824	0.033	0.063	0.000	0.092	0.000	0.138	0.146	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	46	47	0	21	0	49	23	63
N.S.	1	1.00	0.98	1.00	0.00	0.45	0.00	1.04	0.49	1.34
time (sec)	N/A	0.796	0.034	0.064	0.000	0.076	0.000	0.122	0.151	13.684

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	177	101	142	0	238	0	139	105	0
N.S.	1	1.14	0.65	0.92	0.00	1.54	0.00	0.90	0.68	0.00
time (sec)	N/A	1.147	0.182	0.248	0.000	0.093	0.000	0.154	0.141	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	149	93	134	0	222	0	128	87	0
N.S.	1	1.15	0.72	1.03	0.00	1.71	0.00	0.98	0.67	0.00
time (sec)	N/A	1.102	0.202	0.248	0.000	0.096	0.000	0.134	0.152	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	116	84	125	0	204	0	117	68	0
N.S.	1	1.15	0.83	1.24	0.00	2.02	0.00	1.16	0.67	0.00
time (sec)	N/A	0.965	0.178	0.218	0.000	0.102	0.000	0.144	0.155	0.000

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	92	100	114	0	188	0	106	50	0
N.S.	1	1.18	1.28	1.46	0.00	2.41	0.00	1.36	0.64	0.00
time (sec)	N/A	0.818	0.087	0.214	0.000	0.125	0.000	0.155	0.147	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	72	80	196	0	252	0	0	51	0
N.S.	1	0.82	0.91	2.23	0.00	2.86	0.00	0.00	0.58	0.00
time (sec)	N/A	0.744	0.047	0.204	0.000	0.094	0.000	0.000	0.143	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	82	145	0	238	0	127	60	0
N.S.	1	1.00	1.04	1.84	0.00	3.01	0.00	1.61	0.76	0.00
time (sec)	N/A	1.011	0.210	0.228	0.000	0.139	0.000	0.163	0.180	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	97	78	143	0	163	0	194	58	0
N.S.	1	1.23	0.99	1.81	0.00	2.06	0.00	2.46	0.73	0.00
time (sec)	N/A	0.983	0.179	0.223	0.000	0.107	0.000	0.153	0.153	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	120	86	150	0	187	0	231	84	0
N.S.	1	1.17	0.83	1.46	0.00	1.82	0.00	2.24	0.82	0.00
time (sec)	N/A	1.047	0.153	0.240	0.000	0.098	0.000	0.192	0.155	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	151	94	159	0	203	0	316	152	0
N.S.	1	1.14	0.71	1.20	0.00	1.53	0.00	2.38	1.14	0.00
time (sec)	N/A	1.121	0.185	0.234	0.000	0.108	0.000	0.175	0.163	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	174	102	167	0	219	0	362	122	0
N.S.	1	1.18	0.69	1.13	0.00	1.48	0.00	2.45	0.82	0.00
time (sec)	N/A	1.201	0.190	0.247	0.000	0.129	0.000	0.263	0.158	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	72	71	89	0	48	0	94	44	0
N.S.	1	0.39	0.38	0.48	0.00	0.26	0.00	0.51	0.24	0.00
time (sec)	N/A	0.907	0.082	0.088	0.000	0.099	0.000	0.120	0.159	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	66	62	82	0	41	0	80	37	0
N.S.	1	0.44	0.41	0.54	0.00	0.27	0.00	0.53	0.25	0.00
time (sec)	N/A	0.875	0.070	0.070	0.000	0.089	0.000	0.143	0.162	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	56	55	73	0	32	0	62	28	0
N.S.	1	0.50	0.49	0.65	0.00	0.29	0.00	0.55	0.25	0.00
time (sec)	N/A	0.742	0.049	0.064	0.000	0.095	0.000	0.131	0.156	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	47	47	67	0	25	0	55	23	0
N.S.	1	0.44	0.44	0.63	0.00	0.23	0.00	0.51	0.21	0.00
time (sec)	N/A	0.649	0.024	0.063	0.000	0.144	0.000	0.114	0.158	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	53	51	66	0	32	0	59	32	0
N.S.	1	0.49	0.47	0.61	0.00	0.30	0.00	0.55	0.30	0.00
time (sec)	N/A	0.860	0.053	0.067	0.000	0.104	0.000	0.120	0.158	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	65	60	82	0	83	0	69	50	0
N.S.	1	0.45	0.41	0.56	0.00	0.57	0.00	0.47	0.34	0.00
time (sec)	N/A	0.870	0.069	0.079	0.000	0.098	0.000	0.118	0.148	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	75	72	90	0	91	0	87	58	0
N.S.	1	0.40	0.39	0.48	0.00	0.49	0.00	0.47	0.31	0.00
time (sec)	N/A	0.859	0.087	0.075	0.000	0.114	0.000	0.144	0.173	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	80	76	98	0	99	0	107	66	0
N.S.	1	0.36	0.34	0.44	0.00	0.45	0.00	0.48	0.30	0.00
time (sec)	N/A	0.884	0.079	0.078	0.000	0.107	0.000	0.146	0.165	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	93	90	106	0	107	0	123	74	0
N.S.	1	0.35	0.34	0.40	0.00	0.41	0.00	0.47	0.28	0.00
time (sec)	N/A	0.887	0.093	0.076	0.000	0.107	0.000	0.123	0.188	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	114	97	161	206	210	0	0	127	0
N.S.	1	0.60	0.51	0.85	1.09	1.11	0.00	0.00	0.67	0.00
time (sec)	N/A	0.987	0.206	0.076	0.116	0.131	0.000	0.000	0.165	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	64	53	58	50	74	0	0	27	0
N.S.	1	0.74	0.61	0.67	0.57	0.85	0.00	0.00	0.31	0.00
time (sec)	N/A	0.861	0.064	0.065	0.085	0.086	0.000	0.000	0.155	0.000

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	0	0	76	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	1.31	0.00
time (sec)	N/A	0.820	0.048	0.000	0.000	0.000	0.000	0.000	0.148	0.000

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	119	111	0	0	0	0	0	180	0
N.S.	1	0.80	0.74	0.00	0.00	0.00	0.00	0.00	1.21	0.00
time (sec)	N/A	0.974	0.097	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	111	125	161	206	209	0	0	127	0
N.S.	1	0.58	0.65	0.84	1.08	1.09	0.00	0.00	0.66	0.00
time (sec)	N/A	0.998	0.130	0.082	0.121	0.120	0.000	0.000	0.166	0.000

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	66	55	60	52	75	0	0	29	0
N.S.	1	0.75	0.62	0.68	0.59	0.85	0.00	0.00	0.33	0.00
time (sec)	N/A	0.868	0.059	0.064	0.089	0.103	0.000	0.000	0.160	0.000

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	0	0	75	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	1.29	0.00
time (sec)	N/A	0.830	0.047	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	121	112	0	0	0	0	0	183	0
N.S.	1	0.81	0.75	0.00	0.00	0.00	0.00	0.00	1.22	0.00
time (sec)	N/A	0.946	0.083	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	258	123	0	0	0	0	0	63	0
N.S.	1	1.68	0.80	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.733	0.425	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	147	94	0	0	0	0	0	32	0
N.S.	1	0.98	0.63	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.568	0.423	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	286	290	180	0	0	0	0	0	40	0
N.S.	1	1.01	0.63	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.811	1.891	0.000	0.000	0.000	0.000	0.000	0.147	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	295	239	146	0	0	0	0	0	30	0
N.S.	1	0.81	0.49	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.847	0.692	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	183	157	112	0	0	0	0	0	30	0
N.S.	1	0.86	0.61	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.729	0.651	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	116	0	0	0	0	0	0	40	0
N.S.	1	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.653	0.000	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	0	0	0	0	43	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.591	0.000	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	0	0	0	0	0	0	41	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.579	0.000	0.000	0.000	0.000	0.000	0.000	0.150	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [137] had the largest ratio of [1.785709999999999991]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	F	0	0	N/A	0.000	N/A
2	F	0	0	N/A	0.000	N/A
3	F	0	0	N/A	0.000	N/A
4	F	0	0	N/A	0.000	N/A
5	F	0	0	N/A	0.000	N/A
6	F	0	0	N/A	0.000	N/A
7	F	0	0	N/A	0.000	N/A
8	F	0	0	N/A	0.000	N/A
9	A	15	14	1.10	10	1.400
10	A	12	11	1.06	10	1.100
11	A	9	8	1.03	8	1.000
12	A	6	5	1.00	6	0.833
13	A	7	6	1.18	10	0.600
14	A	4	3	1.17	10	0.300
15	A	6	5	1.15	10	0.500
16	A	8	7	1.14	10	0.700
17	A	10	9	1.17	10	0.900
18	A	4	4	1.00	12	0.333
19	A	4	4	1.00	12	0.333
20	A	4	4	1.00	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	4	4	1.00	8	0.500
22	A	4	4	1.00	12	0.333
23	A	4	4	1.00	12	0.333
24	A	4	4	1.00	12	0.333
25	A	4	4	1.00	12	0.333
26	A	4	3	1.00	12	0.250
27	A	4	3	1.00	10	0.300
28	A	4	3	1.00	8	0.375
29	A	12	11	1.80	12	0.917
30	A	7	6	1.08	12	0.500
31	A	15	14	1.22	12	1.167
32	A	17	16	1.24	12	1.333
33	A	4	4	1.00	12	0.333
34	A	4	4	1.00	12	0.333
35	A	4	4	1.00	10	0.400
36	A	4	4	1.00	8	0.500
37	A	4	4	1.00	12	0.333
38	A	4	4	1.00	12	0.333
39	A	4	4	1.00	12	0.333
40	A	4	4	1.00	12	0.333
41	A	12	11	1.10	12	0.917
42	A	10	9	1.06	12	0.750
43	A	8	7	0.98	10	0.700
44	A	6	5	1.00	8	0.625
45	A	7	6	1.20	12	0.500
46	A	4	3	1.16	12	0.250
47	A	7	6	1.13	12	0.500
48	A	10	9	1.14	12	0.750
49	A	13	12	1.17	12	1.000
50	A	4	4	1.00	12	0.333
51	A	4	4	1.00	12	0.333
52	A	4	4	1.00	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	A	4	4	1.00	8	0.500
54	A	4	4	1.00	12	0.333
55	A	4	4	1.00	12	0.333
56	A	4	4	1.00	12	0.333
57	A	4	4	1.00	12	0.333
58	A	4	3	1.00	12	0.250
59	A	4	3	1.00	12	0.250
60	A	4	3	1.00	10	0.300
61	A	4	3	0.95	8	0.375
62	A	12	11	1.69	12	0.917
63	A	7	6	1.15	12	0.500
64	A	13	12	1.19	12	1.000
65	A	15	14	1.25	12	1.167
66	A	17	16	1.28	12	1.333
67	A	16	15	1.08	14	1.071
68	A	14	13	1.06	14	0.929
69	A	12	11	1.04	14	0.786
70	A	8	7	1.02	12	0.583
71	A	7	6	1.06	10	0.600
72	A	17	16	1.16	14	1.143
73	A	13	12	1.14	14	0.857
74	A	14	13	1.07	14	0.929
75	A	16	15	1.08	14	1.071
76	A	17	16	1.08	14	1.143
77	A	15	14	1.05	14	1.000
78	A	13	12	1.04	14	0.857
79	A	9	8	1.02	12	0.667
80	A	8	7	1.04	10	0.700
81	A	18	17	1.17	14	1.214
82	A	13	12	1.13	14	0.857
83	A	14	13	1.07	14	0.929
84	A	16	15	1.08	14	1.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
85	A	18	17	1.09	14	1.214
86	A	16	15	1.08	14	1.071
87	A	14	13	1.07	14	0.929
88	A	9	8	1.02	12	0.667
89	A	8	7	1.05	10	0.700
90	A	20	19	1.15	14	1.357
91	A	14	13	1.13	14	0.929
92	A	15	14	1.06	14	1.000
93	A	17	16	1.07	14	1.143
94	A	17	16	1.08	14	1.143
95	A	15	14	1.06	14	1.000
96	A	13	12	1.04	14	0.857
97	A	9	8	1.02	12	0.667
98	A	8	7	1.05	10	0.700
99	A	18	17	1.16	14	1.214
100	A	13	12	1.15	14	0.857
101	A	14	13	1.06	14	0.929
102	A	16	15	1.07	14	1.071
103	A	16	15	1.08	14	1.071
104	A	14	13	1.05	14	0.929
105	A	12	11	1.04	14	0.786
106	A	8	7	1.02	12	0.583
107	A	7	6	1.04	10	0.600
108	A	17	16	1.17	14	1.143
109	A	13	12	1.13	14	0.857
110	A	14	13	1.06	14	0.929
111	A	16	15	1.07	14	1.071
112	A	19	18	1.08	14	1.286
113	A	17	16	1.07	14	1.143
114	A	15	14	1.06	14	1.000
115	A	10	9	1.02	12	0.750
116	A	9	8	1.05	10	0.800

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
117	A	21	20	1.15	14	1.429
118	A	14	13	1.12	14	0.929
119	A	15	14	1.06	14	1.000
120	A	17	16	1.07	14	1.143
121	A	17	16	1.12	12	1.333
122	A	13	12	1.11	10	1.200
123	A	12	11	1.15	8	1.375
124	A	17	16	1.28	12	1.333
125	A	13	12	1.24	12	1.000
126	A	14	13	1.20	12	1.083
127	A	16	15	1.21	12	1.250
128	A	9	8	1.06	12	0.667
129	A	5	4	1.05	10	0.400
130	A	4	3	1.08	8	0.375
131	A	5	4	1.01	12	0.333
132	A	4	3	1.06	12	0.250
133	A	5	4	1.03	12	0.333
134	A	21	20	1.10	14	1.429
135	A	17	16	1.10	12	1.333
136	A	16	15	1.10	10	1.500
137	A	26	25	1.29	14	1.786
138	A	13	12	1.24	14	0.857
139	A	14	13	1.20	14	0.929
140	A	6	6	1.00	14	0.429
141	A	4	4	1.00	14	0.286
142	A	4	4	1.00	14	0.286
143	A	6	6	1.00	14	0.429
144	A	9	8	1.54	14	0.571
145	A	5	4	1.26	12	0.333
146	A	5	4	1.23	14	0.286
147	A	9	8	1.50	14	0.571
148	A	3	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
149	A	3	2	1.00	16	0.125
150	A	3	2	1.00	16	0.125
151	A	3	2	1.00	16	0.125
152	A	3	2	1.00	16	0.125
153	A	3	2	1.00	14	0.143
154	A	3	2	1.00	14	0.143
155	A	3	2	1.00	16	0.125
156	A	3	2	1.00	14	0.143
157	A	9	8	1.02	12	0.667
158	A	4	3	1.00	10	0.300
159	A	3	2	1.00	8	0.250
160	A	5	4	1.00	12	0.333
161	A	3	2	1.00	12	0.167
162	A	4	3	1.00	12	0.250
163	A	7	6	1.02	12	0.500
164	A	7	6	0.85	12	0.500
165	A	11	10	0.97	16	0.625
166	A	9	8	0.93	16	0.500
167	A	9	8	0.90	16	0.500
168	A	6	5	0.91	14	0.357
169	A	7	6	1.04	16	0.375
170	A	5	4	1.00	16	0.250
171	A	5	4	0.96	16	0.250
172	A	10	9	1.28	16	0.562
173	A	11	10	1.44	16	0.625
174	A	5	5	0.97	18	0.278
175	A	5	5	0.97	18	0.278
176	A	5	5	0.97	18	0.278
177	A	5	5	0.85	18	0.278
178	C	1	1	1.86	16	0.062
179	A	5	5	0.94	18	0.278
180	A	4	4	1.00	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
181	A	5	5	0.97	18	0.278
182	A	5	5	0.97	18	0.278
183	A	10	9	0.96	18	0.500
184	A	7	6	0.97	18	0.333
185	A	10	9	0.99	18	0.500
186	A	10	9	1.08	16	0.562
187	A	11	10	1.00	18	0.556
188	A	5	4	1.00	18	0.222
189	A	5	4	0.99	18	0.222
190	A	9	8	1.01	18	0.444
191	A	9	8	1.01	18	0.444
192	A	5	5	0.91	18	0.278
193	A	6	6	0.81	18	0.333
194	A	5	5	0.94	18	0.278
195	A	4	4	1.00	18	0.222
196	A	5	5	0.96	16	0.312
197	A	5	5	0.90	18	0.278
198	A	4	4	1.00	18	0.222
199	A	5	5	0.90	18	0.278
200	A	5	5	0.91	18	0.278
201	A	11	10	1.02	18	0.556
202	A	10	9	1.02	18	0.500
203	A	8	7	1.08	16	0.438
204	A	5	4	1.00	18	0.222
205	A	5	4	1.00	18	0.222
206	A	5	4	1.06	18	0.222
207	A	10	9	1.37	18	0.500
208	A	10	9	1.30	18	0.500
209	A	5	5	0.86	18	0.278
210	A	5	5	0.86	18	0.278
211	A	5	5	0.85	18	0.278
212	A	5	5	1.00	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
213	A	4	4	1.00	18	0.222
214	A	5	5	1.00	18	0.278
215	A	5	5	0.97	18	0.278
216	A	5	5	0.92	18	0.278
217	A	5	5	0.90	18	0.278
218	A	13	12	1.02	18	0.667
219	A	9	8	1.08	16	0.500
220	A	7	6	0.98	18	0.333
221	A	5	4	1.00	18	0.222
222	A	3	2	1.00	18	0.111
223	A	6	5	1.00	18	0.278
224	A	8	7	0.85	18	0.389
225	A	10	9	0.90	18	0.500
226	A	9	8	0.77	18	0.444
227	A	8	7	0.80	18	0.389
228	A	7	6	1.03	18	0.333
229	A	5	4	1.13	18	0.222
230	A	1	1	1.00	18	0.056
231	A	6	5	0.88	18	0.278
232	A	6	5	0.88	18	0.278
233	A	7	6	0.79	18	0.333
234	A	8	7	0.76	18	0.389
235	A	5	5	1.15	20	0.250
236	A	5	5	1.15	20	0.250
237	A	5	5	1.15	20	0.250
238	A	5	5	1.16	20	0.250
239	A	5	5	1.17	20	0.250
240	A	5	5	1.16	20	0.250
241	A	5	5	1.15	20	0.250
242	A	5	5	1.15	20	0.250
243	A	8	7	0.80	20	0.350
244	A	7	6	0.87	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
245	A	5	4	0.98	20	0.200
246	A	1	1	1.00	20	0.050
247	A	7	6	0.84	20	0.300
248	A	7	6	0.87	20	0.300
249	A	7	6	0.84	20	0.300
250	A	8	7	0.78	20	0.350
251	A	9	8	0.76	20	0.400
252	A	8	7	1.23	20	0.350
253	A	7	6	1.25	20	0.300
254	A	5	4	1.40	20	0.200
255	A	1	1	1.00	20	0.050
256	A	5	4	0.99	20	0.200
257	A	6	5	0.85	20	0.250
258	A	7	6	0.80	20	0.300
259	A	11	10	1.08	20	0.500
260	A	10	9	1.09	20	0.450
261	A	9	8	1.09	20	0.400
262	A	8	7	1.08	20	0.350
263	A	7	6	1.03	20	0.300
264	A	6	5	1.00	20	0.250
265	A	7	6	1.09	20	0.300
266	A	8	7	1.10	20	0.350
267	A	9	8	1.15	20	0.400
268	A	9	8	0.76	20	0.400
269	A	8	7	0.81	20	0.350
270	A	7	6	0.84	20	0.300
271	A	5	4	0.98	20	0.200
272	A	1	1	1.00	20	0.050
273	A	6	5	0.83	20	0.250
274	A	7	6	0.76	20	0.300
275	A	8	7	0.74	20	0.350
276	A	5	5	1.14	18	0.278

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
277	A	5	5	1.12	18	0.278
278	A	4	4	1.00	18	0.222
279	B	5	4	2.48	18	0.222
280	A	4	3	1.87	16	0.188
281	A	3	2	1.00	18	0.111
282	A	3	2	1.00	18	0.111
283	A	3	2	1.00	18	0.111
284	A	3	2	1.00	18	0.111
285	A	3	2	1.00	18	0.111
286	A	3	2	1.00	16	0.125
287	A	3	2	1.00	18	0.111
288	A	3	2	1.00	18	0.111
289	A	4	3	1.01	18	0.167
290	A	8	7	0.93	18	0.389
291	A	3	2	1.08	20	0.100
292	A	3	2	1.08	20	0.100
293	A	3	2	1.08	20	0.100
294	A	3	2	1.08	20	0.100
295	A	3	2	1.08	20	0.100
296	A	4	3	1.91	20	0.150
297	B	5	4	2.54	20	0.200
298	A	8	7	0.81	24	0.292
299	A	5	4	1.02	24	0.167
300	A	1	1	1.00	22	0.045
301	A	4	3	1.20	24	0.125
302	A	4	3	1.19	24	0.125
303	A	7	6	1.81	9	0.667
304	A	6	5	1.63	8	0.625
305	A	3	2	1.00	11	0.182
306	A	6	5	0.89	10	0.500
307	A	8	7	1.76	11	0.636
308	A	7	6	1.62	10	0.600

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
309	A	9	8	1.04	13	0.615
310	A	8	7	0.94	12	0.583
311	A	3	2	1.69	11	0.182
312	A	4	3	1.83	10	0.300
313	A	7	6	1.00	13	0.462
314	A	6	5	1.00	12	0.417
315	A	5	4	1.36	11	0.364
316	A	3	2	1.11	10	0.200
317	A	11	10	0.98	13	0.769
318	A	4	3	1.00	12	0.250
319	A	5	4	0.84	21	0.190
320	A	4	3	0.93	19	0.158
321	A	1	1	1.00	18	0.056
322	A	5	4	0.86	21	0.190
323	A	5	4	0.80	21	0.190
324	A	5	5	1.06	23	0.217
325	A	5	5	1.08	23	0.217
326	A	5	5	1.11	21	0.238
327	A	5	5	1.16	20	0.250
328	A	7	6	1.15	23	0.261
329	A	7	6	1.17	23	0.261
330	A	8	7	1.12	23	0.304
331	A	9	8	1.16	23	0.348
332	A	10	9	1.18	23	0.391
333	A	15	14	0.75	23	0.609
334	A	13	12	0.75	23	0.522
335	A	8	7	0.79	21	0.333
336	A	7	6	0.84	20	0.300
337	A	10	9	0.79	23	0.391
338	A	10	9	0.79	23	0.391
339	A	12	11	0.70	23	0.478
340	A	14	13	0.68	23	0.565

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
341	A	16	15	0.68	23	0.652
342	A	7	6	0.91	13	0.462
343	A	6	5	0.99	12	0.417
344	A	5	4	0.91	15	0.267
345	A	4	3	1.03	14	0.214
346	A	5	4	0.99	13	0.308
347	A	4	3	1.09	12	0.250
348	A	4	3	0.99	15	0.200
349	A	1	1	1.00	14	0.071
350	A	4	3	1.04	13	0.231
351	A	3	2	1.00	12	0.167
352	A	6	5	0.89	15	0.333
353	A	5	4	0.87	14	0.286
354	A	5	4	0.90	13	0.308
355	A	4	3	1.00	12	0.250
356	A	6	5	0.89	15	0.333
357	A	5	4	0.94	14	0.286
358	A	7	6	1.06	23	0.261
359	A	6	5	1.14	21	0.238
360	A	5	4	1.40	20	0.200
361	A	6	5	0.90	23	0.217
362	A	6	5	0.87	23	0.217
363	A	5	5	1.02	23	0.217
364	A	5	5	1.03	23	0.217
365	A	9	8	1.12	21	0.381
366	A	8	7	1.08	20	0.350
367	A	10	9	1.15	23	0.391
368	A	10	9	1.18	23	0.391
369	A	12	11	1.12	23	0.478
370	A	14	13	1.15	23	0.565
371	A	16	15	1.17	23	0.652
372	A	10	9	0.73	23	0.391

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
373	A	9	8	0.76	23	0.348
374	A	8	7	0.79	21	0.333
375	A	7	6	0.84	20	0.300
376	A	8	7	0.78	23	0.304
377	A	8	7	0.80	23	0.304
378	A	9	8	0.74	23	0.348
379	A	10	9	0.72	23	0.391
380	A	11	10	0.71	23	0.435
381	A	4	3	1.07	21	0.143
382	A	4	3	1.07	21	0.143
383	A	5	4	1.14	21	0.190
384	A	4	3	1.00	19	0.158
385	A	4	3	1.14	21	0.143
386	A	4	3	1.37	21	0.143
387	A	4	3	1.31	21	0.143
388	A	7	6	0.92	23	0.261
389	A	5	4	1.02	23	0.174
390	A	3	2	1.00	23	0.087
391	A	4	3	1.00	23	0.130
392	A	4	3	1.00	23	0.130
393	A	5	4	1.04	25	0.160
394	A	12	11	1.01	20	0.550
395	A	10	9	1.00	20	0.450
396	A	9	8	0.90	20	0.400
397	A	4	3	1.26	18	0.167
398	A	9	8	1.08	20	0.400
399	A	12	11	0.87	20	0.550
400	A	14	13	0.93	20	0.650
401	A	16	15	0.95	20	0.750
402	A	6	6	0.79	22	0.273
403	A	6	6	0.85	22	0.273
404	A	6	6	0.82	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
405	A	7	7	1.25	22	0.318
406	A	6	6	1.00	20	0.300
407	A	6	6	0.97	22	0.273
408	A	6	6	0.98	22	0.273
409	A	6	6	0.93	22	0.273
410	A	6	6	0.93	22	0.273
411	A	12	11	0.99	22	0.500
412	A	5	4	1.03	22	0.182
413	A	10	9	1.06	22	0.409
414	A	11	10	1.12	20	0.500
415	A	12	11	0.94	22	0.500
416	A	14	13	0.96	22	0.591
417	A	16	15	0.87	22	0.682
418	A	18	17	0.99	22	0.773
419	A	6	6	0.80	22	0.273
420	A	7	7	0.93	22	0.318
421	A	6	6	0.92	22	0.273
422	A	6	6	0.85	22	0.273
423	A	6	6	0.88	20	0.300
424	A	6	6	0.96	22	0.273
425	A	6	6	0.93	22	0.273
426	A	6	6	0.92	22	0.273
427	A	6	6	0.90	22	0.273
428	A	15	14	1.09	22	0.636
429	A	14	13	1.01	22	0.591
430	A	11	10	1.04	22	0.455
431	A	9	8	1.12	20	0.400
432	A	3	2	1.00	22	0.091
433	A	10	9	1.06	22	0.409
434	A	12	11	0.91	22	0.500
435	A	14	13	0.90	22	0.591
436	A	6	6	0.83	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
437	A	6	6	0.81	22	0.273
438	A	6	6	0.85	22	0.273
439	A	6	6	0.83	20	0.300
440	A	6	6	1.10	22	0.273
441	A	7	7	1.28	22	0.318
442	A	6	6	0.96	22	0.273
443	A	6	6	0.96	22	0.273
444	A	16	15	1.09	22	0.682
445	A	15	14	1.02	22	0.636
446	A	12	11	1.06	22	0.500
447	A	10	9	1.16	20	0.450
448	A	9	8	0.97	22	0.364
449	A	9	8	0.96	22	0.364
450	A	4	3	1.10	22	0.136
451	A	11	10	0.95	22	0.455
452	A	14	13	0.92	22	0.591
453	A	13	12	0.78	22	0.545
454	A	11	10	0.77	22	0.455
455	A	10	9	0.73	22	0.409
456	A	6	5	1.04	22	0.227
457	A	5	4	1.05	22	0.182
458	A	9	8	0.77	22	0.364
459	A	11	10	0.81	22	0.455
460	A	13	12	0.82	22	0.545
461	A	13	12	1.02	24	0.500
462	A	12	11	1.05	24	0.458
463	A	11	10	1.08	24	0.417
464	A	10	9	1.14	24	0.375
465	A	9	8	1.16	24	0.333
466	A	10	9	1.19	24	0.375
467	A	11	10	1.16	24	0.417
468	A	12	11	1.16	24	0.458

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
469	A	13	12	1.13	24	0.500
470	A	12	11	0.71	24	0.458
471	A	11	10	0.68	24	0.417
472	A	7	6	1.04	24	0.250
473	A	6	5	1.04	24	0.208
474	A	9	8	0.77	24	0.333
475	A	12	11	0.79	24	0.458
476	A	14	13	0.83	24	0.542
477	A	16	15	0.84	24	0.625
478	A	11	10	0.76	24	0.417
479	A	9	8	0.72	24	0.333
480	A	9	8	0.80	24	0.333
481	A	5	4	1.10	24	0.167
482	A	5	4	1.08	24	0.167
483	A	10	9	0.82	24	0.375
484	A	11	10	0.78	24	0.417
485	A	13	12	0.81	24	0.500
486	A	17	16	1.06	24	0.667
487	A	15	14	1.09	24	0.583
488	A	13	12	1.13	24	0.500
489	A	11	10	1.17	24	0.417
490	A	11	10	1.11	24	0.417
491	A	11	10	1.23	24	0.417
492	A	14	13	1.14	24	0.542
493	A	15	14	1.10	24	0.583
494	A	17	16	1.12	24	0.667
495	A	13	12	0.76	24	0.500
496	A	11	10	0.75	24	0.417
497	A	9	8	0.72	24	0.333
498	A	9	8	0.80	24	0.333
499	A	6	5	1.06	24	0.208
500	A	6	5	1.03	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
501	A	10	9	0.70	24	0.375
502	A	13	12	0.74	24	0.500
503	A	7	6	1.06	25	0.240
504	A	6	5	1.04	23	0.217
505	A	5	4	1.05	22	0.182
506	A	5	4	1.03	25	0.160
507	A	3	2	1.00	25	0.080
508	A	4	3	1.00	25	0.120
509	A	6	5	1.08	25	0.200
510	A	8	7	1.15	25	0.280
511	A	12	11	1.21	27	0.407
512	A	11	10	1.18	27	0.370
513	A	10	9	1.14	25	0.360
514	A	9	8	1.16	24	0.333
515	A	9	8	1.23	27	0.296
516	A	8	7	1.31	27	0.259
517	A	8	7	1.13	27	0.259
518	A	8	7	1.09	27	0.259
519	A	8	7	1.07	27	0.259
520	A	15	14	0.72	27	0.519
521	A	13	12	0.73	27	0.444
522	A	11	10	0.73	25	0.400
523	A	9	8	0.77	24	0.333
524	A	10	9	0.78	27	0.333
525	A	6	5	1.10	27	0.185
526	A	7	6	1.04	27	0.222
527	A	9	8	1.06	27	0.296
528	A	11	10	1.12	27	0.370
529	A	7	6	1.08	27	0.222
530	A	6	5	1.06	25	0.200
531	A	5	4	1.10	24	0.167
532	A	5	4	1.09	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
533	A	4	3	1.11	27	0.111
534	A	5	4	1.06	27	0.148
535	A	7	6	1.11	27	0.222
536	A	17	16	1.20	27	0.593
537	A	15	14	1.18	27	0.519
538	A	13	12	1.16	25	0.480
539	A	11	10	1.17	24	0.417
540	A	11	10	1.21	27	0.370
541	A	10	9	1.05	27	0.333
542	A	11	10	1.00	27	0.370
543	A	8	7	1.04	27	0.259
544	A	8	7	1.02	27	0.259
545	A	12	11	0.70	27	0.407
546	A	11	10	0.71	27	0.370
547	A	10	9	0.72	25	0.360
548	A	9	8	0.80	24	0.333
549	A	6	5	0.83	27	0.185
550	A	5	4	1.06	27	0.148
551	A	6	5	1.05	27	0.185
552	A	8	7	1.09	27	0.259
553	A	10	9	1.11	27	0.333
554	F	0	0	N/A	0.000	N/A
555	F	0	0	N/A	0.000	N/A
556	A	6	5	1.01	20	0.250
557	A	4	3	1.00	22	0.136
558	A	9	8	0.98	22	0.364
559	A	4	3	1.07	24	0.125
560	A	4	3	1.07	24	0.125
561	A	4	3	1.07	24	0.125
562	A	4	3	1.08	24	0.125
563	A	4	3	1.09	22	0.136
564	A	4	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
565	A	4	3	1.16	23	0.130
566	A	8	7	1.21	22	0.318
567	A	4	3	1.08	20	0.150
568	A	4	3	1.07	22	0.136
569	A	11	10	1.09	22	0.455
570	A	5	5	0.88	22	0.227
571	A	5	5	0.90	22	0.227
572	A	5	5	0.92	22	0.227
573	A	5	5	0.97	22	0.227
574	A	4	4	1.00	20	0.200
575	A	4	4	1.00	22	0.182
576	A	5	5	0.92	22	0.227
577	A	5	5	0.88	22	0.227
578	A	5	5	0.87	22	0.227
579	A	5	5	0.88	22	0.227
580	A	5	5	0.90	22	0.227
581	A	5	5	0.94	22	0.227
582	A	4	4	1.00	22	0.182
583	A	5	5	0.96	20	0.250
584	A	4	4	1.00	22	0.182
585	A	4	4	1.00	22	0.182
586	A	5	5	0.87	22	0.227
587	A	5	5	0.86	22	0.227
588	A	5	5	0.90	22	0.227
589	A	5	5	0.93	22	0.227
590	A	5	5	0.97	22	0.227
591	A	4	4	1.00	20	0.200
592	A	4	4	1.00	22	0.182
593	A	5	5	0.92	22	0.227
594	A	5	5	0.88	22	0.227
595	A	5	5	0.87	22	0.227
596	A	4	4	0.46	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
597	A	5	5	0.50	22	0.227
598	A	4	4	0.56	22	0.182
599	A	5	5	0.71	22	0.227
600	A	3	3	1.00	22	0.136
601	A	3	3	1.00	22	0.136
602	A	4	4	0.69	22	0.182
603	A	5	5	0.51	22	0.227
604	A	4	4	0.44	22	0.182
605	A	11	10	1.17	24	0.417
606	A	10	9	1.18	24	0.375
607	A	9	8	1.19	24	0.333
608	A	8	7	1.20	24	0.292
609	A	7	6	1.23	24	0.250
610	A	6	5	1.12	24	0.208
611	A	4	4	1.12	24	0.167
612	A	5	5	1.19	24	0.208
613	A	6	6	1.23	24	0.250
614	A	7	7	1.25	24	0.292
615	A	5	5	0.51	24	0.208
616	A	4	4	0.57	24	0.167
617	A	5	5	0.71	24	0.208
618	A	3	3	1.00	24	0.125
619	A	5	5	0.54	24	0.208
620	A	4	4	0.72	24	0.167
621	A	3	3	1.00	24	0.125
622	A	4	4	0.50	24	0.167
623	A	5	5	0.44	24	0.208
624	A	5	5	0.47	24	0.208
625	A	4	4	0.51	24	0.167
626	A	5	5	0.58	24	0.208
627	A	4	4	0.71	24	0.167
628	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
629	A	3	3	1.00	24	0.125
630	A	5	5	0.70	24	0.208
631	A	4	4	0.50	24	0.167
632	A	5	5	0.44	24	0.208
633	A	9	8	1.20	24	0.333
634	A	8	7	1.21	24	0.292
635	A	7	6	1.23	24	0.250
636	A	6	5	1.05	24	0.208
637	A	4	4	0.89	24	0.167
638	A	5	5	1.01	24	0.208
639	A	6	6	1.08	24	0.250
640	A	7	7	1.13	24	0.292
641	A	4	4	0.51	24	0.167
642	A	5	5	0.58	24	0.208
643	A	4	4	0.71	24	0.167
644	A	3	3	1.00	24	0.125
645	A	4	4	0.53	24	0.167
646	A	5	5	0.74	24	0.208
647	A	3	3	1.00	24	0.125
648	A	5	5	0.50	24	0.208
649	A	4	4	0.43	24	0.167
650	A	4	4	0.68	25	0.160
651	A	4	4	0.70	23	0.174
652	A	3	3	1.00	22	0.136
653	A	4	4	0.61	25	0.160
654	A	4	4	0.63	25	0.160
655	A	15	14	1.25	27	0.519
656	A	13	12	1.25	27	0.444
657	A	11	10	1.21	25	0.400
658	A	7	6	1.23	24	0.250
659	A	12	11	1.12	27	0.407
660	A	12	11	1.09	27	0.407

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
661	A	10	9	1.13	27	0.333
662	A	12	11	1.15	27	0.407
663	A	16	15	1.15	27	0.556
664	A	5	5	0.39	27	0.185
665	A	5	5	0.41	27	0.185
666	A	5	5	0.47	25	0.200
667	A	5	5	0.54	24	0.208
668	A	5	5	0.46	27	0.185
669	A	5	5	0.51	27	0.185
670	A	5	5	0.46	27	0.185
671	A	5	5	0.41	27	0.185
672	A	5	5	0.38	27	0.185
673	A	4	4	0.45	25	0.160
674	A	4	4	0.49	25	0.160
675	A	4	4	0.61	25	0.160
676	A	4	4	0.72	23	0.174
677	A	4	4	0.69	22	0.182
678	A	4	4	0.41	25	0.160
679	A	4	4	0.38	25	0.160
680	A	4	4	0.39	25	0.160
681	A	5	5	0.43	25	0.200
682	A	5	5	0.50	25	0.200
683	A	5	5	0.53	25	0.200
684	A	5	5	0.51	25	0.200
685	A	5	5	0.57	23	0.217
686	A	5	5	0.51	22	0.227
687	A	5	5	0.35	25	0.200
688	A	5	5	0.35	25	0.200
689	A	5	5	0.70	27	0.185
690	A	5	5	0.72	25	0.200
691	A	3	3	1.00	24	0.125
692	A	5	5	0.63	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
693	A	5	5	0.67	27	0.185
694	A	16	15	1.38	27	0.556
695	A	15	14	1.25	27	0.519
696	A	11	10	1.22	25	0.400
697	A	7	6	1.23	24	0.250
698	A	12	11	1.12	27	0.407
699	A	11	10	1.11	27	0.370
700	A	9	8	1.13	27	0.296
701	A	11	10	1.14	27	0.370
702	A	13	12	1.15	27	0.444
703	A	4	4	0.38	27	0.148
704	A	4	4	0.40	27	0.148
705	A	4	4	0.46	25	0.160
706	A	4	4	0.53	24	0.167
707	A	4	4	0.45	27	0.148
708	A	4	4	0.49	27	0.148
709	A	4	4	0.45	27	0.148
710	A	4	4	0.40	27	0.148
711	A	4	4	0.37	27	0.148
712	A	5	5	0.63	27	0.185
713	A	8	8	1.01	27	0.296
714	A	4	4	0.73	25	0.160
715	A	5	5	0.75	27	0.185
716	A	8	8	1.01	27	0.296
717	A	4	4	0.62	27	0.148
718	A	4	3	1.00	24	0.125
719	A	4	3	1.00	24	0.125
720	A	4	3	1.00	24	0.125
721	A	1	1	1.00	24	0.042
722	A	2	2	1.00	24	0.083
723	A	3	3	0.98	24	0.125
724	A	4	4	0.94	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
725	A	12	11	0.77	27	0.407
726	A	5	4	1.00	27	0.148
727	A	1	1	1.00	25	0.040
728	A	1	1	1.00	24	0.042
729	A	7	6	0.81	27	0.222
730	A	14	13	0.79	27	0.481
731	A	7	6	0.72	27	0.222
732	A	2	2	1.06	27	0.074
733	A	2	2	1.06	25	0.080
734	A	2	2	1.00	24	0.083
735	A	5	4	0.66	27	0.148
736	A	4	3	1.00	22	0.136
737	A	4	3	1.00	23	0.130
738	A	4	3	1.00	23	0.130
739	A	4	4	1.00	22	0.182
740	A	4	4	1.00	22	0.182
741	A	2	2	1.00	13	0.154
742	A	4	4	0.85	22	0.182
743	A	4	4	0.97	22	0.182
744	A	4	3	1.39	22	0.136
745	B	4	3	2.07	20	0.150
746	A	4	3	1.39	22	0.136
747	A	4	3	1.39	22	0.136
748	A	24	23	1.98	20	1.150
749	A	20	19	1.91	20	0.950
750	A	16	15	1.81	20	0.750
751	A	13	12	1.98	18	0.667
752	A	9	8	1.46	20	0.400
753	A	15	14	1.63	20	0.700
754	A	19	18	1.69	20	0.900
755	A	23	22	1.81	20	1.100
756	A	6	6	0.78	22	0.273

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
757	A	6	6	0.81	22	0.273
758	A	6	6	0.79	22	0.273
759	A	6	6	0.90	22	0.273
760	A	6	6	1.00	20	0.300
761	A	6	6	1.03	22	0.273
762	A	6	6	0.96	22	0.273
763	A	6	6	0.90	22	0.273
764	A	6	6	0.90	22	0.273
765	A	25	24	1.87	22	1.091
766	A	21	20	1.79	22	0.909
767	A	17	16	1.67	22	0.727
768	A	9	8	1.16	20	0.400
769	A	11	10	1.33	22	0.455
770	A	13	12	1.25	22	0.545
771	A	20	19	1.43	22	0.864
772	A	24	23	1.53	22	1.045
773	A	6	6	0.80	22	0.273
774	A	6	6	0.78	22	0.273
775	A	6	6	0.84	22	0.273
776	A	6	6	0.80	22	0.273
777	A	6	6	1.00	20	0.300
778	A	6	6	0.98	22	0.273
779	A	6	6	0.93	22	0.273
780	A	6	6	0.92	22	0.273
781	A	6	6	0.88	22	0.273
782	A	26	25	1.94	22	1.136
783	A	21	20	1.90	22	0.909
784	A	17	16	1.79	22	0.727
785	A	11	10	2.00	20	0.500
786	A	8	7	1.44	22	0.318
787	A	13	12	1.60	22	0.545
788	A	17	16	1.72	22	0.727

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
789	A	22	21	1.80	22	0.955
790	A	6	6	0.81	22	0.273
791	A	6	6	0.79	22	0.273
792	A	6	6	0.85	22	0.273
793	A	6	6	1.00	20	0.300
794	A	6	6	1.03	22	0.273
795	A	6	6	0.96	22	0.273
796	A	6	6	0.90	22	0.273
797	A	6	6	0.90	22	0.273
798	A	26	25	1.81	22	1.136
799	A	20	19	1.80	22	0.864
800	A	16	15	1.66	22	0.682
801	A	9	8	1.17	20	0.400
802	A	11	10	1.32	22	0.455
803	A	13	12	1.24	22	0.545
804	A	17	16	1.42	22	0.727
805	A	21	20	1.52	22	0.909
806	A	5	5	0.31	22	0.227
807	A	4	4	0.33	22	0.182
808	A	5	5	0.42	22	0.227
809	A	4	4	0.58	22	0.182
810	A	5	5	0.71	22	0.227
811	A	4	4	0.49	22	0.182
812	A	5	5	0.43	22	0.227
813	A	4	4	0.40	22	0.182
814	A	16	15	0.96	24	0.625
815	A	14	13	0.94	24	0.542
816	A	12	11	0.92	24	0.458
817	A	11	10	0.82	24	0.417
818	A	5	5	1.19	24	0.208
819	A	7	7	1.07	24	0.292
820	A	8	8	1.04	24	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
821	A	9	9	1.02	24	0.375
822	A	5	5	0.32	24	0.208
823	A	4	4	0.31	24	0.167
824	A	5	5	0.34	24	0.208
825	A	4	4	0.41	24	0.167
826	A	5	5	0.46	24	0.208
827	A	4	4	0.54	24	0.167
828	A	5	5	0.49	24	0.208
829	A	4	4	0.43	24	0.167
830	A	5	5	0.40	24	0.208
831	A	4	4	0.31	24	0.167
832	A	5	5	0.34	24	0.208
833	A	4	4	0.40	24	0.167
834	A	5	5	0.60	24	0.208
835	A	4	4	0.67	24	0.167
836	A	5	5	0.49	24	0.208
837	A	4	4	0.42	24	0.167
838	A	5	5	0.40	24	0.208
839	A	16	15	0.96	24	0.625
840	A	14	13	0.94	24	0.542
841	A	12	11	0.92	24	0.458
842	A	12	11	0.82	24	0.458
843	A	5	5	1.16	24	0.208
844	A	9	9	1.06	24	0.375
845	A	10	10	1.03	24	0.417
846	A	11	11	1.01	24	0.458
847	A	4	4	0.31	24	0.167
848	A	5	5	0.32	24	0.208
849	A	4	4	0.33	24	0.167
850	A	5	5	0.42	24	0.208
851	A	4	4	0.44	24	0.167
852	A	5	5	0.55	24	0.208

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
853	A	4	4	0.47	24	0.167
854	A	5	5	0.43	24	0.208
855	A	4	4	0.39	24	0.167
856	A	4	4	0.64	25	0.160
857	A	3	3	1.00	23	0.130
858	A	4	4	0.58	22	0.182
859	A	4	4	0.61	25	0.160
860	A	3	3	1.00	25	0.120
861	A	13	13	1.14	27	0.481
862	A	11	11	1.15	27	0.407
863	A	9	9	1.13	27	0.333
864	A	5	5	1.17	25	0.200
865	A	11	10	0.82	24	0.417
866	A	11	10	0.96	27	0.370
867	A	10	9	1.23	27	0.333
868	A	12	11	1.16	27	0.407
869	A	16	15	1.14	27	0.556
870	A	17	16	1.09	27	0.593
871	A	5	5	0.40	27	0.185
872	A	5	5	0.45	27	0.185
873	A	5	5	0.51	25	0.200
874	A	5	5	0.46	24	0.208
875	A	5	5	0.51	27	0.185
876	A	5	5	0.46	27	0.185
877	A	5	5	0.41	27	0.185
878	A	5	5	0.37	27	0.185
879	A	5	5	0.36	27	0.185
880	A	5	5	0.66	27	0.185
881	A	3	3	1.00	25	0.120
882	A	5	5	0.60	24	0.208
883	A	5	5	0.65	27	0.185
884	A	4	4	1.00	27	0.148

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
885	A	15	15	1.14	27	0.556
886	A	14	14	1.15	27	0.519
887	A	6	6	1.15	27	0.222
888	A	5	5	1.18	25	0.200
889	A	12	11	0.82	24	0.458
890	A	11	10	1.00	27	0.370
891	A	10	9	1.23	27	0.333
892	A	12	11	1.17	27	0.407
893	A	14	13	1.14	27	0.481
894	A	16	15	1.18	27	0.556
895	A	4	4	0.39	27	0.148
896	A	4	4	0.44	27	0.148
897	A	4	4	0.50	25	0.160
898	A	4	4	0.44	24	0.167
899	A	4	4	0.49	27	0.148
900	A	4	4	0.45	27	0.148
901	A	4	4	0.40	27	0.148
902	A	4	4	0.36	27	0.148
903	A	4	4	0.35	27	0.148
904	A	6	6	0.60	27	0.222
905	A	5	5	0.74	27	0.185
906	A	5	5	1.00	27	0.185
907	A	6	6	0.80	27	0.222
908	A	5	5	0.58	29	0.172
909	A	6	6	0.75	29	0.207
910	A	4	4	1.00	29	0.138
911	A	7	7	0.81	29	0.241
912	A	10	9	1.68	20	0.450
913	A	9	8	0.98	22	0.364
914	A	14	13	1.01	22	0.591
915	A	7	6	0.81	24	0.250
916	A	5	4	0.86	24	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
917	A	4	3	0.96	22	0.136
918	A	4	3	1.00	23	0.130
919	A	4	3	1.00	23	0.130

CHAPTER 3

Listing of integrals

3.1 $\int e^{c+4 \coth^{-1}(a+bx)} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 1

$$\int e^{c+4 \coth^{-1}(a+bx)} \, dx = 0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 34.00

$$\int e^{c+4 \coth^{-1}(a+bx)} \, dx = \frac{e^c \left(a+bx-\frac{4}{-1+a+bx}+4 \log(1-a-bx)\right)}{b}$$

input
```
Integrate[E^(c + 4*ArcCoth[a + b*x]),x]
```

output
```
(E^c*(a + b*x - 4/(-1 + a + b*x) + 4*Log[1 - a - b*x]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{4\coth^{-1}(a+bx)+c}\,dx$$

$$\Big\downarrow \; 7281$$

$$\frac{\int e^{c+4\coth^{-1}(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \; 7299$$

$$\frac{\int e^{c+4\coth^{-1}(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c + 4*ArcCoth[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 35.00

method	result	size
risch	$e^{c} x -\frac{4\,e^{c}}{b \left(b x +a -1\right)}+\frac{4\,e^{c} \ln \left(b x +a -1\right)}{b}$	35

input
```
int(exp(c+4*arccoth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
exp(c)*x-4*exp(c)/b/(b*x+a-1)+4*exp(c)/b*ln(b*x+a-1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 50.00

$$\int e^{c+4\coth^{-1}(a+bx)}\,dx = \frac{4\,(bx+a-1)e^{c}\log\left(bx+a-1\right)+(b^{2}x^{2}+(a-1)bx-4)e^{c}}{b^{2}x+(a-1)b}$$

input
```
integrate(exp(c+4*arccoth(b*x+a)),x, algorithm="fricas")
```

output
```
(4*(b*x + a - 1)*e^c*log(b*x + a - 1) + (b^2*x^2 + (a - 1)*b*x - 4)*e^c)/(
b^2*x + (a - 1)*b)
```

Sympy [F]

$$\int e^{c+4\coth^{-1}(a+bx)}\,dx = e^{c}\int e^{4\,\operatorname{acoth}(a+bx)}\,dx$$

input
```
integrate(exp(c+4*acoth(b*x+a)),x)
```

output
```
exp(c)*Integral(exp(4*acoth(a + b*x)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 57.00

$$\int e^{c+4\coth^{-1}(a+bx)}\,dx=\frac{4\,e^{c}\log\left(bx+a-1\right)}{b}+\frac{b^{2}x^{2}e^{c}+\left(abe^{c}-be^{c}\right)x-4\,e^{c}}{b^{2}x+ab-b}$$

input
```
integrate(exp(c+4*arccoth(b*x+a)),x, algorithm="maxima")
```

output
```
4*e^c*log(b*x + a - 1)/b + (b^2*x^2*e^c + (a*b*e^c - b*e^c)*x - 4*e^c)/(b^
2*x + a*b - b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 53.00

$$\int e^{c+4\coth^{-1}(a+bx)}\,dx=\left(\frac{bx+a-1}{b}-\frac{4\,\log\left(\frac{|bx+a-1|}{(bx+a-1)^{2}|b|}\right)}{b}-\frac{4}{(bx+a-1)b}\right)e^{c}$$

input
```
integrate(exp(c+4*arccoth(b*x+a)),x, algorithm="giac")
```

output
```
((b*x + a - 1)/b - 4*log(abs(b*x + a - 1)/((b*x + a - 1)^2*abs(b)))/b - 4/
((b*x + a - 1)*b))*e^c
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 34.00

$$\int e^{c+4\coth^{-1}(a+bx)}\,dx = x\,e^c - \frac{4\,e^c}{b\,(a+b\,x-1)} + \frac{4\,e^c\,\ln(a+b\,x-1)}{b}$$

input
```
int(exp(c + 4*acoth(a + b*x)),x)
```

output
```
x*exp(c) - (4*exp(c))/(b*(a + b*x - 1)) + (4*exp(c)*log(a + b*x - 1))/b
```

Reduce [F]

$$\int e^{c+4\coth^{-1}(a+bx)}\,dx = e^c\left(\int e^{4acoth(bx+a)}dx\right)$$

input
```
int(exp(c+4*acoth(b*x+a)),x)
```

output
```
e**c*int(e**(4*acoth(a + b*x)),x)
```

3.2 $\int e^{c+2 \coth^{-1}(a+b x)} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 1

$$\int e^{c+2 \coth^{-1}(a+b x)} \, dx = 0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 24.00

$$\int e^{c+2 \coth^{-1}(a+b x)} \, dx = \frac{e^c(a+bx+2\log(1-a-bx))}{b}$$

input
```
Integrate[E^(c + 2*ArcCoth[a + b*x]),x]
```

output
```
(E^c*(a + b*x + 2*Log[1 - a - b*x]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\coth^{-1}(a+bx)+c}\,dx$$

$$\Big\downarrow \ 7281$$

$$\frac{\int e^{c+2\coth^{-1}(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \ 7299$$

$$\frac{\int e^{c+2\coth^{-1}(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c + 2*ArcCoth[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 20.00

method	result	size
risch	$\mathrm{e}^{c} x+\frac{2\,\mathrm{e}^{c} \ln\left(b x+a-1\right)}{b}$	20

input
```
int(exp(c+2*arccoth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
exp(c)*x+2*exp(c)/b*ln(b*x+a-1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 21.00

$$\int e^{c+2\coth^{-1}(a+bx)}\,dx=\frac{bxe^c+2\,e^c\log\left(bx+a-1\right)}{b}$$

input
```
integrate(exp(c+2*arccoth(b*x+a)),x, algorithm="fricas")
```

output
```
(b*x*e^c + 2*e^c*log(b*x + a - 1))/b
```

Sympy [F]

$$\int e^{c+2\coth^{-1}(a+bx)}\,dx=e^c\int e^{2\,\mathrm{acoth}\,(a+bx)}\,dx$$

input
```
integrate(exp(c+2*acoth(b*x+a)),x)
```

output
```
exp(c)*Integral(exp(2*acoth(a + b*x)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 19.00

$$\int e^{c+2\coth^{-1}(a+bx)}\,dx = xe^c + \frac{2\,e^c\log(bx+a-1)}{b}$$

input
```
integrate(exp(c+2*arccoth(b*x+a)),x, algorithm="maxima")
```

output
```
x*e^c + 2*e^c*log(b*x + a - 1)/b
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 20.00

$$\int e^{c+2\coth^{-1}(a+bx)}\,dx = xe^c + \frac{2\,e^c\log(|bx+a-1|)}{b}$$

input
```
integrate(exp(c+2*arccoth(b*x+a)),x, algorithm="giac")
```

output
```
x*e^c + 2*e^c*log(abs(b*x + a - 1))/b
```

Mupad [B] (verification not implemented)

Time = 23.87 (sec) , antiderivative size = 19, normalized size of antiderivative = 19.00

$$\int e^{c+2\coth^{-1}(a+bx)}\,dx = \frac{e^c\,(2\,\ln(a+b\,x-1)+b\,x)}{b}$$

input
```
int(exp(c + 2*acoth(a + b*x)),x)
```

output
```
(exp(c)*(2*log(a + b*x - 1) + b*x))/b
```

Reduce [F]

$$\int e^{c+2\coth^{-1}(a+bx)}\,dx = e^c\left(\int e^{2acoth(bx+a)}dx\right)$$

input
```
int(exp(c+2*acoth(b*x+a)),x)
```

output
```
e**c*int(e**(2*acoth(a + b*x)),x)
```

3.3 $\int e^{c-2\coth^{-1}(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 1

$$\int e^{c-2\coth^{-1}(a+bx)}\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 21.00

$$\int e^{c-2\coth^{-1}(a+bx)}\,dx=\frac{e^c(a+bx-2\log(1+a+bx))}{b}$$

input
```
Integrate[E^(c - 2*ArcCoth[a + b*x]),x]
```

output
```
(E^c*(a + b*x - 2*Log[1 + a + b*x]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-2\coth^{-1}(a+bx)}\,dx$$

$$\Big\downarrow \; 7281$$

$$\frac{\int e^{c-2\coth^{-1}(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \; 7299$$

$$\frac{\int e^{c-2\coth^{-1}(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c - 2*ArcCoth[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /; !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 20.00

method	result	size
risch	$\mathrm{e}^{c} x -\frac{2\,\mathrm{e}^{c} \ln \left(b x +a +1\right)}{b}$	20

input
```
int(exp(c-2*arccoth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
exp(c)*x-2*exp(c)*ln(b*x+a+1)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 21.00

$$\int e^{c-2\coth^{-1}(a+bx)}\,dx = \frac{bxe^c - 2\,e^c\log\left(bx+a+1\right)}{b}$$

input
```
integrate(exp(c-2*arccoth(b*x+a)),x, algorithm="fricas")
```

output
```
(b*x*e^c - 2*e^c*log(b*x + a + 1))/b
```

Sympy [F]

$$\int e^{c-2\coth^{-1}(a+bx)}\,dx = e^c\int e^{-2\operatorname{acoth}(a+bx)}\,dx$$

input
```
integrate(exp(c-2*acoth(b*x+a)),x)
```

output
```
exp(c)*Integral(exp(-2*acoth(a + b*x)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 19.00

$$\int e^{c-2\coth^{-1}(a+bx)}\,dx = xe^c - \frac{2\,e^c\log\left(bx+a+1\right)}{b}$$

input
```
integrate(exp(c-2*arccoth(b*x+a)),x, algorithm="maxima")
```

output
```
x*e^c - 2*e^c*log(b*x + a + 1)/b
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 20.00

$$\int e^{c-2\coth^{-1}(a+bx)}\,dx = xe^c - \frac{2\,e^c\log\left(|bx+a+1|\right)}{b}$$

input
```
integrate(exp(c-2*arccoth(b*x+a)),x, algorithm="giac")
```

output
```
x*e^c - 2*e^c*log(abs(b*x + a + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 21, normalized size of antiderivative = 21.00

$$\int e^{c-2\coth^{-1}(a+bx)}\,dx = -\frac{\mathrm{e}^c\left(2\,\ln\left(a+b\,x+1\right)-b\,x\right)}{b}$$

input
```
int(exp(c - 2*acoth(a + b*x)),x)
```

output
```
-(exp(c)*(2*log(a + b*x + 1) - b*x))/b
```

Reduce [F]

$$\int e^{c-2\coth^{-1}(a+bx)}\,dx = e^c\left(\int \frac{1}{e^{2acoth(bx+a)}}dx\right)$$

input
```
int(exp(c-2*acoth(b*x+a)),x)
```

output
```
e**c*int(1/e**(2*acoth(a + b*x)),x)
```

3.4 $\int e^{c-4\coth^{-1}(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 1

$$\int e^{c-4\coth^{-1}(a+bx)}\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 31.00

$$\int e^{c-4\coth^{-1}(a+bx)}\,dx=\frac{e^c\left(a+bx-\frac{4}{1+a+bx}-4\log(1+a+bx)\right)}{b}$$

input
```
Integrate[E^(c - 4*ArcCoth[a + b*x]),x]
```

output
```
(E^c*(a + b*x - 4/(1 + a + b*x) - 4*Log[1 + a + b*x]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-4\coth^{-1}(a+bx)}\,dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int e^{c-4\coth^{-1}(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\int e^{c-4\coth^{-1}(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c - 4*ArcCoth[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 35.00

method	result	size
risch	$\mathrm{e}^{c} x -\frac{4\,\mathrm{e}^{c}}{b \left(b x +a +1\right)}-\frac{4\,\mathrm{e}^{c} \ln \left(b x +a +1\right)}{b}$	35

input
```
int(exp(c-4*arccoth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
exp(c)*x-4*exp(c)/b/(b*x+a+1)-4*exp(c)*ln(b*x+a+1)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 52.00

$$\int e^{c-4\coth^{-1}(a+bx)}\,dx = -\frac{4\,(bx+a+1)e^c\log{(bx+a+1)}-(b^2x^2+(a+1)bx-4)e^c}{b^2x+(a+1)b}$$

input
```
integrate(exp(c-4*arccoth(b*x+a)),x, algorithm="fricas")
```

output
```
-(4*(b*x + a + 1)*e^c*log(b*x + a + 1) - (b^2*x^2 + (a + 1)*b*x - 4)*e^c)/
(b^2*x + (a + 1)*b)
```

Sympy [F]

$$\int e^{c-4\coth^{-1}(a+bx)}\,dx = e^c\int e^{-4\,\mathrm{acoth}\,(a+bx)}\,dx$$

input
```
integrate(exp(c-4*acoth(b*x+a)),x)
```

output
```
exp(c)*Integral(exp(-4*acoth(a + b*x)), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 54.00

$$\int e^{c-4\coth^{-1}(a+bx)}\,dx=-\frac{4\,e^{c}\log\left(bx+a+1\right)}{b}+\frac{b^{2}x^{2}e^{c}+(abe^{c}+be^{c})x-4\,e^{c}}{b^{2}x+ab+b}$$

input
```
integrate(exp(c-4*arccoth(b*x+a)),x, algorithm="maxima")
```

output
```
-4*e^c*log(b*x + a + 1)/b + (b^2*x^2*e^c + (a*b*e^c + b*e^c)*x - 4*e^c)/(b
^2*x + a*b + b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 53.00

$$\int e^{c-4\coth^{-1}(a+bx)}\,dx=\left(\frac{bx+a+1}{b}+\frac{4\log\left(\frac{|bx+a+1|}{(bx+a+1)^{2}|b|}\right)}{b}-\frac{4}{(bx+a+1)b}\right)e^{c}$$

input
```
integrate(exp(c-4*arccoth(b*x+a)),x, algorithm="giac")
```

output
```
((b*x + a + 1)/b + 4*log(abs(b*x + a + 1)/((b*x + a + 1)^2*abs(b)))/b - 4/
((b*x + a + 1)*b))*e^c
```

Mupad [B] (verification not implemented)

Time = 23.88 (sec) , antiderivative size = 34, normalized size of antiderivative = 34.00

$$\int e^{c-4\coth^{-1}(a+bx)}\,dx = x\,e^c - \frac{4\,e^c}{b\,(a+b\,x+1)} - \frac{4\,e^c\,\ln(a+b\,x+1)}{b}$$

input
```
int(exp(c - 4*acoth(a + b*x)),x)
```

output
```
x*exp(c) - (4*exp(c))/(b*(a + b*x + 1)) - (4*exp(c)*log(a + b*x + 1))/b
```

Reduce [F]

$$\int e^{c-4\coth^{-1}(a+bx)}\,dx = e^c\left(\int \frac{1}{e^{4acoth(bx+a)}}dx\right)$$

input
```
int(exp(c-4*acoth(b*x+a)),x)
```

output
```
e**c*int(1/e**(4*acoth(a + b*x)),x)
```

3.5 $\int e^{c+3\coth^{-1}(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 1

$$\int e^{c+3\coth^{-1}(a+bx)}\,dx=0$$

output
```
0
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 78.00

$$\int e^{c+3\coth^{-1}(a+bx)}\,dx = \frac{e^c\left(\frac{\sqrt{1-\frac{1}{(a+bx)^2}}\,(a^2+bx(-5+bx)+a(-5+2bx))}{-1+a+bx}+3\log\left((a+bx)\left(1+\sqrt{1-\frac{1}{(a+bx)^2}}\right)\right)\right)}{b}$$

input
```
Integrate[E^(c + 3*ArcCoth[a + b*x]),x]
```

output
```
(E^c*((Sqrt[1 - (a + b*x)^(-2)]*(a^2 + b*x*(-5 + b*x) + a*(-5 + 2*b*x)))/(
-1 + a + b*x) + 3*Log[(a + b*x)*(1 + Sqrt[1 - (a + b*x)^(-2)])]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3\coth^{-1}(a+bx)+c}\,dx$$

$$\Big\downarrow \; 7281$$

$$\frac{\int e^{c+3\coth^{-1}(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \; 7299$$

$$\frac{\int e^{c+3\coth^{-1}(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c + 3*ArcCoth[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.19 (sec) , antiderivative size = 168, normalized size of antiderivative = 168.00

method	result
risch	$\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,e^{c}}{b}+\frac{\left(\frac{3\ln\left(\frac{\frac{b(a+1)}{2}+\frac{(-1+a)b}{2}+b^{2}x}{\sqrt{b^{2}}}+\sqrt{b^{2}x^{2}+(b(a+1)+(-1+a)b)x+(a+1)(-1+a)}\right)}{\sqrt{b^{2}}}-\frac{4\sqrt{\left(x+\frac{-1+a}{b}\right)^{2}b^{2}+2\left(x+\frac{-1}{b}\right.}}{b^{2}\left(x+\frac{-1+a}{b}\right)}\right.}{\sqrt{bx+a-1}\,\sqrt{bx+a+1}}$

input
```
int(exp(c+3*arccoth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*exp(c)+(3*ln((1/2*b*(a+1)+1/2*(-1+a)*b
+b^2*x)/(b^2)^(1/2)+(b^2*x^2+(b*(a+1)+(-1+a)*b)*x+(a+1)*(-1+a))^(1/2))/(b^
2)^(1/2)-4/b^2/(x+(-1+a)/b)*((x+(-1+a)/b)^2*b^2+2*(x+(-1+a)/b)*b)^(1/2))*e
xp(c)*((b*x+a-1)*(b*x+a+1))^(1/2)/(b*x+a-1)^(1/2)/(b*x+a+1)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 79.00

$$\int e^{c+3\coth^{-1}(a+bx)}\,dx = \frac{(bx+a-5)\sqrt{\frac{bx+a+1}{bx+a-1}}e^{c}+3\,e^{c}\log\left(\sqrt{\frac{bx+a+1}{bx+a-1}}+1\right)-3\,e^{c}\log\left(\sqrt{\frac{bx+a+1}{bx+a-1}}-1\right)}{b}$$

input
```
integrate(exp(c+3*arccoth(b*x+a)),x, algorithm="fricas")
```

output
```
((b*x + a - 5)*sqrt((b*x + a + 1)/(b*x + a - 1))*e^c + 3*e^c*log(sqrt((b*x
 + a + 1)/(b*x + a - 1)) + 1) - 3*e^c*log(sqrt((b*x + a + 1)/(b*x + a - 1)
) - 1))/b
```

Sympy [F]

$$\int e^{c+3\coth^{-1}(a+bx)}\,dx = e^c\int e^{3\,\mathrm{acoth}\,(a+bx)}\,dx$$

input
```
integrate(exp(c+3*acoth(b*x+a)),x)
```

output
```
exp(c)*Integral(exp(3*acoth(a + b*x)), x)
```

Maxima [F]

$$\int e^{c+3\coth^{-1}(a+bx)}\,dx = \int e^{(c+3\,\mathrm{arcoth}(bx+a))}\,dx$$

input
```
integrate(exp(c+3*arccoth(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(e^(c + 3*arccoth(b*x + a)), x)
```

Giac [F]

$$\int e^{c+3\coth^{-1}(a+bx)}\,dx = \int e^{(c+3\,\mathrm{arcoth}(bx+a))}\,dx$$

input
```
integrate(exp(c+3*arccoth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(e^(c + 3*arccoth(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+3\coth^{-1}(a+bx)}\,dx = \int e^{c+3\,\mathrm{acoth}(a+b\,x)}\,dx$$

input
```
int(exp(c + 3*acoth(a + b*x)),x)
```

output
```
int(exp(c + 3*acoth(a + b*x)), x)
```

Reduce [F]

$$\int e^{c+3\coth^{-1}(a+bx)}\,dx = e^c \left(\int e^{3acoth(bx+a)}dx\right)$$

input
```
int(exp(c+3*acoth(b*x+a)),x)
```

output
```
e**c*int(e**(3*acoth(a + b*x)),x)
```

3.6 $\int e^{c+\coth^{-1}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 1

$$\int e^{c+\coth^{-1}(a+bx)}\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 67.00

$$\int e^{c+\coth^{-1}(a+bx)}\,dx=\frac{e^c\left((a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}+\log\left((a+bx)\left(1+\sqrt{1-\frac{1}{(a+bx)^2}}\right)\right)\right)}{b}$$

input
```
Integrate[E^(c + ArcCoth[a + b*x]),x]
```

output
```
(E^c*((a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2] + Log[(a
+ b*x)*(1 + Sqrt[1 - (a + b*x)^(-2)])]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\coth^{-1}(a+bx)+c} \, dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{c+\coth^{-1}(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\int e^{c+\coth^{-1}(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c + ArcCoth[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 119.00

method	result
risch	$\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\mathrm{e}^{c}}{b}+\frac{\ln\left(\frac{\frac{b(a+1)}{2}+\frac{(-1+a)b}{2}+b^{2}x}{\sqrt{b^{2}}}+\sqrt{b^{2}x^{2}+(b(a+1)+(-1+a)b)x+(a+1)(-1+a)}\right)\mathrm{e}^{c}\sqrt{(bx+a-1)(bx+a+1)}}{\sqrt{b^{2}}\,\sqrt{bx+a-1}\,\sqrt{bx+a+1}}$

input
```
int(exp(c+arccoth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*exp(c)+ln((1/2*b*(a+1)+1/2*(-1+a)*b+b^
2*x)/(b^2)^(1/2)+(b^2*x^2+(b*(a+1)+(-1+a)*b)*x+(a+1)*(-1+a))^(1/2))/(b^2)^
(1/2)*exp(c)*((b*x+a-1)*(b*x+a+1))^(1/2)/(b*x+a-1)^(1/2)/(b*x+a+1)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 78.00

$$\int e^{c+\coth^{-1}(a+bx)}\,dx = \frac{(bx+a-1)\sqrt{\frac{bx+a+1}{bx+a-1}}e^{c}+e^{c}\log\left(\sqrt{\frac{bx+a+1}{bx+a-1}}+1\right)-e^{c}\log\left(\sqrt{\frac{bx+a+1}{bx+a-1}}-1\right)}{b}$$

input
```
integrate(exp(c+arccoth(b*x+a)),x, algorithm="fricas")
```

output
```
((b*x + a - 1)*sqrt((b*x + a + 1)/(b*x + a - 1))*e^c + e^c*log(sqrt((b*x +
 a + 1)/(b*x + a - 1)) + 1) - e^c*log(sqrt((b*x + a + 1)/(b*x + a - 1)) -
1))/b
```

Sympy [F]

$$\int e^{c+\coth^{-1}(a+bx)}\,dx = e^c\int e^{\operatorname{acoth}(a+bx)}\,dx$$

input
```
integrate(exp(c+acoth(b*x+a)),x)
```

output
```
exp(c)*Integral(exp(acoth(a + b*x)), x)
```

Maxima [F]

$$\int e^{c+\coth^{-1}(a+bx)}\,dx = \int e^{(c+\operatorname{arcoth}(bx+a))}\,dx$$

input
```
integrate(exp(c+arccoth(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(e^(c + arccoth(b*x + a)), x)
```

Giac [F]

$$\int e^{c+\coth^{-1}(a+bx)}\,dx = \int e^{(c+\operatorname{arcoth}(bx+a))}\,dx$$

input
```
integrate(exp(c+arccoth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(e^(c + arccoth(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+\coth^{-1}(a+bx)}\,dx = \int \mathrm{e}^{c+\mathrm{acoth}(a+b\,x)}\,dx$$

input
```
int(exp(c + acoth(a + b*x)),x)
```

output
```
int(exp(c + acoth(a + b*x)), x)
```

Reduce [F]

$$\int e^{c+\coth^{-1}(a+bx)}\,dx = e^{c}\left(\int e^{acoth(bx+a)}dx\right)$$

input
```
int(exp(c+acoth(b*x+a)),x)
```

output
```
e**c*int(e**acoth(a + b*x),x)
```

3.7 $\int e^{c-\coth^{-1}(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 1

$$\int e^{c-\coth^{-1}(a+bx)}\,dx=0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 69.00

$$\int e^{c-\coth^{-1}(a+bx)}\,dx=\frac{e^c\left((a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}-\log\left((a+bx)\left(1+\sqrt{1-\frac{1}{(a+bx)^2}}\right)\right)\right)}{b}$$

input
```
Integrate[E^(c - ArcCoth[a + b*x]),x]
```

output
```
(E^c*((a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2] - Log[(a
+ b*x)*(1 + Sqrt[1 - (a + b*x)^(-2)])]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-\coth^{-1}(a+bx)}\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{c-\coth^{-1}(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\int e^{c-\coth^{-1}(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c - ArcCoth[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 120.00

method	result
risch	$\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,e^{c}}{b}-\frac{\ln\left(\frac{\frac{b(a+1)}{2}+\frac{(-1+a)b}{2}+b^{2}x}{\sqrt{b^{2}}}+\sqrt{b^{2}x^{2}+(b(a+1)+(-1+a)b)x+(a+1)(-1+a)}\right)e^{c}\sqrt{(bx+a-1)(bx+a+1)}}{\sqrt{b^{2}}\,\sqrt{bx+a-1}\,\sqrt{bx+a+1}}$

input
```
int(exp(c-arccoth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*exp(c)-ln((1/2*b*(a+1)+1/2*(-1+a)*b+b^
2*x)/(b^2)^(1/2)+(b^2*x^2+(b*(a+1)+(-1+a)*b)*x+(a+1)*(-1+a))^(1/2))/(b^2)^
(1/2)*exp(c)*((b*x+a-1)*(b*x+a+1))^(1/2)/(b*x+a-1)^(1/2)/(b*x+a+1)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 78.00

$$\int e^{c-\coth^{-1}(a+bx)}\,dx = \frac{(bx+a-1)\sqrt{\frac{bx+a+1}{bx+a-1}}e^{c}-e^{c}\log\left(\sqrt{\frac{bx+a+1}{bx+a-1}}+1\right)+e^{c}\log\left(\sqrt{\frac{bx+a+1}{bx+a-1}}-1\right)}{b}$$

input
```
integrate(exp(c-arccoth(b*x+a)),x, algorithm="fricas")
```

output
```
((b*x + a - 1)*sqrt((b*x + a + 1)/(b*x + a - 1))*e^c - e^c*log(sqrt((b*x +
 a + 1)/(b*x + a - 1)) + 1) + e^c*log(sqrt((b*x + a + 1)/(b*x + a - 1)) -
1))/b
```

Sympy [F]

$$\int e^{c-\coth^{-1}(a+bx)}\,dx = e^c\int e^{-\operatorname{acoth}(a+bx)}\,dx$$

input
```
integrate(exp(c-acoth(b*x+a)),x)
```

output
```
exp(c)*Integral(exp(-acoth(a + b*x)), x)
```

Maxima [F]

$$\int e^{c-\coth^{-1}(a+bx)}\,dx = \int e^{(c-\operatorname{arcoth}(bx+a))}\,dx$$

input
```
integrate(exp(c-arccoth(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(e^(c - arccoth(b*x + a)), x)
```

Giac [F]

$$\int e^{c-\coth^{-1}(a+bx)}\,dx = \int e^{(c-\operatorname{arcoth}(bx+a))}\,dx$$

input
```
integrate(exp(c-arccoth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(e^(c - arccoth(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c-\coth^{-1}(a+bx)}\,dx = \int e^{c-\mathrm{acoth}(a+b\,x)}\,dx$$

input
```
int(exp(c - acoth(a + b*x)),x)
```

output
```
int(exp(c - acoth(a + b*x)), x)
```

Reduce [F]

$$\int e^{c-\coth^{-1}(a+bx)}\,dx = e^c\left(\int \frac{1}{e^{acoth(bx+a)}}dx\right)$$

input
```
int(exp(c-acoth(b*x+a)),x)
```

output
```
e**c*int(1/e**acoth(a + b*x),x)
```

3.8 $\int e^{c-3\coth^{-1}(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 1

$$\int e^{c-3\coth^{-1}(a+bx)}\,dx=0$$

output
```
0
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 78.00

$$\int e^{c-3\coth^{-1}(a+bx)}\,dx=\frac{e^c\left(\frac{\sqrt{1-\frac{1}{(a+bx)^2}}\,(a^2+bx(5+bx)+a(5+2bx))}{1+a+bx}-3\log\left((a+bx)\left(1+\sqrt{1-\frac{1}{(a+bx)^2}}\right)\right)\right)}{b}$$

input
```
Integrate[E^(c - 3*ArcCoth[a + b*x]),x]
```

output
```
(E^c*((Sqrt[1 - (a + b*x)^(-2)]*(a^2 + b*x*(5 + b*x) + a*(5 + 2*b*x)))/(1
+ a + b*x) - 3*Log[(a + b*x)*(1 + Sqrt[1 - (a + b*x)^(-2)])]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-3\coth^{-1}(a+bx)}\,dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int e^{c-3\coth^{-1}(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\int e^{c-3\coth^{-1}(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c - 3*ArcCoth[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 168.00

method	result
risch	$\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,e^{c}}{b}+\frac{\left(-\frac{3\ln\left(\frac{\frac{b(a+1)}{2}+\frac{(-1+a)b}{2}+b^{2}x}{\sqrt{b^{2}}}+\sqrt{b^{2}x^{2}+(b(a+1)+(-1+a)b)x+(a+1)(-1+a)}\right)}{\sqrt{b^{2}}}+\frac{4\sqrt{\left(x+\frac{a+1}{b}\right)^{2}b^{2}-2\left(x+\frac{a+}{b}\right.}}{b^{2}\left(x+\frac{a+1}{b}\right)}\right.}{\sqrt{bx+a-1}\,\sqrt{bx+a+1}}$

input
```
int(exp(c-3*arccoth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*exp(c)+(-3*ln((1/2*b*(a+1)+1/2*(-1+a)*
b+b^2*x)/(b^2)^(1/2)+(b^2*x^2+(b*(a+1)+(-1+a)*b)*x+(a+1)*(-1+a))^(1/2))/(b
^2)^(1/2)+4/b^2/(x+(a+1)/b)*((x+(a+1)/b)^2*b^2-2*(x+(a+1)/b)*b)^(1/2))*exp
(c)*((b*x+a-1)*(b*x+a+1))^(1/2)/(b*x+a-1)^(1/2)/(b*x+a+1)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 119.00

$$\int e^{c-3\coth^{-1}(a+bx)}\,dx =$$
$$-\frac{3\,(bx+a+1)e^{c}\log\left(\sqrt{\frac{bx+a+1}{bx+a-1}}+1\right)-3\,(bx+a+1)e^{c}\log\left(\sqrt{\frac{bx+a+1}{bx+a-1}}-1\right)-(b^{2}x^{2}+2\,(a+2)bx+a^{2}}{b^{2}x+(a+1)b}$$

input
```
integrate(exp(c-3*arccoth(b*x+a)),x, algorithm="fricas")
```

output
```
-(3*(b*x + a + 1)*e^c*log(sqrt((b*x + a + 1)/(b*x + a - 1)) + 1) - 3*(b*x
+ a + 1)*e^c*log(sqrt((b*x + a + 1)/(b*x + a - 1)) - 1) - (b^2*x^2 + 2*(a
+ 2)*b*x + a^2 + 4*a - 5)*sqrt((b*x + a + 1)/(b*x + a - 1))*e^c)/(b^2*x +
(a + 1)*b)
```

Sympy [F]

$$\int e^{c-3\coth^{-1}(a+bx)}\,dx = e^c\int e^{-3\,\mathrm{acoth}\,(a+bx)}\,dx$$

input
```
integrate(exp(c-3*acoth(b*x+a)),x)
```

output
```
exp(c)*Integral(exp(-3*acoth(a + b*x)), x)
```

Maxima [F]

$$\int e^{c-3\coth^{-1}(a+bx)}\,dx = \int e^{(c-3\,\mathrm{arcoth}(bx+a))}\,dx$$

input
```
integrate(exp(c-3*arccoth(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(e^(c - 3*arccoth(b*x + a)), x)
```

Giac [F]

$$\int e^{c-3\coth^{-1}(a+bx)}\,dx = \int e^{(c-3\,\mathrm{arcoth}(bx+a))}\,dx$$

input
```
integrate(exp(c-3*arccoth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(e^(c - 3*arccoth(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c-3\coth^{-1}(a+bx)}\,dx=\int e^{c-3\,\mathrm{acoth}(a+b\,x)}\,dx$$

input
```
int(exp(c - 3*acoth(a + b*x)),x)
```

output
```
int(exp(c - 3*acoth(a + b*x)), x)
```

Reduce [F]

$$\int e^{c-3\coth^{-1}(a+bx)}\,dx=e^c\left(\int\frac{1}{e^{3acoth(bx+a)}}dx\right)$$

input
```
int(exp(c-3*acoth(b*x+a)),x)
```

output
```
e**c*int(1/e**(3*acoth(a + b*x)),x)
```

3.9 $\int e^{\coth^{-1}(ax)} x^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 114

$$\int e^{\coth^{-1}(ax)} x^3 \, dx = \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}x^2}{8a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}x^4 + \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

output
```
2/3*(1-1/a^2/x^2)^(1/2)*x/a^3+3/8*(1-1/a^2/x^2)^(1/2)*x^2/a^2+1/3*(1-1/a^2
/x^2)^(1/2)*x^3/a+1/4*(1-1/a^2/x^2)^(1/2)*x^4+3/8*arctanh((1-1/a^2/x^2)^(1
/2))/a^4
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(ax)} x^3 \, dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}x(16+9ax+8a^2x^2+6a^3x^3)+9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{24a^4}$$

input
```
Integrate[E^ArcCoth[a*x]*x^3,x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(16 + 9*a*x + 8*a^2*x^2 + 6*a^3*x^3) + 9*Log[(1
 + Sqrt[1 - 1/(a^2*x^2)])*x])/(24*a^4)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.400, Rules used = {6719, 539, 25, 27, 539, 25, 27, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)x^5}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{4}\int -\frac{\left(4a+\frac{3}{x}\right)x^4}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{4}\int \frac{\left(4a+\frac{3}{x}\right)x^4}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\int \frac{\left(4a+\frac{3}{x}\right)x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{4a^2}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{-\frac{1}{3}\int -\frac{\left(9a+\frac{8}{x}\right)x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{1}{3}\int \frac{\left(9a+\frac{8}{x}\right)x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\int \frac{\left(9a+\frac{8}{x}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \; 539$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{-\frac{1}{2}\int -\frac{\left(16a+\frac{9}{x}\right)x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\frac{1}{2}\int \frac{\left(16a+\frac{9}{x}\right)x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\frac{\int \frac{\left(16a+\frac{9}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \; 534$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\frac{9\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \; 243$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\frac{\frac{9}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-16ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \; 73$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\frac{-9a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-16ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\frac{-9\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-16ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}$$

input
```
Int[E^ArcCoth[a*x]*x^3,x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 - ((-4*a*Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 + ((-9
*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (-16*a*Sqrt[1 - 1/(a^2*x^2)]*x - 9*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a))/(3*a))/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

method	result
risch	$\frac{(6a^3x^3+8a^2x^2+9ax+16)(ax-1)}{24a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{(ax-1)(ax+1)}}{8a^3\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)\left(-6\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax-8((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-15\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+15\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-24a\ln\left(\frac{a^2x+\sqrt{(\cdots}{}\right.\right.}{24\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^4\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*a^3*x^3+8*a^2*x^2+9*a*x+16)*(a*x-1)/a^4/((a*x-1)/(a*x+1))^(1/2)+3/
8/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)/((a*x-1)/(a*x+1)
)^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\int e^{\coth^{-1}(ax)}x^3\,dx = \frac{(6\,a^4x^4+14\,a^3x^3+17\,a^2x^2+25\,ax+16)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{24\,a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="fricas")
```

output
```
1/24*((6*a^4*x^4 + 14*a^3*x^3 + 17*a^2*x^2 + 25*a*x + 16)*sqrt((a*x - 1)/(
a*x + 1)) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a
*x + 1)) - 1))/a^4
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}x^3\,dx = \int \frac{x^3}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3,x)
```

output
```
Integral(x**3/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(94) = 188.

Time = 0.04 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.78

$$\int e^{\coth^{-1}(ax)}x^3\,dx = \frac{1}{24}\,a\left(\frac{2\left(9\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-49\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+31\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-39\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4\,(ax-1)a^5}{ax+1}-\frac{6\,(ax-1)^2a^5}{(ax+1)^2}+\frac{4\,(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-\right.}{a^5}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="maxima")
```

output
```
1/24*a*(2*(9*((a*x - 1)/(a*x + 1))^(7/2) - 49*((a*x - 1)/(a*x + 1))^(5/2)
+ 31*((a*x - 1)/(a*x + 1))^(3/2) - 39*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x -
 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(a*x
 + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 9*log(sqrt((a*x - 1)/(a*x +
 1)) + 1)/a^5 - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int e^{\coth^{-1}(ax)}x^3\,dx = \frac{1}{24}\sqrt{a^2x^2-1}\left(\left(2\,x\left(\frac{3\,x}{a\mathrm{sgn}\left(ax+1\right)}+\frac{4}{a^2\mathrm{sgn}\left(ax+1\right)}\right)+\frac{9}{a^3\mathrm{sgn}\left(ax+1\right)}\right)x+\frac{16}{a^4\mathrm{sgn}\left(ax+1\right)}\right) - \frac{3\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,a^3|a|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="giac")
```

output
```
1/24*sqrt(a^2*x^2 - 1)*((2*x*(3*x/(a*sgn(a*x + 1)) + 4/(a^2*sgn(a*x + 1)))
 + 9/(a^3*sgn(a*x + 1)))*x + 16/(a^4*sgn(a*x + 1))) - 3/8*log(abs(-x*abs(a
) + sqrt(a^2*x^2 - 1)))/(a^3*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.50

$$\int e^{\coth^{-1}(ax)}x^3\,dx = \frac{\frac{13\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{31\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{49\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{12}-\frac{3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a^4+\frac{6\,a^4\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a^4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a^4\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a^4\,\left(a\,x-1\right)}{a\,x+1}} + \frac{3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a^4}$$

input
```
int(x^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((13*((a*x - 1)/(a*x + 1))^(1/2))/4 - (31*((a*x - 1)/(a*x + 1))^(3/2))/12
+ (49*((a*x - 1)/(a*x + 1))^(5/2))/12 - (3*((a*x - 1)/(a*x + 1))^(7/2))/4)
/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3
+ (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) + (3*atanh(
((a*x - 1)/(a*x + 1))^(1/2)))/(4*a^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.85

$$\int e^{\coth^{-1}(ax)}x^3\,dx$$
$$= \frac{6\sqrt{ax+1}\,\sqrt{ax-1}\,a^3x^3 + 8\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2 + 9\sqrt{ax+1}\,\sqrt{ax-1}\,ax + 16\sqrt{ax+1}\,\sqrt{ax-1} + 18}{24a^4}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x)
```

output
```
(6*sqrt(a*x + 1)*sqrt(a*x - 1)*a**3*x**3 + 8*sqrt(a*x + 1)*sqrt(a*x - 1)*a
**2*x**2 + 9*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 16*sqrt(a*x + 1)*sqrt(a*x -
 1) + 18*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)))/(24*a**4)
```

3.10 $\int e^{\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 90

$$\int e^{\coth^{-1}(ax)}x^2\,dx=\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

output
```
2/3*(1-1/a^2/x^2)^(1/2)*x/a^2+1/2*(1-1/a^2/x^2)^(1/2)*x^2/a+1/3*(1-1/a^2/x
^2)^(1/2)*x^3+1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int e^{\coth^{-1}(ax)}x^2\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(4+3ax+2a^2x^2)+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{6a^3}$$

input
```
Integrate[E^ArcCoth[a*x]*x^2,x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 + 3*a*x + 2*a^2*x^2) + 3*Log[(1 + Sqrt[1 - 1
/(a^2*x^2)])*x])/(6*a^3)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {6719, 539, 25, 27, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{3}\int -\frac{\left(3a+\frac{2}{x}\right)x^3}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{3}\int \frac{\left(3a+\frac{2}{x}\right)x^3}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\int \frac{\left(3a+\frac{2}{x}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{3a^2}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{-\frac{1}{2}\int -\frac{\left(4a+\frac{3}{x}\right)x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{1}{2}\int \frac{\left(4a+\frac{3}{x}\right)x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\int \frac{\left(4a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}$$

$$\Big\downarrow \text{ 534}$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{3\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{\frac{3}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{-3a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{-3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-4ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}$$

input
```
Int[E^ArcCoth[a*x]*x^2,x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - ((-3*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (-4*
a*Sqrt[1 - 1/(a^2*x^2)]*x - 3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a))/(3*a^
2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.21

method	result
risch	$\frac{(2a^2x^2+3ax+4)(ax-1)}{6a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{(ax-1)(ax+1)}}{2a^2\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)\left(2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}+3\sqrt{a^2}\sqrt{a^2x^2-1}\,ax+6\sqrt{(ax-1)(ax+1)}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a+6a\ln\left(\frac{a^2x+\sqrt{(ax-1)}}{\sqrt{a}}\right.\right.}{6\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(ax+1)}\,a^3\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*x^2+3*a*x+4)*(a*x-1)/a^3/((a*x-1)/(a*x+1))^(1/2)+1/2/a^2*ln(a^2
*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)/((a*x-1)/(a*x+1))^(1/2)*((a*
x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

$$\int e^{\coth^{-1}(ax)}x^2\,dx$$
$$=\frac{(2\,a^3x^3+5\,a^2x^2+7\,ax+4)\sqrt{\frac{ax-1}{ax+1}}+3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{6\,a^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="fricas")
```

output
```
1/6*((2*a^3*x^3 + 5*a^2*x^2 + 7*a*x + 4)*sqrt((a*x - 1)/(a*x + 1)) + 3*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^
3
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)} x^2\,dx = \int \frac{x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2,x)
```

output
```
Integral(x**2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(74) = 148$.

Time $= 0.03$ (sec) , antiderivative size $= 166$, normalized size of antiderivative $= 1.84$

$$\int e^{\coth^{-1}(ax)} x^2\,dx = -\frac{1}{6}\,a\left(\frac{2\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} - 4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 9\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^4}{ax+1} - \frac{3\,(ax-1)^2a^4}{(ax+1)^2} + \frac{(ax-1)^3a^4}{(ax+1)^3} - a^4} - \frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4} + \frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="maxima")
```

output
```
-1/6*a*(2*(3*((a*x - 1)/(a*x + 1))^(5/2) - 4*((a*x - 1)/(a*x + 1))^(3/2) +
 9*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*a
^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 3*log(sqrt((a*x - 1)
/(a*x + 1)) + 1)/a^4 + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.48

$$\int e^{\coth^{-1}(ax)}x^2\,dx = \frac{3\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}+\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}+\frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}$$

input
```
int(x^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(3*((a*x - 1)/(a*x + 1))^(1/2) - (4*((a*x - 1)/(a*x + 1))^(3/2))/3 + ((a*x
 - 1)/(a*x + 1))^(5/2))/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^3*(a*x
 - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + atanh(((a*x - 1)/(a*
x + 1))^(1/2))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

$$\int e^{\coth^{-1}(ax)}x^2\,dx$$
$$= \frac{2\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2+3\sqrt{ax+1}\,\sqrt{ax-1}\,ax+4\sqrt{ax+1}\,\sqrt{ax-1}+6\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)}{6a^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x)
```

output
```
(2*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 + 3*sqrt(a*x + 1)*sqrt(a*x - 1)*a
*x + 4*sqrt(a*x + 1)*sqrt(a*x - 1) + 6*log((sqrt(a*x - 1) + sqrt(a*x + 1))
/sqrt(2)))/(6*a**3)
```

3.11 $\int e^{\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 8, antiderivative size = 63

$$\int e^{\coth^{-1}(ax)}x\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/a+1/2*(1-1/a^2/x^2)^(1/2)*x^2+1/2*arctanh((1-1/a^2/x
^2)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int e^{\coth^{-1}(ax)}x\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(2+ax)+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{2a^2}$$

input
```
Integrate[E^ArcCoth[a*x]*x,x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])
/(2*a^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6719, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{2}\int -\frac{\left(2a+\frac{1}{x}\right)x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{2}\int \frac{\left(2a+\frac{1}{x}\right)x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{\int \frac{\left(2a+\frac{1}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a^2}$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-2ax\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-2ax\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{a^2\left(-\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-2ax\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{-\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-2ax\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}$$

input
```
Int[E^ArcCoth[a*x]*x,x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (-2*a*Sqrt[1 - 1/(a^2*x^2)]*x - ArcTanh[Sq
rt[1 - 1/(a^2*x^2)]])/(2*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.59

method	result	size
risch	$\frac{(ax+2)(ax-1)}{2a^{2}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)\sqrt{(ax-1)(ax+1)}}{2a\sqrt{a^{2}}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	100
default	$\frac{(ax-1)\left(\sqrt{a^{2}}\,\sqrt{a^{2}x^{2}-1}\,ax+2\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}-\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a+2a\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^{2}\sqrt{a^{2}}}$	152

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x+2)*(a*x-1)/a^2/((a*x-1)/(a*x+1))^(1/2)+1/2/a*ln(a^2*x/(a^2)^(1/2)
+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^
(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int e^{\coth^{-1}(ax)}x\,dx = \frac{(a^2x^2+3\,ax+2)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="fricas")
```

output
```
1/2*((a^2*x^2 + 3*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/
(a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}x\,dx = \int \frac{x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x,x)
```

output
```
Integral(x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(53) = 106$.

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.03

$$\int e^{\coth^{-1}(ax)}x\,dx = \frac{1}{2}\,a\left(\frac{2\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="maxima")
```

output
```
1/2*a*(2*(((a*x - 1)/(a*x + 1))^(3/2) - 3*sqrt((a*x - 1)/(a*x + 1)))/(2*(a
*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^3 - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int e^{\coth^{-1}(ax)}x\,dx = \frac{1}{2}\sqrt{a^2x^2-1}\left(\frac{x}{a\mathrm{sgn}\,(ax+1)}+\frac{2}{a^2\mathrm{sgn}\,(ax+1)}\right) - \frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,a|a|\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="giac")
```

output
```
1/2*sqrt(a^2*x^2 - 1)*(x/(a*sgn(a*x + 1)) + 2/(a^2*sgn(a*x + 1))) - 1/2*lo
g(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 23.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.56

$$\int e^{\coth^{-1}(ax)}x\,dx = \frac{3\sqrt{\frac{a\,x-1}{a\,x+1}}-\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}+\frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2}$$

input
```
int(x/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(3*((a*x - 1)/(a*x + 1))^(1/2) - ((a*x - 1)/(a*x + 1))^(3/2))/(a^2 + (a^2*
(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) + atanh(((a*x - 1)
/(a*x + 1))^(1/2))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int e^{\coth^{-1}(ax)}x\,dx = \frac{\sqrt{ax+1}\,\sqrt{ax-1}\,ax + 2\sqrt{ax+1}\,\sqrt{ax-1} + 2\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)}{2a^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x,x)
```

output
```
(sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 2*sqrt(a*x + 1)*sqrt(a*x - 1) + 2*log((
sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)))/(2*a**2)
```

3.12 $\int e^{\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 36

$$\int e^{\coth^{-1}(ax)}\,dx=\sqrt{1-\frac{1}{a^2x^2}}x+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
(1-1/a^2/x^2)^(1/2)*x+arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int e^{\coth^{-1}(ax)}\,dx=\sqrt{1-\frac{1}{a^2x^2}}x+\frac{\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}$$

input
```
Integrate[E^ArcCoth[a*x],x]
```

output
```
Sqrt[1 - 1/(a^2*x^2)]*x + Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]/a
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6718, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6718}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{534}$$

$$x\sqrt{1-\frac{1}{a^2x^2}}-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{243}$$

$$x\sqrt{1-\frac{1}{a^2x^2}}-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a}$$

$$\Big\downarrow \text{73}$$

$$a\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}+x\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+x\sqrt{1-\frac{1}{a^2x^2}}$$

input
```
Int[E^ArcCoth[a*x],x]
```

output
```
Sqrt[1 - 1/(a^2*x^2)]*x + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 6718
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(32) = 64$.

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

method	result	size
risch	$\frac{ax-1}{a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	90
default	$\frac{(ax-1)\left(\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}+a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$	97

input
```
int(1/((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/((a*x-1)/(a*x+1))^(1/2)+ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2)
)/(a^2)^(1/2)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\int e^{\coth^{-1}(ax)}\,dx=\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
((a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/(a*x + 1)) + 1)
- log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\,dx = \int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(1/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(32) = 64$.

Time $= 0.03$ (sec) , antiderivative size $= 90$, normalized size of antiderivative $= 2.50$

$$\int e^{\coth^{-1}(ax)}\,dx = -a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - log(sqrt
((a*x - 1)/(a*x + 1)) + 1)/a^2 + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [A] (verification not implemented)

Time $= 0.15$ (sec) , antiderivative size $= 57$, normalized size of antiderivative $= 1.58$

$$\int e^{\coth^{-1}(ax)}\,dx = -\frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\,(ax+1)} + \frac{\sqrt{a^2x^2-1}}{a\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
-log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*
x^2 - 1)/(a*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int e^{\coth^{-1}(ax)}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int(1/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) + (2*atanh((
(a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int e^{\coth^{-1}(ax)}\,dx = \frac{\sqrt{ax+1}\,\sqrt{ax-1}+2\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)}{a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(a*x + 1)*sqrt(a*x - 1) + 2*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(
2)))/a
```

3.13 $\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 22

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = -\csc^{-1}(ax) + \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

output
```
-arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = -\arcsin\left(\frac{1}{ax}\right) + \log\left(x\left(1+\sqrt{\frac{-1+a^2x^2}{a^2x^2}}\right)\right)$$

input
```
Integrate[E^ArcCoth[a*x]/x,x]
```

output
```
-ArcSin[1/(a*x)] + Log[x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6719, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx$$

$$\Big\downarrow \; 6719$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \; 538$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \; 223$$

$$-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\arcsin\left(\frac{1}{ax}\right)$$

$$\Big\downarrow \; 243$$

$$-\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-\arcsin\left(\frac{1}{ax}\right)$$

$$\Big\downarrow \; 73$$

$$a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-\arcsin\left(\frac{1}{ax}\right)$$

$$\Big\downarrow \; 221$$

$$\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-\arcsin\left(\frac{1}{ax}\right)$$

input
```
Int[E^ArcCoth[a*x]/x,x]
```

output
```
-ArcSin[1/(a*x)] + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(20) = 40.

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 5.95

method	result	size
default	$\frac{(ax-1)\left(\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}+a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)-\sqrt{a^2x^2-1}\,\sqrt{a^2}-\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$	131

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)*(((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+a*ln((a^2*x+((a*x-1)*(a*x+1))
^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)-arctan(1/(a
^2*x^2-1)^(1/2))*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1
/2)/(a^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(20) = 40.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = 2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

output
```
2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(sqrt((a*x - 1)/(a*x + 1)) + 1) -
 log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x,x)
```

output
```
Integral(1/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(20) = 40$.

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.14

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

output
```
a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a + log(sqrt((a*x - 1)/(a*x + 1)) +
 1)/a - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(20) = 40$.

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = \frac{2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{\operatorname{sgn}(ax+1)} - \frac{a\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\operatorname{sgn}(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
```

output
```
2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) - a*log(abs(-x*abs(a)
 + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 23.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

input
```
int(1/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + 2*atanh(((a*x - 1)/(a*x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx = 2atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right) - 2atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)+2\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x,x)
```

output
```
2*(atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1) - atan(sqrt(a*x - 1) + sqrt(a*x
 + 1) + 1) + log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)))
```

3.14 $\int \frac{e^{\coth^{-1}(ax)}}{x^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2} \, dx = a\sqrt{1-\frac{1}{a^2x^2}} - a\csc^{-1}(ax)$$

output
```
a*(1-1/a^2/x^2)^(1/2)-a*arccsc(a*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2} \, dx = a\left(\sqrt{1-\frac{1}{a^2x^2}} - \arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[E^ArcCoth[a*x]/x^2,x]
```

output
```
a*(Sqrt[1 - 1/(a^2*x^2)] - ArcSin[1/(a*x)])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6719, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2} dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{1+\frac{1}{ax}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{455}$$

$$a\sqrt{1-\frac{1}{a^2x^2}}-\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{223}$$

$$a\sqrt{1-\frac{1}{a^2x^2}}-a\arcsin\left(\frac{1}{ax}\right)$$

input

```
Int[E^ArcCoth[a*x]/x^2,x]
```

output

```
a*Sqrt[1 - 1/(a^2*x^2)] - a*ArcSin[1/(a*x)]
```

Defintions of rubi rules used

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(22) = 44$.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.17

method	result
risch	$\frac{ax-1}{x\sqrt{\frac{ax-1}{ax+1}}}-\frac{a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{(ax-1)(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)\left(\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^2x^2+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,ax+\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x-\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax-}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,x}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)/x/((a*x-1)/(a*x+1))^(1/2)-a*arctan(1/(a^2*x^2-1)^(1/2))/((a*x-1)/(
a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx=\frac{2\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(2*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + (a*x + 1)*sqrt((a*x - 1)/(a*x +
 1)))/x
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

output
```
Integral(1/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx = 2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1} + \arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*x + 1) + 1) + arctan(sqrt((a*
x - 1)/(a*x + 1))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(22) = 44$.

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx = \frac{2\,a\arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)}{\operatorname{sgn}\left(ax+1\right)} + \frac{2\,|a|}{\left(\left(x|a| - \sqrt{a^2x^2-1}\right)^2 + 1\right)\operatorname{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

output
```
2*a*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) + 2*abs(a)/(((x*abs
(a) - sqrt(a^2*x^2 - 1))^2 + 1)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 23.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.29

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx = 2\,a\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) + \frac{2\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{a\,x-1}{a\,x+1}+1}$$

input
```
int(1/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
2*a*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (2*a*((a*x - 1)/(a*x + 1))^(1/2))/
((a*x - 1)/(a*x + 1) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{2atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)ax-2atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)ax+\sqrt{ax+1}\,\sqrt{ax-1}+ax}{x}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

output
```
(2*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a*x - 2*atan(sqrt(a*x - 1) + sq
rt(a*x + 1) + 1)*a*x + sqrt(a*x + 1)*sqrt(a*x - 1) + a*x)/x
```

3.15 $\int \frac{e^{\coth^{-1}(ax)}}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 52

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3} dx = a^2\sqrt{1-\frac{1}{a^2x^2}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x} - \frac{1}{2}a^2\csc^{-1}(ax)$$

output
```
a^2*(1-1/a^2/x^2)^(1/2)+1/2*a*(1-1/a^2/x^2)^(1/2)/x-1/2*a^2*arccsc(a*x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3} dx = \frac{a\left(\sqrt{1-\frac{1}{a^2x^2}}(1+2ax) - ax\arcsin\left(\frac{1}{ax}\right)\right)}{2x}$$

input
```
Integrate[E^ArcCoth[a*x]/x^3,x]
```

output
```
(a*(Sqrt[1 - 1/(a^2*x^2)]*(1 + 2*a*x) - a*x*ArcSin[1/(a*x)]))/(2*x)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6719, 533, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3} \, dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{1+\frac{1}{ax}}{\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}$$

$$\Big\downarrow \text{533}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^2\int \frac{a+\frac{2}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}\int \frac{a+\frac{2}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{2}\left(2a^2\sqrt{1-\frac{1}{a^2x^2}}-a\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\left(2a^2\sqrt{1-\frac{1}{a^2x^2}}-a^2\arcsin\left(\frac{1}{ax}\right)\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}$$

input
```
Int[E^ArcCoth[a*x]/x^3,x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)])/(2*x) + (2*a^2*Sqrt[1 - 1/(a^2*x^2)] - a^2*ArcSi
n[1/(a*x)])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.63

method	result
risch	$\frac{(ax-1)(2ax+1)}{2x^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{(ax-1)(ax+1)}}{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)\Big(2\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^3x^3+2\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^2x^2+2\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-2\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax-\sqrt{a^2}\,\sqrt{}}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,x}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x-1)*(2*a*x+1)/x^2/((a*x-1)/(a*x+1))^(1/2)-1/2*a^2*arctan(1/(a^2*x^
2-1)^(1/2))/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx=\frac{2\,a^2x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+(2\,a^2x^2+3\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (2*a^2*x^2 + 3*a*x + 1)
*sqrt((a*x - 1)/(a*x + 1)))/x^2
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

output
```
Integral(1/(x**3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(44) = 88$.

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.75

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx = \left(a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \frac{a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 3\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1} + \frac{(ax-1)^2}{(ax+1)^2} + 1}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
(a*arctan(sqrt((a*x - 1)/(a*x + 1))) + (a*((a*x - 1)/(a*x + 1))^(3/2) + 3*
a*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1
)^2 + 1))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(44) = 88$.

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.75

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx = \frac{a^2\arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)}{\mathrm{sgn}\,(ax+1)} - \frac{\left(x|a| - \sqrt{a^2x^2-1}\right)^3a^2 - 2\left(x|a| - \sqrt{a^2x^2-1}\right)^2a|a| - \left(x|a| - \sqrt{a^2x^2-1}\right)a^2 - 2\,a|a|}{\left(\left(x|a| - \sqrt{a^2x^2-1}\right)^2 + 1\right)^2\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

output
```
a^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) - ((x*abs(a) - sqrt
(a^2*x^2 - 1))^3*a^2 - 2*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*abs(a) - (x*ab
s(a) - sqrt(a^2*x^2 - 1))*a^2 - 2*a*abs(a))/(((x*abs(a) - sqrt(a^2*x^2 - 1
))^2 + 1)^2*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 23.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.56

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx = a^2\sqrt{\frac{a\,x-1}{a\,x+1}} + \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x^2} + a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) + \frac{3\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x}$$

input
```
int(1/(x^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
a^2*((a*x - 1)/(a*x + 1))^(1/2) + ((a*x - 1)/(a*x + 1))^(1/2)/(2*x^2) + a^
2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (3*a*((a*x - 1)/(a*x + 1))^(1/2))/(2
*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.56

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx = \frac{2atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^2x^2 - 2atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^2x^2 + 2\sqrt{ax+1}\,\sqrt{ax-1}\,ax -}{2x^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x)
```

output
```
(2*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**2*x**2 - 2*atan(sqrt(a*x - 1
) + sqrt(a*x + 1) + 1)*a**2*x**2 + 2*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + sqr
t(a*x + 1)*sqrt(a*x - 1))/(2*x**2)
```

3.16 $\int \frac{e^{\coth^{-1}(ax)}}{x^4} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 79

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4} \, dx = \frac{2}{3}a^3\sqrt{1-\frac{1}{a^2x^2}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x} - \frac{1}{2}a^3\csc^{-1}(ax)$$

output
```
2/3*a^3*(1-1/a^2/x^2)^(1/2)+1/3*a*(1-1/a^2/x^2)^(1/2)/x^2+1/2*a^2*(1-1/a^2
/x^2)^(1/2)/x-1/2*a^3*arccsc(a*x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4} \, dx = \frac{1}{6}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(2+3ax+4a^2x^2)}{x^2} - 3a^2\arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[E^ArcCoth[a*x]/x^4,x]
```

output
```
(a*((Sqrt[1 - 1/(a^2*x^2)]*(2 + 3*a*x + 4*a^2*x^2))/x^2 - 3*a^2*ArcSin[1/(
a*x)]))/6
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6719, 533, 27, 533, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{1+\frac{1}{ax}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{533}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a^2\int \frac{2a+\frac{3}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}\int \frac{2a+\frac{3}{x}}{\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^2\int \frac{3a+\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\int \frac{3a+\frac{4}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\left(3a\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-4a^2\sqrt{1-\frac{1}{a^2x^2}}\right)\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \ 223$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\left(3a^2\arcsin\left(\frac{1}{ax}\right)-4a^2\sqrt{1-\frac{1}{a^2x^2}}\right)\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

input
```
Int[E^ArcCoth[a*x]/x^4,x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2) + ((3*a^2*Sqrt[1 - 1/(a^2*x^2)])/(2*x) -
 (a*(-4*a^2*Sqrt[1 - 1/(a^2*x^2)] + 3*a^2*ArcSin[1/(a*x)]))/2)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.18

method	result
risch	$\frac{(ax-1)(4a^2x^2+3ax+2)}{6x^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{(ax-1)(ax+1)}}{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)\left(6\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^4x^4+6\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^4x^3-6\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^2x^2-3\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^3x^3-3a^3\sqrt{a^2}\,x^3}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x-1)*(4*a^2*x^2+3*a*x+2)/x^3/((a*x-1)/(a*x+1))^(1/2)-1/2*a^3*arctan
(1/(a^2*x^2-1)^(1/2))/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x
+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx=\frac{6\,a^3x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+(4\,a^3x^3+7\,a^2x^2+5\,ax+2)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + (4*a^3*x^3 + 7*a^2*x^2
+ 5*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

output
```
Integral(1/(x**4*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(65) = 130.

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.72

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{3}\left(3\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \frac{3\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 4\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 9\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1} + \frac{3\,(ax-1)^2}{(ax+1)^2} + \frac{(ax-1)^3}{(ax+1)^3} + 1}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/3*(3*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (3*a^2*((a*x - 1)/(a*x + 1)
)^(5/2) + 4*a^2*((a*x - 1)/(a*x + 1))^(3/2) + 9*a^2*sqrt((a*x - 1)/(a*x +
1)))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x
 + 1)^3 + 1))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(65) = 130$.

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.87

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4} \, dx = \frac{a^3 \arctan\left(-x|a| + \sqrt{a^2x^2 - 1}\right)}{\operatorname{sgn}(ax+1)} - \frac{3\left(x|a| - \sqrt{a^2x^2-1}\right)^5 a^3 - 12\left(x|a| - \sqrt{a^2x^2-1}\right)^2 a^2|a| - 3\left(x|a| - \sqrt{a^2x^2-1}\right)a^3 - 4a^2|a|}{3\left(\left(x|a| - \sqrt{a^2x^2-1}\right)^2 + 1\right)^3 \operatorname{sgn}(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

output
```
a^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) - 1/3*(3*(x*abs(a)
- sqrt(a^2*x^2 - 1))^5*a^3 - 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a^2*abs(a
) - 3*(x*abs(a) - sqrt(a^2*x^2 - 1))*a^3 - 4*a^2*abs(a))/(((x*abs(a) - sqr
t(a^2*x^2 - 1))^2 + 1)^3*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.33

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4} \, dx = \frac{2a^3\sqrt{\frac{ax-1}{ax+1}}}{3} + \frac{\sqrt{\frac{ax-1}{ax+1}}}{3x^3} + a^3 \operatorname{atan}\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \frac{7a^2\sqrt{\frac{ax-1}{ax+1}}}{6x} + \frac{5a\sqrt{\frac{ax-1}{ax+1}}}{6x^2}$$

input
```
int(1/(x^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(2*a^3*((a*x - 1)/(a*x + 1))^(1/2))/3 + ((a*x - 1)/(a*x + 1))^(1/2)/(3*x^3
) + a^3*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (7*a^2*((a*x - 1)/(a*x + 1))^(
1/2))/(6*x) + (5*a*((a*x - 1)/(a*x + 1))^(1/2))/(6*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.39

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx = \frac{6atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^3x^3-6atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^3x^3+4\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x}{6x^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x)
```

output
```
(6*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**3*x**3 - 6*atan(sqrt(a*x - 1
) + sqrt(a*x + 1) + 1)*a**3*x**3 + 4*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2
 + 3*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 2*sqrt(a*x + 1)*sqrt(a*x - 1) - 4*a
**3*x**3)/(6*x**3)
```

3.17 $\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 103

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = \frac{2}{3}a^4\sqrt{1-\frac{1}{a^2x^2}}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{3a^3\sqrt{1-\frac{1}{a^2x^2}}}{8x}-\frac{3}{8}a^4\csc^{-1}(ax)$$

output
```
2/3*a^4*(1-1/a^2/x^2)^(1/2)+1/4*a*(1-1/a^2/x^2)^(1/2)/x^3+1/3*a^2*(1-1/a^2
/x^2)^(1/2)/x^2+3/8*a^3*(1-1/a^2/x^2)^(1/2)/x-3/8*a^4*arccsc(a*x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.57

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = \frac{1}{24}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(6+8ax+9a^2x^2+16a^3x^3)}{x^3}-9a^3\arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[E^ArcCoth[a*x]/x^5,x]
```

output
```
(a*((Sqrt[1 - 1/(a^2*x^2)]*(6 + 8*a*x + 9*a^2*x^2 + 16*a^3*x^3))/x^3 - 9*a
^3*ArcSin[1/(a*x)]))/24
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6719, 533, 27, 533, 27, 533, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5} \, dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{1+\frac{1}{ax}}{\sqrt{1-\frac{1}{a^2x^2}}x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{533}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}-\frac{1}{4}a^2\int \frac{3a+\frac{4}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}-\frac{1}{4}\int \frac{3a+\frac{4}{x}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a^2\int \frac{8a+\frac{9}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\int \frac{8a+\frac{9}{x}}{\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \textbf{533}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{1}{2}a^2\int\frac{9a+\frac{16}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{1}{2}a\int\frac{9a+\frac{16}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \textbf{455}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{1}{2}a\left(9a\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \textbf{223}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{1}{2}a\left(9a^2\arcsin\left(\frac{1}{ax}\right)-16a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

input

```
Int[E^ArcCoth[a*x]/x^5,x]
```

output

```
(a*Sqrt[1 - 1/(a^2*x^2)])/(4*x^3) + ((4*a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2)
 - (a*((-9*a^2*Sqrt[1 - 1/(a^2*x^2)])/(2*x) + (a*(-16*a^2*Sqrt[1 - 1/(a^2*
x^2)] + 9*a^2*ArcSin[1/(a*x)]))/2))/3)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

method	result
risch	$\frac{(ax-1)(16a^3x^3+9a^2x^2+8ax+6)}{24x^4\sqrt{\frac{ax-1}{ax+1}}}-\frac{3a^4\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{(ax-1)(ax+1)}}{8\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)\left(24\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^5x^5+24\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^4x^4+24\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^5x^4-24\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^3x^3-\right.}{24\sqrt{}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/24*(a*x-1)*(16*a^3*x^3+9*a^2*x^2+8*a*x+6)/x^4/((a*x-1)/(a*x+1))^(1/2)-3/
8*a^4*arctan(1/(a^2*x^2-1)^(1/2))/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1)
)^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = \frac{18\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(16\,a^4x^4+25\,a^3x^3+17\,a^2x^2+14\,ax+6\right)\sqrt{\frac{ax-1}{ax+1}}}{24\,x^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
1/24*(18*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) + (16*a^4*x^4 + 25*a^3*
x^3 + 17*a^2*x^2 + 14*a*x + 6)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{1}{x^5\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**5,x)
```

output
```
Integral(1/(x**5*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(85) = 170$.

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.67

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx$$
$$= \frac{1}{12}\left(9\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \frac{9\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} + 49\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 31\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 39\,a^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4\,(ax-1)}{ax+1} + \frac{6\,(ax-1)^2}{(ax+1)^2} + \frac{4\,(ax-1)^3}{(ax+1)^3} + \frac{(ax-1)^4}{(ax+1)^4} + 1}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
1/12*(9*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + (9*a^3*((a*x - 1)/(a*x + 1
))^(7/2) + 49*a^3*((a*x - 1)/(a*x + 1))^(5/2) + 31*a^3*((a*x - 1)/(a*x + 1
))^(3/2) + 39*a^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(a
*x - 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^3/(a*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^
4 + 1))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(85) = 170$.

Time = 0.15 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.19

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = \frac{3\,a^4\arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)}{4\,\mathrm{sgn}\,(ax+1)}$$
$$- \frac{9\left(x|a| - \sqrt{a^2x^2-1}\right)^7 a^4 + 33\left(x|a| - \sqrt{a^2x^2-1}\right)^5 a^4 - 48\left(x|a| - \sqrt{a^2x^2-1}\right)^4 a^3|a| - 33\left(x|a| - \sqrt{a^2\right.}{12\left(\left(x|a| - \sqrt{a^2x^2-1}\right)^2 + 1\right)^4 \mathrm{sg}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="giac")
```

output
```
3/4*a^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) - 1/12*(9*(x*ab
s(a) - sqrt(a^2*x^2 - 1))^7*a^4 + 33*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*a^4
- 48*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a^3*abs(a) - 33*(x*abs(a) - sqrt(a^2
*x^2 - 1))^3*a^4 - 64*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a^3*abs(a) - 9*(x*a
bs(a) - sqrt(a^2*x^2 - 1))*a^4 - 16*a^3*abs(a))/(((x*abs(a) - sqrt(a^2*x^2
 - 1))^2 + 1)^4*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 23.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.25

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = \frac{2\,a^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3}+\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,x^4}+\frac{3\,a^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4}+\frac{17\,a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x^2}+\frac{25\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x}+\frac{7\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{12\,x^3}$$

input
```
int(1/(x^5*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(2*a^4*((a*x - 1)/(a*x + 1))^(1/2))/3 + ((a*x - 1)/(a*x + 1))^(1/2)/(4*x^4
) + (3*a^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/4 + (17*a^2*((a*x - 1)/(a*x
+ 1))^(1/2))/(24*x^2) + (25*a^3*((a*x - 1)/(a*x + 1))^(1/2))/(24*x) + (7*a
*((a*x - 1)/(a*x + 1))^(1/2))/(12*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.26

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = \frac{18atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^4x^4-18atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^4x^4+16\sqrt{ax+1}\,\sqrt{ax-1}}{24x^4}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x)
```

output
```
(18*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**4*x**4 - 18*atan(sqrt(a*x -
 1) + sqrt(a*x + 1) + 1)*a**4*x**4 + 16*sqrt(a*x + 1)*sqrt(a*x - 1)*a**3*x
**3 + 9*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 + 8*sqrt(a*x + 1)*sqrt(a*x -
 1)*a*x + 6*sqrt(a*x + 1)*sqrt(a*x - 1) - 16*a**4*x**4)/(24*x**4)
```

3.18 $\int e^{2\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int e^{2\coth^{-1}(ax)}x^3\,dx=\frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1-ax)}{a^4}$$

output
```
2*x/a^3+x^2/a^2+2/3*x^3/a+1/4*x^4+2*ln(-a*x+1)/a^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x^3\,dx=\frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1-ax)}{a^4}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x^3,x]
```

output
```
(2*x)/a^3 + x^2/a^2 + (2*x^3)/(3*a) + x^4/4 + (2*Log[1 - a*x])/a^4
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}x^3dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{x^3(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(-x^3-\frac{2x^2}{a}-\frac{2x}{a^2}-\frac{2}{a^3(ax-1)}-\frac{2}{a^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\log(1-ax)}{a^4}+\frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}$$

input
```
Int[E^(2*ArcCoth[a*x])*x^3,x]
```

output
```
(2*x)/a^3 + x^2/a^2 + (2*x^3)/(3*a) + x^4/4 + (2*Log[1 - a*x])/a^4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
norman	$\frac{x^2}{a^2}+\frac{x^4}{4}+\frac{2x}{a^3}+\frac{2x^3}{3a}+\frac{2\ln(ax-1)}{a^4}$	39
risch	$\frac{x^2}{a^2}+\frac{x^4}{4}+\frac{2x}{a^3}+\frac{2x^3}{3a}+\frac{2\ln(ax-1)}{a^4}$	39
default	$\frac{\frac{1}{4}a^3x^4+\frac{2}{3}a^2x^3+a\,x^2+2x}{a^3}+\frac{2\ln(ax-1)}{a^4}$	42
parallelrisch	$\frac{3a^4x^4+8a^3x^3+12a^2x^2+24ax+24\ln(ax-1)}{12a^4}$	43
meijerg	$\frac{\frac{ax\left(15a^3x^3+20a^2x^2+30ax+60\right)}{60}+\ln(-ax+1)}{a^4}-\frac{-\frac{ax\left(4a^2x^2+6ax+12\right)}{12}-\ln(-ax+1)}{a^4}$	73

input
```
int(1/(a*x-1)*(a*x+1)*x^3,x,method=_RETURNVERBOSE)
```

output
```
x^2/a^2+1/4*x^4+2*x/a^3+2/3*x^3/a+2/a^4*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int e^{2\coth^{-1}(ax)}x^3\,dx = \frac{3\,a^4x^4 + 8\,a^3x^3 + 12\,a^2x^2 + 24\,ax + 24\,\log\left(ax-1\right)}{12\,a^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="fricas")
```

output
```
1/12*(3*a^4*x^4 + 8*a^3*x^3 + 12*a^2*x^2 + 24*a*x + 24*log(a*x - 1))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}x^3\,dx = \frac{x^4}{4} + \frac{2x^3}{3a} + \frac{x^2}{a^2} + \frac{2x}{a^3} + \frac{2\log\left(ax-1\right)}{a^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**3,x)
```

output
```
x**4/4 + 2*x**3/(3*a) + x**2/a**2 + 2*x/a**3 + 2*log(a*x - 1)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x^3\,dx = \frac{3\,a^3x^4 + 8\,a^2x^3 + 12\,ax^2 + 24\,x}{12\,a^3} + \frac{2\,\log\left(ax-1\right)}{a^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="maxima")
```

output
```
1/12*(3*a^3*x^4 + 8*a^2*x^3 + 12*a*x^2 + 24*x)/a^3 + 2*log(a*x - 1)/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}x^3\,dx = \frac{3\,a^4x^4 + 8\,a^3x^3 + 12\,a^2x^2 + 24\,ax}{12\,a^4} + \frac{2\,\log\left(|ax-1|\right)}{a^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="giac")
```

output
```
1/12*(3*a^4*x^4 + 8*a^3*x^3 + 12*a^2*x^2 + 24*a*x)/a^4 + 2*log(abs(a*x - 1
))/a^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int e^{2\coth^{-1}(ax)}x^3\,dx = \frac{2\,\ln\left(a\,x-1\right)}{a^4} + \frac{2\,x}{a^3} + \frac{x^4}{4} + \frac{2\,x^3}{3\,a} + \frac{x^2}{a^2}$$

input
```
int((x^3*(a*x + 1))/(a*x - 1),x)
```

output
```
(2*log(a*x - 1))/a^4 + (2*x)/a^3 + x^4/4 + (2*x^3)/(3*a) + x^2/a^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int e^{2\coth^{-1}(ax)}x^3\,dx = \frac{24\log(ax-1) + 3a^4x^4 + 8a^3x^3 + 12a^2x^2 + 24ax}{12a^4}$$

input
```
int(1/(a*x-1)*(a*x+1)*x^3,x)
```

output
```
(24*log(a*x - 1) + 3*a**4*x**4 + 8*a**3*x**3 + 12*a**2*x**2 + 24*a*x)/(12*
a**4)
```

3.19 $\int e^{2\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int e^{2\coth^{-1}(ax)}x^2\,dx=\frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}+\frac{2\log(1-ax)}{a^3}$$

output
```
2*x/a^2+x^2/a+1/3*x^3+2*ln(-a*x+1)/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x^2\,dx=\frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}+\frac{2\log(1-ax)}{a^3}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x^2,x]
```

output
```
(2*x)/a^2 + x^2/a + x^3/3 + (2*Log[1 - a*x])/a^3
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}x^2dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{x^2(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(-x^2-\frac{2x}{a}-\frac{2}{a^2(ax-1)}-\frac{2}{a^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\log(1-ax)}{a^3}+\frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}$$

input
```
Int[E^(2*ArcCoth[a*x])*x^2,x]
```

output
```
(2*x)/a^2 + x^2/a + x^3/3 + (2*Log[1 - a*x])/a^3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

method	result	size
norman	$\frac{x^2}{a}+\frac{x^3}{3}+\frac{2x}{a^2}+\frac{2\ln(ax-1)}{a^3}$	31
risch	$\frac{x^2}{a}+\frac{x^3}{3}+\frac{2x}{a^2}+\frac{2\ln(ax-1)}{a^3}$	31
default	$\frac{\frac{1}{3}a^2x^3+a\,x^2+2x}{a^2}+\frac{2\ln(ax-1)}{a^3}$	34
parallelrisch	$\frac{a^3x^3+3a^2x^2+6ax+6\ln(ax-1)}{3a^3}$	34
meijerg	$-\frac{-\frac{ax\left(4a^2x^2+6ax+12\right)}{12}-\ln(-ax+1)}{a^3}+\frac{\frac{ax(3ax+6)}{6}+\ln(-ax+1)}{a^3}$	57

input
```
int(1/(a*x-1)*(a*x+1)*x^2,x,method=_RETURNVERBOSE)
```

output
```
x^2/a+1/3*x^3+2*x/a^2+2/a^3*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x^2\,dx = \frac{a^3x^3 + 3\,a^2x^2 + 6\,ax + 6\,\log\left(ax - 1\right)}{3\,a^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="fricas")
```

output
```
1/3*(a^3*x^3 + 3*a^2*x^2 + 6*a*x + 6*log(a*x - 1))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int e^{2\coth^{-1}(ax)}x^2\,dx = \frac{x^3}{3} + \frac{x^2}{a} + \frac{2x}{a^2} + \frac{2\log\left(ax - 1\right)}{a^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**2,x)
```

output
```
x**3/3 + x**2/a + 2*x/a**2 + 2*log(a*x - 1)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}x^2\,dx = \frac{a^2x^3 + 3\,ax^2 + 6\,x}{3\,a^2} + \frac{2\,\log\left(ax - 1\right)}{a^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="maxima")
```

output
```
1/3*(a^2*x^3 + 3*a*x^2 + 6*x)/a^2 + 2*log(a*x - 1)/a^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int e^{2\coth^{-1}(ax)}x^2\,dx = \frac{a^3x^3 + 3\,a^2x^2 + 6\,ax}{3\,a^3} + \frac{2\,\log\left(|ax-1|\right)}{a^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="giac")
```

output
```
1/3*(a^3*x^3 + 3*a^2*x^2 + 6*a*x)/a^3 + 2*log(abs(a*x - 1))/a^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int e^{2\coth^{-1}(ax)}x^2\,dx = \frac{2\,\ln\left(a\,x-1\right)}{a^3} + \frac{2\,x}{a^2} + \frac{x^3}{3} + \frac{x^2}{a}$$

input
```
int((x^2*(a*x + 1))/(a*x - 1),x)
```

output
```
(2*log(a*x - 1))/a^3 + (2*x)/a^2 + x^3/3 + x^2/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x^2\,dx = \frac{6\log(ax-1) + a^3x^3 + 3a^2x^2 + 6ax}{3a^3}$$

input
```
int(1/(a*x-1)*(a*x+1)*x^2,x)
```

output
```
(6*log(a*x - 1) + a**3*x**3 + 3*a**2*x**2 + 6*a*x)/(3*a**3)
```

3.20 $\int e^{2\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^{2\coth^{-1}(ax)}x\,dx = \frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1-ax)}{a^2}$$

output
```
2*x/a+1/2*x^2+2*ln(-a*x+1)/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x\,dx = \frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1-ax)}{a^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x,x]
```

output
```
(2*x)/a + x^2/2 + (2*Log[1 - a*x])/a^2
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}xdx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{x(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(-x-\frac{2}{a}-\frac{2}{a(ax-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\log(1-ax)}{a^2}+\frac{2x}{a}+\frac{x^2}{2}$$

input
```
Int[E^(2*ArcCoth[a*x])*x,x]
```

output
```
(2*x)/a + x^2/2 + (2*Log[1 - a*x])/a^2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
norman	$\frac{x^2}{2}+\frac{2x}{a}+\frac{2\ln(ax-1)}{a^2}$	24
risch	$\frac{x^2}{2}+\frac{2x}{a}+\frac{2\ln(ax-1)}{a^2}$	24
parallelrisch	$\frac{a^2x^2+4ax+4\ln(ax-1)}{2a^2}$	26
default	$\frac{\frac{1}{2}a\,x^2+2x}{a}+\frac{2\ln(ax-1)}{a^2}$	27
meijerg	$\frac{\frac{ax(3ax+6)}{6}+\ln(-ax+1)}{a^2}-\frac{-ax-\ln(-ax+1)}{a^2}$	43

input
```
int(1/(a*x-1)*(a*x+1)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+2*x/a+2/a^2*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{a^2x^2+4\,ax+4\,\log{(ax-1)}}{2\,a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="fricas")
```

output
```
1/2*(a^2*x^2 + 4*a*x + 4*log(a*x - 1))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{x^2}{2}+\frac{2x}{a}+\frac{2\log{(ax-1)}}{a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x,x)
```

output
```
x**2/2 + 2*x/a + 2*log(a*x - 1)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{ax^2+4\,x}{2\,a}+\frac{2\,\log{(ax-1)}}{a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="maxima")
```

output
```
1/2*(a*x^2 + 4*x)/a + 2*log(a*x - 1)/a^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{a^2x^2+4\,ax}{2\,a^2}+\frac{2\,\log\left(|ax-1|\right)}{a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="giac")
```

output
```
1/2*(a^2*x^2 + 4*a*x)/a^2 + 2*log(abs(a*x - 1))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{2\,\ln\left(a\,x-1\right)}{a^2}+\frac{2\,x}{a}+\frac{x^2}{2}$$

input
```
int((x*(a*x + 1))/(a*x - 1),x)
```

output
```
(2*log(a*x - 1))/a^2 + (2*x)/a + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int e^{2\coth^{-1}(ax)}x\,dx=\frac{4\log(ax-1)+a^2x^2+4ax}{2a^2}$$

input
```
int(1/(a*x-1)*(a*x+1)*x,x)
```

output
```
(4*log(a*x - 1) + a**2*x**2 + 4*a*x)/(2*a**2)
```

3.21 $\int e^{2\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 14

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\log(1-ax)}{a}$$

output
```
x+2*ln(-a*x+1)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\log(1-ax)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x]),x]
```

output
```
x + (2*Log[1 - a*x])/a
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6717, 6675, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{6675}$$

$$-\int \frac{ax+1}{1-ax}dx$$

$$\Big\downarrow \text{49}$$

$$-\int \left(-1-\frac{2}{ax-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\log(1-ax)}{a}+x$$

input
```
Int[E^(2*ArcCoth[a*x]),x]
```

output
```
x + (2*Log[1 - a*x])/a
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6675
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
default	$x+\frac{2\ln \left(a x -1\right)}{a}$	14
norman	$x+\frac{2\ln \left(a x -1\right)}{a}$	14
risch	$x+\frac{2\ln \left(a x -1\right)}{a}$	14
parallelrisch	$\frac{a x +2\ln \left(a x -1\right)}{a}$	17
meijerg	$-\frac{-a x -\ln \left(-a x +1\right)}{a}+\frac{\ln \left(-a x +1\right)}{a}$	32

input
```
int(1/(a*x-1)*(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
x+2/a*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int e^{2\coth^{-1}(ax)}\,dx = \frac{ax + 2\log(ax-1)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1),x, algorithm="fricas")
```

output
```
(a*x + 2*log(a*x - 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\log(ax-1)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1),x)
```

output
```
x + 2*log(a*x - 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\log(ax-1)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1),x, algorithm="maxima")
```

output
```
x + 2*log(a*x - 1)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\,\log\left(|ax-1|\right)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1),x, algorithm="giac")
```

output
```
x + 2*log(abs(a*x - 1))/a
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\,dx = x + \frac{2\,\ln\left(a\,x-1\right)}{a}$$

input
```
int((a*x + 1)/(a*x - 1),x)
```

output
```
x + (2*log(a*x - 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int e^{2\coth^{-1}(ax)}\,dx = \frac{2\log(ax-1)+ax}{a}$$

input
```
int(1/(a*x-1)*(a*x+1),x)
```

output
```
(2*log(a*x - 1) + a*x)/a
```

3.22 $\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 14

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = -\log(x) + 2\log(1 - ax)$$

output
```
-ln(x)+2*ln(-a*x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = -\log(x) + 2\log(1 - ax)$$

input
```
Integrate[E^(2*ArcCoth[a*x])/x,x]
```

output
```
-Log[x] + 2*Log[1 - a*x]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{x} \, dx$$
$$\Big\downarrow \text{6717}$$
$$-\int \frac{e^{2\text{arctanh}(ax)}}{x}dx$$
$$\Big\downarrow \text{6676}$$
$$-\int \frac{ax+1}{x(1-ax)}dx$$
$$\Big\downarrow \text{86}$$
$$-\int \left(\frac{1}{x}-\frac{2a}{ax-1}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$2\log(1-ax)-\log(x)$$

input
```
Int[E^(2*ArcCoth[a*x])/x,x]
```

output
```
-Log[x] + 2*Log[1 - a*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
default	$2\ln(ax-1)-\ln(x)$	14
norman	$2\ln(ax-1)-\ln(x)$	14
parallelrisch	$2\ln(ax-1)-\ln(x)$	14
risch	$-\ln(x)+2\ln(-ax+1)$	15
meijerg	$-\ln(x)-\ln(-a)+2\ln(-ax+1)$	21

input
```
int((a*x+1)/x/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
2*ln(a*x-1)-ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = 2\,\log\left(ax-1\right) - \log\left(x\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="fricas")
```

output
```
2*log(a*x - 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = -\log\left(x\right) + 2\log\left(x - \frac{1}{a}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x,x)
```

output
```
-log(x) + 2*log(x - 1/a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = 2\,\log\left(ax-1\right) - \log\left(x\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="maxima")
```

output
```
2*log(a*x - 1) - log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = 2\,\log\left(|ax-1|\right) - \log\left(|x|\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="giac")
```

output
```
2*log(abs(a*x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 23.63 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = 2\,\ln\left(3-3\,a\,x\right) - \ln\left(x\right)$$

input
```
int((a*x + 1)/(x*(a*x - 1)),x)
```

output
```
2*log(3 - 3*a*x) - log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx = 2\log(ax-1) - \log(x)$$

input
```
int(1/(a*x-1)*(a*x+1)/x,x)
```

output
```
2*log(a*x - 1) - log(x)
```

3.23 $\int \frac{e^{2\coth^{-1}(ax)}}{x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2} \, dx = \frac{1}{x} - 2a\log(x) + 2a\log(1-ax)$$

output
```
1/x-2*a*ln(x)+2*a*ln(-a*x+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2} \, dx = \frac{1}{x} - 2a\log(x) + 2a\log(1-ax)$$

input
```
Integrate[E^(2*ArcCoth[a*x])/x^2,x]
```

output
```
x^(-1) - 2*a*Log[x] + 2*a*Log[1 - a*x]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6676}$$

$$-\int \frac{ax+1}{x^2(1-ax)}dx$$

$$\Big\downarrow \text{ 86}$$

$$-\int \left(-\frac{2a^2}{ax-1}+\frac{2a}{x}+\frac{1}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-2a\log(x)+2a\log(1-ax)+\frac{1}{x}$$

input
```
Int[E^(2*ArcCoth[a*x])/x^2,x]
```

output
```
x^(-1) - 2*a*Log[x] + 2*a*Log[1 - a*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

method	result	size
default	$2a\ln(ax-1)+\frac{1}{x}-2\ln(x)\,a$	19
norman	$2a\ln(ax-1)+\frac{1}{x}-2\ln(x)\,a$	19
risch	$\frac{1}{x}-2\ln(x)\,a+2a\ln(-ax+1)$	20
parallelrisch	$-\frac{2a\ln(x)x-2a\ln(ax-1)x-1}{x}$	24
meijerg	$-a(\ln(x)+\ln(-a)-\ln(-ax+1))+a\left(\frac{1}{ax}-\ln(x)-\ln(-a)+\ln(-ax+1)\right)$	48

input
```
int(1/(a*x-1)*(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2*a*ln(a*x-1)+1/x-2*ln(x)*a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx = \frac{2\,ax\log\left(ax-1\right) - 2\,ax\log\left(x\right) + 1}{x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="fricas")
```

output
```
(2*a*x*log(a*x - 1) - 2*a*x*log(x) + 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx = 2a\left(-\log\left(x\right) + \log\left(x - \frac{1}{a}\right)\right) + \frac{1}{x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x**2,x)
```

output
```
2*a*(-log(x) + log(x - 1/a)) + 1/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx = 2\,a\log\left(ax-1\right) - 2\,a\log\left(x\right) + \frac{1}{x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="maxima")
```

output
```
2*a*log(a*x - 1) - 2*a*log(x) + 1/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx = 2\,a\log\left(|ax-1|\right) - 2\,a\log\left(|x|\right) + \frac{1}{x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="giac")
```

output
```
2*a*log(abs(a*x - 1)) - 2*a*log(abs(x)) + 1/x
```

Mupad [B] (verification not implemented)

Time = 23.56 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{x} - 4\,a\,\mathrm{atanh}(2\,a\,x-1)$$

input
```
int((a*x + 1)/(x^2*(a*x - 1)),x)
```

output
```
1/x - 4*a*atanh(2*a*x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx = \frac{2\log(ax-1)\,ax - 2\log(x)\,ax + 1}{x}$$

input
```
int(1/(a*x-1)*(a*x+1)/x^2,x)
```

output
```
(2*log(a*x - 1)*a*x - 2*log(x)*a*x + 1)/x
```

3.24 $\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2x^2} + \frac{2a}{x} - 2a^2\log(x) + 2a^2\log(1-ax)$$

output
```
1/2/x^2+2*a/x-2*a^2*ln(x)+2*a^2*ln(-a*x+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2x^2} + \frac{2a}{x} - 2a^2\log(x) + 2a^2\log(1-ax)$$

input
```
Integrate[E^(2*ArcCoth[a*x])/x^3,x]
```

output
```
1/(2*x^2) + (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 - a*x]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{ 6676}$$

$$-\int \frac{ax+1}{x^3(1-ax)}dx$$

$$\Big\downarrow \text{ 86}$$

$$-\int \left(-\frac{2a^3}{ax-1}+\frac{2a^2}{x}+\frac{2a}{x^2}+\frac{1}{x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-2a^2\log(x)+2a^2\log(1-ax)+\frac{2a}{x}+\frac{1}{2x^2}$$

input
```
Int[E^(2*ArcCoth[a*x])/x^3,x]
```

output
```
1/(2*x^2) + (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 - a*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result
norman	$\frac{\frac{1}{2}+2ax}{x^2}-2a^2\ln(x)+2a^2\ln(ax-1)$
default	$2a^2\ln(ax-1)+\frac{1}{2x^2}+\frac{2a}{x}-2a^2\ln(x)$
risch	$\frac{\frac{1}{2}+2ax}{x^2}-2a^2\ln(x)+2a^2\ln(-ax+1)$
parallelrisch	$-\frac{4a^2\ln(x)x^2-4a^2\ln(ax-1)x^2-1-4ax}{2x^2}$
meijerg	$a^2\left(\frac{1}{ax}-\ln(x)-\ln(-a)+\ln(-ax+1)\right)-a^2\left(-\frac{1}{2a^2x^2}-\frac{1}{ax}+\ln(x)+\ln(-a)-\ln(-ax\right.$

input
```
int(1/(a*x-1)*(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2+2*a*x)/x^2-2*a^2*ln(x)+2*a^2*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = \frac{4\,a^2x^2\log(ax-1) - 4\,a^2x^2\log(x) + 4\,ax + 1}{2\,x^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="fricas")
```

output
```
1/2*(4*a^2*x^2*log(a*x - 1) - 4*a^2*x^2*log(x) + 4*a*x + 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = 2a^2\left(-\log(x) + \log\left(x - \frac{1}{a}\right)\right) + \frac{4ax+1}{2x^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x**3,x)
```

output
```
2*a**2*(-log(x) + log(x - 1/a)) + (4*a*x + 1)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = 2\,a^2\log(ax-1) - 2\,a^2\log(x) + \frac{4\,ax+1}{2\,x^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="maxima")
```

output
```
2*a^2*log(a*x - 1) - 2*a^2*log(x) + 1/2*(4*a*x + 1)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = 2\,a^2\log\left(|ax-1|\right) - 2\,a^2\log\left(|x|\right) + \frac{4\,ax+1}{2\,x^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="giac")
```

output
```
2*a^2*log(abs(a*x - 1)) - 2*a^2*log(abs(x)) + 1/2*(4*a*x + 1)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = \frac{2\,a\,x+\frac{1}{2}}{x^2} - 4\,a^2\,\mathrm{atanh}(2\,a\,x-1)$$

input
```
int((a*x + 1)/(x^3*(a*x - 1)),x)
```

output
```
(2*a*x + 1/2)/x^2 - 4*a^2*atanh(2*a*x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = \frac{4\log(ax-1)\,a^2x^2 - 4\log(x)\,a^2x^2 + 4ax + 1}{2x^2}$$

input
```
int(1/(a*x-1)*(a*x+1)/x^3,x)
```

output
```
(4*log(a*x - 1)*a**2*x**2 - 4*log(x)*a**2*x**2 + 4*a*x + 1)/(2*x**2)
```

3.25 $\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(1-ax)$$

output
```
1/3/x^3+a/x^2+2*a^2/x-2*a^3*ln(x)+2*a^3*ln(-a*x+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(1-ax)$$

input
```
Integrate[E^(2*ArcCoth[a*x])/x^4,x]
```

output
```
1/(3*x^3) + a/x^2 + (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 - a*x]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{ax+1}{x^4(1-ax)}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(-\frac{2a^4}{ax-1}+\frac{2a^3}{x}+\frac{2a^2}{x^2}+\frac{2a}{x^3}+\frac{1}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-2a^3\log(x)+2a^3\log(1-ax)+\frac{2a^2}{x}+\frac{a}{x^2}+\frac{1}{3x^3}$$

input
```
Int[E^(2*ArcCoth[a*x])/x^4,x]
```

output
```
1/(3*x^3) + a/x^2 + (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 - a*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

method	result
norman	$\frac{\frac{1}{3}+2a^2x^2+ax}{x^3}-2a^3\ln(x)+2a^3\ln(ax-1)$
default	$2a^3\ln(ax-1)+\frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\ln(x)$
risch	$\frac{\frac{1}{3}+2a^2x^2+ax}{x^3}-2a^3\ln(x)+2a^3\ln(-ax+1)$
parallelrisch	$-\frac{6\ln(x)x^3a^3-6a^3\ln(ax-1)x^3-1-6a^2x^2-3ax}{3x^3}$
meijerg	$-a^3\left(-\frac{1}{2a^2x^2}-\frac{1}{ax}+\ln(x)+\ln(-a)-\ln(-ax+1)\right)+a^3\left(\frac{1}{3x^3a^3}+\frac{1}{2a^2x^2}+\frac{1}{ax}-\ln(x)-\ln\right.$

input
```
int(1/(a*x-1)*(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
(1/3+2*a^2*x^2+a*x)/x^3-2*a^3*ln(x)+2*a^3*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = \frac{6\,a^3x^3\log(ax-1) - 6\,a^3x^3\log(x) + 6\,a^2x^2 + 3\,ax + 1}{3\,x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="fricas")
```

output
```
1/3*(6*a^3*x^3*log(a*x - 1) - 6*a^3*x^3*log(x) + 6*a^2*x^2 + 3*a*x + 1)/x^
3
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = 2a^3\left(-\log(x) + \log\left(x - \frac{1}{a}\right)\right) + \frac{6a^2x^2 + 3ax + 1}{3x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x**4,x)
```

output
```
2*a**3*(-log(x) + log(x - 1/a)) + (6*a**2*x**2 + 3*a*x + 1)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = 2\,a^3\log(ax-1) - 2\,a^3\log(x) + \frac{6\,a^2x^2 + 3\,ax + 1}{3\,x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="maxima")
```

output
```
2*a^3*log(a*x - 1) - 2*a^3*log(x) + 1/3*(6*a^2*x^2 + 3*a*x + 1)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = 2\,a^3\log\left(|ax-1|\right) - 2\,a^3\log\left(|x|\right) + \frac{6\,a^2x^2+3\,ax+1}{3\,x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="giac")
```

output
```
2*a^3*log(abs(a*x - 1)) - 2*a^3*log(abs(x)) + 1/3*(6*a^2*x^2 + 3*a*x + 1)/
x^3
```

Mupad [B] (verification not implemented)

Time = 23.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = \frac{2\,a^2\,x^2+a\,x+\frac{1}{3}}{x^3} - 4\,a^3\,\mathrm{atanh}(2\,a\,x-1)$$

input
```
int((a*x + 1)/(x^4*(a*x - 1)),x)
```

output
```
(a*x + 2*a^2*x^2 + 1/3)/x^3 - 4*a^3*atanh(2*a*x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = \frac{6\log(ax-1)\,a^3x^3-6\log(x)\,a^3x^3+6a^2x^2+3ax+1}{3x^3}$$

input
```
int(1/(a*x-1)*(a*x+1)/x^4,x)
```

output
```
(6*log(a*x - 1)*a**3*x**3 - 6*log(x)*a**3*x**3 + 6*a**2*x**2 + 3*a*x + 1)/
(3*x**3)
```

3.26 $\int e^{3\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 118

$$\int e^{3\coth^{-1}(ax)}x^2\,dx=-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{11\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

output
```
-4*(1-1/a^2/x^2)^(1/2)/a^2/(a-1/x)+14/3*(1-1/a^2/x^2)^(1/2)*x/a^2+3/2*(1-1
/a^2/x^2)^(1/2)*x^2/a+1/3*(1-1/a^2/x^2)^(1/2)*x^3+11/2*arctanh((1-1/a^2/x^
2)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

$$\int e^{3\coth^{-1}(ax)}x^2\,dx=\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-52+19ax+7a^2x^2+2a^3x^3)}{-1+ax}+33\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{6a^3}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x^2,x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-52 + 19*a*x + 7*a^2*x^2 + 2*a^3*x^3))/(-1 +
a*x) + 33*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(6*a^3)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6719, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6719}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^2 x^4}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 2353}$$

$$-\int \left(\frac{x^4}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4}{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{14x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{11\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

input
```
Int[E^(3*ArcCoth[a*x])*x^2,x]
```

output
```
(-4*Sqrt[1 - 1/(a^2*x^2)])/(a^2*(a - x^(-1))) + (14*Sqrt[1 - 1/(a^2*x^2)]*
x)/(3*a^2) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (Sqrt[1 - 1/(a^2*x^2)]*
x^3)/3 + (11*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.28

method	result
risch	$\frac{\left(2 a^{2} x^{2}+9 a x +28\right) \left(a x -1\right)}{6 a^{3} \sqrt{\frac{a x -1}{a x +1}}}+\frac{\left(\frac{11 \ln \left(\frac{a^{2} x}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{2}-1}\right)}{2 a^{2} \sqrt{a^{2}}}-\frac{4 \sqrt{\left(x -\frac{1}{a}\right)^{2} a^{2}+2 a \left(x -\frac{1}{a}\right)}}{a^{4} \left(x -\frac{1}{a}\right)}\right) \sqrt{\left(a x -1\right) \left(a x +1\right)}}{\sqrt{\frac{a x -1}{a x +1}}\, \left(a x +1\right)}$
default	$\frac{9 \sqrt{a^{2}}\, \sqrt{a^{2} x^{2}-1}\, a^{3} x^{3}+2 \sqrt{a^{2}}\, \left(\left(a x -1\right) \left(a x +1\right)\right)^{\frac{3}{2}} a^{2} x^{2}-18 \sqrt{a^{2}}\, \sqrt{a^{2} x^{2}-1}\, a^{2} x^{2}-9 \ln \left(\frac{a^{2} x +\sqrt{a^{2} x^{2}-1}\, \sqrt{a^{2}}}{\sqrt{a^{2}}}\right) a^{3} x^{2}-4 \sqrt{a^{2}}\, ((a x -1)(a x +}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*x^2+9*a*x+28)*(a*x-1)/a^3/((a*x-1)/(a*x+1))^(1/2)+(11/2/a^2*ln(
a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-4/a^4/(x-1/a)*((x-1/a)^2*
a^2+2*a*(x-1/a))^(1/2))/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a
*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.95

$$\int e^{3\coth^{-1}(ax)}x^2\,dx = \frac{33\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-33\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(2\,a^4x^4+9\,a^3x^3+26\,a^2x^2-33\,ax-52}{6\,(a^4x-a^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="fricas")
```

output
```
1/6*(33*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 33*(a*x - 1)*log(sq
rt((a*x - 1)/(a*x + 1)) - 1) + (2*a^4*x^4 + 9*a^3*x^3 + 26*a^2*x^2 - 33*a*
x - 52)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^2\,dx = \int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2,x)
```

output
```
Integral(x**2/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.54

$$\int e^{3\coth^{-1}(ax)}x^2\,dx = -\frac{1}{6}\,a\left(\frac{2\left(\frac{75\,(ax-1)}{ax+1}-\frac{88\,(ax-1)^2}{(ax+1)^2}+\frac{33\,(ax-1)^3}{(ax+1)^3}-12\right)}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^4\sqrt{\frac{ax-1}{ax+1}}}-\frac{33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}+\frac{33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}\right.}{a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="maxima")
```

output
```
-1/6*a*(2*(75*(a*x - 1)/(a*x + 1) - 88*(a*x - 1)^2/(a*x + 1)^2 + 33*(a*x -
 1)^3/(a*x + 1)^3 - 12)/(a^4*((a*x - 1)/(a*x + 1))^(7/2) - 3*a^4*((a*x - 1
)/(a*x + 1))^(5/2) + 3*a^4*((a*x - 1)/(a*x + 1))^(3/2) - a^4*sqrt((a*x - 1
)/(a*x + 1))) - 33*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^4 + 33*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^4)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}x^2\,dx=\int\frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 23.55 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.31

$$\int e^{3\coth^{-1}(ax)}x^2\,dx=\frac{11\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}-\frac{\frac{88\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}-\frac{11\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{25\,\left(a\,x-1\right)}{a\,x+1}+4}{a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}-3\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}+3\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

input
```
int(x^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(11*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^3 - ((88*(a*x - 1)^2)/(3*(a*x +
1)^2) - (11*(a*x - 1)^3)/(a*x + 1)^3 - (25*(a*x - 1))/(a*x + 1) + 4)/(a^3*
((a*x - 1)/(a*x + 1))^(1/2) - 3*a^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*a^3*((
a*x - 1)/(a*x + 1))^(5/2) - a^3*((a*x - 1)/(a*x + 1))^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int e^{3\coth^{-1}(ax)}x^2\,dx$$
$$= \frac{264\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)-153\sqrt{ax-1}+8\sqrt{ax+1}\,a^3x^3+28\sqrt{ax+1}\,a^2x^2+76\sqrt{ax+1}\,ax-2}{24\sqrt{ax-1}\,a^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x)
```

output
```
(264*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) - 153*sqrt
(a*x - 1) + 8*sqrt(a*x + 1)*a**3*x**3 + 28*sqrt(a*x + 1)*a**2*x**2 + 76*sq
rt(a*x + 1)*a*x - 208*sqrt(a*x + 1))/(24*sqrt(a*x - 1)*a**3)
```

3.27 $\int e^{3\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 92

$$\int e^{3\coth^{-1}(ax)}x\,dx = -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{9\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

output
```
-4*(1-1/a^2/x^2)^(1/2)/a/(a-1/x)+3*(1-1/a^2/x^2)^(1/2)*x/a+1/2*(1-1/a^2/x^
2)^(1/2)*x^2+9/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72

$$\int e^{3\coth^{-1}(ax)}x\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}x(-14+5ax+a^2x^2)}{-1+ax}+9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{2a^2}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x,x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-14 + 5*a*x + a^2*x^2))/(-1 + a*x) + 9*Log[(1
 + Sqrt[1 - 1/(a^2*x^2)])*x])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6719, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^2 x^3}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{2353}$$

$$-\int \left(\frac{x^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4}{a^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{9\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}$$

input
```
Int[E^(3*ArcCoth[a*x])*x,x]
```

output
```
(-4*Sqrt[1 - 1/(a^2*x^2)])/(a*(a - x^(-1))) + (3*Sqrt[1 - 1/(a^2*x^2)]*x)/
a + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (9*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*
a^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.54

method	result
risch	$\frac{(ax+6)(ax-1)}{2a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2a\sqrt{a^2}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{a^3\left(x-\frac{1}{a}\right)}\right)\sqrt{(ax-1)(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^3x^3-10\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^2x^2+2\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^2x^2+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-10\ln\left(\frac{a^2x+\sqrt{(ax-}{}\right.}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x+6)*(a*x-1)/a^2/((a*x-1)/(a*x+1))^(1/2)+(9/2/a*ln(a^2*x/(a^2)^(1/2
)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-4/a^3/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))
^(1/2))/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.12

$$\int e^{3\coth^{-1}(ax)}x\,dx = \frac{9\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^3x^3+6\,a^2x^2-9\,ax-14)\sqrt{\frac{ax-1}{ax+1}}}{2\,(a^3x-a^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="fricas")
```

output
```
1/2*(9*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a*x - 1)*log(sqrt
((a*x - 1)/(a*x + 1)) - 1) + (a^3*x^3 + 6*a^2*x^2 - 9*a*x - 14)*sqrt((a*x
- 1)/(a*x + 1)))/(a^3*x - a^2)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x\,dx = \int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x,x)
```

output
```
Integral(x/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.58

$$\int e^{3\coth^{-1}(ax)}x\,dx = \frac{1}{2}\,a\left(\frac{2\left(\frac{15\,(ax-1)}{ax+1}-\frac{9\,(ax-1)^2}{(ax+1)^2}-4\right)}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-2\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="maxima")
```

output
```
1/2*a*(2*(15*(a*x - 1)/(a*x + 1) - 9*(a*x - 1)^2/(a*x + 1)^2 - 4)/(a^3*((a
*x - 1)/(a*x + 1))^(5/2) - 2*a^3*((a*x - 1)/(a*x + 1))^(3/2) + a^3*sqrt((a
*x - 1)/(a*x + 1))) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^3 - 9*log(sqr
t((a*x - 1)/(a*x + 1)) - 1)/a^3)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}x\,dx = \int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.27

$$\int e^{3\coth^{-1}(ax)}x\,dx = \frac{9\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2} - \frac{\frac{9\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{15\,(a\,x-1)}{a\,x+1} + 4}{a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}} - 2\,a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} + a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(x/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(9*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^2 - ((9*(a*x - 1)^2)/(a*x + 1)^2
- (15*(a*x - 1))/(a*x + 1) + 4)/(a^2*((a*x - 1)/(a*x + 1))^(1/2) - 2*a^2*(
(a*x - 1)/(a*x + 1))^(3/2) + a^2*((a*x - 1)/(a*x + 1))^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int e^{3\coth^{-1}(ax)}x\,dx$$
$$= \frac{18\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)-11\sqrt{ax-1}+\sqrt{ax+1}\,a^2x^2+5\sqrt{ax+1}\,ax-14\sqrt{ax+1}}{2\sqrt{ax-1}\,a^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x,x)
```

output
```
(18*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) - 11*sqrt(a
*x - 1) + sqrt(a*x + 1)*a**2*x**2 + 5*sqrt(a*x + 1)*a*x - 14*sqrt(a*x + 1)
)/(2*sqrt(a*x - 1)*a**2)
```

3.28 $\int e^{3\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 62

$$\int e^{3\coth^{-1}(ax)}\,dx=-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-4*(1-1/a^2/x^2)^(1/2)/(a-1/x)+(1-1/a^2/x^2)^(1/2)*x+3*arctanh((1-1/a^2/x^
2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int e^{3\coth^{-1}(ax)}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-5+ax)}{-1+ax}+\frac{3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}$$

input
```
Integrate[E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(-5 + a*x))/(-1 + a*x) + (3*Log[a*(1 + Sqrt[1 - 1
/(a^2*x^2)])*x])/a
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6718, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6718}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^2 x^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{2353}$$

$$-\int \left(\frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4}{a\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+x\sqrt{1-\frac{1}{a^2x^2}}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}$$

input
```
Int[E^(3*ArcCoth[a*x]),x]
```

output
```
(-4*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) + Sqrt[1 - 1/(a^2*x^2)]*x + (3*Arc
Tanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 6718
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(56) = 112$.

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.15

method	result
risch	$\frac{ax-1}{a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{a^2\left(x-\frac{1}{a}\right)}\right)\sqrt{(ax-1)(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-3\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^2x^2-3\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2+2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}+6\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,ax+6\ln}{a\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right.}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/((a*x-1)/(a*x+1))^(1/2)+(3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1
/2))/(a^2)^(1/2)-4/a^2/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2))/((a*x-1)
/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int e^{3\coth^{-1}(ax)}\,dx = \frac{3\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2x^2-4\,ax-5)\sqrt{\frac{ax-1}{ax+1}}}{a^2x-a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
(3*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a*x - 1)*log(sqrt((a*
x - 1)/(a*x + 1)) - 1) + (a^2*x^2 - 4*a*x - 5)*sqrt((a*x - 1)/(a*x + 1)))/
(a^2*x - a)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\,dx = \int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.77

$$\int e^{3\coth^{-1}(ax)}\,dx = -a\left(\frac{2\left(\frac{3\,(ax-1)}{ax+1}-2\right)}{a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-a*(2*(3*(a*x - 1)/(a*x + 1) - 2)/(a^2*((a*x - 1)/(a*x + 1))^(3/2) - a^2*s
qrt((a*x - 1)/(a*x + 1))) - 3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 3*l
og(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\,dx=\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 23.41 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int e^{3\coth^{-1}(ax)}\,dx=\frac{2\,a\,x+12\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-10}{2\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(1/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*a*x + 12*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2)
 - 10)/(2*a*((a*x - 1)/(a*x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int e^{3\coth^{-1}(ax)}\,dx = \frac{12\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)-9\sqrt{ax-1}+2\sqrt{ax+1}\,ax-10\sqrt{ax+1}}{2\sqrt{ax-1}\,a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(12*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) - 9*sqrt(a*
x - 1) + 2*sqrt(a*x + 1)*a*x - 10*sqrt(a*x + 1))/(2*sqrt(a*x - 1)*a)
```

3.29 $\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx=-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\csc^{-1}(ax)+\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

output
```
-4*a*(1-1/a^2/x^2)^(1/2)/(a-1/x)+arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx=-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}x}{-1+ax}+\arcsin\left(\frac{1}{ax}\right)+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)$$

input
```
Integrate[E^(3*ArcCoth[a*x])/x,x]
```

output
```
(-4*a*Sqrt[1 - 1/(a^2*x^2)]*x)/(-1 + a*x) + ArcSin[1/(a*x)] + Log[(1 + Sqr
t[1 - 1/(a^2*x^2)])*x]
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6719, 2351, 27, 564, 25, 27, 243, 73, 221, 671, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^2 x}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{2351}$$

$$-\int \frac{\frac{2}{a}+\frac{1}{xa^2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}-\int \frac{ax}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{\frac{2}{a}+\frac{1}{xa^2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}-a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{564}$$

$$-\int \frac{\frac{2}{a}+\frac{1}{xa^2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\int -\frac{x}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\frac{2}{a}+\frac{1}{xa^2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\int \frac{x}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}\right)$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{\frac{2}{a}+\frac{1}{xa^2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}\right)$$

$$\Big\downarrow \text{243}$$

$$-\int \frac{\frac{2}{a}+\frac{1}{xa^2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}\right)$$

$$\Big\downarrow \text{73}$$

$$-a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-a\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{\frac{2}{a}+\frac{1}{xa^2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{221}$$

$$-\int \frac{\frac{2}{a}+\frac{1}{xa^2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}\right)$$

$$\Big\downarrow \text{671}$$

$$\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\left(a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}\right)\right)-\frac{3a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}$$

$$\Big\downarrow \text{223}$$

$$-\left(a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}\right)\right)-\frac{3a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\arcsin\left(\frac{1}{ax}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])/x,x]
```

output
```
(-3*a*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) + ArcSin[1/(a*x)] - a*(Sqrt[1 -
1/(a^2*x^2)]/(a - x^(-1)) - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(42) = 84$.

Time = 0.09 (sec) , antiderivative size = 372, normalized size of antiderivative = 8.09

method	result
default	$-\frac{-\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^2x^2-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^2x^2-a^2\sqrt{a^2}\,x^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2+2((ax\cdots}{\cdots}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-(-((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*a^2*x^2-(a^2)^(1/2)*(a^2*x^2-1)^(1/
2)*a^2*x^2-a^2*(a^2)^(1/2)*x^2*arctan(1/(a^2*x^2-1)^(1/2))-ln((a^2*x+((a*x
-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^3*x^2+2*((a*x-1)*(a*x+1))^(
3/2)*(a^2)^(1/2)+2*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*a*x+2*(a^2)^(1/2)*(
a^2*x^2-1)^(1/2)*a*x+2*a*(a^2)^(1/2)*x*arctan(1/(a^2*x^2-1)^(1/2))+2*ln((a
^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x-((a*x-1)*(a*x
+1))^(1/2)*(a^2)^(1/2)-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)-arctan(1/(a^2*x^2-1)^
(1/2))*(a^2)^(1/2)-a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^
(1/2)))/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(42) = 84$.

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.26

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = -\frac{2\,(ax-1)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - (ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + (ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right) + 4\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

output
```
-(2*(a*x - 1)*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*x - 1)*log(sqrt((a*x
- 1)/(a*x + 1)) + 1) + (a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + 4*(a
*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/x,x)
```

output
```
Integral(1/(x*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(42) = 84$.

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.96

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = -a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a} + \frac{4}{a\sqrt{\frac{ax-1}{ax+1}}}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

output
```
-a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a - log(sqrt((a*x - 1)/(a*x + 1))
+ 1)/a + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a + 4/(a*sqrt((a*x - 1)/(a*x +
 1))))
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 23.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - 2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - \frac{4}{\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(1/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
2*atanh(((a*x - 1)/(a*x + 1))^(1/2)) - 2*atan(((a*x - 1)/(a*x + 1))^(1/2))
 - 4/((a*x - 1)/(a*x + 1))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.13

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = \frac{-2\sqrt{ax-1}\,atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)+2\sqrt{ax-1}\,atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)+2\sqrt{ax-1}\log}{\sqrt{ax-1}}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/x,x)
```

output
```
(2*( - sqrt(a*x - 1)*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1) + sqrt(a*x -
1)*atan(sqrt(a*x - 1) + sqrt(a*x + 1) + 1) + sqrt(a*x - 1)*log((sqrt(a*x -
 1) + sqrt(a*x + 1))/sqrt(2)) - 2*sqrt(a*x - 1) - 2*sqrt(a*x + 1)))/sqrt(a
*x - 1)
```

3.30 $\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 53

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = -a\sqrt{1-\frac{1}{a^2x^2}} - \frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + 3a\csc^{-1}(ax)$$

output
```
-a*(1-1/a^2/x^2)^(1/2)-4*a^2*(1-1/a^2/x^2)^(1/2)/(a-1/x)+3*a*arccsc(a*x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}(1-5ax)}{-1+ax} + 3a\arcsin\left(\frac{1}{ax}\right)$$

input
```
Integrate[E^(3*ArcCoth[a*x])/x^2,x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*(1 - 5*a*x))/(-1 + a*x) + 3*a*ArcSin[1/(a*x)]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6719, 711, 25, 27, 671, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{711}$$

$$a^4\int -\frac{a+\frac{3}{x}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}-a\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$a^4\left(-\int \frac{a+\frac{3}{x}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}\right)-a\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$a\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{a+\frac{3}{x}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{671}$$

$$3\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-a\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-a\sqrt{1-\frac{1}{a^2x^2}}+3a\arcsin\left(\frac{1}{ax}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])/x^2,x]
```

output
```
-(a*Sqrt[1 - 1/(a^2*x^2)]) - (4*a^2*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) +
3*a*ArcSin[1/(a*x)]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 711
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + c*x^2)^(p + 1)
/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m + n + 2*p + 1))
Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c*e^n*(m + n + 2*p + 1)*(f + g*x)
^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - 2*e*g^n*(m + p + n)*(d + e*x)^(n
 - 2)*(a*e - c*d*x), x], x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && Eq
Q[c*d^2 + a*e^2, 0] && IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(49) = 98$.

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.19

method	result
risch	$-\frac{ax-1}{x\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(3a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}\right)\sqrt{(ax-1)(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^4x^4+\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^2x^2+5\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^3x^3+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^4x^3+3a^3\sqrt{a^2}\,x^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right.}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)/x/((a*x-1)/(a*x+1))^(1/2)+(3*a*arctan(1/(a^2*x^2-1)^(1/2))-4/(x-1
/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2))/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a
*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = -\frac{6\,(a^2x^2-ax)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+(5\,a^2x^2+4\,ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax^2-x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-(6*(a^2*x^2 - a*x)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (5*a^2*x^2 + 4*a*x
 - 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

output
```
Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = -2\,a\left(\frac{\frac{3\,(ax-1)}{ax+1}+2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\sqrt{\frac{ax-1}{ax+1}}}+3\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
-2*a*((3*(a*x - 1)/(a*x + 1) + 2)/(((a*x - 1)/(a*x + 1))^(3/2) + sqrt((a*x
 - 1)/(a*x + 1))) + 3*arctan(sqrt((a*x - 1)/(a*x + 1))))
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 23.44 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{x\sqrt{\frac{a\,x-1}{a\,x+1}}} - 6\,a\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - \frac{5\,a}{\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(1/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
1/(x*((a*x - 1)/(a*x + 1))^(1/2)) - 6*a*atan(((a*x - 1)/(a*x + 1))^(1/2))
- (5*a)/((a*x - 1)/(a*x + 1))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = \frac{-6\sqrt{ax-1}\,atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)ax + 6\sqrt{ax-1}\,atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)ax - 3\sqrt{ax}}{\sqrt{ax-1}\,x}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

output
```
( - 6*sqrt(a*x - 1)*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a*x + 6*sqrt(a
*x - 1)*atan(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a*x - 3*sqrt(a*x - 1)*a*x
- 5*sqrt(a*x + 1)*a*x + sqrt(a*x + 1))/(sqrt(a*x - 1)*x)
```

3.31 $\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 81

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = -3a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{4a^3\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} - \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x} + \frac{9}{2}a^2\csc^{-1}(ax)$$

output
```
-3*a^2*(1-1/a^2/x^2)^(1/2)-4*a^3*(1-1/a^2/x^2)^(1/2)/(a-1/x)-1/2*a*(1-1/a^
2/x^2)^(1/2)/x+9/2*a^2*arccsc(a*x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(1+5ax-14a^2x^2)}{x(-1+ax)} + 9a\arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[E^(3*ArcCoth[a*x])/x^3,x]
```

output
```
(a*((Sqrt[1 - 1/(a^2*x^2)]*(1 + 5*a*x - 14*a^2*x^2))/(x*(-1 + a*x)) + 9*a*
ArcSin[1/(a*x)]))/2
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.22, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {6719, 2164, 25, 27, 2027, 2164, 25, 27, 563, 25, 2346, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \ 6719$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)x}d\frac{1}{x}$$

$$\Big\downarrow \ 2164$$

$$\frac{\int -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x}+\frac{1}{x^2}\right)}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x}+\frac{1}{x^2}\right)}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \ 27$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x}+\frac{1}{x^2}\right)}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \ 2027$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}{\left(a-\frac{1}{x}\right)^2x}d\frac{1}{x}$$

$$\Big\downarrow \ 2164$$

$$a^2\int -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3x}d\frac{1}{x}$$

$$\Big\downarrow \ 25$$

$$-a^2\int \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3 x}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3 x}d\frac{1}{x}$$

$$\Big\downarrow \text{563}$$

$$-a^3\left(\frac{\int -\frac{4a^2+\frac{3a}{x}+\frac{1}{x^2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}\right)$$

$$\Big\downarrow \text{25}$$

$$-a^3\left(\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\int \frac{4a^2+\frac{3a}{x}+\frac{1}{x^2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}\right)$$

$$\Big\downarrow \text{2346}$$

$$-a^3\left(\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{-\frac{1}{2}a^2\int -\frac{3\left(3a+\frac{2}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}}{a^4}\right)$$

$$\Big\downarrow \text{27}$$

$$-a^3\left(\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{3}{2}a\int \frac{3a+\frac{2}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}}{a^4}\right)$$

$$\Big\downarrow \text{455}$$

$$-a^3\left(\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{3}{2}a\left(3a\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-2a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}}{a^4}\right)$$

$$\Big\downarrow \text{223}$$

$$-a^3\left(\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{3}{2}a\left(3a^2\arcsin\left(\frac{1}{ax}\right)-2a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}}{a^4}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])/x^3,x]
```

output
```
-(a^3*((4*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) - (-1/2*(a^2*Sqrt[1 - 1/(a^2
*x^2)])/x + (3*a*(-2*a^2*Sqrt[1 - 1/(a^2*x^2)] + 3*a^2*ArcSin[1/(a*x)]))/2
)/a^4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2164
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + b*d*x, x]*
(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x]
 && EqQ[b*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + b*d*x, x], 0
]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.54

method	result
risch	$-\frac{(ax-1)(6ax+1)}{2x^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{9a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}-\frac{4a\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}\right)\sqrt{(ax-1)(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{6\sqrt{a^2}\sqrt{a^2x^2-1}\,a^5x^5+6\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^4x^4-6\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^3x^3-21\sqrt{a^2}\sqrt{a^2x^2-1}\,a^4x^4-9a^4\sqrt{a^2}\,x^4\arctan\left(\frac{1}{\sqrt{a^2x^2}}\right.}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x-1)*(6*a*x+1)/x^2/((a*x-1)/(a*x+1))^(1/2)+(9/2*a^2*arctan(1/(a^2*
x^2-1)^(1/2))-4*a/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2))/((a*x-1)/(a*x
+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = -\frac{18\,(a^3x^3-a^2x^2)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+(14\,a^3x^3+9\,a^2x^2-6\,ax-1)\sqrt{\frac{ax-1}{ax+1}}}{2\,(ax^3-x^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(18*(a^3*x^3 - a^2*x^2)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (14*a^3*x
^3 + 9*a^2*x^2 - 6*a*x - 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

output
```
Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.36

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = -\left(9\,a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \frac{\frac{15\,(ax-1)a}{ax+1} + \frac{9\,(ax-1)^2a}{(ax+1)^2} + 4\,a}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + \sqrt{\frac{ax-1}{ax+1}}}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-(9*a*arctan(sqrt((a*x - 1)/(a*x + 1))) + (15*(a*x - 1)*a/(a*x + 1) + 9*(a
*x - 1)^2*a/(a*x + 1)^2 + 4*a)/(((a*x - 1)/(a*x + 1))^(5/2) + 2*((a*x - 1)
/(a*x + 1))^(3/2) + sqrt((a*x - 1)/(a*x + 1))))*a
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2\,x^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}} - \frac{7\,a^2}{\sqrt{\frac{a\,x-1}{a\,x+1}}} - 9\,a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) + \frac{5\,a}{2\,x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(1/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
1/(2*x^2*((a*x - 1)/(a*x + 1))^(1/2)) - (7*a^2)/((a*x - 1)/(a*x + 1))^(1/2
) - 9*a^2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (5*a)/(2*x*((a*x - 1)/(a*x +
 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.43

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = \frac{-18\sqrt{ax-1}\,atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^2x^2+18\sqrt{ax-1}\,atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^2x^2-4}{2\sqrt{ax-1}\,x^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

output
```
( - 18*sqrt(a*x - 1)*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**2*x**2 + 1
8*sqrt(a*x - 1)*atan(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**2*x**2 - 4*sqrt
(a*x - 1)*a**2*x**2 - 14*sqrt(a*x + 1)*a**2*x**2 + 5*sqrt(a*x + 1)*a*x + s
qrt(a*x + 1))/(2*sqrt(a*x - 1)*x**2)
```

3.32 $\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 104

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = -5a^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} - \frac{4a^4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+\frac{11}{2}a^3\csc^{-1}(ax)$$

output
```
-5*a^3*(1-1/a^2/x^2)^(1/2)+1/3*a^3*(1-1/a^2/x^2)^(3/2)-4*a^4*(1-1/a^2/x^2)
^(1/2)/(a-1/x)-3/2*a^2*(1-1/a^2/x^2)^(1/2)/x+11/2*a^3*arccsc(a*x)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{6}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(2+7ax+19a^2x^2-52a^3x^3)}{x^2(-1+ax)}+33a^2\arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[E^(3*ArcCoth[a*x])/x^4,x]
```

output
```
(a*((Sqrt[1 - 1/(a^2*x^2)]*(2 + 7*a*x + 19*a^2*x^2 - 52*a^3*x^3))/(x^2*(-1
 + a*x)) + 33*a^2*ArcSin[1/(a*x)]))/6
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.24, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {6719, 2164, 25, 27, 2027, 2164, 25, 27, 563, 2346, 25, 2346, 25, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \; 6719$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)x^2}d\frac{1}{x}$$

$$\Big\downarrow \; 2164$$

$$\frac{\int -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x^2}+\frac{1}{x^3}\right)}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x^2}+\frac{1}{x^3}\right)}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \; 27$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x^2}+\frac{1}{x^3}\right)}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \; 2027$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}{\left(a-\frac{1}{x}\right)^2x^2}d\frac{1}{x}$$

$$\Big\downarrow \; 2164$$

$$a^2 \int -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3 x^2}d\frac{1}{x}$$

$$\Big\downarrow \; 25$$

$$-a^2 \int \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3 x^2}d\frac{1}{x}$$

$$\Big\downarrow \; 27$$

$$-a^3 \int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3 x^2}d\frac{1}{x}$$

$$\Big\downarrow \; 563$$

$$-a^3\left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\int \frac{4a^3+\frac{4a^2}{x}+\frac{3a}{x^2}+\frac{1}{x^3}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}\right)$$

$$\Big\downarrow \; 2346$$

$$-a^3\left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{-\frac{1}{3}a^2\int -\frac{12a+\frac{14}{x}+\frac{9}{x^2a}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}\right)$$

$$\Big\downarrow \; 25$$

$$-a^3\left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{1}{3}a^2\int \frac{12a+\frac{14}{x}+\frac{9}{x^2a}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}\right)$$

$$\Big\downarrow \; 2346$$

$$-a^3\left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{1}{3}a^2\left(-\frac{1}{2}a^2\int -\frac{33a+\frac{28}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}\right)$$

$$\Big\downarrow \; 25$$

$$-a^3\left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{1}{3}a^2\left(\frac{1}{2}a^2\int\frac{33a+\frac{28}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}\right)$$

$$\Big\downarrow \; 27$$

$$-a^3\left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{1}{3}a^2\left(\frac{1}{2}\int\frac{33a+\frac{28}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}\right)$$

$$\Big\downarrow \; 455$$

$$-a^3\left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{1}{3}a^2\left(\frac{1}{2}\left(33a\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-28a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}\right)$$

$$\Big\downarrow \; 223$$

$$-a^3\left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}-\frac{\frac{1}{3}a^2\left(\frac{1}{2}\left(33a^2\arcsin\left(\frac{1}{ax}\right)-28a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}\right)$$

input

```
Int[E^(3*ArcCoth[a*x])/x^4,x]
```

output

```
-(a^3*((4*a*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) - (-1/3*(a^2*Sqrt[1 - 1/(a
^2*x^2)])/x^2 + (a^2*((-9*a*Sqrt[1 - 1/(a^2*x^2)])/(2*x) + (-28*a^2*Sqrt[1
 - 1/(a^2*x^2)] + 33*a^2*ArcSin[1/(a*x)])/2))/3)/a^4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2164
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + b*d*x, x]*
(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x]
 && EqQ[b*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + b*d*x, x], 0
]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.30

method	result
risch	$-\frac{(ax-1)(28a^2x^2+9ax+2)}{6x^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{11a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}-\frac{4a^2\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}\right)\sqrt{(ax-1)(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{-30\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^6x^6+30(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4+93\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^5x^5+33\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}\,a^5x^5+30\ln\left(\frac{a^2x+\sqrt{a^2x^2-}}{\sqrt{a^2}}\right.}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(a*x-1)*(28*a^2*x^2+9*a*x+2)/x^3/((a*x-1)/(a*x+1))^(1/2)+(11/2*a^3*ar
ctan(1/(a^2*x^2-1)^(1/2))-4*a^2/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2))
/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = -\frac{66\left(a^4x^4-a^3x^3\right)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(52\,a^4x^4+33\,a^3x^3-26\,a^2x^2-9\,ax-2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\left(ax^4-x^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(66*(a^4*x^4 - a^3*x^3)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (52*a^4*x
^4 + 33*a^3*x^3 - 26*a^2*x^2 - 9*a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^
4 - x^3)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

output
```
Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.48

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = -\frac{1}{3}\left(33\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{\frac{75\,(ax-1)a^2}{ax+1}+\frac{88\,(ax-1)^2a^2}{(ax+1)^2}+\frac{33\,(ax-1)^3a^2}{(ax+1)^3}+12\,a^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\sqrt{\frac{ax-1}{ax+1}}}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima")
```

output
```
-1/3*(33*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (75*(a*x - 1)*a^2/(a*x +
1) + 88*(a*x - 1)^2*a^2/(a*x + 1)^2 + 33*(a*x - 1)^3*a^2/(a*x + 1)^3 + 12*
a^2)/(((a*x - 1)/(a*x + 1))^(7/2) + 3*((a*x - 1)/(a*x + 1))^(5/2) + 3*((a*
x - 1)/(a*x + 1))^(3/2) + sqrt((a*x - 1)/(a*x + 1))))*a
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 23.59 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.46

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = -\frac{4\,a^3+\frac{88\,a^3\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}+\frac{11\,a^3\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{25\,a^3\,\left(a\,x-1\right)}{a\,x+1}}{\sqrt{\frac{a\,x-1}{a\,x+1}}+3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}+3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}+{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}} - 11\,a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

input
```
int(1/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
- (4*a^3 + (88*a^3*(a*x - 1)^2)/(3*(a*x + 1)^2) + (11*a^3*(a*x - 1)^3)/(a*
x + 1)^3 + (25*a^3*(a*x - 1))/(a*x + 1))/(((a*x - 1)/(a*x + 1))^(1/2) + 3*
((a*x - 1)/(a*x + 1))^(3/2) + 3*((a*x - 1)/(a*x + 1))^(5/2) + ((a*x - 1)/(
a*x + 1))^(7/2)) - 11*a^3*atan(((a*x - 1)/(a*x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.27

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{-66\sqrt{ax-1}\,atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^3x^3+66\sqrt{ax-1}\,atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^3x^3-1}{6\sqrt{ax-1}\,x^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

output
```
( - 66*sqrt(a*x - 1)*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**3*x**3 + 6
6*sqrt(a*x - 1)*atan(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**3*x**3 - 14*sqr
t(a*x - 1)*a**3*x**3 - 52*sqrt(a*x + 1)*a**3*x**3 + 19*sqrt(a*x + 1)*a**2*
x**2 + 7*sqrt(a*x + 1)*a*x + 2*sqrt(a*x + 1))/(6*sqrt(a*x - 1)*x**3)
```

3.33 $\int e^{4\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int e^{4\coth^{-1}(ax)}x^3\,dx=\frac{12x}{a^3}+\frac{4x^2}{a^2}+\frac{4x^3}{3a}+\frac{x^4}{4}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}$$

output
```
12*x/a^3+4*x^2/a^2+4/3*x^3/a+1/4*x^4+4/a^4/(-a*x+1)+16*ln(-a*x+1)/a^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}x^3\,dx=\frac{12x}{a^3}+\frac{4x^2}{a^2}+\frac{4x^3}{3a}+\frac{x^4}{4}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*x^3,x]
```

output
```
(12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16
*Log[1 - a*x])/a^4
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int x^3 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{6676}$$

$$\int \frac{x^3(ax+1)^2}{(1-ax)^2}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(\frac{16}{a^3(ax-1)}+\frac{4}{a^3(ax-1)^2}+\frac{12}{a^3}+\frac{8x}{a^2}+\frac{4x^2}{a}+x^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}+\frac{12x}{a^3}+\frac{4x^2}{a^2}+\frac{4x^3}{3a}+\frac{x^4}{4}$$

input
```
Int[E^(4*ArcCoth[a*x])*x^3,x]
```

output
```
(12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16
*Log[1 - a*x])/a^4
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

method	result
risch	$\frac{x^4}{4}+\frac{4x^3}{3a}+\frac{4x^2}{a^2}+\frac{12x}{a^3}+\frac{16\ln(ax-1)}{a^4}-\frac{4}{a^4(ax-1)}$
norman	$\frac{\frac{13x^4}{12}+\frac{a\,x^5}{4}+\frac{8x^2}{a^2}+\frac{8x^3}{3a}-\frac{16}{a^4}}{ax-1}+\frac{16\ln(ax-1)}{a^4}$
default	$\frac{\frac{1}{4}a^3x^4+\frac{4}{3}a^2x^3+4a\,x^2+12x}{a^3}+\frac{16\ln(ax-1)}{a^4}-\frac{4}{a^4(ax-1)}$
parallelrisch	$\frac{3a^5x^5+13a^4x^4+32a^3x^3-192+96a^2x^2+192a\ln(ax-1)x-192\ln(ax-1)}{12a^4(ax-1)}$
meijerg	$\frac{\frac{xa\left(-3a^4x^4-5a^3x^3-10a^2x^2-30ax+60\right)}{-12ax+12}+5\ln(-ax+1)}{a^4}-\frac{2\left(-\frac{ax\left(-5a^3x^3-10a^2x^2-30ax+60\right)}{15(-ax+1)}-4\ln(-ax+1)\right)}{a^4}+$ $ax(-2a^2x^2$ … $-4a$ …

input
```
int(1/(a*x-1)^2*(a*x+1)^2*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+4/3*x^3/a+4*x^2/a^2+12*x/a^3+16/a^4*ln(a*x-1)-4/a^4/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = \frac{3\,a^5x^5 + 13\,a^4x^4 + 32\,a^3x^3 + 96\,a^2x^2 - 144\,ax + 192\,(ax-1)\log(ax-1) - 48}{12\,(a^5x - a^4)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="fricas")
```

output
```
1/12*(3*a^5*x^5 + 13*a^4*x^4 + 32*a^3*x^3 + 96*a^2*x^2 - 144*a*x + 192*(a*
x - 1)*log(a*x - 1) - 48)/(a^5*x - a^4)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = \frac{x^4}{4} - \frac{4}{a^5x - a^4} + \frac{4x^3}{3a} + \frac{4x^2}{a^2} + \frac{12x}{a^3} + \frac{16\log(ax-1)}{a^4}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*x**3,x)
```

output
```
x**4/4 - 4/(a**5*x - a**4) + 4*x**3/(3*a) + 4*x**2/a**2 + 12*x/a**3 + 16*l
og(a*x - 1)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = -\frac{4}{a^5x - a^4} + \frac{3\,a^3x^4 + 16\,a^2x^3 + 48\,ax^2 + 144\,x}{12\,a^3} + \frac{16\,\log(ax-1)}{a^4}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="maxima")
```

output
```
-4/(a^5*x - a^4) + 1/12*(3*a^3*x^4 + 16*a^2*x^3 + 48*a*x^2 + 144*x)/a^3 +
16*log(a*x - 1)/a^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.37

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = \frac{(ax-1)^4\left(\frac{28}{ax-1}+\frac{114}{(ax-1)^2}+\frac{300}{(ax-1)^3}+3\right)}{12\,a^4} - \frac{16\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a^4} - \frac{4}{(ax-1)a^4}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="giac")
```

output
```
1/12*(a*x - 1)^4*(28/(a*x - 1) + 114/(a*x - 1)^2 + 300/(a*x - 1)^3 + 3)/a^
4 - 16*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a^4 - 4/((a*x - 1)*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = \frac{16\,\ln\left(a\,x-1\right)}{a^4} - \frac{4}{a\,\left(a^4\,x-a^3\right)} + \frac{12\,x}{a^3} + \frac{x^4}{4} + \frac{4\,x^3}{3\,a} + \frac{4\,x^2}{a^2}$$

input
```
int((x^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(16*log(a*x - 1))/a^4 - 4/(a*(a^4*x - a^3)) + (12*x)/a^3 + x^4/4 + (4*x^3)
/(3*a) + (4*x^2)/a^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int e^{4\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{192\log(ax-1)\,ax-192\log(ax-1)+3a^5x^5+13a^4x^4+32a^3x^3+96a^2x^2-192ax}{12a^4\,(ax-1)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*x^3,x)
```

output
```
(192*log(a*x - 1)*a*x - 192*log(a*x - 1) + 3*a**5*x**5 + 13*a**4*x**4 + 32
*a**3*x**3 + 96*a**2*x**2 - 192*a*x)/(12*a**4*(a*x - 1))
```

3.34 $\int e^{4\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 47

$$\int e^{4\coth^{-1}(ax)}x^2\,dx=\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}$$

output
```
8*x/a^2+2*x^2/a+1/3*x^3+4/a^3/(-a*x+1)+12*ln(-a*x+1)/a^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}x^2\,dx=\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*x^2,x]
```

output
```
(8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6676, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{4\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int x^2 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{ 6676}$$

$$\int \frac{x^2(ax+1)^2}{(1-ax)^2}dx$$

$$\Big\downarrow \text{ 99}$$

$$\int \left(\frac{12}{a^2(ax-1)}+\frac{4}{a^2(ax-1)^2}+\frac{8}{a^2}+\frac{4x}{a}+x^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}+\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}$$

input
```
Int[E^(4*ArcCoth[a*x])*x^2,x]
```

output
```
(8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result
risch	$\frac{x^{3}}{3}+\frac{2 x^{2}}{a}+\frac{8 x}{a^{2}}+\frac{12 \ln \left(a x -1\right)}{a^{3}}-\frac{4}{\left(a x -1\right) a^{3}}$
norman	$\frac{\frac{5 x^{3}}{3}+\frac{a \,x^{4}}{3}+\frac{6 x^{2}}{a}-\frac{12}{a^{3}}}{a x -1}+\frac{12 \ln \left(a x -1\right)}{a^{3}}$
default	$\frac{\frac{1}{3} a^{2} x^{3}+2 a \,x^{2}+8 x}{a^{2}}+\frac{12 \ln \left(a x -1\right)}{a^{3}}-\frac{4}{\left(a x -1\right) a^{3}}$
parallelrisch	$\frac{a^{4} x^{4}+5 a^{3} x^{3}-36+18 a^{2} x^{2}+36 a \ln \left(a x -1\right) x -36 \ln \left(a x -1\right)}{3 a^{3} \left(a x -1\right)}$
meijerg	$-\frac{-\frac{a x \left(-5 a^{3} x^{3}-10 a^{2} x^{2}-30 a x +60\right)}{15 \left(-a x +1\right)}-4 \ln \left(-a x +1\right)}{a^{3}}+\frac{\frac{2 a x \left(-2 a^{2} x^{2}-6 a x +12\right)}{-4 a x +4}+6 \ln \left(-a x +1\right)}{a^{3}}-\frac{-\frac{a x \left(-3 a x +6\right)}{3 \left(-a x +1\right)}-2 \ln \left(-a x +1\right)}{a^{3}}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+2*x^2/a+8*x/a^2+12/a^3*ln(a*x-1)-4/(a*x-1)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int e^{4\coth^{-1}(ax)}x^2\,dx = \frac{a^4x^4 + 5\,a^3x^3 + 18\,a^2x^2 - 24\,ax + 36\,(ax-1)\log(ax-1) - 12}{3\,(a^4x - a^3)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="fricas")
```

output
```
1/3*(a^4*x^4 + 5*a^3*x^3 + 18*a^2*x^2 - 24*a*x + 36*(a*x - 1)*log(a*x - 1)
 - 12)/(a^4*x - a^3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int e^{4\coth^{-1}(ax)}x^2\,dx = \frac{x^3}{3} - \frac{4}{a^4x - a^3} + \frac{2x^2}{a} + \frac{8x}{a^2} + \frac{12\log(ax-1)}{a^3}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*x**2,x)
```

output
```
x**3/3 - 4/(a**4*x - a**3) + 2*x**2/a + 8*x/a**2 + 12*log(a*x - 1)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int e^{4\coth^{-1}(ax)}x^2\,dx = -\frac{4}{a^4x - a^3} + \frac{a^2x^3 + 6\,ax^2 + 24\,x}{3\,a^2} + \frac{12\,\log(ax-1)}{a^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="maxima")
```

output
```
-4/(a^4*x - a^3) + 1/3*(a^2*x^3 + 6*a*x^2 + 24*x)/a^2 + 12*log(a*x - 1)/a^
3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.47

$$\int e^{4\coth^{-1}(ax)}x^2\,dx = \frac{(ax-1)^3\left(\frac{9}{ax-1}+\frac{39}{(ax-1)^2}+1\right)}{3\,a^3} - \frac{12\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a^3} - \frac{4}{(ax-1)a^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="giac")
```

output
```
1/3*(a*x - 1)^3*(9/(a*x - 1) + 39/(a*x - 1)^2 + 1)/a^3 - 12*log(abs(a*x -
1)/((a*x - 1)^2*abs(a)))/a^3 - 4/((a*x - 1)*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int e^{4\coth^{-1}(ax)}x^2\,dx = \frac{12\,\ln(a\,x-1)}{a^3} - \frac{4}{a\,(a^3\,x-a^2)} + \frac{8\,x}{a^2} + \frac{x^3}{3} + \frac{2\,x^2}{a}$$

input
```
int((x^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(12*log(a*x - 1))/a^3 - 4/(a*(a^3*x - a^2)) + (8*x)/a^2 + x^3/3 + (2*x^2)/
a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int e^{4\coth^{-1}(ax)}x^2\,dx = \frac{36\log(ax-1)\,ax-36\log(ax-1)+a^4x^4+5a^3x^3+18a^2x^2-36ax}{3a^3\,(ax-1)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*x^2,x)
```

output
```
(36*log(a*x - 1)*a*x - 36*log(a*x - 1) + a**4*x**4 + 5*a**3*x**3 + 18*a**2
*x**2 - 36*a*x)/(3*a**3*(a*x - 1))
```

3.35 $\int e^{4 \coth^{-1}(ax)} x \, dx$

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int e^{4 \coth^{-1}(ax)} x \, dx = \frac{4x}{a} + \frac{x^2}{2} + \frac{4}{a^2(1-ax)} + \frac{8 \log(1-ax)}{a^2}$$

output
```
4*x/a+1/2*x^2+4/a^2/(-a*x+1)+8*ln(-a*x+1)/a^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int e^{4 \coth^{-1}(ax)} x \, dx = \frac{4x}{a} + \frac{x^2}{2} + \frac{4}{a^2(1-ax)} + \frac{8 \log(1-ax)}{a^2}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*x,x]
```

output
```
(4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{4\coth^{-1}(ax)}dx$$
$$\Big\downarrow \text{6717}$$
$$\int xe^{4\text{arctanh}(ax)}dx$$
$$\Big\downarrow \text{6676}$$
$$\int \frac{x(ax+1)^2}{(1-ax)^2}dx$$
$$\Big\downarrow \text{86}$$
$$\int \left(\frac{8}{a(ax-1)}+\frac{4}{a(ax-1)^2}+\frac{4}{a}+x\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}+\frac{4x}{a}+\frac{x^2}{2}$$

input
```
Int[E^(4*ArcCoth[a*x])*x,x]
```

output
```
(4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{x^2}{2}+\frac{4x}{a}+\frac{8\ln(ax-1)}{a^2}-\frac{4}{(ax-1)a^2}$	36
default	$\frac{\frac{1}{2}a\,x^2+4x}{a}+\frac{8\ln(ax-1)}{a^2}-\frac{4}{(ax-1)a^2}$	39
norman	$\frac{\frac{7x^2}{2}+\frac{a\,x^3}{2}-\frac{8x}{a}}{ax-1}+\frac{8\ln(ax-1)}{a^2}$	39
parallelrisch	$\frac{a^3x^3+7a^2x^2+16a\ln(ax-1)x-16ax-16\ln(ax-1)}{2(ax-1)a^2}$	51
meijerg	$\frac{\frac{ax\left(-2a^2x^2-6ax+12\right)}{-4ax+4}+3\ln(-ax+1)}{a^2}-\frac{2\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a^2}+\frac{\frac{ax}{-ax+1}+\ln(-ax+1)}{a^2}$	98

input
```
int(1/(a*x-1)^2*(a*x+1)^2*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+4*x/a+8/a^2*ln(a*x-1)-4/(a*x-1)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int e^{4\coth^{-1}(ax)}x\,dx = \frac{a^3x^3 + 7\,a^2x^2 - 8\,ax + 16\,(ax-1)\log(ax-1) - 8}{2\,(a^3x - a^2)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="fricas")
```

output
```
1/2*(a^3*x^3 + 7*a^2*x^2 - 8*a*x + 16*(a*x - 1)*log(a*x - 1) - 8)/(a^3*x -
 a^2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int e^{4\coth^{-1}(ax)}x\,dx = \frac{x^2}{2} - \frac{4}{a^3x - a^2} + \frac{4x}{a} + \frac{8\log(ax-1)}{a^2}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*x,x)
```

output
```
x**2/2 - 4/(a**3*x - a**2) + 4*x/a + 8*log(a*x - 1)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int e^{4\coth^{-1}(ax)}x\,dx = \frac{ax^2 + 8\,x}{2\,a} - \frac{4}{a^3x - a^2} + \frac{8\,\log(ax-1)}{a^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="maxima")
```

output
```
1/2*(a*x^2 + 8*x)/a - 4/(a^3*x - a^2) + 8*log(a*x - 1)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.64

$$\int e^{4\coth^{-1}(ax)}x\,dx = \frac{\frac{(ax-1)^2\left(\frac{10}{ax-1}+1\right)}{a} - \frac{16\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{8}{(ax-1)a}}{2a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="giac")
```

output
```
1/2*((a*x - 1)^2*(10/(a*x - 1) + 1)/a - 16*log(abs(a*x - 1)/((a*x - 1)^2*a
bs(a)))/a - 8/((a*x - 1)*a))/a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int e^{4\coth^{-1}(ax)}x\,dx = \frac{8\,\ln\left(a\,x-1\right)}{a^2} + \frac{4\,x}{a} + \frac{x^2}{2} + \frac{4}{a\,\left(a-a^2\,x\right)}$$

input
```
int((x*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(8*log(a*x - 1))/a^2 + (4*x)/a + x^2/2 + 4/(a*(a - a^2*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int e^{4\coth^{-1}(ax)}x\,dx = \frac{16\log(ax-1)\,ax - 16\log(ax-1) + a^3x^3 + 7a^2x^2 - 16ax}{2a^2\,(ax-1)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*x,x)
```

output
```
(16*log(a*x - 1)*a*x - 16*log(a*x - 1) + a**3*x**3 + 7*a**2*x**2 - 16*a*x)
/(2*a**2*(a*x - 1))
```

3.36 $\int e^{4\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int e^{4\coth^{-1}(ax)}\,dx=x+\frac{4}{a(1-ax)}+\frac{4\log(1-ax)}{a}$$

output
```
x+4/a/(-a*x+1)+4*ln(-a*x+1)/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\,dx=x-\frac{4}{a(-1+ax)}+\frac{4\log(1-ax)}{a}$$

input
```
Integrate[E^(4*ArcCoth[a*x]),x]
```

output
```
x - 4/(a*(-1 + a*x)) + (4*Log[1 - a*x])/a
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6717, 6675, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{4\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$\int e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{6675}$$

$$\int \frac{(ax+1)^2}{(1-ax)^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{4}{ax-1}+\frac{4}{(ax-1)^2}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{a(1-ax)}+\frac{4\log(1-ax)}{a}+x$$

input
```
Int[E^(4*ArcCoth[a*x]),x]
```

output
```
x + 4/(a*(1 - a*x)) + (4*Log[1 - a*x])/a
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6675
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
default	$x+\frac{4\ln\left(ax-1\right)}{a}-\frac{4}{\left(ax-1\right)a}$	26
risch	$x+\frac{4\ln\left(ax-1\right)}{a}-\frac{4}{\left(ax-1\right)a}$	26
norman	$\frac{a\,x^{2}-5x}{ax-1}+\frac{4\ln\left(ax-1\right)}{a}$	30
parallelrisch	$\frac{a^{2}x^{2}+4a\ln\left(ax-1\right)x-5-4\ln\left(ax-1\right)}{\left(ax-1\right)a}$	39
meijerg	$-\frac{-\frac{ax\left(-3ax+6\right)}{3\left(-ax+1\right)}-2\ln\left(-ax+1\right)}{a}+\frac{\frac{2ax}{-ax+1}+2\ln\left(-ax+1\right)}{a}+\frac{x}{-ax+1}$	69

input
```
int(1/(a*x-1)^2*(a*x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x+4/a*ln(a*x-1)-4/(a*x-1)/a
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int e^{4\coth^{-1}(ax)}\,dx = \frac{a^2x^2 - ax + 4\,(ax-1)\log(ax-1) - 4}{a^2x - a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="fricas")
```

output
```
(a^2*x^2 - a*x + 4*(a*x - 1)*log(a*x - 1) - 4)/(a^2*x - a)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{4\coth^{-1}(ax)}\,dx = x - \frac{4}{a^2x - a} + \frac{4\log(ax-1)}{a}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2,x)
```

output
```
x - 4/(a**2*x - a) + 4*log(a*x - 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\,dx = x + \frac{4\,\log(ax-1)}{a} - \frac{4}{a^2x - a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="maxima")
```

output
```
x + 4*log(a*x - 1)/a - 4/(a^2*x - a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int e^{4\coth^{-1}(ax)}\,dx = \frac{ax-1}{a} - \frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{4}{(ax-1)a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="giac")
```

output
```
(a*x - 1)/a - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - 4/((a*x - 1)*a)
```

Mupad [B] (verification not implemented)

Time = 23.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int e^{4\coth^{-1}(ax)}\,dx = x - \frac{4}{a\,(a\,x-1)} + \frac{4\,\ln(a\,x-1)}{a}$$

input
```
int((a*x + 1)^2/(a*x - 1)^2,x)
```

output
```
x - 4/(a*(a*x - 1)) + (4*log(a*x - 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int e^{4\coth^{-1}(ax)}\,dx = \frac{4\log(ax-1)\,ax - 4\log(ax-1) + a^2x^2 - 5ax}{a\,(ax-1)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2,x)
```

output
```
(4*log(a*x - 1)*a*x - 4*log(a*x - 1) + a**2*x**2 - 5*a*x)/(a*(a*x - 1))
```

3.37 $\int \frac{e^{4\coth^{-1}(ax)}}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{e^{4\coth^{-1}(ax)}}{x} \, dx = \frac{4}{1-ax} + \log(x)$$

output
```
4/(-a*x+1)+ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{x} \, dx = \frac{4}{1-ax} + \log(x)$$

input
```
Integrate[E^(4*ArcCoth[a*x])/x,x]
```

output
```
4/(1 - a*x) + Log[x]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{x} \, dx$$

$$\Big\downarrow \; 6717$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{x} dx$$

$$\Big\downarrow \; 6676$$

$$\int \frac{(ax+1)^2}{x(1-ax)^2} dx$$

$$\Big\downarrow \; 99$$

$$\int \left(\frac{4a}{(ax-1)^2}+\frac{1}{x}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{4}{1-ax}+\log(x)$$

input
```
Int[E^(4*ArcCoth[a*x])/x,x]
```

output
```
4/(1 - a*x) + Log[x]
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{4}{ax-1}+\ln\left(x\right)$	13
norman	$-\frac{4ax}{ax-1}+\ln\left(x\right)$	15
risch	$-\frac{4}{ax-1}+\ln\left(-x\right)$	15
parallelrisch	$\frac{a\ln\left(x\right)x-4ax-\ln\left(x\right)}{ax-1}$	23
meijerg	$\frac{3ax}{-ax+1}+1+\ln\left(x\right)+\ln\left(-a\right)+\frac{2ax}{-2ax+2}$	33

input
```
int(1/(a*x-1)^2*(a*x+1)^2/x,x,method=_RETURNVERBOSE)
```

output
```
-4/(a*x-1)+ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = \frac{(ax-1)\log(x)-4}{ax-1}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="fricas")
```

output
```
((a*x - 1)*log(x) - 4)/(a*x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = \log(x) - \frac{4}{ax-1}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/x,x)
```

output
```
log(x) - 4/(a*x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = -\frac{4}{ax-1} + \log(x)$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="maxima")
```

output
```
-4/(a*x - 1) + log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.38

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = -a\left(\frac{\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a} + \frac{4}{(ax-1)a}\right)$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="giac")
```

output
```
-a*(log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - log(abs(-1/(a*x - 1) - 1))/
a + 4/((a*x - 1)*a))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = \ln(x) - \frac{4}{a\,x-1}$$

input
```
int((a*x + 1)^2/(x*(a*x - 1)^2),x)
```

output
```
log(x) - 4/(a*x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = \frac{\log(x)\,ax - \log(x) - 4ax}{ax-1}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/x,x)
```

output
```
(log(x)*a*x - log(x) - 4*a*x)/(a*x - 1)
```

3.38 $\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = -\frac{1}{x}+\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)$$

output
```
-1/x+4*a/(-a*x+1)+4*a*ln(x)-4*a*ln(-a*x+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = -\frac{1}{x}+\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)$$

input
```
Integrate[E^(4*ArcCoth[a*x])/x^2,x]
```

output
```
-x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6676, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6676}$$

$$\int \frac{(ax+1)^2}{x^2(1-ax)^2}dx$$

$$\Big\downarrow \text{ 99}$$

$$\int \left(-\frac{4a^2}{ax-1}+\frac{4a^2}{(ax-1)^2}+\frac{4a}{x}+\frac{1}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)-\frac{1}{x}$$

input
```
Int[E^(4*ArcCoth[a*x])/x^2,x]
```

output
```
-x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result
default	$-\frac{4a}{ax-1}-4a\ln\left(ax-1\right)-\frac{1}{x}+4\ln\left(x\right)a$
risch	$\frac{-5ax+1}{x(ax-1)}-4a\ln\left(ax-1\right)+4a\ln\left(-x\right)$
norman	$\frac{-5a^2x^2+1}{x(ax-1)}+4\ln\left(x\right)a-4a\ln\left(ax-1\right)$
parallelrisch	$\frac{4a^2\ln(x)x^2-4a^2\ln(ax-1)x^2-5a^2x^2+1-4a\ln(x)x+4a\ln(ax-1)x}{x(ax-1)}$
meijerg	$\frac{a^2x}{-ax+1}+2a\left(1+\ln\left(x\right)+\ln\left(-a\right)+\frac{2ax}{-2ax+2}-\ln\left(-ax+1\right)\right)-a\left(\frac{1}{ax}-1-2\ln\left(x\right)-2\ln\left(-\right.\right.$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-4*a/(a*x-1)-4*a*ln(a*x-1)-1/x+4*ln(x)*a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = -\frac{5\,ax + 4\,(a^2x^2 - ax)\log(ax-1) - 4\,(a^2x^2 - ax)\log(x) - 1}{ax^2 - x}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="fricas")
```

output
```
-(5*a*x + 4*(a^2*x^2 - a*x)*log(a*x - 1) - 4*(a^2*x^2 - a*x)*log(x) - 1)/(
a*x^2 - x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = 4a\left(\log(x) - \log\left(x - \frac{1}{a}\right)\right) + \frac{-5ax + 1}{ax^2 - x}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/x**2,x)
```

output
```
4*a*(log(x) - log(x - 1/a)) + (-5*a*x + 1)/(a*x**2 - x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = -4\,a\log(ax-1) + 4\,a\log(x) - \frac{5\,ax - 1}{ax^2 - x}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="maxima")
```

output
```
-4*a*log(a*x - 1) + 4*a*log(x) - (5*a*x - 1)/(a*x^2 - x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = 4\,a\log\left(\left|-\frac{1}{ax-1}-1\right|\right)-\frac{4\,a}{ax-1}+\frac{a}{\frac{1}{ax-1}+1}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="giac")
```

output
```
4*a*log(abs(-1/(a*x - 1) - 1)) - 4*a/(a*x - 1) + a/(1/(a*x - 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = 8\,a\,\mathrm{atanh}(2\,a\,x-1)+\frac{5\,a\,x-1}{x-a\,x^2}$$

input
```
int((a*x + 1)^2/(x^2*(a*x - 1)^2),x)
```

output
```
8*a*atanh(2*a*x - 1) + (5*a*x - 1)/(x - a*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = \frac{-4\log(ax-1)\,a^2x^2+4\log(ax-1)\,ax+4\log(x)\,a^2x^2-4\log(x)\,ax-5a^2x^2+1}{x\,(ax-1)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/x^2,x)
```

output
```
( - 4*log(a*x - 1)*a**2*x**2 + 4*log(a*x - 1)*a*x + 4*log(x)*a**2*x**2 - 4
*log(x)*a*x - 5*a**2*x**2 + 1)/(x*(a*x - 1))
```

3.39 $\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = -\frac{1}{2x^2} - \frac{4a}{x} + \frac{4a^2}{1-ax} + 8a^2\log(x) - 8a^2\log(1-ax)$$

output
```
-1/2/x^2-4*a/x+4*a^2/(-a*x+1)+8*a^2*ln(x)-8*a^2*ln(-a*x+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = -\frac{1}{2x^2} - \frac{4a}{x} + \frac{4a^2}{1-ax} + 8a^2\log(x) - 8a^2\log(1-ax)$$

input
```
Integrate[E^(4*ArcCoth[a*x])/x^3,x]
```

output
```
-1/2*1/x^2 - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*
x]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6676, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6676}$$

$$\int \frac{(ax+1)^2}{x^3(1-ax)^2}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(-\frac{8a^3}{ax-1}+\frac{4a^3}{(ax-1)^2}+\frac{8a^2}{x}+\frac{4a}{x^2}+\frac{1}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)-\frac{4a}{x}-\frac{1}{2x^2}$$

input
```
Int[E^(4*ArcCoth[a*x])/x^3,x]
```

output
```
-1/2*1/x^2 - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*
x]
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

method	result
default	$-\frac{4a^2}{ax-1}-8a^2\ln\left(ax-1\right)-\frac{1}{2x^2}-\frac{4a}{x}+8a^2\ln\left(x\right)$
norman	$\frac{\frac{1}{2}-8a^3x^3+\frac{7}{2}ax}{x^2(ax-1)}+8a^2\ln\left(x\right)-8a^2\ln\left(ax-1\right)$
risch	$\frac{-8a^2x^2+\frac{7}{2}ax+\frac{1}{2}}{x^2(ax-1)}+8a^2\ln\left(-x\right)-8a^2\ln\left(ax-1\right)$
parallelrisch	$\frac{16\ln(x)x^3a^3-16a^3\ln(ax-1)x^3-16a^3x^3-16a^2\ln(x)x^2+16a^2\ln(ax-1)x^2+1+7ax}{2x^2(ax-1)}$
meijerg	$a^2\left(1+\ln\left(x\right)+\ln\left(-a\right)+\frac{2ax}{-2ax+2}-\ln\left(-ax+1\right)\right)-2a^2\left(\frac{1}{ax}-1-2\ln\left(x\right)-2\ln\left(-a\right)-\right.$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-4*a^2/(a*x-1)-8*a^2*ln(a*x-1)-1/2/x^2-4*a/x+8*a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = -\frac{16\,a^2x^2-7\,ax+16\,(a^3x^3-a^2x^2)\log\left(ax-1\right)-16\,(a^3x^3-a^2x^2)\log\left(x\right)-1}{2\,(ax^3-x^2)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="fricas")
```

output
```
-1/2*(16*a^2*x^2 - 7*a*x + 16*(a^3*x^3 - a^2*x^2)*log(a*x - 1) - 16*(a^3*x
^3 - a^2*x^2)*log(x) - 1)/(a*x^3 - x^2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = 8a^2\left(\log\left(x\right)-\log\left(x-\frac{1}{a}\right)\right)+\frac{-16a^2x^2+7ax+1}{2ax^3-2x^2}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/x**3,x)
```

output
```
8*a**2*(log(x) - log(x - 1/a)) + (-16*a**2*x**2 + 7*a*x + 1)/(2*a*x**3 - 2
*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = -8\,a^2\log\left(ax-1\right)+8\,a^2\log\left(x\right)-\frac{16\,a^2x^2-7\,ax-1}{2\,(ax^3-x^2)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="maxima")
```

output
```
-8*a^2*log(a*x - 1) + 8*a^2*log(x) - 1/2*(16*a^2*x^2 - 7*a*x - 1)/(a*x^3 -
 x^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = 8\,a^2\log\left(\left|-\frac{1}{ax-1}-1\right|\right)-\frac{4\,a^2}{ax-1}+\frac{9\,a^2+\frac{10\,a^2}{ax-1}}{2\left(\frac{1}{ax-1}+1\right)^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="giac")
```

output
```
8*a^2*log(abs(-1/(a*x - 1) - 1)) - 4*a^2/(a*x - 1) + 1/2*(9*a^2 + 10*a^2/(
a*x - 1))/(1/(a*x - 1) + 1)^2
```

Mupad [B] (verification not implemented)

Time = 23.94 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = 16\,a^2\,\mathrm{atanh}(2\,a\,x-1)+\frac{-8\,a^2\,x^2+\frac{7\,a\,x}{2}+\frac{1}{2}}{a\,x^3-x^2}$$

input
```
int((a*x + 1)^2/(x^3*(a*x - 1)^2),x)
```

output
```
16*a^2*atanh(2*a*x - 1) + ((7*a*x)/2 - 8*a^2*x^2 + 1/2)/(a*x^3 - x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = \frac{-16\log(ax-1)\,a^3x^3+16\log(ax-1)\,a^2x^2+16\log(x)\,a^3x^3-16\log(x)\,a^2x^2-16a^3x^3+7ax+1}{2x^2\,(ax-1)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/x^3,x)
```

output
```
( - 16*log(a*x - 1)*a**3*x**3 + 16*log(a*x - 1)*a**2*x**2 + 16*log(x)*a**3
*x**3 - 16*log(x)*a**2*x**2 - 16*a**3*x**3 + 7*a*x + 1)/(2*x**2*(a*x - 1))
```

3.40 $\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 54

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = -\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{8a^2}{x}+\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)$$

output
```
-1/3/x^3-2*a/x^2-8*a^2/x+4*a^3/(-a*x+1)+12*a^3*ln(x)-12*a^3*ln(-a*x+1)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = -\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{8a^2}{x}+\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)$$

input
```
Integrate[E^(4*ArcCoth[a*x])/x^4,x]
```

output
```
-1/3*1/x^3 - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 1
2*a^3*Log[1 - a*x]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{ 6676}$$

$$\int \frac{(ax+1)^2}{x^4(1-ax)^2}dx$$

$$\Big\downarrow \text{ 99}$$

$$\int \left(-\frac{12a^4}{ax-1}+\frac{4a^4}{(ax-1)^2}+\frac{12a^3}{x}+\frac{8a^2}{x^2}+\frac{4a}{x^3}+\frac{1}{x^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)-\frac{8a^2}{x}-\frac{2a}{x^2}-\frac{1}{3x^3}$$

input
```
Int[E^(4*ArcCoth[a*x])/x^4,x]
```

output
```
-1/3*1/x^3 - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 1
2*a^3*Log[1 - a*x]
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result
default	$-\frac{4a^3}{ax-1}-12a^3\ln(ax-1)-\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{8a^2}{x}+12a^3\ln(x)$
norman	$\frac{\frac{1}{3}-12a^4x^4+\frac{5}{3}ax+6a^2x^2}{(ax-1)x^3}+12a^3\ln(x)-12a^3\ln(ax-1)$
risch	$\frac{-12a^3x^3+6a^2x^2+\frac{5}{3}ax+\frac{1}{3}}{x^3(ax-1)}+12a^3\ln(-x)-12a^3\ln(ax-1)$
parallelrisch	$\frac{36\ln(x)x^4a^4-36\ln(ax-1)x^4a^4-36a^4x^4-36\ln(x)x^3a^3+36a^3\ln(ax-1)x^3+1+18a^2x^2+5ax}{3x^3(ax-1)}$
meijerg	$-a^3\left(\frac{1}{ax}-1-2\ln(x)-2\ln(-a)-\frac{3ax}{-3ax+3}+2\ln(-ax+1)\right)+2a^3\left(-\frac{1}{2a^2x^2}-\frac{2}{ax}+1+3\right.$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-4*a^3/(a*x-1)-12*a^3*ln(a*x-1)-1/3/x^3-2*a/x^2-8*a^2/x+12*a^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4} dx = -\frac{36 a^3x^3 - 18 a^2x^2 - 5 ax + 36 (a^4x^4 - a^3x^3) \log (ax - 1) - 36 (a^4x^4 - a^3x^3) \log (x) - 1}{3 (ax^4 - x^3)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="fricas")
```

output
```
-1/3*(36*a^3*x^3 - 18*a^2*x^2 - 5*a*x + 36*(a^4*x^4 - a^3*x^3)*log(a*x - 1
) - 36*(a^4*x^4 - a^3*x^3)*log(x) - 1)/(a*x^4 - x^3)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4} dx = 12a^3 \left(\log (x) - \log \left(x - \frac{1}{a}\right)\right) + \frac{-36a^3x^3 + 18a^2x^2 + 5ax + 1}{3ax^4 - 3x^3}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/x**4,x)
```

output
```
12*a**3*(log(x) - log(x - 1/a)) + (-36*a**3*x**3 + 18*a**2*x**2 + 5*a*x +
1)/(3*a*x**4 - 3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4} dx = -12 a^3 \log (ax - 1) + 12 a^3 \log (x) - \frac{36 a^3x^3 - 18 a^2x^2 - 5 ax - 1}{3 (ax^4 - x^3)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="maxima")
```

output
```
-12*a^3*log(a*x - 1) + 12*a^3*log(x) - 1/3*(36*a^3*x^3 - 18*a^2*x^2 - 5*a*
x - 1)/(a*x^4 - x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = 12\,a^3\log\left(\left|-\frac{1}{ax-1}-1\right|\right)-\frac{4\,a^3}{ax-1}+\frac{31\,a^3+\frac{69\,a^3}{ax-1}+\frac{39\,a^3}{(ax-1)^2}}{3\left(\frac{1}{ax-1}+1\right)^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="giac")
```

output
```
12*a^3*log(abs(-1/(a*x - 1) - 1)) - 4*a^3/(a*x - 1) + 1/3*(31*a^3 + 69*a^3
/(a*x - 1) + 39*a^3/(a*x - 1)^2)/(1/(a*x - 1) + 1)^3
```

Mupad [B] (verification not implemented)

Time = 23.73 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = 24\,a^3\,\mathrm{atanh}(2\,a\,x-1)+\frac{-12\,a^3\,x^3+6\,a^2\,x^2+\frac{5\,a\,x}{3}+\frac{1}{3}}{a\,x^4-x^3}$$

input
```
int((a*x + 1)^2/(x^4*(a*x - 1)^2),x)
```

output
```
24*a^3*atanh(2*a*x - 1) + ((5*a*x)/3 + 6*a^2*x^2 - 12*a^3*x^3 + 1/3)/(a*x^
4 - x^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{-36\log(ax-1)\,a^4x^4 + 36\log(ax-1)\,a^3x^3 + 36\log(x)\,a^4x^4 - 36\log(x)\,a^3x^3 - 36a^4x^4 + 18a^2x^2 + 5ax}{3x^3\,(ax-1)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/x^4,x)
```

output
```
( - 36*log(a*x - 1)*a**4*x**4 + 36*log(a*x - 1)*a**3*x**3 + 36*log(x)*a**4
*x**4 - 36*log(x)*a**3*x**3 - 36*a**4*x**4 + 18*a**2*x**2 + 5*a*x + 1)/(3*
x**3*(a*x - 1))
```

3.41 $\int e^{-\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 114

$$\int e^{-\coth^{-1}(ax)}x^3\,dx=-\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4+\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

output
```
-2/3*(1-1/a^2/x^2)^(1/2)*x/a^3+3/8*(1-1/a^2/x^2)^(1/2)*x^2/a^2-1/3*(1-1/a^
2/x^2)^(1/2)*x^3/a+1/4*(1-1/a^2/x^2)^(1/2)*x^4+3/8*arctanh((1-1/a^2/x^2)^(
1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.60

$$\int e^{-\coth^{-1}(ax)}x^3\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-16+9ax-8a^2x^2+6a^3x^3)+9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{24a^4}$$

input
```
Integrate[x^3/E^ArcCoth[a*x],x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-16 + 9*a*x - 8*a^2*x^2 + 6*a^3*x^3) + 9*Log[(
1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(24*a^4)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6719, 539, 27, 539, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)x^5}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{4}\int \frac{\left(4a-\frac{3}{x}\right)x^4}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(4a-\frac{3}{x}\right)x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$\frac{-\frac{1}{3}\int \frac{\left(9a-\frac{8}{x}\right)x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\left(9a-\frac{8}{x}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{\frac{-\frac{1}{2}\int \frac{\left(16a-\frac{9}{x}\right)x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{-\frac{\int \frac{\left(16a-\frac{9}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \; 534$$

$$-\frac{\frac{-\frac{-9\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \; 243$$

$$-\frac{\frac{-\frac{-\frac{9}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-16ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \; 73$$

$$-\frac{\frac{-\frac{9a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-16ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{-\frac{9\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-16ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{9}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{4}{3}ax^3\sqrt{1-\frac{1}{a^2x^2}}}{4a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}$$

input

```
Int[x^3/E^ArcCoth[a*x],x]
```

output

```
(Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + ((-4*a*Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - ((-9
*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (-16*a*Sqrt[1 - 1/(a^2*x^2)]*x + 9*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a))/(3*a))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

method	result
risch	$\frac{(6a^3x^3-8a^2x^2+9ax-16)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{24a^4}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{8a^3\sqrt{a^2}\,(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(6\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax+15\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax-8((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-15\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a+24a\ln\left(\frac{a^2x-}{}\right.\right.}{24\sqrt{(ax-1)(ax+1)}\,a^4\sqrt{a^2}}$

input
```
int(x^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*a^3*x^3-8*a^2*x^2+9*a*x-16)*(a*x+1)/a^4*((a*x-1)/(a*x+1))^(1/2)+3/
8/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*((a*x-1)/(a*x+1)
)^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.80

$$\int e^{-\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{(6\,a^4x^4-2\,a^3x^3+a^2x^2-7\,ax-16)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{24\,a^4}$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
1/24*((6*a^4*x^4 - 2*a^3*x^3 + a^2*x^2 - 7*a*x - 16)*sqrt((a*x - 1)/(a*x +
 1)) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a*x +
1)) - 1))/a^4
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x^3\,dx = \int x^3\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x**3*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x**3*sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(94) = 188.

Time = 0.03 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.78

$$\int e^{-\coth^{-1}(ax)}x^3\,dx = -\frac{1}{24}a\left(\frac{2\left(39\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-31\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+49\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-9\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}\right.}{a^5}$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/24*a*(2*(39*((a*x - 1)/(a*x + 1))^(7/2) - 31*((a*x - 1)/(a*x + 1))^(5/2
) + 49*((a*x - 1)/(a*x + 1))^(3/2) - 9*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x
- 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(a*
x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 9*log(sqrt((a*x - 1)/(a*x
+ 1)) + 1)/a^5 + 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 23.43 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.51

$$\int e^{-\coth^{-1}(ax)}x^3\,dx = \frac{3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a^4} - \frac{\frac{3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4} - \frac{49\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12} + \frac{31\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{12} - \frac{13\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a^4 + \frac{6\,a^4\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2} - \frac{4\,a^4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3} + \frac{a^4\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4} - \frac{4\,a^4\,\left(a\,x-1\right)}{a\,x+1}}$$

input
```
int(x^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a^4) - ((3*((a*x - 1)/(a*x + 1))
^(1/2))/4 - (49*((a*x - 1)/(a*x + 1))^(3/2))/12 + (31*((a*x - 1)/(a*x + 1)
)^(5/2))/12 - (13*((a*x - 1)/(a*x + 1))^(7/2))/4)/(a^4 + (6*a^4*(a*x - 1)^
2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x
+ 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.85

$$\int e^{-\coth^{-1}(ax)}x^3\,dx$$
$$= \frac{6\sqrt{ax+1}\,\sqrt{ax-1}\,a^3x^3 - 8\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2 + 9\sqrt{ax+1}\,\sqrt{ax-1}\,ax - 16\sqrt{ax+1}\,\sqrt{ax-1} + 18}{24a^4}$$

input
```
int(x^3*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(6*sqrt(a*x + 1)*sqrt(a*x - 1)*a**3*x**3 - 8*sqrt(a*x + 1)*sqrt(a*x - 1)*a
**2*x**2 + 9*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x - 16*sqrt(a*x + 1)*sqrt(a*x -
 1) + 18*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)))/(24*a**4)
```

3.42 $\int e^{-\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int e^{-\coth^{-1}(ax)}x^2\,dx=\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{3a^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

output
```
2/3*(1-1/a^2/x^2)^(1/2)*x/a^2-1/2*(1-1/a^2/x^2)^(1/2)*x^2/a+1/3*(1-1/a^2/x
^2)^(1/2)*x^3-1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int e^{-\coth^{-1}(ax)}x^2\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}x(4-3ax+2a^2x^2)-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{6a^3}$$

input
```
Integrate[x^2/E^ArcCoth[a*x],x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 - 3*a*x + 2*a^2*x^2) - 3*Log[(1 + Sqrt[1 - 1
/(a^2*x^2)])*x])/(6*a^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6719, 539, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right) x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{3}\int \frac{\left(3a-\frac{2}{x}\right) x^3}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(3a-\frac{2}{x}\right) x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$\frac{-\frac{1}{2}\int \frac{\left(4a-\frac{3}{x}\right) x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\left(4a-\frac{3}{x}\right) x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{534}$$

$$-\frac{\frac{-3\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\frac{-\frac{3}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{3a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-4ax\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{3}{2}ax^2\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}$$

input
```
Int[x^2/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 + ((-3*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (-4*
a*Sqrt[1 - 1/(a^2*x^2)]*x + 3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a))/(3*a^
2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.21

method	result
risch	$\frac{\left(2 a^{2} x^{2}-3 a x +4\right) \left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}}{6 a^{3}}-\frac{\ln\left(\frac{a^{2} x}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{2}-1}\right) \sqrt{\frac{a x -1}{a x +1}}\, \sqrt{\left(a x -1\right) \left(a x +1\right)}}{2 a^{2} \sqrt{a^{2}}\, \left(a x -1\right)}$
default	$\frac{\sqrt{\frac{a x -1}{a x +1}}\, \left(a x +1\right) \left(2 \left(\left(a x -1\right) \left(a x +1\right)\right)^{\frac{3}{2}} \sqrt{a^{2}}-3 \sqrt{a^{2}}\, \sqrt{a^{2} x^{2}-1}\, a x +6 \sqrt{\left(a x -1\right) \left(a x +1\right)}\, \sqrt{a^{2}}+3 \ln\left(\frac{a^{2} x +\sqrt{a^{2} x^{2}-1}\, \sqrt{a^{2}}}{\sqrt{a^{2}}}\right) a -6 a \ln\left(\frac{a^{2} x +\cdots\right.\right.}{6 \sqrt{\left(a x -1\right) \left(a x +1\right)}\, a^{3} \sqrt{a^{2}}}$

input
```
int(x^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*x^2-3*a*x+4)*(a*x+1)/a^3*((a*x-1)/(a*x+1))^(1/2)-1/2/a^2*ln(a^2
*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)*((a*
x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int e^{-\coth^{-1}(ax)}x^2\,dx = \frac{(2\,a^3x^3 - a^2x^2 + ax + 4)\sqrt{\frac{ax-1}{ax+1}} - 3\,\log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right) + 3\,\log\left(\sqrt{\frac{ax-1}{ax+1}} - 1\right)}{6\,a^3}$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
1/6*((2*a^3*x^3 - a^2*x^2 + a*x + 4)*sqrt((a*x - 1)/(a*x + 1)) - 3*log(sqr
t((a*x - 1)/(a*x + 1)) + 1) + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^3
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x^2\,dx = \int x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x**2*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x**2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(74) = 148$.

Time = 0.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.84

$$\int e^{-\coth^{-1}(ax)}x^2\,dx = -\frac{1}{6}\,a\left(\frac{2\left(9\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}-\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}\right)$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/6*a*(2*(9*((a*x - 1)/(a*x + 1))^(5/2) - 4*((a*x - 1)/(a*x + 1))^(3/2) +
 3*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*a
^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 3*log(sqrt((a*x - 1)
/(a*x + 1)) + 1)/a^4 - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int e^{-\coth^{-1}(ax)}x^2\,dx = \frac{1}{6}\sqrt{a^2x^2-1}\left(x\left(\frac{2\,x\mathrm{sgn}(ax+1)}{a}-\frac{3\,\mathrm{sgn}(ax+1)}{a^2}\right)+\frac{4\,\mathrm{sgn}(ax+1)}{a^3}\right) + \frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{2\,a^2|a|}$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(a^2*x^2 - 1)*(x*(2*x*sgn(a*x + 1)/a - 3*sgn(a*x + 1)/a^2) + 4*sgn
(a*x + 1)/a^3) + 1/2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/
(a^2*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.49

$$\int e^{-\coth^{-1}(ax)}x^2\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}+3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}$$

input
```
int(x^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(((a*x - 1)/(a*x + 1))^(1/2) - (4*((a*x - 1)/(a*x + 1))^(3/2))/3 + 3*((a*x
 - 1)/(a*x + 1))^(5/2))/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^3*(a*x
 - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - atanh(((a*x - 1)/(a*
x + 1))^(1/2))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

$$\int e^{-\coth^{-1}(ax)}x^2\,dx = \frac{2\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2-3\sqrt{ax+1}\,\sqrt{ax-1}\,ax+4\sqrt{ax+1}\,\sqrt{ax-1}-6\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)}{6a^3}$$

input
```
int(x^2*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(2*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 - 3*sqrt(a*x + 1)*sqrt(a*x - 1)*a
*x + 4*sqrt(a*x + 1)*sqrt(a*x - 1) - 6*log((sqrt(a*x - 1) + sqrt(a*x + 1))
/sqrt(2)))/(6*a**3)
```

3.43 $\int e^{-\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 64

$$\int e^{-\coth^{-1}(ax)}x\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

output
```
-(1-1/a^2/x^2)^(1/2)*x/a+1/2*(1-1/a^2/x^2)^(1/2)*x^2+1/2*arctanh((1-1/a^2/
x^2)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int e^{-\coth^{-1}(ax)}x\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-2+ax)+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{2a^2}$$

input
```
Integrate[x/E^ArcCoth[a*x],x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]
)/(2*a^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6719, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{2}\int \frac{\left(2a-\frac{1}{x}\right)x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(2a-\frac{1}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a^2}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{534}$$

$$\frac{-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-2ax\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{-\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-2ax\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-2ax\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-2ax\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}$$

input
```
Int[x/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (-2*a*Sqrt[1 - 1/(a^2*x^2)]*x + ArcTanh[Sq
rt[1 - 1/(a^2*x^2)]])/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.56

method	result
risch	$\frac{(ax-2)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2a^2}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2a\sqrt{a^2}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+2\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-2a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{2\sqrt{(ax-1)(ax+1)}\,a^2\sqrt{a^2}}$

input
```
int(x*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x-2)*(a*x+1)/a^2*((a*x-1)/(a*x+1))^(1/2)+1/2/a*ln(a^2*x/(a^2)^(1/2)
+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^
(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int e^{-\coth^{-1}(ax)}x\,dx=\frac{(a^2x^2-ax-2)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
1/2*((a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/(a
*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x\,dx = \int x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(54) = 108.

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.03

$$\int e^{-\coth^{-1}(ax)}x\,dx$$
$$= -\frac{1}{2}a\left(\frac{2\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*a*(2*(3*((a*x - 1)/(a*x + 1))^(3/2) - sqrt((a*x - 1)/(a*x + 1)))/(2*(
a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - log(sqrt((a*
x - 1)/(a*x + 1)) + 1)/a^3 + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int e^{-\coth^{-1}(ax)}x\,dx = \frac{1}{2}\sqrt{a^2x^2-1}\left(\frac{x\mathrm{sgn}(ax+1)}{a}-\frac{2\,\mathrm{sgn}(ax+1)}{a^2}\right) - \frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{2\,a|a|}$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(a^2*x^2 - 1)*(x*sgn(a*x + 1)/a - 2*sgn(a*x + 1)/a^2) - 1/2*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.52

$$\int e^{-\coth^{-1}(ax)}x\,dx = \frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2} - \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}-3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}$$

input
```
int(x*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
atanh(((a*x - 1)/(a*x + 1))^(1/2))/a^2 - (((a*x - 1)/(a*x + 1))^(1/2) - 3*
((a*x - 1)/(a*x + 1))^(3/2))/(a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2
*(a*x - 1))/(a*x + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int e^{-\coth^{-1}(ax)}x\,dx=\frac{\sqrt{ax+1}\,\sqrt{ax-1}\,ax-2\sqrt{ax+1}\,\sqrt{ax-1}+2\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)}{2a^2}$$

input
```
int(x*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(a*x + 1)*sqrt(a*x - 1)*a*x - 2*sqrt(a*x + 1)*sqrt(a*x - 1) + 2*log((
sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)))/(2*a**2)
```

3.44 $\int e^{-\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 37

$$\int e^{-\coth^{-1}(ax)}\,dx=\sqrt{1-\frac{1}{a^2x^2}}x-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
(1-1/a^2/x^2)^(1/2)*x-arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int e^{-\coth^{-1}(ax)}\,dx=\sqrt{1-\frac{1}{a^2x^2}}x-\frac{\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}$$

input
```
Integrate[E^(-ArcCoth[a*x]),x]
```

output
```
Sqrt[1 - 1/(a^2*x^2)]*x - Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]/a
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6718, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6718}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{534}$$

$$\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}+x\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a}+x\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$x\sqrt{1-\frac{1}{a^2x^2}}-a\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$x\sqrt{1-\frac{1}{a^2x^2}}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

input
```
Int[E^(-ArcCoth[a*x]),x]
```

output
```
Sqrt[1 - 1/(a^2*x^2)]*x - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 6718
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(33) = 66$.

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.46

method	result	size
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}\,(ax-1)}$	91
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}-a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$	98

input
```
int(((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)*((a*x-1)/(a*x+1))^(1/2)-ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2)
)/(a^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.73

$$\int e^{-\coth^{-1}(ax)}\,dx = \frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}} - \log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + \log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
((a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)
+ log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\,dx = \int \sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(33) = 66$.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int e^{-\coth^{-1}(ax)}\,dx = -a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + log(sqrt
((a*x - 1)/(a*x + 1)) + 1)/a^2 - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int e^{-\coth^{-1}(ax)}\,dx = \frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{a}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2
 - 1)*sgn(a*x + 1)/a
```

Mupad [B] (verification not implemented)

Time = 23.98 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int e^{-\coth^{-1}(ax)}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) - (2*atanh((
(a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int e^{-\coth^{-1}(ax)}\,dx = \frac{\sqrt{ax+1}\,\sqrt{ax-1} - 2\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)}{a}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(a*x + 1)*sqrt(a*x - 1) - 2*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(
2)))/a
```

3.45 $\int \frac{e^{-\coth^{-1}(ax)}}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{e^{-\coth^{-1}(ax)}}{x} \, dx = \csc^{-1}(ax) + \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{a^2x^2}}\right)$$

output

```
arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{e^{-\coth^{-1}(ax)}}{x} \, dx = \arcsin\left(\frac{1}{ax}\right) + \log\left(x\left(1 + \sqrt{\frac{-1 + a^2x^2}{a^2x^2}}\right)\right)$$

input

```
Integrate[1/(E^ArcCoth[a*x]*x),x]
```

output

```
ArcSin[1/(a*x)] + Log[x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6719, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{x} dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{538}$$

$$\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{223}$$

$$\arcsin\left(\frac{1}{ax}\right)-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{243}$$

$$\arcsin\left(\frac{1}{ax}\right)-\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{73}$$

$$a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}+\arcsin\left(\frac{1}{ax}\right)$$

$$\Big\downarrow \text{221}$$

$$\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+\arcsin\left(\frac{1}{ax}\right)$$

input
```
Int[1/(E^ArcCoth[a*x]*x),x]
```

output
```
ArcSin[1/(a*x)] + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(18) = 36$.

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 6.65

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}-\sqrt{a^2x^2-1}\,\sqrt{a^2}-\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}-a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$	133

input
```
int(((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)-(a^2
*x^2-1)^(1/2)*(a^2)^(1/2)-arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)-a*ln((a^
2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x-1)*(a*x+1))^(
1/2)/(a^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(18) = 36$.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.85

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = -2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

output
```
-2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(sqrt((a*x - 1)/(a*x + 1)) + 1)
- log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/x,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(18) = 36$.

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.50

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = -a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

output
```
-a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a - log(sqrt((a*x - 1)/(a*x + 1))
+ 1)/a + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(18) = 36$.

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.95

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = -2\arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1) - \frac{a\log\left(\left|-x|a| + \sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
```

output
```
-2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) - a*log(abs(-x*abs(a
) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - 2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/x,x)
```

output
```
2*atanh(((a*x - 1)/(a*x + 1))^(1/2)) - 2*atan(((a*x - 1)/(a*x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.80

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = -2atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right) + 2atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right) + 2\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/x,x)
```

output
```
2*( - atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1) + atan(sqrt(a*x - 1) + sqrt(
a*x + 1) + 1) + log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)))
```

3.46 $\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = -a\sqrt{1-\frac{1}{a^2x^2}} - a\csc^{-1}(ax)$$

output
```
-a*(1-1/a^2/x^2)^(1/2)-a*arccsc(a*x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = -a\left(\sqrt{1-\frac{1}{a^2x^2}} + \arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[1/(E^ArcCoth[a*x]*x^2),x]
```

output
```
-(a*(Sqrt[1 - 1/(a^2*x^2)] + ArcSin[1/(a*x)]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6719, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}dx$$
$$\Big\downarrow \text{ 6719}$$
$$-\int \frac{1-\frac{1}{ax}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$
$$\Big\downarrow \text{ 455}$$
$$a\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$
$$\Big\downarrow \text{ 223}$$
$$a\left(-\arcsin\left(\frac{1}{ax}\right)\right)-a\sqrt{1-\frac{1}{a^2x^2}}$$

input
```
Int[1/(E^ArcCoth[a*x]*x^2),x]
```

output
```
-(a*Sqrt[1 - 1/(a^2*x^2)]) - a*ArcSin[1/(a*x)]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(23) = 46$.

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.08

method	result
risch	$-\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}-\frac{a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^2x^2+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,ax+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right.}{\sqrt{(ax-1)(ax+1)}\,x\sqrt{a^2}}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(a*x+1)/x*((a*x-1)/(a*x+1))^(1/2)-a*arctan(1/(a^2*x^2-1)^(1/2))*((a*x-1)/
(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(23) = 46$.

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx=\frac{2\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(2*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*x + 1)*sqrt((a*x - 1)/(a*x +
 1)))/x
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/x**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(23) = 46$.

Time $= 0.11$ (sec) , antiderivative size $= 55$, normalized size of antiderivative $= 2.20$

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = -2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1} - \arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*x + 1) + 1) - arctan(sqrt((a
*x - 1)/(a*x + 1))))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 24.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = 2\,a\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-\frac{2\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{a\,x-1}{a\,x+1}+1}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/x^2,x)
```

output
```
2*a*atan(((a*x - 1)/(a*x + 1))^(1/2)) - (2*a*((a*x - 1)/(a*x + 1))^(1/2))/
((a*x - 1)/(a*x + 1) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.44

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = \frac{2atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)ax-2atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)ax-\sqrt{ax+1}\,\sqrt{ax-1}-ax}{x}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

output
```
(2*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a*x - 2*atan(sqrt(a*x - 1) + sq
rt(a*x + 1) + 1)*a*x - sqrt(a*x + 1)*sqrt(a*x - 1) - a*x)/x
```

3.47 $\int \frac{e^{-\coth^{-1}(ax)}}{x^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 52

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3} dx = a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x} + \frac{1}{2}a^2\csc^{-1}(ax)$$

output
```
a^2*(1-1/a^2/x^2)^(1/2)-1/2*a*(1-1/a^2/x^2)^(1/2)/x+1/2*a^2*arccsc(a*x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3} dx = \frac{a\left(\sqrt{1-\frac{1}{a^2x^2}}(-1+2ax)+ax\arcsin\left(\frac{1}{ax}\right)\right)}{2x}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*x^3),x]
```

output
```
(a*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + 2*a*x) + a*x*ArcSin[1/(a*x)]))/(2*x)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6719, 533, 25, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3} \, dx$$

$$\Big\downarrow \text{ 6719}$$

$$-\int \frac{1-\frac{1}{ax}}{\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}$$

$$\Big\downarrow \text{ 533}$$

$$-\frac{1}{2}a^2\int -\frac{a-\frac{2}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}a^2\int \frac{a-\frac{2}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{a-\frac{2}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{1}{2}\left(a\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+2a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{1}{2}\left(a^2\arcsin\left(\frac{1}{ax}\right)+2a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{2x}$$

input
```
Int[1/(E^ArcCoth[a*x]*x^3),x]
```

output
```
-1/2*(a*Sqrt[1 - 1/(a^2*x^2)])/x + (2*a^2*Sqrt[1 - 1/(a^2*x^2)] + a^2*ArcS
in[1/(a*x)])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.63

method	result
risch	$\frac{(ax+1)(2ax-1)\sqrt{\frac{ax-1}{ax+1}}}{2x^2}+\frac{a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2ax-2}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-2\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^3x^3+2\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^2x^2+2\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^2x^2-a^2\sqrt{a^2}\,x^2\,a\right.}{2\sqrt{(ax-1)(ax+1)}\,x^2}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x+1)*(2*a*x-1)/x^2*((a*x-1)/(a*x+1))^(1/2)+1/2*a^2*arctan(1/(a^2*x^
2-1)^(1/2))*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx=-\frac{2\,a^2x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(2\,a^2x^2+ax-1)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (2*a^2*x^2 + a*x - 1)*
sqrt((a*x - 1)/(a*x + 1)))/x^2
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/x**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(44) = 88$.

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.79

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = -\left(a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - \frac{3\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1} + \frac{(ax-1)^2}{(ax+1)^2} + 1}\right)a$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-(a*arctan(sqrt((a*x - 1)/(a*x + 1))) - (3*a*((a*x - 1)/(a*x + 1))^(3/2) +
 a*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x +
1)^2 + 1))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(44) = 88$.

Time = 0.15 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.02

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = -a^2\arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1) + \frac{\left(x|a| - \sqrt{a^2x^2-1}\right)^3 a^2\operatorname{sgn}(ax+1) + 2\left(x|a| - \sqrt{a^2x^2-1}\right)^2 a|a|\operatorname{sgn}(ax+1) - \left(x|a| - \sqrt{a^2x^2-1}\right)a^2\mathrm{s}}{\left(\left(x|a| - \sqrt{a^2x^2-1}\right)^2 + 1\right)^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

output
```
-a^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) + ((x*abs(a) - sqr
t(a^2*x^2 - 1))^3*a^2*sgn(a*x + 1) + 2*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*
abs(a)*sgn(a*x + 1) - (x*abs(a) - sqrt(a^2*x^2 - 1))*a^2*sgn(a*x + 1) + 2*
a*abs(a)*sgn(a*x + 1))/((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^2
```

Mupad [B] (verification not implemented)

Time = 24.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.58

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = a^2\sqrt{\frac{a\,x-1}{a\,x+1}} - \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x^2} - a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) + \frac{a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/x^3,x)
```

output
```
a^2*((a*x - 1)/(a*x + 1))^(1/2) - ((a*x - 1)/(a*x + 1))^(1/2)/(2*x^2) - a^
2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (a*((a*x - 1)/(a*x + 1))^(1/2))/(2*x
)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.58

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = \frac{-2atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^2x^2 + 2atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^2x^2 + 2\sqrt{ax+1}\,\sqrt{ax-1}\,a}{2x^2}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/x^3,x)
```

output
```
( - 2*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**2*x**2 + 2*atan(sqrt(a*x
- 1) + sqrt(a*x + 1) + 1)*a**2*x**2 + 2*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x -
sqrt(a*x + 1)*sqrt(a*x - 1))/(2*x**2)
```

3.48 $\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = -\frac{2}{3}a^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^3\csc^{-1}(ax)$$

output
```
-2/3*a^3*(1-1/a^2/x^2)^(1/2)-1/3*a*(1-1/a^2/x^2)^(1/2)/x^2+1/2*a^2*(1-1/a^
2/x^2)^(1/2)/x-1/2*a^3*arccsc(a*x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = -\frac{a\sqrt{1-\frac{1}{a^2x^2}}(2-3ax+4a^2x^2)}{6x^2}-\frac{1}{2}a^3\arcsin\left(\frac{1}{ax}\right)$$

input
```
Integrate[1/(E^ArcCoth[a*x]*x^4),x]
```

output
```
-1/6*(a*Sqrt[1 - 1/(a^2*x^2)]*(2 - 3*a*x + 4*a^2*x^2))/x^2 - (a^3*ArcSin[1
/(a*x)])/2
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6719, 533, 25, 27, 533, 25, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4} \, dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{1-\frac{1}{ax}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{533}$$

$$-\frac{1}{3}a^2\int -\frac{2a-\frac{3}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}a^2\int \frac{2a-\frac{3}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{2a-\frac{3}{x}}{\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{3}\left(\frac{1}{2}a^2\int -\frac{3a-\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^2\int \frac{3a-\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\int\frac{3a-\frac{4}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\left(3a\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+4a^2\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{3}\left(\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\left(3a^2\arcsin\left(\frac{1}{ax}\right)+4a^2\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}$$

input

```
Int[1/(E^ArcCoth[a*x]*x^4),x]
```

output

```
-1/3*(a*Sqrt[1 - 1/(a^2*x^2)])/x^2 + ((3*a^2*Sqrt[1 - 1/(a^2*x^2)])/(2*x)
- (a*(4*a^2*Sqrt[1 - 1/(a^2*x^2)] + 3*a^2*ArcSin[1/(a*x)]))/2)/3
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455

```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{(ax+1)(4a^2x^2-3ax+2)\sqrt{\frac{ax-1}{ax+1}}}{6x^3}-\frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-6\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^4x^4+6\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^2x^2-3\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^3x^3+6\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^4x^3-3a^3\sqrt{a^2}\right.}{6\sqrt{(ax-1}}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(a*x+1)*(4*a^2*x^2-3*a*x+2)/x^3*((a*x-1)/(a*x+1))^(1/2)-1/2*a^3*arcta
n(1/(a^2*x^2-1)^(1/2))*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*
x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx=\frac{6\,a^3x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(4\,a^3x^3+a^2x^2-ax+2)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^3}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (4*a^3*x^3 + a^2*x^2 -
a*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/x**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(65) = 130$.

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.73

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{1}{3}\left(3\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - \frac{9\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 4\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 3\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1} + \frac{3\,(ax-1)^2}{(ax+1)^2} + \frac{(ax-1)^3}{(ax+1)^3} + 1}\right)a$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/3*(3*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (9*a^2*((a*x - 1)/(a*x + 1)
)^(5/2) + 4*a^2*((a*x - 1)/(a*x + 1))^(3/2) + 3*a^2*sqrt((a*x - 1)/(a*x +
1)))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x
 + 1)^3 + 1))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 23.91 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.33

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,x^3} - \frac{2\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3} - \frac{a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x} + \frac{a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x^2}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/x^4,x)
```

output
```
a^3*atan(((a*x - 1)/(a*x + 1))^(1/2)) - ((a*x - 1)/(a*x + 1))^(1/2)/(3*x^3
) - (2*a^3*((a*x - 1)/(a*x + 1))^(1/2))/3 - (a^2*((a*x - 1)/(a*x + 1))^(1/
2))/(6*x) + (a*((a*x - 1)/(a*x + 1))^(1/2))/(6*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.39

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = \frac{6atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^3x^3 - 6atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^3x^3 - 4\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x}{6x^3}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/x^4,x)
```

output
```
(6*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**3*x**3 - 6*atan(sqrt(a*x - 1
) + sqrt(a*x + 1) + 1)*a**3*x**3 - 4*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2
 + 3*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x - 2*sqrt(a*x + 1)*sqrt(a*x - 1) + 4*a
**3*x**3)/(6*x**3)
```

3.49 $\int \frac{e^{-\coth^{-1}(ax)}}{x^5} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 103

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5} \, dx = \frac{2}{3}a^4\sqrt{1-\frac{1}{a^2x^2}} - \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} - \frac{3a^3\sqrt{1-\frac{1}{a^2x^2}}}{8x} + \frac{3}{8}a^4\csc^{-1}(ax)$$

output
```
2/3*a^4*(1-1/a^2/x^2)^(1/2)-1/4*a*(1-1/a^2/x^2)^(1/2)/x^3+1/3*a^2*(1-1/a^2
/x^2)^(1/2)/x^2-3/8*a^3*(1-1/a^2/x^2)^(1/2)/x+3/8*a^4*arccsc(a*x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.57

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5} \, dx = \frac{1}{24}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(-6+8ax-9a^2x^2+16a^3x^3)}{x^3} + 9a^3\arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[1/(E^ArcCoth[a*x]*x^5),x]
```

output
```
(a*((Sqrt[1 - 1/(a^2*x^2)]*(-6 + 8*a*x - 9*a^2*x^2 + 16*a^3*x^3))/x^3 + 9*
a^3*ArcSin[1/(a*x)]))/24
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6719, 533, 25, 27, 533, 25, 27, 533, 25, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{ 6719}$$

$$-\int \frac{1-\frac{1}{ax}}{\sqrt{1-\frac{1}{a^2x^2}}x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 533}$$

$$-\frac{1}{4}a^2\int -\frac{3a-\frac{4}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4}a^2\int \frac{3a-\frac{4}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\int \frac{3a-\frac{4}{x}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{1}{4}\left(\frac{1}{3}a^2\int -\frac{8a-\frac{9}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a^2\int\frac{8a-\frac{9}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\int\frac{8a-\frac{9}{x}}{\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \; 533$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{1}{2}a^2\int-\frac{9a-\frac{16}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^2\int\frac{9a-\frac{16}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\int\frac{9a-\frac{16}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \; 455$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\left(9a\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+16a^2\sqrt{1-\frac{1}{a^2x^2}}\right)\right)\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

$$\Big\downarrow \; 223$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a\left(\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\left(9a^2\arcsin\left(\frac{1}{ax}\right)+16a^2\sqrt{1-\frac{1}{a^2x^2}}\right)\right)\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}$$

input
```
Int[1/(E^ArcCoth[a*x]*x^5),x]
```

output
```
-1/4*(a*Sqrt[1 - 1/(a^2*x^2)])/x^3 + ((4*a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2
) - (a*((9*a^2*Sqrt[1 - 1/(a^2*x^2)])/(2*x) - (a*(16*a^2*Sqrt[1 - 1/(a^2*x
^2)] + 9*a^2*ArcSin[1/(a*x)]))/2))/3)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

method	result
risch	$\frac{(ax+1)\left(16a^3x^3-9a^2x^2+8ax-6\right)\sqrt{\frac{ax-1}{ax+1}}}{24x^4}+\frac{3a^4\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{8(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-24\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^5x^5+24\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^3x^3-9\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^4x^4-9a^4\sqrt{a^2}\,x^4\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+24\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/24*(a*x+1)*(16*a^3*x^3-9*a^2*x^2+8*a*x-6)/x^4*((a*x-1)/(a*x+1))^(1/2)+3/
8*a^4*arctan(1/(a^2*x^2-1)^(1/2))*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1)
)^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx = -\frac{18\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(16\,a^4x^4+7\,a^3x^3-a^2x^2+2\,ax-6\right)\sqrt{\frac{ax-1}{ax+1}}}{24\,x^4}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/24*(18*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) - (16*a^4*x^4 + 7*a^3*
x^3 - a^2*x^2 + 2*a*x - 6)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x^5}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/x**5,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/x**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(85) = 170$.

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.68

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx = -\frac{1}{12}\left(9\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - \frac{39\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} + 31\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 49\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 9\,a^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4\,(ax-1)}{ax+1} + \frac{6\,(ax-1)^2}{(ax+1)^2} + \frac{4\,(ax-1)^3}{(ax+1)^3} + \frac{(ax-1)^4}{(ax+1)^4} + 1}\right)a$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/12*(9*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (39*a^3*((a*x - 1)/(a*x +
 1))^(7/2) + 31*a^3*((a*x - 1)/(a*x + 1))^(5/2) + 49*a^3*((a*x - 1)/(a*x +
 1))^(3/2) + 9*a^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(
a*x - 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^3/(a*x + 1)^3 + (a*x - 1)^4/(a*x + 1)
^4 + 1))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(85) = 170$.

Time = 0.13 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.50

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx = -\frac{3}{4}\,a^4 \arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)\mathrm{sgn}(ax+1) + \frac{9\left(x|a| - \sqrt{a^2x^2-1}\right)^7 a^4\mathrm{sgn}(ax+1) + 33\left(x|a| - \sqrt{a^2x^2-1}\right)^5 a^4\mathrm{sgn}(ax+1) + 48\left(x|a| - \sqrt{a^2x^2-1\right.}}{}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="giac")
```

output
```
-3/4*a^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) + 1/12*(9*(x*a
bs(a) - sqrt(a^2*x^2 - 1))^7*a^4*sgn(a*x + 1) + 33*(x*abs(a) - sqrt(a^2*x^
2 - 1))^5*a^4*sgn(a*x + 1) + 48*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a^3*abs(a
)*sgn(a*x + 1) - 33*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*a^4*sgn(a*x + 1) + 64
*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a^3*abs(a)*sgn(a*x + 1) - 9*(x*abs(a) -
sqrt(a^2*x^2 - 1))*a^4*sgn(a*x + 1) + 16*a^3*abs(a)*sgn(a*x + 1))/((x*abs(
a) - sqrt(a^2*x^2 - 1))^2 + 1)^4
```

Mupad [B] (verification not implemented)

Time = 23.71 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.25

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx = \frac{2\,a^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3} - \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,x^4} - \frac{3\,a^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4} - \frac{a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x^2} + \frac{7\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x} + \frac{a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{12\,x^3}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/x^5,x)
```

output
```
(2*a^4*((a*x - 1)/(a*x + 1))^(1/2))/3 - ((a*x - 1)/(a*x + 1))^(1/2)/(4*x^4
) - (3*a^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/4 - (a^2*((a*x - 1)/(a*x + 1
))^(1/2))/(24*x^2) + (7*a^3*((a*x - 1)/(a*x + 1))^(1/2))/(24*x) + (a*((a*x
 - 1)/(a*x + 1))^(1/2))/(12*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.26

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx$$
$$= \frac{-18atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^4x^4+18atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^4x^4+16\sqrt{ax+1}\,\sqrt{ax-}}{24x^4}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/x^5,x)
```

output
```
( - 18*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**4*x**4 + 18*atan(sqrt(a*
x - 1) + sqrt(a*x + 1) + 1)*a**4*x**4 + 16*sqrt(a*x + 1)*sqrt(a*x - 1)*a**
3*x**3 - 9*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 + 8*sqrt(a*x + 1)*sqrt(a*
x - 1)*a*x - 6*sqrt(a*x + 1)*sqrt(a*x - 1) - 16*a**4*x**4)/(24*x**4)
```

3.50 $\int e^{-2\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 42

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx=-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1+ax)}{a^4}$$

output
```
-2*x/a^3+x^2/a^2-2/3*x^3/a+1/4*x^4+2*ln(a*x+1)/a^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx=-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1+ax)}{a^4}$$

input
```
Integrate[x^3/E^(2*ArcCoth[a*x]),x]
```

output
```
(-2*x)/a^3 + x^2/a^2 - (2*x^3)/(3*a) + x^4/4 + (2*Log[1 + a*x])/a^4
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x^3dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{x^3(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(-x^3+\frac{2x^2}{a}-\frac{2x}{a^2}-\frac{2}{a^3(ax+1)}+\frac{2}{a^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\log(ax+1)}{a^4}-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}$$

input
```
Int[x^3/E^(2*ArcCoth[a*x]),x]
```

output
```
(-2*x)/a^3 + x^2/a^2 - (2*x^3)/(3*a) + x^4/4 + (2*Log[1 + a*x])/a^4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

method	result	size
norman	$-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\ln(ax+1)}{a^4}$	39
risch	$-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\ln(ax+1)}{a^4}$	39
default	$\frac{\frac{1}{4}a^3x^4-\frac{2}{3}a^2x^3+a\,x^2-2x}{a^3}+\frac{2\ln(ax+1)}{a^4}$	42
parallelrisch	$\frac{3a^4x^4-8a^3x^3+12a^2x^2-24ax+24\ln(ax+1)}{12a^4}$	43
meijerg	$\frac{-\frac{ax\left(-15a^3x^3+20a^2x^2-30ax+60\right)}{60}+\ln(ax+1)}{a^4}-\frac{\frac{ax\left(4a^2x^2-6ax+12\right)}{12}-\ln(ax+1)}{a^4}$	71

input
```
int(x^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-2*x/a^3+x^2/a^2-2/3*x^3/a+1/4*x^4+2*ln(a*x+1)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx=\frac{3\,a^4x^4-8\,a^3x^3+12\,a^2x^2-24\,ax+24\,\log\left(ax+1\right)}{12\,a^4}$$

input
```
integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/12*(3*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 24*a*x + 24*log(a*x + 1))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx=\frac{x^4}{4}-\frac{2x^3}{3a}+\frac{x^2}{a^2}-\frac{2x}{a^3}+\frac{2\log\left(ax+1\right)}{a^4}$$

input
```
integrate(x**3*(a*x-1)/(a*x+1),x)
```

output
```
x**4/4 - 2*x**3/(3*a) + x**2/a**2 - 2*x/a**3 + 2*log(a*x + 1)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx=\frac{3\,a^3x^4-8\,a^2x^3+12\,ax^2-24\,x}{12\,a^3}+\frac{2\,\log\left(ax+1\right)}{a^4}$$

input
```
integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/12*(3*a^3*x^4 - 8*a^2*x^3 + 12*a*x^2 - 24*x)/a^3 + 2*log(a*x + 1)/a^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx = \frac{3\,a^4x^4 - 8\,a^3x^3 + 12\,a^2x^2 - 24\,ax}{12\,a^4} + \frac{2\,\log\left(|ax+1|\right)}{a^4}$$

input
```
integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/12*(3*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 24*a*x)/a^4 + 2*log(abs(a*x + 1
))/a^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx = \frac{2\,\ln\left(a\,x+1\right)}{a^4} - \frac{2\,x}{a^3} + \frac{x^4}{4} - \frac{2\,x^3}{3\,a} + \frac{x^2}{a^2}$$

input
```
int((x^3*(a*x - 1))/(a*x + 1),x)
```

output
```
(2*log(a*x + 1))/a^4 - (2*x)/a^3 + x^4/4 - (2*x^3)/(3*a) + x^2/a^2
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx = \frac{24\log(ax+1) + 3a^4x^4 - 8a^3x^3 + 12a^2x^2 - 24ax}{12a^4}$$

input
```
int(x^3*(a*x-1)/(a*x+1),x)
```

output
```
(24*log(a*x + 1) + 3*a**4*x**4 - 8*a**3*x**3 + 12*a**2*x**2 - 24*a*x)/(12*
a**4)
```

3.51 $\int e^{-2\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx=\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\log(1+ax)}{a^3}$$

output
```
2*x/a^2-x^2/a+1/3*x^3-2*ln(a*x+1)/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx=\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\log(1+ax)}{a^3}$$

input
```
Integrate[x^2/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*x)/a^2 - x^2/a + x^3/3 - (2*Log[1 + a*x])/a^3
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6717$$

$$-\int e^{-2\text{arctanh}(ax)}x^2dx$$

$$\Big\downarrow \; 6676$$

$$-\int \frac{x^2(1-ax)}{ax+1}dx$$

$$\Big\downarrow \; 86$$

$$-\int \left(-x^2+\frac{2x}{a}+\frac{2}{a^2(ax+1)}-\frac{2}{a^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\log(ax+1)}{a^3}+\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}$$

input
```
Int[x^2/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*x)/a^2 - x^2/a + x^3/3 - (2*Log[1 + a*x])/a^3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

method	result	size
norman	$\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\ln(ax+1)}{a^3}$	32
risch	$\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\ln(ax+1)}{a^3}$	32
default	$\frac{\frac{1}{3}a^2x^3-a\,x^2+2x}{a^2}-\frac{2\ln(ax+1)}{a^3}$	35
parallelrisch	$-\frac{-a^3x^3+3a^2x^2-6ax+6\ln(ax+1)}{3a^3}$	35
meijerg	$\frac{\frac{ax\left(4a^2x^2-6ax+12\right)}{12}-\ln(ax+1)}{a^3}-\frac{-\frac{ax(-3ax+6)}{6}+\ln(ax+1)}{a^3}$	55

input
```
int(x^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
2*x/a^2-x^2/a+1/3*x^3-2*ln(a*x+1)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx = \frac{a^3x^3 - 3\,a^2x^2 + 6\,ax - 6\,\log\left(ax+1\right)}{3\,a^3}$$

input
```
integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/3*(a^3*x^3 - 3*a^2*x^2 + 6*a*x - 6*log(a*x + 1))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx = \frac{x^3}{3} - \frac{x^2}{a} + \frac{2x}{a^2} - \frac{2\log\left(ax+1\right)}{a^3}$$

input
```
integrate(x**2*(a*x-1)/(a*x+1),x)
```

output
```
x**3/3 - x**2/a + 2*x/a**2 - 2*log(a*x + 1)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx = \frac{a^2x^3 - 3\,ax^2 + 6\,x}{3\,a^2} - \frac{2\,\log\left(ax+1\right)}{a^3}$$

input
```
integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/3*(a^2*x^3 - 3*a*x^2 + 6*x)/a^2 - 2*log(a*x + 1)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx = \frac{a^3x^3 - 3\,a^2x^2 + 6\,ax}{3\,a^3} - \frac{2\,\log\left(|ax+1|\right)}{a^3}$$

input
```
integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/3*(a^3*x^3 - 3*a^2*x^2 + 6*a*x)/a^3 - 2*log(abs(a*x + 1))/a^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx = \frac{2\,x}{a^2} - \frac{2\,\ln\left(a\,x+1\right)}{a^3} + \frac{x^3}{3} - \frac{x^2}{a}$$

input
```
int((x^2*(a*x - 1))/(a*x + 1),x)
```

output
```
(2*x)/a^2 - (2*log(a*x + 1))/a^3 + x^3/3 - x^2/a
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^2\,dx = \frac{-6\log(ax+1) + a^3x^3 - 3a^2x^2 + 6ax}{3a^3}$$

input
```
int(x^2*(a*x-1)/(a*x+1),x)
```

output
```
( - 6*log(a*x + 1) + a**3*x**3 - 3*a**2*x**2 + 6*a*x)/(3*a**3)
```

3.52 $\int e^{-2\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 25

$$\int e^{-2\coth^{-1}(ax)}x\,dx=-\frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1+ax)}{a^2}$$

output
```
-2*x/a+1/2*x^2+2*ln(a*x+1)/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x\,dx=-\frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1+ax)}{a^2}$$

input
```
Integrate[x/E^(2*ArcCoth[a*x]),x]
```

output
```
(-2*x)/a + x^2/2 + (2*Log[1 + a*x])/a^2
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}xdx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{x(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(-x+\frac{2}{a}-\frac{2}{a(ax+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\log(ax+1)}{a^2}-\frac{2x}{a}+\frac{x^2}{2}$$

input
```
Int[x/E^(2*ArcCoth[a*x]),x]
```

output
```
(-2*x)/a + x^2/2 + (2*Log[1 + a*x])/a^2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
norman	$-\frac{2x}{a}+\frac{x^2}{2}+\frac{2\ln(ax+1)}{a^2}$	24
risch	$-\frac{2x}{a}+\frac{x^2}{2}+\frac{2\ln(ax+1)}{a^2}$	24
parallelrisch	$\frac{a^2x^2-4ax+4\ln(ax+1)}{2a^2}$	26
default	$\frac{\frac{1}{2}a\,x^2-2x}{a}+\frac{2\ln(ax+1)}{a^2}$	27
meijerg	$\frac{-\frac{ax(-3ax+6)}{6}+\ln(ax+1)}{a^2}-\frac{ax-\ln(ax+1)}{a^2}$	40

input
```
int(x*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-2*x/a+1/2*x^2+2*ln(a*x+1)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x\,dx = \frac{a^2x^2 - 4\,ax + 4\,\log(ax+1)}{2\,a^2}$$

input
```
integrate(x*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/2*(a^2*x^2 - 4*a*x + 4*log(a*x + 1))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}x\,dx = \frac{x^2}{2} - \frac{2x}{a} + \frac{2\log(ax+1)}{a^2}$$

input
```
integrate(x*(a*x-1)/(a*x+1),x)
```

output
```
x**2/2 - 2*x/a + 2*log(a*x + 1)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int e^{-2\coth^{-1}(ax)}x\,dx = \frac{ax^2 - 4\,x}{2\,a} + \frac{2\,\log(ax+1)}{a^2}$$

input
```
integrate(x*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/2*(a*x^2 - 4*x)/a + 2*log(a*x + 1)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int e^{-2\coth^{-1}(ax)}x\,dx = \frac{a^2x^2-4\,ax}{2\,a^2}+\frac{2\,\log\left(|ax+1|\right)}{a^2}$$

input
```
integrate(x*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/2*(a^2*x^2 - 4*a*x)/a^2 + 2*log(abs(a*x + 1))/a^2
```

Mupad [B] (verification not implemented)

Time = 23.78 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}x\,dx = \frac{2\,\ln\left(a\,x+1\right)}{a^2}-\frac{2\,x}{a}+\frac{x^2}{2}$$

input
```
int((x*(a*x - 1))/(a*x + 1),x)
```

output
```
(2*log(a*x + 1))/a^2 - (2*x)/a + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x\,dx = \frac{4\log(ax+1)+a^2x^2-4ax}{2a^2}$$

input
```
int(x*(a*x-1)/(a*x+1),x)
```

output
```
(4*log(a*x + 1) + a**2*x**2 - 4*a*x)/(2*a**2)
```

3.53 $\int e^{-2\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 13

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\log(1+ax)}{a}$$

output
```
x-2*ln(a*x+1)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\log(1+ax)}{a}$$

input
```
Integrate[E^(-2*ArcCoth[a*x]),x]
```

output
```
x - (2*Log[1 + a*x])/a
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6717, 6675, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{6675}$$

$$-\int \frac{1-ax}{ax+1}dx$$

$$\Big\downarrow \text{49}$$

$$-\int \left(\frac{2}{ax+1}-1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x-\frac{2\log(ax+1)}{a}$$

input
```
Int[E^(-2*ArcCoth[a*x]),x]
```

output
```
x - (2*Log[1 + a*x])/a
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6675
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$x-\frac{2\ln\left(ax+1\right)}{a}$	14
norman	$x-\frac{2\ln\left(ax+1\right)}{a}$	14
risch	$x-\frac{2\ln\left(ax+1\right)}{a}$	14
parallelrisch	$-\frac{-ax+2\ln\left(ax+1\right)}{a}$	19
meijerg	$\frac{ax-\ln\left(ax+1\right)}{a}-\frac{\ln\left(ax+1\right)}{a}$	29

input
```
int((a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
x-2*ln(a*x+1)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int e^{-2\coth^{-1}(ax)}\,dx = \frac{ax - 2\,\log\left(ax+1\right)}{a}$$

input
```
integrate((a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
(a*x - 2*log(a*x + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\log\left(ax+1\right)}{a}$$

input
```
integrate((a*x-1)/(a*x+1),x)
```

output
```
x - 2*log(a*x + 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\,\log\left(ax+1\right)}{a}$$

input
```
integrate((a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
x - 2*log(a*x + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\log(|ax+1|)}{a}$$

input
```
integrate((a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
x - 2*log(abs(a*x + 1))/a
```

Mupad [B] (verification not implemented)

Time = 23.67 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\,dx = x - \frac{2\ln(a\,x+1)}{a}$$

input
```
int((a*x - 1)/(a*x + 1),x)
```

output
```
x - (2*log(a*x + 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int e^{-2\coth^{-1}(ax)}\,dx = \frac{-2\log(ax+1)+ax}{a}$$

input
```
int((a*x-1)/(a*x+1),x)
```

output
```
( - 2*log(a*x + 1) + a*x)/a
```

3.54 $\int \frac{e^{-2\coth^{-1}(ax)}}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x} \, dx = -\log(x) + 2\log(1+ax)$$

output
```
-ln(x)+2*ln(a*x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x} \, dx = -\log(x) + 2\log(1+ax)$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
-Log[x] + 2*Log[1 + a*x]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x} \, dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{x}dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{1-ax}{x(ax+1)}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(\frac{1}{x}-\frac{2a}{ax+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\log(ax+1)-\log(x)$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
-Log[x] + 2*Log[1 + a*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-\ln \left(x \right)+2 \ln \left(a x +1\right)$	14
norman	$-\ln \left(x \right)+2 \ln \left(a x +1\right)$	14
parallelrisch	$-\ln \left(x \right)+2 \ln \left(a x +1\right)$	14
risch	$-\ln \left(x \right)+2 \ln \left(-a x -1\right)$	15
meijerg	$2 \ln \left(a x +1\right)-\ln \left(x \right)-\ln \left(a \right)$	18

input
```
int((a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

output
```
-ln(x)+2*ln(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = 2\,\log\left(ax+1\right) - \log\left(x\right)$$

input
```
integrate((a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

output
```
2*log(a*x + 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = -\log\left(x\right) + 2\log\left(x+\frac{1}{a}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/x,x)
```

output
```
-log(x) + 2*log(x + 1/a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = 2\,\log\left(ax+1\right) - \log\left(x\right)$$

input
```
integrate((a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

output
```
2*log(a*x + 1) - log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = 2\,\log\left(|ax+1|\right) - \log\left(|x|\right)$$

input
```
integrate((a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

output
```
2*log(abs(a*x + 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = 2\,\ln\left(-3\,a\,x-3\right) - \ln\left(x\right)$$

input
```
int((a*x - 1)/(x*(a*x + 1)),x)
```

output
```
2*log(- 3*a*x - 3) - log(x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = 2\log(ax+1) - \log(x)$$

input
```
int((a*x-1)/(a*x+1)/x,x)
```

output
```
2*log(a*x + 1) - log(x)
```

3.55 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 18

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2} dx = \frac{1}{x} + 2a\log(x) - 2a\log(1 + ax)$$

output
```
1/x+2*a*ln(x)-2*a*ln(a*x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2} dx = \frac{1}{x} + 2a\log(x) - 2a\log(1 + ax)$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
x^(-1) + 2*a*Log[x] - 2*a*Log[1 + a*x]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \ 6717$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{x^2}dx$$

$$\Big\downarrow \ 6676$$

$$-\int \frac{1-ax}{x^2(ax+1)}dx$$

$$\Big\downarrow \ 86$$

$$-\int \left(\frac{2a^2}{ax+1}-\frac{2a}{x}+\frac{1}{x^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$2a\log(x)-2a\log(ax+1)+\frac{1}{x}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
x^(-1) + 2*a*Log[x] - 2*a*Log[1 + a*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{1}{x}+2\ln(x)\,a-2a\ln(ax+1)$	19
norman	$\frac{1}{x}+2\ln(x)\,a-2a\ln(ax+1)$	19
risch	$\frac{1}{x}+2a\ln(-x)-2a\ln(ax+1)$	21
parallelrisch	$\frac{2a\ln(x)x-2\ln(ax+1)xa+1}{x}$	23
meijerg	$a(\ln(x)+\ln(a)-\ln(ax+1))-a\left(-\frac{1}{ax}-\ln(x)-\ln(a)+\ln(ax+1)\right)$	43

input
```
int((a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x+2*ln(x)*a-2*a*ln(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = -\frac{2\,ax\log\left(ax+1\right) - 2\,ax\log\left(x\right) - 1}{x}$$

input
```
integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

output
```
-(2*a*x*log(a*x + 1) - 2*a*x*log(x) - 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = 2a\left(\log\left(x\right) - \log\left(x + \frac{1}{a}\right)\right) + \frac{1}{x}$$

input
```
integrate((a*x-1)/(a*x+1)/x**2,x)
```

output
```
2*a*(log(x) - log(x + 1/a)) + 1/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = -2\,a\log\left(ax+1\right) + 2\,a\log\left(x\right) + \frac{1}{x}$$

input
```
integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

output
```
-2*a*log(a*x + 1) + 2*a*log(x) + 1/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = -2\,a\log\left(|ax+1|\right) + 2\,a\log\left(|x|\right) + \frac{1}{x}$$

input
```
integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

output
```
-2*a*log(abs(a*x + 1)) + 2*a*log(abs(x)) + 1/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{x} - 4\,a\,\mathrm{atanh}(2\,a\,x+1)$$

input
```
int((a*x - 1)/(x^2*(a*x + 1)),x)
```

output
```
1/x - 4*a*atanh(2*a*x + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = \frac{-2\log(ax+1)\,ax + 2\log(x)\,ax + 1}{x}$$

input
```
int((a*x-1)/(a*x+1)/x^2,x)
```

output
```
( - 2*log(a*x + 1)*a*x + 2*log(x)*a*x + 1)/x
```

3.56 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3} dx = \frac{1}{2x^2} - \frac{2a}{x} - 2a^2\log(x) + 2a^2\log(1+ax)$$

output
```
1/2/x^2-2*a/x-2*a^2*ln(x)+2*a^2*ln(a*x+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3} dx = \frac{1}{2x^2} - \frac{2a}{x} - 2a^2\log(x) + 2a^2\log(1+ax)$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
1/(2*x^2) - (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 + a*x]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{1-ax}{x^3(ax+1)}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(-\frac{2a^3}{ax+1}+\frac{2a^2}{x}-\frac{2a}{x^2}+\frac{1}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-2a^2\log(x)+2a^2\log(ax+1)-\frac{2a}{x}+\frac{1}{2x^2}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
1/(2*x^2) - (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 + a*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

method	result
norman	$\frac{\frac{1}{2}-2ax}{x^2}-2a^2\ln(x)+2a^2\ln(ax+1)$
default	$\frac{1}{2x^2}-\frac{2a}{x}-2a^2\ln(x)+2a^2\ln(ax+1)$
risch	$\frac{\frac{1}{2}-2ax}{x^2}+2a^2\ln(-ax-1)-2a^2\ln(x)$
parallelrisch	$-\frac{4a^2\ln(x)x^2-4\ln(ax+1)x^2a^2-1+4ax}{2x^2}$
meijerg	$a^2\left(-\frac{1}{ax}-\ln(x)-\ln(a)+\ln(ax+1)\right)-a^2\left(-\frac{1}{2a^2x^2}+\frac{1}{ax}+\ln(x)+\ln(a)-\ln(ax+1)\right)$

input
```
int((a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2-2*a*x)/x^2-2*a^2*ln(x)+2*a^2*ln(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = \frac{4\,a^2x^2\log(ax+1) - 4\,a^2x^2\log(x) - 4\,ax + 1}{2\,x^2}$$

input
```
integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

output
```
1/2*(4*a^2*x^2*log(a*x + 1) - 4*a^2*x^2*log(x) - 4*a*x + 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = 2a^2\left(-\log(x) + \log\left(x + \frac{1}{a}\right)\right) + \frac{-4ax + 1}{2x^2}$$

input
```
integrate((a*x-1)/(a*x+1)/x**3,x)
```

output
```
2*a**2*(-log(x) + log(x + 1/a)) + (-4*a*x + 1)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = 2\,a^2\log(ax+1) - 2\,a^2\log(x) - \frac{4\,ax - 1}{2\,x^2}$$

input
```
integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

output
```
2*a^2*log(a*x + 1) - 2*a^2*log(x) - 1/2*(4*a*x - 1)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = 2\,a^2\log\left(|ax+1|\right) - 2\,a^2\log\left(|x|\right) - \frac{4\,ax-1}{2\,x^2}$$

input
```
integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

output
```
2*a^2*log(abs(a*x + 1)) - 2*a^2*log(abs(x)) - 1/2*(4*a*x - 1)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = 4\,a^2\,\mathrm{atanh}(2\,a\,x+1) - \frac{2\,a\,x-\frac{1}{2}}{x^2}$$

input
```
int((a*x - 1)/(x^3*(a*x + 1)),x)
```

output
```
4*a^2*atanh(2*a*x + 1) - (2*a*x - 1/2)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx = \frac{4\log(ax+1)\,a^2x^2 - 4\log(x)\,a^2x^2 - 4ax + 1}{2x^2}$$

input
```
int((a*x-1)/(a*x+1)/x^3,x)
```

output
```
(4*log(a*x + 1)*a**2*x**2 - 4*log(x)*a**2*x**2 - 4*a*x + 1)/(2*x**2)
```

3.57 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{3x^3} - \frac{a}{x^2} + \frac{2a^2}{x} + 2a^3\log(x) - 2a^3\log(1+ax)$$

output
```
1/3/x^3-a/x^2+2*a^2/x+2*a^3*ln(x)-2*a^3*ln(a*x+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{3x^3} - \frac{a}{x^2} + \frac{2a^2}{x} + 2a^3\log(x) - 2a^3\log(1+ax)$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
1/(3*x^3) - a/x^2 + (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 + a*x]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6676, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{1-ax}{x^4(ax+1)}dx$$

$$\Big\downarrow \text{86}$$

$$-\int \left(\frac{2a^4}{ax+1}-\frac{2a^3}{x}+\frac{2a^2}{x^2}-\frac{2a}{x^3}+\frac{1}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a^3\log(x)-2a^3\log(ax+1)+\frac{2a^2}{x}-\frac{a}{x^2}+\frac{1}{3x^3}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
1/(3*x^3) - a/x^2 + (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 + a*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

method	result
norman	$\frac{\frac{1}{3}+2a^2x^2-ax}{x^3}+2a^3\ln(x)-2a^3\ln(ax+1)$
default	$\frac{1}{3x^3}-\frac{a}{x^2}+\frac{2a^2}{x}+2a^3\ln(x)-2a^3\ln(ax+1)$
risch	$\frac{\frac{1}{3}+2a^2x^2-ax}{x^3}+2a^3\ln(-x)-2a^3\ln(ax+1)$
parallelrisch	$\frac{6\ln(x)x^3a^3-6\ln(ax+1)x^3a^3+1+6a^2x^2-3ax}{3x^3}$
meijerg	$a^3\left(-\frac{1}{2a^2x^2}+\frac{1}{ax}+\ln(x)+\ln(a)-\ln(ax+1)\right)-a^3\left(-\frac{1}{3x^3a^3}+\frac{1}{2a^2x^2}-\frac{1}{ax}-\ln(x)-\ln(a)\right.$

input
```
int((a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
(1/3+2*a^2*x^2-a*x)/x^3+2*a^3*ln(x)-2*a^3*ln(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = -\frac{6\,a^3x^3\log\left(ax+1\right)-6\,a^3x^3\log\left(x\right)-6\,a^2x^2+3\,ax-1}{3\,x^3}$$

input
```
integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(6*a^3*x^3*log(a*x + 1) - 6*a^3*x^3*log(x) - 6*a^2*x^2 + 3*a*x - 1)/x
^3
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = 2a^3\left(\log\left(x\right)-\log\left(x+\frac{1}{a}\right)\right)+\frac{6a^2x^2-3ax+1}{3x^3}$$

input
```
integrate((a*x-1)/(a*x+1)/x**4,x)
```

output
```
2*a**3*(log(x) - log(x + 1/a)) + (6*a**2*x**2 - 3*a*x + 1)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = -2\,a^3\log\left(ax+1\right)+2\,a^3\log\left(x\right)+\frac{6\,a^2x^2-3\,ax+1}{3\,x^3}$$

input
```
integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

output
```
-2*a^3*log(a*x + 1) + 2*a^3*log(x) + 1/3*(6*a^2*x^2 - 3*a*x + 1)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = -2\,a^3\log\left(|ax+1|\right) + 2\,a^3\log\left(|x|\right) + \frac{6\,a^2x^2 - 3\,ax + 1}{3\,x^3}$$

input
```
integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

output
```
-2*a^3*log(abs(a*x + 1)) + 2*a^3*log(abs(x)) + 1/3*(6*a^2*x^2 - 3*a*x + 1)
/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = \frac{2\,a^2\,x^2 - a\,x + \frac{1}{3}}{x^3} - 4\,a^3\,\mathrm{atanh}(2\,a\,x + 1)$$

input
```
int((a*x - 1)/(x^4*(a*x + 1)),x)
```

output
```
(2*a^2*x^2 - a*x + 1/3)/x^3 - 4*a^3*atanh(2*a*x + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = \frac{-6\log(ax+1)\,a^3x^3 + 6\log(x)\,a^3x^3 + 6a^2x^2 - 3ax + 1}{3x^3}$$

input
```
int((a*x-1)/(a*x+1)/x^4,x)
```

output
```
( - 6*log(a*x + 1)*a**3*x**3 + 6*log(x)*a**3*x**3 + 6*a**2*x**2 - 3*a*x +
1)/(3*x**3)
```

3.58 $\int e^{-3\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 136

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx=-\frac{4\left(a-\frac{1}{x}\right)}{a^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3}+\frac{19\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4+\frac{51\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

output
```
(-4*a+4/x)/a^5/(1-1/a^2/x^2)^(1/2)-6*(1-1/a^2/x^2)^(1/2)*x/a^3+19/8*(1-1/a
^2/x^2)^(1/2)*x^2/a^2-(1-1/a^2/x^2)^(1/2)*x^3/a+1/4*(1-1/a^2/x^2)^(1/2)*x^
4+51/8*arctanh((1-1/a^2/x^2)^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.61

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx=\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-80-29ax+11a^2x^2-6a^3x^3+2a^4x^4\right)}{1+ax}+51\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{8a^4}$$

input
```
Integrate[x^3/E^(3*ArcCoth[a*x]),x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-80 - 29*a*x + 11*a^2*x^2 - 6*a^3*x^3 + 2*a^4
*x^4))/(1 + a*x) + 51*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(8*a^4)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6719, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2 x^5}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{2353}$$

$$-\int \left(\frac{x^5}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^4}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^3}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4x^2}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4}{a^4\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{19x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{51\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}-\frac{6x\sqrt{1-\frac{1}{a^2x^2}}}{a^3}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)}$$

input
```
Int[x^3/E^(3*ArcCoth[a*x]),x]
```

output
```
(-4*Sqrt[1 - 1/(a^2*x^2)])/(a^3*(a + x^(-1))) - (6*Sqrt[1 - 1/(a^2*x^2)]*x
)/a^3 + (19*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2) - (Sqrt[1 - 1/(a^2*x^2)]*x^
3)/a + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (51*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])
/(8*a^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

method	result
risch	$\frac{(2a^3x^3-8a^2x^2+19ax-48)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{8a^4}+\frac{\left(\frac{51\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{8a^3\sqrt{a^2}}-\frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^5\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{\left(-2\sqrt{a^2}\left(a^2x^2-1\right)^{\frac{3}{2}}a^3x^3+8\sqrt{a^2}\left((ax-1)(ax+1)\right)^{\frac{3}{2}}a^2x^2-4\sqrt{a^2}\left(a^2x^2-1\right)^{\frac{3}{2}}a^2x^2-21\sqrt{a^2}\sqrt{a^2x^2-1}a^3x^3+16\sqrt{a^2}\left((ax-1)(ax\right.\right.}{}$

input
```
int(x^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*a^3*x^3-8*a^2*x^2+19*a*x-48)*(a*x+1)/a^4*((a*x-1)/(a*x+1))^(1/2)+(5
1/8/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-4/a^5/(x+1/a)*
(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1
))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.68

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx = \frac{(2a^4x^4 - 6a^3x^3 + 11a^2x^2 - 29ax - 80)\sqrt{\frac{ax-1}{ax+1}} + 51\log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right) - 51\log\left(\sqrt{\frac{ax-1}{ax+1}} - 1\right)}{8a^4}$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
1/8*((2*a^4*x^4 - 6*a^3*x^3 + 11*a^2*x^2 - 29*a*x - 80)*sqrt((a*x - 1)/(a*
x + 1)) + 51*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 51*log(sqrt((a*x - 1)/(a
*x + 1)) - 1))/a^4
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx = \int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**3*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Integral(x**3*((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.64

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx =$$

$$-\frac{1}{8}\,a\left(\frac{2\left(77\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-149\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+123\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-35\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4\,(ax-1)a^5}{ax+1}-\frac{6\,(ax-1)^2a^5}{(ax+1)^2}+\frac{4\,(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{51\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}+\frac{51\,\log\left(\sqrt{\right.}{a}\right.$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-1/8*a*(2*(77*((a*x - 1)/(a*x + 1))^(7/2) - 149*((a*x - 1)/(a*x + 1))^(5/2
) + 123*((a*x - 1)/(a*x + 1))^(3/2) - 35*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*
x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(
a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 51*log(sqrt((a*x - 1)/(a
*x + 1)) + 1)/a^5 + 51*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5 + 32*sqrt((a
*x - 1)/(a*x + 1))/a^5)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx = \int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 23.42 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.41

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx = \frac{51\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a^4} - \frac{\frac{35\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4} - \frac{123\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{4} + \frac{149\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{4} - \frac{77\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{4}}{a^4 + \frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3} + \frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4} - \frac{4\,a^4\,(a\,x-1)}{a\,x+1}} - \frac{4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a^4}$$

input
```
int(x^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(51*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a^4) - ((35*((a*x - 1)/(a*x + 1
))^(1/2))/4 - (123*((a*x - 1)/(a*x + 1))^(3/2))/4 + (149*((a*x - 1)/(a*x +
 1))^(5/2))/4 - (77*((a*x - 1)/(a*x + 1))^(7/2))/4)/(a^4 + (6*a^4*(a*x - 1
)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*
x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) - (4*((a*x - 1)/(a*x + 1))^(1/2))/
a^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.12

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx = \frac{2\sqrt{ax+1}\,\sqrt{ax-1}\,a^4x^4 - 6\sqrt{ax+1}\,\sqrt{ax-1}\,a^3x^3 + 11\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2 - 29\sqrt{ax+1}\,\sqrt{ax-1}\,a}{8a^4\,(ax+1)}$$

input
```
int(x^3*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(2*sqrt(a*x + 1)*sqrt(a*x - 1)*a**4*x**4 - 6*sqrt(a*x + 1)*sqrt(a*x - 1)*a
**3*x**3 + 11*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 - 29*sqrt(a*x + 1)*sqr
t(a*x - 1)*a*x - 80*sqrt(a*x + 1)*sqrt(a*x - 1) + 102*log((sqrt(a*x - 1) +
 sqrt(a*x + 1))/sqrt(2))*a*x + 102*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqr
t(2)) - 57*a*x - 57)/(8*a**4*(a*x + 1))
```

3.59 $\int e^{-3\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 116

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx=\frac{4\left(a-\frac{1}{x}\right)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{11\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

output
```
4*(a-1/x)/a^4/(1-1/a^2/x^2)^(1/2)+14/3*(1-1/a^2/x^2)^(1/2)*x/a^2-3/2*(1-1/
a^2/x^2)^(1/2)*x^2/a+1/3*(1-1/a^2/x^2)^(1/2)*x^3-11/2*arctanh((1-1/a^2/x^2
)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx=\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(52+19ax-7a^2x^2+2a^3x^3)}{1+ax}-33\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{6a^3}$$

input
```
Integrate[x^2/E^(3*ArcCoth[a*x]),x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(52 + 19*a*x - 7*a^2*x^2 + 2*a^3*x^3))/(1 + a*
x) - 33*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(6*a^3)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6719, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2 x^4}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{2353}$$

$$-\int \left(\frac{x^4}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4}{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{14x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{11\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

input
```
Int[x^2/E^(3*ArcCoth[a*x]),x]
```

output
```
(4*Sqrt[1 - 1/(a^2*x^2)])/(a^2*(a + x^(-1))) + (14*Sqrt[1 - 1/(a^2*x^2)]*x
)/(3*a^2) - (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (Sqrt[1 - 1/(a^2*x^2)]*x
^3)/3 - (11*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.25

method	result
risch	$\frac{(2a^2x^2-9ax+28)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{6a^3}+\frac{\left(-\frac{11\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2a^2\sqrt{a^2}}+\frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^4\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{\left(9\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^3x^3-2\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2+18\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^2x^2-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-4\sqrt{a^2}\,((ax-1)\right.}{}$

input
```
int(x^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*x^2-9*a*x+28)*(a*x+1)/a^3*((a*x-1)/(a*x+1))^(1/2)+(-11/2/a^2*ln
(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+4/a^4/(x+1/a)*(a^2*(x+1/
a)^2-2*a*(x+1/a))^(1/2))*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(
a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx = \frac{(2a^3x^3-7a^2x^2+19ax+52)\sqrt{\frac{ax-1}{ax+1}}-33\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+33\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{6a^3}$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
1/6*((2*a^3*x^3 - 7*a^2*x^2 + 19*a*x + 52)*sqrt((a*x - 1)/(a*x + 1)) - 33*
log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 33*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
)/a^3
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx = \int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Integral(x**2*((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.60

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx = -\frac{1}{6}a\left(\frac{2\left(39\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-52\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+21\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{33\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}-\frac{33\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}-\frac{24}{}\right.$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-1/6*a*(2*(39*((a*x - 1)/(a*x + 1))^(5/2) - 52*((a*x - 1)/(a*x + 1))^(3/2)
 + 21*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^
2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 33*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^4 - 33*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4 - 24*
sqrt((a*x - 1)/(a*x + 1))/a^4)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx = \int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 24.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.34

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx = \frac{7\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{52\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}+13\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a^3+\frac{3\,a^3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a^3\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{3\,a^3\,\left(a\,x-1\right)}{a\,x+1}} + \frac{4\sqrt{\frac{a\,x-1}{a\,x+1}}}{a^3}-\frac{11\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}$$

input
```
int(x^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(7*((a*x - 1)/(a*x + 1))^(1/2) - (52*((a*x - 1)/(a*x + 1))^(3/2))/3 + 13*(
(a*x - 1)/(a*x + 1))^(5/2))/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^3*
(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (4*((a*x - 1)/(a
*x + 1))^(1/2))/a^3 - (11*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.14

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx$$

$$= \frac{8\sqrt{ax+1}\,\sqrt{ax-1}\,a^3x^3 - 28\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2 + 76\sqrt{ax+1}\,\sqrt{ax-1}\,ax + 208\sqrt{ax+1}\,\sqrt{ax-1} -}{24a^3\,(ax+1)}$$

input
```
int(x^2*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(8*sqrt(a*x + 1)*sqrt(a*x - 1)*a**3*x**3 - 28*sqrt(a*x + 1)*sqrt(a*x - 1)*
a**2*x**2 + 76*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 208*sqrt(a*x + 1)*sqrt(a*
x - 1) - 264*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x - 264*log((s
qrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 153*a*x + 153)/(24*a**3*(a*x + 1)
)
```

3.60 $\int e^{-3\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 90

$$\int e^{-3\coth^{-1}(ax)}x\,dx=-\frac{4\left(a-\frac{1}{x}\right)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{9\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

output
```
(-4*a+4/x)/a^3/(1-1/a^2/x^2)^(1/2)-3*(1-1/a^2/x^2)^(1/2)*x/a+1/2*(1-1/a^2/
x^2)^(1/2)*x^2+9/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int e^{-3\coth^{-1}(ax)}x\,dx=\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-14-5ax+a^2x^2)}{1+ax}+9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{2a^2}$$

input
```
Integrate[x/E^(3*ArcCoth[a*x]),x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-14 - 5*a*x + a^2*x^2))/(1 + a*x) + 9*Log[(1
+ Sqrt[1 - 1/(a^2*x^2)])*x])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6719, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2 x^3}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{2353}$$

$$-\int \left(\frac{x^3}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4}{a^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{9\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}+\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)}$$

input
```
Int[x/E^(3*ArcCoth[a*x]),x]
```

output
```
(-4*Sqrt[1 - 1/(a^2*x^2)])/(a*(a + x^(-1))) - (3*Sqrt[1 - 1/(a^2*x^2)]*x)/
a + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (9*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*
a^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.51

method	result
risch	$\frac{(ax-6)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2a^{2}}+\frac{\left(\frac{9\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{2a\sqrt{a^{2}}}-\frac{4\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{a^{3}\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$\frac{\left(\sqrt{a^{2}}\,\sqrt{a^{2}x^{2}-1}\,a^{3}x^{3}-10\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,a^{2}x^{2}+2\sqrt{a^{2}}\,\sqrt{a^{2}x^{2}-1}\,a^{2}x^{2}-\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{3}x^{2}+10\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(}}{\sqrt{a^{2}}}\right.\right.}{}$

input
```
int(x*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x-6)*(a*x+1)/a^2*((a*x-1)/(a*x+1))^(1/2)+(9/2/a*ln(a^2*x/(a^2)^(1/2
)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-4/a^3/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))
^(1/2))*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

$$\int e^{-3\coth^{-1}(ax)}x\,dx=\frac{(a^2x^2-5\,ax-14)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
1/2*((a^2*x^2 - 5*a*x - 14)*sqrt((a*x - 1)/(a*x + 1)) + 9*log(sqrt((a*x -
1)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}x\,dx=\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Integral(x*((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.68

$$\int e^{-3\coth^{-1}(ax)}x\,dx=$$
$$-\frac{1}{2}\,a\left(\frac{2\left(7\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}+\frac{8\sqrt{\frac{ax-1}{ax+1}}}{a^3}\right)$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-1/2*a*(2*(7*((a*x - 1)/(a*x + 1))^(3/2) - 5*sqrt((a*x - 1)/(a*x + 1)))/(2
*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 9*log(sqrt
((a*x - 1)/(a*x + 1)) + 1)/a^3 + 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3
+ 8*sqrt((a*x - 1)/(a*x + 1))/a^3)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}x\,dx = \int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 23.90 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.33

$$\int e^{-3\coth^{-1}(ax)}x\,dx = \frac{9\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2} - \frac{4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a^2} - \frac{5\,\sqrt{\frac{a\,x-1}{a\,x+1}} - 7\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a^2 + \frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{2\,a^2\,(a\,x-1)}{a\,x+1}}$$

input
```
int(x*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(9*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^2 - (4*((a*x - 1)/(a*x + 1))^(1/2
))/a^2 - (5*((a*x - 1)/(a*x + 1))^(1/2) - 7*((a*x - 1)/(a*x + 1))^(3/2))/(
a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.23

$$\int e^{-3\coth^{-1}(ax)}x\,dx$$

$$=\frac{\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2-5\sqrt{ax+1}\,\sqrt{ax-1}\,ax-14\sqrt{ax+1}\,\sqrt{ax-1}+18\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax+18}{2a^2\,(ax+1)}$$

input
```
int(x*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 - 5*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x
 - 14*sqrt(a*x + 1)*sqrt(a*x - 1) + 18*log((sqrt(a*x - 1) + sqrt(a*x + 1))
/sqrt(2))*a*x + 18*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) - 11*a*x -
 11)/(2*a**2*(a*x + 1))
```

3.61 $\int e^{-3\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 63

$$\int e^{-3\coth^{-1}(ax)}\,dx=\frac{4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
4*(a-1/x)/a^2/(1-1/a^2/x^2)^(1/2)+(1-1/a^2/x^2)^(1/2)*x-3*arctanh((1-1/a^2
/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int e^{-3\coth^{-1}(ax)}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(5+ax)}{1+ax}-\frac{3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}$$

input
```
Integrate[E^(-3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(5 + a*x))/(1 + a*x) - (3*Log[a*(1 + Sqrt[1 - 1/(
a^2*x^2)])*x])/a
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6718, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \; 6718$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2 x^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \; 2353$$

$$-\int \left(\frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4}{a\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \; 2009$$

$$-\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+x\sqrt{1-\frac{1}{a^2x^2}}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}$$

input
```
Int[E^(-3*ArcCoth[a*x]),x]
```

output
```
(4*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) + Sqrt[1 - 1/(a^2*x^2)]*x - (3*ArcT
anh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 6718
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(57) = 114$.

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.02

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a}+\frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^2\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{\left(-3\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^2x^2+3\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2+2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-6\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,ax+6\cdots\right.}{a\sqrt{a^2}\,(ax-1)\sqrt{(ax-1)(a\cdots}}$

input
```
int(((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)*((a*x-1)/(a*x+1))^(1/2)+(-3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(
1/2))/(a^2)^(1/2)+4/a^2/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*((a*x-1
)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int e^{-3\coth^{-1}(ax)}\,dx = \frac{(ax+5)\sqrt{\frac{ax-1}{ax+1}} - 3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + 3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
((a*x + 5)*sqrt((a*x - 1)/(a*x + 1)) - 3*log(sqrt((a*x - 1)/(a*x + 1)) + 1
) + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\,dx = \int \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.76

$$\int e^{-3\coth^{-1}(ax)}\,dx = -a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2} + \frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} - \frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2} - \frac{4\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + 3*log(sq
rt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^
2 - 4*sqrt((a*x - 1)/(a*x + 1))/a^2)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\,dx = \int \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.24

$$\int e^{-3\coth^{-1}(ax)}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}} + \frac{4\sqrt{\frac{a\,x-1}{a\,x+1}}}{a} - \frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) + (4*((a*x -
 1)/(a*x + 1))^(1/2))/a - (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.46

$$\int e^{-3\coth^{-1}(ax)}\,dx$$
$$= \frac{2\sqrt{ax+1}\,\sqrt{ax-1}\,ax+10\sqrt{ax+1}\,\sqrt{ax-1}-12\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax-12\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)+9ax}{2a\,(ax+1)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(2*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 10*sqrt(a*x + 1)*sqrt(a*x - 1) - 12*l
og((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x - 12*log((sqrt(a*x - 1) +
sqrt(a*x + 1))/sqrt(2)) + 9*a*x + 9)/(2*a*(a*x + 1))
```

3.62 $\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 48

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = -\frac{4\left(a-\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}} - \csc^{-1}(ax) + \operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

output
```
(-4*a+4/x)/a/(1-1/a^2/x^2)^(1/2)-arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}\,x}{1+ax} - \arcsin\left(\frac{1}{ax}\right) + \log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
(-4*a*Sqrt[1 - 1/(a^2*x^2)]*x)/(1 + a*x) - ArcSin[1/(a*x)] + Log[(1 + Sqrt
[1 - 1/(a^2*x^2)])*x]
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.69, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6719, 2351, 27, 564, 25, 27, 243, 73, 221, 671, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2 x}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{2351}$$

$$-\int \frac{\frac{1}{a^2x}-\frac{2}{a}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}-\int \frac{ax}{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{\frac{1}{a^2x}-\frac{2}{a}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}-a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{564}$$

$$-\int \frac{\frac{1}{a^2x}-\frac{2}{a}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\int -\frac{x}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\frac{1}{a^2x}-\frac{2}{a}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\int \frac{x}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{\frac{1}{a^2x}-\frac{2}{a}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \ 243$$

$$-\int \frac{\frac{1}{a^2x}-\frac{2}{a}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \ 73$$

$$-a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-a\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{\frac{1}{a^2x}-\frac{2}{a}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \ 221$$

$$-\int \frac{\frac{1}{a^2x}-\frac{2}{a}}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}-a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}\right)$$

$$\Big\downarrow \ 671$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\left(a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}\right)\right)-\frac{3a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}$$

$$\Big\downarrow \ 223$$

$$-\left(a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}\right)\right)-\frac{3a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\arcsin\left(\frac{1}{ax}\right)$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
(-3*a*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) - ArcSin[1/(a*x)] - a*(Sqrt[1 -
1/(a^2*x^2)]/(a + x^(-1)) - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(45) = 90$.

Time $= 0.09$ (sec) , antiderivative size $= 369$, normalized size of antiderivative $= 7.69$

method	result
default	$\frac{\left(\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^2x^2-a^2\sqrt{a^2}\,x^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^2x^2+2\ln\left(\frac{a^2x\cdots}{\cdots}\right.\right.}{\cdots}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^3*x^2-(a^2)
^(1/2)*(a^2*x^2-1)^(1/2)*a^2*x^2-a^2*(a^2)^(1/2)*x^2*arctan(1/(a^2*x^2-1)^
(1/2))-((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*a^2*x^2+2*ln((a^2*x+((a*x-1)*(a
*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x-2*(a^2)^(1/2)*(a^2*x^2-1)^(1/
2)*a*x-2*a*(a^2)^(1/2)*x*arctan(1/(a^2*x^2-1)^(1/2))+2*((a*x-1)*(a*x+1))^(
3/2)*(a^2)^(1/2)-2*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*a*x+a*ln((a^2*x+((a
*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))-(a^2*x^2-1)^(1/2)*(a^2)^(1/
2)-arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)-((a*x-1)*(a*x+1))^(1/2)*(a^2)^(
1/2))*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.54

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = -4\sqrt{\frac{ax-1}{ax+1}} + 2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) - \log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

output
```
-4*sqrt((a*x - 1)/(a*x + 1)) + 2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(s
qrt((a*x - 1)/(a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/x,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(44) = 88$.

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.85

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a} - \frac{4\sqrt{\frac{ax-1}{ax+1}}}{a}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

output
```
a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a + log(sqrt((a*x - 1)/(a*x + 1)) +
 1)/a - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a - 4*sqrt((a*x - 1)/(a*x + 1))
/a)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) + 2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - 4\,\sqrt{\frac{a\,x-1}{a\,x+1}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/x,x)
```

output
```
2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + 2*atanh(((a*x - 1)/(a*x + 1))^(1/2))
 - 4*((a*x - 1)/(a*x + 1))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.90

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx$$
$$= \frac{2atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)ax+2atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)-2atan\left(\sqrt{ax-1}+\sqrt{ax+1}+\right.}{}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/x,x)
```

output
```
(2*(atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a*x + atan(sqrt(a*x - 1) + sqr
t(a*x + 1) - 1) - atan(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a*x - atan(sqrt(
a*x - 1) + sqrt(a*x + 1) + 1) - 2*sqrt(a*x + 1)*sqrt(a*x - 1) + log((sqrt(
a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x + log((sqrt(a*x - 1) + sqrt(a*x + 1
))/sqrt(2)) - 2*a*x - 2))/(a*x + 1)
```

3.63 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 47

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = a\sqrt{1-\frac{1}{a^2x^2}}+\frac{4\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+3a\csc^{-1}(ax)$$

output
```
a*(1-1/a^2/x^2)^(1/2)+4*(a-1/x)/(1-1/a^2/x^2)^(1/2)+3*a*arccsc(a*x)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}(1+5ax)}{1+ax}+3a\arcsin\left(\frac{1}{ax}\right)$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*(1 + 5*a*x))/(1 + a*x) + 3*a*ArcSin[1/(a*x)]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6719, 711, 25, 27, 671, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 711}$$

$$a^4\int -\frac{a-\frac{3}{x}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}+a\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{ 25}$$

$$a\sqrt{1-\frac{1}{a^2x^2}}-a^4\int \frac{a-\frac{3}{x}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$a\sqrt{1-\frac{1}{a^2x^2}}-\int \frac{a-\frac{3}{x}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 671}$$

$$3\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}+a\sqrt{1-\frac{1}{a^2x^2}}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}+a\sqrt{1-\frac{1}{a^2x^2}}+3a\arcsin\left(\frac{1}{ax}\right)$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
a*Sqrt[1 - 1/(a^2*x^2)] + (4*a^2*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) + 3*a
*ArcSin[1/(a*x)]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 711
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + c*x^2)^(p + 1)
/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m + n + 2*p + 1))
Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c*e^n*(m + n + 2*p + 1)*(f + g*x)
^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - 2*e*g^n*(m + p + n)*(d + e*x)^(n
 - 2)*(a*e - c*d*x), x], x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && Eq
Q[c*d^2 + a*e^2, 0] && IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(43) = 86$.

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.32

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}+\frac{\left(3a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{\left(-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^4x^4+\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^2x^2-5\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^3x^3+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^4x^3-3a^3\sqrt{a^2}\,x^3\arctan\left(\frac{1}{\sqrt{a^2x}}\right.\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(a*x+1)/x*((a*x-1)/(a*x+1))^(1/2)+(3*a*arctan(1/(a^2*x^2-1)^(1/2))+4/(x+1/
a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*
x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = -\frac{6\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(5\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-(6*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - (5*a*x + 1)*sqrt((a*x - 1)/(a*
x + 1)))/x
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.53

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = 2\,a\left(2\sqrt{\frac{ax-1}{ax+1}} + \frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1} - 3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
2*a*(2*sqrt((a*x - 1)/(a*x + 1)) + sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a
*x + 1) + 1) - 3*arctan(sqrt((a*x - 1)/(a*x + 1))))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 23.82 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}+5\,a\,x\,\sqrt{\frac{a\,x-1}{a\,x+1}}-6\,a\,x\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{x}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/x^2,x)
```

output
```
(((a*x - 1)/(a*x + 1))^(1/2) + 5*a*x*((a*x - 1)/(a*x + 1))^(1/2) - 6*a*x*a
tan(((a*x - 1)/(a*x + 1))^(1/2)))/x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.91

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{-6atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^2x^2-6atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)ax+6atan\left(\sqrt{ax-1}+\sqrt{ax\right.}{}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

output
```
( - 6*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**2*x**2 - 6*atan(sqrt(a*x
- 1) + sqrt(a*x + 1) - 1)*a*x + 6*atan(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*
a**2*x**2 + 6*atan(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a*x + 5*sqrt(a*x + 1
)*sqrt(a*x - 1)*a*x + sqrt(a*x + 1)*sqrt(a*x - 1) + 3*a**2*x**2 + 3*a*x)/(
x*(a*x + 1))
```

3.64 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 81

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = -6a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{\left(a-\frac{1}{x}\right)^3}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{3a\sqrt{1-\frac{1}{a^2x^2}}}{2x} - \frac{9}{2}a^2\csc^{-1}(ax)$$

output
```
-6*a^2*(1-1/a^2/x^2)^(1/2)-(a-1/x)^3/a/(1-1/a^2/x^2)^(1/2)+3/2*a*(1-1/a^2/
x^2)^(1/2)/x-9/2*a^2*arccsc(a*x)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(1-5ax-14a^2x^2)}{x(1+ax)} - 9a\arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*x^3),x]
```

output
```
(a*((Sqrt[1 - 1/(a^2*x^2)]*(1 - 5*a*x - 14*a^2*x^2))/(x*(1 + a*x)) - 9*a*A
rcSin[1/(a*x)]))/2
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6719, 2164, 27, 2027, 2164, 27, 563, 25, 2346, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)x}d\frac{1}{x}$$

$$\Big\downarrow \text{2164}$$

$$-\frac{\int \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x}-\frac{1}{x^2}\right)}{\left(a+\frac{1}{x}\right)^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{27}$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x}-\frac{1}{x^2}\right)}{\left(a+\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{2027}$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{\left(a+\frac{1}{x}\right)^2x}d\frac{1}{x}$$

$$\Big\downarrow \text{2164}$$

$$-a^2\int \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a+\frac{1}{x}\right)^3x}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a+\frac{1}{x}\right)^3x}d\frac{1}{x}$$

$$\Big\downarrow \text{563}$$

$$-a^3\left(\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{\int -\frac{4a^2-\frac{3a}{x}+\frac{1}{x^2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}\right)$$

$$\Big\downarrow \; 25$$

$$-a^3\left(\frac{\int \frac{4a^2-\frac{3a}{x}+\frac{1}{x^2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \; 2346$$

$$-a^3\left(\frac{-\frac{1}{2}a^2\int -\frac{3\left(3a-\frac{2}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}}{a^4}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \; 27$$

$$-a^3\left(\frac{\frac{3}{2}a\int \frac{3a-\frac{2}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}}{a^4}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \; 455$$

$$-a^3\left(\frac{\frac{3}{2}a\left(3a\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+2a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}}{a^4}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \; 223$$

$$-a^3\left(\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}+\frac{\frac{3}{2}a\left(3a^2\arcsin\left(\frac{1}{ax}\right)+2a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}}{a^4}\right)$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*x^3),x]
```

output
```
-(a^3*((4*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) + (-1/2*(a^2*Sqrt[1 - 1/(a^2
*x^2)])/x + (3*a*(2*a^2*Sqrt[1 - 1/(a^2*x^2)] + 3*a^2*ArcSin[1/(a*x)]))/2)
/a^4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2164
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + b*d*x, x]*
(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x]
 && EqQ[b*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + b*d*x, x], 0
]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.47

method	result
risch	$-\frac{(ax+1)(6ax-1)\sqrt{\frac{ax-1}{ax+1}}}{2x^{2}}+\frac{\left(-\frac{9a^{2}\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)}{2}-\frac{4a\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$\left(-6\sqrt{a^{2}}\,\sqrt{a^{2}x^{2}-1}\,a^{5}x^{5}+6\sqrt{a^{2}}\,\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}a^{3}x^{3}-21\sqrt{a^{2}}\,\sqrt{a^{2}x^{2}-1}\,a^{4}x^{4}-9a^{4}\sqrt{a^{2}}\,x^{4}\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)+6\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{}}{\sqrt{a^{2}}}\right.\right.$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x+1)*(6*a*x-1)/x^2*((a*x-1)/(a*x+1))^(1/2)+(-9/2*a^2*arctan(1/(a^2
*x^2-1)^(1/2))-4*a/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*((a*x-1)/(a*
x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = \frac{18\,a^2x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - (14\,a^2x^2 + 5\,ax - 1)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(18*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (14*a^2*x^2 + 5*a*x -
1)*sqrt((a*x - 1)/(a*x + 1)))/x^2
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = \left(9\,a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - 4\,a\sqrt{\frac{ax-1}{ax+1}} - \frac{7\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 5\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1} + \frac{(ax-1)^2}{(ax+1)^2} + 1}\right)a$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
(9*a*arctan(sqrt((a*x - 1)/(a*x + 1))) - 4*a*sqrt((a*x - 1)/(a*x + 1)) - (
7*a*((a*x - 1)/(a*x + 1))^(3/2) + 5*a*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x -
 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 + 1))*a
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.46

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = 9\,a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) - \frac{5\,a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}} + 7\,a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\frac{(a\,x-1)^2}{(a\,x+1)^2} + \frac{2\,(a\,x-1)}{a\,x+1} + 1} - 4\,a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/x^3,x)
```

output
```
9*a^2*atan(((a*x - 1)/(a*x + 1))^(1/2)) - (5*a^2*((a*x - 1)/(a*x + 1))^(1/
2) + 7*a^2*((a*x - 1)/(a*x + 1))^(3/2))/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x
 - 1))/(a*x + 1) + 1) - 4*a^2*((a*x - 1)/(a*x + 1))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.10

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx$$
$$= \frac{18atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^3x^3+18atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^2x^2-18atan\left(\sqrt{ax-1}+\sqrt{}\right.}{}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

output
```
(18*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**3*x**3 + 18*atan(sqrt(a*x -
 1) + sqrt(a*x + 1) - 1)*a**2*x**2 - 18*atan(sqrt(a*x - 1) + sqrt(a*x + 1)
 + 1)*a**3*x**3 - 18*atan(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**2*x**2 - 1
4*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 - 5*sqrt(a*x + 1)*sqrt(a*x - 1)*a*
x + sqrt(a*x + 1)*sqrt(a*x - 1) - 4*a**3*x**3 - 4*a**2*x**2)/(2*x**2*(a*x
+ 1))
```

3.65 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 102

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = 5a^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{4a^2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+\frac{11}{2}a^3\csc^{-1}(ax)$$

output
```
5*a^3*(1-1/a^2/x^2)^(1/2)-1/3*a^3*(1-1/a^2/x^2)^(3/2)+4*a^2*(a-1/x)/(1-1/a
^2/x^2)^(1/2)-3/2*a^2*(1-1/a^2/x^2)^(1/2)/x+11/2*a^3*arccsc(a*x)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{6}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(2-7ax+19a^2x^2+52a^3x^3\right)}{x^2(1+ax)}+33a^2\arcsin\left(\frac{1}{ax}\right)\right)$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
(a*((Sqrt[1 - 1/(a^2*x^2)]*(2 - 7*a*x + 19*a^2*x^2 + 52*a^3*x^3))/(x^2*(1
+ a*x)) + 33*a^2*ArcSin[1/(a*x)]))/6
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.25, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {6719, 2164, 27, 2027, 2164, 27, 563, 2346, 25, 2346, 25, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \ 6719$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)x^2}d\frac{1}{x}$$

$$\Big\downarrow \ 2164$$

$$-\frac{\int \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x^2}-\frac{1}{x^3}\right)}{\left(a+\frac{1}{x}\right)^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \ 27$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a}{x^2}-\frac{1}{x^3}\right)}{\left(a+\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \ 2027$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{\left(a+\frac{1}{x}\right)^2x^2}d\frac{1}{x}$$

$$\Big\downarrow \ 2164$$

$$-a^2\int \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a+\frac{1}{x}\right)^3x^2}d\frac{1}{x}$$

$$\Big\downarrow \ 27$$

$$-a^3 \int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a+\frac{1}{x}\right)^3 x^2}d\frac{1}{x}$$

$\Big\downarrow$ 563

$$-a^3 \left(-\frac{\int \frac{4a^3-\frac{4a^2}{x}+\frac{3a}{x^2}-\frac{1}{x^3}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$\Big\downarrow$ 2346

$$-a^3 \left(-\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{3}a^2\int -\frac{12a-\frac{14}{x}+\frac{9}{x^2a}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$\Big\downarrow$ 25

$$-a^3 \left(-\frac{\frac{1}{3}a^2\int \frac{12a-\frac{14}{x}+\frac{9}{x^2a}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$\Big\downarrow$ 2346

$$-a^3 \left(-\frac{\frac{1}{3}a^2\left(-\frac{1}{2}a^2\int -\frac{33a-\frac{28}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$\Big\downarrow$ 25

$$-a^3 \left(-\frac{\frac{1}{3}a^2\left(\frac{1}{2}a^2\int \frac{33a-\frac{28}{x}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$\Big\downarrow$ 27

$$-a^3 \left(-\frac{\frac{1}{3}a^2\left(\frac{1}{2}\int \frac{33a-\frac{28}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \; 455$$

$$-a^3\left(-\frac{\frac{1}{3}a^2\left(\frac{1}{2}\left(33a\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+28a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \; 223$$

$$-a^3\left(-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{\frac{1}{3}a^2\left(\frac{1}{2}\left(33a^2\arcsin\left(\frac{1}{ax}\right)+28a^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{9a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}}{a^4}\right)$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
-(a^3*((-4*a*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) - ((a^2*Sqrt[1 - 1/(a^2*x
^2)])/(3*x^2) + (a^2*((-9*a*Sqrt[1 - 1/(a^2*x^2)])/(2*x) + (28*a^2*Sqrt[1
- 1/(a^2*x^2)] + 33*a^2*ArcSin[1/(a*x)])/2))/3)/a^4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2164
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + b*d*x, x]*
(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x]
 && EqQ[b*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + b*d*x, x], 0
]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.26

method	result
risch	$\frac{(ax+1)(28a^2x^2-9ax+2)\sqrt{\frac{ax-1}{ax+1}}}{6x^3}+\frac{\left(\frac{11a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+\frac{4a^2\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{\left(-30\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^6x^6+30\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^5x^5+30\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4-93\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^5x^5-33\arctan\left(\frac{1}{\sqrt{a^2x^2-}}\right.\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x+1)*(28*a^2*x^2-9*a*x+2)/x^3*((a*x-1)/(a*x+1))^(1/2)+(11/2*a^3*arc
tan(1/(a^2*x^2-1)^(1/2))+4*a^2/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*
((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.68

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx=-\frac{66\,a^3x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(52\,a^3x^3+19\,a^2x^2-7\,ax+2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^3}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(66*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (52*a^3*x^3 + 19*a^2*
x^2 - 7*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.54

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = -\frac{1}{3}\left(33\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - 12\,a^2\sqrt{\frac{ax-1}{ax+1}} - \frac{39\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 52\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 21\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1} + \frac{3\,(ax-1)^2}{(ax+1)^2} + \frac{(ax-1)^3}{(ax+1)^3} + 1}\right)a$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima")
```

output
```
-1/3*(33*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - 12*a^2*sqrt((a*x - 1)/(a*
x + 1)) - (39*a^2*((a*x - 1)/(a*x + 1))^(5/2) + 52*a^2*((a*x - 1)/(a*x + 1
))^(3/2) + 21*a^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)/(a*x + 1) + 3*(a
*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1))*a
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 23.62 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.50

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = \frac{7\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}} + \frac{52\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3} + 13\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{\frac{3\,(a\,x-1)^2}{(a\,x+1)^2} + \frac{(a\,x-1)^3}{(a\,x+1)^3} + \frac{3\,(a\,x-1)}{a\,x+1} + 1} + 4\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}} - 11\,a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/x^4,x)
```

output
```
(7*a^3*((a*x - 1)/(a*x + 1))^(1/2) + (52*a^3*((a*x - 1)/(a*x + 1))^(3/2))/
3 + 13*a^3*((a*x - 1)/(a*x + 1))^(5/2))/((3*(a*x - 1)^2)/(a*x + 1)^2 + (a*
x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) + 4*a^3*((a*x - 1)/(a*
x + 1))^(1/2) - 11*a^3*atan(((a*x - 1)/(a*x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.87

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{-66atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^4x^4-66atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^3x^3+66atan\left(\sqrt{ax-1}+\right.}{}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

output
```
( - 66*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**4*x**4 - 66*atan(sqrt(a*
x - 1) + sqrt(a*x + 1) - 1)*a**3*x**3 + 66*atan(sqrt(a*x - 1) + sqrt(a*x +
 1) + 1)*a**4*x**4 + 66*atan(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**3*x**3
+ 52*sqrt(a*x + 1)*sqrt(a*x - 1)*a**3*x**3 + 19*sqrt(a*x + 1)*sqrt(a*x - 1
)*a**2*x**2 - 7*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 2*sqrt(a*x + 1)*sqrt(a*x
 - 1) + 14*a**4*x**4 + 14*a**3*x**3)/(6*x**3*(a*x + 1))
```

3.66 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 121

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = -7a^4\sqrt{1-\frac{1}{a^2x^2}}+a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{4a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{19a^3\sqrt{1-\frac{1}{a^2x^2}}}{8x}-\frac{51}{8}a^4\csc^{-1}(ax)$$

output
```
-7*a^4*(1-1/a^2/x^2)^(1/2)+a^4*(1-1/a^2/x^2)^(3/2)-4*a^3*(a-1/x)/(1-1/a^2/
x^2)^(1/2)+1/4*a*(1-1/a^2/x^2)^(1/2)/x^3+19/8*a^3*(1-1/a^2/x^2)^(1/2)/x-51
/8*a^4*arccsc(a*x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.62

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = -\frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(-2+6ax-11a^2x^2+29a^3x^3+80a^4x^4\right)}{8x^3(1+ax)}-\frac{51}{8}a^4\arcsin\left(\frac{1}{ax}\right)$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
-1/8*(a*Sqrt[1 - 1/(a^2*x^2)]*(-2 + 6*a*x - 11*a^2*x^2 + 29*a^3*x^3 + 80*a
^4*x^4))/(x^3*(1 + a*x)) - (51*a^4*ArcSin[1/(a*x)])/8
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.28, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {6719, 2164, 27, 2027, 2164, 27, 563, 25, 2346, 25, 2346, 27, 2346, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{ 6719}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(1+\frac{1}{ax}\right)x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 2164}$$

$$-\frac{\int \frac{a^2\left(\frac{a}{x^3}-\frac{1}{x^4}\right)\sqrt{1-\frac{1}{a^2x^2}}}{\left(a+\frac{1}{x}\right)^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 27}$$

$$-a\int \frac{\left(\frac{a}{x^3}-\frac{1}{x^4}\right)\sqrt{1-\frac{1}{a^2x^2}}}{\left(a+\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 2027}$$

$$-a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{\left(a+\frac{1}{x}\right)^2x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 2164}$$

$$-a^2 \int \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a+\frac{1}{x}\right)^3 x^3}d\frac{1}{x}$$

$$\Big\downarrow \textbf{27}$$

$$-a^3 \int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a+\frac{1}{x}\right)^3 x^3}d\frac{1}{x}$$

$$\Big\downarrow \textbf{563}$$

$$-a^3\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{\int -\frac{4a^4-\frac{4a^3}{x}+\frac{4a^2}{x^2}-\frac{3a}{x^3}+\frac{1}{x^4}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}\right)$$

$$\Big\downarrow \textbf{25}$$

$$-a^3\left(\frac{\int \frac{4a^4-\frac{4a^3}{x}+\frac{4a^2}{x^2}-\frac{3a}{x^3}+\frac{1}{x^4}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \textbf{2346}$$

$$-a^3\left(\frac{-\frac{1}{4}a^2\int -\frac{16a^2-\frac{16a}{x}+\frac{19}{x^2}-\frac{12}{x^3a}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}}{a^4}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \textbf{25}$$

$$-a^3\left(\frac{\frac{1}{4}a^2\int \frac{16a^2-\frac{16a}{x}+\frac{19}{x^2}-\frac{12}{x^3a}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}}{a^4}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \textbf{2346}$$

$$-a^3\left(\frac{\frac{1}{4}a^2\left(\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{x^2}-\frac{1}{3}a^2\int -\frac{3\left(16-\frac{24}{ax}+\frac{19}{a^2x^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}}{a^4}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \textbf{27}$$

$$-a^3\left(\frac{\frac{1}{4}a^2\left(a^2\int\frac{16-\frac{24}{ax}+\frac{19}{a^2x^2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}}{a^4}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{2346}$$

$$-a^3\left(\frac{\frac{1}{4}a^2\left(a^2\left(-\frac{1}{2}a^2\int-\frac{3\left(17a-\frac{16}{x}\right)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{19\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}}{a^4}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{27}$$

$$-a^3\left(\frac{\frac{1}{4}a^2\left(a^2\left(\frac{3\int\frac{17a-\frac{16}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{19\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}}{a^4}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{455}$$

$$-a^3\left(\frac{\frac{1}{4}a^2\left(a^2\left(\frac{3\left(17a\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+16a^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}-\frac{19\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}}{a^4}+\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}\right)$$

$$\Big\downarrow \text{223}$$

$$-a^3\left(\frac{4a^2\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}+\frac{\frac{1}{4}a^2\left(a^2\left(\frac{3\left(17a^2\arcsin\left(\frac{1}{ax}\right)+16a^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}-\frac{19\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}}{a^4}\right)$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
-(a^3*((4*a^2*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) + (-1/4*(a^2*Sqrt[1 - 1/
(a^2*x^2)])/x^3 + (a^2*((4*a*Sqrt[1 - 1/(a^2*x^2)])/x^2 + a^2*((-19*Sqrt[1
 - 1/(a^2*x^2)])/(2*x) + (3*(16*a^2*Sqrt[1 - 1/(a^2*x^2)] + 17*a^2*ArcSin[
1/(a*x)]))/(2*a))))/4)/a^4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2164
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + b*d*x, x]*
(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x]
 && EqQ[b*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + b*d*x, x], 0
]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 6719
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{(ax+1)\left(48a^3x^3-19a^2x^2+8ax-2\right)\sqrt{\frac{ax-1}{ax+1}}}{8x^4}+\frac{\left(-\frac{51a^4\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{8}-\frac{4a^3\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(ax-}}{ax-1}$
default	$\frac{\left(-56\sqrt{a^2x^2-1}\sqrt{a^2}a^7x^7+56\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}a^5x^5-163\sqrt{a^2x^2-1}\sqrt{a^2}a^6x^6-51\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}a^6x^6+56\ln\left(\frac{a^2x+\sqrt{a^2x}}{\sqrt{a}}\right.\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(a*x+1)*(48*a^3*x^3-19*a^2*x^2+8*a*x-2)/x^4*((a*x-1)/(a*x+1))^(1/2)+(
-51/8*a^4*arctan(1/(a^2*x^2-1)^(1/2))-4*a^3/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+
1/a))^(1/2))*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.64

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = \frac{102\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - (80\,a^4x^4 + 29\,a^3x^3 - 11\,a^2x^2 + 6\,ax - 2)\sqrt{\frac{ax-1}{ax+1}}}{8\,x^4}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fricas")
```

output
```
1/8*(102*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) - (80*a^4*x^4 + 29*a^3*
x^3 - 11*a^2*x^2 + 6*a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.60

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = \frac{1}{4}\left(51\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - 16\,a^3\sqrt{\frac{ax-1}{ax+1}} - \frac{77\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} + 149\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 123\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 35}{\frac{4\,(ax-1)}{ax+1} + \frac{6\,(ax-1)^2}{(ax+1)^2} + \frac{4\,(ax-1)^3}{(ax+1)^3} + \frac{(ax-1)^4}{(ax+1)^4} + 1}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxima")
```

output
```
1/4*(51*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 16*a^3*sqrt((a*x - 1)/(a*x
 + 1)) - (77*a^3*((a*x - 1)/(a*x + 1))^(7/2) + 149*a^3*((a*x - 1)/(a*x + 1
))^(5/2) + 123*a^3*((a*x - 1)/(a*x + 1))^(3/2) + 35*a^3*sqrt((a*x - 1)/(a*
x + 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(a*x - 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^
3/(a*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 + 1))*a
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.57

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = \frac{51\,a^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4} - 4\,a^4\,\sqrt{\frac{a\,x-1}{a\,x+1}} - \frac{\frac{35\,a^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4} + \frac{123\,a^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{4} + \frac{149\,a^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{4} + \frac{77\,a^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{\frac{6\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2} + \frac{4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3} + \frac{{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4} + \frac{4\,\left(a\,x-1\right)}{a\,x+1} + 1}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/x^5,x)
```

output
```
(51*a^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/4 - 4*a^4*((a*x - 1)/(a*x + 1))
^(1/2) - ((35*a^4*((a*x - 1)/(a*x + 1))^(1/2))/4 + (123*a^4*((a*x - 1)/(a*
x + 1))^(3/2))/4 + (149*a^4*((a*x - 1)/(a*x + 1))^(5/2))/4 + (77*a^4*((a*x
 - 1)/(a*x + 1))^(7/2))/4)/((6*(a*x - 1)^2)/(a*x + 1)^2 + (4*(a*x - 1)^3)/
(a*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 + (4*(a*x - 1))/(a*x + 1) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.74

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = \frac{102atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^5x^5+102atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^4x^4-102atan\left(\sqrt{ax-1}\right.}{}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/x^5,x)
```

output
```
(102*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**5*x**5 + 102*atan(sqrt(a*x
 - 1) + sqrt(a*x + 1) - 1)*a**4*x**4 - 102*atan(sqrt(a*x - 1) + sqrt(a*x +
 1) + 1)*a**5*x**5 - 102*atan(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**4*x**4
 - 80*sqrt(a*x + 1)*sqrt(a*x - 1)*a**4*x**4 - 29*sqrt(a*x + 1)*sqrt(a*x -
1)*a**3*x**3 + 11*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 - 6*sqrt(a*x + 1)*
sqrt(a*x - 1)*a*x + 2*sqrt(a*x + 1)*sqrt(a*x - 1) - 22*a**5*x**5 - 22*a**4
*x**4)/(8*x**4*(a*x + 1))
```

3.67 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 253

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4}+\frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3}+\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{48a^2}+\frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a}+\frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5+\frac{31\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{31\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

output
```
611/1920*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^4+269/960*(1-1/a/x)^(3/4)*(1+
1/a/x)^(1/4)*x^2/a^3+11/48*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a^2+9/40*(1
-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^4/a+1/5*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^
5+31/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5+31/128*arctanh((1+1/a
/x)^(1/4)/(1-1/a/x)^(1/4))/a^5
```

Mathematica [A] (verified)

Time = 5.30 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.68

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$$
$$=\frac{\frac{24576e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^5}+\frac{62976e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{64640e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{34000e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{9620e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+930\arctan}{3840a^5}$$

input
```
Integrate[E^(ArcCoth[a*x]/2)*x^4,x]
```

output
```
((24576*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^5 + (62976*E^(ArcCot
h[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (64640*E^(ArcCoth[a*x]/2))/(-1 +
E^(2*ArcCoth[a*x]))^3 + (34000*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]
))^2 + (9620*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + 930*ArcTan[E^
(ArcCoth[a*x]/2)] - 465*Log[1 - E^(ArcCoth[a*x]/2)] + 465*Log[1 + E^(ArcCo
th[a*x]/2)])/(3840*a^5)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {6721, 110, 27, 168, 27, 168, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4e^{\frac{1}{2}\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{6721}$$
$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}x^6}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$
$$\Big\downarrow \text{110}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{1}{5}\int\frac{\left(9a+\frac{8}{x}\right)x^5}{2a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\int\frac{\left(9a+\frac{8}{x}\right)x^5}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{10a^2}$$

$$\Big\downarrow \; 168$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{-\frac{1}{4}\int-\frac{\left(55a+\frac{54}{x}\right)x^4}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{9}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\int\frac{\left(55a+\frac{54}{x}\right)x^4}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{8a}-\frac{9}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}$$

$$\Big\downarrow \; 168$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{-\frac{1}{3}\int-\frac{\left(269a+\frac{220}{x}\right)x^3}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a}-\frac{9}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{\int\frac{\left(269a+\frac{220}{x}\right)x^3}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a}-\frac{9}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}$$

$$\Big\downarrow \; 168$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{-\frac{1}{2}\int -\frac{\left(611a+\frac{538}{x}\right)x^2}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{269}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a}-\frac{9}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{\frac{\int \frac{\left(611a+\frac{538}{x}\right)x^2}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{269}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a}-\frac{9}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}$$

$$\Big\downarrow \; 168$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{\frac{-\int -\frac{465x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-611ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{269}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a}-\frac{9}{4}ax^4(1\cdots}{10a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{\frac{\frac{465}{2}\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-611ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{269}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a}-\frac{9}{4}ax^4(1\cdots}{10a^2}$$

$$\Big\downarrow \; 104$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{\frac{930\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-611ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{269}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a}-\frac{9}{4}ax^4\left(1-\frac{1}{ax}\right)}{10a^2}$$

$\Big\downarrow$ 756

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{930\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-611ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{269}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{}}{8a}}{10a^2}$$

$\Big\downarrow$ 216

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{930\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-611ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{269}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{}}{8a}}{10a^2}$$

$\Big\downarrow$ 219

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{930\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-611ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{269}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{55}{3}ax^3\left(1-\frac{1}{ax}\right)^{3}}{8a}}{10a^2}$$

input
```
Int[E^(ArcCoth[a*x]/2)*x^4,x]
```

output
```
((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^5)/5 - ((-9*a*(1 - 1/(a*x))^(3/
4)*(1 + 1/(a*x))^(1/4)*x^4)/4 + ((-55*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^
(1/4)*x^3)/3 + ((-269*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/2 + (
-611*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x + 930*(-1/2*ArcTan[(1 + 1
/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a
*x))^(1/4)]/2))/(4*a))/(6*a))/(8*a))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.47

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$$
$$= \frac{2\left(384\,a^5x^5 + 816\,a^4x^4 + 872\,a^3x^3 + 978\,a^2x^2 + 1149\,ax + 611\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} - 930\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 465\,\ldots}{3840\,a^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="fricas")
```

output
```
1/3840*(2*(384*a^5*x^5 + 816*a^4*x^4 + 872*a^3*x^3 + 978*a^2*x^2 + 1149*a*
x + 611)*((a*x - 1)/(a*x + 1))^(3/4) - 930*arctan(((a*x - 1)/(a*x + 1))^(1
/4)) + 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 465*log(((a*x - 1)/(a*x
+ 1))^(1/4) - 1))/a^5
```

Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \int \frac{x^4}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)*x***4,x)
```

output
```
Integral(x**4/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{3840}a\left(\frac{4\left(465\left(\frac{ax-1}{ax+1}\right)^{\frac{19}{4}}-696\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}+5090\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-1120\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+2405\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1}-\frac{10(ax-1)^2a^6}{(ax+1)^2}+\frac{10(ax-1)^3a^6}{(ax+1)^3}-\frac{5(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}+\frac{930\,\mathrm{arc}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="maxima")
```

output
```
-1/3840*a*(4*(465*((a*x - 1)/(a*x + 1))^(19/4) - 696*((a*x - 1)/(a*x + 1))
^(15/4) + 5090*((a*x - 1)/(a*x + 1))^(11/4) - 1120*((a*x - 1)/(a*x + 1))^(
7/4) + 2405*((a*x - 1)/(a*x + 1))^(3/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(
a*x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^
4*a^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 930*arctan(((a*x
- 1)/(a*x + 1))^(1/4))/a^6 - 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6
+ 465*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.92

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{3840}a\left(\frac{930\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{465\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{465\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-\frac{4\left(\frac{1120(ax-1}{ax-}\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="giac")
```

output
```
-1/3840*a*(930*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 465*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^6 + 465*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^6 - 4*(1120*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 5090*(a*
x - 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 696*(a*x - 1)^3*((a*x -
 1)/(a*x + 1))^(3/4)/(a*x + 1)^3 - 465*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(
3/4)/(a*x + 1)^4 - 2405*((a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a*x
+ 1) - 1)^5))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.91

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \frac{\frac{481\left(\frac{ax-1}{ax+1}\right)^{3/4}}{192}-\frac{7\left(\frac{ax-1}{ax+1}\right)^{7/4}}{6}+\frac{509\left(\frac{ax-1}{ax+1}\right)^{11/4}}{96}-\frac{29\left(\frac{ax-1}{ax+1}\right)^{15/4}}{40}+\frac{31\left(\frac{ax-1}{ax+1}\right)^{19/4}}{64}}{a^5+\frac{10a^5(ax-1)^2}{(ax+1)^2}-\frac{10a^5(ax-1)^3}{(ax+1)^3}+\frac{5a^5(ax-1)^4}{(ax+1)^4}-\frac{a^5(ax-1)^5}{(ax+1)^5}-\frac{5a^5(ax-1)}{ax+1}} - \frac{31\,\mathrm{atan}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{128\,a^5}+\frac{31\,\mathrm{atanh}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{128\,a^5}$$

input
```
int(x^4/((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
((481*((a*x - 1)/(a*x + 1))^(3/4))/192 - (7*((a*x - 1)/(a*x + 1))^(7/4))/6
 + (509*((a*x - 1)/(a*x + 1))^(11/4))/96 - (29*((a*x - 1)/(a*x + 1))^(15/4
))/40 + (31*((a*x - 1)/(a*x + 1))^(19/4))/64)/(a^5 + (10*a^5*(a*x - 1)^2)/
(a*x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x
+ 1)^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) - (3
1*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) + (31*atanh(((a*x - 1)/(a*x
 + 1))^(1/4)))/(128*a^5)
```

Reduce [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \int \frac{(ax+1)^{\frac{1}{4}}\,x^4}{(ax-1)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x)
```

output
```
int(((a*x + 1)**(1/4)*x***4)/(a*x - 1)**(1/4),x)
```

3.68 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 216

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3}+\frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2}+\frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4+\frac{11\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

output
```
83/192*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^3+29/96*(1-1/a/x)^(3/4)*(1+1/a/
x)^(1/4)*x^2/a^2+7/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a+1/4*(1-1/a/x)^
(3/4)*(1+1/a/x)^(1/4)*x^4+11/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^
4+11/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Mathematica [A] (verified)

Time = 5.21 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \frac{\frac{1536e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{3200e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{2512e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{980e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+66\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-33}{384a^4}$$

input
```
Integrate[E^(ArcCoth[a*x]/2)*x^3,x]
```

output
```
((1536*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (3200*E^(ArcCoth[
a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (2512*E^(ArcCoth[a*x]/2))/(-1 + E^(
2*ArcCoth[a*x]))^2 + (980*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) +
66*ArcTan[E^(ArcCoth[a*x]/2)] - 33*Log[1 - E^(ArcCoth[a*x]/2)] + 33*Log[1
+ E^(ArcCoth[a*x]/2)])/(384*a^4)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6721, 110, 27, 168, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{\frac{1}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}x^5}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{1}{4}\int\frac{\left(7a+\frac{6}{x}\right)x^4}{2a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\int\frac{\left(7a+\frac{6}{x}\right)x^4}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{8a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{-\frac{1}{3}\int-\frac{\left(29a+\frac{28}{x}\right)x^3}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\int\frac{\left(29a+\frac{28}{x}\right)x^3}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{-\frac{1}{2}\int-\frac{\left(83a+\frac{58}{x}\right)x^2}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{29}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{\int\frac{\left(83a+\frac{58}{x}\right)x^2}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{29}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{-\int -\frac{33x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-83ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{29}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{\frac{33}{2}\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-83ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{29}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a^2}$$

$$\Big\downarrow \; 104$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{66\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-83ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{29}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a^2}$$

$$\Big\downarrow \; 756$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{66\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-83ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{29}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/}}{8a^2}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{66\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-83ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{29}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/}}{8a^2}$$

$\downarrow$ 219

$$\frac{\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{66\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-83ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{29}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{7}{3}ax^3\left(1-\frac{1}{ax}\right.}{8a^2}$$

input
```
Int[E^(ArcCoth[a*x]/2)*x^3,x]
```

output
```
((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^4)/4 - ((-7*a*(1 - 1/(a*x))^(3/
4)*(1 + 1/(a*x))^(1/4)*x^3)/3 + ((-29*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^
(1/4)*x^2)/2 + (-83*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x + 66*(-1/2
*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(
1/4)/(1 - 1/(a*x))^(1/4)]/2))/(4*a))/(6*a))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.51

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \frac{2\left(48\,a^4x^4+104\,a^3x^3+114\,a^2x^2+141\,ax+83\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{384\,a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="fricas")
```

output
```
1/384*(2*(48*a^4*x^4 + 104*a^3*x^3 + 114*a^2*x^2 + 141*a*x + 83)*((a*x - 1
)/(a*x + 1))^(3/4) - 66*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 33*log(((a*x
 - 1)/(a*x + 1))^(1/4) + 1) - 33*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \int \frac{x^3}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**3,x)
```

output
```
Integral(x**3/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$$
$$= \frac{1}{384}a\left(\frac{4\left(33\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}} - 279\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}} + 107\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}} - 245\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1} - \frac{6(ax-1)^2a^5}{(ax+1)^2} + \frac{4(ax-1)^3a^5}{(ax+1)^3} - \frac{(ax-1)^4a^5}{(ax+1)^4} - a^5} - \frac{66\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5} + \frac{33\log}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="maxima")
```

output
```
1/384*a*(4*(33*((a*x - 1)/(a*x + 1))^(15/4) - 279*((a*x - 1)/(a*x + 1))^(1
1/4) + 107*((a*x - 1)/(a*x + 1))^(7/4) - 245*((a*x - 1)/(a*x + 1))^(3/4))/
(4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3
*a^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 66*arctan(((a*x -
1)/(a*x + 1))^(1/4))/a^5 + 33*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 - 3
3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{1}{384}a\left(\frac{66\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5} - \frac{33\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5} + \frac{33\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5} + \frac{4\left(\frac{107\,(ax-1)\left(\frac{ax-1}{ax+1}\right.}{ax+1}\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="giac")
```

output
```
-1/384*a*(66*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 33*log(((a*x - 1)/(
a*x + 1))^(1/4) + 1)/a^5 + 33*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^
5 + 4*(107*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 279*(a*x - 1)
^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 33*(a*x - 1)^3*((a*x - 1)/(a*
x + 1))^(3/4)/(a*x + 1)^3 - 245*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x -
1)/(a*x + 1) - 1)^4))
```

Mupad [B] (verification not implemented)

Time = 23.70 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.89

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \frac{\frac{245\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{96} - \frac{107\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{96} + \frac{93\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{32} - \frac{11\left(\frac{a\,x-1}{a\,x+1}\right)^{15/4}}{32}}{a^4 + \frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3} + \frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4} - \frac{4\,a^4\,(a\,x-1)}{a\,x+1}} - \frac{11\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4} + \frac{11\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}$$

input
```
int(x^3/((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
((245*((a*x - 1)/(a*x + 1))^(3/4))/96 - (107*((a*x - 1)/(a*x + 1))^(7/4))/
96 + (93*((a*x - 1)/(a*x + 1))^(11/4))/32 - (11*((a*x - 1)/(a*x + 1))^(15/
4))/32)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x
+ 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) - (1
1*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) + (11*atanh(((a*x - 1)/(a*x
+ 1))^(1/4)))/(64*a^4)
```

Reduce [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \int \frac{(ax+1)^{\frac{1}{4}}\,x^3}{(ax-1)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x)
```

output
```
int(((a*x + 1)**(1/4)*x**3)/(a*x - 1)**(1/4),x)
```

3.69 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 179

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2}+\frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3+\frac{3\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

output
```
11/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^2+5/12*(1-1/a/x)^(3/4)*(1+1/a/x)
^(1/4)*x^2/a+1/3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3+3/8*arctan((1+1/a/x)^
(1/4)/(1-1/a/x)^(1/4))/a^3+3/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^
3
```

Mathematica [A] (verified)

Time = 5.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$$
$$= \frac{\frac{128e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{208e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{116e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+18\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-9\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right.}{48a^3}$$

input
```
Integrate[E^(ArcCoth[a*x]/2)*x^2,x]
```

output
```
((128*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (208*E^(ArcCoth[a*
x]/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (116*E^(ArcCoth[a*x]/2))/(-1 + E^(2*A
rcCoth[a*x])) + 18*ArcTan[E^(ArcCoth[a*x]/2)] - 9*Log[1 - E^(ArcCoth[a*x]/
2)] + 9*Log[1 + E^(ArcCoth[a*x]/2)])/(48*a^3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6721, 110, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\frac{1}{2}\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{ 6721}$$
$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}x^4}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$
$$\Big\downarrow \text{ 110}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{1}{3}\int\frac{\left(5a+\frac{4}{x}\right)x^3}{2a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\int\frac{\left(5a+\frac{4}{x}\right)x^3}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{6a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{-\frac{1}{2}\int-\frac{\left(11a+\frac{10}{x}\right)x^2}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\int\frac{\left(11a+\frac{10}{x}\right)x^2}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{-\int-\frac{9x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-11ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\frac{\frac{9}{2}\int\frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-11ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}$$

$$\Big\downarrow \text{104}$$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{18\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-11ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}$$

$$\Big\downarrow \text{756}$$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{18\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-11ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{18\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-11ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{18\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-11ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}$$

input

```
Int[E^(ArcCoth[a*x]/2)*x^2,x]
```

output
```
((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^3)/3 - ((-5*a*(1 - 1/(a*x))^(3/
4)*(1 + 1/(a*x))^(1/4)*x^2)/2 + (-11*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(
1/4)*x + 18*(-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTan
h[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/(4*a))/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.58

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = \frac{2\,(8\,a^3x^3+18\,a^2x^2+21\,ax+11)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-9\,\log\left(\left(\frac{ax-1}{ax+1}\right.\right.}{48\,a^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="fricas")
```

output
```
1/48*(2*(8*a^3*x^3 + 18*a^2*x^2 + 21*a*x + 11)*((a*x - 1)/(a*x + 1))^(3/4)
 - 18*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1
/4) + 1) - 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = \int \frac{x^2}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**2,x)
```

output
```
Integral(x**2/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{48}\,a\left(\frac{4\left(9\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-6\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+29\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{9}{\ }\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="maxima")
```

output
```
-1/48*a*(4*(9*((a*x - 1)/(a*x + 1))^(11/4) - 6*((a*x - 1)/(a*x + 1))^(7/4)
 + 29*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1
)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 18*arctan(((a*x
 - 1)/(a*x + 1))^(1/4))/a^4 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 +
 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{1}{48}a\left(\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4} - \frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4} + \frac{9\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4} - \frac{4\left(\frac{6\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1} - \right.}{a^4}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="giac")
```

output
```
-1/48*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 9*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^4 + 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 -
 4*(6*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 9*(a*x - 1)^2*((a*
x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 - 29*((a*x - 1)/(a*x + 1))^(3/4))/(a^4
*((a*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B] (verification not implemented)

Time = 23.95 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = \frac{\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}} - \frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}+\frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

input
```
int(x^2/((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
((29*((a*x - 1)/(a*x + 1))^(3/4))/12 - ((a*x - 1)/(a*x + 1))^(7/4)/2 + (3*
((a*x - 1)/(a*x + 1))^(11/4))/4)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 -
(a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - (3*atan(((a
*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) + (3*atanh(((a*x - 1)/(a*x + 1))^(1/4))
)/(8*a^3)
```

Reduce [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = \int \frac{(ax+1)^{\frac{1}{4}}\,x^2}{(ax-1)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x)
```

output
```
int(((a*x + 1)**(1/4)*x**2)/(a*x - 1)**(1/4),x)
```

3.70 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 142

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx=\frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2+\frac{\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

output
```
1/4*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a+1/2*(1-1/a/x)^(3/4)*(1+1/a/x)^(5/4
)*x^2+1/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+1/4*arctanh((1+1/a/x
)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.46

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(-1+5e^{2\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+\operatorname{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

input
```
Integrate[E^(ArcCoth[a*x]/2)*x,x]
```

output
```
((2*E^(ArcCoth[a*x]/2)*(-1 + 5*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*x
]))^2 + ArcTan[E^(ArcCoth[a*x]/2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6721, 107, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{1}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}x^3}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{\int \frac{\sqrt[4]{1+\frac{1}{ax}}x^2}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}}{4a}$$

$$\Big\downarrow \; 105$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{\frac{\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}-x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

$$\Big\downarrow \; 104$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{\frac{2\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}-x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

$$\Big\downarrow \; 756$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{\frac{2\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{\frac{2\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}-x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{\frac{2\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}-x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}}{}$$

input
```
Int[E^(ArcCoth[a*x]/2)*x,x]
```

output
```
((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(5/4)*x^2)/2 - (-((1 - 1/(a*x))^(3/4)*(
1 + 1/(a*x))^(1/4)*x) + (2*(-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^
(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/a)/(4*a)
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.65

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{2\,(2\,a^2x^2+5\,ax+3)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^2*x^2 + 5*a*x + 3)*((a*x - 1)/(a*x + 1))^(3/4) - 2*arctan(((a*
x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*
x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = \int \frac{x}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)*x,x)
```

output
```
Integral(x/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.05

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{1}{8}\,a\left(\frac{4\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="maxima")
```

output
```
1/8*a*(4*(((a*x - 1)/(a*x + 1))^(7/4) - 5*((a*x - 1)/(a*x + 1))^(3/4))/(2*
(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 2*arctan(((
a*x - 1)/(a*x + 1))^(1/4))/a^3 + log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3
- log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.98

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = -\frac{1}{8}a\left(\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3} - \frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3} + \frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3} + \frac{4\left(\frac{(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1} - 5\left(\frac{ax-1}{ax+1}\right.\right.}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="giac")
```

output
```
-1/8*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - log(((a*x - 1)/(a*x +
1))^(1/4) + 1)/a^3 + log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 + 4*((a
*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 5*((a*x - 1)/(a*x + 1))^(3
/4))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{2} - \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}}{a^2 + \frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{2\,a^2\,(a\,x-1)}{a\,x+1}} - \frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2} + \frac{\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}$$

input
```
int(x/((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
((5*((a*x - 1)/(a*x + 1))^(3/4))/2 - ((a*x - 1)/(a*x + 1))^(7/4)/2)/(a^2 +
 (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) - atan(((a*x
 - 1)/(a*x + 1))^(1/4))/(4*a^2) + atanh(((a*x - 1)/(a*x + 1))^(1/4))/(4*a^
2)
```

Reduce [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx=\int\frac{(ax+1)^{\frac{1}{4}}x}{(ax-1)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)*x,x)
```

output
```
int(((a*x + 1)**(1/4)*x)/(a*x - 1)**(1/4),x)
```

3.71 $\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 96

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx=\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x+\frac{\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

output
```
(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x+arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/
a+arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx=\frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+\operatorname{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

input
```
Integrate[E^(ArcCoth[a*x]/2),x]
```

output
```
((2*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + ArcTan[E^(ArcCoth[a*x]
/2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6720, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6720}$$

$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}x^2}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{104}$$

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{2\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}$$

$\Big\downarrow$ 756

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{2\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$\Big\downarrow$ 216

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{2\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}$$

$\Big\downarrow$ 219

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{2\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}$$

input
```
Int[E^(ArcCoth[a*x]/2),x]
```

output
```
(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x - (2*(-1/2*ArcTan[(1 + 1/(a*x))^
(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/
4)]/2))/a
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx = \frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} - 2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

output
```
1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4) - 2*arctan(((a*x - 1)/(a*x +
1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x +
1))^(1/4) - 1))/a
```

Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx = \int \frac{1}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(-1/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.16

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

output
```
-1/2*a*(4*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 2*
arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - log(((a*x - 1)/(a*x + 1))^(1/4)
+ 1)/a^2 + log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.12

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}\,a\left(\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

output
```
-1/2*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - log(((a*x - 1)/(a*x +
1))^(1/4) + 1)/a^2 + log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 + 4*((a
*x - 1)/(a*x + 1))^(3/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.81

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx=\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{\operatorname{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}+\frac{\operatorname{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

input
```
int(1/((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(3/4))/(a - (a*(a*x - 1))/(a*x + 1)) - atan(((a*x
 - 1)/(a*x + 1))^(1/4))/a + atanh(((a*x - 1)/(a*x + 1))^(1/4))/a
```

Reduce [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx=\int\frac{(ax+1)^{\frac{1}{4}}}{(ax-1)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int((a*x + 1)**(1/4)/(a*x - 1)**(1/4),x)
```

3.72 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 222

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = -\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

output
```
2^(1/2)*arctan(-1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))+2^(1/2)*arctan(
1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))+2*arctan((1+1/a/x)^(1/4)/(1-1/a
/x)^(1/4))-2^(1/2)*arctanh(2^(1/2)*(1-1/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(1+1
/a/x)^(1/2))/(1+1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.14

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \frac{8}{5}e^{\frac{5}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{5}{8},1,\frac{13}{8},e^{4\coth^{-1}(ax)}\right)$$

input
```
Integrate[E^(ArcCoth[a*x]/2)/x,x]
```

output
```
(8*E^((5*ArcCoth[a*x])/2)*Hypergeometric2F1[5/8, 1, 13/8, E^(4*ArcCoth[a*x
])])/5
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.16, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {6721, 140, 73, 104, 756, 216, 219, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}\,x}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{140}$$

$$-\frac{\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{a}-\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{73}$$

$$4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4} x^2}d\sqrt[4]{1-\frac{1}{ax}}-\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{104}$$

$$4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4} x^2}d\sqrt[4]{1-\frac{1}{ax}}-4\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{756}$$

$$4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4} x^2}d\sqrt[4]{1-\frac{1}{ax}}-4\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$\Big\downarrow \text{216}$$

$$4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4} x^2}d\sqrt[4]{1-\frac{1}{ax}}-4\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4} x^2}d\sqrt[4]{1-\frac{1}{ax}}-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{854}$$

$$4\int \frac{1}{\left(1+\frac{1}{x^4}\right) x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{826}$$

$$4\left(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \ 1476$$

$$4\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \ 1082$$

$$4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \ 217$$

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{1479}$$

$$4\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{a}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \; 1103$$

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\text{lo}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

input
```
Int[E^(ArcCoth[a*x]/2)/x,x]
```

output
```
-4*(-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2) + 4*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a
*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x
))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))
^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1
/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} x}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.87

$$\begin{aligned}\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}dx &= \sqrt{2}\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+\sqrt{2}\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)\\&\quad-\frac{1}{2}\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)\\&\quad+\frac{1}{2}\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)\\&\quad-2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\\&\quad+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)\end{aligned}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="fricas")
```

output
```
sqrt(2)*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + sqrt(2)*arctan(s
qrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - 1/2*sqrt(2)*log(sqrt(2)*((a*x -
1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 1/2*sqrt(2)*log(-sq
rt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 2*arc
tan(((a*x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) -
log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)/x,x)
```

output
```
Integral(1/(x*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.01

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$= \frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}(\right.}{a}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="maxima")
```

output
```
1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1))/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4
) - 1)/a)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$= \frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="giac")
```

output
```
1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
sqrt((a*x - 1)/(a*x + 1)) + 1)/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a
 + 2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x +
1))^(1/4) - 1))/a)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.45

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = -\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)\,2\mathrm{i}-2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)$$
$$+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,(1-\mathrm{i})+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,(1+1\mathrm{i})$$

input
```
int(1/(x*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

output
```
2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 - 1i) -
2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)
*2i + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 +
1i)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{(ax+1)^{\frac{1}{4}}}{(ax-1)^{\frac{1}{4}}\,x}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x,x)
```

output
```
int((a*x + 1)**(1/4)/((a*x - 1)**(1/4)*x),x)
```

3.73 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 192

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

output
```
a*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/2*a*arctan(-1+2^(1/2)*(1-1/a/x)^(1/4)/
(1+1/a/x)^(1/4))*2^(1/2)+1/2*a*arctan(1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^
(1/4))*2^(1/2)-1/2*a*arctanh(2^(1/2)*(1-1/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(1
+1/a/x)^(1/2))/(1+1/a/x)^(1/4))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.77

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = a\left(\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}} + \frac{\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}\right)$$

input
```
Integrate[E^(ArcCoth[a*x]/2)/x^2,x]
```

output
```
a*((2*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + ArcTan[1 - Sqrt[2]*E^
(ArcCoth[a*x]/2)]/Sqrt[2] - ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)]/Sqrt[2]
 + Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]/(2*Sqrt[2]) - Log[
1 + Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]/(2*Sqrt[2]))
```

Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6721, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$\big\downarrow$ 60

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{1}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$\big\downarrow$ 73

$$2a\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}+a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\big\downarrow$ 854

$$2a\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\big\downarrow$ 826

$$2a\left(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\big\downarrow$ 1476

$$2a\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1}}{\sqrt[4]{2}}\right.$$

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\big\downarrow$ 1082

$$\frac{2a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+}{a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 217$$

$$\frac{2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+}{a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 1479$$

$$\frac{2a\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\cdots}}{\sqrt[4]{2-\frac{1}{x^4}}}\cdots\right)}{\sqrt{2}}\cdots\right.\right.}{a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 25

$$\frac{2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\cdots\right)}{\sqrt{2}}\cdots\right.\right.}{a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 27

$$\frac{2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\ldots}}{\sqrt[4]{2-\ldots}}\right.}{\sqrt{2}}\right.}{a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\ldots\right.}{a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}$$

input

```
Int[E^(ArcCoth[a*x]/2)/x^2,x]
```

output

```
a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4) + 2*a*((-(ArcTan[1 - (Sqrt[2]*(1
 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 -
 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1
/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]
*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} x^2} dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.91

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{2\sqrt{2}ax\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+2\sqrt{2}ax\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-\sqrt{2}ax\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}\right.}{4x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*a*x*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + 2*sqr
t(2)*a*x*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - sqrt(2)*a*x*log
(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + sq
rt(2)*a*x*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) + 4*(a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4))/x
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**2,x)
```

output
```
Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.97

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\right.$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="maxima")
```

output

```
1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1
) + 1))*a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.97

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\right.$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="giac")
```

output

```
1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1
) + 1))*a
```

Mupad [B] (verification not implemented)

Time = 24.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.45

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= (-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right) - (-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right) + \frac{2\,a\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{\frac{a\,x-1}{a\,x+1}+1}$$

input
```
int(1/(x^2*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

output
```
(-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - (-1)^(1/4)*a*a
tanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) + (2*a*((a*x - 1)/(a*x + 1))^
(3/4))/((a*x - 1)/(a*x + 1) + 1)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{(ax+1)^{\frac{1}{4}}}{(ax-1)^{\frac{1}{4}}\,x^2}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

output
```
int((a*x + 1)**(1/4)/((a*x - 1)**(1/4)*x**2),x)
```

3.74 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 244

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3} dx = \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{a^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}$$

output

```
1/4*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/2*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^
(5/4)+1/8*a^2*arctan(-1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)+1
/8*a^2*arctan(1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)-1/8*a^2*a
rctanh(2^(1/2)*(1-1/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1+1/a/
x)^(1/4))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.71

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{16}a^2\left(-\frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{40e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}+2\sqrt{2}\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)-2\sqrt{2}\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)+\sqrt{2}\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)-\sqrt{2}\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[E^(ArcCoth[a*x]/2)/x^3,x]
```

output
```
(a^2*((-32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x]))^2 + (40*E^(ArcCoth
[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^(ArcCo
th[a*x]/2)] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)] + Sqrt[2]*L
og[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]] - Sqrt[2]*Log[1 + Sqrt
[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]))/16
```

Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6721, 90, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}x}d\frac{1}{x}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{1}{4}a\int \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{1}{4}a\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{1}{4}a\left(-2a\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{ 854}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{1}{4}a\left(-2a\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{ 826}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{1}{4}a\left(-2a\left(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-$$

$$\frac{1}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\int\frac{1-}{1+}\right.\right.$$

$\downarrow$ 1082

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-$$

$$\frac{1}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\right.$$

$\downarrow$ 217

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-$$

$$\frac{1}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\left(1\right.\right.$$

$\downarrow$ 1479

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-$$

$$\frac{1}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\cdots}{\sqrt[4]{\cdots}}\right.}{\cdots}\right.\right.\right.$$

$$\Big\downarrow 25$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-$$

$$\frac{1}{4}a\left(-2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{2\cdots}}\right.}{\sqrt{\cdots}}\right.\right.\right.$$

$$\Big\downarrow 27$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-$$

$$\frac{1}{4}a\left(-2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\cdots\right)}{\cdots}\right.\right.\right.$$

$\downarrow$ 1103

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-$$

$$\frac{1}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+\cdots\right)}{2\sqrt{2}}\right.\right.\right.$$

input

```
Int[E^(ArcCoth[a*x]/2)/x^3,x]
```

output

```
(a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(5/4))/2 - (a*(-(a*(1 - 1/(a*x))^(3
/4)*(1 + 1/(a*x))^(1/4)) - 2*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4)
)/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2
 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1
/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} x^3} dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.82

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3} dx = \frac{2\sqrt{2}a^2x^2 \arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+2\sqrt{2}a^2x^2 \arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-\sqrt{2}a^2x^2 \log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\right.}{16\,x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="fricas")
```

output
```
1/16*(2*sqrt(2)*a^2*x^2*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) +
2*sqrt(2)*a^2*x^2*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - sqrt(2
)*a^2*x^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
1)) + 1) + sqrt(2)*a^2*x^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) + 4*(3*a^2*x^2 + 5*a*x + 2)*((a*x - 1)/(a*x + 1
))^(3/4))/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**3,x)
```

output
```
Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.93

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$$
$$= \frac{1}{16}\left(\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="maxima")
```

output
```
1/16*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1))*a + 8*(a*((a*x - 1)/(a*x + 1))^(7/4) + 5*a*((a*x
 - 1)/(a*x + 1))^(3/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 +
 1))*a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.91

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$$
$$= \frac{1}{16}\left(2\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="giac")
```

output
```
1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) - sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
 1)/(a*x + 1)) + 1) + sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)*a*((a*x - 1)/(a*x + 1))^(3/
4)/(a*x + 1) + 5*a*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^
2)*a
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.54

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{\frac{5\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/4}}{2}+\frac{a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/4}}{2}}{\frac{{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{2\,\left(a\,x-1\right)}{a\,x+1}+1}+\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4}-\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4}$$

input
```
int(1/(x^3*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

output
```
((5*a^2*((a*x - 1)/(a*x + 1))^(3/4))/2 + (a^2*((a*x - 1)/(a*x + 1))^(7/4))
/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) + ((-1)^(1/4)*
a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 - ((-1)^(1/4)*a^2*atan
h((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4
```

Reduce [F]

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{(ax+1)^{\frac{1}{4}}}{(ax-1)^{\frac{1}{4}}\,x^3}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

output
```
int((a*x + 1)**(1/4)/((a*x - 1)**(1/4)*x**3),x)
```

3.75 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 281

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} - \frac{3a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{3a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \frac{3a^3\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

output
```
3/8*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/12*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)
^(5/4)+1/3*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(5/4)/x+3/16*a^3*arctan(-1+2^(1/2
)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)+3/16*a^3*arctan(1+2^(1/2)*(1-1/
a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)-3/16*a^3*arctanh(2^(1/2)*(1-1/a/x)^(1/
4)/(1+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1+1/a/x)^(1/4))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.33

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{96}a^3\left(\frac{8e^{\frac{1}{2}\coth^{-1}(ax)}\left(9+6e^{2\coth^{-1}(ax)}+29e^{4\coth^{-1}(ax)}\right)}{\left(1+e^{2\coth^{-1}(ax)}\right)^3} + 9\text{RootSum}\left[1 + \#1^4\&, \frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

input
```
Integrate[E^(ArcCoth[a*x]/2)/x^4,x]
```

output
```
(a^3*((8*E^(ArcCoth[a*x]/2)*(9 + 6*E^(2*ArcCoth[a*x]) + 29*E^(4*ArcCoth[a*
x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 9*RootSum[1 + #1^4 & , (ArcCoth[a*x] -
2*Log[E^(ArcCoth[a*x]/2) - #1])/#1^3 & ]))/96
```

Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {6721, 101, 27, 90, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \; 6721$$

$$-\int \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \; 101$$

$$\frac{1}{3}a^2\int -\frac{\left(2a+\frac{1}{x}\right)\sqrt[4]{1+\frac{1}{ax}}}{2a\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}$$

$$\Big\downarrow \; 27$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-\frac{1}{6}a\int \frac{\left(2a+\frac{1}{x}\right)\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \; 90$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\int \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-$$
$$\frac{1}{6}a\left(\frac{9}{4}a\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-$$
$$\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)$$

$$\Big\downarrow \; 854$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\int\frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)$$

$\big\downarrow$ 826

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\left(\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a^2\left(1-\frac{1}{ax}\right.\right.$$

$\big\downarrow$ 1476

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\int\right.\right.\right.$$

$\big\downarrow$ 1082

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.\right.\right.$$

$\big\downarrow$ 217

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\right.\right.$$

$\downarrow$ 1479

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan}{}\right.\right.\right.\right.$$

$\downarrow$ 25

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arctan}\left(\right.}{}\right.\right.\right.\right.$$

$$\Big\downarrow \ 27$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\text{arcta}}{}\right.\right.\right.\right.$$

$$\Big\downarrow \ 1103$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{9}{4}a\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\right.}{2\sqrt{2}}\right.\right.\right.\right.$$

input
```
Int[E^(ArcCoth[a*x]/2)/x^4,x]
```

output
```
(a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(5/4))/(3*x) - (a*(-1/2*(a^2*(1 - 1
/(a*x))^(3/4)*(1 + 1/(a*x))^(5/4)) + (9*a*(-(a*(1 - 1/(a*x))^(3/4)*(1 + 1/
(a*x))^(1/4)) - 2*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-
4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4)
)^(1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^
(1/4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x
^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} x^4}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.74

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{18\sqrt{2}a^3x^3\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+18\sqrt{2}a^3x^3\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-9\sqrt{2}a^3x^3\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{96}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="fricas")
```

output
```
1/96*(18*sqrt(2)*a^3*x^3*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) +
 18*sqrt(2)*a^3*x^3*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - 9*sq
rt(2)*a^3*x^3*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) + 9*sqrt(2)*a^3*x^3*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
+ sqrt((a*x - 1)/(a*x + 1)) + 1) + 4*(11*a^3*x^3 + 21*a^2*x^2 + 18*a*x + 8
)*((a*x - 1)/(a*x + 1))^(3/4))/x^3
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)/x***4,x)
```

output
```
Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.96

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{1}{96}\left(9\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="maxima")
```

output
```
1/96*(9*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1))*a^2 + 8*(9*a^2*((a*x - 1)/(a*x + 1))^(11/4) +
6*a^2*((a*x - 1)/(a*x + 1))^(7/4) + 29*a^2*((a*x - 1)/(a*x + 1))^(3/4))/(3
*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3
 + 1))*a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.96

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{1}{96}\left(18\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="giac")
```

output
```
1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) - 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) + 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(6*(a*x - 1)*a^2*((a*x -
 1)/(a*x + 1))^(3/4)/(a*x + 1) + 9*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(
3/4)/(a*x + 1)^2 + 29*a^2*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1
) + 1)^3)*a
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.60

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{\frac{29\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}+\frac{a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{3\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{\frac{3\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{(a\,x-1)^3}{(a\,x+1)^3}+\frac{3\,(a\,x-1)}{a\,x+1}+1} + \frac{3\,(-1)^{1/4}\,a^3\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8} - \frac{3\,(-1)^{1/4}\,a^3\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8}$$

input
```
int(1/(x^4*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

output
```
((29*a^3*((a*x - 1)/(a*x + 1))^(3/4))/12 + (a^3*((a*x - 1)/(a*x + 1))^(7/4
))/2 + (3*a^3*((a*x - 1)/(a*x + 1))^(11/4))/4)/((3*(a*x - 1)^2)/(a*x + 1)^
2 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) + (3*(-1)^(1/4)
*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - (3*(-1)^(1/4)*a^3*a
tanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8
```

Reduce [F]

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{(ax+1)^{\frac{1}{4}}}{(ax-1)^{\frac{1}{4}}\,x^4}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

output
```
int((a*x + 1)**(1/4)/((a*x - 1)**(1/4)*x**4),x)
```

3.76 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 253

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4}+\frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2}+\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5-\frac{237\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{237\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

output
```
557/640*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^4+157/320*(1-1/a/x)^(1/4)*(1+1
/a/x)^(3/4)*x^2/a^3+5/16*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a^2+11/40*(1-
1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^4/a+1/5*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^5
-237/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5+237/128*arctanh((1+1/
a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5
```

Mathematica [A] (verified)

Time = 5.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.68

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$$

$$= \frac{\frac{8192e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^5}+\frac{22016e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{23936e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{14032e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{5500e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-2370\,\text{arcta}}{1280a^5}$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)*x^4,x]
```

output
```
((8192*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 + (22016*E^((3*
ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (23936*E^((3*ArcCoth[a*x])
/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (14032*E^((3*ArcCoth[a*x])/2))/(-1 + E^
(2*ArcCoth[a*x]))^2 + (5500*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x
])) - 2370*ArcTan[E^(ArcCoth[a*x]/2)] - 1185*Log[1 - E^(ArcCoth[a*x]/2)] +
 1185*Log[1 + E^(ArcCoth[a*x]/2)])/(1280*a^5)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6721, 110, 27, 168, 27, 168, 27, 168, 27, 168, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4e^{\frac{3}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}x^6}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 110}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{1}{5}\int \frac{\left(11a+\frac{8}{x}\right)x^5}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\int \frac{\left(11a+\frac{8}{x}\right)x^5}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{10a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{-\frac{1}{4}\int -\frac{3\left(25a+\frac{22}{x}\right)x^4}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{11}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{3\int \frac{\left(25a+\frac{22}{x}\right)x^4}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{8a}-\frac{11}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{3\left(-\frac{1}{3}\int -\frac{\left(157a+\frac{100}{x}\right)x^3}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a}-\frac{11}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{3\left(\frac{\int \frac{\left(157a+\frac{100}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a}-\frac{11}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{-\frac{1}{2}\int -\frac{\left(557a+\frac{314}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a}-\frac{11}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{\int \frac{\left(557a+\frac{314}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a}-\frac{11}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{-\int -\frac{1185x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-557ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a}-\frac{11}{4}a}{10a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{\frac{1185}{2}\int\frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-557ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a}-\frac{11}{4}a}{10a^2}$$

$\downarrow$ 104

$$\frac{\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{2370\int-\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-557ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a}-\frac{11}{4}a}{10a^2}$$

$\downarrow$ 25

$$\frac{\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{-2370\int\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-557ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a}-\frac{11}{4}a}{10a^2}$$

$\downarrow$ 827

$$\frac{\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{2370\left(\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-557ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\cdots}\right.}{8a}}{10a^2}$$

$\downarrow$ 216

$$\frac{\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{2370\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-557ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\cdots}\right.}{8a}}{10a^2}$$

$\downarrow$ 219

$$\frac{\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{2370\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-557ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{157}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{25}{3}ax^3\sqrt[4]{1-\cdots}\right.}{8a}}{10a^2}$$

input
```
Int[E^((3*ArcCoth[a*x])/2)*x^4,x]
```

output
```
((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^5)/5 - ((-11*a*(1 - 1/(a*x))^(1
/4)*(1 + 1/(a*x))^(3/4)*x^4)/4 + (3*((-25*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*
x))^(3/4)*x^3)/3 + ((-157*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/2
 + (-557*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x + 2370*(ArcTan[(1 + 1
/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/
(a*x))^(1/4)]/2))/(4*a))/(6*a)))/(8*a))/(10*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.47

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \frac{2\left(128\,a^5x^5+304\,a^4x^4+376\,a^3x^3+514\,a^2x^2+871\,ax+557\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2370\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+1185}{1280\,a^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="fricas")
```

output
```
1/1280*(2*(128*a^5*x^5 + 304*a^4*x^4 + 376*a^3*x^3 + 514*a^2*x^2 + 871*a*x
 + 557)*((a*x - 1)/(a*x + 1))^(1/4) + 2370*arctan(((a*x - 1)/(a*x + 1))^(1
/4)) + 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1185*log(((a*x - 1)/(a*
x + 1))^(1/4) - 1))/a^5
```

Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**4,x)
```

output
```
Integral(x**4/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{1280}\,a\left(\frac{4\left(395\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-1440\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+3710\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-1992\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+1375\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1}-\frac{10(ax-1)^2a^6}{(ax+1)^2}+\frac{10(ax-1)^3a^6}{(ax+1)^3}-\frac{5(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}-\frac{2370\,a}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="maxima")
```

output
```
-1/1280*a*(4*(395*((a*x - 1)/(a*x + 1))^(17/4) - 1440*((a*x - 1)/(a*x + 1)
)^(13/4) + 3710*((a*x - 1)/(a*x + 1))^(9/4) - 1992*((a*x - 1)/(a*x + 1))^(
5/4) + 1375*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(
a*x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^
4*a^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) - 2370*arctan(((a*x
 - 1)/(a*x + 1))^(1/4))/a^6 - 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^
6 + 1185*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.92

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$$

$$= \frac{1}{1280}a\left(\frac{2370\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6} + \frac{1185\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6} - \frac{1185\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6} + \frac{4\left(\frac{1992\,(ax-\cdots}{ax\cdots}\cdots\right.}{\cdots}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="giac")
```

output
```
1/1280*a*(2370*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 1185*log(((a*x -
1)/(a*x + 1))^(1/4) + 1)/a^6 - 1185*log(abs(((a*x - 1)/(a*x + 1))^(1/4) -
1))/a^6 + 4*(1992*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 3710*(
a*x - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 1440*(a*x - 1)^3*((a*
x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^3 - 395*(a*x - 1)^4*((a*x - 1)/(a*x + 1)
)^(1/4)/(a*x + 1)^4 - 1375*((a*x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a
*x + 1) - 1)^5))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.91

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \frac{\frac{275\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{64}-\frac{249\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{40}+\frac{371\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{32}-\frac{9\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{2}+\frac{79\left(\frac{a\,x-1}{a\,x+1}\right)^{17/4}}{64}}{a^5+\frac{10\,a^5\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a^5\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a^5\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a^5\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{5\,a^5\,(a\,x-1)}{a\,x+1}} + \frac{237\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}+\frac{237\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}$$

input
```
int(x^4/((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
((275*((a*x - 1)/(a*x + 1))^(1/4))/64 - (249*((a*x - 1)/(a*x + 1))^(5/4))/
40 + (371*((a*x - 1)/(a*x + 1))^(9/4))/32 - (9*((a*x - 1)/(a*x + 1))^(13/4
))/2 + (79*((a*x - 1)/(a*x + 1))^(17/4))/64)/(a^5 + (10*a^5*(a*x - 1)^2)/(
a*x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x +
 1)^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) + (23
7*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) + (237*atanh(((a*x - 1)/(a*
x + 1))^(1/4)))/(128*a^5)
```

Reduce [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \int \frac{(ax+1)^{\frac{3}{4}}\,x^4}{(ax-1)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x)
```

output
```
int(((a*x + 1)**(3/4)*x**4)/(a*x - 1)**(3/4),x)
```

3.77 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 216

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3}+\frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2}+\frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a}+\frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4-\frac{123\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\,\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

output
```
63/64*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^3+15/32*(1-1/a/x)^(1/4)*(1+1/a/x
)^(3/4)*x^2/a^2+3/8*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a+1/4*(1-1/a/x)^(1
/4)*(1+1/a/x)^(3/4)*x^4-123/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
+123/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Mathematica [A] (verified)

Time = 5.21 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$$
$$= \frac{\frac{512e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{1152e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{1008e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{532e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-246\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-12}{128a^4}$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)*x^3,x]
```

output
```
((512*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (1152*E^((3*Ar
cCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (1008*E^((3*ArcCoth[a*x])/2)
)/(-1 + E^(2*ArcCoth[a*x]))^2 + (532*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*Ar
cCoth[a*x])) - 246*ArcTan[E^(ArcCoth[a*x]/2)] - 123*Log[1 - E^(ArcCoth[a*x
]/2)] + 123*Log[1 + E^(ArcCoth[a*x]/2)])/(128*a^4)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6721, 110, 27, 168, 27, 168, 27, 168, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3e^{\frac{3}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}x^5}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 110}$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{1}{4}\int \frac{3\left(3a+\frac{2}{x}\right)x^4}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\int \frac{\left(3a+\frac{2}{x}\right)x^4}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{8a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(-\frac{1}{3}\int -\frac{3\left(5a+\frac{4}{x}\right)x^3}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\int \frac{\left(5a+\frac{4}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{-\frac{1}{2}\int -\frac{\left(21a+\frac{10}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-3\left(\frac{\frac{\int\frac{\left(21a+\frac{10}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a^2}$$

$\Big\downarrow$ 168

$$\frac{\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-3\left(\frac{\frac{-\int-\frac{41x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-21ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-3\left(\frac{\frac{\frac{41}{2}\int\frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-21ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a^2}$$

$\Big\downarrow$ 104

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{82\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-21ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a^2}$$

$\downarrow$ 25

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{-82\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-21ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{8a^2}$$

$\downarrow$ 827

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{82\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-21ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\right.\right.}{8a^2}$$

$\downarrow$ 216

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{82\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-21ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\cdots\right.}{8a^2}$$

$\downarrow$ 219

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\left(\frac{\frac{82\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-21ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{2a}-ax^3\sqrt[4]{1-\cdots}\right.}{8a^2}$$

input
```
Int[E^((3*ArcCoth[a*x])/2)*x^3,x]
```

output
```
((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^4)/4 - (3*(-(a*(1 - 1/(a*x))^(1
/4)*(1 + 1/(a*x))^(3/4)*x^3) + ((-5*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3
/4)*x^2)/2 + (-21*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x + 82*(ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/
(1 - 1/(a*x))^(1/4)]/2))/(4*a))/(2*a)))/(8*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.51

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \frac{2\,(16\,a^4x^4+40\,a^3x^3+54\,a^2x^2+93\,ax+63)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+246\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+123\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{128\,a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="fricas")
```

output
```
1/128*(2*(16*a^4*x^4 + 40*a^3*x^3 + 54*a^2*x^2 + 93*a*x + 63)*((a*x - 1)/(
a*x + 1))^(1/4) + 246*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 123*log(((a*x
- 1)/(a*x + 1))^(1/4) + 1) - 123*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx=\int\frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**3,x)
```

output
```
Integral(x**3/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$$
$$=\frac{1}{128}a\left(\frac{4\left(41\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-183\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+147\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-133\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}+\frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{123\ \mathrm{lo}}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="maxima")
```

output
```
1/128*a*(4*(41*((a*x - 1)/(a*x + 1))^(13/4) - 183*((a*x - 1)/(a*x + 1))^(9
/4) + 147*((a*x - 1)/(a*x + 1))^(5/4) - 133*((a*x - 1)/(a*x + 1))^(1/4))/(
4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*
a^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 246*arctan(((a*x -
1)/(a*x + 1))^(1/4))/a^5 + 123*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 -
123*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \frac{1}{128}a\left(\frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{123\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}-\frac{4\left(\frac{147(ax-1)\left(\frac{ax-}{ax+}\right.}{ax+1}\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="giac")
```

output
```
1/128*a*(246*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 123*log(((a*x - 1)/
(a*x + 1))^(1/4) + 1)/a^5 - 123*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/
a^5 - 4*(147*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 183*(a*x -
1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 41*(a*x - 1)^3*((a*x - 1)/(
a*x + 1))^(1/4)/(a*x + 1)^3 - 133*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x
- 1)/(a*x + 1) - 1)^4))
```

Mupad [B] (verification not implemented)

Time = 23.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.89

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \frac{\frac{133\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{32}-\frac{147\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{32}+\frac{183\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{32}-\frac{41\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{32}}{a^4+\frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a^4\,(a\,x-1)}{a\,x+1}}+\frac{123\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}+\frac{123\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}$$

input
```
int(x^3/((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
((133*((a*x - 1)/(a*x + 1))^(1/4))/32 - (147*((a*x - 1)/(a*x + 1))^(5/4))/
32 + (183*((a*x - 1)/(a*x + 1))^(9/4))/32 - (41*((a*x - 1)/(a*x + 1))^(13/
4))/32)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x
+ 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) + (1
23*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) + (123*atanh(((a*x - 1)/(a*
x + 1))^(1/4)))/(64*a^4)
```

Reduce [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \int \frac{(ax+1)^{\frac{3}{4}}x^3}{(ax-1)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x)
```

output
```
int(((a*x + 1)**(3/4)*x**3)/(a*x - 1)**(3/4),x)
```

3.78 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 179

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx=\frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2}+\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a}+\frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3-\frac{17\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{17\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

output
```
23/24*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^2+7/12*(1-1/a/x)^(1/4)*(1+1/a/x)
^(3/4)*x^2/a+1/3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3-17/8*arctan((1+1/a/x)
^(1/4)/(1-1/a/x)^(1/4))/a^3+17/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/
a^3
```

Mathematica [A] (verified)

Time = 5.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{\frac{128e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{240e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{180e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-102\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-51\log\left(1-e^{\frac{1}{2}\coth^{-1}(a}\right.}{48a^3}$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)*x^2,x]
```

output
```
((128*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (240*E^((3*Arc
Coth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (180*E^((3*ArcCoth[a*x])/2))/
(-1 + E^(2*ArcCoth[a*x])) - 102*ArcTan[E^(ArcCoth[a*x]/2)] - 51*Log[1 - E^
(ArcCoth[a*x]/2)] + 51*Log[1 + E^(ArcCoth[a*x]/2)])/(48*a^3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6721, 110, 27, 168, 27, 168, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{\frac{3}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}x^4}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 110}$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{1}{3}\int \frac{\left(7a+\frac{4}{x}\right)x^3}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\int\frac{\left(7a+\frac{4}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{6a^2}$$

$$\Big\downarrow \text{ 168}$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{-\frac{1}{2}\int-\frac{\left(23a+\frac{14}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{\int\frac{\left(23a+\frac{14}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}$$

$$\Big\downarrow \text{ 168}$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{-\int-\frac{51x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-23ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{\frac{51}{2}\int\frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-23ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}$$

$$\Big\downarrow \text{ 104}$$

$$\frac{\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{102\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-23ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}}{}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{-102\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-23ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}$$

$$\Big\downarrow \text{ 827}$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{102\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-23ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{102\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-23ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\frac{102\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-23ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{7}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}}{}$$

input
```
Int[E^((3*ArcCoth[a*x])/2)*x^2,x]
```

output
```
((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^3)/3 - ((-7*a*(1 - 1/(a*x))^(1/
4)*(1 + 1/(a*x))^(3/4)*x^2)/2 + (-23*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(
3/4)*x + 102*(ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[
(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/(4*a))/(6*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.58

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{2\left(8\,a^3x^3+22\,a^2x^2+37\,ax+23\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+102\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-51\,\log\left(\left(\right.\right.}{48\,a^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="fricas")
```

output
```
1/48*(2*(8*a^3*x^3 + 22*a^2*x^2 + 37*a*x + 23)*((a*x - 1)/(a*x + 1))^(1/4)
 + 102*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 51*log(((a*x - 1)/(a*x + 1))^
(1/4) + 1) - 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**2,x)
```

output
```
Integral(x**2/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{1}{48}\,a\left(\frac{4\left(17\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-30\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+45\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}-\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="maxima")
```

output
```
-1/48*a*(4*(17*((a*x - 1)/(a*x + 1))^(9/4) - 30*((a*x - 1)/(a*x + 1))^(5/4
) + 45*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x -
1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 102*arctan(((a
*x - 1)/(a*x + 1))^(1/4))/a^4 - 51*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^
4 + 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$$

$$=\frac{1}{48}\,a\left(\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{51\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}+\frac{4\left(\frac{30(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="giac")
```

output
```
1/48*a*(102*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 51*log(((a*x - 1)/(a
*x + 1))^(1/4) + 1)/a^4 - 51*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
 + 4*(30*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 17*(a*x - 1)^2*
((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 - 45*((a*x - 1)/(a*x + 1))^(1/4))/
(a^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{\frac{15\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{17\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}} + \frac{17\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3} + \frac{17\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

input
```
int(x^2/((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
((15*((a*x - 1)/(a*x + 1))^(1/4))/4 - (5*((a*x - 1)/(a*x + 1))^(5/4))/2 +
(17*((a*x - 1)/(a*x + 1))^(9/4))/12)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^
2 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (17*ata
n(((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) + (17*atanh(((a*x - 1)/(a*x + 1))^
(1/4)))/(8*a^3)
```

Reduce [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \int \frac{(ax+1)^{\frac{3}{4}}\,x^2}{(ax-1)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x)
```

output
```
int(((a*x + 1)**(3/4)*x**2)/(a*x - 1)**(3/4),x)
```

3.79 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 142

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx=\frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a}+\frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2-\frac{9\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

output
```
3/4*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a+1/2*(1-1/a/x)^(1/4)*(1+1/a/x)^(7/4
)*x^2-9/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+9/4*arctanh((1+1/a/x
)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.49

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx = \frac{\frac{2e^{\frac{3}{2}\coth^{-1}(ax)}\left(-3+7e^{2\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}-9\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+9\text{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)*x,x]
```

output
```
((2*E^((3*ArcCoth[a*x])/2)*(-3 + 7*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth
[a*x]))^2 - 9*ArcTan[E^(ArcCoth[a*x]/2)] + 9*ArcTanh[E^(ArcCoth[a*x]/2)])/
(4*a^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6721, 107, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{3}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}x^3}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}x^2}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3\left(\frac{3\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{4a}$$

$\downarrow$ 104

$$\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3\left(\frac{6\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}-x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{4a}$$

$\downarrow$ 25

$$\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3\left(x\left(-\sqrt[4]{1-\frac{1}{ax}}\right)\left(\frac{1}{ax}+1\right)^{3/4}-\frac{6\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}\right)}{4a}$$

$\downarrow$ 827

$$\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3\left(\frac{6\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{4a}$$

$\downarrow$ 216

$$\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3\left(\frac{6\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{4a}$$

$\downarrow$ 219

$$\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3\left(\frac{6\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}-x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)}{4a}$$

input
```
Int[E^((3*ArcCoth[a*x])/2)*x,x]
```

output
```
((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(7/4)*x^2)/2 - (3*(-((1 - 1/(a*x))^(1/4
)*(1 + 1/(a*x))^(3/4)*x) + (6*(ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1
/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/a))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx = \frac{2\left(2a^2x^2+7ax+5\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^2*x^2 + 7*a*x + 5)*((a*x - 1)/(a*x + 1))^(1/4) + 18*arctan(((a
*x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 9*log
(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx = \int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)*x,x)
```

output
```
Integral(x/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.07

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx = \frac{1}{8}a\left(\frac{4\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-7\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}+\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="maxima")
```

output
```
1/8*a*(4*(3*((a*x - 1)/(a*x + 1))^(5/4) - 7*((a*x - 1)/(a*x + 1))^(1/4))/(
2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + 18*arctan
(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)
/a^3 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx$$
$$=\frac{1}{8}a\left(\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{9\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}-\frac{4\left(\frac{3(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-7\left(\frac{a}{a}\right.\right.}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="giac")
```

output
```
1/8*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 9*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^3 - 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 - 4
*(3*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 7*((a*x - 1)/(a*x +
1))^(1/4))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx=\frac{\frac{7\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}-\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}+\frac{9\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}+\frac{9\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}$$

input
```
int(x/((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
((7*((a*x - 1)/(a*x + 1))^(1/4))/2 - (3*((a*x - 1)/(a*x + 1))^(5/4))/2)/(a
^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) + (9*ata
n(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) + (9*atanh(((a*x - 1)/(a*x + 1))^(
1/4)))/(4*a^2)
```

Reduce [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx=\int\frac{(ax+1)^{\frac{3}{4}}x}{(ax-1)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)*x,x)
```

output
```
int(((a*x + 1)**(3/4)*x)/(a*x - 1)**(3/4),x)
```

3.80 $\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 98

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx=\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x-\frac{3\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

output
```
(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x-3*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)
)/a+3*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx = \frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(1+\left(-1+e^{2\coth^{-1}(ax)}\right)\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},e^{2\coth^{-1}(ax)}\right)\right)}{a\left(-1+e^{2\coth^{-1}(ax)}\right)}$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2),x]
```

output
```
(8*E^((3*ArcCoth[a*x])/2)*(1 + (-1 + E^(2*ArcCoth[a*x]))*Hypergeometric2F1
[3/4, 2, 7/4, E^(2*ArcCoth[a*x])]))/(a*(-1 + E^(2*ArcCoth[a*x])))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6720, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6720}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}x^2}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{104}$$

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{6\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{6\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}+x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{827}$$

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{6\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$\Big\downarrow \text{216}$$

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{6\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$\Big\downarrow \text{219}$$

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{6\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}$$

input
```
Int[E^((3*ArcCoth[a*x])/2),x]
```

output
```
(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x - (6*(ArcTan[(1 + 1/(a*x))^(1/4)
/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]
/2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx = \frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) - 3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right)}{2\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

output
```
1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4) + 6*arctan(((a*x - 1)/(a*x +
1))^(1/4)) + 3*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 3*log(((a*x - 1)/(a*
x + 1))^(1/4) - 1))/a
```

Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx = \int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(-3/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{6\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

output
```
-1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) - 6*
arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 3*log(((a*x - 1)/(a*x + 1))^(1/4
) + 1)/a^2 + 3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx = \frac{1}{2}\,a\left(\frac{6\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{3\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

output
```
1/2*a*(6*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 3*log(((a*x - 1)/(a*x +
 1))^(1/4) + 1)/a^2 - 3*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 - 4*
((a*x - 1)/(a*x + 1))^(1/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

Mupad [B] (verification not implemented)

Time = 23.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx=\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}+\frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

input
```
int(1/((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/4))/(a - (a*(a*x - 1))/(a*x + 1)) + (3*atan(((
a*x - 1)/(a*x + 1))^(1/4)))/a + (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/a
```

Reduce [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx=\int\frac{(ax+1)^{\frac{3}{4}}}{(ax-1)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int((a*x + 1)**(3/4)/(a*x - 1)**(3/4),x)
```

3.81 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 221

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

output
```
2^(1/2)*arctan(-1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))+2^(1/2)*arctan(
1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))-2*arctan((1+1/a/x)^(1/4)/(1-1/a
/x)^(1/4))+2^(1/2)*arctanh(2^(1/2)*(1-1/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(1+1
/a/x)^(1/2))/(1+1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.14

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx=\frac{8}{7}e^{\frac{7}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{7}{8},1,\frac{15}{8},e^{4\coth^{-1}(ax)}\right)$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)/x,x]
```

output
```
(8*E^((7*ArcCoth[a*x])/2)*Hypergeometric2F1[7/8, 1, 15/8, E^(4*ArcCoth[a*x
])])/7
```

Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.17, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.214, Rules used = {6721, 140, 73, 104, 25, 770, 755, 827, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}x}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 140}$$

$$-\frac{\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{73}$$

$$4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{104}$$

$$4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-4\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{25}$$

$$4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}+4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{770}$$

$$4\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{755}$$

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{827}$$

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$
$$4\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$\Big\downarrow \text{216}$$

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$\downarrow$ 219

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$\downarrow$ 1476

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$\downarrow$ 1082

$$4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{ 217}$$

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)\right)-$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{ 1479}$$

$$4\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\ldots}}{\sqrt[4]{2-\ldots}}\ldots\right.}{\sqrt{2}}\ldots\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$\downarrow$ 25

$$4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\ldots\right.}{\sqrt{2}}\ldots\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$\Big\downarrow$ 27

$$4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{a\ldots}}}{\sqrt[4]{2-\frac{1}{x^4}}}\ldots\right.}{\sqrt{2}\ldots}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$\Big\downarrow$ 1103

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\frac{\log\ldots}{\ldots}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

input
```
Int[E^((3*ArcCoth[a*x])/2)/x,x]
```

output
```
-4*(ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*
x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2) + 4*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^
(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x
))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(
a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.88

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = \sqrt{2}\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+\sqrt{2}\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right) + \frac{1}{2}\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right) - \frac{1}{2}\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right) + 2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="fricas")
```

output
```
sqrt(2)*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + sqrt(2)*arctan(s
qrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + 1/2*sqrt(2)*log(sqrt(2)*((a*x -
1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 1/2*sqrt(2)*log(-sq
rt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 2*arc
tan(((a*x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) -
log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)/x,x)
```

output
```
Integral(1/(x*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.01

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$= \frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\right(}{a}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="maxima")
```

output
```
1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4
) - 1)/a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$= \frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\log\left(\sqrt{2}\right(}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="giac")
```

output
```
1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
sqrt((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a
 + 2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x +
1))^(1/4) - 1))/a)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.46

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right) - \mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)2\mathrm{i} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)(1+1\mathrm{i}) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)(1-\mathrm{i})$$

input
```
int(1/(x*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

output
```
2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)
*2i + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 +
1i) + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 -
1i)
```

Reduce [F]

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{(ax+1)^{\frac{3}{4}}}{(ax-1)^{\frac{3}{4}}\,x}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x,x)
```

output
```
int((a*x + 1)**(3/4)/((a*x - 1)**(3/4)*x),x)
```

3.82 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 193

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{3a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

output
```
a*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+3/2*a*arctan(-1+2^(1/2)*(1-1/a/x)^(1/4)/
(1+1/a/x)^(1/4))*2^(1/2)+3/2*a*arctan(1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^
(1/4))*2^(1/2)+3/2*a*arctanh(2^(1/2)*(1-1/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(1
+1/a/x)^(1/2))/(1+1/a/x)^(1/4))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.24

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = -8ae^{\frac{3}{2}\coth^{-1}(ax)}\left(-\frac{1}{1+e^{2\coth^{-1}(ax)}} + \text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)/x^2,x]
```

output
```
-8*a*E^((3*ArcCoth[a*x])/2)*(-(1 + E^(2*ArcCoth[a*x]))^(-1) + Hypergeometr
ic2F1[3/4, 2, 7/4, -E^(2*ArcCoth[a*x])])
```

Rubi [A] (warning: unable to verify)

Time = 0.67 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6721, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 60}$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 73}$$

$$6a\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 770$$

$$6a\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 755$$

$$6a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 1476$$

$$6a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\cdots}}{\sqrt[4]{2-\cdots}}\cdots\right.\right.$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 1082$$

$$6a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 217$$

$$\frac{6a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)\right)+}{a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}$$

$$\Big\downarrow \; 1479$$

$$\frac{6a\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.}{\sqrt{2}}\right.\right.}{a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}$$

$$\Big\downarrow \; 25$$

$$\frac{6a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}} \cdots\right.}{\sqrt{2}} \cdots\right.\right.}{a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}$$

$$\Big\downarrow \; 27$$

$$\frac{6a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\cdots}}{\sqrt[4]{2-\cdots}} \cdots\right.}{\sqrt{2}} \cdots\right.\right.}{a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}$$

$$\Big\downarrow \; 1103$$

$$6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\right.\right.$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

input
```
Int[E^((3*ArcCoth[a*x])/2)/x^2,x]
```

output
```
a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4) + 6*a*((-(ArcTan[1 - (Sqrt[2]*(1
 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 -
 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(
1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2
]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x^2}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.91

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{6\sqrt{2}ax\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+6\sqrt{2}ax\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+3\sqrt{2}ax\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-}{ax-}}\right.}{4x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="fricas")
```

output
```
1/4*(6*sqrt(2)*a*x*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + 6*sqr
t(2)*a*x*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + 3*sqrt(2)*a*x*l
og(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) -
3*sqrt(2)*a*x*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) + 4*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4))/x
```

Sympy [F]

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**2,x)
```

output
```
Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.97

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+3\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="maxima")
```

output
```
1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
)) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) - 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x
 + 1) + 1))*a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.97

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$=\frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+3\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="giac")
```

output
```
1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
)) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) - 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x
 + 1) + 1))*a
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.46

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx=\frac{2\,a\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{\frac{a\,x-1}{a\,x+1}+1}$$
$$-{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,3{}\mathrm{i}-{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,3{}\mathrm{i}$$

input
```
int(1/(x^2*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

output
```
(2*a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1) - (-1)^(1/4)*a
*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i - (-1)^(1/4)*a*atan((-1)
^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i
```

Reduce [F]

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{(ax+1)^{\frac{3}{4}}}{(ax-1)^{\frac{3}{4}}\,x^2}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

output
```
int((a*x + 1)**(3/4)/((a*x - 1)**(3/4)*x**2),x)
```

3.83 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 244

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} - \frac{9a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{9a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{9a^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}$$

output
```
3/4*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/2*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^
(7/4)+9/8*a^2*arctan(-1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)+9
/8*a^2*arctan(1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)+9/8*a^2*a
rctanh(2^(1/2)*(1-1/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1+1/a/
x)^(1/4))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.31

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{8}{3}a^2e^{\frac{3}{2}\coth^{-1}(ax)}\left(\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)\right. \\ -3\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right) \\ \left.+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},3,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)/x^3,x]
```

output
```
(-8*a^2*E^((3*ArcCoth[a*x])/2)*(Hypergeometric2F1[3/4, 1, 7/4, -E^(2*ArcCo
th[a*x])] - 3*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCoth[a*x])] + 2*Hype
rgeometric2F1[3/4, 3, 7/4, -E^(2*ArcCoth[a*x])]))/3
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6721, 90, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}}{\left(1-\frac{1}{ax}\right)^{3/4}x}d\frac{1}{x}$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3}{4}a\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3}{4}a\left(\frac{3}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3}{4}a\left(-6a\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)$$

$$\Big\downarrow \text{770}$$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-\frac{3}{4}a\left(-6a\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-$$
$$\frac{3}{4}a\left(-6a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-$$
$$\frac{3}{4}a\left(-6a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}\right.\right.\right.$$

$\downarrow$ 1082

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-$$
$$\frac{3}{4}a\left(-6a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\right.$$

$\downarrow$ 217

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-$$
$$\frac{3}{4}a\left(-6a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)\right)-a\sqrt[4]{1\right.$$

$\downarrow$ 1479

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-$$

$$\frac{3}{4}a\left(-6a\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\cdots\right.}{\cdots}\right.\right.\right.$$

$\downarrow$ 25

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-$$

$$\frac{3}{4}a\left(-6a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1\cdots}}{\sqrt[4]{2-\cdots}}\right.}{\sqrt{2}\cdots}\right.\right.\right.$$

$\downarrow$ 27

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-$$
$$\frac{3}{4}a\left(-6a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{}\right.\right.\right.$$

$\downarrow$ 1103

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}-$$
$$\frac{3}{4}a\left(-6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}\right.\right.\right.$$

input
```
Int[E^((3*ArcCoth[a*x])/2)/x^3,x]
```

output
```
(a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(7/4))/2 - (3*a*(-(a*(1 - 1/(a*x))^
(1/4)*(1 + 1/(a*x))^(3/4)) - 6*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/
4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4)
)/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1
/4))/(2 - x^(-4))^(1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(a*x))
^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x^3} dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.82

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3} dx$$
$$= \frac{18\sqrt{2}a^2x^2 \arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+18\sqrt{2}a^2x^2 \arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+9\sqrt{2}a^2x^2 \log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{16\,x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="fricas")
```

output
```
1/16*(18*sqrt(2)*a^2*x^2*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) +
 18*sqrt(2)*a^2*x^2*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + 9*sq
rt(2)*a^2*x^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) - 9*sqrt(2)*a^2*x^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
+ sqrt((a*x - 1)/(a*x + 1)) + 1) + 4*(5*a^2*x^2 + 7*a*x + 2)*((a*x - 1)/(a
*x + 1))^(1/4))/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**3,x)
```

output
```
Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.94

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$$
$$= \frac{1}{16}\left(18\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="maxima")
```

output
```
1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1
))^(1/4))) + 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(
1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(3*a*((a*x - 1)/(a*x + 1))^(5/4)
 + 7*a*((a*x - 1)/(a*x + 1))^(1/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(
a*x + 1)^2 + 1))*a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.92

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$$
$$= \frac{1}{16}\left(18\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="giac")
```

output
```
1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1
))^(1/4))) + 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(
1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(3*(a*x - 1)*a*((a*x - 1)/(a*x +
 1))^(1/4)/(a*x + 1) + 7*a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x +
1) + 1)^2)*a
```

Mupad [B] (verification not implemented)

Time = 23.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.54

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{\frac{7\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{2}+\frac{3\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}}{2}}{\frac{{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{2\,\left(a\,x-1\right)}{a\,x+1}+1} - \frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,9{}\mathrm{i}}{4} - \frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,9{}\mathrm{i}}{4}$$

input
```
int(1/(x^3*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

output
```
((7*a^2*((a*x - 1)/(a*x + 1))^(1/4))/2 + (3*a^2*((a*x - 1)/(a*x + 1))^(5/4
))/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4
)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*9i)/4 - ((-1)^(1/4)*a^2
*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*9i)/4
```

Reduce [F]

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{(ax+1)^{\frac{3}{4}}}{(ax-1)^{\frac{3}{4}}\,x^3}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

output
```
int((a*x + 1)**(3/4)/((a*x - 1)**(3/4)*x**3),x)
```

3.84 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 281

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}+\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x}-\frac{17a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{17a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{17a^3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

output
```
17/24*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/4*a^3*(1-1/a/x)^(1/4)*(1+1/a/x
)^(7/4)+1/3*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(7/4)/x+17/16*a^3*arctan(-1+2^(1
/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)+17/16*a^3*arctan(1+2^(1/2)*(1
-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)+17/16*a^3*arctanh(2^(1/2)*(1-1/a/x)
^(1/4)/(1+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1+1/a/x)^(1/4))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.33

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{96}a^3\left(\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(17+30e^{2\coth^{-1}(ax)}+45e^{4\coth^{-1}(ax)}\right)}{\left(1+e^{2\coth^{-1}(ax)}\right)^3} + 51\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1}\&\right]\right)$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)/x^4,x]
```

output
```
(a^3*((8*E^((3*ArcCoth[a*x])/2)*(17 + 30*E^(2*ArcCoth[a*x]) + 45*E^(4*ArcC
oth[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 51*RootSum[1 + #1^4 & , (ArcCoth[
a*x] - 2*Log[E^(ArcCoth[a*x]/2) - #1])/#1 & ]))/96
```

Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {6721, 101, 27, 90, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}}{\left(1-\frac{1}{ax}\right)^{3/4}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 101}$$

$$\frac{1}{3}a^2\int -\frac{\left(2a+\frac{3}{x}\right)\left(1+\frac{1}{ax}\right)^{3/4}}{2a\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-\frac{1}{6}a\int \frac{\left(2a+\frac{3}{x}\right)\left(1+\frac{1}{ax}\right)^{3/4}}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{3}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(\frac{3}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)-\frac{3}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(-6a\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)-\frac{3}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}\right)$$

$$\Big\downarrow \text{ 770}$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$
$$\frac{1}{6}a\left(\frac{17}{4}a\left(-6a\int\frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)-\frac{3}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}\right)$$

$\downarrow$ 755

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$
$$\frac{1}{6}a\left(\frac{17}{4}a\left(-6a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)-\frac{3}{2}a^2\sqrt[4]{1-\cdots}\right.$$

$\downarrow$ 1476

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$
$$\frac{1}{6}a\left(\frac{17}{4}a\left(-6a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\cdots}\right.\right.\right.\right.$$

$\downarrow$ 1082

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$
$$\frac{1}{6}a\left(\frac{17}{4}a\left(-6a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\cdots}}{\sqrt[4]{2-\cdots}}\right.\right.\right.$$

$\downarrow$ 217

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{17}{4}a\left(-6a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)\right)\right)\right.$$

$$\downarrow \text{1479}$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{17}{4}a\left(-6a\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\text{arcta}\right.\right.\right.\right.$$

$$\downarrow \text{25}$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{17}{4}a\left(-6a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\cdots\right.}{\cdots}\right.\right.\right.\right.$$

$$\Big\downarrow \; 27$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{17}{4}a\left(-6a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\cdots}{\cdots}\right.\right.\right.\right.$$

$$\Big\downarrow \; 1103$$

$$\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-$$
$$\frac{1}{6}a\left(\frac{17}{4}a\left(-6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x}\right.}{2\sqrt{2}}\right.\right.\right.\right.$$

input
```
Int[E^((3*ArcCoth[a*x])/2)/x^4,x]
```

output
```
(a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(7/4))/(3*x) - (a*((-3*a^2*(1 - 1/(
a*x))^(1/4)*(1 + 1/(a*x))^(7/4))/2 + (17*a*(-(a*(1 - 1/(a*x))^(1/4)*(1 + 1
/(a*x))^(3/4)) - 6*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(
-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4
))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^
(-4))^(1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 -
 x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x^4} dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.74

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{102\sqrt{2}a^3x^3\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+102\sqrt{2}a^3x^3\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+51\sqrt{2}a^3x^3\log\left(\sqrt{2}\left(\frac{ax-}{ax+}\right.\right.}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="fricas")
```

output
```
1/96*(102*sqrt(2)*a^3*x^3*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1)
+ 102*sqrt(2)*a^3*x^3*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + 51
*sqrt(2)*a^3*x^3*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1) - 51*sqrt(2)*a^3*x^3*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1
/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 4*(23*a^3*x^3 + 37*a^2*x^2 + 22*a*x
 + 8)*((a*x - 1)/(a*x + 1))^(1/4))/x^3
```

Sympy [F]

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**4,x)
```

output
```
Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.99

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{1}{96}\left(102\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+102\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)\right.\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="maxima")
```

output
```
1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a
*x + 1))^(1/4))) + 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
+ sqrt((a*x - 1)/(a*x + 1)) + 1) - 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/
(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(17*a^2*((a*x - 1)/(
a*x + 1))^(9/4) + 30*a^2*((a*x - 1)/(a*x + 1))^(5/4) + 45*a^2*((a*x - 1)/(
a*x + 1))^(1/4))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x
 - 1)^3/(a*x + 1)^3 + 1))*a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.96

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{1}{96}\left(102\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+102\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)\right.\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="giac")
```

output
```
1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a
*x + 1))^(1/4))) + 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
+ sqrt((a*x - 1)/(a*x + 1)) + 1) - 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/
(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(30*(a*x - 1)*a^2*((
a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) + 17*(a*x - 1)^2*a^2*((a*x - 1)/(a*x +
 1))^(1/4)/(a*x + 1)^2 + 45*a^2*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a
*x + 1) + 1)^3)*a
```

Mupad [B] (verification not implemented)

Time = 23.38 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.60

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{\frac{15\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}+\frac{5\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{17\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{\frac{3\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{(a\,x-1)^3}{(a\,x+1)^3}+\frac{3\,(a\,x-1)}{a\,x+1}+1} - \frac{(-1)^{1/4}\,a^3\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,17\mathrm{i}}{8} - \frac{(-1)^{1/4}\,a^3\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,17\mathrm{i}}{8}$$

input
```
int(1/(x^4*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

output
```
((15*a^3*((a*x - 1)/(a*x + 1))^(1/4))/4 + (5*a^3*((a*x - 1)/(a*x + 1))^(5/
4))/2 + (17*a^3*((a*x - 1)/(a*x + 1))^(9/4))/12)/((3*(a*x - 1)^2)/(a*x + 1
)^2 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)
*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*17i)/8 - ((-1)^(1/4)*a^3
*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*17i)/8
```

Reduce [F]

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{(ax+1)^{\frac{3}{4}}}{(ax-1)^{\frac{3}{4}}\,x^4}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

output
```
int((a*x + 1)**(3/4)/((a*x - 1)**(3/4)*x**4),x)
```

3.85 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 285

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx=\frac{26111\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{1920a^4}+\frac{10289\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{960a^3}+\frac{455\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{48a^2}+\frac{349\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4}{40a}-\frac{8\sqrt[4]{1+\frac{1}{ax}}x^5}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{41}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^5+\frac{1003\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{1003\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

output
```
26111/1920*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^4+10289/960*(1-1/a/x)^(3/4)
*(1+1/a/x)^(1/4)*x^2/a^3+455/48*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a^2+34
9/40*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^4/a-8*(1+1/a/x)^(1/4)*x^5/(1-1/a/x)
^(1/4)+41/5*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^5+1003/128*arctan((1+1/a/x)^
(1/4)/(1-1/a/x)^(1/4))/a^5+1003/128*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4
))/a^5
```

Mathematica [A] (verified)

Time = 5.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.69

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = \frac{-8e^{\frac{1}{2}\coth^{-1}(ax)}+\frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{5\left(-1+e^{2\coth^{-1}(ax)}\right)^5}+\frac{122e^{\frac{1}{2}\coth^{-1}(ax)}}{5\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{233e^{\frac{1}{2}\coth^{-1}(ax)}}{6\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{1661e^{\frac{1}{2}\coth^{-1}(ax)}}{48\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{411}{192(-}}{a^5}$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)*x^4,x]
```

output
```
(-8*E^(ArcCoth[a*x]/2) + (32*E^(ArcCoth[a*x]/2))/(5*(-1 + E^(2*ArcCoth[a*x
]))^5) + (122*E^(ArcCoth[a*x]/2))/(5*(-1 + E^(2*ArcCoth[a*x]))^4) + (233*E
^(ArcCoth[a*x]/2))/(6*(-1 + E^(2*ArcCoth[a*x]))^3) + (1661*E^(ArcCoth[a*x]
/2))/(48*(-1 + E^(2*ArcCoth[a*x]))^2) + (4117*E^(ArcCoth[a*x]/2))/(192*(-1
 + E^(2*ArcCoth[a*x]))) + (1003*ArcTan[E^(ArcCoth[a*x]/2)])/128 - (1003*Lo
g[1 - E^(ArcCoth[a*x]/2)])/256 + (1003*Log[1 + E^(ArcCoth[a*x]/2)])/256)/a
^5
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.214$, Rules used = {6721, 109, 27, 168, 27, 168, 27, 168, 27, 168, 27, 172, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{\frac{5}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \ 6721$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}x^6}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \ 109$$

$$\frac{1}{5}\int -\frac{\left(21a+\frac{20}{x}\right)x^5}{2a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 27$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\int \frac{\left(21a+\frac{20}{x}\right)x^5}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{10a^2}$$

$$\Big\downarrow \ 168$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{-\frac{1}{4}\int -\frac{\left(181a+\frac{168}{x}\right)x^4}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\int \frac{\left(181a+\frac{168}{x}\right)x^4}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow \ 168$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{-\frac{1}{3}\int -\frac{\left(1189a+\frac{1086}{x}\right)x^3}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\int \frac{\left(1189a+\frac{1086}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow \; 168$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{-\frac{1}{2}\int -\frac{\left(5533a+\frac{4756}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{\int \frac{\left(5533a+\frac{4756}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow \; 168$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{-\int -\frac{\left(15045a+\frac{11066}{x}\right)x}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{5533ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$\downarrow$ 27

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{\frac{\int \frac{\left(15045a+\frac{11066}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}-\frac{5533ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$\downarrow$ 172

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{\frac{\frac{52222a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-2a\int -\frac{15045x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}-\frac{5533ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$\big\downarrow$ 27

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{\frac{15045a\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{52222a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{5533ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$\big\downarrow$ 104

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{\frac{60180a\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{52222a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{5533ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow \text{ 756}$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{\frac{60180a\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{52222a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{5533ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{21ax^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}}{10a^2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{60180a\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{2a}+\frac{52222a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{5533ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{181ax^3\sqrt[4]{\frac{1}{ax}}}{3\sqrt[4]{1-}}}{10a^2}$$

$\Big\downarrow$ 219

$$\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{60180a\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{2a}+\frac{52222a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{5533ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{1189ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{8a}-\frac{181ax^3\sqrt[4]{}}{3\sqrt[4]{1}}}{10a^2}$$

input
```
Int[E^((5*ArcCoth[a*x])/2)*x^4,x]
```

output
```
((1 + 1/(a*x))^(1/4)*x^5)/(5*(1 - 1/(a*x))^(1/4)) - ((-21*a*(1 + 1/(a*x))^
(1/4)*x^4)/(4*(1 - 1/(a*x))^(1/4)) + ((-181*a*(1 + 1/(a*x))^(1/4)*x^3)/(3*
(1 - 1/(a*x))^(1/4)) + ((-1189*a*(1 + 1/(a*x))^(1/4)*x^2)/(2*(1 - 1/(a*x))
^(1/4)) + ((-5533*a*(1 + 1/(a*x))^(1/4)*x)/(1 - 1/(a*x))^(1/4) + ((52222*a
*(1 + 1/(a*x))^(1/4))/(1 - 1/(a*x))^(1/4) + 60180*a*(-1/2*ArcTan[(1 + 1/(a
*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x)
)^(1/4)]/2))/(2*a))/(4*a))/(6*a))/(8*a))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.53

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = -\frac{30090\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 15045\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) + 15045\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{3840\,(a^6x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="fricas")
```

output
```
-1/3840*(30090*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 15045*(a*x
- 1)*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 15045*(a*x - 1)*log(((a*x - 1)
/(a*x + 1))^(1/4) - 1) - 2*(384*a^6*x^6 + 1392*a^5*x^5 + 2456*a^4*x^4 + 38
26*a^3*x^3 + 7911*a^2*x^2 - 20578*a*x - 26111)*((a*x - 1)/(a*x + 1))^(3/4)
)/(a^6*x - a^5)
```

Sympy [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = \int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**4,x)
```

output
```
Integral(x**4/((a*x - 1)/(a*x + 1))**(5/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.96

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{3840}\,a\left(\frac{4\left(\frac{58985\,(ax-1)}{ax+1}-\frac{125920\,(ax-1)^2}{(ax+1)^2}+\frac{137930\,(ax-1)^3}{(ax+1)^3}-\frac{72216\,(ax-1)^4}{(ax+1)^4}+\frac{15045\,(ax-1)^5}{(ax+1)^5}-7680\right)}{a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{21}{4}}-5\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}+10\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-10\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+5\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="maxima")
```

output
```
-1/3840*a*(4*(58985*(a*x - 1)/(a*x + 1) - 125920*(a*x - 1)^2/(a*x + 1)^2 +
 137930*(a*x - 1)^3/(a*x + 1)^3 - 72216*(a*x - 1)^4/(a*x + 1)^4 + 15045*(a
*x - 1)^5/(a*x + 1)^5 - 7680)/(a^6*((a*x - 1)/(a*x + 1))^(21/4) - 5*a^6*((
a*x - 1)/(a*x + 1))^(17/4) + 10*a^6*((a*x - 1)/(a*x + 1))^(13/4) - 10*a^6*
((a*x - 1)/(a*x + 1))^(9/4) + 5*a^6*((a*x - 1)/(a*x + 1))^(5/4) - a^6*((a*
x - 1)/(a*x + 1))^(1/4)) + 30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 -
 15045*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 + 15045*log(((a*x - 1)/(a*
x + 1))^(1/4) - 1)/a^6)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.89

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{3840}\,a\left(\frac{30090\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{15045\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{15045\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}+\frac{307}{a^6\left(\frac{ax}{ax}\right.}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="giac")
```

output
```
-1/3840*a*(30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 15045*log(((a*x
 - 1)/(a*x + 1))^(1/4) + 1)/a^6 + 15045*log(abs(((a*x - 1)/(a*x + 1))^(1/4
) - 1))/a^6 + 30720/(a^6*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(49120*(a*x - 1)
*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 61130*(a*x - 1)^2*((a*x - 1)/(a*x
 + 1))^(3/4)/(a*x + 1)^2 + 33816*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(3/4)/(
a*x + 1)^3 - 7365*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^4 - 20
585*((a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a*x + 1) - 1)^5))
```

Mupad [B] (verification not implemented)

Time = 23.57 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.87

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = \frac{1003\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}-\frac{1003\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5} - \frac{\frac{787\,{\left(a\,x-1\right)}^2}{6\,{\left(a\,x+1\right)}^2}-\frac{13793\,{\left(a\,x-1\right)}^3}{96\,{\left(a\,x+1\right)}^3}+\frac{3009\,{\left(a\,x-1\right)}^4}{40\,{\left(a\,x+1\right)}^4}-\frac{1003\,{\left(a\,x-1\right)}^5}{64\,{\left(a\,x+1\right)}^5}-\frac{11797\,\left(a\,x-1\right)}{192\,\left(a\,x+1\right)}+8}{a^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}-5\,a^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}+10\,a^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}-10\,a^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/4}+5\,a^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{17/4}-a^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{21/4}}$$

input
```
int(x^4/((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
(1003*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) - (1003*atan(((a*x - 1
)/(a*x + 1))^(1/4)))/(128*a^5) - ((787*(a*x - 1)^2)/(6*(a*x + 1)^2) - (137
93*(a*x - 1)^3)/(96*(a*x + 1)^3) + (3009*(a*x - 1)^4)/(40*(a*x + 1)^4) - (
1003*(a*x - 1)^5)/(64*(a*x + 1)^5) - (11797*(a*x - 1))/(192*(a*x + 1)) + 8
)/(a^5*((a*x - 1)/(a*x + 1))^(1/4) - 5*a^5*((a*x - 1)/(a*x + 1))^(5/4) + 1
0*a^5*((a*x - 1)/(a*x + 1))^(9/4) - 10*a^5*((a*x - 1)/(a*x + 1))^(13/4) +
5*a^5*((a*x - 1)/(a*x + 1))^(17/4) - a^5*((a*x - 1)/(a*x + 1))^(21/4))
```

Reduce [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = \left(\int \frac{(ax+1)^{\frac{1}{4}}\,x^5}{(ax-1)^{\frac{1}{4}}\,ax-(ax-1)^{\frac{1}{4}}}dx\right)a + \int \frac{(ax+1)^{\frac{1}{4}}\,x^4}{(ax-1)^{\frac{1}{4}}\,ax-(ax-1)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x)
```

output
```
int(((a*x + 1)**(1/4)*x***5)/((a*x - 1)**(1/4)*a*x - (a*x - 1)**(1/4)),x)*a
 + int(((a*x + 1)**(1/4)*x***4)/((a*x - 1)**(1/4)*a*x - (a*x - 1)**(1/4)),x
)
```

3.86 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 248

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx=\frac{2467\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3}+\frac{973\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2}+\frac{215\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a}-\frac{8\sqrt[4]{1+\frac{1}{ax}}\,x^4}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{33}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4+\frac{475\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{475\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

output
```
2467/192*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^3+973/96*(1-1/a/x)^(3/4)*(1+1
/a/x)^(1/4)*x^2/a^2+215/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a-8*(1+1/a/
x)^(1/4)*x^4/(1-1/a/x)^(1/4)+33/4*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^4+475/
64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+475/64*arctanh((1+1/a/x)^(1
/4)/(1-1/a/x)^(1/4))/a^4
```

Mathematica [A] (verified)

Time = 5.25 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.65

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \frac{-3072e^{\frac{1}{2}\coth^{-1}(ax)}+\frac{1536e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{5248e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{7376e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{6292e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+2850\,\mathrm{arc}}{384a^4}$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)*x^3,x]
```

output
```
(-3072*E^(ArcCoth[a*x]/2) + (1536*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a
*x]))^4 + (5248*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (7376*E^
(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (6292*E^(ArcCoth[a*x]/2))/
(-1 + E^(2*ArcCoth[a*x])) + 2850*ArcTan[E^(ArcCoth[a*x]/2)] - 1425*Log[1 -
 E^(ArcCoth[a*x]/2)] + 1425*Log[1 + E^(ArcCoth[a*x]/2)])/(384*a^4)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {6721, 109, 27, 168, 27, 168, 27, 168, 27, 172, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{\frac{5}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6721$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}x^5}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \; 109$$

$$\frac{1}{4}\int -\frac{\left(17a+\frac{16}{x}\right)x^4}{2a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\int \frac{\left(17a+\frac{16}{x}\right)x^4}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{8a^2}$$

$$\Big\downarrow \text{ 168}$$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{-\frac{1}{3}\int -\frac{\left(113a+\frac{102}{x}\right)x^3}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\int \frac{\left(113a+\frac{102}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$$\Big\downarrow \text{ 168}$$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{-\frac{1}{2}\int -\frac{\left(521a+\frac{452}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\int \frac{\left(521a+\frac{452}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$\Big\downarrow$ 168

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{-\int -\frac{\left(1425a+\frac{1042}{x}\right)x}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{521ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$\Big\downarrow$ 27

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{\int \frac{\left(1425a+\frac{1042}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}-\frac{521ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$\Big\downarrow$ 172

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{\frac{4934a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-2a\int -\frac{1425x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}-\frac{521ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$\Big\downarrow$ 27

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{1425a\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{4934a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{521ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$\Big\downarrow$ 104

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{5700a\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{4934a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{521ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$\downarrow$ 756

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{5700a\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4934a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{521ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$\downarrow$ 216

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{5700a\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4934a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{521ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

$$\Big\downarrow \; 219$$

$$\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{5700a\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4934a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{521ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{113ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a}-\frac{17ax^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}}{8a^2}$$

input
```
Int[E^((5*ArcCoth[a*x])/2)*x^3,x]
```

output
```
((1 + 1/(a*x))^(1/4)*x^4)/(4*(1 - 1/(a*x))^(1/4)) - ((-17*a*(1 + 1/(a*x))^
(1/4)*x^3)/(3*(1 - 1/(a*x))^(1/4)) + ((-113*a*(1 + 1/(a*x))^(1/4)*x^2)/(2*
(1 - 1/(a*x))^(1/4)) + ((-521*a*(1 + 1/(a*x))^(1/4)*x)/(1 - 1/(a*x))^(1/4)
 + ((4934*a*(1 + 1/(a*x))^(1/4))/(1 - 1/(a*x))^(1/4) + 5700*a*(-1/2*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1
 - 1/(a*x))^(1/4)]/2))/(2*a))/(4*a))/(6*a))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.58

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx =$$

$$-\frac{2850\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-1425\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+1425\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1}{384\,(a^5x-a^4)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="fricas")
```

output
```
-1/384*(2850*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 1425*(a*x - 1
)*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1425*(a*x - 1)*log(((a*x - 1)/(a*
x + 1))^(1/4) - 1) - 2*(48*a^5*x^5 + 184*a^4*x^4 + 362*a^3*x^3 + 747*a^2*x
^2 - 1946*a*x - 2467)*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*x - a^4)
```

Sympy [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)*x***3,x)
```

output
```
Integral(x**3/((a*x - 1)/(a*x + 1))**(5/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.96

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \frac{1}{384}a\left(\frac{4\left(\frac{4645\,(ax-1)}{ax+1}-\frac{7483\,(ax-1)^2}{(ax+1)^2}+\frac{5415\,(ax-1)^3}{(ax+1)^3}-\frac{1425\,(ax-1)^4}{(ax+1)^4}-768\right)}{a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-4\,a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+6\,a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-4\,a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{2850\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="maxima")
```

output
```
1/384*a*(4*(4645*(a*x - 1)/(a*x + 1) - 7483*(a*x - 1)^2/(a*x + 1)^2 + 5415
*(a*x - 1)^3/(a*x + 1)^3 - 1425*(a*x - 1)^4/(a*x + 1)^4 - 768)/(a^5*((a*x
- 1)/(a*x + 1))^(17/4) - 4*a^5*((a*x - 1)/(a*x + 1))^(13/4) + 6*a^5*((a*x
- 1)/(a*x + 1))^(9/4) - 4*a^5*((a*x - 1)/(a*x + 1))^(5/4) + a^5*((a*x - 1)
/(a*x + 1))^(1/4)) - 2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 1425*l
og(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 - 1425*log(((a*x - 1)/(a*x + 1))^(
1/4) - 1)/a^5)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.90

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{1}{384}a\left(\frac{2850\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{1425\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{1425\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}+\frac{3072}{a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="giac")
```

output
```
-1/384*a*(2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 1425*log(((a*x -
1)/(a*x + 1))^(1/4) + 1)/a^5 + 1425*log(abs(((a*x - 1)/(a*x + 1))^(1/4) -
1))/a^5 + 3072/(a^5*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(2875*(a*x - 1)*((a*x
 - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 2343*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^
(3/4)/(a*x + 1)^2 + 657*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^
3 - 1573*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x - 1)/(a*x + 1) - 1)^4))
```

Mupad [B] (verification not implemented)

Time = 23.84 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.85

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \frac{475\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{475\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{\frac{7483\,(a\,x-1)^2}{96\,(a\,x+1)^2}-\frac{1805\,(a\,x-1)^3}{32\,(a\,x+1)^3}+\frac{475\,(a\,x-1)^4}{32\,(a\,x+1)^4}-\frac{4645\,(a\,x-1)}{96\,(a\,x+1)}+8}{a^4\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}-4\,a^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}+6\,a^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}-4\,a^4\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}+a^4\left(\frac{a\,x-1}{a\,x+1}\right)^{17/4}}$$

input
```
int(x^3/((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
(475*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - (475*atan(((a*x - 1)/(
a*x + 1))^(1/4)))/(64*a^4) - ((7483*(a*x - 1)^2)/(96*(a*x + 1)^2) - (1805*
(a*x - 1)^3)/(32*(a*x + 1)^3) + (475*(a*x - 1)^4)/(32*(a*x + 1)^4) - (4645
*(a*x - 1))/(96*(a*x + 1)) + 8)/(a^4*((a*x - 1)/(a*x + 1))^(1/4) - 4*a^4*(
(a*x - 1)/(a*x + 1))^(5/4) + 6*a^4*((a*x - 1)/(a*x + 1))^(9/4) - 4*a^4*((a
*x - 1)/(a*x + 1))^(13/4) + a^4*((a*x - 1)/(a*x + 1))^(17/4))
```

Reduce [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \left(\int \frac{(ax+1)^{\frac{1}{4}}x^4}{(ax-1)^{\frac{1}{4}}ax-(ax-1)^{\frac{1}{4}}}dx\right)a + \int \frac{(ax+1)^{\frac{1}{4}}x^3}{(ax-1)^{\frac{1}{4}}ax-(ax-1)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x)
```

output
```
int(((a*x + 1)**(1/4)*x**4)/((a*x - 1)**(1/4)*a*x - (a*x - 1)**(1/4)),x)*a
 + int(((a*x + 1)**(1/4)*x**3)/((a*x - 1)**(1/4)*a*x - (a*x - 1)**(1/4)),x
)
```

3.87 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 211

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx=\frac{287\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2}+\frac{113\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a}-\frac{8\sqrt[4]{1+\frac{1}{ax}}\,x^3}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{25}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3+\frac{55\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{55\,\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

output
```
287/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^2+113/12*(1-1/a/x)^(3/4)*(1+1/a
/x)^(1/4)*x^2/a-8*(1+1/a/x)^(1/4)*x^3/(1-1/a/x)^(1/4)+25/3*(1-1/a/x)^(3/4)
*(1+1/a/x)^(1/4)*x^3+55/8*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3+55/8
*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Mathematica [A] (verified)

Time = 5.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.65

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \frac{-384e^{\frac{1}{2}\coth^{-1}(ax)}+\frac{128e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{400e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{548e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+330\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-1\ldots}{48a^3}$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)*x^2,x]
```

output
```
(-384*E^(ArcCoth[a*x]/2) + (128*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x
]))^3 + (400*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (548*E^(Arc
Coth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + 330*ArcTan[E^(ArcCoth[a*x]/2)] -
 165*Log[1 - E^(ArcCoth[a*x]/2)] + 165*Log[1 + E^(ArcCoth[a*x]/2)])/(48*a^
3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6721, 109, 27, 168, 27, 168, 27, 172, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{\frac{5}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}x^4}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{3}\int -\frac{\left(13a+\frac{12}{x}\right)x^3}{2a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 27$$

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\int \frac{\left(13a+\frac{12}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{6a^2}$$

$$\Big\downarrow \ 168$$

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{-\frac{1}{2}\int -\frac{\left(61a+\frac{52}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\int \frac{\left(61a+\frac{52}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$$\Big\downarrow \ 168$$

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{-\int -\frac{\left(165a+\frac{122}{x}\right)x}{2a\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{61ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\int \frac{\left(165a+\frac{122}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}-\frac{61ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$\Big\downarrow$ 172

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{\frac{574a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-2a\int -\frac{165x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}-\frac{61ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$\Big\downarrow$ 27

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{165a\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{574a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{61ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$\Big\downarrow$ 104

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{660a\int\frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{574a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{61ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$\Big\downarrow$ 756

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{660a\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{574a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{61ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$\Big\downarrow$ 216

$$\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{660a\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{574a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{61ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}-\frac{\frac{\frac{660a\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{574a\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{61ax\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}}{4a}-\frac{13ax^2\sqrt[4]{\frac{1}{ax}+1}}{2\sqrt[4]{1-\frac{1}{ax}}}}{6a^2}}{}$$

input
```
Int[E^((5*ArcCoth[a*x])/2)*x^2,x]
```

output
```
((1 + 1/(a*x))^(1/4)*x^3)/(3*(1 - 1/(a*x))^(1/4)) - ((-13*a*(1 + 1/(a*x))^
(1/4)*x^2)/(2*(1 - 1/(a*x))^(1/4)) + ((-61*a*(1 + 1/(a*x))^(1/4)*x)/(1 - 1
/(a*x))^(1/4) + ((574*a*(1 + 1/(a*x))^(1/4))/(1 - 1/(a*x))^(1/4) + 660*a*(
-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x
))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/(2*a))/(4*a))/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
])))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.64

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{330\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-165\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+165\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-}{48\,(a^4x-a^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="fricas")
```

output
```
-1/48*(330*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 165*(a*x - 1)*l
og(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 165*(a*x - 1)*log(((a*x - 1)/(a*x +
1))^(1/4) - 1) - 2*(8*a^4*x^4 + 34*a^3*x^3 + 87*a^2*x^2 - 226*a*x - 287)*(
(a*x - 1)/(a*x + 1))^(3/4))/(a^4*x - a^3)
```

Sympy [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)*x***2,x)
```

output
```
Integral(x**2/((a*x - 1)/(a*x + 1))**(5/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.96

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = -\frac{1}{48}\,a\left(\frac{4\left(\frac{425\,(ax-1)}{ax+1}-\frac{462\,(ax-1)^2}{(ax+1)^2}+\frac{165\,(ax-1)^3}{(ax+1)^3}-96\right)}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{330\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{165\,\log\left(\left(\right.\right.}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="maxima")
```

output
```
-1/48*a*(4*(425*(a*x - 1)/(a*x + 1) - 462*(a*x - 1)^2/(a*x + 1)^2 + 165*(a
*x - 1)^3/(a*x + 1)^3 - 96)/(a^4*((a*x - 1)/(a*x + 1))^(13/4) - 3*a^4*((a*
x - 1)/(a*x + 1))^(9/4) + 3*a^4*((a*x - 1)/(a*x + 1))^(5/4) - a^4*((a*x -
1)/(a*x + 1))^(1/4)) + 330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 165*l
og(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 165*log(((a*x - 1)/(a*x + 1))^(1
/4) - 1)/a^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.91

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{48}a\left(\frac{330\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{165\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{165\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}+\frac{384}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="giac")
```

output
```
-1/48*a*(330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 165*log(((a*x - 1)/
(a*x + 1))^(1/4) + 1)/a^4 + 165*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/
a^4 + 384/(a^4*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(174*(a*x - 1)*((a*x - 1)/
(a*x + 1))^(3/4)/(a*x + 1) - 69*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a
*x + 1)^2 - 137*((a*x - 1)/(a*x + 1))^(3/4))/(a^4*((a*x - 1)/(a*x + 1) - 1
)^3))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.83

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \frac{55\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}-\frac{55\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}-\frac{\frac{77\,{\left(a\,x-1\right)}^2}{2\,{\left(a\,x+1\right)}^2}-\frac{55\,{\left(a\,x-1\right)}^3}{4\,{\left(a\,x+1\right)}^3}-\frac{425\,\left(a\,x-1\right)}{12\,\left(a\,x+1\right)}+8}{a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}-3\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}+3\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}-a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/4}}$$

input
```
int(x^2/((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
(55*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) - (55*atan(((a*x - 1)/(a*x
 + 1))^(1/4)))/(8*a^3) - ((77*(a*x - 1)^2)/(2*(a*x + 1)^2) - (55*(a*x - 1)
^3)/(4*(a*x + 1)^3) - (425*(a*x - 1))/(12*(a*x + 1)) + 8)/(a^3*((a*x - 1)/
(a*x + 1))^(1/4) - 3*a^3*((a*x - 1)/(a*x + 1))^(5/4) + 3*a^3*((a*x - 1)/(a
*x + 1))^(9/4) - a^3*((a*x - 1)/(a*x + 1))^(13/4))
```

Reduce [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \left(\int \frac{(ax+1)^{\frac{1}{4}}x^3}{(ax-1)^{\frac{1}{4}}ax-(ax-1)^{\frac{1}{4}}}dx\right)a + \int \frac{(ax+1)^{\frac{1}{4}}x^2}{(ax-1)^{\frac{1}{4}}ax-(ax-1)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x)
```

output
```
int(((a*x + 1)**(1/4)*x**3)/((a*x - 1)**(1/4)*a*x - (a*x - 1)**(1/4)),x)*a
 + int(((a*x + 1)**(1/4)*x**2)/((a*x - 1)**(1/4)*a*x - (a*x - 1)**(1/4)),x
)
```

3.88 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 176

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx = -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}}+\frac{25\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

output

```
-25/2*(1+1/a/x)^(1/4)/a^2/(1-1/a/x)^(1/4)+5/4*(1+1/a/x)^(5/4)*x/a/(1-1/a/x
)^(1/4)+1/2*(1+1/a/x)^(9/4)*x^2/(1-1/a/x)^(1/4)+25/4*arctan((1+1/a/x)^(1/4
)/(1-1/a/x)^(1/4))/a^2+25/4*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.45

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx = \frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(25-45e^{2\coth^{-1}(ax)}+16e^{4\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+25\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+25\text{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)*x,x]
```

output
```
((-2*E^(ArcCoth[a*x]/2)*(25 - 45*E^(2*ArcCoth[a*x]) + 16*E^(4*ArcCoth[a*x]
)))/(-1 + E^(2*ArcCoth[a*x]))^2 + 25*ArcTan[E^(ArcCoth[a*x]/2)] + 25*ArcTa
nh[E^(ArcCoth[a*x]/2)])/(4*a^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6721, 107, 105, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{5}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}x^3}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{107}$$

$$\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}x^2}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{4a}$$

$$\Big\downarrow \text{105}$$

$$\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(\frac{5\int\frac{\sqrt[4]{1+\frac{1}{ax}}x}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{2a}-\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a}$$

$$\Big\downarrow \text{105}$$

$$\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(\frac{5\left(\int\frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}-\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a}$$

$$\Big\downarrow \text{104}$$

$$\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(\frac{5\left(4\int\frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}-\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a}$$

$$\Big\downarrow \text{756}$$

$$\frac{\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}-5\left(\frac{5\left(4\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}-\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a}$$

$\Big\downarrow$ 216

$$\frac{\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}-5\left(\frac{5\left(4\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}-\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a}$$

$\Big\downarrow$ 219

$$\frac{\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}-5\left(\frac{5\left(4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}-\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a}$$

input
```
Int[E^((5*ArcCoth[a*x])/2)*x,x]
```

output
```
((1 + 1/(a*x))^(9/4)*x^2)/(2*(1 - 1/(a*x))^(1/4)) - (5*(-(((1 + 1/(a*x))^(
5/4)*x)/(1 - 1/(a*x))^(1/4)) + (5*((4*(1 + 1/(a*x))^(1/4))/(1 - 1/(a*x))^(
1/4) + 4*(-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(
1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2)))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.73

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx = -\frac{50\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 25\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) + 25\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right) - 2\,(}{8\,(a^3x-a^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="fricas")
```

output
```
-1/8*(50*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 25*(a*x - 1)*log(
((a*x - 1)/(a*x + 1))^(1/4) + 1) + 25*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^
(1/4) - 1) - 2*(2*a^3*x^3 + 11*a^2*x^2 - 34*a*x - 43)*((a*x - 1)/(a*x + 1)
)^(3/4))/(a^3*x - a^2)
```

Sympy [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx = \int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)*x,x)
```

output
```
Integral(x/((a*x - 1)/(a*x + 1))**(5/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.94

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx$$
$$= \frac{1}{8}a\left(\frac{4\left(\frac{45(ax-1)}{ax+1} - \frac{25(ax-1)^2}{(ax+1)^2} - 16\right)}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}} - 2a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} + a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}} - \frac{50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3} + \frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3} - \frac{25\log}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="maxima")
```

output
```
1/8*a*(4*(45*(a*x - 1)/(a*x + 1) - 25*(a*x - 1)^2/(a*x + 1)^2 - 16)/(a^3*(
(a*x - 1)/(a*x + 1))^(9/4) - 2*a^3*((a*x - 1)/(a*x + 1))^(5/4) + a^3*((a*x
 - 1)/(a*x + 1))^(1/4)) - 50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 25*
log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 - 25*log(((a*x - 1)/(a*x + 1))^(1
/4) - 1)/a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.91

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx =$$

$$-\frac{1}{8}a\left(\frac{50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{25\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}+\frac{64}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{4\left(\frac{9(\cdots}{}\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="giac")
```

output
```
-1/8*a*(50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 25*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^3 + 25*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
+ 64/(a^3*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(9*(a*x - 1)*((a*x - 1)/(a*x +
1))^(3/4)/(a*x + 1) - 13*((a*x - 1)/(a*x + 1))^(3/4))/(a^3*((a*x - 1)/(a*x
 + 1) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.79

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx = \frac{25\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{25\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{\frac{25\,{\left(a\,x-1\right)}^2}{2\,{\left(a\,x+1\right)}^2}-\frac{45\,\left(a\,x-1\right)}{2\,\left(a\,x+1\right)}+8}{a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}-2\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}+a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}}$$

input
```
int(x/((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
(25*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) - (25*atan(((a*x - 1)/(a*x
 + 1))^(1/4)))/(4*a^2) - ((25*(a*x - 1)^2)/(2*(a*x + 1)^2) - (45*(a*x - 1)
)/(2*(a*x + 1)) + 8)/(a^2*((a*x - 1)/(a*x + 1))^(1/4) - 2*a^2*((a*x - 1)/(
a*x + 1))^(5/4) + a^2*((a*x - 1)/(a*x + 1))^(9/4))
```

Reduce [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx$$
$$= \left(\int \frac{(ax+1)^{\frac{1}{4}}x^{2}}{(ax-1)^{\frac{1}{4}}ax-(ax-1)^{\frac{1}{4}}}dx\right)a+\int \frac{(ax+1)^{\frac{1}{4}}x}{(ax-1)^{\frac{1}{4}}ax-(ax-1)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)*x,x)
```

output
```
int(((a*x + 1)**(1/4)*x**2)/((a*x - 1)**(1/4)*a*x - (a*x - 1)**(1/4)),x)*a
 + int(((a*x + 1)**(1/4)*x)/((a*x - 1)**(1/4)*a*x - (a*x - 1)**(1/4)),x)
```

3.89 $\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 130

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{5\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

output
```
-10*(1+1/a/x)^(1/4)/a/(1-1/a/x)^(1/4)+(1+1/a/x)^(5/4)*x/(1-1/a/x)^(1/4)+5*
arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a+5*arctanh((1+1/a/x)^(1/4)/(1-1/a
/x)^(1/4))/a
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.52

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = \frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(-5+4e^{2\coth^{-1}(ax)}\right)}{-1+e^{2\coth^{-1}(ax)}}+5\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+5\text{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2),x]
```

output
```
((-2*E^(ArcCoth[a*x]/2)*(-5 + 4*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*
x])) + 5*ArcTan[E^(ArcCoth[a*x]/2)] + 5*ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6720, 105, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6720}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}x^2}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 105}$$

$$\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\int \frac{\sqrt[4]{1+\frac{1}{ax}}x}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{105}$$

$$\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}$$

$$\Big\downarrow \text{104}$$

$$\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(4\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}$$

$$\Big\downarrow \text{756}$$

$$\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(4\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}$$

$$\Big\downarrow \text{216}$$

$$\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(4\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\left(4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}$$

input
```
Int[E^((5*ArcCoth[a*x])/2),x]
```

output
```
((1 + 1/(a*x))^(5/4)*x)/(1 - 1/(a*x))^(1/4) - (5*((4*(1 + 1/(a*x))^(1/4))/
(1 - 1/(a*x))^(1/4) + 4*(-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/
4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2)))/(2*a)
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] && !IntegerQ[n]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}} dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int e^{\frac{5}{2}\coth^{-1}(ax)} dx = -\frac{10\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 5\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) + 5\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right) - 2\,(a^2}{2\,(a^2x-a)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

output
```
-1/2*(10*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 5*(a*x - 1)*log((
(a*x - 1)/(a*x + 1))^(1/4) + 1) + 5*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1
/4) - 1) - 2*(a^2*x^2 - 8*a*x - 9)*((a*x - 1)/(a*x + 1))^(3/4))/(a^2*x - a
)
```

Sympy [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = \int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(-5/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.01

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}a\left(\frac{4\left(\frac{5(ax-1)}{ax+1}-4\right)}{a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

output
```
-1/2*a*(4*(5*(a*x - 1)/(a*x + 1) - 4)/(a^2*((a*x - 1)/(a*x + 1))^(5/4) - a
^2*((a*x - 1)/(a*x + 1))^(1/4)) + 10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a
^2 - 5*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^2 + 5*log(((a*x - 1)/(a*x +
1))^(1/4) - 1)/a^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}a\left(\frac{10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2} - \frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2} + \frac{5\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2} + \frac{4\left(\frac{5(ax-1)}{ax+1}-4\right)}{a^2\left(\frac{(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

output
```
-1/2*a*(10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 5*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^2 + 5*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 +
4*(5*(a*x - 1)/(a*x + 1) - 4)/(a^2*((a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/
(a*x + 1) - ((a*x - 1)/(a*x + 1))^(1/4))))
```

Mupad [B] (verification not implemented)

Time = 23.43 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = \frac{\frac{10\,(a\,x-1)}{a\,x+1}-8}{a\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}-a\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}} - \frac{5\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a} + \frac{5\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

input
```
int(1/((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
((10*(a*x - 1))/(a*x + 1) - 8)/(a*((a*x - 1)/(a*x + 1))^(1/4) - a*((a*x -
1)/(a*x + 1))^(5/4)) - (5*atan(((a*x - 1)/(a*x + 1))^(1/4)))/a + (5*atanh(
((a*x - 1)/(a*x + 1))^(1/4)))/a
```

Reduce [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx$$
$$= \int \frac{(ax+1)^{\frac{1}{4}}}{(ax-1)^{\frac{1}{4}}\,ax-(ax-1)^{\frac{1}{4}}}dx + \left(\int \frac{(ax+1)^{\frac{1}{4}}\,x}{(ax-1)^{\frac{1}{4}}\,ax-(ax-1)^{\frac{1}{4}}}dx\right)a$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int((a*x + 1)**(1/4)/((a*x - 1)**(1/4)*a*x - (a*x - 1)**(1/4)),x) + int(((
a*x + 1)**(1/4)*x)/((a*x - 1)**(1/4)*a*x - (a*x - 1)**(1/4)),x)*a
```

3.90 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 250

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\,\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

output
```
-8*(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-2^(1/2)*arctan(-1+2^(1/2)*(1-1/a/x)^(1/
4)/(1+1/a/x)^(1/4))-2^(1/2)*arctan(1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/
4))+2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2^(1/2)*arctanh(2^(1/2)*(1-1
/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1+1/a/x)^(1/4))+2*arctanh
((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.12

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = 8e^{\frac{1}{2}\coth^{-1}(ax)}\left(-1+\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},e^{4\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)/x,x]
```

output
```
8*E^(ArcCoth[a*x]/2)*(-1 + Hypergeometric2F1[1/8, 1, 9/8, E^(4*ArcCoth[a*x
])])
```

Rubi [A] (warning: unable to verify)

Time = 0.82 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.15, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.357, Rules used = {6721, 109, 27, 35, 140, 73, 104, 756, 216, 219, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 109}$$

$$4a\int -\frac{\left(a-\frac{1}{x}\right)x}{4a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{\left(a-\frac{1}{x}\right)x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{a}-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\downarrow$ 35

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}x}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\downarrow$ 140

$$\frac{\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{a}-\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\downarrow$ 73

$$-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\downarrow$ 104

$$-4\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\downarrow$ 756

$$-4\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\downarrow$ 216

$$-4\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 219}$$

$$-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 854}$$

$$-4\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 826}$$

$$-4\left(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 1476}$$

$$-4\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{1082}$$

$$-4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{217}$$

$$-4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 1479$$

$$-4\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\ldots}}{\sqrt[4]{2-\ldots}}\ldots\right.}{\sqrt{2}}\ldots\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 25$$

$$-4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{a\ldots}}}{\sqrt[4]{2-\frac{1}{x^4}}}\ldots\right.}{\sqrt{2}}\ldots\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\ldots}}{\sqrt[4]{2-\ldots}}\ldots\right.}{\sqrt{2}}\ldots\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{1103}$$

$$-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-$$

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\frac{\text{lo}}{}\right.\right.$$

$$\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}$$

input
```
Int[E^((5*ArcCoth[a*x])/2)/x,x]
```

output
```
(-8*(1 + 1/(a*x))^(1/4))/(1 - 1/(a*x))^(1/4) - 4*(-1/2*ArcTan[(1 + 1/(a*x)
)^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(
1/4)]/2) - 4*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/
4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)
]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) +
 x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^
(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx =$$
$$-\frac{2\sqrt{2}(ax-1)\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+2\sqrt{2}(ax-1)\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-\sqrt{2}(ax-1)\log\left(\cdots\right.}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(2)*(a*x - 1)*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1)
+ 2*sqrt(2)*(a*x - 1)*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - sq
rt(2)*(a*x - 1)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + sqrt(2)*(a*x - 1)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4
) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 4*(a*x - 1)*arctan(((a*x - 1)/(a*x +
1))^(1/4)) - 2*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 2*(a*x - 1
)*log(((a*x - 1)/(a*x + 1))^(1/4) - 1) + 16*(a*x + 1)*((a*x - 1)/(a*x + 1)
)^(3/4))/(a*x - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)/x,x)
```

output
```
Integral(1/(x*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.98

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx =$$

$$-\frac{1}{2}\,a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{\right.}}{a}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="maxima")
```

output
```
-1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a -
2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/
4) - 1)/a + 16/(a*((a*x - 1)/(a*x + 1))^(1/4)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.01

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx =$$

$$-\frac{1}{2}\,a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{\right.}}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="giac")
```

output
```
-1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
 1)/(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/
a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x +
 1))^(1/4) - 1))/a + 16/(a*((a*x - 1)/(a*x + 1))^(1/4)))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.47

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right) - \frac{8}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}} - \mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,2\mathrm{i} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)(-1+1\mathrm{i}) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-}{a\,x+}\right.\right.$$

input
```
int(1/(x*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

output
```
- atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*2i - 2*atan(((a*x - 1)/(a*x + 1))^(
1/4)) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1
- 1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1
+ 1i) - 8/((a*x - 1)/(a*x + 1))^(1/4)
```

Reduce [F]

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{(ax+1)^{\frac{1}{4}}}{(ax-1)^{\frac{1}{4}}\,a\,x^2-(ax-1)^{\frac{1}{4}}\,x}dx + \left(\int \frac{(ax+1)^{\frac{1}{4}}}{(ax-1)^{\frac{1}{4}}\,ax-(ax-1)^{\frac{1}{4}}}dx\right)a$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x,x)
```

output

```
int((a*x + 1)**(1/4)/((a*x - 1)**(1/4)*a*x**2 - (a*x - 1)**(1/4)*x),x) + i
nt((a*x + 1)**(1/4)/((a*x - 1)**(1/4)*a*x - (a*x - 1)**(1/4)),x)*a
```

3.91 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 224

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{5a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{5a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{5a\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

output

```
-5*a*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-4*a*(1+1/a/x)^(5/4)/(1-1/a/x)^(1/4)-5
/2*a*arctan(-1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)-5/2*a*arct
an(1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)+5/2*a*arctanh(2^(1/2
)*(1-1/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1+1/a/x)^(1/4))*2^(
1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.77

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = a\left(-\frac{10e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}} - \frac{8e^{\frac{5}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}} - \frac{5\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}} + \frac{5\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}} - \frac{5\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}} + \frac{5\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}\right)$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)/x^2,x]
```

output
```
a*((-10*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) - (8*E^((5*ArcCoth[a*
x])/2))/(1 + E^(2*ArcCoth[a*x])) - (5*ArcTan[1 - Sqrt[2]*E^(ArcCoth[a*x]/2
)])/Sqrt[2] + (5*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)])/Sqrt[2] - (5*Log[
1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[2]) + (5*Log[1 +
 Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[2]))
```

Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6721, 57, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{57}$$

$$5\int \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{60}$$

$$5\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{73}$$

$$5\left(-2a\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{854}$$

$$5\left(-2a\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{826}$$

$$5\left(-2a\left(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{1476}$$

$$5\left(-2a\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}\right.\right.$$

$$\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\big\downarrow$ 1082

$$5\left(-2a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\right.$$

$$\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\big\downarrow$ 217

$$5\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\left(1-\right.\right.$$

$$\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\big\downarrow$ 1479

$$5\left(-2a\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{\ldots}}{\sqrt[4]{2\ldots}}\right.}{\sqrt{\ldots}}\right.\right.\right.$$

$$\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 25$$

$$5\left(-2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1\ldots}}{\sqrt[4]{2-\ldots}}\right.}{\sqrt{2}\ldots}\right.\right.\right.$$

$$\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$5\left(-2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{}\right.\right.\right.$$

$$\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 1103

$$5\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right.}{2\sqrt{2}}\right.\right.\right.$$

$$\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

input
```
Int[E^((5*ArcCoth[a*x])/2)/x^2,x]
```

output
```
(-4*a*(1 + 1/(a*x))^(5/4))/(1 - 1/(a*x))^(1/4) + 5*(-(a*(1 - 1/(a*x))^(3/4
)*(1 + 1/(a*x))^(1/4)) - 2*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2
 - x^(-4))^(1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 -
 x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x^2} dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.04

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2} dx = -\frac{10\sqrt{2}(a^2x^2-ax)\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+10\sqrt{2}(a^2x^2-ax)\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-5\sqrt{2}(a^2x}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="fricas")
```

output
```
-1/4*(10*sqrt(2)*(a^2*x^2 - a*x)*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4
) + 1) + 10*sqrt(2)*(a^2*x^2 - a*x)*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(
1/4) - 1) - 5*sqrt(2)*(a^2*x^2 - a*x)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1
/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 5*sqrt(2)*(a^2*x^2 - a*x)*log(-sqrt
(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 4*(9*a^
2*x^2 + 8*a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4))/(a*x^2 - x)
```

Sympy [F]

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**2,x)
```

output
```
Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.91

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx =$$
$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="maxima")
```

output
```
-1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) + 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) + 8*(5*(a*x - 1)/(a*x + 1) + 4)/(((a*x - 1)/
(a*x + 1))^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.97

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx =$$

$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="giac")
```

output
```
-1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) + 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) + 8*(5*(a*x - 1)/(a*x + 1) + 4)/((a*x - 1)*(
(a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.48

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$$

$$=5\,(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-5\,(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-\frac{8\,a+\frac{10\,a\,(a\,x}{a\,x+}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}+\left(\frac{a}{a}\right.}$$

input
```
int(1/(x^2*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

output
```
5*(-1)^(1/4)*a*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - 5*(-1)^(1/4
)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - (8*a + (10*a*(a*x - 1))
/(a*x + 1))/(((a*x - 1)/(a*x + 1))^(1/4) + ((a*x - 1)/(a*x + 1))^(5/4))
```

Reduce [F]

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{(ax+1)^{\frac{1}{4}}}{(ax-1)^{\frac{1}{4}}\,a\,x^3-(ax-1)^{\frac{1}{4}}\,x^2}dx + \left(\int \frac{(ax+1)^{\frac{1}{4}}}{(ax-1)^{\frac{1}{4}}\,a\,x^2-(ax-1)^{\frac{1}{4}}\,x}dx\right)a$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

output
```
int((a*x + 1)**(1/4)/((a*x - 1)**(1/4)*a*x**3 - (a*x - 1)**(1/4)*x**2),x)
+ int((a*x + 1)**(1/4)/((a*x - 1)**(1/4)*a*x**2 - (a*x - 1)**(1/4)*x),x)*a
```

3.92 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 276

$$\begin{aligned}\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx &= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}\\&\quad-\frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{25a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}\\&\quad-\frac{25a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}\\&\quad+\frac{25a^2\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}\end{aligned}$$

output
```
-25/4*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-5/2*a^2*(1-1/a/x)^(3/4)*(1+1/a/x
)^(5/4)-2*a^2*(1+1/a/x)^(9/4)/(1-1/a/x)^(1/4)-25/8*a^2*arctan(-1+2^(1/2)*(
1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)-25/8*a^2*arctan(1+2^(1/2)*(1-1/a/x
)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)+25/8*a^2*arctanh(2^(1/2)*(1-1/a/x)^(1/4)/
(1+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1+1/a/x)^(1/4))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.67

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{16}a^2\left(-128e^{\frac{1}{2}\coth^{-1}(ax)} + \frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(1+e^{2\coth^{-1}(ax)}\right)^2} - \frac{104e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}} - 50\sqrt{2}\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 50\sqrt{2}\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right) - 25\sqrt{2}\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right) + 25\sqrt{2}\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)/x^3,x]
```

output
```
(a^2*(-128*E^(ArcCoth[a*x]/2) + (32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[
a*x]))^2 - (104*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) - 50*Sqrt[2]*
ArcTan[1 - Sqrt[2]*E^(ArcCoth[a*x]/2)] + 50*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(
ArcCoth[a*x]/2)] - 25*Sqrt[2]*Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCo
th[a*x]] + 25*Sqrt[2]*Log[1 + Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]
))/16
```

Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6721, 87, 60, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}}{\left(1-\frac{1}{ax}\right)^{5/4}x}d\frac{1}{x}$$

$$\Big\downarrow \text{87}$$

$$5a\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{60}$$

$$5a\left(\frac{5}{4}\int \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{60}$$

$$5a\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{73}$$

$$5a\left(\frac{5}{4}\left(-2a\int\frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{854}$$

$$5a\left(\frac{5}{4}\left(-2a\int\frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{826}$$

$$5a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a\left(1-\frac{1}{ax}\right)\right.$$

$$\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{1476}$$

$$5a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\int\frac{1}{1}\right.\right.\right.$$

$$\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{1082}$$

$$5a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.\right.\right.$$

$$\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\downarrow$ 217

$$5a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\right.\right.$$

$$\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\downarrow$ 1479

$$5a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\cdots\right.}{\cdots}\right.\right.\right.\right.$$

$$\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 25$$

$$5a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\cdots\right.}{\cdots}\right.\right.\right.\right.$$

$$\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$5a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan}{\quad}\right.\right.\right.\right.$$

$$\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\downarrow$ 1103

$$5a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}\right.}{2\sqrt{2}}\right.\right.\right.\right.$$

$$\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

input
```
Int[E^((5*ArcCoth[a*x])/2)/x^3,x]
```

output
```
(-2*a^2*(1 + 1/(a*x))^(9/4))/(1 - 1/(a*x))^(1/4) + 5*a*(-1/2*(a*(1 - 1/(a*
x))^(3/4)*(1 + 1/(a*x))^(5/4)) + (5*(-(a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))
^(1/4)) - 2*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1
/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4
)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)
+ x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))
^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x^3} dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.93

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3} dx = -\frac{50\sqrt{2}(a^3x^3 - a^2x^2)\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) + 50\sqrt{2}(a^3x^3 - a^2x^2)\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right) - 25\sqrt{}}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="fricas")
```

output
```
-1/16*(50*sqrt(2)*(a^3*x^3 - a^2*x^2)*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))
^(1/4) + 1) + 50*sqrt(2)*(a^3*x^3 - a^2*x^2)*arctan(sqrt(2)*((a*x - 1)/(a*
x + 1))^(1/4) - 1) - 25*sqrt(2)*(a^3*x^3 - a^2*x^2)*log(sqrt(2)*((a*x - 1)
/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 25*sqrt(2)*(a^3*x^3 -
 a^2*x^2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) + 4*(43*a^3*x^3 + 34*a^2*x^2 - 11*a*x - 2)*((a*x - 1)/(a*x + 1))
^(3/4))/(a*x^3 - x^2)
```

Sympy [F]

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**3,x)
```

output
```
Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.88

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx =$$
$$-\frac{1}{16}\left(25\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="maxima")
```

output
```
-1/16*(25*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sq
rt((a*x - 1)/(a*x + 1)) + 1))*a + 8*(45*(a*x - 1)*a/(a*x + 1) + 25*(a*x -
1)^2*a/(a*x + 1)^2 + 16*a)/(((a*x - 1)/(a*x + 1))^(9/4) + 2*((a*x - 1)/(a*
x + 1))^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.88

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{1}{16}\left(50\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="giac")
```

output
```
-1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
1))^(1/4))) - 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1) + 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1)
)^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 128*a/((a*x - 1)/(a*x + 1))^(1/
4) + 8*(9*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) + 13*a*((a*x -
 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.55

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{25\,(-1)^{1/4}\,a^2\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4} - \frac{25\,(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4} - \frac{8\,a^2+\frac{25\,a^2\,{(a\,x-1)}^2}{2\,{(a\,x+1)}^2}+\frac{45\,a^2\,(a\,x-1)}{2\,(a\,x+1)}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}+2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}+{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}}$$

input
```
int(1/(x^3*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

output
```
(25*(-1)^(1/4)*a^2*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 - (25*
(-1)^(1/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 - (8*a^2 +
(25*a^2*(a*x - 1)^2)/(2*(a*x + 1)^2) + (45*a^2*(a*x - 1))/(2*(a*x + 1)))/(
((a*x - 1)/(a*x + 1))^(1/4) + 2*((a*x - 1)/(a*x + 1))^(5/4) + ((a*x - 1)/(
a*x + 1))^(9/4))
```

Reduce [F]

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{(ax+1)^{\frac{1}{4}}}{(ax-1)^{\frac{1}{4}}\,a\,x^4-(ax-1)^{\frac{1}{4}}\,x^3}dx + \left(\int \frac{(ax+1)^{\frac{1}{4}}}{(ax-1)^{\frac{1}{4}}\,a\,x^3-(ax-1)^{\frac{1}{4}}\,x^2}dx\right)a$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

output
```
int((a*x + 1)**(1/4)/((a*x - 1)**(1/4)*a*x**4 - (a*x - 1)**(1/4)*x**3),x)
+ int((a*x + 1)**(1/4)/((a*x - 1)**(1/4)*a*x**3 - (a*x - 1)**(1/4)*x**2),x
)*a
```

3.93 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 310

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}-\frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} -\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4}+\frac{55a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}-\frac{55a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+$$

output
```
-55/8*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-11/4*a^3*(1-1/a/x)^(3/4)*(1+1/a/
x)^(5/4)-2*a^3*(1+1/a/x)^(9/4)/(1-1/a/x)^(1/4)-1/3*a^3*(1-1/a/x)^(3/4)*(1+
1/a/x)^(9/4)-55/16*a^3*arctan(-1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*
2^(1/2)-55/16*a^3*arctan(1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2
)+55/16*a^3*arctanh(2^(1/2)*(1-1/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(1+1/a/x)^(
1/2))/(1+1/a/x)^(1/4))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.34

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = a^3\left(-\frac{e^{\frac{1}{2}\coth^{-1}(ax)}\left(165+462e^{2\coth^{-1}(ax)}+425e^{4\coth^{-1}(ax)}+96e^{6\coth^{-1}(ax)}\right)}{12\left(1+e^{2\coth^{-1}(ax)}\right)^3} - \frac{55}{32}\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)/x^4,x]
```

output
```
a^3*(-1/12*(E^(ArcCoth[a*x]/2)*(165 + 462*E^(2*ArcCoth[a*x]) + 425*E^(4*Ar
cCoth[a*x]) + 96*E^(6*ArcCoth[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 - (55*Roo
tSum[1 + #1^4 & , (ArcCoth[a*x] - 2*Log[E^(ArcCoth[a*x]/2) - #1])/#1^3 & ]
)/32)
```

Rubi [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6721, 100, 27, 90, 60, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \ 6721$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}}{\left(1-\frac{1}{ax}\right)^{5/4}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{100}$$

$$2a^3\int \frac{\left(5a+\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{5/4}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$a\int \frac{\left(5a+\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{90}$$

$$a\left(\frac{11}{2}a\int \frac{\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{3}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{9/4}\right)-\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{60}$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\int \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)-\frac{1}{3}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{9/4}\right)-\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{60}$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)-\frac{1}{3}\cdots\right.$$

$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{73}$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)-\right.$$
$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\big\downarrow$ 854

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}\right)-\right.$$
$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\big\downarrow$ 826

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)-\frac{1}{2}a\left(1-\right.\right.\right.$$
$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\big\downarrow$ 1476

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\right.\right.\right.\right.$$
$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\big\downarrow$ 1082

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.\right.\right.\right.$$

$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{217}$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right.\right.\right.$$

$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{1479}$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arct}}{}\right.\right.\right.\right.\right.$$

$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 25$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arctan}}{}\right.\right.\right.\right.\right.$$

$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\text{ar}\ldots \right.\right.\right.\right.\right.$$

$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$\downarrow$ 1103

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\ldots\right.}{2\sqrt{2}}\ldots\right.\right.\right.\right.$$

$$\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}$$

input

```
Int[E^((5*ArcCoth[a*x])/2)/x^4,x]
```

output
```
(-2*a^3*(1 + 1/(a*x))^(9/4))/(1 - 1/(a*x))^(1/4) + a*(-1/3*(a^2*(1 - 1/(a*
x))^(3/4)*(1 + 1/(a*x))^(9/4)) + (11*a*(-1/2*(a*(1 - 1/(a*x))^(3/4)*(1 + 1
/(a*x))^(5/4)) + (5*(-(a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)) - 2*a*((
-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) +
ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 +
(Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqr
t[2]) - Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]
/(2*Sqrt[2]))/2)))/4))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x^4}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.86

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\frac{330\sqrt{2}(a^4x^4-a^3x^3)\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+330\sqrt{2}(a^4x^4-a^3x^3)\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-16}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="fricas")
```

output
```
-1/96*(330*sqrt(2)*(a^4*x^4 - a^3*x^3)*arctan(sqrt(2)*((a*x - 1)/(a*x + 1)
)^(1/4) + 1) + 330*sqrt(2)*(a^4*x^4 - a^3*x^3)*arctan(sqrt(2)*((a*x - 1)/(
a*x + 1))^(1/4) - 1) - 165*sqrt(2)*(a^4*x^4 - a^3*x^3)*log(sqrt(2)*((a*x -
 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 165*sqrt(2)*(a^4*x
^4 - a^3*x^3)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) + 4*(287*a^4*x^4 + 226*a^3*x^3 - 87*a^2*x^2 - 34*a*x - 8)*((
a*x - 1)/(a*x + 1))^(3/4))/(a*x^4 - x^3)
```

Sympy [F]

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**4,x)
```

output
```
Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.93

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\frac{1}{96}\left(165\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="maxima")
```

output
```
-1/96*(165*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1
))^(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
 1)/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1))*a^2 + 8*(425*(a*x - 1)*a^2/(a*x + 1) + 462*
(a*x - 1)^2*a^2/(a*x + 1)^2 + 165*(a*x - 1)^3*a^2/(a*x + 1)^3 + 96*a^2)/((
(a*x - 1)/(a*x + 1))^(13/4) + 3*((a*x - 1)/(a*x + 1))^(9/4) + 3*((a*x - 1)
/(a*x + 1))^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.94

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx =$$

$$-\frac{1}{96}\left(330\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right.\right.\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="giac")
```

output
```
-1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1
))^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(
a*x + 1))^(1/4))) - 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4
) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x -
1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 768*a^2/((a*x - 1)/
(a*x + 1))^(1/4) + 8*(174*(a*x - 1)*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x +
 1) + 69*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 137*a^2
*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^3)*a
```

Mupad [B] (verification not implemented)

Time = 23.56 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.61

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{55\,(-1)^{1/4}\,a^3\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8} - \frac{55\,(-1)^{1/4}\,a^3\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8} - \frac{8\,a^3+\frac{77\,a^3\,(a\,x-1)^2}{2\,(a\,x+1)^2}+\frac{55\,a^3\,(a\,x-1)^3}{4\,(a\,x+1)^3}+\frac{425\,a^3\,(a\,x-1)}{12\,(a\,x+1)}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}+3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}+3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}+\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}$$

input
```
int(1/(x^4*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

output
```
(55*(-1)^(1/4)*a^3*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - (55*
(-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - (8*a^3 +
(77*a^3*(a*x - 1)^2)/(2*(a*x + 1)^2) + (55*a^3*(a*x - 1)^3)/(4*(a*x + 1)^3
) + (425*a^3*(a*x - 1))/(12*(a*x + 1)))/(((a*x - 1)/(a*x + 1))^(1/4) + 3*(
(a*x - 1)/(a*x + 1))^(5/4) + 3*((a*x - 1)/(a*x + 1))^(9/4) + ((a*x - 1)/(a
*x + 1))^(13/4))
```

Reduce [F]

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{(ax+1)^{\frac{1}{4}}}{(ax-1)^{\frac{1}{4}}\,a\,x^5-(ax-1)^{\frac{1}{4}}\,x^4}dx + \left(\int \frac{(ax+1)^{\frac{1}{4}}}{(ax-1)^{\frac{1}{4}}\,a\,x^4-(ax-1)^{\frac{1}{4}}\,x^3}dx\right)a$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

output
```
int((a*x + 1)**(1/4)/((a*x - 1)**(1/4)*a*x**5 - (a*x - 1)**(1/4)*x**4),x)
+ int((a*x + 1)**(1/4)/((a*x - 1)**(1/4)*a*x**4 - (a*x - 1)**(1/4)*x**3),x
)*a
```

3.94 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 253

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4}-\frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3}+\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2}-\frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a}+\frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5+\frac{31\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{31\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

output
```
611/1920*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^4-269/960*(1-1/a/x)^(1/4)*(1+
1/a/x)^(3/4)*x^2/a^3+11/48*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a^2-9/40*(1
-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^4/a+1/5*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^
5+31/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5-31/128*arctanh((1+1/a
/x)^(1/4)/(1-1/a/x)^(1/4))/a^5
```

Mathematica [A] (verified)

Time = 5.33 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.68

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$$
$$= \frac{\frac{24576e^{\frac{19}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^5} - \frac{62976e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4} + \frac{64640e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} - \frac{34000e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \frac{9620e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} - 930\arctan}{3840a^5}$$

input
```
Integrate[x^4/E^(ArcCoth[a*x]/2),x]
```

output
```
((24576*E^((19*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 - (62976*E^((
15*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (64640*E^((11*ArcCoth[a
*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 - (34000*E^((7*ArcCoth[a*x])/2))/(-1
+ E^(2*ArcCoth[a*x]))^2 + (9620*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth
[a*x])) - 930*ArcTan[E^(-1/2*ArcCoth[a*x])] + 465*Log[1 - E^(-1/2*ArcCoth[
a*x])] - 465*Log[1 + E^(-1/2*ArcCoth[a*x])])/(3840*a^5)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6721, 110, 27, 168, 27, 168, 27, 168, 27, 168, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \ 6721$$
$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}x^6}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$
$$\Big\downarrow \ 110$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{1}{5}\int -\frac{\left(9a-\frac{8}{x}\right)x^5}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\left(9a-\frac{8}{x}\right)x^5}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{10a^2}+\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{ 168}$$

$$\frac{-\frac{1}{4}\int \frac{\left(55a-\frac{54}{x}\right)x^4}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{9}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}+\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{\int \frac{\left(55a-\frac{54}{x}\right)x^4}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{8a}-\frac{9}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}+\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{ 168}$$

$$\frac{-\frac{-\frac{1}{3}\int \frac{\left(269a-\frac{220}{x}\right)x^3}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{55}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}-\frac{9}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}+\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{-\frac{\int \frac{\left(269a-\frac{220}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}-\frac{9}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}+\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{ 168}$$

$$-\frac{-\frac{-\frac{-\frac{1}{2}\int \frac{\left(611a-\frac{538}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}-\frac{9}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}+$$
$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{-\frac{\int \frac{\left(611a-\frac{538}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}-\frac{9}{4}ax^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{10a^2}+$$
$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 168$$

$$-\frac{-\frac{-\frac{-\frac{-\int \frac{465x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-611ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}-\frac{9}{4}ax}{10a^2}$$
$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{-\frac{-\frac{465}{2}\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-611ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}-\frac{9}{4}a}{10a^2}$$
$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 104

$$\frac{-\frac{-\frac{-\frac{-930\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-611ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}-\frac{9}{4}a\cdots}{10a^2}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 25

$$\frac{-\frac{-\frac{-\frac{930\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-611ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}-\frac{9}{4}ax^4\cdots}{10a^2}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 827

$$\frac{-\frac{-\frac{-\frac{-930\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-611ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-\cdots}}{8a}\cdots}{10a^2}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 216

$$-\frac{-\frac{-\frac{-\frac{-930\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-611ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{1-\cdots}}{8a}\cdots}{10a^2}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{-\frac{-\frac{-930\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-611ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{269}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{55}{3}ax^3\sqrt[4]{\cdots}}{8a}\cdots}{10a^2}$$

$$\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

input
```
Int[x^4/E^(ArcCoth[a*x]/2),x]
```

output
```
((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^5)/5 + ((-9*a*(1 - 1/(a*x))^(1/
4)*(1 + 1/(a*x))^(3/4)*x^4)/4 - ((-55*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^
(3/4)*x^3)/3 - ((-269*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/2 - (
-611*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x - 930*(ArcTan[(1 + 1/(a*x
))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x)
)^(1/4)]/2))/(4*a))/(6*a))/(8*a))/(10*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int x^4 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} dx$$

input
```
int(x^4*((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(x^4*((a*x-1)/(a*x+1))^(1/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.47

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)} x^4 \, dx = \frac{2\,(384\,a^5x^5 - 48\,a^4x^4 + 8\,a^3x^3 - 98\,a^2x^2 + 73\,ax + 611)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 930\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 465\,\log\left(\left(\frac{a}{a}\right.\right.}{3840\,a^5}$$

input
```
integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

output
```
1/3840*(2*(384*a^5*x^5 - 48*a^4*x^4 + 8*a^3*x^3 - 98*a^2*x^2 + 73*a*x + 61
1)*((a*x - 1)/(a*x + 1))^(1/4) - 930*arctan(((a*x - 1)/(a*x + 1))^(1/4)) -
 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 465*log(((a*x - 1)/(a*x + 1))^
(1/4) - 1))/a^5
```

Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx=\int x^4\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x**4*((a*x-1)/(a*x+1))**(1/4),x)
```

output
```
Integral(x**4*((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx =$$
$$-\frac{1}{3840}a\left(\frac{4\left(2405\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-1120\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+5090\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-696\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+465\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1}-\frac{10(ax-1)^2a^6}{(ax+1)^2}+\frac{10(ax-1)^3a^6}{(ax+1)^3}-\frac{5(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}+\frac{930\,\mathrm{arc}}{}\right.$$

input
```
integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

output
```
-1/3840*a*(4*(2405*((a*x - 1)/(a*x + 1))^(17/4) - 1120*((a*x - 1)/(a*x + 1
))^(13/4) + 5090*((a*x - 1)/(a*x + 1))^(9/4) - 696*((a*x - 1)/(a*x + 1))^(
5/4) + 465*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a
*x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4
*a^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 930*arctan(((a*x -
 1)/(a*x + 1))^(1/4))/a^6 + 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 -
 465*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.92

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx =$$

$$-\frac{1}{3840}a\left(\frac{930\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{465\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{465\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-\frac{4\left(\frac{696\,(ax-1)(}{ax+}\right.}{}\right.$$

input
```
integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

output
```
-1/3840*a*(930*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 465*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^6 - 465*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^6 - 4*(696*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 5090*(a*x
 - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 1120*(a*x - 1)^3*((a*x -
 1)/(a*x + 1))^(1/4)/(a*x + 1)^3 - 2405*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^
(1/4)/(a*x + 1)^4 - 465*((a*x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a*x
+ 1) - 1)^5))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.91

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$$

$$= \frac{\frac{31\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{64}-\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{40}+\frac{509\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{96}-\frac{7\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{6}+\frac{481\left(\frac{a\,x-1}{a\,x+1}\right)^{17/4}}{192}}{a^5+\frac{10\,a^5\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a^5\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a^5\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a^5\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{5\,a^5\,(a\,x-1)}{a\,x+1}}$$
$$-\frac{31\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}-\frac{31\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}$$

input
```
int(x^4*((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
((31*((a*x - 1)/(a*x + 1))^(1/4))/64 - (29*((a*x - 1)/(a*x + 1))^(5/4))/40
 + (509*((a*x - 1)/(a*x + 1))^(9/4))/96 - (7*((a*x - 1)/(a*x + 1))^(13/4))
/6 + (481*((a*x - 1)/(a*x + 1))^(17/4))/192)/(a^5 + (10*a^5*(a*x - 1)^2)/(
a*x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x +
 1)^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) - (31
*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) - (31*atanh(((a*x - 1)/(a*x
+ 1))^(1/4)))/(128*a^5)
```

Reduce [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx=\int\frac{(ax-1)^{\frac{1}{4}}x^4}{(ax+1)^{\frac{1}{4}}}dx$$

input
```
int(x^4*((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(((a*x - 1)**(1/4)*x**4)/(a*x + 1)**(1/4),x)
```

3.95 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 216

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3}+\frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2}-\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a}+\frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4-\frac{11\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

output

```
-83/192*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^3+29/96*(1-1/a/x)^(1/4)*(1+1/a
/x)^(3/4)*x^2/a^2-7/24*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a+1/4*(1-1/a/x)
^(1/4)*(1+1/a/x)^(3/4)*x^4-11/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
^4+11/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Mathematica [A] (verified)

Time = 5.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$$
$$= \frac{\frac{1536e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4} - \frac{3200e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} + \frac{2512e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} - \frac{980e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} + 66\arctan\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right) - 3}{384a^4}$$

input
```
Integrate[x^3/E^(ArcCoth[a*x]/2),x]
```

output
```
((1536*E^((15*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 - (3200*E^((11
*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (2512*E^((7*ArcCoth[a*x])
/2))/(-1 + E^(2*ArcCoth[a*x]))^2 - (980*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2
*ArcCoth[a*x])) + 66*ArcTan[E^(-1/2*ArcCoth[a*x])] - 33*Log[1 - E^(-1/2*Ar
cCoth[a*x])] + 33*Log[1 + E^(-1/2*ArcCoth[a*x])])/(384*a^4)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6721, 110, 27, 168, 27, 168, 27, 168, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}x^5}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{1}{4}\int -\frac{\left(7a-\frac{6}{x}\right)x^4}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$\downarrow$ 27

$$\frac{\int \frac{\left(7a-\frac{6}{x}\right)x^4}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 168

$$\frac{-\frac{1}{3}\int \frac{\left(29a-\frac{28}{x}\right)x^3}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 27

$$\frac{-\frac{\int \frac{\left(29a-\frac{28}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 168

$$\frac{-\frac{-\frac{1}{2}\int \frac{\left(83a-\frac{58}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 27

$$\frac{-\frac{-\frac{\int \frac{\left(83a-\frac{58}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 168

$$\frac{-\frac{-\frac{-\int \frac{33x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-83ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{-\frac{-\frac{33}{2}\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-83ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 104$$

$$\frac{-\frac{-\frac{-66\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-83ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{-\frac{66\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-83ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 827

$$-\frac{-\frac{-\frac{-66\left(\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-83ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\cdots}{8a^2}$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 216

$$-\frac{-\frac{-\frac{-66\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-83ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1-\frac{1}{ax}}\cdots}{8a^2}$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 219

$$-\frac{-\frac{-\frac{-66\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-83ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{29}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a}-\frac{7}{3}ax^3\sqrt[4]{1\cdots}}{8a^2}$$

$$\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

input
```
Int[x^3/E^(ArcCoth[a*x]/2),x]
```

output
```
((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^4)/4 + ((-7*a*(1 - 1/(a*x))^(1/
4)*(1 + 1/(a*x))^(3/4)*x^3)/3 - ((-29*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^
(3/4)*x^2)/2 - (-83*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x - 66*(ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4
)/(1 - 1/(a*x))^(1/4)]/2))/(4*a))/(6*a))/(8*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int x^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} dx$$

input
```
int(x^3*((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(x^3*((a*x-1)/(a*x+1))^(1/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.51

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \frac{2\left(48\,a^4x^4-8\,a^3x^3+2\,a^2x^2-25\,ax-83\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-33}{384\,a^4}$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

output
```
1/384*(2*(48*a^4*x^4 - 8*a^3*x^3 + 2*a^2*x^2 - 25*a*x - 83)*((a*x - 1)/(a*
x + 1))^(1/4) + 66*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 33*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1) - 33*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \int x^3\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x**3*((a*x-1)/(a*x+1))**(1/4),x)
```

output
```
Integral(x**3*((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{1}{384}\,a\left(\frac{4\left(245\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-107\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+279\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-33\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{33\,\mathrm{lo}}{}\right.$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

output
```
-1/384*a*(4*(245*((a*x - 1)/(a*x + 1))^(13/4) - 107*((a*x - 1)/(a*x + 1))^
(9/4) + 279*((a*x - 1)/(a*x + 1))^(5/4) - 33*((a*x - 1)/(a*x + 1))^(1/4))/
(4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3
*a^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 66*arctan(((a*x -
1)/(a*x + 1))^(1/4))/a^5 - 33*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 + 3
3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$$

$$= \frac{1}{384}a\left(\frac{66\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5} + \frac{33\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5} - \frac{33\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5} + \frac{4\left(\frac{279\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}\right.}{}\right.$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

output
```
1/384*a*(66*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 33*log(((a*x - 1)/(a
*x + 1))^(1/4) + 1)/a^5 - 33*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^5
 + 4*(279*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 107*(a*x - 1)^
2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 245*(a*x - 1)^3*((a*x - 1)/(a*
x + 1))^(1/4)/(a*x + 1)^3 - 33*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x - 1
)/(a*x + 1) - 1)^4))
```

Mupad [B] (verification not implemented)

Time = 23.52 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.89

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \frac{11\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4} - \frac{\frac{11\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{32}-\frac{93\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{32}+\frac{107\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{96}-\frac{245\,\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{96}}{a^4+\frac{6\,a^4\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a^4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a^4\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a^4\,\left(a\,x-1\right)}{a\,x+1}} + \frac{11\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}$$

input
```
int(x^3*((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
(11*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - ((11*((a*x - 1)/(a*x + 1
))^(1/4))/32 - (93*((a*x - 1)/(a*x + 1))^(5/4))/32 + (107*((a*x - 1)/(a*x
+ 1))^(9/4))/96 - (245*((a*x - 1)/(a*x + 1))^(13/4))/96)/(a^4 + (6*a^4*(a*
x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4
)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) + (11*atanh(((a*x - 1)/(a*x +
 1))^(1/4)))/(64*a^4)
```

Reduce [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = \int \frac{(ax-1)^{\frac{1}{4}}\,x^3}{(ax+1)^{\frac{1}{4}}}dx$$

input
```
int(x^3*((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(((a*x - 1)**(1/4)*x***3)/(a*x + 1)**(1/4),x)
```

3.96 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 179

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx=\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2}-\frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a}+\frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3+\frac{3\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{3\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

output
```
11/24*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^2-5/12*(1-1/a/x)^(1/4)*(1+1/a/x)
^(3/4)*x^2/a+1/3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3+3/8*arctan((1+1/a/x)^
(1/4)/(1-1/a/x)^(1/4))/a^3-3/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^
3
```

Mathematica [A] (verified)

Time = 5.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$$
$$= \frac{\frac{128e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} - \frac{208e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \frac{116e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} - 18\arctan\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right) + 9\log\left(1-e^{-\frac{1}{2}\coth^{-1}(\cdots}\right.}{48a^3}$$

input
```
Integrate[x^2/E^(ArcCoth[a*x]/2),x]
```

output
```
((128*E^((11*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 - (208*E^((7*Ar
cCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (116*E^((3*ArcCoth[a*x])/2))
/(-1 + E^(2*ArcCoth[a*x])) - 18*ArcTan[E^(-1/2*ArcCoth[a*x])] + 9*Log[1 -
E^(-1/2*ArcCoth[a*x])] - 9*Log[1 + E^(-1/2*ArcCoth[a*x])])/(48*a^3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6721, 110, 27, 168, 27, 168, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \; 6721$$
$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}x^4}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$
$$\Big\downarrow \; 110$$

$$\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{1}{3}\int -\frac{\left(5a-\frac{4}{x}\right)x^3}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(5a-\frac{4}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 168$$

$$\frac{-\frac{1}{2}\int \frac{\left(11a-\frac{10}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\int \frac{\left(11a-\frac{10}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 168$$

$$\frac{-\frac{-\int \frac{9x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-11ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{-\frac{9}{2}\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-11ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 104$$

$$\frac{-\frac{-18\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-11ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 25

$$\frac{-\frac{18\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-11ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 827

$$\frac{-\frac{-18\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-11ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 216

$$\frac{-\frac{-18\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-11ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 219

$$-\frac{-\frac{-18\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-11ax\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}-\frac{5}{2}ax^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{6a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

input
```
Int[x^2/E^(ArcCoth[a*x]/2),x]
```

output
```
((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^3)/3 + ((-5*a*(1 - 1/(a*x))^(1/
4)*(1 + 1/(a*x))^(3/4)*x^2)/2 - (-11*a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(
3/4)*x - 18*(ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(
1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/(4*a))/(6*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} dx$$

input
```
int(x^2*((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(x^2*((a*x-1)/(a*x+1))^(1/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.57

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)} x^2 \, dx = \frac{2\left(8\,a^3x^3 - 2\,a^2x^2 + ax + 11\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 18 \arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 9 \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) + 9 \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{48\,a^3}$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

output
```
1/48*(2*(8*a^3*x^3 - 2*a^2*x^2 + a*x + 11)*((a*x - 1)/(a*x + 1))^(1/4) - 1
8*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 9*log(((a*x - 1)/(a*x + 1))^(1/4)
+ 1) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)} x^2 \, dx = \int x^2 \sqrt[4]{\frac{ax-1}{ax+1}} \, dx$$

input
```
integrate(x**2*((a*x-1)/(a*x+1))**(1/4),x)
```

output
```
Integral(x**2*((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{48}a\left(\frac{4\left(29\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-6\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{9\,\mathrm{l}}{}\right.$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

output
```
-1/48*a*(4*(29*((a*x - 1)/(a*x + 1))^(9/4) - 6*((a*x - 1)/(a*x + 1))^(5/4)
 + 9*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)
^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 18*arctan(((a*x
- 1)/(a*x + 1))^(1/4))/a^4 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 -
9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{48}a\left(\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{9\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}-\frac{4\left(\frac{6(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\right.}{a^4}\right.$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

output
```
-1/48*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 9*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^4 - 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 -
 4*(6*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 29*(a*x - 1)^2*((a
*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 - 9*((a*x - 1)/(a*x + 1))^(1/4))/(a^4
*((a*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B] (verification not implemented)

Time = 23.56 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = \frac{\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}} - \frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}-\frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

input
```
int(x^2*((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
((3*((a*x - 1)/(a*x + 1))^(1/4))/4 - ((a*x - 1)/(a*x + 1))^(5/4)/2 + (29*(
(a*x - 1)/(a*x + 1))^(9/4))/12)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (
a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - (3*atan(((a*
x - 1)/(a*x + 1))^(1/4)))/(8*a^3) - (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))
/(8*a^3)
```

Reduce [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = \int \frac{(ax-1)^{\frac{1}{4}}\,x^2}{(ax+1)^{\frac{1}{4}}}dx$$

input
```
int(x^2*((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(((a*x - 1)**(1/4)*x**2)/(a*x + 1)**(1/4),x)
```

3.97 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 142

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx=-\frac{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2-\frac{\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

output

```
-1/4*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a+1/2*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/
4)*x^2-1/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+1/4*arctanh((1+1/a/
x)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.46

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{-\frac{2e^{\frac{3}{2}\coth^{-1}(ax)}\left(-5+e^{2\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}-\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+\operatorname{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

input
```
Integrate[x/E^(ArcCoth[a*x]/2),x]
```

output
```
((-2*E^((3*ArcCoth[a*x])/2)*(-5 + E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[
a*x]))^2 - ArcTan[E^(ArcCoth[a*x]/2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^
2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6721, 107, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-\frac{1}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{107}$$

$$\frac{\int \frac{\sqrt[4]{1-\frac{1}{ax}}x^2}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 105$$

$$\frac{x\left(-\sqrt[4]{1-\frac{1}{ax}}\right)\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 104$$

$$\frac{x\left(-\sqrt[4]{1-\frac{1}{ax}}\right)\left(\frac{1}{ax}+1\right)^{3/4}-\frac{2\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}-x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 827$$

$$\frac{x\left(-\sqrt[4]{1-\frac{1}{ax}}\right)\left(\frac{1}{ax}+1\right)^{3/4}-\frac{2\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{x\left(-\sqrt[4]{1-\frac{1}{ax}}\right)\left(\frac{1}{ax}+1\right)^{3/4}-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\left(-\sqrt[4]{1-\frac{1}{ax}}\right)\left(\frac{1}{ax}+1\right)^{3/4}-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

input
```
Int[x/E^(ArcCoth[a*x]/2),x]
```

output
```
((1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4)*x^2)/2 + (-((1 - 1/(a*x))^(1/4)*(
1 + 1/(a*x))^(3/4)*x) - (2*(ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)
]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/a)/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}dx$$

input
```
int(x*((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(x*((a*x-1)/(a*x+1))^(1/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.65

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{2\,(2\,a^2x^2-ax-3)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^2*x^2 - a*x - 3)*((a*x - 1)/(a*x + 1))^(1/4) + 2*arctan(((a*x
- 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x
- 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = \int x\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x*((a*x-1)/(a*x+1))**(1/4),x)
```

output
```
Integral(x*((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = -\frac{1}{8}\,a\left(\frac{4\left(5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

output
```
-1/8*a*(4*(5*((a*x - 1)/(a*x + 1))^(5/4) - ((a*x - 1)/(a*x + 1))^(1/4))/(2
*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 2*arctan((
(a*x - 1)/(a*x + 1))^(1/4))/a^3 - log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3
 + log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{1}{8}\,a\left(\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}+\frac{4\left(\frac{5\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

output
```
1/8*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + log(((a*x - 1)/(a*x + 1
))^(1/4) + 1)/a^3 - log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 + 4*(5*(
a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - ((a*x - 1)/(a*x + 1))^(1/
4))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 23.55 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = \frac{\operatorname{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2} - \frac{\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2} - \frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{a^2 + \frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{2\,a^2\,(a\,x-1)}{a\,x+1}} + \frac{\operatorname{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}$$

input
```
int(x*((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
atan(((a*x - 1)/(a*x + 1))^(1/4))/(4*a^2) - (((a*x - 1)/(a*x + 1))^(1/4)/2
 - (5*((a*x - 1)/(a*x + 1))^(5/4))/2)/(a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2
 - (2*a^2*(a*x - 1))/(a*x + 1)) + atanh(((a*x - 1)/(a*x + 1))^(1/4))/(4*a^
2)
```

Reduce [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = \int \frac{(ax-1)^{\frac{1}{4}}\,x}{(ax+1)^{\frac{1}{4}}}dx$$

input
```
int(x*((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(((a*x - 1)**(1/4)*x)/(a*x + 1)**(1/4),x)
```

3.98 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 97

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx=\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x+\frac{\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

output
```
(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x+arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/
a-arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.34

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = -\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},e^{2\coth^{-1}(ax)}\right)}{3a}$$

input
```
Integrate[E^(-1/2*ArcCoth[a*x]),x]
```

output
```
(-8*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/4, 2, 7/4, E^(2*ArcCoth[a*x
])])/(3*a)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6720, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6720}$$

$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^2}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 105}$$

$$\frac{\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}+x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{ 104}$$

$$\frac{2\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}+x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\Big\downarrow$ 25

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{2\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}$$

$\Big\downarrow$ 827

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\Big\downarrow$ 216

$$\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\Big\downarrow$ 219

$$\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}+x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

input
```
Int[E^(-1/2*ArcCoth[a*x]),x]
```

output
```
(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x + (2*(ArcTan[(1 + 1/(a*x))^(1/4)
/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]
/2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} dx$$

input
```
int(((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int(((a*x-1)/(a*x+1))^(1/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)} dx = \frac{2(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right)}{2a}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

output
```
1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4) - 2*arctan(((a*x - 1)/(a*x +
1))^(1/4)) - log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + log(((a*x - 1)/(a*x +
1))^(1/4) - 1))/a
```

Sympy [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)} dx = \int \sqrt[4]{\frac{ax-1}{ax+1}} dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/4),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

output
```
-1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 2*
arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + log(((a*x - 1)/(a*x + 1))^(1/4)
+ 1)/a^2 - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}\,a\left(\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

output
```
-1/2*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + log(((a*x - 1)/(a*x +
1))^(1/4) + 1)/a^2 - log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 + 4*((a
*x - 1)/(a*x + 1))^(1/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

Mupad [B] (verification not implemented)

Time = 23.56 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = \frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}} - \frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a} - \frac{\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/4))/(a - (a*(a*x - 1))/(a*x + 1)) - atan(((a*x
 - 1)/(a*x + 1))^(1/4))/a - atanh(((a*x - 1)/(a*x + 1))^(1/4))/a
```

Reduce [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = \int \frac{(ax-1)^{\frac{1}{4}}}{(ax+1)^{\frac{1}{4}}}dx$$

input
```
int(((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int((a*x - 1)**(1/4)/(a*x + 1)**(1/4),x)
```

3.99 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 222

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x} \, dx = \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

output
```
-2^(1/2)*arctan(-1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))-2^(1/2)*arctan
(1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))-2*arctan((1+1/a/x)^(1/4)/(1-1/
a/x)^(1/4))-2^(1/2)*arctanh(2^(1/2)*(1-1/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(1+
1/a/x)^(1/2))/(1+1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.14

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \frac{8}{3}e^{\frac{3}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{8},1,\frac{11}{8},e^{4\coth^{-1}(ax)}\right)$$

input
```
Integrate[1/(E^(ArcCoth[a*x]/2)*x),x]
```

output
```
(8*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/8, 1, 11/8, E^(4*ArcCoth[a*x
])])/3
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.16, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.214$, Rules used = {6721, 140, 73, 104, 25, 770, 755, 827, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}x}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \; 140$$

$$\frac{\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \; 73$$

$$-4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \; 104$$

$$-4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-4\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 25$$

$$4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}$$

$$\Big\downarrow \; 770$$

$$4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-4\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}$$

$$\Big\downarrow \; 755$$

$$4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)$$

$$\Big\downarrow \; 827$$

$$-4\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)$$

$$\Big\downarrow \text{216}$$

$$-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)$$

$$\Big\downarrow \text{219}$$

$$-4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{1476}$$

$$-4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{1082}$$

$$-4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$-4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)\right)-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{1479}$$

$$-4\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\cdots}}{\sqrt[4]{2-\cdots}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)\Bigg)$$

$$\Big\downarrow \; 25$$

$$-4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)\Bigg)$$

$\Big\downarrow$ 27

$$-4\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\right.}{\sqrt[4]{2-}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$\Big\downarrow$ 1103

$$-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-$$

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\frac{\log}{}\right.\right.$$

input
```
Int[1/(E^(ArcCoth[a*x]/2)*x),x]
```

output
```
-4*(ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*
x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2) - 4*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^
(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x
))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(
a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x}dx$$

input
```
int(((a*x-1)/(a*x+1))^(1/4)/x,x)
```

output
```
int(((a*x-1)/(a*x+1))^(1/4)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = -\sqrt{2}\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - \sqrt{2}\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right) - \frac{1}{2}\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right) + \frac{1}{2}\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right) + 2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="fricas")
```

output
```
-sqrt(2)*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) - sqrt(2)*arctan(
sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - 1/2*sqrt(2)*log(sqrt(2)*((a*x -
 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 1/2*sqrt(2)*log(-s
qrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 2*ar
ctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) -
 log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/4)/x,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(1/4)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.01

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx =$$

$$-\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\right.}{a}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="maxima")
```

output
```
-1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1))/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a -
2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/
4) - 1)/a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.05

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx =$$

$$-\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\log\left(\sqrt{\right.}}{}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="giac")
```

output
```
-1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
 1)/(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1)/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/
a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x +
 1))^(1/4) - 1))/a)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.45

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right) - \mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,2\mathrm{i} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,(-1-\mathrm{i}) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,(-1+1\mathrm{i})$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/4)/x,x)
```

output
```
2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)
*2i - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 +
1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 -
1i)
```

Reduce [F]

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{(ax-1)^{\frac{1}{4}}}{(ax+1)^{\frac{1}{4}}\,x}dx$$

input
```
int(((a*x-1)/(a*x+1))^(1/4)/x,x)
```

output
```
int((a*x - 1)**(1/4)/((a*x + 1)**(1/4)*x),x)
```

3.100 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 192

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

output
```
-a*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/2*a*arctan(-1+2^(1/2)*(1-1/a/x)^(1/4)
/(1+1/a/x)^(1/4))*2^(1/2)+1/2*a*arctan(1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)
^(1/4))*2^(1/2)+1/2*a*arctanh(2^(1/2)*(1-1/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(
1+1/a/x)^(1/2))/(1+1/a/x)^(1/4))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.17

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx=-\frac{8}{3}ae^{\frac{3}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)$$

input
```
Integrate[1/(E^(ArcCoth[a*x]/2)*x^2),x]
```

output
```
(-8*a*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCoth[
a*x])])/3
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.857$, Rules used = {6721, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$

$$\Big\downarrow \ 6721$$

$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \ 60$$

$$a\left(-\sqrt[4]{1-\frac{1}{ax}}\right)\left(\frac{1}{ax}+1\right)^{3/4}-\frac{1}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \ 73$$

$$2a\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\Big\downarrow$ 770

$$2a\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\Big\downarrow$ 755

$$2a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\Big\downarrow$ 1476

$$2a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.\right.$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\Big\downarrow$ 1082

$$2a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\Big\downarrow$ 217

$$\frac{2a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)\right)-}{a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}$$

$\downarrow$ 1479

$$\frac{2a\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.}{\sqrt{2}}\right.\right.}{a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}$$

$\downarrow$ 25

$$2a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}} \cdots\right)}{\sqrt{2}} \cdots\right.\right.$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 27$$

$$2a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\cdots}}{\sqrt[4]{2-\cdots}} \cdots\right)}{\sqrt{2}} \cdots\right.\right.$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \; 1103$$

$$2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\text{lo}\right.\right.$$

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}$$

input
```
Int[1/(E^(ArcCoth[a*x]/2)*x^2),x]
```

output
```
-(a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)) + 2*a*((-(ArcTan[1 - (Sqrt[2]
*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(
1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2
]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqr
t[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x^2}dx$$

input
```
int(((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

output
```
int(((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.91

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{2\sqrt{2}ax\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+2\sqrt{2}ax\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+\sqrt{2}ax\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}\right.}{4x}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*a*x*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + 2*sqr
t(2)*a*x*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + sqrt(2)*a*x*log
(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - sq
rt(2)*a*x*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) - 4*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4))/x
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/4)/x**2,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.97

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$=\frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1
) + 1))*a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.97

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$=\frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="giac")
```

output
```
1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1
) + 1))*a
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.46

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx=-\frac{2\,a\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{\frac{a\,x-1}{a\,x+1}+1}$$
$$-(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,1\mathrm{i}-(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,1\mathrm{i}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/4)/x^2,x)
```

output
```
- (-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i - (-1)^(1/4
)*a*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i - (2*a*((a*x - 1)/(a*
x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)
```

Reduce [F]

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{(ax-1)^{\frac{1}{4}}}{(ax+1)^{\frac{1}{4}}\,x^2}dx$$

input
```
int(((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

output
```
int((a*x - 1)**(1/4)/((a*x + 1)**(1/4)*x**2),x)
```

3.101 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 244

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{a^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}$$

output
```
1/4*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/2*a^2*(1-1/a/x)^(5/4)*(1+1/a/x)^
(3/4)-1/8*a^2*arctan(-1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)-1
/8*a^2*arctan(1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)-1/8*a^2*a
rctanh(2^(1/2)*(1-1/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1+1/a/
x)^(1/4))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.23

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{8}{3}a^2e^{\frac{3}{2}\coth^{-1}(ax)}\left(\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right) - 2\,\text{Hypergeometric2F1}\left(\frac{3}{4},3,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[1/(E^(ArcCoth[a*x]/2)*x^3),x]
```

output
```
(-8*a^2*E^((3*ArcCoth[a*x])/2)*(Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCo
th[a*x])] - 2*Hypergeometric2F1[3/4, 3, 7/4, -E^(2*ArcCoth[a*x])]))/3
```

Rubi [A] (warning: unable to verify)

Time = 0.74 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6721, 90, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}x}d\frac{1}{x}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{4}a\int \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}a\left(\frac{1}{2}\int \frac{1}{(1-\frac{1}{ax})^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)+ \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{770}$$

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right)+ \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\right.\right.\right.}{\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}$$

$\Big\downarrow$ 1082

$$\frac{\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\right.\right.}{\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}$$

$\Big\downarrow$ 217

$$\frac{\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\cdots\right.}{\sqrt{\cdots}}\right.\right.\right.}{\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}$$

$\Big\downarrow$ 1479

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1 \cdots}}{\sqrt[4]{2 \cdots}}}{2\sqrt{2}} \cdots\right.\right.\right.$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 25

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}} \cdots\right.\right.\right.$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$\downarrow$ 27

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.\right.\right.$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}$$

$$\downarrow 1103$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}+$$

$$\frac{1}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log}{}\right.\right.\right.$$

input
```
Int[1/(E^(ArcCoth[a*x]/2)*x^3),x]
```

output
```
(a^2*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4))/2 + (a*(a*(1 - 1/(a*x))^(1/4
)*(1 + 1/(a*x))^(3/4) - 2*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(
2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2
- x^(-4))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(2 - x^(-4))^(1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x^3}\,dx$$

input
```
int(((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

output
```
int(((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.81

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{2\sqrt{2}a^2x^2\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+2\sqrt{2}a^2x^2\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+\sqrt{2}a^2x^2\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} -\right.}{16\,x^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="fricas")
```

output
```
-1/16*(2*sqrt(2)*a^2*x^2*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) +
 2*sqrt(2)*a^2*x^2*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + sqrt(
2)*a^2*x^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) - sqrt(2)*a^2*x^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1) - 4*(3*a^2*x^2 + a*x - 2)*((a*x - 1)/(a*x + 1)
)^(1/4))/x^2
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/4)/x**3,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.93

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{1}{16}\left(2\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="maxima")
```

output
```
-1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
- 1)/(a*x + 1)) + 1) - sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
+ sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(5*a*((a*x - 1)/(a*x + 1))^(5/4) + a*
((a*x - 1)/(a*x + 1))^(1/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1
)^2 + 1))*a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.91

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx =$$

$$-\frac{1}{16}\left(2\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="giac")
```

output
```
-1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
- 1)/(a*x + 1)) + 1) - sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
+ sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(5*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^
(1/4)/(a*x + 1) + a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)
^2)*a
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.54

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{\frac{a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}+\frac{5\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{\frac{(a\,x-1)^2}{(a\,x+1)^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1} + \frac{(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,1\mathrm{i}}{4} + \frac{(-1)^{1/4}\,a^2\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,1\mathrm{i}}{4}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/4)/x^3,x)
```

output
```
((a^2*((a*x - 1)/(a*x + 1))^(1/4))/2 + (5*a^2*((a*x - 1)/(a*x + 1))^(5/4))
/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) + ((-1)^(1/4)*
a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i)/4 + ((-1)^(1/4)*a^2*a
tanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i)/4
```

Reduce [F]

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{(ax-1)^{\frac{1}{4}}}{(ax+1)^{\frac{1}{4}}\,x^3}dx$$

input
```
int(((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

output
```
int((a*x - 1)**(1/4)/((a*x + 1)**(1/4)*x**3),x)
```

3.102 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 281

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} - \frac{3a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{3a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + \frac{3a^3\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

output
```
-3/8*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)-1/12*a^3*(1-1/a/x)^(5/4)*(1+1/a/x
)^(3/4)+1/3*a^2*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4)/x+3/16*a^3*arctan(-1+2^(1/
2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)+3/16*a^3*arctan(1+2^(1/2)*(1-1
/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)+3/16*a^3*arctanh(2^(1/2)*(1-1/a/x)^(1
/4)/(1+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1+1/a/x)^(1/4))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.33

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{96}a^3\left(-\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(29+6e^{2\coth^{-1}(ax)}+9e^{4\coth^{-1}(ax)}\right)}{\left(1+e^{2\coth^{-1}(ax)}\right)^3} + 9\text{RootSum}\left[1 + \#1^4\&,\frac{\coth^{-1}(ax)+2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

input
```
Integrate[1/(E^(ArcCoth[a*x]/2)*x^4),x]
```

output
```
(a^3*((-8*E^((3*ArcCoth[a*x])/2)*(29 + 6*E^(2*ArcCoth[a*x]) + 9*E^(4*ArcCo
th[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 9*RootSum[1 + #1^4 & , (ArcCoth[a*
x] + 2*Log[E^(-1/2*ArcCoth[a*x]) - #1])/#1^3 & ]))/96
```

Rubi [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {6721, 101, 27, 90, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 101}$$

$$\frac{1}{3}a^2\int -\frac{\left(2a-\frac{1}{x}\right)\sqrt[4]{1-\frac{1}{ax}}}{2a\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\int \frac{\left(2a-\frac{1}{x}\right)\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\int \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(\frac{1}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}\right)$$

$$\Big\downarrow \text{770}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int\frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}\right)$$

$$\Big\downarrow \text{755}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{1}{6}a\left(\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)\right.\right.\right.\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(\right.}{}\right.\right.\right.\right.$$

$\big\downarrow$ 1479

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d}{2\sqrt{2}}\right.\right.\right.\right.$$

$\big\downarrow$ 25

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1}}{\sqrt[4]{2}}}{2\sqrt{2}}\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 27

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 1103

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-$$

$$\frac{1}{6}a\left(\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{9}{4}a\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\right.\right.\right.\right.$$

input
```
Int[1/(E^(ArcCoth[a*x]/2)*x^4),x]
```

output
```
(a^2*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4))/(3*x) - (a*((a^2*(1 - 1/(a*x
))^(5/4)*(1 + 1/(a*x))^(3/4))/2 + (9*a*(a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x)
)^(3/4) - 2*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1
/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4
)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(
1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4)
)^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x^4}dx$$

input
```
int(((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

output
```
int(((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.74

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{18\sqrt{2}a^3x^3\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+18\sqrt{2}a^3x^3\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+9\sqrt{2}a^3x^3\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{96x}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="fricas")
```

output
```
1/96*(18*sqrt(2)*a^3*x^3*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) +
 18*sqrt(2)*a^3*x^3*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + 9*sq
rt(2)*a^3*x^3*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) - 9*sqrt(2)*a^3*x^3*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
+ sqrt((a*x - 1)/(a*x + 1)) + 1) - 4*(11*a^3*x^3 + a^2*x^2 - 2*a*x + 8)*((
a*x - 1)/(a*x + 1))^(1/4))/x^3
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/4)/x**4,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.99

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{1}{96}\left(18\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="maxima")
```

output
```
1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) + 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(29*a^2*((a*x - 1)/(a*x
+ 1))^(9/4) + 6*a^2*((a*x - 1)/(a*x + 1))^(5/4) + 9*a^2*((a*x - 1)/(a*x +
1))^(1/4))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^
3/(a*x + 1)^3 + 1))*a
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.96

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{1}{96}\left(18\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="giac")
```

output
```
1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) + 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(6*(a*x - 1)*a^2*((a*x -
 1)/(a*x + 1))^(1/4)/(a*x + 1) + 29*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^
(1/4)/(a*x + 1)^2 + 9*a^2*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1
) + 1)^3)*a
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\frac{\frac{3a^3\left(\frac{ax-1}{ax+1}\right)^{1/4}}{4}+\frac{a^3\left(\frac{ax-1}{ax+1}\right)^{5/4}}{2}+\frac{29a^3\left(\frac{ax-1}{ax+1}\right)^{9/4}}{12}}{\frac{3(ax-1)^2}{(ax+1)^2}+\frac{(ax-1)^3}{(ax+1)^3}+\frac{3(ax-1)}{ax+1}+1} - \frac{(-1)^{1/4}a^3\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)3\mathrm{i}}{8} - \frac{(-1)^{1/4}a^3\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)3\mathrm{i}}{8}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/4)/x^4,x)
```

output
```
- ((3*a^3*((a*x - 1)/(a*x + 1))^(1/4))/4 + (a^3*((a*x - 1)/(a*x + 1))^(5/4
))/2 + (29*a^3*((a*x - 1)/(a*x + 1))^(9/4))/12)/((3*(a*x - 1)^2)/(a*x + 1)
^2 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)*
a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i)/8 - ((-1)^(1/4)*a^3*a
tanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i)/8
```

Reduce [F]

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{(ax-1)^{\frac{1}{4}}}{(ax+1)^{\frac{1}{4}}\,x^4}dx$$

input
```
int(((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

output
```
int((a*x - 1)**(1/4)/((a*x + 1)**(1/4)*x**4),x)
```

3.103 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 253

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^5 - \frac{237\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5} - \frac{237\operatorname{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

output

```
557/640*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^4-157/320*(1-1/a/x)^(3/4)*(1+1
/a/x)^(1/4)*x^2/a^3+5/16*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a^2-11/40*(1-
1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^4/a+1/5*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^5
-237/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5-237/128*arctanh((1+1/
a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5
```

Mathematica [A] (verified)

Time = 5.37 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.68

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$$
$$= \frac{\frac{8192e^{\frac{17}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^5} - \frac{22016e^{\frac{13}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4} + \frac{23936e^{\frac{9}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} - \frac{14032e^{\frac{5}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \frac{5500e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} + 2370\,\text{arcta}}{1280a^5}$$

input
```
Integrate[x^4/E^((3*ArcCoth[a*x])/2),x]
```

output
```
((8192*E^((17*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 - (22016*E^((1
3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (23936*E^((9*ArcCoth[a*x
])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 - (14032*E^((5*ArcCoth[a*x])/2))/(-1 +
E^(2*ArcCoth[a*x]))^2 + (5500*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])
) + 2370*ArcTan[E^(-1/2*ArcCoth[a*x])] + 1185*Log[1 - E^(-1/2*ArcCoth[a*x]
)] - 1185*Log[1 + E^(-1/2*ArcCoth[a*x])])/(1280*a^5)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {6721, 110, 27, 168, 27, 168, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \; 6721$$
$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}x^6}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$
$$\Big\downarrow \; 110$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{1}{5}\int -\frac{\left(11a-\frac{8}{x}\right)x^5}{2a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(11a-\frac{8}{x}\right)x^5}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{10a^2}+\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; 168$$

$$\frac{-\frac{1}{4}\int \frac{3\left(25a-\frac{22}{x}\right)x^4}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{11}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}+\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{3\int \frac{\left(25a-\frac{22}{x}\right)x^4}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{8a}-\frac{11}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}+\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; 168$$

$$\frac{-\frac{3\left(-\frac{1}{3}\int \frac{\left(157a-\frac{100}{x}\right)x^3}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{25}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a}-\frac{11}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}+\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{3\left(-\frac{\int \frac{\left(157a-\frac{100}{x}\right)x^3}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{6a}-\frac{25}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a}-\frac{11}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{10a^2}+\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{168}$$

$$\frac{-\frac{3\left(-\frac{-\frac{1}{2}\int \frac{\left(557a-\frac{314}{x}\right)x^2}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{157}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{25}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a}-\frac{11}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}}}{10a^2}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{3\left(-\frac{-\frac{\int \frac{\left(557a-\frac{314}{x}\right)x^2}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{157}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{25}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a}-\frac{11}{4}ax^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+}}{10a^2}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{168}$$

$$\frac{-\frac{3\left(-\frac{-\frac{-\int \frac{1185x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-557ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{157}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{25}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a}-}{10a^2}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(-\frac{-\frac{-\frac{1185}{2}\int\frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-557ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{157}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{25}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a}}{10a^2}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 104

$$-\frac{3\left(-\frac{-\frac{-2370\int\frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-557ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{157}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{25}{3}ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a}-\frac{11}{4}a}{10a^2}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 756

$$-\frac{-\frac{3\left(-\frac{-\frac{-2370\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-557ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{157}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{25}{3}ax^3\left(\cdots\right.}{8a}\cdots}{10a^2}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; 216$$

$$-\frac{-\frac{3\left(-\frac{-\frac{-2370\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-557ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{157}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{25}{3}ax^3\left(\cdots\right.}{8a}\cdots}{10a^2}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; 219$$

$$-\frac{3\left(-\frac{-\frac{-2370\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-557ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{157}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a}-\frac{25}{3}a \cdots\right)}{8a}}{10a^2}$$

$$\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

input
```
Int[x^4/E^((3*ArcCoth[a*x])/2),x]
```

output
```
((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^5)/5 + ((-11*a*(1 - 1/(a*x))^(3
/4)*(1 + 1/(a*x))^(1/4)*x^4)/4 - (3*((-25*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*
x))^(1/4)*x^3)/3 - ((-157*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/2
 - (-557*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x - 2370*(-1/2*ArcTan[(
1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 -
 1/(a*x))^(1/4)]/2))/(4*a))/(6*a)))/(8*a))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int x^4 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

input
```
int(x^4*((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int(x^4*((a*x-1)/(a*x+1))^(3/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.47

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)} x^4 \, dx = \frac{2\,(128\,a^5x^5 - 48\,a^4x^4 + 24\,a^3x^3 - 114\,a^2x^2 + 243\,ax + 557)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} + 2370\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 1185\,\mathrm{lo}}{1280\,a^5}$$

input
```
integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

output
```
1/1280*(2*(128*a^5*x^5 - 48*a^4*x^4 + 24*a^3*x^3 - 114*a^2*x^2 + 243*a*x +
 557)*((a*x - 1)/(a*x + 1))^(3/4) + 2370*arctan(((a*x - 1)/(a*x + 1))^(1/4
)) - 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1185*log(((a*x - 1)/(a*x
+ 1))^(1/4) - 1))/a^5
```

Sympy [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)} x^4 \, dx = \int x^4 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

input
```
integrate(x**4*((a*x-1)/(a*x+1))**(3/4),x)
```

output
```
Integral(x**4*((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = -\frac{1}{1280}a\left(\frac{4\left(1375\left(\frac{ax-1}{ax+1}\right)^{\frac{19}{4}}-1992\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}+3710\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-1440\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+395\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1}-\frac{10(ax-1)^2a^6}{(ax+1)^2}+\frac{10(ax-1)^3a^6}{(ax+1)^3}-\frac{5(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}-\frac{2370}{\ldots}\right.$$

input
```
integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

output
```
-1/1280*a*(4*(1375*((a*x - 1)/(a*x + 1))^(19/4) - 1992*((a*x - 1)/(a*x + 1
))^(15/4) + 3710*((a*x - 1)/(a*x + 1))^(11/4) - 1440*((a*x - 1)/(a*x + 1))
^(7/4) + 395*((a*x - 1)/(a*x + 1))^(3/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*
(a*x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)
^4*a^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) - 2370*arctan(((a*
x - 1)/(a*x + 1))^(1/4))/a^6 + 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a
^6 - 1185*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.92

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \frac{1}{1280}a\left(\frac{2370\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{1185\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{1185\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}+\frac{4\left(\frac{1440(ax-\ldots}{ax\ldots}\right.}{\ldots}\right.$$

input
```
integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

output
```
1/1280*a*(2370*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 1185*log(((a*x -
1)/(a*x + 1))^(1/4) + 1)/a^6 + 1185*log(abs(((a*x - 1)/(a*x + 1))^(1/4) -
1))/a^6 + 4*(1440*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 3710*(
a*x - 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 1992*(a*x - 1)^3*((a*
x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^3 - 1375*(a*x - 1)^4*((a*x - 1)/(a*x + 1
))^(3/4)/(a*x + 1)^4 - 395*((a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a
*x + 1) - 1)^5))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.91

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \frac{\frac{79\left(\frac{ax-1}{ax+1}\right)^{3/4}}{64}-\frac{9\left(\frac{ax-1}{ax+1}\right)^{7/4}}{2}+\frac{371\left(\frac{ax-1}{ax+1}\right)^{11/4}}{32}-\frac{249\left(\frac{ax-1}{ax+1}\right)^{15/4}}{40}+\frac{275\left(\frac{ax-1}{ax+1}\right)^{19/4}}{64}}{a^5+\frac{10a^5(ax-1)^2}{(ax+1)^2}-\frac{10a^5(ax-1)^3}{(ax+1)^3}+\frac{5a^5(ax-1)^4}{(ax+1)^4}-\frac{a^5(ax-1)^5}{(ax+1)^5}-\frac{5a^5(ax-1)}{ax+1}} + \frac{237\,\mathrm{atan}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{128a^5}-\frac{237\,\mathrm{atanh}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{128a^5}$$

input
```
int(x^4*((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
((79*((a*x - 1)/(a*x + 1))^(3/4))/64 - (9*((a*x - 1)/(a*x + 1))^(7/4))/2 +
 (371*((a*x - 1)/(a*x + 1))^(11/4))/32 - (249*((a*x - 1)/(a*x + 1))^(15/4)
)/40 + (275*((a*x - 1)/(a*x + 1))^(19/4))/64)/(a^5 + (10*a^5*(a*x - 1)^2)/
(a*x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x
+ 1)^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) + (2
37*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) - (237*atanh(((a*x - 1)/(a
*x + 1))^(1/4)))/(128*a^5)
```

Reduce [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = \int \frac{(ax-1)^{\frac{3}{4}}\,x^4}{(ax+1)^{\frac{3}{4}}}dx$$

input
```
int(x^4*((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int(((a*x - 1)**(3/4)*x**4)/(a*x + 1)**(3/4),x)
```

3.104 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 216

$$\begin{aligned}\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx &= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{64a^3}+\frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{32a^2}-\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{8a}\\&\quad+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4+\frac{123\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}\end{aligned}$$

output
```
-63/64*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^3+15/32*(1-1/a/x)^(3/4)*(1+1/a/
x)^(1/4)*x^2/a^2-3/8*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a+1/4*(1-1/a/x)^(
3/4)*(1+1/a/x)^(1/4)*x^4+123/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^
4+123/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Mathematica [A] (verified)

Time = 5.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$$
$$= \frac{\frac{512e^{\frac{13}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4} - \frac{1152e^{\frac{9}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3} + \frac{1008e^{\frac{5}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} - \frac{532e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} - 246\arctan\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right) -}{128a^4}$$

input
```
Integrate[x^3/E^((3*ArcCoth[a*x])/2),x]
```

output
```
((512*E^((13*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 - (1152*E^((9*A
rcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (1008*E^((5*ArcCoth[a*x])/2
))/(-1 + E^(2*ArcCoth[a*x]))^2 - (532*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCo
th[a*x])) - 246*ArcTan[E^(-1/2*ArcCoth[a*x])] - 123*Log[1 - E^(-1/2*ArcCot
h[a*x])] + 123*Log[1 + E^(-1/2*ArcCoth[a*x])])/(128*a^4)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6721, 110, 27, 168, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}x^5}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{1}{4}\int -\frac{3\left(3a-\frac{2}{x}\right)x^4}{2a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\left(3a-\frac{2}{x}\right)x^4}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{8a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{168}$$

$$\frac{3\left(-\frac{1}{3}\int \frac{3\left(5a-\frac{4}{x}\right)x^3}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(-\frac{\int \frac{\left(5a-\frac{4}{x}\right)x^3}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}-ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{168}$$

$$\frac{3\left(-\frac{-\frac{1}{2}\int \frac{\left(21a-\frac{10}{x}\right)x^2}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{2a}-ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a^2}+$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(-\frac{-\frac{\int \frac{\left(21a-\frac{10}{x}\right)x^2}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{2a}-ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\Big\downarrow$ 168

$$\frac{3\left(-\frac{-\frac{-\int \frac{41x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-21ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{2a}-ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\Big\downarrow$ 27

$$\frac{3\left(-\frac{-\frac{-\frac{41}{2}\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-21ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{2a}-ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\Big\downarrow$ 104

$$\frac{3\left(-\frac{-\frac{-82\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-21ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{2a}-ax^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)}{8a^2}+$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 756$$

$$\frac{3\left(-\frac{-\frac{-82\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-21ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{2a}-ax^3\left(1-\right.}{8a^2}$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \ 216$$

$$\frac{3\left(-\frac{-\frac{-82\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-21ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{2a}-ax^3\left(1-\right.}{8a^2}$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \; 219$$

$$\frac{3\left(-\frac{-\frac{-82\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-21ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{5}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{2a}-ax^3\left(1\right.\right.}{8a^2}$$

$$\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

input
```
Int[x^3/E^((3*ArcCoth[a*x])/2),x]
```

output
```
((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^4)/4 + (3*(-(a*(1 - 1/(a*x))^(3
/4)*(1 + 1/(a*x))^(1/4)*x^3) - ((-5*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1
/4)*x^2)/2 - (-21*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x - 82*(-1/2*A
rcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/
4)/(1 - 1/(a*x))^(1/4)]/2))/(4*a))/(2*a)))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}dx$$

input
```
int(x^3*((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int(x^3*((a*x-1)/(a*x+1))^(3/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.51

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \frac{2\left(16\,a^4x^4-8\,a^3x^3+6\,a^2x^2-33\,ax-63\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-}{128\,a^4}$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

output
```
1/128*(2*(16*a^4*x^4 - 8*a^3*x^3 + 6*a^2*x^2 - 33*a*x - 63)*((a*x - 1)/(a*
x + 1))^(3/4) - 246*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 123*log(((a*x -
1)/(a*x + 1))^(1/4) + 1) - 123*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

Sympy [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

input
```
integrate(x**3*((a*x-1)/(a*x+1))**(3/4),x)
```

output
```
Integral(x**3*((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{1}{128}\,a\left(\frac{4\left(133\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}-147\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}+183\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-41\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}+\frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{12}{}\right.$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

output
```
-1/128*a*(4*(133*((a*x - 1)/(a*x + 1))^(15/4) - 147*((a*x - 1)/(a*x + 1))^
(11/4) + 183*((a*x - 1)/(a*x + 1))^(7/4) - 41*((a*x - 1)/(a*x + 1))^(3/4))
/(4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^
3*a^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 246*arctan(((a*x
- 1)/(a*x + 1))^(1/4))/a^5 - 123*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5
+ 123*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{1}{128}\,a\left(\frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{123\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}-\frac{4\left(\frac{183(ax-1)(\cdots}{ax+1}\right.}{}\right.$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

output
```
-1/128*a*(246*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 123*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1)/a^5 + 123*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))
/a^5 - 4*(183*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 147*(a*x -
 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 133*(a*x - 1)^3*((a*x - 1)
/(a*x + 1))^(3/4)/(a*x + 1)^3 - 41*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x
 - 1)/(a*x + 1) - 1)^4))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.89

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \frac{123\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4} - \frac{123\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4} - \frac{\frac{41\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{32} - \frac{183\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{32} + \frac{147\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{32} - \frac{133\left(\frac{a\,x-1}{a\,x+1}\right)^{15/4}}{32}}{a^4 + \frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3} + \frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4} - \frac{4\,a^4\,(a\,x-1)}{a\,x+1}}$$

input
```
int(x^3*((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
(123*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - (123*atan(((a*x - 1)/(
a*x + 1))^(1/4)))/(64*a^4) - ((41*((a*x - 1)/(a*x + 1))^(3/4))/32 - (183*(
(a*x - 1)/(a*x + 1))^(7/4))/32 + (147*((a*x - 1)/(a*x + 1))^(11/4))/32 - (
133*((a*x - 1)/(a*x + 1))^(15/4))/32)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)
^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*
a^4*(a*x - 1))/(a*x + 1))
```

Reduce [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = \int \frac{(ax-1)^{\frac{3}{4}}\,x^3}{(ax+1)^{\frac{3}{4}}}dx$$

input
```
int(x^3*((a*x-1)/(a*x+1))^(3/4),x)
```

output

```
int(((a*x - 1)**(3/4)*x**3)/(a*x + 1)**(3/4),x)
```

3.105 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 179

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx=\frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{24a^2}-\frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{12a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3-\frac{17\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{17\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

output
```
23/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^2-7/12*(1-1/a/x)^(3/4)*(1+1/a/x)
^(1/4)*x^2/a+1/3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3-17/8*arctan((1+1/a/x)
^(1/4)/(1-1/a/x)^(1/4))/a^3-17/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/
a^3
```

Mathematica [A] (verified)

Time = 5.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{\frac{128e^{\frac{9}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}-\frac{240e^{\frac{5}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{180e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+102\arctan\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right)+51\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right)\ldots}{48a^3}$$

input
```
Integrate[x^2/E^((3*ArcCoth[a*x])/2),x]
```

output
```
((128*E^((9*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 - (240*E^((5*Arc
Coth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (180*E^(ArcCoth[a*x]/2))/(-1
+ E^(2*ArcCoth[a*x])) + 102*ArcTan[E^(-1/2*ArcCoth[a*x])] + 51*Log[1 - E^(
-1/2*ArcCoth[a*x])] - 51*Log[1 + E^(-1/2*ArcCoth[a*x])])/(48*a^3)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6721, 110, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}x^4}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{1}{3}\int -\frac{\left(7a-\frac{4}{x}\right)x^3}{2a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(7a-\frac{4}{x}\right)x^3}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{168}$$

$$\frac{-\frac{1}{2}\int \frac{\left(23a-\frac{14}{x}\right)x^2}{2a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{7}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\left(23a-\frac{14}{x}\right)x^2}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}-\frac{7}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{168}$$

$$\frac{-\frac{-\int \frac{51x}{2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-23ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{7}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{-\frac{51}{2}\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-23ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{7}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{104}$$

$$\frac{-\frac{-102\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-23ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{7}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 756

$$\frac{-\frac{-102\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-23ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{7}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 216

$$\frac{-\frac{-102\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-23ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{7}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 219

$$\frac{-\frac{-102\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-23ax\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}-\frac{7}{2}ax^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{6a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

input
```
Int[x^2/E^((3*ArcCoth[a*x])/2),x]
```

output
```
((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^3)/3 + ((-7*a*(1 - 1/(a*x))^(3/
4)*(1 + 1/(a*x))^(1/4)*x^2)/2 - (-23*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(
1/4)*x - 102*(-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTa
nh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/(4*a))/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

input
```
int(x^2*((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int(x^2*((a*x-1)/(a*x+1))^(3/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.58

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$$
$$= \frac{2\,(8\,a^3x^3 - 6\,a^2x^2 + 9\,ax + 23)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} + 102\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) + 51\,\log\left(\left(\frac{ax-}{ax+}\right.\right.}{48\,a^3}$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

output
```
1/48*(2*(8*a^3*x^3 - 6*a^2*x^2 + 9*a*x + 23)*((a*x - 1)/(a*x + 1))^(3/4) +
 102*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 51*log(((a*x - 1)/(a*x + 1))^(1
/4) + 1) + 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

input
```
integrate(x**2*((a*x-1)/(a*x+1))**(3/4),x)
```

output
```
Integral(x**2*((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx =$$
$$-\frac{1}{48}\,a\left(\frac{4\left(45\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}} - 30\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}} + 17\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{3\,(ax-1)a^4}{ax+1} - \frac{3\,(ax-1)^2a^4}{(ax+1)^2} + \frac{(ax-1)^3a^4}{(ax+1)^3} - a^4} - \frac{102\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4} + \frac{51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right)}{a^4}\right.$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

output
```
-1/48*a*(4*(45*((a*x - 1)/(a*x + 1))^(11/4) - 30*((a*x - 1)/(a*x + 1))^(7/
4) + 17*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x -
 1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 102*arctan(((
a*x - 1)/(a*x + 1))^(1/4))/a^4 + 51*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a
^4 - 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{1}{48}a\left(\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4} - \frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4} + \frac{51\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4} + \frac{4\left(\frac{30(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1} \cdots\right.}{\cdots}\right.$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

output
```
1/48*a*(102*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 51*log(((a*x - 1)/(a
*x + 1))^(1/4) + 1)/a^4 + 51*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
 + 4*(30*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 45*(a*x - 1)^2*
((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 - 17*((a*x - 1)/(a*x + 1))^(3/4))/
(a^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \frac{\frac{17\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{15\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}} + \frac{17\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3} - \frac{17\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

input
```
int(x^2*((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
((17*((a*x - 1)/(a*x + 1))^(3/4))/12 - (5*((a*x - 1)/(a*x + 1))^(7/4))/2 +
 (15*((a*x - 1)/(a*x + 1))^(11/4))/4)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)
^2 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (17*at
an(((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) - (17*atanh(((a*x - 1)/(a*x + 1))
^(1/4)))/(8*a^3)
```

Reduce [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = \int \frac{(ax-1)^{\frac{3}{4}}\,x^2}{(ax+1)^{\frac{3}{4}}}dx$$

input
```
int(x^2*((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int(((a*x - 1)**(3/4)*x**2)/(a*x + 1)**(3/4),x)
```

3.106 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 142

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx=-\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2+\frac{9\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

output
```
-3/4*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a+1/2*(1-1/a/x)^(7/4)*(1+1/a/x)^(1/
4)*x^2+9/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+9/4*arctanh((1+1/a/
x)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.49

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx$$
$$=\frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(-7+3e^{2\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+9\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+9\text{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

input
```
Integrate[x/E^((3*ArcCoth[a*x])/2),x]
```

output
```
((-2*E^(ArcCoth[a*x]/2)*(-7 + 3*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*
x]))^2 + 9*ArcTan[E^(ArcCoth[a*x]/2)] + 9*ArcTanh[E^(ArcCoth[a*x]/2)])/(4*
a^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6721, 107, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-\frac{3}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}x^3}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 107}$$

$$\frac{3\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}x^2}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{ 105}$$

$$\frac{3\left(x\left(-\left(1-\frac{1}{ax}\right)^{3/4}\right)\sqrt[4]{\frac{1}{ax}+1}-\frac{3\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}\right)}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\Big\downarrow$ 104

$$\frac{3\left(x\left(-\left(1-\frac{1}{ax}\right)^{3/4}\right)\sqrt[4]{\frac{1}{ax}+1}-\frac{6\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}\right)}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\Big\downarrow$ 756

$$\frac{3\left(x\left(-\left(1-\frac{1}{ax}\right)^{3/4}\right)\sqrt[4]{\frac{1}{ax}+1}-\frac{6\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}\right)}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\Big\downarrow$ 216

$$\frac{3\left(x\left(-\left(1-\frac{1}{ax}\right)^{3/4}\right)\sqrt[4]{\frac{1}{ax}+1}-\frac{6\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}\right)}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\downarrow \text{219}$$

$$\frac{3\left(x\left(-\left(1-\frac{1}{ax}\right)^{3/4}\right)\sqrt[4]{\frac{1}{ax}+1}-\frac{6\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}\right)}{4a}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

input
```
Int[x/E^((3*ArcCoth[a*x])/2),x]
```

output
```
((1 - 1/(a*x))^(7/4)*(1 + 1/(a*x))^(1/4)*x^2)/2 + (3*(-((1 - 1/(a*x))^(3/4
)*(1 + 1/(a*x))^(1/4)*x) - (6*(-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x
))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2))/a))/(4*a)
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}dx$$

input
```
int(x*((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int(x*((a*x-1)/(a*x+1))^(3/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx = \frac{2\,(2\,a^2x^2-3\,ax-5)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^2*x^2 - 3*a*x - 5)*((a*x - 1)/(a*x + 1))^(3/4) - 18*arctan(((a
*x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 9*log
(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx = \int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

input
```
integrate(x*((a*x-1)/(a*x+1))**(3/4),x)
```

output
```
Integral(x*((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.07

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx = -\frac{1}{8}\,a\left(\frac{4\left(7\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}+\frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\right.}{a^3}$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

output
```
-1/8*a*(4*(7*((a*x - 1)/(a*x + 1))^(7/4) - 3*((a*x - 1)/(a*x + 1))^(3/4))/
(2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + 18*arcta
n(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1
)/a^3 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx = -\frac{1}{8}a\left(\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3} - \frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3} + \frac{9\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3} - \frac{4\left(\frac{7(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1} - 3\right.}{a^3\left(\frac{ax-1}{ax+1}-1\right)}\right.$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

output
```
-1/8*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 9*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^3 + 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 -
4*(7*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 3*((a*x - 1)/(a*x +
 1))^(3/4))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 23.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx = \frac{9\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2} - \frac{9\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2} - \frac{\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{2} - \frac{7\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}}{a^2 + \frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{2\,a^2\,(a\,x-1)}{a\,x+1}}$$

input
```
int(x*((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
(9*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) - (9*atan(((a*x - 1)/(a*x +
 1))^(1/4)))/(4*a^2) - ((3*((a*x - 1)/(a*x + 1))^(3/4))/2 - (7*((a*x - 1)/
(a*x + 1))^(7/4))/2)/(a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x -
1))/(a*x + 1))
```

Reduce [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx=\int\frac{(ax-1)^{\frac{3}{4}}\,x}{(ax+1)^{\frac{3}{4}}}dx$$

input
```
int(x*((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int(((a*x - 1)**(3/4)*x)/(a*x + 1)**(3/4),x)
```

3.107 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 98

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx = \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x-\frac{3\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{3\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

output

```
(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x-3*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)
)/a-3*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx=\frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-3\arctan\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-3\text{arctanh}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

input
```
Integrate[E^((-3*ArcCoth[a*x])/2),x]
```

output
```
((2*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) - 3*ArcTan[E^(ArcCoth[a*
x]/2)] - 3*ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6720, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6720}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}x^2}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\frac{3\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{2a}+x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{104}$$

$$\frac{6\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{a}+x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 756

$$\frac{6\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 216

$$\frac{6\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}+x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 219

$$\frac{6\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)}{a}+x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

input
```
Int[E^((-3*ArcCoth[a*x])/2),x]
```

output
```
(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x + (6*(-1/2*ArcTan[(1 + 1/(a*x))^
(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/
4)]/2))/a
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

input
```
int(((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int(((a*x-1)/(a*x+1))^(3/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)} dx = \frac{2(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} + 6\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) + 3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right)}{2a}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

output
```
1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4) + 6*arctan(((a*x - 1)/(a*x +
1))^(1/4)) - 3*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 3*log(((a*x - 1)/(a*
x + 1))^(1/4) - 1))/a
```

Sympy [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)} dx = \int \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/4),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx =$$

$$-\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{6\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

output
```
-1/2*a*(4*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) - 6*
arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 3*log(((a*x - 1)/(a*x + 1))^(1/4
) + 1)/a^2 - 3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx$$

$$=\frac{1}{2}\,a\left(\frac{6\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{3\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{3\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

output
```
1/2*a*(6*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 3*log(((a*x - 1)/(a*x +
 1))^(1/4) + 1)/a^2 + 3*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 - 4*
((a*x - 1)/(a*x + 1))^(3/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

Mupad [B] (verification not implemented)

Time = 23.48 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx = \frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}} + \frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a} - \frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(3/4))/(a - (a*(a*x - 1))/(a*x + 1)) + (3*atan(((
a*x - 1)/(a*x + 1))^(1/4)))/a - (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/a
```

Reduce [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx = \int \frac{(ax-1)^{\frac{3}{4}}}{(ax+1)^{\frac{3}{4}}}dx$$

input
```
int(((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int((a*x - 1)**(3/4)/(a*x + 1)**(3/4),x)
```

3.108 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 221

$$\begin{aligned}\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx &= \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)\\
&\quad-\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\\
&\quad+\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)\\
&\quad+2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\end{aligned}$$

output
```
-2^(1/2)*arctan(-1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))-2^(1/2)*arctan
(1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))+2*arctan((1+1/a/x)^(1/4)/(1-1/
a/x)^(1/4))+2^(1/2)*arctanh(2^(1/2)*(1-1/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(1+
1/a/x)^(1/2))/(1+1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.13

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = 8e^{\frac{1}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},e^{4\coth^{-1}(ax)}\right)$$

input
```
Integrate[1/(E^((3*ArcCoth[a*x])/2)*x),x]
```

output
```
8*E^(ArcCoth[a*x]/2)*Hypergeometric2F1[1/8, 1, 9/8, E^(4*ArcCoth[a*x])]
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.17, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {6721, 140, 73, 104, 756, 216, 219, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}x}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{140}$$

$$\frac{\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}}{a}-\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$\downarrow$ 73

$$-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-\int \frac{x}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$\downarrow$ 104

$$-4\int \frac{1}{\frac{1}{x^4}-1}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}$$

$\downarrow$ 756

$$-4\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}$$

$\downarrow$ 216

$$-4\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}$$

$\downarrow$ 219

$$-4\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$\downarrow$ 854

$$-4\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$\downarrow$ 826

$$-4\left(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$\downarrow$ 1476

$$-4\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$\downarrow$ 1082

$$-4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$\downarrow$ 217

$$-4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$\Big\downarrow$ 1479

$$-4\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\ldots}}{\sqrt[4]{2-\ldots}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$\Big\downarrow$ 25

$$-4\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{a\ldots}}}{\sqrt[4]{2-\frac{1}{x^4}}}\ldots\right.}{\sqrt{2}}\ldots\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \; 27$$

$$-4\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1\ldots}}{\sqrt[4]{2-\ldots}}\ldots\right.}{\sqrt{2}}\ldots\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$-4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-$$

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\frac{\text{lo}}{}\right.\right.$$

input
```
Int[1/(E^((3*ArcCoth[a*x])/2)*x),x]
```

output
```
-4*(-1/2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - ArcTanh[(1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2) - 4*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a
*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x
))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))
^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1
/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x}dx$$

input
```
int(((a*x-1)/(a*x+1))^(3/4)/x,x)
```

output
```
int(((a*x-1)/(a*x+1))^(3/4)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.89

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\sqrt{2}\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - \sqrt{2}\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right) + \frac{1}{2}\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right) - \frac{1}{2}\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right) - 2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="fricas")
```

output
```
-sqrt(2)*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) - sqrt(2)*arctan(
sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + 1/2*sqrt(2)*log(sqrt(2)*((a*x -
 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 1/2*sqrt(2)*log(-s
qrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 2*ar
ctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) -
 log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/4)/x,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/4)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.01

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{\right.}{a}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="maxima")
```

output
```
-1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a -
2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/
4) - 1)/a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.05

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{}\right.}{}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="giac")
```

output
```
-1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
 1)/(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/
a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x +
 1))^(1/4) - 1))/a)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.46

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)\,2\mathrm{i}-2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)$$
$$+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-1+1\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-1-\mathrm{i}\right)$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/4)/x,x)
```

output
```
- atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*2i - 2*atan(((a*x - 1)/(a*x + 1))^(
1/4)) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1
- 1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1
+ 1i)
```

Reduce [F]

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{(ax-1)^{\frac{3}{4}}}{(ax+1)^{\frac{3}{4}}\,x}dx$$

input
```
int(((a*x-1)/(a*x+1))^(3/4)/x,x)
```

output
```
int((a*x - 1)**(3/4)/((a*x + 1)**(3/4)*x),x)
```

3.109 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 194

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{3a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{3a\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

output

```
-a*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+3/2*a*arctan(-1+2^(1/2)*(1-1/a/x)^(1/4)
/(1+1/a/x)^(1/4))*2^(1/2)+3/2*a*arctan(1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)
^(1/4))*2^(1/2)-3/2*a*arctanh(2^(1/2)*(1-1/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(
1+1/a/x)^(1/2))/(1+1/a/x)^(1/4))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.77

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = a\left(-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}+\frac{3\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}} - \frac{3\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}} + \frac{3\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}} - \frac{3\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}\right)$$

input
```
Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^2),x]
```

output
```
a*((-2*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + (3*ArcTan[1 - Sqrt[2
]*E^(ArcCoth[a*x]/2)])/Sqrt[2] - (3*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)]
)/Sqrt[2] + (3*Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sq
rt[2]) - (3*Log[1 + Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[
2]))
```

Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6721, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$\downarrow$ 60

$$a\left(-\left(1-\frac{1}{ax}\right)^{3/4}\right)\sqrt[4]{\frac{1}{ax}+1}-\frac{3}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$\downarrow$ 73

$$6a\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 854

$$6a\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 826

$$6a\left(\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 1476

$$6a\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1 \ldots}}{\sqrt[4]{2 \ldots}}\right.$$

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 1082

$$\frac{6a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-}{a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{217}$$

$$\frac{6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-}{a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{1479}$$

$$6a\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\cdots}}{\sqrt[4]{2-\frac{1}{x^{\cdots}}}}\cdots\right.}{\sqrt{2}}\cdots\right.\right.$$

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 25

$$6a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\cdots\right.}{\sqrt{2}}\cdots\right.\right.$$

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 27

$$6a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\cdots}}{\sqrt[4]{2-\cdots}}\cdots\right.}{\sqrt{2}}\cdots\right.\right.$$

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 1103

$$6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\cdots\right.\right.$$

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}$$

input
```
Int[1/(E^((3*ArcCoth[a*x])/2)*x^2),x]
```

output
```
-(a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)) + 6*a*((-(ArcTan[1 - (Sqrt[2]
*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(
1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt
[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^2}dx$$

input
```
int(((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

output
```
int(((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.90

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{6\sqrt{2}ax\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+6\sqrt{2}ax\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-3\sqrt{2}ax\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-}{ax-}}\right.}{4x}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="fricas")
```

output
```
1/4*(6*sqrt(2)*a*x*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + 6*sqr
t(2)*a*x*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - 3*sqrt(2)*a*x*l
og(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) +
3*sqrt(2)*a*x*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) - 4*(a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4))/x
```

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/4)/x**2,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/4)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.96

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-3\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="maxima")
```

output
```
1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
)) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) - 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x
 + 1) + 1))*a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.96

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$= \frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-3\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="giac")
```

output
```
1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4)
)) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) - 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x
 + 1) + 1))*a
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.45

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$$
$$=3\,(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-3\,(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-\frac{2\,a\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{\frac{a\,x-1}{a\,x+1}+1}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/4)/x^2,x)
```

output
```
3*(-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - 3*(-1)^(1/4)
*a*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - (2*a*((a*x - 1)/(a*x +
1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)
```

Reduce [F]

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx=\int \frac{(ax-1)^{\frac{3}{4}}}{(ax+1)^{\frac{3}{4}}\,x^2}dx$$

input
```
int(((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

output
```
int((a*x - 1)**(3/4)/((a*x + 1)**(3/4)*x**2),x)
```

3.110 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 244

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{9a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{9a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{9a^2\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}$$

output
```
3/4*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/2*a^2*(1-1/a/x)^(7/4)*(1+1/a/x)^
(1/4)-9/8*a^2*arctan(-1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)-9
/8*a^2*arctan(1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)+9/8*a^2*a
rctanh(2^(1/2)*(1-1/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1+1/a/
x)^(1/4))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.71

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{16}a^2\left(\frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{24e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}} - 18\sqrt{2}\arctan\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 18\sqrt{2}\arctan\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}\right) - 9\sqrt{2}\log\left(1-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right) + 9\sqrt{2}\log\left(1+\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^3),x]
```

output
```
(a^2*((32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x]))^2 + (24*E^(ArcCoth[
a*x]/2))/(1 + E^(2*ArcCoth[a*x])) - 18*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^(ArcCo
th[a*x]/2)] + 18*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)] - 9*Sqrt[2
]*Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]] + 9*Sqrt[2]*Log[1 +
 Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]))/16
```

Rubi [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6721, 90, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}}{\left(1+\frac{1}{ax}\right)^{3/4}x}d\frac{1}{x}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{3}{4}a\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{3}{4}a\left(\frac{3}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{ 854}$$

$$\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\int \frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right)+}{\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 1476

$$\frac{\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}\right.\right.\right.}{\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 1082

$$\frac{\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\right.\right.}{\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 217

$$\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1}{1\ldots}\right.\right.$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 1479

$$\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\ldots}}{\sqrt[4]{2-\ldots}}}{2\sqrt{2}}\right.\right.\right.$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 25

$$\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\cdots}}{\sqrt[4]{2-\cdots}}}{2\sqrt{2}}\cdots$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 27

$$\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1\cdots}}{\sqrt[4]{2\cdots}}\cdots$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}$$

$\downarrow$ 1103

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}+$$
$$\frac{3}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\right.\right.\right.\right.$$

input
```
Int[1/(E^((3*ArcCoth[a*x])/2)*x^3),x]
```

output
```
(a^2*(1 - 1/(a*x))^(7/4)*(1 + 1/(a*x))^(1/4))/2 + (3*a*(a*(1 - 1/(a*x))^(3
/4)*(1 + 1/(a*x))^(1/4) - 6*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))
/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(
2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2
- x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/
4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^3}dx$$

input
```
int(((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

output
```
int(((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.82

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx =$$
$$-\frac{18\sqrt{2}a^2x^2\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+18\sqrt{2}a^2x^2\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-9\sqrt{2}a^2x^2\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right.\right.}{16\,x^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="fricas")
```

output
```
-1/16*(18*sqrt(2)*a^2*x^2*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1)
+ 18*sqrt(2)*a^2*x^2*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - 9*s
qrt(2)*a^2*x^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) + 9*sqrt(2)*a^2*x^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
 + sqrt((a*x - 1)/(a*x + 1)) + 1) - 4*(5*a^2*x^2 + 3*a*x - 2)*((a*x - 1)/(
a*x + 1))^(3/4))/x^2
```

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/4)/x**3,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/4)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.93

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx =$$

$$-\frac{1}{16}\left(9\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right.\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="maxima")
```

output
```
-1/16*(9*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1))*a - 8*(7*a*((a*x - 1)/(a*x + 1))^(7/4) + 3*a*
((a*x - 1)/(a*x + 1))^(3/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1
)^2 + 1))*a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.92

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx =$$

$$-\frac{1}{16}\left(18\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="giac")
```

output
```
-1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
1))^(1/4))) - 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1) + 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^
(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(7*(a*x - 1)*a*((a*x - 1)/(a*x
+ 1))^(3/4)/(a*x + 1) + 3*a*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x +
 1) + 1)^2)*a
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.54

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{\frac{3a^2\left(\frac{ax-1}{ax+1}\right)^{3/4}}{2}+\frac{7a^2\left(\frac{ax-1}{ax+1}\right)^{7/4}}{2}}{\frac{(ax-1)^2}{(ax+1)^2}+\frac{2(ax-1)}{ax+1}+1} - \frac{9(-1)^{1/4}a^2\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{4} + \frac{9(-1)^{1/4}a^2\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{4}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/4)/x^3,x)
```

output
```
((3*a^2*((a*x - 1)/(a*x + 1))^(3/4))/2 + (7*a^2*((a*x - 1)/(a*x + 1))^(7/4
))/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) - (9*(-1)^(1
/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 + (9*(-1)^(1/4)*a^
2*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4
```

Reduce [F]

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{(ax-1)^{\frac{3}{4}}}{(ax+1)^{\frac{3}{4}}\,x^3}dx$$

input
```
int(((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

output
```
int((a*x - 1)**(3/4)/((a*x + 1)**(3/4)*x**3),x)
```

3.111 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 281

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x}-\frac{17a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}+\frac{17a^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}-\frac{17a^3\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

output
```
-17/24*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-1/4*a^3*(1-1/a/x)^(7/4)*(1+1/a/
x)^(1/4)+1/3*a^2*(1-1/a/x)^(7/4)*(1+1/a/x)^(1/4)/x+17/16*a^3*arctan(-1+2^(
1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)+17/16*a^3*arctan(1+2^(1/2)*(
1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)-17/16*a^3*arctanh(2^(1/2)*(1-1/a/x
)^(1/4)/(1+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1+1/a/x)^(1/4))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.33

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{1}{96}a^3\left(-\frac{8e^{\frac{1}{2}\coth^{-1}(ax)}\left(45+30e^{2\coth^{-1}(ax)}+17e^{4\coth^{-1}(ax)}\right)}{\left(1+e^{2\coth^{-1}(ax)}\right)^3} + 51\text{RootSum}\left[1 + \#1^4\&,\frac{\coth^{-1}(ax)+2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1}\&\right]\right)$$

input
```
Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^4),x]
```

output
```
(a^3*((-8*E^(ArcCoth[a*x]/2)*(45 + 30*E^(2*ArcCoth[a*x]) + 17*E^(4*ArcCoth
[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 51*RootSum[1 + #1^4 & , (ArcCoth[a*x
] + 2*Log[E^(-1/2*ArcCoth[a*x]) - #1])/#1 & ]))/96
```

Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {6721, 101, 27, 90, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}}{\left(1+\frac{1}{ax}\right)^{3/4}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 101}$$

$$\frac{1}{3}a^2\int -\frac{\left(2a-\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{3/4}}{2a\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\int \frac{\left(2a-\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{3/4}}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+\frac{3}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(\frac{3}{2}\int \frac{1}{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}+a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}\right)+\frac{3}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\int \frac{1}{\left(2-\frac{1}{x^4}\right)^{3/4}x^2}d\sqrt[4]{1-\frac{1}{ax}}\right)+\frac{3}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{ 854}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\int\frac{1}{\left(1+\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{3}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}\right)$$

$\downarrow$ 826

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right)+\frac{3}{2}a^2\left(1-\frac{1}{ax}\right.\right.$$

$\downarrow$ 1476

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\ldots}\right.\right.\right.\right.$$

$\downarrow$ 1082

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right.\right.\right.\right.$$

$\downarrow$ 217

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-$$

$$\frac{1}{6}a\left(\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)-\right.\right.\right.$$

$\big\downarrow$ 1479

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-$$

$$\frac{1}{6}a\left(\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d}{2\sqrt{2}}\right.\right.\right.\right.$$

$\big\downarrow$ 25

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{}}{\sqrt[4]{}}}{2\sqrt{2}}\right.\right.\right.\right.$$

$\downarrow$ 27

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-\frac{1}{6}a\left(\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+}\right.\right.\right.\right.$$

$\downarrow$ 1103

$$\frac{a^2(1-\frac{1}{ax})^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}-$$

$$\frac{1}{6}a\left(\frac{3}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{17}{4}a\left(a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-6a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\text{arc}}{}\right.\right.\right.\right.\right.$$

input
```
Int[1/(E^((3*ArcCoth[a*x])/2)*x^4),x]
```

output
```
(a^2*(1 - 1/(a*x))^(7/4)*(1 + 1/(a*x))^(1/4))/(3*x) - (a*((3*a^2*(1 - 1/(a
*x))^(7/4)*(1 + 1/(a*x))^(1/4))/2 + (17*a*(a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a
*x))^(1/4) - 6*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))
^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(
1/4)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/
4) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-
4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2)))/4))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^4}dx$$

input
```
int(((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

output
```
int(((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.74

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{102\sqrt{2}a^3x^3\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+102\sqrt{2}a^3x^3\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-51\sqrt{2}a^3x^3\log\left(\sqrt{2}\left(\frac{ax-}{ax+}\right.\right.}{9}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="fricas")
```

output
```
1/96*(102*sqrt(2)*a^3*x^3*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1)
+ 102*sqrt(2)*a^3*x^3*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - 51
*sqrt(2)*a^3*x^3*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1) + 51*sqrt(2)*a^3*x^3*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1
/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 4*(23*a^3*x^3 + 9*a^2*x^2 - 6*a*x +
 8)*((a*x - 1)/(a*x + 1))^(3/4))/x^3
```

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/4)/x**4,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/4)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.96

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$= \frac{1}{96}\left(51\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="maxima")
```

output
```
1/96*(51*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1))*a^2 - 8*(45*a^2*((a*x - 1)/(a*x + 1))^(11/4)
+ 30*a^2*((a*x - 1)/(a*x + 1))^(7/4) + 17*a^2*((a*x - 1)/(a*x + 1))^(3/4))
/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1
)^3 + 1))*a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.96

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$$

$$= \frac{1}{96}\left(102\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+102\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)\right.\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="giac")
```

output
```
1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a
*x + 1))^(1/4))) - 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
+ sqrt((a*x - 1)/(a*x + 1)) + 1) + 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/
(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(30*(a*x - 1)*a^2*((
a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) + 45*(a*x - 1)^2*a^2*((a*x - 1)/(a*x +
 1))^(3/4)/(a*x + 1)^2 + 17*a^2*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a
*x + 1) + 1)^3)*a
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{17\,(-1)^{1/4}\,a^3\,\mathrm{atan}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8} - \frac{\frac{17\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}+\frac{5\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{15\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{\frac{3\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{(a\,x-1)^3}{(a\,x+1)^3}+\frac{3\,(a\,x-1)}{a\,x+1}+1} - \frac{17\,(-1)^{1/4}\,a^3\,\mathrm{atanh}\left((-1)^{1/4}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/4)/x^4,x)
```

output
```
(17*(-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - ((17*
a^3*((a*x - 1)/(a*x + 1))^(3/4))/12 + (5*a^3*((a*x - 1)/(a*x + 1))^(7/4))/
2 + (15*a^3*((a*x - 1)/(a*x + 1))^(11/4))/4)/((3*(a*x - 1)^2)/(a*x + 1)^2
+ (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) - (17*(-1)^(1/4)*
a^3*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8
```

Reduce [F]

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{(ax-1)^{\frac{3}{4}}}{(ax+1)^{\frac{3}{4}}\,x^4}dx$$

input
```
int(((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

output
```
int((a*x - 1)**(3/4)/((a*x + 1)**(3/4)*x**4),x)
```

3.112 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 287

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx=\frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}}+\frac{5533\sqrt[4]{1-\frac{1}{ax}}x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}}-\frac{1189\sqrt[4]{1-\frac{1}{ax}}x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}}+\frac{181\sqrt[4]{1-\frac{1}{ax}}x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{21\sqrt[4]{1-\frac{1}{ax}}x^4}{40a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt[4]{1-\frac{1}{ax}}x^5}{5\sqrt[4]{1+\frac{1}{ax}}}+\frac{1003\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{1003\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

output
```
26111/1920*(1-1/a/x)^(1/4)/a^5/(1+1/a/x)^(1/4)+5533/1920*(1-1/a/x)^(1/4)*x
/a^4/(1+1/a/x)^(1/4)-1189/960*(1-1/a/x)^(1/4)*x^2/a^3/(1+1/a/x)^(1/4)+181/
240*(1-1/a/x)^(1/4)*x^3/a^2/(1+1/a/x)^(1/4)-21/40*(1-1/a/x)^(1/4)*x^4/a/(1
+1/a/x)^(1/4)+1/5*(1-1/a/x)^(1/4)*x^5/(1+1/a/x)^(1/4)+1003/128*arctan((1+1
/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5-1003/128*arctanh((1+1/a/x)^(1/4)/(1-1/a/x
)^(1/4))/a^5
```

Mathematica [A] (verified)

Time = 5.43 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.69

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$$
$$= \frac{8e^{-\frac{1}{2}\coth^{-1}(ax)} - \frac{32e^{-\frac{1}{2}\coth^{-1}(ax)}}{5\left(-1+e^{-2\coth^{-1}(ax)}\right)^5} - \frac{122e^{-\frac{1}{2}\coth^{-1}(ax)}}{5\left(-1+e^{-2\coth^{-1}(ax)}\right)^4} - \frac{233e^{-\frac{1}{2}\coth^{-1}(ax)}}{6\left(-1+e^{-2\coth^{-1}(ax)}\right)^3} - \frac{1661e^{-\frac{1}{2}\coth^{-1}(ax)}}{48\left(-1+e^{-2\coth^{-1}(ax)}\right)^2} - \dots}{\dots}$$

input
```
Integrate[x^4/E^((5*ArcCoth[a*x])/2),x]
```

output
```
(8/E^(ArcCoth[a*x]/2) - 32/(5*E^(ArcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x])
)^5) - 122/(5*E^(ArcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x]))^4) - 233/(6*E^
(ArcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x]))^3) - 1661/(48*E^(ArcCoth[a*x]/
2)*(-1 + E^(-2*ArcCoth[a*x]))^2) - 4117/(192*E^(ArcCoth[a*x]/2)*(-1 + E^(-
2*ArcCoth[a*x]))) - (1003*ArcTan[E^(-1/2*ArcCoth[a*x])])/128 + (1003*Log[1
 - E^(-1/2*ArcCoth[a*x])])/256 - (1003*Log[1 + E^(-1/2*ArcCoth[a*x])])/256
)/a^5
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.08, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.286$, Rules used = {6721, 109, 27, 168, 27, 168, 27, 168, 27, 168, 27, 172, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{6721}$$
$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}x^6}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$
$$\Big\downarrow \text{109}$$

$$\frac{1}{5}\int \frac{\left(21a-\frac{20}{x}\right)x^5}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(21a-\frac{20}{x}\right)x^5}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 168$$

$$\frac{-\frac{1}{4}\int \frac{\left(181a-\frac{168}{x}\right)x^4}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{21ax^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{\left(181a-\frac{168}{x}\right)x^4}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{8a}-\frac{21ax^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 168$$

$$\frac{-\frac{-\frac{1}{3}\int \frac{\left(1189a-\frac{1086}{x}\right)x^3}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{-\frac{\int \frac{\left(1189a-\frac{1086}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{168}$$

$$\frac{-\frac{-\frac{-\frac{1}{2}\int \frac{\left(5533a-\frac{4756}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{-\frac{-\frac{\int \frac{\left(5533a-\frac{4756}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{168}$$

$$\frac{-\frac{-\frac{-\frac{-\int \frac{\left(15045a-\frac{11066}{x}\right)x}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{-\frac{-\frac{-\frac{\int \frac{\left(15045a-\frac{11066}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{2a}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}}{10a^2}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$\Big\downarrow$ 172

$$\frac{-\frac{-\frac{-\frac{-\frac{2a\int \frac{15045x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{52222a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{1\cdots}}{4\sqrt[4]{\frac{1}{ax}\cdots}}}{10a^2}$$

$$\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow\ 27$$

$$\frac{-\frac{-\frac{-\frac{-\frac{15045a\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{52222a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\cdots}{4\sqrt[4]{\cdots}}}{10a^2}$$

$$\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow\ 104$$

$$\frac{-\frac{-\frac{-\frac{-\frac{60180a\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{52222a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{}}{4\sqrt[4]{}}}{10a^2}$$

$$\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{-\frac{-\frac{-\frac{\frac{52222a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-60180a\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a}-\frac{21ax^4\sqrt[4]{}}{4\sqrt[4]{\frac{1}{ax}}}}{10a^2}$$

$$\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 827$$

$$-\frac{-\frac{-\frac{-\frac{60180a\left(\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{52222a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{\cdots}}{3\sqrt[4]{\cdots}}\cdots}{8a}\cdots}{10a^2}\cdots$$

$$\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 216$$

$$-\frac{-\frac{-\frac{-\frac{60180a\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{52222a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181ax^3\sqrt[4]{\cdots}}{3\sqrt[4]{\cdots}}\cdots}{8a}\cdots}{10a^2}\cdots$$

$$\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 219$$

$$
\frac{-\frac{-\frac{-\frac{-\frac{60180a\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{52222a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{5533ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{1189ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{181a\cdots}{3\cdots}}{8a}\cdots}{10a^2}
$$

$$
\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}
$$

input
```
Int[x^4/E^((5*ArcCoth[a*x])/2),x]
```

output
```
((1 - 1/(a*x))^(1/4)*x^5)/(5*(1 + 1/(a*x))^(1/4)) + ((-21*a*(1 - 1/(a*x))^
(1/4)*x^4)/(4*(1 + 1/(a*x))^(1/4)) - ((-181*a*(1 - 1/(a*x))^(1/4)*x^3)/(3*
(1 + 1/(a*x))^(1/4)) - ((-1189*a*(1 - 1/(a*x))^(1/4)*x^2)/(2*(1 + 1/(a*x))
^(1/4)) - ((-5533*a*(1 - 1/(a*x))^(1/4)*x)/(1 + 1/(a*x))^(1/4) - ((52222*a
*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4) + 60180*a*(ArcTan[(1 + 1/(a*x))^
(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(
1/4)]/2))/(2*a))/(4*a))/(6*a))/(8*a))/(10*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int x^4 \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} dx$$

input
```
int(x^4*((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int(x^4*((a*x-1)/(a*x+1))^(5/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.41

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)} x^4 \, dx = \frac{2\left(384\,a^5x^5 - 1008\,a^4x^4 + 1448\,a^3x^3 - 2378\,a^2x^2 + 5533\,ax + 26111\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 30090\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{3840\,a^5}$$

input
```
integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

output
```
1/3840*(2*(384*a^5*x^5 - 1008*a^4*x^4 + 1448*a^3*x^3 - 2378*a^2*x^2 + 5533
*a*x + 26111)*((a*x - 1)/(a*x + 1))^(1/4) - 30090*arctan(((a*x - 1)/(a*x +
 1))^(1/4)) - 15045*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 15045*log(((a*x
 - 1)/(a*x + 1))^(1/4) - 1))/a^5
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = \text{Timed out}$$

input
```
integrate(x**4*((a*x-1)/(a*x+1))**(5/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.97

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx =$$
$$-\frac{1}{3840}a\left(\frac{4\left(20585\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}} - 49120\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}} + 61130\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}} - 33816\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} + 7365\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1} - \frac{10(ax-1)^2a^6}{(ax+1)^2} + \frac{10(ax-1)^3a^6}{(ax+1)^3} - \frac{5(ax-1)^4a^6}{(ax+1)^4} + \frac{(ax-1)^5a^6}{(ax+1)^5} - a^6} + \cdots\right.$$

input
```
integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

output
```
-1/3840*a*(4*(20585*((a*x - 1)/(a*x + 1))^(17/4) - 49120*((a*x - 1)/(a*x +
 1))^(13/4) + 61130*((a*x - 1)/(a*x + 1))^(9/4) - 33816*((a*x - 1)/(a*x +
1))^(5/4) + 7365*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) -
 10*(a*x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x
- 1)^4*a^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 30090*arctan
(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 15045*log(((a*x - 1)/(a*x + 1))^(1/4)
+ 1)/a^6 - 15045*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6 - 30720*((a*x -
1)/(a*x + 1))^(1/4)/a^6)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.89

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = -\frac{1}{3840}a\left(\frac{30090\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{15045\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{15045\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-\frac{30720}{}\right.$$

input
```
integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

output
```
-1/3840*a*(30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 15045*log(((a*x
 - 1)/(a*x + 1))^(1/4) + 1)/a^6 - 15045*log(abs(((a*x - 1)/(a*x + 1))^(1/4
) - 1))/a^6 - 30720*((a*x - 1)/(a*x + 1))^(1/4)/a^6 - 4*(33816*(a*x - 1)*(
(a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 61130*(a*x - 1)^2*((a*x - 1)/(a*x +
 1))^(1/4)/(a*x + 1)^2 + 49120*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(1/4)/(a*
x + 1)^3 - 20585*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^4 - 736
5*((a*x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a*x + 1) - 1)^5))
```

Mupad [B] (verification not implemented)

Time = 24.06 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.88

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = \frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^5} + \frac{\frac{491\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{64}-\frac{1409\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{40}+\frac{6113\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{96}-\frac{307\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{6}+\frac{4117\left(\frac{a\,x-1}{a\,x+1}\right)^{17/4}}{192}}{a^5+\frac{10\,a^5\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a^5\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a^5\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a^5\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{5\,a^5\,(a\,x-1)}{a\,x+1}} - \frac{1003\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5} + \frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,1003\mathrm{i}}{128\,a^5}$$

input
```
int(x^4*((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
(atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*1003i)/(128*a^5) + (8*((a*x - 1)/(a*
x + 1))^(1/4))/a^5 + ((491*((a*x - 1)/(a*x + 1))^(1/4))/64 - (1409*((a*x -
 1)/(a*x + 1))^(5/4))/40 + (6113*((a*x - 1)/(a*x + 1))^(9/4))/96 - (307*((
a*x - 1)/(a*x + 1))^(13/4))/6 + (4117*((a*x - 1)/(a*x + 1))^(17/4))/192)/(
a^5 + (10*a^5*(a*x - 1)^2)/(a*x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3
+ (5*a^5*(a*x - 1)^4)/(a*x + 1)^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5
*(a*x - 1))/(a*x + 1)) - (1003*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5
)
```

Reduce [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = \left(\int \frac{(ax-1)^{\frac{1}{4}}x^5}{(ax+1)^{\frac{1}{4}}ax+(ax+1)^{\frac{1}{4}}}dx\right)a - \left(\int \frac{(ax-1)^{\frac{1}{4}}x^4}{(ax+1)^{\frac{1}{4}}ax+(ax+1)^{\frac{1}{4}}}dx\right)$$

input
```
int(x^4*((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int(((a*x - 1)**(1/4)*x**5)/((a*x + 1)**(1/4)*a*x + (a*x + 1)**(1/4)),x)*a
 - int(((a*x - 1)**(1/4)*x**4)/((a*x + 1)**(1/4)*a*x + (a*x + 1)**(1/4)),x
)
```

3.113 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 250

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}}-\frac{521\sqrt[4]{1-\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1+\frac{1}{ax}}}+\frac{113\sqrt[4]{1-\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt[4]{1-\frac{1}{ax}}\,x^4}{4\sqrt[4]{1+\frac{1}{ax}}}-\frac{475\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{475\,\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

output
```
-2467/192*(1-1/a/x)^(1/4)/a^4/(1+1/a/x)^(1/4)-521/192*(1-1/a/x)^(1/4)*x/a^
3/(1+1/a/x)^(1/4)+113/96*(1-1/a/x)^(1/4)*x^2/a^2/(1+1/a/x)^(1/4)-17/24*(1-
1/a/x)^(1/4)*x^3/a/(1+1/a/x)^(1/4)+1/4*(1-1/a/x)^(1/4)*x^4/(1+1/a/x)^(1/4)
-475/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+475/64*arctanh((1+1/a/
x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Mathematica [A] (verified)

Time = 5.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.64

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \frac{-3072e^{-\frac{1}{2}\coth^{-1}(ax)}+\frac{1536e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}-\frac{5248e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{7376e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}-\frac{6292e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+2850\,\mathrm{ar}}{384a^4}$$

input
```
Integrate[x^3/E^((5*ArcCoth[a*x])/2),x]
```

output
```
(-3072/E^(ArcCoth[a*x]/2) + (1536*E^((15*ArcCoth[a*x])/2))/(-1 + E^(2*ArcC
oth[a*x]))^4 - (5248*E^((11*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3
+ (7376*E^((7*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 - (6292*E^((3*
ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x])) + 2850*ArcTan[E^(-1/2*ArcCoth[
a*x])] - 1425*Log[1 - E^(-1/2*ArcCoth[a*x])] + 1425*Log[1 + E^(-1/2*ArcCot
h[a*x])])/(384*a^4)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6721, 109, 27, 168, 27, 168, 27, 168, 27, 172, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}x^5}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{4}\int \frac{\left(17a-\frac{16}{x}\right)x^4}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(17a-\frac{16}{x}\right)x^4}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{168}$$

$$\frac{-\frac{1}{3}\int \frac{\left(113a-\frac{102}{x}\right)x^3}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\left(113a-\frac{102}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{168}$$

$$\frac{-\frac{-\frac{1}{2}\int \frac{\left(521a-\frac{452}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{-\frac{\int \frac{\left(521a-\frac{452}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 168$$

$$\frac{-\frac{-\frac{-\int \frac{\left(1425a-\frac{1042}{x}\right)x}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{-\frac{-\frac{\int \frac{\left(1425a-\frac{1042}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{2a}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 172$$

$$\frac{-\frac{-\frac{-\frac{2a\int \frac{1425x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4934a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 27

$$\frac{-\frac{-\frac{-\frac{1425a\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4934a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 104

$$\frac{-\frac{-\frac{-\frac{5700a\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{4934a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{-\frac{-\frac{-\frac{\frac{4934a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-5700a\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}}{8a^2}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{ 827}$$

$$\frac{-\frac{-\frac{-\frac{5700a\left(\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4934a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1}}{3\sqrt[4]{\frac{1}{ax}}}}{8a^2}$$

$$\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 216

$$\frac{-\frac{-\frac{-\frac{5700a\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4934a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{1}}{3\sqrt[4]{\frac{1}{ax}}}}{8a^2}$$

$$\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 219

$$-\frac{-\frac{-\frac{-\frac{5700a\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4934a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{521ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{113ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a}-\frac{17ax^3\sqrt[4]{\cdots}}{3\sqrt[4]{\cdots}}}{8a^2}$$

$$\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}$$

input
```
Int[x^3/E^((5*ArcCoth[a*x])/2),x]
```

output
```
((1 - 1/(a*x))^(1/4)*x^4)/(4*(1 + 1/(a*x))^(1/4)) + ((-17*a*(1 - 1/(a*x))^
(1/4)*x^3)/(3*(1 + 1/(a*x))^(1/4)) - ((-113*a*(1 - 1/(a*x))^(1/4)*x^2)/(2*
(1 + 1/(a*x))^(1/4)) - ((-521*a*(1 - 1/(a*x))^(1/4)*x)/(1 + 1/(a*x))^(1/4)
 - ((4934*a*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4) + 5700*a*(ArcTan[(1 +
 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 -
1/(a*x))^(1/4)]/2))/(2*a))/(4*a))/(6*a))/(8*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}dx$$

input
```
int(x^3*((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int(x^3*((a*x-1)/(a*x+1))^(5/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.44

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \frac{2\left(48\,a^4x^4-136\,a^3x^3+226\,a^2x^2-521\,ax-2467\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+1425\,\log\left(\left(\frac{ax-1}{ax+1}\right.\right.}{384\,a^4}$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

output
```
1/384*(2*(48*a^4*x^4 - 136*a^3*x^3 + 226*a^2*x^2 - 521*a*x - 2467)*((a*x -
 1)/(a*x + 1))^(1/4) + 2850*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 1425*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1425*log(((a*x - 1)/(a*x + 1))^(1/4) -
 1))/a^4
```

Sympy [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

input
```
integrate(x**3*((a*x-1)/(a*x+1))**(5/4),x)
```

output
```
Integral(x**3*((a*x - 1)/(a*x + 1))**(5/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.98

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = -\frac{1}{384}\,a\left(\frac{4\left(1573\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-2875\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+2343\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-657\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{2850\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}\right.$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

output
```
-1/384*a*(4*(1573*((a*x - 1)/(a*x + 1))^(13/4) - 2875*((a*x - 1)/(a*x + 1)
)^(9/4) + 2343*((a*x - 1)/(a*x + 1))^(5/4) - 657*((a*x - 1)/(a*x + 1))^(1/
4))/(4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x -
1)^3*a^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 2850*arctan(((
a*x - 1)/(a*x + 1))^(1/4))/a^5 - 1425*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)
/a^5 + 1425*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5 + 3072*((a*x - 1)/(a*
x + 1))^(1/4)/a^5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.89

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$$
$$= \frac{1}{384}a\left(\frac{2850\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5} + \frac{1425\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5} - \frac{1425\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5} - \frac{3072\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^5}\right.$$

input
```
integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

output
```
1/384*a*(2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 1425*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^5 - 1425*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1
))/a^5 - 3072*((a*x - 1)/(a*x + 1))^(1/4)/a^5 + 4*(2343*(a*x - 1)*((a*x -
1)/(a*x + 1))^(1/4)/(a*x + 1) - 2875*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(1/
4)/(a*x + 1)^2 + 1573*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^3
- 657*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x - 1)/(a*x + 1) - 1)^4))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.87

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \frac{475\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4} - \frac{\frac{219\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{32} - \frac{781\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{32} + \frac{2875\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{96} - \frac{1573\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{96}}{a^4 + \frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3} + \frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4} - \frac{4\,a^4\,(a\,x-1)}{a\,x+1}} - \frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^4} - \frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)\,475\mathrm{i}}{64\,a^4}$$

input
```
int(x^3*((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
(475*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - (8*((a*x - 1)/(a*x + 1)
)^(1/4))/a^4 - ((219*((a*x - 1)/(a*x + 1))^(1/4))/32 - (781*((a*x - 1)/(a*
x + 1))^(5/4))/32 + (2875*((a*x - 1)/(a*x + 1))^(9/4))/96 - (1573*((a*x -
1)/(a*x + 1))^(13/4))/96)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*
(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1
))/(a*x + 1)) - (atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*475i)/(64*a^4)
```

Reduce [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = \left(\int \frac{(ax-1)^{\frac{1}{4}}\,x^4}{(ax+1)^{\frac{1}{4}}\,ax+(ax+1)^{\frac{1}{4}}}dx\right)a - \left(\int \frac{(ax-1)^{\frac{1}{4}}\,x^3}{(ax+1)^{\frac{1}{4}}\,ax+(ax+1)^{\frac{1}{4}}}dx\right)$$

input
```
int(x^3*((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int(((a*x - 1)**(1/4)*x**4)/((a*x + 1)**(1/4)*a*x + (a*x + 1)**(1/4)),x)*a
 - int(((a*x - 1)**(1/4)*x**3)/((a*x + 1)**(1/4)*a*x + (a*x + 1)**(1/4)),x
)
```

3.114 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 213

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx=\frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}}+\frac{61\sqrt[4]{1-\frac{1}{ax}}x}{24a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{13\sqrt[4]{1-\frac{1}{ax}}x^2}{12a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt[4]{1-\frac{1}{ax}}x^3}{3\sqrt[4]{1+\frac{1}{ax}}}+\frac{55\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{55\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

output

```
287/24*(1-1/a/x)^(1/4)/a^3/(1+1/a/x)^(1/4)+61/24*(1-1/a/x)^(1/4)*x/a^2/(1+
1/a/x)^(1/4)-13/12*(1-1/a/x)^(1/4)*x^2/a/(1+1/a/x)^(1/4)+1/3*(1-1/a/x)^(1/
4)*x^3/(1+1/a/x)^(1/4)+55/8*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3-55
/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Mathematica [A] (verified)

Time = 5.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.64

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$$
$$=\frac{384e^{-\frac{1}{2}\coth^{-1}(ax)}+\frac{128e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}-\frac{400e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{548e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-330\arctan\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right)+1}{48a^3}$$

input
```
Integrate[x^2/E^((5*ArcCoth[a*x])/2),x]
```

output
```
(384/E^(ArcCoth[a*x]/2) + (128*E^((11*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth
[a*x]))^3 - (400*E^((7*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (54
8*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x])) - 330*ArcTan[E^(-1/2*A
rcCoth[a*x])] + 165*Log[1 - E^(-1/2*ArcCoth[a*x])] - 165*Log[1 + E^(-1/2*A
rcCoth[a*x])])/(48*a^3)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6721, 109, 27, 168, 27, 168, 27, 172, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{ 6721}$$
$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}x^4}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$
$$\Big\downarrow \text{ 109}$$

$$\frac{1}{3}\int \frac{\left(13a-\frac{12}{x}\right)x^3}{2a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\downarrow 27$$

$$\frac{\int \frac{\left(13a-\frac{12}{x}\right)x^3}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\downarrow 168$$

$$\frac{-\frac{1}{2}\int \frac{\left(61a-\frac{52}{x}\right)x^2}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\downarrow 27$$

$$\frac{-\frac{\int \frac{\left(61a-\frac{52}{x}\right)x^2}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\downarrow 168$$

$$\frac{-\frac{-\int \frac{\left(165a-\frac{122}{x}\right)x}{2a\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\downarrow 27$$

$$\frac{-\frac{-\frac{\int \frac{\left(165a-\frac{122}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{2a}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{172}$$

$$\frac{-\frac{-\frac{2a\int \frac{165x}{2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{574a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{-\frac{165a\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{574a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{104}$$

$$\frac{-\frac{-\frac{660a\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{574a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$\big\downarrow$ 25

$$\frac{-\frac{-\frac{\frac{574a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-660a\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}}{2a}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$\big\downarrow$ 827

$$\frac{-\frac{-\frac{660a\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{574a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$\big\downarrow$ 216

$$\frac{-\frac{-\frac{660a\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{574a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

$\Big\downarrow$ 219

$$\frac{-\frac{-\frac{660a\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{574a\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{2a}-\frac{61ax\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}}{4a}-\frac{13ax^2\sqrt[4]{1-\frac{1}{ax}}}{2\sqrt[4]{\frac{1}{ax}+1}}}{6a^2}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}$$

input
```
Int[x^2/E^((5*ArcCoth[a*x])/2),x]
```

output
```
((1 - 1/(a*x))^(1/4)*x^3)/(3*(1 + 1/(a*x))^(1/4)) + ((-13*a*(1 - 1/(a*x))^
(1/4)*x^2)/(2*(1 + 1/(a*x))^(1/4)) - ((-61*a*(1 - 1/(a*x))^(1/4)*x)/(1 + 1
/(a*x))^(1/4) - ((574*a*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4) + 660*a*(
ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^
(1/4)/(1 - 1/(a*x))^(1/4)]/2))/(2*a))/(4*a))/(6*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
])))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} dx$$

input
```
int(x^2*((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int(x^2*((a*x-1)/(a*x+1))^(5/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.48

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \frac{2\left(8a^3x^3-26a^2x^2+61ax+287\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-330\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-165\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+165\log}{48a^3}$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

output
```
1/48*(2*(8*a^3*x^3 - 26*a^2*x^2 + 61*a*x + 287)*((a*x - 1)/(a*x + 1))^(1/4
) - 330*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 165*log(((a*x - 1)/(a*x + 1)
)^(1/4) + 1) + 165*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

input
```
integrate(x**2*((a*x-1)/(a*x+1))**(5/4),x)
```

output
```
Integral(x**2*((a*x - 1)/(a*x + 1))**(5/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.97

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{48}\,a\left(\frac{4\left(137\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-174\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+69\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{330\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\right.}{a^4}\right.$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

output
```
-1/48*a*(4*(137*((a*x - 1)/(a*x + 1))^(9/4) - 174*((a*x - 1)/(a*x + 1))^(5
/4) + 69*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x
- 1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 330*arctan((
(a*x - 1)/(a*x + 1))^(1/4))/a^4 + 165*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)
/a^4 - 165*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4 - 384*((a*x - 1)/(a*x
+ 1))^(1/4)/a^4)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.90

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{48}\,a\left(\frac{330\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{165\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}-\frac{384\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^4}-\right.$$

input
```
integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

output
```
-1/48*a*(330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 165*log(((a*x - 1)/
(a*x + 1))^(1/4) + 1)/a^4 - 165*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/
a^4 - 384*((a*x - 1)/(a*x + 1))^(1/4)/a^4 - 4*(174*(a*x - 1)*((a*x - 1)/(a
*x + 1))^(1/4)/(a*x + 1) - 137*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*
x + 1)^2 - 69*((a*x - 1)/(a*x + 1))^(1/4))/(a^4*((a*x - 1)/(a*x + 1) - 1)^
3))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.85

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \frac{\frac{23\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}-\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{137\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}+\frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^3} - \frac{55\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}+\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,55\mathrm{i}}{8\,a^3}$$

input
```
int(x^2*((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
((23*((a*x - 1)/(a*x + 1))^(1/4))/4 - (29*((a*x - 1)/(a*x + 1))^(5/4))/2 +
 (137*((a*x - 1)/(a*x + 1))^(9/4))/12)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1
)^2 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (atan
(((a*x - 1)/(a*x + 1))^(1/4)*1i)*55i)/(8*a^3) + (8*((a*x - 1)/(a*x + 1))^(
1/4))/a^3 - (55*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3)
```

Reduce [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = \left(\int \frac{(ax-1)^{\frac{1}{4}}x^3}{(ax+1)^{\frac{1}{4}}ax+(ax+1)^{\frac{1}{4}}}dx\right)a - \left(\int \frac{(ax-1)^{\frac{1}{4}}x^2}{(ax+1)^{\frac{1}{4}}ax+(ax+1)^{\frac{1}{4}}}dx\right)$$

input
```
int(x^2*((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int(((a*x - 1)**(1/4)*x**3)/((a*x + 1)**(1/4)*a*x + (a*x + 1)**(1/4)),x)*a
 - int(((a*x - 1)**(1/4)*x**2)/((a*x + 1)**(1/4)*a*x + (a*x + 1)**(1/4)),x
)
```

3.115 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 176

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx = -\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}}-\frac{25\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

output

```
-25/2*(1-1/a/x)^(1/4)/a^2/(1+1/a/x)^(1/4)-5/4*(1-1/a/x)^(5/4)*x/a/(1+1/a/x
)^(1/4)+1/2*(1-1/a/x)^(9/4)*x^2/(1+1/a/x)^(1/4)-25/4*arctan((1+1/a/x)^(1/4
)/(1-1/a/x)^(1/4))/a^2+25/4*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.69

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx =$$

$$-\frac{e^{-\frac{1}{2}\coth^{-1}(ax)}\left(32-90e^{2\coth^{-1}(ax)}+50e^{4\coth^{-1}(ax)}+25e^{\frac{1}{2}\coth^{-1}(ax)}\left(-1+e^{2\coth^{-1}(ax)}\right)^2\arctan\left(e^{\frac{1}{2}\coth^{-1}\cdots}\right.\right.}{4a^2\left(-1+e^{2\coth^{-1}(ax)}\right)^2}$$

input
```
Integrate[x/E^((5*ArcCoth[a*x])/2),x]
```

output
```
-1/4*(32 - 90*E^(2*ArcCoth[a*x]) + 50*E^(4*ArcCoth[a*x]) + 25*E^(ArcCoth[a
*x]/2)*(-1 + E^(2*ArcCoth[a*x]))^2*ArcTan[E^(ArcCoth[a*x]/2)] - 25*E^(ArcC
oth[a*x]/2)*(-1 + E^(2*ArcCoth[a*x]))^2*ArcTanh[E^(ArcCoth[a*x]/2)])/(a^2*
E^(ArcCoth[a*x]/2)*(-1 + E^(2*ArcCoth[a*x]))^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6721, 107, 105, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-\frac{5}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6721$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}x^3}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \; 107$$

$$\frac{5\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}x^2}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{4a}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{105}$$

$$\frac{5\left(-\frac{5\int \frac{\sqrt[4]{1-\frac{1}{ax}}\,x}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{2a}-\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4a}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{105}$$

$$\frac{5\left(-\frac{5\left(\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}-\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4a}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{104}$$

$$\frac{5\left(-\frac{5\left(4\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}-\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4a}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{5\left(-\frac{5\left(\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}-\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4a}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 827

$$\frac{5\left(-\frac{5\left(4\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}-\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4a}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 216

$$\frac{5\left(-\frac{5\left(4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}-\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4a}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 219

$$\frac{5\left(-\frac{5\left(4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}-\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4a}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}$$

input
```
Int[x/E^((5*ArcCoth[a*x])/2),x]
```

output
```
((1 - 1/(a*x))^(9/4)*x^2)/(2*(1 + 1/(a*x))^(1/4)) + (5*(-(((1 - 1/(a*x))^(
5/4)*x)/(1 + 1/(a*x))^(1/4)) - (5*((4*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(
1/4) + 4*(ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 +
 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2)))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int x \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} dx$$

input
```
int(x*((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int(x*((a*x-1)/(a*x+1))^(5/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.54

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx = \frac{2\,(2\,a^2x^2-9\,ax-43)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+50\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^2*x^2 - 9*a*x - 43)*((a*x - 1)/(a*x + 1))^(1/4) + 50*arctan(((
a*x - 1)/(a*x + 1))^(1/4)) + 25*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 25*
log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx = \int x\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

input
```
integrate(x*((a*x-1)/(a*x+1))**(5/4),x)
```

output
```
Integral(x*((a*x - 1)/(a*x + 1))**(5/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.98

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx = -\frac{1}{8}\,a\left(\frac{4\left(13\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{50\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{25\,\log\left(\left(\frac{ax-1}{ax+1}\right)\right.}{a^3}\right.$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

output
```
-1/8*a*(4*(13*((a*x - 1)/(a*x + 1))^(5/4) - 9*((a*x - 1)/(a*x + 1))^(1/4))
/(2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 50*arct
an(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 25*log(((a*x - 1)/(a*x + 1))^(1/4) +
 1)/a^3 + 25*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3 + 64*((a*x - 1)/(a*x
 + 1))^(1/4)/a^3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.91

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx$$
$$=\frac{1}{8}a\left(\frac{50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{25\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}-\frac{64\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^3}+\frac{4\left(\frac{13(a\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

output
```
1/8*a*(50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 25*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^3 - 25*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 -
 64*((a*x - 1)/(a*x + 1))^(1/4)/a^3 + 4*(13*(a*x - 1)*((a*x - 1)/(a*x + 1)
)^(1/4)/(a*x + 1) - 9*((a*x - 1)/(a*x + 1))^(1/4))/(a^3*((a*x - 1)/(a*x +
1) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 24.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.82

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx = \frac{25\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2} - \frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^2} - \frac{\frac{9\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2} - \frac{13\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{a^2 + \frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{2\,a^2\,(a\,x-1)}{a\,x+1}} - \frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,25\mathrm{i}}{4\,a^2}$$

input
```
int(x*((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
(25*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) - ((9*((a*x - 1)/(a*x + 1))
^(1/4))/2 - (13*((a*x - 1)/(a*x + 1))^(5/4))/2)/(a^2 + (a^2*(a*x - 1)^2)/(
a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) - (8*((a*x - 1)/(a*x + 1))^(1/4)
)/a^2 - (atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*25i)/(4*a^2)
```

Reduce [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx = \left(\int \frac{(ax-1)^{\frac{1}{4}}\,x^2}{(ax+1)^{\frac{1}{4}}\,ax+(ax+1)^{\frac{1}{4}}}dx\right)a - \left(\int \frac{(ax-1)^{\frac{1}{4}}\,x}{(ax+1)^{\frac{1}{4}}\,ax+(ax+1)^{\frac{1}{4}}}dx\right)$$

input
```
int(x*((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int(((a*x - 1)**(1/4)*x**2)/((a*x + 1)**(1/4)*a*x + (a*x + 1)**(1/4)),x)*a
 - int(((a*x - 1)**(1/4)*x)/((a*x + 1)**(1/4)*a*x + (a*x + 1)**(1/4)),x)
```

3.116 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 130

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx=\frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}}+\frac{5\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{5\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

output
```
10*(1-1/a/x)^(1/4)/a/(1+1/a/x)^(1/4)+(1-1/a/x)^(5/4)*x/(1+1/a/x)^(1/4)+5*a
rctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a-5*arctanh((1+1/a/x)^(1/4)/(1-1/a/
x)^(1/4))/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.24

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx=\frac{8e^{-\frac{1}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},2,\frac{3}{4},e^{2\coth^{-1}(ax)}\right)}{a}$$

input
```
Integrate[E^((-5*ArcCoth[a*x])/2),x]
```

output
```
(8*Hypergeometric2F1[-1/4, 2, 3/4, E^(2*ArcCoth[a*x])])/(a*E^(ArcCoth[a*x]
/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6720, 105, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6720$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}x^2}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \; 105$$

$$\frac{5\int \frac{\sqrt[4]{1-\frac{1}{ax}}x}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}}{2a}+\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 105$$

$$\frac{5\left(\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}+\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 104$$

$$\frac{5\left(4\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}+\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 25$$

$$\frac{5\left(\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a}+\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 827$$

$$\frac{5\left(4\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}+\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 216$$

$$\frac{5\left(4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}+\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 219$$

$$\frac{5\left(4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+\frac{4\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{2a}+\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

input
```
Int[E^((-5*ArcCoth[a*x])/2),x]
```

output
```
((1 - 1/(a*x))^(5/4)*x)/(1 + 1/(a*x))^(1/4) + (5*((4*(1 - 1/(a*x))^(1/4))/
(1 + 1/(a*x))^(1/4) + 4*(ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2
 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/2)))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} dx$$

input
```
int(((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
int(((a*x-1)/(a*x+1))^(5/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.66

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)} dx = \frac{2\,(ax+9)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) + 5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

output
```
1/2*(2*(a*x + 9)*((a*x - 1)/(a*x + 1))^(1/4) - 10*arctan(((a*x - 1)/(a*x +
 1))^(1/4)) - 5*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 5*log(((a*x - 1)/(a
*x + 1))^(1/4) - 1))/a
```

Sympy [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx=\int\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(5/4),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(5/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx=$$
$$-\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}-\frac{16\left(\frac{ax-}{ax+}\right.}{a^2}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

output
```
-1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 10
*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 5*log(((a*x - 1)/(a*x + 1))^(1/
4) + 1)/a^2 - 5*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2 - 16*((a*x - 1)/(
a*x + 1))^(1/4)/a^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.99

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx = -\frac{1}{2}\,a\left(\frac{10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{5\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

output
```
-1/2*a*(10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 5*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^2 - 5*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 -
16*((a*x - 1)/(a*x + 1))^(1/4)/a^2 + 4*((a*x - 1)/(a*x + 1))^(1/4)/(a^2*((
a*x - 1)/(a*x + 1) - 1)))
```

Mupad [B] (verification not implemented)

Time = 23.78 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx = \frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a} - \frac{5\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}+\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,5\mathrm{i}}{a}$$

input
```
int(((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/4))/(a - (a*(a*x - 1))/(a*x + 1)) + (atan(((a*
x - 1)/(a*x + 1))^(1/4)*1i)*5i)/a + (8*((a*x - 1)/(a*x + 1))^(1/4))/a - (5
*atan(((a*x - 1)/(a*x + 1))^(1/4)))/a
```

Reduce [F]

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx = -\left(\int \frac{(ax-1)^{\frac{1}{4}}}{(ax+1)^{\frac{1}{4}}\,ax+(ax+1)^{\frac{1}{4}}}dx\right) + \left(\int \frac{(ax-1)^{\frac{1}{4}}\,x}{(ax+1)^{\frac{1}{4}}\,ax+(ax+1)^{\frac{1}{4}}}dx\right)a$$

input
```
int(((a*x-1)/(a*x+1))^(5/4),x)
```

output
```
 - int((a*x - 1)**(1/4)/((a*x + 1)**(1/4)*a*x + (a*x + 1)**(1/4)),x) + int
(((a*x - 1)**(1/4)*x)/((a*x + 1)**(1/4)*a*x + (a*x + 1)**(1/4)),x)*a
```

3.117 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 250

$$\begin{aligned}\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx &= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)\\
&\quad+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\arctan\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\\
&\quad+\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)\\
&\quad+2\text{arctanh}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\end{aligned}$$

output
```
-8*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)+2^(1/2)*arctan(-1+2^(1/2)*(1-1/a/x)^(1/
4)/(1+1/a/x)^(1/4))+2^(1/2)*arctan(1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/
4))-2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2^(1/2)*arctanh(2^(1/2)*(1-1
/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1+1/a/x)^(1/4))+2*arctanh
((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.11

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -8e^{-\frac{1}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},1,\frac{7}{8},e^{4\coth^{-1}(ax)}\right)$$

input
```
Integrate[1/(E^((5*ArcCoth[a*x])/2)*x),x]
```

output
```
(-8*Hypergeometric2F1[-1/8, 1, 7/8, E^(4*ArcCoth[a*x])])/E^(ArcCoth[a*x]/2
)
```

Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.15, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.429$, Rules used = {6721, 109, 27, 35, 140, 73, 104, 25, 770, 755, 827, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$-4a\int \frac{\left(a+\frac{1}{x}\right)x}{4a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{35}$$

$$-\int \frac{\left(1+\frac{1}{ax}\right)^{3/4}x}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{140}$$

$$-\frac{\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{73}$$

$$4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-\int \frac{x}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{104}$$

$$4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}-4\int -\frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{25}$$

$$4\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}+4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{770}$$

$$4\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{755}$$

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+4\int \frac{1}{\left(1-\frac{1}{x^4}\right)x^2}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{827}$$

$$-4\left(\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{216}$$

$$-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{219}$$

$$4 \left(\frac{1}{2} \int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}} d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2} \int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}} d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-4 \left(\frac{1}{2} \arctan \left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2} \text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8 \sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 1476$$

$$4 \left(\frac{1}{2} \int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}} d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2} \left(\frac{1}{2} \int \frac{1}{-\frac{\sqrt{2} \sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1} d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2} \int \frac{1}{\frac{\sqrt{2} \sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1} d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}} \right. \right.$$

$$4 \left(\frac{1}{2} \arctan \left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2} \text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8 \sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 1082$$

$$4 \left(\frac{1}{2} \left(\frac{\int \frac{1}{-1-\frac{1}{x^2}} d\left(1-\frac{\sqrt{2} \sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}} d\left(\frac{\sqrt{2} \sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2} \int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}} d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)-4 \left(\frac{1}{2} \arctan \left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2} \text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8 \sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 217$$

$$4\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)\right)-$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 1479$$

$$4\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\cdots}}{\sqrt[4]{2-\cdots}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \ 25$$

$$4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\right.}{\sqrt{2}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow 27$$

$$4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 1103

$$-4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\right)+4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2}}-\frac{\log}{}\right.\right.$$

$$\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}$$

input
```
Int[1/(E^((5*ArcCoth[a*x])/2)*x),x]
```

output
```
(-8*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4) - 4*(ArcTan[(1 + 1/(a*x))^(1/
4)/(1 - 1/(a*x))^(1/4)]/2 - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4
)]/2) + 4*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]
/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/S
qrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)
 + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1
/4) + x^(-2)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x}dx$$

input
```
int(((a*x-1)/(a*x+1))^(5/4)/x,x)
```

output
```
int(((a*x-1)/(a*x+1))^(5/4)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.84

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = \sqrt{2}\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) + \sqrt{2}\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right) + \frac{1}{2}\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right) - \frac{1}{2}\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right) - 8\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="fricas")
```

output
```
sqrt(2)*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + sqrt(2)*arctan(s
qrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + 1/2*sqrt(2)*log(sqrt(2)*((a*x -
1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 1/2*sqrt(2)*log(-sq
rt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*((a
*x - 1)/(a*x + 1))^(1/4) + 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(((a
*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F]

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(5/4)/x,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(5/4)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.98

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$= \frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}(\right.}{a}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="maxima")
```

output
```
1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4
) - 1)/a - 16*((a*x - 1)/(a*x + 1))^(1/4)/a)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.01

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$$
$$= \frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\log\left(\sqrt{2}(\right.}{}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="giac")
```

output
```
1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
sqrt((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a
 + 2*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x +
1))^(1/4) - 1))/a - 16*((a*x - 1)/(a*x + 1))^(1/4)/a)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.47

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = 2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right) - 8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4} - \mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)2\mathrm{i} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)(1+1\mathrm{i}) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right.\right.$$

input
```
int(((a*x - 1)/(a*x + 1))^(5/4)/x,x)
```

output
```
2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)
*2i + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 +
1i) + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 -
1i) - 8*((a*x - 1)/(a*x + 1))^(1/4)
```

Reduce [F]

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -\left(\int \frac{(ax-1)^{\frac{1}{4}}}{(ax+1)^{\frac{1}{4}}a\,x^2+(ax+1)^{\frac{1}{4}}x}dx\right) + \left(\int \frac{(ax-1)^{\frac{1}{4}}}{(ax+1)^{\frac{1}{4}}ax+(ax+1)^{\frac{1}{4}}}dx\right)a$$

input
```
int(((a*x-1)/(a*x+1))^(5/4)/x,x)
```

output

```
 - int((a*x - 1)**(1/4)/((a*x + 1)**(1/4)*a*x**2 + (a*x + 1)**(1/4)*x),x)
+ int((a*x - 1)**(1/4)/((a*x + 1)**(1/4)*a*x + (a*x + 1)**(1/4)),x)*a
```

3.118 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 224

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{5a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{5a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{5a\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$

output

```
4*a*(1-1/a/x)^(5/4)/(1+1/a/x)^(1/4)+5*a*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)-5/
2*a*arctan(-1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)-5/2*a*arcta
n(1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)-5/2*a*arctanh(2^(1/2)
*(1-1/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1+1/a/x)^(1/4))*2^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.14

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = 8ae^{-\frac{1}{2}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},2,\frac{3}{4},-e^{2\coth^{-1}(ax)}\right)$$

input
```
Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^2),x]
```

output
```
(8*a*Hypergeometric2F1[-1/4, 2, 3/4, -E^(2*ArcCoth[a*x])])/E^(ArcCoth[a*x]
/2)
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6721, 57, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}}{\left(1+\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{57}$$

$$5\int \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{60}$$

$$5\left(\frac{1}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)+\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{73}$$

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}\right)+\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{770}$$

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{755}$$

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right)+$$
$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{1476}$$

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\right.\right.\right.$$
$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\right.\right.}{\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}}$$

$\downarrow$ 217

$$\frac{5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{\sqrt{2}}\right.\right.\right.}{\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}}$$

$\downarrow$ 1479

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}}{2\sqrt{2}}\right.\right.\right.$$

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 25

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}\right)\right.\right.$$

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 27

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\ldots}}{\sqrt[4]{2-\ldots}}\right.\right.\right.$$

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 1103

$$5\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\ldots\right.}{\ldots}\right.\right.\right.$$

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

input
```
Int[1/(E^((5*ArcCoth[a*x])/2)*x^2),x]
```

output
```
(4*a*(1 - 1/(a*x))^(5/4))/(1 + 1/(a*x))^(1/4) + 5*(a*(1 - 1/(a*x))^(1/4)*(
1 + 1/(a*x))^(3/4) - 2*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 -
 x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x
^(-4))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2
- x^(-4))^(1/4) + x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[2]))/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^2}dx$$

input
```
int(((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

output
```
int(((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.79

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{10\sqrt{2}ax\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+10\sqrt{2}ax\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+5\sqrt{2}ax\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\right.}{4\,x}$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="fricas")
```

output
```
-1/4*(10*sqrt(2)*a*x*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + 10*
sqrt(2)*a*x*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + 5*sqrt(2)*a*
x*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1)
 - 5*sqrt(2)*a*x*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) - 4*(9*a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4))/x
```

Sympy [F]

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(5/4)/x**2,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(5/4)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.91

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="maxima")
```

output
```
-1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) - 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) - 32*((a*x - 1)/(a*x + 1))^(1/4) - 8*((a*x -
 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1))*a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.91

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="giac")
```

output
```
-1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) - 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) - 32*((a*x - 1)/(a*x + 1))^(1/4) - 8*((a*x -
 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1))*a
```

Mupad [B] (verification not implemented)

Time = 23.66 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.47

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = 8\,a\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4} +5\,(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)+\frac{2\,a\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{\frac{a\,x-1}{a\,x+1}+1}+(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)$$

input
```
int(((a*x - 1)/(a*x + 1))^(5/4)/x^2,x)
```

output
```
8*a*((a*x - 1)/(a*x + 1))^(1/4) + (-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/
(a*x + 1))^(1/4))*5i + 5*(-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1)
)^(1/4)*1i) + (2*a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)
```

Reduce [F]

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\left(\int \frac{(ax-1)^{\frac{1}{4}}}{(ax+1)^{\frac{1}{4}}\,a\,x^3+(ax+1)^{\frac{1}{4}}\,x^2}dx\right) + \left(\int \frac{(ax-1)^{\frac{1}{4}}}{(ax+1)^{\frac{1}{4}}\,a\,x^2+(ax+1)^{\frac{1}{4}}\,x}dx\right)a$$

input
```
int(((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

output
```
 - int((a*x - 1)**(1/4)/((a*x + 1)**(1/4)*a*x**3 + (a*x + 1)**(1/4)*x**2),
x) + int((a*x - 1)**(1/4)/((a*x + 1)**(1/4)*a*x**2 + (a*x + 1)**(1/4)*x),x
)*a
```

3.119 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 276

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{25a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{25a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} +$$

output
```
-2*a^2*(1-1/a/x)^(9/4)/(1+1/a/x)^(1/4)-25/4*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^
(3/4)-5/2*a^2*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4)+25/8*a^2*arctan(-1+2^(1/2)*(
1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)+25/8*a^2*arctan(1+2^(1/2)*(1-1/a/x
)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)+25/8*a^2*arctanh(2^(1/2)*(1-1/a/x)^(1/4)/
(1+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1+1/a/x)^(1/4))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.37

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{8}{3}a^2e^{-\frac{1}{2}\coth^{-1}(ax)}\Big(3 + e^{2\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right) + e^{2\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right) + 2e^{2\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},3,\frac{7}{4},-e^{2\coth^{-1}(ax)}\right)\Big)$$

input
```
Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^3),x]
```

output
```
(-8*a^2*(3 + E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, -E^(2*ArcCo
th[a*x])] + E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCot
h[a*x])] + 2*E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 3, 7/4, -E^(2*ArcCo
th[a*x])]))/(3*E^(ArcCoth[a*x]/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6721, 87, 60, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}}{\left(1+\frac{1}{ax}\right)^{5/4}x}\,d\frac{1}{x}$$

$$\Big\downarrow \text{87}$$

$$-5a\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{60}$$

$$-5a\left(\frac{5}{4}\int \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}\right)-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{60}$$

$$-5a\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}\right)-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{73}$$

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int \frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}\right)+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}\right)-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{770}$$

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}\right)-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{755}$$

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right)+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3}$$

$$\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 1476

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right.\right.\right.\right.$$

$$\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 1082

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right)\right.\right.\right.$$

$$\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 217

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(\right.}{}\right.\right.\right.\right.$$

$$\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 1479

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d}{2\sqrt{2}}\right.\right.\right.\right.$$

$$\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$\downarrow$ 25

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\cdots}}{\sqrt[4]{2-\cdots}}}{2\sqrt{2}}\cdots\right.\right.\right.\right.$$

$$\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$\Big\downarrow$ 27

$$-5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\cdots\right.\right.\right.\right.$$

$$\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$\Big\downarrow$ 1103

$$-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}-$$

$$5a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\right.\right.\right.\right.$$

input
```
Int[1/(E^((5*ArcCoth[a*x])/2)*x^3),x]
```

output
```
(-2*a^2*(1 - 1/(a*x))^(9/4))/(1 + 1/(a*x))^(1/4) - 5*a*((a*(1 - 1/(a*x))^(
5/4)*(1 + 1/(a*x))^(3/4))/2 + (5*(a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4
) - 2*a*((-(ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/S
qrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqr
t[2])/2 + (-1/2*Log[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) +
 x^(-2)]/Sqrt[2] + Log[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4
) + x^(-2)]/(2*Sqrt[2]))/2)))/4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^3}dx$$

input
```
int(((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

output
```
int(((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.72

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = \frac{50\sqrt{2}a^2x^2\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+50\sqrt{2}a^2x^2\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+25\sqrt{2}a^2x^2\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)\right.}{16\,x^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="fricas")
```

output
```
1/16*(50*sqrt(2)*a^2*x^2*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) +
 50*sqrt(2)*a^2*x^2*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + 25*s
qrt(2)*a^2*x^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) - 25*sqrt(2)*a^2*x^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4
) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 4*(43*a^2*x^2 + 9*a*x - 2)*((a*x - 1)
/(a*x + 1))^(1/4))/x^2
```

Sympy [F]

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(5/4)/x**3,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(5/4)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.89

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$$
$$= \frac{1}{16}\left(50\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="maxima")
```

output
```
1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1
))^(1/4))) + 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1) - 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))
^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 128*a*((a*x - 1)/(a*x + 1))^(1/4
) - 8*(13*a*((a*x - 1)/(a*x + 1))^(5/4) + 9*a*((a*x - 1)/(a*x + 1))^(1/4))
/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 + 1))*a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.88

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$$
$$= \frac{1}{16}\left(50\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="giac")
```

output
```
1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1
))^(1/4))) + 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1) - 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))
^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 128*a*((a*x - 1)/(a*x + 1))^(1/4
) - 8*(13*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) + 9*a*((a*x -
1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -8\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4} - \frac{\frac{9\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}+\frac{13\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{\frac{(a\,x-1)^2}{(a\,x+1)^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1} - \frac{(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,25\mathrm{i}}{4} - \frac{25\,(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)}{4}$$

input
```
int(((a*x - 1)/(a*x + 1))^(5/4)/x^3,x)
```

output
```
- 8*a^2*((a*x - 1)/(a*x + 1))^(1/4) - ((9*a^2*((a*x - 1)/(a*x + 1))^(1/4))
/2 + (13*a^2*((a*x - 1)/(a*x + 1))^(5/4))/2)/((a*x - 1)^2/(a*x + 1)^2 + (2
*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a
*x + 1))^(1/4))*25i)/4 - (25*(-1)^(1/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*
x + 1))^(1/4)*1i))/4
```

Reduce [F]

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\left(\int \frac{(ax-1)^{\frac{1}{4}}}{(ax+1)^{\frac{1}{4}}\,a\,x^4+(ax+1)^{\frac{1}{4}}\,x^3}dx\right) + \left(\int \frac{(ax-1)^{\frac{1}{4}}}{(ax+1)^{\frac{1}{4}}\,a\,x^3+(ax+1)^{\frac{1}{4}}\,x^2}dx\right)a$$

input
```
int(((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

output
```
 - int((a*x - 1)**(1/4)/((a*x + 1)**(1/4)*a*x**4 + (a*x + 1)**(1/4)*x**3),
x) + int((a*x - 1)**(1/4)/((a*x + 1)**(1/4)*a*x**3 + (a*x + 1)**(1/4)*x**2
),x)*a
```

3.120 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 310

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{55a^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} - \cdots$$

output
```
2*a^3*(1-1/a/x)^(9/4)/(1+1/a/x)^(1/4)+55/8*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)^(
3/4)+11/4*a^3*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4)+1/3*a^3*(1-1/a/x)^(9/4)*(1+1
/a/x)^(3/4)-55/16*a^3*arctan(-1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2
^(1/2)-55/16*a^3*arctan(1+2^(1/2)*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))*2^(1/2)
-55/16*a^3*arctanh(2^(1/2)*(1-1/a/x)^(1/4)/(1+(1-1/a/x)^(1/2)/(1+1/a/x)^(1
/2))/(1+1/a/x)^(1/4))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.34

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$= a^3\left(\frac{e^{-\frac{1}{2}\coth^{-1}(ax)}\left(96+425e^{2\coth^{-1}(ax)}+462e^{4\coth^{-1}(ax)}+165e^{6\coth^{-1}(ax)}\right)}{12\left(1+e^{2\coth^{-1}(ax)}\right)^3} - \frac{55}{32}\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)+2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

input
```
Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^4),x]
```

output
```
a^3*((96 + 425*E^(2*ArcCoth[a*x]) + 462*E^(4*ArcCoth[a*x]) + 165*E^(6*ArcC
oth[a*x]))/(12*E^(ArcCoth[a*x]/2)*(1 + E^(2*ArcCoth[a*x]))^3) - (55*RootSu
m[1 + #1^4 & , (ArcCoth[a*x] + 2*Log[E^(-1/2*ArcCoth[a*x]) - #1])/#1^3 & ]
)/32)
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6721, 100, 27, 90, 60, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$$
$$\Big\downarrow \text{6721}$$
$$-\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}}{\left(1+\frac{1}{ax}\right)^{5/4}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{100}$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}-2a^3\int -\frac{\left(5a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{5/4}}{2a^2\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$a\int \frac{\left(5a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{90}$$

$$a\left(\frac{11}{2}a\int \frac{\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{1}{3}a^2\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{9/4}\right)+\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{60}$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\int \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}\right)+\frac{1}{3}a^2\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{9/4}\right)+\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{60}$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}\right)+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}\right)+\frac{1}{3}\right.$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{73}$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int\frac{1}{\sqrt[4]{2-\frac{1}{x^4}}}d\sqrt[4]{1-\frac{1}{ax}}\right)+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}\right)+\frac{1}{3}a^2\left(\right.\right.$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 770$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\int\frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)+\frac{1}{2}a\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}\right)+\frac{1}{3}a^2\left(\frac{1}{a}\right.\right.$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 755$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)\right)+\frac{1}{2}a\left(\frac{1}{ax}+\right.\right.\right.$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 1476$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{2-}}\right.\right.\right.\right.\right.$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{1082}$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right.}{\sqrt{2}}\right.\right.\right.\right.\right.$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{217}$$

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}-\frac{\text{arctan}}{}\right.\right.\right.\right.\right.$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{a\left(\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\cdots\right.}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+\cdots}\cdots}{2\sqrt{2}}\cdots\right.\right.\right.\right.\right.}{\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}}$$

$\downarrow$ 25

$$\frac{a\left(\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\cdots}{2\sqrt{2}}\cdots\right.\right.\right.\right.\right.}{\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}}$$

$\downarrow$ 27

$$a\left(\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+1}d\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1}{\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+\frac{1}{x^2}+\cdots}\right.\right.\right.\right.\right.$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}$$

$$\Big\downarrow \; 1103$$

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}+$$

$$a\left(\frac{1}{3}a^2\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{9/4}+\frac{11}{2}a\left(\frac{5}{4}\left(a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{2-\frac{1}{x^4}}}+1\right)}{\sqrt{2}}\right.\right.\right.\right.\right.$$

input
```
Int[1/(E^((5*ArcCoth[a*x])/2)*x^4),x]
```

output
```
(2*a^3*(1 - 1/(a*x))^(9/4))/(1 + 1/(a*x))^(1/4) + a*((a^2*(1 - 1/(a*x))^(9
/4)*(1 + 1/(a*x))^(3/4))/3 + (11*a*((a*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(
3/4))/2 + (5*(a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4) - 2*a*((-(ArcTan[1
 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2]) + ArcTan[1 +
 (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4)]/Sqrt[2])/2 + (-1/2*Log[
1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/Sqrt[2] + L
og[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(2 - x^(-4))^(1/4) + x^(-2)]/(2*Sqrt[
2]))/2)))/4))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^4}dx$$

input
```
int(((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

output
```
int(((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.67

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\frac{330\sqrt{2}a^3x^3\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+330\sqrt{2}a^3x^3\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+165\sqrt{2}a^3x^3\log\left(\sqrt{2}\right.}{}$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="fricas")
```

output
```
-1/96*(330*sqrt(2)*a^3*x^3*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1)
 + 330*sqrt(2)*a^3*x^3*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + 1
65*sqrt(2)*a^3*x^3*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) - 165*sqrt(2)*a^3*x^3*log(-sqrt(2)*((a*x - 1)/(a*x + 1))
^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 4*(287*a^3*x^3 + 61*a^2*x^2 - 26
*a*x + 8)*((a*x - 1)/(a*x + 1))^(1/4))/x^3
```

Sympy [F]

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(5/4)/x**4,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(5/4)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.96

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx =$$
$$-\frac{1}{96}\left(330\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right.\right.\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="maxima")
```

output
```
-1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1
))^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(
a*x + 1))^(1/4))) + 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4
) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x -
1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 768*a^2*((a*x - 1)/
(a*x + 1))^(1/4) - 8*(137*a^2*((a*x - 1)/(a*x + 1))^(9/4) + 174*a^2*((a*x
- 1)/(a*x + 1))^(5/4) + 69*a^2*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)/(
a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1))*a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.94

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx =$$

$$-\frac{1}{96}\left(330\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right.\right.\right.\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="giac")
```

output
```
-1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1
))^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(
a*x + 1))^(1/4))) + 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4
) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x -
1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 768*a^2*((a*x - 1)/
(a*x + 1))^(1/4) - 8*(174*(a*x - 1)*a^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x +
 1) + 137*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 69*a^2
*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^3)*a
```

Mupad [B] (verification not implemented)

Time = 23.59 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.61

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = \frac{\frac{23\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}+\frac{29\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{137\,a^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{\frac{3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{3\,\left(a\,x-1\right)}{a\,x+1}+1} + 8\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}+\frac{{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,55{}\mathrm{i}}{8} + \frac{55\,{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,1{}\mathrm{i}\right)}{8}$$

input
```
int(((a*x - 1)/(a*x + 1))^(5/4)/x^4,x)
```

output
```
((23*a^3*((a*x - 1)/(a*x + 1))^(1/4))/4 + (29*a^3*((a*x - 1)/(a*x + 1))^(5
/4))/2 + (137*a^3*((a*x - 1)/(a*x + 1))^(9/4))/12)/((3*(a*x - 1)^2)/(a*x +
 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) + 8*a^3*((a
*x - 1)/(a*x + 1))^(1/4) + ((-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x
 + 1))^(1/4))*55i)/8 + (55*(-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x
+ 1))^(1/4)*1i))/8
```

Reduce [F]

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\left(\int \frac{(ax-1)^{\frac{1}{4}}}{(ax+1)^{\frac{1}{4}}\,a\,x^5+(ax+1)^{\frac{1}{4}}\,x^4}dx\right) + \left(\int \frac{(ax-1)^{\frac{1}{4}}}{(ax+1)^{\frac{1}{4}}\,a\,x^4+(ax+1)^{\frac{1}{4}}\,x^3}dx\right)a$$

input
```
int(((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

output
```
 - int((a*x - 1)**(1/4)/((a*x + 1)**(1/4)*a*x**5 + (a*x + 1)**(1/4)*x**4),
x) + int((a*x - 1)**(1/4)/((a*x + 1)**(1/4)*a*x**4 + (a*x + 1)**(1/4)*x**3
),x)*a
```

3.121 $\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 237

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = \frac{11}{27}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{1+\frac{1}{x}}x+\frac{7}{18}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{1+\frac{1}{x}}x^2+\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{1+\frac{1}{x}}x^3-\frac{19\arctan\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{\sqrt{3}}\right)}{54\sqrt{3}}+\frac{19\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{\sqrt{3}}\right)}{54\sqrt{3}}+\frac{19}{81}\operatorname{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)+\frac{19}{162}\operatorname{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{1-\frac{1}{x}}}\right)\sqrt[6]{1-\frac{1}{x}}}\right)$$

output
```
11/27*(1-1/x)^(5/6)*(1+1/x)^(1/6)*x+7/18*(1-1/x)^(5/6)*(1+1/x)^(1/6)*x^2+1
/3*(1-1/x)^(5/6)*(1+1/x)^(1/6)*x^3-19/162*arctan(1/3*(1-2*(1+1/x)^(1/6)/(1
-1/x)^(1/6))*3^(1/2))*3^(1/2)+19/162*arctan(1/3*(1+2*(1+1/x)^(1/6)/(1-1/x)
^(1/6))*3^(1/2))*3^(1/2)+19/81*arctanh((1+1/x)^(1/6)/(1-1/x)^(1/6))+19/162
*arctanh((1+1/x)^(1/6)/(1+(1+1/x)^(1/3)/(1-1/x)^(1/3))/(1-1/x)^(1/6))
```

Mathematica [A] (verified)

Time = 5.29 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.80

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = \frac{1}{324}\left(\frac{864e^{\frac{1}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^3}+\frac{1368e^{\frac{1}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^2}+\frac{732e^{\frac{1}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}}+38\sqrt{3}\arctan\left(\frac{-1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)+38\sqrt{3}\arctan\left(\frac{1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-38\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right)+38\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}\right)-19\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)+19\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)\right)$$

input
```
Integrate[E^(ArcCoth[x]/3)*x^2,x]
```

output
```
((864*E^(ArcCoth[x]/3))/(-1 + E^(2*ArcCoth[x]))^3 + (1368*E^(ArcCoth[x]/3)
)/(-1 + E^(2*ArcCoth[x]))^2 + (732*E^(ArcCoth[x]/3))/(-1 + E^(2*ArcCoth[x]
)) + 38*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] + 38*Sqrt[3]*Arc
Tan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 38*Log[1 - E^(ArcCoth[x]/3)] + 38*
Log[1 + E^(ArcCoth[x]/3)] - 19*Log[1 - E^(ArcCoth[x]/3) + E^((2*ArcCoth[x]
)/3)] + 19*Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/324
```

Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.12, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {6721, 110, 27, 168, 27, 168, 27, 104, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\frac{1}{3}\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[6]{1+\frac{1}{x}}x^4}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3-\frac{1}{3}\int \frac{\left(7+\frac{6}{x}\right)x^3}{3\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3-\frac{1}{9}\int \frac{\left(7+\frac{6}{x}\right)x^3}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{9}\left(\frac{1}{2}\int -\frac{\left(22+\frac{21}{x}\right)x^2}{3\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}+\frac{7}{2}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^2\right)+\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(\frac{7}{2}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^2-\frac{1}{6}\int \frac{\left(22+\frac{21}{x}\right)x^2}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}\right)+\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{9}\left(\frac{1}{6}\left(\int -\frac{19x}{3\sqrt[6]{1-\frac{1}{x}}\,(1+\frac{1}{x})^{5/6}}d\frac{1}{x}+22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x\right)+\frac{7}{2}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^2\right)+$$
$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$\Big\downarrow$ 27

$$\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-\frac{19}{3}\int\frac{x}{\sqrt[6]{1-\frac{1}{x}}\,(1+\frac{1}{x})^{5/6}}d\frac{1}{x}\right)+\frac{7}{2}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^2\right)+$$
$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$\Big\downarrow$ 104

$$\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\int\frac{1}{\frac{1}{x^6}-1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)+\frac{7}{2}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^2\right)+$$
$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$\Big\downarrow$ 754

$$\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\left(-\frac{1}{3}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\int\frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{2\left(-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\right.\right.\right.$$
$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$\Big\downarrow$ 27

$$\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\left(-\frac{1}{3}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int\frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int\right.\right.\right.$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$\downarrow$ 219

$$\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\left(-\frac{1}{6}\int\frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int\frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+}}{\sqrt[6]{1-}}\right.\right.\right.$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$\downarrow$ 1142

$$\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\left(\frac{1}{6}\left(\frac{1}{2}\int-\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{3}{2}\int\frac{1}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\right.\right.\right.\right.$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$\downarrow$ 25

$$\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\left(\frac{1}{6}\left(-\frac{3}{2}\int\frac{1}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\right.\right.\right.\right.$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$\downarrow$ 1083

$$\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\left(\frac{1}{6}\left(3\int\frac{1}{-3-\frac{1}{x^2}}d\left(\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-1\right)-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+}}{\sqrt[6]{1-}}\right.\right.\right.\right.$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$\downarrow$ 217

$$\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\left(\frac{1}{6}\left(-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}-1}{\sqrt{3}}\right)\right.\right.\right.\right.$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

$\downarrow$ 1103

$$\frac{1}{9}\left(\frac{1}{6}\left(22\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-38\left(\frac{1}{6}\left(\frac{1}{2}\log\left(\frac{1}{x^2}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\right.\right.\right.\right.$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x^3$$

input
```
Int[E^(ArcCoth[x]/3)*x^2,x]
```

output
```
((1 - x^(-1))^(5/6)*(1 + x^(-1))^(1/6)*x^3)/3 + ((7*(1 - x^(-1))^(5/6)*(1
+ x^(-1))^(1/6)*x^2)/2 + (22*(1 - x^(-1))^(5/6)*(1 + x^(-1))^(1/6)*x - 38*
(-1/3*ArcTanh[(1 + x^(-1))^(1/6)/(1 - x^(-1))^(1/6)] + (-(Sqrt[3]*ArcTan[(
-1 + (2*(1 + x^(-1))^(1/6))/(1 - x^(-1))^(1/6))/Sqrt[3]]) + Log[1 - (1 + x
^(-1))^(1/6)/(1 - x^(-1))^(1/6) + x^(-2)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + (2
*(1 + x^(-1))^(1/6))/(1 - x^(-1))^(1/6))/Sqrt[3]]) - Log[1 + (1 + x^(-1))^
(1/6)/(1 - x^(-1))^(1/6) + x^(-2)]/2)/6))/6)/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 7.39 (sec) , antiderivative size = 1163, normalized size of antiderivative = 4.91

method	result	size
trager	Expression too large to display	1163
risch	Expression too large to display	2895

input
```
int(1/((x-1)/(1+x))^(1/6)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/54*(1+x)*(18*x^2+21*x+22)*(-(1-x)/(1+x))^(5/6)+19/162*ln(9*RootOf(9*_Z^2
-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)*x+9*RootOf(9*_Z^2-3
*_Z+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*
x+3*(-(1-x)/(1+x))^(5/6)+3*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_Z^2-3*_Z+1)
*(-(1-x)/(1+x))^(1/2)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x+3*(-
(1-x)/(1+x))^(2/3)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)+9*RootOf(
9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/3)*x+9*RootOf(9*
_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x))^
(1/6)*x+3*RootOf(9*_Z^2-3*_Z+1)-3*(-(1-x)/(1+x))^(1/6)-2)-19/54*ln(9*RootO
f(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)*x+9*RootOf(
9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x)
)^(1/2)*x+3*(-(1-x)/(1+x))^(5/6)+3*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_Z^2
-3*_Z+1)*(-(1-x)/(1+x))^(1/2)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3
)*x+3*(-(1-x)/(1+x))^(2/3)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)+9
*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/3)*x+9*R
ootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)
/(1+x))^(1/6)*x+3*RootOf(9*_Z^2-3*_Z+1)-3*(-(1-x)/(1+x))^(1/6)-2)*RootOf(9
*_Z^2-3*_Z+1)+19/54*RootOf(9*_Z^2-3*_Z+1)*ln(-9*RootOf(9*_Z^2-3*_Z+1)*(-(1
-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)*x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x
)/(1+x))^(2/3)-18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+3*(-(1-x...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.73

$$\begin{aligned}\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx &= \frac{1}{54}\left(18\,x^3+39\,x^2+43\,x+22\right)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}} \\ &\quad -\frac{19}{162}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\,\sqrt{3}\right) \\ &\quad -\frac{19}{162}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\,\sqrt{3}\right) \\ &\quad +\frac{19}{324}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\ &\quad -\frac{19}{324}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\ &\quad +\frac{19}{162}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{19}{162}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\end{aligned}$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)*x^2,x, algorithm="fricas")
```

output
```
1/54*(18*x^3 + 39*x^2 + 43*x + 22)*((x - 1)/(x + 1))^(5/6) - 19/162*sqrt(3
)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - 19/162*sqrt(
3)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) + 19/324*log(
((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 19/324*log(((x -
1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/162*log(((x - 1)/(x
+ 1))^(1/6) + 1) - 19/162*log(((x - 1)/(x + 1))^(1/6) - 1)
```

Sympy [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx=\int\frac{x^2}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/6)*x**2,x)
```

output
```
Integral(x**2/((x - 1)/(x + 1))**(1/6), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.93

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = -\frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right) - \frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right) - \frac{19\left(\frac{x-1}{x+1}\right)^{\frac{17}{6}} - 8\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}} + 61\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{27\left(\frac{3(x-1)}{x+1} - \frac{3(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1\right)} + \frac{19}{324}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) - \frac{19}{324}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) + \frac{19}{162}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) - \frac{19}{162}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} - 1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)*x^2,x, algorithm="maxima")
```

output
```
-19/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 19/1
62*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/27*(19*
((x - 1)/(x + 1))^(17/6) - 8*((x - 1)/(x + 1))^(11/6) + 61*((x - 1)/(x + 1
))^(5/6))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^3
 - 1) + 19/324*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1)
- 19/324*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/1
62*log(((x - 1)/(x + 1))^(1/6) + 1) - 19/162*log(((x - 1)/(x + 1))^(1/6) -
 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.91

$$
\begin{aligned}
\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = & -\frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right) \\
& -\frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right) \\
& +\frac{\frac{8(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1}-\frac{19(x-1)^2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{(x+1)^2}-61\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{27\left(\frac{x-1}{x+1}-1\right)^3} \\
& +\frac{19}{324}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\
& -\frac{19}{324}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\
& +\frac{19}{162}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{19}{162}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right|\right)
\end{aligned}
$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)*x^2,x, algorithm="giac")
```

output
```
-19/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 19/1
62*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) + 1/27*(8*(
x - 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) - 19*(x - 1)^2*((x - 1)/(x + 1))^(5
/6)/(x + 1)^2 - 61*((x - 1)/(x + 1))^(5/6))/((x - 1)/(x + 1) - 1)^3 + 19/3
24*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 19/324*log
(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/162*log(((x -
 1)/(x + 1))^(1/6) + 1) - 19/162*log(abs(((x - 1)/(x + 1))^(1/6) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.71

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = -\frac{\operatorname{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}1\mathrm{i}\right)19\mathrm{i}}{81} - \frac{\frac{61\left(\frac{x-1}{x+1}\right)^{5/6}}{27} - \frac{8\left(\frac{x-1}{x+1}\right)^{11/6}}{27} + \frac{19\left(\frac{x-1}{x+1}\right)^{17/6}}{27}}{\frac{3(x-1)}{x+1} - \frac{3(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1} - \operatorname{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}4952198\mathrm{i}}{14348907\left(-\frac{2476099}{14348907} + \frac{\sqrt{3}\,2476099\mathrm{i}}{14348907}\right)}\right)\left(\frac{19\sqrt{3}}{162} - \frac{19}{162}\mathrm{i}\right) - \operatorname{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}4952198\mathrm{i}}{14348907\left(\frac{2476099}{14348907} + \frac{\sqrt{3}\,2476099}{14348907}\right.}\right.$$

input
```
int(x^2/((x - 1)/(x + 1))^(1/6),x)
```

output
```
- (atan(((x - 1)/(x + 1))^(1/6)*1i)*19i)/81 - ((61*((x - 1)/(x + 1))^(5/6
)/27 - (8*((x - 1)/(x + 1))^(11/6))/27 + (19*((x - 1)/(x + 1))^(17/6))/27)
/((3*(x - 1))/(x + 1) - (3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
 - atan((((x - 1)/(x + 1))^(1/6)*4952198i)/(14348907*((3^(1/2)*2476099i)/1
4348907 - 2476099/14348907)))*((19*3^(1/2))/162 - 19i/162) - atan((((x - 1
)/(x + 1))^(1/6)*4952198i)/(14348907*((3^(1/2)*2476099i)/14348907 + 247609
9/14348907)))*((19*3^(1/2))/162 + 19i/162)
```

Reduce [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = \int \frac{(x+1)^{\frac{1}{6}}\,x^2}{(x-1)^{\frac{1}{6}}}dx$$

input
```
int(1/((x-1)/(1+x))^(1/6)*x^2,x)
```

output
```
int(((x + 1)**(1/6)*x**2)/(x - 1)**(1/6),x)
```

3.122 $\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 210

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx = \frac{1}{6}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{1+\frac{1}{x}}x+\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(1+\frac{1}{x}\right)^{7/6}x^2 - \frac{\arctan\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{1}{9}\operatorname{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right) + \frac{1}{18}\operatorname{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{1-\frac{1}{x}}}\right)\sqrt[6]{1-\frac{1}{x}}}\right)$$

output
```
1/6*(1-1/x)^(5/6)*(1+1/x)^(1/6)*x+1/2*(1-1/x)^(5/6)*(1+1/x)^(7/6)*x^2-1/18
*arctan(1/3*(1-2*(1+1/x)^(1/6)/(1-1/x)^(1/6))*3^(1/2))*3^(1/2)+1/18*arctan
(1/3*(1+2*(1+1/x)^(1/6)/(1-1/x)^(1/6))*3^(1/2))*3^(1/2)+1/9*arctanh((1+1/x
)^(1/6)/(1-1/x)^(1/6))+1/18*arctanh((1+1/x)^(1/6)/(1+(1+1/x)^(1/3)/(1-1/x)
^(1/3))/(1-1/x)^(1/6))
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.80

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx=\frac{1}{36}\left(\frac{72e^{\frac{1}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^2}+\frac{84e^{\frac{1}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}}+2\sqrt{3}\arctan\left(\frac{-1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)+2\sqrt{3}\arctan\left(\frac{1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-2\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right)+2\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}\right)-\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)+\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)\right)$$

input
```
Integrate[E^(ArcCoth[x]/3)*x,x]
```

output
```
((72*E^(ArcCoth[x]/3))/(-1 + E^(2*ArcCoth[x]))^2 + (84*E^(ArcCoth[x]/3))/(
-1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/Sqrt[3
]] + 2*Sqrt[3]*ArcTan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 2*Log[1 - E^(Arc
Coth[x]/3)] + 2*Log[1 + E^(ArcCoth[x]/3)] - Log[1 - E^(ArcCoth[x]/3) + E^(
(2*ArcCoth[x])/3)] + Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/36
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {6721, 107, 105, 104, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{1}{3}\coth^{-1}(x)}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[6]{1+\frac{1}{x}}x^3}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2-\frac{1}{6}\int \frac{\sqrt[6]{1+\frac{1}{x}}x^2}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-\frac{1}{3}\int \frac{x}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}\right)+\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\int \frac{1}{\frac{1}{x^6}-1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)+\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2$$

$$\Big\downarrow \text{754}$$

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(-\frac{1}{3}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\int\frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{2\left(-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\int\frac{}{2\left(\right.}\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2$$

$\Big\downarrow$ 27

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(-\frac{1}{3}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int\frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int\frac{\frac{\sqrt[6]{1+}}{\sqrt[6]{1-}}}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2$$

$\Big\downarrow$ 219

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(-\frac{1}{6}\int\frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int\frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\mathrm{a}\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2$$

$\Big\downarrow$ 1142

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(\frac{1}{6}\left(\frac{1}{2}\int -\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{3}{2}\int \frac{1}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right.\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2$$

$$\Big\downarrow \; 25$$

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right.\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(\frac{1}{6}\left(3\int \frac{1}{-3-\frac{1}{x^2}}d\left(\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-1\right)-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)+\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(\frac{1}{6}\left(-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\right.\right.}{\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2}$$

$\downarrow$ 1103

$$\frac{\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(\frac{1}{6}\left(\frac{1}{2}\log\left(\frac{1}{x^2}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\sqrt{3}\text{ar}\right.\right.\right.}{\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}x^2}$$

input

```
Int[E^(ArcCoth[x]/3)*x,x]
```

output

```
((1 - x^(-1))^(5/6)*(1 + x^(-1))^(7/6)*x^2)/2 + ((1 - x^(-1))^(5/6)*(1 + x
^(-1))^(1/6)*x - 2*(-1/3*ArcTanh[(1 + x^(-1))^(1/6)/(1 - x^(-1))^(1/6)] +
(-(Sqrt[3]*ArcTan[(-1 + (2*(1 + x^(-1))^(1/6))/(1 - x^(-1))^(1/6))/Sqrt[3]
]) + Log[1 - (1 + x^(-1))^(1/6)/(1 - x^(-1))^(1/6) + x^(-2)]/2)/6 + (-(Sqr
t[3]*ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/(1 - x^(-1))^(1/6))/Sqrt[3]]) - Lo
g[1 + (1 + x^(-1))^(1/6)/(1 - x^(-1))^(1/6) + x^(-2)]/2)/6))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.85 (sec) , antiderivative size = 1158, normalized size of antiderivative = 5.51

method	result	size
trager	Expression too large to display	1158
risch	Expression too large to display	2890

input
```
int(1/((x-1)/(1+x))^(1/6)*x,x,method=_RETURNVERBOSE)
```

output
```
1/6*(1+x)*(3*x+4)*(-(1-x)/(1+x))^(5/6)+1/18*ln(9*RootOf(9*_Z^2-3*_Z+1)*(-(
1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-
x)/(1+x))^(2/3)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+3*(-(1-x)/
(1+x))^(5/6)+3*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+
x))^(1/2)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x+3*(-(1-x)/(1+x))
^(2/3)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)+9*RootOf(9*_Z^2-3*_Z+
1)*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/3)*x+9*RootOf(9*_Z^2-3*_Z+1)
*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x))^(1/6)*x+3*Ro
otOf(9*_Z^2-3*_Z+1)-3*(-(1-x)/(1+x))^(1/6)-2)-1/6*ln(9*RootOf(9*_Z^2-3*_Z+
1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)*x+9*RootOf(9*_Z^2-3*_Z+1)
*(-(1-x)/(1+x))^(2/3)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+3*(-
(1-x)/(1+x))^(5/6)+3*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-
x)/(1+x))^(1/2)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x+3*(-(1-x)/
(1+x))^(2/3)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)+9*RootOf(9*_Z^2
-3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/3)*x+9*RootOf(9*_Z^2-3
*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x))^(1/6)*
x+3*RootOf(9*_Z^2-3*_Z+1)-3*(-(1-x)/(1+x))^(1/6)-2)*RootOf(9*_Z^2-3*_Z+1)+
1/6*RootOf(9*_Z^2-3*_Z+1)*ln(-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)
*x+3*(-(1-x)/(1+x))^(5/6)*x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-1
8*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+3*(-(1-x)/(1+x))^(5/6)+6...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

$$\begin{aligned}\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx &= \frac{1}{6}\left(3\,x^2+7\,x+4\right)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}} \\ &\quad -\frac{1}{18}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\,\sqrt{3}\right) \\ &\quad -\frac{1}{18}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\,\sqrt{3}\right) \\ &\quad +\frac{1}{36}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\ &\quad -\frac{1}{36}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\ &\quad +\frac{1}{18}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{18}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\end{aligned}$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)*x,x, algorithm="fricas")
```

output
```
1/6*(3*x^2 + 7*x + 4)*((x - 1)/(x + 1))^(5/6) - 1/18*sqrt(3)*arctan(2/3*sq
rt(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - 1/18*sqrt(3)*arctan(2/3*sqr
t(3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) + 1/36*log(((x - 1)/(x + 1))^(
1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 1/36*log(((x - 1)/(x + 1))^(1/3) - (
(x - 1)/(x + 1))^(1/6) + 1) + 1/18*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/18
*log(((x - 1)/(x + 1))^(1/6) - 1)
```

Sympy [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx = \int \frac{x}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/6)*x,x)
```

output
```
Integral(x/((x - 1)/(x + 1))**(1/6), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.92

$$\begin{aligned}\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx = & -\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right) \\ & -\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right) \\ & +\frac{\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}-7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{2(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1\right)}+\frac{1}{36}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\ & -\frac{1}{36}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) \\ & +\frac{1}{18}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{18}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\end{aligned}$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)*x,x, algorithm="maxima")
```

output
```
-1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/18*s
qrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) + 1/3*(((x - 1)
/(x + 1))^(11/6) - 7*((x - 1)/(x + 1))^(5/6))/(2*(x - 1)/(x + 1) - (x - 1)
^2/(x + 1)^2 - 1) + 1/36*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(
1/6) + 1) - 1/36*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1
) + 1/18*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/18*log(((x - 1)/(x + 1))^(1/
6) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.91

$$\begin{aligned}\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx =& -\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)\\ &-\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)\\ &-\frac{\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1}-7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{x-1}{x+1}-1\right)^{2}}+\frac{1}{36}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\ &-\frac{1}{36}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\ &+\frac{1}{18}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{18}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right|\right)\end{aligned}$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)*x,x, algorithm="giac")
```

output
```
-1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/18*s
qrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/3*((x - 1)*
((x - 1)/(x + 1))^(5/6)/(x + 1) - 7*((x - 1)/(x + 1))^(5/6))/((x - 1)/(x +
 1) - 1)^2 + 1/36*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) +
1) - 1/36*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 1/1
8*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/18*log(abs(((x - 1)/(x + 1))^(1/6)
- 1))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.68

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx=\frac{\frac{7\left(\frac{x-1}{x+1}\right)^{5/6}}{3}-\frac{\left(\frac{x-1}{x+1}\right)^{11/6}}{3}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2(x-1)}{x+1}+1}-\frac{\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}1\mathrm{i}\right)1\mathrm{i}}{9}$$
$$-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}2\mathrm{i}}{243\left(-\frac{1}{243}+\frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\left(\frac{\sqrt{3}}{18}-\frac{1}{18}\mathrm{i}\right)-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}2\mathrm{i}}{243\left(\frac{1}{243}+\frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\left(\frac{\sqrt{3}}{18}+\frac{1}{18}\mathrm{i}\right)$$

input
```
int(x/((x - 1)/(x + 1))^(1/6),x)
```

output
```
((7*((x - 1)/(x + 1))^(5/6))/3 - ((x - 1)/(x + 1))^(11/6)/3)/((x - 1)^2/(x
 + 1)^2 - (2*(x - 1))/(x + 1) + 1) - (atan(((x - 1)/(x + 1))^(1/6)*1i)*1i)
/9 - atan((((x - 1)/(x + 1))^(1/6)*2i)/(243*((3^(1/2)*1i)/243 - 1/243)))*(
3^(1/2)/18 - 1i/18) - atan((((x - 1)/(x + 1))^(1/6)*2i)/(243*((3^(1/2)*1i)
/243 + 1/243)))*(3^(1/2)/18 + 1i/18)
```

Reduce [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx=\int\frac{(x+1)^{\frac{1}{6}}x}{(x-1)^{\frac{1}{6}}}dx$$

input
```
int(1/((x-1)/(1+x))^(1/6)*x,x)
```

output
```
int(((x + 1)**(1/6)*x)/(x - 1)**(1/6),x)
```

3.123 $\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 175

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx=\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{1+\frac{1}{x}}\,x-\frac{\arctan\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2}{3}\text{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)+\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{1-\frac{1}{x}}}\right)\sqrt[6]{1-\frac{1}{x}}}\right)$$

output
```
(1-1/x)^(5/6)*(1+1/x)^(1/6)*x-1/3*arctan(1/3*(1-2*(1+1/x)^(1/6)/(1-1/x)^(1
/6))*3^(1/2))*3^(1/2)+1/3*arctan(1/3*(1+2*(1+1/x)^(1/6)/(1-1/x)^(1/6))*3^(
1/2))*3^(1/2)+2/3*arctanh((1+1/x)^(1/6)/(1-1/x)^(1/6))+1/3*arctanh((1+1/x)
^(1/6)/(1+(1+1/x)^(1/3)/(1-1/x)^(1/3))/(1-1/x)^(1/6))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.20

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx = 2e^{\frac{1}{3}\coth^{-1}(x)}\left(\frac{1}{-1+e^{2\coth^{-1}(x)}} + \text{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},e^{2\coth^{-1}(x)}\right)\right)$$

input
```
Integrate[E^(ArcCoth[x]/3),x]
```

output
```
2*E^(ArcCoth[x]/3)*((-1 + E^(2*ArcCoth[x]))^(-1) + Hypergeometric2F1[1/6,
1, 7/6, E^(2*ArcCoth[x])])
```

Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.375, Rules used = {6720, 105, 104, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6720}$$

$$-\int \frac{\sqrt[6]{1+\frac{1}{x}}x^2}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-\frac{1}{3}\int \frac{x}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}$$

$$\Big\downarrow \ 104$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\int \frac{1}{\frac{1}{x^6}-1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}$$

$$\Big\downarrow \ 754$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(-\frac{1}{3}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{2\left(-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{2\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)$$

$$\Big\downarrow \ 27$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(-\frac{1}{3}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)$$

$$\Big\downarrow \ 219$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}\right)\right)$$

$$\Big\downarrow \text{1142}$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(\frac{1}{6}\left(\frac{1}{2}\int -\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{3}{2}\int \frac{1}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)+\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}\cdots}\right.\right.$$

$$\Big\downarrow \text{25}$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)+\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}\cdots}\right.\right.$$

$$\Big\downarrow \text{1083}$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-2\left(\frac{1}{6}\left(3\int \frac{1}{-3-\frac{1}{x^2}}d\left(\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-1\right)-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)+\frac{1}{6}\left(3\int \frac{1}{-3-\frac{1}{x^2}}d\left(\frac{2\sqrt[6]{1+\cdots}}{\sqrt[6]{1-\cdots}}\right.\right.\right.$$

$$\Big\downarrow \text{217}$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-$$
$$2\left(\frac{1}{6}\left(-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\frac{1}{2}\int\frac{\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+1}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\right.\right.$$

$\downarrow$ 1103

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}x-$$
$$2\left(\frac{1}{6}\left(\frac{1}{2}\log\left(\frac{1}{x^2}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}+1}{\sqrt{3}}\right)-\frac{1}{2}\right.\right.$$

input
```
Int[E^(ArcCoth[x]/3),x]
```

output
```
(1 - x^(-1))^(5/6)*(1 + x^(-1))^(1/6)*x - 2*(-1/3*ArcTanh[(1 + x^(-1))^(1/
6)/(1 - x^(-1))^(1/6)] + (-(Sqrt[3]*ArcTan[(-1 + (2*(1 + x^(-1))^(1/6))/(1
 - x^(-1))^(1/6))/Sqrt[3]]) + Log[1 - (1 + x^(-1))^(1/6)/(1 - x^(-1))^(1/6
) + x^(-2)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/(1 - x^(-
1))^(1/6))/Sqrt[3]]) - Log[1 + (1 + x^(-1))^(1/6)/(1 - x^(-1))^(1/6) + x^(
-2)]/2)/6)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 7.30 (sec) , antiderivative size = 1151, normalized size of antiderivative = 6.58

method	result	size
trager	Expression too large to display	1151
risch	Expression too large to display	2884

input
```
int(1/((x-1)/(1+x))^(1/6),x,method=_RETURNVERBOSE)
```

output
```
(1+x)*(-(1-x)/(1+x))^(5/6)+RootOf(9*_Z^2+3*_Z+1)*ln(-9*RootOf(9*_Z^2+3*_Z+
1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)*x-9*RootOf(9*_Z^2+3*_Z+1)
*(-(1-x)/(1+x))^(2/3)+18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+3*(-
(1-x)/(1+x))^(5/6)-6*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_Z^2+3*_Z+1)*(-(1-
x)/(1+x))^(1/2)-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x-6*(-(1-x)/
(1+x))^(2/3)+6*(-(1-x)/(1+x))^(1/2)*x-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+
x))^(1/3)+9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x+6*(-(1-x)/(1+x))^
(1/2)-3*(-(1-x)/(1+x))^(1/3)*x+9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/6
)-3*(-(1-x)/(1+x))^(1/3)-3*RootOf(9*_Z^2+3*_Z+1)+1)-1/3*ln(9*RootOf(9*_Z^2
+3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)*x+9*RootOf(9*_Z^2+3
*_Z+1)*(-(1-x)/(1+x))^(2/3)-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/2)*
x+3*(-(1-x)/(1+x))^(5/6)-3*(-(1-x)/(1+x))^(2/3)*x-18*RootOf(9*_Z^2+3*_Z+1)
*(-(1-x)/(1+x))^(1/2)+18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x-3*(-
(1-x)/(1+x))^(2/3)+18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)-9*RootOf(
9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x+3*(-(1-x)/(1+x))^(1/3)*x-9*RootOf(9*
_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/6)+3*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x))^
(1/6)*x+3*RootOf(9*_Z^2+3*_Z+1)-3*(-(1-x)/(1+x))^(1/6)+2)-ln(9*RootOf(9*_Z
^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)*x+9*RootOf(9*_Z^2
+3*_Z+1)*(-(1-x)/(1+x))^(2/3)-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/2
)*x+3*(-(1-x)/(1+x))^(5/6)-3*(-(1-x)/(1+x))^(2/3)*x-18*RootOf(9*_Z^2+3*...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.91

$$\begin{aligned}\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx &= (x+1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}} - \frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \frac{1}{3}\sqrt{3}\right)\\ &\quad - \frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} - \frac{1}{3}\sqrt{3}\right)\\ &\quad + \frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right)\\ &\quad - \frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right)\\ &\quad + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) - \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} - 1\right)\end{aligned}$$

input
```
integrate(1/((x-1)/(1+x))^(1/6),x, algorithm="fricas")
```

output
```
(x + 1)*((x - 1)/(x + 1))^(5/6) - 1/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/
(x + 1))^(1/6) + 1/3*sqrt(3)) - 1/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x
 + 1))^(1/6) - 1/3*sqrt(3)) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(
x + 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(
1/6) + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(((x - 1)/(x + 1
))^(1/6) - 1)
```

Sympy [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx = \int \frac{1}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/6),x)
```

output
```
Integral(((x - 1)/(x + 1))**(-1/6), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.95

$$\begin{aligned}\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx = &-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)\\ &-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)\\ &-\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}-1}+\frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\ &-\frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\ &+\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\end{aligned}$$

input
```
integrate(1/((x-1)/(1+x))^(1/6),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/3*sqr
t(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 2*((x - 1)/(x +
 1))^(5/6)/((x - 1)/(x + 1) - 1) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x -
 1)/(x + 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x +
1))^(1/6) + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(((x - 1)/(
x + 1))^(1/6) - 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.96

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right) - \frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}-1} + \frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) - \frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) - \frac{1}{3}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right|\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/6),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/3*sqr
t(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 2*((x - 1)/(x +
 1))^(5/6)/((x - 1)/(x + 1) - 1) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x -
 1)/(x + 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x +
1))^(1/6) + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(abs(((x -
1)/(x + 1))^(1/6) - 1))
```

Mupad [B] (verification not implemented)

Time = 23.64 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.66

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx$$
$$= -\frac{\operatorname{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6} 1\mathrm{i}\right) 2\mathrm{i}}{3} - \frac{2\left(\frac{x-1}{x+1}\right)^{5/6}}{\frac{x-1}{x+1}-1}$$
$$- \operatorname{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6} 64\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}-\frac{1}{3}\mathrm{i}\right) - \operatorname{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6} 64\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}+\frac{1}{3}\mathrm{i}\right)$$

input
```
int(1/((x - 1)/(x + 1))^(1/6),x)
```

output
```
- (atan(((x - 1)/(x + 1))^(1/6)*1i)*2i)/3 - (2*((x - 1)/(x + 1))^(5/6))/((
x - 1)/(x + 1) - 1) - atan((((x - 1)/(x + 1))^(1/6)*64i)/(3^(1/2)*32i - 32
))*(3^(1/2)/3 - 1i/3) - atan((((x - 1)/(x + 1))^(1/6)*64i)/(3^(1/2)*32i +
32))*(3^(1/2)/3 + 1i/3)
```

Reduce [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx = \int \frac{(x+1)^{\frac{1}{6}}}{(x-1)^{\frac{1}{6}}}dx$$

input
```
int(1/((x-1)/(1+x))^(1/6),x)
```

output
```
int((x + 1)**(1/6)/(x - 1)**(1/6),x)
```

3.124 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 290

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = -\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{\sqrt{3}}\right)+\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{\sqrt{3}}\right)+2\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)-\sqrt{3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\left(1+\frac{\sqrt[3]{1-\frac{1}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)\sqrt[6]{1+\frac{1}{x}}}\right)+2\text{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)+\text{arctanh}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{1-\frac{1}{x}}}\right)\sqrt[6]{1-\frac{1}{x}}}\right)$$

output
```
arctan(-3^(1/2)+2*(1-1/x)^(1/6)/(1+1/x)^(1/6))+arctan(3^(1/2)+2*(1-1/x)^(1
/6)/(1+1/x)^(1/6))-arctan(1/3*(1-2*(1+1/x)^(1/6)/(1-1/x)^(1/6))*3^(1/2))*3
^(1/2)+arctan(1/3*(1+2*(1+1/x)^(1/6)/(1-1/x)^(1/6))*3^(1/2))*3^(1/2)+2*arc
tan((1-1/x)^(1/6)/(1+1/x)^(1/6))-3^(1/2)*arctanh(3^(1/2)*(1-1/x)^(1/6)/(1+
(1-1/x)^(1/3)/(1+1/x)^(1/3))/(1+1/x)^(1/6))+2*arctanh((1+1/x)^(1/6)/(1-1/x
)^(1/6))+arctanh((1+1/x)^(1/6)/(1+(1+1/x)^(1/3)/(1-1/x)^(1/3))/(1-1/x)^(1/
6))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.09

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx=\frac{12}{7}e^{\frac{7}{3}\coth^{-1}(x)}\,\text{Hypergeometric2F1}\left(\frac{7}{12},1,\frac{19}{12},e^{4\coth^{-1}(x)}\right)$$

input
```
Integrate[E^(ArcCoth[x]/3)/x,x]
```

output
```
(12*E^((7*ArcCoth[x])/3)*Hypergeometric2F1[7/12, 1, 19/12, E^(4*ArcCoth[x]
)])/7
```

Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.28, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {6721, 140, 73, 104, 754, 27, 219, 854, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[6]{1+\frac{1}{x}}x}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{140}$$

$$-\int \frac{1}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}-\int \frac{x}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}$$

$$\Big\downarrow \text{73}$$

$$6\int \frac{1}{\left(2-\frac{1}{x^6}\right)^{5/6}x^4}d\sqrt[6]{1-\frac{1}{x}}-\int \frac{x}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}$$

$$\Big\downarrow \text{104}$$

$$6\int \frac{1}{\left(2-\frac{1}{x^6}\right)^{5/6}x^4}d\sqrt[6]{1-\frac{1}{x}}-6\int \frac{1}{\frac{1}{x^6}-1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}$$

$$\Big\downarrow \text{754}$$

$$6\int \frac{1}{\left(2-\frac{1}{x^6}\right)^{5/6}x^4}d\sqrt[6]{1-\frac{1}{x}}-$$
$$6\left(-\frac{1}{3}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{2\left(-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{2\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1+}}{\sqrt[6]{1-}}\right.$$

$$\Big\downarrow \text{27}$$

$$6\int \frac{1}{\left(2-\frac{1}{x^6}\right)^{5/6} x^4}d\sqrt[6]{1-\frac{1}{x}}-$$
$$6\left(-\frac{1}{3}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)$$

$\downarrow$ 219

$$6\int \frac{1}{\left(2-\frac{1}{x^6}\right)^{5/6} x^4}d\sqrt[6]{1-\frac{1}{x}}-$$
$$6\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}\right)\right)$$

$\downarrow$ 854

$$6\int \frac{1}{\left(1+\frac{1}{x^6}\right) x^4}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-$$
$$6\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}\right)\right)$$

$\downarrow$ 824

$$6\left(\frac{1}{3}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\int -\frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{2\left(-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\int -\frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{2\left(\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)}\right.$$

$$6\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}\right)\right)$$

$$\downarrow \text{27}$$

$$6\left(\frac{1}{3}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)-$$

$$6\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}\right)\right)$$

$$\downarrow \text{216}$$

$$6\left(-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\right)$$

$$6\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+2}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}\right)\right)$$

$\downarrow$ 1142

$$6\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \cdots\right.\right.$$

$$6\left(\frac{1}{6}\left(\frac{1}{2}\int -\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{3}{2}\int \frac{1}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)+\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\cdots}\right.\right.$$

$\downarrow$ 25

$$6\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{}{\frac{\sqrt{3}\sqrt[6]{1}}{\sqrt[6]{2-}}}\right.\right.$$

$$6\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)+\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x}}\right.\right.$$

$$\Big\downarrow \text{1083}$$

$$6\left(\frac{1}{6}\left(-\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(-\int \frac{1}{-1-\frac{1}{x^2}}d\left(\right.\right.\right.$$

$$6\left(\frac{1}{6}\left(3\int \frac{1}{-3-\frac{1}{x^2}}d\left(\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-1\right)-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right)+\frac{1}{6}\left(3\int \frac{1}{-3-\frac{1}{x^2}}d\left(\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}\right.\right.\right.$$

$$\Big\downarrow \text{217}$$

$$6\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\right)+\frac{1}{6}\left(\arctan\left(\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\right.\right.\right.$$

$$6\left(\frac{1}{6}\left(-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}}{-\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}-1}{\sqrt{3}}\right)\right)\right)+\frac{1}{6}\left(-\frac{1}{2}\int\frac{\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+1}{\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}+\frac{1}{x^2}+1}d\right.$$

$\downarrow$ 1103

$$6\left(\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\right)+\frac{1}{6}\left(\text{arct}\right.\right.$$

$$6\left(\frac{1}{6}\left(\frac{1}{2}\log\left(\frac{1}{x^2}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}-1}{\sqrt{3}}\right)\right)\right)+\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{1-\frac{1}{x}}}+1}{\sqrt{3}}\right)-\frac{1}{2}\right.$$

input
```
Int[E^(ArcCoth[x]/3)/x,x]
```

output
```
6*(ArcTan[(1 - x^(-1))^(1/6)/(2 - x^(-6))^(1/6)]/3 + (-ArcTan[Sqrt[3] - (2
*(1 - x^(-1))^(1/6))/(2 - x^(-6))^(1/6)] + (Sqrt[3]*Log[1 - (Sqrt[3]*(1 -
x^(-1))^(1/6))/(2 - x^(-6))^(1/6) + x^(-2)])/2)/6 + (ArcTan[Sqrt[3] + (2*(
1 - x^(-1))^(1/6))/(2 - x^(-6))^(1/6)] - (Sqrt[3]*Log[1 + (Sqrt[3]*(1 - x^
(-1))^(1/6))/(2 - x^(-6))^(1/6) + x^(-2)])/2)/6) - 6*(-1/3*ArcTanh[(1 + x^
(-1))^(1/6)/(1 - x^(-1))^(1/6)] + (-(Sqrt[3]*ArcTan[(-1 + (2*(1 + x^(-1))^
(1/6))/(1 - x^(-1))^(1/6))/Sqrt[3]]) + Log[1 - (1 + x^(-1))^(1/6)/(1 - x^(
-1))^(1/6) + x^(-2)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/
(1 - x^(-1))^(1/6))/Sqrt[3]]) - Log[1 + (1 + x^(-1))^(1/6)/(1 - x^(-1))^(1
/6) + x^(-2)]/2)/6)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 10.05 (sec) , antiderivative size = 2202, normalized size of antiderivative = 7.59

method	result	size
trager	Expression too large to display	2202

input
```
int(1/((x-1)/(1+x))^(1/6)/x,x,method=_RETURNVERBOSE)
```

output
```
27*RootOf(81*_Z^4-9*_Z^2+1)^3*ln(-(27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(
1+x))^(2/3)*x+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)+27*RootOf
(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)*x+18*RootOf(81*_Z^4-9*_Z^2+1)^2*
(-(1-x)/(1+x))^(1/2)*x+3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+(
-(1-x)/(1+x))^(5/6)*x+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)+1
8*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)+3*RootOf(81*_Z^4-9*_Z^2+
1)*(-(1-x)/(1+x))^(2/3)+(-(1-x)/(1+x))^(5/6)-18*RootOf(81*_Z^4-9*_Z^2+1)^3
*x-9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x-6*RootOf(81*_Z^4-9*
_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x-2*(-(1-x)/(1+x))^(1/2)*x-9*RootOf(81*_Z^4-9
*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)-6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^
(1/3)-2*(-(1-x)/(1+x))^(1/2)+RootOf(81*_Z^4-9*_Z^2+1)*x)/x)-9*RootOf(81*_Z
^4-9*_Z^2+1)^2*ln(-54*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(2/3)*x-54
*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(2/3)-3*(-(1-x)/(1+x))^(5/6)*x+
54*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/3)*x-3*(-(1-x)/(1+x))^(5/6
)+3*(-(1-x)/(1+x))^(2/3)*x+54*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1
/3)+3*(-(1-x)/(1+x))^(2/3)+6*(-(1-x)/(1+x))^(1/2)*x+6*(-(1-x)/(1+x))^(1/2)
-3*(-(1-x)/(1+x))^(1/3)*x-18*RootOf(81*_Z^4-9*_Z^2+1)^2-3*(-(1-x)/(1+x))^(
1/3)-3*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/6)+1)-3*RootOf(81*_Z^4-9
*_Z^2+1)*ln((54*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)*x+54*RootO
f(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)-27*RootOf(81*_Z^4-9*_Z^2+1)^...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx &= -\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\sqrt{3}\right)\\ &\quad-\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right)\\ &\quad-\frac{1}{2}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\ &\quad+\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\ &\quad+\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\\ &\quad+2\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\ &\quad-\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\\ &\quad+\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\end{aligned}$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)/x,x, algorithm="fricas")
```

output
```
-sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - sqrt(
3)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) - 1/2*sqrt(3)
*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) + 1/2*
sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1
) + arctan(sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + arctan(-sqrt(3) + 2*((x
- 1)/(x + 1))^(1/6)) + 2*arctan(((x - 1)/(x + 1))^(1/6)) + 1/2*log(((x - 1
)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 1/2*log(((x - 1)/(x + 1)
)^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + log(((x - 1)/(x + 1))^(1/6) + 1)
- log(((x - 1)/(x + 1))^(1/6) - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = \int \frac{1}{x\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/6)/x,x)
```

output
```
Integral(1/(x*((x - 1)/(x + 1))**(1/6)), x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = \int \frac{1}{x\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)/x,x, algorithm="maxima")
```

output
```
integrate(1/(x*((x - 1)/(x + 1))^(1/6)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.90

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = -\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right) - \sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right) - \frac{1}{2}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) + \frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) + \arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + 2\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) - \frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) + \log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right) - \log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right|\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)/x,x, algorithm="giac")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - sqrt(3)*arc
tan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/2*sqrt(3)*log(sqrt(3)
*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) + 1/2*sqrt(3)*log(
-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) + arctan(s
qrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + arctan(-sqrt(3) + 2*((x - 1)/(x + 1)
)^(1/6)) + 2*arctan(((x - 1)/(x + 1))^(1/6)) + 1/2*log(((x - 1)/(x + 1))^(
1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 1/2*log(((x - 1)/(x + 1))^(1/3) - ((
x - 1)/(x + 1))^(1/6) + 1) + log(((x - 1)/(x + 1))^(1/6) + 1) - log(abs(((
x - 1)/(x + 1))^(1/6) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.58

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = 2\,\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right) - \mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6} 1\mathrm{i}\right) 2\mathrm{i} - \mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6} 1486016741376\mathrm{i}}{-743008370688+\sqrt{3}\,743008370688\mathrm{i}}\right)\left(\sqrt{3}-\mathrm{i}\right) - \mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6} 1486016741376\mathrm{i}}{743008370688+\sqrt{3}\,743008370688\mathrm{i}}\right)$$

input
```
int(1/(x*((x - 1)/(x + 1))^(1/6)),x)
```

output
```
2*atan(((x - 1)/(x + 1))^(1/6)) - atan(((x - 1)/(x + 1))^(1/6)*1i)*2i - at
an((((x - 1)/(x + 1))^(1/6)*1486016741376i)/(3^(1/2)*743008370688i - 74300
8370688))*(3^(1/2) - 1i) - atan((((x - 1)/(x + 1))^(1/6)*1486016741376i)/(
3^(1/2)*743008370688i + 743008370688))*(3^(1/2) + 1i) - atan((148601674137
6*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*743008370688i - 743008370688))*(3^(1/2
)*1i + 1) - atan((1486016741376*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*74300837
0688i + 743008370688))*(3^(1/2)*1i - 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.31

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = \frac{(x+1)^{\frac{7}{6}}\left(4atan\left((x-1)^{\frac{1}{6}}\right)+2atan\left(2(x-1)^{\frac{1}{6}}-\sqrt{3}\right)+2atan\left(2(x-1)^{\frac{1}{6}}+\sqrt{3}\right)+\sqrt{3}\log\left(-(x-1)^{\frac{1}{6}}\right.\right.}{2x+2}$$

input
```
int(1/((x-1)/(1+x))^(1/6)/x,x)
```

output
```
((x + 1)**(7/6)*(4*atan((x - 1)**(1/6)) + 2*atan(2*(x - 1)**(1/6) - sqrt(3
)) + 2*atan(2*(x - 1)**(1/6) + sqrt(3)) + sqrt(3)*log( - (x - 1)**(1/6)*sq
rt(3) + (x - 1)**(1/3) + 1) - sqrt(3)*log((x - 1)**(1/6)*sqrt(3) + (x - 1)
**(1/3) + 1)))/(2*(x + 1))
```

3.125 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 173

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = \left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{1+\frac{1}{x}} - \frac{1}{3}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{3}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{2}{3}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\left(1+\frac{\sqrt[3]{1-\frac{1}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)\sqrt[6]{1+\frac{1}{x}}}\right)}{\sqrt{3}}$$

output
```
(1-1/x)^(5/6)*(1+1/x)^(1/6)+1/3*arctan(-3^(1/2)+2*(1-1/x)^(1/6)/(1+1/x)^(1
/6))+1/3*arctan(3^(1/2)+2*(1-1/x)^(1/6)/(1+1/x)^(1/6))+2/3*arctan((1-1/x)^
(1/6)/(1+1/x)^(1/6))-1/3*3^(1/2)*arctanh(3^(1/2)*(1-1/x)^(1/6)/(1+(1-1/x)^
(1/3)/(1+1/x)^(1/3))/(1+1/x)^(1/6))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.23

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = -2e^{\frac{1}{3}\coth^{-1}(x)}\left(-\frac{1}{1+e^{2\coth^{-1}(x)}} + \text{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},-e^{2\coth^{-1}(x)}\right)\right)$$

input
```
Integrate[E^(ArcCoth[x]/3)/x^2,x]
```

output
```
-2*E^(ArcCoth[x]/3)*(-(1 + E^(2*ArcCoth[x]))^(-1) + Hypergeometric2F1[1/6,
 1, 7/6, -E^(2*ArcCoth[x])])
```

Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6721, 60, 73, 854, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx$$

$$\Big\downarrow \; 6721$$

$$-\int \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 60}$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}-\frac{1}{3}\int \frac{1}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 73}$$

$$2\int \frac{1}{\left(2-\frac{1}{x^6}\right)^{5/6}x^4}d\sqrt[6]{1-\frac{1}{x}}+\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}$$

$$\Big\downarrow \text{ 854}$$

$$2\int \frac{1}{\left(1+\frac{1}{x^6}\right)x^4}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}$$

$$\Big\downarrow \text{ 824}$$

$$2\left(\frac{1}{3}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\int -\frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{2\left(-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\int -\frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{2\left(\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)}\right.$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\left(\frac{1}{3}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+}{\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}}$$

$\downarrow$ 216

$$\frac{2\left(-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\right)}{\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}}$$

$\downarrow$ 1142

$$\frac{2\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{}{\sqrt{3}\sqrt[6]{}\ldots\sqrt[6]{2}}\right.\right.}{\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}}$$

$\downarrow$ 25

$$\frac{2\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{}{\frac{\sqrt{3}\sqrt[6]{1}}{\sqrt[6]{2-}}} \cdots\right.\right.}{\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}}$$

$$\Big\downarrow \; 1083$$

$$\frac{2\left(\frac{1}{6}\left(-\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(-\int \frac{1}{-1-\frac{1}{x^2}}d\left(\cdots\right.\right.\right.}{\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}}$$

$$\Big\downarrow \; 217$$

$$\frac{2\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\right)+\frac{1}{6}\left(\arctan\left(\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\cdots\right.\right.\right.}{\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}}$$

$$\Big\downarrow \; 1103$$

$$2\left(\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\right)+\frac{1}{6}\left(\text{arct}\right.\right.$$

$$\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}$$

input
```
Int[E^(ArcCoth[x]/3)/x^2,x]
```

output
```
(1 - x^(-1))^(5/6)*(1 + x^(-1))^(1/6) + 2*(ArcTan[(1 - x^(-1))^(1/6)/(2 -
x^(-6))^(1/6)]/3 + (-ArcTan[Sqrt[3] - (2*(1 - x^(-1))^(1/6))/(2 - x^(-6))^
(1/6)] + (Sqrt[3]*Log[1 - (Sqrt[3]*(1 - x^(-1))^(1/6))/(2 - x^(-6))^(1/6)
+ x^(-2)])/2)/6 + (ArcTan[Sqrt[3] + (2*(1 - x^(-1))^(1/6))/(2 - x^(-6))^(1
/6)] - (Sqrt[3]*Log[1 + (Sqrt[3]*(1 - x^(-1))^(1/6))/(2 - x^(-6))^(1/6) +
x^(-2)])/2)/6)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 26.56 (sec) , antiderivative size = 889, normalized size of antiderivative = 5.14

method	result	size
trager	Expression too large to display	889
risch	Expression too large to display	2157

input
```
int(1/((x-1)/(1+x))^(1/6)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(1+x)*(-(1-x)/(1+x))^(5/6)/x-1/3*RootOf(_Z^2+1)*ln((18*RootOf(-3*_Z*RootOf
(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/2)*x-3*RootOf(_Z^2+1)*
(-(1-x)/(1+x))^(2/3)*x-9*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+
x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)*x+18*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2
-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/2)-3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2
/3)-9*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)
/(1+x))^(5/6)-9*RootOf(_Z^2+1)*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-
x)/(1+x))^(1/6)*x-3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x+18*RootOf(-3*_Z*
RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)*x-9*RootOf(_Z^2+1)*RootOf(-3
*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/6)-3*RootOf(_Z^2+1)*(-(1-x)
/(1+x))^(1/3)+18*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3
)+2*x*RootOf(_Z^2+1)-3*x*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)-3*(-(1-x)/(
1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/6))/x)+RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^
2-1)*ln((3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x-18*RootOf(-3*_Z*RootOf(_Z
^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)*x+3*RootOf(_
Z^2+1)*(-(1-x)/(1+x))^(2/3)-18*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-
x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(5/6)+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/
3)*x-18*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)*x+6*(-(
1-x)/(1+x))^(1/2)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)-18*RootOf(-3*_Z*
RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)+6*(-(1-x)/(1+x))^(1/2)+x*...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.87

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = -\frac{\sqrt{3}x\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \sqrt{3}x\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - 2\,x\arctan\left(\sqrt{3} + 2\left(\frac{x}{x}\right.\right.}{6\,x}$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)/x^2,x, algorithm="fricas")
```

output
```
-1/6*(sqrt(3)*x*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1
/3) + 1) - sqrt(3)*x*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x +
1))^(1/3) + 1) - 2*x*arctan(sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) - 2*x*arc
tan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) - 4*x*arctan(((x - 1)/(x + 1))^(
1/6)) - 6*(x + 1)*((x - 1)/(x + 1))^(5/6))/x
```

Sympy [F]

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx=\int \frac{1}{x^2\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/6)/x**2,x)
```

output
```
Integral(1/(x**2*((x - 1)/(x + 1))**(1/6)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx=&-\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\&+\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\\&+\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}+1}+\frac{1}{3}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\\&+\frac{1}{3}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{2}{3}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\end{aligned}$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)/x^2,x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
 + 1) + 1/6*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1
))^(1/3) + 1) + 2*((x - 1)/(x + 1))^(5/6)/((x - 1)/(x + 1) + 1) + 1/3*arct
an(sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 1/3*arctan(-sqrt(3) + 2*((x - 1)
/(x + 1))^(1/6)) + 2/3*arctan(((x - 1)/(x + 1))^(1/6))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.88

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = -\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) + \frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) + \frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}+1} + \frac{1}{3}\arctan\left(\sqrt{3} + 2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{1}{3}\arctan\left(-\sqrt{3} + 2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{2}{3}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)/x^2,x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
 + 1) + 1/6*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1
))^(1/3) + 1) + 2*((x - 1)/(x + 1))^(5/6)/((x - 1)/(x + 1) + 1) + 1/3*arct
an(sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 1/3*arctan(-sqrt(3) + 2*((x - 1)
/(x + 1))^(1/6)) + 2/3*arctan(((x - 1)/(x + 1))^(1/6))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.63

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = \frac{2\,\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)}{3} + \frac{2\left(\frac{x-1}{x+1}\right)^{5/6}}{\frac{x-1}{x+1}+1} - \mathrm{atan}\left(\frac{64\left(\frac{x-1}{x+1}\right)^{1/6}}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right) - \mathrm{atan}\left(\frac{64\left(\frac{x-1}{x+1}\right)^{1/6}}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)$$

input
```
int(1/(x^2*((x - 1)/(x + 1))^(1/6)),x)
```

output
```
(2*atan(((x - 1)/(x + 1))^(1/6)))/3 + (2*((x - 1)/(x + 1))^(5/6))/((x - 1)
/(x + 1) + 1) - atan((64*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*32i - 32))*((3^
(1/2)*1i)/3 + 1/3) - atan((64*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*32i + 32))
*((3^(1/2)*1i)/3 - 1/3)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.29

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = \frac{(x-1)^{\frac{5}{6}}\left(-(x+1)^{\frac{3}{2}}x+(x+1)^{\frac{3}{2}}+6\sqrt{x+1}\log\left((x-1)^{\frac{1}{6}}\right)x-\sqrt{x+1}\log(x)\,x\right)}{(x+1)^{\frac{1}{3}}x}$$

input
```
int(1/((x-1)/(1+x))^(1/6)/x^2,x)
```

output
```
((x - 1)**(5/6)*( - (x + 1)**(3/2)*x + (x + 1)**(3/2) + 6*sqrt(x + 1)*log(
(x - 1)**(1/6))*x - sqrt(x + 1)*log(x)*x))/((x + 1)**(1/3)*x)
```

3.126 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 202

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{1}{6}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{1+\frac{1}{x}}+\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(1+\frac{1}{x}\right)^{7/6}$$

$$-\frac{1}{18}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{1}{18}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{1}{9}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)-$$

arctanh

output
```
1/6*(1-1/x)^(5/6)*(1+1/x)^(1/6)+1/2*(1-1/x)^(5/6)*(1+1/x)^(7/6)+1/18*arcta
n(-3^(1/2)+2*(1-1/x)^(1/6)/(1+1/x)^(1/6))+1/18*arctan(3^(1/2)+2*(1-1/x)^(1
/6)/(1+1/x)^(1/6))+1/9*arctan((1-1/x)^(1/6)/(1+1/x)^(1/6))-1/18*3^(1/2)*ar
ctanh(3^(1/2)*(1-1/x)^(1/6)/(1+(1-1/x)^(1/3)/(1+1/x)^(1/3))/(1+1/x)^(1/6))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.93 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.61

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx$$
$$= \frac{1}{54}\left(\frac{18e^{\frac{1}{3}\coth^{-1}(x)}\left(1+7e^{2\coth^{-1}(x)}\right)}{\left(1+e^{2\coth^{-1}(x)}\right)^2} - 6\arctan\left(e^{\frac{1}{3}\coth^{-1}(x)}\right) + \text{RootSum}\left[1-\#1^2 + \#1^4\&, \frac{2\coth^{-1}(x) - 6\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right) - \coth^{-1}(x)\#1^2 + 3\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)\#1^2}{-\#1+2\#1^3}\&\right]\right)$$

input
```
Integrate[E^(ArcCoth[x]/3)/x^3,x]
```

output
```
((18*E^(ArcCoth[x]/3)*(1 + 7*E^(2*ArcCoth[x])))/(1 + E^(2*ArcCoth[x]))^2 -
 6*ArcTan[E^(ArcCoth[x]/3)] + RootSum[1 - #1^2 + #1^4 & , (2*ArcCoth[x] -
6*Log[E^(ArcCoth[x]/3) - #1] - ArcCoth[x]*#1^2 + 3*Log[E^(ArcCoth[x]/3) -
#1]*#1^2)/(-#1 + 2*#1^3) & ])/54
```

Rubi [A] (warning: unable to verify)

Time = 0.67 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.20, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {6721, 90, 60, 73, 854, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx$$
$$\Big\downarrow \; 6721$$

$$-\int \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}x}d\frac{1}{x}$$

$$\Big\downarrow \ 90$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{1}{6}\int \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \ 60$$

$$\frac{1}{6}\left(\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}-\frac{1}{3}\int \frac{1}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}\right)+\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\Big\downarrow \ 73$$

$$\frac{1}{6}\left(2\int \frac{1}{\left(2-\frac{1}{x^6}\right)^{5/6}x^4}d\sqrt[6]{1-\frac{1}{x}}+\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}\right)+\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\Big\downarrow \ 854$$

$$\frac{1}{6}\left(2\int \frac{1}{\left(1+\frac{1}{x^6}\right)x^4}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}\right)+\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\Big\downarrow \ 824$$

$$\frac{1}{6}\left(2\left(\frac{1}{3}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\int -\frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{2\left(-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\int -\frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{2\left(\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+\right.}\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{6}\left(2\left(\frac{1}{3}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\downarrow 216$$

$$\frac{1}{6}\left(2\left(-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right.\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\downarrow 1142$$

$$\frac{1}{6}\left(2\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\int -\right.\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\downarrow 25$$

$$\frac{1}{6}\left(2\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{}{\frac{\sqrt{3}}{\sqrt[6]{}}}\right.\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{6}\left(2\left(\frac{1}{6}\left(-\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(-\int \frac{1}{-1-\frac{1}{x}}\right.\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{6}\left(2\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\right)+\frac{1}{6}\left(\arctan\left(\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right.\right.\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{6}\left(2\left(\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)\right)+\frac{1}{6}\left(\right.\right.\right.$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}$$

input
```
Int[E^(ArcCoth[x]/3)/x^3,x]
```

output
```
((1 - x^(-1))^(5/6)*(1 + x^(-1))^(7/6))/2 + ((1 - x^(-1))^(5/6)*(1 + x^(-1
))^(1/6) + 2*(ArcTan[(1 - x^(-1))^(1/6)/(2 - x^(-6))^(1/6)]/3 + (-ArcTan[S
qrt[3] - (2*(1 - x^(-1))^(1/6))/(2 - x^(-6))^(1/6)] + (Sqrt[3]*Log[1 - (Sq
rt[3]*(1 - x^(-1))^(1/6))/(2 - x^(-6))^(1/6) + x^(-2)])/2)/6 + (ArcTan[Sqr
t[3] + (2*(1 - x^(-1))^(1/6))/(2 - x^(-6))^(1/6)] - (Sqrt[3]*Log[1 + (Sqrt
[3]*(1 - x^(-1))^(1/6))/(2 - x^(-6))^(1/6) + x^(-2)])/2)/6))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 11.20 (sec) , antiderivative size = 1502, normalized size of antiderivative = 7.44

method	result	size
trager	Expression too large to display	1502
risch	Expression too large to display	3480

input
```
int(1/((x-1)/(1+x))^(1/6)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*(1+x)*(3+4*x)/x^2*(-(1-x)/(1+x))^(5/6)-3/2*RootOf(81*_Z^4-9*_Z^2+1)^3*
ln(-(27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)*x+27*RootOf(81*_Z^
4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(
1+x))^(1/3)*x-18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)*x+3*RootO
f(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x-(-(1-x)/(1+x))^(5/6)*x+27*RootO
f(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)-18*RootOf(81*_Z^4-9*_Z^2+1)^2*(
-(1-x)/(1+x))^(1/2)+3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)-(-(1-x
)/(1+x))^(5/6)-18*RootOf(81*_Z^4-9*_Z^2+1)^3*x+9*RootOf(81*_Z^4-9*_Z^2+1)^
2*(-(1-x)/(1+x))^(1/6)*x-6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x
+2*(-(1-x)/(1+x))^(1/2)*x+9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6
)-6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)+2*(-(1-x)/(1+x))^(1/2)+R
ootOf(81*_Z^4-9*_Z^2+1)*x)/x)+1/6*RootOf(81*_Z^4-9*_Z^2+1)*ln(-(27*RootOf(
81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)*x+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(
-(1-x)/(1+x))^(2/3)+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)*x-1
8*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)*x+3*RootOf(81*_Z^4-9*_Z^
2+1)*(-(1-x)/(1+x))^(2/3)*x-(-(1-x)/(1+x))^(5/6)*x+27*RootOf(81*_Z^4-9*_Z^
2+1)^3*(-(1-x)/(1+x))^(1/3)-18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(
1/2)+3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)-(-(1-x)/(1+x))^(5/6)-
18*RootOf(81*_Z^4-9*_Z^2+1)^3*x+9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x)
)^(1/6)*x-6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x+2*(-(1-x)/(...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.83

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = -\frac{\sqrt{3}x^2\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \sqrt{3}x^2\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - 2\,x^2\arctan\left(\sqrt{3} + 2\right.}{36\,x^2}$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)/x^3,x, algorithm="fricas")
```

output
```
-1/36*(sqrt(3)*x^2*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))
^(1/3) + 1) - sqrt(3)*x^2*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/
(x + 1))^(1/3) + 1) - 2*x^2*arctan(sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) -
2*x^2*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) - 4*x^2*arctan(((x - 1)
/(x + 1))^(1/6)) - 6*(4*x^2 + 7*x + 3)*((x - 1)/(x + 1))^(5/6))/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3} \, dx = \int \frac{1}{x^3 \sqrt[6]{\frac{x-1}{x+1}}} \, dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/6)/x**3,x)
```

output
```
Integral(1/(x**3*((x - 1)/(x + 1))**(1/6)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3} \, dx = & -\frac{1}{36}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) \\ & + \frac{1}{36}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) \\ & + \frac{\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}} + 7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{2(x-1)}{x+1} + \frac{(x-1)^2}{(x+1)^2} + 1\right)} + \frac{1}{18}\arctan\left(\sqrt{3} + 2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) \\ & + \frac{1}{18}\arctan\left(-\sqrt{3} + 2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{1}{9}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\end{aligned}$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)/x^3,x, algorithm="maxima")
```

output
```
-1/36*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3
) + 1) + 1/36*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x +
 1))^(1/3) + 1) + 1/3*(((x - 1)/(x + 1))^(11/6) + 7*((x - 1)/(x + 1))^(5/6
))/(2*(x - 1)/(x + 1) + (x - 1)^2/(x + 1)^2 + 1) + 1/18*arctan(sqrt(3) + 2
*((x - 1)/(x + 1))^(1/6)) + 1/18*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/
6)) + 1/9*arctan(((x - 1)/(x + 1))^(1/6))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = -\frac{1}{36}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) + \frac{1}{36}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) + \frac{\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1} + 7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{x-1}{x+1}+1\right)^2} + \frac{1}{18}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{1}{18}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{1}{9}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)/x^3,x, algorithm="giac")
```

output
```
-1/36*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3
) + 1) + 1/36*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x +
 1))^(1/3) + 1) + 1/3*((x - 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) + 7*((x - 1
)/(x + 1))^(5/6))/((x - 1)/(x + 1) + 1)^2 + 1/18*arctan(sqrt(3) + 2*((x -
1)/(x + 1))^(1/6)) + 1/18*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 1
/9*arctan(((x - 1)/(x + 1))^(1/6))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.67

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)}{9} + \frac{\frac{7\left(\frac{x-1}{x+1}\right)^{5/6}}{3} + \frac{\left(\frac{x-1}{x+1}\right)^{11/6}}{3}}{\frac{2(x-1)}{x+1} + \frac{(x-1)^2}{(x+1)^2} + 1} - \mathrm{atan}\left(\frac{2\left(\frac{x-1}{x+1}\right)^{1/6}}{243\left(-\frac{1}{243}+\frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right) - \mathrm{atan}\left(\frac{2\left(\frac{x-1}{x+1}\right)^{1/6}}{243\left(\frac{1}{243}+\frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)$$

input
```
int(1/(x^3*((x - 1)/(x + 1))^(1/6)),x)
```

output
```
atan(((x - 1)/(x + 1))^(1/6))/9 + ((7*((x - 1)/(x + 1))^(5/6))/3 + ((x - 1
)/(x + 1))^(11/6)/3)/((2*(x - 1))/(x + 1) + (x - 1)^2/(x + 1)^2 + 1) - ata
n((2*((x - 1)/(x + 1))^(1/6))/(243*((3^(1/2)*1i)/243 - 1/243)))*((3^(1/2)*
1i)/18 + 1/18) - atan((2*((x - 1)/(x + 1))^(1/6))/(243*((3^(1/2)*1i)/243 +
 1/243)))*((3^(1/2)*1i)/18 - 1/18)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.32

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{(x-1)^{\frac{5}{6}}\left(-(x+1)^{\frac{3}{2}}x^2+(x+1)^{\frac{3}{2}}x+(x+1)^{\frac{3}{2}}+18\sqrt{x+1}\log\left((x-1)^{\frac{1}{6}}\right)x^2-3\sqrt{x+1}\log(x)x^2\right)}{2(x+1)^{\frac{1}{3}}x^2}$$

input
```
int(1/((x-1)/(1+x))^(1/6)/x^3,x)
```

output
```
((x - 1)**(5/6)*( - (x + 1)**(3/2)*x**2 + (x + 1)**(3/2)*x + (x + 1)**(3/2
) + 18*sqrt(x + 1)*log((x - 1)**(1/6))*x**2 - 3*sqrt(x + 1)*log(x)*x**2))/
(2*(x + 1)**(1/3)*x**2)
```

3.127 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 229

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx$$
$$= \frac{19}{54}\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{1+\frac{1}{x}}+\frac{1}{18}\left(1-\frac{1}{x}\right)^{5/6}\left(1+\frac{1}{x}\right)^{7/6}+\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(1+\frac{1}{x}\right)^{7/6}}{3x}$$

19arctanh

$$-\frac{19}{162}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{19}{162}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{19}{81}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)-$$

output
```
19/54*(1-1/x)^(5/6)*(1+1/x)^(1/6)+1/18*(1-1/x)^(5/6)*(1+1/x)^(7/6)+1/3*(1-
1/x)^(5/6)*(1+1/x)^(7/6)/x+19/162*arctan(-3^(1/2)+2*(1-1/x)^(1/6)/(1+1/x)^
(1/6))+19/162*arctan(3^(1/2)+2*(1-1/x)^(1/6)/(1+1/x)^(1/6))+19/81*arctan((
1-1/x)^(1/6)/(1+1/x)^(1/6))-19/162*3^(1/2)*arctanh(3^(1/2)*(1-1/x)^(1/6)/(
1+(1-1/x)^(1/3)/(1+1/x)^(1/3))/(1+1/x)^(1/6))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.58

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = \frac{1}{486}\left(\frac{18e^{\frac{1}{3}\coth^{-1}(x)}\left(19+8e^{2\coth^{-1}(x)}+61e^{4\coth^{-1}(x)}\right)}{\left(1+e^{2\coth^{-1}(x)}\right)^3} - 114\arctan\left(e^{\frac{1}{3}\coth^{-1}(x)}\right) - 19\text{RootSum}\left[1-\#1^2 +\#1^4\&, \frac{-2\coth^{-1}(x)+6\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)+\coth^{-1}(x)\#1^2-3\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)\#1^2}{-\#1+2\#1^3}\&\right]\right)$$

input
```
Integrate[E^(ArcCoth[x]/3)/x^4,x]
```

output
```
((18*E^(ArcCoth[x]/3)*(19 + 8*E^(2*ArcCoth[x]) + 61*E^(4*ArcCoth[x])))/(1
+ E^(2*ArcCoth[x]))^3 - 114*ArcTan[E^(ArcCoth[x]/3)] - 19*RootSum[1 - #1^2
 + #1^4 & , (-2*ArcCoth[x] + 6*Log[E^(ArcCoth[x]/3) - #1] + ArcCoth[x]*#1^
2 - 3*Log[E^(ArcCoth[x]/3) - #1]*#1^2)/(-#1 + 2*#1^3) & ])/486
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.21, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {6721, 101, 27, 90, 60, 73, 854, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}x^2}d\frac{1}{x}$$

$\downarrow$ 101

$$\frac{1}{3}\int -\frac{\sqrt[6]{1+\frac{1}{x}}\left(3+\frac{1}{x}\right)}{3\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}+\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$\downarrow$ 27

$$\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}-\frac{1}{9}\int \frac{\sqrt[6]{1+\frac{1}{x}}\left(3+\frac{1}{x}\right)}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$\downarrow$ 90

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\int \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}\right)+\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$\downarrow$ 60

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(\frac{1}{3}\int \frac{1}{\sqrt[6]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{5/6}}d\frac{1}{x}-\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}\right)\right)+\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$\downarrow$ 73

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\int \frac{1}{\left(2-\frac{1}{x^6}\right)^{5/6}x^4}d\sqrt[6]{1-\frac{1}{x}}-\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}\right)\right)+\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$\downarrow$ 854

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\int\frac{1}{\left(1+\frac{1}{x^6}\right)x^4}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\left(1-\frac{1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}\right)\right)+\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$\downarrow$ 824

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\left(\frac{1}{3}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{3}\int-\frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{2\left(-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)}d\frac{\sqrt[6]{1-}}{\sqrt[6]{2-}}\right.\right.\right.$$

$$+\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$\downarrow$ 27

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\left(\frac{1}{3}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int\frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\right.\right.\right.$$

$$+\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$\downarrow$ 216

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\left(-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+1}{\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+\right.\right.\right.$$

$$\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$\downarrow$ 1142

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-\frac{2\sqrt[6]{}}{\sqrt[6]{2}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}\right.\right.\right.\right.$$

$$\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$\downarrow$ 25

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1}}{\sqrt[6]{2-}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+}\right.\right.\right.\right.$$

$$\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$\downarrow$ 1083

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\left(\frac{1}{6}\left(-\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}\cdots}\right.\right.\right.\right.$$

$$\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$\big\downarrow$ 217

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}}{-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1}d\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{\cdots}}{\sqrt[6]{\cdots}}\right.\right.\right.\right.\right.$$

$$\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

$\big\downarrow$ 1103

$$\frac{1}{9}\left(\frac{1}{2}\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}-\frac{19}{6}\left(-2\left(\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(-\frac{\sqrt{3}\sqrt[6]{1-\frac{1}{x}}}{\sqrt[6]{2-\frac{1}{x^6}}}+\frac{1}{x^2}+1\right)-\cdots\right.\right.\right.\right.$$

$$\frac{\left(1-\frac{1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}$$

input

```
Int[E^(ArcCoth[x]/3)/x^4,x]
```

output
```
((1 - x^(-1))^(5/6)*(1 + x^(-1))^(7/6))/(3*x) + (((1 - x^(-1))^(5/6)*(1 +
x^(-1))^(7/6))/2 - (19*(-((1 - x^(-1))^(5/6)*(1 + x^(-1))^(1/6)) - 2*(ArcT
an[(1 - x^(-1))^(1/6)/(2 - x^(-6))^(1/6)]/3 + (-ArcTan[Sqrt[3] - (2*(1 - x
^(-1))^(1/6))/(2 - x^(-6))^(1/6)] + (Sqrt[3]*Log[1 - (Sqrt[3]*(1 - x^(-1))
^(1/6))/(2 - x^(-6))^(1/6) + x^(-2)])/2)/6 + (ArcTan[Sqrt[3] + (2*(1 - x^(
-1))^(1/6))/(2 - x^(-6))^(1/6)] - (Sqrt[3]*Log[1 + (Sqrt[3]*(1 - x^(-1))^(
1/6))/(2 - x^(-6))^(1/6) + x^(-2)])/2)/6)))/6)/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 17.21 (sec) , antiderivative size = 901, normalized size of antiderivative = 3.93

method	result	size
trager	Expression too large to display	901
risch	Expression too large to display	3483

input
```
int(1/((x-1)/(1+x))^(1/6)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/54*(1+x)*(22*x^2+21*x+18)/x^3*(-(1-x)/(1+x))^(5/6)+19/162*RootOf(_Z^2+1)
*ln((18*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x)
)^(1/2)*x+9*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)*x+3
*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)*x+18*RootOf(
-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/2)+9*RootO
f(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)+3*RootOf(_Z^2+1)*(-(
1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(5/6)-9*RootOf(_Z^2+1)*RootOf(-3*_Z*Roo
tOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/6)*x-18*RootOf(-3*_Z*RootOf(_Z^2+1
)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x
-9*RootOf(_Z^2+1)*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/
6)-18*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(1/3)+3*RootOf(
_Z^2+1)*(-(1-x)/(1+x))^(1/3)+3*x*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)-2*x
*RootOf(_Z^2+1)-3*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/6))/x)+19/54*
RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*ln((3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^
(2/3)*x-18*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)*x+3*
(-(1-x)/(1+x))^(5/6)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)-18*RootOf(-3*
_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(5/6)+3*
RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x-18*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^
2-1)*(-(1-x)/(1+x))^(1/3)*x+6*(-(1-x)/(1+x))^(1/2)*x+3*RootOf(_Z^2+1)*(-(1
-x)/(1+x))^(1/3)-18*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x)...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.76

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx =$$
$$-\frac{19\sqrt{3}x^3\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-19\sqrt{3}x^3\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-38\,x^3\arctan\left(\right.}{}$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)/x^4,x, algorithm="fricas")
```

output
```
-1/324*(19*sqrt(3)*x^3*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x +
 1))^(1/3) + 1) - 19*sqrt(3)*x^3*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((
x - 1)/(x + 1))^(1/3) + 1) - 38*x^3*arctan(sqrt(3) + 2*((x - 1)/(x + 1))^(
1/6)) - 38*x^3*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) - 76*x^3*arcta
n(((x - 1)/(x + 1))^(1/6)) - 6*(22*x^3 + 43*x^2 + 39*x + 18)*((x - 1)/(x +
 1))^(5/6))/x^3
```

Sympy [F]

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = \int \frac{1}{x^4\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/6)/x**4,x)
```

output
```
Integral(1/(x**4*((x - 1)/(x + 1))**(1/6)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx =& -\frac{19}{324}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right)\\&+ \frac{19}{324}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right)\\&+ \frac{19\left(\frac{x-1}{x+1}\right)^{\frac{17}{6}} + 8\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}} + 61\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{27\left(\frac{3(x-1)}{x+1} + \frac{3(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} + 1\right)}\\&+ \frac{19}{162}\arctan\left(\sqrt{3} + 2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\\&+ \frac{19}{162}\arctan\left(-\sqrt{3} + 2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{19}{81}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)\end{aligned}$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)/x^4,x, algorithm="maxima")
```

output
```
-19/324*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1
/3) + 1) + 19/324*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/
(x + 1))^(1/3) + 1) + 1/27*(19*((x - 1)/(x + 1))^(17/6) + 8*((x - 1)/(x +
1))^(11/6) + 61*((x - 1)/(x + 1))^(5/6))/(3*(x - 1)/(x + 1) + 3*(x - 1)^2/
(x + 1)^2 + (x - 1)^3/(x + 1)^3 + 1) + 19/162*arctan(sqrt(3) + 2*((x - 1)/
(x + 1))^(1/6)) + 19/162*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 19
/81*arctan(((x - 1)/(x + 1))^(1/6))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.87

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = -\frac{19}{324}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) + \frac{19}{324}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) + \frac{\frac{8\,(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1}+\frac{19\,(x-1)^2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{(x+1)^2}+61\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{27\left(\frac{x-1}{x+1}+1\right)^3} + \frac{19}{162}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{19}{162}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right) + \frac{19}{81}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)/x^4,x, algorithm="giac")
```

output
```
-19/324*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1
/3) + 1) + 19/324*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/
(x + 1))^(1/3) + 1) + 1/27*(8*(x - 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) + 19
*(x - 1)^2*((x - 1)/(x + 1))^(5/6)/(x + 1)^2 + 61*((x - 1)/(x + 1))^(5/6))
/((x - 1)/(x + 1) + 1)^3 + 19/162*arctan(sqrt(3) + 2*((x - 1)/(x + 1))^(1/
6)) + 19/162*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 19/81*arctan((
(x - 1)/(x + 1))^(1/6))
```

Mupad [B] (verification not implemented)

Time = 23.45 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.70

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = \frac{19\,\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)}{81} + \frac{\frac{61\left(\frac{x-1}{x+1}\right)^{5/6}}{27} + \frac{8\left(\frac{x-1}{x+1}\right)^{11/6}}{27} + \frac{19\left(\frac{x-1}{x+1}\right)^{17/6}}{27}}{\frac{3\,(x-1)}{x+1} + \frac{3\,(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} + 1} - \mathrm{atan}\left(\frac{4952198\left(\frac{x-1}{x+1}\right)^{1/6}}{14348907\left(-\frac{2476099}{14348907} + \frac{\sqrt{3}\,2476099\mathrm{i}}{14348907}\right)}\right)\left(\frac{19}{162} + \frac{\sqrt{3}\,19\mathrm{i}}{162}\right) - \mathrm{atan}\left(\frac{4952198\left(\frac{x-1}{x+1}\right)^{1/6}}{14348907\left(\frac{2476099}{14348907} + \frac{\sqrt{3}\,2476099}{14348907}\right.}\right.$$

input
```
int(1/(x^4*((x - 1)/(x + 1))^(1/6)),x)
```

output
```
(19*atan(((x - 1)/(x + 1))^(1/6)))/81 + ((61*((x - 1)/(x + 1))^(5/6))/27 +
 (8*((x - 1)/(x + 1))^(11/6))/27 + (19*((x - 1)/(x + 1))^(17/6))/27)/((3*(
x - 1))/(x + 1) + (3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 + 1) - ata
n((4952198*((x - 1)/(x + 1))^(1/6))/(14348907*((3^(1/2)*2476099i)/14348907
 - 2476099/14348907)))*((3^(1/2)*19i)/162 + 19/162) - atan((4952198*((x -
1)/(x + 1))^(1/6))/(14348907*((3^(1/2)*2476099i)/14348907 + 2476099/143489
07)))*((3^(1/2)*19i)/162 - 19/162)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.33

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx$$
$$= \frac{(x-1)^{\frac{5}{6}}\left(-7(x+1)^{\frac{3}{2}}x^3+7(x+1)^{\frac{3}{2}}x^2+(x+1)^{\frac{3}{2}}x+2(x+1)^{\frac{3}{2}}+54\sqrt{x+1}\log\left((x-1)^{\frac{1}{6}}\right)x^3-9\sqrt{x}\right.}{6\,(x+1)^{\frac{1}{3}}x^3}$$

input
```
int(1/((x-1)/(1+x))^(1/6)/x^4,x)
```

output
```
((x - 1)**(5/6)*( - 7*(x + 1)**(3/2)*x**3 + 7*(x + 1)**(3/2)*x**2 + (x + 1
)**(3/2)*x + 2*(x + 1)**(3/2) + 54*sqrt(x + 1)*log((x - 1)**(1/6))*x**3 -
9*sqrt(x + 1)*log(x)*x**3))/(6*(x + 1)**(1/3)*x**3)
```

3.128 $\int e^{\frac{2}{3}\coth^{-1}(x)} x^2 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 157

$$\int e^{\frac{2}{3}\coth^{-1}(x)} x^2 \, dx = \frac{14}{27}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{1+\frac{1}{x}}\,x+\frac{4}{9}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{1+\frac{1}{x}}\,x^2+\frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{1+\frac{1}{x}}\,x^3-\frac{22\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{27\sqrt{3}}-\frac{11}{27}\log\left(\sqrt[3]{1-\frac{1}{x}}-\sqrt[3]{1+\frac{1}{x}}\right)-\frac{11\log(x)}{81}$$

output

```
14/27*(1-1/x)^(2/3)*(1+1/x)^(1/3)*x+4/9*(1-1/x)^(2/3)*(1+1/x)^(1/3)*x^2+1/
3*(1-1/x)^(2/3)*(1+1/x)^(1/3)*x^3-22/81*arctan(1/3*3^(1/2)+2/3*(1-1/x)^(1/
3)*3^(1/2)/(1+1/x)^(1/3))*3^(1/2)-11/27*ln((1-1/x)^(1/3)-(1+1/x)^(1/3))-11
/81*ln(x)
```

Mathematica [A] (verified)

Time = 5.34 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.20

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = \frac{1}{81}\left(\frac{216e^{\frac{2}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^3}+\frac{360e^{\frac{2}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^2}+\frac{210e^{\frac{2}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}}+22\sqrt{3}\arctan\left(\frac{-1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-22\sqrt{3}\arctan\left(\frac{1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-22\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right)-22\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}\right)+11\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)+11\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)\right)$$

input
```
Integrate[E^((2*ArcCoth[x])/3)*x^2,x]
```

output
```
((216*E^((2*ArcCoth[x])/3))/(-1 + E^(2*ArcCoth[x]))^3 + (360*E^((2*ArcCoth
[x])/3))/(-1 + E^(2*ArcCoth[x]))^2 + (210*E^((2*ArcCoth[x])/3))/(-1 + E^(2
*ArcCoth[x])) + 22*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 22*
Sqrt[3]*ArcTan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 22*Log[1 - E^(ArcCoth[x
]/3)] - 22*Log[1 + E^(ArcCoth[x]/3)] + 11*Log[1 - E^(ArcCoth[x]/3) + E^((2
*ArcCoth[x])/3)] + 11*Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/81
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6721, 110, 27, 168, 27, 168, 27, 102}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\frac{2}{3}\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[3]{1+\frac{1}{x}}x^4}{\sqrt[3]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^3-\frac{1}{3}\int \frac{2\left(4+\frac{3}{x}\right)x^3}{3\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^3-\frac{2}{9}\int \frac{\left(4+\frac{3}{x}\right)x^3}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^3-\frac{2}{9}\left(-\frac{1}{2}\int -\frac{2\left(7+\frac{6}{x}\right)x^2}{3\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}-2\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^3-\frac{2}{9}\left(\frac{1}{3}\int \frac{\left(7+\frac{6}{x}\right)x^2}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}-2\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^2\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^3-$$
$$\frac{2}{9}\left(\frac{1}{3}\left(-\int -\frac{11x}{3\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}-7\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x\right)-2\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^3-$$
$$\frac{2}{9}\left(\frac{1}{3}\left(\frac{11}{3}\int\frac{x}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}-7\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x\right)-2\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^2\right)$$

$$\Big\downarrow \text{102}$$

$$\frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x^3-$$
$$\frac{2}{9}\left(\frac{1}{3}\left(\frac{11}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)+\frac{3}{2}\log\left(\sqrt[3]{1-\frac{1}{x}}-\sqrt[3]{\frac{1}{x}+1}\right)-\frac{1}{2}\log\left(\frac{1}{x}\right)\right)-7\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{}\right.\right.$$

input
```
Int[E^((2*ArcCoth[x])/3)*x^2,x]
```

output
```
((1 - x^(-1))^(2/3)*(1 + x^(-1))^(1/3)*x^3)/3 - (2*(-2*(1 - x^(-1))^(2/3)*
(1 + x^(-1))^(1/3)*x^2 + (-7*(1 - x^(-1))^(2/3)*(1 + x^(-1))^(1/3)*x + (11
*(Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 - x^(-1))^(1/3))/(Sqrt[3]*(1 + x^(-1))^
(1/3))] + (3*Log[(1 - x^(-1))^(1/3) - (1 + x^(-1))^(1/3)])/2 - Log[x^(-1)]
/2))/3)/3))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 102
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqrt[3])*
q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))]/(d*e
 - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*(Log[q
*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{a, b,
c, d, e, f}, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.60 (sec) , antiderivative size = 616, normalized size of antiderivative = 3.92

method	result
risch	$\frac{(9x^2+12x+14)(x-1)}{27\left(\frac{x-1}{1+x}\right)^{\frac{1}{3}}}+\frac{\left(\frac{22\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\ln\left(-\frac{-2\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x^2+3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3+x^2-x-1\right.}{}\right.}{}\right.}{}$
trager	$\frac{(1+x)(9x^2+12x+14)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}}{27}+\frac{22\ln\left(9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x+18\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)^2x+9\,\mathrm{RootO}\right.}{}$

input
```
int(1/((x-1)/(1+x))^(1/3)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/27*(9*x^2+12*x+14)*(x-1)/((x-1)/(1+x))^(1/3)+(22/81*RootOf(_Z^2-_Z+1)*ln
(-(-2*RootOf(_Z^2-_Z+1)^2*x^2+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2/3)+3*Ro
otOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)*x-2*RootOf(_Z^2-_Z+1)^2*x+5*RootOf(_Z^
2-_Z+1)*x^2+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)+4*RootOf(_Z^2-_Z+1)*x-
2*x^2-RootOf(_Z^2-_Z+1)+2)/(1+x))-22/81*ln((2*RootOf(_Z^2-_Z+1)^2*x^2+3*Ro
otOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2/3)+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3
)*x+2*RootOf(_Z^2-_Z+1)^2*x+RootOf(_Z^2-_Z+1)*x^2-3*(x^3+x^2-x-1)^(2/3)+3*
RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)-3*(x^3+x^2-x-1)^(1/3)*x-x^2-3*(x^3+x
^2-x-1)^(1/3)-RootOf(_Z^2-_Z+1)-2*x-1)/(1+x))*RootOf(_Z^2-_Z+1)+22/81*ln((
2*RootOf(_Z^2-_Z+1)^2*x^2+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2/3)+3*RootOf
(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)*x+2*RootOf(_Z^2-_Z+1)^2*x+RootOf(_Z^2-_Z+1
)*x^2-3*(x^3+x^2-x-1)^(2/3)+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)-3*(x^3
+x^2-x-1)^(1/3)*x-x^2-3*(x^3+x^2-x-1)^(1/3)-RootOf(_Z^2-_Z+1)-2*x-1)/(1+x)
))/((x-1)/(1+x))^(1/3)*((x-1)*(1+x)^2)^(1/3)/(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.64

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = \frac{1}{27}\left(9\,x^3+21\,x^2+26\,x+14\right)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - \frac{22}{81}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right) + \frac{11}{81}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) - \frac{22}{81}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)*x^2,x, algorithm="fricas")
```

output
```
1/27*(9*x^3 + 21*x^2 + 26*x + 14)*((x - 1)/(x + 1))^(2/3) - 22/81*sqrt(3)*
arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) + 1/3*sqrt(3)) + 11/81*log(((x
- 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 22/81*log(((x - 1)/(x
 + 1))^(1/3) - 1)
```

Sympy [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = \int \frac{x^2}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/3)*x**2,x)
```

output
```
Integral(x**2/((x - 1)/(x + 1))**(1/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.95

$$\begin{aligned}\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx =& -\frac{22}{81}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right) \\ &- \frac{2\left(11\left(\frac{x-1}{x+1}\right)^{\frac{8}{3}} - 10\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}} + 35\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{27\left(\frac{3(x-1)}{x+1} - \frac{3(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1\right)} \\ &+ \frac{11}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) \\ &- \frac{22}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - 1\right)\end{aligned}$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)*x^2,x, algorithm="maxima")
```

output
```
-22/81*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2/27*
(11*((x - 1)/(x + 1))^(8/3) - 10*((x - 1)/(x + 1))^(5/3) + 35*((x - 1)/(x
+ 1))^(2/3))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1
)^3 - 1) + 11/81*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1
) - 22/81*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = -\frac{22}{81}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right) + \frac{2\left(\frac{10\,(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1} - \frac{11\,(x-1)^2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{(x+1)^2} - 35\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{27\left(\frac{x-1}{x+1}-1\right)^3} + \frac{11}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \frac{22}{81}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - 1\right|\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)*x^2,x, algorithm="giac")
```

output
```
-22/81*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + 2/27*
(10*(x - 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) - 11*(x - 1)^2*((x - 1)/(x + 1
))^(2/3)/(x + 1)^2 - 35*((x - 1)/(x + 1))^(2/3))/((x - 1)/(x + 1) - 1)^3 +
 11/81*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 22/81*
log(abs(((x - 1)/(x + 1))^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.09

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = -\frac{22\ln\left(\frac{484\left(\frac{x-1}{x+1}\right)^{1/3}}{729}-\frac{484}{729}\right)}{81}-\frac{\frac{70\left(\frac{x-1}{x+1}\right)^{2/3}}{27}-\frac{20\left(\frac{x-1}{x+1}\right)^{5/3}}{27}+\frac{22\left(\frac{x-1}{x+1}\right)^{8/3}}{27}}{\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1}$$
$$-\ln\left(\frac{484\left(\frac{x-1}{x+1}\right)^{1/3}}{729}-9\left(-\frac{11}{81}+\frac{\sqrt{3}\,11i}{81}\right)^2\right)\left(-\frac{11}{81}+\frac{\sqrt{3}\,11i}{81}\right)+\ln\left(\frac{484\left(\frac{x-1}{x+1}\right)^{1/3}}{729}-9\left(\frac{11}{81}+\frac{\sqrt{3}\,11i}{81}\right)^2\right.$$

input
```
int(x^2/((x - 1)/(x + 1))^(1/3),x)
```

output
```
log((484*((x - 1)/(x + 1))^(1/3))/729 - 9*((3^(1/2)*11i)/81 + 11/81)^2)*((
3^(1/2)*11i)/81 + 11/81) - ((70*((x - 1)/(x + 1))^(2/3))/27 - (20*((x - 1)
/(x + 1))^(5/3))/27 + (22*((x - 1)/(x + 1))^(8/3))/27)/((3*(x - 1))/(x + 1
) - (3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1) - log((484*((x - 1)
/(x + 1))^(1/3))/729 - 9*((3^(1/2)*11i)/81 - 11/81)^2)*((3^(1/2)*11i)/81 -
 11/81) - (22*log((484*((x - 1)/(x + 1))^(1/3))/729 - 484/729))/81
```

Reduce [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = \int \frac{(x+1)^{\frac{1}{3}}x^2}{(x-1)^{\frac{1}{3}}}dx$$

input
```
int(1/((x-1)/(1+x))^(1/3)*x^2,x)
```

output
```
int(((x + 1)**(1/3)*x**2)/(x - 1)**(1/3),x)
```

3.129 $\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 130

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx=\frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{1+\frac{1}{x}}x+\frac{1}{2}\left(1-\frac{1}{x}\right)^{2/3}\left(1+\frac{1}{x}\right)^{4/3}x^2-\frac{2\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{3\sqrt{3}}-\frac{1}{3}\log\left(\sqrt[3]{1-\frac{1}{x}}-\sqrt[3]{1+\frac{1}{x}}\right)-\frac{\log(x)}{9}$$

output
```
1/3*(1-1/x)^(2/3)*(1+1/x)^(1/3)*x+1/2*(1-1/x)^(2/3)*(1+1/x)^(4/3)*x^2-2/9*
arctan(1/3*3^(1/2)+2/3*(1-1/x)^(1/3)*3^(1/2)/(1+1/x)^(1/3))*3^(1/2)-1/3*ln
((1-1/x)^(1/3)-(1+1/x)^(1/3))-1/9*ln(x)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.27

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx = \frac{1}{9}\left(\frac{18e^{\frac{2}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^2}+\frac{24e^{\frac{2}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}} + 2\sqrt{3}\arctan\left(\frac{-1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right) - 2\sqrt{3}\arctan\left(\frac{1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right) - 2\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right) - 2\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}\right) + \log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right) + \log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)\right)$$

input
```
Integrate[E^((2*ArcCoth[x])/3)*x,x]
```

output
```
((18*E^((2*ArcCoth[x])/3))/(-1 + E^(2*ArcCoth[x]))^2 + (24*E^((2*ArcCoth[x
])/3))/(-1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3)
)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 2*Log[1
- E^(ArcCoth[x]/3)] - 2*Log[1 + E^(ArcCoth[x]/3)] + Log[1 - E^(ArcCoth[x]/
3) + E^((2*ArcCoth[x])/3)] + Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/
3)])/9
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6721, 107, 105, 102}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{2}{3}\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[3]{1+\frac{1}{x}}x^3}{\sqrt[3]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}x^2-\frac{1}{3}\int \frac{\sqrt[3]{1+\frac{1}{x}}x^2}{\sqrt[3]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{3}\left(\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x-\frac{2}{3}\int \frac{x}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}\right)+\frac{1}{2}\left(1-\frac{1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}x^2$$

$$\Big\downarrow \text{102}$$

$$\frac{1}{3}\left(\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x-\frac{2}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)+\frac{3}{2}\log\left(\sqrt[3]{1-\frac{1}{x}}-\sqrt[3]{\frac{1}{x}+1}\right)-\frac{1}{2}\log\left(\frac{1}{x}\right)\right)\right)+\frac{1}{2}\left(1-\frac{1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}x^2$$

input
```
Int[E^((2*ArcCoth[x])/3)*x,x]
```

output
```
((1 - x^(-1))^(2/3)*(1 + x^(-1))^(4/3)*x^2)/2 + ((1 - x^(-1))^(2/3)*(1 + x
^(-1))^(1/3)*x - (2*(Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 - x^(-1))^(1/3))/(Sq
rt[3]*(1 + x^(-1))^(1/3))] + (3*Log[(1 - x^(-1))^(1/3) - (1 + x^(-1))^(1/3
)])/2 - Log[x^(-1)]/2))/3)/3
```

Defintions of rubi rules used

rule 102
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqrt[3])*
q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))]/(d*e
 - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*(Log[q
*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{a, b,
c, d, e, f}, x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.60 (sec) , antiderivative size = 509, normalized size of antiderivative = 3.92

method	result
trager	$\frac{(1+x)(5+3x)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}}{6} - \frac{2\ln\left(9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x-36\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)^2x+9\,\mathrm{RootOf}\left(9\_Z^2 \ldots\right.\right.}{\ldots}$
risch	$\frac{(5+3x)(x-1)}{6\left(\frac{x-1}{1+x}\right)^{\frac{1}{3}}} + \frac{\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\ln\left(-\frac{-2\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x^2+3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3+x^2-x-1\right)^{\frac{2}{3}}+3\,\mathrm{Ro}\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}$

input
```
int(1/((x-1)/(1+x))^(1/3)*x,x,method=_RETURNVERBOSE)
```

output
```
1/6*(1+x)*(5+3*x)*(-(1-x)/(1+x))^(2/3)-2/9*ln(9*RootOf(9*_Z^2-3*_Z+1)*(-(1
-x)/(1+x))^(2/3)*x-36*RootOf(9*_Z^2-3*_Z+1)^2*x+9*RootOf(9*_Z^2-3*_Z+1)*(-
(1-x)/(1+x))^(2/3)-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x-3*(-(1-x
)/(1+x))^(2/3)*x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)+12*RootOf(9*
_Z^2-3*_Z+1)*x-3*(-(1-x)/(1+x))^(2/3)+6*RootOf(9*_Z^2-3*_Z+1)-x-1)+2/9*ln(
-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_Z^2-3*_Z+1)^2
*x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*RootOf(9*_Z^2-3*_Z+1)*x+
3*(-(1-x)/(1+x))^(1/3)*x+3*RootOf(9*_Z^2-3*_Z+1)+3*(-(1-x)/(1+x))^(1/3)-x+
1)-2/3*ln(-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_Z^2
-3*_Z+1)^2*x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*RootOf(9*_Z^2-
3*_Z+1)*x+3*(-(1-x)/(1+x))^(1/3)*x+3*RootOf(9*_Z^2-3*_Z+1)+3*(-(1-x)/(1+x)
)^(1/3)-x+1)*RootOf(9*_Z^2-3*_Z+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx = \frac{1}{6}\left(3x^2+8x+5\right)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - \frac{2}{9}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right) + \frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) - \frac{2}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)*x,x, algorithm="fricas")
```

output
```
1/6*(3*x^2 + 8*x + 5)*((x - 1)/(x + 1))^(2/3) - 2/9*sqrt(3)*arctan(2/3*sqr
t(3)*((x - 1)/(x + 1))^(1/3) + 1/3*sqrt(3)) + 1/9*log(((x - 1)/(x + 1))^(2
/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Sympy [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx = \int \frac{x}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/3)*x,x)
```

output
```
Integral(x/((x - 1)/(x + 1))**(1/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx = -\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right) + \frac{2\left(\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}}-4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{2(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1\right)} + \frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) - \frac{2}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)*x,x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + 2/3*(((
x - 1)/(x + 1))^(5/3) - 4*((x - 1)/(x + 1))^(2/3))/(2*(x - 1)/(x + 1) - (x
 - 1)^2/(x + 1)^2 - 1) + 1/9*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1
))^(1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.92

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx = -\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right) - \frac{2\left(\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1}-4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{x-1}{x+1}-1\right)^2} + \frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) - \frac{2}{9}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)*x,x, algorithm="giac")
```

output
```
-2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2/3*((x
 - 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) - 4*((x - 1)/(x + 1))^(2/3))/((x - 1
)/(x + 1) - 1)^2 + 1/9*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/
3) + 1) - 2/9*log(abs(((x - 1)/(x + 1))^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx=\frac{\frac{8\left(\frac{x-1}{x+1}\right)^{2/3}}{3}-\frac{2\left(\frac{x-1}{x+1}\right)^{5/3}}{3}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2(x-1)}{x+1}+1}-\frac{2\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}-\frac{4}{9}\right)}{9}$$
$$-\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}-9\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)+\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}-9\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2\right)\left(\frac{1}{9}+\right.$$

input
```
int(x/((x - 1)/(x + 1))^(1/3),x)
```

output
```
((8*((x - 1)/(x + 1))^(2/3))/3 - (2*((x - 1)/(x + 1))^(5/3))/3)/((x - 1)^2
/(x + 1)^2 - (2*(x - 1))/(x + 1) + 1) - (2*log((4*((x - 1)/(x + 1))^(1/3))
/9 - 4/9))/9 - log((4*((x - 1)/(x + 1))^(1/3))/9 - 9*((3^(1/2)*1i)/9 - 1/9
)^2)*((3^(1/2)*1i)/9 - 1/9) + log((4*((x - 1)/(x + 1))^(1/3))/9 - 9*((3^(1
/2)*1i)/9 + 1/9)^2)*((3^(1/2)*1i)/9 + 1/9)
```

Reduce [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx=\int\frac{(x+1)^{\frac{1}{3}}x}{(x-1)^{\frac{1}{3}}}dx$$

input
```
int(1/((x-1)/(1+x))^(1/3)*x,x)
```

output
```
int(((x + 1)**(1/3)*x)/(x - 1)**(1/3),x)
```

3.130 $\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 96

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx=\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{1+\frac{1}{x}}x-\frac{2\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{\sqrt{3}}-\log\left(\sqrt[3]{1-\frac{1}{x}}-\sqrt[3]{1+\frac{1}{x}}\right)-\frac{\log(x)}{3}$$

output
```
(1-1/x)^(2/3)*(1+1/x)^(1/3)*x-2/3*arctan(1/3*3^(1/2)+2/3*(1-1/x)^(1/3)*3^(
1/2)/(1+1/x)^(1/3))*3^(1/2)-ln((1-1/x)^(1/3)-(1+1/x)^(1/3))-1/3*ln(x)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx=\frac{1}{3}\left(\frac{6e^{\frac{2}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}}+2\sqrt{3}\arctan\left(\frac{1+2e^{\frac{2}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-2\log\left(1-e^{\frac{2}{3}\coth^{-1}(x)}\right)+\log\left(1+e^{\frac{2}{3}\coth^{-1}(x)}+e^{\frac{4}{3}\coth^{-1}(x)}\right)\right)$$

input
```
Integrate[E^((2*ArcCoth[x])/3),x]
```

output
```
((6*E^((2*ArcCoth[x])/3))/(-1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(1 +
2*E^((2*ArcCoth[x])/3))/Sqrt[3]] - 2*Log[1 - E^((2*ArcCoth[x])/3)] + Log[1
 + E^((2*ArcCoth[x])/3) + E^((4*ArcCoth[x])/3)])/3
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6720, 105, 102}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{2}{3}\coth^{-1}(x)}dx$$

$$\Big\downarrow \text{6720}$$

$$-\int \frac{\sqrt[3]{1+\frac{1}{x}}x^2}{\sqrt[3]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x-\frac{2}{3}\int \frac{x}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}$$

$$\Big\downarrow \text{102}$$

$$\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}x-\frac{2}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)+\frac{3}{2}\log\left(\sqrt[3]{1-\frac{1}{x}}-\sqrt[3]{\frac{1}{x}+1}\right)-\frac{1}{2}\log\left(\frac{1}{x}\right)\right)$$

input
```
Int[E^((2*ArcCoth[x])/3),x]
```

output
```
(1 - x^(-1))^(2/3)*(1 + x^(-1))^(1/3)*x - (2*(Sqrt[3]*ArcTan[1/Sqrt[3] + (
2*(1 - x^(-1))^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))] + (3*Log[(1 - x^(-1))^
(1/3) - (1 + x^(-1))^(1/3)])/2 - Log[x^(-1)]/2))/3
```

Defintions of rubi rules used

rule 102
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqrt[3])*
q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))]/(d*e
 - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*(Log[q
*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{a, b,
c, d, e, f}, x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 397, normalized size of antiderivative = 4.14

method	result
risch	$\frac{x-1}{\left(\frac{x-1}{1+x}\right)^{\frac{1}{3}}}+\frac{\left(-\frac{2\ln\left(-\frac{4\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x^2+4\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x+3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3+x^2-x-1\right)^{\frac{2}{3}}-3\,\mathrm{Ro}}{}\right.}{}\right.}{}$
trager	$(1+x)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}-\frac{2\ln\left(9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x-36\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)^2x+9\,\mathrm{RootOf}\left(9\_Z^2-\right.\right.}{}$

input
```
int(1/((x-1)/(1+x))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
(x-1)/((x-1)/(1+x))^(1/3)+(-2/3*ln(-(4*RootOf(_Z^2-_Z+1)^2*x^2+4*RootOf(_Z
^2-_Z+1)^2*x+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2/3)-3*RootOf(_Z^2-_Z+1)*(
x^3+x^2-x-1)^(1/3)*x-4*RootOf(_Z^2-_Z+1)*x^2-3*RootOf(_Z^2-_Z+1)*(x^3+x^2-
x-1)^(1/3)-2*RootOf(_Z^2-_Z+1)*x+3*(x^3+x^2-x-1)^(1/3)*x+x^2+2*RootOf(_Z^2
-_Z+1)+3*(x^3+x^2-x-1)^(1/3)-1)/(1+x))+2/3*RootOf(_Z^2-_Z+1)*ln((2*RootOf(
_Z^2-_Z+1)^2*x^2+2*RootOf(_Z^2-_Z+1)^2*x+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)
^(2/3)-5*RootOf(_Z^2-_Z+1)*x^2-6*RootOf(_Z^2-_Z+1)*x-3*(x^3+x^2-x-1)^(2/3)
+3*(x^3+x^2-x-1)^(1/3)*x+2*x^2-RootOf(_Z^2-_Z+1)+3*(x^3+x^2-x-1)^(1/3)+4*x
+2)/(1+x)))/((x-1)/(1+x))^(1/3)*((x-1)*(1+x)^2)^(1/3)/(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx=(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right)+\frac{1}{3}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/3),x, algorithm="fricas")
```

output
```
(x + 1)*((x - 1)/(x + 1))^(2/3) - 2/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/
(x + 1))^(1/3) + 1/3*sqrt(3)) + 1/3*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)
/(x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Sympy [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx = \int \frac{1}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/3),x)
```

output
```
Integral(((x - 1)/(x + 1))**(-1/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right) - \frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}-1} + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \frac{2}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - 1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/3),x, algorithm="maxima")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2*((x -
 1)/(x + 1))^(2/3)/((x - 1)/(x + 1) - 1) + 1/3*log(((x - 1)/(x + 1))^(2/3)
 + ((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}-1}+\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/3),x, algorithm="giac")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2*((x -
 1)/(x + 1))^(2/3)/((x - 1)/(x + 1) - 1) + 1/3*log(((x - 1)/(x + 1))^(2/3)
 + ((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(abs(((x - 1)/(x + 1))^(1/3) - 1)
)
```

Mupad [B] (verification not implemented)

Time = 23.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.23

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx = -\frac{2\ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}-4\right)}{3}-\ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}-9\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)+\ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}-9\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)-\frac{2\left(\frac{x-1}{x+1}\right)^{2/3}}{\frac{x-1}{x+1}-1}$$

input
```
int(1/((x - 1)/(x + 1))^(1/3),x)
```

output
```
log(4*((x - 1)/(x + 1))^(1/3) - 9*((3^(1/2)*1i)/3 + 1/3)^2)*((3^(1/2)*1i)/
3 + 1/3) - log(4*((x - 1)/(x + 1))^(1/3) - 9*((3^(1/2)*1i)/3 - 1/3)^2)*((3
^(1/2)*1i)/3 - 1/3) - (2*log(4*((x - 1)/(x + 1))^(1/3) - 4))/3 - (2*((x -
1)/(x + 1))^(2/3))/((x - 1)/(x + 1) - 1)
```

Reduce [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)} dx = \int \frac{(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{1}{3}}} dx$$

input
```
int(1/((x-1)/(1+x))^(1/3),x)
```

output
```
int((x + 1)**(1/3)/(x - 1)**(1/3),x)
```

3.131 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 155

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = -\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right) - \sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right) - \frac{3}{2}\log\left(1+\frac{\sqrt[3]{1-\frac{1}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right) - \frac{3}{2}\log\left(\sqrt[3]{1-\frac{1}{x}}-\sqrt[3]{1+\frac{1}{x}}\right) - \frac{1}{2}\log\left(1+\frac{1}{x}\right) - \frac{\log(x)}{2}$$

output
```
3^(1/2)*arctan(-1/3*3^(1/2)+2/3*(1-1/x)^(1/3)*3^(1/2)/(1+1/x)^(1/3))-arcta
n(1/3*3^(1/2)+2/3*(1-1/x)^(1/3)*3^(1/2)/(1+1/x)^(1/3))*3^(1/2)-3/2*ln(1+(1
-1/x)^(1/3)/(1+1/x)^(1/3))-3/2*ln((1-1/x)^(1/3)-(1+1/x)^(1/3))-1/2*ln(1+1/
x)-1/2*ln(x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.17

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = \frac{3}{2}e^{\frac{8}{3}\coth^{-1}(x)}\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},e^{4\coth^{-1}(x)}\right)$$

input
```
Integrate[E^((2*ArcCoth[x])/3)/x,x]
```

output
```
(3*E^((8*ArcCoth[x])/3)*Hypergeometric2F1[2/3, 1, 5/3, E^(4*ArcCoth[x])])/
2
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6721, 140, 72, 102}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[3]{1+\frac{1}{x}}x}{\sqrt[3]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{140}$$

$$-\int \frac{1}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}-\int \frac{x}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}$$

$$\Big\downarrow \text{72}$$

$$-\int \frac{x}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}-\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)-\frac{3}{2}\log\left(\frac{\sqrt[3]{1-\frac{1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{2}\log\left(\frac{1}{x}+1\right)$$

$\Big\downarrow$ 102

$$-\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)-\frac{3}{2}\log\left(\frac{\sqrt[3]{1-\frac{1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{3}{2}\log\left(\sqrt[3]{1-\frac{1}{x}}-\sqrt[3]{\frac{1}{x}+1}\right)-\frac{1}{2}\log\left(\frac{1}{x}+1\right)+\frac{1}{2}\log\left(\frac{1}{x}\right)$$

input
```
Int[E^((2*ArcCoth[x])/3)/x,x]
```

output
```
-(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(1 - x^(-1))^(1/3))/(Sqrt[3]*(1 + x^(-1))^
(1/3))]) - Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 - x^(-1))^(1/3))/(Sqrt[3]*(1 +
 x^(-1))^(1/3))] - (3*Log[1 + (1 - x^(-1))^(1/3)/(1 + x^(-1))^(1/3)])/2 -
(3*Log[(1 - x^(-1))^(1/3) - (1 + x^(-1))^(1/3)])/2 - Log[1 + x^(-1)]/2 + L
og[x^(-1)]/2
```

Defintions of rubi rules used

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 102
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqrt[3])*
q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))]/(d*e
 - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*(Log[q
*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{a, b,
c, d, e, f}, x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.58 (sec) , antiderivative size = 1039, normalized size of antiderivative = 6.70

method	result	size
trager	Expression too large to display	1039

input
```
int(1/((x-1)/(1+x))^(1/3)/x,x,method=_RETURNVERBOSE)
```

output
```
3*RootOf(9*_Z^2-3*_Z+1)*ln(-(-945*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/
3)*x^2+1224*RootOf(9*_Z^2-3*_Z+1)^2*x^2-1890*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)
/(1+x))^(2/3)*x-945*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x^2+168*(-(
1-x)/(1+x))^(2/3)*x^2-3060*RootOf(9*_Z^2-3*_Z+1)^2*x-945*RootOf(9*_Z^2-3*_
Z+1)*(-(1-x)/(1+x))^(2/3)-1353*RootOf(9*_Z^2-3*_Z+1)*x^2+336*(-(1-x)/(1+x)
)^(2/3)*x+168*(-(1-x)/(1+x))^(1/3)*x^2+1224*RootOf(9*_Z^2-3*_Z+1)^2+945*Ro
otOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)+1062*RootOf(9*_Z^2-3*_Z+1)*x+168*
(-(1-x)/(1+x))^(2/3)+304*x^2-1353*RootOf(9*_Z^2-3*_Z+1)-168*(-(1-x)/(1+x))
^(1/3)-32*x+304)/x)-3*ln(-(945*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*
x^2+1224*RootOf(9*_Z^2-3*_Z+1)^2*x^2+1890*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1
+x))^(2/3)*x+945*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x^2-147*(-(1-x
)/(1+x))^(2/3)*x^2-3060*RootOf(9*_Z^2-3*_Z+1)^2*x+945*RootOf(9*_Z^2-3*_Z+1
)*(-(1-x)/(1+x))^(2/3)+537*RootOf(9*_Z^2-3*_Z+1)*x^2-294*(-(1-x)/(1+x))^(2
/3)*x-147*(-(1-x)/(1+x))^(1/3)*x^2+1224*RootOf(9*_Z^2-3*_Z+1)^2-945*RootOf
(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)+978*RootOf(9*_Z^2-3*_Z+1)*x-147*(-(1-
x)/(1+x))^(2/3)-11*x^2+537*RootOf(9*_Z^2-3*_Z+1)+147*(-(1-x)/(1+x))^(1/3)-
18*x-11)/x)*RootOf(9*_Z^2-3*_Z+1)+ln(-(945*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(
1+x))^(2/3)*x^2+1224*RootOf(9*_Z^2-3*_Z+1)^2*x^2+1890*RootOf(9*_Z^2-3*_Z+1
)*(-(1-x)/(1+x))^(2/3)*x+945*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x^
2-147*(-(1-x)/(1+x))^(2/3)*x^2-3060*RootOf(9*_Z^2-3*_Z+1)^2*x+945*RootO...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.55

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = \sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + \frac{1}{3}\sqrt{3}\right) - \log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - 1\right) + \frac{1}{2}\log\left(\frac{(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + (x-1)\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + x + 1}{x+1}\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)/x,x, algorithm="fricas")
```

output
```
sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(2/3) + 1/3*sqrt(3)) - log(((
x - 1)/(x + 1))^(2/3) - 1) + 1/2*log(((x + 1)*((x - 1)/(x + 1))^(2/3) + (x
 - 1)*((x - 1)/(x + 1))^(1/3) + x + 1)/(x + 1))
```

Sympy [F]

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = \int \frac{1}{x\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/3)/x,x)
```

output
```
Integral(1/(x*((x - 1)/(x + 1))**(1/3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = & -\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right) \\ & +\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right) \\ & +\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) \\ & +\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) \\ & -\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\end{aligned}$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)/x,x, algorithm="maxima")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + sqrt(3)*arc
tan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 1/2*log(((x - 1)/(x + 1
))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) + 1/2*log(((x - 1)/(x + 1))^(2/3)
- ((x - 1)/(x + 1))^(1/3) + 1) - log(((x - 1)/(x + 1))^(1/3) + 1) - log(((
x - 1)/(x + 1))^(1/3) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.51

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = \sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+1\right)\right) + \frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}{x+1}+1\right) - \log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-1\right|\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)/x,x, algorithm="giac")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(2/3) + 1)) + 1/2*log(((x
- 1)/(x + 1))^(2/3) + (x - 1)*((x - 1)/(x + 1))^(1/3)/(x + 1) + 1) - log(a
bs(((x - 1)/(x + 1))^(2/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.53

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = -\ln\left(1296\left(\frac{x-1}{x+1}\right)^{2/3}-1296\right) - \ln\left(1296\left(\frac{x-1}{x+1}\right)^{2/3}+648-\sqrt{3}\,648\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) + \ln\left(1296\left(\frac{x-1}{x+1}\right)^{2/3}+648+\sqrt{3}\,648\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)$$

input
```
int(1/(x*((x - 1)/(x + 1))^(1/3)),x)
```

output
```
log(3^(1/2)*648i + 1296*((x - 1)/(x + 1))^(2/3) + 648)*((3^(1/2)*1i)/2 + 1
/2) - log(1296*((x - 1)/(x + 1))^(2/3) - 3^(1/2)*648i + 648)*((3^(1/2)*1i)
/2 - 1/2) - log(1296*((x - 1)/(x + 1))^(2/3) - 1296)
```

Reduce [F]

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = \int \frac{(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{1}{3}}\,x}dx$$

input
```
int(1/((x-1)/(1+x))^(1/3)/x,x)
```

output
```
int((x + 1)**(1/3)/((x - 1)**(1/3)*x),x)
```

3.132 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 99

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{1+\frac{1}{x}} - \frac{2\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{\sqrt{3}} - \log\left(1+\frac{\sqrt[3]{1-\frac{1}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right) - \frac{1}{3}\log\left(1+\frac{1}{x}\right)$$

output
```
(1-1/x)^(2/3)*(1+1/x)^(1/3)+2/3*3^(1/2)*arctan(-1/3*3^(1/2)+2/3*(1-1/x)^(1
/3)*3^(1/2)/(1+1/x)^(1/3))-ln(1+(1-1/x)^(1/3)/(1+1/x)^(1/3))-1/3*ln(1+1/x)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \frac{2e^{\frac{2}{3}\coth^{-1}(x)}}{1+e^{2\coth^{-1}(x)}} - \frac{2\arctan\left(\frac{-1+2e^{\frac{2}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\log\left(1+e^{\frac{2}{3}\coth^{-1}(x)}\right) + \frac{1}{3}\log\left(1-e^{\frac{2}{3}\coth^{-1}(x)}+e^{\frac{4}{3}\coth^{-1}(x)}\right)$$

input
```
Integrate[E^((2*ArcCoth[x])/3)/x^2,x]
```

output
```
(2*E^((2*ArcCoth[x])/3))/(1 + E^(2*ArcCoth[x])) - (2*ArcTan[(-1 + 2*E^((2*
ArcCoth[x])/3))/Sqrt[3]])/Sqrt[3] - (2*Log[1 + E^((2*ArcCoth[x])/3)])/3 +
Log[1 - E^((2*ArcCoth[x])/3) + E^((4*ArcCoth[x])/3)]/3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6721, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}-\frac{2}{3}\int \frac{1}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}$$

$$\Big\downarrow \; 72$$

$$\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}-\frac{2}{3}\left(\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)+\frac{3}{2}\log\left(\frac{\sqrt[3]{1-\frac{1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)+\frac{1}{2}\log\left(\frac{1}{x}+1\right)\right)$$

input
```
Int[E^((2*ArcCoth[x])/3)/x^2,x]
```

output
```
(1 - x^(-1))^(2/3)*(1 + x^(-1))^(1/3) - (2*(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*
(1 - x^(-1))^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))] + (3*Log[1 + (1 - x^(-1)
)^(1/3)/(1 + x^(-1))^(1/3)])/2 + Log[1 + x^(-1)]/2))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 501, normalized size of antiderivative = 5.06

method	result
risch	$\frac{x-1}{x\left(\frac{x-1}{1+x}\right)^{\frac{1}{3}}}+\frac{\left(-\frac{2\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)^2x^2+27\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)\left(x^3+x^2-x-1\right)^{\frac{2}{3}}-45\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)\left(x^3\right.}{}\right.}{}\right.}{}$
trager	$\frac{(1+x)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}}{x}+2\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\ln\left(\frac{-9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x-9\,\mathrm{RootOf}\left(9\_Z^2\right.}{}\right.$

input
```
int(1/((x-1)/(1+x))^(1/3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(x-1)/x/((x-1)/(1+x))^(1/3)+(-2/3*ln((8*RootOf(_Z^2-3*_Z+9)^2*x^2+27*RootO
f(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)-45*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1
/3)*x-8*RootOf(_Z^2-3*_Z+9)^2*x-30*RootOf(_Z^2-3*_Z+9)*x^2-216*(x^3+x^2-x-
1)^(2/3)-45*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)-81*(x^3+x^2-x-1)^(1/3)
*x-16*RootOf(_Z^2-3*_Z+9)^2-54*RootOf(_Z^2-3*_Z+9)*x-27*x^2-81*(x^3+x^2-x-
1)^(1/3)-24*RootOf(_Z^2-3*_Z+9)-36*x-9)/x/(1+x))+2/9*RootOf(_Z^2-3*_Z+9)*l
n(-(-2*RootOf(_Z^2-3*_Z+9)^2*x^2+27*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3
)+72*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x+2*RootOf(_Z^2-3*_Z+9)^2*x-2
7*RootOf(_Z^2-3*_Z+9)*x^2+135*(x^3+x^2-x-1)^(2/3)+72*RootOf(_Z^2-3*_Z+9)*(
x^3+x^2-x-1)^(1/3)-81*(x^3+x^2-x-1)^(1/3)*x+4*RootOf(_Z^2-3*_Z+9)^2+6*Root
Of(_Z^2-3*_Z+9)*x-36*x^2-81*(x^3+x^2-x-1)^(1/3)+33*RootOf(_Z^2-3*_Z+9)-216
*x-180)/x/(1+x)))/((x-1)/(1+x))^(1/3)*((x-1)*(1+x)^2)^(1/3)/(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx$$
$$= \frac{2\sqrt{3}x\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) + x\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - 2\,x\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) + 3\,(x+1}{3\,x}$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)/x^2,x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(3)*x*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) - 1/3*sqrt(3))
 + x*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) + 1) - 2*x*log(
((x - 1)/(x + 1))^(1/3) + 1) + 3*(x + 1)*((x - 1)/(x + 1))^(2/3))/x
```

Sympy [F]

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/3)/x**2,x)
```

output
```
Integral(1/(x**2*((x - 1)/(x + 1))**(1/3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - 1\right)\right) + \frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1} + 1}$$
$$+ \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \frac{2}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)/x^2,x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2*((x -
1)/(x + 1))^(2/3)/((x - 1)/(x + 1) + 1) + 1/3*log(((x - 1)/(x + 1))^(2/3)
- ((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}+1} + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right|\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)/x^2,x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2*((x -
1)/(x + 1))^(2/3)/((x - 1)/(x + 1) + 1) + 1/3*log(((x - 1)/(x + 1))^(2/3)
- ((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(abs(((x - 1)/(x + 1))^(1/3) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.19

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \frac{2\left(\frac{x-1}{x+1}\right)^{2/3}}{\frac{x-1}{x+1}+1}-\ln\left(9\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2+4\left(\frac{x-1}{x+1}\right)^{1/3}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right) + \ln\left(9\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2+4\left(\frac{x-1}{x+1}\right)^{1/3}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)-\frac{2\ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}+4\right)}{3}$$

input
```
int(1/(x^2*((x - 1)/(x + 1))^(1/3)),x)
```

output
```
log(9*((3^(1/2)*1i)/3 + 1/3)^2 + 4*((x - 1)/(x + 1))^(1/3))*((3^(1/2)*1i)/
3 + 1/3) - log(9*((3^(1/2)*1i)/3 - 1/3)^2 + 4*((x - 1)/(x + 1))^(1/3))*((3
^(1/2)*1i)/3 - 1/3) - (2*log(4*((x - 1)/(x + 1))^(1/3) + 4))/3 + (2*((x -
1)/(x + 1))^(2/3))/((x - 1)/(x + 1) + 1)
```

Reduce [F]

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = \int \frac{(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{1}{3}}\,x^2}dx$$

input
```
int(1/((x-1)/(1+x))^(1/3)/x^2,x)
```

output
```
int((x + 1)**(1/3)/((x - 1)**(1/3)*x**2),x)
```

3.133 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 130

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{1}{3}\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{1+\frac{1}{x}} + \frac{1}{2}\left(1-\frac{1}{x}\right)^{2/3}\left(1+\frac{1}{x}\right)^{4/3} - \frac{2\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{3\sqrt{3}} - \frac{1}{3}\log\left(1+\frac{\sqrt[3]{1-\frac{1}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right) - \frac{1}{9}\log\left(1+\frac{1}{x}\right)$$

output
```
1/3*(1-1/x)^(2/3)*(1+1/x)^(1/3)+1/2*(1-1/x)^(2/3)*(1+1/x)^(4/3)+2/9*3^(1/2
)*arctan(-1/3*3^(1/2)+2/3*(1-1/x)^(1/3)*3^(1/2)/(1+1/x)^(1/3))-1/3*ln(1+(1
-1/x)^(1/3)/(1+1/x)^(1/3))-1/9*ln(1+1/x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = -\frac{2}{27}\left(\frac{27e^{\frac{2}{3}\coth^{-1}(x)}}{\left(1+e^{2\coth^{-1}(x)}\right)^2} - \frac{36e^{\frac{2}{3}\coth^{-1}(x)}}{1+e^{2\coth^{-1}(x)}} - 2\coth^{-1}(x) + 3\log\left(1+e^{\frac{2}{3}\coth^{-1}(x)}\right) - \text{RootSum}\left[1-\#1^2 + \#1^4\&, \frac{\coth^{-1}(x) - 3\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right) + \coth^{-1}(x)\#1^2 - 3\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)\#1^2}{-2+\#1^2}\&\right]\right)$$

input
```
Integrate[E^((2*ArcCoth[x])/3)/x^3,x]
```

output
```
(-2*((27*E^((2*ArcCoth[x])/3))/(1 + E^(2*ArcCoth[x]))^2 - (36*E^((2*ArcCot
h[x])/3))/(1 + E^(2*ArcCoth[x])) - 2*ArcCoth[x] + 3*Log[1 + E^((2*ArcCoth[
x])/3)] - RootSum[1 - #1^2 + #1^4 & , (ArcCoth[x] - 3*Log[E^(ArcCoth[x]/3)
 - #1] + ArcCoth[x]*#1^2 - 3*Log[E^(ArcCoth[x]/3) - #1]*#1^2)/(-2 + #1^2)
& ]))/27
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6721, 90, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{1-\frac{1}{x}}x}d\frac{1}{x}$$

$\Big\downarrow$ 90

$$\frac{1}{2}\left(1-\frac{1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}-\frac{1}{3}\int \frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{1-\frac{1}{x}}}d\frac{1}{x}$$

$\Big\downarrow$ 60

$$\frac{1}{3}\left(\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}-\frac{2}{3}\int \frac{1}{\sqrt[3]{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{2/3}}d\frac{1}{x}\right)+\frac{1}{2}\left(1-\frac{1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}$$

$\Big\downarrow$ 72

$$\frac{1}{3}\left(\left(1-\frac{1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}-\frac{2}{3}\left(\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-\frac{1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)+\frac{3}{2}\log\left(\frac{\sqrt[3]{1-\frac{1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)+\frac{1}{2}\log\left(\frac{1}{x}+1\right)\right)\right)+\frac{1}{2}\left(1-\frac{1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}$$

input
```
Int[E^((2*ArcCoth[x])/3)/x^3,x]
```

output
```
((1 - x^(-1))^(2/3)*(1 + x^(-1))^(4/3))/2 + ((1 - x^(-1))^(2/3)*(1 + x^(-1
))^(1/3) - (2*(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(1 - x^(-1))^(1/3))/(Sqrt[3]*
(1 + x^(-1))^(1/3))] + (3*Log[1 + (1 - x^(-1))^(1/3)/(1 + x^(-1))^(1/3)])/
2 + Log[1 + x^(-1)]/2))/3)/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 675, normalized size of antiderivative = 5.19

method	result
trager	$\frac{(1+x)(5x+3)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}}{6x^2}+\frac{2\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\ln\left(\frac{-9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x-9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(1/((x-1)/(1+x))^(1/3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*(1+x)*(5*x+3)/x^2*(-(1-x)/(1+x))^(2/3)+2/3*RootOf(9*_Z^2-3*_Z+1)*ln((-
9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-9*RootOf(9*_Z^2-3*_Z+1)*(-(
1-x)/(1+x))^(2/3)+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x+18*RootOf
(9*_Z^2-3*_Z+1)^2+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)+3*RootOf(9*
_Z^2-3*_Z+1)*x-15*RootOf(9*_Z^2-3*_Z+1)-2*x+2)/x)+2/9*ln((9*RootOf(9*_Z^2-
3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3
)-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x-3*(-(1-x)/(1+x))^(2/3)*x+
18*RootOf(9*_Z^2-3*_Z+1)^2-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)-3*
RootOf(9*_Z^2-3*_Z+1)*x-3*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(1/3)*x+3*
RootOf(9*_Z^2-3*_Z+1)+3*(-(1-x)/(1+x))^(1/3)-x-1)/x)-2/3*ln((9*RootOf(9*_Z
^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(
2/3)-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x-3*(-(1-x)/(1+x))^(2/3)
*x+18*RootOf(9*_Z^2-3*_Z+1)^2-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)
-3*RootOf(9*_Z^2-3*_Z+1)*x-3*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(1/3)*x
+3*RootOf(9*_Z^2-3*_Z+1)+3*(-(1-x)/(1+x))^(1/3)-x-1)/x)*RootOf(9*_Z^2-3*_Z
+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{4\sqrt{3}x^2\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+2\,x^2\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-4\,x^2\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+3\,(5}{18\,x^2}$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)/x^3,x, algorithm="fricas")
```

output
```
1/18*(4*sqrt(3)*x^2*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) - 1/3*sqrt(
3)) + 2*x^2*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) + 1) - 4
*x^2*log(((x - 1)/(x + 1))^(1/3) + 1) + 3*(5*x^2 + 8*x + 3)*((x - 1)/(x +
1))^(2/3))/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \int \frac{1}{x^3\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/3)/x**3,x)
```

output
```
Integral(1/(x**3*((x - 1)/(x + 1))**(1/3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right) + \frac{2\left(\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}}+4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{2(x-1)}{x+1}+\frac{(x-1)^2}{(x+1)^2}+1\right)} + \frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) - \frac{2}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)/x^3,x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2/3*(((x
 - 1)/(x + 1))^(5/3) + 4*((x - 1)/(x + 1))^(2/3))/(2*(x - 1)/(x + 1) + (x
- 1)^2/(x + 1)^2 + 1) + 1/9*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1)
)^(1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.94

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right) + \frac{2\left(\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1}+4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{x-1}{x+1}+1\right)^2} + \frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right) - \frac{2}{9}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right|\right)$$

input

```
integrate(1/((x-1)/(1+x))^(1/3)/x^3,x, algorithm="giac")
```

output

```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2/3*((x
- 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) + 4*((x - 1)/(x + 1))^(2/3))/((x - 1)
/(x + 1) + 1)^2 + 1/9*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3
) + 1) - 2/9*log(abs(((x - 1)/(x + 1))^(1/3) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \frac{\frac{8\left(\frac{x-1}{x+1}\right)^{2/3}}{3}+\frac{2\left(\frac{x-1}{x+1}\right)^{5/3}}{3}}{\frac{2(x-1)}{x+1}+\frac{(x-1)^2}{(x+1)^2}+1} - \frac{2\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}+\frac{4}{9}\right)}{9} - \ln\left(9\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2+\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)+\ln\left(9\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2+\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}\right)\left(\frac{1}{9}+\frac{\sqrt{3}}{9}\right.$$

input

```
int(1/(x^3*((x - 1)/(x + 1))^(1/3)),x)
```

output
```
((8*((x - 1)/(x + 1))^(2/3))/3 + (2*((x - 1)/(x + 1))^(5/3))/3)/((2*(x - 1
))/(x + 1) + (x - 1)^2/(x + 1)^2 + 1) - (2*log((4*((x - 1)/(x + 1))^(1/3))
/9 + 4/9))/9 - log(9*((3^(1/2)*1i)/9 - 1/9)^2 + (4*((x - 1)/(x + 1))^(1/3)
)/9)*((3^(1/2)*1i)/9 - 1/9) + log(9*((3^(1/2)*1i)/9 + 1/9)^2 + (4*((x - 1)
/(x + 1))^(1/3))/9)*((3^(1/2)*1i)/9 + 1/9)
```

Reduce [F]

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = \int \frac{(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{1}{3}}\,x^3}dx$$

input
```
int(1/((x-1)/(1+x))^(1/3)/x^3,x)
```

output
```
int((x + 1)**(1/3)/((x - 1)**(1/3)*x**3),x)
```

3.134 $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 354

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx = \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{96a^2}+\frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^2}{8a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^3-\frac{11\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64\sqrt{2}a^3}+\frac{11\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64\sqrt{2}a^3}+\frac{11\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3}+\frac{11\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3}+\frac{11\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\left(1+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\sqrt[8]{1-\frac{1}{ax}}}\right)}{64\sqrt{2}a^3}$$

output

```
37/96*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x/a^2+3/8*(1-1/a/x)^(7/8)*(1+1/a/x)^
(1/8)*x^2/a+1/3*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x^3-11/128*arctan(1-2^(1/2
)*(1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))*2^(1/2)/a^3+11/128*arctan(1+2^(1/2)*(1+
1/a/x)^(1/8)/(1-1/a/x)^(1/8))*2^(1/2)/a^3+11/64*arctan((1+1/a/x)^(1/8)/(1-
1/a/x)^(1/8))/a^3+11/64*arctanh((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))/a^3+11/12
8*arctanh(2^(1/2)*(1+1/a/x)^(1/8)/(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/(1-1
/a/x)^(1/8))*2^(1/2)/a^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.47

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$$
$$= \frac{-4\left(-\frac{1024e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}-\frac{1600e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}-\frac{840e^{\frac{1}{4}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-66\arctan\left(e^{\frac{1}{4}\coth^{-1}(ax)}\right)+33\log\left(1-e^{\frac{1}{4}\mathrm{c}}\right.\right.}{1536a}$$

input
```
Integrate[E^(ArcCoth[a*x]/4)*x^2,x]
```

output
```
(-4*((-1024*E^(ArcCoth[a*x]/4))/(-1 + E^(2*ArcCoth[a*x]))^3 - (1600*E^(Arc
Coth[a*x]/4))/(-1 + E^(2*ArcCoth[a*x]))^2 - (840*E^(ArcCoth[a*x]/4))/(-1 +
 E^(2*ArcCoth[a*x])) - 66*ArcTan[E^(ArcCoth[a*x]/4)] + 33*Log[1 - E^(ArcCo
th[a*x]/4)] - 33*Log[1 + E^(ArcCoth[a*x]/4)]) - 33*RootSum[1 + #1^4 & , (A
rcCoth[a*x] - 4*Log[E^(ArcCoth[a*x]/4) - #1])/#1^3 & ])/(1536*a^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.10, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.429, Rules used = {6721, 110, 27, 168, 27, 168, 27, 104, 758, 755, 756, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \ 6721$$
$$-\int \frac{\sqrt[8]{1+\frac{1}{ax}}x^4}{\sqrt[8]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{1}{3}\int \frac{\left(9a+\frac{8}{x}\right)x^3}{4a^2\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\int \frac{\left(9a+\frac{8}{x}\right)x^3}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}}{12a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{-\frac{1}{2}\int -\frac{\left(37a+\frac{36}{x}\right)x^2}{4a\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}-\frac{9}{2}ax^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{12a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\frac{\int \frac{\left(37a+\frac{36}{x}\right)x^2}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}}{8a}-\frac{9}{2}ax^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{12a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\frac{-\int -\frac{33x}{4\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}-37ax\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}-\frac{9}{2}ax^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{12a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\frac{\frac{33}{4}\int \frac{x}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}-37ax\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}-\frac{9}{2}ax^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{12a^2}$$

$$\Big\downarrow \text{104}$$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{66\int\frac{1}{\frac{1}{x^8}-1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-37ax\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}-\frac{9}{2}ax^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{12a^2}$$

$$\Big\downarrow \text{758}$$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{66\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-37ax\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}-\frac{9}{2}ax^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{12a^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{66\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-37ax\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}-\frac{9}{2}ax^2\left(1-\frac{1}{ax}\right.}{12a^2}$$

$$\Big\downarrow \text{756}$$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{66\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)-37ax\left(1-\frac{1}{ax}\right)^{7/}}{8a}}{12a^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{66\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)-37ax\left(1-\frac{1}{ax}\right)^{7}}{8a}}{12a^2}$$

$\downarrow$ 219

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{66\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)\right)-37ax\left(1-\right.}{8a}}{12a^2}$$

$\downarrow$ 1476

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{66\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\text{ar}\right.}{8a}}{12a^2}$$

$\downarrow$ 1082

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{66\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right.}{8a}}{12a^2}$$

$\downarrow$ 217

$$\frac{\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{66\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arcta}\right.}{8a}}{12a^2}$$

$\downarrow$ 1479

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{66\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\cdots}{\cdots}\right.\cdots}{8a\cdots}$$

$\downarrow$ 25

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{66\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\cdots}{\cdots}\right.\cdots}{8a\cdots}$$

$\downarrow$ 27

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-$$

$$66\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\ldots}{\ldots}\right.\right.\right.$$

$$\overline{\qquad 8a \qquad}$$

$\downarrow$ 1103

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-$$

$$66\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\ldots\right.\right.\right.$$

$$\overline{\qquad 8a \qquad}$$

$$\overline{\qquad 12a^2 \qquad}$$

input
```
Int[E^(ArcCoth[a*x]/4)*x^2,x]
```

output
```
((1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x^3)/3 - ((-9*a*(1 - 1/(a*x))^(7/
8)*(1 + 1/(a*x))^(1/8)*x^2)/2 + (-37*a*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(
1/8)*x + 66*((-1/2*ArcTan[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)] - ArcTa
nh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)]/2)/2 + ((ArcTan[1 - (Sqrt[2]*(
1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/Sqrt[2] - ArcTan[1 + (Sqrt[2]*(1
+ 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 +
 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt
[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + x^(-2)]/(2*Sqrt[2]))/2)/2))
/(8*a))/(12*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.68

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$$
$$= \frac{8\left(32a^3x^3+68a^2x^2+73ax+37\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}-66\sqrt{2}\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right)-66\sqrt{2}\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right.\right.}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="fricas")
```

output
```
1/768*(8*(32*a^3*x^3 + 68*a^2*x^2 + 73*a*x + 37)*((a*x - 1)/(a*x + 1))^(7/
8) - 66*sqrt(2)*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + 1) - 66*sqrt(
2)*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) - 1) + 33*sqrt(2)*log(sqrt(2
)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) - 33*sqrt
(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4)
 + 1) - 132*arctan(((a*x - 1)/(a*x + 1))^(1/8)) + 66*log(((a*x - 1)/(a*x +
 1))^(1/8) + 1) - 66*log(((a*x - 1)/(a*x + 1))^(1/8) - 1))/a^3
```

Sympy [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx=\int\frac{x^2}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/8)*x**2,x)
```

output
```
Integral(x**2/((a*x - 1)/(a*x + 1))**(1/8), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.96

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx=$$
$$-\frac{1}{768}a\left(\frac{16\left(33\left(\frac{ax-1}{ax+1}\right)^{\frac{23}{8}}-10\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{8}}+105\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{33\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="maxima")
```

output
```
-1/768*a*(16*(33*((a*x - 1)/(a*x + 1))^(23/8) - 10*((a*x - 1)/(a*x + 1))^(
15/8) + 105*((a*x - 1)/(a*x + 1))^(7/8))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a
*x - 1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 33*(2*sqr
t(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) + 2*sqr
t(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) - sqrt
(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4)
+ 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x
+ 1))^(1/4) + 1))/a^4 + 132*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^4 - 66*l
og(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^4 + 66*log(((a*x - 1)/(a*x + 1))^(1/
8) - 1)/a^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.87

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx =$$

$$-\frac{1}{768}\,a\left(\frac{66\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^4}+\frac{66\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^4}-\frac{33\,\sqrt{\cdots}}{\cdots}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="giac")
```

output
```
-1/768*a*(66*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/8)))/a^4 + 66*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/8)))/a^4 - 33*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) +
((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 33*sqrt(2)*log(-sqrt(2)*((a*x - 1)/
(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 132*arctan(((a*x
 - 1)/(a*x + 1))^(1/8))/a^4 - 66*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^4
+ 66*log(-((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^4 + 16*(33*((a*x - 1)/(a*x +
1))^(23/8) - 10*((a*x - 1)/(a*x + 1))^(15/8) + 105*((a*x - 1)/(a*x + 1))^(
7/8))/(a^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B] (verification not implemented)

Time = 24.13 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.64

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx=\frac{\frac{35\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{16}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{15/8}}{24}+\frac{11\left(\frac{a\,x-1}{a\,x+1}\right)^{23/8}}{16}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,1\mathrm{i}\right)\,11\mathrm{i}}{64\,a^3}-\frac{11\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{64\,a^3}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{11}{128}+\frac{11}{128}\mathrm{i}\right)}{a^3}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{11}{128}-\frac{11}{128}\mathrm{i}\right)}{a^3}$$

input
```
int(x^2/((a*x - 1)/(a*x + 1))^(1/8),x)
```

output
```
((35*((a*x - 1)/(a*x + 1))^(7/8))/16 - (5*((a*x - 1)/(a*x + 1))^(15/8))/24
 + (11*((a*x - 1)/(a*x + 1))^(23/8))/16)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x +
 1)^2 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - (at
an(((a*x - 1)/(a*x + 1))^(1/8)*1i)*11i)/(64*a^3) - (11*atan(((a*x - 1)/(a*
x + 1))^(1/8)))/(64*a^3) - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/
8)*(1/2 - 1i/2))*(11/128 - 11i/128))/a^3 - (2^(1/2)*atan(2^(1/2)*((a*x - 1
)/(a*x + 1))^(1/8)*(1/2 + 1i/2))*(11/128 + 11i/128))/a^3
```

Reduce [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx = \int \frac{(ax+1)^{\frac{1}{8}}\,x^2}{(ax-1)^{\frac{1}{8}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x)
```

output
```
int(((a*x + 1)**(1/8)*x**2)/(a*x - 1)**(1/8),x)
```

3.135 $\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 317

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx=\frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2}+\frac{\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\left(1+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2}$$

output
```
1/8*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x/a+1/2*(1-1/a/x)^(7/8)*(1+1/a/x)^(9/8
)*x^2-1/32*arctan(1-2^(1/2)*(1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))*2^(1/2)/a^2+1
/32*arctan(1+2^(1/2)*(1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))*2^(1/2)/a^2+1/16*arc
tan((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))/a^2+1/16*arctanh((1+1/a/x)^(1/8)/(1-1
/a/x)^(1/8))/a^2+1/32*arctanh(2^(1/2)*(1+1/a/x)^(1/8)/(1+(1+1/a/x)^(1/4)/(
1-1/a/x)^(1/4))/(1-1/a/x)^(1/8))*2^(1/2)/a^2
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.01

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx = \frac{\frac{2}{\left(-1+e^{\frac{1}{4}\coth^{-1}(ax)}\right)^2}+\frac{6}{-1+e^{\frac{1}{4}\coth^{-1}(ax)}}-\frac{2}{\left(1+e^{\frac{1}{4}\coth^{-1}(ax)}\right)^2}+\frac{6}{1+e^{\frac{1}{4}\coth^{-1}(ax)}}+\frac{8e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(1+e^{\frac{1}{2}\coth^{-1}(ax)}\right)^2}-\frac{12e^{\frac{1}{4}\coth^{-1}(ax)}}{1+e^{\frac{1}{2}\coth^{-1}(ax)}}+}{}$$

input
```
Integrate[E^(ArcCoth[a*x]/4)*x,x]
```

output
```
(2/(-1 + E^(ArcCoth[a*x]/4))^2 + 6/(-1 + E^(ArcCoth[a*x]/4)) - 2/(1 + E^(A
rcCoth[a*x]/4))^2 + 6/(1 + E^(ArcCoth[a*x]/4)) + (8*E^(ArcCoth[a*x]/4))/(1
 + E^(ArcCoth[a*x]/2))^2 - (12*E^(ArcCoth[a*x]/4))/(1 + E^(ArcCoth[a*x]/2)
) + (32*E^(ArcCoth[a*x]/4))/(1 + E^ArcCoth[a*x])^2 - (40*E^(ArcCoth[a*x]/4
))/(1 + E^ArcCoth[a*x]) + 4*ArcTan[E^(ArcCoth[a*x]/4)] - 2*Sqrt[2]*ArcTan[
1 - Sqrt[2]*E^(ArcCoth[a*x]/4)] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[
a*x]/4)] + 4*ArcTanh[E^(ArcCoth[a*x]/4)] - Sqrt[2]*Log[1 - Sqrt[2]*E^(ArcC
oth[a*x]/4) + E^(ArcCoth[a*x]/2)] + Sqrt[2]*Log[1 + Sqrt[2]*E^(ArcCoth[a*x
]/4) + E^(ArcCoth[a*x]/2)])/(64*a^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.10, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {6721, 107, 105, 104, 758, 755, 756, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{1}{4}\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[8]{1+\frac{1}{ax}}x^3}{\sqrt[8]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{\int \frac{\sqrt[8]{1+\frac{1}{ax}}x^2}{\sqrt[8]{1-\frac{1}{ax}}}d\frac{1}{x}}{8a}$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{\frac{\int \frac{x}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}}{4a}-x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{\frac{2\int \frac{1}{\frac{1}{x^8}-1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{a}-x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}$$

$$\Big\downarrow \text{758}$$

$$\frac{\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{2\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a}-x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}$$

$\downarrow$ 755

$$\frac{\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a}-x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}$$

$\downarrow$ 756

$$\frac{\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)}{a}-x\left(1-\frac{1}{ax}\right)^{7}}{8a}$$

$\downarrow$ 216

$$\frac{\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)}{a}-x\left(1-\frac{1}{ax}\right)}{8a}$$

$\downarrow$ 219

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)\right)}{a}-x(1-}{8a}$$

$\downarrow$ 1476

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\text{arc}\right.}{a}}{8a}$$

$\downarrow$ 1082

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\right.}{a}}{8a}$$

$\downarrow$ 217

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arcta}\right.}{a}}{8a}$$

$\downarrow$ 1479

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$

$$\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\text{arctan}}{}\right.\right.}{a}}{8a}$$

$\downarrow$ 25

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\cdots\right)}{\cdots}\cdots\right)\cdots\right)\cdots\right)}{a\cdots}}{8a\cdots}$$

$\downarrow$ 27

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\cdots\right)}{\cdots}\cdots\right)\cdots\right)\cdots\right)}{a\cdots}}{8a\cdots}$$

$\downarrow$ 1103

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\cdots\right.\right.\right.}{a}}{8a}$$

input
```
Int[E^(ArcCoth[a*x]/4)*x,x]
```

output
```
((1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(9/8)*x^2)/2 - (-((1 - 1/(a*x))^(7/8)*(
1 + 1/(a*x))^(1/8)*x) + (2*((-1/2*ArcTan[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))
^(1/8)] - ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)]/2)/2 + ((ArcTan
[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/Sqrt[2] - ArcTan[1
 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/Sqrt[2])/2 + (Log[1
- (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + x^(-2)]/(2*Sqrt[2])
- Log[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + x^(-2)]/(2*S
qrt[2]))/2)/2))/a)/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)*x,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/8)*x,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.74

$$\int e^{\frac{1}{4}\coth^{-1}(ax)} x\,dx = \frac{8\left(4\,a^2x^2+9\,ax+5\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}} - 2\sqrt{2}\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right) - 2\sqrt{2}\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}-1\right) + \sqrt{2}\,\mathrm{lo}}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="fricas")
```

output
```
1/64*(8*(4*a^2*x^2 + 9*a*x + 5)*((a*x - 1)/(a*x + 1))^(7/8) - 2*sqrt(2)*ar
ctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + 1) - 2*sqrt(2)*arctan(sqrt(2)*(
(a*x - 1)/(a*x + 1))^(1/8) - 1) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1)
)^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) - sqrt(2)*log(-sqrt(2)*((a*x -
1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) - 4*arctan(((a*x -
1)/(a*x + 1))^(1/8)) + 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1) - 2*log(((a*
x - 1)/(a*x + 1))^(1/8) - 1))/a^2
```

Sympy [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx=\int\frac{x}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/8)*x,x)
```

output
```
Integral(x/((a*x - 1)/(a*x + 1))**(1/8), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.96

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx$$
$$=\frac{1}{64}a\left(\frac{16\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{8}}-9\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}\right.\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="maxima")
```

output
```
1/64*a*(16*(((a*x - 1)/(a*x + 1))^(15/8) - 9*((a*x - 1)/(a*x + 1))^(7/8))/
(2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - (2*sqrt(
2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) - sqrt(2
)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) +
1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x +
1))^(1/4) + 1))/a^3 - 4*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^3 + 2*log(((
a*x - 1)/(a*x + 1))^(1/8) + 1)/a^3 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) - 1
)/a^3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.91

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx =$$

$$-\frac{1}{64}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^3}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^3}-\frac{\sqrt{2}\log\left(\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="giac")
```

output
```
-1/64*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/8)))/a^3 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
1))^(1/8)))/a^3 - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x
- 1)/(a*x + 1))^(1/4) + 1)/a^3 + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1)
)^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 + 4*arctan(((a*x - 1)/(a*x
+ 1))^(1/8))/a^3 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^3 + 2*log(-((a
*x - 1)/(a*x + 1))^(1/8) + 1)/a^3 + 16*(((a*x - 1)/(a*x + 1))^(15/8) - 9*(
(a*x - 1)/(a*x + 1))^(7/8))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.60

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx = \frac{\frac{9\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{4}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{15/8}}{4}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}} - \frac{\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{16\,a^2} - \frac{\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\right)}{16\,a^2} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{32}+\frac{1}{32}{}\mathrm{i}\right)}{a^2} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{32}-\frac{1}{32}{}\mathrm{i}\right)}{a^2}$$

input
```
int(x/((a*x - 1)/(a*x + 1))^(1/8),x)
```

output
```
((9*((a*x - 1)/(a*x + 1))^(7/8))/4 - ((a*x - 1)/(a*x + 1))^(15/8)/4)/(a^2
+ (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) - (atan(((a
*x - 1)/(a*x + 1))^(1/8)*1i)*1i)/(16*a^2) - atan(((a*x - 1)/(a*x + 1))^(1/
8))/(16*a^2) - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i
/2))*(1/32 - 1i/32))/a^2 - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/
8)*(1/2 + 1i/2))*(1/32 + 1i/32))/a^2
```

Reduce [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx=\int\frac{(ax+1)^{\frac{1}{8}}x}{(ax-1)^{\frac{1}{8}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)*x,x)
```

output
```
int(((a*x + 1)**(1/8)*x)/(a*x - 1)**(1/8),x)
```

3.136 $\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 277

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}a} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}a} + \frac{\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\left(1+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}a}$$

output
```
(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x-1/4*arctan(1-2^(1/2)*(1+1/a/x)^(1/8)/(1-
1/a/x)^(1/8))*2^(1/2)/a+1/4*arctan(1+2^(1/2)*(1+1/a/x)^(1/8)/(1-1/a/x)^(1/
8))*2^(1/2)/a+1/2*arctan((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))/a+1/2*arctanh((1
+1/a/x)^(1/8)/(1-1/a/x)^(1/8))/a+1/4*arctanh(2^(1/2)*(1+1/a/x)^(1/8)/(1+(1
+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/(1-1/a/x)^(1/8))*2^(1/2)/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.20

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = \frac{2e^{\frac{1}{4}\coth^{-1}(ax)}\left(1+\left(-1+e^{2\coth^{-1}(ax)}\right)\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},e^{2\coth^{-1}(ax)}\right)\right)}{a\left(-1+e^{2\coth^{-1}(ax)}\right)}$$

input
```
Integrate[E^(ArcCoth[a*x]/4),x]
```

output
```
(2*E^(ArcCoth[a*x]/4)*(1 + (-1 + E^(2*ArcCoth[a*x]))*Hypergeometric2F1[1/8
, 1, 9/8, E^(2*ArcCoth[a*x])]))/(a*(-1 + E^(2*ArcCoth[a*x])))
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {6720, 105, 104, 758, 755, 756, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6720}$$

$$-\int \frac{\sqrt[8]{1+\frac{1}{ax}}x^2}{\sqrt[8]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$\downarrow$ 105

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\int \frac{x}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}}{4a}$$

$\downarrow$ 104

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{2\int \frac{1}{\frac{1}{x^8}-1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{a}$$

$\downarrow$ 758

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{2\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a}$$

$\downarrow$ 755

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a}$$

$\downarrow$ 756

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)}{a}$$

$\downarrow$ 216

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right.}{a}$$

$\downarrow$ 219

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right.\right.}{a}$$

$\downarrow$ 1476

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-$$

$$\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int\frac{1}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right.}{a}$$

$\downarrow$ 1082

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-$$

$$\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\right.\right.}{a}$$

$$\Big\downarrow \; 217$$

$$\frac{x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}}}{\sqrt[8]{1-}}\right.\right.\right.}{a}$$

$$\Big\downarrow \; 1479$$

$$\frac{x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right.}{\sqrt{2}}\right.\right.\right.}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-2\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}\cdots\right.\right.\right.}{a}$$

$\downarrow$ 27

$$\frac{x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-2\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}\cdots\right.\right.\right.}{a}$$

$\downarrow$ 1103

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{2\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}}}{\sqrt[8]{1-}}\right)}{\sqrt{2}}\right.\right.\right.}{a}$$

input
```
Int[E^(ArcCoth[a*x]/4),x]
```

output
```
(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x - (2*((-1/2*ArcTan[(1 + 1/(a*x))
^(1/8)/(1 - 1/(a*x))^(1/8)] - ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1
/8)]/2)/2 + ((ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)
]/Sqrt[2] - ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/
Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) +
x^(-2)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^
(1/8) + x^(-2)]/(2*Sqrt[2]))/2)/2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/8),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.81

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$$
$$= \frac{8\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}} - 2\sqrt{2}\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right) - 2\sqrt{2}\arctan\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}-1\right) + \sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-}{ax+}\right.\right.}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="fricas")
```

output
```
1/8*(8*(a*x + 1)*((a*x - 1)/(a*x + 1))^(7/8) - 2*sqrt(2)*arctan(sqrt(2)*((
a*x - 1)/(a*x + 1))^(1/8) + 1) - 2*sqrt(2)*arctan(sqrt(2)*((a*x - 1)/(a*x
+ 1))^(1/8) - 1) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x
 - 1)/(a*x + 1))^(1/4) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(
1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) - 4*arctan(((a*x - 1)/(a*x + 1))^(
1/8)) + 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1) - 2*log(((a*x - 1)/(a*x + 1
))^(1/8) - 1))/a
```

Sympy [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = \int \frac{1}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/8),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(-1/8), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.96

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = -\frac{1}{8}\,a\left(\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right.\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="maxima")
```

output
```
-1/8*a*(16*((a*x - 1)/(a*x + 1))^(7/8)/((a*x - 1)*a^2/(a*x + 1) - a^2) + (
2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) -
 sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(
1/4) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/
(a*x + 1))^(1/4) + 1))/a^2 + 4*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^2 - 2
*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^2 + 2*log(((a*x - 1)/(a*x + 1))^(1
/8) - 1)/a^2)
```

Giac [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = \int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(-1/8), x)
```

Mupad [B] (verification not implemented)

Time = 23.95 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.54

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = \frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{a-\frac{a\,(a\,x-1)}{a\,x+1}} - \frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,a} - \frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{2\,a} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{a} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)}{a}$$

input
```
int(1/((a*x - 1)/(a*x + 1))^(1/8),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(7/8))/(a - (a*(a*x - 1))/(a*x + 1)) - (atan(((a*
x - 1)/(a*x + 1))^(1/8)*1i)*1i)/(2*a) - atan(((a*x - 1)/(a*x + 1))^(1/8))/
(2*a) - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1
/4 - 1i/4))/a - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 + 1
i/2))*(1/4 + 1i/4))/a
```

Reduce [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = \int \frac{(ax+1)^{\frac{1}{8}}}{(ax-1)^{\frac{1}{8}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8),x)
```

output
```
int((a*x + 1)**(1/8)/(a*x - 1)**(1/8),x)
```

3.137 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 673

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = -\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)$$

$$-\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right)$$

$$+\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)$$

$$+\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right)$$

$$-\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+2\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

output
```
-(2+2^(1/2))^(1/2)*arctan(((2-2^(1/2))^(1/2)-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(
1/8))/(2+2^(1/2))^(1/2))-(2-2^(1/2))^(1/2)*arctan(((2+2^(1/2))^(1/2)-2*(1-
1/a/x)^(1/8)/(1+1/a/x)^(1/8))/(2-2^(1/2))^(1/2))+(2+2^(1/2))^(1/2)*arctan(
((2-2^(1/2))^(1/2)+2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8))/(2+2^(1/2))^(1/2))+(
2-2^(1/2))^(1/2)*arctan(((2+2^(1/2))^(1/2)+2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/
8))/(2-2^(1/2))^(1/2))-2^(1/2)*arctan(1-2^(1/2)*(1+1/a/x)^(1/8)/(1-1/a/x)^
(1/8))+2^(1/2)*arctan(1+2^(1/2)*(1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))+2*arctan(
(1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))-(2-2^(1/2))^(1/2)*arctanh((2-2^(1/2))^(1/
2)*(1-1/a/x)^(1/8)/(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))/(1+1/a/x)^(1/8))-(2
+2^(1/2))^(1/2)*arctanh((2+2^(1/2))^(1/2)*(1-1/a/x)^(1/8)/(1+(1-1/a/x)^(1/
4)/(1+1/a/x)^(1/4))/(1+1/a/x)^(1/8))+2*arctanh((1+1/a/x)^(1/8)/(1-1/a/x)^(
1/8))+2^(1/2)*arctanh(2^(1/2)*(1+1/a/x)^(1/8)/(1+(1+1/a/x)^(1/4)/(1-1/a/x)
^(1/4))/(1-1/a/x)^(1/8))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.04

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = \frac{16}{9}e^{\frac{9}{4}\coth^{-1}(ax)}\,\text{Hypergeometric2F1}\left(\frac{9}{16},1,\frac{25}{16},e^{4\coth^{-1}(ax)}\right)$$

input
```
Integrate[E^(ArcCoth[a*x]/4)/x,x]
```

output
```
(16*E^((9*ArcCoth[a*x])/4)*Hypergeometric2F1[9/16, 1, 25/16, E^(4*ArcCoth[
a*x])])/9
```

Rubi [A] (warning: unable to verify)

Time = 1.99 (sec) , antiderivative size = 866, normalized size of antiderivative = 1.29, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.786, Rules used = {6721, 140, 73, 104, 758, 755, 756, 216, 219, 854, 828, 1442, 1476, 1082, 217, 1479, 25, 27, 1103, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[8]{1+\frac{1}{ax}}x}{\sqrt[8]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{140}$$

$$-\frac{\int \frac{1}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}}{a}-\int \frac{x}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}$$

$$\Big\downarrow \text{73}$$

$$8\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}-\int \frac{x}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}$$

$$\Big\downarrow \text{104}$$

$$8\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}-8\int \frac{1}{\frac{1}{x^8}-1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{758}$$

$$8\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}-8\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$\Big\downarrow \text{755}$$

$$8\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}-$$
$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$\Big\downarrow \text{756}$$

$$8\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}-$$
$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{1+\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1}}{\sqrt[8]{1}}\right.\right.$$

$$\Big\downarrow \text{216}$$

$$8\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}-$$
$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1}}{\sqrt[8]{1}}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$8\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}-$$
$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}}}{\sqrt[8]{1-}}\right.\right.\right.$$

$$\Big\downarrow \text{854}$$

$$8\int \frac{1}{\left(1+\frac{1}{x^8}\right)x^6}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-$$
$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}}}{\sqrt[8]{1-}}\right.\right.\right.$$

$$\Big\downarrow \text{828}$$

$$8\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}\right)x^4}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\int \frac{1}{\left(1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}\right)x^4}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}}}{\sqrt[8]{1-}}\right.\right.\right.$$

$$\downarrow 1442$$

$$8\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1+\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}}}{\sqrt[8]{1-}}\right.\right.\right.$$

$$\downarrow 1476$$

$$8\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\right.\right.$$

$$\downarrow 1082$$

$$8\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-\frac{1}{x^2}}d\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(\right.\right.$$

$$\downarrow 217$$

$$8\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^4}}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(-\frac{1}{2}\text{a}\right.\right.$$

$\downarrow$ 1479

$$8\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\cdots}}{\sqrt[8]{\cdots}}\right.}{\sqrt{2}}\right.\right.\right.$$

$\downarrow$ 25

$$8\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\ldots}}{\sqrt[8]{1\ldots}}\right.}{\sqrt{2}}\right.\right.\right.$$

$\downarrow$ 27

$$8\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int\frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int\frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+1}{\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{\ }}{\ }\right.}{\sqrt{\ }}\right.\right.\right.$$

$\downarrow$ 1103

$$8\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int\frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int\frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\right.}{}\right.\right.\right.$$

$\downarrow$ 1483

$$8\left(\frac{-\frac{\int \frac{\sqrt{2+\sqrt{2}}-\frac{\left(1+\sqrt{2}\right)\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\int \frac{\frac{\left(1+\sqrt{2}\right)\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\sqrt{2+\sqrt{2}}}{\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}+\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{-\frac{\int \frac{\sqrt{2-\sqrt{2}}-\frac{\left(1-\cdots\right.}{\cdots}}{-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{\cdots}}{\sqrt[8]{2-\cdots}}\cdots}\cdots}{\cdots}\cdots}{\cdots}\cdots\right.$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\cdots\right.}{\cdots}\cdots\right.\right.\right.$$

↓ 1142

$$8\left(\frac{-\frac{-\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\frac{1}{2}\left(1+\sqrt{2}\right)\int -\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}} - \frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int}{\cdots}}{2\sqrt{2}}\right.$$

$$\downarrow 25$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\cdots\right.}{\cdots}\right.\right.\right.$$

$$8\left(\frac{-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \cdots}{\cdots}}{2\sqrt{2}}\cdots\right.$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\cdots\right)}{\cdots}\cdots\right.\right.\right.$$

$\downarrow$ 1083

$$8\left(\frac{-\frac{\sqrt{2-\sqrt{2}}\int\frac{1}{\sqrt{2}-2-\frac{1}{x^2}}d\left(\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}\right)+\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\sqrt{2-\sqrt{2}}\int\frac{1}{\sqrt{2}-2-\frac{1}{x^2}}d}{}}{2\sqrt{2}}\right.$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\right.}{}\right.\right.\right.$$

↓ 217

$$8\left(\frac{-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\sqrt{2+\sqrt{2}}}{\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+}}{}}{2\sqrt{2}}\right.$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\operatorname{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\right.}{}\right.\right.\right.$$

$\downarrow$ 1103

$$8\left(\frac{-\frac{-\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}}\right.$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\right.}{}\right.\right.\right.$$

input
```
Int[E^(ArcCoth[a*x]/4)/x,x]
```

output
```
8*(-1/2*((1 - 1/(a*x))^(1/8)/(2 - x^(-8))^(1/8) - (ArcTan[(-Sqrt[2 - Sqrt[
2]] + (2*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8))/Sqrt[2 + Sqrt[2]]] - ((1
 - Sqrt[2])*Log[1 - (Sqrt[2 - Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(
1/8) + x^(-2)])/2)/(2*Sqrt[2 - Sqrt[2]]) - (ArcTan[(Sqrt[2 - Sqrt[2]] + (2
*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8))/Sqrt[2 + Sqrt[2]]] + ((1 - Sqrt[
2])*Log[1 + (Sqrt[2 - Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8) + x
^(-2)])/2)/(2*Sqrt[2 - Sqrt[2]]))/Sqrt[2] + ((1 - 1/(a*x))^(1/8)/(2 - x^(-
8))^(1/8) - (-ArcTan[(-Sqrt[2 + Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(2 - x^
(-8))^(1/8))/Sqrt[2 - Sqrt[2]]] - ((1 + Sqrt[2])*Log[1 - (Sqrt[2 + Sqrt[2]
]*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8) + x^(-2)])/2)/(2*Sqrt[2 + Sqrt[2
]]) - (-ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(
1/8))/Sqrt[2 - Sqrt[2]]] + ((1 + Sqrt[2])*Log[1 + (Sqrt[2 + Sqrt[2]]*(1 -
1/(a*x))^(1/8))/(2 - x^(-8))^(1/8) + x^(-2)])/2)/(2*Sqrt[2 + Sqrt[2]]))/(2
*Sqrt[2])) - 8*((-1/2*ArcTan[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)] - Ar
cTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)]/2)/2 + ((ArcTan[1 - (Sqrt[2
]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/Sqrt[2] - ArcTan[1 + (Sqrt[2]*
(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(
1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + x^(-2)]/(2*Sqrt[2]) - Log[1 + (S
qrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + x^(-2)]/(2*Sqrt[2]))/2)/
2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 828
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Simp[s^3/(2*Sqrt[2]*b*r)   Int[x^
(m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Simp[s^3/(2*S
qrt[2]*b*r)   Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] &&
 GtQ[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} x} dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)/x,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/8)/x,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.66

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x} dx = \text{Too large to display}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="fricas")
```

output
```
-(1/2*I - 1/2)*sqrt(2)*(-1)^(1/8)*log((I + 1)*sqrt(2)*(-1)^(7/8) + 2*((a*x
 - 1)/(a*x + 1))^(1/8)) + (1/2*I + 1/2)*sqrt(2)*(-1)^(1/8)*log(-(I - 1)*sq
rt(2)*(-1)^(7/8) + 2*((a*x - 1)/(a*x + 1))^(1/8)) - (1/2*I + 1/2)*sqrt(2)*
(-1)^(1/8)*log((I - 1)*sqrt(2)*(-1)^(7/8) + 2*((a*x - 1)/(a*x + 1))^(1/8))
 + (1/2*I - 1/2)*sqrt(2)*(-1)^(1/8)*log(-(I + 1)*sqrt(2)*(-1)^(7/8) + 2*((
a*x - 1)/(a*x + 1))^(1/8)) - sqrt(2)*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^
(1/8) + 1) - sqrt(2)*arctan(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) - 1) + 1/2
*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(
1/4) + 1) - 1/2*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x -
 1)/(a*x + 1))^(1/4) + 1) + (-1)^(1/8)*log((-1)^(7/8) + ((a*x - 1)/(a*x +
1))^(1/8)) - I*(-1)^(1/8)*log(I*(-1)^(7/8) + ((a*x - 1)/(a*x + 1))^(1/8))
+ I*(-1)^(1/8)*log(-I*(-1)^(7/8) + ((a*x - 1)/(a*x + 1))^(1/8)) - (-1)^(1/
8)*log(-(-1)^(7/8) + ((a*x - 1)/(a*x + 1))^(1/8)) - 2*arctan(((a*x - 1)/(a
*x + 1))^(1/8)) + log(((a*x - 1)/(a*x + 1))^(1/8) + 1) - log(((a*x - 1)/(a
*x + 1))^(1/8) - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/8)/x,x)
```

output
```
Integral(1/(x*((a*x - 1)/(a*x + 1))**(1/8)), x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="maxima")
```

output
```
integrate(1/(x*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

Giac [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 661, normalized size of antiderivative = 0.98

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = \text{Too large to display}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="giac")
```

output
```
-1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/8)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/8)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/
(a*x + 1))^(1/4) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8)
 + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/
8))/a - 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a + 2*log(-((a*x - 1)/(a*x
+ 1))^(1/8) + 1)/a - 4*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))
^(1/8))/sqrt(-sqrt(2) + 2))/(a*sqrt(2*sqrt(2) + 4)) - 4*arctan(-(sqrt(sqrt
(2) + 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2))/(a*sqrt(2*sq
rt(2) + 4)) - 4*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8)
)/sqrt(sqrt(2) + 2))/(a*sqrt(-2*sqrt(2) + 4)) - 4*arctan(-(sqrt(-sqrt(2) +
 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(sqrt(2) + 2))/(a*sqrt(-2*sqrt(2)
 + 4)) + 2*log(sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/
(a*x + 1))^(1/4) + 1)/(a*sqrt(-2*sqrt(2) + 4)) - 2*log(-sqrt(sqrt(2) + 2)*
((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/(a*sqrt(-2*
sqrt(2) + 4)) + 2*log(sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a
*x - 1)/(a*x + 1))^(1/4) + 1)/(a*sqrt(2*sqrt(2) + 4)) - 2*log(-sqrt(-sqrt(
2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/(a*
sqrt(2*sqrt(2) + 4)))
```

Mupad [B] (verification not implemented)

Time = 23.91 (sec) , antiderivative size = 648, normalized size of antiderivative = 0.96

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = \text{Too large to display}$$

input
```
int(1/(x*((a*x - 1)/(a*x + 1))^(1/8)),x)
```

output
```
atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2^(1/2) + 2)^(1/2) - (((a*x - 1)/(a
*x + 1))^(1/8)*1i)/(2^(1/2) - 2)^(1/2) + (2^(1/2)*((a*x - 1)/(a*x + 1))^(1
/8)*1i)/(2*(2^(1/2) - 2)^(1/2)) + (2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*1i)
/(2*(2^(1/2) + 2)^(1/2)))*((2^(1/2) - 2)^(1/2)*1i + (2^(1/2) + 2)^(1/2)*1i
) - 2*atan(((a*x - 1)/(a*x + 1))^(1/8)) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/
(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1 - 1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/
(a*x + 1))^(1/8)*(1/2 + 1i/2))*(1 + 1i) - atan(((a*x - 1)/(a*x + 1))^(1/8)
*1i)*2i - atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2^(1/2) - 2)^(1/2) + (((a
*x - 1)/(a*x + 1))^(1/8)*1i)/(2^(1/2) + 2)^(1/2) - (2^(1/2)*((a*x - 1)/(a*
x + 1))^(1/8)*1i)/(2*(2^(1/2) - 2)^(1/2)) + (2^(1/2)*((a*x - 1)/(a*x + 1))
^(1/8)*1i)/(2*(2^(1/2) + 2)^(1/2)))*((2^(1/2) - 2)^(1/2)*1i - (2^(1/2) + 2
)^(1/2)*1i) - atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(- 2^(1/2) - 2)^(1/2)
- (((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2 - 2^(1/2))^(1/2) + (2^(1/2)*((a*x -
1)/(a*x + 1))^(1/8)*1i)/(2*(- 2^(1/2) - 2)^(1/2)) + (2^(1/2)*((a*x - 1)/(a
*x + 1))^(1/8)*1i)/(2*(2 - 2^(1/2))^(1/2)))*((- 2^(1/2) - 2)^(1/2)*1i + (2
 - 2^(1/2))^(1/2)*1i) - atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(- 2^(1/2) -
 2)^(1/2) + (((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2 - 2^(1/2))^(1/2) + (2^(1/2
)*((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2*(- 2^(1/2) - 2)^(1/2)) - (2^(1/2)*((a
*x - 1)/(a*x + 1))^(1/8)*1i)/(2*(2 - 2^(1/2))^(1/2)))*((- 2^(1/2) - 2)^(1/
2)*1i - (2 - 2^(1/2))^(1/2)*1i)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx=\int \frac{(ax+1)^{\frac{1}{8}}}{(ax-1)^{\frac{1}{8}}\,x}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)/x,x)
```

output
```
int((a*x + 1)**(1/8)/((a*x - 1)**(1/8)*x),x)
```

3.138 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 502

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}$$

$$-\frac{1}{4}\sqrt{2+\sqrt{2}}a\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)$$

$$-\frac{1}{4}\sqrt{2-\sqrt{2}}a\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right)$$

$$+\frac{1}{4}\sqrt{2+\sqrt{2}}a\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)$$

$$+\frac{1}{4}\sqrt{2-\sqrt{2}}a\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right)$$

$$-\frac{1}{4}\sqrt{2-\sqrt{2}}a\,\text{arctanh}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{}}}\right)\sqrt[8]{1+\frac{1}{ax}}}\right.$$

output
```
a*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)-1/4*(2+2^(1/2))^(1/2)*a*arctan(((2-2^(1/
2))^(1/2)-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8))/(2+2^(1/2))^(1/2))-1/4*(2-2^(
1/2))^(1/2)*a*arctan(((2+2^(1/2))^(1/2)-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8))
/(2-2^(1/2))^(1/2))+1/4*(2+2^(1/2))^(1/2)*a*arctan(((2-2^(1/2))^(1/2)+2*(1
-1/a/x)^(1/8)/(1+1/a/x)^(1/8))/(2+2^(1/2))^(1/2))+1/4*(2-2^(1/2))^(1/2)*a*
arctan(((2+2^(1/2))^(1/2)+2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8))/(2-2^(1/2))^(
1/2))-1/4*(2-2^(1/2))^(1/2)*a*arctanh((2-2^(1/2))^(1/2)*(1-1/a/x)^(1/8)/(1
+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))/(1+1/a/x)^(1/8))-1/4*(2+2^(1/2))^(1/2)*a
*arctanh((2+2^(1/2))^(1/2)*(1-1/a/x)^(1/8)/(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1
/4))/(1+1/a/x)^(1/8))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.09

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = -2ae^{\frac{1}{4}\coth^{-1}(ax)}\left(-\frac{1}{1+e^{2\coth^{-1}(ax)}} + \text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},-e^{2\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[E^(ArcCoth[a*x]/4)/x^2,x]
```

output
```
-2*a*E^(ArcCoth[a*x]/4)*(-(1 + E^(2*ArcCoth[a*x]))^(-1) + Hypergeometric2F
1[1/8, 1, 9/8, -E^(2*ArcCoth[a*x])])
```

Rubi [A] (warning: unable to verify)

Time = 1.30 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6721, 60, 73, 854, 828, 1442, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 60}$$

$$a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{1}{4}\int \frac{1}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 73}$$

$$2a\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}+a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{ 854}$$

$$2a\int \frac{1}{\left(1+\frac{1}{x^8}\right)x^6}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{ 828}$$

$$2a\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}\right)x^4}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\int \frac{1}{\left(1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}\right)x^4}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)+a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}$$

$$\Big\downarrow \text{ 1442}$$

$$\frac{2a\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)+}{a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}$$

$\big\downarrow$ 1483

$$\frac{2a\left(\frac{-\frac{\int \frac{\sqrt{2+\sqrt{2}}-\frac{\left(1+\sqrt{2}\right)\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\int \frac{\frac{\left(1+\sqrt{2}\right)\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\sqrt{2+\sqrt{2}}}{\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}+\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{-\frac{\int \frac{\sqrt{2-\sqrt{2}}-\frac{\left(1\cdots\right.}{\cdots}}{-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{\cdots}}{\sqrt[8]{2\cdots}}\cdots}\cdots}{\cdots}\cdots}{\cdots}\cdots\right.}{a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}$$

$\big\downarrow$ 1142

$$2\left(\frac{-\frac{-\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\frac{1}{2}\left(1+\sqrt{2}\right)\int -\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int}{}}{2\sqrt{2}}\right.$$

$$\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}a$$

$$\Big\downarrow 25$$

$$2\left(\frac{-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\frac{1}{2}\sqrt{2-\sqrt{2}}\int\frac{1}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \cdots}{\cdots}}{2\sqrt{2}}\right.$$

$$\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}a$$

$\downarrow$ 1083

$$\frac{2\left(\frac{-\frac{\sqrt{2-\sqrt{2}}\int \frac{1}{\sqrt{2}-2-\frac{1}{x^2}}d\left(\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}\right)+\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}} - \frac{\sqrt{2-\sqrt{2}}\int \frac{1}{\sqrt{2}-2-\frac{1}{x^2}}d}{}}{2\sqrt{2}}\right.}{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}a}$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\sqrt{2+\sqrt{2}}}{\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+}}{}}{2\sqrt{2}}\right.$$

$$\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}a$$

$$\Big\downarrow \; 1103$$

$$2a\left(\frac{-\frac{-\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2+}}}{2\sqrt{2}}\right.$$

$$a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}$$

input
```
Int[E^(ArcCoth[a*x]/4)/x^2,x]
```

output
```
a*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8) + 2*a*(-1/2*((1 - 1/(a*x))^(1/8)
/(2 - x^(-8))^(1/8) - (ArcTan[(-Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8)
)/(2 - x^(-8))^(1/8))/Sqrt[2 + Sqrt[2]]] - ((1 - Sqrt[2])*Log[1 - (Sqrt[2
- Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8) + x^(-2)])/2)/(2*Sqrt[2
 - Sqrt[2]]) - (ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(2 - x
^(-8))^(1/8))/Sqrt[2 + Sqrt[2]]] + ((1 - Sqrt[2])*Log[1 + (Sqrt[2 - Sqrt[2
]]*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8) + x^(-2)])/2)/(2*Sqrt[2 - Sqrt[
2]]))/Sqrt[2] + ((1 - 1/(a*x))^(1/8)/(2 - x^(-8))^(1/8) - (-ArcTan[(-Sqrt[
2 + Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8))/Sqrt[2 - Sqrt[2
]]] - ((1 + Sqrt[2])*Log[1 - (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(2 -
x^(-8))^(1/8) + x^(-2)])/2)/(2*Sqrt[2 + Sqrt[2]]) - (-ArcTan[(Sqrt[2 + Sqr
t[2]] + (2*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8))/Sqrt[2 - Sqrt[2]]] + (
(1 + Sqrt[2])*Log[1 + (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))
^(1/8) + x^(-2)])/2)/(2*Sqrt[2 + Sqrt[2]]))/(2*Sqrt[2]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 828
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Simp[s^3/(2*Sqrt[2]*b*r)   Int[x^
(m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Simp[s^3/(2*S
qrt[2]*b*r)   Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] &&
 GtQ[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} x^2} dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.75

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2} dx = \frac{-(i-1)\sqrt{2}(-a^8)^{\frac{1}{8}} x \log\left(2a^7\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} + (i+1)\sqrt{2}(-a^8)^{\frac{7}{8}}\right) + (i+1)\sqrt{2}(-a^8)^{\frac{1}{8}} x \log\left(2a^7\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} -\right.}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="fricas")
```

output
```
1/8*(-(I - 1)*sqrt(2)*(-a^8)^(1/8)*x*log(2*a^7*((a*x - 1)/(a*x + 1))^(1/8)
 + (I + 1)*sqrt(2)*(-a^8)^(7/8)) + (I + 1)*sqrt(2)*(-a^8)^(1/8)*x*log(2*a^
7*((a*x - 1)/(a*x + 1))^(1/8) - (I - 1)*sqrt(2)*(-a^8)^(7/8)) - (I + 1)*sq
rt(2)*(-a^8)^(1/8)*x*log(2*a^7*((a*x - 1)/(a*x + 1))^(1/8) + (I - 1)*sqrt(
2)*(-a^8)^(7/8)) + (I - 1)*sqrt(2)*(-a^8)^(1/8)*x*log(2*a^7*((a*x - 1)/(a*
x + 1))^(1/8) - (I + 1)*sqrt(2)*(-a^8)^(7/8)) + 2*(-a^8)^(1/8)*x*log(a^7*(
(a*x - 1)/(a*x + 1))^(1/8) + (-a^8)^(7/8)) - 2*I*(-a^8)^(1/8)*x*log(a^7*((
a*x - 1)/(a*x + 1))^(1/8) + I*(-a^8)^(7/8)) + 2*I*(-a^8)^(1/8)*x*log(a^7*(
(a*x - 1)/(a*x + 1))^(1/8) - I*(-a^8)^(7/8)) - 2*(-a^8)^(1/8)*x*log(a^7*((
a*x - 1)/(a*x + 1))^(1/8) - (-a^8)^(7/8)) + 8*(a*x + 1)*((a*x - 1)/(a*x +
1))^(7/8))/x
```

Sympy [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/8)/x**2,x)
```

output
```
Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(1/8)), x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="maxima")
```

output
```
integrate(1/(x^2*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.86

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = \frac{1}{8}\left(2\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="giac")
```

output
```
1/8*(2*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x +
1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqrt(2
) + 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*sqrt(sqrt(
2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(s
qrt(2) + 2)) + 2*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*((a*x -
 1)/(a*x + 1))^(1/8))/sqrt(sqrt(2) + 2)) - sqrt(sqrt(2) + 2)*log(sqrt(sqrt
(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) +
sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a
*x - 1)/(a*x + 1))^(1/4) + 1) - sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*
((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + sqrt(-sqr
t(2) + 2)*log(-sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)
/(a*x + 1))^(1/4) + 1) + 16*((a*x - 1)/(a*x + 1))^(7/8)/((a*x - 1)/(a*x +
1) + 1))*a
```

Mupad [B] (verification not implemented)

Time = 16.70 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.32

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = \frac{(-1)^{1/8}\,a\,\mathrm{atan}\left((-1)^{1/8}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{2} + \frac{(-1)^{1/8}\,a\,\mathrm{atan}\left((-1)^{1/8}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,1\mathrm{i}\right)\,1\mathrm{i}}{2} + \frac{2\,a\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{\frac{a\,x-1}{a\,x+1}+1} + (-1)^{1/8}\,\sqrt{2}\,a\,\mathrm{atan}\left((-1)^{1/8}\,\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + (-1)^{1/8}\,\sqrt{2}\,a\,\mathrm{atan}\Bigg((-1)^{1/8}\,\sqrt{2}\,\Bigg($$

input
```
int(1/(x^2*((a*x - 1)/(a*x + 1))^(1/8)),x)
```

output
```
((-1)^(1/8)*a*atan((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)))/2 + ((-1)^(1/8
)*a*atan((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)*1i)*1i)/2 + (2*a*((a*x - 1
)/(a*x + 1))^(7/8))/((a*x - 1)/(a*x + 1) + 1) + (-1)^(1/8)*2^(1/2)*a*atan(
(-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1/4 - 1i/4)
+ (-1)^(1/8)*2^(1/2)*a*atan((-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)
*(1/2 + 1i/2))*(1/4 + 1i/4)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$$

$$= \frac{-36(ax+1)^{\frac{7}{8}}(ax-1)^{\frac{5}{8}}a^3x^3+16(ax+1)^{\frac{7}{8}}(ax-1)^{\frac{5}{8}}ax-16(ax+1)^{\frac{7}{8}}(ax-1)^{\frac{5}{8}}+36(ax+1)^{\frac{7}{8}}(ax-}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x)
```

output
```
( - 36*(a*x + 1)**(7/8)*(a*x - 1)**(5/8)*a**3*x**3 + 52*(a*x + 1)**(7/8)*(
a*x - 1)**(5/8)*a*x - 16*(a*x + 1)**(7/8)*(a*x - 1)**(5/8) + 36*(a*x + 1)*
*(7/8)*(a*x - 1)**(5/8)*a**2*x**2 - 36*(a*x + 1)**(7/8)*(a*x - 1)**(5/8)*a
*x + 36*(a*x + 1)**(3/4)*(a*x - 1)**(3/4)*int(((a*x + 1)**(3/4)*sqrt(a*x -
 1)*x**2)/((a*x + 1)**(5/8)*(a*x - 1)**(5/8)*a*x + (a*x + 1)**(5/8)*(a*x -
 1)**(5/8)),x)*a**4*x - 9*(a*x + 1)**(3/4)*(a*x - 1)**(3/4)*int(((a*x + 1)
**(3/4)*sqrt(a*x - 1)*x)/((a*x + 1)**(5/8)*(a*x - 1)**(5/8)*a*x + (a*x + 1
)**(5/8)*(a*x - 1)**(5/8)),x)*a**3*x + 16*(a*x + 1)**(3/4)*(a*x - 1)**(3/4
)*int(((a*x + 1)**(3/4)*sqrt(a*x - 1))/((a*x + 1)**(5/8)*(a*x - 1)**(5/8)*
a*x**2 + (a*x + 1)**(5/8)*(a*x - 1)**(5/8)*x),x)*a*x)/(16*(a*x + 1)**(3/4)
*(a*x - 1)**(3/4)*x)
```

3.139 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 553

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}$$
$$-\frac{1}{32}\sqrt{2+\sqrt{2}}a^2\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{32}\sqrt{2-\sqrt{2}}a^2\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right.$$

output
```
1/8*a^2*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)+1/2*a^2*(1-1/a/x)^(7/8)*(1+1/a/x)^
(9/8)-1/32*(2+2^(1/2))^(1/2)*a^2*arctan(((2-2^(1/2))^(1/2)-2*(1-1/a/x)^(1/
8)/(1+1/a/x)^(1/8))/(2+2^(1/2))^(1/2))-1/32*(2-2^(1/2))^(1/2)*a^2*arctan((
(2+2^(1/2))^(1/2)-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8))/(2-2^(1/2))^(1/2))+1/
32*(2+2^(1/2))^(1/2)*a^2*arctan(((2-2^(1/2))^(1/2)+2*(1-1/a/x)^(1/8)/(1+1/
a/x)^(1/8))/(2+2^(1/2))^(1/2))+1/32*(2-2^(1/2))^(1/2)*a^2*arctan(((2+2^(1/
2))^(1/2)+2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8))/(2-2^(1/2))^(1/2))-1/32*(2-2^
(1/2))^(1/2)*a^2*arctanh((2-2^(1/2))^(1/2)*(1-1/a/x)^(1/8)/(1+(1-1/a/x)^(1
/4)/(1+1/a/x)^(1/4))/(1+1/a/x)^(1/8))-1/32*(2+2^(1/2))^(1/2)*a^2*arctanh((
2+2^(1/2))^(1/2)*(1-1/a/x)^(1/8)/(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4))/(1+1/
a/x)^(1/8))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.13

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = -\frac{a^2 e^{\frac{1}{4}\coth^{-1}(ax)}\left(-1-9e^{2\coth^{-1}(ax)}+\left(1+e^{2\coth^{-1}(ax)}\right)^2 \text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},-e^{2\coth^{-1}(ax)}\right)\right)}{4\left(1+e^{2\coth^{-1}(ax)}\right)^2}$$

input
```
Integrate[E^(ArcCoth[a*x]/4)/x^3,x]
```

output
```
-1/4*(a^2*E^(ArcCoth[a*x]/4)*(-1 - 9*E^(2*ArcCoth[a*x]) + (1 + E^(2*ArcCot
h[a*x]))^2*Hypergeometric2F1[1/8, 1, 9/8, -E^(2*ArcCoth[a*x])]))/(1 + E^(2
*ArcCoth[a*x]))^2
```

Rubi [A] (warning: unable to verify)

Time = 1.44 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.20, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6721, 90, 60, 73, 854, 828, 1442, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}\,x}d\frac{1}{x}$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{1}{8}a\int \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{1}{8}a\left(\frac{1}{4}\int \frac{1}{\sqrt[8]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/8}}d\frac{1}{x}-a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-\frac{1}{8}a\left(-2a\int \frac{1}{\left(2-\frac{1}{x^8}\right)^{7/8}x^6}d\sqrt[8]{1-\frac{1}{ax}}-a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{854}$$

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$
$$\frac{1}{8}a\left(-2a\int \frac{1}{\left(1+\frac{1}{x^8}\right)x^6}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}\right)$$

$\downarrow$ 828

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$
$$\frac{1}{8}a\left(-2a\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}\right)x^4}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\int \frac{1}{\left(1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}\right)x^4}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}\right)$$

$\downarrow$ 1442

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$
$$\frac{1}{8}a\left(-2a\left(\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1-\frac{\sqrt{2}}{x^2}}{1-\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\int \frac{1+\frac{\sqrt{2}}{x^2}}{1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4}}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2}}\right)-a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}\right.$$

$\downarrow$ 1483

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$

$$\frac{1}{8}a\left(-2a\left(\frac{-\frac{\int \frac{\sqrt{2+\sqrt{2}}-\frac{\left(1+\sqrt{2}\right)\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\int \frac{\frac{\left(1+\sqrt{2}\right)\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\sqrt{2+\sqrt{2}}}{\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}+\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\cdots}{2\sqrt{2}}-\cdots\right.\right.$$

↓ 1142

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}-$$

$$\frac{1}{8}a\left(-2\left(\frac{-\frac{-\frac{1}{2}\sqrt{2-\sqrt{2}}\int\frac{1}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\frac{1}{2}\left(1+\sqrt{2}\right)\int-\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}\quad\frac{1}{2}\left(1\right.}{2\sqrt{2+\sqrt{2}}}-\;—}{2\sqrt{2}}\right.\right.$$

↓ 25

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}-$$

$$\frac{1}{8}a\left(-2\left(\frac{-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\frac{1}{2}\sqrt{2-\sqrt{2}}\int\frac{1}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{\cdots}\right.}{\cdots}}{2\sqrt{2}}\cdots\right.\right.$$

↓ 1083

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}-$$

$$\frac{1}{8}a\left(-2\left(\frac{-\frac{\sqrt{2-\sqrt{2}}\int\frac{1}{\sqrt{2}-2-\frac{1}{x^2}}\,d\left(\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}\right)+\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}\,d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{2\sqrt{2+\sqrt{2}}}-\frac{\sqrt{2-\sqrt{2}}\int\cdots}{\cdots}}{2\sqrt{2}}\cdots\right.\right.$$

↓ 217

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}-$$

$$\frac{1}{8}a\left(-2\left(\frac{-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}}{-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1}d\frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}\cdots}{\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\cdots}}{\sqrt[8]{2-\frac{1}{x^8}}}\cdots}\cdots}{\cdots}}{2\sqrt{2}}\right.\right.$$

↓ 1103

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}-$$

$$\frac{1}{8}a\left(-2a\left(\frac{-\frac{-\arctan\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}+\frac{1}{x^2}+1\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{2-\frac{1}{x^8}}}\right.}{}}{2\sqrt{2}}\right.\right.$$

input
```
Int[E^(ArcCoth[a*x]/4)/x^3,x]
```

output
```
(a^2*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(9/8))/2 - (a*(-(a*(1 - 1/(a*x))^(7
/8)*(1 + 1/(a*x))^(1/8)) - 2*a*(-1/2*((1 - 1/(a*x))^(1/8)/(2 - x^(-8))^(1/
8) - (ArcTan[(-Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1
/8))/Sqrt[2 + Sqrt[2]]] - ((1 - Sqrt[2])*Log[1 - (Sqrt[2 - Sqrt[2]]*(1 - 1
/(a*x))^(1/8))/(2 - x^(-8))^(1/8) + x^(-2)])/2)/(2*Sqrt[2 - Sqrt[2]]) - (A
rcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8))/Sqr
t[2 + Sqrt[2]]] + ((1 - Sqrt[2])*Log[1 + (Sqrt[2 - Sqrt[2]]*(1 - 1/(a*x))^
(1/8))/(2 - x^(-8))^(1/8) + x^(-2)])/2)/(2*Sqrt[2 - Sqrt[2]]))/Sqrt[2] + (
(1 - 1/(a*x))^(1/8)/(2 - x^(-8))^(1/8) - (-ArcTan[(-Sqrt[2 + Sqrt[2]] + (2
*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8))/Sqrt[2 - Sqrt[2]]] - ((1 + Sqrt[
2])*Log[1 - (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8) + x
^(-2)])/2)/(2*Sqrt[2 + Sqrt[2]]) - (-ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - 1
/(a*x))^(1/8))/(2 - x^(-8))^(1/8))/Sqrt[2 - Sqrt[2]]] + ((1 + Sqrt[2])*Log
[1 + (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(2 - x^(-8))^(1/8) + x^(-2)])
/2)/(2*Sqrt[2 + Sqrt[2]]))/(2*Sqrt[2]))))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 828
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Simp[s^3/(2*Sqrt[2]*b*r)   Int[x^
(m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Simp[s^3/(2*S
qrt[2]*b*r)   Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] &&
 GtQ[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} x^3}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.72

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = \frac{-(i-1)\sqrt{2}(-a^{16})^{\frac{1}{8}}x^2\log\left(2a^{14}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+(i+1)\sqrt{2}(-a^{16})^{\frac{7}{8}}\right)+(i+1)\sqrt{2}(-a^{16})^{\frac{1}{8}}x^2\log\left(2a^{14}\left(\frac{ax-}{ax+}\right.\right.}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="fricas")
```

output
```
1/64*(-(I - 1)*sqrt(2)*(-a^16)^(1/8)*x^2*log(2*a^14*((a*x - 1)/(a*x + 1))^
(1/8) + (I + 1)*sqrt(2)*(-a^16)^(7/8)) + (I + 1)*sqrt(2)*(-a^16)^(1/8)*x^2
*log(2*a^14*((a*x - 1)/(a*x + 1))^(1/8) - (I - 1)*sqrt(2)*(-a^16)^(7/8)) -
 (I + 1)*sqrt(2)*(-a^16)^(1/8)*x^2*log(2*a^14*((a*x - 1)/(a*x + 1))^(1/8)
+ (I - 1)*sqrt(2)*(-a^16)^(7/8)) + (I - 1)*sqrt(2)*(-a^16)^(1/8)*x^2*log(2
*a^14*((a*x - 1)/(a*x + 1))^(1/8) - (I + 1)*sqrt(2)*(-a^16)^(7/8)) + 2*(-a
^16)^(1/8)*x^2*log(a^14*((a*x - 1)/(a*x + 1))^(1/8) + (-a^16)^(7/8)) - 2*I
*(-a^16)^(1/8)*x^2*log(a^14*((a*x - 1)/(a*x + 1))^(1/8) + I*(-a^16)^(7/8))
 + 2*I*(-a^16)^(1/8)*x^2*log(a^14*((a*x - 1)/(a*x + 1))^(1/8) - I*(-a^16)^
(7/8)) - 2*(-a^16)^(1/8)*x^2*log(a^14*((a*x - 1)/(a*x + 1))^(1/8) - (-a^16
)^(7/8)) + 8*(5*a^2*x^2 + 9*a*x + 4)*((a*x - 1)/(a*x + 1))^(7/8))/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/8)/x**3,x)
```

output
```
Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(1/8)), x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="maxima")
```

output
```
integrate(1/(x^3*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.83

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{64}\left(2\,a\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2\,a\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\left(\frac{ax-1}{ax+1}\right)}{\sqrt{-\sqrt{2}+2}}\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="giac")
```

output
```
1/64*(2*a*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x
 + 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*a*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(s
qrt(2) + 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*a*sqr
t(sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))
/sqrt(sqrt(2) + 2)) + 2*a*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) -
2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(sqrt(2) + 2)) - a*sqrt(sqrt(2) + 2)*lo
g(sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1
/4) + 1) + a*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1)
)^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) - a*sqrt(-sqrt(2) + 2)*log(sqrt
(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) +
 1) + a*sqrt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(
1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + 16*(a*((a*x - 1)/(a*x + 1))^(15/
8) + 9*a*((a*x - 1)/(a*x + 1))^(7/8))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.38

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = \frac{\frac{9\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{4}+\frac{a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{15/8}}{4}}{\frac{(a\,x-1)^2}{(a\,x+1)^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1} + \frac{(-1)^{1/8}\,a^2\,\mathrm{atan}\left((-1)^{1/8}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{16} + \frac{(-1)^{1/8}\,a^2\,\mathrm{atan}\left((-1)^{1/8}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,1\mathrm{i}\right)\,1\mathrm{i}}{16} + (-1)^{1/8}\,\sqrt{2}\,a^2\,\mathrm{atan}\left((-1)^{1/8}\,\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{32}-\frac{1}{32}\mathrm{i}\right) + (-1)^{1/8}\,\sqrt{2}\,a^2\,\mathrm{atan}\left((-1)^{1/8}\right.$$

input
```
int(1/(x^3*((a*x - 1)/(a*x + 1))^(1/8)),x)
```

output
```
((9*a^2*((a*x - 1)/(a*x + 1))^(7/8))/4 + (a^2*((a*x - 1)/(a*x + 1))^(15/8)
)/4)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) + ((-1)^(1/8)
*a^2*atan((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)))/16 + ((-1)^(1/8)*a^2*at
an((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)*1i)*1i)/16 + (-1)^(1/8)*2^(1/2)*
a^2*atan((-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1/3
2 - 1i/32) + (-1)^(1/8)*2^(1/2)*a^2*atan((-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*
x + 1))^(1/8)*(1/2 + 1i/2))*(1/32 + 1i/32)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$$

$$= \frac{-36(ax+1)^{\frac{7}{8}}(ax-1)^{\frac{5}{8}}a^4x^4+16(ax+1)^{\frac{7}{8}}(ax-1)^{\frac{5}{8}}a^2x^2-8(ax+1)^{\frac{7}{8}}(ax-1)^{\frac{5}{8}}ax-8(ax+1)^{\frac{7}{8}}(a}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x)
```

output
```
( - 36*(a*x + 1)**(7/8)*(a*x - 1)**(5/8)*a**4*x**4 + 52*(a*x + 1)**(7/8)*(
a*x - 1)**(5/8)*a**2*x**2 - 8*(a*x + 1)**(7/8)*(a*x - 1)**(5/8)*a*x - 8*(a
*x + 1)**(7/8)*(a*x - 1)**(5/8) + 36*(a*x + 1)**(7/8)*(a*x - 1)**(5/8)*a**
3*x**3 - 36*(a*x + 1)**(7/8)*(a*x - 1)**(5/8)*a**2*x**2 + 36*(a*x + 1)**(3
/4)*(a*x - 1)**(3/4)*int(((a*x + 1)**(3/4)*sqrt(a*x - 1)*x**2)/((a*x + 1)*
*(5/8)*(a*x - 1)**(5/8)*a*x + (a*x + 1)**(5/8)*(a*x - 1)**(5/8)),x)*a**5*x
**2 - 9*(a*x + 1)**(3/4)*(a*x - 1)**(3/4)*int(((a*x + 1)**(3/4)*sqrt(a*x -
 1)*x)/((a*x + 1)**(5/8)*(a*x - 1)**(5/8)*a*x + (a*x + 1)**(5/8)*(a*x - 1)
**(5/8)),x)*a**4*x**2 + 26*(a*x + 1)**(3/4)*(a*x - 1)**(3/4)*int(((a*x + 1
)**(3/4)*sqrt(a*x - 1))/((a*x + 1)**(5/8)*(a*x - 1)**(5/8)*a*x**2 + (a*x +
 1)**(5/8)*(a*x - 1)**(5/8)*x),x)*a**2*x**2)/(16*(a*x + 1)**(3/4)*(a*x - 1
)**(3/4)*x**2)
```

3.140 $\int e^{4\coth^{-1}(ax)}(cx)^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 60

$$\int e^{4\coth^{-1}(ax)}(cx)^m\,dx = \frac{(cx)^{1+m}}{c(1+m)} + \frac{4(cx)^{1+m}}{c(1-ax)} - \frac{4(cx)^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,ax)}{c}$$

output
```
(c*x)^(1+m)/c/(1+m)+4*(c*x)^(1+m)/c/(-a*x+1)-4*(c*x)^(1+m)*hypergeom([1, 1
+m],[2+m],a*x)/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int e^{4\coth^{-1}(ax)}(cx)^m\,dx = -\frac{x(cx)^m(5+4m-ax+4(1+m)(-1+ax)\,\text{Hypergeometric2F1}(1,1+m,2+m,ax))}{(1+m)(-1+ax)}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c*x)^m,x]
```

output
```
-((x*(c*x)^m*(5 + 4*m - a*x + 4*(1 + m)*(-1 + a*x)*Hypergeometric2F1[1, 1
+ m, 2 + m, a*x]))/((1 + m)*(-1 + a*x)))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6717, 6676, 100, 27, 90, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{4\coth^{-1}(ax)}(cx)^m\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int e^{4\text{arctanh}(ax)}(cx)^m dx$$

$$\Big\downarrow \text{6676}$$

$$\int \frac{(ax+1)^2(cx)^m}{(1-ax)^2}dx$$

$$\Big\downarrow \text{100}$$

$$\frac{4(cx)^{m+1}}{c(1-ax)}-\frac{\int \frac{a^2c(cx)^m(4m+ax+3)}{1-ax}dx}{a^2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{4(cx)^{m+1}}{c(1-ax)}-\int \frac{(cx)^m(4m+ax+3)}{1-ax}dx$$

$$\Big\downarrow \text{90}$$

$$-4(m+1)\int \frac{(cx)^m}{1-ax}dx+\frac{4(cx)^{m+1}}{c(1-ax)}+\frac{(cx)^{m+1}}{c(m+1)}$$

$$\Big\downarrow \text{74}$$

$$-\frac{4(cx)^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{c}+\frac{4(cx)^{m+1}}{c(1-ax)}+\frac{(cx)^{m+1}}{c(m+1)}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c*x)^m,x]
```

output
```
(c*x)^(1 + m)/(c*(1 + m)) + (4*(c*x)^(1 + m))/(c*(1 - a*x)) - (4*(c*x)^(1
+ m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.34 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.85

method	result
meijerg	$-\frac{(-a)^{-m}(xc)^{m}x^{-m}\left(\frac{x^{m}(-a)^{m}\left(a^{2}m\,x^{2}+amx+2ax-m^{2}-3m-2\right)}{(1+m)m(-ax+1)}+x^{m}(-a)^{m}(2+m)\,\mathrm{LerchPhi}(ax,1,m)\right)}{a}+\frac{2(-a)^{-m}(xc)^{m}x^{-m}}{}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(x*c)^m,x,method=_RETURNVERBOSE)
```

output
```
-(-a)^(-m)*(x*c)^m*x^(-m)/a*(x^m*(-a)^m*(a^2*m*x^2+a*m*x+2*a*x-m^2-3*m-2)/
(1+m)/m/(-a*x+1)+x^m*(-a)^m*(2+m)*LerchPhi(a*x,1,m))+2*(-a)^(-m)*(x*c)^m*x
^(-m)/a*(-x^m*(-a)^m*(a*x-m-1)/m/(-a*x+1)-x^m*(-a)^m*(1+m)*LerchPhi(a*x,1,
m))-(-a)^(-m)*(x*c)^m*x^(-m)/a*(1/(1+m)*x^m*(-a)^m*(-1-m)/(-a*x+1)+x^m*(-a
)^m*m*LerchPhi(a*x,1,m))
```

Fricas [F]

$$\int e^{4\coth^{-1}(ax)}(cx)^m\,dx=\int\frac{(ax+1)^2(cx)^m}{(ax-1)^2}\,dx$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c*x)^m,x, algorithm="fricas")
```

output
```
integral((a^2*x^2 + 2*a*x + 1)*(c*x)^m/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F]

$$\int e^{4\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(cx)^m\,(ax+1)^2}{(ax-1)^2}\,dx$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c*x)**m,x)
```

output
```
Integral((c*x)**m*(a*x + 1)**2/(a*x - 1)**2, x)
```

Maxima [F]

$$\int e^{4\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(ax+1)^2(cx)^m}{(ax-1)^2}\,dx$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c*x)^m,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)^2*(c*x)^m/(a*x - 1)^2, x)
```

Giac [F]

$$\int e^{4\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(ax+1)^2(cx)^m}{(ax-1)^2}\,dx$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((a*x + 1)^2*(c*x)^m/(a*x - 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{4\coth^{-1}(ax)}(cx)^m\,dx=\int \frac{(c\,x)^m\,(a\,x+1)^2}{(a\,x-1)^2}\,dx$$

input
```
int(((c*x)^m*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
int(((c*x)^m*(a*x + 1)^2)/(a*x - 1)^2, x)
```

Reduce [F]

$$\int e^{4\coth^{-1}(ax)}(cx)^m\,dx = \frac{c^m\left(x^m a^2 m^2 x^2 - x^m a^2 m\,x^2 + 3x^m a\,m^2 x + x^m a m x - 4x^m a x + 4x^m m^2 + 8x^m m + 4x^m + 4\left(\int \frac{}{a^2 m\,x^3 - a^2 x^3 \ldots}\right.\right.}{}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c*x)^m,x)
```

output
```
(c**m*(x**m*a**2*m**2*x**2 - x**m*a**2*m*x**2 + 3*x**m*a*m**2*x + x**m*a*m
*x - 4*x**m*a*x + 4*x**m*m**2 + 8*x**m*m + 4*x**m + 4*int(x**m/(a**2*m*x**
3 - a**2*x**3 - 2*a*m*x**2 + 2*a*x**2 + m*x - x),x)*a*m**4*x + 4*int(x**m/
(a**2*m*x**3 - a**2*x**3 - 2*a*m*x**2 + 2*a*x**2 + m*x - x),x)*a*m**3*x -
4*int(x**m/(a**2*m*x**3 - a**2*x**3 - 2*a*m*x**2 + 2*a*x**2 + m*x - x),x)*
a*m**2*x - 4*int(x**m/(a**2*m*x**3 - a**2*x**3 - 2*a*m*x**2 + 2*a*x**2 + m
*x - x),x)*a*m*x - 4*int(x**m/(a**2*m*x**3 - a**2*x**3 - 2*a*m*x**2 + 2*a*
x**2 + m*x - x),x)*m**4 - 4*int(x**m/(a**2*m*x**3 - a**2*x**3 - 2*a*m*x**2
 + 2*a*x**2 + m*x - x),x)*m**3 + 4*int(x**m/(a**2*m*x**3 - a**2*x**3 - 2*a
*m*x**2 + 2*a*x**2 + m*x - x),x)*m**2 + 4*int(x**m/(a**2*m*x**3 - a**2*x**
3 - 2*a*m*x**2 + 2*a*x**2 + m*x - x),x)*m))/(a*m*(a*m**2*x - a*x - m**2 +
1))
```

3.141 $\int e^{2\coth^{-1}(ax)}(cx)^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int e^{2\coth^{-1}(ax)}(cx)^m\,dx=\frac{(cx)^{1+m}}{c(1+m)}-\frac{2(cx)^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,ax)}{c(1+m)}$$

output
```
(c*x)^(1+m)/c/(1+m)-2*(c*x)^(1+m)*hypergeom([1, 1+m],[2+m],a*x)/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.62

$$\int e^{2\coth^{-1}(ax)}(cx)^m\,dx=-\frac{x(cx)^m(-1+2\,\text{Hypergeometric2F1}(1,1+m,2+m,ax))}{1+m}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c*x)^m,x]
```

output
```
-((x*(c*x)^m*(-1 + 2*Hypergeometric2F1[1, 1 + m, 2 + m, a*x]))/(1 + m))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6717, 6676, 90, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\coth^{-1}(ax)}(cx)^m\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}(cx)^m dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{(cx)^m(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(cx)^{m+1}}{c(m+1)}-2\int \frac{(cx)^m}{1-ax}dx$$

$$\Big\downarrow \text{74}$$

$$\frac{(cx)^{m+1}}{c(m+1)}-\frac{2(cx)^{m+1}\,\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{c(m+1)}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c*x)^m,x]
```

output
```
(c*x)^(1 + m)/(c*(1 + m)) - (2*(c*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2
 + m, a*x])/(c*(1 + m))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.80

method	result
meijerg	$-\frac{(-a)^{-m} (xc)^{m} x^{-m} \left(-\frac{x^{m} (-a)^{m} (amx+m+1)}{(1+m)m}+x^{m} (-a)^{m} \operatorname{LerchPhi}(ax,1,m)\right)}{a}+\frac{(-a)^{-m} (xc)^{m} x^{-m} \left(-\frac{x^{m} (-a)^{m} (-1-m)}{(1+m)m}-x^{m} (\right.}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(x*c)^m,x,method=_RETURNVERBOSE)
```

output
```
-(-a)^(-m)*(x*c)^m*x^(-m)/a*(-x^m*(-a)^m*(a*m*x+m+1)/(1+m)/m+x^m*(-a)^m*Le
rchPhi(a*x,1,m))+(-a)^(-m)*(x*c)^m*x^(-m)/a*(-1/(1+m)*x^m*(-a)^m*(-1-m)/m-
x^m*(-a)^m*LerchPhi(a*x,1,m))
```

Fricas [F]

$$\int e^{2\coth^{-1}(ax)}(cx)^m\,dx=\int\frac{(ax+1)(cx)^m}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c*x)^m,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*(c*x)^m/(a*x - 1), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(32) = 64.

Time = 1.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.44

$$\int e^{2\coth^{-1}(ax)}(cx)^m\,dx=-\frac{ac^mmx^{m+2}\Phi(ax,1,m+2)\,\Gamma(m+2)}{\Gamma\,(m+3)}-\frac{2ac^mx^{m+2}\Phi(ax,1,m+2)\,\Gamma(m+2)}{\Gamma\,(m+3)}-\frac{c^mmx^{m+1}\Phi(ax,1,m+1)\,\Gamma(m+1)}{\Gamma\,(m+2)}-\frac{c^mx^{m+1}\Phi(ax,1,m+1)\,\Gamma(m+1)}{\Gamma\,(m+2)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c*x)**m,x)
```

output
```
-a*c**m*m*x**(m + 2)*lerchphi(a*x, 1, m + 2)*gamma(m + 2)/gamma(m + 3) - 2
*a*c**m*x**(m + 2)*lerchphi(a*x, 1, m + 2)*gamma(m + 2)/gamma(m + 3) - c**
m*m*x**(m + 1)*lerchphi(a*x, 1, m + 1)*gamma(m + 1)/gamma(m + 2) - c**m*x*
*(m + 1)*lerchphi(a*x, 1, m + 1)*gamma(m + 1)/gamma(m + 2)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}(cx)^m\,dx=\int\frac{(ax+1)(cx)^m}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c*x)^m,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c*x)^m/(a*x - 1), x)
```

Giac [F]

$$\int e^{2\coth^{-1}(ax)}(cx)^m\,dx=\int\frac{(ax+1)(cx)^m}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((a*x + 1)*(c*x)^m/(a*x - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}(cx)^m\,dx=\int\frac{(c\,x)^m\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c*x)^m*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c*x)^m*(a*x + 1))/(a*x - 1), x)
```

Reduce [F]

$$\int e^{2\coth^{-1}(ax)}(cx)^m\,dx$$
$$= \frac{c^m\left(x^m amx + 2x^m m + 2x^m + 2\left(\int \frac{x^m}{a\,x^2-x}dx\right)m^2 + 2\left(\int \frac{x^m}{a\,x^2-x}dx\right)m\right)}{am\,(m+1)}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c*x)^m,x)
```

output
```
(c**m*(x**m*a*m*x + 2*x**m*m + 2*x**m + 2*int(x**m/(a*x**2 - x),x)*m**2 +
2*int(x**m/(a*x**2 - x),x)*m))/(a*m*(m + 1))
```

3.142 $\int e^{-2\coth^{-1}(ax)}(cx)^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int e^{-2\coth^{-1}(ax)}(cx)^m\,dx = \frac{(cx)^{1+m}}{c(1+m)} - \frac{2(cx)^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,-ax)}{c(1+m)}$$

output
```
(c*x)^(1+m)/c/(1+m)-2*(c*x)^(1+m)*hypergeom([1, 1+m],[2+m],-a*x)/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

$$\int e^{-2\coth^{-1}(ax)}(cx)^m\,dx = -\frac{x(cx)^m(-1+2\,\text{Hypergeometric2F1}(1,1+m,2+m,-ax))}{1+m}$$

input
```
Integrate[(c*x)^m/E^(2*ArcCoth[a*x]),x]
```

output
```
-((x*(c*x)^m*(-1 + 2*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)]))/(1 + m))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6717, 6676, 90, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2\coth^{-1}(ax)}(cx)^m\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}(cx)^m dx$$

$$\Big\downarrow \text{6676}$$

$$-\int \frac{(cx)^m(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(cx)^{m+1}}{c(m+1)}-2\int \frac{(cx)^m}{ax+1}dx$$

$$\Big\downarrow \text{74}$$

$$\frac{(cx)^{m+1}}{c(m+1)}-\frac{2(cx)^{m+1}\,\text{Hypergeometric2F1}(1,m+1,m+2,-ax)}{c(m+1)}$$

input
```
Int[(c*x)^m/E^(2*ArcCoth[a*x]),x]
```

output
```
(c*x)^(1 + m)/(c*(1 + m)) - (2*(c*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2
 + m, -(a*x)])/(c*(1 + m))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.46

method	result
meijerg	$a^{-1-m}(xc)^m\, x^{-m}\left(\frac{x^m a^m (amx-m-1)}{(1+m)m}+x^m a^m \operatorname{LerchPhi}(-ax,1,m)\right)-a^{-1-m}(xc)^m\, x^{-m}\left(\frac{x^m a^m}{m}+\right.$

input
```
int((x*c)^m*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
a^(-1-m)*(x*c)^m*x^(-m)*(x^m*a^m*(a*m*x-m-1)/(1+m)/m+x^m*a^m*LerchPhi(-a*x
,1,m))-a^(-1-m)*(x*c)^m*x^(-m)*(x^m*a^m/m+1/(1+m)*x^m*a^m*(-1-m)*LerchPhi(
-a*x,1,m))
```

Fricas [F]

$$\int e^{-2\coth^{-1}(ax)}(cx)^m\,dx=\int \frac{(ax-1)(cx)^m}{ax+1}\,dx$$

input
```
integrate((c*x)^m*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
integral((a*x - 1)*(c*x)^m/(a*x + 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.80

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}(cx)^m\,dx=&\ \frac{ac^m m x^{m+2}\Phi(axe^{i\pi},1,m+2)\,\Gamma(m+2)}{\Gamma(m+3)}\\&+\frac{2ac^m x^{m+2}\Phi(axe^{i\pi},1,m+2)\,\Gamma(m+2)}{\Gamma(m+3)}\\&-\frac{c^m m x^{m+1}\Phi(axe^{i\pi},1,m+1)\,\Gamma(m+1)}{\Gamma(m+2)}\\&-\frac{c^m x^{m+1}\Phi(axe^{i\pi},1,m+1)\,\Gamma(m+1)}{\Gamma(m+2)}\end{aligned}$$

input
```
integrate((c*x)**m*(a*x-1)/(a*x+1),x)
```

output
```
a*c**m*m*x**(m + 2)*lerchphi(a*x*exp_polar(I*pi), 1, m + 2)*gamma(m + 2)/g
amma(m + 3) + 2*a*c**m*x**(m + 2)*lerchphi(a*x*exp_polar(I*pi), 1, m + 2)*
gamma(m + 2)/gamma(m + 3) - c**m*m*x**(m + 1)*lerchphi(a*x*exp_polar(I*pi)
, 1, m + 1)*gamma(m + 1)/gamma(m + 2) - c**m*x**(m + 1)*lerchphi(a*x*exp_p
olar(I*pi), 1, m + 1)*gamma(m + 1)/gamma(m + 2)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(ax-1)(cx)^m}{ax+1}\,dx$$

input
```
integrate((c*x)^m*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(c*x)^m/(a*x + 1), x)
```

Giac [F]

$$\int e^{-2\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(ax-1)(cx)^m}{ax+1}\,dx$$

input
```
integrate((c*x)^m*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
integrate((a*x - 1)*(c*x)^m/(a*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(c\,x)^m\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c*x)^m*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c*x)^m*(a*x - 1))/(a*x + 1), x)
```

Reduce [F]

$$\int e^{-2\coth^{-1}(ax)}(cx)^m\,dx$$
$$=\frac{c^m\left(x^m a m x-2x^m m-2x^m+2\left(\int\frac{x^m}{a\,x^2+x}dx\right)m^2+2\left(\int\frac{x^m}{a\,x^2+x}dx\right)m\right)}{am\,(m+1)}$$

input
```
int((c*x)^m*(a*x-1)/(a*x+1),x)
```

output
```
(c**m*(x**m*a*m*x - 2*x**m*m - 2*x**m + 2*int(x**m/(a*x**2 + x),x)*m**2 +
2*int(x**m/(a*x**2 + x),x)*m))/(a*m*(m + 1))
```

3.143 $\int e^{-4\coth^{-1}(ax)}(cx)^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 60

$$\int e^{-4\coth^{-1}(ax)}(cx)^m\,dx=\frac{(cx)^{1+m}}{c(1+m)}+\frac{4(cx)^{1+m}}{c(1+ax)}-\frac{4(cx)^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,-ax)}{c}$$

output
```
(c*x)^(1+m)/c/(1+m)+4*(c*x)^(1+m)/c/(a*x+1)-4*(c*x)^(1+m)*hypergeom([1, 1+
m],[2+m],-a*x)/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int e^{-4\coth^{-1}(ax)}(cx)^m\,dx=-\frac{x(cx)^m(-5-4m-ax+4(1+m)(1+ax)\,\text{Hypergeometric2F1}(1,1+m,2+m,-ax))}{(1+m)(1+ax)}$$

input
```
Integrate[(c*x)^m/E^(4*ArcCoth[a*x]),x]
```

output
```
-((x*(c*x)^m*(-5 - 4*m - a*x + 4*(1 + m)*(1 + a*x)*Hypergeometric2F1[1, 1
+ m, 2 + m, -(a*x)]))/((1 + m)*(1 + a*x)))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6717, 6676, 100, 27, 90, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-4\coth^{-1}(ax)}(cx)^m\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int e^{-4\text{arctanh}(ax)}(cx)^m dx$$

$$\Big\downarrow \text{6676}$$

$$\int \frac{(1-ax)^2(cx)^m}{(ax+1)^2}dx$$

$$\Big\downarrow \text{100}$$

$$\frac{4(cx)^{m+1}}{c(ax+1)}-\frac{\int \frac{a^2c(cx)^m(4m-ax+3)}{ax+1}dx}{a^2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{4(cx)^{m+1}}{c(ax+1)}-\int \frac{(cx)^m(4m-ax+3)}{ax+1}dx$$

$$\Big\downarrow \text{90}$$

$$-4(m+1)\int \frac{(cx)^m}{ax+1}dx+\frac{4(cx)^{m+1}}{c(ax+1)}+\frac{(cx)^{m+1}}{c(m+1)}$$

$$\Big\downarrow \text{74}$$

$$-\frac{4(cx)^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-ax)}{c}+\frac{4(cx)^{m+1}}{c(ax+1)}+\frac{(cx)^{m+1}}{c(m+1)}$$

input
```
Int[(c*x)^m/E^(4*ArcCoth[a*x]),x]
```

output
```
(c*x)^(1 + m)/(c*(1 + m)) + (4*(c*x)^(1 + m))/(c*(1 + a*x)) - (4*(c*x)^(1
+ m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6676
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)
^m*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c, m, n}, x] &&  !Int
egerQ[(n - 1)/2]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.33 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.43

method	result
meijerg	$a^{-1-m}(xc)^m x^{-m}\left(\frac{x^m a^m (a^2 m x^2-amx-2ax-m^2-3m-2)}{(1+m)m(ax+1)}+x^m a^m(2+m)\,\text{LerchPhi}\left(-ax,1,m\right)\right)-2a$

input
```
int((x*c)^m*(a*x-1)^2/(a*x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
a^(-1-m)*(x*c)^m*x^(-m)*(x^m*a^m*(a^2*m*x^2-a*m*x-2*a*x-m^2-3*m-2)/(1+m)/m
/(a*x+1)+x^m*a^m*(2+m)*LerchPhi(-a*x,1,m))-2*a^(-1-m)*(x*c)^m*x^(-m)*(x^m*
a^m*(a*x+m+1)/m/(a*x+1)-x^m*a^m*(1+m)*LerchPhi(-a*x,1,m))+a^(-1-m)*(x*c)^m
*x^(-m)*(1/(1+m)*x^m*a^m*(-1-m)/(a*x+1)+x^m*a^m*m*LerchPhi(-a*x,1,m))
```

Fricas [F]

$$\int e^{-4\coth^{-1}(ax)}(cx)^m\,dx=\int \frac{(ax-1)^2(cx)^m}{(ax+1)^2}\,dx$$

input
```
integrate((c*x)^m*(a*x-1)^2/(a*x+1)^2,x, algorithm="fricas")
```

output
```
integral((a^2*x^2 - 2*a*x + 1)*(c*x)^m/(a^2*x^2 + 2*a*x + 1), x)
```

Sympy [F]

$$\int e^{-4\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(cx)^m\,(ax-1)^2}{(ax+1)^2}\,dx$$

input
```
integrate((c*x)**m*(a*x-1)**2/(a*x+1)**2,x)
```

output
```
Integral((c*x)**m*(a*x - 1)**2/(a*x + 1)**2, x)
```

Maxima [F]

$$\int e^{-4\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(ax-1)^2(cx)^m}{(ax+1)^2}\,dx$$

input
```
integrate((c*x)^m*(a*x-1)^2/(a*x+1)^2,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)^2*(c*x)^m/(a*x + 1)^2, x)
```

Giac [F]

$$\int e^{-4\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(ax-1)^2(cx)^m}{(ax+1)^2}\,dx$$

input
```
integrate((c*x)^m*(a*x-1)^2/(a*x+1)^2,x, algorithm="giac")
```

output
```
integrate((a*x - 1)^2*(c*x)^m/(a*x + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-4\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(c\,x)^m\,(a\,x-1)^2}{(a\,x+1)^2}\,dx$$

input
```
int(((c*x)^m*(a*x - 1)^2)/(a*x + 1)^2,x)
```

output
```
int(((c*x)^m*(a*x - 1)^2)/(a*x + 1)^2, x)
```

Reduce [F]

$$\int e^{-4\coth^{-1}(ax)}(cx)^m\,dx$$
$$= \frac{c^m\left(x^m a^2 m^2 x^2 - x^m a^2 m\,x^2 - 3x^m a\,m^2 x - x^m a m x + 4x^m a x + 4x^m m^2 + 8x^m m + 4x^m - 4\left(\int \frac{}{a^2 m\,x^3 - a^2 x^{\cdots}}\right.\right.}{}$$

input
```
int((c*x)^m*(a*x-1)^2/(a*x+1)^2,x)
```

output
```
(c**m*(x**m*a**2*m**2*x**2 - x**m*a**2*m*x**2 - 3*x**m*a*m**2*x - x**m*a*m
*x + 4*x**m*a*x + 4*x**m*m**2 + 8*x**m*m + 4*x**m - 4*int(x**m/(a**2*m*x**
3 - a**2*x**3 + 2*a*m*x**2 - 2*a*x**2 + m*x - x),x)*a*m**4*x - 4*int(x**m/
(a**2*m*x**3 - a**2*x**3 + 2*a*m*x**2 - 2*a*x**2 + m*x - x),x)*a*m**3*x +
4*int(x**m/(a**2*m*x**3 - a**2*x**3 + 2*a*m*x**2 - 2*a*x**2 + m*x - x),x)*
a*m**2*x + 4*int(x**m/(a**2*m*x**3 - a**2*x**3 + 2*a*m*x**2 - 2*a*x**2 + m
*x - x),x)*a*m*x - 4*int(x**m/(a**2*m*x**3 - a**2*x**3 + 2*a*m*x**2 - 2*a*
x**2 + m*x - x),x)*m**4 - 4*int(x**m/(a**2*m*x**3 - a**2*x**3 + 2*a*m*x**2
 - 2*a*x**2 + m*x - x),x)*m**3 + 4*int(x**m/(a**2*m*x**3 - a**2*x**3 + 2*a
*m*x**2 - 2*a*x**2 + m*x - x),x)*m**2 + 4*int(x**m/(a**2*m*x**3 - a**2*x**
3 + 2*a*m*x**2 - 2*a*x**2 + m*x - x),x)*m))/(a*m*(a*m**2*x - a*x + m**2 -
1))
```

3.144 $\int e^{3\coth^{-1}(ax)}(cx)^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 117

$$\int e^{3\coth^{-1}(ax)}(cx)^m\,dx = -\frac{4\left(a+\frac{1}{x}\right)x(cx)^m}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{(5+4m)x(cx)^m \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m} + \frac{(3+4m)(cx)^m \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}$$

output
```
-4*(a+1/x)*x*(c*x)^m/a/(1-1/a^2/x^2)^(1/2)+(5+4*m)*x*(c*x)^m*hypergeom([1/
2, -1/2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)+(3+4*m)*(c*x)^m*hypergeom([1/2
, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.39 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.96

$$\int e^{3\coth^{-1}(ax)}(cx)^m\,dx$$
$$= \frac{x(cx)^m \left(3(1+m)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{1-ax}\sqrt{\frac{1+ax}{a^2}}\,\text{AppellF1}\left(m,-\frac{1}{2},\frac{1}{2},1+m,-ax,ax\right)-2(1+m)\sqrt{1-\frac{1}{a^2x}}\cdots\right.}{m\cdots}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c*x)^m,x]
```

output
```
(x*(c*x)^m*(3*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 - a*x]*Sqrt[(1 + a*x)/a
^2]*AppellF1[m, -1/2, 1/2, 1 + m, -(a*x), a*x] - 2*(1 + m)*Sqrt[1 - 1/(a^2
*x^2)]*Sqrt[1 - a*x]*Sqrt[(1 + a*x)/a^2]*AppellF1[m, -1/2, 3/2, 1 + m, -(a
*x), a*x] + m*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[-a^(-2) + x^2]*Hypergeomet
ric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]))/(m*(1 + m)*Sqrt[-1 + a*
x]*Sqrt[1 + a*x]*Sqrt[-a^(-2) + x^2])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.54, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6722, 27, 2355, 557, 278, 583, 557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3\coth^{-1}(ax)}(cx)^m\,dx$$

$$\Big\downarrow \text{6722}$$

$$-\left(\frac{1}{x}\right)^m (cx)^m \int \frac{\left(a+\frac{1}{x}\right)^2 \left(\frac{1}{x}\right)^{-m-2}}{a\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \int \frac{\left(a+\frac{1}{x}\right)^2 \left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{a}$$

$\downarrow$ 2355

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \left(4a^2\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}+\int \frac{\left(-3a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a}$$

$\downarrow$ 557

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \left(4a^2\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}-3a\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a}$$

$\downarrow$ 278

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \left(4a^2\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}+\frac{3a\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}+\frac{\left(\frac{1}{x}\right)^{-m}\text{Hypergeometric2}}{m}\right.}{a}$$

$\downarrow$ 583

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \left(4\int \frac{\left(a+\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{3a\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}+\frac{\left(\frac{1}{x}\right)^{-m}\text{Hypergeometric2F1}(}{m}\right.}{a}$$

$\downarrow$ 557

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \left(4\left(a\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)+\frac{3a\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}+\right.}{a}$$

$\downarrow$ 278

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \left(\frac{3a\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}+\frac{\left(\frac{1}{x}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{m}+4\left(-\right.\right.}{a}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c*x)^m,x]
```

output
```
-(((x^(-1))^m*(c*x)^m*((3*a*(x^(-1))^(-1 - m)*Hypergeometric2F1[1/2, (-1 -
 m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 + m) + Hypergeometric2F1[1/2, -1/2*m, 1
 - m/2, 1/(a^2*x^2)]/(m*(x^(-1))^m) + 4*(-((a*(x^(-1))^(-1 - m)*Hypergeome
tric2F1[3/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 + m)) - Hypergeometri
c2F1[3/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)]/(m*(x^(-1))^m))))/a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 583
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, 0]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 6722
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c
*x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n -
 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, c, m}, x] && Intege
rQ[(n - 1)/2] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(xc)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(x*c)^m,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(x*c)^m,x)
```

Fricas [F]

$$\int e^{3\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(cx)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c*x)^m,x, algorithm="fricas")
```

output
```
integral((a^2*x^2 + 2*a*x + 1)*(c*x)^m*sqrt((a*x - 1)/(a*x + 1))/(a^2*x^2
- 2*a*x + 1), x)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(cx)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c*x)**m,x)
```

output
```
Integral((c*x)**m/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(cx)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c*x)^m,x, algorithm="maxima")
```

output
```
integrate((c*x)^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}(cx)^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c*x)^m,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(c\,x)^m}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c*x)^m/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c*x)^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [F]

$$\int e^{3\coth^{-1}(ax)}(cx)^m\,dx = c^m\left(\left(\int \frac{x^m\sqrt{ax+1}\,x}{\sqrt{ax-1}\,ax-\sqrt{ax-1}}dx\right)a + \int \frac{x^m\sqrt{ax+1}}{\sqrt{ax-1}\,ax-\sqrt{ax-1}}dx\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c*x)^m,x)
```

output
```
c**m*(int((x**m*sqrt(a*x + 1)*x)/(sqrt(a*x - 1)*a*x - sqrt(a*x - 1)),x)*a
+ int((x**m*sqrt(a*x + 1))/(sqrt(a*x - 1)*a*x - sqrt(a*x - 1)),x))
```

3.145 $\int e^{\coth^{-1}(ax)}(cx)^m\,dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int e^{\coth^{-1}(ax)}(cx)^m\,dx=\frac{x(cx)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m}+\frac{(cx)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}$$

output
```
x*(c*x)^m*hypergeom([1/2, -1/2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)+(c*x)^m
*hypergeom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.68

$$\int e^{\coth^{-1}(ax)}(cx)^m\,dx = x(cx)^m\left(-\frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{-\frac{1}{a^2}+x^2}\,\text{AppellF1}\left(m,-\frac{1}{2},\frac{1}{2},1+m,-ax,ax\right)}{m\sqrt{-1+ax}\sqrt{\frac{1+ax}{a^2}}\sqrt{1-a^2x^2}}+\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{a^2x^2}\right)}{1+m}\right)$$

input
```
Integrate[E^ArcCoth[a*x]*(c*x)^m,x]
```

output
```
x*(c*x)^m*(-((Sqrt[1 - 1/(a^2*x^2)]*Sqrt[-a^(-2) + x^2]*AppellF1[m, -1/2,
1/2, 1 + m, -(a*x), a*x])/(m*Sqrt[-1 + a*x]*Sqrt[(1 + a*x)/a^2]*Sqrt[1 - a
^2*x^2])) + Hypergeometric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]/(1
 + m))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6722, 27, 557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\coth^{-1}(ax)}(cx)^m\,dx$$

$$\Big\downarrow \text{6722}$$

$$-\left(\frac{1}{x}\right)^m (cx)^m \int \frac{\left(a+\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-2}}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \int \frac{\left(a+\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{557}$$

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \left(a\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \left(-\frac{a\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}-\frac{\left(\frac{1}{x}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{m}\right)}{a}$$

input
```
Int[E^ArcCoth[a*x]*(c*x)^m,x]
```

output
```
-(((x^(-1))^m*(c*x)^m*(-((a*(x^(-1))^(-1 - m)*Hypergeometric2F1[1/2, (-1 -
 m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 + m)) - Hypergeometric2F1[1/2, -1/2*m,
1 - m/2, 1/(a^2*x^2)]/(m*(x^(-1))^m)))/a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 6722
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c
*x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n -
 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, c, m}, x] && Intege
rQ[(n - 1)/2] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(xc)^m}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(x*c)^m,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(x*c)^m,x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(cx)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c*x)^m,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*(c*x)^m*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1), x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(cx)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c*x)**m,x)
```

output
```
Integral((c*x)**m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(cx)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c*x)^m,x, algorithm="maxima")
```

output
```
integrate((c*x)^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}(cx)^m\,dx=\int\frac{(cx)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((c*x)^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(cx)^m\,dx=\int\frac{(c\,x)^m}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c*x)^m/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c*x)^m/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{\coth^{-1}(ax)}(cx)^m\,dx=c^m\left(\int\frac{x^m\sqrt{ax+1}}{\sqrt{ax-1}}dx\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c*x)^m,x)
```

output
```
c**m*int((x**m*sqrt(a*x + 1))/sqrt(a*x - 1),x)
```

3.146 $\int e^{-\coth^{-1}(ax)}(cx)^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 78

$$\int e^{-\coth^{-1}(ax)}(cx)^m\,dx = \frac{x(cx)^m \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m} - \frac{(cx)^m \operatorname{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}$$

output
```
x*(c*x)^m*hypergeom([1/2, -1/2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)-(c*x)^m
*hypergeom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int e^{-\coth^{-1}(ax)}(cx)^m\,dx = x(cx)^m\left(-\frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{-1+ax}{a^2}}\operatorname{AppellF1}\left(m,-\frac{1}{2},\frac{1}{2},1+m,ax,-ax\right)}{m\sqrt{1-ax}\sqrt{-\frac{1}{a^2}+x^2}} + \frac{\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{a^2x^2}\right)}{1+m}\right)$$

input
```
Integrate[(c*x)^m/E^ArcCoth[a*x],x]
```

output
```
x*(c*x)^m*(-((Sqrt[1 - 1/(a^2*x^2)]*Sqrt[(-1 + a*x)/a^2]*AppellF1[m, -1/2,
 1/2, 1 + m, a*x, -(a*x)])/(m*Sqrt[1 - a*x]*Sqrt[-a^(-2) + x^2])) + Hyperg
eometric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]/(1 + m))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6722, 27, 557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\coth^{-1}(ax)}(cx)^m\,dx$$

$$\Big\downarrow \text{6722}$$

$$-\left(\frac{1}{x}\right)^m (cx)^m \int \frac{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-2}}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \int \frac{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{557}$$

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \left(a\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \left(\frac{\left(\frac{1}{x}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{m}-\frac{a\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}\right)}{a}$$

input
```
Int[(c*x)^m/E^ArcCoth[a*x],x]
```

output
```
-(((x^(-1))^m*(c*x)^m*(-((a*(x^(-1))^(-1 - m)*Hypergeometric2F1[1/2, (-1 -
 m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 + m)) + Hypergeometric2F1[1/2, -1/2*m,
1 - m/2, 1/(a^2*x^2)]/(m*(x^(-1))^m)))/a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 6722
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c
*x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n -
 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, c, m}, x] && Intege
rQ[(n - 1)/2] &&  !IntegerQ[m]
```

Maple [F]

$$\int (xc)^m \sqrt{\frac{ax-1}{ax+1}}dx$$

input
```
int((x*c)^m*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
int((x*c)^m*((a*x-1)/(a*x+1))^(1/2),x)
```

Fricas [F]

$$\int e^{-\coth^{-1}(ax)}(cx)^m\,dx = \int (cx)^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c*x)^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
integral((c*x)^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}(cx)^m\,dx = \int (cx)^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c*x)**m*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral((c*x)**m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}(cx)^m\,dx = \int (cx)^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c*x)^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}(cx)^m\,dx = \int (cx)^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c*x)^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(cx)^m\,dx = \int (c\,x)^m\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c*x)^m*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c*x)^m*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{-\coth^{-1}(ax)}(cx)^m\,dx = c^m\left(\int \frac{x^m\sqrt{ax-1}}{\sqrt{ax+1}}dx\right)$$

input
```
int((c*x)^m*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
c**m*int((x**m*sqrt(a*x - 1))/sqrt(a*x + 1),x)
```

3.147 $\int e^{-3\coth^{-1}(ax)}(cx)^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 120

$$\int e^{-3\coth^{-1}(ax)}(cx)^m\,dx = -\frac{4\left(a-\frac{1}{x}\right)x(cx)^m}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{(5+4m)x(cx)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{1+m} - \frac{(3+4m)(cx)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{am}$$

output
```
-4*(a-1/x)*x*(c*x)^m/a/(1-1/a^2/x^2)^(1/2)+(5+4*m)*x*(c*x)^m*hypergeom([1/
2, -1/2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)-(3+4*m)*(c*x)^m*hypergeom([1/2
, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.32 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.61

$$\int e^{-3\coth^{-1}(ax)}(cx)^m\,dx$$
$$= \frac{x(cx)^m\left(-3(1+m)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{-1+ax}{a^2}}\operatorname{AppellF1}\left(m,-\frac{1}{2},\frac{1}{2},1+m,ax,-ax\right)+2(1+m)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{-1}{}}\right.}{m(1+m}$$

input
```
Integrate[(c*x)^m/E^(3*ArcCoth[a*x]),x]
```

output
```
(x*(c*x)^m*(-3*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[(-1 + a*x)/a^2]*AppellF1
[m, -1/2, 1/2, 1 + m, a*x, -(a*x)] + 2*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[
(-1 + a*x)/a^2]*AppellF1[m, -1/2, 3/2, 1 + m, a*x, -(a*x)] + m*Sqrt[1 - a*
x]*Sqrt[-a^(-2) + x^2]*Hypergeometric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a
^2*x^2)]))/(m*(1 + m)*Sqrt[1 - a*x]*Sqrt[-a^(-2) + x^2])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.50, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6722, 27, 2355, 557, 278, 583, 557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-3\coth^{-1}(ax)}(cx)^m\,dx$$

$$\Big\downarrow \text{ 6722}$$

$$-\left(\frac{1}{x}\right)^m(cx)^m\int \frac{\left(a-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{-m-2}}{a\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \int \frac{\left(a-\frac{1}{x}\right)^2 \left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}}{a}$$

$\downarrow$ 2355

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \left(4a^2 \int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}+\int \frac{\left(\frac{1}{x}-3a\right)\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a}$$

$\downarrow$ 557

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \left(4a^2 \int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}-3a\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a}$$

$\downarrow$ 278

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \left(4a^2 \int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}d\frac{1}{x}+\frac{3a\left(\frac{1}{x}\right)^{-m-1} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}-\frac{\left(\frac{1}{x}\right)^{-m} \text{Hypergeometric2F}}{m}\right.}{a}$$

$\downarrow$ 583

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \left(4\int \frac{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{3a\left(\frac{1}{x}\right)^{-m-1} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}-\frac{\left(\frac{1}{x}\right)^{-m} \text{Hypergeometric2F1}(}{m}\right.}{a}$$

$\downarrow$ 557

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \left(4\left(a\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}-\int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)+\frac{3a\left(\frac{1}{x}\right)^{-m-1} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}-\right.}{a}$$

$\downarrow$ 278

$$-\frac{\left(\frac{1}{x}\right)^m (cx)^m \left(\frac{3a\left(\frac{1}{x}\right)^{-m-1} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{1}{a^2x^2}\right)}{m+1}-\frac{\left(\frac{1}{x}\right)^{-m} \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{m}+4\left(\right.\right.}{a}$$

input
```
Int[(c*x)^m/E^(3*ArcCoth[a*x]),x]
```

output
```
-(((x^(-1))^m*(c*x)^m*((3*a*(x^(-1))^(-1 - m)*Hypergeometric2F1[1/2, (-1 -
 m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 + m) - Hypergeometric2F1[1/2, -1/2*m, 1
 - m/2, 1/(a^2*x^2)]/(m*(x^(-1))^m) + 4*(-((a*(x^(-1))^(-1 - m)*Hypergeome
tric2F1[3/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 + m)) + Hypergeometri
c2F1[3/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)]/(m*(x^(-1))^m))))/a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 583
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, 0]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 6722
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c
*x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n -
 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, c, m}, x] && Intege
rQ[(n - 1)/2] &&  !IntegerQ[m]
```

Maple [F]

$$\int (xc)^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

input
```
int((x*c)^m*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
int((x*c)^m*((a*x-1)/(a*x+1))^(3/2),x)
```

Fricas [F]

$$\int e^{-3\coth^{-1}(ax)}(cx)^m \, dx = \int (cx)^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

input
```
integrate((c*x)^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
integral((a*x - 1)*(c*x)^m*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(cx)^m \, dx = \text{Timed out}$$

input
```
integrate((c*x)**m*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}(cx)^m\,dx = \int (cx)^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x)^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}(cx)^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(cx)^m\,dx = \int (c\,x)^m \left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c*x)^m*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c*x)^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [F]

$$\int e^{-3\coth^{-1}(ax)}(cx)^m\,dx = c^m\left(\left(\int \frac{x^m\sqrt{ax-1}\,x}{\sqrt{ax+1}\,ax+\sqrt{ax+1}}dx\right)a - \left(\int \frac{x^m\sqrt{ax-1}}{\sqrt{ax+1}\,ax+\sqrt{ax+1}}dx\right)\right)$$

input
```
int((c*x)^m*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
c**m*(int((x**m*sqrt(a*x - 1)*x)/(sqrt(a*x + 1)*a*x + sqrt(a*x + 1)),x)*a
- int((x**m*sqrt(a*x - 1))/(sqrt(a*x + 1)*a*x + sqrt(a*x + 1)),x))
```

3.148 $\int e^{\frac{5}{2}\coth^{-1}(ax)}(cx)^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}(cx)^m\,dx=\frac{x(cx)^m\,\text{AppellF1}\left(-1-m,\frac{5}{4},-\frac{5}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

output
```
x*(c*x)^m*AppellF1(-1-m,5/4,-5/4,-m,1/a/x,-1/a/x)/(1+m)
```

Mathematica [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}(cx)^m\,dx=\int e^{\frac{5}{2}\coth^{-1}(ax)}(cx)^m\,dx$$

input
```
Integrate[E^((5*ArcCoth[a*x])/2)*(c*x)^m,x]
```

output
```
Integrate[E^((5*ArcCoth[a*x])/2)*(c*x)^m, x]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}(cx)^m\,dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m (cx)^m \int \frac{\left(1+\frac{1}{ax}\right)^{5/4}\left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{1}{ax}\right)^{5/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x(cx)^m \text{AppellF1}\left(-m-1,\frac{5}{4},-\frac{5}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

input
```
Int[E^((5*ArcCoth[a*x])/2)*(c*x)^m,x]
```

output
```
(x*(c*x)^m*AppellF1[-1 - m, 5/4, -5/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Maple [F]

$$\int \frac{(xc)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)*(x*c)^m,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(5/4)*(x*c)^m,x)
```

Fricas [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(cx)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*(c*x)^m,x, algorithm="fricas")
```

output
```
integral((a^2*x^2 + 2*a*x + 1)*(c*x)^m*((a*x - 1)/(a*x + 1))^(3/4)/(a^2*x^
2 - 2*a*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}(cx)^m\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(5/4)*(c*x)**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(cx)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*(c*x)^m,x, algorithm="maxima")
```

output
```
integrate((c*x)^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

Giac [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(cx)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(5/4)*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((c*x)^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(c\,x)^m}{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}\,dx$$

input
```
int((c*x)^m/((a*x - 1)/(a*x + 1))^(5/4),x)
```

output
```
int((c*x)^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

Reduce [F]

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}(cx)^m\,dx = c^m\left(\left(\int \frac{x^m(ax+1)^{\frac{1}{4}}x}{(ax-1)^{\frac{1}{4}}ax-(ax-1)^{\frac{1}{4}}}dx\right)a + \int \frac{x^m(ax+1)^{\frac{1}{4}}}{(ax-1)^{\frac{1}{4}}ax-(ax-1)^{\frac{1}{4}}}dx\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(5/4)*(c*x)^m,x)
```

output
```
c**m*(int((x**m*(a*x + 1)**(1/4)*x)/((a*x - 1)**(1/4)*a*x - (a*x - 1)**(1/
4)),x)*a + int((x**m*(a*x + 1)**(1/4))/((a*x - 1)**(1/4)*a*x - (a*x - 1)**
(1/4)),x))
```

3.149 $\int e^{\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx=\frac{x(cx)^m\operatorname{AppellF1}\left(-1-m,\frac{3}{4},-\frac{3}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

output
```
x*(c*x)^m*AppellF1(-1-m,3/4,-3/4,-m,1/a/x,-1/a/x)/(1+m)
```

Mathematica [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx=\int e^{\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx$$

input
```
Integrate[E^((3*ArcCoth[a*x])/2)*(c*x)^m,x]
```

output
```
Integrate[E^((3*ArcCoth[a*x])/2)*(c*x)^m, x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m (cx)^m \int \frac{\left(1+\frac{1}{ax}\right)^{3/4}\left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x(cx)^m \operatorname{AppellF1}\left(-m-1,\frac{3}{4},-\frac{3}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

input
```
Int[E^((3*ArcCoth[a*x])/2)*(c*x)^m,x]
```

output
```
(x*(c*x)^m*AppellF1[-1 - m, 3/4, -3/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(xc)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)*(x*c)^m,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/4)*(x*c)^m,x)
```

Fricas [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(cx)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*(c*x)^m,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*(c*x)^m*((a*x - 1)/(a*x + 1))^(1/4)/(a*x - 1), x)
```

Sympy [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(cx)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/4)*(c*x)**m,x)
```

output
```
Integral((c*x)**m/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Maxima [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx=\int\frac{(cx)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*(c*x)^m,x, algorithm="maxima")
```

output
```
integrate((c*x)^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

Giac [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx=\int\frac{(cx)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/4)*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((c*x)^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx=\int\frac{(c\,x)^m}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}\,dx$$

input
```
int((c*x)^m/((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
int((c*x)^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

Reduce [F]

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx = c^m\left(\int \frac{x^m(ax+1)^{\frac{3}{4}}}{(ax-1)^{\frac{3}{4}}}dx\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/4)*(c*x)^m,x)
```

output
```
c**m*int((x**m*(a*x + 1)**(3/4))/(a*x - 1)**(3/4),x)
```

3.150 $\int e^{\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx=\frac{x(cx)^m\operatorname{AppellF1}\left(-1-m,\frac{1}{4},-\frac{1}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

output
```
x*(c*x)^m*AppellF1(-1-m,1/4,-1/4,-m,1/a/x,-1/a/x)/(1+m)
```

Mathematica [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx=\int e^{\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx$$

input
```
Integrate[E^(ArcCoth[a*x]/2)*(c*x)^m,x]
```

output
```
Integrate[E^(ArcCoth[a*x]/2)*(c*x)^m, x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m (cx)^m \int \frac{\sqrt[4]{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-2}}{\sqrt[4]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x(cx)^m \operatorname{AppellF1}\left(-m-1,\frac{1}{4},-\frac{1}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

input
```
Int[E^(ArcCoth[a*x]/2)*(c*x)^m,x]
```

output
```
(x*(c*x)^m*AppellF1[-1 - m, 1/4, -1/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Maple [F]

$$\int \frac{(xc)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)*(x*c)^m,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/4)*(x*c)^m,x)
```

Fricas [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(cx)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*(c*x)^m,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*(c*x)^m*((a*x - 1)/(a*x + 1))^(3/4)/(a*x - 1), x)
```

Sympy [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(cx)^m}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/4)*(c*x)**m,x)
```

output
```
Integral((c*x)**m/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Maxima [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx=\int \frac{(cx)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*(c*x)^m,x, algorithm="maxima")
```

output
```
integrate((c*x)^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

Giac [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx=\int \frac{(cx)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/4)*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((c*x)^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx=\int \frac{(c\,x)^m}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}\,dx$$

input
```
int((c*x)^m/((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
int((c*x)^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

Reduce [F]

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx = c^m\left(\int \frac{x^m(ax+1)^{\frac{1}{4}}}{(ax-1)^{\frac{1}{4}}}dx\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/4)*(c*x)^m,x)
```

output
```
c**m*int((x**m*(a*x + 1)**(1/4))/(a*x - 1)**(1/4),x)
```

3.151 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx=\frac{x(cx)^m\operatorname{AppellF1}\left(-1-m,-\frac{1}{4},\frac{1}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

output
```
x*(c*x)^m*AppellF1(-1-m,-1/4,1/4,-m,1/a/x,-1/a/x)/(1+m)
```

Mathematica [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx=\int e^{-\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx$$

input
```
Integrate[(c*x)^m/E^(ArcCoth[a*x]/2),x]
```

output
```
Integrate[(c*x)^m/E^(ArcCoth[a*x]/2), x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m (cx)^m \int \frac{\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-2}}{\sqrt[4]{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x(cx)^m \operatorname{AppellF1}\left(-m-1,-\frac{1}{4},\frac{1}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

input
```
Int[(c*x)^m/E^(ArcCoth[a*x]/2),x]
```

output
```
(x*(c*x)^m*AppellF1[-1 - m, -1/4, 1/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Maple [F]

$$\int (xc)^m \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} dx$$

input
```
int((x*c)^m*((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
int((x*c)^m*((a*x-1)/(a*x+1))^(1/4),x)
```

Fricas [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}(cx)^m\, dx = \int (cx)^m \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} dx$$

input
```
integrate((c*x)^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

output
```
integral((c*x)^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}(cx)^m\, dx = \text{Timed out}$$

input
```
integrate((c*x)**m*((a*x-1)/(a*x+1))**(1/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx = \int (cx)^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((c*x)^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

Giac [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx = \int (cx)^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((c*x)^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

output
```
integrate((c*x)^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx = \int (c\,x)^m\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,dx$$

input
```
int((c*x)^m*((a*x - 1)/(a*x + 1))^(1/4),x)
```

output
```
int((c*x)^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

Reduce [F]

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}(cx)^m\,dx = c^m\left(\int \frac{x^m(ax-1)^{\frac{1}{4}}}{(ax+1)^{\frac{1}{4}}}dx\right)$$

input
```
int((c*x)^m*((a*x-1)/(a*x+1))^(1/4),x)
```

output
```
c**m*int((x**m*(a*x - 1)**(1/4))/(a*x + 1)**(1/4),x)
```

3.152 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx=\frac{x(cx)^m\operatorname{AppellF1}\left(-1-m,-\frac{3}{4},\frac{3}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

output
```
x*(c*x)^m*AppellF1(-1-m,-3/4,3/4,-m,1/a/x,-1/a/x)/(1+m)
```

Mathematica [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx=\int e^{-\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx$$

input
```
Integrate[(c*x)^m/E^((3*ArcCoth[a*x])/2),x]
```

output
```
Integrate[(c*x)^m/E^((3*ArcCoth[a*x])/2), x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m(cx)^m\int \frac{\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{x}\right)^{-m-2}}{\left(1+\frac{1}{ax}\right)^{3/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x(cx)^m \operatorname{AppellF1}\left(-m-1,-\frac{3}{4},\frac{3}{4},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

input
```
Int[(c*x)^m/E^((3*ArcCoth[a*x])/2),x]
```

output
```
(x*(c*x)^m*AppellF1[-1 - m, -3/4, 3/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Maple [F]

$$\int (xc)^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

input
```
int((x*c)^m*((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
int((x*c)^m*((a*x-1)/(a*x+1))^(3/4),x)
```

Fricas [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}(cx)^m \, dx = \int (cx)^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

input
```
integrate((c*x)^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

output
```
integral((c*x)^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}(cx)^m \, dx = \text{Timed out}$$

input
```
integrate((c*x)**m*((a*x-1)/(a*x+1))**(3/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx = \int (cx)^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((c*x)^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

Giac [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx = \int (cx)^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((c*x)^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

output
```
integrate((c*x)^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx = \int (c\,x)^m\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}\,dx$$

input
```
int((c*x)^m*((a*x - 1)/(a*x + 1))^(3/4),x)
```

output
```
int((c*x)^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

Reduce [F]

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}(cx)^m\,dx = c^m\left(\int \frac{x^m(ax-1)^{\frac{3}{4}}}{(ax+1)^{\frac{3}{4}}}dx\right)$$

input
```
int((c*x)^m*((a*x-1)/(a*x+1))^(3/4),x)
```

output
```
c**m*int((x**m*(a*x - 1)**(3/4))/(a*x + 1)**(3/4),x)
```

3.153 $\int e^{\frac{2}{3}\coth^{-1}(x)}(cx)^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int e^{\frac{2}{3}\coth^{-1}(x)}(cx)^m\,dx=\frac{x(cx)^m\operatorname{AppellF1}\left(-1-m,\frac{1}{3},-\frac{1}{3},-m,\frac{1}{x},-\frac{1}{x}\right)}{1+m}$$

output
```
x*(c*x)^m*AppellF1(-1-m,1/3,-1/3,-m,1/x,-1/x)/(1+m)
```

Mathematica [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}(cx)^m\,dx=\int e^{\frac{2}{3}\coth^{-1}(x)}(cx)^m\,dx$$

input
```
Integrate[E^((2*ArcCoth[x])/3)*(c*x)^m,x]
```

output
```
Integrate[E^((2*ArcCoth[x])/3)*(c*x)^m, x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{2}{3}\coth^{-1}(x)}(cx)^m\,dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m (cx)^m \int \frac{\sqrt[3]{1+\frac{1}{x}}\left(\frac{1}{x}\right)^{-m-2}}{\sqrt[3]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x\,\text{AppellF1}\left(-m-1,\frac{1}{3},-\frac{1}{3},-m,\frac{1}{x},-\frac{1}{x}\right)(cx)^m}{m+1}$$

input
```
Int[E^((2*ArcCoth[x])/3)*(c*x)^m,x]
```

output
```
(x*(c*x)^m*AppellF1[-1 - m, 1/3, -1/3, -m, x^(-1), -x^(-1)])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Maple [F]

$$\int \frac{(xc)^m}{\left(\frac{x-1}{1+x}\right)^{\frac{1}{3}}}dx$$

input
```
int(1/((x-1)/(1+x))^(1/3)*(x*c)^m,x)
```

output
```
int(1/((x-1)/(1+x))^(1/3)*(x*c)^m,x)
```

Fricas [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}(cx)^m\,dx = \int \frac{(cx)^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)*(c*x)^m,x, algorithm="fricas")
```

output
```
integral((c*x)^m*(x + 1)*((x - 1)/(x + 1))^(2/3)/(x - 1), x)
```

Sympy [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}(cx)^m\,dx = \int \frac{(cx)^m}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/3)*(c*x)**m,x)
```

output
```
Integral((c*x)**m/((x - 1)/(x + 1))**(1/3), x)
```

Maxima [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}(cx)^m\,dx=\int\frac{(cx)^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)*(c*x)^m,x, algorithm="maxima")
```

output
```
integrate((c*x)^m/((x - 1)/(x + 1))^(1/3), x)
```

Giac [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}(cx)^m\,dx=\int\frac{(cx)^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))^(1/3)*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((c*x)^m/((x - 1)/(x + 1))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{2}{3}\coth^{-1}(x)}(cx)^m\,dx=\int\frac{(c\,x)^m}{\left(\frac{x-1}{x+1}\right)^{1/3}}\,dx$$

input
```
int((c*x)^m/((x - 1)/(x + 1))^(1/3),x)
```

output
```
int((c*x)^m/((x - 1)/(x + 1))^(1/3), x)
```

Reduce [F]

$$\int e^{\frac{2}{3}\coth^{-1}(x)}(cx)^m\,dx = c^m\left(\int \frac{x^m(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{1}{3}}}dx\right)$$

input
```
int(1/((x-1)/(1+x))^(1/3)*(c*x)^m,x)
```

output
```
c**m*int((x**m*(x + 1)**(1/3))/(x - 1)**(1/3),x)
```

3.154 $\int e^{\frac{1}{3}\coth^{-1}(x)}(cx)^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int e^{\frac{1}{3}\coth^{-1}(x)}(cx)^m\,dx=\frac{x(cx)^m\operatorname{AppellF1}\left(-1-m,\frac{1}{6},-\frac{1}{6},-m,\frac{1}{x},-\frac{1}{x}\right)}{1+m}$$

output
```
x*(c*x)^m*AppellF1(-1-m,1/6,-1/6,-m,1/x,-1/x)/(1+m)
```

Mathematica [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}(cx)^m\,dx=\int e^{\frac{1}{3}\coth^{-1}(x)}(cx)^m\,dx$$

input
```
Integrate[E^(ArcCoth[x]/3)*(c*x)^m,x]
```

output
```
Integrate[E^(ArcCoth[x]/3)*(c*x)^m, x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{1}{3}\coth^{-1}(x)}(cx)^m\,dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m (cx)^m \int \frac{\sqrt[6]{1+\frac{1}{x}}\left(\frac{1}{x}\right)^{-m-2}}{\sqrt[6]{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x\,\text{AppellF1}\left(-m-1,\frac{1}{6},-\frac{1}{6},-m,\frac{1}{x},-\frac{1}{x}\right)(cx)^m}{m+1}$$

input
```
Int[E^(ArcCoth[x]/3)*(c*x)^m,x]
```

output
```
(x*(c*x)^m*AppellF1[-1 - m, 1/6, -1/6, -m, x^(-1), -x^(-1)])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Maple [F]

$$\int \frac{(xc)^m}{\left(\frac{x-1}{1+x}\right)^{\frac{1}{6}}} dx$$

input
```
int(1/((x-1)/(1+x))^(1/6)*(x*c)^m,x)
```

output
```
int(1/((x-1)/(1+x))^(1/6)*(x*c)^m,x)
```

Fricas [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}(cx)^m\,dx = \int \frac{(cx)^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)*(c*x)^m,x, algorithm="fricas")
```

output
```
integral((c*x)^m*(x + 1)*((x - 1)/(x + 1))^(5/6)/(x - 1), x)
```

Sympy [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}(cx)^m\,dx = \int \frac{(cx)^m}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/6)*(c*x)**m,x)
```

output
```
Integral((c*x)**m/((x - 1)/(x + 1))**(1/6), x)
```

Maxima [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}(cx)^m\,dx = \int \frac{(cx)^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)*(c*x)^m,x, algorithm="maxima")
```

output
```
integrate((c*x)^m/((x - 1)/(x + 1))^(1/6), x)
```

Giac [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}(cx)^m\,dx = \int \frac{(cx)^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))^(1/6)*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((c*x)^m/((x - 1)/(x + 1))^(1/6), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{3}\coth^{-1}(x)}(cx)^m\,dx = \int \frac{(c\,x)^m}{\left(\frac{x-1}{x+1}\right)^{1/6}}\,dx$$

input
```
int((c*x)^m/((x - 1)/(x + 1))^(1/6),x)
```

output
```
int((c*x)^m/((x - 1)/(x + 1))^(1/6), x)
```

Reduce [F]

$$\int e^{\frac{1}{3}\coth^{-1}(x)}(cx)^m\,dx = c^m\left(\int \frac{x^m(x+1)^{\frac{1}{6}}}{(x-1)^{\frac{1}{6}}}dx\right)$$

input
```
int(1/((x-1)/(1+x))^(1/6)*(c*x)^m,x)
```

output
```
c**m*int((x**m*(x + 1)**(1/6))/(x - 1)**(1/6),x)
```

3.155 $\int e^{\frac{1}{4}\coth^{-1}(ax)}(cx)^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}(cx)^m\,dx=\frac{x(cx)^m\operatorname{AppellF1}\left(-1-m,\frac{1}{8},-\frac{1}{8},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

output
```
x*(c*x)^m*AppellF1(-1-m,1/8,-1/8,-m,1/a/x,-1/a/x)/(1+m)
```

Mathematica [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}(cx)^m\,dx=\int e^{\frac{1}{4}\coth^{-1}(ax)}(cx)^m\,dx$$

input
```
Integrate[E^(ArcCoth[a*x]/4)*(c*x)^m,x]
```

output
```
Integrate[E^(ArcCoth[a*x]/4)*(c*x)^m, x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}(cx)^m\,dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m (cx)^m \int \frac{\sqrt[8]{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-2}}{\sqrt[8]{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x(cx)^m \operatorname{AppellF1}\left(-m-1,\frac{1}{8},-\frac{1}{8},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

input
```
Int[E^(ArcCoth[a*x]/4)*(c*x)^m,x]
```

output
```
(x*(c*x)^m*AppellF1[-1 - m, 1/8, -1/8, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(xc)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)*(x*c)^m,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/8)*(x*c)^m,x)
```

Fricas [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}(cx)^m\,dx = \int \frac{(cx)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*(c*x)^m,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*(c*x)^m*((a*x - 1)/(a*x + 1))^(7/8)/(a*x - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}(cx)^m\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/8)*(c*x)**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}(cx)^m\,dx=\int \frac{(cx)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*(c*x)^m,x, algorithm="maxima")
```

output
```
integrate((c*x)^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

Giac [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}(cx)^m\,dx=\int \frac{(cx)^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/8)*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((c*x)^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}(cx)^m\,dx=\int \frac{(c\,x)^m}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}}\,dx$$

input
```
int((c*x)^m/((a*x - 1)/(a*x + 1))^(1/8),x)
```

output
```
int((c*x)^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

Reduce [F]

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}(cx)^m\,dx = c^m\left(\int \frac{x^m(ax+1)^{\frac{1}{8}}}{(ax-1)^{\frac{1}{8}}}dx\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/8)*(c*x)^m,x)
```

output
```
c**m*int((x**m*(a*x + 1)**(1/8))/(a*x - 1)**(1/8),x)
```

3.156 $\int e^{n\coth^{-1}(ax)}(cx)^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int e^{n\coth^{-1}(ax)}(cx)^m\,dx=\frac{x(cx)^m\operatorname{AppellF1}\left(-1-m,\frac{n}{2},-\frac{n}{2},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

output
```
x*(c*x)^m*AppellF1(-1-m,1/2*n,-1/2*n,-m,1/a/x,-1/a/x)/(1+m)
```

Mathematica [F]

$$\int e^{n\coth^{-1}(ax)}(cx)^m\,dx=\int e^{n\coth^{-1}(ax)}(cx)^m\,dx$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c*x)^m,x]
```

output
```
Integrate[E^(n*ArcCoth[a*x])*(c*x)^m, x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6723, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6723}$$

$$-\left(\frac{1}{x}\right)^m (cx)^m \int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-m-2}d\frac{1}{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x(cx)^m \operatorname{AppellF1}\left(-m-1,\frac{n}{2},-\frac{n}{2},-m,\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c*x)^m,x]
```

output
```
(x*(c*x)^m*AppellF1[-1 - m, n/2, -1/2*n, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6723
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*((c_.)*(x_))^(m_), x_Symbol] :> Simp[(-(c*
x)^m)*(1/x)^m   Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x],
x, 1/x], x] /; FreeQ[{a, c, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Maple [F]

$$\int e^{n \operatorname{arccoth}(ax)} (xc)^m \, dx$$

input
```
int(exp(n*arccoth(a*x))*(x*c)^m,x)
```

output
```
int(exp(n*arccoth(a*x))*(x*c)^m,x)
```

Fricas [F]

$$\int e^{n \coth^{-1}(ax)} (cx)^m \, dx = \int (cx)^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c*x)^m,x, algorithm="fricas")
```

output
```
integral((c*x)^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n \coth^{-1}(ax)} (cx)^m \, dx = \int (cx)^m e^{n \operatorname{acoth}(ax)} \, dx$$

input
```
integrate(exp(n*acoth(a*x))*(c*x)**m,x)
```

output
```
Integral((c*x)**m*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(cx)^m\,dx = \int (cx)^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c*x)^m,x, algorithm="maxima")
```

output
```
integrate((c*x)^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(cx)^m\,dx = \int (cx)^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((c*x)^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(cx)^m\,dx = \int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c\,x)^m\,dx$$

input
```
int(exp(n*acoth(a*x))*(c*x)^m,x)
```

output
```
int(exp(n*acoth(a*x))*(c*x)^m, x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}(cx)^m\,dx = c^m\left(\int x^m e^{acoth(ax)n}dx\right)$$

input
```
int(exp(n*acoth(a*x))*(c*x)^m,x)
```

output
```
c**m*int(x**m*e**(acoth(a*x)*n),x)
```

3.157 $\int e^{n\coth^{-1}(ax)}x^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 174

$$\int e^{n\coth^{-1}(ax)}x^2\,dx = \frac{n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{6a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3 + \frac{2(2+n^2)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{3a^3(2-n)}$$

output
```
1/6*n*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2/a+1/3*(1-1/a/x)^(1-1/2*n
)*(1+1/a/x)^(1+1/2*n)*x^3+2/3*(n^2+2)*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/
2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(a-1/x)/(a+1/x))/a^3/(2-n)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.68

$$\int e^{n\coth^{-1}(ax)}x^2\,dx = \frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n(2+n^2)\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(an^2x+2\right.\right.}{6a^3(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*x^2,x]
```

output
```
(E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(2 + n^2)*Hypergeometric2F1[1, 1
 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*n^2*x + 2*a^3*x^3 + n*(-
1 + a^2*x^2) + (2 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a
*x])])))/(6*a^3*(2 + n))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6721, 114, 25, 27, 168, 25, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{n \coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \left(1-\frac{1}{ax}\right)^{-n/2} \left(1+\frac{1}{ax}\right)^{n/2} x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{3} \int -\frac{\left(an+\frac{1}{x}\right) \left(1-\frac{1}{ax}\right)^{-n/2} \left(1+\frac{1}{ax}\right)^{n/2} x^3}{a^2} d\frac{1}{x} + \frac{1}{3} x^3 \left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3} x^3 \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} - \frac{1}{3} \int \frac{\left(an+\frac{1}{x}\right) \left(1-\frac{1}{ax}\right)^{-n/2} \left(1+\frac{1}{ax}\right)^{n/2} x^3}{a^2} d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} x^3 \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} - \frac{\int \left(an+\frac{1}{x}\right) \left(1-\frac{1}{ax}\right)^{-n/2} \left(1+\frac{1}{ax}\right)^{n/2} x^3 d\frac{1}{x}}{3a^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}-\frac{-\frac{1}{2}\int -\left(\left(n^2+2\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x^2\right)d\frac{1}{x}-\frac{1}{2}anx^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{3a^2}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}-\frac{\frac{1}{2}\int \left(n^2+2\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}-\frac{1}{2}anx^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{3a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}-\frac{\frac{1}{2}\left(n^2+2\right)\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}-\frac{1}{2}anx^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{3a^2}$$

$$\Big\downarrow \; 141$$

$$\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}-\frac{-\frac{2\left(n^2+2\right)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}-\frac{1}{2}anx^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{3a^2}$$

input
```
Int[E^(n*ArcCoth[a*x])*x^2,x]
```

output
```
((1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^3)/3 - (-1/2*(a*n*(1
- 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2) - (2*(2 + n^2)*(1 - 1/
(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2,
2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n)))/(3*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int e^{n\ \mathrm{arccoth}(ax)} x^2 dx$$

input
```
int(exp(n*arccoth(a*x))*x^2,x)
```

output
```
int(exp(n*arccoth(a*x))*x^2,x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}x^2\,dx=\int x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="fricas")
```

output
```
integral(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}x^2\,dx=\int x^2 e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*x**2,x)
```

output
```
Integral(x**2*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}x^2\,dx=\int x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="maxima")
```

output
```
integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}x^2\,dx=\int x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="giac")
```

output
```
integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}x^2\,dx=\int x^2\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,dx$$

input
```
int(x^2*exp(n*acoth(a*x)),x)
```

output
```
int(x^2*exp(n*acoth(a*x)), x)
```

Reduce [F]

$$\int e^{n \coth^{-1}(ax)} x^2 \, dx = \int e^{acoth(ax)n} x^2 dx$$

input
```
int(exp(n*acoth(a*x))*x^2,x)
```

output
```
int(e**(acoth(a*x)*n)*x**2,x)
```

3.158 $\int e^{n\coth^{-1}(ax)}x\,dx$

Optimal result

Integrand size = 10, antiderivative size = 122

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}x\,dx &= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2 \\ &\quad+\frac{2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(2-n)}\end{aligned}$$

output
```
1/2*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2+2*n*(1-1/a/x)^(1-1/2*n)*(1
+1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(a-1/x)/(a+1/x))/a^2/(
2-n)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int e^{n\coth^{-1}(ax)}x\,dx = \frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n^2\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+anx+a^2x\right.\right.}{2a^2(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*x,x]
```

output
```
(E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1, 1 + n/2,
2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + a^2*x^2 + n*Hypergeom
etric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(2*a^2*(2 + n))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6721, 107, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x^3d\frac{1}{x}$$

$$\Big\downarrow \text{ 107}$$

$$\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}-\frac{n\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{ 141}$$

$$\frac{2n\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(2-n)}+\frac{1}{2}x^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

input
```
Int[E^(n*ArcCoth[a*x])*x,x]
```

output
```
((1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/2 + (2*n*(1 - 1/(a
*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2
- n/2, (a - x^(-1))/(a + x^(-1))])/(a^2*(2 - n))
```

Defintions of rubi rules used

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int e^{n \operatorname{arccoth}(ax)} x dx$$

input
```
int(exp(n*arccoth(a*x))*x,x)
```

output
```
int(exp(n*arccoth(a*x))*x,x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}x\,dx=\int x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x,x, algorithm="fricas")
```

output
```
integral(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}x\,dx=\int xe^{n\operatorname{acoth}(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*x,x)
```

output
```
Integral(x*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}x\,dx=\int x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x,x, algorithm="maxima")
```

output
```
integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n \coth^{-1}(ax)} x \, dx = \int x \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

input
```
integrate(exp(n*arccoth(a*x))*x,x, algorithm="giac")
```

output
```
integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n \coth^{-1}(ax)} x \, dx = \int x \, e^{n \, \mathrm{acoth}(a \, x)} \, dx$$

input
```
int(x*exp(n*acoth(a*x)),x)
```

output
```
int(x*exp(n*acoth(a*x)), x)
```

Reduce [F]

$$\int e^{n \coth^{-1}(ax)} x \, dx = \int e^{acoth(ax)n} x dx$$

input
```
int(exp(n*acoth(a*x))*x,x)
```

output
```
int(e**(acoth(a*x)*n)*x,x)
```

3.159 $\int e^{n\coth^{-1}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 78

$$\int e^{n\coth^{-1}(ax)}\,dx = \frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

output
```
4*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n
],(a-1/x)/(a+1/x))/a/(2-n)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05

$$\int e^{n\coth^{-1}(ax)}\,dx = \frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(ax+\text{Hypergeom}\right.\right.}{a(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x]),x]
```

output
```
(E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2
+ n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*x + Hypergeometric2F1[1, n/2, 1 +
n/2, E^(2*ArcCoth[a*x])])))/(a*(2 + n))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6720, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6720}$$

$$-\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{141}$$

$$\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

input
```
Int[E^(n*ArcCoth[a*x]),x]
```

output
```
(4*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2,
 1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n))
```

Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6720
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int e^{n\ \mathrm{arccoth}(ax)}dx$$

input
```
int(exp(n*arccoth(a*x)),x)
```

output
```
int(exp(n*arccoth(a*x)),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\,dx=\int\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x)),x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\,dx=\int e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x)),x)
```

output
```
Integral(exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\,dx=\int\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x)),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\,dx=\int\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x)),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,dx$$

input
```
int(exp(n*acoth(a*x)),x)
```

output
```
int(exp(n*acoth(a*x)), x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}\,dx=\int e^{acoth(ax)n}dx$$

input
```
int(exp(n*acoth(a*x)),x)
```

output
```
int(e**(acoth(a*x)*n),x)
```

3.160 $\int \frac{e^{n \coth^{-1}(ax)}}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 127

$$\int \frac{e^{n \coth^{-1}(ax)}}{x} dx = -\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2} \text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n} + \frac{2^{1+\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2} \text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{n}$$

output
```
-2*(1+1/a/x)^(1/2*n)*hypergeom([1, -1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/n/((
1-1/a/x)^(1/2*n))+2^(1+1/2*n)*hypergeom([-1/2*n, -1/2*n],[1-1/2*n],1/2*(a-
1/x)/a)/n/((1-1/a/x)^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.12

$$\int \frac{e^{n \coth^{-1}(ax)}}{x} dx = \frac{e^{n \coth^{-1}(ax)}\left(e^{2 \coth^{-1}(ax)} n \text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},-e^{2 \coth^{-1}(ax)}\right) + e^{2 \coth^{-1}(ax)} n \text{Hypergeom}\right.}{}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/x,x]
```

output
```
(E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2
+ n/2, -E^(2*ArcCoth[a*x])] + E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1
+ n/2, 2 + n/2, E^(2*ArcCoth[a*x])] - (2 + n)*(Hypergeometric2F1[1, n/2, 1
 + n/2, -E^(2*ArcCoth[a*x])] - Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*Arc
Coth[a*x])])))/(n*(2 + n))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6721, 140, 79, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}$$

$$\Big\downarrow \text{140}$$

$$\frac{\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}d\frac{1}{x}}{a}-\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}$$

$$\Big\downarrow \text{79}$$

$$\frac{2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{n}-\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}$$

$$\Big\downarrow \text{141}$$

$$\frac{2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{n}-\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n}$$

input
```
Int[E^(n*ArcCoth[a*x])/x,x]
```

output
```
(-2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (a - x^(-1))
/(a + x^(-1))])/(n*(1 - 1/(a*x))^(n/2)) + (2^(1 + n/2)*Hypergeometric2F1[-
1/2*n, -1/2*n, 1 - n/2, (a - x^(-1))/(2*a)])/(n*(1 - 1/(a*x))^(n/2))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) && !ILtQ[m, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{x}dx$$

input
```
int(exp(n*arccoth(a*x))/x,x)
```

output
```
int(exp(n*arccoth(a*x))/x,x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x}\,dx=\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/x,x)
```

output
```
Integral(exp(n*acoth(a*x))/x, x)
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{x}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{x}\,dx$$

input
```
int(exp(n*acoth(a*x))/x,x)
```

output
```
int(exp(n*acoth(a*x))/x, x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x}\,dx = \int \frac{e^{acoth(ax)n}}{x}dx$$

input
```
int(exp(n*acoth(a*x))/x,x)
```

output
```
int(e**(acoth(a*x)*n)/x,x)
```

3.161 $\int \frac{e^{n \coth^{-1}(ax)}}{x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 70

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^2} \, dx = \frac{2^{1+\frac{n}{2}} a \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

output
```
2^(1+1/2*n)*a*(1-1/a/x)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/
2*(a-1/x)/a)/(2-n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^2} \, dx = -\frac{4ae^{(2+n) \coth^{-1}(ax)} \text{Hypergeometric2F1}\left(2,1+\frac{n}{2},2+\frac{n}{2},-e^{2 \coth^{-1}(ax)}\right)}{2+n}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/x^2,x]
```

output
```
(-4*a*E^((2 + n)*ArcCoth[a*x])*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E^(
2*ArcCoth[a*x])])/(2 + n)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6721, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx$$

$$\Big\downarrow \text{6721}$$

$$-\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}d\frac{1}{x}$$

$$\Big\downarrow \text{79}$$

$$\frac{a2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

input
```
Int[E^(n*ArcCoth[a*x])/x^2,x]
```

output
```
(2^(1 + n/2)*a*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n,
2 - n/2, (a - x^(-1))/(2*a)])/(2 - n)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{x^2}dx$$

input
```
int(exp(n*arccoth(a*x))/x^2,x)
```

output
```
int(exp(n*arccoth(a*x))/x^2,x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x^2}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/x**2,x)
```

output
```
Integral(exp(n*acoth(a*x))/x**2, x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{x^2}\,dx$$

input
```
int(exp(n*acoth(a*x))/x^2,x)
```

output
```
int(exp(n*acoth(a*x))/x^2, x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx = \frac{-e^{acoth(ax)n} + \left(\int \frac{e^{acoth(ax)n}}{a^2x^3-x}dx\right)anx}{x}$$

input
```
int(exp(n*acoth(a*x))/x^2,x)
```

output
```
( - e**(acoth(a*x)*n) + int(e**(acoth(a*x)*n)/(a**2*x**3 - x),x)*a*n*x)/x
```

3.162 $\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 114

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx = \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} + \frac{2^{n/2}a^2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

output
```
1/2*a^2*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)+2^(1/2*n)*a^2*n*(1-1/a/x)^
(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/(2-n)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx = -\frac{a^2e^{n\coth^{-1}(ax)}\left(-e^{2\coth^{-1}(ax)}n^2\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+ \right.\right.}{2(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/x^3,x]
```

output
```
-1/2*(a^2*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1
, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]) + (2 + n)*(-1 + 1/(a^2*x^2) + n/
(a*x) + n*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])])))/(2 +
n)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6721, 90, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3}dx$$
$$\Big\downarrow \text{ 6721}$$
$$-\int \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{x}d\frac{1}{x}$$
$$\Big\downarrow \text{ 90}$$
$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}-\frac{1}{2}an\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}d\frac{1}{x}$$
$$\Big\downarrow \text{ 79}$$
$$\frac{a^2 2^{n/2} n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n}+\frac{1}{2}a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/x^3,x]
```

output
```
(a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/2 + (2^(n/2)*a^2*n
*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (a -
x^(-1))/(2*a)])/(2 - n)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{x^3}dx$$

input
```
int(exp(n*arccoth(a*x))/x^3,x)
```

output
```
int(exp(n*arccoth(a*x))/x^3,x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x^3}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x^3}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/x**3,x)
```

output
```
Integral(exp(n*acoth(a*x))/x**3, x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^3}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^3}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{x^3}\,dx$$

input
```
int(exp(n*acoth(a*x))/x^3,x)
```

output
```
int(exp(n*acoth(a*x))/x^3, x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx = \frac{-e^{acoth(ax)n} + \left(\int \frac{e^{acoth(ax)n}}{a^2x^4-x^2}dx\right) an\,x^2}{2x^2}$$

input
```
int(exp(n*acoth(a*x))/x^3,x)
```

output
```
( - e**(acoth(a*x)*n) + int(e**(acoth(a*x)*n)/(a**2*x**4 - x**2),x)*a*n*x*
*2)/(2*x**2)
```

3.163 $\int \frac{e^{n\coth^{-1}(ax)}}{x^4} dx$

Optimal result

Integrand size = 12, antiderivative size = 167

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^4} dx = \frac{1}{6}a^3n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{3x} + \frac{2^{n/2}a^3(2+n^2)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}$$

output
```
1/6*a^3*n*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)+1/3*a^2*(1-1/a/x)^(1-1/2
*n)*(1+1/a/x)^(1+1/2*n)/x+2^(1/2*n)*a^3*(n^2+2)*(1-1/a/x)^(1-1/2*n)*hyperg
eom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/(6-3*n)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.79

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx = -\frac{a^3e^{n\coth^{-1}(ax)}\left(-e^{2\coth^{-1}(ax)}n(2+n^2)\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)+(2+n)\left(\right.\right.}{6(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/x^4,x]
```

output
```
-1/6*(a^3*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n*(2 + n^2)*Hypergeomet
ric2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]) + (2 + n)*(-((1 - 1/(a^2
*x^2))*(n + 2/(a*x))) + (2 + n^2)/(a*x) + (2 + n^2)*Hypergeometric2F1[1, n
/2, 1 + n/2, -E^(2*ArcCoth[a*x])])))/(2 + n)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6721, 101, 25, 27, 90, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \text{ 6721}$$

$$-\int \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 101}$$

$$\frac{1}{3}a^2\int -\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(a+\frac{n}{x}\right)}{a}d\frac{1}{x}+\frac{a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{3x}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{3x}-\frac{1}{3}a^2\int\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(a+\frac{n}{x}\right)}{a}d\frac{1}{x}$$

$$\Big\downarrow 27$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{3x}-\frac{1}{3}a\int\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(a+\frac{n}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow 90$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{3x}-\frac{1}{3}a\left(\frac{1}{2}a\left(n^2+2\right)\int\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}d\frac{1}{x}-\frac{1}{2}a^2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\right)$$

$$\Big\downarrow 79$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{3x}-\frac{1}{3}a\left(-\frac{a^2 2^{n/2}\left(n^2+2\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{2-n}-\frac{1}{2}a^2n\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{a\right.$$

input
```
Int[E^(n*ArcCoth[a*x])/x^4,x]
```

output
```
(a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(3*x) - (a*(-1/2*(
a^2*n*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)) - (2^(n/2)*a^2*(2
 + n^2)*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2
, (a - x^(-1))/(2*a)]))/(2 - n)))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{e^{n\ \mathrm{arccoth}(ax)}}{x^4} dx$$

input
```
int(exp(n*arccoth(a*x))/x^4,x)
```

output
```
int(exp(n*arccoth(a*x))/x^4,x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x^4}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x^4}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/x**4,x)
```

output
```
Integral(exp(n*acoth(a*x))/x**4, x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x^4}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^4}\, dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\, n}}{x^4}\, dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^4}\, dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{x^4}\, dx$$

input
```
int(exp(n*acoth(a*x))/x^4,x)
```

output
```
int(exp(n*acoth(a*x))/x^4, x)
```

Reduce [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^4}\, dx = \frac{-e^{acoth(ax)n} + \left(\int \frac{e^{acoth(ax)n}}{a^2n^2x^5+2a^2x^5-n^2x^3-2x^3} dx\right) a\, n^3 x^3 + 2\left(\int \frac{e^{acoth(ax)n}}{a^2n^2x^5+2a^2x^5-n^2x^3-2x^3} dx\right) an\, x^3}{3x^3}$$

input
```
int(exp(n*acoth(a*x))/x^4,x)
```

output
```
( - e**(acoth(a*x)*n) + int(e**(acoth(a*x)*n)/(a**2*n**2*x**5 + 2*a**2*x**
5 - n**2*x**3 - 2*x**3),x)*a*n**3*x**3 + 2*int(e**(acoth(a*x)*n)/(a**2*n**
2*x**5 + 2*a**2*x**5 - n**2*x**3 - 2*x**3),x)*a*n*x**3)/(3*x**3)
```

3.164 $\int \frac{e^{n \coth^{-1}(ax)}}{x^5} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 220

$$
\begin{aligned}
\int \frac{e^{n \coth^{-1}(ax)}}{x^5} \, dx
&= \frac{1}{24} a^4 (6 + 2n + n^2) \left(1 - \frac{1}{ax}\right)^{1-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{2+n}{2}} \\
&\quad - \frac{1}{12} a^4 n \left(1 - \frac{1}{ax}\right)^{2-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{2+n}{2}} + \frac{a^2 \left(1 - \frac{1}{ax}\right)^{1-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{2+n}{2}}}{4x^2} \\
&\quad + \frac{2^{-2+\frac{n}{2}} a^4 n (8 + n^2) \left(1 - \frac{1}{ax}\right)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1 - \frac{n}{2}, -\frac{n}{2}, 2 - \frac{n}{2}, \frac{a - \frac{1}{x}}{2a}\right)}{3(2 - n)}
\end{aligned}
$$

output

```
1/24*a^4*(n^2+2*n+6)*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)-1/12*a^4*n*(1
-1/a/x)^(2-1/2*n)*(1+1/a/x)^(1+1/2*n)+1/4*a^2*(1-1/a/x)^(1-1/2*n)*(1+1/a/x
)^(1+1/2*n)/x^2+2^(-2+1/2*n)*a^4*n*(n^2+8)*(1-1/a/x)^(1-1/2*n)*hypergeom([
-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/(6-3*n)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.67

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx$$
$$= -\frac{1}{24}a^4e^{n\coth^{-1}(ax)}\left(-6-n^2+\frac{6}{a^4x^4}+\frac{2n}{a^3x^3}+\frac{n^2}{a^2x^2}+\frac{6n}{ax}+\frac{n^3}{ax} - \frac{e^{2\coth^{-1}(ax)}n^2(8+n^2)\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)}{2+n} + n(8+n^2)\,\text{Hypergeometric2F1}\left(1,\frac{n}{2},1+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)\right)$$

input
```
Integrate[E^(n*ArcCoth[a*x])/x^5,x]
```

output
```
-1/24*(a^4*E^(n*ArcCoth[a*x])*(-6 - n^2 + 6/(a^4*x^4) + (2*n)/(a^3*x^3) +
n^2/(a^2*x^2) + (6*n)/(a*x) + n^3/(a*x) - (E^(2*ArcCoth[a*x])*n^2*(8 + n^2
)*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])])/(2 + n) + n
*(8 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])]))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6721, 111, 25, 27, 164, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx$$
$$\Big\downarrow \text{6721}$$
$$-\int \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{111}$$

$$\frac{1}{4}a^2\int -\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(2a+\frac{n}{x}\right)}{ax}d\frac{1}{x}+\frac{a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{4x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{4x^2}-\frac{1}{4}a^2\int \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(2a+\frac{n}{x}\right)}{ax}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{4x^2}-\frac{1}{4}a\int \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(2a+\frac{n}{x}\right)}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{164}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{4x^2}-$$
$$\frac{1}{4}a\left(\frac{1}{6}a^2n\left(n^2+8\right)\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}d\frac{1}{x}-\frac{1}{6}a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(a\left(n^2+6\right)+\frac{2n}{x}\right)\right)$$

$$\Big\downarrow \text{79}$$

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{4x^2}-$$
$$\frac{1}{4}a\left(-\frac{a^3 2^{n/2}n\left(n^2+8\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}-\frac{1}{6}a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(a\left(n^2\right.\right.\right.$$

input
```
Int[E^(n*ArcCoth[a*x])/x^5,x]
```

output
```
(a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(4*x^2) - (a*(-1/6
*(a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(a*(6 + n^2) + (2*
n)/x)) - (2^(n/2)*a^3*n*(8 + n^2)*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F
1[1 - n/2, -1/2*n, 2 - n/2, (a - x^(-1))/(2*a)])/(3*(2 - n))))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 6721
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\,\operatorname{arccoth}(ax)}}{x^5}dx$$

input
```
int(exp(n*arccoth(a*x))/x^5,x)
```

output
```
int(exp(n*arccoth(a*x))/x^5,x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x^5}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^5, x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx=\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x^5}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/x**5,x)
```

output
```
Integral(exp(n*acoth(a*x))/x**5, x)
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^5} dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^5} dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^5, x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^5} dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^5} dx$$

input
```
integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{x^5} dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{x^5} dx$$

input
```
int(exp(n*acoth(a*x))/x^5,x)
```

output
```
int(exp(n*acoth(a*x))/x^5, x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx$$
$$= \frac{-e^{acoth(ax)n} + \left(\int \frac{e^{acoth(ax)n}}{a^2n^2x^6+8a^2x^6-n^2x^4-8x^4}dx\right) a\,n^3x^4 + 8\left(\int \frac{e^{acoth(ax)n}}{a^2n^2x^6+8a^2x^6-n^2x^4-8x^4}dx\right) an\,x^4}{4x^4}$$

input
```
int(exp(n*acoth(a*x))/x^5,x)
```

output
```
( - e**(acoth(a*x)*n) + int(e**(acoth(a*x)*n)/(a**2*n**2*x**6 + 8*a**2*x**
6 - n**2*x**4 - 8*x**4),x)*a*n**3*x**4 + 8*int(e**(acoth(a*x)*n)/(a**2*n**
2*x**6 + 8*a**2*x**6 - n**2*x**4 - 8*x**4),x)*a*n*x**4)/(4*x**4)
```

3.165 $\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal result

Integrand size = 16, antiderivative size = 132

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx=-\frac{7}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5+\frac{7c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

output
```
-7/8*a*c^4*(1-1/a^2/x^2)^(1/2)*x^2+17/15*a^2*c^4*(1-1/a^2/x^2)^(3/2)*x^3-3
/4*a^3*c^4*(1-1/a^2/x^2)^(3/2)*x^4+1/5*a^4*c^4*(1-1/a^2/x^2)^(3/2)*x^5+7/8
*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-136-15ax+112a^2x^2-90a^3x^3+24a^4x^4)+105\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{120a}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x)^4,x]
```

output
```
(c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3
+ 24*a^4*x^4) + 105*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a)
```

Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6724, 25, 27, 540, 2338, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^4e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \ 6724$$

$$ac\int -c^3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3x^6d\frac{1}{x}$$

$$\Big\downarrow \ 25$$

$$-ac\int c^3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3x^6d\frac{1}{x}$$

$$\Big\downarrow \ 27$$

$$-ac^4\int \sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3x^6d\frac{1}{x}$$

$$\Big\downarrow \text{540}$$

$$-ac^4\left(-\frac{1}{5}\int \sqrt{1-\frac{1}{a^2x^2}}\left(15a^2-\frac{17a}{x}+\frac{5}{x^2}\right)x^5d\frac{1}{x}-\frac{1}{5}a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{2338}$$

$$-ac^4\left(\frac{1}{5}\left(\frac{1}{4}\int \sqrt{1-\frac{1}{a^2x^2}}\left(68a-\frac{35}{x}\right)x^4d\frac{1}{x}+\frac{15}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{5}a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{534}$$

$$-ac^4\left(\frac{1}{5}\left(\frac{1}{4}\left(-35\int \sqrt{1-\frac{1}{a^2x^2}}x^3d\frac{1}{x}-\frac{68}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)+\frac{15}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{5}a^3x^5\left(1-\frac{1}{a^2x^2}\right.\right.$$

$$\Big\downarrow \text{243}$$

$$-ac^4\left(\frac{1}{5}\left(\frac{1}{4}\left(-\frac{35}{2}\int \sqrt{1-\frac{1}{a^2x^2}}x^2d\frac{1}{x^2}-\frac{68}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)+\frac{15}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{5}a^3x^5\left(1-\frac{1}{a^2}\right.\right.$$

$$\Big\downarrow \text{51}$$

$$-ac^4\left(\frac{1}{5}\left(\frac{1}{4}\left(-\frac{35}{2}\left(x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}\right)-\frac{68}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)+\frac{15}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right.\right.$$

$$\Big\downarrow \text{73}$$

$$-ac^4\left(\frac{1}{5}\left(\frac{1}{4}\left(-\frac{35}{2}\left(\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-x\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{68}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)+\frac{15}{4}a^2x^4\right.\right.$$

$$\Big\downarrow \text{221}$$

$$-ac^4\left(\frac{1}{5}\left(\frac{1}{4}\left(-\frac{35}{2}\left(\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}-x\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{68}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)+\frac{15}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3}\right.\right.$$

input

```
Int[E^ArcCoth[a*x]*(c - a*c*x)^4,x]
```

output
```
-(a*c^4*(-1/5*(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^5) + ((15*a^2*(1 - 1/(a^2*x^2
))^(3/2)*x^4)/4 + ((-68*a*(1 - 1/(a^2*x^2))^(3/2)*x^3)/3 - (35*(-(Sqrt[1 -
 1/(a^2*x^2)]*x) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a^2))/2)/4)/5))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

method	result
risch	$\frac{\left(24 a^{4} x^{4}-90 a^{3} x^{3}+112 a^{2} x^{2}-15 a x -136\right) \left(a x -1\right) c^{4}}{120 a \sqrt{\frac{a x -1}{a x +1}}}+\frac{7 \ln \left(\frac{a^{2} x}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{2}-1}\right) c^{4} \sqrt{\left(a x -1\right) \left(a x +1\right)}}{8 \sqrt{a^{2}}\, \sqrt{\frac{a x -1}{a x +1}}\, \left(a x +1\right)}$
default	$\frac{\left(a x -1\right) c^{4} \left(24 \sqrt{a^{2}}\, \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} a^{2} x^{2}-90 \sqrt{a^{2}}\, \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} a x +120 \left(\left(a x -1\right) \left(a x +1\right)\right)^{\frac{3}{2}} \sqrt{a^{2}}+16 \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} \sqrt{a^{2}}-105 \sqrt{a^{2}}\, \sqrt{a^{2} x^{2}-1}\, a x}{120 a \sqrt{\frac{a x -1}{a x +1}}\, \sqrt{\left(a x -1\right) \left(a x +1\right)}\, \sqrt{a^{2}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/120*(24*a^4*x^4-90*a^3*x^3+112*a^2*x^2-15*a*x-136)*(a*x-1)/a*c^4/((a*x-1
)/(a*x+1))^(1/2)+7/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c
^4/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(24\,a^5c^4x^5-66\,a^4c^4x^4+22\,a^3c^4x^3+97\,a^2c^4x^2-1}{120\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
1/120*(105*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^4*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) + (24*a^5*c^4*x^5 - 66*a^4*c^4*x^4 + 22*a^3*c^4*x^3 +
 97*a^2*c^4*x^2 - 151*a*c^4*x - 136*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = c^4\left(\int\left(-\frac{4ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^3x^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**4,x)
```

output
```
c**4*(Integral(-4*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(6*a
**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-4*a**3*x**3/sqr
t(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**4*x**4/sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(112) = 224$.

Time = 0.04 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.96

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx$$
$$= \frac{1}{120}\left(\frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} - \frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2} - \frac{2\left(105\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}} + 790\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} - 896\,c^4\right.}{\frac{5\,(ax-1)a^2}{ax+1} - \frac{10\,(ax-1)^2a^2}{(ax+1)^2} + \frac{10\,(ax-1)}{(ax+1)}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
1/120*(105*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^4*((a*x - 1)/(a*x + 1))^(9/2) + 79
0*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 896*c^4*((a*x - 1)/(a*x + 1))^(5/2) +
490*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^4*sqrt((a*x - 1)/(a*x + 1)))/(
5*(a*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^
3*a^2/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x +
 1)^5 - a^2))*a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = -\frac{7\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\mathrm{sgn}\,(ax+1)} - \frac{1}{120}\,\sqrt{a^2x^2-1}\left(\left(\frac{15\,c^4}{\mathrm{sgn}\,(ax+1)}-2\left(\frac{56\,ac^4}{\mathrm{sgn}\,(ax+1)}+3\left(\frac{4\,a^3c^4x}{\mathrm{sgn}\,(ax+1)}-\frac{15\,a^2c^4}{\mathrm{sgn}\,(ax+1)}\right)x\right)x\right)x+\frac{}{a\mathrm{sg}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
-7/8*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) - 1
/120*sqrt(a^2*x^2 - 1)*((15*c^4/sgn(a*x + 1) - 2*(56*a*c^4/sgn(a*x + 1) +
3*(4*a^3*c^4*x/sgn(a*x + 1) - 15*a^2*c^4/sgn(a*x + 1))*x)*x)*x + 136*c^4/(
a*sgn(a*x + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.62

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{\frac{49\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6}-\frac{7\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{224\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{15}+\frac{79\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{6}+\frac{7\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}}{a-\frac{5\,a\,(a\,x-1)}{a\,x+1}+\frac{10\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a\,(a\,x-1)^5}{(a\,x+1)^5}} + \frac{7\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

input
```
int((c - a*c*x)^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((49*c^4*((a*x - 1)/(a*x + 1))^(3/2))/6 - (7*c^4*((a*x - 1)/(a*x + 1))^(1/
2))/4 - (224*c^4*((a*x - 1)/(a*x + 1))^(5/2))/15 + (79*c^4*((a*x - 1)/(a*x
 + 1))^(7/2))/6 + (7*c^4*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x -
1))/(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x +
 1)^3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) + (7*
c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.91

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx$$
$$=\frac{c^4\left(24\sqrt{ax+1}\,\sqrt{ax-1}\,a^4x^4-90\sqrt{ax+1}\,\sqrt{ax-1}\,a^3x^3+112\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2-15\sqrt{ax+1}\,\sqrt{ax}\right.}{120a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x)
```

output
```
(c**4*(24*sqrt(a*x + 1)*sqrt(a*x - 1)*a**4*x**4 - 90*sqrt(a*x + 1)*sqrt(a*
x - 1)*a**3*x**3 + 112*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 - 15*sqrt(a*x
 + 1)*sqrt(a*x - 1)*a*x - 136*sqrt(a*x + 1)*sqrt(a*x - 1) + 210*log((sqrt(
a*x - 1) + sqrt(a*x + 1))/sqrt(2))))/(120*a)
```

3.166 $\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{5}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{5c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

output
```
-5/8*a*c^3*(1-1/a^2/x^2)^(1/2)*x^2+2/3*a^2*c^3*(1-1/a^2/x^2)^(3/2)*x^3-1/4
*a^3*c^3*(1-1/a^2/x^2)^(3/2)*x^4+5/8*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{c^3\left(-a\sqrt{1-\frac{1}{a^2x^2}}x(16+9ax-16a^2x^2+6a^3x^3)+15\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{24a}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x)^3,x]
```

output
```
(c^3*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x*(16 + 9*a*x - 16*a^2*x^2 + 6*a^3*x^3)) +
 15*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(24*a)
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6724, 27, 540, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^3e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6724}$$

$$ac\int c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2x^5d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$ac^3\int \sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2x^5d\frac{1}{x}$$

$$\Big\downarrow \text{540}$$

$$ac^3\left(-\frac{1}{4}\int \sqrt{1-\frac{1}{a^2x^2}}\left(8a-\frac{5}{x}\right)x^4d\frac{1}{x}-\frac{1}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{534}$$

$$ac^3\left(\frac{1}{4}\left(5\int \sqrt{1-\frac{1}{a^2x^2}}x^3d\frac{1}{x}+\frac{8}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{243}$$

$$ac^3\left(\frac{1}{4}\left(\frac{5}{2}\int \sqrt{1-\frac{1}{a^2x^2}}x^2d\frac{1}{x^2}+\frac{8}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{51}$$

$$ac^3\left(\frac{1}{4}\left(\frac{5}{2}\left(x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}\right)+\frac{8}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow 73$$

$$ac^3\left(\frac{1}{4}\left(\frac{5}{2}\left(\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-x\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{8}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)\right.$$

$$\Big\downarrow 221$$

$$ac^3\left(\frac{1}{4}\left(\frac{5}{2}\left(\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}-x\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{8}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{4}a^2x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

input
```
Int[E^ArcCoth[a*x]*(c - a*c*x)^3,x]
```

output
```
a*c^3*(-1/4*(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^4) + ((8*a*(1 - 1/(a^2*x^2))^(3
/2)*x^3)/3 + (5*(-(Sqrt[1 - 1/(a^2*x^2)]*x) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)
]]/a^2))/2)/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14

method	result	size
risch	$-\frac{(6a^3x^3-16a^2x^2+9ax+16)(ax-1)c^3}{24a\sqrt{\frac{ax-1}{ax+1}}}+\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	120
default	$\frac{(ax-1)c^3\left(-6\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax+16((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-15\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+15\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{24a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$	141

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*a^3*x^3-16*a^2*x^2+9*a*x+16)*(a*x-1)/a*c^3/((a*x-1)/(a*x+1))^(1/2
)+5/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^3/((a*x-1)/(a*
x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx$$
$$=\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(6\,a^4c^3x^4-10\,a^3c^3x^3-7\,a^2c^3x^2+25\,ac^3x+16\,c^3)\sqrt{}}{24\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
1/24*(15*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^3*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (6*a^4*c^3*x^4 - 10*a^3*c^3*x^3 - 7*a^2*c^3*x^2 + 25*a
*c^3*x + 16*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx = -c^3\left(\int \frac{3ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx + \int \left(-\frac{3a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx + \int \frac{a^3x^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx + \int \left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**3,x)
```

output
```
-c**3*(Integral(3*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-3*
a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**3*x**3/sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(a*x/(a*x + 1) - 1/(a*x
 + 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(89) = 178$.

Time = 0.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.10

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx$$
$$=\frac{1}{24}\left(\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{2\left(15\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+73\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-55\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\right.}{\frac{4\,(ax-1)a^2}{ax+1}-\frac{6\,(ax-1)^2a^2}{(ax+1)^2}+\frac{4\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-}{(ax}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
1/24*(15*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1)/a^2 + 2*(15*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 73*c^3
*((a*x - 1)/(a*x + 1))^(5/2) - 55*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^3
*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2
/(a*x + 1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4
 - a^2))*a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{5\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\mathrm{sgn}\,(ax+1)} - \frac{1}{24}\sqrt{a^2x^2-1}\left(\left(2\left(\frac{3\,a^2c^3x}{\mathrm{sgn}\,(ax+1)}-\frac{8\,ac^3}{\mathrm{sgn}\,(ax+1)}\right)x+\frac{9\,c^3}{\mathrm{sgn}\,(ax+1)}\right)x+\frac{16\,c^3}{a\mathrm{sgn}\,(ax+1)}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
-5/8*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) - 1
/24*sqrt(a^2*x^2 - 1)*((2*(3*a^2*c^3*x/sgn(a*x + 1) - 8*a*c^3/sgn(a*x + 1)
)*x + 9*c^3/sgn(a*x + 1))*x + 16*c^3/(a*sgn(a*x + 1)))
```

Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.69

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx = \frac{5\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a} - \frac{\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{55\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{73\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{12}+\frac{5\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a-\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{6\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}$$

input
```
int((c - a*c*x)^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(5*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a) - ((5*c^3*((a*x - 1)/(a*x
 + 1))^(1/2))/4 - (55*c^3*((a*x - 1)/(a*x + 1))^(3/2))/12 + (73*c^3*((a*x
- 1)/(a*x + 1))^(5/2))/12 + (5*c^3*((a*x - 1)/(a*x + 1))^(7/2))/4)/(a - (4
*a*(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*x + 1)^2 - (4*a*(a*x - 1)^3
)/(a*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx$$
$$=\frac{c^3\left(-6\sqrt{ax+1}\,\sqrt{ax-1}\,a^3x^3+16\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2-9\sqrt{ax+1}\,\sqrt{ax-1}\,ax-16\sqrt{ax+1}\,\sqrt{ax-\right.}{24a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x)
```

output
```
(c**3*( - 6*sqrt(a*x + 1)*sqrt(a*x - 1)*a**3*x**3 + 16*sqrt(a*x + 1)*sqrt(
a*x - 1)*a**2*x**2 - 9*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x - 16*sqrt(a*x + 1)*
sqrt(a*x - 1) + 30*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))))/(24*a)
```

3.167 $\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 + \frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

output
```
-1/2*a*c^2*(1-1/a^2/x^2)^(1/2)*x^2+1/3*a^2*c^2*(1-1/a^2/x^2)^(3/2)*x^3+1/2
*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-2-3ax+2a^2x^2)+3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x)^2,x]
```

output
```
(c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 - 3*a*x + 2*a^2*x^2) + 3*Log[a*(1 + Sq
rt[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6724, 25, 27, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2 e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6724}$$

$$ac\int -c\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^4d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-ac\int c\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^4d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-ac^2\int \sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^4d\frac{1}{x}$$

$$\Big\downarrow \text{534}$$

$$-ac^2\left(-\int \sqrt{1-\frac{1}{a^2x^2}}x^3d\frac{1}{x}-\frac{1}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{243}$$

$$-ac^2\left(-\frac{1}{2}\int \sqrt{1-\frac{1}{a^2x^2}}x^2d\frac{1}{x^2}-\frac{1}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{51}$$

$$-ac^2\left(\frac{1}{2}\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}+x\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \textbf{73}$$

$$-ac^2\left(\frac{1}{2}\left(x\sqrt{1-\frac{1}{a^2x^2}}-\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \textbf{221}$$

$$-ac^2\left(\frac{1}{2}\left(x\sqrt{1-\frac{1}{a^2x^2}}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}\right)-\frac{1}{3}ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

input
```
Int[E^ArcCoth[a*x]*(c - a*c*x)^2,x]
```

output
```
-(a*c^2*(-1/3*(a*(1 - 1/(a^2*x^2))^(3/2)*x^3) + (Sqrt[1 - 1/(a^2*x^2)]*x -
 ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a^2)/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

method	result	size
risch	$\frac{(2a^2x^2-3ax-2)(ax-1)c^2}{6a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$	112
default	$-\frac{(ax-1)c^2\left(3\sqrt{a^2}\sqrt{a^2x^2-1}\,ax-2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$	121

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*x^2-3*a*x-2)*(a*x-1)/a*c^2/((a*x-1)/(a*x+1))^(1/2)+1/2*ln(a^2*x
/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^2/((a*x-1)/(a*x+1))^(1/2)*((
a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.32

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(2\,a^3c^2x^3-a^2c^2x^2-5\,ac^2x-2\,c^2)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
1/6*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^2*log(sqrt((a*x - 1)/(
a*x + 1)) - 1) + (2*a^3*c^2*x^3 - a^2*c^2*x^2 - 5*a*c^2*x - 2*c^2)*sqrt((a
*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx = c^2\left(\int\left(-\frac{2ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**2,x)
```

output
```
c**2*(Integral(-2*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**
2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*x +
1) - 1/(a*x + 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(66) = 132$.

Time = 0.03 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.32

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx$$
$$=\frac{1}{6}a\left(\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+8\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
1/6*a*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x
- 1)/(a*x + 1)) - 1)/a^2 - 2*(3*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 8*c^2*((
a*x - 1)/(a*x + 1))^(3/2) - 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*
a^2/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^
3 - a^2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.26

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx$$
$$=\frac{1}{6}\sqrt{a^2x^2-1}\left(\left(\frac{2\,ac^2x}{\mathrm{sgn}\,(ax+1)}-\frac{3\,c^2}{\mathrm{sgn}\,(ax+1)}\right)x-\frac{2\,c^2}{a\mathrm{sgn}\,(ax+1)}\right)$$
$$-\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
1/6*sqrt(a^2*x^2 - 1)*((2*a*c^2*x/sgn(a*x + 1) - 3*c^2/sgn(a*x + 1))*x - 2
*c^2/(a*sgn(a*x + 1))) - 1/2*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(
abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.77

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{\frac{8\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}-c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a-\frac{3\,a\,(a\,x-1)}{a\,x+1}+\frac{3\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a\,(a\,x-1)^3}{(a\,x+1)^3}}+\frac{c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - a*c*x)^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((8*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 - c^2*((a*x - 1)/(a*x + 1))^(1/2) +
 c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a
*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (c^2*atanh(((a*x -
 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{c^2\left(2\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2-3\sqrt{ax+1}\,\sqrt{ax-1}\,ax-2\sqrt{ax+1}\,\sqrt{ax-1}+6\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{6a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x)
```

output
```
(c**2*(2*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 - 3*sqrt(a*x + 1)*sqrt(a*x
- 1)*a*x - 2*sqrt(a*x + 1)*sqrt(a*x - 1) + 6*log((sqrt(a*x - 1) + sqrt(a*x
 + 1))/sqrt(2))))/(6*a)
```

3.168 $\int e^{\coth^{-1}(ax)}(c-acx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{\operatorname{carctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

output
```
-1/2*a*c*(1-1/a^2/x^2)^(1/2)*x^2+1/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx=\frac{c\left(-a^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{2a}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x),x]
```

output
```
(c*(-(a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x
]))/(2*a)
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6724, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6724}$$

$$ac\int \sqrt{1-\frac{1}{a^2x^2}}x^3d\frac{1}{x}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}ac\int \sqrt{1-\frac{1}{a^2x^2}}x^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}ac\left(x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}ac\left(\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-x\sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}ac\left(\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}-x\sqrt{1-\frac{1}{a^2x^2}}\right)$$

input
```
Int[E^ArcCoth[a*x]*(c - a*c*x),x]
```

output
```
(a*c*(-(Sqrt[1 - 1/(a^2*x^2)]*x) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a^2))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(39) = 78$.

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.96

method	result	size
risch	$-\frac{x(ax-1)c}{2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	92
default	$-\frac{(ax-1)c\left(x\sqrt{a^2x^2-1}\,\sqrt{a^2}-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$	93

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(a*x-1)*c/((a*x-1)/(a*x+1))^(1/2)+1/2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2
-1)^(1/2))/(a^2)^(1/2)*c*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx = \frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2cx^2+acx)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="fricas")
```

output
```
1/2*(c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - c*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) - (a^2*c*x^2 + a*c*x)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx = -c\left(\int \frac{ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx + \int \left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c),x)
```

output
```
-c*(Integral(a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(39) = 78$.

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.81

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx = \frac{1}{2}a\left(\frac{2\left(c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="maxima")
```

output
```
1/2*a*(2*(c*((a*x - 1)/(a*x + 1))^(3/2) + c*sqrt((a*x - 1)/(a*x + 1)))/(2*
(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + c*log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx=-\frac{\sqrt{a^2x^2-1}cx}{2\,\mathrm{sgn}\,(ax+1)}-\frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="giac")
```

output
```
-1/2*sqrt(a^2*x^2 - 1)*c*x/sgn(a*x + 1) - 1/2*c*log(abs(-x*abs(a) + sqrt(a
^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.00

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx=\frac{c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{c\,\sqrt{\frac{a\,x-1}{a\,x+1}}+c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{2\,a\,(a\,x-1)}{a\,x+1}+\frac{a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}}$$

input
```
int((c - a*c*x)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (c*((a*x - 1)/(a*x + 1))^(1/2)
+ c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (2*a*(a*x - 1))/(a*x + 1) + (a*(a*x
- 1)^2)/(a*x + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx=\frac{c\left(-\sqrt{ax+1}\,\sqrt{ax-1}\,ax+2\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{2a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x)
```

output
```
(c*( - sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 2*log((sqrt(a*x - 1) + sqrt(a*x +
 1))/sqrt(2))))/(2*a)
```

3.169 $\int \frac{e^{\coth^{-1}(ax)}}{c-acx} dx$

Optimal result

Integrand size = 16, antiderivative size = 52

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx} dx = \frac{2\sqrt{1-\frac{1}{a^2x^2}}}{c\left(a-\frac{1}{x}\right)} - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
2*(1-1/a^2/x^2)^(1/2)/c/(a-1/x)-arctanh((1-1/a^2/x^2)^(1/2))/a/c
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx} dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}x + (1-ax)\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac(-1+ax)}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a*c*x),x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*x + (1 - a*x)*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])
*x])/(a*c*(-1 + a*x))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6724, 27, 564, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}dx$$

$$\Big\downarrow \text{6724}$$

$$ac\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^2\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{564}$$

$$\frac{a\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^2}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}\right)}{c}$$

$$\Big\downarrow \text{243}$$

$$\frac{a\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}\right)}{c}$$

$$\Big\downarrow \text{73}$$

$$\frac{a\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}-\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{a\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}\right)}{c}$$

input
```
Int[E^ArcCoth[a*x]/(c - a*c*x),x]
```

output
```
(a*((2*Sqrt[1 - 1/(a^2*x^2)])/(a*(a - x^(-1))) - ArcTanh[Sqrt[1 - 1/(a^2*x
^2)]]/a^2))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(48) = 96$.

Time = 0.13 (sec) , antiderivative size = 249, normalized size of antiderivative = 4.79

method	result
default	$\frac{-\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^2x^2-\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}+2\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,ax+2\ln\left(\frac{a^2x}{}\right.}{a\sqrt{a^2}\,(ax-1)c\sqrt{(ax-1)(ax+1)}\,\sqrt{\frac{ax-1}{ax+1}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*a^2*x^2-ln((a^2*x+((a*x-1)*(a*x+
1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^3*x^2+((a*x-1)*(a*x+1))^(3/2)*(a^2)^
(1/2)+2*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*a*x+2*ln((a^2*x+((a*x-1)*(a*x+
1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x-((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1
/2)-a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/(a^2)^(
1/2)/(a*x-1)/c/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.67

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-2\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="fricas")
```

output
```
-((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) - 2*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c*x - a*
c)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{1}{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c),x)
```

output
```
-Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) -
1/(a*x + 1))), x)/c
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.50

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = -a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{2}{a^2c\sqrt{\frac{ax-1}{ax+1}}}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
 1)) - 1)/(a^2*c) - 2/(a^2*c*sqrt((a*x - 1)/(a*x + 1))))
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = \int -\frac{1}{(acx-c)\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = \frac{2}{a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}} - \frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

input
```
int(1/((c - a*c*x)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
2/(a*c*((a*x - 1)/(a*x + 1))^(1/2)) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)
))/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = \frac{-2\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)+2\sqrt{ax-1}+2\sqrt{ax+1}}{\sqrt{ax-1}\,ac}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x)
```

output
```
(2*( - sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + sqrt(a
*x - 1) + sqrt(a*x + 1)))/(sqrt(a*x - 1)*a*c)
```

3.170 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx=-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c^2\left(a-\frac{1}{x}\right)^3}$$

output
```
-1/3*a^2*(1-1/a^2/x^2)^(3/2)/c^2/(a-1/x)^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}x(1+ax)}{3c^2(-1+ax)^2}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a*c*x)^2,x]
```

output
```
-1/3*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x))/(c^2*(-1 + a*x)^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6724, 25, 27, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}dx$$

$$\Big\downarrow \text{6724}$$

$$ac\int -\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^3\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-ac\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^3\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{460}$$

$$-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c^2\left(a-\frac{1}{x}\right)^3}$$

input
```
Int[E^ArcCoth[a*x]/(c - a*c*x)^2,x]
```

output
```
-1/3*(a^2*(1 - 1/(a^2*x^2))^(3/2))/(c^2*(a - x^(-1))^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{ax+1}{3(ax-1)c^{2}\sqrt{\frac{ax-1}{ax+1}}\,a}$	36
default	$-\frac{ax+1}{3(ax-1)c^{2}\sqrt{\frac{ax-1}{ax+1}}\,a}$	36
trager	$-\frac{\left(ax+1\right)^{2}\sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^{2}\left(ax-1\right)^{2}}$	40
orering	$-\frac{(ax-1)(ax+1)}{3a\sqrt{\frac{ax-1}{ax+1}}\,\left(-acx+c\right)^{2}}$	40

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a*x+1)/(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{(a^2x^2+2\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
-1/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 - 2*a^2*
c^2*x + a*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{1}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**2,x)
```

output
```
Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a*x*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{3\,ac^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
-1/3/(a*c^2*((a*x - 1)/(a*x + 1))^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^3ac^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
-2/3*(3*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1
)^3*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{3\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

input
```
int(1/((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
-1/(3*a*c^2*((a*x - 1)/(a*x + 1))^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{-\sqrt{ax-1}\,ax+\sqrt{ax-1}-\sqrt{ax+1}\,ax-\sqrt{ax+1}}{3\sqrt{ax-1}\,a\,c^2\,(ax-1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x)
```

output
```
( - sqrt(a*x - 1)*a*x + sqrt(a*x - 1) - sqrt(a*x + 1)*a*x - sqrt(a*x + 1))
/(3*sqrt(a*x - 1)*a*c**2*(a*x - 1))
```

3.171 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx=\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4}-\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15c^3\left(a-\frac{1}{x}\right)^3}$$

output
```
1/5*a^3*(1-1/a^2/x^2)^(3/2)/c^3/(a-1/x)^4-4/15*a^2*(1-1/a^2/x^2)^(3/2)/c^3
/(a-1/x)^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}x(-4-3ax+a^2x^2)}{15c^3(-1+ax)^3}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a*c*x)^3,x]
```

output
```
-1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(-4 - 3*a*x + a^2*x^2))/(c^3*(-1 + a*x)^3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6724, 27, 571, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3} \, dx$$

$$\Big\downarrow \text{6724}$$

$$ac \int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^4 \left(a-\frac{1}{x}\right)^4 x} d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{a \int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^4 x} d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{571}$$

$$\frac{a \left(\frac{a^2 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\left(a-\frac{1}{x}\right)^4} - \frac{4}{5} \int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^3} d\frac{1}{x}\right)}{c^3}$$

$$\Big\downarrow \text{460}$$

$$\frac{a \left(\frac{a^2 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\left(a-\frac{1}{x}\right)^4} - \frac{4a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(a-\frac{1}{x}\right)^3}\right)}{c^3}$$

input
```
Int[E^ArcCoth[a*x]/(c - a*c*x)^3,x]
```

output
```
(a*((a^2*(1 - 1/(a^2*x^2))^(3/2))/(5*(a - x^(-1))^4) - (4*a*(1 - 1/(a^2*x^
2))^(3/2))/(15*(a - x^(-1))^3)))/c^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 571
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*(n + p + 1))), x] + Simp[n/(2*d*
(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ((LtQ[n, -1] &&  !IGtQ[n + p +
1, 0]) || (LtQ[n, 0] && LtQ[p, -1]) || EqQ[n + 2*p + 2, 0]) && NeQ[n + p +
1, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{(ax-4)(ax+1)}{15(ax-1)^{2} c^{3} \sqrt{\frac{ax-1}{ax+1}}\, a}$	41
default	$-\frac{(ax-4)(ax+1)}{15(ax-1)^{2} c^{3} \sqrt{\frac{ax-1}{ax+1}}\, a}$	41
orering	$\frac{(ax-4)(ax-1)(ax+1)}{15a\sqrt{\frac{ax-1}{ax+1}}\,(-acx+c)^{3}}$	45
trager	$-\frac{(ax+1)\left(a^{2}x^{2}-3ax-4\right)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^{3}(ax-1)^{3}}$	51

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(a*x-4)*(a*x+1)/(a*x-1)^2/c^3/((a*x-1)/(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{(a^3x^3-2\,a^2x^2-7\,ax-4)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
-1/15*(a^3*x^3 - 2*a^2*x^2 - 7*a*x - 4)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^3
*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \frac{1}{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**3,x)
```

output
```
-Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**2*x**2*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) -
 sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{5\,(ax-1)}{ax+1}-3}{30\,ac^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
-1/30*(5*(a*x - 1)/(a*x + 1) - 3)/(a*c^3*((a*x - 1)/(a*x + 1))^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.27

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{2\left(15\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3+5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^5ac^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
2/15*(15*(a + sqrt(a^2 - 1/x^2))^3*x^3 + 5*(a + sqrt(a^2 - 1/x^2))^2*x^2 +
 5*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^3
)
```

Mupad [B] (verification not implemented)

Time = 13.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{a\,x-1}{3\,(a\,x+1)}-\frac{1}{5}}{2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(1/((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
-((a*x - 1)/(3*(a*x + 1)) - 1/5)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.39

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\sqrt{ax-1}\,a^2x^2-2\sqrt{ax-1}\,ax+\sqrt{ax-1}-\sqrt{ax+1}\,a^2x^2+3\sqrt{ax+1}\,ax+4\sqrt{ax+1}}{15\sqrt{ax-1}\,a\,c^3\,(a^2x^2-2ax+1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x)
```

output
```
(sqrt(a*x - 1)*a**2*x**2 - 2*sqrt(a*x - 1)*a*x + sqrt(a*x - 1) - sqrt(a*x
+ 1)*a**2*x**2 + 3*sqrt(a*x + 1)*a*x + 4*sqrt(a*x + 1))/(15*sqrt(a*x - 1)*
a*c**3*(a**2*x**2 - 2*a*x + 1))
```

3.172 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5}+\frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4}-\frac{23a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^4\left(a-\frac{1}{x}\right)^3}$$

output
```
-1/7*a^4*(1-1/a^2/x^2)^(3/2)/c^4/(a-1/x)^5+12/35*a^3*(1-1/a^2/x^2)^(3/2)/c
^4/(a-1/x)^4-23/105*a^2*(1-1/a^2/x^2)^(3/2)/c^4/(a-1/x)^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.51

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(23+13ax-8a^2x^2+2a^3x^3)}{105c^4(-1+ax)^4}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a*c*x)^4,x]
```

output
```
-1/105*(Sqrt[1 - 1/(a^2*x^2)]*x*(23 + 13*a*x - 8*a^2*x^2 + 2*a^3*x^3))/(c^
4*(-1 + a*x)^4)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6724, 25, 27, 581, 25, 27, 671, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}dx$$

$$\Big\downarrow \text{6724}$$

$$ac\int -\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^5\left(a-\frac{1}{x}\right)^5x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-ac\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^5\left(a-\frac{1}{x}\right)^5x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^5x^2}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{581}$$

$$-\frac{a\left(-\int -\frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(4a-\frac{3}{x}\right)}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}\right)}{c^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\left(\int \frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(4a-\frac{3}{x}\right)}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}\right)}{c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(4a-\frac{3}{x}\right)}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}\right)}{c^4}$$

$$\Big\downarrow \text{671}$$

$$-\frac{a\left(a\left(\frac{23}{7}\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(a-\frac{1}{x}\right)^5}\right)-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}\right)}{c^4}$$

$$\Big\downarrow \text{461}$$

$$-\frac{a\left(a\left(\frac{23}{7}\left(\frac{\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{5a}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\left(a-\frac{1}{x}\right)^4}\right)+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(a-\frac{1}{x}\right)^5}\right)-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}\right)}{c^4}$$

$$\Big\downarrow \text{460}$$

$$-\frac{a\left(a\left(\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(a-\frac{1}{x}\right)^5}+\frac{23}{7}\left(\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(a-\frac{1}{x}\right)^3}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\left(a-\frac{1}{x}\right)^4}\right)\right)-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}\right)}{c^4}$$

input
```
Int[E^ArcCoth[a*x]/(c - a*c*x)^4,x]
```

output
```
-((a*(a*((23*((a*(1 - 1/(a^2*x^2))^(3/2))/(5*(a - x^(-1))^4) + (1 - 1/(a^2
*x^2))^(3/2)/(15*(a - x^(-1))^3)))/7 + (a^2*(1 - 1/(a^2*x^2))^(3/2))/(7*(a
 - x^(-1))^5)) - (a^2*(1 - 1/(a^2*x^2))^(3/2))/(a - x^(-1))^4))/c^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{(2a^2x^2-10ax+23)(ax+1)}{105(ax-1)^3c^4\sqrt{\frac{ax-1}{ax+1}}\,a}$	50
default	$-\frac{(2a^2x^2-10ax+23)(ax+1)}{105(ax-1)^3c^4\sqrt{\frac{ax-1}{ax+1}}\,a}$	50
orering	$-\frac{(2a^2x^2-10ax+23)(ax-1)(ax+1)}{105a\sqrt{\frac{ax-1}{ax+1}}\,(-acx+c)^4}$	54
trager	$-\frac{(ax+1)(2a^3x^3-8a^2x^2+13ax+23)\sqrt{-\frac{-ax+1}{ax+1}}}{105a\,c^4(ax-1)^4}$	60

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/105*(2*a^2*x^2-10*a*x+23)*(a*x+1)/(a*x-1)^3/c^4/((a*x-1)/(a*x+1))^(1/2)
/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{(2\,a^4x^4-6\,a^3x^3+5\,a^2x^2+36\,ax+23)\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
-1/105*(2*a^4*x^4 - 6*a^3*x^3 + 5*a^2*x^2 + 36*a*x + 23)*sqrt((a*x - 1)/(a
*x + 1))/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^
4)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{1}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**4,x)
```

output
```
Integral(1/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**3*x**3*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x +
1)) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))), x)/c**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.55

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\frac{42\,(ax-1)}{ax+1} - \frac{35\,(ax-1)^2}{(ax+1)^2} - 15}{420\,ac^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
1/420*(42*(a*x - 1)/(a*x + 1) - 35*(a*x - 1)^2/(a*x + 1)^2 - 15)/(a*c^4*((
a*x - 1)/(a*x + 1))^(7/2))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{4\left(70\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4x^4+35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3+21\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2-7\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x\right.}{105\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^7ac^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
-4/105*(70*(a + sqrt(a^2 - 1/x^2))^4*x^4 + 35*(a + sqrt(a^2 - 1/x^2))^3*x^
3 + 21*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 7*(a + sqrt(a^2 - 1/x^2))*x + 1)/((
(a + sqrt(a^2 - 1/x^2))*x - 1)^7*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 13.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.56

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\frac{(a\,x-1)^2}{3\,(a\,x+1)^2}-\frac{2\,(a\,x-1)}{5\,(a\,x+1)}+\frac{1}{7}}{4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

input
```
int(1/((c - a*c*x)^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
-((a*x - 1)^2/(3*(a*x + 1)^2) - (2*(a*x - 1))/(5*(a*x + 1)) + 1/7)/(4*a*c^
4*((a*x - 1)/(a*x + 1))^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.32

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{2\sqrt{ax-1}\,a^3x^3-6\sqrt{ax-1}\,a^2x^2+6\sqrt{ax-1}\,ax-2\sqrt{ax-1}-2\sqrt{ax+1}\,a^3x^3+8\sqrt{ax+1}\,a^2x^2-13\sqrt{}}{105\sqrt{ax-1}\,a\,c^4\left(a^3x^3-3a^2x^2+3ax-1\right)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x)
```

output
```
(2*sqrt(a*x - 1)*a**3*x**3 - 6*sqrt(a*x - 1)*a**2*x**2 + 6*sqrt(a*x - 1)*a
*x - 2*sqrt(a*x - 1) - 2*sqrt(a*x + 1)*a**3*x**3 + 8*sqrt(a*x + 1)*a**2*x*
*2 - 13*sqrt(a*x + 1)*a*x - 23*sqrt(a*x + 1))/(105*sqrt(a*x - 1)*a*c**4*(a
**3*x**3 - 3*a**2*x**2 + 3*a*x - 1))
```

3.173 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5} dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5} dx = \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} - \frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5} + \frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4} - \frac{58a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315c^5\left(a-\frac{1}{x}\right)^3}$$

output
```
1/9*a^5*(1-1/a^2/x^2)^(3/2)/c^5/(a-1/x)^6-8/21*a^4*(1-1/a^2/x^2)^(3/2)/c^5
/(a-1/x)^5+47/105*a^3*(1-1/a^2/x^2)^(3/2)/c^5/(a-1/x)^4-58/315*a^2*(1-1/a^
2/x^2)^(3/2)/c^5/(a-1/x)^3
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.44

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5} dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-58-25ax+21a^2x^2-10a^3x^3+2a^4x^4)}{315c^5(-1+ax)^5}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a*c*x)^5,x]
```

output
```
-1/315*(Sqrt[1 - 1/(a^2*x^2)]*x*(-58 - 25*a*x + 21*a^2*x^2 - 10*a^3*x^3 +
2*a^4*x^4))/(c^5*(-1 + a*x)^5)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.44, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6724, 27, 581, 25, 2170, 27, 671, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx$$

$$\Big\downarrow \ 6724$$

$$ac\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^6\left(a-\frac{1}{x}\right)^6x^3}d\frac{1}{x}$$

$$\Big\downarrow \ 27$$

$$\frac{a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^6x^3}d\frac{1}{x}}{c^5}$$

$$\Big\downarrow \ 581$$

$$\frac{a\left(\int -\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(4a^3-\frac{7a^2}{x}+\frac{2a}{x^2}\right)}{\left(a-\frac{1}{x}\right)^6}d\frac{1}{x}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}\right)}{c^5}$$

$$\Big\downarrow \ 25$$

$$\frac{a\left(\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}-\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(4a^3-\frac{7a^2}{x}+\frac{2a}{x^2}\right)}{\left(a-\frac{1}{x}\right)^6}d\frac{1}{x}\right)}{c^5}$$

$$\Big\downarrow \ 2170$$

$$\frac{a\left(-\frac{1}{2}a^2\int \frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(9a-\frac{10}{x}\right)}{\left(a-\frac{1}{x}\right)^6}d\frac{1}{x}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}\right)}{c^5}$$

$$\Big\downarrow \; 27$$

$$\frac{a\left(-a^2\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(9a-\frac{10}{x}\right)}{\left(a-\frac{1}{x}\right)^6}d\frac{1}{x}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}\right)}{c^5}$$

$$\Big\downarrow \; 671$$

$$\frac{a\left(-a^2\left(\frac{29}{3}\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(a-\frac{1}{x}\right)^6}\right)+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}\right)}{c^5}$$

$$\Big\downarrow \; 461$$

$$\frac{a\left(-a^2\left(\frac{29}{3}\left(\frac{2\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{7a}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(a-\frac{1}{x}\right)^5}\right)-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(a-\frac{1}{x}\right)^6}\right)+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}\right)}{c^5}$$

$$\Big\downarrow \; 461$$

$$\frac{a\left(-a^2\left(\frac{29}{3}\left(\frac{2\left(\frac{\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{5a}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\left(a-\frac{1}{x}\right)^4}\right)}{7a}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(a-\frac{1}{x}\right)^5}\right)-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(a-\frac{1}{x}\right)^6}\right)+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}\right)}{c^5}$$

$$\Big\downarrow \; 460$$

$$\frac{a\left(-a^2\left(\frac{29}{3}\left(\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(a-\frac{1}{x}\right)^5}+\frac{2\left(\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(a-\frac{1}{x}\right)^3}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\left(a-\frac{1}{x}\right)^4}\right)}{7a}\right)-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(a-\frac{1}{x}\right)^6}\right)+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}\right)}{c^5}$$

input
```
Int[E^ArcCoth[a*x]/(c - a*c*x)^5,x]
```

output
```
(a*(-(a^2*((29*((2*((a*(1 - 1/(a^2*x^2))^(3/2))/(5*(a - x^(-1))^4) + (1 -
1/(a^2*x^2))^(3/2)/(15*(a - x^(-1))^3)))/(7*a) + (a*(1 - 1/(a^2*x^2))^(3/2
))/(7*(a - x^(-1))^5)))/3 - (a^2*(1 - 1/(a^2*x^2))^(3/2))/(9*(a - x^(-1))^
6))) + (a^3*(1 - 1/(a^2*x^2))^(3/2))/(a - x^(-1))^5 + (a^2*(1 - 1/(a^2*x^2
))^(3/2))/(a - x^(-1))^4))/c^5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 2170
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{(2a^3x^3-12a^2x^2+33ax-58)(ax+1)}{315(ax-1)^4c^5\sqrt{\frac{ax-1}{ax+1}}\,a}$	58
default	$-\frac{(2a^3x^3-12a^2x^2+33ax-58)(ax+1)}{315(ax-1)^4c^5\sqrt{\frac{ax-1}{ax+1}}\,a}$	58
orering	$\frac{(2a^3x^3-12a^2x^2+33ax-58)(ax-1)(ax+1)}{315a\sqrt{\frac{ax-1}{ax+1}}\,(-acx+c)^5}$	62
trager	$-\frac{(ax+1)(2a^4x^4-10a^3x^3+21a^2x^2-25ax-58)\sqrt{-\frac{-ax+1}{ax+1}}}{315a\,c^5(ax-1)^5}$	68

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(2*a^3*x^3-12*a^2*x^2+33*a*x-58)*(a*x+1)/(a*x-1)^4/c^5/((a*x-1)/(a*
x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{(2\,a^5x^5-8\,a^4x^4+11\,a^3x^3-4\,a^2x^2-83\,ax-58)\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^6c^5x^5-5\,a^5c^5x^4+10\,a^4c^5x^3-10\,a^3c^5x^2+5\,a^2c^5x-ac^5)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

output
```
-1/315*(2*a^5*x^5 - 8*a^4*x^4 + 11*a^3*x^3 - 4*a^2*x^2 - 83*a*x - 58)*sqrt
((a*x - 1)/(a*x + 1))/(a^6*c^5*x^5 - 5*a^5*c^5*x^4 + 10*a^4*c^5*x^3 - 10*a
^3*c^5*x^2 + 5*a^2*c^5*x - a*c^5)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{1}{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-5a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+10a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-10a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+5ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**5,x)
```

output
```
-Integral(1/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 5*a**4*x**4*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1)) + 10*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1)) - 10*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 5*a*x*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\frac{135\,(ax-1)}{ax+1} - \frac{189\,(ax-1)^2}{(ax+1)^2} + \frac{105\,(ax-1)^3}{(ax+1)^3} - 35}{2520\,ac^5\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

output
```
-1/2520*(135*(a*x - 1)/(a*x + 1) - 189*(a*x - 1)^2/(a*x + 1)^2 + 105*(a*x
- 1)^3/(a*x + 1)^3 - 35)/(a*c^5*((a*x - 1)/(a*x + 1))^(9/2))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.94

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx$$
$$= \frac{4\left(315\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^5 x^5 + 189\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4 x^4 + 84\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3 x^3 - 36\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2\right.}{315\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^9 ac^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

output
```
4/315*(315*(a + sqrt(a^2 - 1/x^2))^5*x^5 + 189*(a + sqrt(a^2 - 1/x^2))^4*x
^4 + 84*(a + sqrt(a^2 - 1/x^2))^3*x^3 - 36*(a + sqrt(a^2 - 1/x^2))^2*x^2 +
 9*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^9*a*c^5
)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.54

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\frac{3\,(a\,x-1)^2}{5\,(a\,x+1)^2} - \frac{(a\,x-1)^3}{3\,(a\,x+1)^3} - \frac{3\,(a\,x-1)}{7\,(a\,x+1)} + \frac{1}{9}}{8\,a\,c^5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

input
```
int(1/((c - a*c*x)^5*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
((3*(a*x - 1)^2)/(5*(a*x + 1)^2) - (a*x - 1)^3/(3*(a*x + 1)^3) - (3*(a*x -
 1))/(7*(a*x + 1)) + 1/9)/(8*a*c^5*((a*x - 1)/(a*x + 1))^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.26

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx$$
$$= \frac{2\sqrt{ax-1}\,a^4x^4 - 8\sqrt{ax-1}\,a^3x^3 + 12\sqrt{ax-1}\,a^2x^2 - 8\sqrt{ax-1}\,ax + 2\sqrt{ax-1} - 2\sqrt{ax+1}\,a^4x^4 + 10}{315\sqrt{ax-1}\,a\,c^5\,(a^4x^4 - 4a^3x^3 + 6a^2x^2 - 4ax}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x)
```

output
```
(2*sqrt(a*x - 1)*a**4*x**4 - 8*sqrt(a*x - 1)*a**3*x**3 + 12*sqrt(a*x - 1)*
a**2*x**2 - 8*sqrt(a*x - 1)*a*x + 2*sqrt(a*x - 1) - 2*sqrt(a*x + 1)*a**4*x
**4 + 10*sqrt(a*x + 1)*a**3*x**3 - 21*sqrt(a*x + 1)*a**2*x**2 + 25*sqrt(a*
x + 1)*a*x + 58*sqrt(a*x + 1))/(315*sqrt(a*x - 1)*a*c**5*(a**4*x**4 - 4*a*
*3*x**3 + 6*a**2*x**2 - 4*a*x + 1))
```

3.174 $\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=\frac{2c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}$$

output
```
2/5*c^5*(-a*x+1)^5/a-1/6*c^5*(-a*x+1)^6/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{c^5(-1+ax)^5(7+5ax)}{30a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

output
```
-1/30*(c^5*(-1 + a*x)^5*(7 + 5*a*x))/a
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^5 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int c^5 e^{2\text{arctanh}(ax)}(1-ax)^5dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^5\int e^{2\text{arctanh}(ax)}(1-ax)^5dx$$

$$\Big\downarrow \text{ 6679}$$

$$-c^5\int (1-ax)^4(ax+1)dx$$

$$\Big\downarrow \text{ 49}$$

$$-c^5\int \left(2(1-ax)^4-(1-ax)^5\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-c^5\left(\frac{(1-ax)^6}{6a}-\frac{2(1-ax)^5}{5a}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

output
```
-(c^5*((-2*(1 - a*x)^5)/(5*a) + (1 - a*x)^6/(6*a)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

method	result
gosper	$-\frac{x\left(5a^5x^5-18a^4x^4+15a^3x^3+20a^2x^2-45ax+30\right)c^5}{30}$
default	$c^5\left(-\frac{1}{6}a^5x^6+\frac{3}{5}x^5a^4-\frac{1}{2}a^3x^4-\frac{2}{3}a^2x^3+\frac{3}{2}a\,x^2-x\right)$
orering	$\frac{x\left(5a^5x^5-18a^4x^4+15a^3x^3+20a^2x^2-45ax+30\right)\left(-acx+c\right)^5}{30\left(ax-1\right)^5}$
norman	$-c^5x-\frac{2}{3}a^2c^5x^3-\frac{1}{2}a^3c^5x^4+\frac{3}{5}a^4c^5x^5-\frac{1}{6}a^5c^5x^6+\frac{3}{2}c^5a\,x^2$
risch	$-c^5x-\frac{2}{3}a^2c^5x^3-\frac{1}{2}a^3c^5x^4+\frac{3}{5}a^4c^5x^5-\frac{1}{6}a^5c^5x^6+\frac{3}{2}c^5a\,x^2$
parallelrisch	$-c^5x-\frac{2}{3}a^2c^5x^3-\frac{1}{2}a^3c^5x^4+\frac{3}{5}a^4c^5x^5-\frac{1}{6}a^5c^5x^6+\frac{3}{2}c^5a\,x^2$
meijerg	$-\frac{c^5\left(\frac{ax\left(70a^5x^5+84a^4x^4+105a^3x^3+140a^2x^2+210ax+420\right)}{420}+\ln\left(-ax+1\right)\right)}{a}-\frac{4c^5\left(-\frac{xa\left(12a^4x^4+15a^3x^3+20a^2x^2+30ax+60\right)}{60}-\text{l}\right.}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/30*x*(5*a^5*x^5-18*a^4*x^4+15*a^3*x^3+20*a^2*x^2-45*a*x+30)*c^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{1}{6}\,a^5c^5x^6+\frac{3}{5}\,a^4c^5x^5-\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3+\frac{3}{2}\,ac^5x^2-c^5x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="fricas")
```

output
```
-1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3
/2*a*c^5*x^2 - c^5*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(27) = 54.

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.78

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{a^5c^5x^6}{6}+\frac{3a^4c^5x^5}{5}-\frac{a^3c^5x^4}{2}-\frac{2a^2c^5x^3}{3}+\frac{3ac^5x^2}{2}-c^5x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**5,x)
```

output
```
-a**5*c**5*x**6/6 + 3*a**4*c**5*x**5/5 - a**3*c**5*x**4/2 - 2*a**2*c**5*x*
*3/3 + 3*a*c**5*x**2/2 - c**5*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{1}{6}\,a^5c^5x^6+\frac{3}{5}\,a^4c^5x^5-\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3+\frac{3}{2}\,ac^5x^2-c^5x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="maxima")
```

output
```
-1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3
/2*a*c^5*x^2 - c^5*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{1}{6}\,a^5c^5x^6+\frac{3}{5}\,a^4c^5x^5-\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3+\frac{3}{2}\,ac^5x^2-c^5x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="giac")
```

output
```
-1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3
/2*a*c^5*x^2 - c^5*x
```

Mupad [B] (verification not implemented)

Time = 13.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{a^5\,c^5\,x^6}{6}+\frac{3\,a^4\,c^5\,x^5}{5}-\frac{a^3\,c^5\,x^4}{2}-\frac{2\,a^2\,c^5\,x^3}{3}+\frac{3\,a\,c^5\,x^2}{2}-c^5\,x$$

input
```
int(((c - a*c*x)^5*(a*x + 1))/(a*x - 1),x)
```

output
```
(3*a*c^5*x^2)/2 - c^5*x - (2*a^2*c^5*x^3)/3 - (a^3*c^5*x^4)/2 + (3*a^4*c^5
*x^5)/5 - (a^5*c^5*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx=\frac{c^5x(-5a^5x^5+18a^4x^4-15a^3x^3-20a^2x^2+45ax-30)}{30}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x)
```

output
```
(c**5*x*( - 5*a**5*x**5 + 18*a**4*x**4 - 15*a**3*x**3 - 20*a**2*x**2 + 45*
a*x - 30))/30
```

3.175 $\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}$$

output
```
1/2*c^4*(-a*x+1)^4/a-1/5*c^4*(-a*x+1)^5/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{1}{10}c^4x\left(-10+10ax-5a^3x^3+2a^4x^4\right)$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

output
```
(c^4*x*(-10 + 10*a*x - 5*a^3*x^3 + 2*a^4*x^4))/10
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^4e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int c^4e^{2\text{arctanh}(ax)}(1-ax)^4dx$$

$$\Big\downarrow \text{27}$$

$$-c^4\int e^{2\text{arctanh}(ax)}(1-ax)^4dx$$

$$\Big\downarrow \text{6679}$$

$$-c^4\int (1-ax)^3(ax+1)dx$$

$$\Big\downarrow \text{49}$$

$$-c^4\int \left(2(1-ax)^3-(1-ax)^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c^4\left(\frac{(1-ax)^5}{5a}-\frac{(1-ax)^4}{2a}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

output
```
-(c^4*(-1/2*(1 - a*x)^4/a + (1 - a*x)^5/(5*a)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result
gosper	$\frac{x\left(2a^4x^4-5a^3x^3+10ax-10\right)c^4}{10}$
default	$c^4\left(\frac{1}{5}x^5a^4-\frac{1}{2}a^3x^4+a\,x^2-x\right)$
norman	$a\,c^4x^2-c^4x-\frac{1}{2}a^3c^4x^4+\frac{1}{5}a^4c^4x^5$
risch	$a\,c^4x^2-c^4x-\frac{1}{2}a^3c^4x^4+\frac{1}{5}a^4c^4x^5$
parallelrisch	$a\,c^4x^2-c^4x-\frac{1}{2}a^3c^4x^4+\frac{1}{5}a^4c^4x^5$
orering	$\frac{x\left(2a^4x^4-5a^3x^3+10ax-10\right)\left(-acx+c\right)^4}{10\left(ax-1\right)^4}$
meijerg	$-\frac{c^4\left(-\frac{xa\left(12a^4x^4+15a^3x^3+20a^2x^2+30ax+60\right)}{60}-\ln\left(-ax+1\right)\right)}{a}-\frac{3c^4\left(\frac{ax\left(15a^3x^3+20a^2x^2+30ax+60\right)}{60}+\ln\left(-ax+1\right)\right)}{a}-\frac{2c^4}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/10*x*(2*a^4*x^4-5*a^3*x^3+10*a*x-10)*c^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{1}{5}\,a^4c^4x^5 - \frac{1}{2}\,a^3c^4x^4 + ac^4x^2 - c^4x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{a^4c^4x^5}{5} - \frac{a^3c^4x^4}{2} + ac^4x^2 - c^4x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**4,x)
```

output
```
a**4*c**4*x**5/5 - a**3*c**4*x**4/2 + a*c**4*x**2 - c**4*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{1}{5}a^4c^4x^5 - \frac{1}{2}a^3c^4x^4 + ac^4x^2 - c^4x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{1}{5}a^4c^4x^5 - \frac{1}{2}a^3c^4x^4 + ac^4x^2 - c^4x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{a^4\,c^4\,x^5}{5} - \frac{a^3\,c^4\,x^4}{2} + a\,c^4\,x^2 - c^4\,x$$

input
```
int(((c - a*c*x)^4*(a*x + 1))/(a*x - 1),x)
```

output
```
a*c^4*x^2 - c^4*x - (a^3*c^4*x^4)/2 + (a^4*c^4*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{c^4x(2a^4x^4-5a^3x^3+10ax-10)}{10}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x)
```

output
```
(c**4*x*(2*a**4*x**4 - 5*a**3*x**3 + 10*a*x - 10))/10
```

3.176 $\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}$$

output
```
2/3*c^3*(-a*x+1)^3/a-1/4*c^3*(-a*x+1)^4/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{1}{12}c^3x\left(12-6ax-4a^2x^2+3a^3x^3\right)$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

output
```
-1/12*(c^3*x*(12 - 6*a*x - 4*a^2*x^2 + 3*a^3*x^3))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^3 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int c^3 e^{2\text{arctanh}(ax)}(1-ax)^3dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^3\int e^{2\text{arctanh}(ax)}(1-ax)^3dx$$

$$\Big\downarrow \text{ 6679}$$

$$-c^3\int (1-ax)^2(ax+1)dx$$

$$\Big\downarrow \text{ 49}$$

$$-c^3\int \left(2(1-ax)^2-(1-ax)^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-c^3\left(\frac{(1-ax)^4}{4a}-\frac{2(1-ax)^3}{3a}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

output
```
-(c^3*((-2*(1 - a*x)^3)/(3*a) + (1 - a*x)^4/(4*a)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result
gosper	$-\frac{x\left(3a^{3}x^{3}-4a^{2}x^{2}-6ax+12\right)c^{3}}{12}$
default	$c^{3}\left(-\frac{1}{4}a^{3}x^{4}+\frac{1}{3}a^{2}x^{3}+\frac{1}{2}a\,x^{2}-x\right)$
norman	$-c^{3}x+\frac{1}{2}a\,c^{3}x^{2}+\frac{1}{3}a^{2}c^{3}x^{3}-\frac{1}{4}a^{3}c^{3}x^{4}$
risch	$-c^{3}x+\frac{1}{2}a\,c^{3}x^{2}+\frac{1}{3}a^{2}c^{3}x^{3}-\frac{1}{4}a^{3}c^{3}x^{4}$
parallelrisch	$-c^{3}x+\frac{1}{2}a\,c^{3}x^{2}+\frac{1}{3}a^{2}c^{3}x^{3}-\frac{1}{4}a^{3}c^{3}x^{4}$
orering	$\frac{x\left(3a^{3}x^{3}-4a^{2}x^{2}-6ax+12\right)\left(-acx+c\right)^{3}}{12\left(ax-1\right)^{3}}$
meijerg	$-\frac{c^{3}\left(\frac{ax\left(15a^{3}x^{3}+20a^{2}x^{2}+30ax+60\right)}{60}+\ln\left(-ax+1\right)\right)}{a}-\frac{2c^{3}\left(-\frac{ax\left(4a^{2}x^{2}+6ax+12\right)}{12}-\ln\left(-ax+1\right)\right)}{a}+\frac{2c^{3}\left(-ax-\ln\left(-ax+1\right)\right.}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/12*x*(3*a^3*x^3-4*a^2*x^2-6*a*x+12)*c^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{4}\,a^3c^3x^4 + \frac{1}{3}\,a^2c^3x^3 + \frac{1}{2}\,ac^3x^2 - c^3x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{a^3c^3x^4}{4} + \frac{a^2c^3x^3}{3} + \frac{ac^3x^2}{2} - c^3x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**3,x)
```

output
```
-a**3*c**3*x**4/4 + a**2*c**3*x**3/3 + a*c**3*x**2/2 - c**3*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{4}a^3c^3x^4+\frac{1}{3}a^2c^3x^3+\frac{1}{2}ac^3x^2-c^3x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{4}a^3c^3x^4+\frac{1}{3}a^2c^3x^3+\frac{1}{2}ac^3x^2-c^3x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{a^3\,c^3\,x^4}{4}+\frac{a^2\,c^3\,x^3}{3}+\frac{a\,c^3\,x^2}{2}-c^3\,x$$

input
```
int(((c - a*c*x)^3*(a*x + 1))/(a*x - 1),x)
```

output
```
(a*c^3*x^2)/2 - c^3*x + (a^2*c^3*x^3)/3 - (a^3*c^3*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{c^3x(-3a^3x^3+4a^2x^2+6ax-12)}{12}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x)
```

output
```
(c**3*x*( - 3*a**3*x**3 + 4*a**2*x**2 + 6*a*x - 12))/12
```

3.177 $\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 20

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=-c^2x+\frac{1}{3}a^2c^2x^3$$

output
```
-c^2*x+1/3*a^2*c^2*x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=-c^2\left(x-\frac{a^2x^3}{3}\right)$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

output
```
-(c^2*(x - (a^2*x^3)/3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 39, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int c^2 e^{2\text{arctanh}(ax)}(1-ax)^2dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^2\int e^{2\text{arctanh}(ax)}(1-ax)^2dx$$

$$\Big\downarrow \text{ 6679}$$

$$-c^2\int (1-ax)(ax+1)dx$$

$$\Big\downarrow \text{ 39}$$

$$-c^2\int \left(1-a^2x^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-c^2\left(x-\frac{a^2x^3}{3}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

output
```
-(c^2*(x - (a^2*x^3)/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

method	result
gosper	$\frac{x\left(a^{2} x^{2}-3\right) c^{2}}{3}$
default	$c^{2}\left(\frac{1}{3} a^{2} x^{3}-x\right)$
norman	$-c^{2} x+\frac{1}{3} a^{2} c^{2} x^{3}$
risch	$-c^{2} x+\frac{1}{3} a^{2} c^{2} x^{3}$
parallelrisch	$-c^{2} x+\frac{1}{3} a^{2} c^{2} x^{3}$
orering	$\frac{x\left(a^{2} x^{2}-3\right)\left(-a c x+c\right)^{2}}{3\left(a x-1\right)^{2}}$
meijerg	$-\frac{c^{2}\left(-\frac{a x\left(4 a^{2} x^{2}+6 a x+12\right)}{12}-\ln\left(-a x+1\right)\right)}{a}-\frac{c^{2}\left(\frac{a x\left(3 a x+6\right)}{6}+\ln\left(-a x+1\right)\right)}{a}+\frac{c^{2}\left(-a x-\ln\left(-a x+1\right)\right)}{a}+\frac{c^{2}\ln\left(-a x+1\right)}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(a^2*x^2-3)*c^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{3}\,a^2c^2x^3-c^2x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
1/3*a^2*c^2*x^3 - c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{a^2c^2x^3}{3}-c^2x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**2,x)
```

output
```
a**2*c**2*x**3/3 - c**2*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{3}\,a^2c^2x^3-c^2x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
1/3*a^2*c^2*x^3 - c^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{3}\,a^2c^2x^3-c^2x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
1/3*a^2*c^2*x^3 - c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2\,x\,(a^2\,x^2-3)}{3}$$

input
```
int(((c - a*c*x)^2*(a*x + 1))/(a*x - 1),x)
```

output
```
(c^2*x*(a^2*x^2 - 3))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2x(a^2x^2-3)}{3}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x)
```

output
```
(c**2*x*(a**2*x**2 - 3))/3
```

3.178 $\int e^{2\coth^{-1}(ax)}(c-acx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 14

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-cx-\frac{1}{2}acx^2$$

output
```
-c*x-1/2*a*c*x^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=\frac{ce^{2\coth^{-1}(ax)}(1-a^2x^2)}{2a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x),x]
```

output
```
(c*E^(2*ArcCoth[a*x])*(1 - a^2*x^2))/(2*a)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{2726\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 2726$$

$$\frac{c(1-a^2x^2)\,e^{2\coth^{-1}(ax)}}{2a}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x),x]
```

output
```
(c*E^(2*ArcCoth[a*x])*(1 - a^2*x^2))/(2*a)
```

Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{cx(ax+2)}{2}$	10
default	$c\left(-\frac{1}{2}a\,x^2-x\right)$	13
norman	$-xc-\frac{1}{2}ac\,x^2$	13
risch	$-xc-\frac{1}{2}ac\,x^2$	13
parallelrisch	$-xc-\frac{1}{2}ac\,x^2$	13
orering	$\frac{x(ax+2)(-acx+c)}{2ax-2}$	23
meijerg	$-\frac{c\left(\frac{ax(3ax+6)}{6}+\ln(-ax+1)\right)}{a}+\frac{c\ln(-ax+1)}{a}$	38

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c*x*(a*x+2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}\,acx^2-cx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="fricas")
```

output
```
-1/2*a*c*x^2 - c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{acx^2}{2}-cx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x)
```

output
```
-a*c*x**2/2 - c*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}\,acx^2-cx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="maxima")
```

output
```
-1/2*a*c*x^2 - c*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}\,acx^2-cx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="giac")
```

output
```
-1/2*a*c*x^2 - c*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{c\,x\,(a\,x+2)}{2}$$

input
```
int(((c - a*c*x)*(a*x + 1))/(a*x - 1),x)
```

output
```
-(c*x*(a*x + 2))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=\frac{cx(-ax-2)}{2}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x)
```

output
```
(c*x*( - a*x - 2))/2
```

3.179 $\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 32

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{2}{ac(1-ax)}-\frac{\log(1-ax)}{ac}$$

output
```
-2/a/c/(-a*x+1)-ln(-a*x+1)/a/c
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{\frac{2}{a(1-ax)}+\frac{\log(1-ax)}{a}}{c}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x),x]
```

output
```
-((2/(a*(1 - a*x)) + Log[1 - a*x]/a)/c)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{c(1-ax)}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{e^{2\text{arctanh}(ax)}}{1-ax}dx}{c}$$

$$\Big\downarrow \text{ 6679}$$

$$-\frac{\int \frac{ax+1}{(1-ax)^2}dx}{c}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{\int \left(\frac{1}{ax-1}+\frac{2}{(ax-1)^2}\right)dx}{c}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{2}{a(1-ax)}+\frac{\log(1-ax)}{a}}{c}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a*c*x),x]
```

output
```
-((2/(a*(1 - a*x)) + Log[1 - a*x]/a)/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\frac{2}{(ax-1)a}-\frac{\ln(ax-1)}{a}}{c}$	29
norman	$\frac{2x}{c(ax-1)}-\frac{\ln(ax-1)}{ac}$	29
risch	$\frac{2}{ac(ax-1)}-\frac{\ln(ax-1)}{ac}$	31
parallelrisch	$\frac{-a\ln(ax-1)x+2ax+\ln(ax-1)}{c(ax-1)a}$	36

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/c*(2/(a*x-1)/a-1/a*ln(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{(ax-1)\log(ax-1)-2}{a^2cx-ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="fricas")
```

output
```
-((a*x - 1)*log(a*x - 1) - 2)/(a^2*c*x - a*c)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2}{a^2cx-ac}-\frac{\log(ax-1)}{ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x)
```

output
```
2/(a**2*c*x - a*c) - log(a*x - 1)/(a*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2}{a^2cx-ac}-\frac{\log(ax-1)}{ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="maxima")
```

output
```
2/(a^2*c*x - a*c) - log(a*x - 1)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log(|ax-1|)}{ac} + \frac{2}{(ax-1)ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="giac")
```

output
```
-log(abs(a*x - 1))/(a*c) + 2/((a*x - 1)*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{2}{a\,(c-a\,c\,x)} - \frac{\ln(a\,x-1)}{a\,c}$$

input
```
int((a*x + 1)/((c - a*c*x)*(a*x - 1)),x)
```

output
```
- 2/(a*(c - a*c*x)) - log(a*x - 1)/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = \frac{-\log(ax-1)\,ax + \log(ax-1) + 2ax}{ac\,(ax-1)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x)
```

output
```
( - log(a*x - 1)*a*x + log(a*x - 1) + 2*a*x)/(a*c*(a*x - 1))
```

3.180 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{x}{c^2(1-ax)^2}$$

output
```
-x/c^2/(-a*x+1)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{(1+ax)^2}{4ac^2(1-ax)^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

output
```
-1/4*(1 + a*x)^2/(a*c^2*(1 - a*x)^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6717, 27, 6679, 38}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}dx$$
$$\Big\downarrow \text{ 6717}$$
$$-\int \frac{e^{2\text{arctanh}(ax)}}{c^2(1-ax)^2}dx$$
$$\Big\downarrow \text{ 27}$$
$$-\frac{\int \frac{e^{2\text{arctanh}(ax)}}{(1-ax)^2}dx}{c^2}$$
$$\Big\downarrow \text{ 6679}$$
$$-\frac{\int \frac{ax+1}{(1-ax)^3}dx}{c^2}$$
$$\Big\downarrow \text{ 38}$$
$$-\frac{x}{c^2(1-ax)^2}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

output
```
-(x/(c^2*(1 - a*x)^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 38
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
orering	$-\frac{x}{\left(-acx+c\right)^{2}}$	13
gosper	$-\frac{x}{c^{2}\left(ax-1\right)^{2}}$	14
norman	$-\frac{x}{c^{2}\left(ax-1\right)^{2}}$	14
risch	$-\frac{x}{c^{2}\left(ax-1\right)^{2}}$	14
parallelrisch	$-\frac{x}{c^{2}\left(ax-1\right)^{2}}$	14
default	$\frac{-\frac{1}{\left(ax-1\right)a}-\frac{1}{a\left(ax-1\right)^{2}}}{c^{2}}$	30

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-x/(-a*c*x+c)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{x}{a^2c^2x^2-2\,ac^2x+c^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
-x/(a^2*c^2*x^2 - 2*a*c^2*x + c^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{x}{a^2c^2x^2-2ac^2x+c^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**2,x)
```

output
```
-x/(a**2*c**2*x**2 - 2*a*c**2*x + c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{x}{a^2c^2x^2-2\,ac^2x+c^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
-x/(a^2*c^2*x^2 - 2*a*c^2*x + c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(13) = 26$.

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.43

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{(acx-c)^2a} - \frac{1}{(acx-c)ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
-1/((a*c*x - c)^2*a) - 1/((a*c*x - c)*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{x}{c^2\,(a\,x-1)^2}$$

input
```
int((a*x + 1)/((c - a*c*x)^2*(a*x - 1)),x)
```

output
```
-x/(c^2*(a*x - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{-a^2x^2-1}{2a\,c^2\,(a^2x^2-2ax+1)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x)
```

output
```
( - (a**2*x**2 + 1))/(2*a*c**2*(a**2*x**2 - 2*a*x + 1))
```

3.181 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{2}{3ac^3(1-ax)^3}+\frac{1}{2ac^3(1-ax)^2}$$

output
```
-2/3/a/c^3/(-a*x+1)^3+1/2/a/c^3/(-a*x+1)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{1+3ax}{6ac^3(-1+ax)^3}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

output
```
(1 + 3*a*x)/(6*a*c^3*(-1 + a*x)^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{c^3(1-ax)^3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{2\text{arctanh}(ax)}}{(1-ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{\int \frac{ax+1}{(1-ax)^4}dx}{c^3}$$

$$\Big\downarrow \text{53}$$

$$-\frac{\int \left(\frac{1}{(ax-1)^3}+\frac{2}{(ax-1)^4}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{2}{3a(1-ax)^3}-\frac{1}{2a(1-ax)^2}}{c^3}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

output
```
-((2/(3*a*(1 - a*x)^3) - 1/(2*a*(1 - a*x)^2))/c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{\frac{x}{2}+\frac{1}{6a}}{c^{3}\left(a x -1\right)^{3}}$	21
gosper	$\frac{3 a x +1}{6 a \,c^{3} \left(a x -1\right)^{3}}$	22
orering	$-\frac{x \left(a^{2} x^{2}-3 a x +6\right)}{6 \left(-a c x +c \right)^{3}}$	26
default	$\frac{\frac{1}{2 a \left(a x -1\right)^{2}}+\frac{2}{3 a \left(a x -1\right)^{3}}}{c^{3}}$	30
parallelrisch	$\frac{a^{2} x^{3}-3 a \,x^{2}+6 x}{6 c^{3} \left(a x -1\right)^{3}}$	30
norman	$\frac{\frac{x}{c}-\frac{a \,x^{2}}{2 c}+\frac{a^{2} x^{3}}{6 c}}{\left(a x -1\right)^{3} c^{2}}$	38

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2*x+1/6/a)/c^3/(a*x-1)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,ax+1}{6\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
1/6*(3*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{-3ax-1}{6a^4c^3x^3-18a^3c^3x^2+18a^2c^3x-6ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**3,x)
```

output
```
-(-3*a*x - 1)/(6*a**4*c**3*x**3 - 18*a**3*c**3*x**2 + 18*a**2*c**3*x - 6*a
*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,ax+1}{6\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
1/6*(3*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,ax+1}{6\,(ax-1)^3ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
1/6*(3*a*x + 1)/((a*x - 1)^3*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{x}{2}+\frac{1}{6\,a}}{-a^3\,c^3\,x^3+3\,a^2\,c^3\,x^2-3\,a\,c^3\,x+c^3}$$

input
```
int((a*x + 1)/((c - a*c*x)^3*(a*x - 1)),x)
```

output
```
-(x/2 + 1/(6*a))/(c^3 + 3*a^2*c^3*x^2 - a^3*c^3*x^3 - 3*a*c^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3ax+1}{6a\,c^3\left(a^3x^3-3a^2x^2+3ax-1\right)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x)
```

output
```
(3*a*x + 1)/(6*a*c**3*(a**3*x**3 - 3*a**2*x**2 + 3*a*x - 1))
```

3.182 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{1}{2ac^4(1-ax)^4}+\frac{1}{3ac^4(1-ax)^3}$$

output
```
-1/2/a/c^4/(-a*x+1)^4+1/3/a/c^4/(-a*x+1)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{1+2ax}{6ac^4(-1+ax)^4}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

output
```
-1/6*(1 + 2*a*x)/(a*c^4*(-1 + a*x)^4)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{c^4(1-ax)^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{2\text{arctanh}(ax)}}{(1-ax)^4}dx}{c^4}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{\int \frac{ax+1}{(1-ax)^5}dx}{c^4}$$

$$\Big\downarrow \text{53}$$

$$-\frac{\int \left(-\frac{1}{(ax-1)^4}-\frac{2}{(ax-1)^5}\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2a(1-ax)^4}-\frac{1}{3a(1-ax)^3}}{c^4}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

output
```
-((1/(2*a*(1 - a*x)^4) - 1/(3*a*(1 - a*x)^3))/c^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{-\frac{x}{3}-\frac{1}{6a}}{c^{4}\left(ax-1\right)^{4}}$	21
gosper	$-\frac{2ax+1}{6a\,c^{4}\left(ax-1\right)^{4}}$	22
default	$\frac{-\frac{1}{2a\left(ax-1\right)^{4}}-\frac{1}{3a\left(ax-1\right)^{3}}}{c^{4}}$	30
orering	$\frac{x\left(a^{3}x^{3}-4a^{2}x^{2}+6ax-6\right)}{6\left(-acx+c\right)^{4}}$	34
parallelrisch	$\frac{a^{3}x^{4}-4a^{2}x^{3}+6a\,x^{2}-6x}{6c^{4}\left(ax-1\right)^{4}}$	38
norman	$\frac{\frac{a\,x^{2}}{c}-\frac{x}{c}-\frac{2a^{2}x^{3}}{3c}+\frac{a^{3}x^{4}}{6c}}{\left(ax-1\right)^{4}c^{3}}$	49

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*x-1/6/a)/c^4/(a*x-1)^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{2\,ax+1}{6\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
-1/6*(2*a*x + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*
x + a*c^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(29) = 58.

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{-2ax-1}{6a^5c^4x^4-24a^4c^4x^3+36a^3c^4x^2-24a^2c^4x+6ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**4,x)
```

output
```
(-2*a*x - 1)/(6*a**5*c**4*x**4 - 24*a**4*c**4*x**3 + 36*a**3*c**4*x**2 - 2
4*a**2*c**4*x + 6*a*c**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{2\,ax+1}{6\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
-1/6*(2*a*x + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*
x + a*c^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{2\,ax+1}{6\,(ax-1)^4ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
-1/6*(2*a*x + 1)/((a*x - 1)^4*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\frac{x}{3}+\frac{1}{6\,a}}{a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3+6\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}$$

input
```
int((a*x + 1)/((c - a*c*x)^4*(a*x - 1)),x)
```

output
```
-(x/3 + 1/(6*a))/(c^4 + 6*a^2*c^4*x^2 - 4*a^3*c^4*x^3 + a^4*c^4*x^4 - 4*a*
c^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{-2ax-1}{6a\,c^4\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x)
```

output
```
( - 2*a*x - 1)/(6*a*c**4*(a**4*x**4 - 4*a**3*x**3 + 6*a**2*x**2 - 4*a*x +
1))
```

3.183 $\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal result

Integrand size = 18, antiderivative size = 105

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{3}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5-\frac{3c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

output
```
3/8*a*c^4*(1-1/a^2/x^2)^(1/2)*x^2-1/4*a^3*c^4*(1-1/a^2/x^2)^(3/2)*x^4+1/5*
a^4*c^4*(1-1/a^2/x^2)^(5/2)*x^5-3/8*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}x(8+25ax-16a^2x^2-10a^3x^3+8a^4x^4)-15\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{40a}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

output
```
(c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 + 25*a*x - 16*a^2*x^2 - 10*a^3*x^3 + 8*
a^4*x^4) - 15*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(40*a)
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6724, 25, 27, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^4 e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6724}$$

$$a^3c^3\int -c\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)x^6d\frac{1}{x}$$

$$\Big\downarrow \text{ 25}$$

$$-a^3c^3\int c\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)x^6d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-a^3c^4\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)x^6d\frac{1}{x}$$

$$\Big\downarrow \text{ 534}$$

$$-a^3c^4\left(-\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5d\frac{1}{x}-\frac{1}{5}ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\right)$$

$$\Big\downarrow \text{ 243}$$

$$-a^3c^4\left(-\frac{1}{2}\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3d\frac{1}{x^2}-\frac{1}{5}ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\right)$$

$$\Big\downarrow \text{ 51}$$

$$-a^3c^4\left(\frac{1}{2}\left(\frac{3\int \sqrt{1-\frac{1}{a^2x^2}}x^2d\frac{1}{x^2}}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{5}ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\right)$$

$$\Big\downarrow \ 51$$

$$-a^3c^4\left(\frac{1}{2}\left(\frac{3\left(x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{5}ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\right)$$

$$\Big\downarrow \ 73$$

$$-a^3c^4\left(\frac{1}{2}\left(\frac{3\left(\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-x\sqrt{1-\frac{1}{a^2x^2}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{5}ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\right)$$

$$\Big\downarrow \ 221$$

$$-a^3c^4\left(\frac{1}{2}\left(\frac{3\left(\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}-x\sqrt{1-\frac{1}{a^2x^2}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{5}ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

output
```
-(a^3*c^4*(-1/5*(a*(1 - 1/(a^2*x^2))^(5/2)*x^5) + (((1 - 1/(a^2*x^2))^(3/2
)*x^2)/2 + (3*(-(Sqrt[1 - 1/(a^2*x^2)]*x) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
/a^2))/(4*a^2))/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.22

method	result
risch	$\frac{(8a^4x^4-10a^3x^3-16a^2x^2+25ax+8)(ax-1)c^4}{40a\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^4\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$
default	$-\frac{(ax-1)^2c^4\left(-24\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^2x^2+30\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax+40((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-16(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-45\sqrt{a^2}\,\sqrt{a^2x^2-1}\right.}{120a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/40*(8*a^4*x^4-10*a^3*x^3-16*a^2*x^2+25*a*x+8)*(a*x-1)/a*c^4/((a*x-1)/(a*
x+1))^(1/2)-3/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^4/((
a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.20

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx =$$
$$-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(8\,a^5c^4x^5-2\,a^4c^4x^4-26\,a^3c^4x^3+9\,a^2c^4x^2+33\,a}{40\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
-1/40*(15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^4*log(sqrt((a*x -
1)/(a*x + 1)) - 1) - (8*a^5*c^4*x^5 - 2*a^4*c^4*x^4 - 26*a^3*c^4*x^3 + 9*a
^2*c^4*x^2 + 33*a*c^4*x + 8*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx = c^4\left(\int\left(-\frac{4ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx + \int\frac{6a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\left(-\frac{4a^3x^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx + \int\frac{a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**4,x)
```

output
```
c**4*(Integral(-4*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**2*x**2/(a*
x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))/(a*x + 1)), x) + Integral(-4*a**3*x**3/(a*x*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) +
 Integral(a**4*x**4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
 + 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(89) = 178$.

Time = 0.03 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.47

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx = -\frac{1}{40}\left(\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(15\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-70\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-128\,c^4\left(\frac{ax-}{ax-}\right.\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
-1/40*(15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^4*log(sqrt((a*
x - 1)/(a*x + 1)) - 1)/a^2 - 2*(15*c^4*((a*x - 1)/(a*x + 1))^(9/2) - 70*c^
4*((a*x - 1)/(a*x + 1))^(7/2) - 128*c^4*((a*x - 1)/(a*x + 1))^(5/2) + 70*c
^4*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(5*(a*x
 - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2/
(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5
- a^2))*a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.31

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx = \frac{3\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\mathrm{sgn}\,(ax+1)} + \frac{1}{40}\sqrt{a^2x^2-1}\left(\left(\left(\frac{25\,c^4}{\mathrm{sgn}\,(ax+1)}-2\left(\frac{8\,ac^4}{\mathrm{sgn}\,(ax+1)}-\left(\frac{4\,a^3c^4x}{\mathrm{sgn}\,(ax+1)}-\frac{5\,a^2c^4}{\mathrm{sgn}\,(ax+1)}\right)x\right)x\right)x+\frac{8}{a\mathrm{sgn}\,(\right.\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
3/8*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + 1/
40*sqrt(a^2*x^2 - 1)*((25*c^4/sgn(a*x + 1) - 2*(8*a*c^4/sgn(a*x + 1) - (4*
a^3*c^4*x/sgn(a*x + 1) - 5*a^2*c^4/sgn(a*x + 1))*x)*x)*x + 8*c^4/(a*sgn(a*
x + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.04

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx$$
$$=\frac{\frac{3c^4\sqrt{\frac{ax-1}{ax+1}}}{4}-\frac{7c^4\left(\frac{ax-1}{ax+1}\right)^{3/2}}{2}+\frac{32c^4\left(\frac{ax-1}{ax+1}\right)^{5/2}}{5}+\frac{7c^4\left(\frac{ax-1}{ax+1}\right)^{7/2}}{2}-\frac{3c^4\left(\frac{ax-1}{ax+1}\right)^{9/2}}{4}}{a-\frac{5a(ax-1)}{ax+1}+\frac{10a(ax-1)^2}{(ax+1)^2}-\frac{10a(ax-1)^3}{(ax+1)^3}+\frac{5a(ax-1)^4}{(ax+1)^4}-\frac{a(ax-1)^5}{(ax+1)^5}}$$
$$-\frac{3c^4\operatorname{atanh}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{4a}$$

input
```
int((c - a*c*x)^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((3*c^4*((a*x - 1)/(a*x + 1))^(1/2))/4 - (7*c^4*((a*x - 1)/(a*x + 1))^(3/2
))/2 + (32*c^4*((a*x - 1)/(a*x + 1))^(5/2))/5 + (7*c^4*((a*x - 1)/(a*x + 1
))^(7/2))/2 - (3*c^4*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))/
(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^
3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) - (3*c^4*
atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx$$
$$=\frac{c^4\left(8\sqrt{ax+1}\,\sqrt{ax-1}\,a^4x^4-10\sqrt{ax+1}\,\sqrt{ax-1}\,a^3x^3-16\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2+25\sqrt{ax+1}\,\sqrt{ax-\right.}{40a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x)
```

output
```
(c**4*(8*sqrt(a*x + 1)*sqrt(a*x - 1)*a**4*x**4 - 10*sqrt(a*x + 1)*sqrt(a*x
 - 1)*a**3*x**3 - 16*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 + 25*sqrt(a*x +
 1)*sqrt(a*x - 1)*a*x + 8*sqrt(a*x + 1)*sqrt(a*x - 1) - 30*log((sqrt(a*x -
 1) + sqrt(a*x + 1))/sqrt(2))))/(40*a)
```

3.184 $\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 78

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{3}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4-\frac{3c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

output
```
3/8*a*c^3*(1-1/a^2/x^2)^(1/2)*x^2-1/4*a^3*c^3*(1-1/a^2/x^2)^(3/2)*x^4-3/8*
c^3*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{c^3\left(a^2\sqrt{1-\frac{1}{a^2x^2}}x^2(5-2a^2x^2)-3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{8a}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

output
```
(c^3*(a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*(5 - 2*a^2*x^2) - 3*Log[a*(1 + Sqrt[1
- 1/(a^2*x^2)])*x]))/(8*a)
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6724, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^3 e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6724}$$

$$a^3c^3\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5d\frac{1}{x}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a^3c^3\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3d\frac{1}{x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}a^3c^3\left(-\frac{3\int \sqrt{1-\frac{1}{a^2x^2}}x^2d\frac{1}{x^2}}{4a^2}-\frac{1}{2}x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}a^3c^3\left(-\frac{3\left(x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}\right)}{4a^2}-\frac{1}{2}x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a^3c^3\left(-\frac{3\left(\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-x\sqrt{1-\frac{1}{a^2x^2}}\right)}{4a^2}-\frac{1}{2}x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}a^3c^3\left(-\frac{3\left(\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}-x\sqrt{1-\frac{1}{a^2x^2}}\right)}{4a^2}-\frac{1}{2}x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

output
```
(a^3*c^3*(-1/2*((1 - 1/(a^2*x^2))^(3/2)*x^2) - (3*(-(Sqrt[1 - 1/(a^2*x^2)]
*x) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a^2))/(4*a^2)))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.36

method	result	size
risch	$-\frac{x(2a^2x^2-5)(ax-1)c^3}{8\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{(ax-1)(ax+1)}}{8\sqrt{a^2}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	106
default	$-\frac{(ax-1)^2c^3\left(2x(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-3x\sqrt{a^2x^2-1}\,\sqrt{a^2}+3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$	124

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*x*(2*a^2*x^2-5)*(a*x-1)*c^3/((a*x-1)/(a*x+1))^(1/2)-3/8*ln(a^2*x/(a^2
)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^3/((a*x-1)/(a*x+1))^(1/2)*((a*x-1
)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.40

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^4c^3x^4+2\,a^3c^3x^3-5\,a^2c^3x^2-5\,ac^3x\right)\sqrt{\frac{ax-1}{ax+1}}}{8\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
-1/8*(3*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^3*log(sqrt((a*x - 1)/
(a*x + 1)) - 1) + (2*a^4*c^3*x^4 + 2*a^3*c^3*x^3 - 5*a^2*c^3*x^2 - 5*a*c^3
*x)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx = -c^3\left(\int\frac{3ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\left(-\frac{3a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx + \int\frac{a^3x^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx + \int\left(-\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**3,x)
```

output
```
-c**3*(Integral(3*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**2*x**2/(a
*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a
*x + 1))/(a*x + 1)), x) + Integral(a**3*x**3/(a*x*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + I
ntegral(-1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(66) = 132$.

Time = 0.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.83

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{8}\left(\frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} - \frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2} + \frac{2\left(3\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} - 11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} - 11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} \cdots\right.}{\frac{4\,(ax-1)a^2}{ax+1} - \frac{6\,(ax-1)^2a^2}{(ax+1)^2} + \frac{4\,(ax-1)^3a^2}{(ax+1)^3} - \frac{(ax-1)\cdots}{(ax+\cdots}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^3*log(sqrt((a*x -
 1)/(a*x + 1)) - 1)/a^2 + 2*(3*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 11*c^3*((
a*x - 1)/(a*x + 1))^(5/2) - 11*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^3*sqr
t((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2/(a*
x + 1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 - a
^2))*a
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{8}\left(\frac{2\,a^2c^3x^2}{\mathrm{sgn}\,(ax+1)} - \frac{5\,c^3}{\mathrm{sgn}\,(ax+1)}\right)\sqrt{a^2x^2-1}x + \frac{3\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
-1/8*(2*a^2*c^3*x^2/sgn(a*x + 1) - 5*c^3/sgn(a*x + 1))*sqrt(a^2*x^2 - 1)*x
 + 3/8*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.26

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{\frac{3\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{11\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{4}-\frac{11\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{4}+\frac{3\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a-\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{6\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{3\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

input
```
int((c - a*c*x)^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((3*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 - (11*c^3*((a*x - 1)/(a*x + 1))^(3/
2))/4 - (11*c^3*((a*x - 1)/(a*x + 1))^(5/2))/4 + (3*c^3*((a*x - 1)/(a*x +
1))^(7/2))/4)/(a - (4*a*(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*x + 1)
^2 - (4*a*(a*x - 1)^3)/(a*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4) - (3*c^3
*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{c^3\left(-2\sqrt{ax+1}\,\sqrt{ax-1}\,a^3x^3+5\sqrt{ax+1}\,\sqrt{ax-1}\,ax-6\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{8a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x)
```

output
```
(c**3*( - 2*sqrt(a*x + 1)*sqrt(a*x - 1)*a**3*x**3 + 5*sqrt(a*x + 1)*sqrt(a
*x - 1)*a*x - 6*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))))/(8*a)
```

3.185 $\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 78

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

output
```
1/2*a*c^2*(1-1/a^2/x^2)^(1/2)*x^2+1/3*a^2*c^2*(1-1/a^2/x^2)^(3/2)*x^3-1/2*
c^2*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-2+3ax+2a^2x^2)-3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

output
```
(c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + 3*a*x + 2*a^2*x^2) - 3*Log[a*(1 + Sq
rt[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6724, 25, 27, 566, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2 e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6724}$$

$$a^3c^3\int -\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{c\left(a-\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-a^3c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{c\left(a-\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-a^3c^2\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{a-\frac{1}{x}}d\frac{1}{x}$$

$$\Big\downarrow \text{566}$$

$$-a^3c^2\int \sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{a}+\frac{1}{xa^2}\right)x^4d\frac{1}{x}$$

$$\Big\downarrow \text{534}$$

$$-a^3c^2\left(\frac{\int \sqrt{1-\frac{1}{a^2x^2}}x^3d\frac{1}{x}}{a^2}-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}\right)$$

$$\Big\downarrow \text{243}$$

$$-a^3c^2\left(\frac{\int \sqrt{1-\frac{1}{a^2x^2}}x^2d\frac{1}{x^2}}{2a^2}-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}\right)$$

$\Big\downarrow$ 51

$$-a^3c^2\left(\frac{x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}}{2a^2}-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}\right)$$

$\Big\downarrow$ 73

$$-a^3c^2\left(\frac{\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-x\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}\right)$$

$\Big\downarrow$ 221

$$-a^3c^2\left(\frac{\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}-x\sqrt{1-\frac{1}{a^2x^2}}}{2a^2}-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

output
```
-(a^3*c^2*(-1/3*((1 - 1/(a^2*x^2))^(3/2)*x^3)/a + (-(Sqrt[1 - 1/(a^2*x^2)]
*x) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a^2)/(2*a^2)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 566
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

method	result	size
risch	$\frac{(2a^2x^2+3ax-2)(ax-1)c^2}{6a\sqrt{\frac{ax-1}{ax+1}}}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	112
default	$\frac{(ax-1)^2c^2\left(3\sqrt{a^2}\sqrt{a^2x^2-1}\,ax+2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\sqrt{a^2}\,a}$	130

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*x^2+3*a*x-2)*(a*x-1)/a*c^2/((a*x-1)/(a*x+1))^(1/2)-1/2*ln(a^2*x
/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^2/((a*x-1)/(a*x+1))^(1/2)*((
a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.32

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(2\,a^3c^2x^3+5\,a^2c^2x^2+ac^2x-2\,c^2)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
-1/6*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^2*log(sqrt((a*x - 1)/
(a*x + 1)) - 1) - (2*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + a*c^2*x - 2*c^2)*sqrt((
a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx=c^2\left(\int\left(-\frac{2ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**2,x)
```

output
```
c**2*(Integral(-2*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**2*x**2/(a*x*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(
a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(66) = 132$.

Time = 0.04 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.32

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx=-\frac{1}{6}a\left(\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-8c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3(ax-1)a^2}{ax+1}-\frac{3(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
-1/6*a*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1)/a^2 - 2*(3*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 8*c^2*(
(a*x - 1)/(a*x + 1))^(3/2) - 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)
*a^2/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)
^3 - a^2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.26

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{1}{6}\sqrt{a^2x^2-1}\left(\left(\frac{2\,ac^2x}{\operatorname{sgn}(ax+1)}+\frac{3\,c^2}{\operatorname{sgn}(ax+1)}\right)x-\frac{2\,c^2}{a\operatorname{sgn}(ax+1)}\right) + \frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\operatorname{sgn}(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
1/6*sqrt(a^2*x^2 - 1)*((2*a*c^2*x/sgn(a*x + 1) + 3*c^2/sgn(a*x + 1))*x - 2
*c^2/(a*sgn(a*x + 1))) + 1/2*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(
abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.61 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.78

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{c^2\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{8\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}-c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a-\frac{3\,a\,(a\,x-1)}{a\,x+1}+\frac{3\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a\,(a\,x-1)^3}{(a\,x+1)^3}}-\frac{c^2\operatorname{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - a*c*x)^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(c^2*((a*x - 1)/(a*x + 1))^(1/2) + (8*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 -
 c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a
*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (c^2*atanh(((a*x -
 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{c^2\left(2\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2+3\sqrt{ax+1}\,\sqrt{ax-1}\,ax-2\sqrt{ax+1}\,\sqrt{ax-1}-6\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{6a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x)
```

output
```
(c**2*(2*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 + 3*sqrt(a*x + 1)*sqrt(a*x
- 1)*a*x - 2*sqrt(a*x + 1)*sqrt(a*x - 1) - 6*log((sqrt(a*x - 1) + sqrt(a*x
 + 1))/sqrt(2))))/(6*a)
```

3.186 $\int e^{3\coth^{-1}(ax)}(c-acx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx=-2c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{3c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

output
```
-2*c*(1-1/a^2/x^2)^(1/2)*x-1/2*a*c*(1-1/a^2/x^2)^(1/2)*x^2-3/2*c*arctanh((
1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx=-\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x(4+ax)+3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{2a}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x),x]
```

output
```
-1/2*(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 + a*x) + 3*Log[a*(1 + Sqrt[1 - 1/(a^
2*x^2)])*x]))/a
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6724, 27, 570, 540, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6724}$$

$$a^3c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{c^2\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$a^3c\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{570}$$

$$\frac{c\int \frac{\left(a+\frac{1}{x}\right)^2x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{540}$$

$$\frac{c\left(-\frac{1}{2}\int -\frac{\left(4a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{c\left(\frac{1}{2}\int \frac{\left(4a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{534}$$

$$\frac{c\left(\frac{1}{2}\left(3\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-4ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{243}$$

$$\frac{c\left(\frac{1}{2}\left(\frac{3}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-4ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{c\left(\frac{1}{2}\left(-3a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-4ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{221}$$

$$\frac{c\left(\frac{1}{2}\left(-3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-4ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x),x]
```

output
```
(c*(-1/2*(a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (-4*a*Sqrt[1 - 1/(a^2*x^2)]*x -
 3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.52

method	result	size
risch	$-\frac{(ax+4)(ax-1)c}{2a\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	99
default	$-\frac{(ax-1)^2c\left(\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+4\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a+4a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$	162

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x+4)*(a*x-1)/a*c/((a*x-1)/(a*x+1))^(1/2)-3/2*ln(a^2*x/(a^2)^(1/2)+
(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/
(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx$$
$$=-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2cx^2+5\,acx+4\,c)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="fricas")
```

output
```
-1/2*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) + (a^2*c*x^2 + 5*a*c*x + 4*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx=-c\left(\int\frac{ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c),x)
```

output
```
-c*(Integral(a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(55) = 110$.

Time = 0.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.08

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx = -\frac{1}{2}a\left(\frac{2\left(3c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{3c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="maxima")
```

output
```
-1/2*a*(2*(3*c*((a*x - 1)/(a*x + 1))^(3/2) - 5*c*sqrt((a*x - 1)/(a*x + 1))
)/(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 3*c*lo
g(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1)) -
 1)/a^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx = -\frac{1}{2}\sqrt{a^2x^2-1}\left(\frac{cx}{\operatorname{sgn}(ax+1)}+\frac{4c}{a\operatorname{sgn}(ax+1)}\right) + \frac{3c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2|a|\operatorname{sgn}(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="giac")
```

output
```
-1/2*sqrt(a^2*x^2 - 1)*(c*x/sgn(a*x + 1) + 4*c/(a*sgn(a*x + 1))) + 3/2*c*l
og(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx = -\frac{5c\sqrt{\frac{ax-1}{ax+1}}-3c\left(\frac{ax-1}{ax+1}\right)^{3/2}}{a-\frac{2a(ax-1)}{ax+1}+\frac{a(ax-1)^2}{(ax+1)^2}} - \frac{3c\operatorname{atanh}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}$$

input
```
int((c - a*c*x)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
- (5*c*((a*x - 1)/(a*x + 1))^(1/2) - 3*c*((a*x - 1)/(a*x + 1))^(3/2))/(a -
 (2*a*(a*x - 1))/(a*x + 1) + (a*(a*x - 1)^2)/(a*x + 1)^2) - (3*c*atanh(((a
*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx$$
$$=\frac{c\left(-\sqrt{ax+1}\,\sqrt{ax-1}\,ax-4\sqrt{ax+1}\,\sqrt{ax-1}-6\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{2a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x)
```

output
```
(c*( - sqrt(a*x + 1)*sqrt(a*x - 1)*a*x - 4*sqrt(a*x + 1)*sqrt(a*x - 1) - 6
*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))))/(2*a)
```

3.187 $\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx=\frac{8}{3c\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}+\frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
8/3/c/(1-1/a^2/x^2)^(1/2)/(a-1/x)+4/3/a^2/c/(1-1/a^2/x^2)^(1/2)/x-arctanh(
(1-1/a^2/x^2)^(1/2))/a/c
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.78

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx=\frac{\frac{4\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+2ax)}{(-1+ax)^2}-\frac{3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{3c}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x),x]
```

output
```
((4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + 2*a*x))/(-1 + a*x)^2 - (3*Log[a*(1 + Sqr
t[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6724, 27, 570, 532, 25, 2336, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}dx$$

$$\Big\downarrow \; 6724$$

$$a^3c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{c^4\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}$$

$$\Big\downarrow \; 27$$

$$\frac{a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{c}$$

$$\Big\downarrow \; 570$$

$$\frac{\int \frac{\left(a+\frac{1}{x}\right)^4x}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^5c}$$

$$\Big\downarrow \; 532$$

$$\frac{\frac{8a^3\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(3a^4+\frac{4a^3}{x}-\frac{3a^2}{x^2}\right)x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^5c}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{3}\int \frac{\left(3a^4+\frac{4a^3}{x}-\frac{3a^2}{x^2}\right)x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{8a^3\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^5c}$$

$$\Big\downarrow \; 2336$$

$$\frac{\frac{1}{3}\left(\frac{4a^3}{x\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{3a^4x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{8a^3\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^5c}$$

$$\downarrow 27$$

$$\frac{\frac{1}{3}\left(3a^4\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{4a^3}{x\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{8a^3\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^5c}$$

$$\downarrow 243$$

$$\frac{\frac{1}{3}\left(\frac{3}{2}a^4\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+\frac{4a^3}{x\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{8a^3\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^5c}$$

$$\downarrow 73$$

$$\frac{\frac{1}{3}\left(\frac{4a^3}{x\sqrt{1-\frac{1}{a^2x^2}}}-3a^6\int\frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{8a^3\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^5c}$$

$$\downarrow 221$$

$$\frac{\frac{8a^3\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{1}{3}\left(\frac{4a^3}{x\sqrt{1-\frac{1}{a^2x^2}}}-3a^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^5c}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a*c*x),x]
```

output
```
((8*a^3*(a + x^(-1)))/(3*(1 - 1/(a^2*x^2))^(3/2)) + ((4*a^3)/(Sqrt[1 - 1/(
a^2*x^2)]*x) - 3*a^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/3)/(a^5*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(71) = 142$.

Time = 0.13 (sec) , antiderivative size = 345, normalized size of antiderivative = 4.26

method	result
default	$-\frac{3\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^4x^3+3\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^3x^3-9\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-3\sqrt{a^2}\,((ax-1)(ax+$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a*(3*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^4*
x^3+3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*a^3*x^3-9*ln((a^2*x+((a*x-1)*(a*
x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^3*x^2-3*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(3/2)*a*x-9*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*a^2*x^2+9*ln((a^2*x+((
a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x+((a*x-1)*(a*x+1))^(3
/2)*(a^2)^(1/2)+9*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*a*x-3*a*ln((a^2*x+((
a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))-3*((a*x-1)*(a*x+1))^(1/2)*
(a^2)^(1/2))/(a^2)^(1/2)/(a*x-1)/c/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1
)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.48

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-4\left(2\,a^2x^2+ax-1\right)\sqrt{}}{3\left(a^3cx^2-2\,a^2cx+ac\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="fricas")
```

output
```
-1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*
x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - 4*(2*a^2*x^2 + a*x -
 1)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{1}{\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c),x)
```

output
```
-Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a*x
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x
 + 1))/(a*x + 1)), x)/c
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{1}{3}\,a\left(\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{2\left(\frac{3\,(ax-1)}{ax+1}+1\right)}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-1/3*a*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 3*log(sqrt((a*x - 1
)/(a*x + 1)) - 1)/(a^2*c) - 2*(3*(a*x - 1)/(a*x + 1) + 1)/(a^2*c*((a*x - 1
)/(a*x + 1))^(3/2)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.43

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = \frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c|a|\text{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="giac")
```

output
```
log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.78

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = \frac{\frac{2\,(a\,x-1)}{a\,x+1}+\frac{2}{3}}{a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}} - \frac{2\,\text{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

input
```
int(1/((c - a*c*x)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
((2*(a*x - 1))/(a*x + 1) + 2/3)/(a*c*((a*x - 1)/(a*x + 1))^(3/2)) - (2*ata
nh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.21

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = \frac{-2\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax+2\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)+\frac{8\sqrt{ax+1}\,ax}{3}-\frac{4\sqrt{ax+1}}{3}}{\sqrt{ax-1}\,ac\,(ax-1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x)
```

output
```
(2*( - 3*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x +
3*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 4*sqrt(a*x
+ 1)*a*x - 2*sqrt(a*x + 1)))/(3*sqrt(a*x - 1)*a*c*(a*x - 1))
```

3.188 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5c^2\left(a-\frac{1}{x}\right)^5}$$

output
```
-1/5*a^4*(1-1/a^2/x^2)^(5/2)/c^2/(a-1/x)^5
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(1+ax)^2}{5c^2(-1+ax)^3}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

output
```
-1/5*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2)/(c^2*(-1 + a*x)^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6724, 25, 27, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}dx$$

$$\Big\downarrow \text{6724}$$

$$a^3c^3\int -\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-a^3c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{460}$$

$$-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5c^2\left(a-\frac{1}{x}\right)^5}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

output
```
-1/5*(a^4*(1 - 1/(a^2*x^2))^(5/2))/(c^2*(a - x^(-1))^5)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{ax+1}{5(ax-1)c^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	36
default	$-\frac{ax+1}{5(ax-1)c^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	36
orering	$-\frac{(ax-1)(ax+1)}{5a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(-acx+c)^{2}}$	40
trager	$-\frac{(ax+1)\left(a^{2}x^{2}+2ax+1\right)\sqrt{-\frac{-ax+1}{ax+1}}}{5a\,c^{2}(ax-1)^{3}}$	51

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(a*x+1)/(a*x-1)/c^2/((a*x-1)/(a*x+1))^(3/2)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(29) = 58$.

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{(a^3x^3+3\,a^2x^2+3\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{5\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
-1/5*(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^2*
x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{1}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**2,x)
```

output
```
Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3*a**2
*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a*x*sqrt(a*x/(a*x +
1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1))
, x)/c**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{5\,ac^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
-1/5/(a*c^2*((a*x - 1)/(a*x + 1))^(5/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(29) = 58.

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.09

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{2\left(5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4x^4+10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+1\right)}{5\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^5ac^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
-2/5*(5*(a + sqrt(a^2 - 1/x^2))^4*x^4 + 10*(a + sqrt(a^2 - 1/x^2))^2*x^2 +
 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{5\,a\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(1/((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
-1/(5*a*c^2*((a*x - 1)/(a*x + 1))^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{-\sqrt{ax-1}\,a^2x^2+2\sqrt{ax-1}\,ax-\sqrt{ax-1}-\sqrt{ax+1}\,a^2x^2-2\sqrt{ax+1}\,ax-\sqrt{ax+1}}{5\sqrt{ax-1}\,a\,c^2\,(a^2x^2-2ax+1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x)
```

output
```
( - sqrt(a*x - 1)*a**2*x**2 + 2*sqrt(a*x - 1)*a*x - sqrt(a*x - 1) - sqrt(a
*x + 1)*a**2*x**2 - 2*sqrt(a*x + 1)*a*x - sqrt(a*x + 1))/(5*sqrt(a*x - 1)*
a*c**2*(a**2*x**2 - 2*a*x + 1))
```

3.189 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx=\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6}-\frac{6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{35c^3\left(a-\frac{1}{x}\right)^5}$$

output
```
1/7*a^5*(1-1/a^2/x^2)^(5/2)/c^3/(a-1/x)^6-6/35*a^4*(1-1/a^2/x^2)^(5/2)/c^3
/(a-1/x)^5
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}x(-6+ax)(1+ax)^2}{35c^3(-1+ax)^4}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

output
```
-1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-6 + a*x)*(1 + a*x)^2)/(c^3*(-1 + a*x)^4)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6724, 27, 571, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}dx$$

$$\Big\downarrow \text{6724}$$

$$a^3c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^6\left(a-\frac{1}{x}\right)^6x}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^6x}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{571}$$

$$\frac{a^3\left(\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(a-\frac{1}{x}\right)^6}-\frac{6}{7}\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}\right)}{c^3}$$

$$\Big\downarrow \text{460}$$

$$\frac{a^3\left(\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(a-\frac{1}{x}\right)^6}-\frac{6a\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{35\left(a-\frac{1}{x}\right)^5}\right)}{c^3}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

output
```
(a^3*((a^2*(1 - 1/(a^2*x^2))^(5/2))/(7*(a - x^(-1))^6) - (6*a*(1 - 1/(a^2*
x^2))^(5/2))/(35*(a - x^(-1))^5)))/c^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 571
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*(n + p + 1))), x] + Simp[n/(2*d*
(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ((LtQ[n, -1] &&  !IGtQ[n + p +
1, 0]) || (LtQ[n, 0] && LtQ[p, -1]) || EqQ[n + 2*p + 2, 0]) && NeQ[n + p +
1, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{(ax-6)(ax+1)}{35(ax-1)^{2}c^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	41
default	$-\frac{(ax-6)(ax+1)}{35(ax-1)^{2}c^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	41
orering	$\frac{(ax-6)(ax-1)(ax+1)}{35a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(-acx+c)^{3}}$	45
trager	$-\frac{(ax+1)\left(a^{3}x^{3}-4a^{2}x^{2}-11ax-6\right)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^{3}(ax-1)^{4}}$	59

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/35*(a*x-6)*(a*x+1)/(a*x-1)^2/c^3/((a*x-1)/(a*x+1))^(3/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{(a^4x^4-3\,a^3x^3-15\,a^2x^2-17\,ax-6)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
-1/35*(a^4*x^4 - 3*a^3*x^3 - 15*a^2*x^2 - 17*a*x - 6)*sqrt((a*x - 1)/(a*x
+ 1))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \frac{1}{\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**3,x)
```

output
```
-Integral(1/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a**
3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 6*a**2*x**2*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1
))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{7(ax-1)}{ax+1}-5}{70\,ac^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
-1/70*(7*(a*x - 1)/(a*x + 1) - 5)/(a*c^3*((a*x - 1)/(a*x + 1))^(7/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(59) = 118.

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.87

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{2\left(35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^5x^5+35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4x^4+70\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3+14\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2\right.}{35\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^7ac^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
2/35*(35*(a + sqrt(a^2 - 1/x^2))^5*x^5 + 35*(a + sqrt(a^2 - 1/x^2))^4*x^4
+ 70*(a + sqrt(a^2 - 1/x^2))^3*x^3 + 14*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 7*
(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^7*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 14.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{a\,x-1}{5\,(a\,x+1)}-\frac{1}{7}}{2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

input
```
int(1/((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
-((a*x - 1)/(5*(a*x + 1)) - 1/7)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.96

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\sqrt{ax-1}\,a^3x^3-3\sqrt{ax-1}\,a^2x^2+3\sqrt{ax-1}\,ax-\sqrt{ax-1}-\sqrt{ax+1}\,a^3x^3+4\sqrt{ax+1}\,a^2x^2+11\sqrt{ax}}{35\sqrt{ax-1}\,a\,c^3\,(a^3x^3-3a^2x^2+3ax-1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x)
```

output
```
(sqrt(a*x - 1)*a**3*x**3 - 3*sqrt(a*x - 1)*a**2*x**2 + 3*sqrt(a*x - 1)*a*x
 - sqrt(a*x - 1) - sqrt(a*x + 1)*a**3*x**3 + 4*sqrt(a*x + 1)*a**2*x**2 + 1
1*sqrt(a*x + 1)*a*x + 6*sqrt(a*x + 1))/(35*sqrt(a*x - 1)*a*c**3*(a**3*x**3
 - 3*a**2*x**2 + 3*a*x - 1))
```

3.190 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 100

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx=-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{9c^4\left(a-\frac{1}{x}\right)^7}+\frac{16a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{63c^4\left(a-\frac{1}{x}\right)^6}-\frac{47a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{315c^4\left(a-\frac{1}{x}\right)^5}$$

output
```
-1/9*a^6*(1-1/a^2/x^2)^(5/2)/c^4/(a-1/x)^7+16/63*a^5*(1-1/a^2/x^2)^(5/2)/c
^4/(a-1/x)^6-47/315*a^4*(1-1/a^2/x^2)^(5/2)/c^4/(a-1/x)^5
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.50

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(1+ax)^2\left(47-14ax+2a^2x^2\right)}{315c^4(-1+ax)^5}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

output
```
-1/315*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2*(47 - 14*a*x + 2*a^2*x^2))/(c^
4*(-1 + a*x)^5)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6724, 25, 27, 570, 529, 27, 669, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}dx$$

$$\Big\downarrow \text{6724}$$

$$a^3c^3\int -\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^7\left(a-\frac{1}{x}\right)^7x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-a^3c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^7\left(a-\frac{1}{x}\right)^7x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^7x^2}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^7}{\left(1-\frac{1}{a^2x^2}\right)^{11/2}x^2}d\frac{1}{x}}{a^{11}c^4}$$

$$\Big\downarrow \text{529}$$

$$-\frac{\frac{a^3\left(a+\frac{1}{x}\right)^7}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{9}a\int \frac{a\left(a+\frac{1}{x}\right)^6\left(7a+\frac{9}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}d\frac{1}{x}}{a^{11}c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{a^3\left(a+\frac{1}{x}\right)^7}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{9}a^2\int \frac{\left(a+\frac{1}{x}\right)^6\left(7a+\frac{9}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}d\frac{1}{x}}{a^{11}c^4}$$

$$\Big\downarrow \text{669}$$

$$-\frac{\frac{a^3\left(a+\frac{1}{x}\right)^7}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{9}a^2\left(\frac{16a^2\left(a+\frac{1}{x}\right)^6}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{47}{7}a^2\int\frac{\left(a+\frac{1}{x}\right)^5}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}\right)}{a^{11}c^4}$$

$$\Big\downarrow \text{460}$$

$$-\frac{\frac{a^3\left(a+\frac{1}{x}\right)^7}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{9}a^2\left(\frac{16a^2\left(a+\frac{1}{x}\right)^6}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{47a^3\left(a+\frac{1}{x}\right)^5}{35\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)}{a^{11}c^4}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

output
```
-((-1/9*(a^2*((-47*a^3*(a + x^(-1))^5)/(35*(1 - 1/(a^2*x^2))^(5/2)) + (16*
a^2*(a + x^(-1))^6)/(7*(1 - 1/(a^2*x^2))^(7/2)))) + (a^3*(a + x^(-1))^7)/(
9*(1 - 1/(a^2*x^2))^(9/2)))/(a^11*c^4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 669
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*
(p + 1))), x] - Simp[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1)))
Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{\left(2 a^{2} x^{2}-14 a x +47\right) \left(a x +1\right)}{315 \left(a x -1\right)^{3} c^{4} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} a}$	50
default	$-\frac{\left(2 a^{2} x^{2}-14 a x +47\right) \left(a x +1\right)}{315 \left(a x -1\right)^{3} c^{4} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} a}$	50
orering	$-\frac{\left(2 a^{2} x^{2}-14 a x +47\right) \left(a x -1\right) \left(a x +1\right)}{315 a \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(-a c x +c \right)^{4}}$	54
trager	$-\frac{\left(a x +1\right) \left(2 a^{4} x^{4}-10 a^{3} x^{3}+21 a^{2} x^{2}+80 a x +47\right) \sqrt{-\frac{-a x +1}{a x +1}}}{315 a \,c^{4} \left(a x -1\right)^{5}}$	68

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(2*a^2*x^2-14*a*x+47)*(a*x+1)/(a*x-1)^3/c^4/((a*x-1)/(a*x+1))^(3/2)
/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\left(2\,a^5x^5-8\,a^4x^4+11\,a^3x^3+101\,a^2x^2+127\,ax+47\right)\sqrt{\frac{ax-1}{ax+1}}}{315\,\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}$$

input

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

output

```
-1/315*(2*a^5*x^5 - 8*a^4*x^4 + 11*a^3*x^3 + 101*a^2*x^2 + 127*a*x + 47)*s
qrt((a*x - 1)/(a*x + 1))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 1
0*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{1}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{5a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{10a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{10a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{5ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^4}$$

input

```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**4,x)
```

output

```
Integral(1/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 5*a**4
*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 10*a**3*x**3*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 10*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1))/(a*x + 1) + 5*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.55

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\frac{90\,(ax-1)}{ax+1}-\frac{63\,(ax-1)^2}{(ax+1)^2}-35}{1260\,ac^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
1/1260*(90*(a*x - 1)/(a*x + 1) - 63*(a*x - 1)^2/(a*x + 1)^2 - 35)/(a*c^4*(
(a*x - 1)/(a*x + 1))^(9/2))
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.45

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx =$$
$$-\frac{4\left(210\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^6x^6+315\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^5x^5+441\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4x^4+126\left(a+\sqrt{a^2-\right.\right.}{315\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^9ac^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
-4/315*(210*(a + sqrt(a^2 - 1/x^2))^6*x^6 + 315*(a + sqrt(a^2 - 1/x^2))^5*
x^5 + 441*(a + sqrt(a^2 - 1/x^2))^4*x^4 + 126*(a + sqrt(a^2 - 1/x^2))^3*x^
3 + 36*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 9*(a + sqrt(a^2 - 1/x^2))*x + 1)/((
(a + sqrt(a^2 - 1/x^2))*x - 1)^9*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.56

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\frac{(a\,x-1)^2}{5\,(a\,x+1)^2}-\frac{2\,(a\,x-1)}{7\,(a\,x+1)}+\frac{1}{9}}{4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

input
```
int(1/((c - a*c*x)^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
-((a*x - 1)^2/(5*(a*x + 1)^2) - (2*(a*x - 1))/(7*(a*x + 1)) + 1/9)/(4*a*c^
4*((a*x - 1)/(a*x + 1))^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.68

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$$
$$= \frac{2\sqrt{ax-1}\,a^4x^4-8\sqrt{ax-1}\,a^3x^3+12\sqrt{ax-1}\,a^2x^2-8\sqrt{ax-1}\,ax+2\sqrt{ax-1}-2\sqrt{ax+1}\,a^4x^4+10}{315\sqrt{ax-1}\,a\,c^4\,(a^4x^4-4a^3x^3+6a^2x^2-4ax}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x)
```

output
```
(2*sqrt(a*x - 1)*a**4*x**4 - 8*sqrt(a*x - 1)*a**3*x**3 + 12*sqrt(a*x - 1)*
a**2*x**2 - 8*sqrt(a*x - 1)*a*x + 2*sqrt(a*x - 1) - 2*sqrt(a*x + 1)*a**4*x
**4 + 10*sqrt(a*x + 1)*a**3*x**3 - 21*sqrt(a*x + 1)*a**2*x**2 - 80*sqrt(a*
x + 1)*a*x - 47*sqrt(a*x + 1))/(315*sqrt(a*x - 1)*a*c**4*(a**4*x**4 - 4*a*
*3*x**3 + 6*a**2*x**2 - 4*a*x + 1))
```

3.191 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 133

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx=\frac{a^7\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{11c^5\left(a-\frac{1}{x}\right)^8}-\frac{10a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{33c^5\left(a-\frac{1}{x}\right)^7}+\frac{79a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{231c^5\left(a-\frac{1}{x}\right)^6}-\frac{152a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{1155c^5\left(a-\frac{1}{x}\right)^5}$$

output
```
1/11*a^7*(1-1/a^2/x^2)^(5/2)/c^5/(a-1/x)^8-10/33*a^6*(1-1/a^2/x^2)^(5/2)/c
^5/(a-1/x)^7+79/231*a^5*(1-1/a^2/x^2)^(5/2)/c^5/(a-1/x)^6-152/1155*a^4*(1-
1/a^2/x^2)^(5/2)/c^5/(a-1/x)^5
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.44

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(1+ax)^2\left(-152+61ax-16a^2x^2+2a^3x^3\right)}{1155c^5(-1+ax)^6}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^5,x]
```

output
```
-1/1155*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2*(-152 + 61*a*x - 16*a^2*x^2 +
 2*a^3*x^3))/(c^5*(-1 + a*x)^6)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6724, 27, 570, 529, 2166, 27, 669, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}dx$$

$$\Big\downarrow \text{6724}$$

$$a^3c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^8\left(a-\frac{1}{x}\right)^8x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^8x^3}d\frac{1}{x}}{c^5}$$

$$\Big\downarrow \text{570}$$

$$\frac{\int \frac{\left(a+\frac{1}{x}\right)^8}{\left(1-\frac{1}{a^2x^2}\right)^{13/2}x^3}d\frac{1}{x}}{a^{13}c^5}$$

$$\Big\downarrow \text{529}$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^8}{11\left(1-\frac{1}{a^2x^2}\right)^{11/2}}-\frac{1}{11}a\int \frac{\left(a+\frac{1}{x}\right)^7\left(8a^3+\frac{11a^2}{x}+\frac{11a}{x^2}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{11/2}}d\frac{1}{x}}{a^{13}c^5}$$

$$\Big\downarrow \text{2166}$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^8}{11\left(1-\frac{1}{a^2x^2}\right)^{11/2}}-\frac{1}{11}a\left(\frac{10a^4\left(a+\frac{1}{x}\right)^7}{3\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{9}a\int \frac{3a^2\left(a+\frac{1}{x}\right)^6\left(46a+\frac{33}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}d\frac{1}{x}\right)}{a^{13}c^5}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^8}{11\left(1-\frac{1}{a^2x^2}\right)^{11/2}}-\frac{1}{11}a\left(\frac{10a^4\left(a+\frac{1}{x}\right)^7}{3\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{3}a^3\int \frac{\left(a+\frac{1}{x}\right)^6\left(46a+\frac{33}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}d\frac{1}{x}\right)}{a^{13}c^5}$$

$$\Big\downarrow \; 669$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^8}{11\left(1-\frac{1}{a^2x^2}\right)^{11/2}}-\frac{1}{11}a\left(\frac{10a^4\left(a+\frac{1}{x}\right)^7}{3\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{3}a^3\left(\frac{79a^2\left(a+\frac{1}{x}\right)^6}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{152}{7}a^2\int \frac{\left(a+\frac{1}{x}\right)^5}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}\right)\right)}{a^{13}c^5}$$

$$\Big\downarrow \; 460$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^8}{11\left(1-\frac{1}{a^2x^2}\right)^{11/2}}-\frac{1}{11}a\left(\frac{10a^4\left(a+\frac{1}{x}\right)^7}{3\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{3}a^3\left(\frac{79a^2\left(a+\frac{1}{x}\right)^6}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{152a^3\left(a+\frac{1}{x}\right)^5}{35\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)\right)}{a^{13}c^5}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^5,x]
```

output
```
(-1/11*(a*(-1/3*(a^3*((-152*a^3*(a + x^(-1))^5)/(35*(1 - 1/(a^2*x^2))^(5/2
)) + (79*a^2*(a + x^(-1))^6)/(7*(1 - 1/(a^2*x^2))^(7/2)))) + (10*a^4*(a +
x^(-1))^7)/(3*(1 - 1/(a^2*x^2))^(9/2)))) + (a^4*(a + x^(-1))^8)/(11*(1 - 1
/(a^2*x^2))^(11/2)))/(a^13*c^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 669
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*
(p + 1))), x] - Simp[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1)))
Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{(2a^3x^3-16a^2x^2+61ax-152)(ax+1)}{1155(ax-1)^4c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	58
default	$-\frac{(2a^3x^3-16a^2x^2+61ax-152)(ax+1)}{1155(ax-1)^4c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	58
orering	$\frac{(2a^3x^3-16a^2x^2+61ax-152)(ax-1)(ax+1)}{1155a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(-acx+c)^5}$	62
trager	$-\frac{(ax+1)(2a^5x^5-12a^4x^4+31a^3x^3-46a^2x^2-243ax-152)\sqrt{-\frac{-ax+1}{ax+1}}}{1155a\,c^5(ax-1)^6}$	76

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/1155*(2*a^3*x^3-16*a^2*x^2+61*a*x-152)*(a*x+1)/(a*x-1)^4/c^5/((a*x-1)/(
a*x+1))^(3/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{(2\,a^6x^6-10\,a^5x^5+19\,a^4x^4-15\,a^3x^3-289\,a^2x^2-395\,ax-152)\sqrt{\frac{ax-1}{ax+1}}}{1155\,(a^7c^5x^6-6\,a^6c^5x^5+15\,a^5c^5x^4-20\,a^4c^5x^3+15\,a^3c^5x^2-6\,a^2c^5x+ac^5)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

output
```
-1/1155*(2*a^6*x^6 - 10*a^5*x^5 + 19*a^4*x^4 - 15*a^3*x^3 - 289*a^2*x^2 -
395*a*x - 152)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^5*x^6 - 6*a^6*c^5*x^5 + 15
*a^5*c^5*x^4 - 20*a^4*c^5*x^3 + 15*a^3*c^5*x^2 - 6*a^2*c^5*x + a*c^5)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{1}{\frac{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{6a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{15a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{20a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{15a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{6ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\cdots}}{c^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**5,x)
```

output
```
-Integral(1/(a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 6*a**
5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 15*a**4*x**4*sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 20*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1) + 15*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
 + 1) - 6*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\frac{385\,(ax-1)}{ax+1}-\frac{495\,(ax-1)^2}{(ax+1)^2}+\frac{231\,(ax-1)^3}{(ax+1)^3}-105}{9240\,ac^5\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

output
```
-1/9240*(385*(a*x - 1)/(a*x + 1) - 495*(a*x - 1)^2/(a*x + 1)^2 + 231*(a*x
- 1)^3/(a*x + 1)^3 - 105)/(a*c^5*((a*x - 1)/(a*x + 1))^(11/2))
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.24

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{4\left(1155\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^7x^7+2079\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^6x^6+2541\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^5x^5+825\left(a+\sqrt{a^2-\right.\right.}{1155\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x\right.}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

output
```
4/1155*(1155*(a + sqrt(a^2 - 1/x^2))^7*x^7 + 2079*(a + sqrt(a^2 - 1/x^2))^
6*x^6 + 2541*(a + sqrt(a^2 - 1/x^2))^5*x^5 + 825*(a + sqrt(a^2 - 1/x^2))^4
*x^4 + 165*(a + sqrt(a^2 - 1/x^2))^3*x^3 - 55*(a + sqrt(a^2 - 1/x^2))^2*x^
2 + 11*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^11*
a*c^5)
```

Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.54

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\frac{3\,(a\,x-1)^2}{7\,(a\,x+1)^2}-\frac{(a\,x-1)^3}{5\,(a\,x+1)^3}-\frac{a\,x-1}{3\,(a\,x+1)}+\frac{1}{11}}{8\,a\,c^5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}$$

input
```
int(1/((c - a*c*x)^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
((3*(a*x - 1)^2)/(7*(a*x + 1)^2) - (a*x - 1)^3/(5*(a*x + 1)^3) - (a*x - 1)
/(3*(a*x + 1)) + 1/11)/(8*a*c^5*((a*x - 1)/(a*x + 1))^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.53

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx$$
$$= \frac{2\sqrt{ax-1}\,a^5x^5-10\sqrt{ax-1}\,a^4x^4+20\sqrt{ax-1}\,a^3x^3-20\sqrt{ax-1}\,a^2x^2+10\sqrt{ax-1}\,ax-2\sqrt{ax-1}-}{1155\sqrt{ax-1}\,a\,c^5\left(a^5x^5-5a^4x\right.}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x)
```

output
```
(2*sqrt(a*x - 1)*a**5*x**5 - 10*sqrt(a*x - 1)*a**4*x**4 + 20*sqrt(a*x - 1)
*a**3*x**3 - 20*sqrt(a*x - 1)*a**2*x**2 + 10*sqrt(a*x - 1)*a*x - 2*sqrt(a*
x - 1) - 2*sqrt(a*x + 1)*a**5*x**5 + 12*sqrt(a*x + 1)*a**4*x**4 - 31*sqrt(
a*x + 1)*a**3*x**3 + 46*sqrt(a*x + 1)*a**2*x**2 + 243*sqrt(a*x + 1)*a*x +
152*sqrt(a*x + 1))/(1155*sqrt(a*x - 1)*a*c**5*(a**5*x**5 - 5*a**4*x**4 + 1
0*a**3*x**3 - 10*a**2*x**2 + 5*a*x - 1))
```

3.192 $\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx$

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{c^5(1-ax)^4}{a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}$$

output
```
-c^5*(-a*x+1)^4/a+4/5*c^5*(-a*x+1)^5/a-1/6*c^5*(-a*x+1)^6/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{c^5(-1+ax)^4\,(11+14ax+5a^2x^2)}{30a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

output
```
-1/30*(c^5*(-1 + a*x)^4*(11 + 14*a*x + 5*a^2*x^2))/a
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^5 e^{4\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6717}$$

$$\int c^5(1-ax)^5 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{27}$$

$$c^5 \int e^{4\text{arctanh}(ax)}(1-ax)^5dx$$

$$\Big\downarrow \text{6679}$$

$$c^5 \int (1-ax)^3(ax+1)^2dx$$

$$\Big\downarrow \text{49}$$

$$c^5 \int \left((1-ax)^5-4(1-ax)^4+4(1-ax)^3\right) dx$$

$$\Big\downarrow \text{2009}$$

$$c^5\left(-\frac{(1-ax)^6}{6a}+\frac{4(1-ax)^5}{5a}-\frac{(1-ax)^4}{a}\right)$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

output
```
c^5*(-((1 - a*x)^4/a) + (4*(1 - a*x)^5)/(5*a) - (1 - a*x)^6/(6*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{x\left(5a^5x^5-6a^4x^4-15a^3x^3+20a^2x^2+15ax-30\right)c^5}{30}$
default	$c^5\left(-\frac{1}{6}a^5x^6+\frac{1}{5}x^5a^4+\frac{1}{2}a^3x^4-\frac{2}{3}a^2x^3-\frac{1}{2}a\,x^2+x\right)$
orering	$\frac{x\left(5a^5x^5-6a^4x^4-15a^3x^3+20a^2x^2+15ax-30\right)\left(-acx+c\right)^5}{30\left(ax-1\right)^5}$
risch	$-\frac{1}{6}a^5c^5x^6+\frac{1}{5}a^4c^5x^5+\frac{1}{2}a^3c^5x^4-\frac{2}{3}a^2c^5x^3-\frac{1}{2}c^5a\,x^2+c^5x$
parallelrisch	$-\frac{1}{6}a^5c^5x^6+\frac{1}{5}a^4c^5x^5+\frac{1}{2}a^3c^5x^4-\frac{2}{3}a^2c^5x^3-\frac{1}{2}c^5a\,x^2+c^5x$
norman	$\frac{-c^5x+\frac{1}{6}a^2c^5x^3-\frac{7}{6}a^3c^5x^4+\frac{3}{10}a^4c^5x^5+\frac{11}{30}a^5c^5x^6-\frac{1}{6}a^6c^5x^7+\frac{3}{2}c^5a\,x^2}{ax-1}$
meijerg	$-\frac{c^5\left(\frac{ax\left(-20x^6a^6-28a^5x^5-42a^4x^4-70a^3x^3-140a^2x^2-420ax+840\right)}{-120ax+120}+7\ln\left(-ax+1\right)\right)}{a}-\frac{3c^5\left(-\frac{ax\left(-14a^5x^5-21a^4x^4-35a^3x^3-\right.}{70\left(-ax+1\right)}\right.}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/30*x*(5*a^5*x^5-6*a^4*x^4-15*a^3*x^3+20*a^2*x^2+15*a*x-30)*c^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{1}{6}\,a^5c^5x^6+\frac{1}{5}\,a^4c^5x^5+\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3-\frac{1}{2}\,ac^5x^2+c^5x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="fricas")
```

output
```
-1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1
/2*a*c^5*x^2 + c^5*x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx=-\frac{a^5c^5x^6}{6}+\frac{a^4c^5x^5}{5}+\frac{a^3c^5x^4}{2}-\frac{2a^2c^5x^3}{3}-\frac{ac^5x^2}{2}+c^5x$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**5,x)
```

output
```
-a**5*c**5*x**6/6 + a**4*c**5*x**5/5 + a**3*c**5*x**4/2 - 2*a**2*c**5*x**3
/3 - a*c**5*x**2/2 + c**5*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx = -\frac{1}{6}\,a^5c^5x^6+\frac{1}{5}\,a^4c^5x^5+\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3-\frac{1}{2}\,ac^5x^2+c^5x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="maxima")
```

output
```
-1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1
/2*a*c^5*x^2 + c^5*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx = -\frac{\left(5\,c^5+\frac{24\,c^5}{ax-1}+\frac{30\,c^5}{(ax-1)^2}\right)(ax-1)^6}{30\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="giac")
```

output
```
-1/30*(5*c^5 + 24*c^5/(a*x - 1) + 30*c^5/(a*x - 1)^2)*(a*x - 1)^6/a
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx = -\frac{a^5\,c^5\,x^6}{6}+\frac{a^4\,c^5\,x^5}{5}+\frac{a^3\,c^5\,x^4}{2}-\frac{2\,a^2\,c^5\,x^3}{3}-\frac{a\,c^5\,x^2}{2}+c^5\,x$$

input
```
int(((c - a*c*x)^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
c^5*x - (a*c^5*x^2)/2 - (2*a^2*c^5*x^3)/3 + (a^3*c^5*x^4)/2 + (a^4*c^5*x^5
)/5 - (a^5*c^5*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx = \frac{c^5x(-5a^5x^5+6a^4x^4+15a^3x^3-20a^2x^2-15ax+30)}{30}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x)
```

output
```
(c**5*x*( - 5*a**5*x**5 + 6*a**4*x**4 + 15*a**3*x**3 - 20*a**2*x**2 - 15*a
*x + 30))/30
```

3.193 $\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal result

Integrand size = 18, antiderivative size = 32

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=c^4x-\frac{2}{3}a^2c^4x^3+\frac{1}{5}a^4c^4x^5$$

output
```
c^4*x-2/3*a^2*c^4*x^3+1/5*a^4*c^4*x^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=c^4\left(x-\frac{2a^2x^3}{3}+\frac{a^4x^5}{5}\right)$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

output
```
c^4*(x - (2*a^2*x^3)/3 + (a^4*x^5)/5)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 27, 6679, 39, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^4 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int c^4(1-ax)^4 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{ 27}$$

$$c^4\int e^{4\text{arctanh}(ax)}(1-ax)^4dx$$

$$\Big\downarrow \text{ 6679}$$

$$c^4\int (1-ax)^2(ax+1)^2dx$$

$$\Big\downarrow \text{ 39}$$

$$c^4\int \left(1-a^2x^2\right)^2dx$$

$$\Big\downarrow \text{ 210}$$

$$c^4\int \left(a^4x^4-2a^2x^2+1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$c^4\left(\frac{a^4x^5}{5}-\frac{2a^2x^3}{3}+x\right)$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

output
```
c^4*(x - (2*a^2*x^3)/3 + (a^4*x^5)/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

method	result
default	$c^4\left(\frac{1}{5}x^5a^4-\frac{2}{3}a^2x^3+x\right)$
gosper	$\frac{x(3a^4x^4-10a^2x^2+15)c^4}{15}$
risch	$c^4x-\frac{2}{3}a^2c^4x^3+\frac{1}{5}a^4c^4x^5$
parallelrisch	$c^4x-\frac{2}{3}a^2c^4x^3+\frac{1}{5}a^4c^4x^5$
orering	$\frac{x(3a^4x^4-10a^2x^2+15)(-acx+c)^4}{15(ax-1)^4}$
norman	$\frac{-c^4x+a\,c^4x^2+\frac{2}{3}a^2c^4x^3-\frac{2}{3}a^3c^4x^4-\frac{1}{5}a^4c^4x^5+\frac{1}{5}a^5c^4x^6}{ax-1}$
meijerg	$-\frac{c^4\left(-\frac{ax\left(-14a^5x^5-21a^4x^4-35a^3x^3-70a^2x^2-210ax+420\right)}{70(-ax+1)}-6\ln(-ax+1)\right)}{a}-\frac{2c^4\left(\frac{xa\left(-3a^4x^4-5a^3x^3-10a^2x^2-30ax+60\right)}{-12ax+12}+\right.}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
c^4*(1/5*x^5*a^4-2/3*a^2*x^3+x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{1}{5}\,a^4c^4x^5-\frac{2}{3}\,a^2c^4x^3+c^4x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{a^4c^4x^5}{5}-\frac{2a^2c^4x^3}{3}+c^4x$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**4,x)
```

output
```
a**4*c**4*x**5/5 - 2*a**2*c**4*x**3/3 + c**4*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{1}{5}\,a^4c^4x^5-\frac{2}{3}\,a^2c^4x^3+c^4x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{\left(3\,c^4+\frac{15\,c^4}{ax-1}+\frac{20\,c^4}{(ax-1)^2}\right)(ax-1)^5}{15\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
1/15*(3*c^4 + 15*c^4/(a*x - 1) + 20*c^4/(a*x - 1)^2)*(a*x - 1)^5/a
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{c^4\,x\,\left(3\,a^4\,x^4-10\,a^2\,x^2+15\right)}{15}$$

input
```
int(((c - a*c*x)^4*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(c^4*x*(3*a^4*x^4 - 10*a^2*x^2 + 15))/15
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{c^4x(3a^4x^4-10a^2x^2+15)}{15}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x)
```

output
```
(c**4*x*(3*a**4*x**4 - 10*a**2*x**2 + 15))/15
```

3.194 $\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 35

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{2c^3(1+ax)^3}{3a}-\frac{c^3(1+ax)^4}{4a}$$

output
```
2/3*c^3*(a*x+1)^3/a-1/4*c^3*(a*x+1)^4/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{1}{12}c^3x\left(-12-6ax+4a^2x^2+3a^3x^3\right)$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

output
```
-1/12*(c^3*x*(-12 - 6*a*x + 4*a^2*x^2 + 3*a^3*x^3))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^3 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int c^3(1-ax)^3 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{27}$$

$$c^3\int e^{4\text{arctanh}(ax)}(1-ax)^3dx$$

$$\Big\downarrow \text{6679}$$

$$c^3\int (1-ax)(ax+1)^2dx$$

$$\Big\downarrow \text{49}$$

$$c^3\int \left(2(ax+1)^2-(ax+1)^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c^3\left(\frac{2(ax+1)^3}{3a}-\frac{(ax+1)^4}{4a}\right)$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

output
```
c^3*((2*(1 + a*x)^3)/(3*a) - (1 + a*x)^4/(4*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result
gosper	$-\frac{x\left(3a^{3}x^{3}+4a^{2}x^{2}-6ax-12\right)c^{3}}{12}$
default	$c^{3}\left(-\frac{1}{4}a^{3}x^{4}-\frac{1}{3}a^{2}x^{3}+\frac{1}{2}a\,x^{2}+x\right)$
risch	$-\frac{1}{4}a^{3}c^{3}x^{4}-\frac{1}{3}a^{2}c^{3}x^{3}+\frac{1}{2}a\,c^{3}x^{2}+c^{3}x$
parallelrisch	$-\frac{1}{4}a^{3}c^{3}x^{4}-\frac{1}{3}a^{2}c^{3}x^{3}+\frac{1}{2}a\,c^{3}x^{2}+c^{3}x$
orering	$\frac{x\left(3a^{3}x^{3}+4a^{2}x^{2}-6ax-12\right)\left(-acx+c\right)^{3}}{12\left(ax-1\right)^{3}}$
norman	$\frac{-c^{3}x+\frac{1}{2}a\,c^{3}x^{2}+\frac{5}{6}a^{2}c^{3}x^{3}-\frac{1}{12}a^{3}c^{3}x^{4}-\frac{1}{4}c^{3}a^{4}x^{5}}{ax-1}$
meijerg	$-\frac{c^{3}\left(\frac{xa\left(-3a^{4}x^{4}-5a^{3}x^{3}-10a^{2}x^{2}-30ax+60\right)}{-12ax+12}+5\ln\left(-ax+1\right)\right)}{a}-\frac{c^{3}\left(-\frac{ax\left(-5a^{3}x^{3}-10a^{2}x^{2}-30ax+60\right)}{15\left(-ax+1\right)}-4\ln\left(-ax+1\right)\right)}{a}+$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/12*x*(3*a^3*x^3+4*a^2*x^2-6*a*x-12)*c^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{1}{4}\,a^3c^3x^4-\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2+c^3x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{a^3c^3x^4}{4}-\frac{a^2c^3x^3}{3}+\frac{ac^3x^2}{2}+c^3x$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**3,x)
```

output
```
-a**3*c**3*x**4/4 - a**2*c**3*x**3/3 + a*c**3*x**2/2 + c**3*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{4}\,a^3c^3x^4-\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2+c^3x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{\left(3\,c^3+\frac{16\,c^3}{ax-1}+\frac{24\,c^3}{(ax-1)^2}\right)(ax-1)^4}{12\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
-1/12*(3*c^3 + 16*c^3/(a*x - 1) + 24*c^3/(a*x - 1)^2)*(a*x - 1)^4/a
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{a^3\,c^3\,x^4}{4}-\frac{a^2\,c^3\,x^3}{3}+\frac{a\,c^3\,x^2}{2}+c^3\,x$$

input
```
int(((c - a*c*x)^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
c^3*x + (a*c^3*x^2)/2 - (a^2*c^3*x^3)/3 - (a^3*c^3*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{c^3x(-3a^3x^3-4a^2x^2+6ax+12)}{12}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x)
```

output
```
(c**3*x*( - 3*a**3*x**3 - 4*a**2*x**2 + 6*a*x + 12))/12
```

3.195 $\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 17

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2(1+ax)^3}{3a}$$

output
```
1/3*c^2*(a*x+1)^3/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx=c^2\left(x+ax^2+\frac{a^2x^3}{3}\right)$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

output
```
c^2*(x + a*x^2 + (a^2*x^3)/3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6717, 27, 6679, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int c^2(1-ax)^2 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{ 27}$$

$$c^2\int e^{4\text{arctanh}(ax)}(1-ax)^2dx$$

$$\Big\downarrow \text{ 6679}$$

$$c^2\int (ax+1)^2dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{c^2(ax+1)^3}{3a}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

output
```
(c^2*(1 + a*x)^3)/(3*a)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{c^{2}\left(ax+1\right)^{3}}{3a}$	16
gosper	$\frac{x\left(a^{2}x^{2}+3ax+3\right)c^{2}}{3}$	20
parallelrisch	$\frac{1}{3}a^{2}c^{2}x^{3}+a\,c^{2}x^{2}+c^{2}x$	26
orering	$\frac{x\left(a^{2}x^{2}+3ax+3\right)\left(-acx+c\right)^{2}}{3\left(ax-1\right)^{2}}$	33
risch	$\frac{a^{2}c^{2}x^{3}}{3}+a\,c^{2}x^{2}+c^{2}x+\frac{c^{2}}{3a}$	34
norman	$\frac{-\frac{c^{2}}{a}+\frac{2a^{2}c^{2}x^{3}}{3}+\frac{a^{3}c^{2}x^{4}}{3}}{ax-1}$	40
meijerg	$-\frac{c^{2}\left(-\frac{ax\left(-5a^{3}x^{3}-10a^{2}x^{2}-30ax+60\right)}{15\left(-ax+1\right)}-4\ln\left(-ax+1\right)\right)}{a}+\frac{2c^{2}\left(-\frac{ax\left(-3ax+6\right)}{3\left(-ax+1\right)}-2\ln\left(-ax+1\right)\right)}{a}+\frac{c^{2}x}{-ax+1}$	103

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*c^2*(a*x+1)^3/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{3}\,a^2c^2x^3+ac^2x^2+c^2x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{a^2c^2x^3}{3}+ac^2x^2+c^2x$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**2,x)
```

output
```
a**2*c**2*x**3/3 + a*c**2*x**2 + c**2*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{3}\,a^2c^2x^3+ac^2x^2+c^2x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(15) = 30$.

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.35

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{\left(c^2 + \frac{6\,c^2}{ax-1} + \frac{12\,c^2}{(ax-1)^2}\right)(ax-1)^3}{3\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
1/3*(c^2 + 6*c^2/(a*x - 1) + 12*c^2/(a*x - 1)^2)*(a*x - 1)^3/a
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{c^2\,x\,(a^2\,x^2 + 3\,a\,x + 3)}{3}$$

input
```
int(((c - a*c*x)^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(c^2*x*(3*a*x + a^2*x^2 + 3))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{c^2x(a^2x^2 + 3ax + 3)}{3}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x)
```

output
```
(c**2*x*(a**2*x**2 + 3*a*x + 3))/3
```

3.196 $\int e^{4\coth^{-1}(ax)}(c-acx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=-3cx-\frac{1}{2}acx^2-\frac{4c\log(1-ax)}{a}$$

output
```
-3*c*x-1/2*a*c*x^2-4*c*ln(-a*x+1)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=c\left(-3x-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}\right)$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x),x]
```

output
```
c*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{4\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int c(1-ax)e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{ 27}$$

$$c\int e^{4\text{arctanh}(ax)}(1-ax)dx$$

$$\Big\downarrow \text{ 6679}$$

$$c\int \frac{(ax+1)^2}{1-ax}dx$$

$$\Big\downarrow \text{ 49}$$

$$c\int \left(-ax+\frac{4}{1-ax}-3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$c\left(-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}-3x\right)$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a*c*x),x]
```

output
```
c*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$c\left(-\frac{a\,x^{2}}{2}-3x-\frac{4\ln\left(ax-1\right)}{a}\right)$
risch	$-\frac{ac\,x^{2}}{2}-3xc-\frac{4c\ln\left(ax-1\right)}{a}$
parallelrisch	$-\frac{a^{2}c\,x^{2}+6acx+8c\ln\left(ax-1\right)}{2a}$
norman	$\frac{3xc-\frac{5}{2}ac\,x^{2}-\frac{1}{2}a^{2}c\,x^{3}}{ax-1}-\frac{4c\ln\left(ax-1\right)}{a}$
meijerg	$-\frac{c\left(\frac{ax\left(-2a^{2}x^{2}-6ax+12\right)}{-4ax+4}+3\ln\left(-ax+1\right)\right)}{a}+\frac{c\left(-\frac{ax\left(-3ax+6\right)}{3\left(-ax+1\right)}-2\ln\left(-ax+1\right)\right)}{a}+\frac{c\left(\frac{ax}{-ax+1}+\ln\left(-ax+1\right)\right)}{a}+\frac{cx}{-ax+1}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
c*(-1/2*a*x^2-3*x-4/a*ln(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx = -\frac{a^2cx^2+6\,acx+8\,c\log{(ax-1)}}{2\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="fricas")
```

output
```
-1/2*(a^2*c*x^2 + 6*a*c*x + 8*c*log(a*x - 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx = -\frac{acx^2}{2}-3cx-\frac{4c\log{(ax-1)}}{a}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c),x)
```

output
```
-a*c*x**2/2 - 3*c*x - 4*c*log(a*x - 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=-\frac{1}{2}\,acx^2-3\,cx-\frac{4\,c\log\left(ax-1\right)}{a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="maxima")
```

output
```
-1/2*a*c*x^2 - 3*c*x - 4*c*log(a*x - 1)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=-\frac{(ax-1)^2\left(c+\frac{8\,c}{ax-1}\right)}{2\,a}+\frac{4\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="giac")
```

output
```
-1/2*(a*x - 1)^2*(c + 8*c/(a*x - 1))/a + 4*c*log(abs(a*x - 1)/((a*x - 1)^2
*abs(a)))/a
```

Mupad [B] (verification not implemented)

Time = 13.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=-\frac{c\left(8\,\ln\left(a\,x-1\right)+6\,a\,x+a^2\,x^2\right)}{2\,a}$$

input
```
int(((c - a*c*x)*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
-(c*(8*log(a*x - 1) + 6*a*x + a^2*x^2))/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}(c-acx)\,dx=\frac{c(-8\log(ax-1)-a^2x^2-6ax)}{2a}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x)
```

output
```
(c*( - 8*log(a*x - 1) - a**2*x**2 - 6*a*x))/(2*a)
```

3.197 $\int \frac{e^{4\coth^{-1}(ax)}}{c-acx} dx$

Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx} dx = \frac{2}{ac(1-ax)^2} - \frac{4}{ac(1-ax)} - \frac{\log(1-ax)}{ac}$$

output
```
2/a/c/(-a*x+1)^2-4/a/c/(-a*x+1)-ln(-a*x+1)/a/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx} dx = \frac{-2+4ax-(-1+ax)^2\log(1-ax)}{ac(-1+ax)^2}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x),x]
```

output
```
(-2 + 4*a*x - (-1 + a*x)^2*Log[1 - a*x])/(a*c*(-1 + a*x)^2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{c(1-ax)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{e^{4\text{arctanh}(ax)}}{1-ax}dx}{c}$$

$$\Big\downarrow \text{ 6679}$$

$$\frac{\int \frac{(ax+1)^2}{(1-ax)^3}dx}{c}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(-\frac{4}{(ax-1)^2}-\frac{4}{(ax-1)^3}+\frac{1}{1-ax}\right)dx}{c}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{4}{a(1-ax)}+\frac{2}{a(1-ax)^2}-\frac{\log(1-ax)}{a}}{c}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - a*c*x),x]
```

output
```
(2/(a*(1 - a*x)^2) - 4/(a*(1 - a*x)) - Log[1 - a*x]/a)/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

method	result	size
norman	$\frac{2a\,x^2}{c(ax-1)^2}-\frac{\ln(ax-1)}{ac}$	32
risch	$\frac{4x-\frac{2}{a}}{c(ax-1)^2}-\frac{\ln(ax-1)}{ac}$	36
default	$\frac{\frac{4}{(ax-1)a}-\frac{\ln(ax-1)}{a}+\frac{2}{a(ax-1)^2}}{c}$	41
parallelrisch	$\frac{-a^2\ln(ax-1)x^2+2a^2x^2+2a\ln(ax-1)x-\ln(ax-1)}{(ax-1)^2ca}$	56

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
2*a/c*x^2/(a*x-1)^2-1/a/c*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \frac{4\,ax-(a^2x^2-2\,ax+1)\log{(ax-1)}-2}{a^3cx^2-2\,a^2cx+ac}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="fricas")
```

output
```
(4*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x - 1) - 2)/(a^3*c*x^2 - 2*a^2*c*x +
a*c)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{-4ax+2}{a^3cx^2-2a^2cx+ac}-\frac{\log{(ax-1)}}{ac}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c),x)
```

output
```
-(-4*a*x + 2)/(a**3*c*x**2 - 2*a**2*c*x + a*c) - log(a*x - 1)/(a*c)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \frac{2\,(2\,ax-1)}{a^3cx^2-2\,a^2cx+ac}-\frac{\log{(ax-1)}}{ac}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="maxima")
```

output
```
2*(2*a*x - 1)/(a^3*c*x^2 - 2*a^2*c*x + a*c) - log(a*x - 1)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \frac{\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac} + \frac{2\left(\frac{2\,ac}{ax-1} + \frac{ac}{(ax-1)^2}\right)}{a^2c^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="giac")
```

output
```
log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) + 2*(2*a*c/(a*x - 1) + a*c/(a
*x - 1)^2)/(a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \frac{4\,x-\frac{2}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c} - \frac{\ln\left(a\,x-1\right)}{a\,c}$$

input
```
int((a*x + 1)^2/((c - a*c*x)*(a*x - 1)^2),x)
```

output
```
(4*x - 2/a)/(c + a^2*c*x^2 - 2*a*c*x) - log(a*x - 1)/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.31

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \frac{-\log(ax-1)\,a^2x^2 + 2\log(ax-1)\,ax - \log(ax-1) + 2a^2x^2}{ac\,(a^2x^2-2ax+1)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x)
```

output
```
( - log(a*x - 1)*a**2*x**2 + 2*log(a*x - 1)*a*x - log(a*x - 1) + 2*a**2*x*
*2)/(a*c*(a**2*x**2 - 2*a*x + 1))
```

3.198 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 25

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{(1+ax)^3}{6ac^2(1-ax)^3}$$

output
```
1/6*(a*x+1)^3/a/c^2/(-a*x+1)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{(1+ax)^3}{6ac^2(1-ax)^3}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

output
```
(1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6717, 27, 6679, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}dx$$

$$\Big\downarrow \ 6717$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{c^2(1-ax)^2}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{e^{4\text{arctanh}(ax)}}{(1-ax)^2}dx}{c^2}$$

$$\Big\downarrow \ 6679$$

$$\frac{\int \frac{(ax+1)^2}{(1-ax)^4}dx}{c^2}$$

$$\Big\downarrow \ 48$$

$$\frac{(ax+1)^3}{6ac^2(1-ax)^3}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

output
```
(1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{-a\,x^{2}-\frac{1}{3a}}{\left(ax-1\right)^{3} c^{2}}$	24
parallelrisch	$\frac{-a^{2} x^{3}-3x}{3\left(ax-1\right)^{3} c^{2}}$	25
gosper	$-\frac{3a^{2} x^{2}+1}{3\left(ax-1\right)^{3} a\,c^{2}}$	26
orering	$-\frac{x\left(a^{2} x^{2}+3\right)}{3\left(ax-1\right)\left(-acx+c\right)^{2}}$	29
norman	$\frac{-\frac{x}{c}-\frac{a^{2} x^{3}}{3c}}{\left(ax-1\right)^{3} c}$	30
default	$\frac{-\frac{1}{\left(ax-1\right)a}-\frac{2}{a\left(ax-1\right)^{2}}-\frac{4}{3a\left(ax-1\right)^{3}}}{c^{2}}$	42

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
(-a*x^2-1/3/a)/(a*x-1)^3/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(22) = 44$.

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{3\,a^2x^2+1}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
-1/3*(3*a^2*x^2 + 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(19) = 38$.

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{-3a^2x^2-1}{3a^4c^2x^3-9a^3c^2x^2+9a^2c^2x-3ac^2}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**2,x)
```

output
```
(-3*a**2*x**2 - 1)/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2*c**2*x -
3*a*c**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(22) = 44$.

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{3\,a^2x^2+1}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
-1/3*(3*a^2*x^2 + 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(22) = 44$.

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{2}{(acx-c)^2a}-\frac{1}{(acx-c)ac}-\frac{4\,c}{3\,(acx-c)^3a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
-2/((a*c*x - c)^2*a) - 1/((a*c*x - c)*a*c) - 4/3*c/((a*c*x - c)^3*a)
```

Mupad [B] (verification not implemented)

Time = 13.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{3\,a^2\,x^2+1}{3\,a\,c^2\,(a\,x-1)^3}$$

input
```
int((a*x + 1)^2/((c - a*c*x)^2*(a*x - 1)^2),x)
```

output
```
-(3*a^2*x^2 + 1)/(3*a*c^2*(a*x - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{x(-a^2x^2-3)}{3c^2(a^3x^3-3a^2x^2+3ax-1)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x)
```

output
```
(x*( - a**2*x**2 - 3))/(3*c**2*(a**3*x**3 - 3*a**2*x**2 + 3*a*x - 1))
```

3.199 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 52

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx=\frac{1}{ac^3(1-ax)^4}-\frac{4}{3ac^3(1-ax)^3}+\frac{1}{2ac^3(1-ax)^2}$$

output
```
1/a/c^3/(-a*x+1)^4-4/3/a/c^3/(-a*x+1)^3+1/2/a/c^3/(-a*x+1)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx=\frac{1+2ax+3a^2x^2}{6ac^3(-1+ax)^4}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

output
```
(1 + 2*a*x + 3*a^2*x^2)/(6*a*c^3*(-1 + a*x)^4)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{c^3(1-ax)^3}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{e^{4\text{arctanh}(ax)}}{(1-ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{ 6679}$$

$$\frac{\int \frac{(ax+1)^2}{(1-ax)^5}dx}{c^3}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\int \left(-\frac{1}{(ax-1)^3}-\frac{4}{(ax-1)^4}-\frac{4}{(ax-1)^5}\right)dx}{c^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2a(1-ax)^2}-\frac{4}{3a(1-ax)^3}+\frac{1}{a(1-ax)^4}}{c^3}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

output
```
(1/(a*(1 - a*x)^4) - 4/(3*a*(1 - a*x)^3) + 1/(2*a*(1 - a*x)^2))/c^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.52

method	result	size
risch	$\frac{\frac{a\,x^{2}}{2}+\frac{x}{3}+\frac{1}{6a}}{\left(ax-1\right)^{4}c^{3}}$	27
gosper	$\frac{3a^{2}x^{2}+2ax+1}{6\left(ax-1\right)^{4}a\,c^{3}}$	30
parallelrisch	$\frac{-a^{3}x^{4}+4a^{2}x^{3}-3a\,x^{2}+6x}{6\left(ax-1\right)^{4}c^{3}}$	39
default	$\frac{\frac{1}{a\left(ax-1\right)^{4}}+\frac{1}{2a\left(ax-1\right)^{2}}+\frac{4}{3a\left(ax-1\right)^{3}}}{c^{3}}$	41
orering	$\frac{x\left(a^{3}x^{3}-4a^{2}x^{2}+3ax-6\right)}{6\left(ax-1\right)\left(-acx+c\right)^{3}}$	41
norman	$\frac{\frac{x}{c}-\frac{a\,x^{2}}{2c}+\frac{2a^{2}x^{3}}{3c}-\frac{a^{3}x^{4}}{6c}}{\left(ax-1\right)^{4}c^{2}}$	49

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2*a*x^2+1/3*x+1/6/a)/(a*x-1)^4/c^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,a^2x^2+2\,ax+1}{6\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
1/6*(3*a^2*x^2 + 2*a*x + 1)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 -
 4*a^2*c^3*x + a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{-3a^2x^2-2ax-1}{6a^5c^3x^4-24a^4c^3x^3+36a^3c^3x^2-24a^2c^3x+6ac^3}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**3,x)
```

output
```
-(-3*a**2*x**2 - 2*a*x - 1)/(6*a**5*c**3*x**4 - 24*a**4*c**3*x**3 + 36*a**
3*c**3*x**2 - 24*a**2*c**3*x + 6*a*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,a^2x^2+2\,ax+1}{6\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
1/6*(3*a^2*x^2 + 2*a*x + 1)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 -
 4*a^2*c^3*x + a*c^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\frac{3}{(ax-1)^2a}+\frac{8}{(ax-1)^3a}+\frac{6}{(ax-1)^4a}}{6\,c^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
1/6*(3/((a*x - 1)^2*a) + 8/((a*x - 1)^3*a) + 6/((a*x - 1)^4*a))/c^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.56

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3\,a^2\,x^2+2\,a\,x+1}{6\,a\,c^3\,(a\,x-1)^4}$$

input
```
int((a*x + 1)^2/((c - a*c*x)^3*(a*x - 1)^2),x)
```

output
```
(2*a*x + 3*a^2*x^2 + 1)/(6*a*c^3*(a*x - 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{3a^2x^2+2ax+1}{6a\,c^3\,(a^4x^4-4a^3x^3+6a^2x^2-4ax+1)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x)
```

output
```
(3*a**2*x**2 + 2*a*x + 1)/(6*a*c**3*(a**4*x**4 - 4*a**3*x**3 + 6*a**2*x**2
 - 4*a*x + 1))
```

3.200 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{4}{5ac^4(1-ax)^5} - \frac{1}{ac^4(1-ax)^4} + \frac{1}{3ac^4(1-ax)^3}$$

output
```
4/5/a/c^4/(-a*x+1)^5-1/a/c^4/(-a*x+1)^4+1/3/a/c^4/(-a*x+1)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{2+5ax+5a^2x^2}{15ac^4(-1+ax)^5}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

output
```
-1/15*(2 + 5*a*x + 5*a^2*x^2)/(a*c^4*(-1 + a*x)^5)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{c^4(1-ax)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{e^{4\text{arctanh}(ax)}}{(1-ax)^4}dx}{c^4}$$

$$\Big\downarrow \text{ 6679}$$

$$\frac{\int \frac{(ax+1)^2}{(1-ax)^6}dx}{c^4}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\int \left(\frac{1}{(ax-1)^4}+\frac{4}{(ax-1)^5}+\frac{4}{(ax-1)^6}\right)dx}{c^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{3a(1-ax)^3}-\frac{1}{a(1-ax)^4}+\frac{4}{5a(1-ax)^5}}{c^4}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

output
```
(4/(5*a*(1 - a*x)^5) - 1/(a*(1 - a*x)^4) + 1/(3*a*(1 - a*x)^3))/c^4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.51

method	result	size
risch	$\frac{-\frac{a\,x^{2}}{3}-\frac{x}{3}-\frac{2}{15 a}}{\left(a x -1\right)^{5} c^{4}}$	27
gosper	$-\frac{5 a^{2} x^{2}+5 a x +2}{15 \left(a x -1\right)^{5} a \,c^{4}}$	30
default	$\frac{-\frac{1}{a \left(a x -1\right)^{4}}-\frac{4}{5 a \left(a x -1\right)^{5}}-\frac{1}{3 a \left(a x -1\right)^{3}}}{c^{4}}$	42
parallelrisch	$\frac{-2 x^{5} a^{4}+10 a^{3} x^{4}-20 a^{2} x^{3}+15 a \,x^{2}-15 x}{15 \left(a x -1\right)^{5} c^{4}}$	47
orering	$-\frac{x \left(2 a^{4} x^{4}-10 a^{3} x^{3}+20 a^{2} x^{2}-15 a x +15\right)}{15 \left(a x -1\right) \left(-a c x +c \right)^{4}}$	50
norman	$\frac{\frac{a \,x^{2}}{c}-\frac{x}{c}-\frac{4 a^{2} x^{3}}{3 c}+\frac{2 a^{3} x^{4}}{3 c}-\frac{2 a^{4} x^{5}}{15 c}}{\left(a x -1\right)^{5} c^{3}}$	60

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*a*x^2-1/3*x-2/15/a)/(a*x-1)^5/c^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{5\,a^2x^2+5\,ax+2}{15\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
-1/15*(5*a^2*x^2 + 5*a*x + 2)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^
3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{-5a^2x^2-5ax-2}{15a^6c^4x^5-75a^5c^4x^4+150a^4c^4x^3-150a^3c^4x^2+75a^2c^4x-15ac^4}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**4,x)
```

output
```
(-5*a**2*x**2 - 5*a*x - 2)/(15*a**6*c**4*x**5 - 75*a**5*c**4*x**4 + 150*a*
*4*c**4*x**3 - 150*a**3*c**4*x**2 + 75*a**2*c**4*x - 15*a*c**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{5\,a^2x^2+5\,ax+2}{15\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
-1/15*(5*a^2*x^2 + 5*a*x + 2)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^
3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\frac{5}{(ax-1)^3a}+\frac{15}{(ax-1)^4a}+\frac{12}{(ax-1)^5a}}{15\,c^4}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
-1/15*(5/((a*x - 1)^3*a) + 15/((a*x - 1)^4*a) + 12/((a*x - 1)^5*a))/c^4
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.55

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{5\,a^2\,x^2+5\,a\,x+2}{15\,a\,c^4\,{\left(a\,x-1\right)}^5}$$

input
```
int((a*x + 1)^2/((c - a*c*x)^4*(a*x - 1)^2),x)
```

output
```
-(5*a*x + 5*a^2*x^2 + 2)/(15*a*c^4*(a*x - 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{-5a^2x^2-5ax-2}{15a\,c^4\left(a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1\right)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x)
```

output
```
( - 5*a**2*x**2 - 5*a*x - 2)/(15*a*c**4*(a**5*x**5 - 5*a**4*x**4 + 10*a**3
*x**3 - 10*a**2*x**2 + 5*a*x - 1))
```

3.201 $\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 127

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}x-\frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}x^4-\frac{35c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

output
```
20/3*c^3*(1-1/a^2/x^2)^(1/2)*x-27/8*a*c^3*(1-1/a^2/x^2)^(1/2)*x^2+4/3*a^2*
c^3*(1-1/a^2/x^2)^(1/2)*x^3-1/4*a^3*c^3*(1-1/a^2/x^2)^(1/2)*x^4-35/8*c^3*a
rctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.57

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx=\frac{c^3\left(a\sqrt{1-\frac{1}{a^2x^2}}x(160-81ax+32a^2x^2-6a^3x^3)-105\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{24a}$$

input
```
Integrate[(c - a*c*x)^3/E^ArcCoth[a*x],x]
```

output
```
(c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(160 - 81*a*x + 32*a^2*x^2 - 6*a^3*x^3) -
105*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(24*a)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6724, 27, 540, 2338, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^3 e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6724$$

$$\frac{\int \frac{c^4\left(a-\frac{1}{x}\right)^4 x^5}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \; 27$$

$$\frac{c^3\int \frac{\left(a-\frac{1}{x}\right)^4 x^5}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \; 540$$

$$\frac{c^3\left(-\frac{1}{4}\int \frac{\left(16a^3-\frac{27a^2}{x}+\frac{16a}{x^2}-\frac{4}{x^3}\right)x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{4}a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \; 2338$$

$$\frac{c^3\left(\frac{1}{4}\left(\frac{1}{3}\int \frac{\left(81a^2-\frac{80a}{x}+\frac{12}{x^2}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{16}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{4}a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \; 2338$$

$$\frac{c^3\left(\frac{1}{4}\left(\frac{1}{3}\left(-\frac{1}{2}\int \frac{5\left(32a-\frac{21}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{81}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{16}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{4}a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^3\left(\frac{1}{4}\left(\frac{1}{3}\left(-\frac{5}{2}\int \frac{\left(32a-\frac{21}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{81}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{16}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{4}a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{534}$$

$$\frac{c^3\left(\frac{1}{4}\left(\frac{1}{3}\left(-\frac{5}{2}\left(-21\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-32ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{81}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{16}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{4}a^4x^4\sqrt{1\right.}{a}$$

$$\Big\downarrow \text{243}$$

$$\frac{c^3\left(\frac{1}{4}\left(\frac{1}{3}\left(-\frac{5}{2}\left(-\frac{21}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-32ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{81}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{16}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{4}a^4x^4\sqrt{1\right.}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{c^3\left(\frac{1}{4}\left(\frac{1}{3}\left(-\frac{5}{2}\left(21a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-32ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{81}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{16}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right.\right.}{a}$$

$$\Big\downarrow \text{221}$$

$$\frac{c^3\left(\frac{1}{4}\left(\frac{1}{3}\left(-\frac{5}{2}\left(21\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-32ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{81}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{16}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{4}a^4x^4\sqrt{\right.}{a}$$

input
```
Int[(c - a*c*x)^3/E^ArcCoth[a*x],x]
```

output
```
(c^3*(-1/4*(a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4) + ((16*a^3*Sqrt[1 - 1/(a^2*x^2)
]*x^3)/3 + ((-81*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (5*(-32*a*Sqrt[1 - 1/(
a^2*x^2)]*x + 21*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/2)/3)/4))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\left(6 a^{3} x^{3}-32 a^{2} x^{2}+81 a x -160\right) \left(a x +1\right) c^{3} \sqrt{\frac{a x -1}{a x +1}}}{24 a}-\frac{35 \ln \left(\frac{a^{2} x}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{2}-1}\right) c^{3} \sqrt{\frac{a x -1}{a x +1}}\, \sqrt{\left(a x -1\right) \left(a x +1\right)}}{8 \sqrt{a^{2}}\, \left(a x -1\right)}$
default	$\frac{\sqrt{\frac{a x -1}{a x +1}}\, \left(a x +1\right) c^{3} \left(-6 \sqrt{a^{2}}\, \left(a^{2} x^{2}-1\right)^{\frac{3}{2}} a x +32 \left(\left(a x -1\right) \left(a x +1\right)\right)^{\frac{3}{2}} \sqrt{a^{2}}-87 \sqrt{a^{2}}\, \sqrt{a^{2} x^{2}-1}\, a x +192 \sqrt{\left(a x -1\right) \left(a x +1\right)}\, \sqrt{a^{2}}+87 \ln \left(\frac{a^{2} x}{\cdots}\right.\right.}{24 a \sqrt{\left(a x -1\right) \left(a x +1\right)}\, \sqrt{a^{2}}}$

input
```
int((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*a^3*x^3-32*a^2*x^2+81*a*x-160)*(a*x+1)/a*c^3*((a*x-1)/(a*x+1))^(1
/2)-35/8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^3*((a*x-1)/
(a*x+1))^(1/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx =$$

$$-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^3x^4-26\,a^3c^3x^3+49\,a^2c^3x^2-79\,ac^3x-1\right.}{24\,a}$$

input
```
integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) + (6*a^4*c^3*x^4 - 26*a^3*c^3*x^3 + 49*a^2*c^3*x^2 -
79*a*c^3*x - 160*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = -c^3\left(\int 3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\left(-3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx + \int a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\left(-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx\right)$$

input
```
integrate((-a*c*x+c)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
-c**3*(Integral(3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-3*
a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**3*x**3*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x +
 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(107) = 214$.

Time = 0.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.74

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{1}{24}\left(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} - \frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2} - \frac{2\left(279\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}} - 511\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 385\,c\right.}{\frac{4\,(ax-1)a^2}{ax+1} - \frac{6\,(ax-1)^2a^2}{(ax+1)^2} + \frac{4\,(ax-1)^3a}{(ax+1)^3}}\right.$$

input
```
integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/24*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(279*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 51
1*c^3*((a*x - 1)/(a*x + 1))^(5/2) + 385*c^3*((a*x - 1)/(a*x + 1))^(3/2) -
105*c^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1
)^2*a^2/(a*x + 1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x
 + 1)^4 - a^2))*a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.86

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = \frac{35\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{8\,|a|} + \frac{1}{24}\sqrt{a^2x^2-1}\left(\frac{160\,c^3\mathrm{sgn}(ax+1)}{a} - \left(81\,c^3\mathrm{sgn}(ax+1) + 2\left(3\,a^2c^3x\mathrm{sgn}(ax+1) - 16\,ac^3\mathrm{sgn}(ax+1)\right)\right.\right.$$

input
```
integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
35/8*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/2
4*sqrt(a^2*x^2 - 1)*(160*c^3*sgn(a*x + 1)/a - (81*c^3*sgn(a*x + 1) + 2*(3*
a^2*c^3*x*sgn(a*x + 1) - 16*a*c^3*sgn(a*x + 1))*x)*x)
```

Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.39

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx$$
$$=\frac{\frac{35\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{385\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{511\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{12}-\frac{93\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a-\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{6\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{35\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

input
```
int((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((35*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 - (385*c^3*((a*x - 1)/(a*x + 1))^(
3/2))/12 + (511*c^3*((a*x - 1)/(a*x + 1))^(5/2))/12 - (93*c^3*((a*x - 1)/(
a*x + 1))^(7/2))/4)/(a - (4*a*(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*
x + 1)^2 - (4*a*(a*x - 1)^3)/(a*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4) -
(35*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.79

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx$$
$$=\frac{c^3\Big(-6\sqrt{ax+1}\,\sqrt{ax-1}\,a^3x^3+32\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2-81\sqrt{ax+1}\,\sqrt{ax-1}\,ax+160\sqrt{ax+1}\,\sqrt{ax}}{24a}$$

input
```
int((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(c**3*( - 6*sqrt(a*x + 1)*sqrt(a*x - 1)*a**3*x**3 + 32*sqrt(a*x + 1)*sqrt(
a*x - 1)*a**2*x**2 - 81*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 160*sqrt(a*x + 1
)*sqrt(a*x - 1) - 210*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))))/(24*a
)
```

3.202 $\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 100

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{5c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

output
```
11/3*c^2*(1-1/a^2/x^2)^(1/2)*x-3/2*a*c^2*(1-1/a^2/x^2)^(1/2)*x^2+1/3*a^2*c
^2*(1-1/a^2/x^2)^(1/2)*x^3-5/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.64

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}x(22-9ax+2a^2x^2)-15\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

input
```
Integrate[(c - a*c*x)^2/E^ArcCoth[a*x],x]
```

output
```
(c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(22 - 9*a*x + 2*a^2*x^2) - 15*Log[a*(1 + S
qrt[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6724, 25, 27, 540, 2338, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2 e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int -\frac{c^3\left(a-\frac{1}{x}\right)^3x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{c^3\left(a-\frac{1}{x}\right)^3x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\int \frac{\left(a-\frac{1}{x}\right)^3x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{540}$$

$$-\frac{c^2\left(-\frac{1}{3}\int \frac{\left(9a^2-\frac{11a}{x}+\frac{3}{x^2}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{c^2\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{\left(22a-\frac{15}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{9}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{534}$$

$$-\frac{c^2\left(\frac{1}{3}\left(\frac{1}{2}\left(-15\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-22ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{9}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{243}$$

$$-\frac{c^2\left(\frac{1}{3}\left(\frac{1}{2}\left(-\frac{15}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-22ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{9}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c^2\left(\frac{1}{3}\left(\frac{1}{2}\left(15a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-22ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{9}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c^2\left(\frac{1}{3}\left(\frac{1}{2}\left(15\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-22ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{9}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

input
```
Int[(c - a*c*x)^2/E^ArcCoth[a*x],x]
```

output
```
-((c^2*(-1/3*(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^3) + ((9*a^2*Sqrt[1 - 1/(a^2*x^2
)]*x^2)/2 + (-22*a*Sqrt[1 - 1/(a^2*x^2)]*x + 15*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]])/2)/3))/a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

method	result
risch	$\frac{(2a^2x^2-9ax+22)(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{6a}-\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^2\left(2((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-9\sqrt{a^2}\sqrt{a^2x^2-1}\,ax+24\sqrt{(ax-1)(ax+1)}\sqrt{a^2}+9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a-24a\ln\left(\right.\right.}{6\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$

input
```
int((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*x^2-9*a*x+22)*(a*x+1)/a*c^2*((a*x-1)/(a*x+1))^(1/2)-5/2*ln(a^2*
x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c^2*((a*x-1)/(a*x+1))^(1/2)/(
a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3c^2x^3-7\,a^2c^2x^2+13\,ac^2x+22\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

input
```
integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^2*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (2*a^3*c^2*x^3 - 7*a^2*c^2*x^2 + 13*a*c^2*x + 22*c^2)*
sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = c^2\left(\int\left(-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx + \int a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx\right)$$

input
```
integrate((-a*c*x+c)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c**2*(Integral(-2*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**
2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(84) = 168.

Time = 0.03 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.81

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{1}{6}a\left(\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{2\left(33\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-40\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15\,c^2\sqrt{\frac{ax}{ax}}\right.}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right.$$

input
```
integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/6*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 + 2*(33*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 40*c
^2*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x
 - 1)*a^2/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x
 + 1)^3 - a^2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{5\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{2\,|a|} + \frac{1}{6}\sqrt{a^2x^2-1}\left(\left(2\,ac^2x\operatorname{sgn}(ax+1)-9\,c^2\operatorname{sgn}(ax+1)\right)x+\frac{22\,c^2\operatorname{sgn}(ax+1)}{a}\right)$$

input
```
integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
5/2*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/6*
sqrt(a^2*x^2 - 1)*((2*a*c^2*x*sgn(a*x + 1) - 9*c^2*sgn(a*x + 1))*x + 22*c^
2*sgn(a*x + 1)/a)
```

Mupad [B] (verification not implemented)

Time = 13.73 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.40

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{5\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{40\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+11\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}} - \frac{5\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(5*c^2*((a*x - 1)/(a*x + 1))^(1/2) - (40*c^2*((a*x - 1)/(a*x + 1))^(3/2))/
3 + 11*c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (
3*a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (5*c^2*atanh
(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{c^2\left(2\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2-9\sqrt{ax+1}\,\sqrt{ax-1}\,ax+22\sqrt{ax+1}\,\sqrt{ax-1}-30\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{6a}$$

input
```
int((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(c**2*(2*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 - 9*sqrt(a*x + 1)*sqrt(a*x
- 1)*a*x + 22*sqrt(a*x + 1)*sqrt(a*x - 1) - 30*log((sqrt(a*x - 1) + sqrt(a
*x + 1))/sqrt(2))))/(6*a)
```

3.203 $\int e^{-\coth^{-1}(ax)}(c-acx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx=2c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}x^2-\frac{3c\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

output
```
2*c*(1-1/a^2/x^2)^(1/2)*x-1/2*a*c*(1-1/a^2/x^2)^(1/2)*x^2-3/2*c*arctanh((1
-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx=-\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-4+ax)+3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{2a}$$

input
```
Integrate[(c - a*c*x)/E^ArcCoth[a*x],x]
```

output
```
-1/2*(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-4 + a*x) + 3*Log[a*(1 + Sqrt[1 - 1/(a
^2*x^2)])*x]))/a
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6724, 27, 540, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int \frac{c^2\left(a-\frac{1}{x}\right)^2x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{\left(a-\frac{1}{x}\right)^2x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{540}$$

$$\frac{c\left(-\frac{1}{2}\int \frac{\left(4a-\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{534}$$

$$\frac{c\left(\frac{1}{2}\left(3\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+4ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{243}$$

$$\frac{c\left(\frac{1}{2}\left(\frac{3}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+4ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{c\left(\frac{1}{2}\left(4ax\sqrt{1-\frac{1}{a^2x^2}}-3a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

$$\Big\downarrow \text{221}$$

$$\frac{c\left(\frac{1}{2}\left(4ax\sqrt{1-\frac{1}{a^2x^2}}-3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

input
```
Int[(c - a*c*x)/E^ArcCoth[a*x],x]
```

output
```
(c*(-1/2*(a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a*Sqrt[1 - 1/(a^2*x^2)]*x -
3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/2))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{(ax-4)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{2a}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{2\sqrt{a^2}\,(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c\left(-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+4\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-4a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{2\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$

input
```
int((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x-4)*(a*x+1)/a*c*((a*x-1)/(a*x+1))^(1/2)-3/2*ln(a^2*x/(a^2)^(1/2)+
(a^2*x^2-1)^(1/2))/(a^2)^(1/2)*c*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))
^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx = -\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2cx^2-3\,acx-4\,c)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

input
```
integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) + (a^2*c*x^2 - 3*a*c*x - 4*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx = -c\left(\int ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx + \int\left(-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx\right)$$

input
```
integrate((-a*c*x+c)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
-c*(Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*
x/(a*x + 1) - 1/(a*x + 1)), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(55) = 110$.

Time = 0.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.08

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx = \frac{1}{2}\,a\left(\frac{2\left(5\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} - 3\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1} - \frac{(ax-1)^2a^2}{(ax+1)^2} - a^2} - \frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} + \frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
1/2*a*(2*(5*c*((a*x - 1)/(a*x + 1))^(3/2) - 3*c*sqrt((a*x - 1)/(a*x + 1)))
/(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 3*c*log
(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) -
1)/a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx = \frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{2\,|a|} - \frac{1}{2}\sqrt{a^2x^2-1}\left(cx\operatorname{sgn}(ax+1) - \frac{4\,c\operatorname{sgn}(ax+1)}{a}\right)$$

input
```
integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
3/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/2*sq
rt(a^2*x^2 - 1)*(c*x*sgn(a*x + 1) - 4*c*sgn(a*x + 1)/a)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.48

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx=\frac{3\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-5\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a-\frac{2\,a\,(a\,x-1)}{a\,x+1}+\frac{a\,(a\,x-1)^2}{(a\,x+1)^2}}-\frac{3\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - a*c*x)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(3*c*((a*x - 1)/(a*x + 1))^(1/2) - 5*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (
2*a*(a*x - 1))/(a*x + 1) + (a*(a*x - 1)^2)/(a*x + 1)^2) - (3*c*atanh(((a*x
 - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx =\frac{c\left(-\sqrt{ax+1}\,\sqrt{ax-1}\,ax+4\sqrt{ax+1}\,\sqrt{ax-1}-6\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{2a}$$

input
```
int((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(c*( - sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 4*sqrt(a*x + 1)*sqrt(a*x - 1) - 6
*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))))/(2*a)
```

3.204 $\int \frac{e^{-\coth^{-1}(ax)}}{c-acx} dx$

Optimal result

Integrand size = 18, antiderivative size = 23

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx} dx = -\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
-arctanh((1-1/a^2/x^2)^(1/2))/a/c
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx} dx = -\frac{\log\left(ax\left(1+\sqrt{\frac{-1+a^2x^2}{a^2x^2}}\right)\right)}{ac}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)),x]
```

output
```
-(Log[a*x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]/(a*c))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6724, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}dx$$
$$\Big\downarrow \text{6724}$$
$$\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{ac}$$
$$\Big\downarrow \text{243}$$
$$\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2ac}$$
$$\Big\downarrow \text{73}$$
$$-\frac{a\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}}{c}$$
$$\Big\downarrow \text{221}$$
$$-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a*c*x)),x]
```

output
```
-(ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(21) = 42$.

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.30

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)}{\sqrt{(ax-1)(ax+1)}\,c\sqrt{a^{2}}}$	76

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
-((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(
1/2))/(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/c/(a^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(21) = 42.

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="fricas")
```

output
```
-(log(sqrt((a*x - 1)/(a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))
/(a*c)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax-1}\,dx}{c}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c),x)
```

output
```
-Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x - 1), x)/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(21) = 42.

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = -a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
 1)) - 1)/(a^2*c))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = \frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{c|a|}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="giac")
```

output
```
log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c*abs(a))
```

Mupad [B] (verification not implemented)

Time = 13.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x),x)
```

output
```
-(2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{2\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)}{ac}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x)
```

output
```
( - 2*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)))/(a*c)
```

3.205 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2} dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}$$

output
```
-(1-1/a^2/x^2)^(1/2)/c^2/(a-1/x)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2} dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^2(-1+ax)}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^2),x]
```

output
```
-((Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(-1 + a*x)))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6724, 25, 27, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2} \, dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int -\frac{1}{c\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{1}{c\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{ac^2}$$

$$\Big\downarrow \text{460}$$

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^2),x]
```

output
```
-(Sqrt[1 - 1/(a^2*x^2)]/(c^2*(a - x^(-1))))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{(ax-1)a\,c^{2}}$	36
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{(ax-1)a\,c^{2}}$	36
trager	$-\frac{(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{a\,c^{2}(ax-1)}$	38
orering	$-\frac{(ax-1)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a(-acx+c)^{2}}$	40

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/(a*x-1)/a/c^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2x-ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
-(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*x - a*c^2)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-2ax+1}\,dx}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**2,x)
```

output
```
Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 2*a*x + 1), x)/c**
2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{ac^2\sqrt{\frac{ax-1}{ax+1}}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
-1/(a*c^2*sqrt((a*x - 1)/(a*x + 1)))
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(acx-c)^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{1}{a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^2,x)
```

output
```
-1/(a*c^2*((a*x - 1)/(a*x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{-\sqrt{ax-1}-\sqrt{ax+1}}{\sqrt{ax-1}\,a\,c^2}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x)
```

output
```
( - (sqrt(a*x - 1) + sqrt(a*x + 1)))/(sqrt(a*x - 1)*a*c**2)
```

3.206 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3} dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2} - \frac{2\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)}$$

output
```
1/3*a*(1-1/a^2/x^2)^(1/2)/c^3/(a-1/x)^2-2/3*(1-1/a^2/x^2)^(1/2)/c^3/(a-1/x
)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3} dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}x(-2+ax)}{3c^3(-1+ax)^2}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^3),x]
```

output
```
-1/3*(Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x))/(c^3*(-1 + a*x)^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6724, 27, 571, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int \frac{1}{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2x}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2x}d\frac{1}{x}}{ac^3}$$

$$\Big\downarrow \text{571}$$

$$\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)^2}-\frac{2}{3}\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{ac^3}$$

$$\Big\downarrow \text{460}$$

$$\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)^2}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)}}{ac^3}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^3),x]
```

output
```
((a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*(a - x^(-1))^2) - (2*a*Sqrt[1 - 1/(a^2*x^2
)])/(3*(a - x^(-1))))/(a*c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 571
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*(n + p + 1))), x] + Simp[n/(2*d*
(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ((LtQ[n, -1] &&  !IGtQ[n + p +
1, 0]) || (LtQ[n, 0] && LtQ[p, -1]) || EqQ[n + 2*p + 2, 0]) && NeQ[n + p +
1, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{\sqrt{\frac{a x -1}{a x +1}}\, \left(a x -2\right) \left(a x +1\right)}{3 \left(a x -1\right)^{2} a \,c^{3}}$	41
default	$-\frac{\sqrt{\frac{a x -1}{a x +1}}\, \left(a x -2\right) \left(a x +1\right)}{3 \left(a x -1\right)^{2} a \,c^{3}}$	41
trager	$-\frac{\left(a x -2\right) \left(a x +1\right) \sqrt{-\frac{-a x +1}{a x +1}}}{3 a \,c^{3} \left(a x -1\right)^{2}}$	43
orering	$\frac{\left(a x -2\right) \left(a x -1\right) \left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}}{3 a \left(-a c x +c \right)^{3}}$	45

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*((a*x-1)/(a*x+1))^(1/2)*(a*x-2)*(a*x+1)/(a*x-1)^2/a/c^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{(a^2x^2-ax-2)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^3x^2-2\,a^2c^3x+ac^3)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
-1/3*(a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^3*x^2 - 2*a^2*c^
3*x + a*c^3)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-3a^2x^2+3ax-1}\,dx}{c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**3,x)
```

output
```
-Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - 3*a**2*x**2 + 3*a
*x - 1), x)/c**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.63

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{3\,(ax-1)}{ax+1}-1}{6\,ac^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
-1/6*(3*(a*x - 1)/(a*x + 1) - 1)/(a*c^3*((a*x - 1)/(a*x + 1))^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^3 ac^3}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
2/3*(3*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^3*a
*c^3)
```

Mupad [B] (verification not implemented)

Time = 13.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.61

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\frac{a\,x-1}{a\,x+1}-\frac{1}{3}}{2\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^3,x)
```

output
```
-((a*x - 1)/(a*x + 1) - 1/3)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\sqrt{ax-1}\,ax-\sqrt{ax-1}-\sqrt{ax+1}\,ax+2\sqrt{ax+1}}{3\sqrt{ax-1}\,a\,c^3\,(ax-1)}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x)
```

output
```
(sqrt(a*x - 1)*a*x - sqrt(a*x - 1) - sqrt(a*x + 1)*a*x + 2*sqrt(a*x + 1))/
(3*sqrt(a*x - 1)*a*c**3*(a*x - 1))
```

3.207 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 95

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx=-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3}+\frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2}-\frac{7\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)}$$

output
```
-1/5*a^2*(1-1/a^2/x^2)^(1/2)/c^4/(a-1/x)^3+8/15*a*(1-1/a^2/x^2)^(1/2)/c^4/
(a-1/x)^2-7/15*(1-1/a^2/x^2)^(1/2)/c^4/(a-1/x)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.45

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}x(7-6ax+2a^2x^2)}{15c^4(-1+ax)^3}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^4),x]
```

output
```
-1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(7 - 6*a*x + 2*a^2*x^2))/(c^4*(-1 + a*x)^3)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.37, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6724, 25, 27, 581, 25, 27, 671, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4} \, dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int -\frac{1}{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3x^2}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{1}{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3x^2}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3x^2}d\frac{1}{x}}{ac^4}$$

$$\Big\downarrow \text{581}$$

$$-\frac{-\int -\frac{a\left(2a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}}{ac^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{a\left(2a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}}{ac^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{2a-\frac{1}{x}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}}{ac^4}$$

$$\Big\downarrow \text{671}$$

$$-\frac{a\left(\frac{7}{5}\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5\left(a-\frac{1}{x}\right)^3}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}}{ac^4}$$

$$\Big\downarrow \text{461}$$

$$-\frac{a\left(\frac{7}{5}\left(\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{3a}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)^2}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5\left(a-\frac{1}{x}\right)^3}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}}{ac^4}$$

$$\Big\downarrow \text{460}$$

$$-\frac{a\left(\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5\left(a-\frac{1}{x}\right)^3}+\frac{7}{5}\left(\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)}\right)\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}}{ac^4}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^4),x]
```

output
```
-((a*((7*((a*Sqrt[1 - 1/(a^2*x^2)])/(3*(a - x^(-1))^2) + Sqrt[1 - 1/(a^2*x
^2)]/(3*(a - x^(-1)))))/5 + (a^2*Sqrt[1 - 1/(a^2*x^2)])/(5*(a - x^(-1))^3)
) - (a^2*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1))^2)/(a*c^4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^2x^2-6ax+7)(ax+1)}{15(ax-1)^3 a\,c^4}$	50
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^2x^2-6ax+7)(ax+1)}{15(ax-1)^3 a\,c^4}$	50
trager	$-\frac{(2a^2x^2-6ax+7)(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^4(ax-1)^3}$	52
orering	$-\frac{(2a^2x^2-6ax+7)(ax-1)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{15a(-acx+c)^4}$	54

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/15*((a*x-1)/(a*x+1))^(1/2)*(2*a^2*x^2-6*a*x+7)*(a*x+1)/(a*x-1)^3/a/c^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{(2\,a^3x^3-4\,a^2x^2+ax+7)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^4c^4x^3-3\,a^3c^4x^2+3\,a^2c^4x-ac^4)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
-1/15*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 7)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^4
*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-4a^3x^3+6a^2x^2-4ax+1}\,dx}{c^4}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**4,x)
```

output
```
Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 4*a**3*x**3 + 6*a*
*2*x**2 - 4*a*x + 1), x)/c**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\frac{10\,(ax-1)}{ax+1}-\frac{15\,(ax-1)^2}{(ax+1)^2}-3}{60\,ac^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
1/60*(10*(a*x - 1)/(a*x + 1) - 15*(a*x - 1)^2/(a*x + 1)^2 - 3)/(a*c^4*((a*
x - 1)/(a*x + 1))^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{4\left(10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2-5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^5ac^4}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
-4/15*(10*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 5*(a + sqrt(a^2 - 1/x^2))*x + 1)
/(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\frac{(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,(a\,x-1)}{3\,(a\,x+1)}+\frac{1}{5}}{4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^4,x)
```

output
```
-((a*x - 1)^2/(a*x + 1)^2 - (2*(a*x - 1))/(3*(a*x + 1)) + 1/5)/(4*a*c^4*((
a*x - 1)/(a*x + 1))^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{2\sqrt{ax-1}\,a^2x^2-4\sqrt{ax-1}\,ax+2\sqrt{ax-1}-2\sqrt{ax+1}\,a^2x^2+6\sqrt{ax+1}\,ax-7\sqrt{ax+1}}{15\sqrt{ax-1}\,a\,c^4\,(a^2x^2-2ax+1)}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x)
```

output
```
(2*sqrt(a*x - 1)*a**2*x**2 - 4*sqrt(a*x - 1)*a*x + 2*sqrt(a*x - 1) - 2*sqr
t(a*x + 1)*a**2*x**2 + 6*sqrt(a*x + 1)*a*x - 7*sqrt(a*x + 1))/(15*sqrt(a*x
 - 1)*a*c**4*(a**2*x**2 - 2*a*x + 1))
```

3.208 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 128

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx=\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4}-\frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3}+\frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2}-\frac{12\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)}$$

output
```
1/7*a^3*(1-1/a^2/x^2)^(1/2)/c^5/(a-1/x)^4-18/35*a^2*(1-1/a^2/x^2)^(1/2)/c^
5/(a-1/x)^3+23/35*a*(1-1/a^2/x^2)^(1/2)/c^5/(a-1/x)^2-12/35*(1-1/a^2/x^2)^
(1/2)/c^5/(a-1/x)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-12+13ax-8a^2x^2+2a^3x^3)}{35c^5(-1+ax)^4}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^5),x]
```

output
```
-1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-12 + 13*a*x - 8*a^2*x^2 + 2*a^3*x^3))/(c^
5*(-1 + a*x)^4)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6724, 27, 581, 25, 27, 671, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int \frac{1}{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^4x^3}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^4x^3}d\frac{1}{x}}{ac^5}$$

$$\Big\downarrow \text{581}$$

$$\frac{\int -\frac{a^2\left(2a-\frac{3}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}}{ac^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}-\int \frac{a^2\left(2a-\frac{3}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{ac^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}-a^2\int \frac{2a-\frac{3}{x}}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{ac^5}$$

$$\Big\downarrow \text{671}$$

$$\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}-a^2\left(\frac{18}{7}\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{7\left(a-\frac{1}{x}\right)^4}\right)}{ac^5}$$

$\Big\downarrow$ 461

$$\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}-a^2\left(\frac{18}{7}\left(\frac{2\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{5a}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{5\left(a-\frac{1}{x}\right)^3}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{7\left(a-\frac{1}{x}\right)^4}\right)}{ac^5}$$

$\Big\downarrow$ 461

$$\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}-a^2\left(\frac{18}{7}\left(\frac{2\left(\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{3a}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)^2}\right)}{5a}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{5\left(a-\frac{1}{x}\right)^3}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{7\left(a-\frac{1}{x}\right)^4}\right)}{ac^5}$$

$\Big\downarrow$ 460

$$\frac{\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{\left(a-\frac{1}{x}\right)^2}-a^2\left(\frac{18}{7}\left(\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{5\left(a-\frac{1}{x}\right)^3}+\frac{2\left(\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{3\left(a-\frac{1}{x}\right)}\right)}{5a}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{7\left(a-\frac{1}{x}\right)^4}\right)}{ac^5}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^5),x]
```

output
```
(-(a^2*((18*((2*((a*Sqrt[1 - 1/(a^2*x^2)])/(3*(a - x^(-1))^2) + Sqrt[1 - 1
/(a^2*x^2)]/(3*(a - x^(-1)))))/(5*a) + (a*Sqrt[1 - 1/(a^2*x^2)])/(5*(a - x
^(-1))^3)))/7 - (a^2*Sqrt[1 - 1/(a^2*x^2)])/(7*(a - x^(-1))^4))) + (a^2*Sq
rt[1 - 1/(a^2*x^2)])/(a - x^(-1))^2)/(a*c^5)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.45

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^3x^3-8a^2x^2+13ax-12)(ax+1)}{35(ax-1)^4c^5a}$	58
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^3x^3-8a^2x^2+13ax-12)(ax+1)}{35(ax-1)^4c^5a}$	58
trager	$-\frac{(2a^3x^3-8a^2x^2+13ax-12)(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^5(ax-1)^4}$	60
orering	$\frac{(2a^3x^3-8a^2x^2+13ax-12)(ax-1)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{35a(-acx+c)^5}$	62

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/35*((a*x-1)/(a*x+1))^(1/2)*(2*a^3*x^3-8*a^2*x^2+13*a*x-12)*(a*x+1)/(a*x
-1)^4/c^5/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\left(2\,a^4x^4-6\,a^3x^3+5\,a^2x^2+ax-12\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^5c^5x^4-4\,a^4c^5x^3+6\,a^3c^5x^2-4\,a^2c^5x+ac^5\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

output
```
-1/35*(2*a^4*x^4 - 6*a^3*x^3 + 5*a^2*x^2 + a*x - 12)*sqrt((a*x - 1)/(a*x +
 1))/(a^5*c^5*x^4 - 4*a^4*c^5*x^3 + 6*a^3*c^5*x^2 - 4*a^2*c^5*x + a*c^5)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1}\,dx}{c^5}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**5,x)
```

output
```
-Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 5*a**4*x**4 + 10*
a**3*x**3 - 10*a**2*x**2 + 5*a*x - 1), x)/c**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\frac{21\,(ax-1)}{ax+1}-\frac{35\,(ax-1)^2}{(ax+1)^2}+\frac{35\,(ax-1)^3}{(ax+1)^3}-5}{280\,ac^5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

output
```
-1/280*(21*(a*x - 1)/(a*x + 1) - 35*(a*x - 1)^2/(a*x + 1)^2 + 35*(a*x - 1)
^3/(a*x + 1)^3 - 5)/(a*c^5*((a*x - 1)/(a*x + 1))^(7/2))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.66

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{4\left(35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3-21\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+7\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)}{35\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^7ac^5}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

output
```
4/35*(35*(a + sqrt(a^2 - 1/x^2))^3*x^3 - 21*(a + sqrt(a^2 - 1/x^2))^2*x^2
+ 7*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^7*a*c^
5)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\frac{{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{3\,\left(a\,x-1\right)}{5\,\left(a\,x+1\right)}+\frac{1}{7}}{8\,a\,c^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^5,x)
```

output
```
((a*x - 1)^2/(a*x + 1)^2 - (a*x - 1)^3/(a*x + 1)^3 - (3*(a*x - 1))/(5*(a*x
 + 1)) + 1/7)/(8*a*c^5*((a*x - 1)/(a*x + 1))^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.03

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx$$
$$= \frac{2\sqrt{ax-1}\,a^3x^3 - 6\sqrt{ax-1}\,a^2x^2 + 6\sqrt{ax-1}\,ax - 2\sqrt{ax-1} - 2\sqrt{ax+1}\,a^3x^3 + 8\sqrt{ax+1}\,a^2x^2 - 13\sqrt{}}{35\sqrt{ax-1}\,a\,c^5\,(a^3x^3 - 3a^2x^2 + 3ax - 1)}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x)
```

output
```
(2*sqrt(a*x - 1)*a**3*x**3 - 6*sqrt(a*x - 1)*a**2*x**2 + 6*sqrt(a*x - 1)*a
*x - 2*sqrt(a*x - 1) - 2*sqrt(a*x + 1)*a**3*x**3 + 8*sqrt(a*x + 1)*a**2*x*
*2 - 13*sqrt(a*x + 1)*a*x + 12*sqrt(a*x + 1))/(35*sqrt(a*x - 1)*a*c**5*(a*
*3*x**3 - 3*a**2*x**2 + 3*a*x - 1))
```

3.209 $\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal result

Integrand size = 18, antiderivative size = 91

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx=16c^4x-\frac{4c^4(1-ax)^2}{a}-\frac{4c^4(1-ax)^3}{3a}-\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}-\frac{32c^4\log(1+ax)}{a}$$

output
```
16*c^4*x-4*c^4*(-a*x+1)^2/a-4/3*c^4*(-a*x+1)^3/a-1/2*c^4*(-a*x+1)^4/a-1/5*
c^4*(-a*x+1)^5/a-32*c^4*ln(a*x+1)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{c^4(-181+930ax-390a^2x^2+160a^3x^3-45a^4x^4+6a^5x^5-960\log(1+ax))}{30a}$$

input
```
Integrate[(c - a*c*x)^4/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^4*(-181 + 930*a*x - 390*a^2*x^2 + 160*a^3*x^3 - 45*a^4*x^4 + 6*a^5*x^5
- 960*Log[1 + a*x]))/(30*a)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^4 e^{-2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int c^4 e^{-2\text{arctanh}(ax)}(1-ax)^4 dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^4 \int e^{-2\text{arctanh}(ax)}(1-ax)^4 dx$$

$$\Big\downarrow \text{ 6679}$$

$$-c^4 \int \frac{(1-ax)^5}{ax+1} dx$$

$$\Big\downarrow \text{ 49}$$

$$-c^4 \int \left(-(1-ax)^4-2(1-ax)^3-4(1-ax)^2-8(1-ax)+\frac{32}{ax+1}-16\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-c^4 \left(\frac{(1-ax)^5}{5a}+\frac{(1-ax)^4}{2a}+\frac{4(1-ax)^3}{3a}+\frac{4(1-ax)^2}{a}+\frac{32\log(ax+1)}{a}-16x\right)$$

input
```
Int[(c - a*c*x)^4/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c^4*(-16*x + (4*(1 - a*x)^2)/a + (4*(1 - a*x)^3)/(3*a) + (1 - a*x)^4/(2*
a) + (1 - a*x)^5/(5*a) + (32*Log[1 + a*x])/a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.55

method	result
default	$c^{4}\left(\frac{x^{5}a^{4}}{5}-\frac{3a^{3}x^{4}}{2}+\frac{16a^{2}x^{3}}{3}-13a\,x^{2}+31x-\frac{32\ln(ax+1)}{a}\right)$
norman	$31c^{4}x-13a\,c^{4}x^{2}+\frac{16a^{2}c^{4}x^{3}}{3}-\frac{3a^{3}c^{4}x^{4}}{2}+\frac{a^{4}c^{4}x^{5}}{5}-\frac{32c^{4}\ln(ax+1)}{a}$
risch	$31c^{4}x-13a\,c^{4}x^{2}+\frac{16a^{2}c^{4}x^{3}}{3}-\frac{3a^{3}c^{4}x^{4}}{2}+\frac{a^{4}c^{4}x^{5}}{5}-\frac{32c^{4}\ln(ax+1)}{a}$
parallelrisch	$-\frac{-6a^{5}c^{4}x^{5}+45a^{4}c^{4}x^{4}-160a^{3}c^{4}x^{3}+390a^{2}c^{4}x^{2}-930a\,c^{4}x+960c^{4}\ln(ax+1)}{30a}$
meijerg	$\frac{c^{4}\left(\frac{xa\left(12a^{4}x^{4}-15a^{3}x^{3}+20a^{2}x^{2}-30ax+60\right)}{60}-\ln(ax+1)\right)}{a}-\frac{5c^{4}\left(-\frac{ax\left(-15a^{3}x^{3}+20a^{2}x^{2}-30ax+60\right)}{60}+\ln(ax+1)\right)}{a}+\frac{10c^{4}\left(\right.}{}$

input
```
int((-a*c*x+c)^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^4*(1/5*x^5*a^4-3/2*a^3*x^4+16/3*a^2*x^3-13*a*x^2+31*x-32*ln(a*x+1)/a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{4}\,dx$$
$$=\frac{6\,a^{5}c^{4}x^{5}-45\,a^{4}c^{4}x^{4}+160\,a^{3}c^{4}x^{3}-390\,a^{2}c^{4}x^{2}+930\,ac^{4}x-960\,c^{4}\log\left(ax+1\right)}{30\,a}$$

input
```
integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/30*(6*a^5*c^4*x^5 - 45*a^4*c^4*x^4 + 160*a^3*c^4*x^3 - 390*a^2*c^4*x^2 +
 930*a*c^4*x - 960*c^4*log(a*x + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{a^4c^4x^5}{5}-\frac{3a^3c^4x^4}{2}+\frac{16a^2c^4x^3}{3}-13ac^4x^2+31c^4x-\frac{32c^4\log(ax+1)}{a}$$

input
```
integrate((-a*c*x+c)**4*(a*x-1)/(a*x+1),x)
```

output
```
a**4*c**4*x**5/5 - 3*a**3*c**4*x**4/2 + 16*a**2*c**4*x**3/3 - 13*a*c**4*x**
*2 + 31*c**4*x - 32*c**4*log(a*x + 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx=\frac{1}{5}a^4c^4x^5-\frac{3}{2}a^3c^4x^4+\frac{16}{3}a^2c^4x^3-13\,ac^4x^2+31\,c^4x-\frac{32\,c^4\log(ax+1)}{a}$$

input
```
integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/5*a^4*c^4*x^5 - 3/2*a^3*c^4*x^4 + 16/3*a^2*c^4*x^3 - 13*a*c^4*x^2 + 31*c
^4*x - 32*c^4*log(a*x + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx$$
$$=-\frac{32\,c^4\log(|ax+1|)}{a}+\frac{6\,a^9c^4x^5-45\,a^8c^4x^4+160\,a^7c^4x^3-390\,a^6c^4x^2+930\,a^5c^4x}{30\,a^5}$$

input
```
integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-32*c^4*log(abs(a*x + 1))/a + 1/30*(6*a^9*c^4*x^5 - 45*a^8*c^4*x^4 + 160*a
^7*c^4*x^3 - 390*a^6*c^4*x^2 + 930*a^5*c^4*x)/a^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx=31\,c^4\,x-13\,a\,c^4\,x^2+\frac{16\,a^2\,c^4\,x^3}{3}-\frac{3\,a^3\,c^4\,x^4}{2}+\frac{a^4\,c^4\,x^5}{5}-\frac{32\,c^4\,\ln(a\,x+1)}{a}$$

input
```
int(((c - a*c*x)^4*(a*x - 1))/(a*x + 1),x)
```

output
```
31*c^4*x - 13*a*c^4*x^2 + (16*a^2*c^4*x^3)/3 - (3*a^3*c^4*x^4)/2 + (a^4*c^
4*x^5)/5 - (32*c^4*log(a*x + 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx$$
$$=\frac{c^4(-960\log(ax+1)+6a^5x^5-45a^4x^4+160a^3x^3-390a^2x^2+930ax)}{30a}$$

input
```
int((-a*c*x+c)^4*(a*x-1)/(a*x+1),x)
```

output
```
(c**4*( - 960*log(a*x + 1) + 6*a**5*x**5 - 45*a**4*x**4 + 160*a**3*x**3 -
390*a**2*x**2 + 930*a*x))/(30*a)
```

3.210 $\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 73

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx=8c^3x-\frac{2c^3(1-ax)^2}{a}-\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}-\frac{16c^3\log(1+ax)}{a}$$

output
```
8*c^3*x-2*c^3*(-a*x+1)^2/a-2/3*c^3*(-a*x+1)^3/a-1/4*c^3*(-a*x+1)^4/a-16*c^
3*ln(a*x+1)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{c^3(35-180ax+66a^2x^2-20a^3x^3+3a^4x^4+192\log(1+ax))}{12a}$$

input
```
Integrate[(c - a*c*x)^3/E^(2*ArcCoth[a*x]),x]
```

output
```
-1/12*(c^3*(35 - 180*a*x + 66*a^2*x^2 - 20*a^3*x^3 + 3*a^4*x^4 + 192*Log[1
 + a*x]))/a
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^3 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int c^3 e^{-2\text{arctanh}(ax)}(1-ax)^3dx$$

$$\Big\downarrow \text{27}$$

$$-c^3\int e^{-2\text{arctanh}(ax)}(1-ax)^3dx$$

$$\Big\downarrow \text{6679}$$

$$-c^3\int \frac{(1-ax)^4}{ax+1}dx$$

$$\Big\downarrow \text{49}$$

$$-c^3\int \left(-(1-ax)^3-2(1-ax)^2-4(1-ax)+\frac{16}{ax+1}-8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c^3\left(\frac{(1-ax)^4}{4a}+\frac{2(1-ax)^3}{3a}+\frac{2(1-ax)^2}{a}+\frac{16\log(ax+1)}{a}-8x\right)$$

input
```
Int[(c - a*c*x)^3/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c^3*(-8*x + (2*(1 - a*x)^2)/a + (2*(1 - a*x)^3)/(3*a) + (1 - a*x)^4/(4*a
) + (16*Log[1 + a*x])/a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58

method	result
default	$c^3\left(-\frac{a^3x^4}{4}+\frac{5a^2x^3}{3}-\frac{11a\,x^2}{2}+15x-\frac{16\ln(ax+1)}{a}\right)$
norman	$15c^3x-\frac{11a\,c^3x^2}{2}+\frac{5a^2c^3x^3}{3}-\frac{a^3c^3x^4}{4}-\frac{16c^3\ln(ax+1)}{a}$
risch	$15c^3x-\frac{11a\,c^3x^2}{2}+\frac{5a^2c^3x^3}{3}-\frac{a^3c^3x^4}{4}-\frac{16c^3\ln(ax+1)}{a}$
parallelrisch	$-\frac{3a^4c^3x^4-20a^3c^3x^3+66a^2c^3x^2-180a\,c^3x+192c^3\ln(ax+1)}{12a}$
meijerg	$-\frac{c^3\left(-\frac{ax\left(-15a^3x^3+20a^2x^2-30ax+60\right)}{60}+\ln(ax+1)\right)}{a}+\frac{4c^3\left(\frac{ax\left(4a^2x^2-6ax+12\right)}{12}-\ln(ax+1)\right)}{a}-\frac{6c^3\left(-\frac{ax(-3ax+6)}{6}+\ln(\right.}{a}$

input
```
int((-a*c*x+c)^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/4*a^3*x^4+5/3*a^2*x^3-11/2*a*x^2+15*x-16*ln(a*x+1)/a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{3\,a^4c^3x^4-20\,a^3c^3x^3+66\,a^2c^3x^2-180\,ac^3x+192\,c^3\log(ax+1)}{12\,a}$$

input
```
integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
-1/12*(3*a^4*c^3*x^4 - 20*a^3*c^3*x^3 + 66*a^2*c^3*x^2 - 180*a*c^3*x + 192
*c^3*log(a*x + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{a^3c^3x^4}{4}+\frac{5a^2c^3x^3}{3}-\frac{11ac^3x^2}{2}+15c^3x-\frac{16c^3\log(ax+1)}{a}$$

input
```
integrate((-a*c*x+c)**3*(a*x-1)/(a*x+1),x)
```

output
```
-a**3*c**3*x**4/4 + 5*a**2*c**3*x**3/3 - 11*a*c**3*x**2/2 + 15*c**3*x - 16
*c**3*log(a*x + 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{1}{4}\,a^3c^3x^4+\frac{5}{3}\,a^2c^3x^3-\frac{11}{2}\,ac^3x^2+15\,c^3x-\frac{16\,c^3\log(ax+1)}{a}$$

input
```
integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-1/4*a^3*c^3*x^4 + 5/3*a^2*c^3*x^3 - 11/2*a*c^3*x^2 + 15*c^3*x - 16*c^3*lo
g(a*x + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx=-\frac{16\,c^3\log(|ax+1|)}{a}-\frac{3\,a^7c^3x^4-20\,a^6c^3x^3+66\,a^5c^3x^2-180\,a^4c^3x}{12\,a^4}$$

input
```
integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-16*c^3*log(abs(a*x + 1))/a - 1/12*(3*a^7*c^3*x^4 - 20*a^6*c^3*x^3 + 66*a^
5*c^3*x^2 - 180*a^4*c^3*x)/a^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx = 15\,c^3\,x-\frac{11\,a\,c^3\,x^2}{2}+\frac{5\,a^2\,c^3\,x^3}{3}-\frac{a^3\,c^3\,x^4}{4}-\frac{16\,c^3\,\ln\left(a\,x+1\right)}{a}$$

input
```
int(((c - a*c*x)^3*(a*x - 1))/(a*x + 1),x)
```

output
```
15*c^3*x - (11*a*c^3*x^2)/2 + (5*a^2*c^3*x^3)/3 - (a^3*c^3*x^4)/4 - (16*c^
3*log(a*x + 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx = \frac{c^3(-192\log(ax+1)-3a^4x^4+20a^3x^3-66a^2x^2+180ax)}{12a}$$

input
```
int((-a*c*x+c)^3*(a*x-1)/(a*x+1),x)
```

output
```
(c**3*( - 192*log(a*x + 1) - 3*a**4*x***4 + 20*a**3*x***3 - 66*a**2*x***2 + 1
80*a*x))/(12*a)
```

3.211 $\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 55

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx = 4c^2x-\frac{c^2(1-ax)^2}{a}-\frac{c^2(1-ax)^3}{3a}-\frac{8c^2\log(1+ax)}{a}$$

output
```
4*c^2*x-c^2*(-a*x+1)^2/a-1/3*c^2*(-a*x+1)^3/a-8*c^2*ln(a*x+1)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{c^2(-4+21ax-6a^2x^2+a^3x^3-24\log(1+ax))}{3a}$$

input
```
Integrate[(c - a*c*x)^2/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^2*(-4 + 21*a*x - 6*a^2*x^2 + a^3*x^3 - 24*Log[1 + a*x]))/(3*a)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int c^2 e^{-2\text{arctanh}(ax)}(1-ax)^2dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^2\int e^{-2\text{arctanh}(ax)}(1-ax)^2dx$$

$$\Big\downarrow \text{ 6679}$$

$$-c^2\int \frac{(1-ax)^3}{ax+1}dx$$

$$\Big\downarrow \text{ 49}$$

$$-c^2\int \left(-(1-ax)^2-2(1-ax)+\frac{8}{ax+1}-4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-c^2\left(\frac{(1-ax)^3}{3a}+\frac{(1-ax)^2}{a}+\frac{8\log(ax+1)}{a}-4x\right)$$

input
```
Int[(c - a*c*x)^2/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c^2*(-4*x + (1 - a*x)^2/a + (1 - a*x)^3/(3*a) + (8*Log[1 + a*x])/a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

method	result	size
default	$c^{2}\left(\frac{a^{2} x^{3}}{3}-2 a \,x^{2}+7 x -\frac{8 \ln \left(a x +1\right)}{a}\right)$	34
norman	$7 c^{2} x -2 a \,c^{2} x^{2}+\frac{a^{2} c^{2} x^{3}}{3}-\frac{8 c^{2} \ln \left(a x +1\right)}{a}$	42
risch	$7 c^{2} x -2 a \,c^{2} x^{2}+\frac{a^{2} c^{2} x^{3}}{3}-\frac{8 c^{2} \ln \left(a x +1\right)}{a}$	42
parallelrisch	$-\frac{-a^{3} c^{2} x^{3}+6 c^{2} a^{2} x^{2}-21 a \,c^{2} x +24 c^{2} \ln \left(a x +1\right)}{3 a}$	47
meijerg	$\frac{c^{2}\left(\frac{a x \left(4 a^{2} x^{2}-6 a x +12\right)}{12}-\ln \left(a x +1\right)\right)}{a}-\frac{3 c^{2}\left(-\frac{a x \left(-3 a x +6\right)}{6}+\ln \left(a x +1\right)\right)}{a}+\frac{3 c^{2}\left(a x -\ln \left(a x +1\right)\right)}{a}-\frac{c^{2} \ln \left(a x +1\right)}{a}$	95

input
```
int((-a*c*x+c)^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^2*(1/3*a^2*x^3-2*a*x^2+7*x-8*ln(a*x+1)/a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{a^3c^2x^3-6\,a^2c^2x^2+21\,ac^2x-24\,c^2\log\left(ax+1\right)}{3\,a}$$

input
```
integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/3*(a^3*c^2*x^3 - 6*a^2*c^2*x^2 + 21*a*c^2*x - 24*c^2*log(a*x + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{a^2c^2x^3}{3}-2ac^2x^2+7c^2x-\frac{8c^2\log\left(ax+1\right)}{a}$$

input
```
integrate((-a*c*x+c)**2*(a*x-1)/(a*x+1),x)
```

output
```
a**2*c**2*x**3/3 - 2*a*c**2*x**2 + 7*c**2*x - 8*c**2*log(a*x + 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{3}\,a^2c^2x^3-2\,ac^2x^2+7\,c^2x-\frac{8\,c^2\log\left(ax+1\right)}{a}$$

input
```
integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/3*a^2*c^2*x^3 - 2*a*c^2*x^2 + 7*c^2*x - 8*c^2*log(a*x + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{8\,c^2\log\left(|ax+1|\right)}{a}+\frac{a^5c^2x^3-6\,a^4c^2x^2+21\,a^3c^2x}{3\,a^3}$$

input
```
integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-8*c^2*log(abs(a*x + 1))/a + 1/3*(a^5*c^2*x^3 - 6*a^4*c^2*x^2 + 21*a^3*c^2
*x)/a^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx = 7\,c^2\,x-2\,a\,c^2\,x^2+\frac{a^2\,c^2\,x^3}{3}-\frac{8\,c^2\,\ln\left(a\,x+1\right)}{a}$$

input
```
int(((c - a*c*x)^2*(a*x - 1))/(a*x + 1),x)
```

output
```
7*c^2*x - 2*a*c^2*x^2 + (a^2*c^2*x^3)/3 - (8*c^2*log(a*x + 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{c^2(-24\log(ax+1)+a^3x^3-6a^2x^2+21ax)}{3a}$$

input
```
int((-a*c*x+c)^2*(a*x-1)/(a*x+1),x)
```

output
```
(c**2*( - 24*log(a*x + 1) + a**3*x**3 - 6*a**2*x**2 + 21*a*x))/(3*a)
```

3.212 $\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 26

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=3cx-\frac{1}{2}acx^2-\frac{4c\log(1+ax)}{a}$$

output
```
3*c*x-1/2*a*c*x^2-4*c*ln(a*x+1)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=3cx-\frac{1}{2}acx^2-\frac{4c\log(1+ax)}{a}$$

input
```
Integrate[(c - a*c*x)/E^(2*ArcCoth[a*x]),x]
```

output
```
3*c*x - (a*c*x^2)/2 - (4*c*Log[1 + a*x])/a
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6717, 27, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int ce^{-2\text{arctanh}(ax)}(1-ax)dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{-2\text{arctanh}(ax)}(1-ax)dx$$

$$\Big\downarrow \text{6679}$$

$$-c\int \frac{(1-ax)^2}{ax+1}dx$$

$$\Big\downarrow \text{49}$$

$$-c\int \left(ax+\frac{4}{ax+1}-3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{ax^2}{2}+\frac{4\log(ax+1)}{a}-3x\right)$$

input
```
Int[(c - a*c*x)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(-3*x + (a*x^2)/2 + (4*Log[1 + a*x])/a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
default	$c\left(-\frac{a\,x^{2}}{2}+3x-\frac{4\ln\left(ax+1\right)}{a}\right)$	24
norman	$3xc-\frac{ac\,x^{2}}{2}-\frac{4c\ln\left(ax+1\right)}{a}$	25
risch	$3xc-\frac{ac\,x^{2}}{2}-\frac{4c\ln\left(ax+1\right)}{a}$	25
parallelrisch	$-\frac{a^{2}c\,x^{2}-6acx+8c\ln\left(ax+1\right)}{2a}$	29
meijerg	$-\frac{c\left(-\frac{ax\left(-3ax+6\right)}{6}+\ln\left(ax+1\right)\right)}{a}+\frac{2c\left(ax-\ln\left(ax+1\right)\right)}{a}-\frac{c\ln\left(ax+1\right)}{a}$	55

input
```
int((-a*c*x+c)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c*(-1/2*a*x^2+3*x-4*ln(a*x+1)/a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx = -\frac{a^2cx^2-6\,acx+8\,c\log(ax+1)}{2\,a}$$

input
```
integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
-1/2*(a^2*c*x^2 - 6*a*c*x + 8*c*log(a*x + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx = -\frac{acx^2}{2}+3cx-\frac{4c\log(ax+1)}{a}$$

input
```
integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x)
```

output
```
-a*c*x**2/2 + 3*c*x - 4*c*log(a*x + 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx = -\frac{1}{2}\,acx^2+3\,cx-\frac{4\,c\log(ax+1)}{a}$$

input
```
integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-1/2*a*c*x^2 + 3*c*x - 4*c*log(a*x + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{4\,c\log\left(|ax+1|\right)}{a}-\frac{a^3cx^2-6\,a^2cx}{2\,a^2}$$

input
```
integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-4*c*log(abs(a*x + 1))/a - 1/2*(a^3*c*x^2 - 6*a^2*c*x)/a^2
```

Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=-\frac{c\left(8\,\ln\left(a\,x+1\right)-6\,a\,x+a^2\,x^2\right)}{2\,a}$$

input
```
int(((c - a*c*x)*(a*x - 1))/(a*x + 1),x)
```

output
```
-(c*(8*log(a*x + 1) - 6*a*x + a^2*x^2))/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx=\frac{c(-8\log(ax+1)-a^2x^2+6ax)}{2a}$$

input
```
int((-a*c*x+c)*(a*x-1)/(a*x+1),x)
```

output
```
(c*( - 8*log(a*x + 1) - a**2*x**2 + 6*a*x))/(2*a)
```

3.213 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{\log(1+ax)}{ac}$$

output
```
-ln(a*x+1)/a/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{\log(1+ax)}{ac}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)),x]
```

output
```
-(Log[1 + a*x]/(a*c))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6717, 27, 6679, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}dx$$

$$\Big\downarrow \; 6717$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c(1-ax)}dx$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{1-ax}dx}{c}$$

$$\Big\downarrow \; 6679$$

$$-\frac{\int \frac{1}{ax+1}dx}{c}$$

$$\Big\downarrow \; 16$$

$$-\frac{\log(ax+1)}{ac}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)),x]
```

output
```
-(Log[1 + a*x]/(a*c))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{\ln(ax+1)}{ac}$	15
norman	$-\frac{\ln(ax+1)}{ac}$	15
risch	$-\frac{\ln(ax+1)}{ac}$	15
parallelrisch	$-\frac{\ln(ax+1)}{ac}$	15

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
-ln(a*x+1)/a/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log(ax+1)}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="fricas")
```

output
```
-log(a*x + 1)/(a*c)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log(acx+c)}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x)
```

output
```
-log(a*c*x + c)/(a*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\log(ax+1)}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-log(a*x + 1)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{\log(|ax+1|)}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="giac")
```

output
```
-log(abs(a*x + 1))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{\ln(a\,x+1)}{a\,c}$$

input
```
int((a*x - 1)/((c - a*c*x)*(a*x + 1)),x)
```

output
```
-log(a*x + 1)/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{\log(ax+1)}{ac}$$

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c),x)
```

output
```
( - log(a*x + 1))/(a*c)
```

3.214 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\operatorname{arctanh}(ax)}{ac^2}$$

output
```
-arctanh(a*x)/a/c^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\operatorname{arctanh}(ax)}{ac^2}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^2),x]
```

output
```
-(ArcTanh[a*x]/(a*c^2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 39, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}dx$$
$$\Big\downarrow \text{6717}$$
$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c^2(1-ax)^2}dx$$
$$\Big\downarrow \text{27}$$
$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{(1-ax)^2}dx}{c^2}$$
$$\Big\downarrow \text{6679}$$
$$-\frac{\int \frac{1}{(1-ax)(ax+1)}dx}{c^2}$$
$$\Big\downarrow \text{39}$$
$$-\frac{\int \frac{1}{1-a^2x^2}dx}{c^2}$$
$$\Big\downarrow \text{219}$$
$$-\frac{\text{arctanh}(ax)}{ac^2}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^2),x]
```

output
```
-(ArcTanh[a*x]/(a*c^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

method	result	size
parallelrisch	$\frac{\ln(ax-1)-\ln(ax+1)}{2a\,c^2}$	24
default	$\frac{-\frac{\ln(ax+1)}{2a}+\frac{\ln(ax-1)}{2a}}{c^2}$	28
norman	$\frac{\ln(ax-1)}{2a\,c^2}-\frac{\ln(ax+1)}{2a\,c^2}$	30
risch	$-\frac{\ln(ax+1)}{2a\,c^2}+\frac{\ln(-ax+1)}{2a\,c^2}$	31

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(ln(a*x-1)-ln(a*x+1))/a/c^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\log(ax+1)-\log(ax-1)}{2\,ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(log(a*x + 1) - log(a*x - 1))/(a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\frac{\log\left(x-\frac{1}{a}\right)}{2}-\frac{\log\left(x+\frac{1}{a}\right)}{2}}{ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**2,x)
```

output
```
(log(x - 1/a)/2 - log(x + 1/a)/2)/(a*c**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\log(ax+1)}{2\,ac^2}+\frac{\log(ax-1)}{2\,ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*log(a*x + 1)/(a*c^2) + 1/2*log(a*x - 1)/(a*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\log\left(\left|-\frac{2\,c}{acx-c}-1\right|\right)}{2\,ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*log(abs(-2*c/(a*c*x - c) - 1))/(a*c^2)
```

Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{\operatorname{atanh}(a\,x)}{a\,c^2}$$

input
```
int((a*x - 1)/((c - a*c*x)^2*(a*x + 1)),x)
```

output
```
-atanh(a*x)/(a*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\frac{\log(a^2x-a)-\log(a^2x+a)}{2a\,c^2}$$

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x)
```

output
```
(log(a**2*x - a) - log(a**2*x + a))/(2*a*c**2)
```

3.215 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx=-\frac{1}{2ac^3(1-ax)}-\frac{\operatorname{arctanh}(ax)}{2ac^3}$$

output
```
-1/2/a/c^3/(-a*x+1)-1/2*arctanh(a*x)/a/c^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx=-\frac{\frac{1}{2a(1-ax)}+\frac{\operatorname{arctanh}(ax)}{2a}}{c^3}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

output
```
-((1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a))/c^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx$$

$$\Big\downarrow \; 6717$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c^3(1-ax)^3}dx$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{(1-ax)^3}dx}{c^3}$$

$$\Big\downarrow \; 6679$$

$$-\frac{\int \frac{1}{(1-ax)^2(ax+1)}dx}{c^3}$$

$$\Big\downarrow \; 54$$

$$-\frac{\int \left(\frac{1}{2(ax-1)^2}-\frac{1}{2(a^2x^2-1)}\right)dx}{c^3}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{\text{arctanh}(ax)}{2a}+\frac{1}{2a(1-ax)}}{c^3}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

output
```
-((1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a))/c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

method	result	size
default	$\frac{-\frac{\ln(ax+1)}{4a}+\frac{1}{2(ax-1)a}+\frac{\ln(ax-1)}{4a}}{c^3}$	40
risch	$\frac{1}{2a(ax-1)c^3}+\frac{\ln(-ax+1)}{4a\,c^3}-\frac{\ln(ax+1)}{4a\,c^3}$	46
parallelrisch	$\frac{a\ln(ax-1)x-\ln(ax+1)xa+2ax-\ln(ax-1)+\ln(ax+1)}{4c^3(ax-1)a}$	54
norman	$\frac{-\frac{x}{2c}+\frac{a\,x^2}{2c}}{c^2(ax-1)^2}+\frac{\ln(ax-1)}{4a\,c^3}-\frac{\ln(ax+1)}{4a\,c^3}$	57

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-1/4*ln(a*x+1)/a+1/2/(a*x-1)/a+1/4/a*ln(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{(ax-1)\log(ax+1)-(ax-1)\log(ax-1)-2}{4\,(a^2c^3x-ac^3)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*((a*x - 1)*log(a*x + 1) - (a*x - 1)*log(a*x - 1) - 2)/(a^2*c^3*x - a*
c^3)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{1}{2a^2c^3x-2ac^3} - \frac{-\frac{\log\left(x-\frac{1}{a}\right)}{4}+\frac{\log\left(x+\frac{1}{a}\right)}{4}}{ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**3,x)
```

output
```
1/(2*a**2*c**3*x - 2*a*c**3) - (-log(x - 1/a)/4 + log(x + 1/a)/4)/(a*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{1}{2\,(a^2c^3x-ac^3)} - \frac{\log(ax+1)}{4\,ac^3} + \frac{\log(ax-1)}{4\,ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
1/2/(a^2*c^3*x - a*c^3) - 1/4*log(a*x + 1)/(a*c^3) + 1/4*log(a*x - 1)/(a*c
^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\log(|ax+1|)}{4\,ac^3} + \frac{\log(|ax-1|)}{4\,ac^3} + \frac{1}{2\,(ax-1)ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*log(abs(a*x + 1))/(a*c^3) + 1/4*log(abs(a*x - 1))/(a*c^3) + 1/2/((a*x
 - 1)*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{1}{2\,a\,\left(c^3-a\,c^3\,x\right)} - \frac{\operatorname{atanh}(a\,x)}{2\,a\,c^3}$$

input
```
int((a*x - 1)/((c - a*c*x)^3*(a*x + 1)),x)
```

output
```
- 1/(2*a*(c^3 - a*c^3*x)) - atanh(a*x)/(2*a*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.61

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\log(ax-1)\,ax-\log(ax-1)-\log(ax+1)\,ax+\log(ax+1)+2ax}{4a\,c^3\,(ax-1)}$$

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x)
```

output
```
(log(a*x - 1)*a*x - log(a*x - 1) - log(a*x + 1)*a*x + log(a*x + 1) + 2*a*x
)/(4*a*c**3*(a*x - 1))
```

3.216 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 51

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4} dx = -\frac{1}{4ac^4(1-ax)^2} - \frac{1}{4ac^4(1-ax)} - \frac{\operatorname{arctanh}(ax)}{4ac^4}$$

output
```
-1/4/a/c^4/(-a*x+1)^2-1/4/a/c^4/(-a*x+1)-1/4*arctanh(a*x)/a/c^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4} dx = \frac{-2+ax-(-1+ax)^2\operatorname{arctanh}(ax)}{4ac^4(-1+ax)^2}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

output
```
(-2 + a*x - (-1 + a*x)^2*ArcTanh[a*x])/(4*a*c^4*(-1 + a*x)^2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6717, 27, 6679, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c^4(1-ax)^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{(1-ax)^4}dx}{c^4}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{\int \frac{1}{(1-ax)^3(ax+1)}dx}{c^4}$$

$$\Big\downarrow \text{54}$$

$$-\frac{\int \left(\frac{1}{4(ax-1)^2}-\frac{1}{2(ax-1)^3}-\frac{1}{4(a^2x^2-1)}\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\text{arctanh}(ax)}{4a}+\frac{1}{4a(1-ax)}+\frac{1}{4a(1-ax)^2}}{c^4}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

output
```
-((1/(4*a*(1 - a*x)^2) + 1/(4*a*(1 - a*x)) + ArcTanh[a*x]/(4*a))/c^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{\frac{x}{4}-\frac{1}{2a}}{(ax-1)^2 c^4}-\frac{\ln(ax+1)}{8a\,c^4}+\frac{\ln(-ax+1)}{8a\,c^4}$	51
default	$\frac{-\frac{\ln(ax+1)}{8a}-\frac{1}{4a(ax-1)^2}+\frac{1}{4(ax-1)a}+\frac{\ln(ax-1)}{8a}}{c^4}$	52
norman	$\frac{\frac{3x}{4c}-\frac{5a\,x^2}{4c}+\frac{a^2x^3}{2c}}{c^3(ax-1)^3}+\frac{\ln(ax-1)}{8a\,c^4}-\frac{\ln(ax+1)}{8a\,c^4}$	68
parallelrisch	$\frac{a^2\ln(ax-1)x^2-\ln(ax+1)x^2a^2+4a^2x^2-2a\ln(ax-1)x+2\ln(ax+1)xa-6ax+\ln(ax-1)-\ln(ax+1)}{8c^4(ax-1)^2a}$	90

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/4*x-1/2/a)/(a*x-1)^2/c^4-1/8*ln(a*x+1)/a/c^4+1/8*ln(-a*x+1)/a/c^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{2\,ax-(a^2x^2-2\,ax+1)\log(ax+1)+(a^2x^2-2\,ax+1)\log(ax-1)-4}{8\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*lo
g(a*x - 1) - 4)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{ax-2}{4a^3c^4x^2-8a^2c^4x+4ac^4} + \frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**4,x)
```

output
```
(a*x - 2)/(4*a**3*c**4*x**2 - 8*a**2*c**4*x + 4*a*c**4) + (log(x - 1/a)/8
- log(x + 1/a)/8)/(a*c**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{ax-2}{4\,(a^3c^4x^2-2\,a^2c^4x+ac^4)} - \frac{\log(ax+1)}{8\,ac^4} + \frac{\log(ax-1)}{8\,ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
1/4*(a*x - 2)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4) - 1/8*log(a*x + 1)/(a*c^
4) + 1/8*log(a*x - 1)/(a*c^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{\log(|ax+1|)}{8\,ac^4} + \frac{\log(|ax-1|)}{8\,ac^4} + \frac{ax-2}{4\,(ax-1)^2ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
-1/8*log(abs(a*x + 1))/(a*c^4) + 1/8*log(abs(a*x - 1))/(a*c^4) + 1/4*(a*x
- 2)/((a*x - 1)^2*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\frac{x}{4}-\frac{1}{2\,a}}{a^2\,c^4\,x^2-2\,a\,c^4\,x+c^4} - \frac{\mathrm{atanh}(a\,x)}{4\,a\,c^4}$$

input
```
int((a*x - 1)/((c - a*c*x)^4*(a*x + 1)),x)
```

output
```
(x/4 - 1/(2*a))/(c^4 + a^2*c^4*x^2 - 2*a*c^4*x) - atanh(a*x)/(4*a*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.82

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx$$
$$= \frac{\log(ax-1)\,a^2x^2 - 2\log(ax-1)\,ax + \log(ax-1) - \log(ax+1)\,a^2x^2 + 2\log(ax+1)\,ax - \log(ax+1)}{8a\,c^4\left(a^2x^2-2ax+1\right)}$$

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x)
```

output
```
(log(a*x - 1)*a**2*x**2 - 2*log(a*x - 1)*a*x + log(a*x - 1) - log(a*x + 1)
*a**2*x**2 + 2*log(a*x + 1)*a*x - log(a*x + 1) + a**2*x**2 - 3)/(8*a*c**4*
(a**2*x**2 - 2*a*x + 1))
```

3.217 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{1}{6ac^5(1-ax)^3} - \frac{1}{8ac^5(1-ax)^2} - \frac{1}{8ac^5(1-ax)} - \frac{\operatorname{arctanh}(ax)}{8ac^5}$$

output
```
-1/6/a/c^5/(-a*x+1)^3-1/8/a/c^5/(-a*x+1)^2-1/8/a/c^5/(-a*x+1)-1/8*arctanh(
a*x)/a/c^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{10-9ax+3a^2x^2-3(-1+ax)^3\operatorname{arctanh}(ax)}{24ac^5(-1+ax)^3}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

output
```
(10 - 9*a*x + 3*a^2*x^2 - 3*(-1 + a*x)^3*ArcTanh[a*x])/(24*a*c^5*(-1 + a*x
)^3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 27, 6679, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c^5(1-ax)^5}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{(1-ax)^5}dx}{c^5}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{\int \frac{1}{(1-ax)^4(ax+1)}dx}{c^5}$$

$$\Big\downarrow \text{54}$$

$$-\frac{\int \left(\frac{1}{8(ax-1)^2}-\frac{1}{4(ax-1)^3}+\frac{1}{2(ax-1)^4}-\frac{1}{8(a^2x^2-1)}\right)dx}{c^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\text{arctanh}(ax)}{8a}+\frac{1}{8a(1-ax)}+\frac{1}{8a(1-ax)^2}+\frac{1}{6a(1-ax)^3}}{c^5}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

output
```
-((1/(6*a*(1 - a*x)^3) + 1/(8*a*(1 - a*x)^2) + 1/(8*a*(1 - a*x)) + ArcTanh
[a*x]/(8*a))/c^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

method	result
risch	$\frac{\frac{a\,x^{2}}{8}-\frac{3x}{8}+\frac{5}{12a}}{\left(ax-1\right)^{3} c^{5}}+\frac{\ln\left(-ax+1\right)}{16c^{5} a}-\frac{\ln\left(ax+1\right)}{16c^{5} a}$
default	$\frac{-\frac{\ln\left(ax+1\right)}{16a}+\frac{1}{6a\left(ax-1\right)^{3}}-\frac{1}{8a\left(ax-1\right)^{2}}+\frac{1}{8\left(ax-1\right)a}+\frac{\ln\left(ax-1\right)}{16a}}{c^{5}}$
norman	$\frac{-\frac{7x}{8c}+\frac{2a\,x^{2}}{c}-\frac{37a^{2}x^{3}}{24c}+\frac{5a^{3}x^{4}}{12c}}{c^{4}\left(ax-1\right)^{4}}+\frac{\ln\left(ax-1\right)}{16a\,c^{5}}-\frac{\ln\left(ax+1\right)}{16c^{5} a}$
parallelrisch	$\frac{3a^{3}\ln\left(ax-1\right)x^{3}-3\ln\left(ax+1\right)x^{3}a^{3}+20a^{3}x^{3}-9a^{2}\ln\left(ax-1\right)x^{2}+9\ln\left(ax+1\right)x^{2}a^{2}-54a^{2}x^{2}+9a\ln\left(ax-1\right)x-9\ln\left(ax+1\right)xa+42a}{48c^{5}\left(ax-1\right)^{3}a}$

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
(1/8*a*x^2-3/8*x+5/12/a)/(a*x-1)^3/c^5+1/16/c^5/a*ln(-a*x+1)-1/16/c^5/a*ln
(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.64

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{6\,a^2x^2 - 18\,ax - 3\,(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1)\log(ax+1) + 3\,(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1)\log(ax-1) +}{48\,(a^4c^5x^3 - 3\,a^3c^5x^2 + 3\,a^2c^5x - ac^5)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="fricas")
```

output
```
1/48*(6*a^2*x^2 - 18*a*x - 3*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a*x + 1
) + 3*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a*x - 1) + 20)/(a^4*c^5*x^3 -
3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{-3a^2x^2 + 9ax - 10}{24a^4c^5x^3 - 72a^3c^5x^2 + 72a^2c^5x - 24ac^5} - \frac{-\frac{\log\left(x-\frac{1}{a}\right)}{16} + \frac{\log\left(x+\frac{1}{a}\right)}{16}}{ac^5}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**5,x)
```

output
```
-(-3*a**2*x**2 + 9*a*x - 10)/(24*a**4*c**5*x**3 - 72*a**3*c**5*x**2 + 72*a
**2*c**5*x - 24*a*c**5) - (-log(x - 1/a)/16 + log(x + 1/a)/16)/(a*c**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{3\,a^2x^2-9\,ax+10}{24\,(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5)} - \frac{\log{(ax+1)}}{16\,ac^5} + \frac{\log{(ax-1)}}{16\,ac^5}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="maxima")
```

output
```
1/24*(3*a^2*x^2 - 9*a*x + 10)/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x -
 a*c^5) - 1/16*log(a*x + 1)/(a*c^5) + 1/16*log(a*x - 1)/(a*c^5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\log\left(\left|-\frac{2\,c}{acx-c}-1\right|\right)}{16\,ac^5} + \frac{\frac{3\,a^2c^2}{acx-c} - \frac{3\,a^2c^3}{(acx-c)^2} + \frac{4\,a^2c^4}{(acx-c)^3}}{24\,a^3c^6}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="giac")
```

output
```
-1/16*log(abs(-2*c/(a*c*x - c) - 1))/(a*c^5) + 1/24*(3*a^2*c^2/(a*c*x - c)
 - 3*a^2*c^3/(a*c*x - c)^2 + 4*a^2*c^4/(a*c*x - c)^3)/(a^3*c^6)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\frac{a\,x^2}{8} - \frac{3\,x}{8} + \frac{5}{12\,a}}{-a^3\,c^5\,x^3 + 3\,a^2\,c^5\,x^2 - 3\,a\,c^5\,x + c^5} - \frac{\mathrm{atanh}(a\,x)}{8\,a\,c^5}$$

input
```
int((a*x - 1)/((c - a*c*x)^5*(a*x + 1)),x)
```

output
```
- ((a*x^2)/8 - (3*x)/8 + 5/(12*a))/(c^5 + 3*a^2*c^5*x^2 - a^3*c^5*x^3 - 3*
a*c^5*x) - atanh(a*x)/(8*a*c^5)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.99

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx$$
$$= \frac{3\log(ax-1)\,a^3x^3-9\log(ax-1)\,a^2x^2+9\log(ax-1)\,ax-3\log(ax-1)-3\log(ax+1)\,a^3x^3+9\log(}{48a\,c^5\,(a^3x^3-3a^2x^2+3ax-1)}$$

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x)
```

output
```
(3*log(a*x - 1)*a**3*x**3 - 9*log(a*x - 1)*a**2*x**2 + 9*log(a*x - 1)*a*x
- 3*log(a*x - 1) - 3*log(a*x + 1)*a**3*x**3 + 9*log(a*x + 1)*a**2*x**2 - 9
*log(a*x + 1)*a*x + 3*log(a*x + 1) + 2*a**3*x**3 - 12*a*x + 18)/(48*a*c**5
*(a**3*x**3 - 3*a**2*x**2 + 3*a*x - 1))
```

3.218 $\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 129

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}x-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}x^3-\frac{35c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

output
```
16*c^2*(a-1/x)/a^2/(1-1/a^2/x^2)^(1/2)+35/3*c^2*(1-1/a^2/x^2)^(1/2)*x-5/2*
a*c^2*(1-1/a^2/x^2)^(1/2)*x^2+1/3*a^2*c^2*(1-1/a^2/x^2)^(1/2)*x^3-35/2*c^2
*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.60

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx=\frac{1}{6}c^2\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}x(166+55ax-13a^2x^2+2a^3x^3)}{1+ax}-\frac{105\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}\right)$$

input
```
Integrate[(c - a*c*x)^2/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^2*((Sqrt[1 - 1/(a^2*x^2)]*x*(166 + 55*a*x - 13*a^2*x^2 + 2*a^3*x^3))/(1
 + a*x) - (105*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/6
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6724, 25, 27, 528, 2338, 27, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2 e^{-3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \ 6724$$

$$\frac{\int -\frac{c^5\left(a-\frac{1}{x}\right)^5 x^4}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{c^5\left(a-\frac{1}{x}\right)^5 x^4}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{c^2\int \frac{\left(a-\frac{1}{x}\right)^5 x^4}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3}$$

$$\Big\downarrow \ 528$$

$$-\frac{c^2\left(a^2\int \frac{\left(a^3-\frac{5a^2}{x}+\frac{11a}{x^2}-\frac{15}{x^3}\right)x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{16a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \ 2338$$

$$-\frac{c^2\left(a^2\left(-\frac{1}{3}\int\frac{5\left(3a^2-\frac{7a}{x}+\frac{9}{x^2}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{16a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\left(a^2\left(-\frac{5}{3}\int\frac{\left(3a^2-\frac{7a}{x}+\frac{9}{x^2}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{16a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{c^2\left(a^2\left(-\frac{5}{3}\left(-\frac{1}{2}\int\frac{7(2a-\frac{3}{x})x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{3}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{16a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\left(a^2\left(-\frac{5}{3}\left(-\frac{7}{2}\int\frac{(2a-\frac{3}{x})x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{3}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{16a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{534}$$

$$-\frac{c^2\left(a^2\left(-\frac{5}{3}\left(-\frac{7}{2}\left(-3\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-2ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{3}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{16a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{243}$$

$$-\frac{c^2\left(a^2\left(-\frac{5}{3}\left(-\frac{7}{2}\left(-\frac{3}{2}\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-2ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{3}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{16a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right.}{a^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c^2\left(a^2\left(-\frac{5}{3}\left(-\frac{7}{2}\left(3a^2\int\frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-2ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{3}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right.\right.}{a^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c^2\left(a^2\left(-\frac{5}{3}\left(-\frac{7}{2}\left(3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-2ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{3}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{3}a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{16a(a-\ldots}{\sqrt{1-\frac{1}{a^2\ldots}}}\right.}{a^3}$$

input
```
Int[(c - a*c*x)^2/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c^2*((-16*a*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] + a^2*(-1/3*(a^3*Sqrt[1
 - 1/(a^2*x^2)]*x^3) - (5*((-3*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (7*(-2*a
*Sqrt[1 - 1/(a^2*x^2)]*x + 3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/2))/3)))/a^3
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.15

method	result
risch	$\frac{\left(2a^2x^2-15ax+70\right)(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{6a}+\frac{\left(-\frac{35\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2\sqrt{a^2}}+\frac{16\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^2\left(x+\frac{1}{a}\right)}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{\left(15\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^3x^3-2\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2+30\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^2x^2-15\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-4\sqrt{a^2}\,((ax-\right.}{}$

input
```
int((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*x^2-15*a*x+70)*(a*x+1)/a*c^2*((a*x-1)/(a*x+1))^(1/2)+(-35/2*ln(
a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+16/a^2/(x+1/a)*(a^2*(x+1/
a)^2-2*a*(x+1/a))^(1/2))*c^2/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x
+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.81

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(2\,a^3c^2x^3-13\,a^2c^2x^2+55\,ac^2x+166\,c^2)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

input
```
integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) - (2*a^3*c^2*x^3 - 13*a^2*c^2*x^2 + 55*a*c^2*x + 166*c
^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx = c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx + \int\frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx + \int\left(-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx + \int\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

input
```
integrate((-a*c*x+c)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral
(3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-3*a**2*
x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x**3*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.58

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx = -\frac{1}{6}a\left(\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{96\,c^2\sqrt{\frac{ax-1}{ax+1}}}{a^2}+\frac{2\left(87\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-136\,c^2\left(\right.\right.}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}}\right.$$

input
```
integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-1/6*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1)/a^2 - 96*c^2*sqrt((a*x - 1)/(a*x + 1))/a^2 + 2*(
87*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 136*c^2*((a*x - 1)/(a*x + 1))^(3/2) +
 57*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x - 1
)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2))
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx = \int (acx-c)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.26

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{19\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{136\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+29\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}} + \frac{16\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{35\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(19*c^2*((a*x - 1)/(a*x + 1))^(1/2) - (136*c^2*((a*x - 1)/(a*x + 1))^(3/2)
)/3 + 29*c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) +
 (3*a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (16*c^2*((
a*x - 1)/(a*x + 1))^(1/2))/a - (35*c^2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))
/a
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx = \frac{c^2\left(8\sqrt{ax+1}\,\sqrt{ax-1}\,a^3x^3-52\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2+220\sqrt{ax+1}\,\sqrt{ax-1}\,ax+664\sqrt{ax+1}\,\sqrt{ax-1}\right.}{24a\,(ax+1)}$$

input
```
int((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(c**2*(8*sqrt(a*x + 1)*sqrt(a*x - 1)*a**3*x**3 - 52*sqrt(a*x + 1)*sqrt(a*x
 - 1)*a**2*x**2 + 220*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 664*sqrt(a*x + 1)*
sqrt(a*x - 1) - 840*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x - 840
*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 525*a*x + 525))/(24*a*(a*x
 + 1))
```

3.219 $\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+4c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{15c\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

output
```
8*c*(a-1/x)/a^2/(1-1/a^2/x^2)^(1/2)+4*c*(1-1/a^2/x^2)^(1/2)*x-1/2*a*c*(1-1
/a^2/x^2)^(1/2)*x^2-15/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.74

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx=\frac{1}{2}c\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(24+7ax-a^2x^2)}{1+ax}-\frac{15\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}\right)$$

input
```
Integrate[(c - a*c*x)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c*((Sqrt[1 - 1/(a^2*x^2)]*x*(24 + 7*a*x - a^2*x^2))/(1 + a*x) - (15*Log[a
*(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/2
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6724, 27, 528, 2338, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int \frac{c^4\left(a-\frac{1}{x}\right)^4x^3}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{\left(a-\frac{1}{x}\right)^4x^3}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3}$$

$$\Big\downarrow \text{528}$$

$$\frac{c\left(a^2\int \frac{\left(a^2-\frac{4a}{x}+\frac{7}{x^2}\right)x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{8a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{2338}$$

$$\frac{c\left(a^2\left(-\frac{1}{2}\int \frac{\left(8a-\frac{15}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{8a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{534}$$

$$\frac{c\left(a^2\left(\frac{1}{2}\left(15\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+8ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{8a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{c\left(a^2\left(\frac{1}{2}\left(\frac{15}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+8ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{8a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{c\left(a^2\left(\frac{1}{2}\left(8ax\sqrt{1-\frac{1}{a^2x^2}}-15a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{8a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{c\left(a^2\left(\frac{1}{2}\left(8ax\sqrt{1-\frac{1}{a^2x^2}}-15\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{1}{2}a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{8a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^3}$$

input
```
Int[(c - a*c*x)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c*((8*a*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] + a^2*(-1/2*(a^2*Sqrt[1 - 1/(
a^2*x^2)]*x^2) + (8*a*Sqrt[1 - 1/(a^2*x^2)]*x - 15*ArcTanh[Sqrt[1 - 1/(a^2
*x^2)]])/2)))/a^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.48

method	result
risch	$-\frac{(ax-8)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{2a}-\frac{\left(\frac{15\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2\sqrt{a^2}}-\frac{8\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^2\left(x+\frac{1}{a}\right)}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{ax-1}$
default	$-\frac{\left(\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^3x^3+2\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^2x^2-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-16\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^2x^2+16\ln\left(\frac{a^2x+\sqrt{(ax-}{\sqrt{}}\right.\right.}{}$

input
```
int((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x-8)*(a*x+1)/a*c*((a*x-1)/(a*x+1))^(1/2)-(15/2*ln(a^2*x/(a^2)^(1/2
)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)-8/a^2/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))
^(1/2))*c*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx = -\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2cx^2-7\,acx-24\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

input
```
integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) + (a^2*c*x^2 - 7*a*c*x - 24*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx = -c\left(\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx + \int \left(-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx + \int \frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

input
```
integrate((-a*c*x+c)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
-c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-2
*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**2*x**2*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.70

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx = \frac{1}{2}a\left(\frac{2\left(9\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-7\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{16\,c\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

input
```
integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
1/2*a*(2*(9*c*((a*x - 1)/(a*x + 1))^(3/2) - 7*c*sqrt((a*x - 1)/(a*x + 1)))
/(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 15*c*lo
g(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 15*c*log(sqrt((a*x - 1)/(a*x + 1))
- 1)/a^2 + 16*c*sqrt((a*x - 1)/(a*x + 1))/a^2)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx=\int -(acx-c)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.27

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx=\frac{7\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-9\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{15\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{8\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}$$

input
```
int((c - a*c*x)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(7*c*((a*x - 1)/(a*x + 1))^(1/2) - 9*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (
2*a*(a*x - 1))/(a*x + 1) + (a*(a*x - 1)^2)/(a*x + 1)^2) - (15*c*atanh(((a*
x - 1)/(a*x + 1))^(1/2)))/a + (8*c*((a*x - 1)/(a*x + 1))^(1/2))/a
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.23

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx$$
$$=\frac{c\left(-\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2+7\sqrt{ax+1}\,\sqrt{ax-1}\,ax+24\sqrt{ax+1}\,\sqrt{ax-1}-30\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax\right.}{2a\left(ax+1\right)}$$

input
```
int((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(c*( - sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 + 7*sqrt(a*x + 1)*sqrt(a*x -
1)*a*x + 24*sqrt(a*x + 1)*sqrt(a*x - 1) - 30*log((sqrt(a*x - 1) + sqrt(a*x
 + 1))/sqrt(2))*a*x - 30*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 20
*a*x + 20))/(2*a*(a*x + 1))
```

3.220 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
2*(a-1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)-arctanh((1-1/a^2/x^2)^(1/2))/a/c
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx=\frac{\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{1+ax}-\frac{\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{c}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)),x]
```

output
```
((2*Sqrt[1 - 1/(a^2*x^2)]*x)/(1 + a*x) - Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])
*x]/a)/c
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6724, 27, 528, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int \frac{c^2\left(a-\frac{1}{x}\right)^2x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(a-\frac{1}{x}\right)^2x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c}$$

$$\Big\downarrow \text{528}$$

$$\frac{a^2\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}}{a^3c}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}a^2\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+\frac{2a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}}{a^3c}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-a^4\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}}{a^3c}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2a\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-a^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^3c}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)),x]
```

output
```
((2*a*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] - a^2*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]])/(a^3*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(49) = 98$.

Time = 0.12 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.68

method	result
default	$-\frac{\left(-\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^2x^2+\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-2\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,ax+2\ln\left(\right.}{a\sqrt{a^2}\,c(ax-1)\sqrt{(ax-1)(ax+}}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

output
```
-(-((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*a^2*x^2+ln((a^2*x+((a*x-1)*(a*x+1))
^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^3*x^2+((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/
2)-2*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*a*x+2*ln((a^2*x+((a*x-1)*(a*x+1))
^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x-((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)
+a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a*((a*x-1)
/(a*x+1))^(3/2)/(a^2)^(1/2)/c/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2\sqrt{\frac{ax-1}{ax+1}}-\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="fricas")
```

output
```
(2*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) + log(sq
rt((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\right)dx + \int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx}{c}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c),x)
```

output
```
-(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x) + Integr
al(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x))/c
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx = -a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c} - \frac{2\sqrt{\frac{ax-1}{ax+1}}}{a^2c}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
 1)) - 1)/(a^2*c) - 2*sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx = \int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{acx-c}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 13.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/
2)))/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx =\frac{2\sqrt{ax+1}\,\sqrt{ax-1}-2\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax-2\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)+2ax+2}{ac\,(ax+1)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x)
```

output
```
(2*(sqrt(a*x + 1)*sqrt(a*x - 1) - log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt
(2))*a*x - log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + a*x + 1))/(a*c*(
a*x + 1))
```

3.221 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(a-1/x)/a^2/c^2/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2(1+ax)}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^2),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(1 + a*x))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6724, 25, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int -\frac{c\left(a-\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{c\left(a-\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{a-\frac{1}{x}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^2}$$

$$\Big\downarrow \text{453}$$

$$\frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^2),x]
```

output
```
(a - x^(-1))/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
trager	$\frac{\sqrt{-\frac{-ax+1}{ax+1}}}{a\,c^{2}}$	25
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)}{(ax-1)a\,c^{2}}$	35
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)}{(ax-1)a\,c^{2}}$	35
orering	$\frac{(ax-1)(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{a(-acx+c)^{2}}$	39

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a/c^2*(-(-a*x+1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
sqrt((a*x - 1)/(a*x + 1))/(a*c^2)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\,dx}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**2,x)
```

output
```
(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**2*x**2 - a*x
+ 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**
2*x**2 - a*x + 1), x))/c**2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx=\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
sqrt((a*x - 1)/(a*x + 1))/(a*c^2)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^2,x)
```

output
```
((a*x - 1)/(a*x + 1))^(1/2)/(a*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\sqrt{ax+1}\,\sqrt{ax-1}+ax+1}{a\,c^2\,(ax+1)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x)
```

output
```
(sqrt(a*x + 1)*sqrt(a*x - 1) + a*x + 1)/(a*c**2*(a*x + 1))
```

3.222 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{1}{ac^3\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/a/c^3/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}x^2}{c^3(-1+a^2x^2)}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(c^3*(-1 + a^2*x^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6724, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}dx$$

$$\Big\downarrow \text{6724}$$

$$\frac{\int \frac{1}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \text{241}$$

$$\frac{1}{ac^3\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

output
```
1/(a*c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

method	result	size
trager	$\frac{x\sqrt{-\frac{-ax+1}{ax+1}}}{c^3(ax-1)}$	30
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}x(ax+1)}{(ax-1)^2c^3}$	33
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}x(ax+1)}{(ax-1)^2c^3}$	33
orering	$-\frac{(ax-1)(ax+1)x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-acx+c)^3}$	38

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*x/(a*x-1)*(-(-a*x+1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{x\sqrt{\frac{ax-1}{ax+1}}}{ac^3x-c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
x*sqrt((a*x - 1)/(a*x + 1))/(a*c^3*x - c^3)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\right)dx + \int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\,dx}{c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**3,x)
```

output
```
-(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**3*x**3 + 2
*a*x - 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4
- 2*a**3*x**3 + 2*a*x - 1), x))/c**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(19) = 38$.

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{1}{2}a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3} + \frac{1}{a^2c^3\sqrt{\frac{ax-1}{ax+1}}}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
1/2*a*(sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3) + 1/(a^2*c^3*sqrt((a*x - 1)/(a*
x + 1))))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{x\text{sgn}(ax+1)}{\sqrt{a^2x^2-1}c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
x*sgn(a*x + 1)/(sqrt(a^2*x^2 - 1)*c^3)
```

Mupad [B] (verification not implemented)

Time = 13.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\frac{a\,x-1}{a\,x+1}+1}{2\,a\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^3,x)
```

output
```
((a*x - 1)/(a*x + 1) + 1)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.24

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\sqrt{ax-1}\,ax+\sqrt{ax-1}+\sqrt{ax+1}\,ax}{\sqrt{ax-1}\,a\,c^3\,(ax+1)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x)
```

output
```
(sqrt(a*x - 1)*a*x + sqrt(a*x - 1) + sqrt(a*x + 1)*a*x)/(sqrt(a*x - 1)*a*c
**3*(a*x + 1))
```

3.223 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx=\frac{2}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{1}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2}$$

output
```
2/3/a/c^4/(1-1/a^2/x^2)^(1/2)-1/3/a^2/c^4/(1-1/a^2/x^2)^(1/2)/(a-1/x)/x^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x(-1-2ax+2a^2x^2)}{3c^4(-1+ax)^2(1+ax)}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(-1 - 2*a*x + 2*a^2*x^2))/(3*c^4*(-1 + a*x)^2*(1
+ a*x))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6724, 25, 27, 567, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$$

$$\Big\downarrow \; 6724$$

$$\frac{\int -\frac{1}{c\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)x^2}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{1}{c\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)x^2}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{1}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)x^2}d\frac{1}{x}}{a^3c^4}$$

$$\Big\downarrow \; 567$$

$$-\frac{\frac{a}{3x^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}-\frac{2}{3}\int \frac{1}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}d\frac{1}{x}}{a^3c^4}$$

$$\Big\downarrow \; 241$$

$$-\frac{\frac{a}{3x^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}-\frac{2a^2}{3\sqrt{1-\frac{1}{a^2x^2}}}}{a^3c^4}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

output
```
-(((-2*a^2)/(3*Sqrt[1 - 1/(a^2*x^2)]) + a/(3*Sqrt[1 - 1/(a^2*x^2)]*(a - x^
(-1))*x^2))/(a^3*c^4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 567
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[c*x^m*((a + b*x^2)^(p + 1)/(2*a*d*p*(c + d*x))), x] - Simp[m/(2*d*p)
   Int[x^(m - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
^2 + a*d^2, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[m + 2*p + 1, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} \left(2a^{3}x^{3}-3ax-1\right)}{3\left(ax-1\right)^{3} a\,c^{4}}$	45
trager	$\frac{\left(2a^{2}x^{2}-2ax-1\right) \sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^{4} \left(ax-1\right)^{2}}$	47
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} \left(2a^{2}x^{2}-2ax-1\right) \left(ax+1\right)}{3\left(ax-1\right)^{3} a\,c^{4}}$	50
orering	$\frac{\left(2a^{2}x^{2}-2ax-1\right) \left(ax-1\right) \left(ax+1\right) \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3a\left(-acx+c\right)^{4}}$	54

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*((a*x-1)/(a*x+1))^(3/2)*(2*a^3*x^3-3*a*x-1)/(a*x-1)^3/a/c^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{(2\,a^2x^2 - 2\,ax - 1)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^4x^2 - 2\,a^2c^4x + ac^4)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*(2*a^2*x^2 - 2*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^4*x^2 - 2*a^2
*c^4*x + a*c^4)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\right)dx + \int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\,dx}{c^4}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**4,x)
```

output
```
(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 3*a**4*x**4 + 2*
a**3*x**3 + 2*a**2*x**2 - 3*a*x + 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))/(a**5*x**5 - 3*a**4*x**4 + 2*a**3*x**3 + 2*a**2*x**2 - 3*a
*x + 1), x))/c**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{1}{12}a\left(\frac{3\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4} + \frac{\frac{6\,(ax-1)}{ax+1}-1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
1/12*a*(3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^4) + (6*(a*x - 1)/(a*x + 1) - 1
)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2)))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 13.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{-2\,a^2\,x^2+2\,a\,x+1}{\left(3\,a\,c^4-3\,a^3\,c^4\,x^2\right)\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^4,x)
```

output
```
(2*a*x - 2*a^2*x^2 + 1)/((3*a*c^4 - 3*a^3*c^4*x^2)*((a*x - 1)/(a*x + 1))^(
1/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{-2\sqrt{ax-1}\,a^2x^2+2\sqrt{ax-1}+2\sqrt{ax+1}\,a^2x^2-2\sqrt{ax+1}\,ax-\sqrt{ax+1}}{3\sqrt{ax-1}\,a\,c^4\,(a^2x^2-1)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x)
```

output
```
( - 2*sqrt(a*x - 1)*a**2*x**2 + 2*sqrt(a*x - 1) + 2*sqrt(a*x + 1)*a**2*x**
2 - 2*sqrt(a*x + 1)*a*x - sqrt(a*x + 1))/(3*sqrt(a*x - 1)*a*c**4*(a**2*x**
2 - 1))
```

3.224 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 110

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx=\frac{1}{ac^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{a}{5c^5\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2}-\frac{4}{5c^5\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}+\frac{2}{5a^2c^5\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
1/a/c^5/(1-1/a^2/x^2)^(1/2)+1/5*a/c^5/(1-1/a^2/x^2)^(1/2)/(a-1/x)^2-4/5/c^
5/(1-1/a^2/x^2)^(1/2)/(a-1/x)+2/5/a^2/c^5/(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.52

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x(2+ax-4a^2x^2+2a^3x^3)}{5c^5(-1+ax)^3(1+ax)}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x - 4*a^2*x^2 + 2*a^3*x^3))/(5*c^5*(-1 + a
*x)^3*(1 + a*x))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6724, 27, 570, 529, 2166, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5} \, dx$$

$$\Big\downarrow \; 6724$$

$$\frac{\int \frac{1}{c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)^2x^3}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{1}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)^2x^3}d\frac{1}{x}}{a^3c^5}$$

$$\Big\downarrow \; 570$$

$$\frac{\int \frac{\left(a+\frac{1}{x}\right)^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^3}d\frac{1}{x}}{a^7c^5}$$

$$\Big\downarrow \; 529$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^2}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}a\int \frac{\left(a+\frac{1}{x}\right)\left(2a^3+\frac{5a^2}{x}+\frac{5a}{x^2}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^7c^5}$$

$$\Big\downarrow \; 2166$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^2}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}a\left(\frac{4a^4\left(a+\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}a\int \frac{3a^2\left(2a+\frac{5}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)}{a^7c^5}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^2}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}a\left(\frac{4a^4\left(a+\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-a^3\int\frac{2a+\frac{5}{x}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)}{a^7c^5}$$

$$\Big\downarrow \textbf{453}$$

$$\frac{\frac{a^4\left(a+\frac{1}{x}\right)^2}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}a\left(\frac{4a^4\left(a+\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{a^4\left(5a+\frac{2}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^7c^5}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

output
```
(-1/5*(a*((4*a^4*(a + x^(-1)))/(1 - 1/(a^2*x^2))^(3/2) - (a^4*(5*a + 2/x))
/Sqrt[1 - 1/(a^2*x^2)])) + (a^4*(a + x^(-1))^2)/(5*(1 - 1/(a^2*x^2))^(5/2)
))/(a^7*c^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.49

method	result	size
trager	$\frac{\left(2 a^{3} x^{3}-4 a^{2} x^{2}+a x +2\right) \sqrt{-\frac{-a x +1}{a x +1}}}{5 a \,c^{5} \left(a x -1\right)^{3}}$	54
gosper	$\frac{\left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(2 a^{3} x^{3}-4 a^{2} x^{2}+a x +2\right) \left(a x +1\right)}{5 \left(a x -1\right)^{4} c^{5} a}$	57
default	$\frac{\left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(2 a^{4} x^{4}-2 a^{3} x^{3}-3 a^{2} x^{2}+3 a x +2\right)}{5 \left(a x -1\right)^{4} c^{5} a}$	61
orering	$-\frac{\left(2 a^{3} x^{3}-4 a^{2} x^{2}+a x +2\right) \left(a x -1\right) \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{5 a \left(-a c x +c \right)^{5}}$	61

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/5/a/c^5*(2*a^3*x^3-4*a^2*x^2+a*x+2)/(a*x-1)^3*(-(-a*x+1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.70

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{(2\,a^3x^3-4\,a^2x^2+ax+2)\sqrt{\frac{ax-1}{ax+1}}}{5\,(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

output
```
1/5*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 2)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^5*x
^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\right)dx+\int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\,dx}{c^5}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**5,x)
```

output
```
-(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 4*a**5*x**5 + 5
*a**4*x**4 - 5*a**2*x**2 + 4*a*x - 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/(a**6*x**6 - 4*a**5*x**5 + 5*a**4*x**4 - 5*a**2*x**2 + 4*
a*x - 1), x))/c**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{1}{40}\,a\left(\frac{5\sqrt{\frac{ax-1}{ax+1}}}{a^2c^5} - \frac{\frac{5\,(ax-1)}{ax+1} - \frac{15\,(ax-1)^2}{(ax+1)^2} - 1}{a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

output
```
1/40*a*(5*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^5) - (5*(a*x - 1)/(a*x + 1) - 1
5*(a*x - 1)^2/(a*x + 1)^2 - 1)/(a^2*c^5*((a*x - 1)/(a*x + 1))^(5/2)))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^5}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

output
```
integrate(-((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^5, x)
```

Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.46

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{2\,a^3\,x^3 - 4\,a^2\,x^2 + a\,x + 2}{5\,a\,c^5\,(a\,x+1)^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^5,x)
```

output
```
(a*x - 4*a^2*x^2 + 2*a^3*x^3 + 2)/(5*a*c^5*(a*x + 1)^3*((a*x - 1)/(a*x + 1
))^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.19

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx$$
$$= \frac{-2\sqrt{ax-1}\,a^3x^3+2\sqrt{ax-1}\,a^2x^2+2\sqrt{ax-1}\,ax-2\sqrt{ax-1}+2\sqrt{ax+1}\,a^3x^3-4\sqrt{ax+1}\,a^2x^2+\sqrt{}}{5\sqrt{ax-1}\,a\,c^5\left(a^3x^3-a^2x^2-ax+1\right)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x)
```

output
```
( - 2*sqrt(a*x - 1)*a**3*x**3 + 2*sqrt(a*x - 1)*a**2*x**2 + 2*sqrt(a*x - 1
)*a*x - 2*sqrt(a*x - 1) + 2*sqrt(a*x + 1)*a**3*x**3 - 4*sqrt(a*x + 1)*a**2
*x**2 + sqrt(a*x + 1)*a*x + 2*sqrt(a*x + 1))/(5*sqrt(a*x - 1)*a*c**5*(a**3
*x**3 - a**2*x**2 - a*x + 1))
```

3.225 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 143

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx=\frac{1}{ac^6\sqrt{1-\frac{1}{a^2x^2}}}-\frac{a^2}{7c^6\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3}+\frac{24a}{35c^6\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2}-\frac{46}{35c^6\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}+\frac{13}{35a^2c^6\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

output
```
1/a/c^6/(1-1/a^2/x^2)^(1/2)-1/7*a^2/c^6/(1-1/a^2/x^2)^(1/2)/(a-1/x)^3+24/3
5*a/c^6/(1-1/a^2/x^2)^(1/2)/(a-1/x)^2-46/35/c^6/(1-1/a^2/x^2)^(1/2)/(a-1/x
)+13/35/a^2/c^6/(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.46

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-13+4ax+20a^2x^2-24a^3x^3+8a^4x^4)}{35c^6(-1+ax)^4(1+ax)}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^6),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(-13 + 4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^
4))/(35*c^6*(-1 + a*x)^4*(1 + a*x))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6724, 25, 27, 570, 529, 2166, 2166, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx$$

$$\Big\downarrow \ 6724$$

$$\frac{\int -\frac{1}{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)^3x^4}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{1}{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)^3x^4}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{1}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)^3x^4}d\frac{1}{x}}{a^3c^6}$$

$$\Big\downarrow \ 570$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^3}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^4}d\frac{1}{x}}{a^9c^6}$$

$$\Big\downarrow \ 529$$

$$-\frac{\frac{a^5\left(a+\frac{1}{x}\right)^3}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{1}{7}a\int \frac{\left(a+\frac{1}{x}\right)^2\left(3a^4+\frac{7a^3}{x}+\frac{7a^2}{x^2}+\frac{7a}{x^3}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}}{a^9c^6}$$

$$\Big\downarrow \ 2166$$

$$-\frac{\frac{a^5\left(a+\frac{1}{x}\right)^3}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{1}{7}a\left(\frac{24a^5\left(a+\frac{1}{x}\right)^2}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}a\int\frac{\left(a+\frac{1}{x}\right)\left(33a^4+\frac{70a^3}{x}+\frac{35a^2}{x^2}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}\right)}{a^9c^6}$$

$$\Big\downarrow \text{2166}$$

$$-\frac{\frac{a^5\left(a+\frac{1}{x}\right)^3}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{1}{7}a\left(\frac{24a^5\left(a+\frac{1}{x}\right)^2}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}a\left(\frac{46a^5\left(a+\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}a\int\frac{3a^3\left(13a+\frac{35}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)\right)}{a^9c^6}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{a^5\left(a+\frac{1}{x}\right)^3}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{1}{7}a\left(\frac{24a^5\left(a+\frac{1}{x}\right)^2}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}a\left(\frac{46a^5\left(a+\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-a^4\int\frac{13a+\frac{35}{x}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)\right)}{a^9c^6}$$

$$\Big\downarrow \text{453}$$

$$-\frac{\frac{a^5\left(a+\frac{1}{x}\right)^3}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{1}{7}a\left(\frac{24a^5\left(a+\frac{1}{x}\right)^2}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}a\left(\frac{46a^5\left(a+\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{a^5\left(35a+\frac{13}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)\right)}{a^9c^6}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^6),x]
```

output
```
-((-1/7*(a*(-1/5*(a*((46*a^5*(a + x^(-1)))/(1 - 1/(a^2*x^2))^(3/2) - (a^5*
(35*a + 13/x))/Sqrt[1 - 1/(a^2*x^2)])) + (24*a^5*(a + x^(-1))^2)/(5*(1 - 1
/(a^2*x^2))^(5/2)))) + (a^5*(a + x^(-1))^3)/(7*(1 - 1/(a^2*x^2))^(7/2)))/(
a^9*c^6))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.44

method	result	size
trager	$\frac{(8a^4x^4-24a^3x^3+20a^2x^2+4ax-13)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^6(ax-1)^4}$	63
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(8a^4x^4-24a^3x^3+20a^2x^2+4ax-13)(ax+1)}{35(ax-1)^5c^6a}$	66
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(8a^5x^5-16a^4x^4-4a^3x^3+24a^2x^2-9ax-13)}{35(ax-1)^5c^6a}$	69
orering	$\frac{(8a^4x^4-24a^3x^3+20a^2x^2+4ax-13)(ax-1)(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{35a(-acx+c)^6}$	70

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/35/a/c^6*(8*a^4*x^4-24*a^3*x^3+20*a^2*x^2+4*a*x-13)/(a*x-1)^4*(-(-a*x+1)
/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.67

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = \frac{(8\,a^4x^4-24\,a^3x^3+20\,a^2x^2+4\,ax-13)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^5c^6x^4-4\,a^4c^6x^3+6\,a^3c^6x^2-4\,a^2c^6x+ac^6)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="fricas")
```

output
```
1/35*(8*a^4*x^4 - 24*a^3*x^3 + 20*a^2*x^2 + 4*a*x - 13)*sqrt((a*x - 1)/(a*
x + 1))/(a^5*c^6*x^4 - 4*a^4*c^6*x^3 + 6*a^3*c^6*x^2 - 4*a^2*c^6*x + a*c^6
)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = \frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7-5a^6x^6+9a^5x^5-5a^4x^4-5a^3x^3+9a^2x^2-5ax+1}\right)dx+\int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7-5a^6x^6+9a^5x^5-5a^4x^4-5a^3x^3+9a^2x^2-5ax+1}\,dx}{c^6}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**6,x)
```

output
```
(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 - 5*a**6*x**6 + 9*
a**5*x**5 - 5*a**4*x**4 - 5*a**3*x**3 + 9*a**2*x**2 - 5*a*x + 1), x) + Int
egral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 - 5*a**6*x**6 + 9*a
**5*x**5 - 5*a**4*x**4 - 5*a**3*x**3 + 9*a**2*x**2 - 5*a*x + 1), x))/c**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.68

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = \frac{1}{560}\,a\left(\frac{35\sqrt{\frac{ax-1}{ax+1}}}{a^2c^6}+\frac{\frac{28\,(ax-1)}{ax+1}-\frac{70\,(ax-1)^2}{(ax+1)^2}+\frac{140\,(ax-1)^3}{(ax+1)^3}-5}{a^2c^6\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="maxima")
```

output
```
1/560*a*(35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^6) + (28*(a*x - 1)/(a*x + 1)
- 70*(a*x - 1)^2/(a*x + 1)^2 + 140*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a^2*c^6*(
(a*x - 1)/(a*x + 1))^(7/2)))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^6}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^6, x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx=\frac{8\,a^4\,x^4-24\,a^3\,x^3+20\,a^2\,x^2+4\,a\,x-13}{35\,a\,c^6\,{\left(a\,x+1\right)}^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^6,x)
```

output
```
(4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4 - 13)/(35*a*c^6*(a*x + 1)^4*(
(a*x - 1)/(a*x + 1))^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.02

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx =\frac{-8\sqrt{ax-1}\,a^4x^4+16\sqrt{ax-1}\,a^3x^3-16\sqrt{ax-1}\,ax+8\sqrt{ax-1}+8\sqrt{ax+1}\,a^4x^4-24\sqrt{ax+1}\,a^3x^3+}{35\sqrt{ax-1}\,a\,c^6\left(a^4x^4-2a^3x^3+2ax-1\right)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x)
```

output
```
( - 8*sqrt(a*x - 1)*a**4*x**4 + 16*sqrt(a*x - 1)*a**3*x**3 - 16*sqrt(a*x -
 1)*a*x + 8*sqrt(a*x - 1) + 8*sqrt(a*x + 1)*a**4*x**4 - 24*sqrt(a*x + 1)*a
**3*x**3 + 20*sqrt(a*x + 1)*a**2*x**2 + 4*sqrt(a*x + 1)*a*x - 13*sqrt(a*x
+ 1))/(35*sqrt(a*x - 1)*a*c**6*(a**4*x**4 - 2*a**3*x**3 + 2*a*x - 1))
```

3.226 $\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 254

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=-\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9088\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{3465a^4\left(1-\frac{1}{ax}\right)^{9/2}x^3}-\frac{768\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{385a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2}+\frac{128\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{231a^2\left(1-\frac{1}{ax}\right)^{9/2}x}+\frac{2\left(a-\frac{1}{x}\right)^4\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

output
```
-32/99*(a-1/x)^3*(1+1/a/x)^(3/2)*(-a*c*x+c)^(9/2)/a^4/(1-1/a/x)^(9/2)+9088
/3465*(1+1/a/x)^(3/2)*(-a*c*x+c)^(9/2)/a^4/(1-1/a/x)^(9/2)/x^3-768/385*(1+
1/a/x)^(3/2)*(-a*c*x+c)^(9/2)/a^3/(1-1/a/x)^(9/2)/x^2+128/231*(1+1/a/x)^(3
/2)*(-a*c*x+c)^(9/2)/a^2/(1-1/a/x)^(9/2)/x+2/11*(a-1/x)^4*(1+1/a/x)^(3/2)*
x*(-a*c*x+c)^(9/2)/a^4/(1-1/a/x)^(9/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.34

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2c^4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(5419-977ax-1866a^2x^2+2710a^3x^3-1505a^4x^4+315a^5x^5)}{3465a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(9/2),x]
```

output
```
(2*c^4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(5419 - 977*a*x - 1866*a^2*x^2 +
2710*a^3*x^3 - 1505*a^4*x^4 + 315*a^5*x^5))/(3465*a*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.77, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6727, 27, 105, 105, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{9/2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\int \frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}}{a^4\left(\frac{1}{x}\right)^{13/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\int \frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{13/2}}d\frac{1}{x}}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{16}{11}\int \frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^4}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{16}{11}\left(-\frac{4}{3}\int \frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^4}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{16}{11}\left(-\frac{4}{3}\left(\frac{2}{7}\int -\frac{\left(18a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}}{2\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\right.}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{16}{11}\left(-\frac{4}{3}\left(-\frac{1}{7}\int \frac{\left(18a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\right.}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{16}{11}\left(-\frac{4}{3}\left(\frac{1}{7}\left(\frac{71}{5}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}+\frac{36a\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\right)-2(\right)}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}\left(-\frac{16}{11}\left(-\frac{4}{3}\left(\frac{1}{7}\left(\frac{36a\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}-\frac{142\left(\frac{1}{ax}+1\right)^{3/2}}{15\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^4}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

input
```
Int[E^ArcCoth[a*x]*(c - a*c*x)^(9/2),x]
```

output
```
-((((-16*((-4*(((36*a*(1 + 1/(a*x))^(3/2))/(5*(x^(-1))^(5/2)) - (142*(1 +
1/(a*x))^(3/2))/(15*(x^(-1))^(3/2)))/7 - (2*a^2*(1 + 1/(a*x))^(3/2))/(7*(x
^(-1))^(7/2))))/3 - (2*(a - x^(-1))^3*(1 + 1/(a*x))^(3/2))/(9*(x^(-1))^(9/
2))))/11 - (2*(a - x^(-1))^4*(1 + 1/(a*x))^(3/2))/(11*(x^(-1))^(11/2)))*(x
^(-1))^(9/2)*(c - a*c*x)^(9/2))/(a^4*(1 - 1/(a*x))^(9/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.27

method	result	size
default	$\frac{2\sqrt{-c(ax-1)}\, c^{4}(ax+1)\left(315a^{4}x^{4}-1820a^{3}x^{3}+4530a^{2}x^{2}-6396ax+5419\right)}{3465\sqrt{\frac{ax-1}{ax+1}}\, a}$	69
gosper	$\frac{2(ax+1)\left(315a^{4}x^{4}-1820a^{3}x^{3}+4530a^{2}x^{2}-6396ax+5419\right)(-acx+c)^{\frac{9}{2}}}{3465a\left(ax-1\right)^{4}\sqrt{\frac{ax-1}{ax+1}}}$	72
orering	$\frac{2(ax+1)\left(315a^{4}x^{4}-1820a^{3}x^{3}+4530a^{2}x^{2}-6396ax+5419\right)(-acx+c)^{\frac{9}{2}}}{3465a\left(ax-1\right)^{4}\sqrt{\frac{ax-1}{ax+1}}}$	72
risch	$-\frac{2c^{5}(ax-1)\left(315a^{5}x^{5}-1505a^{4}x^{4}+2710a^{3}x^{3}-1866a^{2}x^{2}-977ax+5419\right)}{3465\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\, a}$	77

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/3465/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*c^4*(a*x+1)*(315*a^4*x^4
-1820*a^3*x^3+4530*a^2*x^2-6396*a*x+5419)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\,(315\,a^6c^4x^6-1190\,a^5c^4x^5+1205\,a^4c^4x^4+844\,a^3c^4x^3-2843\,a^2c^4x^2+4442\,ac^4x+5419\,c}{3465\,(a^2x-a)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="fricas"
)
```

output
```
2/3465*(315*a^6*c^4*x^6 - 1190*a^5*c^4*x^5 + 1205*a^4*c^4*x^4 + 844*a^3*c^
4*x^3 - 2843*a^2*c^4*x^2 + 4442*a*c^4*x + 5419*c^4)*sqrt(-a*c*x + c)*sqrt(
(a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\,(315\,a^5\sqrt{-c}c^4x^5-1505\,a^4\sqrt{-c}c^4x^4+2710\,a^3\sqrt{-c}c^4x^3-1866\,a^2\sqrt{-c}c^4x^2-977\,a\sqrt{-c}c^4}{3465\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="maxima"
)
```

output
```
2/3465*(315*a^5*sqrt(-c)*c^4*x^5 - 1505*a^4*sqrt(-c)*c^4*x^4 + 2710*a^3*sq
rt(-c)*c^4*x^3 - 1866*a^2*sqrt(-c)*c^4*x^2 - 977*a*sqrt(-c)*c^4*x + 5419*s
qrt(-c)*c^4)*sqrt(a*x + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\left(4096\sqrt{2}\sqrt{-c}c^3-\frac{315\,(acx+c)^5\sqrt{-acx-c}-3080\,(acx+c)^4\sqrt{-acx-c}c+11880\,(acx+c)^3\sqrt{-acx-c}c^2-22176\,(acx+}{c^2}}{3465\,a|c|\text{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="giac")
```

output
```
2/3465*(4096*sqrt(2)*sqrt(-c)*c^3 - (315*(a*c*x + c)^5*sqrt(-a*c*x - c) -
3080*(a*c*x + c)^4*sqrt(-a*c*x - c)*c + 11880*(a*c*x + c)^3*sqrt(-a*c*x -
c)*c^2 - 22176*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^3 - 18480*(-a*c*x - c)^(3/
2)*c^4)/c^2)*c^2/(a*abs(c)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.47 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.30

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\,c^4\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(315\,a^4\,x^4-1820\,a^3\,x^3+4530\,a^2\,x^2-6396\,a\,x+5419\right)}{3465\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(9/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*c^4*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2)*(4530*a^2
*x^2 - 6396*a*x - 1820*a^3*x^3 + 315*a^4*x^4 + 5419))/(3465*a*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.22

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\sqrt{c}\,\sqrt{ax+1}\,c^4 i(-315a^5x^5+1505a^4x^4-2710a^3x^3+1866a^2x^2+977ax-5419)}{3465a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x)
```

output
```
(2*sqrt(c)*sqrt(a*x + 1)*c**4*i*( - 315*a**5*x**5 + 1505*a**4*x**4 - 2710*
a**3*x**3 + 1866*a**2*x**2 + 977*a*x - 5419))/(3465*a)
```

3.227 $\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 197

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{8\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{568\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{315a^3\left(1-\frac{1}{ax}\right)^{7/2}x^2}+\frac{48\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{35a^2\left(1-\frac{1}{ax}\right)^{7/2}x}+\frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

output
```
-8/21*(1+1/a/x)^(3/2)*(-a*c*x+c)^(7/2)/a/(1-1/a/x)^(7/2)-568/315*(1+1/a/x)
^(3/2)*(-a*c*x+c)^(7/2)/a^3/(1-1/a/x)^(7/2)/x^2+48/35*(1+1/a/x)^(3/2)*(-a*
c*x+c)^(7/2)/a^2/(1-1/a/x)^(7/2)/x+2/9*(a-1/x)^3*(1+1/a/x)^(3/2)*x*(-a*c*x
+c)^(7/2)/a^3/(1-1/a/x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.40

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2c^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(-319+2ax+156a^2x^2-130a^3x^3+35a^4x^4)}{315a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(7/2),x]
```

output
```
(-2*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-319 + 2*a*x + 156*a^2*x^2 - 13
0*a^3*x^3 + 35*a^4*x^4))/(315*a*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6727, 27, 105, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{7/2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\int \frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}{a^3\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\int \frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \; 105$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{4}{3}\int \frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \; 100$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{4}{3}\left(\frac{2}{7}\int -\frac{\left(18a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}}{2\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{4}{3}\left(-\frac{1}{7}\int \frac{\left(18a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \; 87$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{4}{3}\left(\frac{1}{7}\left(\frac{71}{5}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}+\frac{36a\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \; 48$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}\left(-\frac{4}{3}\left(\frac{1}{7}\left(\frac{36a\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}-\frac{142\left(\frac{1}{ax}+1\right)^{3/2}}{15\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^3}{9\left(\frac{1}{x}\right)^{9/2}}\right)(c-acx)^{7/2}}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

input
```
Int[E^ArcCoth[a*x]*(c - a*c*x)^(7/2),x]
```

output
```
-((((-4*(((36*a*(1 + 1/(a*x))^(3/2))/(5*(x^(-1))^(5/2)) - (142*(1 + 1/(a*x
))^(3/2))/(15*(x^(-1))^(3/2)))/7 - (2*a^2*(1 + 1/(a*x))^(3/2))/(7*(x^(-1))
^(7/2))))/3 - (2*(a - x^(-1))^3*(1 + 1/(a*x))^(3/2))/(9*(x^(-1))^(9/2)))*(
x^(-1))^(7/2)*(c - a*c*x)^(7/2))/(a^3*(1 - 1/(a*x))^(7/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.31

method	result	size
default	$-\frac{2\sqrt{-c(ax-1)}\, c^{3}(ax+1)(35a^{3}x^{3}-165a^{2}x^{2}+321ax-319)}{315\sqrt{\frac{ax-1}{ax+1}}\, a}$	61
gosper	$\frac{2(ax+1)(35a^{3}x^{3}-165a^{2}x^{2}+321ax-319)(-acx+c)^{\frac{7}{2}}}{315a(ax-1)^{3}\sqrt{\frac{ax-1}{ax+1}}}$	64
orering	$\frac{2(ax+1)(35a^{3}x^{3}-165a^{2}x^{2}+321ax-319)(-acx+c)^{\frac{7}{2}}}{315a(ax-1)^{3}\sqrt{\frac{ax-1}{ax+1}}}$	64
risch	$\frac{2c^{4}(ax-1)(35a^{4}x^{4}-130a^{3}x^{3}+156a^{2}x^{2}+2ax-319)}{315\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\, a}$	69

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*c^3*(a*x+1)*(35*a^3*x^3-
165*a^2*x^2+321*a*x-319)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.48

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\,(35\,a^{5}c^{3}x^{5}-95\,a^{4}c^{3}x^{4}+26\,a^{3}c^{3}x^{3}+158\,a^{2}c^{3}x^{2}-317\,ac^{3}x-319\,c^{3})\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^{2}x-a)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="fricas"
)
```

output
```
-2/315*(35*a^5*c^3*x^5 - 95*a^4*c^3*x^4 + 26*a^3*c^3*x^3 + 158*a^2*c^3*x^2
 - 317*a*c^3*x - 319*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*
x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.42

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\left(35\,a^4\sqrt{-c}c^3x^4-130\,a^3\sqrt{-c}c^3x^3+156\,a^2\sqrt{-c}c^3x^2+2\,a\sqrt{-c}c^3x-319\,\sqrt{-c}c^3\right)\sqrt{ax+1}}{315\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="maxima"
)
```

output
```
-2/315*(35*a^4*sqrt(-c)*c^3*x^4 - 130*a^3*sqrt(-c)*c^3*x^3 + 156*a^2*sqrt(
-c)*c^3*x^2 + 2*a*sqrt(-c)*c^3*x - 319*sqrt(-c)*c^3)*sqrt(a*x + 1)/a
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.46 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.52

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{2\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(-35\,a^4\,x^4+60\,a^3\,x^3+34\,a^2\,x^2-124\,a\,x+193\right)}{315\,a} + \frac{1024\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{315\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(34*a^2*x^2 - 124*a*x
 + 60*a^3*x^3 - 35*a^4*x^4 + 193))/(315*a) + (1024*c^3*(c - a*c*x)^(1/2)*(
(a*x - 1)/(a*x + 1))^(1/2))/(315*a*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.24

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{2\sqrt{c}\,\sqrt{ax+1}\,c^3 i(35a^4x^4-130a^3x^3+156a^2x^2+2ax-319)}{315a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x)
```

output
```
(2*sqrt(c)*sqrt(a*x + 1)*c**3*i*(35*a**4*x**4 - 130*a**3*x**3 + 156*a**2*x
**2 + 2*a*x - 319))/(315*a)
```

3.228 $\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 115

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{64a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{105(c-acx)^{3/2}} + \frac{16a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{35\sqrt{c-acx}} + \frac{2}{7}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\sqrt{c-acx}$$

output
```
64/105*a^2*c^4*(1-1/a^2/x^2)^(3/2)*x^3/(-a*c*x+c)^(3/2)+16/35*a^2*c^3*(1-1
/a^2/x^2)^(3/2)*x^3/(-a*c*x+c)^(1/2)+2/7*a^2*c^2*(1-1/a^2/x^2)^(3/2)*x^3*(
-a*c*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2c^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(71+17ax-39a^2x^2+15a^3x^3)}{105a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(5/2),x]
```

output
```
(2*c^2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(71 + 17*a*x - 39*a^2*x^2 + 15*a^
3*x^3))/(105*a*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6727, 27, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{5/2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}{a^2\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{ 100}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(\frac{2}{7}\int -\frac{\left(18a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}}{2\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{1}{7}\int \frac{\left(18a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(\frac{1}{7}\left(\frac{71}{5}\int\frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}+\frac{36a\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow 48$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}\left(\frac{1}{7}\left(\frac{36a\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}-\frac{142\left(\frac{1}{ax}+1\right)^{3/2}}{15\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)(c-acx)^{5/2}}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

input
```
Int[E^ArcCoth[a*x]*(c - a*c*x)^(5/2),x]
```

output
```
-(((((36*a*(1 + 1/(a*x))^(3/2))/(5*(x^(-1))^(5/2)) - (142*(1 + 1/(a*x))^(3
/2))/(15*(x^(-1))^(3/2)))/7 - (2*a^2*(1 + 1/(a*x))^(3/2))/(7*(x^(-1))^(7/2
)))*(x^(-1))^(5/2)*(c - a*c*x)^(5/2))/(a^2*(1 - 1/(a*x))^(5/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.46

method	result	size
default	$\frac{2\sqrt{-c(ax-1)}\, c^{2}(ax+1)\left(15a^{2}x^{2}-54ax+71\right)}{105\sqrt{\frac{ax-1}{ax+1}}\, a}$	53
gosper	$\frac{2(ax+1)\left(15a^{2}x^{2}-54ax+71\right)\left(-acx+c\right)^{\frac{5}{2}}}{105a\left(ax-1\right)^{2}\sqrt{\frac{ax-1}{ax+1}}}$	56
orering	$\frac{2(ax+1)\left(15a^{2}x^{2}-54ax+71\right)\left(-acx+c\right)^{\frac{5}{2}}}{105a\left(ax-1\right)^{2}\sqrt{\frac{ax-1}{ax+1}}}$	56
risch	$-\frac{2c^{3}(ax-1)\left(15a^{3}x^{3}-39a^{2}x^{2}+17ax+71\right)}{105\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\, a}$	61

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*c^2*(a*x+1)*(15*a^2*x^2-5
4*a*x+71)/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\,(15\,a^4c^2x^4-24\,a^3c^2x^3-22\,a^2c^2x^2+88\,ac^2x+71\,c^2)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^2x-a)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="fricas"
)
```

output
```
2/105*(15*a^4*c^2*x^4 - 24*a^3*c^2*x^3 - 22*a^2*c^2*x^2 + 88*a*c^2*x + 71*
c^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(15\,a^3\sqrt{-c}c^2x^3-39\,a^2\sqrt{-c}c^2x^2+17\,a\sqrt{-c}c^2x+71\,\sqrt{-c}c^2\right)\sqrt{ax+1}}{105\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="maxima"
)
```

output
```
2/105*(15*a^3*sqrt(-c)*c^2*x^3 - 39*a^2*sqrt(-c)*c^2*x^2 + 17*a*sqrt(-c)*c
^2*x + 71*sqrt(-c)*c^2)*sqrt(a*x + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(64\sqrt{2}\sqrt{-c}c-\frac{15\,(acx+c)^3\sqrt{-acx-c}-84\,(acx+c)^2\sqrt{-acx-c}c-140\,(-acx-c)^{\frac{3}{2}}c^2}{c^2}\right)c^2}{105\,a|c|\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
2/105*(64*sqrt(2)*sqrt(-c)*c - (15*(a*c*x + c)^3*sqrt(-a*c*x - c) - 84*(a*
c*x + c)^2*sqrt(-a*c*x - c)*c - 140*(-a*c*x - c)^(3/2)*c^2)/c^2)*c^2/(a*ab
s(c)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\,c^2\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^2\,x^2-54\,a\,x+71\right)}{105\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*c^2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2)*(15*a^2*x
^2 - 54*a*x + 71))/(105*a*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.34

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\sqrt{c}\,\sqrt{ax+1}\,c^2i(-15a^3x^3+39a^2x^2-17ax-71)}{105a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x)
```

output
```
(2*sqrt(c)*sqrt(a*x + 1)*c**2*i*( - 15*a**3*x**3 + 39*a**2*x**2 - 17*a*x -
 71))/(105*a)
```

3.229 $\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 77

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{8a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{15(c-acx)^{3/2}}+\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{5\sqrt{c-acx}}$$

output
```
8/15*a^2*c^3*(1-1/a^2/x^2)^(3/2)*x^3/(-a*c*x+c)^(3/2)+2/5*a^2*c^2*(1-1/a^2
/x^2)^(3/2)*x^3/(-a*c*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2c\sqrt{1+\frac{1}{ax}}(1+ax)(-7+3ax)\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(3/2),x]
```

output
```
(-2*c*Sqrt[1 + 1/(a*x)]*(1 + a*x)*(-7 + 3*a*x)*Sqrt[c - a*c*x])/(15*a*Sqrt
[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6727, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{3/2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\int \frac{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}{a\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\int \frac{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\left(-\frac{7}{5}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{14\left(\frac{1}{ax}+1\right)^{3/2}}{15\left(\frac{1}{x}\right)^{3/2}}-\frac{2a\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)(c-acx)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

input
```
Int[E^ArcCoth[a*x]*(c - a*c*x)^(3/2),x]
```

output
```
-((((-2*a*(1 + 1/(a*x))^(3/2))/(5*(x^(-1))^(5/2)) + (14*(1 + 1/(a*x))^(3/2
))/(15*(x^(-1))^(3/2)))*(x^(-1))^(3/2)*(c - a*c*x)^(3/2))/(a*(1 - 1/(a*x))
^(3/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{2\sqrt{-c(ax-1)}\, c(ax+1)(3ax-7)}{15\sqrt{\frac{ax-1}{ax+1}}\, a}$	43
gosper	$\frac{2(ax+1)(3ax-7)(-acx+c)^{\frac{3}{2}}}{15a(ax-1)\sqrt{\frac{ax-1}{ax+1}}}$	48
orering	$\frac{2(ax+1)(3ax-7)(-acx+c)^{\frac{3}{2}}}{15a(ax-1)\sqrt{\frac{ax-1}{ax+1}}}$	48
risch	$\frac{2c^{2}(ax-1)(3a^{2}x^{2}-4ax-7)}{15\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\, a}$	53

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*c*(a*x+1)*(3*a*x-7)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\,(3\,a^3cx^3 - a^2cx^2 - 11\,acx - 7\,c)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^2x-a)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="fricas"
)
```

output
```
-2/15*(3*a^3*c*x^3 - a^2*c*x^2 - 11*a*c*x - 7*c)*sqrt(-a*c*x + c)*sqrt((a*
x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \int \frac{(-c(ax-1))^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(3/2),x)
```

output
```
Integral((-c*(a*x - 1))**(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\left(3\,a^2\sqrt{-c}cx^2-4\,a\sqrt{-c}cx-7\,\sqrt{-c}c\right)\sqrt{ax+1}}{15\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="maxima"
)
```

output
```
-2/15*(3*a^2*sqrt(-c)*c*x^2 - 4*a*sqrt(-c)*c*x - 7*sqrt(-c)*c)*sqrt(a*x +
1)/a
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.65

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\,c\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\left(3\,a\,x-7\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
-(2*c*(c - a*c*x)^(1/2)*(a*x + 1)^2*(3*a*x - 7)*((a*x - 1)/(a*x + 1))^(1/2
))/(15*a*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.38

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{2\sqrt{c}\,\sqrt{ax+1}\,ci(3a^2x^2-4ax-7)}{15a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt(a*x + 1)*c*i*(3*a**2*x**2 - 4*a*x - 7))/(15*a)
```

3.230 $\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2e^{\coth^{-1}(ax)}(1+ax)\sqrt{c-acx}}{3a}$$

output
```
2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

output
```
(2*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {6726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6726}$$

$$\frac{2(ax+1)\sqrt{c-acx}e^{\coth^{-1}(ax)}}{3a}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

output
```
(2*E^ArcCoth[a*x]*(1 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Defintions of rubi rules used

rule 6726
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] &&  !IntegerQ[p] && EqQ[p, n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

method	result	size
gosper	$\frac{2(ax+1)\sqrt{-acx+c}}{3\sqrt{\frac{ax-1}{ax+1}}\, a}$	35
orering	$\frac{2(ax+1)\sqrt{-acx+c}}{3\sqrt{\frac{ax-1}{ax+1}}\, a}$	35
default	$\frac{2\sqrt{-c(ax-1)}\,(ax+1)}{3\sqrt{\frac{ax-1}{ax+1}}\, a}$	36
risch	$-\frac{2c(ax+1)(ax-1)}{3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\, a}$	42

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\,(a^2x^2+2\,ax+1)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^2x-a)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
2/3*(a^2*x^2 + 2*a*x + 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*
x - a)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\frac{\sqrt{-c(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(a\sqrt{-c}x+\sqrt{-c}\right)\sqrt{ax+1}}{3a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
2/3*(a*sqrt(-c)*x + sqrt(-c))*sqrt(a*x + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2c^2\left(\frac{2\sqrt{2}\sqrt{-c}}{c}+\frac{(-acx-c)^{\frac{3}{2}}}{c^2}\right)}{3a|c|\mathrm{sgn}(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
2/3*c^2*(2*sqrt(2)*sqrt(-c)/c + (-a*c*x - c)^(3/2)/c^2)/(a*abs(c)*sgn(a*x
+ 1))
```

Mupad [B] (verification not implemented)

Time = 13.90 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x -
1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=-\frac{2\sqrt{c}\,\sqrt{ax+1}\,i(ax+1)}{3a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x)
```

output
```
( - 2*sqrt(c)*sqrt(a*x + 1)*i*(a*x + 1))/(3*a)
```

3.231 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 119

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\sqrt{c-acx}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a\sqrt{\frac{1}{ax}}\sqrt{c-acx}}$$

output
```
2*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)*x/(-a*c*x+c)^(1/2)-2*2^(1/2)*(1-1/a/x)^(
1/2)*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))/a/(1/a/x)^(1/2)/(-a*c*
x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{2\sqrt{1-\frac{1}{ax}}x\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}-\sqrt{2}\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{\sqrt{a}\sqrt{c-acx}}$$

input
```
Integrate[E^ArcCoth[a*x]/Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[1 - 1/(a*x)]*x*(Sqrt[a]*Sqrt[1 + 1/(a*x)] - Sqrt[2]*Sqrt[x^(-1)]*A
rcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(Sqrt[a]*Sqrt
[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6727, 27, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{2\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{4\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

input
```
Int[E^ArcCoth[a*x]/Sqrt[c - a*c*x],x]
```

output
```
-((a*Sqrt[1 - 1/(a*x)]*((-2*Sqrt[1 + 1/(a*x)])/(a*Sqrt[x^(-1)]) + (2*Sqrt[
2]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/a^(3/2)))/
(Sqrt[x^(-1)]*Sqrt[c - a*c*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

method	result	size
default	$-\frac{2\sqrt{-c(ax-1)}\,\left(-\sqrt{c}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\sqrt{-c(ax+1)}\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}\,ca}$	82
risch	$\frac{2ax-2}{a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}+\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{a\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}$	115

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-c*(a*x-1))^(1/2)*(-c^(1/2)*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1
/2)/c^(1/2))+(-c*(a*x+1))^(1/2))/((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x+1))^(1/2
)/c/a
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.06

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$$
$$= \left[\frac{\sqrt{2}(acx-c)\sqrt{-\frac{1}{c}}\log\left(-\frac{a^2x^2-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}+2\,ax-3}{a^2x^2-2\,ax+1}\right)-2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac},\right.$$
$$\left.-\frac{2\left(\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}-\frac{\sqrt{2}(acx-c)\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\,(ax-1)\sqrt{c}}\right)}{\sqrt{c}}\right)}{a^2cx-ac}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
[(sqrt(2)*(a*c*x - c)*sqrt(-1/c)*log(-(a^2*x^2 - 2*sqrt(2)*sqrt(-a*c*x + c
)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a*x - 3)/(a^2*x^2 - 2
*a*x + 1)) - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*
c*x - a*c), -2*(sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) - sqr
t(2)*(a*c*x - c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x -
 1)/(a*x + 1))/((a*x - 1)*sqrt(c)))/sqrt(c))/(a^2*c*x - a*c)]
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(1/2),x)
```

output
```
Integral(1/(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{1}{\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.79

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\,c\left(\frac{\sqrt{2}\left(\sqrt{c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{c}}\right)-\sqrt{-c}\right)}{c}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)-\sqrt{-acx-c}}{c}\right)}{a|c|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
2*c*(sqrt(2)*(sqrt(c)*arctan(sqrt(-c)/sqrt(c)) - sqrt(-c))/c - (sqrt(2)*sq
rt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) - sqrt(-a*c*x - c))/c)/
(a*abs(c)*sgn(a*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{1}{\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.35

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\sqrt{c}\,i\left(\sqrt{ax+1}+\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)-\sqrt{2}\right)}{ac}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x)
```

output
```
(2*sqrt(c)*i*(sqrt(a*x + 1) + sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2
))/2)) - sqrt(2)))/(a*c)
```

3.232 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 129

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=-\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}}-\frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{2}a\left(\frac{1}{ax}\right)^{3/2}(c-acx)^{3/2}}$$

output
```
-a*(1-1/a/x)^(3/2)*(1+1/a/x)^(1/2)*x/(a-1/x)/(-a*c*x+c)^(3/2)-1/2*(1-1/a/x
)^(3/2)*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)/a/(1/a/x)^(
3/2)/(-a*c*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.90

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\,x\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}+\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{2\sqrt{a}c(-1+ax)\sqrt{c-acx}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(3/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[2]*Sqrt[x^(-1)]*(
-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(2
*Sqrt[a]*c*(-1 + a*x)*Sqrt[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6727, 27, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{a^2\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}a^{3/2}}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

input
```
Int[E^ArcCoth[a*x]/(c - a*c*x)^(3/2),x]
```

output
```
-((a^2*(1 - 1/(a*x))^(3/2)*((Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)])/(a*(a - x^(-1
))) + ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]/(Sqrt[2]
*a^(3/2))))/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{\sqrt{-c(ax-1)}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx-\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)c+2\sqrt{-c(ax+1)}\,\sqrt{c}\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)\sqrt{-c(ax+1)}\,c^{\frac{5}{2}}a}$	118

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a*x-1))^(1/2)*(2^(1/2)*arctan(1/
2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-2^(1/2)*arctan(1/2*(-c*(a*x+1)
)^(1/2)*2^(1/2)/c^(1/2))*c+2*(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2
)/c^(5/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.22

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \left[-\frac{\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+4\,\sqrt{}}{4\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}\right.$$

$$\left.-\frac{\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2\,(acx-c)}\right)+2\,\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas"
)
```

output
```
[-1/4*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x -
2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) -
3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/
(a*x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/2*(sqrt(2)*(a^2*x^2 -
2*a*x + 1)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*s
qrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) + 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(
(a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\,(-c\,(ax-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(3/2),x)
```

output
```
Integral(1/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{1}{(-acx+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(1/((-a*c*x + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.47

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{2\sqrt{-acx-c}}{acx-c}}{2\,a|c|}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) + 2*sqrt
(-a*c*x - c)/(a*c*x - c))/(a*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{1}{(c-a\,c\,x)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.54

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\sqrt{c}\,i\left(2\sqrt{ax+1}-\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)ax+\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)\right.}{2a\,c^{2}\,(ax-1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x)
```

output
```
(sqrt(c)*i*(2*sqrt(a*x + 1) - sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2
))/2))*a*x + sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2))/2))))/(2*a*c**
2*(a*x - 1))
```

3.233 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 194

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{8\sqrt{2}a\left(\frac{1}{ax}\right)^{5/2}(c-acx)^{5/2}}$$

output
```
1/8*a^2*(1-1/a/x)^(5/2)*(1+1/a/x)^(1/2)*x^2/(a-1/x)/(-a*c*x+c)^(5/2)-1/4*a
^3*(1-1/a/x)^(5/2)*(1+1/a/x)^(3/2)*x^2/(a-1/x)^2/(-a*c*x+c)^(5/2)+1/16*(1-
1/a/x)^(5/2)*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)/a/(1/a
/x)^(5/2)/(-a*c*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.63

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}x\left(-2\sqrt{a}\sqrt{1+\frac{1}{ax}}(3+ax)+\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{16\sqrt{a}c^2(-1+ax)^2\sqrt{c-acx}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(5/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*x*(-2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(3 + a*x) + Sqrt[2]*Sqr
t[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/
(a*x)])]))/(16*Sqrt[a]*c^2*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.79, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6727, 27, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{a^3\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2}-\frac{1}{8}\int\frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{8}\left(-\frac{\int\frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2a}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{8}\left(-\frac{\int\frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{8}\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}a^{3/2}}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

input

```
Int[E^ArcCoth[a*x]/(c - a*c*x)^(5/2),x]
```

output

```
-((a^3*(1 - 1/(a*x))^(5/2)*(((1 + 1/(a*x))^(3/2)*Sqrt[x^(-1)])/(4*(a - x^(
-1))^2) + (-((Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)])/(a*(a - x^(-1)))) - ArcTanh[
(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]/(Sqrt[2]*a^(3/2)))/8))
/((x^(-1))^(5/2)*(c - a*c*x)^(5/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.85

method	result
default	$-\frac{\sqrt{-c(ax-1)}\left(\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^2cx^2-2\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)acx-2ax\sqrt{-c(ax+1)}\sqrt{c}+\sqrt{2}\arctan\left(\frac{\sqrt{-c(a}}{2}\right.}{16\sqrt{\frac{ax-1}{ax+1}}(ax-1)^2c^{\frac{7}{2}}\sqrt{-c(ax+1)}a}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^2*(-c*(a*x-1))^(1/2)/c^(7/2)*(2^(1/2
)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2-2*2^(1/2)*arcta
n(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-2*a*x*(-c*(a*x+1))^(1/2)*c
^(1/2)+2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-6*(-c*(a*x
+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.77

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \left[-\frac{\sqrt{2}(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)}{32\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)} \right.$$
$$-\frac{\sqrt{2}(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2\,(acx-c)}\right)-2\,(a^2x^2+4\,ax+3)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{16\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas"
)
```

output
```
[-1/32*(sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2
 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/
(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(a^2*x^2 + 4*a*x + 3)*sqrt(-a
*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*
c^3*x - a*c^3), -1/16*(sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*a
rctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x +
 1))/(a*c*x - c)) - 2*(a^2*x^2 + 4*a*x + 3)*sqrt(-a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{1}{(-acx+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima"
)
```

output
```
integrate(1/((-a*c*x + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.40

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2\left((-acx-c)^{\frac{3}{2}}-2\sqrt{-acx-c}c\right)}{(acx-c)^2c}}{16\,a|c|}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
1/16*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) + 2*((-
a*c*x - c)^(3/2) - 2*sqrt(-a*c*x - c)*c)/((a*c*x - c)^2*c))/(a*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{1}{(c-a\,c\,x)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\sqrt{c}\,\sqrt{2}\,i\left(-8\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)^4+\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)^8 -}{128\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)^4 a\,c^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x)
```

output
```
(sqrt(c)*sqrt(2)*i*( - 8*log(tan(asin(sqrt( - a*x + 1)/sqrt(2))/2))*tan(as
in(sqrt( - a*x + 1)/sqrt(2))/2)**4 + tan(asin(sqrt( - a*x + 1)/sqrt(2))/2)
**8 - 1))/(128*tan(asin(sqrt( - a*x + 1)/sqrt(2))/2)**4*a*c**3)
```

3.234 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}} dx$

Optimal result

Integrand size = 18, antiderivative size = 249

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}} dx = -\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{3\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{32\sqrt{2}a\left(\frac{1}{ax}\right)^{7/2}(c-acx)^{7/2}}$$

output
```
-1/3*a^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(1/2)*x/(a-1/x)^3/(-a*c*x+c)^(7/2)-1/24
*a^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(1/2)*x^2/(a-1/x)^2/(-a*c*x+c)^(7/2)+1/32*a
^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(1/2)*x^3/(a-1/x)/(-a*c*x+c)^(7/2)-1/64*(1-1/
a/x)^(7/2)*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)/a/(1/a/x
)^(7/2)/(-a*c*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.56

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(\frac{2\sqrt{a}\sqrt{1+\frac{1}{ax}}(25+10ax-3a^2x^2)}{\sqrt{\frac{1}{x}}}+3\sqrt{2}(-1+ax)^3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{192\sqrt{a}c^3\sqrt{\frac{1}{x}}(-1+ax)^3\sqrt{c-acx}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(7/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*((2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(25 + 10*a*x - 3*a^2*x^2)
)/Sqrt[x^(-1)] + 3*Sqrt[2]*(-1 + a*x)^3*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sq
rt[a]*Sqrt[1 + 1/(a*x)])]))/(192*Sqrt[a]*c^3*Sqrt[x^(-1)]*(-1 + a*x)^3*Sqr
t[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.76, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6727, 27, 105, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{a^4\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \ 105$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{6\left(a-\frac{1}{x}\right)^3}-\frac{1}{4}\int \frac{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \ 105$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{4}\left(\frac{1}{8}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2}\right)+\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{6\left(a-\frac{1}{x}\right)^3}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \ 105$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2}\right)+\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{6\left(a-\frac{1}{x}\right)^3}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \ 104$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2}\right)+\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{6\left(a-\frac{1}{x}\right)^3}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \ 219$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}a^{3/2}}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2}\right)+\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{6\left(a-\frac{1}{x}\right)^3}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

input
```
Int[E^ArcCoth[a*x]/(c - a*c*x)^(7/2),x]
```

output
```
-((a^4*(1 - 1/(a*x))^(7/2)*(((1 + 1/(a*x))^(3/2)*(x^(-1))^(3/2))/(6*(a - x
^(-1))^3) + (-1/4*((1 + 1/(a*x))^(3/2)*Sqrt[x^(-1)])/(a - x^(-1))^2 + ((Sq
rt[1 + 1/(a*x)]*Sqrt[x^(-1)])/(a*(a - x^(-1))) + ArcTanh[(Sqrt[2]*Sqrt[x^(
-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]/(Sqrt[2]*a^(3/2)))/8)/4))/((x^(-1))^(7/
2)*(c - a*c*x)^(7/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.88

method	result
default	$-\frac{\sqrt{-c(ax-1)}\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{3}c\,x^{3}-9\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{2}c\,x^{2}-6a^{2}x^{2}\sqrt{-c(ax+1)}\,\sqrt{c}+9\sqrt{2}\,\arctan\left(\right.\right.}{192\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)^{3}c^{\frac{9}{2}}\sqrt{-c(ax+}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^3*(-c*(a*x-1))^(1/2)/c^(9/2)*(3*2^(
1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^3*c*x^3-9*2^(1/2)*ar
ctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2-6*a^2*x^2*(-c*(a*x+
1))^(1/2)*c^(1/2)+9*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))
*a*c*x+20*a*x*(-c*(a*x+1))^(1/2)*c^(1/2)-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))
^(1/2)*2^(1/2)/c^(1/2))*c+50*(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2
)/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.60

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \left[-\frac{3\sqrt{2}(a^4x^4-4a^3x^3+6a^2x^2-4ax+1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{}}{a^2x^2-2ax+1}\right.}{384\,(a^5c^4x^4-4a^4c^4x^3+6a^3c^4x^2}\right.$$

$$-\frac{3\sqrt{2}(a^4x^4-4a^3x^3+6a^2x^2-4ax+1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2(acx-c)}\right)-2(3a^3x^3-7a^2x^2-35}{192\,(a^5c^4x^4-4a^4c^4x^3+6a^3c^4x^2-4a^2c^4x+ac^4)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas"
)
```

output
```
[-1/384*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*
log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*
sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^3*x^3 - 7
*a^2*x^2 - 35*a*x - 25)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c
^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4), -1/192*(3*s
qrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*arctan(1/2*sq
rt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x
- c)) - 2*(3*a^3*x^3 - 7*a^2*x^2 - 35*a*x - 25)*sqrt(-a*c*x + c)*sqrt((a*x
 - 1)/(a*x + 1)))/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4
*x + a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{1}{(-acx+c)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima"
)
```

output
```
integrate(1/((-a*c*x + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.42

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}} - \frac{2\left(3(acx+c)^2\sqrt{-acx-c}+16(-acx-c)^{\frac{3}{2}}c-12\sqrt{-acx-c}c^2\right)}{(acx-c)^3c^2}}{192\,a|c|}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
1/192*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) - 2*
(3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 16*(-a*c*x - c)^(3/2)*c - 12*sqrt(-a*c
*x - c)*c^2)/((a*c*x - c)^3*c^2))/(a*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{1}{(c-a\,c\,x)^{7/2}\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.66

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\sqrt{c}\,i\left(-6\sqrt{ax+1}\,a^2x^2+20\sqrt{ax+1}\,ax+50\sqrt{ax+1}-3\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x)
```

output

```
(sqrt(c)*i*( - 6*sqrt(a*x + 1)*a**2*x**2 + 20*sqrt(a*x + 1)*a*x + 50*sqrt(
a*x + 1) - 3*sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2))/2))*a**3*x**3
+ 9*sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2))/2))*a**2*x**2 - 9*sqrt(
2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2))/2))*a*x + 3*sqrt(2)*log(tan(asin
(sqrt( - a*x + 1)/sqrt(2))/2))))/(192*a*c**4*(a**3*x**3 - 3*a**2*x**2 + 3*
a*x - 1))
```

3.235 $\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 40

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}$$

output
```
4/7*(-a*c*x+c)^(7/2)/a-2/9*(-a*c*x+c)^(9/2)/a/c
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{2c^3(-1+ax)^3(11+7ax)\sqrt{c-acx}}{63a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

output
```
(-2*c^3*(-1 + a*x)^3*(11 + 7*a*x)*Sqrt[c - a*c*x])/(63*a)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{7/2}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}(c-acx)^{7/2}dx$$

$$\Big\downarrow \text{ 6680}$$

$$-\int \frac{(ax+1)(c-acx)^{7/2}}{1-ax}dx$$

$$\Big\downarrow \text{ 35}$$

$$-c\int (ax+1)(c-acx)^{5/2}dx$$

$$\Big\downarrow \text{ 53}$$

$$-c\int \left(2(c-acx)^{5/2}-\frac{(c-acx)^{7/2}}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-c\left(\frac{2(c-acx)^{9/2}}{9ac^2}-\frac{4(c-acx)^{7/2}}{7ac}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

output
```
-(c*((-4*(c - a*c*x)^(7/2))/(7*a*c) + (2*(c - a*c*x)^(9/2))/(9*a*c^2)))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
gosper	$\frac{2(-acx+c)^{\frac{7}{2}}(7ax+11)}{63a}$	21
orering	$\frac{2(-acx+c)^{\frac{7}{2}}(7ax+11)}{63a}$	21
pseudoelliptic	$-\frac{2\left(ax+\frac{11}{7}\right)(ax-1)^{3}c^{3}\sqrt{-c(ax-1)}}{9a}$	31
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{9}{2}}}{9}-\frac{2c(-acx+c)^{\frac{7}{2}}}{7}\right)}{ca}$	33
default	$\frac{-\frac{2(-acx+c)^{\frac{9}{2}}}{9}+\frac{4c(-acx+c)^{\frac{7}{2}}}{7}}{ac}$	33
trager	$-\frac{2c^{3}(7a^{4}x^{4}-10a^{3}x^{3}-12a^{2}x^{2}+26ax-11)\sqrt{-acx+c}}{63a}$	48
risch	$\frac{2c^{4}(7a^{4}x^{4}-10a^{3}x^{3}-12a^{2}x^{2}+26ax-11)(ax-1)}{63a\sqrt{-c(ax-1)}}$	54

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/63*(-a*c*x+c)^(7/2)*(7*a*x+11)/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\,(7\,a^{4}c^{3}x^{4}-10\,a^{3}c^{3}x^{3}-12\,a^{2}c^{3}x^{2}+26\,ac^{3}x-11\,c^{3})\sqrt{-acx+c}}{63\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/63*(7*a^4*c^3*x^4 - 10*a^3*c^3*x^3 - 12*a^2*c^3*x^2 + 26*a*c^3*x - 11*c
^3)*sqrt(-a*c*x + c)/a
```

Sympy [A] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \begin{cases} -\frac{2\left(-\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{ac} & \text{for } ac\neq 0 \\ c^{\frac{7}{2}}\left(\begin{cases} -x & \text{for } a=0 \\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(7/2),x)
```

output
```
Piecewise((-2*(-2*c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/(a*c),
Ne(a*c, 0)), (c**(7/2)*Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) -
1)/a, True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\left(7(-acx+c)^{\frac{9}{2}}-18(-acx+c)^{\frac{7}{2}}c\right)}{63\,ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-2/63*(7*(-a*c*x + c)^(9/2) - 18*(-a*c*x + c)^(7/2)*c)/(a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(32) = 64$.

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 5.12

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{2\left(90\,(acx-c)^3\sqrt{-acx+c}+378\,(acx-c)^2\sqrt{-acx+c}c-630\,(-acx+c)^{\frac{3}{2}}c^2+945\,\sqrt{-a\right.}{}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
2/315*(90*(a*c*x - c)^3*sqrt(-a*c*x + c) + 378*(a*c*x - c)^2*sqrt(-a*c*x +
 c)*c - 630*(-a*c*x + c)^(3/2)*c^2 + 945*sqrt(-a*c*x + c)*c^3 + 210*((-a*c
*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)*c^2 - (35*(a*c*x - c)^4*sqrt(-a*c*x
+ c) + 180*(a*c*x - c)^3*sqrt(-a*c*x + c)*c + 378*(a*c*x - c)^2*sqrt(-a*c*
x + c)*c^2 - 420*(-a*c*x + c)^(3/2)*c^3 + 315*sqrt(-a*c*x + c)*c^4)/c)/a
```

Mupad [B] (verification not implemented)

Time = 13.71 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{4\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a}-\frac{2\,{\left(c-a\,c\,x\right)}^{9/2}}{9\,a\,c}$$

input
```
int(((c - a*c*x)^(7/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
(4*(c - a*c*x)^(7/2))/(7*a) - (2*(c - a*c*x)^(9/2))/(9*a*c)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{2\sqrt{c}\,\sqrt{-ax+1}\,c^3(-7a^4x^4+10a^3x^3+12a^2x^2-26ax+11)}{63a}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x)
```

output
```
(2*sqrt(c)*sqrt( - a*x + 1)*c**3*( - 7*a**4*x**4 + 10*a**3*x**3 + 12*a**2*
x**2 - 26*a*x + 11))/(63*a)
```

3.236 $\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 40

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}$$

output
```
4/5*(-a*c*x+c)^(5/2)/a-2/7*(-a*c*x+c)^(7/2)/a/c
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2c^2(-1+ax)^2(9+5ax)\sqrt{c-acx}}{35a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

output
```
(2*c^2*(-1 + a*x)^2*(9 + 5*a*x)*Sqrt[c - a*c*x])/(35*a)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{5/2}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}(c-acx)^{5/2}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(ax+1)(c-acx)^{5/2}}{1-ax}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int (ax+1)(c-acx)^{3/2}dx$$

$$\Big\downarrow \text{53}$$

$$-c\int \left(2(c-acx)^{3/2}-\frac{(c-acx)^{5/2}}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{2(c-acx)^{7/2}}{7ac^2}-\frac{4(c-acx)^{5/2}}{5ac}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

output
```
-(c*((-4*(c - a*c*x)^(5/2))/(5*a*c) + (2*(c - a*c*x)^(7/2))/(7*a*c^2)))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
gosper	$\frac{2(-acx+c)^{\frac{5}{2}}(5ax+9)}{35a}$	21
orering	$\frac{2(-acx+c)^{\frac{5}{2}}(5ax+9)}{35a}$	21
pseudoelliptic	$\frac{2\left(ax+\frac{9}{5}\right)(ax-1)^{2} c^{2} \sqrt{-c(ax-1)}}{7a}$	31
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}-\frac{2(-acx+c)^{\frac{5}{2}} c}{5}\right)}{ca}$	33
default	$\frac{-\frac{2(-acx+c)^{\frac{7}{2}}}{7}+\frac{4(-acx+c)^{\frac{5}{2}} c}{5}}{ac}$	33
trager	$\frac{2c^{2}\left(5a^{3}x^{3}-a^{2}x^{2}-13ax+9\right)\sqrt{-acx+c}}{35a}$	40
risch	$-\frac{2c^{3}\left(5a^{3}x^{3}-a^{2}x^{2}-13ax+9\right)(ax-1)}{35a\sqrt{-c(ax-1)}}$	46

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(-a*c*x+c)^(5/2)*(5*a*x+9)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(5\,a^{3}c^{2}x^{3}-a^{2}c^{2}x^{2}-13\,ac^{2}x+9\,c^{2}\right)\sqrt{-acx+c}}{35\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/35*(5*a^3*c^2*x^3 - a^2*c^2*x^2 - 13*a*c^2*x + 9*c^2)*sqrt(-a*c*x + c)/a
```

Sympy [A] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\begin{cases} -\frac{2\left(-\frac{2c(-acx+c)^{\frac{5}{2}}}{5}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{ac} & \text{for } ac\neq 0\\ c^{\frac{5}{2}}\left(\begin{cases} -x & \text{for } a=0\\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(5/2),x)
```

output
```
Piecewise((-2*(-2*c*(-a*c*x + c)**(5/2)/5 + (-a*c*x + c)**(7/2)/7)/(a*c),
Ne(a*c, 0)), (c**(5/2)*Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) -
1)/a, True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=-\frac{2\left(5\,(-acx+c)^{\frac{7}{2}}-14\,(-acx+c)^{\frac{5}{2}}c\right)}{35\,ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-2/35*(5*(-a*c*x + c)^(7/2) - 14*(-a*c*x + c)^(5/2)*c)/(a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(32) = 64$.

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.52

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx =$$

$$-\frac{2\left(21\,(acx-c)^2\sqrt{-acx+c}-70\,(-acx+c)^{\frac{3}{2}}c-35\left((-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}c\right)c-\frac{3\left(5\,(acx-c)^3\sqrt{-ac\cdots}\right.}{\cdots}\right.}{105\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
-2/105*(21*(a*c*x - c)^2*sqrt(-a*c*x + c) - 70*(-a*c*x + c)^(3/2)*c - 35*(
(-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)*c - 3*(5*(a*c*x - c)^3*sqrt(-a*
c*x + c) + 21*(a*c*x - c)^2*sqrt(-a*c*x + c)*c - 35*(-a*c*x + c)^(3/2)*c^2
 + 35*sqrt(-a*c*x + c)*c^3)/c)/a
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{4\,(c-a\,c\,x)^{5/2}}{5\,a}-\frac{2\,(c-a\,c\,x)^{7/2}}{7\,a\,c}$$

input
```
int(((c - a*c*x)^(5/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
(4*(c - a*c*x)^(5/2))/(5*a) - (2*(c - a*c*x)^(7/2))/(7*a*c)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\sqrt{c}\,\sqrt{-ax+1}\,c^2(5a^3x^3-a^2x^2-13ax+9)}{35a}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x)
```

output
```
(2*sqrt(c)*sqrt( - a*x + 1)*c**2*(5*a**3*x**3 - a**2*x**2 - 13*a*x + 9))/(
35*a)
```

3.237 $\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 40

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}$$

output
```
4/3*(-a*c*x+c)^(3/2)/a-2/5*(-a*c*x+c)^(5/2)/a/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2c(-1+ax)(7+3ax)\sqrt{c-acx}}{15a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

output
```
(-2*c*(-1 + a*x)*(7 + 3*a*x)*Sqrt[c - a*c*x])/(15*a)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{3/2}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}(c-acx)^{3/2}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(ax+1)(c-acx)^{3/2}}{1-ax}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int (ax+1)\sqrt{c-acx}dx$$

$$\Big\downarrow \text{53}$$

$$-c\int \left(2\sqrt{c-acx}-\frac{(c-acx)^{3/2}}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{2(c-acx)^{5/2}}{5ac^2}-\frac{4(c-acx)^{3/2}}{3ac}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

output
```
-(c*((-4*(c - a*c*x)^(3/2))/(3*a*c) + (2*(c - a*c*x)^(5/2))/(5*a*c^2)))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
gosper	$\frac{2(-acx+c)^{\frac{3}{2}}(3ax+7)}{15a}$	21
orering	$\frac{2(-acx+c)^{\frac{3}{2}}(3ax+7)}{15a}$	21
pseudoelliptic	$-\frac{2(ax-1)c\left(ax+\frac{7}{3}\right)\sqrt{-c(ax-1)}}{5a}$	27
trager	$-\frac{2c(3a^2x^2+4ax-7)\sqrt{-acx+c}}{15a}$	30
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{5}{2}}}{5}-\frac{2(-acx+c)^{\frac{3}{2}}c}{3}\right)}{ca}$	33
default	$\frac{-\frac{2(-acx+c)^{\frac{5}{2}}}{5}+\frac{4(-acx+c)^{\frac{3}{2}}c}{3}}{ac}$	33
risch	$\frac{2c^2(3a^2x^2+4ax-7)(ax-1)}{15a\sqrt{-c(ax-1)}}$	38

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(-a*c*x+c)^(3/2)*(3*a*x+7)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\,(3\,a^2cx^2+4\,acx-7\,c)\sqrt{-acx+c}}{15\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(3*a^2*c*x^2 + 4*a*c*x - 7*c)*sqrt(-a*c*x + c)/a
```

Sympy [A] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\begin{cases}-\frac{2\left(-\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{ac} & \text{for } ac\neq 0\\ c^{\frac{3}{2}}\left(\begin{cases}-x & \text{for } a=0\\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(3/2),x)
```

output
```
Piecewise((-2*(-2*c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**(5/2)/5)/(a*c),
Ne(a*c, 0)), (c**(3/2)*Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) -
1)/a, True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\left(3(-acx+c)^{\frac{5}{2}}-10(-acx+c)^{\frac{3}{2}}c\right)}{15\,ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-2/15*(3*(-a*c*x + c)^(5/2) - 10*(-a*c*x + c)^(3/2)*c)/(a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(32) = 64$.

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$\int e^{2\coth^{-1}(ax)}(c - acx)^{3/2}\,dx = \frac{2\left(15\sqrt{-acx+c}c - \frac{3(acx-c)^2\sqrt{-acx+c}-10(-acx+c)^{\frac{3}{2}}c+15\sqrt{-acx+c}c^2}{c}\right)}{15a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
2/15*(15*sqrt(-a*c*x + c)*c - (3*(a*c*x - c)^2*sqrt(-a*c*x + c) - 10*(-a*c
*x + c)^(3/2)*c + 15*sqrt(-a*c*x + c)*c^2)/c)/a
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}(c - acx)^{3/2}\,dx = \frac{4\,(c-a\,c\,x)^{3/2}}{3\,a} - \frac{2\,(c-a\,c\,x)^{5/2}}{5\,a\,c}$$

input
```
int(((c - a*c*x)^(3/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
(4*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(5/2))/(5*a*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{2\sqrt{c}\,\sqrt{-ax+1}\,c(-3a^2x^2-4ax+7)}{15a}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt( - a*x + 1)*c*( - 3*a**2*x**2 - 4*a*x + 7))/(15*a)
```

3.238 $\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

output
```
4*(-a*c*x+c)^(1/2)/a-2/3*(-a*c*x+c)^(3/2)/a/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2(5+ax)\sqrt{c-acx}}{3a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
(2*(5 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-acx}dx$$

$$\Big\downarrow \text{ 6680}$$

$$-\int \frac{(ax+1)\sqrt{c-acx}}{1-ax}dx$$

$$\Big\downarrow \text{ 35}$$

$$-c\int \frac{ax+1}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{ 53}$$

$$-c\int \left(\frac{2}{\sqrt{c-acx}}-\frac{\sqrt{c-acx}}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-c\left(\frac{2(c-acx)^{3/2}}{3ac^2}-\frac{4\sqrt{c-acx}}{ac}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
-(c*((-4*Sqrt[c - a*c*x])/(a*c) + (2*(c - a*c*x)^(3/2))/(3*a*c^2)))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
trager	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
orering	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(ax+5)}{3a}$	21
risch	$-\frac{2c(ax+5)(ax-1)}{3a\sqrt{-c(ax-1)}}$	27
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}-2\sqrt{-acx+c}\,c\right)}{ca}$	33
default	$\frac{-\frac{2(-acx+c)^{\frac{3}{2}}}{3}+4\sqrt{-acx+c}\,c}{ac}$	33

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-a*c*x+c)^(1/2)*(a*x+5)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.50

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{-acx+c}(ax+5)}{3\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(-a*c*x + c)*(a*x + 5)/a
```

Sympy [A] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \begin{cases} -\frac{2\left(-2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\begin{cases} -x & \text{for } a=0 \\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2),x)
```

output
```
Piecewise((-2*(-2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c), Ne(a*
c, 0)), (sqrt(c)*Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a,
True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{2\left((-acx+c)^{\frac{3}{2}}-6\sqrt{-acx+c}c\right)}{3\,ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\left(3\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}c}{c}\right)}{3a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*sqrt(-a*c*x + c) - ((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/c)/a
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{4\sqrt{c-a\,c\,x}}{a}-\frac{2\,(c-a\,c\,x)^{3/2}}{3\,a\,c}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
(4*(c - a*c*x)^(1/2))/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.50

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{c}\,\sqrt{-ax+1}\,(ax+5)}{3a}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt( - a*x + 1)*(a*x + 5))/(3*a)
```

3.239 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 36

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{4}{a\sqrt{c-acx}} - \frac{2\sqrt{c-acx}}{ac}$$

output
```
-4/a/(-a*c*x+c)^(1/2)-2*(-a*c*x+c)^(1/2)/a/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{-6+2ax}{a\sqrt{c-acx}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

output
```
(-6 + 2*a*x)/(a*Sqrt[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{ 6680}$$

$$-\int \frac{ax+1}{(1-ax)\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{ 35}$$

$$-c\int \frac{ax+1}{(c-acx)^{3/2}}dx$$

$$\Big\downarrow \text{ 53}$$

$$-c\int \left(\frac{2}{(c-acx)^{3/2}}-\frac{1}{c\sqrt{c-acx}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-c\left(\frac{2\sqrt{c-acx}}{ac^2}+\frac{4}{ac\sqrt{c-acx}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

output
```
-(c*(4/(a*c*Sqrt[c - a*c*x]) + (2*Sqrt[c - a*c*x])/(a*c^2)))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{2ax-6}{a\sqrt{-acx+c}}$	20
orering	$\frac{2ax-6}{a\sqrt{-acx+c}}$	20
pseudoelliptic	$\frac{2ax-6}{a\sqrt{-c(ax-1)}}$	21
trager	$-\frac{2(ax-3)\sqrt{-acx+c}}{ca(ax-1)}$	30
derivativedivides	$-\frac{2\left(\sqrt{-acx+c}+\frac{2c}{\sqrt{-acx+c}}\right)}{ca}$	31
default	$\frac{-2\sqrt{-acx+c}-\frac{4c}{\sqrt{-acx+c}}}{ac}$	33
risch	$\frac{2ax-2}{a\sqrt{-c(ax-1)}}-\frac{4}{a\sqrt{-c(ax-1)}}$	37

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*x-3)/a/(-a*c*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=-\frac{2\sqrt{-acx+c}(ax-3)}{a^2cx-ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-a*c*x + c)*(a*x - 3)/(a^2*c*x - a*c)
```

Sympy [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2c}{\sqrt{-acx+c}}+\sqrt{-acx+c}\right)}{ac} & \text{for } ac\neq 0 \\ \frac{\begin{cases} -x & \text{for } a=0 \\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(1/2),x)
```

output
```
Piecewise((-2*(2*c/sqrt(-a*c*x + c) + sqrt(-a*c*x + c))/(a*c), Ne(a*c, 0))
, (Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a, True))/sqrt(c)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\left(\sqrt{-acx+c}+\frac{2c}{\sqrt{-acx+c}}\right)}{ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-2*(sqrt(-a*c*x + c) + 2*c/sqrt(-a*c*x + c))/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{4}{\sqrt{-acx+c}a} - \frac{2\sqrt{-acx+c}}{ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
-4/(sqrt(-a*c*x + c)*a) - 2*sqrt(-a*c*x + c)/(a*c)
```

Mupad [B] (verification not implemented)

Time = 13.71 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\,a\,x-6}{a\,\sqrt{c-a\,c\,x}}$$

input
```
int((a*x + 1)/((c - a*c*x)^(1/2)*(a*x - 1)),x)
```

output
```
(2*a*x - 6)/(a*(c - a*c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\sqrt{c}\,(ax-3)}{\sqrt{-ax+1}\,ac}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x)
```

output
```
(2*sqrt(c)*(a*x - 3))/(sqrt( - a*x + 1)*a*c)
```

3.240 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{4}{3a(c-acx)^{3/2}}+\frac{2}{ac\sqrt{c-acx}}$$

output
```
-4/3/a/(-a*c*x+c)^(3/2)+2/a/c/(-a*c*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{-2+6ax}{3ac(-1+ax)\sqrt{c-acx}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

output
```
(-2 + 6*a*x)/(3*a*c*(-1 + a*x)*Sqrt[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}dx$$
$$\Big\downarrow \text{ 6717}$$
$$-\int \frac{e^{2\text{arctanh}(ax)}}{(c-acx)^{3/2}}dx$$
$$\Big\downarrow \text{ 6680}$$
$$-\int \frac{ax+1}{(1-ax)(c-acx)^{3/2}}dx$$
$$\Big\downarrow \text{ 35}$$
$$-c\int \frac{ax+1}{(c-acx)^{5/2}}dx$$
$$\Big\downarrow \text{ 53}$$
$$-c\int \left(\frac{2}{(c-acx)^{5/2}}-\frac{1}{c(c-acx)^{3/2}}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-c\left(\frac{4}{3ac(c-acx)^{3/2}}-\frac{2}{ac^2\sqrt{c-acx}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

output
```
-(c*(4/(3*a*c*(c - a*c*x)^(3/2)) - 2/(a*c^2*Sqrt[c - a*c*x])))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{2(3ax-1)}{3a(-acx+c)^{\frac{3}{2}}}$	21
orering	$-\frac{2(3ax-1)}{3a(-acx+c)^{\frac{3}{2}}}$	21
default	$\frac{\frac{2}{\sqrt{-acx+c}}-\frac{4c}{3(-acx+c)^{\frac{3}{2}}}}{ac}$	31
trager	$-\frac{2(3ax-1)\sqrt{-acx+c}}{3c^2(ax-1)^2a}$	31
pseudoelliptic	$\frac{6ax-2}{3ac(ax-1)\sqrt{-c(ax-1)}}$	32
derivativedivides	$-\frac{2\left(\frac{2c}{3(-acx+c)^{\frac{3}{2}}}-\frac{1}{\sqrt{-acx+c}}\right)}{ca}$	33

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*a*x-1)/a/(-a*c*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2\sqrt{-acx+c}(3\,ax-1)}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(-a*c*x + c)*(3*a*x - 1)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [A] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2c}{3(-acx+c)^{\frac{3}{2}}}-\frac{1}{\sqrt{-acx+c}}\right)}{ac} & \text{for } ac\neq 0 \\ \frac{\begin{cases} -x & \text{for } a=0 \\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise} \end{cases}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(3/2),x)
```

output
```
Piecewise((-2*(2*c/(3*(-a*c*x + c)**(3/2)) - 1/sqrt(-a*c*x + c))/(a*c), Ne
(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a, True)
)/c**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2\,(3\,acx-c)}{3\,(-acx+c)^{\frac{3}{2}}ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-2/3*(3*a*c*x - c)/((-a*c*x + c)^(3/2)*a*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{2\,(3\,acx-c)}{3\,(acx-c)\sqrt{-acx+c}ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
2/3*(3*a*c*x - c)/((a*c*x - c)*sqrt(-a*c*x + c)*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{6\,a\,x-2}{3\,a\,(c-a\,c\,x)^{3/2}}$$

input
```
int((a*x + 1)/((c - a*c*x)^(3/2)*(a*x - 1)),x)
```

output
```
-(6*a*x - 2)/(3*a*(c - a*c*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{2\sqrt{c}\,(3ax-1)}{3\sqrt{-ax+1}\,a\,c^2\,(ax-1)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x)
```

output
```
(2*sqrt(c)*(3*a*x - 1))/(3*sqrt( - a*x + 1)*a*c**2*(a*x - 1))
```

3.241 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 40

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{4}{5a(c-acx)^{5/2}} + \frac{2}{3ac(c-acx)^{3/2}}$$

output
```
-4/5/a/(-a*c*x+c)^(5/2)+2/3/a/c/(-a*c*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{2(1+5ax)}{15ac^2(-1+ax)^2\sqrt{c-acx}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

output
```
(-2*(1 + 5*a*x))/(15*a*c^2*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{(c-acx)^{5/2}}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{ax+1}{(1-ax)(c-acx)^{5/2}}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{ax+1}{(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{53}$$

$$-c\int \left(\frac{2}{(c-acx)^{7/2}}-\frac{1}{c(c-acx)^{5/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{4}{5ac(c-acx)^{5/2}}-\frac{2}{3ac^2(c-acx)^{3/2}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

output
```
-(c*(4/(5*a*c*(c - a*c*x)^(5/2)) - 2/(3*a*c^2*(c - a*c*x)^(3/2))))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{2(5ax+1)}{15a(-acx+c)^{\frac{5}{2}}}$	21
orering	$-\frac{2(5ax+1)}{15a(-acx+c)^{\frac{5}{2}}}$	21
trager	$\frac{2(5ax+1)\sqrt{-acx+c}}{15c^3(ax-1)^3 a}$	31
pseudoelliptic	$\frac{-\frac{2ax}{3}-\frac{2}{15}}{a\,c^2(ax-1)^2\sqrt{-c(ax-1)}}$	32
derivativedivides	$-\frac{2\left(\frac{2c}{5(-acx+c)^{\frac{5}{2}}}-\frac{1}{3(-acx+c)^{\frac{3}{2}}}\right)}{ca}$	33
default	$\frac{\frac{2}{3(-acx+c)^{\frac{3}{2}}}-\frac{4c}{5(-acx+c)^{\frac{5}{2}}}}{ac}$	33

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(5*a*x+1)/a/(-a*c*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{2\sqrt{-acx+c}(5\,ax+1)}{15\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/15*sqrt(-a*c*x + c)*(5*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3
*x - a*c^3)
```

Sympy [A] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2c}{5(-acx+c)^{\frac{5}{2}}}-\frac{1}{3(-acx+c)^{\frac{3}{2}}}\right)}{ac} & \text{for } ac\neq 0 \\ \frac{\begin{cases} -x & \text{for } a=0 \\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise} \end{cases}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(5/2),x)
```

output
```
Piecewise((-2*(2*c/(5*(-a*c*x + c)**(5/2)) - 1/(3*(-a*c*x + c)**(3/2)))/(a
*c), Ne(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a
, True))/c**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{2\,(5\,acx+c)}{15\,(-acx+c)^{\frac{5}{2}}ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-2/15*(5*a*c*x + c)/((-a*c*x + c)^(5/2)*a*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{2\,(5\,acx+c)}{15\,(acx-c)^2\sqrt{-acx+c}ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
-2/15*(5*a*c*x + c)/((a*c*x - c)^2*sqrt(-a*c*x + c)*a*c)
```

Mupad [B] (verification not implemented)

Time = 13.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.50

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{10\,a\,x+2}{15\,a\,(c-a\,c\,x)^{5/2}}$$

input
```
int((a*x + 1)/((c - a*c*x)^(5/2)*(a*x - 1)),x)
```

output
```
-(10*a*x + 2)/(15*a*(c - a*c*x)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{2\sqrt{c}\,(-5ax-1)}{15\sqrt{-ax+1}\,a\,c^3\,(a^2x^2-2ax+1)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x)
```

output
```
(2*sqrt(c)*( - 5*a*x - 1))/(15*sqrt( - a*x + 1)*a*c**3*(a**2*x**2 - 2*a*x
+ 1))
```

3.242 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 40

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{4}{7a(c-acx)^{7/2}}+\frac{2}{5ac(c-acx)^{5/2}}$$

output
```
-4/7/a/(-a*c*x+c)^(7/2)+2/5/a/c/(-a*c*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{6+14ax}{35ac^3(-1+ax)^3\sqrt{c-acx}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

output
```
(6 + 14*a*x)/(35*a*c^3*(-1 + a*x)^3*Sqrt[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{ax+1}{(1-ax)(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{ax+1}{(c-acx)^{9/2}}dx$$

$$\Big\downarrow \text{53}$$

$$-c\int \left(\frac{2}{(c-acx)^{9/2}}-\frac{1}{c(c-acx)^{7/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{4}{7ac(c-acx)^{7/2}}-\frac{2}{5ac^2(c-acx)^{5/2}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

output
```
-(c*(4/(7*a*c*(c - a*c*x)^(7/2)) - 2/(5*a*c^2*(c - a*c*x)^(5/2))))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{2(7ax+3)}{35a(-acx+c)^{\frac{7}{2}}}$	21
orering	$-\frac{2(7ax+3)}{35a(-acx+c)^{\frac{7}{2}}}$	21
trager	$-\frac{2(7ax+3)\sqrt{-acx+c}}{35c^4(ax-1)^4 a}$	31
pseudoelliptic	$\frac{\frac{2ax}{5}+\frac{6}{35}}{a\,c^3(ax-1)^3\sqrt{-c(ax-1)}}$	32
derivativedivides	$-\frac{2\left(-\frac{1}{5(-acx+c)^{\frac{5}{2}}}+\frac{2c}{7(-acx+c)^{\frac{7}{2}}}\right)}{ca}$	33
default	$\frac{-\frac{4c}{7(-acx+c)^{\frac{7}{2}}}+\frac{2}{5(-acx+c)^{\frac{5}{2}}}}{ac}$	33

input
```
int((a*x+1)/(-a*c*x+c)^(7/2)/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(7*a*x+3)/a/(-a*c*x+c)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{2\sqrt{-acx+c}(7\,ax+3)}{35\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/35*sqrt(-a*c*x + c)*(7*a*x + 3)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^
4*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2c}{7(-acx+c)^{\frac{7}{2}}}-\frac{1}{5(-acx+c)^{\frac{5}{2}}}\right)}{ac} & \text{for } ac\neq 0 \\ \frac{\begin{cases} -x & \text{for } a=0 \\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise} \end{cases}}{c^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(7/2),x)
```

output
```
Piecewise((-2*(2*c/(7*(-a*c*x + c)**(7/2)) - 1/(5*(-a*c*x + c)**(5/2)))/(a
*c), Ne(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a
, True))/c**(7/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{2\,(7\,acx+3\,c)}{35\,(-acx+c)^{\frac{7}{2}}ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-2/35*(7*a*c*x + 3*c)/((-a*c*x + c)^(7/2)*a*c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{2\,(7\,acx+3\,c)}{35\,(acx-c)^3\sqrt{-acx+c}ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
2/35*(7*a*c*x + 3*c)/((a*c*x - c)^3*sqrt(-a*c*x + c)*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.50

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{14\,a\,x+6}{35\,a\,(c-a\,c\,x)^{7/2}}$$

input
```
int((a*x + 1)/((c - a*c*x)^(7/2)*(a*x - 1)),x)
```

output
```
-(14*a*x + 6)/(35*a*(c - a*c*x)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{2\sqrt{c}\,(7ax+3)}{35\sqrt{-ax+1}\,a\,c^4\,(a^3x^3-3a^2x^2+3ax-1)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x)
```

output
```
(2*sqrt(c)*(7*a*x + 3))/(35*sqrt( - a*x + 1)*a*c**4*(a**3*x**3 - 3*a**2*x*
*2 + 3*a*x - 1))
```

3.243 $\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 197

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=-\frac{8\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{856\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{1155a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2}+\frac{16\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{21a^2\left(1-\frac{1}{ax}\right)^{9/2}x}+\frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{9/2}}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

output

```
-8/33*(1+1/a/x)^(5/2)*(-a*c*x+c)^(9/2)/a/(1-1/a/x)^(9/2)-856/1155*(1+1/a/x
)^(5/2)*(-a*c*x+c)^(9/2)/a^3/(1-1/a/x)^(9/2)/x^2+16/21*(1+1/a/x)^(5/2)*(-a
*c*x+c)^(9/2)/a^2/(1-1/a/x)^(9/2)/x+2/11*(a-1/x)^3*(1+1/a/x)^(5/2)*x*(-a*c
*x+c)^(9/2)/a^3/(1-1/a/x)^(9/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.39

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{2c^4\sqrt{1+\frac{1}{ax}}(1+ax)^2\sqrt{c-acx}(-533+755ax-455a^2x^2+105a^3x^3)}{1155a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(9/2),x]
```

output
```
(2*c^4*Sqrt[1 + 1/(a*x)]*(1 + a*x)^2*Sqrt[c - a*c*x]*(-533 + 755*a*x - 455
*a^2*x^2 + 105*a^3*x^3))/(1155*a*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6727, 27, 105, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{9/2}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\int \frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}}{a^3\left(\frac{1}{x}\right)^{13/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\int \frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{13/2}}d\frac{1}{x}}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{12}{11}\int\frac{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(a-\frac{1}{x}\right)^3}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \ 100$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{12}{11}\left(\frac{2}{9}\int-\frac{\left(22a-\frac{9}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}{2\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{5/2}}{9\left(\frac{1}{x}\right)^{9/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(a-\frac{1}{x}\right)^3}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{12}{11}\left(-\frac{1}{9}\int\frac{\left(22a-\frac{9}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{5/2}}{9\left(\frac{1}{x}\right)^{9/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(a-\frac{1}{x}\right)^3}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \ 87$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\left(-\frac{12}{11}\left(\frac{1}{9}\left(\frac{107}{7}\int\frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}+\frac{44a\left(\frac{1}{ax}+1\right)^{5/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{5/2}}{9\left(\frac{1}{x}\right)^{9/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(a-\frac{1}{x}\right)^3}{11\left(\frac{1}{x}\right)^{11/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$\Big\downarrow \ 48$$

$$-\frac{\left(\frac{1}{x}\right)^{9/2}\left(-\frac{12}{11}\left(\frac{1}{9}\left(\frac{44a\left(\frac{1}{ax}+1\right)^{5/2}}{7\left(\frac{1}{x}\right)^{7/2}}-\frac{214\left(\frac{1}{ax}+1\right)^{5/2}}{35\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{5/2}}{9\left(\frac{1}{x}\right)^{9/2}}\right)-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(a-\frac{1}{x}\right)^3}{11\left(\frac{1}{x}\right)^{11/2}}\right)(c-acx)^{9/2}}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(9/2),x]
```

output
```
-((((-12*(((44*a*(1 + 1/(a*x))^(5/2))/(7*(x^(-1))^(7/2)) - (214*(1 + 1/(a*
x))^(5/2))/(35*(x^(-1))^(5/2)))/9 - (2*a^2*(1 + 1/(a*x))^(5/2))/(9*(x^(-1)
)^(9/2))))/11 - (2*(a - x^(-1))^3*(1 + 1/(a*x))^(5/2))/(11*(x^(-1))^(11/2)
))*(x^(-1))^(9/2)*(c - a*c*x)^(9/2))/(a^3*(1 - 1/(a*x))^(9/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.32

method	result	size
gosper	$\frac{2(ax+1)(105a^3x^3-455a^2x^2+755ax-533)(-acx+c)^{\frac{9}{2}}}{1155a(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	64
orering	$\frac{2(ax+1)(105a^3x^3-455a^2x^2+755ax-533)(-acx+c)^{\frac{9}{2}}}{1155a(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	64
default	$\frac{2(ax-1)(ax+1)\sqrt{-c(ax-1)}\,c^4(105a^3x^3-455a^2x^2+755ax-533)}{1155\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	66
risch	$-\frac{2c^5(ax-1)(105a^5x^5-245a^4x^4-50a^3x^3+522a^2x^2-311ax-533)}{1155\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}\,a}$	77

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155*(a*x+1)*(105*a^3*x^3-455*a^2*x^2+755*a*x-533)*(-a*c*x+c)^(9/2)/a/(a
*x-1)^3/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.53

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\left(105\,a^6c^4x^6-140\,a^5c^4x^5-295\,a^4c^4x^4+472\,a^3c^4x^3+211\,a^2c^4x^2-844\,ac^4x-533\,c^4\right)\sqrt{-}}{1155\,(a^2x-a)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="fricas"
)
```

output
```
2/1155*(105*a^6*c^4*x^6 - 140*a^5*c^4*x^5 - 295*a^4*c^4*x^4 + 472*a^3*c^4*
x^3 + 211*a^2*c^4*x^2 - 844*a*c^4*x - 533*c^4)*sqrt(-a*c*x + c)*sqrt((a*x
- 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.54

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{2\left(105\,a^5\sqrt{-c}c^4x^5 - 455\,a^4\sqrt{-c}c^4x^4 + 650\,a^3\sqrt{-c}c^4x^3 - 78\,a^2\sqrt{-c}c^4x^2 - 755\,a\sqrt{-c}c^4x + \cdots\right.}{1155\,(ax-1)a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="maxima"
)
```

output
```
2/1155*(105*a^5*sqrt(-c)*c^4*x^5 - 455*a^4*sqrt(-c)*c^4*x^4 + 650*a^3*sqrt
(-c)*c^4*x^3 - 78*a^2*sqrt(-c)*c^4*x^2 - 755*a*sqrt(-c)*c^4*x + 533*sqrt(-
c)*c^4)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.66

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = -\frac{2\left(512\sqrt{2}\sqrt{-c}c^3 + \frac{105\,(acx+c)^5\sqrt{-acx-c}-770\,(acx+c)^4\sqrt{-acx-c}c+1980\,(acx+c)^3\sqrt{-acx-c}c^2-1848\,(acx+c)^2\sqrt{-acx-c}c^3}{c^2}\right)c^2}{1155\,a|c|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="giac")
```

output
```
-2/1155*(512*sqrt(2)*sqrt(-c)*c^3 + (105*(a*c*x + c)^5*sqrt(-a*c*x - c) -
770*(a*c*x + c)^4*sqrt(-a*c*x - c)*c + 1980*(a*c*x + c)^3*sqrt(-a*c*x - c)
*c^2 - 1848*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^3)/c^2)*c^2/(a*abs(c)*sgn(a*x
 + 1))
```

Mupad [B] (verification not implemented)

Time = 13.95 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.56

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{2\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(105\,a^5\,x^5-35\,a^4\,x^4-330\,a^3\,x^3+142\,a^2\,x^2+353\,a\,x-491\right)}{1155\,a}-\frac{2048\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{1155\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*c^4*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(353*a*x + 142*a^2*x^
2 - 330*a^3*x^3 - 35*a^4*x^4 + 105*a^5*x^5 - 491))/(1155*a) - (2048*c^4*(c
 - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(1155*a*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.28

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{2\sqrt{c}\,\sqrt{ax+1}\,c^4i(-105a^5x^5+245a^4x^4+50a^3x^3-522a^2x^2+311ax+533)}{1155a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x)
```

output
```
(2*sqrt(c)*sqrt(a*x + 1)*c**4*i*( - 105*a**5*x**5 + 245*a**4*x**4 + 50*a**
3*x**3 - 522*a**2*x**2 + 311*a*x + 533))/(1155*a)
```

3.244 $\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{44\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{214\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{315a^2\left(1-\frac{1}{ax}\right)^{7/2}x}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

output
```
-44/63*(1+1/a/x)^(5/2)*(-a*c*x+c)^(7/2)/a/(1-1/a/x)^(7/2)+214/315*(1+1/a/x
)^(5/2)*(-a*c*x+c)^(7/2)/a^2/(1-1/a/x)^(7/2)/x+2/9*(1+1/a/x)^(5/2)*x*(-a*c
*x+c)^(7/2)/(1-1/a/x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.50

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{2c^3\sqrt{1+\frac{1}{ax}}(1+ax)^2\sqrt{c-acx}(107-110ax+35a^2x^2)}{315a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

output
```
(-2*c^3*Sqrt[1 + 1/(a*x)]*(1 + a*x)^2*Sqrt[c - a*c*x]*(107 - 110*a*x + 35*
a^2*x^2))/(315*a*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6727, 27, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{7/2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\int \frac{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}}{a^2\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\int \frac{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{ 100}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(\frac{2}{9}\int -\frac{\left(22a-\frac{9}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}{2\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{5/2}}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(-\frac{1}{9}\int \frac{\left(22a-\frac{9}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{5/2}}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\left(\frac{1}{9}\left(\frac{107}{7}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}+\frac{44a\left(\frac{1}{ax}+1\right)^{5/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{5/2}}{9\left(\frac{1}{x}\right)^{9/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$\Big\downarrow \; 48$$

$$-\frac{\left(\frac{1}{x}\right)^{7/2}\left(\frac{1}{9}\left(\frac{44a\left(\frac{1}{ax}+1\right)^{5/2}}{7\left(\frac{1}{x}\right)^{7/2}}-\frac{214\left(\frac{1}{ax}+1\right)^{5/2}}{35\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2a^2\left(\frac{1}{ax}+1\right)^{5/2}}{9\left(\frac{1}{x}\right)^{9/2}}\right)(c-acx)^{7/2}}{a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

output
```
-(((((44*a*(1 + 1/(a*x))^(5/2))/(7*(x^(-1))^(7/2)) - (214*(1 + 1/(a*x))^(5
/2))/(35*(x^(-1))^(5/2)))/9 - (2*a^2*(1 + 1/(a*x))^(5/2))/(9*(x^(-1))^(9/2
)))*(x^(-1))^(7/2)*(c - a*c*x)^(7/2))/(a^2*(1 - 1/(a*x))^(7/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.41

method	result	size
gosper	$\frac{2(ax+1)\left(35a^{2}x^{2}-110ax+107\right)\left(-acx+c\right)^{\frac{7}{2}}}{315a\left(ax-1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	56
orering	$\frac{2(ax+1)\left(35a^{2}x^{2}-110ax+107\right)\left(-acx+c\right)^{\frac{7}{2}}}{315a\left(ax-1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	56
default	$-\frac{2(ax-1)(ax+1)\sqrt{-c(ax-1)}\,c^{3}\left(35a^{2}x^{2}-110ax+107\right)}{315\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	58
risch	$\frac{2c^{4}(ax-1)\left(35a^{4}x^{4}-40a^{3}x^{3}-78a^{2}x^{2}+104ax+107\right)}{315\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	69

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(a*x+1)*(35*a^2*x^2-110*a*x+107)*(-a*c*x+c)^(7/2)/a/(a*x-1)^2/((a*x-
1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\,(35\,a^5c^3x^5-5\,a^4c^3x^4-118\,a^3c^3x^3+26\,a^2c^3x^2+211\,ac^3x+107\,c^3)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^2x-a)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="fricas"
)
```

output
```
-2/315*(35*a^5*c^3*x^5 - 5*a^4*c^3*x^4 - 118*a^3*c^3*x^3 + 26*a^2*c^3*x^2
+ 211*a*c^3*x + 107*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.66

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\,\left(35\,a^4\sqrt{-c}c^3x^4-110\,a^3\sqrt{-c}c^3x^3+72\,a^2\sqrt{-c}c^3x^2+110\,a\sqrt{-c}c^3x-107\,\sqrt{-c}c^3\right)(ax+1)^{\frac{3}{2}}}{315\,(ax-1)a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="maxima"
)
```

output
```
-2/315*(35*a^4*sqrt(-c)*c^3*x^4 - 110*a^3*sqrt(-c)*c^3*x^3 + 72*a^2*sqrt(-
c)*c^3*x^2 + 110*a*sqrt(-c)*c^3*x - 107*sqrt(-c)*c^3)*(a*x + 1)^(3/2)/((a*
x - 1)*a)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.96 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.74

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{2\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,(35\,a^4\,x^4+30\,a^3\,x^3-88\,a^2\,x^2-62\,a\,x+149)}{315\,a} - \frac{512\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{315\,a\,(a\,x-1)}$$

input
```
int((c - a*c*x)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
- (2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(30*a^3*x^3 - 88*a^
2*x^2 - 62*a*x + 35*a^4*x^4 + 149))/(315*a) - (512*c^3*(c - a*c*x)^(1/2)*(
(a*x - 1)/(a*x + 1))^(1/2))/(315*a*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.34

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{2\sqrt{c}\,\sqrt{ax+1}\,c^3 i(35a^4x^4 - 40a^3x^3 - 78a^2x^2 + 104ax + 107)}{315a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x)
```

output
```
(2*sqrt(c)*sqrt(a*x + 1)*c**3*i*(35*a**4*x**4 - 40*a**3*x**3 - 78*a**2*x**
2 + 104*a*x + 107))/(315*a)
```

3.245 $\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=-\frac{18\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}$$

output
```
-18/35*(1+1/a/x)^(5/2)*(-a*c*x+c)^(5/2)/a/(1-1/a/x)^(5/2)+2/7*(1+1/a/x)^(5
/2)*x*(-a*c*x+c)^(5/2)/(1-1/a/x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}(-9+5ax)\sqrt{c-acx}(c+acx)^2}{35a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

output
```
(2*Sqrt[1 + 1/(a*x)]*(-9 + 5*a*x)*Sqrt[c - a*c*x]*(c + a*c*x)^2)/(35*a*Sqr
t[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6727, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{5/2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}{a\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{9}{7}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a\left(\frac{1}{ax}+1\right)^{5/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}\left(\frac{18\left(\frac{1}{ax}+1\right)^{5/2}}{35\left(\frac{1}{x}\right)^{5/2}}-\frac{2a\left(\frac{1}{ax}+1\right)^{5/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)(c-acx)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

output
```
-((((-2*a*(1 + 1/(a*x))^(5/2))/(7*(x^(-1))^(7/2)) + (18*(1 + 1/(a*x))^(5/2
))/(35*(x^(-1))^(5/2)))*(x^(-1))^(5/2)*(c - a*c*x)^(5/2))/(a*(1 - 1/(a*x))
^(5/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.54

method	result	size
gosper	$\frac{2(ax+1)(5ax-9)(-acx+c)^{\frac{5}{2}}}{35a(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	48
orering	$\frac{2(ax+1)(5ax-9)(-acx+c)^{\frac{5}{2}}}{35a(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	48
default	$\frac{2(ax-1)(ax+1)\sqrt{-c(ax-1)}\,c^{2}(5ax-9)}{35\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	50
risch	$-\frac{2c^{3}(ax-1)(5a^{3}x^{3}+a^{2}x^{2}-13ax-9)}{35\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	60

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(a*x+1)*(5*a*x-9)*(-a*c*x+c)^(5/2)/a/(a*x-1)/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(5\,a^4c^2x^4+6\,a^3c^2x^3-12\,a^2c^2x^2-22\,ac^2x-9\,c^2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^2x-a)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="fricas"
)
```

output
```
2/35*(5*a^4*c^2*x^4 + 6*a^3*c^2*x^3 - 12*a^2*c^2*x^2 - 22*a*c^2*x - 9*c^2)
*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2\left(5a^3\sqrt{-c}c^2x^3 - 9a^2\sqrt{-c}c^2x^2 - 5a\sqrt{-c}c^2x + 9\sqrt{-c}c^2\right)(ax+1)^{\frac{3}{2}}}{35(ax-1)a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="maxima"
)
```

output
```
2/35*(5*a^3*sqrt(-c)*c^2*x^3 - 9*a^2*sqrt(-c)*c^2*x^2 - 5*a*sqrt(-c)*c^2*x
 + 9*sqrt(-c)*c^2)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = -\frac{2\left(16\sqrt{2}\sqrt{-c}c + \frac{5(acx+c)^3\sqrt{-acx-c}-14(acx+c)^2\sqrt{-acx-c}c}{c^2}\right)c^2}{35a|c|\mathrm{sgn}(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
-2/35*(16*sqrt(2)*sqrt(-c)*c + (5*(a*c*x + c)^3*sqrt(-a*c*x - c) - 14*(a*c
*x + c)^2*sqrt(-a*c*x - c)*c)/c^2)*c^2/(a*abs(c)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 14.01 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = -\frac{2\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(-5\,a^3\,x^3-11\,a^2\,x^2+a\,x+23\right)}{35\,a}-\frac{64\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{35\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
- (2*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(a*x - 11*a^2*x^2 -
 5*a^3*x^3 + 23))/(35*a) - (64*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))
^(1/2))/(35*a*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2\sqrt{c}\,\sqrt{ax+1}\,c^2 i(-5a^3x^3-a^2x^2+13ax+9)}{35a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x)
```

output
```
(2*sqrt(c)*sqrt(a*x + 1)*c**2*i*( - 5*a**3*x**3 - a**2*x**2 + 13*a*x + 9))
/(35*a)
```

3.246 $\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 31

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{2e^{3\coth^{-1}(ax)}(1+ax)(c-acx)^{3/2}}{5a}$$

output
```
2/5/((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(-a*c*x+c)^(3/2)/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

output
```
(2*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(3/2))/(5*(1 - 1/(a*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{3/2}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6726}$$

$$\frac{2(ax+1)(c-acx)^{3/2}e^{3\coth^{-1}(ax)}}{5a}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

output
```
(2*E^(3*ArcCoth[a*x])*(1 + a*x)*(c - a*c*x)^(3/2))/(5*a)
```

Defintions of rubi rules used

rule 6726
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] &&  !IntegerQ[p] && EqQ[p, n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

method	result	size
gosper	$\frac{2(ax+1)(-acx+c)^{\frac{3}{2}}}{5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	35
orering	$\frac{2(ax+1)(-acx+c)^{\frac{3}{2}}}{5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	35
default	$-\frac{2(ax-1)(ax+1)\sqrt{-c(ax-1)}\,c}{5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	42
risch	$\frac{2c^{2}(ax-1)\left(a^{2}x^{2}+2ax+1\right)}{5\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	52

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(-a*c*x+c)^(3/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\left(a^{3}cx^{3}+3\,a^{2}cx^{2}+3\,acx+c\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^{2}x-a\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="fricas"
)
```

output
```
-2/5*(a^3*c*x^3 + 3*a^2*c*x^2 + 3*a*c*x + c)*sqrt(-a*c*x + c)*sqrt((a*x -
1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\left(a^2\sqrt{-c}cx^2-\sqrt{-c}c\right)(ax+1)^{\frac{3}{2}}}{5\,(ax-1)a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="maxima"
)
```

output
```
-2/5*(a^2*sqrt(-c)*c*x^2 - sqrt(-c)*c)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.61

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(a^2\,x^2+4\,a\,x+7\right)}{5\,a}-\frac{16\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{5\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
- (2*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(4*a*x + a^2*x^2 + 7)
)/(5*a) - (16*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(5*a*(a*x -
 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{2\sqrt{c}\,\sqrt{ax+1}\,ci(a^2x^2+2ax+1)}{5a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt(a*x + 1)*c*i*(a**2*x**2 + 2*a*x + 1))/(5*a)
```

3.247 $\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 164

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

output
```
4*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/3*(1+1/a/x)^(3/2)*x
*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*2^(1/2)*(1/a/x)^(1/2)*(-a*c*x+c)^(1/2)
*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))/a/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.64

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}(7+ax)-6\sqrt{2}\sqrt{\frac{1}{x}}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{3a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 + a*x) - 6*Sqrt[2]*Sqrt[x
^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(3*a^
(3/2)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6727, 27, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6727}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2\left(\frac{2\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 104

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2\left(\frac{4\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 219

$$-\frac{a\sqrt{\frac{1}{x}}\left(\frac{2\left(\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*(1 + 1/(a*x))^(3/2))/(3*a*(x^(-1))^
(3/2)) + (2*((-2*Sqrt[1 + 1/(a*x)])/(a*Sqrt[x^(-1)]) + (2*Sqrt[2]*ArcTanh[
(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/a^(3/2)))/a))/Sqrt[1
- 1/(a*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(6\sqrt{c}\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)-ax\sqrt{-c(ax+1)}-7\sqrt{-c(ax+1)}\right)}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a}$	107
risch	$-\frac{2(ax+7)c(ax-1)}{3a\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}(ax-1)}{a\sqrt{\frac{ax-1}{ax+1}}(ax+1)\sqrt{-c(ax-1)}}$	121

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(6*c^(1/2)
*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-a*x*(-c*(a*x+1))^(
1/2)-7*(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.56

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$$

$$= \left[\frac{2\left(3\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(a^2x^2+8\,ax+7)\sqrt{-acx+c}\right.}{3\,(a^2x-a)}\right.$$

$$\left.-\frac{2\left(6\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2\,(acx-c)}\right)-(a^2x^2+8\,ax+7)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}\right)}{3\,(a^2x-a)}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
[2/3*(3*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*s
qrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x
^2 - 2*a*x + 1)) + (a^2*x^2 + 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(
a*x + 1)))/(a^2*x - a), -2/3*(6*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(1/2*sqrt(
2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c
)) - (a^2*x^2 + 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^
2*x - a)]
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\frac{\sqrt{-c(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\frac{\sqrt{-acx+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a*c*x + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.32

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{c}\,i\left(-\sqrt{ax+1}\,ax-7\sqrt{ax+1}-6\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)+8\sqrt{2}\right)}{3a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x)
```

output
```
(2*sqrt(c)*i*( - sqrt(a*x + 1)*a*x - 7*sqrt(a*x + 1) - 6*sqrt(2)*log(tan(a
sin(sqrt( - a*x + 1)/sqrt(2))/2)) + 8*sqrt(2)))/(3*a)
```

3.248 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 170

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{\sqrt{c-acx}}-\frac{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}-\frac{3\sqrt{2}\sqrt{1-\frac{1}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a\sqrt{\frac{1}{ax}}\sqrt{c-acx}}$$

output
```
3*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)*x/(-a*c*x+c)^(1/2)-a*(1-1/a/x)^(1/2)*(1+
1/a/x)^(3/2)*x/(a-1/x)/(-a*c*x+c)^(1/2)-3*2^(1/2)*(1-1/a/x)^(1/2)*arctanh(
2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))/a/(1/a/x)^(1/2)/(-a*c*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.68

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}x\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}(-2+ax)-3\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{\sqrt{a}(-1+ax)\sqrt{c-acx}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

output
```
(Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(-2 + a*x) - 3*Sqrt[2]*S
qrt[x^(-1)]*(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/
(a*x)])]))/(Sqrt[a]*(-1 + a*x)*Sqrt[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6727, 27, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^2\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{2a}+\frac{\left(\frac{1}{ax}+1\right)^{3/2}}{a\sqrt{\frac{1}{x}}\left(a-\frac{1}{x}\right)}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\left(\frac{2\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{2a}+\frac{\left(\frac{1}{ax}+1\right)^{3/2}}{a\sqrt{\frac{1}{x}}\left(a-\frac{1}{x}\right)}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{ 104}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\left(\frac{4\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{2a}+\frac{\left(\frac{1}{ax}+1\right)^{3/2}}{a\sqrt{\frac{1}{x}}\left(a-\frac{1}{x}\right)}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\left(\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{2a}+\frac{\left(\frac{1}{ax}+1\right)^{3/2}}{a\sqrt{\frac{1}{x}}\left(a-\frac{1}{x}\right)}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

input
```
Int[E^(3*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

output
```
-((a^2*Sqrt[1 - 1/(a*x)]*((1 + 1/(a*x))^(3/2)/(a*(a - x^(-1))*Sqrt[x^(-1)]
) + (3*((-2*Sqrt[1 + 1/(a*x)])/(a*Sqrt[x^(-1)]) + (2*Sqrt[2]*ArcTanh[(Sqrt
[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/a^(3/2)))/(2*a)))/(Sqrt[x^
(-1)]*Sqrt[c - a*c*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.79

method	result	siz
default	$\frac{\sqrt{-c(ax-1)}\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx-2ax\sqrt{-c(ax+1)}\,\sqrt{c}-3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)c+4\sqrt{-c(ax+1)}\,\sqrt{c}\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)c^{\frac{3}{2}}\sqrt{-c(ax+1)}\,a}$	13
risch	$\frac{2ax-2}{a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}+\frac{\left(-\frac{2\sqrt{-acx-c}}{a(-acx+c)}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}$	14

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)*(-c*(a*x-1))^(1/2)*(3*2^(1/2)*arctan(1/2
*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-2*a*x*(-c*(a*x+1))^(1/2)*c^(1/2
)-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+4*(-c*(a*x+1)
)^(1/2)*c^(1/2))/c^(3/2)/(-c*(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.73

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$$

$$= \left[\frac{3\sqrt{2}(a^2cx^2-2\,acx+c)\sqrt{-\frac{1}{c}}\log\left(-\frac{a^2x^2-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}+2\,ax-3}{a^2x^2-2\,ax+1}\right)-4\,(a^2x^2-ax-2)\sqrt{-c}}{2\,(a^3cx^2-2\,a^2cx+ac)}\right.$$

$$\left.-\frac{2\,(a^2x^2-ax-2)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}-\frac{3\sqrt{2}(a^2cx^2-2\,acx+c)\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\,(ax-1)\sqrt{c}}\right)}{\sqrt{c}}}{a^3cx^2-2\,a^2cx+ac}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/2*(3*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*sqrt(-1/c)*log(-(a^2*x^2 - 2*sqr
t(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a
*x - 3)/(a^2*x^2 - 2*a*x + 1)) - 4*(a^2*x^2 - a*x - 2)*sqrt(-a*c*x + c)*sq
rt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c), -(2*(a^2*x^2 - a*x
 - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)) - 3*sqrt(2)*(a^2*c*x^2 -
2*a*c*x + c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/
(a*x + 1))/((a*x - 1)*sqrt(c)))/sqrt(c))/(a^3*c*x^2 - 2*a^2*c*x + a*c)]
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\,(ax-1)}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(1/2),x)
```

output
```
Integral(1/(((a*x - 1)/(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1))), x)
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\int \frac{1}{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.44

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{3\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)-2\sqrt{-acx-c}+\frac{2\sqrt{-acx-c}c}{acx-c}}{a|c|}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
-(3*sqrt(2)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) - 2*sqrt(
-a*c*x - c) + 2*sqrt(-a*c*x - c)*c/(a*c*x - c))/(a*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{1}{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.54

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\sqrt{c}\,i\left(8\sqrt{ax+1}\,ax-16\sqrt{ax+1}+12\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)ax-12\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right.\right.}{4ac\,(ax-1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x)
```

output
```
(sqrt(c)*i*(8*sqrt(a*x + 1)*a*x - 16*sqrt(a*x + 1) + 12*sqrt(2)*log(tan(as
in(sqrt( - a*x + 1)/sqrt(2))/2))*a*x - 12*sqrt(2)*log(tan(asin(sqrt( - a*x
 + 1)/sqrt(2))/2)) - 9*sqrt(2)*a*x + 9*sqrt(2)))/(4*a*c*(a*x - 1))
```

3.249 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 188

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{3a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{4\sqrt{2}a\left(\frac{1}{ax}\right)^{3/2}(c-acx)^{3/2}}$$

output
```
-3/4*a*(1-1/a/x)^(3/2)*(1+1/a/x)^(1/2)*x/(a-1/x)/(-a*c*x+c)^(3/2)-1/2*a^2*
(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)*x/(a-1/x)^2/(-a*c*x+c)^(3/2)-3/8*(1-1/a/x)
^(3/2)*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)/a/(1/a/x)^(3
/2)/(-a*c*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.66

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}x\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}(-1+5ax)+3\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{8\sqrt{a}c(-1+ax)^2\sqrt{c-acx}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(-1 + 5*a*x) + 3*Sqrt[2]
*Sqrt[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1
+ 1/(a*x)])]))/(8*Sqrt[a]*c*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6727, 27, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}dx$$

$$\Big\downarrow \text{ 6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{a^3\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{3\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}d\frac{1}{x}}{4a}+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{3\left(\frac{\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{3\left(\frac{\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}a^{3/2}}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

output
```
-((a^3*(1 - 1/(a*x))^(3/2)*(((1 + 1/(a*x))^(3/2)*Sqrt[x^(-1)])/(2*a*(a - x
^(-1))^2) + (3*((Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)])/(a*(a - x^(-1))) + ArcTan
h[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]/(Sqrt[2]*a^(3/2))))/
(4*a)))/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\sqrt{-c(ax-1)}\left(3\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^{2}c\,x^{2}-6\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)acx+10ax\sqrt{-c(ax+1)}\sqrt{c}+3\sqrt{2}\arctan\left(\frac{\sqrt{-}}{}\right.\right.}{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-1)(ax+1)c^{\frac{5}{2}}\sqrt{-c(ax+1)}\,a}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)/c^(5/2)*(3
*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2-6*2^(1/2
)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+10*a*x*(-c*(a*x+1))
^(1/2)*c^(1/2)+3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-
2*(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.85

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \left[-\frac{3\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-}{a^2x^2-2ax+1}\right.}{16\left(a^4c^2x^3-3a^3c^2x^2+3a^2c^2x-ac^2\right.}\right.$$

$$-\frac{3\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2(acx-c)}\right)+2\left(5a^2x^2+4ax-1\right)\sqrt{-acx+c}}{8\left(a^4c^2x^3-3a^3c^2x^2+3a^2c^2x-ac^2\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas"
)
```

output
```
[-1/16*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x
^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1
)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*(5*a^2*x^2 + 4*a*x - 1)*sqr
t(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*
a^2*c^2*x - a*c^2), -1/8*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt
(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(
a*x + 1))/(a*c*x - c)) + 2*(5*a^2*x^2 + 4*a*x - 1)*sqrt(-a*c*x + c)*sqrt((
a*x - 1)/(a*x + 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{1}{(-acx+c)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(1/((-a*c*x + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\sqrt{c}} - \frac{2\left(5(-acx-c)^{\frac{3}{2}}+6\sqrt{-acx-c}c\right)}{(acx-c)^2}}{8a|c|}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - 2*(5
*(-a*c*x - c)^(3/2) + 6sqrt(-a*c*x - c)*c)/(a*c*x - c)^2)/(a*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{1}{(c-a\,c\,x)^{3/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.62

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\sqrt{c}\,i\left(10\sqrt{ax+1}\,ax-2\sqrt{ax+1}-3\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)a^2x^2+6\sqrt{2}\log\left(t\right.\right.}{8a\,c^2\left(a^2x^2-2ax+1\right)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x)
```

output
```
(sqrt(c)*i*(10*sqrt(a*x + 1)*a*x - 2*sqrt(a*x + 1) - 3*sqrt(2)*log(tan(asi
n(sqrt( - a*x + 1)/sqrt(2))/2))*a**2*x**2 + 6*sqrt(2)*log(tan(asin(sqrt( -
 a*x + 1)/sqrt(2))/2))*a*x - 3*sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(
2))/2))))/(8*a*c**2*(a**2*x**2 - 2*a*x + 1))
```

3.250 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 251

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}+\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}+\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{16\sqrt{2}a\left(\frac{1}{ax}\right)^{5/2}(c-acx)^{5/2}}$$

output
```
1/16*a^2*(1-1/a/x)^(5/2)*(1+1/a/x)^(1/2)*x^2/(a-1/x)/(-a*c*x+c)^(5/2)+1/24
*a^3*(1-1/a/x)^(5/2)*(1+1/a/x)^(3/2)*x^2/(a-1/x)^2/(-a*c*x+c)^(5/2)-1/6*a^
4*(1-1/a/x)^(5/2)*(1+1/a/x)^(5/2)*x^2/(a-1/x)^3/(-a*c*x+c)^(5/2)+1/32*(1-1
/a/x)^(5/2)*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)/a/(1/a/
x)^(5/2)/(-a*c*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.57

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{\sqrt{1-\frac{1}{ax}}\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}(7+22ax+3a^2x^2)-\frac{3\sqrt{2}(-1+ax)^3\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{x}\right)}{96\sqrt{a}c^2\left(\frac{1}{x}\right)^{3/2}(-1+ax)^3\sqrt{c-acx}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

output
```
-1/96*(Sqrt[1 - 1/(a*x)]*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]*(7 + 22
*a*x + 3*a^2*x^2) - (3*Sqrt[2]*(-1 + a*x)^3*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])
/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/x))/(Sqrt[a]*c^2*(x^(-1))^(3/2)*(-1 + a*x)^
3*Sqrt[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6727, 27, 105, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \; 105$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}-\frac{1}{12}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \; 105$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{12}\left(-\frac{3\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}d\frac{1}{x}}{4a}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \; 105$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{12}\left(-\frac{3\left(\frac{\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \; 104$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{12}\left(-\frac{3\left(\frac{\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \; 219$$

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{12}\left(-\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}a^{3/2}}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

output
```
-((a^4*(1 - 1/(a*x))^(5/2)*(((1 + 1/(a*x))^(5/2)*Sqrt[x^(-1)])/(6*(a - x^(
-1))^3) + (-1/2*((1 + 1/(a*x))^(3/2)*Sqrt[x^(-1)])/(a*(a - x^(-1))^2) - (3
*((Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)])/(a*(a - x^(-1))) + ArcTanh[(Sqrt[2]*Sqr
t[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]/(Sqrt[2]*a^(3/2))))/(4*a))/12))/((
x^(-1))^(5/2)*(c - a*c*x)^(5/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.90

method	result
default	$-\frac{\sqrt{-c(ax-1)}\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^3 c\,x^3-9\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^2 c\,x^2-6a^2x^2\sqrt{-c(ax+1)}\,\sqrt{c}+9\sqrt{2}\,\arctan\left(\right.\right.}{96\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-1)^2(ax+1)c^{\frac{7}{2}}\sqrt{-c(}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)*(-c*(a*x-1))^(1/2)/c^(7/2)
*(3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^3*c*x^3-9*2^(
1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2-6*a^2*x^2*(-
c*(a*x+1))^(1/2)*c^(1/2)+9*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c
^(1/2))*a*c*x-44*a*x*(-c*(a*x+1))^(1/2)*c^(1/2)-3*2^(1/2)*arctan(1/2*(-c*(
a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-14*(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1
))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.59

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \left[-\frac{3\sqrt{2}(a^4x^4-4a^3x^3+6a^2x^2-4ax+1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{}}{a^2x^2-2ax+1}\right.}{192\left(a^5c^3x^4-4a^4c^3x^3+6a^3c^3x^2\right.}\right.$$

$$-\frac{3\sqrt{2}(a^4x^4-4a^3x^3+6a^2x^2-4ax+1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2(acx-c)}\right)-2\left(3a^3x^3+25a^2x^2+29\right.}{96\left(a^5c^3x^4-4a^4c^3x^3+6a^3c^3x^2-4a^2c^3x+ac^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas"
)
```

output
```
[-1/192*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*
log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*
sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^3*x^3 + 2
5*a^2*x^2 + 29*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c
^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3), -1/96*(3*sq
rt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*arctan(1/2*sqr
t(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x -
 c)) - 2*(3*a^3*x^3 + 25*a^2*x^2 + 29*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x
- 1)/(a*x + 1)))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*
x + a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{1}{(-acx+c)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima"
)
```

output
```
integrate(1/((-a*c*x + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{2\left(3(acx+c)^2\sqrt{-acx-c}-16(-acx-c)^{\frac{3}{2}}c-12\sqrt{-acx-c}c^2\right)}{(acx-c)^3c}}{96\,a|c|}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
1/96*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) - 2*(
3*(a*c*x + c)^2*sqrt(-a*c*x - c) - 16*(-a*c*x - c)^(3/2)*c - 12*sqrt(-a*c*
x - c)*c^2)/((a*c*x - c)^3*c))/(a*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{1}{(c-a\,c\,x)^{5/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.66

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\sqrt{c}\,i\left(-6\sqrt{ax+1}\,a^2x^2-44\sqrt{ax+1}\,ax-14\sqrt{ax+1}-3\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x)
```

output
```
(sqrt(c)*i*( - 6*sqrt(a*x + 1)*a**2*x**2 - 44*sqrt(a*x + 1)*a*x - 14*sqrt(
a*x + 1) - 3*sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2))/2))*a**3*x**3
+ 9*sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2))/2))*a**2*x**2 - 9*sqrt(
2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2))/2))*a*x + 3*sqrt(2)*log(tan(asin
(sqrt( - a*x + 1)/sqrt(2))/2))))/(96*a*c**3*(a**3*x**3 - 3*a**2*x**2 + 3*a
*x - 1))
```

3.251 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 308

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^3}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{256\sqrt{2}a\left(\frac{1}{ax}\right)^{7/2}(c-acx)^{7/2}}$$

output

```
-1/8*a^5*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x^2/(a-1/x)^4/(-a*c*x+c)^(7/2)-3/
256*a^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(1/2)*x^3/(a-1/x)/(-a*c*x+c)^(7/2)-1/128
*a^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^3/(a-1/x)^2/(-a*c*x+c)^(7/2)+1/32*a
^5*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x^3/(a-1/x)^3/(-a*c*x+c)^(7/2)-3/512*(1
-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)/a/(1/
a/x)^(7/2)/(-a*c*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.48

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(\frac{2\sqrt{a}\sqrt{1+\frac{1}{ax}}\,(39+79ax+13a^2x^2-3a^3x^3)}{\sqrt{\frac{1}{x}}}+3\sqrt{2}(-1+ax)^4\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{512\sqrt{a}c^3\sqrt{\frac{1}{x}}(-1+ax)^4\sqrt{c-acx}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*((2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(39 + 79*a*x + 13*a^2*x^2
 - 3*a^3*x^3))/Sqrt[x^(-1)] + 3*Sqrt[2]*(-1 + a*x)^4*ArcTanh[(Sqrt[2]*Sqrt
[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(512*Sqrt[a]*c^3*Sqrt[x^(-1)]*(-1
 + a*x)^4*Sqrt[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.76, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6727, 27, 105, 105, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{a^5\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4}-\frac{3}{16}\int\frac{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4}-\frac{3}{16}\left(\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}-\frac{1}{12}\int\frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4}-\frac{3}{16}\left(\frac{1}{12}\left(-\frac{3\int\frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}d\frac{1}{x}}{4a}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4}-\frac{3}{16}\left(\frac{1}{12}\left(-\frac{3\left(\frac{\int\frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4}-\frac{3}{16}\left(\frac{1}{12}\left(-\frac{3\left(\frac{\int\frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4}-\frac{3}{16}\left(\frac{1}{12}\left(-\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}a^{3/2}}+\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{4a}-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}}{2a\left(a-\frac{1}{x}\right)^2}\right)+\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

output
```
-((a^5*(1 - 1/(a*x))^(7/2)*(((1 + 1/(a*x))^(5/2)*(x^(-1))^(3/2))/(8*(a - x
^(-1))^4) - (3*(((1 + 1/(a*x))^(5/2)*Sqrt[x^(-1)])/(6*(a - x^(-1))^3) + (-
1/2*((1 + 1/(a*x))^(3/2)*Sqrt[x^(-1)])/(a*(a - x^(-1))^2) - (3*((Sqrt[1 +
1/(a*x)]*Sqrt[x^(-1)])/(a*(a - x^(-1))) + ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(
Sqrt[a]*Sqrt[1 + 1/(a*x)])]/(Sqrt[2]*a^(3/2))))/(4*a))/12))/16))/((x^(-1))
^(7/2)*(c - a*c*x)^(7/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.90

method	result
default	$-\frac{\sqrt{-c(ax-1)}\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{4}c\,x^{4}-12\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{3}c\,x^{3}-6a^{3}x^{3}\sqrt{-c(ax+1)}\,\sqrt{c}+18\sqrt{2}\,\arctan\right.}{512(}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/512/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3/(a*x+1)*(-c*(a*x-1))^(1/2)/c^(9/2
)*(3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^4*c*x^4-12*2
^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^3*c*x^3-6*a^3*x^3*
(-c*(a*x+1))^(1/2)*c^(1/2)+18*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2
)/c^(1/2))*a^2*c*x^2+26*a^2*x^2*(-c*(a*x+1))^(1/2)*c^(1/2)-12*2^(1/2)*arct
an(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+158*a*x*(-c*(a*x+1))^(1/2
)*c^(1/2)+3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+78*(-
c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.48

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \left[-\frac{3\sqrt{2}(a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-\cdots}}{a^2x\cdots}\right.}{1024\,(a^6c^4x^5-5a^5c^4x^4+10\cdots}\right.$$

$$-\frac{3\sqrt{2}(a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2\,(acx-c)}\right)-2\,(3\,a^4x^4-\cdots}{512\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-\cdots}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas"
)
```

output
```
[-1/1024*(3*sqrt(2)*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x
 - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x
 + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4
*(3*a^4*x^4 - 10*a^3*x^3 - 92*a^2*x^2 - 118*a*x - 39)*sqrt(-a*c*x + c)*sqr
t((a*x - 1)/(a*x + 1)))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10
*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4), -1/512*(3*sqrt(2)*(a^5*x^5 - 5*a^4*x^
4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-
a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*(3
*a^4*x^4 - 10*a^3*x^3 - 92*a^2*x^2 - 118*a*x - 39)*sqrt(-a*c*x + c)*sqrt((
a*x - 1)/(a*x + 1)))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^
3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\int \frac{1}{(-acx+c)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima"
)
```

output
```
integrate(1/((-a*c*x + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}-\frac{2\left(3(acx+c)^3\sqrt{-acx-c}-22(acx+c)^2\sqrt{-acx-c}c+44(-acx-c)^{\frac{3}{2}}c^2+24\sqrt{-acx-c}c^3\right.}{(acx-c)^4c^2}}{512\,a|c|}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
1/512*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) - 2*
(3*(a*c*x + c)^3*sqrt(-a*c*x - c) - 22*(a*c*x + c)^2*sqrt(-a*c*x - c)*c +
44*(-a*c*x - c)^(3/2)*c^2 + 24*sqrt(-a*c*x - c)*c^3)/((a*c*x - c)^4*c^2))/
(a*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\int \frac{1}{{\left(c-a\,c\,x\right)}^{7/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,dx$$

input
```
int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\frac{\sqrt{c}\,\sqrt{2}\,i\left(-48\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)^{8}-\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)^{16}\cdots\right.}{8192\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)^{8}a\,c\cdots}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x)
```

output
```
(sqrt(c)*sqrt(2)*i*( - 48*log(tan(asin(sqrt( - a*x + 1)/sqrt(2))/2))*tan(a
sin(sqrt( - a*x + 1)/sqrt(2))/2)**8 - tan(asin(sqrt( - a*x + 1)/sqrt(2))/2
)**16 + 8*tan(asin(sqrt( - a*x + 1)/sqrt(2))/2)**12 - 8*tan(asin(sqrt( - a
*x + 1)/sqrt(2))/2)**4 + 1))/(8192*tan(asin(sqrt( - a*x + 1)/sqrt(2))/2)**
8*a*c**4)
```

3.252 $\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 128

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{256c^3\sqrt{1-\frac{1}{a^2x^2}}x}{35\sqrt{c-acx}}+\frac{64}{35}c^2\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-acx}+\frac{24}{35}c\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{3/2}+\frac{2}{7}\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{5/2}$$

output
```
256/35*c^3*(1-1/a^2/x^2)^(1/2)*x/(-a*c*x+c)^(1/2)+64/35*c^2*(1-1/a^2/x^2)^
(1/2)*x*(-a*c*x+c)^(1/2)+24/35*c*(1-1/a^2/x^2)^(1/2)*x*(-a*c*x+c)^(3/2)+2/
7*(1-1/a^2/x^2)^(1/2)*x*(-a*c*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.55

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2c^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(-177+71ax-27a^2x^2+5a^3x^3)}{35a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(c - a*c*x)^(5/2)/E^ArcCoth[a*x],x]
```

output
```
(2*c^2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-177 + 71*a*x - 27*a^2*x^2 + 5*a
^3*x^3))/(35*a*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6727, 27, 105, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{5/2}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(a-\frac{1}{x}\right)^3}{a^3\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{12}{7}\int \frac{\left(a-\frac{1}{x}\right)^2}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{12}{7}\left(\frac{2}{5}\int -\frac{14a-\frac{5}{x}}{2\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{12}{7}\left(-\frac{1}{5}\int \frac{14a-\frac{5}{x}}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \; 87$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{12}{7}\left(\frac{1}{5}\left(\frac{43}{3}\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}+\frac{28a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \; 48$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}\left(-\frac{12}{7}\left(\frac{1}{5}\left(\frac{28a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}-\frac{86\sqrt{\frac{1}{ax}+1}}{3\sqrt{\frac{1}{x}}}\right)-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{7\left(\frac{1}{x}\right)^{7/2}}\right)(c-acx)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}}$$

input
```
Int[(c - a*c*x)^(5/2)/E^ArcCoth[a*x],x]
```

output
```
-((((-12*(((28*a*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) - (86*Sqrt[1 + 1/(a
*x)])/(3*Sqrt[x^(-1)]))/5 - (2*a^2*Sqrt[1 + 1/(a*x)])/(5*(x^(-1))^(5/2))))
/7 - (2*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)])/(7*(x^(-1))^(7/2)))*(x^(-1))^(5/
2)*(c - a*c*x)^(5/2))/(a^3*(1 - 1/(a*x))^(5/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.48

method	result	size
risch	$-\frac{2c^{3}\sqrt{\frac{ax-1}{ax+1}}\,(5a^{3}x^{3}-27a^{2}x^{2}+71ax-177)\,(ax+1)}{35\sqrt{-c(ax-1)}\,a}$	61
gosper	$\frac{2(ax+1)\,(5a^{3}x^{3}-27a^{2}x^{2}+71ax-177)\,(-acx+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}{35a(ax-1)^{3}}$	64
orering	$\frac{2(ax+1)\,(5a^{3}x^{3}-27a^{2}x^{2}+71ax-177)\,(-acx+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}{35a(ax-1)^{3}}$	64
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,c^{2}\,(5a^{3}x^{3}-27a^{2}x^{2}+71ax-177)}{35(ax-1)a}$	68

input
```
int((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*c^3*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(5*a^3*x^3-27*a^2*x^2
+71*a*x-177)/a*(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\left(5\,a^{4}c^{2}x^{4}-22\,a^{3}c^{2}x^{3}+44\,a^{2}c^{2}x^{2}-106\,ac^{2}x-177\,c^{2}\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^{2}x-a\right)}$$

input
```
integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
2/35*(5*a^4*c^2*x^4 - 22*a^3*c^2*x^3 + 44*a^2*c^2*x^2 - 106*a*c^2*x - 177*
c^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.75

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2\left(5a^4\sqrt{-c}c^2x^4-22a^3\sqrt{-c}c^2x^3+44a^2\sqrt{-c}c^2x^2-106a\sqrt{-c}c^2x-177\sqrt{-c}c^2\right)(ax-1)}{35\left(a^2x-a\right)\sqrt{ax+1}}$$

input
```
integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
2/35*(5*a^4*sqrt(-c)*c^2*x^4 - 22*a^3*sqrt(-c)*c^2*x^3 + 44*a^2*sqrt(-c)*c
^2*x^2 - 106*a*sqrt(-c)*c^2*x - 177*sqrt(-c)*c^2)*(a*x - 1)/((a^2*x - a)*s
qrt(a*x + 1))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.73

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^3\,x^3-17\,a^2\,x^2+27\,a\,x-79\right)}{35\,a}-\frac{512\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{35\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(27*a*x - 17*a^2*x^2
+ 5*a^3*x^3 - 79))/(35*a) - (512*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1
))^(1/2))/(35*a*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.30

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2\sqrt{c}\,\sqrt{ax+1}\,c^2i(-5a^3x^3+27a^2x^2-71ax+177)}{35a}$$

input
```
int((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a*x + 1)*c**2*i*( - 5*a**3*x**3 + 27*a**2*x**2 - 71*a*x +
177))/(35*a)
```

3.253 $\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 95

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{64c^2\sqrt{1-\frac{1}{a^2x^2}}x}{15\sqrt{c-acx}}+\frac{16}{15}c\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-acx}+\frac{2}{5}\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{3/2}$$

output
```
64/15*c^2*(1-1/a^2/x^2)^(1/2)*x/(-a*c*x+c)^(1/2)+16/15*c*(1-1/a^2/x^2)^(1/
2)*x*(-a*c*x+c)^(1/2)+2/5*(1-1/a^2/x^2)^(1/2)*x*(-a*c*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.63

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2c\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(43-14ax+3a^2x^2)}{15a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(c - a*c*x)^(3/2)/E^ArcCoth[a*x],x]
```

output
```
(-2*c*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(43 - 14*a*x + 3*a^2*x^2))/(15*a*S
qrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6727, 27, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{3/2}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\int \frac{\left(a-\frac{1}{x}\right)^2}{a^2\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\int \frac{\left(a-\frac{1}{x}\right)^2}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\left(\frac{2}{5}\int -\frac{14a-\frac{5}{x}}{2\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\left(-\frac{1}{5}\int \frac{14a-\frac{5}{x}}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\left(\frac{1}{5}\left(\frac{43}{3}\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}+\frac{28a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{a^2\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \; 48$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{5}\left(\frac{28a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}-\frac{86\sqrt{\frac{1}{ax}+1}}{3\sqrt{\frac{1}{x}}}\right)-\frac{2a^2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)(c-acx)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}}$$

input
```
Int[(c - a*c*x)^(3/2)/E^ArcCoth[a*x],x]
```

output
```
-(((((28*a*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) - (86*Sqrt[1 + 1/(a*x)])/
(3*Sqrt[x^(-1)]))/5 - (2*a^2*Sqrt[1 + 1/(a*x)])/(5*(x^(-1))^(5/2)))*(x^(-1
))^(3/2)*(c - a*c*x)^(3/2))/(a^2*(1 - 1/(a*x))^(3/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.56

method	result	size
risch	$\frac{2c^2\sqrt{\frac{ax-1}{ax+1}}\,(3a^2x^2-14ax+43)(ax+1)}{15\sqrt{-c(ax-1)}\,a}$	53
gosper	$\frac{2(ax+1)(3a^2x^2-14ax+43)(-acx+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}{15a(ax-1)^2}$	56
orering	$\frac{2(ax+1)(3a^2x^2-14ax+43)(-acx+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}{15a(ax-1)^2}$	56
default	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,c(3a^2x^2-14ax+43)}{15(ax-1)a}$	58

input
```
int((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*c^2*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(3*a^2*x^2-14*a*x+43)/
a*(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\,(3\,a^3cx^3-11\,a^2cx^2+29\,acx+43\,c)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^2x-a)}$$

input
```
integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(3*a^3*c*x^3 - 11*a^2*c*x^2 + 29*a*c*x + 43*c)*sqrt(-a*c*x + c)*sqrt
((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.76

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\left(3\,a^3\sqrt{-c}cx^3-11\,a^2\sqrt{-c}cx^2+29\,a\sqrt{-c}cx+43\,\sqrt{-c}c\right)(ax-1)}{15\,(a^2x-a)\sqrt{ax+1}}$$

input
```
integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-2/15*(3*a^3*sqrt(-c)*c*x^3 - 11*a^2*sqrt(-c)*c*x^2 + 29*a*sqrt(-c)*c*x +
43*sqrt(-c)*c)*(a*x - 1)/((a^2*x - a)*sqrt(a*x + 1))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 14.55 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.86

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(3\,a^2\,x^2-8\,a\,x+21\right)}{15\,a}-\frac{128\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
- (2*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3*a^2*x^2 - 8*a*x +
21))/(15*a) - (128*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(15*a*
(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.31

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{2\sqrt{c}\,\sqrt{ax+1}\,ci(3a^2x^2-14ax+43)}{15a}$$

input
```
int((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a*x + 1)*c*i*(3*a**2*x**2 - 14*a*x + 43))/(15*a)
```

3.254 $\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{8c\sqrt{1-\frac{1}{a^2x^2}}x}{3\sqrt{c-acx}}+\frac{2}{3}\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-acx}$$

output
```
8/3*c*(1-1/a^2/x^2)^(1/2)*x/(-a*c*x+c)^(1/2)+2/3*(1-1/a^2/x^2)^(1/2)*x*(-a
*c*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}(-5+ax)\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

output
```
(2*Sqrt[1 + 1/(a*x)]*(-5 + a*x)*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6727, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{a\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{5}{3}\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{10\sqrt{\frac{1}{ax}+1}}{3\sqrt{\frac{1}{x}}}-\frac{2a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

output
```
-((((-2*a*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) + (10*Sqrt[1 + 1/(a*x)])/(
3*Sqrt[x^(-1)]))*Sqrt[x^(-1)]*Sqrt[c - a*c*x])/(a*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{2c\sqrt{\frac{ax-1}{ax+1}}\,(ax-5)(ax+1)}{3\sqrt{-c(ax-1)}\,a}$	42
gosper	$\frac{2(ax+1)(ax-5)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3(ax-1)a}$	47
orering	$\frac{2(ax+1)(ax-5)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3(ax-1)a}$	47
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,(ax-5)}{3(ax-1)a}$	48

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*c*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(a*x-5)/a*(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\,(a^2x^2-4\,ax-5)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^2x-a)}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(a^2*x^2 - 4*a*x - 5)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*
x - a)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\left(a^2\sqrt{-c}x^2 - 4\,a\sqrt{-c}x - 5\,\sqrt{-c}\right)(ax-1)}{3\,(a^2x-a)\sqrt{ax+1}}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x - 5*sqrt(-c))*(a*x - 1)/((a^2*x - a
)*sqrt(a*x + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.69

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\,(-acx-c)^{\frac{3}{2}}\,|c|}{3\,ac^2} + \frac{4\,\sqrt{-acx-c}|c|}{ac}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
2/3*(-a*c*x - c)^(3/2)*abs(c)/(a*c^2) + 4*sqrt(-a*c*x - c)*abs(c)/(a*c)
```

Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\,\sqrt{c-a\,c\,x}\,(a\,x-3)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a} - \frac{16\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,(a\,x-1)}$$

input
```
int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*(a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (16*(c
 - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.32

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{c}\,\sqrt{ax+1}\,i(-ax+5)}{3a}$$

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a*x + 1)*i*( - a*x + 5))/(3*a)
```

3.255 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2e^{-\coth^{-1}(ax)}(1+ax)}{a\sqrt{c-acx}}$$

output
```
2*(a*x+1)/a*((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-acx}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - a*c*x]),x]
```

output
```
(2*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - a*c*x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$$

$$\Big\downarrow \text{ 6726}$$

$$\frac{2(ax+1)e^{-\coth^{-1}(ax)}}{a\sqrt{c-acx}}$$

input
```
Int[1/(E^ArcCoth[a*x]*Sqrt[c - a*c*x]),x]
```

output
```
(2*(1 + a*x))/(a*E^ArcCoth[a*x]*Sqrt[c - a*c*x])
```

Defintions of rubi rules used

rule 6726
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] &&  !IntegerQ[p] && EqQ[p, n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

method	result	size
gosper	$\frac{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-acx+c}}$	35
orering	$\frac{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-acx+c}}$	35
risch	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{\sqrt{-c(ax-1)}\,a}$	36
default	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}{(ax-1)ca}$	46

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*x+1)/a*((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\,(ax-1)}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\left(a\sqrt{-c}x+\sqrt{-c}\right)}{\sqrt{ax+1}ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-2*(a*sqrt(-c)*x + sqrt(-c))/(sqrt(a*x + 1)*a*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 14.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\left(2\,x+\frac{2}{a}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-a\,c\,x}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(1/2),x)
```

output
```
((2*x + 2/a)*((a*x - 1)/(a*x + 1))^(1/2))/(c - a*c*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.59

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\sqrt{c}\,\sqrt{ax+1}\,i}{ac}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a*x + 1)*i)/(a*c)
```

3.256 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 77

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=-\frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a\left(\frac{1}{ax}\right)^{3/2}(c-acx)^{3/2}}$$

output
```
-(1-1/a/x)^(3/2)*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)/a/
(1/a/x)^(3/2)/(-a*c*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=-\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(3/2)),x]
```

output
```
-((Sqrt[2]*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqr
t[a]*Sqrt[1 + 1/(a*x)])])/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6727, 27, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$$

$$\Big\downarrow \; 6727$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{a}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \; 104$$

$$-\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$-\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(3/2)),x]
```

output
```
-((Sqrt[2]*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqr
t[a]*Sqrt[1 + 1/(a*x)])])/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)}{(ax-1)\sqrt{-c(ax+1)}\,c^{\frac{3}{2}}a}$	78

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*2^(1/2)*arctan(1/2*(-c
*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/2)/c^(3/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.91

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \left[\frac{\sqrt{2}\sqrt{-\frac{1}{c}}\log\left(-\frac{a^2x^2+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}+2ax-3}{a^2x^2-2ax+1}\right)}{2ac},\right.$$
$$\left.-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2(ax-1)\sqrt{c}}\right)}{ac^{\frac{3}{2}}}\right]$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*sqrt(-1/c)*log(-(a^2*x^2 + 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x +
1)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a*x - 3)/(a^2*x^2 - 2*a*x + 1)
)/(a*c), -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x
- 1)/(a*x + 1))/((a*x - 1)*sqrt(c)))/(a*c^(3/2))]
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-c(ax-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(3/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/(-c*(a*x - 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{a\sqrt{c}} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{-c}}{\sqrt{c}}\right)}{a\sqrt{c}}\right)|c|\mathrm{sgn}(ax+1)}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(a*sqrt(c)) - sqrt(2
)*arctan(sqrt(-c)/sqrt(c))/(a*sqrt(c)))*abs(c)*sgn(a*x + 1)/c^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{(c-a\,c\,x)^{3/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(3/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.39

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{\sqrt{c}\,\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)i}{a\,c^2}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x)
```

output
```
( - sqrt(c)*sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2))/2))*i)/(a*c**2)
```

3.257 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=-\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}+\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{2\sqrt{2}a\left(\frac{1}{ax}\right)^{5/2}(c-acx)^{5/2}}$$

output
```
-1/2*a^2*(1-1/a/x)^(5/2)*(1+1/a/x)^(1/2)*x^2/(a-1/x)/(-a*c*x+c)^(5/2)+1/4*
(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)/a/(
1/a/x)^(5/2)/(-a*c*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\,x\left(-2\sqrt{a}\sqrt{1+\frac{1}{ax}}+\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{4\sqrt{a}c^2(-1+ax)\sqrt{c-acx}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(5/2)),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*x*(-2*Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[2]*Sqrt[x^(-1)]*
(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(
4*Sqrt[a]*c^2*(-1 + a*x)*Sqrt[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6727, 27, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{a^2\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{1}{4}\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{1}{2}\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{2\sqrt{2}\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(5/2)),x]
```

output
```
-((a^2*(1 - 1/(a*x))^(5/2)*((Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)])/(2*(a - x^(-1
))) - ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]/(2*Sqrt[
2]*Sqrt[a])))/((x^(-1))^(5/2)*(c - a*c*x)^(5/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\left(\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx-\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)c-2\sqrt{-c(ax+1)}\,\sqrt{c}\right)}{4c^{\frac{7}{2}}\,(ax-1)^{2}\sqrt{-c(ax+1)}\,a}$	123

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)/c^(7/2)*(2^(1/2)*a
rctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-2^(1/2)*arctan(1/2*(-c
*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-2*(-c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)^2
/(-c*(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.09

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \left[-\frac{\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)-4\,\sqrt{}}{8\,(a^3c^3x^2-2\,a^2c^3x+ac^3)}\right.$$

$$\left.-\frac{\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2\,(acx-c)}\right)-2\,\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{4\,(a^3c^3x^2-2\,a^2c^3x+ac^3)}\right]$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x -
2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) -
3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/
(a*x + 1)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3), -1/4*(sqrt(2)*(a^2*x^2 -
2*a*x + 1)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*s
qrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(
(a*x - 1)/(a*x + 1)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{(c-a\,c\,x)^{5/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(5/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.51

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\sqrt{c}\,i\left(-2\sqrt{ax+1}-\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)ax+\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right.\right.}{4a\,c^{3}\,(ax-1)}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x)
```

output
```
(sqrt(c)*i*( - 2*sqrt(a*x + 1) - sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqr
t(2))/2))*a*x + sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2))/2))))/(4*a*
c**3*(a*x - 1))
```

3.258 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 194

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^2}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^3}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{16\sqrt{2}a\left(\frac{1}{ax}\right)^{7/2}(c-acx)^{7/2}}$$

output

```
-1/4*a^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(1/2)*x^2/(a-1/x)^2/(-a*c*x+c)^(7/2)+3/
16*a^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(1/2)*x^3/(a-1/x)/(-a*c*x+c)^(7/2)-3/32*(
1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)/a/(1
/a/x)^(7/2)/(-a*c*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.64

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}x\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}(7-3ax)+3\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{32\sqrt{a}c^3(-1+ax)^2\sqrt{c-acx}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(7/2)),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 - 3*a*x) + 3*Sqrt[2]*
Sqrt[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 +
 1/(a*x)])]))/(32*Sqrt[a]*c^3*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6727, 27, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{a^3\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}-\frac{3}{8}\int\frac{\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}-\frac{3}{8}\left(\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{1}{4}\int\frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}-\frac{3}{8}\left(\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{1}{2}\int\frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}-\frac{3}{8}\left(\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{2\sqrt{2}\sqrt{a}}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(7/2)),x]
```

output
```
-((a^3*(1 - 1/(a*x))^(7/2)*((Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2))/(4*(a - x^(
-1))^2) - (3*((Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)])/(2*(a - x^(-1))) - ArcTanh[
(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]/(2*Sqrt[2]*Sqrt[a])))/
8))/((x^(-1))^(7/2)*(c - a*c*x)^(7/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.89

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\left(-3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{2}c\,x^{2}+6\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx+6ax\sqrt{-c(ax+1)}\,\sqrt{c}-3\sqrt{}\right.}{32c^{\frac{9}{2}}\,(ax-1)^{3}\sqrt{-c(ax+1)}\,a}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan
(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2+6*2^(1/2)*arctan(1/2*(-
c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+6*a*x*(-c*(a*x+1))^(1/2)*c^(1/2)-3
*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-14*(-c*(a*x+1))^
(1/2)*c^(1/2))/c^(9/2)/(a*x-1)^3/(-c*(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.79

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \left[-\frac{3\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-}{a^2x^2-2ax+1}\right.}{64\left(a^4c^4x^3-3a^3c^4x^2+3a^2c^4x-ac^4\right.}\right.$$

$$-\frac{3\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2(acx-c)}\right)-2\left(3a^2x^2-4ax-7\right)\sqrt{-acx+c}}{32\left(a^4c^4x^3-3a^3c^4x^2+3a^2c^4x-ac^4\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

output
```
[-1/64*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x
^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1
)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^2*x^2 - 4*a*x - 7)*sqr
t(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*
a^2*c^4*x - a*c^4), -1/32*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqr
t(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/
(a*x + 1))/(a*c*x - c)) - 2*(3*a^2*x^2 - 4*a*x - 7)*sqrt(-a*c*x + c)*sqrt(
(a*x - 1)/(a*x + 1)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.45

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}} + \frac{2\left(3\,(-acx-c)^{\frac{3}{2}}+10\sqrt{-acx-c}c\right)}{(acx-c)^2c^2}\right)|c|\mathrm{sgn}(ax+1)}{32\,ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
1/32*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) + 2*(
3*(-a*c*x - c)^(3/2) + 10*sqrt(-a*c*x - c)*c)/((a*c*x - c)^2*c^2))*abs(c)*
sgn(a*x + 1)/(a*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{(c-a\,c\,x)^{7/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(7/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\sqrt{c}\,i\left(-6\sqrt{ax+1}\,ax+14\sqrt{ax+1}-3\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)a^2x^2+6\sqrt{2}\log\left(\right.\right.}{32a\,c^4\left(a^2x^2-2ax+1\right)}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x)
```

output
```
(sqrt(c)*i*( - 6*sqrt(a*x + 1)*a*x + 14*sqrt(a*x + 1) - 3*sqrt(2)*log(tan(
asin(sqrt( - a*x + 1)/sqrt(2))/2))*a**2*x**2 + 6*sqrt(2)*log(tan(asin(sqrt
( - a*x + 1)/sqrt(2))/2))*a*x - 3*sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sq
rt(2))/2))))/(32*a*c**4*(a**2*x**2 - 2*a*x + 1))
```

3.259 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=-\frac{32c^3\sqrt{c-acx}}{a}-\frac{16c^2(c-acx)^{3/2}}{3a}-\frac{8c(c-acx)^{5/2}}{5a}-\frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}+\frac{32\sqrt{2}c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
-32*c^3*(-a*c*x+c)^(1/2)/a-16/3*c^2*(-a*c*x+c)^(3/2)/a-8/5*c*(-a*c*x+c)^(5
/2)/a-4/7*(-a*c*x+c)^(7/2)/a-2/9*(-a*c*x+c)^(9/2)/a/c+32*2^(1/2)*c^(7/2)*a
rctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.64

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{2c^3\left(\sqrt{c-acx}(-6257+1754ax-732a^2x^2+230a^3x^3-35a^4x^4)+5040\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-}}{\sqrt{2}}\right.\right.}{315a}$$

input
```
Integrate[(c - a*c*x)^(7/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*c^3*(Sqrt[c - a*c*x]*(-6257 + 1754*a*x - 732*a^2*x^2 + 230*a^3*x^3 - 35
*a^4*x^4) + 5040*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])
]))/(315*a)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6717, 6680, 35, 60, 60, 60, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{7/2}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}(c-acx)^{7/2}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(1-ax)(c-acx)^{7/2}}{ax+1}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{(c-acx)^{9/2}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\int \frac{(c-acx)^{7/2}}{ax+1}dx+\frac{2(c-acx)^{9/2}}{9a}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\left(2c\int \frac{(c-acx)^{5/2}}{ax+1}dx+\frac{2(c-acx)^{7/2}}{7a}\right)+\frac{2(c-acx)^{9/2}}{9a}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\left(2c\left(2c\int \frac{(c-acx)^{3/2}}{ax+1}dx+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}\right)+\frac{2(c-acx)^{9/2}}{9a}}{c}$$

$$\Big\downarrow 60$$

$$-\frac{2c\left(2c\left(2c\left(2c\int\frac{\sqrt{c-acx}}{ax+1}dx+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}\right)+\frac{2(c-acx)^{9/2}}{9a}}{c}$$

$$\Big\downarrow 60$$

$$-\frac{2c\left(2c\left(2c\left(2c\left(2c\int\frac{1}{(ax+1)\sqrt{c-acx}}dx+\frac{2\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}\right)+\frac{2(c-acx)^{9/2}}{9a}}{c}$$

$$\Big\downarrow 73$$

$$-\frac{2c\left(2c\left(2c\left(2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{4\int\frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}\right)+\frac{2(c-acx)^{9/2}}{9a}}{c}$$

$$\Big\downarrow 219$$

$$-\frac{2c\left(2c\left(2c\left(2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{2\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}\right)+\frac{2(c-acx)^{9/2}}{9a}}{c}$$

input
```
Int[(c - a*c*x)^(7/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((2*(c - a*c*x)^(9/2))/(9*a) + 2*c*((2*(c - a*c*x)^(7/2))/(7*a) + 2*c*((
2*(c - a*c*x)^(5/2))/(5*a) + 2*c*((2*(c - a*c*x)^(3/2))/(3*a) + 2*c*((2*Sq
rt[c - a*c*x])/a - (2*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqr
t[c])])/a)))))/c)
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$-\frac{2c^3\left(-144\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\frac{\sqrt{-c(ax-1)}\,\left(35a^4x^4-230a^3x^3+732a^2x^2-1754ax+6257\right)}{35}\right)}{9a}$
risch	$\frac{2(35a^4x^4-230a^3x^3+732a^2x^2-1754ax+6257)(ax-1)c^4}{315a\sqrt{-c(ax-1)}}+\frac{32\sqrt{2}\,c^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a}$
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{9}{2}}}{9}+\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{4c^2(-acx+c)^{\frac{5}{2}}}{5}+\frac{8c^3(-acx+c)^{\frac{3}{2}}}{3}+16\sqrt{-acx+c}\,c^4-16c^{\frac{9}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right.}{ca}$
default	$\frac{-\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{4c(-acx+c)^{\frac{7}{2}}}{7}-\frac{8c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{16c^3(-acx+c)^{\frac{3}{2}}}{3}-32\sqrt{-acx+c}\,c^4+32c^{\frac{9}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$

input
```
int((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-2/9*c^3*(-144*c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1
/2))+1/35*(-c*(a*x-1))^(1/2)*(35*a^4*x^4-230*a^3*x^3+732*a^2*x^2-1754*a*x+
6257))/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.54

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\left[\frac{2\left(2520\,\sqrt{2}c^{\frac{7}{2}}\log\left(\frac{acx-2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)-(35\,a^4c^3x^4-230\,a^3c^3x^3+732\,a^2c^3x^2-1754\,a\right.}{315\,a}\right.$$

input
```
integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[2/315*(2520*sqrt(2)*c^(7/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(
c) - 3*c)/(a*x + 1)) - (35*a^4*c^3*x^4 - 230*a^3*c^3*x^3 + 732*a^2*c^3*x^2
 - 1754*a*c^3*x + 6257*c^3)*sqrt(-a*c*x + c))/a, 2/315*(5040*sqrt(2)*sqrt(
-c)*c^3*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x - c)) - (35*a^4*c^
3*x^4 - 230*a^3*c^3*x^3 + 732*a^2*c^3*x^2 - 1754*a*c^3*x + 6257*c^3)*sqrt(
-a*c*x + c))/a]
```

Sympy [A] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\begin{cases}-\frac{2\cdot\left(\frac{16\sqrt{2}c^5\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+16c^4\sqrt{-acx+c}+\frac{8c^3(-acx+c)^{\frac{3}{2}}}{3}+\frac{4c^2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{ac} & \text{for } a \\ c^{\frac{7}{2}}\left(\begin{cases}-x & \text{for } a=0\\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise}\end{cases}\right) & \text{othe}\end{cases}$$

input
```
integrate((-a*c*x+c)**(7/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise((-2*(16*sqrt(2)*c**5*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))
/sqrt(-c) + 16*c**4*sqrt(-a*c*x + c) + 8*c**3*(-a*c*x + c)**(3/2)/3 + 4*c*
*2*(-a*c*x + c)**(5/2)/5 + 2*c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)
/9)/(a*c), Ne(a*c, 0)), (c**(7/2)*Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(
a*x + 1) + 1)/a, True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=$$

$$-\frac{2\left(2520\sqrt{2}c^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+35(-acx+c)^{\frac{9}{2}}+90(-acx+c)^{\frac{7}{2}}c+252(-acx+c)^{\frac{5}{2}}c^2+840(-\right.}{315\,ac}$$

input
```
integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-2/315*(2520*sqrt(2)*c^(9/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sq
rt(2)*sqrt(c) + sqrt(-a*c*x + c))) + 35*(-a*c*x + c)^(9/2) + 90*(-a*c*x +
c)^(7/2)*c + 252*(-a*c*x + c)^(5/2)*c^2 + 840*(-a*c*x + c)^(3/2)*c^3 + 504
0*sqrt(-a*c*x + c)*c^4)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{32\sqrt{2}c^4\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2\left(35\,(acx-c)^4\sqrt{-acx+c}a^8c^8 - 90\,(acx-c)^3\sqrt{-acx+c}a^8c^9 + 252\,(acx-c)^2\sqrt{-acx+c}a^8c^{10} + 840\right.}{315\,a^9c^9}$$

input
```
integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-32*sqrt(2)*c^4*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c))
 - 2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c)*a^8*c^8 - 90*(a*c*x - c)^3*sqr
t(-a*c*x + c)*a^8*c^9 + 252*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^8*c^10 + 840*
(-a*c*x + c)^(3/2)*a^8*c^11 + 5040*sqrt(-a*c*x + c)*a^8*c^12)/(a^9*c^9)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\frac{4\,(c-a\,c\,x)^{7/2}}{7\,a} - \frac{8\,c\,(c-a\,c\,x)^{5/2}}{5\,a} - \frac{32\,c^3\,\sqrt{c-a\,c\,x}}{a} - \frac{16\,c^2\,(c-a\,c\,x)^{3/2}}{3\,a} - \frac{2\,(c-a\,c\,x)^{9/2}}{9\,a\,c} - \frac{\sqrt{2}\,c^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,32\mathrm{i}}{a}$$

input
```
int(((c - a*c*x)^(7/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
- (4*(c - a*c*x)^(7/2))/(7*a) - (8*c*(c - a*c*x)^(5/2))/(5*a) - (32*c^3*(c
 - a*c*x)^(1/2))/a - (16*c^2*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(9/
2))/(9*a*c) - (2^(1/2)*c^(7/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1
/2)))*32i)/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{2\sqrt{c}\,c^3\left(-35\sqrt{-ax+1}\,a^4x^4+230\sqrt{-ax+1}\,a^3x^3-732\sqrt{-ax+1}\,a^2x^2+1754\sqrt{-ax+1}\,a\right.}{315a}$$

input
```
int((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x)
```

output
```
(2*sqrt(c)*c**3*( - 35*sqrt( - a*x + 1)*a**4*x**4 + 230*sqrt( - a*x + 1)*a
**3*x**3 - 732*sqrt( - a*x + 1)*a**2*x**2 + 1754*sqrt( - a*x + 1)*a*x - 62
57*sqrt( - a*x + 1) - 2520*sqrt(2)*log(sqrt( - a*x + 1) - sqrt(2)) + 2520*
sqrt(2)*log(sqrt( - a*x + 1) + sqrt(2))))/(315*a)
```

3.260 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 116

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=-\frac{16c^2\sqrt{c-acx}}{a}-\frac{8c(c-acx)^{3/2}}{3a}-\frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}+\frac{16\sqrt{2}c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
-16*c^2*(-a*c*x+c)^(1/2)/a-8/3*c*(-a*c*x+c)^(3/2)/a-4/5*(-a*c*x+c)^(5/2)/a
-2/7*(-a*c*x+c)^(7/2)/a/c+16*2^(1/2)*c^(5/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*
2^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2c^2\left(\sqrt{c-acx}(-1037+269ax-87a^2x^2+15a^3x^3)+840\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{105a}$$

input
```
Integrate[(c - a*c*x)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*c^2*(Sqrt[c - a*c*x]*(-1037 + 269*a*x - 87*a^2*x^2 + 15*a^3*x^3) + 840*
Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]))/(105*a)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6717, 6680, 35, 60, 60, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{5/2}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}(c-acx)^{5/2}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(1-ax)(c-acx)^{5/2}}{ax+1}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{(c-acx)^{7/2}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\int \frac{(c-acx)^{5/2}}{ax+1}dx+\frac{2(c-acx)^{7/2}}{7a}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\left(2c\int \frac{(c-acx)^{3/2}}{ax+1}dx+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\left(2c\left(2c\int \frac{\sqrt{c-acx}}{ax+1}dx+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\left(2c\left(2c\left(2c\int \frac{1}{(ax+1)\sqrt{c-acx}}dx+\frac{2\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2c\left(2c\left(2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{4\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2c\left(2c\left(2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{2\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}\right)+\frac{2(c-acx)^{7/2}}{7a}}{c}$$

input
```
Int[(c - a*c*x)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((2*(c - a*c*x)^(7/2))/(7*a) + 2*c*((2*(c - a*c*x)^(5/2))/(5*a) + 2*c*((
2*(c - a*c*x)^(3/2))/(3*a) + 2*c*((2*Sqrt[c - a*c*x])/a - (2*Sqrt[2]*Sqrt[
c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a))))/c)
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$\frac{2\left(56\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\frac{\left(15a^{3}x^{3}-87a^{2}x^{2}+269ax-1037\right)\sqrt{-c(ax-1)}}{15}\right)c^{2}}{7a}$	71
risch	$-\frac{2(15a^{3}x^{3}-87a^{2}x^{2}+269ax-1037)(ax-1)c^{3}}{105a\sqrt{-c(ax-1)}}+\frac{16\sqrt{2}\,c^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a}$	76
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}+\frac{2(-acx+c)^{\frac{5}{2}}c}{5}+\frac{4c^{2}(-acx+c)^{\frac{3}{2}}}{3}+8\sqrt{-acx+c}\,c^{3}-8c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	87
default	$\frac{-\frac{2(-acx+c)^{\frac{7}{2}}}{7}-\frac{4(-acx+c)^{\frac{5}{2}}c}{5}-\frac{8c^{2}(-acx+c)^{\frac{3}{2}}}{3}-16\sqrt{-acx+c}\,c^{3}+16c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$	87

input
```
int((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
2/7*(56*c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))+1/
15*(15*a^3*x^3-87*a^2*x^2+269*a*x-1037)*(-c*(a*x-1))^(1/2))*c^2/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.61

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \left[\frac{2\left(420\sqrt{2}c^{\frac{5}{2}}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+(15a^3c^2x^3-87a^2c^2x^2+269ac^2x-1037c^2)\sqrt{}\right.}{105a}\right.$$

input
```
integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[2/105*(420*sqrt(2)*c^(5/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c
) - 3*c)/(a*x + 1)) + (15*a^3*c^2*x^3 - 87*a^2*c^2*x^2 + 269*a*c^2*x - 103
7*c^2)*sqrt(-a*c*x + c))/a, 2/105*(840*sqrt(2)*sqrt(-c)*c^2*arctan(sqrt(2)
*sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x - c)) + (15*a^3*c^2*x^3 - 87*a^2*c^2*x^2
 + 269*a*c^2*x - 1037*c^2)*sqrt(-a*c*x + c))/a]
```

Sympy [A] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{8\sqrt{2}c^4\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+8c^3\sqrt{-acx+c}+\frac{4c^2(-acx+c)^{\frac{3}{2}}}{3}+\frac{2c(-acx+c)^{\frac{5}{2}}}{5}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{ac} & \text{for } ac\neq 0 \\ c^{\frac{5}{2}}\left(\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((-a*c*x+c)**(5/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise((-2*(8*sqrt(2)*c**4*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/
sqrt(-c) + 8*c**3*sqrt(-a*c*x + c) + 4*c**2*(-a*c*x + c)**(3/2)/3 + 2*c*(-
a*c*x + c)**(5/2)/5 + (-a*c*x + c)**(7/2)/7)/(a*c), Ne(a*c, 0)), (c**(5/2)
*Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/a, True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = -\frac{2\left(420\sqrt{2}c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+15(-acx+c)^{\frac{7}{2}}+42(-acx+c)^{\frac{5}{2}}c+140(-acx+c)^{\frac{3}{2}}c^2+840\sqrt{-}\right.}{105\,ac}$$

input
```
integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-2/105*(420*sqrt(2)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqr
t(2)*sqrt(c) + sqrt(-a*c*x + c))) + 15*(-a*c*x + c)^(7/2) + 42*(-a*c*x + c
)^(5/2)*c + 140*(-a*c*x + c)^(3/2)*c^2 + 840*sqrt(-a*c*x + c)*c^3)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.16

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = -\frac{16\sqrt{2}c^3\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} + \frac{2\left(15(acx-c)^3\sqrt{-acx+c}a^6c^6-42(acx-c)^2\sqrt{-acx+c}a^6c^7-140(-acx+c)^{\frac{3}{2}}a^6c^8-840\sqrt{-acx+}\right.}{105\,a^7c^7}$$

input
```
integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-16*sqrt(2)*c^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c))
 + 2/105*(15*(a*c*x - c)^3*sqrt(-a*c*x + c)*a^6*c^6 - 42*(a*c*x - c)^2*sqr
t(-a*c*x + c)*a^6*c^7 - 140*(-a*c*x + c)^(3/2)*a^6*c^8 - 840*sqrt(-a*c*x +
 c)*a^6*c^9)/(a^7*c^7)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = -\frac{4\,(c-a\,c\,x)^{5/2}}{5\,a}-\frac{8\,c\,(c-a\,c\,x)^{3/2}}{3\,a}-\frac{16\,c^2\,\sqrt{c-a\,c\,x}}{a}-\frac{2\,(c-a\,c\,x)^{7/2}}{7\,a\,c}-\frac{\sqrt{2}\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,16\mathrm{i}}{a}$$

input
```
int(((c - a*c*x)^(5/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
- (4*(c - a*c*x)^(5/2))/(5*a) - (8*c*(c - a*c*x)^(3/2))/(3*a) - (16*c^2*(c
 - a*c*x)^(1/2))/a - (2*(c - a*c*x)^(7/2))/(7*a*c) - (2^(1/2)*c^(5/2)*atan
((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*16i)/a
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2\sqrt{c}\,c^2\left(15\sqrt{-ax+1}\,a^3x^3-87\sqrt{-ax+1}\,a^2x^2+269\sqrt{-ax+1}\,ax-1037\sqrt{-ax+1}-420\right.}{105a}$$

input
```
int((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x)
```

output
```
(2*sqrt(c)*c**2*(15*sqrt( - a*x + 1)*a**3*x**3 - 87*sqrt( - a*x + 1)*a**2*
x**2 + 269*sqrt( - a*x + 1)*a*x - 1037*sqrt( - a*x + 1) - 420*sqrt(2)*log(
sqrt( - a*x + 1) - sqrt(2)) + 420*sqrt(2)*log(sqrt( - a*x + 1) + sqrt(2)))
)/(105*a)
```

3.261 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 95

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{8c\sqrt{c-acx}}{a}-\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}+\frac{8\sqrt{2}c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
-8*c*(-a*c*x+c)^(1/2)/a-4/3*(-a*c*x+c)^(3/2)/a-2/5*(-a*c*x+c)^(5/2)/a/c+8*
2^(1/2)*c^(3/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.75

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{-2c\sqrt{c-acx}(73-16ax+3a^2x^2)+120\sqrt{2}c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{15a}$$

input
```
Integrate[(c - a*c*x)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(-2*c*Sqrt[c - a*c*x]*(73 - 16*a*x + 3*a^2*x^2) + 120*Sqrt[2]*c^(3/2)*ArcT
anh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(15*a)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6717, 6680, 35, 60, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{3/2}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}(c-acx)^{3/2}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(1-ax)(c-acx)^{3/2}}{ax+1}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{(c-acx)^{5/2}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\int \frac{(c-acx)^{3/2}}{ax+1}dx+\frac{2(c-acx)^{5/2}}{5a}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\left(2c\int \frac{\sqrt{c-acx}}{ax+1}dx+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\left(2c\left(2c\int \frac{1}{(ax+1)\sqrt{c-acx}}dx+\frac{2\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}}{c}$$

$$\Big\downarrow \, 73$$

$$-\frac{2c\left(2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{4\int\frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}}{c}$$

$$\Big\downarrow \, 219$$

$$-\frac{2c\left(2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{2\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}\right)+\frac{2(c-acx)^{5/2}}{5a}}{c}$$

input
```
Int[(c - a*c*x)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((2*(c - a*c*x)^(5/2))/(5*a) + 2*c*((2*(c - a*c*x)^(3/2))/(3*a) + 2*c*((
2*Sqrt[c - a*c*x])/a - (2*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]
*Sqrt[c])])/a)))/c)
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$-\frac{2c\left(-20\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\frac{\left(3a^{2}x^{2}-16ax+73\right)\sqrt{-c(ax-1)}}{3}\right)}{5a}$	61
risch	$\frac{2\left(3a^{2}x^{2}-16ax+73\right)(ax-1)c^{2}}{15a\sqrt{-c(ax-1)}}+\frac{8\sqrt{2}\,c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a}$	68
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{5}{2}}}{5}+\frac{2(-acx+c)^{\frac{3}{2}}c}{3}+4\sqrt{-acx+c}\,c^{2}-4c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	73
default	$\frac{-\frac{2(-acx+c)^{\frac{5}{2}}}{5}-\frac{4(-acx+c)^{\frac{3}{2}}c}{3}-8\sqrt{-acx+c}\,c^{2}+8c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$	73

input
```
int((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-2/5*c*(-20*c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2)
)+1/3*(3*a^2*x^2-16*a*x+73)*(-c*(a*x-1))^(1/2))/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.61

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \left[\frac{2\left(30\sqrt{2}c^{\frac{3}{2}}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-(3a^2cx^2-16acx+73c)\sqrt{-acx+c}\right)}{15a}, \frac{2\left(60\sqrt{2}\right.}{}\right.$$

input
```
integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[2/15*(30*sqrt(2)*c^(3/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
- 3*c)/(a*x + 1)) - (3*a^2*c*x^2 - 16*a*c*x + 73*c)*sqrt(-a*c*x + c))/a, 2
/15*(60*sqrt(2)*sqrt(-c)*c*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x
 - c)) - (3*a^2*c*x^2 - 16*a*c*x + 73*c)*sqrt(-a*c*x + c))/a]
```

Sympy [A] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.22

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{4\sqrt{2}c^3\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+4c^2\sqrt{-acx+c}+\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{ac} & \text{for } ac\neq 0 \\ c^{\frac{3}{2}}\left(\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((-a*c*x+c)**(3/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise((-2*(4*sqrt(2)*c**3*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/
sqrt(-c) + 4*c**2*sqrt(-a*c*x + c) + 2*c*(-a*c*x + c)**(3/2)/3 + (-a*c*x +
 c)**(5/2)/5)/(a*c), Ne(a*c, 0)), (c**(3/2)*Piecewise((-x, Eq(a, 0)), ((a*
x - 2*log(a*x + 1) + 1)/a, True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\left(30\sqrt{2}c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+3(-acx+c)^{\frac{5}{2}}+10(-acx+c)^{\frac{3}{2}}c+60\sqrt{-acx+c}c^2\right)}{15\,ac}$$

input
```
integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-2/15*(30*sqrt(2)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(
2)*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(5/2) + 10*(-a*c*x + c)^(
3/2)*c + 60*sqrt(-a*c*x + c)*c^2)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{8\sqrt{2}c^2\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2\left(3(acx-c)^2\sqrt{-acx+c}a^4c^4+10(-acx+c)^{\frac{3}{2}}a^4c^5+60\sqrt{-acx+c}a^4c^6\right)}{15\,a^5c^5}$$

input
```
integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-8*sqrt(2)*c^2*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c))
- 2/15*(3*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c^4 + 10*(-a*c*x + c)^(3/2)*a
^4*c^5 + 60*sqrt(-a*c*x + c)*a^4*c^6)/(a^5*c^5)
```

Mupad [B] (verification not implemented)

Time = 13.74 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{4\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a}-\frac{8\,c\,\sqrt{c-a\,c\,x}}{a}-\frac{2\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a\,c}-\frac{\sqrt{2}\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right)\,8{}\mathrm{i}}{a}$$

input
```
int(((c - a*c*x)^(3/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
- (4*(c - a*c*x)^(3/2))/(3*a) - (8*c*(c - a*c*x)^(1/2))/a - (2*(c - a*c*x)
^(5/2))/(5*a*c) - (2^(1/2)*c^(3/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*
c^(1/2)))*8i)/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{2\sqrt{c}\,c\left(-3\sqrt{-ax+1}\,a^2x^2+16\sqrt{-ax+1}\,ax-73\sqrt{-ax+1}-30\sqrt{2}\log\left(\sqrt{-ax+1}-\sqrt{2}\right)\right.}{15a}$$

input
```
int((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x)
```

output
```
(2*sqrt(c)*c*( - 3*sqrt( - a*x + 1)*a**2*x**2 + 16*sqrt( - a*x + 1)*a*x -
73*sqrt( - a*x + 1) - 30*sqrt(2)*log(sqrt( - a*x + 1) - sqrt(2)) + 30*sqrt
(2)*log(sqrt( - a*x + 1) + sqrt(2))))/(15*a)
```

3.262 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=-\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
-4*(-a*c*x+c)^(1/2)/a-2/3*(-a*c*x+c)^(3/2)/a/c+4*2^(1/2)*c^(1/2)*arctanh(1
/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2(-7+ax)\sqrt{c-acx}+12\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{3a}$$

input
```
Integrate[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*(-7 + a*x)*Sqrt[c - a*c*x] + 12*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]
/(Sqrt[2]*Sqrt[c])])/(3*a)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6717, 6680, 35, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(1-ax)\sqrt{c-acx}}{ax+1}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{(c-acx)^{3/2}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\int \frac{\sqrt{c-acx}}{ax+1}dx+\frac{2(c-acx)^{3/2}}{3a}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\left(2c\int \frac{1}{(ax+1)\sqrt{c-acx}}dx+\frac{2\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{4\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{2\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{c}$$

input
```
Int[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((2*(c - a*c*x)^(3/2))/(3*a) + 2*c*((2*Sqrt[c - a*c*x])/a - (2*Sqrt[2]*S
qrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a))/c)
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{2(ax-7)(ax-1)c}{3a\sqrt{-c(ax-1)}}+\frac{4\sqrt{2}\,\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a}$	57
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}+2\sqrt{-acx+c}\,c-2c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	59
default	$\frac{-\frac{2(-acx+c)^{\frac{3}{2}}}{3}-4\sqrt{-acx+c}\,c+4c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$	59
pseudoelliptic	$\frac{4\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\frac{2ax\sqrt{-c(ax-1)}}{3}-\frac{14\sqrt{-c(ax-1)}}{3}}{a}$	59

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(a*x-7)*(a*x-1)/a/(-c*(a*x-1))^(1/2)*c+4*2^(1/2)*c^(1/2)*arctanh(1/2*
(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.63

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$$
$$=\left[\frac{2\left(3\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{-acx+c}(ax-7)\right)}{3a},\ \frac{2\left(6\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{acx-c}\right)-\right.}{3a}\right.$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[2/3*(3*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
3*c)/(a*x + 1)) + sqrt(-a*c*x + c)*(a*x - 7))/a, 2/3*(6*sqrt(2)*sqrt(-c)*a
rctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x - c)) + sqrt(-a*c*x + c)*(a
*x - 7))/a]
```

Sympy [A] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2\sqrt{2}c^2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise((-2*(2*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/
sqrt(-c) + 2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c), Ne(a*c, 0)
), (sqrt(c)*Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/a, True)
), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{2\left(3\sqrt{2}c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+(-acx+c)^{\frac{3}{2}}+6\sqrt{-acx+c}c\right)}{3ac}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-2/3*(3*sqrt(2)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)
*sqrt(c) + sqrt(-a*c*x + c))) + (-a*c*x + c)^(3/2) + 6*sqrt(-a*c*x + c)*c)
/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2\left((-acx+c)^{\frac{3}{2}}a^2c^2+6\sqrt{-acx+c}a^2c^3\right)}{3a^3c^3}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) -
2/3*((-a*c*x + c)^(3/2)*a^2*c^2 + 6*sqrt(-a*c*x + c)*a^2*c^3)/(a^3*c^3)
```

Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{4\sqrt{2}\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)}{a} - \frac{2\,(c-a\,c\,x)^{3/2}}{3\,a\,c} - \frac{4\sqrt{c-a\,c\,x}}{a}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
(4*2^(1/2)*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/a - (2*
(c - a*c*x)^(3/2))/(3*a*c) - (4*(c - a*c*x)^(1/2))/a
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$$
$$=\frac{2\sqrt{c}\left(\sqrt{-ax+1}\,ax-7\sqrt{-ax+1}-3\sqrt{2}\log\left(\sqrt{-ax+1}-\sqrt{2}\right)+3\sqrt{2}\log\left(\sqrt{-ax+1}+\sqrt{2}\right)\right)}{3a}$$

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
(2*sqrt(c)*(sqrt( - a*x + 1)*a*x - 7*sqrt( - a*x + 1) - 3*sqrt(2)*log(sqrt
( - a*x + 1) - sqrt(2)) + 3*sqrt(2)*log(sqrt( - a*x + 1) + sqrt(2))))/(3*a
)
```

3.263 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\sqrt{c-acx}}{ac}+\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

output
```
-2*(-a*c*x+c)^(1/2)/a/c+2*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(
1/2))/a/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\sqrt{c-acx}}{ac}+\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

output
```
(-2*Sqrt[c - a*c*x])/(a*c) + (2*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*S
qrt[c])])/(a*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6717, 6680, 35, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{ 6680}$$

$$-\int \frac{1-ax}{(ax+1)\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{ 35}$$

$$-\frac{\int \frac{\sqrt{c-acx}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{2c\int \frac{1}{(ax+1)\sqrt{c-acx}}dx+\frac{2\sqrt{c-acx}}{a}}{c}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\frac{2\sqrt{c-acx}}{a}-\frac{4\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{a}}{c}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\frac{2\sqrt{c-acx}}{a}-\frac{2\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

output
```
-(((2*Sqrt[c - a*c*x])/a - (2*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqr
t[2]*Sqrt[c])])/a)/c)
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2\left(\sqrt{-acx+c}-\sqrt{2}\,\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	45
default	$\frac{-2\sqrt{-acx+c}+2\sqrt{2}\,\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$	46
pseudoelliptic	$\frac{-2\sqrt{-c(ax-1)}+2\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$	48
risch	$\frac{2ax-2}{a\sqrt{-c(ax-1)}}+\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a\sqrt{c}}$	51

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/c/a*((-a*c*x+c)^(1/2)-2^(1/2)*c^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1
/2)/c^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.90

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{ax-\frac{2\sqrt{2}\sqrt{-acx+c}}{\sqrt{c}}-3}{ax+1}\right)-2\sqrt{-acx+c}}{ac},\ -\frac{2\left(\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-acx+c}\sqrt{-\frac{1}{c}}\right)+\sqrt{-acx+c}\right)}{ac}\right]$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[(sqrt(2)*sqrt(c)*log((a*x - 2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(c) - 3)/(a*x
+ 1)) - 2*sqrt(-a*c*x + c))/(a*c), -2*(sqrt(2)*c*sqrt(-1/c)*arctan(1/2*sqr
t(2)*sqrt(-a*c*x + c)*sqrt(-1/c)) + sqrt(-a*c*x + c))/(a*c)]
```

Sympy [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \begin{cases} -\frac{2\left(\frac{\sqrt{2}c\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+\sqrt{-acx+c}\right)}{ac} & \text{for } ac\neq 0 \\ \frac{\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(1/2),x)
```

output
```
Piecewise((-2*(sqrt(2)*c*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(
-c) + sqrt(-a*c*x + c))/(a*c), Ne(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a
*x - 2*log(a*x + 1) + 1)/a, True))/sqrt(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{\sqrt{2}\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+2\sqrt{-acx+c}}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(
c) + sqrt(-a*c*x + c))) + 2*sqrt(-a*c*x + c))/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = -\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2\sqrt{-acx+c}}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) - 2*
sqrt(-a*c*x + c)/(a*c)
```

Mupad [B] (verification not implemented)

Time = 13.74 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)}{a\sqrt{c}} - \frac{2\sqrt{c-a\,c\,x}}{a\,c}$$

input
```
int((a*x - 1)/((c - a*c*x)^(1/2)*(a*x + 1)),x)
```

output
```
(2*2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(a*c^(1/2)) - (
2*(c - a*c*x)^(1/2))/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\sqrt{c}\left(-2\sqrt{-ax+1}-\sqrt{2}\log\left(\sqrt{-ax+1}-\sqrt{2}\right)+\sqrt{2}\log\left(\sqrt{-ax+1}+\sqrt{2}\right)\right)}{ac}$$

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x)
```

output
```
(sqrt(c)*( - 2*sqrt( - a*x + 1) - sqrt(2)*log(sqrt( - a*x + 1) - sqrt(2))
+ sqrt(2)*log(sqrt( - a*x + 1) + sqrt(2))))/(a*c)
```

3.264 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

output
```
2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

output
```
(Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6680, 35, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{(c-acx)^{3/2}}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{1-ax}{(ax+1)(c-acx)^{3/2}}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{1}{(ax+1)\sqrt{c-acx}}dx}{c}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{ac^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

output
```
(Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a\,c^{\frac{3}{2}}}$	29
default	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a\,c^{\frac{3}{2}}}$	29
pseudoelliptic	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}\,a}$	30

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a/c^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.24

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \left[\frac{\sqrt{2}\log\left(\frac{ax-\frac{2\sqrt{2}\sqrt{-acx+c}}{\sqrt{c}}-3}{ax+1}\right)}{2\,ac^{\frac{3}{2}}},\ -\frac{\sqrt{2}\sqrt{-\frac{1}{c}}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-acx+c}\sqrt{-\frac{1}{c}}\right)}{ac}\right]$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*log((a*x - 2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(c) - 3)/(a*x + 1))
/(a*c^(3/2)), -sqrt(2)*sqrt(-1/c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt
(-1/c))/(a*c)]
```

Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.78

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \begin{cases} -\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{ac\sqrt{-c}} & \text{for}\ ac\neq 0 \\ \frac{\begin{cases} -x & \text{for}\ a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(3/2),x)
```

output
```
Piecewise((-sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*c*sqrt(
-c)), Ne(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/
a, True))/c**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{2\,ac^{\frac{3}{2}}}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) +
sqrt(-a*c*x + c)))/(a*c^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{a\sqrt{-c}c}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
-sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{a\,c^{3/2}}$$

input
```
int((a*x - 1)/((c - a*c*x)^(3/2)*(a*x + 1)),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(a*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{2}\,\left(-\log\left(\sqrt{-ax+1}-\sqrt{2}\right)+\log\left(\sqrt{-ax+1}+\sqrt{2}\right)\right)}{2a\,c^2}$$

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x)
```

output
```
(sqrt(c)*sqrt(2)*( - log(sqrt( - a*x + 1) - sqrt(2)) + log(sqrt( - a*x + 1
) + sqrt(2))))/(2*a*c**2)
```

3.265 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 57

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{1}{ac^2\sqrt{c-acx}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

output
```
-1/a/c^2/(-a*c*x+c)^(1/2)+1/2*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)
)*2^(1/2)/a/c^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-ax)\right)}{ac^2\sqrt{c-acx}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

output
```
-(Hypergeometric2F1[-1/2, 1, 1/2, (1 - a*x)/2]/(a*c^2*Sqrt[c - a*c*x]))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6717, 6680, 35, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{(c-acx)^{5/2}}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{1-ax}{(ax+1)(c-acx)^{5/2}}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{1}{(ax+1)(c-acx)^{3/2}}dx}{c}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\frac{\int \frac{1}{(ax+1)\sqrt{c-acx}}dx}{2c}+\frac{1}{ac\sqrt{c-acx}}}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{1}{ac\sqrt{c-acx}}-\frac{\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{ac^2}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{1}{ac\sqrt{c-acx}}-\frac{\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{3/2}}}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

output
```
-((1/(a*c*Sqrt[c - a*c*x]) - ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(S
qrt[2]*a*c^(3/2)))/c)
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{2\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{4c^{\frac{3}{2}}}+\frac{1}{2c\sqrt{-acx+c}}\right)}{ca}$	50
default	$\frac{-\frac{1}{c\sqrt{-acx+c}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{2c^{\frac{3}{2}}}}{ac}$	50
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{-c(ax-1)}-2\sqrt{c}}{2c^{\frac{5}{2}}\,\sqrt{-c(ax-1)}\,a}$	58

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/c/a*(-1/4/c^(3/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))
+1/2/c/(-a*c*x+c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.67

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=\left[\frac{\sqrt{2}(ax-1)\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+4\sqrt{-acx+c}}{4\left(a^2c^3x-ac^3\right)},\ \frac{\sqrt{2}(ax-1)\sqrt{-c}\arctan$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(2)*(a*x - 1)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sq
rt(c) - 3*c)/(a*x + 1)) + 4*sqrt(-a*c*x + c))/(a^2*c^3*x - a*c^3), 1/2*(sq
rt(2)*(a*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x -
 c)) + 2*sqrt(-a*c*x + c))/(a^2*c^3*x - a*c^3)]
```

Sympy [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.46

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{1}{2c\sqrt{-acx+c}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{4c\sqrt{-c}}\right)}{ac} & \text{for } ac\neq 0 \\ \frac{\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(5/2),x)
```

output
```
Piecewise((-2*(1/(2*c*sqrt(-a*c*x + c)) + sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x
 + c)/(2*sqrt(-c)))/(4*c*sqrt(-c)))/(a*c), Ne(a*c, 0)), (Piecewise((-x, Eq
(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/a, True))/c**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{3}{2}}}+\frac{4}{\sqrt{-acx+c}c}}{4\,ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/4*(sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) +
 sqrt(-a*c*x + c)))/c^(3/2) + 4/(sqrt(-a*c*x + c)*c))/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{2\,a\sqrt{-c}c^2} - \frac{1}{\sqrt{-acx+c}ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^2
) - 1/(sqrt(-a*c*x + c)*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{2\,a\,c^{5/2}} - \frac{1}{a\,c^2\,\sqrt{c-a\,c\,x}}$$

input
```
int((a*x - 1)/((c - a*c*x)^(5/2)*(a*x + 1)),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(2*a*c^(5/2)) - 1
/(a*c^2*(c - a*c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\sqrt{c}\left(-\sqrt{-ax+1}\,\sqrt{2}\log\left(\sqrt{-ax+1}-\sqrt{2}\right)+\sqrt{-ax+1}\,\sqrt{2}\log\left(\sqrt{-ax+1}+\sqrt{2}\right)-\right.}{4\sqrt{-ax+1}\,a\,c^3}$$

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x)
```

output
```
(sqrt(c)*( - sqrt( - a*x + 1)*sqrt(2)*log(sqrt( - a*x + 1) - sqrt(2)) + sq
rt( - a*x + 1)*sqrt(2)*log(sqrt( - a*x + 1) + sqrt(2)) - 4))/(4*sqrt( - a*
x + 1)*a*c**3)
```

3.266 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 83

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{1}{3ac^2(c-acx)^{3/2}} - \frac{1}{2ac^3\sqrt{c-acx}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

output
```
-1/3/a/c^2/(-a*c*x+c)^(3/2)-1/2/a/c^3/(-a*c*x+c)^(1/2)+1/4*arctanh(1/2*(-a
*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(7/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.47

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1-ax)\right)}{3ac^2(c-acx)^{3/2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

output
```
-1/3*Hypergeometric2F1[-3/2, 1, -1/2, (1 - a*x)/2]/(a*c^2*(c - a*c*x)^(3/2
))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6717, 6680, 35, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{ 6680}$$

$$-\int \frac{1-ax}{(ax+1)(c-acx)^{7/2}}dx$$

$$\Big\downarrow \text{ 35}$$

$$-\frac{\int \frac{1}{(ax+1)(c-acx)^{5/2}}dx}{c}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{\frac{\int \frac{1}{(ax+1)(c-acx)^{3/2}}dx}{2c}+\frac{1}{3ac(c-acx)^{3/2}}}{c}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{\frac{\frac{\int \frac{1}{(ax+1)\sqrt{c-acx}}dx}{2c}+\frac{1}{ac\sqrt{c-acx}}}{2c}+\frac{1}{3ac(c-acx)^{3/2}}}{c}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\frac{\frac{1}{ac\sqrt{c-acx}}-\frac{\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{ac^2}}{2c}+\frac{1}{3ac(c-acx)^{3/2}}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\frac{1}{ac\sqrt{c-acx}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{3/2}}}{2c}+\frac{1}{3ac(c-acx)^{3/2}}}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

output
```
-((1/(3*a*c*(c - a*c*x)^(3/2)) + (1/(a*c*Sqrt[c - a*c*x]) - ArcTanh[Sqrt[c
 - a*c*x]/(Sqrt[2]*Sqrt[c])]/(Sqrt[2]*a*c^(3/2)))/(2*c))/c)
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{2\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{8c^{\frac{5}{2}}}+\frac{1}{4c^{2}\sqrt{-acx+c}}+\frac{1}{6c\left(-acx+c\right)^{\frac{3}{2}}}\right)}{ca}$	64
default	$\frac{\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{4c^{\frac{5}{2}}}-\frac{1}{2c^{2}\sqrt{-acx+c}}-\frac{1}{3c\left(-acx+c\right)^{\frac{3}{2}}}}{ac}$	64
pseudoelliptic	$\frac{\sqrt{2}\,\sqrt{-c(ax-1)}\,(ax-1)\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\sqrt{c}\left(-2ax+\frac{10}{3}\right)}{4\sqrt{-c(ax-1)}\,c^{\frac{7}{2}}(ax-1)a}$	75

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/c/a*(-1/8/c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))
+1/4/c^2/(-a*c*x+c)^(1/2)+1/6/c/(-a*c*x+c)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.43

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \left[\frac{3\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+4\sqrt{-acx+c}(3\,ax-5)}{24\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}, \; 3\sqrt{}\ldots\right.$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt
(-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) + 4*sqrt(-a*c*x + c)*(3*a*x - 5))/(
a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), 1/12*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*
sqrt(-c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x - c)) + 2*sqrt(-a
*c*x + c)*(3*a*x - 5))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [A] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{1}{6c(-acx+c)^{\frac{3}{2}}}+\frac{1}{4c^2\sqrt{-acx+c}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{8c^2\sqrt{-c}}\right)}{ac} & \text{for}\ ac\neq 0 \\ \frac{\begin{cases} -x & \text{for}\ a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}}{c^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(7/2),x)
```

output
```
Piecewise((-2*(1/(6*c*(-a*c*x + c)**(3/2)) + 1/(4*c**2*sqrt(-a*c*x + c)) +
 sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(8*c**2*sqrt(-c)))/(a
*c), Ne(a*c, 0)), (Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/a
, True))/c**(7/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{\frac{3\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{5}{2}}}-\frac{4\,(3\,acx-5\,c)}{(-acx+c)^{\frac{3}{2}}c^2}}{24\,ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-1/24*(3*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c
) + sqrt(-a*c*x + c)))/c^(5/2) - 4*(3*a*c*x - 5*c)/((-a*c*x + c)^(3/2)*c^2
))/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{4\,a\sqrt{-c}c^3}-\frac{3\,acx-5\,c}{6\,(acx-c)\sqrt{-acx+c}ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^3
) - 1/6*(3*a*c*x - 5*c)/((a*c*x - c)*sqrt(-a*c*x + c)*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 13.82 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{4\,a\,c^{7/2}}-\frac{\frac{c-a\,c\,x}{2\,c^2}+\frac{1}{3\,c}}{a\,c\,{\left(c-a\,c\,x\right)}^{3/2}}$$

input
```
int((a*x - 1)/((c - a*c*x)^(7/2)*(a*x + 1)),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(4*a*c^(7/2)) - (
(c - a*c*x)/(2*c^2) + 1/(3*c))/(a*c*(c - a*c*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.54

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\sqrt{c}\left(-3\sqrt{-ax+1}\,\sqrt{2}\log\left(\sqrt{-ax+1}-\sqrt{2}\right)ax+3\sqrt{-ax+1}\,\sqrt{2}\log\left(\sqrt{-ax+1}-\sqrt{\right.}\right.}{24\sqrt{}}$$

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x)
```

output
```
(sqrt(c)*( - 3*sqrt( - a*x + 1)*sqrt(2)*log(sqrt( - a*x + 1) - sqrt(2))*a*
x + 3*sqrt( - a*x + 1)*sqrt(2)*log(sqrt( - a*x + 1) - sqrt(2)) + 3*sqrt( -
 a*x + 1)*sqrt(2)*log(sqrt( - a*x + 1) + sqrt(2))*a*x - 3*sqrt( - a*x + 1)
*sqrt(2)*log(sqrt( - a*x + 1) + sqrt(2)) - 12*a*x + 20))/(24*sqrt( - a*x +
 1)*a*c**4*(a*x - 1))
```

3.267 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{1}{4ac^4\sqrt{c-acx}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

output
```
-1/5/a/c^2/(-a*c*x+c)^(5/2)-1/6/a/c^3/(-a*c*x+c)^(3/2)-1/4/a/c^4/(-a*c*x+c
)^(1/2)+1/8*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(9/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.38

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1-ax)\right)}{5ac^2(c-acx)^{5/2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(9/2)),x]
```

output
```
-1/5*Hypergeometric2F1[-5/2, 1, -3/2, (1 - a*x)/2]/(a*c^2*(c - a*c*x)^(5/2
))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6717, 6680, 35, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{(c-acx)^{9/2}}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{1-ax}{(ax+1)(c-acx)^{9/2}}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{1}{(ax+1)(c-acx)^{7/2}}dx}{c}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\frac{\int \frac{1}{(ax+1)(c-acx)^{5/2}}dx}{2c}+\frac{1}{5ac(c-acx)^{5/2}}}{c}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\frac{\frac{\int \frac{1}{(ax+1)(c-acx)^{3/2}}dx}{2c}+\frac{1}{3ac(c-acx)^{3/2}}}{2c}+\frac{1}{5ac(c-acx)^{5/2}}}{c}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\frac{\frac{\frac{\int \frac{1}{(ax+1)\sqrt{c-acx}}dx}{2c}+\frac{1}{ac\sqrt{c-acx}}}{2c}+\frac{1}{3ac(c-acx)^{3/2}}}{2c}+\frac{1}{5ac(c-acx)^{5/2}}}{c}$$

$$\Big\downarrow \ 73$$

$$-\frac{\frac{\frac{\frac{1}{ac\sqrt{c-acx}}-\frac{\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{ac^2}}{2c}+\frac{1}{3ac(c-acx)^{3/2}}}{2c}+\frac{1}{5ac(c-acx)^{5/2}}}{c}$$

$$\Big\downarrow \ 219$$

$$-\frac{\frac{\frac{\frac{1}{ac\sqrt{c-acx}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{3/2}}}{2c}+\frac{1}{3ac(c-acx)^{3/2}}}{2c}+\frac{1}{5ac(c-acx)^{5/2}}}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(9/2)),x]
```

output
```
-((1/(5*a*c*(c - a*c*x)^(5/2)) + (1/(3*a*c*(c - a*c*x)^(3/2)) + (1/(a*c*Sq
rt[c - a*c*x]) - ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(Sqrt[2]*a*c^(
3/2)))/(2*c))/(2*c))/c)
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{2\left(\frac{1}{8c^{3}\sqrt{-acx+c}}+\frac{1}{12c^{2}(-acx+c)^{\frac{3}{2}}}+\frac{1}{10c(-acx+c)^{\frac{5}{2}}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{16c^{\frac{7}{2}}}\right)}{ca}$	78
default	$\frac{\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{8c^{\frac{7}{2}}}-\frac{1}{4c^{3}\sqrt{-acx+c}}-\frac{1}{6c^{2}(-acx+c)^{\frac{3}{2}}}-\frac{1}{5c(-acx+c)^{\frac{5}{2}}}}{ac}$	78
pseudoelliptic	$\frac{\sqrt{2}\,\sqrt{-c(ax-1)}\,(ax-1)^{2}\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)-2\left(a^{2}x^{2}-\frac{8}{3}ax+\frac{37}{15}\right)\sqrt{c}}{8\sqrt{-c(ax-1)}\,c^{\frac{9}{2}}(ax-1)^{2}a}$	85

input
```
int((a*x-1)/(-a*c*x+c)^(9/2)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-2/c/a*(1/8/c^3/(-a*c*x+c)^(1/2)+1/12/c^2/(-a*c*x+c)^(3/2)+1/10/c/(-a*c*x+
c)^(5/2)-1/16/c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)
))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.48

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \left[\frac{15\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+4\left(15a^2x^2-40ax\right.}{240\left(a^4c^5x^3-3a^3c^5x^2+3a^2c^5x-ac^5\right)}\right.$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

output
```
[1/240*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log((a*c*x -
2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) + 4*(15*a^2*x^2 - 40*
a*x + 37)*sqrt(-a*c*x + c))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a
*c^5), 1/120*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan
(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x - c)) + 2*(15*a^2*x^2 - 40*a*x +
 37)*sqrt(-a*c*x + c))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
]
```

Sympy [A] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{1}{10c(-acx+c)^{\frac{5}{2}}}+\frac{1}{12c^2(-acx+c)^{\frac{3}{2}}}+\frac{1}{8c^3\sqrt{-acx+c}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{16c^3\sqrt{-c}}\right)}{ac} & \text{for}\ ac\neq 0 \\ \frac{\begin{cases} -x & \text{for}\ a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}}{c^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(9/2),x)
```

output
```
Piecewise((-2*(1/(10*c*(-a*c*x + c)**(5/2)) + 1/(12*c**2*(-a*c*x + c)**(3/
2)) + 1/(8*c**3*sqrt(-a*c*x + c)) + sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/
(2*sqrt(-c)))/(16*c**3*sqrt(-c)))/(a*c), Ne(a*c, 0)), (Piecewise((-x, Eq(a
, 0)), ((a*x - 2*log(a*x + 1) + 1)/a, True))/c**(9/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = -\frac{\frac{15\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{7}{2}}}+\frac{4\left(15\,(acx-c)^2-10\,(acx-c)c+12\,c^2\right)}{(-acx+c)^{\frac{5}{2}}c^3}}{240\,ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="maxima")
```

output
```
-1/240*(15*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt
(c) + sqrt(-a*c*x + c)))/c^(7/2) + 4*(15*(a*c*x - c)^2 - 10*(a*c*x - c)*c
+ 12*c^2)/((-a*c*x + c)^(5/2)*c^3))/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{8\,a\sqrt{-c}c^4}-\frac{15\,(acx-c)^2-10\,(acx-c)c+12\,c^2}{60\,(acx-c)^2\sqrt{-acx+c}ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^4
) - 1/60*(15*(a*c*x - c)^2 - 10*(a*c*x - c)*c + 12*c^2)/((a*c*x - c)^2*sqr
t(-a*c*x + c)*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{8\,a\,c^{9/2}} - \frac{\frac{c-a\,c\,x}{6\,c^2}+\frac{1}{5\,c}+\frac{(c-a\,c\,x)^2}{4\,c^3}}{a\,c\,(c-a\,c\,x)^{5/2}}$$

input
```
int((a*x - 1)/((c - a*c*x)^(9/2)*(a*x + 1)),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(8*a*c^(9/2)) - (
(c - a*c*x)/(6*c^2) + 1/(5*c) + (c - a*c*x)^2/(4*c^3))/(a*c*(c - a*c*x)^(5
/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \frac{\sqrt{c}\,\left(-15\sqrt{-ax+1}\,\sqrt{2}\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^2x^2+30\sqrt{-ax+1}\,\sqrt{2}\log\left(\sqrt{-ax+1}\right.\right.}{}$$

input
```
int((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x)
```

output
```
(sqrt(c)*( - 15*sqrt( - a*x + 1)*sqrt(2)*log(sqrt( - a*x + 1) - sqrt(2))*a
**2*x**2 + 30*sqrt( - a*x + 1)*sqrt(2)*log(sqrt( - a*x + 1) - sqrt(2))*a*x
 - 15*sqrt( - a*x + 1)*sqrt(2)*log(sqrt( - a*x + 1) - sqrt(2)) + 15*sqrt(
- a*x + 1)*sqrt(2)*log(sqrt( - a*x + 1) + sqrt(2))*a**2*x**2 - 30*sqrt( -
a*x + 1)*sqrt(2)*log(sqrt( - a*x + 1) + sqrt(2))*a*x + 15*sqrt( - a*x + 1)
*sqrt(2)*log(sqrt( - a*x + 1) + sqrt(2)) - 60*a**2*x**2 + 160*a*x - 148))/
(240*sqrt( - a*x + 1)*a*c**5*(a**2*x**2 - 2*a*x + 1))
```

3.268 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 250

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=-\frac{192\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{2752\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}{35a^3\left(1-\frac{1}{ax}\right)^{5/2}x^2}+\frac{128\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}{5a^2\left(1-\frac{1}{ax}\right)^{5/2}x}-\frac{2\left(a-\frac{1}{x}\right)^4x(c-acx)^{5/2}}{a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{16\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}x(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}}$$

output
```
-192/35*(1+1/a/x)^(1/2)*(-a*c*x+c)^(5/2)/a/(1-1/a/x)^(5/2)-2752/35*(1+1/a/
x)^(1/2)*(-a*c*x+c)^(5/2)/a^3/(1-1/a/x)^(5/2)/x^2+128/5*(1+1/a/x)^(1/2)*(-
a*c*x+c)^(5/2)/a^2/(1-1/a/x)^(5/2)/x-2*(a-1/x)^4*x*(-a*c*x+c)^(5/2)/a^4/(1
-1/a/x)^(5/2)/(1+1/a/x)^(1/2)+16/7*(a-1/x)^3*(1+1/a/x)^(1/2)*x*(-a*c*x+c)^
(5/2)/a^3/(1-1/a/x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.27

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2c^2\sqrt{c-acx}(-1451-708ax+142a^2x^2-36a^3x^3+5a^4x^4)}{35a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a*c*x)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(2*c^2*Sqrt[c - a*c*x]*(-1451 - 708*a*x + 142*a^2*x^2 - 36*a^3*x^3 + 5*a^4
*x^4))/(35*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.76, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6727, 27, 105, 105, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{5/2}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(a-\frac{1}{x}\right)^4}{a^4\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(a-\frac{1}{x}\right)^4}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{a^4\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{16}{7}\int \frac{\left(a-\frac{1}{x}\right)^3}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)^4}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \ 105$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{16}{7}\left(-\frac{12}{5}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^4}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \ 100$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{16}{7}\left(-\frac{12}{5}\left(\frac{2}{3}\int -\frac{10a-\frac{3}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^4}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{16}{7}\left(-\frac{12}{5}\left(-\frac{1}{3}\int \frac{10a-\frac{3}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^4}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^4\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \ 87$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\left(-\frac{16}{7}\left(-\frac{12}{5}\left(\frac{1}{3}\left(23\int \frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}+\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)- \cdots\right)}{a^4\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \ 48$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}\left(-\frac{16}{7}\left(-\frac{12}{5}\left(\frac{1}{3}\left(\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}+\frac{46\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^4}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)(c-acx \cdots}{a^4\left(1-\frac{1}{ax}\right)^{5/2}}$$

input
```
Int[(c - a*c*x)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((((-16*((-12*(((20*a)/(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + (46*Sqrt[x^(-1)
])/Sqrt[1 + 1/(a*x)])/3 - (2*a^2)/(3*Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2))))/5
 - (2*(a - x^(-1))^3)/(5*Sqrt[1 + 1/(a*x)]*(x^(-1))^(5/2))))/7 - (2*(a - x
^(-1))^4)/(7*Sqrt[1 + 1/(a*x)]*(x^(-1))^(7/2)))*(x^(-1))^(5/2)*(c - a*c*x)
^(5/2))/(a^4*(1 - 1/(a*x))^(5/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.29

method	result	size
gosper	$\frac{2(ax+1)(5a^4x^4-36a^3x^3+142a^2x^2-708ax-1451)(-acx+c)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{35a(ax-1)^4}$	72
orering	$\frac{2(ax+1)(5a^4x^4-36a^3x^3+142a^2x^2-708ax-1451)(-acx+c)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{35a(ax-1)^4}$	72
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,c^2(5a^4x^4-36a^3x^3+142a^2x^2-708ax-1451)}{35(ax-1)^2a}$	76
risch	$-\frac{2(5a^3x^3-41a^2x^2+183ax-891)(ax+1)c^3\sqrt{\frac{ax-1}{ax+1}}}{35a\sqrt{-c(ax-1)}}+\frac{32c^3\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	95

input
```
int((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(a*x+1)*(5*a^4*x^4-36*a^3*x^3+142*a^2*x^2-708*a*x-1451)*(-a*c*x+c)^(5
/2)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2\,(5\,a^4c^2x^4-36\,a^3c^2x^3+142\,a^2c^2x^2-708\,ac^2x-1451\,c^2)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^2x-a)}$$

input
```
integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
2/35*(5*a^4*c^2*x^4 - 36*a^3*c^2*x^3 + 142*a^2*c^2*x^2 - 708*a*c^2*x - 145
1*c^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.48

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2\,(5\,a^5\sqrt{-c}c^2x^5-31\,a^4\sqrt{-c}c^2x^4+106\,a^3\sqrt{-c}c^2x^3-566\,a^2\sqrt{-c}c^2x^2-2159\,a\sqrt{-c}c^2x-1}{35\,(a^3x^2-2\,a^2x+a)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
2/35*(5*a^5*sqrt(-c)*c^2*x^5 - 31*a^4*sqrt(-c)*c^2*x^4 + 106*a^3*sqrt(-c)*
c^2*x^3 - 566*a^2*sqrt(-c)*c^2*x^2 - 2159*a*sqrt(-c)*c^2*x - 1451*sqrt(-c)
*c^2)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.89 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.38

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^3\,x^3-31\,a^2\,x^2+111\,a\,x-597\right)}{35\,a} - \frac{4096\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{35\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(111*a*x - 31*a^2*x^2
 + 5*a^3*x^3 - 597))/(35*a) - (4096*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x
+ 1))^(1/2))/(35*a*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.20

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{2\sqrt{c}\,c^2 i(-5a^4x^4+36a^3x^3-142a^2x^2+708ax+1451)}{35\sqrt{ax+1}\,a}$$

input
```
int((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(2*sqrt(c)*c**2*i*( - 5*a**4*x**4 + 36*a**3*x**3 - 142*a**2*x**2 + 708*a*x
 + 1451))/(35*sqrt(a*x + 1)*a)
```

3.269 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 190

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{56\sqrt{1+\frac{1}{ax}}(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{172\sqrt{1+\frac{1}{ax}}(c-acx)^{3/2}}{5a^2\left(1-\frac{1}{ax}\right)^{3/2}x}-\frac{2\left(a-\frac{1}{x}\right)^3x(c-acx)^{3/2}}{a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{12\sqrt{1+\frac{1}{ax}}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}$$

output
```
-56/5*(1+1/a/x)^(1/2)*(-a*c*x+c)^(3/2)/a/(1-1/a/x)^(3/2)+172/5*(1+1/a/x)^(
1/2)*(-a*c*x+c)^(3/2)/a^2/(1-1/a/x)^(3/2)/x-2*(a-1/x)^3*x*(-a*c*x+c)^(3/2)
/a^3/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)+12/5*(1+1/a/x)^(1/2)*x*(-a*c*x+c)^(3/
2)/(1-1/a/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2c\sqrt{c-acx}(91+43ax-7a^2x^2+a^3x^3)}{5a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a*c*x)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(-2*c*Sqrt[c - a*c*x]*(91 + 43*a*x - 7*a^2*x^2 + a^3*x^3))/(5*a^2*Sqrt[1 -
 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6727, 27, 105, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{3/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\int \frac{\left(a-\frac{1}{x}\right)^3}{a^3\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\int \frac{\left(a-\frac{1}{x}\right)^3}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{a^3\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\left(-\frac{12}{5}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\left(-\frac{12}{5}\left(\frac{2}{3}\int -\frac{10a-\frac{3}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\left(-\frac{12}{5}\left(-\frac{1}{3}\int \frac{10a-\frac{3}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \; 87$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\left(-\frac{12}{5}\left(\frac{1}{3}\left(23\int \frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}+\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^3\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \; 48$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}\left(-\frac{12}{5}\left(\frac{1}{3}\left(\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}+\frac{46\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2\left(a-\frac{1}{x}\right)^3}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)(c-acx)^{3/2}}{a^3\left(1-\frac{1}{ax}\right)^{3/2}}$$

input
```
Int[(c - a*c*x)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((((-12*(((20*a)/(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + (46*Sqrt[x^(-1)])/Sqr
t[1 + 1/(a*x)])/3 - (2*a^2)/(3*Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2))))/5 - (2*
(a - x^(-1))^3)/(5*Sqrt[1 + 1/(a*x)]*(x^(-1))^(5/2)))*(x^(-1))^(3/2)*(c -
a*c*x)^(3/2))/(a^3*(1 - 1/(a*x))^(3/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.33

method	result	size
gosper	$\frac{2(ax+1)(a^3x^3-7a^2x^2+43ax+91)(-acx+c)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{5a(ax-1)^3}$	63
orering	$\frac{2(ax+1)(a^3x^3-7a^2x^2+43ax+91)(-acx+c)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{5a(ax-1)^3}$	63
default	$-\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,c(a^3x^3-7a^2x^2+43ax+91)}{5(ax-1)^2a}$	65
risch	$\frac{2(a^2x^2-8ax+51)(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{5a\sqrt{-c(ax-1)}}+\frac{16c^2\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	86

input
```
int((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(a*x+1)*(a^3*x^3-7*a^2*x^2+43*a*x+91)*(-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1
))^(3/2)/a/(a*x-1)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.33

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2\left(a^3cx^3-7\,a^2cx^2+43\,acx+91\,c\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^2x-a\right)}$$

input
```
integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-2/5*(a^3*c*x^3 - 7*a^2*c*x^2 + 43*a*c*x + 91*c)*sqrt(-a*c*x + c)*sqrt((a*
x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((-a*c*x+c)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.49

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\left(a^4\sqrt{-c}cx^4-6\,a^3\sqrt{-c}cx^3+36\,a^2\sqrt{-c}cx^2+134\,a\sqrt{-c}cx+91\,\sqrt{-c}c\right)(ax-1)^2}{5\left(a^3x^2-2\,a^2x+a\right)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-2/5*(a^4*sqrt(-c)*c*x^4 - 6*a^3*sqrt(-c)*c*x^3 + 36*a^2*sqrt(-c)*c*x^2 +
134*a*sqrt(-c)*c*x + 91*sqrt(-c)*c)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(
a*x + 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.83 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.43

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = -\frac{2\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(a^2\,x^2-6\,a\,x+37\right)}{5\,a}-\frac{256\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{5\,a\,\left(a\,x-1\right)}$$

input
```
int((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
- (2*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(a^2*x^2 - 6*a*x + 37
))/(5*a) - (256*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(5*a*(a*x
 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.20

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{2\sqrt{c}\,ci(a^3x^3-7a^2x^2+43ax+91)}{5\sqrt{ax+1}\,a}$$

input
```
int((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(2*sqrt(c)*c*i*(a**3*x**3 - 7*a**2*x**2 + 43*a*x + 91))/(5*sqrt(a*x + 1)*a
)
```

3.270 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=-\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{46\sqrt{c-acx}}{3a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

output
```
-20/3*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-46/3*(-a*c*x+c)^(
1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)/x+2/3*x*(-a*c*x+c)^(1/2)/(1-1/a/x
)^(1/2)/(1+1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.35

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}(-23-10ax+a^2x^2)}{3a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

output
```
(2*Sqrt[c - a*c*x]*(-23 - 10*a*x + a^2*x^2))/(3*a^2*Sqrt[1 - 1/(a^2*x^2)]*
x)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6727, 27, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2}{3}\int -\frac{10a-\frac{3}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{1}{3}\int \frac{10a-\frac{3}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{3}\left(23\int\frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}+\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{3}\left(\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}+\frac{46\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

output
```
-(((((20*a)/(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + (46*Sqrt[x^(-1)])/Sqrt[1 +
1/(a*x)])/3 - (2*a^2)/(3*Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2)))*Sqrt[x^(-1)]*S
qrt[c - a*c*x])/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.40

method	result	size
gosper	$\frac{2(ax+1)\left(a^{2}x^{2}-10ax-23\right)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3a\left(ax-1\right)^{2}}$	55
orering	$\frac{2(ax+1)\left(a^{2}x^{2}-10ax-23\right)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3a\left(ax-1\right)^{2}}$	55
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,\left(a^{2}x^{2}-10ax-23\right)}{3\left(ax-1\right)^{2}a}$	56
risch	$-\frac{2(ax-11)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{3a\sqrt{-c(ax-1)}}+\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	74

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a*x+1)*(a^2*x^2-10*a*x-23)*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/a
/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.36

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\,(a^2x^2-10\,ax-23)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^2x-a)}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(a^2*x^2 - 10*a*x - 23)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^
2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.55

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\,\left(a^3\sqrt{-c}x^3-9\,a^2\sqrt{-c}x^2-33\,a\sqrt{-c}x-23\,\sqrt{-c}\right)(ax-1)^2}{3\,(a^3x^2-2\,a^2x+a)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
2/3*(a^3*sqrt(-c)*x^3 - 9*a^2*sqrt(-c)*x^2 - 33*a*sqrt(-c)*x - 23*sqrt(-c)
)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.80 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.52

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,(a\,x-9)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a} - \frac{64\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,(a\,x-1)}$$

input
```
int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*(a*x - 9)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (64*(c
 - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.22

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{c}\,i(-a^2x^2+10ax+23)}{3\sqrt{ax+1}\,a}$$

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(2*sqrt(c)*i*( - a**2*x**2 + 10*a*x + 23))/(3*sqrt(a*x + 1)*a)
```

3.271 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{6\sqrt{1-\frac{1}{ax}}}{a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}+\frac{2\sqrt{1-\frac{1}{ax}}\,x}{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}$$

output
```
6*(1-1/a/x)^(1/2)/a/(1+1/a/x)^(1/2)/(-a*c*x+c)^(1/2)+2*(1-1/a/x)^(1/2)*x/(
1+1/a/x)^(1/2)/(-a*c*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{2\sqrt{1-\frac{1}{ax}}(3+ax)}{a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

output
```
(2*Sqrt[1 - 1/(a*x)]*(3 + a*x))/(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6727, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a-\frac{1}{x}}{a\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a-\frac{1}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\left(-3\int \frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(-\frac{2a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}-\frac{6\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)\sqrt{1-\frac{1}{ax}}}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

output
```
-((((-2*a)/(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) - (6*Sqrt[x^(-1)])/Sqrt[1 + 1/
(a*x)])*Sqrt[1 - 1/(a*x)])/(a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{2(ax+1)(ax+3)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{a(ax-1)\sqrt{-acx+c}}$	47
orering	$\frac{2(ax+1)(ax+3)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{a(ax-1)\sqrt{-acx+c}}$	47
default	$-\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,(ax+3)}{(ax-1)^{2}ca}$	51
risch	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{\sqrt{-c(ax-1)}\,a}+\frac{4\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	67

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*x+1)*(a*x+3)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)/(-a*c*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=-\frac{2\sqrt{-acx+c}(ax+3)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-a*c*x + c)*(a*x + 3)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\,(a^2x^2+4\,ax+3)(ax-1)}{\left(a^2\sqrt{-c}x-a\sqrt{-c}\right)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
2*(a^2*x^2 + 4*a*x + 3)*(a*x - 1)/((a^2*sqrt(-c)*x - a*sqrt(-c))*(a*x + 1)
^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.49

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = 2\left(\frac{\sqrt{-acx-c}}{ac^2}-\frac{2}{\sqrt{-acx-c}ac}\right)|c|$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
2*(sqrt(-a*c*x - c)/(a*c^2) - 2/(sqrt(-a*c*x - c)*a*c))*abs(c)
```

Mupad [B] (verification not implemented)

Time = 13.91 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.40

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\left(2\,x+\frac{6}{a}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-a\,c\,x}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(1/2),x)
```

output
```
((2*x + 6/a)*((a*x - 1)/(a*x + 1))^(1/2))/(c - a*c*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.28

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\sqrt{c}\,i(ax+3)}{\sqrt{ax+1}\,ac}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x)
```

output
```
(2*sqrt(c)*i*(a*x + 3))/(sqrt(a*x + 1)*a*c)
```

3.272 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=-\frac{2e^{-3\coth^{-1}(ax)}(1+ax)}{a(c-acx)^{3/2}}$$

output
```
(-2*a*x-2)/a*((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx=-\frac{2\left(1-\frac{1}{ax}\right)^{3/2}x}{\sqrt{1+\frac{1}{ax}}(c-acx)^{3/2}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

output
```
(-2*(1 - 1/(a*x))^(3/2)*x)/(Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6726}$$

$$-\frac{2(ax+1)e^{-3\coth^{-1}(ax)}}{a(c-acx)^{3/2}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

output
```
(-2*(1 + a*x))/(a*E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2))
```

Defintions of rubi rules used

rule 6726
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] &&  !IntegerQ[p] && EqQ[p, n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{a(-acx+c)^{\frac{3}{2}}}$	35
orering	$-\frac{2(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{a(-acx+c)^{\frac{3}{2}}}$	35
default	$-\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}}{(ax-1)^{2}c^{2}a}$	46

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a/(-a*c*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{a^{2}c^{2}x-ac^{2}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*x - a*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2\left(a\sqrt{-c}x+\sqrt{-c}\right)(ax-1)}{(a^2c^2x-ac^2)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-2*(a*sqrt(-c)*x + sqrt(-c))*(a*x - 1)/((a^2*c^2*x - a*c^2)*(a*x + 1)^(3/2
))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\left(\frac{\sqrt{2}}{a\sqrt{-c}}-\frac{2}{\sqrt{-acx-ca}}\right)|c|\mathrm{sgn}(ax+1)}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
(sqrt(2)/(a*sqrt(-c)) - 2/(sqrt(-a*c*x - c)*a))*abs(c)*sgn(a*x + 1)/c^2
```

Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c\,\sqrt{c-a\,c\,x}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(3/2),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c*(c - a*c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{2\sqrt{c}\,i}{\sqrt{ax+1}\,a\,c^2}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x)
```

output
```
(2*sqrt(c)*i)/(sqrt(a*x + 1)*a*c**2)
```

3.273 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 121

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=\frac{a\left(1-\frac{1}{ax}\right)^{5/2}x^2}{\sqrt{1+\frac{1}{ax}}(c-acx)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{2}a\left(\frac{1}{ax}\right)^{5/2}(c-acx)^{5/2}}$$

output
```
a*(1-1/a/x)^(5/2)*x^2/(1+1/a/x)^(1/2)/(-a*c*x+c)^(5/2)-1/2*(1-1/a/x)^(5/2)
*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)/a/(1/a/x)^(5/2)/(-
a*c*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.01

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(2\sqrt{\frac{1}{x}}-\sqrt{2}\sqrt{a}\sqrt{1+\frac{1}{ax}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{2ac^2\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(2*Sqrt[x^(-1)] - Sqrt[2]*Sqrt[a]*Sqrt[1 + 1/(a*x)]*Arc
Tanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(2*a*c^2*Sqrt[1
 + 1/(a*x)]*Sqrt[x^(-1)]*Sqrt[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6727, 27, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{a\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{2}a\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a\left(1-\frac{1}{ax}\right)^{5/2}\left(a\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}}-\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

output
```
-((a*(1 - 1/(a*x))^(5/2)*(-(Sqrt[x^(-1)]/Sqrt[1 + 1/(a*x)]) + (Sqrt[a]*Arc
Tanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[2]))/((x^(-
1))^(5/2)*(c - a*c*x)^(5/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{-c(ax+1)}+2\sqrt{c}\right)}{2(ax-1)^{2}c^{\frac{7}{2}}a}$	85

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)/c^(7/2)*
(arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(-c*(a*x+1))^(1/2)
+2*c^(1/2))/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.99

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \left[-\frac{\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^{2}cx^{2}+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^{2}x^{2}-2\,ax+1}\right)+4\,\sqrt{-acx+c}}{4\left(a^{2}c^{3}x-ac^{3}\right)}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sq
rt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^
2 - 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3*x
 - a*c^3), 1/2*(sqrt(2)*(a*x - 1)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x +
 c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*sqrt(-a*c
*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3*x - a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a*c*x + c)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{(c-a\,c\,x)^{5/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(5/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\sqrt{c}\,i\left(\sqrt{ax+1}\,\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)-\sqrt{ax+1}\,\sqrt{2}+2\right)}{2\sqrt{ax+1}\,a\,c^3}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x)
```

output
```
(sqrt(c)*i*(sqrt(a*x + 1)*sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2))/2
)) - sqrt(a*x + 1)*sqrt(2) + 2))/(2*sqrt(a*x + 1)*a*c**3)
```

3.274 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 185

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} - \frac{3a^2\left(1-\frac{1}{ax}\right)^{7/2}x^3}{4\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}} + \frac{3\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{4\sqrt{2}a\left(\frac{1}{ax}\right)^{7/2}(c-acx)^{7/2}}$$

output
```
-1/2*a^2*(1-1/a/x)^(7/2)*x^2/(a-1/x)/(1+1/a/x)^(1/2)/(-a*c*x+c)^(7/2)-3/4*
a^2*(1-1/a/x)^(7/2)*x^3/(1+1/a/x)^(1/2)/(-a*c*x+c)^(7/2)+3/8*(1-1/a/x)^(7/
2)*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)/a/(1/a/x)^(7/2)/
(-a*c*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.70

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(-2+6ax-\frac{3\sqrt{2}\sqrt{a}\sqrt{1+\frac{1}{ax}}(-1+ax)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{\frac{1}{x}}}\right)}{8ac^3\sqrt{1+\frac{1}{ax}}(-1+ax)\sqrt{c-acx}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(-2 + 6*a*x - (3*Sqrt[2]*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(-1
+ a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[x
^(-1)]))/(8*a*c^3*Sqrt[1 + 1/(a*x)]*(-1 + a*x)*Sqrt[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.76, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6727, 27, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{a^2\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}-\frac{3}{4}\int\frac{\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}-\frac{3}{4}\left(\frac{1}{2}a\int\frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}-\frac{3}{4}\left(a\int\frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{\left(\frac{1}{x}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}-\frac{3}{4}\left(\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}}-\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

output
```
-((a^2*(1 - 1/(a*x))^(7/2)*((x^(-1))^(3/2)/(2*(a - x^(-1))*Sqrt[1 + 1/(a*x
)]) - (3*(-(Sqrt[x^(-1)]/Sqrt[1 + 1/(a*x)]) + (Sqrt[a]*ArcTanh[(Sqrt[2]*Sq
rt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[2]))/4))/((x^(-1))^(7/2)*(c
 - a*c*x)^(7/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.70

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)ax\sqrt{-c(ax+1)}-3\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{-c(ax+1)}+6\sqrt{c}}{8(ax-1)^{3}c^{\frac{9}{2}}a}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3*(a*x+1)*(-c*(a*x-1))^(1/2)/c^(9/2)*
(3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*x*(-c*(a*x+1))
^(1/2)-3*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(-c*(a*x+1
))^(1/2)+6*c^(1/2)*a*x-2*c^(1/2))/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.57

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \left[-\frac{3\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+4\cdots}{16\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

output
```
[-1/16*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x
 + 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))
 - 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*(3*a*x - 1)*sqrt((a*x
- 1)/(a*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), 1/8*(3*sqrt(2)*(a^2*
x^2 - 2*a*x + 1)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqr
t(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*(3*a*x -
1)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a*c*x + c)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.49

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{ac^{\frac{5}{2}}} - \frac{2\,(3\,acx-c)}{\left((-acx-c)^{\frac{3}{2}}+2\sqrt{-acx-c}c\right)ac^{2}}\right)|c|\mathrm{sgn}(ax+1)}{8\,c^{2}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
-1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(a*c^(5/2)) -
 2*(3*a*c*x - c)/(((-a*c*x - c)^(3/2) + 2*sqrt(-a*c*x - c)*c)*a*c^2))*abs(
c)*sgn(a*x + 1)/c^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-a\,c\,x\right)^{7/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(7/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.59

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\sqrt{c}\,i\left(12\sqrt{ax+1}\,\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)ax-12\sqrt{ax+1}\,\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-}}{}\right.}{2}\right.\right.\right.}{32\sqrt{ax+1}\,a\,c^4\,(ax-1)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x)
```

output
```
(sqrt(c)*i*(12*sqrt(a*x + 1)*sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2)
)/2))*a*x - 12*sqrt(a*x + 1)*sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2)
)/2)) - 9*sqrt(a*x + 1)*sqrt(2)*a*x + 9*sqrt(a*x + 1)*sqrt(2) + 24*a*x - 8
))/(32*sqrt(a*x + 1)*a*c**4*(a*x - 1))
```

3.275 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 242

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = -\frac{a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2}{4\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}} + \frac{5a^3\left(1-\frac{1}{ax}\right)^{9/2}x^3}{16\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}} + \frac{15a^3\left(1-\frac{1}{ax}\right)^{9/2}x^4}{32\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}} - \frac{15\left(1-\frac{1}{ax}\right)^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{32\sqrt{2}a\left(\frac{1}{ax}\right)^{9/2}(c-acx)^{9/2}}$$

output

```
-1/4*a^3*(1-1/a/x)^(9/2)*x^2/(a-1/x)^2/(1+1/a/x)^(1/2)/(-a*c*x+c)^(9/2)+5/
16*a^3*(1-1/a/x)^(9/2)*x^3/(a-1/x)/(1+1/a/x)^(1/2)/(-a*c*x+c)^(9/2)+15/32*
a^3*(1-1/a/x)^(9/2)*x^4/(1+1/a/x)^(1/2)/(-a*c*x+c)^(9/2)-15/64*(1-1/a/x)^(
9/2)*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)/a/(1/a/x)^(9/2
)/(-a*c*x+c)^(9/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.58

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = -\frac{\sqrt{1-\frac{1}{ax}}\left(6+40ax-30a^2x^2+\frac{15\sqrt{2}\sqrt{a}\sqrt{1+\frac{1}{ax}}(-1+ax)^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{\frac{1}{x}}}\right)}{64ac^4\sqrt{1+\frac{1}{ax}}(-1+ax)^2\sqrt{c-acx}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(9/2)),x]
```

output
```
-1/64*(Sqrt[1 - 1/(a*x)]*(6 + 40*a*x - 30*a^2*x^2 + (15*Sqrt[2]*Sqrt[a]*Sq
rt[1 + 1/(a*x)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[
1 + 1/(a*x)])]])/Sqrt[x^(-1)]))/(a*c^4*Sqrt[1 + 1/(a*x)]*(-1 + a*x)^2*Sqrt[
c - a*c*x])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6727, 27, 105, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx$$

$$\Big\downarrow \; 6727$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{9/2}\int \frac{a^3\left(\frac{1}{x}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{9/2}\int\frac{\left(\frac{1}{x}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}}$$

$\Big\downarrow$ 105

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{\left(\frac{1}{x}\right)^{5/2}}{4\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}}-\frac{5}{8}\int\frac{\left(\frac{1}{x}\right)^{3/2}}{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}}$$

$\Big\downarrow$ 105

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{\left(\frac{1}{x}\right)^{5/2}}{4\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}}-\frac{5}{8}\left(\frac{\left(\frac{1}{x}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}-\frac{3}{4}\int\frac{\sqrt{\frac{1}{x}}}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}\right)\right)}{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}}$$

$\Big\downarrow$ 105

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{\left(\frac{1}{x}\right)^{5/2}}{4\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}}-\frac{5}{8}\left(\frac{\left(\frac{1}{x}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}-\frac{3}{4}\left(\frac{1}{2}a\int\frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)\right)\right)}{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}}$$

$\Big\downarrow$ 104

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{\left(\frac{1}{x}\right)^{5/2}}{4\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}}-\frac{5}{8}\left(\frac{\left(\frac{1}{x}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}-\frac{3}{4}\left(a\int\frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)\right)\right)}{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}}$$

$\Big\downarrow$ 219

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{\left(\frac{1}{x}\right)^{5/2}}{4\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}}-\frac{5}{8}\left(\frac{\left(\frac{1}{x}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}-\frac{3}{4}\left(\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}}-\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)\right)\right)}{\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(9/2)),x]
```

output
```
-((a^3*(1 - 1/(a*x))^(9/2)*((x^(-1))^(5/2)/(4*(a - x^(-1))^2*Sqrt[1 + 1/(a
*x)]) - (5*((x^(-1))^(3/2)/(2*(a - x^(-1))*Sqrt[1 + 1/(a*x)]) - (3*(-(Sqrt
[x^(-1)]/Sqrt[1 + 1/(a*x)]) + (Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqr
t[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[2]))/4))/8))/((x^(-1))^(9/2)*(c - a*c*x)^(9
/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.74

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(15\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^2x^2\sqrt{-c(ax+1)}-30\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)ax\sqrt{-c(ax+1}\right.}{64(ax-1)^4c^{\frac{11}{2}}a}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^4*(a*x+1)*(-c*(a*x-1))^(1/2)/c^(11/2
)*(15*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*x^2*(-c*(
a*x+1))^(1/2)-30*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*
x*(-c*(a*x+1))^(1/2)+30*c^(1/2)*a^2*x^2+15*arctan(1/2*(-c*(a*x+1))^(1/2)*2
^(1/2)/c^(1/2))*2^(1/2)*(-c*(a*x+1))^(1/2)-40*c^(1/2)*a*x-6*c^(1/2))/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.43

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \left[-\frac{15\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}}{a^2x^2-2ax+1}\right.}{128\left(a^4c^5x^3-3a^3c^5x^2+3a^2c^5x-\right.}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

output
```
[-1/128*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c
*x^2 + 2*a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x -
 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*(15*a^2*x^2 - 20*a*x - 3)
*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2
+ 3*a^2*c^5*x - a*c^5), 1/64*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)
*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x -
 1)/(a*x + 1))/(a*c*x - c)) - 2*(15*a^2*x^2 - 20*a*x - 3)*sqrt(-a*c*x + c)
*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a
*c^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-acx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a*c*x + c)^(9/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(9/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{(c-a\,c\,x)^{9/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(9/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.86

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \frac{\sqrt{c}\,\sqrt{2}\,i\left(120\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)^{6} - 120\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-}}{}\right.}{2}\right.\right.\right.}{51}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(9/2),x)
```

output
```
(sqrt(c)*sqrt(2)*i*(120*log(tan(asin(sqrt( - a*x + 1)/sqrt(2))/2))*tan(asi
n(sqrt( - a*x + 1)/sqrt(2))/2)**6 - 120*log(tan(asin(sqrt( - a*x + 1)/sqrt
(2))/2))*tan(asin(sqrt( - a*x + 1)/sqrt(2))/2)**4 + tan(asin(sqrt( - a*x +
 1)/sqrt(2))/2)**10 + 15*tan(asin(sqrt( - a*x + 1)/sqrt(2))/2)**8 - 160*ta
n(asin(sqrt( - a*x + 1)/sqrt(2))/2)**6 + 15*tan(asin(sqrt( - a*x + 1)/sqrt
(2))/2)**2 + 1))/(512*tan(asin(sqrt( - a*x + 1)/sqrt(2))/2)**4*a*c**5*(tan
(asin(sqrt( - a*x + 1)/sqrt(2))/2)**2 - 1))
```

3.276 $\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{4c(c-acx)^{-1+p}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{1+p}}{ac(1+p)}$$

output
```
4*c*(-a*c*x+c)^(-1+p)/a/(1-p)+4*(-a*c*x+c)^p/a/p-(-a*c*x+c)^(p+1)/a/c/(p+1
)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{(c-acx)^p\left(\frac{4+3p}{p(1+p)}+\frac{ax}{1+p}+\frac{4}{(-1+p)(-1+ax)}\right)}{a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

output
```
((c - a*c*x)^p*((4 + 3*p)/(p*(1 + p)) + (a*x)/(1 + p) + 4/((-1 + p)*(-1 +
a*x))))/a
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int e^{4\text{arctanh}(ax)}(c-acx)^pdx$$

$$\Big\downarrow \text{6680}$$

$$\int \frac{(ax+1)^2(c-acx)^p}{(1-ax)^2}dx$$

$$\Big\downarrow \text{35}$$

$$c^2\int (ax+1)^2(c-acx)^{p-2}dx$$

$$\Big\downarrow \text{53}$$

$$c^2\int \left(4(c-acx)^{p-2}-\frac{4(c-acx)^{p-1}}{c}+\frac{(c-acx)^p}{c^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c^2\left(-\frac{(c-acx)^{p+1}}{ac^3(p+1)}+\frac{4(c-acx)^p}{ac^2p}+\frac{4(c-acx)^{p-1}}{ac(1-p)}\right)$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

output
```
c^2*((4*(c - a*c*x)^(-1 + p))/(a*c*(1 - p)) + (4*(c - a*c*x)^p)/(a*c^2*p)
- (c - a*c*x)^(1 + p)/(a*c^3*(1 + p)))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

method	result
gosper	$\frac{(-acx+c)^{p}\left(a^{2}p^{2}x^{2}-a^{2}x^{2}p+2a\,p^{2}x+2apx-4ax+p^{2}+3p+4\right)}{(p^{2}-1)ap(ax-1)}$
orering	$\frac{(-acx+c)^{p}\left(a^{2}p^{2}x^{2}-a^{2}x^{2}p+2a\,p^{2}x+2apx-4ax+p^{2}+3p+4\right)}{(p^{2}-1)ap(ax-1)}$
risch	$\frac{\left(a^{2}p^{2}x^{2}-a^{2}x^{2}p+2a\,p^{2}x+2apx-4ax+p^{2}+3p+4\right)(-acx+c)^{p}}{ap(p+1)(p-1)(ax-1)}$
norman	$\frac{\frac{a\,x^{2}\mathrm{e}^{p\ln(-acx+c)}}{p+1}+\frac{\left(p^{2}+3p+4\right)\mathrm{e}^{p\ln(-acx+c)}}{ap\left(p^{2}-1\right)}+\frac{2(2+p)x\,\mathrm{e}^{p\ln(-acx+c)}}{p(p+1)}}{ax-1}$
parallelrisch	$\frac{x^{2}(-acx+c)^{p}a^{2}p^{2}-x^{2}(-acx+c)^{p}a^{2}p+2x(-acx+c)^{p}a\,p^{2}+2x(-acx+c)^{p}ap-4(-acx+c)^{p}xa+(-acx+c)^{p}p^{2}+3(-acx+c)^{p}p}{(ax-1)ap(p^{2}-1)}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x,method=_RETURNVERBOSE)
```

output
```
(-a*c*x+c)^p*(a^2*p^2*x^2-a^2*p*x^2+2*a*p^2*x+2*a*p*x-4*a*x+p^2+3*p+4)/(p^
2-1)/a/p/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx = -\frac{((a^2p^2-a^2p)x^2+p^2+2\,(ap^2+ap-2\,a)x+3\,p+4)(-acx+c)^p}{ap^3-ap-(a^2p^3-a^2p)x}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="fricas")
```

output
```
-((a^2*p^2 - a^2*p)*x^2 + p^2 + 2*(a*p^2 + a*p - 2*a)*x + 3*p + 4)*(-a*c*x
 + c)^p/(a*p^3 - a*p - (a^2*p^3 - a^2*p)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(48) = 96.

Time = 0.54 (sec) , antiderivative size = 530, normalized size of antiderivative = 8.03

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx = \begin{cases} c^p x \\ -\frac{a^2x^2\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac}+\frac{2ax\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac}+\frac{4ax}{a^3cx^2-2a^2cx+ac}-\frac{\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac}-\frac{2}{a^3cx^2-2a^2cx+ac} \\ \frac{a^2x^2}{a^2x-a}+\frac{4ax\log\left(x-\frac{1}{a}\right)}{a^2x-a}-\frac{4\log\left(x-\frac{1}{a}\right)}{a^2x-a}-\frac{5}{a^2x-a} \\ -\frac{acx^2}{2}-3cx-\frac{4c\log\left(x-\frac{1}{a}\right)}{a} \\ \frac{a^2p^2x^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}-\frac{a^2px^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+\frac{2ap^2x(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+\frac{2apx(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}-\frac{4ax(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+\frac{}{a^2p} \end{cases}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**p,x)
```

output
```
Piecewise((c**p*x, Eq(a, 0)), (-a**2*x**2*log(x - 1/a)/(a**3*c*x**2 - 2*a*
*2*c*x + a*c) + 2*a*x*log(x - 1/a)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + 4*a*
x/(a**3*c*x**2 - 2*a**2*c*x + a*c) - log(x - 1/a)/(a**3*c*x**2 - 2*a**2*c*
x + a*c) - 2/(a**3*c*x**2 - 2*a**2*c*x + a*c), Eq(p, -1)), (a**2*x**2/(a**
2*x - a) + 4*a*x*log(x - 1/a)/(a**2*x - a) - 4*log(x - 1/a)/(a**2*x - a) -
 5/(a**2*x - a), Eq(p, 0)), (-a*c*x**2/2 - 3*c*x - 4*c*log(x - 1/a)/a, Eq(
p, 1)), (a**2*p**2*x**2*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 +
 a*p) - a**2*p*x**2*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p
) + 2*a*p**2*x*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + 2
*a*p*x*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) - 4*a*x*(-a
*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + p**2*(-a*c*x + c)**
p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + 3*p*(-a*c*x + c)**p/(a**2*p**3
*x - a**2*p*x - a*p**3 + a*p) + 4*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x
- a*p**3 + a*p), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(64) = 128$.

Time = 0.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.32

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{((p^2-p)a^2c^px^2+2\,ac^p(p-1)x+2\,c^p)(-ax+1)^pa^2}{(p^3-p)a^4x-(p^3-p)a^3} + \frac{2\,(ac^p(p-1)x+c^p)(-ax+1)^pa}{(p^2-p)a^3x-(p^2-p)a^2} + \frac{(-ax+1)^pc^p}{a^2(p-1)x-a(p-1)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="maxima")
```

output
```
((p^2 - p)*a^2*c^p*x^2 + 2*a*c^p*(p - 1)*x + 2*c^p)*(-a*x + 1)^p*a^2/((p^3
 - p)*a^4*x - (p^3 - p)*a^3) + 2*(a*c^p*(p - 1)*x + c^p)*(-a*x + 1)^p*a/((
p^2 - p)*a^3*x - (p^2 - p)*a^2) + (-a*x + 1)^p*c^p/(a^2*(p - 1)*x - a*(p -
 1))
```

Giac [F]

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(ax+1)^2(-acx+c)^p}{(ax-1)^2}\,dx$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="giac")
```

output
```
integrate((a*x + 1)^2*(-a*c*x + c)^p/(a*x - 1)^2, x)
```

Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{4\,(c-a\,c\,x)^p}{a\,\left(a\,x-1\right)\,\left(p-1\right)}+\frac{(c-a\,c\,x)^p\,\left(3\,p+a\,p\,x+4\right)}{a\,p\,\left(p+1\right)}$$

input
```
int(((c - a*c*x)^p*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(4*(c - a*c*x)^p)/(a*(a*x - 1)*(p - 1)) + ((c - a*c*x)^p*(3*p + a*p*x + 4)
)/(a*p*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx =\frac{(-acx+c)^p\,(a^2p^2x^2-a^2p\,x^2+2a\,p^2x+2apx-4ax+p^2+3p+4)}{ap\,(a\,p^2x-ax-p^2+1)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x)
```

output
```
(( - a*c*x + c)**p*(a**2*p**2*x**2 - a**2*p*x**2 + 2*a*p**2*x + 2*a*p*x -
4*a*x + p**2 + 3*p + 4))/(a*p*(a*p**2*x - a*x - p**2 + 1))
```

3.277 $\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 42

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{2(c-acx)^p}{ap}-\frac{(c-acx)^{1+p}}{ac(1+p)}$$

output
```
2*(-a*c*x+c)^p/a/p-(-a*c*x+c)^(p+1)/a/c/(p+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{(c-acx)^p(2+p+apx)}{ap(1+p)}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

output
```
((c - a*c*x)^p*(2 + p + a*p*x))/(a*p*(1 + p))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}(c-acx)^p dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(ax+1)(c-acx)^p}{1-ax}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int (ax+1)(c-acx)^{p-1}dx$$

$$\Big\downarrow \text{53}$$

$$-c\int \left(2(c-acx)^{p-1}-\frac{(c-acx)^p}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{(c-acx)^{p+1}}{ac^2(p+1)}-\frac{2(c-acx)^p}{acp}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

output
```
-(c*((-2*(c - a*c*x)^p)/(a*c*p) + (c - a*c*x)^(1 + p)/(a*c^2*(1 + p))))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{(apx+p+2)(-acx+c)^{p}}{ap(p+1)}$	29
risch	$\frac{(apx+p+2)(-acx+c)^{p}}{ap(p+1)}$	29
orering	$\frac{(apx+p+2)(-acx+c)^{p}}{ap(p+1)}$	29
norman	$\frac{x\,{\mathrm e}^{p\ln(-acx+c)}}{p+1}+\frac{(2+p)\,{\mathrm e}^{p\ln(-acx+c)}}{ap(p+1)}$	46
parallelrisch	$\frac{x(-acx+c)^{p}ap+(-acx+c)^{p}p+2(-acx+c)^{p}}{ap(p+1)}$	49

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x,method=_RETURNVERBOSE)
```

output
```
(a*p*x+p+2)*(-a*c*x+c)^p/a/p/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{(apx+p+2)(-acx+c)^p}{ap^2+ap}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="fricas")
```

output
```
(a*p*x + p + 2)*(-a*c*x + c)^p/(a*p^2 + a*p)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(29) = 58.

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.95

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx=\begin{cases}-c^px & \text{for } a=0\\ -\frac{ax\log\left(x-\frac{1}{a}\right)}{a^2cx-ac}+\frac{\log\left(x-\frac{1}{a}\right)}{a^2cx-ac}+\frac{2}{a^2cx-ac} & \text{for } p=-1\\ x+\frac{2\log\left(x-\frac{1}{a}\right)}{a} & \text{for } p=0\\ \frac{apx(-acx+c)^p}{ap^2+ap}+\frac{p(-acx+c)^p}{ap^2+ap}+\frac{2(-acx+c)^p}{ap^2+ap} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**p,x)
```

output
```
Piecewise((-c**p*x, Eq(a, 0)), (-a*x*log(x - 1/a)/(a**2*c*x - a*c) + log(x
 - 1/a)/(a**2*c*x - a*c) + 2/(a**2*c*x - a*c), Eq(p, -1)), (x + 2*log(x -
1/a)/a, Eq(p, 0)), (a*p*x*(-a*c*x + c)**p/(a*p**2 + a*p) + p*(-a*c*x + c)*
*p/(a*p**2 + a*p) + 2*(-a*c*x + c)**p/(a*p**2 + a*p), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{(ac^ppx+c^p)(-ax+1)^p}{(p^2+p)a} + \frac{(-ax+1)^pc^p}{ap}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="maxima")
```

output
```
(a*c^p*p*x + c^p)*(-a*x + 1)^p/((p^2 + p)*a) + (-a*x + 1)^p*c^p/(a*p)
```

Giac [F]

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx = \int \frac{(ax+1)(-acx+c)^p}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="giac")
```

output
```
integrate((a*x + 1)*(-a*c*x + c)^p/(a*x - 1), x)
```

Mupad [B] (verification not implemented)

Time = 13.86 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{(c-a\,c\,x)^p\,(p+a\,p\,x+2)}{a\,p\,(p+1)}$$

input
```
int(((c - a*c*x)^p*(a*x + 1))/(a*x - 1),x)
```

output
```
((c - a*c*x)^p*(p + a*p*x + 2))/(a*p*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx=\frac{(-acx+c)^p\,(apx+p+2)}{ap\,(p+1)}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x)
```

output
```
(( - a*c*x + c)**p*(a*p*x + p + 2))/(a*p*(p + 1))
```

3.278 $\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 44

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{(c-acx)^{2+p}\,\text{Hypergeometric2F1}\left(1,2+p,3+p,\frac{1}{2}(1-ax)\right)}{2ac^2(2+p)}$$

output
```
1/2*(-a*c*x+c)^(2+p)*hypergeom([1, 2+p],[3+p],-1/2*a*x+1/2)/a/c^2/(2+p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx = -\frac{(-1+ax)(c-acx)^p\left(-1+\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{1}{2}(1-ax)\right)\right)}{a(1+p)}$$

input
```
Integrate[(c - a*c*x)^p/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((-1 + a*x)*(c - a*c*x)^p*(-1 + Hypergeometric2F1[1, 1 + p, 2 + p, (1 -
a*x)/2]))/(a*(1 + p)))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6717, 6680, 35, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}(c-acx)^pdx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(1-ax)(c-acx)^p}{ax+1}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{(c-acx)^{p+1}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{78}$$

$$\frac{(c-acx)^{p+2}\text{Hypergeometric2F1}\left(1,p+2,p+3,\frac{1}{2}(1-ax)\right)}{2ac^2(p+2)}$$

input
```
Int[(c - a*c*x)^p/E^(2*ArcCoth[a*x]),x]
```

output
```
((c - a*c*x)^(2 + p)*Hypergeometric2F1[1, 2 + p, 3 + p, (1 - a*x)/2])/(2*a
*c^2*(2 + p))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [F]

$$\int \frac{(-acx+c)^p (ax-1)}{ax+1} dx$$

input
```
int((-a*c*x+c)^p*(a*x-1)/(a*x+1),x)
```

output
```
int((-a*c*x+c)^p*(a*x-1)/(a*x+1),x)
```

Fricas [F]

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(ax-1)(-acx+c)^p}{ax+1}\,dx$$

input
```
integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
integral((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(-c(ax-1))^p\,(ax-1)}{ax+1}\,dx$$

input
```
integrate((-a*c*x+c)**p*(a*x-1)/(a*x+1),x)
```

output
```
Integral((-c*(a*x - 1))**p*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(ax-1)(-acx+c)^p}{ax+1}\,dx$$

input
```
integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

Giac [F]

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(ax-1)(-acx+c)^p}{ax+1}\,dx$$

input
```
integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
integrate((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(c-a\,c\,x)^p\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - a*c*x)^p*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - a*c*x)^p*(a*x - 1))/(a*x + 1), x)
```

Reduce [F]

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx$$
$$=\frac{(-acx+c)^p\,apx+(-acx+c)^p\,p+2(-acx+c)^p-4\left(\int\frac{(-acx+c)^p x}{a^2x^2-1}dx\right)a^2p^2-4\left(\int\frac{(-acx+c)^p x}{a^2x^2-1}dx\right)a^2p}{ap\,(p+1)}$$

input
```
int((-a*c*x+c)^p*(a*x-1)/(a*x+1),x)
```

output
```
(( - a*c*x + c)**p*a*p*x + ( - a*c*x + c)**p*p + 2*( - a*c*x + c)**p - 4*i
nt((( - a*c*x + c)**p*x)/(a**2*x**2 - 1),x)*a**2*p**2 - 4*int((( - a*c*x +
 c)**p*x)/(a**2*x**2 - 1),x)*a**2*p)/(a*p*(p + 1))
```

3.279 $\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 94

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^p\,\text{Hypergeometric2F1}\left(-1-p,\frac{3}{2}-p,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(1+p)\left(1-\frac{1}{ax}\right)^{3/2}}$$

output
```
((a-1/x)/(a+1/x))^(3/2-p)*(1+1/a/x)^(5/2)*x*(-a*c*x+c)^p*hypergeom([-1-p,
3/2-p],[-p],2/(a+1/x)/x)/(p+1)/(1-1/a/x)^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.47

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\left(\frac{-1+ax}{1+ax}\right)^{-p}(c-acx)^p\left((-1+p)\left(\frac{-1+ax}{1+ax}\right)^p(1+ax)(3+p+apx)+3\sqrt{\frac{-1+ax}{1+ax}}\,\text{Hypergeometric2F1}\left(1-p\right.\right.}{a^2(-1+p)p(1+p)\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

output
```
((c - a*c*x)^p*((-1 + p)*((-1 + a*x)/(1 + a*x))^p*(1 + a*x)*(3 + p + a*p*x
) + 3*Sqrt[(-1 + a*x)/(1 + a*x)]*Hypergeometric2F1[1 - p, 3/2 - p, 2 - p,
2/(1 + a*x)]))/(a^2*(-1 + p)*p*(1 + p)*Sqrt[1 - 1/(a^2*x^2)]*x*((-1 + a*x)
/(1 + a*x))^p)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(94) = 188.

Time = 0.58 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.48, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6727, 105, 105, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx$$

$$\Big\downarrow \text{6727}$$

$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\int\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{-p-2}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\left(\frac{3\int\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-p-1}d\frac{1}{x}}{a(p+1)}-\frac{\left(\frac{1}{ax}+1\right)^{3/2}\left(\frac{1}{x}\right)^{-p-1}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}}{p+1}\right)$$

$$\Big\downarrow \text{105}$$

$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\left(\frac{3\left(\frac{\int\frac{\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\left(\frac{1}{x}\right)^{-p}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{ap}-\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)^{-p}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}}{p}\right)}{a(p+1)}-\frac{\left(\frac{1}{ax}+1\right)^{3/2}\left(\frac{1}{x}\right)^{-p-1}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}}{p+1}\right)$$

$$\Big\downarrow \text{142}$$

$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\left(\frac{3\left(\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)^{1-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\text{Hypergeometric2F1}\left(1-p,\frac{3}{2}-p,2-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{a(1-p)p}-\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)^{-p}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}}{p}\right)}{a(p+1)}-\right.$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

output
```
-(((x^(-1))^p*(c - a*c*x)^p*(-(((1 - 1/(a*x))^(-1/2 + p)*(1 + 1/(a*x))^(3/
2)*(x^(-1))^(-1 - p))/(1 + p)) + (3*(-(((1 - 1/(a*x))^(-1/2 + p)*Sqrt[1 +
1/(a*x)])/(p*(x^(-1))^p)) + (((a - x^(-1))/(a + x^(-1)))^(3/2 - p)*(1 - 1/
(a*x))^(-3/2 + p)*Sqrt[1 + 1/(a*x)]*(x^(-1))^(1 - p)*Hypergeometric2F1[1 -
 p, 3/2 - p, 2 - p, 2/((a + x^(-1))*x)])/(a*(1 - p)*p)))/(a*(1 + p))))/(1
- 1/(a*x))^p)
```

Defintions of rubi rules used

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{(-acx+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x)
```

Fricas [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(-acx+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="fricas")
```

output
```
integral((a^2*x^2 + 2*a*x + 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a
^2*x^2 - 2*a*x + 1), x)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(-c(ax-1))^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**p,x)
```

output
```
Integral((-c*(a*x - 1))**p/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(-acx+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx = \int \frac{(c-a\,c\,x)^p}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [F]

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx = \left(\int \frac{\sqrt{ax+1}\,(-acx+c)^p\,x}{\sqrt{ax-1}\,ax-\sqrt{ax-1}}dx\right)a + \int \frac{\sqrt{ax+1}\,(-acx+c)^p}{\sqrt{ax-1}\,ax-\sqrt{ax-1}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x)
```

output
```
int((sqrt(a*x + 1)*( - a*c*x + c)**p*x)/(sqrt(a*x - 1)*a*x - sqrt(a*x - 1)
),x)*a + int((sqrt(a*x + 1)*( - a*c*x + c)**p)/(sqrt(a*x - 1)*a*x - sqrt(a
*x - 1)),x)
```

3.280 $\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal result

Integrand size = 16, antiderivative size = 94

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^p\,\text{Hypergeometric2F1}\left(-1-p,\frac{1}{2}-p,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(1+p)\sqrt{1-\frac{1}{ax}}}$$

output
```
((a-1/x)/(a+1/x))^(1/2-p)*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^p*hypergeom([-1-p,
1/2-p],[-p],2/(a+1/x)/x)/(p+1)/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.39

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\sqrt{1+\frac{1}{ax}}\left(\frac{-1+ax}{1+ax}\right)^{-p}(c-acx)^p\left(p(-1+ax)\left(\frac{-1+ax}{1+ax}\right)^p+\sqrt{\frac{-1+ax}{1+ax}}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-p,-p,1-p,\right.\right.}{ap(1+p)\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a*c*x)^p,x]
```

output
```
(Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*(p*(-1 + a*x)*((-1 + a*x)/(1 + a*x))^p +
Sqrt[(-1 + a*x)/(1 + a*x)]*Hypergeometric2F1[1/2 - p, -p, 1 - p, 2/(1 + a*
x)]))/(a*p*(1 + p)*Sqrt[1 - 1/(a*x)]*((-1 + a*x)/(1 + a*x))^p)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {6727, 105, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx$$

$$\Big\downarrow \text{6727}$$

$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\int\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-p-2}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\left(\frac{\int\frac{\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\left(\frac{1}{x}\right)^{-p-1}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a(p+1)}-\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)^{-p-1}\left(1-\frac{1}{ax}\right)^{p+\frac{1}{2}}}{p+1}\right)$$

$$\Big\downarrow \text{142}$$

$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\left(-\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2}-p,-p,1-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{ap(p+1)}-\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right.}{}\right.$$

input
```
Int[E^ArcCoth[a*x]*(c - a*c*x)^p,x]
```

output
```
-(((x^(-1))^p*(c - a*c*x)^p*(-(((1 - 1/(a*x))^(1/2 + p)*Sqrt[1 + 1/(a*x)]*
(x^(-1))^(-1 - p))/(1 + p)) - (((a - x^(-1))/(a + x^(-1)))^(1/2 - p)*(1 -
1/(a*x))^(-1/2 + p)*Sqrt[1 + 1/(a*x)]*Hypergeometric2F1[1/2 - p, -p, 1 - p
, 2/((a + x^(-1))*x)])/(a*p*(1 + p)*(x^(-1))^p)))/(1 - 1/(a*x))^p)
```

Defintions of rubi rules used

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{(-acx+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(-acx+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1), x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(-c(ax-1))^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**p,x)
```

output
```
Integral((-c*(a*x - 1))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(-acx+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(-acx+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{(c-a\,c\,x)^p}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{\sqrt{ax+1}\,(-acx+c)^p}{\sqrt{ax-1}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x)
```

output
```
int((sqrt(a*x + 1)*( - a*c*x + c)**p)/sqrt(a*x - 1),x)
```

3.281 $\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 94

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{1}{2}-p}\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x(c-acx)^p \text{Hypergeometric2F1}\left(-1-p,-\frac{1}{2}-p,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+p}$$

output
```
((a-1/x)/(a+1/x))^(-1/2-p)*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)*x*(-a*c*x+c)^p*
hypergeom([-1-p, -1/2-p],[-p],2/(a+1/x)/x)/(p+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x\left(\frac{-1+ax}{1+ax}\right)^{-\frac{1}{2}-p}(c-acx)^p \text{Hypergeometric2F1}\left(-1-p,-\frac{1}{2}-p,-p,\frac{2}{1+ax}\right)}{1+p}$$

input
```
Integrate[(c - a*c*x)^p/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*((-1 + a*x)/(1 + a*x))^(-1/2 - p)*(c - a*c*x)^p*H
ypergeometric2F1[-1 - p, -1/2 - p, -p, 2/(1 + a*x)])/(1 + p)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6727, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx$$

$$\Big\downarrow \text{6727}$$

$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\int\frac{\left(1-\frac{1}{ax}\right)^{p+\frac{1}{2}}\left(\frac{1}{x}\right)^{-p-2}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{142}$$

$$\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{1}{2}}(c-acx)^p\,\text{Hypergeometric2F1}\left(-p-1,-p-\frac{1}{2},-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{p+1}$$

input
```
Int[(c - a*c*x)^p/E^ArcCoth[a*x],x]
```

output
```
(((a - x^(-1))/(a + x^(-1)))^(-1/2 - p)*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)
]*x*(c - a*c*x)^p*Hypergeometric2F1[-1 - p, -1/2 - p, -p, 2/((a + x^(-1))*
x)])/(1 + p)
```

Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] && !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[p]
```

Maple [F]

$$\int (-acx+c)^p \sqrt{\frac{ax-1}{ax+1}}dx$$

input
```
int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

Fricas [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx = \int (-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
integral((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx=\int\sqrt{\frac{ax-1}{ax+1}}(-c(ax-1))^p\,dx$$

input
```
integrate((-a*c*x+c)**p*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1))**p, x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx=\int(-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx=\int(-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx=\int (c-a\,c\,x)^p\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx=\int \frac{\sqrt{ax-1}\,(-acx+c)^p}{\sqrt{ax+1}}dx$$

input
```
int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
int((sqrt(a*x - 1)*( - a*c*x + c)**p)/sqrt(a*x + 1),x)
```

3.282 $\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 94

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{3/2}x(c-acx)^p\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-p,-1-p,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(1+p)\sqrt{1+\frac{1}{ax}}}$$

output
```
((a-1/x)/(a+1/x))^(-3/2-p)*(1-1/a/x)^(3/2)*x*(-a*c*x+c)^p*hypergeom([-1-p,
 -3/2-p],[-p],2/(a+1/x)/x)/(p+1)/(1+1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(\frac{-1+ax}{1+ax}\right)^{-\frac{1}{2}-p}(1+ax)(c-acx)^p\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-p,-1-p,-p,\frac{2}{1+ax}\right)}{a(1+p)\sqrt{1+\frac{1}{ax}}}$$

input
```
Integrate[(c - a*c*x)^p/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*((-1 + a*x)/(1 + a*x))^(-1/2 - p)*(1 + a*x)*(c - a*c*x)
^p*Hypergeometric2F1[-3/2 - p, -1 - p, -p, 2/(1 + a*x)])/(a*(1 + p)*Sqrt[1
 + 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6727, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx$$

$$\Big\downarrow \text{6727}$$

$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\int\frac{\left(1-\frac{1}{ax}\right)^{p+\frac{3}{2}}\left(\frac{1}{x}\right)^{-p-2}}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{142}$$

$$\frac{x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{3}{2}}(c-acx)^p\,\text{Hypergeometric2F1}\left(-p-\frac{3}{2},-p-1,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(p+1)\sqrt{\frac{1}{ax}+1}}$$

input
```
Int[(c - a*c*x)^p/E^(3*ArcCoth[a*x]),x]
```

output
```
(((a - x^(-1))/(a + x^(-1)))^(-3/2 - p)*(1 - 1/(a*x))^(3/2)*x*(c - a*c*x)^
p*Hypergeometric2F1[-3/2 - p, -1 - p, -p, 2/((a + x^(-1))*x)])/((1 + p)*Sq
rt[1 + 1/(a*x)])
```

Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int (-acx+c)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

input
```
int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

Fricas [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx = \int (-acx+c)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

input
```
integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
integral((a*x - 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx=\text{Timed out}$$

input
```
integrate((-a*c*x+c)**p*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx=\int(-acx+c)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx=\int (c-a\,c\,x)^p\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [F]

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx=\left(\int\frac{\sqrt{ax-1}\,(-acx+c)^p\,x}{\sqrt{ax+1}\,ax+\sqrt{ax+1}}dx\right)a-\left(\int\frac{\sqrt{ax-1}\,(-acx+c)^p}{\sqrt{ax+1}\,ax+\sqrt{ax+1}}dx\right)$$

input
```
int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
int((sqrt(a*x - 1)*( - a*c*x + c)**p*x)/(sqrt(a*x + 1)*a*x + sqrt(a*x + 1)
),x)*a - int((sqrt(a*x - 1)*( - a*c*x + c)**p)/(sqrt(a*x + 1)*a*x + sqrt(a
*x + 1)),x)
```

3.283 $\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 104

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2}(n-2p),-1-p,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+p}$$

output
```
((a-1/x)/(a+1/x))^(1/2*n-p)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^p*hypergeom([
-1-p, 1/2*n-p],[-p],2/(a+1/x)/x)/(p+1)/((1-1/a/x)^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx = \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1}{2}(n-2p)}(1+ax)(c-acx)^p\,\text{Hypergeometric2F1}\left(-1-p,\frac{n}{2}-p,-p,\frac{2}{1+ax}\right)}{a(1+p)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

output
```
((1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((n - 2*p)/2)*(1 + a*x)*(c - a
*c*x)^p*Hypergeometric2F1[-1 - p, n/2 - p, -p, 2/(1 + a*x)])/(a*(1 + p)*(1
 - 1/(a*x))^(n/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6727, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^p e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$\left(\frac{1}{x}\right)^p\left(-\left(1-\frac{1}{ax}\right)^{-p}\right)(c-acx)^p\int \left(1-\frac{1}{ax}\right)^{p-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-p-2}d\frac{1}{x}$$

$$\Big\downarrow \text{142}$$

$$\frac{x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^p\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\text{Hypergeometric2F1}\left(\frac{1}{2}(n-2p),-p-1,-p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{p+1}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

output
```
(((a - x^(-1))/(a + x^(-1)))^((n - 2*p)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c
- a*c*x)^p*Hypergeometric2F1[(n - 2*p)/2, -1 - p, -p, 2/((a + x^(-1))*x)])
/((1 + p)*(1 - 1/(a*x))^(n/2))
```

Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] && !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[p]
```

Maple [F]

$$\int e^{n\ \mathrm{arccoth}(ax)}(-acx+c)^p\,dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx=\int(-acx+c)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="fricas")
```

output
```
integral((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx=\int(-c(ax-1))^p\,e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**p,x)
```

output
```
Integral((-c*(a*x - 1))**p*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx=\int(-acx+c)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx=\int(-acx+c)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^p\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^p,x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x)^p, x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx=\int e^{acoth(ax)n}(-acx+c)^p\,dx$$

input
```
int(exp(n*acoth(a*x))*(-a*c*x+c)^p,x)
```

output
```
int(e**(acoth(a*x)*n)*( - a*c*x + c)**p,x)
```

3.284 $\int e^{n \coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int e^{n \coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{32c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-8+n)}\text{Hypergeometric2F1}\left(5,4-\frac{n}{2},5-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

output
```
-32*c^3*(1-1/a/x)^(4-1/2*n)*(1+1/a/x)^(-4+1/2*n)*hypergeom([5, 4-1/2*n],[5
-1/2*n],(a-1/x)/(a+1/x))/a/(8-n)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(81) = 162.

Time = 1.98 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.35

$$\int e^{n \coth^{-1}(ax)}(c-acx)^3\,dx = -\frac{c^3 e^{n \coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n(-48+44n-12n^2+n^3)\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right.\right.}{}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

output
```
-1/24*(c^3*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(-48 + 44*n - 12*n^2 +
 n^3)*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)
*(a*n^3*x + n^2*(-1 - 12*a*x + a^2*x^2) + 2*n*(6 + 21*a*x - 6*a^2*x^2 + a^
3*x^3) + 6*(-7 - 4*a*x + 6*a^2*x^2 - 4*a^3*x^3 + a^4*x^4) + (-48 + 44*n -
12*n^2 + n^3)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a
*(2 + n))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6725, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^3 e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6725}$$

$$a^3c^3\int \left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x^5d\frac{1}{x}$$

$$\Big\downarrow \text{141}$$

$$-\frac{32c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-8}{2}}\text{Hypergeometric2F1}\left(5,4-\frac{n}{2},5-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

output
```
(-32*c^3*(1 - 1/(a*x))^(4 - n/2)*(1 + 1/(a*x))^((-8 + n)/2)*Hypergeometric
2F1[5, 4 - n/2, 5 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(8 - n))
```

Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

Maple [F]

$$\int e^{n\ \mathrm{arccoth}(ax)}(-acx+c)^3\, dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\, dx = \int -(acx-c)^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\, dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
integral(-(a^3*c^3*x^3 - 3*a^2*c^3*x^2 + 3*a*c^3*x - c^3)*((a*x + 1)/(a*x
- 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx=-c^3\left(\int 3axe^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-3a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\right)\,dx+\int a^3x^3e^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-e^{n\,\mathrm{acoth}\,(ax)}\right)\,dx\right)$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**3,x)
```

output
```
-c**3*(Integral(3*a*x*exp(n*acoth(a*x)), x) + Integral(-3*a**2*x**2*exp(n*
acoth(a*x)), x) + Integral(a**3*x**3*exp(n*acoth(a*x)), x) + Integral(-exp
(n*acoth(a*x)), x))
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx=\int-(acx-c)^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
-integrate((a*c*x - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx=\int-(acx-c)^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
integrate(-(a*c*x - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^3\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^3,x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x)^3, x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx=c^3\left(\int e^{acoth(ax)n}dx-\left(\int e^{acoth(ax)n}x^3dx\right)a^3+3\left(\int e^{acoth(ax)n}x^2dx\right)a^2-3\left(\int e^{acoth(ax)n}xdx\right)a\right)$$

input
```
int(exp(n*acoth(a*x))*(-a*c*x+c)^3,x)
```

output
```
c**3*(int(e**(acoth(a*x)*n),x) - int(e**(acoth(a*x)*n)*x**3,x)*a**3 + 3*in
t(e**(acoth(a*x)*n)*x**2,x)*a**2 - 3*int(e**(acoth(a*x)*n)*x,x)*a)
```

3.285 $\int e^{n \coth^{-1}(ax)}(c-acx)^2 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int e^{n \coth^{-1}(ax)}(c-acx)^2 \, dx = \frac{16c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-6+n)}\text{Hypergeometric2F1}\left(4,3-\frac{n}{2},4-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

output
```
16*c^2*(1-1/a/x)^(3-1/2*n)*(1+1/a/x)^(-3+1/2*n)*hypergeom([4, 3-1/2*n],[4-
1/2*n],(a-1/x)/(a+1/x))/a/(6-n)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.78

$$\int e^{n \coth^{-1}(ax)}(c-acx)^2 \, dx = \frac{c^2e^{n \coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n(8-6n+n^2)\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(6\right.\right.}{}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

output
```
(c^2*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(8 - 6*n + n^2)*Hypergeometr
ic2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(6 + 6*a*x + a*n^
2*x - 6*a^2*x^2 + 2*a^3*x^3 + n*(-1 - 6*a*x + a^2*x^2) + (8 - 6*n + n^2)*H
ypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(6*a*(2 + n))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6725, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2 e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6725$$

$$-a^2c^2\int \left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x^4d\frac{1}{x}$$

$$\Big\downarrow \; 141$$

$$\frac{16c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-6}{2}}\text{Hypergeometric2F1}\left(4,3-\frac{n}{2},4-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

output
```
(16*c^2*(1 - 1/(a*x))^(3 - n/2)*(1 + 1/(a*x))^((-6 + n)/2)*Hypergeometric2
F1[4, 3 - n/2, 4 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(6 - n))
```

Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

Maple [F]

$$\int e^{n \operatorname{arccoth}(ax)}(-acx+c)^2 \, dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x)
```

Fricas [F]

$$\int e^{n \coth^{-1}(ax)}(c-acx)^2 \, dx = \int (acx-c)^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx=c^2\left(\int\left(-2axe^{n\operatorname{acoth}(ax)}\right)dx+\int a^2x^2e^{n\operatorname{acoth}(ax)}\,dx+\int e^{n\operatorname{acoth}(ax)}\,dx\right)$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**2,x)
```

output
```
c**2*(Integral(-2*a*x*exp(n*acoth(a*x)), x) + Integral(a**2*x**2*exp(n*aco
th(a*x)), x) + Integral(exp(n*acoth(a*x)), x))
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx=\int (acx-c)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((a*c*x - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx=\int (acx-c)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
integrate((a*c*x - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^2\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^2,x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x)^2, x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx=c^2\left(\int e^{acoth(ax)n}dx+\left(\int e^{acoth(ax)n}x^2dx\right)a^2-2\left(\int e^{acoth(ax)n}xdx\right)a\right)$$

input
```
int(exp(n*acoth(a*x))*(-a*c*x+c)^2,x)
```

output
```
c**2*(int(e**(acoth(a*x)*n),x) + int(e**(acoth(a*x)*n)*x**2,x)*a**2 - 2*in
t(e**(acoth(a*x)*n)*x,x)*a)
```

3.286 $\int e^{n \coth^{-1}(ax)}(c-acx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int e^{n \coth^{-1}(ax)}(c-acx)\,dx = -\frac{8c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-4+n)}\text{Hypergeometric2F1}\left(3,2-\frac{n}{2},3-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

output
```
-8*c*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(-2+1/2*n)*hypergeom([3, 2-1/2*n],[3-1/
2*n],(a-1/x)/(a+1/x))/a/(4-n)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.32

$$\int e^{n \coth^{-1}(ax)}(c-acx)\,dx = -\frac{ce^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}(-2+n)n\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1\right.\right.}{2a(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x),x]
```

output
```
-1/2*(c*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*(-2 + n)*n*Hypergeometric2F
1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*(-2 + n)*x +
a^2*x^2 + (-2 + n)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])
))/(a*(2 + n))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6725, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6725}$$

$$ac\int \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x^3d\frac{1}{x}$$

$$\Big\downarrow \text{141}$$

$$-\frac{8c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-4}{2}}\text{Hypergeometric2F1}\left(3,2-\frac{n}{2},3-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x),x]
```

output
```
(-8*c*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-4 + n)/2)*Hypergeometric2F1
[3, 2 - n/2, 3 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(4 - n))
```

Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)\,dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx=\int-(acx-c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="fricas")
```

output
```
integral(-(a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx=-c\left(\int axe^{n\operatorname{acoth}(ax)}\,dx+\int\left(-e^{n\operatorname{acoth}(ax)}\right)\,dx\right)$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c),x)
```

output
```
-c*(Integral(a*x*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x)), x))
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx=\int-(acx-c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="maxima")
```

output
```
-integrate((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx=\int-(acx-c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="giac")
```

output
```
integrate(-(a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x),x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x), x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx=c\left(\int e^{acoth(ax)n}dx-\left(\int e^{acoth(ax)n}xdx\right)a\right)$$

input
```
int(exp(n*acoth(a*x))*(-a*c*x+c),x)
```

output
```
c*(int(e**(acoth(a*x)*n),x) - int(e**(acoth(a*x)*n)*x,x)*a)
```

3.287 $\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx=\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

output
```
2*(1+1/a/x)^(1/2*n)*hypergeom([1, -1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/a/c/n
/((1-1/a/x)^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.23

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+\text{Hyperge}\right.\right.}{acn(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2,
2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + Hypergeometric2F1[1, n/2, 1 +
 n/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n)))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6725, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n \coth^{-1}(ax)}}{c-acx} \, dx$$
$$\Big\downarrow \text{6725}$$
$$\frac{\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1} \left(1+\frac{1}{ax}\right)^{n/2} x d\frac{1}{x}}{ac}$$
$$\Big\downarrow \text{141}$$
$$\frac{2\left(1-\frac{1}{ax}\right)^{-n/2} \left(\frac{1}{ax}+1\right)^{n/2} \text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a*c*x),x]
```

output
```
(2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (a - x^(-1))/
(a + x^(-1))])/(a*c*n*(1 - 1/(a*x))^(n/2))
```

Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{-acx+c}dx$$

input
```
int(exp(n*arccoth(a*x))/(-a*c*x+c),x)
```

output
```
int(exp(n*arccoth(a*x))/(-a*c*x+c),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx=\int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx-c}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="fricas")
```

output
```
integral(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Sympy [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{c - acx} \, dx = -\frac{\int \frac{e^{n \operatorname{acoth}(ax)}}{ax-1} \, dx}{c}$$

input
```
integrate(exp(n*acoth(a*x))/(-a*c*x+c),x)
```

output
```
-Integral(exp(n*acoth(a*x))/(a*x - 1), x)/c
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{c - acx} \, dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx - c} \, dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="maxima")
```

output
```
-integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{c - acx} \, dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx - c} \, dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="giac")
```

output
```
integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx=\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{c-a\,c\,x}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - a*c*x),x)
```

output
```
int(exp(n*acoth(a*x))/(c - a*c*x), x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx=-\frac{\int \frac{e^{acoth(ax)n}}{ax-1}dx}{c}$$

input
```
int(exp(n*acoth(a*x))/(-a*c*x+c),x)
```

output
```
( - int(e**(acoth(a*x)*n)/(a*x - 1),x))/c
```

3.288 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^2} \, dx = -\frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}} \left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)}$$

output
```
-(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^2/(2+n)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^2} \, dx = -\frac{e^{n \coth^{-1}(ax)}(1+ax)}{ac^2(2+n)(-1+ax)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

output
```
-((E^(n*ArcCoth[a*x])*(1 + a*x))/(a*c^2*(2 + n)*(-1 + a*x)))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6725, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}dx$$

$$\Big\downarrow \text{6725}$$

$$-\frac{\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{n/2}d\frac{1}{x}}{a^2c^2}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^2(n+2)}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

output
```
-(((1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^2*(2 + n)))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69

method	result	size
gosper	$-\frac{e^{n\,\mathrm{arccoth}(ax)}(ax+1)}{(ax-1)c^2(2+n)a}$	33
orering	$-\frac{(ax-1)(ax+1)e^{n\,\mathrm{arccoth}(ax)}}{(2+n)a(-acx+c)^2}$	37
parallelrisch	$\frac{-x\,e^{n\,\mathrm{arccoth}(ax)}a-e^{n\,\mathrm{arccoth}(ax)}}{c^2(ax-1)(2+n)a}$	41

input
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-exp(n*arccoth(a*x))*(a*x+1)/(a*x-1)/c^2/(2+n)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{(ax+1)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^2n+2\,ac^2-(a^2c^2n+2\,a^2c^2)x}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="fricas")
```

output
```
(a*x + 1)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n + 2*a*c^2 - (a^2*c^2*n +
2*a^2*c^2)*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.58 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.90

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \begin{cases} -\frac{x}{c^2} & \text{for } a=0 \wedge n=-2 \\ \frac{xe^{\frac{i\pi n}{2}}}{c^2} & \text{for } a=0 \\ -\frac{ax\,\mathrm{acoth}\,(ax)}{a^2c^2xe^{2\,\mathrm{acoth}\,(ax)}-ac^2e^{2\,\mathrm{acoth}\,(ax)}} - \frac{\mathrm{acoth}\,(ax)}{a^2c^2xe^{2\,\mathrm{acoth}\,(ax)}-ac^2e^{2\,\mathrm{acoth}\,(ax)}} & \text{for } n=-2 \\ -\frac{axe^{n\,\mathrm{acoth}\,(ax)}}{a^2c^2nx+2a^2c^2x-ac^2n-2ac^2} - \frac{e^{n\,\mathrm{acoth}\,(ax)}}{a^2c^2nx+2a^2c^2x-ac^2n-2ac^2} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(n*acoth(a*x))/(-a*c*x+c)**2,x)
```

output
```
Piecewise((-x/c**2, Eq(a, 0) & Eq(n, -2)), (x*exp(I*pi*n/2)/c**2, Eq(a, 0)
), (-a*x*acoth(a*x)/(a**2*c**2*x*exp(2*acoth(a*x)) - a*c**2*exp(2*acoth(a*
x))) - acoth(a*x)/(a**2*c**2*x*exp(2*acoth(a*x)) - a*c**2*exp(2*acoth(a*x)
)), Eq(n, -2)), (-a*x*exp(n*acoth(a*x))/(a**2*c**2*n*x + 2*a**2*c**2*x - a
*c**2*n - 2*a*c**2) - exp(n*acoth(a*x))/(a**2*c**2*n*x + 2*a**2*c**2*x - a
*c**2*n - 2*a*c**2), True))
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^2, x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 13.83 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\frac{e^{n\,\mathrm{acoth}(a\,x)}\,(a\,x+1)}{a\,c^2\,(a\,x-1)\,(n+2)}$$

input
```
int(exp(n*acoth(a*x))/(c - a*c*x)^2,x)
```

output
```
-(exp(n*acoth(a*x))*(a*x + 1))/(a*c^2*(a*x - 1)*(n + 2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{e^{acoth(ax)n}(ax+1)}{a\,c^2\,(anx-2ax-n+2)}$$

input
```
int(exp(n*acoth(a*x))/(-a*c*x+c)^2,x)
```

output
```
(e**(acoth(a*x)*n)*(a*x + 1))/(a*c**2*(a*n*x - 2*a*x - n + 2))
```

3.289 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 104

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^3} \, dx = \frac{\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}} \left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(4+n)} - \frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}} \left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(2+n)(4+n)}$$

output
```
(1-1/a/x)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^3/(4+n)-(3+n)*(1-1/a/x)^(-1-1
/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^3/(2+n)/(4+n)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^3} \, dx = \frac{e^{n \coth^{-1}(ax)}(3+n-ax)\left(\cosh\left(3\coth^{-1}(ax)\right)+\sinh\left(3\coth^{-1}(ax)\right)\right)}{a^2c^3(2+n)(4+n)\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

output
```
(E^(n*ArcCoth[a*x])*(3 + n - a*x)*(Cosh[3*ArcCoth[a*x]] + Sinh[3*ArcCoth[a
*x]]))/(a^2*c^3*(2 + n)*(4 + n)*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6725, 88, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}dx$$

$$\Big\downarrow \text{6725}$$

$$\frac{\int \frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}\left(1+\frac{1}{ax}\right)^{n/2}}{x}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \text{88}$$

$$\frac{\frac{a^2\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+4}-\frac{a(n+3)\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{n/2}d\frac{1}{x}}{n+4}}{a^3c^3}$$

$$\Big\downarrow \text{48}$$

$$\frac{\frac{a^2\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+4}-\frac{a^2(n+3)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{(n+2)(n+4)}}{a^3c^3}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

output
```
((a^2*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(4 + n) - (a^2*(
3 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/((2 + n)*(4 + n
)))/(a^3*c^3)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 5.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{{\mathrm e}^{n \,\operatorname{arccoth}\left(a x \right)} \left(a x -n -3\right) \left(a x +1\right)}{\left(a x -1\right)^{2} c^{3} \left(n^{2}+6 n +8\right) a}$	46
orering	$\frac{\left(a x -n -3\right) \left(a x -1\right) \left(a x +1\right) {\mathrm e}^{n \,\operatorname{arccoth}\left(a x \right)}}{\left(n^{2}+6 n +8\right) a \left(-a c x +c \right)^{3}}$	49
parallelrisch	$\frac{3 \,{\mathrm e}^{n \,\operatorname{arccoth}\left(a x \right)}+{\mathrm e}^{n \,\operatorname{arccoth}\left(a x \right)} n +2 x \,{\mathrm e}^{n \,\operatorname{arccoth}\left(a x \right)} a +x \,{\mathrm e}^{n \,\operatorname{arccoth}\left(a x \right)} a n -x^{2} {\mathrm e}^{n \,\operatorname{arccoth}\left(a x \right)} a^{2}}{c^{3} \left(a x -1\right)^{2} \left(n^{2}+6 n +8\right) a}$	81

input
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-exp(n*arccoth(a*x))*(a*x-n-3)*(a*x+1)/(a*x-1)^2/c^3/(n^2+6*n+8)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.23

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^3} \, dx = -\frac{(a^2x^2 - (an+2\,a)x - n - 3)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^3n^2 + 6\,ac^3n + 8\,ac^3 + (a^3c^3n^2 + 6\,a^3c^3n + 8\,a^3c^3)x^2 - 2\,(a^2c^3n^2 + 6\,a^2c^3n + 8\,a^2c^3)x}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="fricas")
```

output
```
-(a^2*x^2 - (a*n + 2*a)*x - n - 3)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^
2 + 6*a*c^3*n + 8*a*c^3 + (a^3*c^3*n^2 + 6*a^3*c^3*n + 8*a^3*c^3)*x^2 - 2*
(a^2*c^3*n^2 + 6*a^2*c^3*n + 8*a^2*c^3)*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 55.94 (sec) , antiderivative size = 1112, normalized size of antiderivative = 10.69

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^3} \, dx = \text{Too large to display}$$

input
```
integrate(exp(n*acoth(a*x))/(-a*c*x+c)**3,x)
```

output
```
Piecewise((x*exp(I*pi*n/2)/c**3, Eq(a, 0)), (a**2*x**2*acoth(a*x)/(2*a**3*
c**3*x**2*exp(4*acoth(a*x)) - 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*e
xp(4*acoth(a*x))) + 2*a*x*acoth(a*x)/(2*a**3*c**3*x**2*exp(4*acoth(a*x)) -
 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp(4*acoth(a*x))) - a*x/(2*a*
*3*c**3*x**2*exp(4*acoth(a*x)) - 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**
3*exp(4*acoth(a*x))) + acoth(a*x)/(2*a**3*c**3*x**2*exp(4*acoth(a*x)) - 4*
a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp(4*acoth(a*x))) - 1/(2*a**3*c*
*3*x**2*exp(4*acoth(a*x)) - 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp
(4*acoth(a*x))), Eq(n, -4)), (-a**2*x**2*acoth(a*x)/(2*a**3*c**3*x**2*exp(
2*acoth(a*x)) - 4*a**2*c**3*x*exp(2*acoth(a*x)) + 2*a*c**3*exp(2*acoth(a*x
))) + a*x/(2*a**3*c**3*x**2*exp(2*acoth(a*x)) - 4*a**2*c**3*x*exp(2*acoth(
a*x)) + 2*a*c**3*exp(2*acoth(a*x))) + acoth(a*x)/(2*a**3*c**3*x**2*exp(2*a
coth(a*x)) - 4*a**2*c**3*x*exp(2*acoth(a*x)) + 2*a*c**3*exp(2*acoth(a*x)))
 + 1/(2*a**3*c**3*x**2*exp(2*acoth(a*x)) - 4*a**2*c**3*x*exp(2*acoth(a*x))
 + 2*a*c**3*exp(2*acoth(a*x))), Eq(n, -2)), (-a**2*x**2*exp(n*acoth(a*x))/
(a**3*c**3*n**2*x**2 + 6*a**3*c**3*n*x**2 + 8*a**3*c**3*x**2 - 2*a**2*c**3
*n**2*x - 12*a**2*c**3*n*x - 16*a**2*c**3*x + a*c**3*n**2 + 6*a*c**3*n + 8
*a*c**3) + a*n*x*exp(n*acoth(a*x))/(a**3*c**3*n**2*x**2 + 6*a**3*c**3*n*x*
*2 + 8*a**3*c**3*x**2 - 2*a**2*c**3*n**2*x - 12*a**2*c**3*n*x - 16*a**2*c*
*3*x + a*c**3*n**2 + 6*a*c**3*n + 8*a*c**3) + 2*a*x*exp(n*acoth(a*x))/(...
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^3}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="maxima")
```

output
```
-integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^3, x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^3}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="giac")
```

output
```
integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 13.93 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{n+3}{a^3\,c^3\,(n^2+6\,n+8)}-\frac{x^2}{a\,c^3\,(n^2+6\,n+8)}+\frac{x\,(n+2)}{a^2\,c^3\,(n^2+6\,n+8)}\right)}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\left(\frac{1}{a^2}-\frac{2\,x}{a}+x^2\right)}$$

input
```
int(exp(n*acoth(a*x))/(c - a*c*x)^3,x)
```

output
```
(((a*x + 1)/(a*x))^(n/2)*((n + 3)/(a^3*c^3*(6*n + n^2 + 8)) - x^2/(a*c^3*(
6*n + n^2 + 8)) + (x*(n + 2))/(a^2*c^3*(6*n + n^2 + 8))))/(((a*x - 1)/(a*x
))^(n/2)*(1/a^2 - (2*x)/a + x^2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{e^{acoth(ax)n}(-a^2x^2-anx+2ax-n+3)}{a\,c^3\,(a^2n^2x^2-6a^2n\,x^2+8a^2x^2-2a\,n^2x+12anx-16ax+n^2-6n+8)}$$

input
```
int(exp(n*acoth(a*x))/(-a*c*x+c)^3,x)
```

output
```
(e**(acoth(a*x)*n)*( - a**2*x**2 - a*n*x + 2*a*x - n + 3))/(a*c**3*(a**2*n
**2*x**2 - 6*a**2*n*x**2 + 8*a**2*x**2 - 2*a*n**2*x + 12*a*n*x - 16*a*x +
n**2 - 6*n + 8))
```

3.290 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 224

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx &= \frac{(5+n)\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)}\\ &\quad-\frac{(14+8n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(4+n)(6+n)}\\ &\quad-\frac{(14+8n+n^2)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)\left(8+6n+n^2\right)}\\ &\quad-\frac{\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{a^2c^4x}\end{aligned}$$

output

```
(5+n)*(1-1/a/x)^(-3-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^4/(6+n)-(n^2+8*n+14)*(1
-1/a/x)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^4/(4+n)/(6+n)-(n^2+8*n+14)*(1-1
/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^4/(6+n)/(n^2+6*n+8)-(1-1/a/x)^(-3
-1/2*n)*(1+1/a/x)^(1+1/2*n)/a^2/c^4/x
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.37

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\frac{e^{n\coth^{-1}(ax)}\left(-12-8n-n^2+(4+n)^2\cosh\left(2\coth^{-1}(ax)\right)-2(4+n)\sinh\left(2\coth^{-1}(ax)\right)\right)\left(\cosh\left(4\coth^{-1}(ax)\right)\right.}{2ac^4(2+n)(4+n)(6+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

output
```
-1/2*(E^(n*ArcCoth[a*x])*(-12 - 8*n - n^2 + (4 + n)^2*Cosh[2*ArcCoth[a*x]]
 - 2*(4 + n)*Sinh[2*ArcCoth[a*x]])*(Cosh[4*ArcCoth[a*x]] + Sinh[4*ArcCoth[
a*x]]))/(a*c^4*(2 + n)*(4 + n)*(6 + n))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6725, 101, 25, 27, 88, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}dx$$

$$\Big\downarrow \text{6725}$$

$$-\frac{\int \frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-4}\left(1+\frac{1}{ax}\right)^{n/2}}{x^2}d\frac{1}{x}}{a^4c^4}$$

$$\Big\downarrow \text{101}$$

$$-\frac{a^2\int -\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-4}\left(1+\frac{1}{ax}\right)^{n/2}\left(a+\frac{n+4}{x}\right)}{a}d\frac{1}{x}+\frac{a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}}{x}}{a^4c^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{a^2\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{x}-a^2\int\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-4}\left(1+\frac{1}{ax}\right)^{n/2}\left(a+\frac{n+4}{x}\right)}{a}d\frac{1}{x}}{a^4c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{a^2\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{x}-a\int\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-4}\left(1+\frac{1}{ax}\right)^{n/2}\left(a+\frac{n+4}{x}\right)d\frac{1}{x}}{a^4c^4}$$

$$\Big\downarrow \; 88$$

$$-\frac{\frac{a^2\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{x}-a\left(\frac{a^2(n+5)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+6}-\frac{a\left(n^2+8n+14\right)\int\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}\left(1+\frac{1}{ax}\right)^{n/2}d\frac{1}{x}}{n+6}\right)}{a^4c^4}$$

$$\Big\downarrow \; 55$$

$$-\frac{\frac{a^2\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{x}-a\left(\frac{a^2(n+5)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+6}-\frac{a\left(n^2+8n+14\right)\left(\frac{\int\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{n/2}d\frac{1}{x}}{n+4}+\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n-}\right.}{n+6}\right.}{a^4c^4}$$

$$\Big\downarrow \; 48$$

$$-\frac{\frac{a^2\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{x}-a\left(\frac{a^2(n+5)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+6}-\frac{a\left(n^2+8n+14\right)\left(\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}}{n+4}+\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1\right.}{(n+2)(n}\right.}{n+6}\right.}{a^4c^4}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

output
```
-((-(a*(-((a*(14 + 8*n + n^2)*((a*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^(
(2 + n)/2))/(4 + n) + (a*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2
))/((2 + n)*(4 + n))))/(6 + n)) + (a^2*(5 + n)*(1 - 1/(a*x))^(-3 - n/2)*(1
 + 1/(a*x))^((2 + n)/2))/(6 + n))) + (a^2*(1 - 1/(a*x))^(-3 - n/2)*(1 + 1/
(a*x))^((2 + n)/2))/x)/(a^4*c^4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 13.99 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.30

method	result
gosper	$-\frac{(ax+1)(2a^2x^2-2nax-8ax+n^2+8n+14)e^{n\ \mathrm{arccoth}(ax)}}{(ax-1)^3c^4a(n^2+8n+12)(4+n)}$
orering	$-\frac{(2a^2x^2-2nax-8ax+n^2+8n+14)(ax-1)(ax+1)e^{n\ \mathrm{arccoth}(ax)}}{a(n^3+12n^2+44n+48)(-acx+c)^4}$
parallelrisch	$\frac{-14\,e^{n\ \mathrm{arccoth}(ax)}+2x^2e^{n\ \mathrm{arccoth}(ax)}a^2n-x\,e^{n\ \mathrm{arccoth}(ax)}a\,n^2-2a^3e^{n\ \mathrm{arccoth}(ax)}x^3-8\,e^{n\ \mathrm{arccoth}(ax)}n-6x\,e^{n\ \mathrm{arccoth}(ax)}a-}{c^4(ax-1)^3a(n^3+12n^2+44n+48)}$

input
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-(a*x+1)*(2*a^2*x^2-2*a*n*x-8*a*x+n^2+8*n+14)*exp(n*arccoth(a*x))/(a*x-1)^
3/c^4/a/(n^2+8*n+12)/(4+n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.02

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx$$
$$= \frac{(2\,a^3x^3 - 2\,(a^2n + 3\,a^2)x^2 + n^2 + (an^2 + 6\,an + 6\,a)}{ac^4n^3 + 12\,ac^4n^2 + 44\,ac^4n + 48\,ac^4 - (a^4c^4n^3 + 12\,a^4c^4n^2 + 44\,a^4c^4n + 48\,a^4c^4)x^3 + 3\,(a^3c^4n^3 + 12\,a^3c}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="fricas")
```

output
```
(2*a^3*x^3 - 2*(a^2*n + 3*a^2)*x^2 + n^2 + (a*n^2 + 6*a*n + 6*a)*x + 8*n +
 14)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^4*n^3 + 12*a*c^4*n^2 + 44*a*c^4*n
+ 48*a*c^4 - (a^4*c^4*n^3 + 12*a^4*c^4*n^2 + 44*a^4*c^4*n + 48*a^4*c^4)*x^
3 + 3*(a^3*c^4*n^3 + 12*a^3*c^4*n^2 + 44*a^3*c^4*n + 48*a^3*c^4)*x^2 - 3*(
a^2*c^4*n^3 + 12*a^2*c^4*n^2 + 44*a^2*c^4*n + 48*a^2*c^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^4} \, dx = \text{Timed out}$$

input
```
integrate(exp(n*acoth(a*x))/(-a*c*x+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^4} \, dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^4} \, dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^4, x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^4} \, dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^4} \, dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 13.95 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.80

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^4} \, dx = -\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2} \left(\frac{2\,x^3}{a\,c^4\,(n^3+12\,n^2+44\,n+48)} + \frac{n^2+8\,n+14}{a^4\,c^4\,(n^3+12\,n^2+44\,n+48)} - \frac{x^2\,(2\,n+6)}{a^2\,c^4\,(n^3+12\,n^2+44\,n+48)} + \frac{x\,(n^2+6\,n+6)}{a^3\,c^4\,(n^3+12\,n^2+44\,n+48)}\right)}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2} \left(\frac{3\,x}{a^2} - \frac{1}{a^3} + x^3 - \frac{3\,x^2}{a}\right)}$$

input
```
int(exp(n*acoth(a*x))/(c - a*c*x)^4,x)
```

output
```
-(((a*x + 1)/(a*x))^(n/2)*((2*x^3)/(a*c^4*(44*n + 12*n^2 + n^3 + 48)) + (8
*n + n^2 + 14)/(a^4*c^4*(44*n + 12*n^2 + n^3 + 48)) - (x^2*(2*n + 6))/(a^2
*c^4*(44*n + 12*n^2 + n^3 + 48)) + (x*(6*n + n^2 + 6))/(a^3*c^4*(44*n + 12
*n^2 + n^3 + 48))))/(((a*x - 1)/(a*x))^(n/2)*((3*x)/a^2 - 1/a^3 + x^3 - (3
*x^2)/a))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.80

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx$$
$$= \frac{e^{acoth(ax)n}(2a^3x^3+2a^2n\,x^2-6a^2x^2+a\,n^2x-6anx+6ax+n}{a\,c^4\,(a^3n^3x^3-12a^3n^2x^3+44a^3n\,x^3-3a^2n^3x^2-48a^3x^3+36a^2n^2x^2-132a^2n\,x^2+3a\,n^3x+144a^2x^2-}$$

input
```
int(exp(n*acoth(a*x))/(-a*c*x+c)^4,x)
```

output
```
(e**(acoth(a*x)*n)*(2*a**3*x**3 + 2*a**2*n*x**2 - 6*a**2*x**2 + a*n**2*x -
 6*a*n*x + 6*a*x + n**2 - 8*n + 14))/(a*c**4*(a**3*n**3*x**3 - 12*a**3*n**
2*x**3 + 44*a**3*n*x**3 - 48*a**3*x**3 - 3*a**2*n**3*x**2 + 36*a**2*n**2*x
**2 - 132*a**2*n*x**2 + 144*a**2*x**2 + 3*a*n**3*x - 36*a*n**2*x + 132*a*n
*x - 144*a*x - n**3 + 12*n**2 - 44*n + 48))
```

3.291 $\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2}{7}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-5+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{5/2}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},\frac{1}{2}(-5+n),-\frac{5}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

output
```
2/7*((a-1/x)/(a+1/x))^(-5/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(5/2)*
hypergeom([-7/2, -5/2+1/2*n],[-5/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{2c^2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1}{2}(-1+n)}(1+ax)^3\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},\frac{1}{2}\right.}{7a}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

output
```
(2*c^2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)^3
*Sqrt[c - a*c*x]*Hypergeometric2F1[-7/2, (-5 + n)/2, -5/2, 2/(1 + a*x)])/(
7*a*(1 - 1/(a*x))^(n/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6727, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{5/2}e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{ 142}$$

$$\frac{2}{7}x(c-acx)^{5/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-5}{2}}\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}-\frac{5}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(-\frac{7}{2},\frac{n-5}{2},-\frac{5}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

output
```
(2*((a - x^(-1))/(a + x^(-1)))^((-5 + n)/2)*(1 - 1/(a*x))^(-5/2 + (5 - n)/
2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c - a*c*x)^(5/2)*Hypergeometric2F1[-7/2, (
-5 + n)/2, -5/2, 2/((a + x^(-1))*x)])/7
```

Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(ax)}(-acx+c)^{\frac{5}{2}}\,dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\int(-acx+c)^{\frac{5}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*sqrt(-a*c*x + c)*((a*x + 1)/(a*x
- 1))^(1/2*n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(5/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \int (-acx+c)^{\frac{5}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^(5/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \int (-acx+c)^{\frac{5}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^(5/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^{5/2}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(5/2),x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(5/2), x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\sqrt{c}\,c^2\left(\left(\int e^{acoth(ax)n}\sqrt{-ax+1}\,x^2dx\right)a^2-2\left(\int e^{acoth(ax)n}\sqrt{-ax+1}\,xdx\right)a+\int e^{acoth(ax)n}\sqrt{-ax+1}dx\right)$$

input
```
int(exp(n*acoth(a*x))*(-a*c*x+c)^(5/2),x)
```

output
```
sqrt(c)*c**2*(int(e**(acoth(a*x)*n)*sqrt( - a*x + 1)*x**2,x)*a**2 - 2*int(
e**(acoth(a*x)*n)*sqrt( - a*x + 1)*x,x)*a + int(e**(acoth(a*x)*n)*sqrt( -
a*x + 1),x))
```

3.292 $\int e^{n \coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{2}{5}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-3+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}(-3+n),-\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

output
```
2/5*((a-1/x)/(a+1/x))^(-3/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(3/2)*
hypergeom([-5/2, -3/2+1/2*n],[-3/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.03

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{3/2}\,dx=-\frac{2c\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1}{2}(-1+n)}(1+ax)^2\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}(-3+n),-\frac{3}{2},\right.}{5a}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

output
```
(-2*c*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)^2*
Sqrt[c - a*c*x]*Hypergeometric2F1[-5/2, (-3 + n)/2, -3/2, 2/(1 + a*x)])/(5
*a*(1 - 1/(a*x))^(n/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6727, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{3/2}e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{142}$$

$$\frac{2}{5}x(c-acx)^{3/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}-\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{n-3}{2},-\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

output
```
(2*((a - x^(-1))/(a + x^(-1)))^((-3 + n)/2)*(1 - 1/(a*x))^(-3/2 + (3 - n)/
2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c - a*c*x)^(3/2)*Hypergeometric2F1[-5/2, (
-3 + n)/2, -3/2, 2/((a + x^(-1))*x)])/5
```

Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \mathrm{e}^{n\ \mathrm{arccoth}(ax)}(-acx+c)^{\frac{3}{2}}\,dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\int(-acx+c)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(a*c*x - c)*sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\int(-c(ax-1))^{\frac{3}{2}}\,e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(3/2),x)
```

output
```
Integral((-c*(a*x - 1))**(3/2)*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\int(-acx+c)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\int(-acx+c)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^{3/2}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(3/2),x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(3/2), x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\sqrt{c}\,c\left(-\left(\int e^{acoth(ax)n}\sqrt{-ax+1}\,xdx\right)a+\int e^{acoth(ax)n}\sqrt{-ax+1}dx\right)$$

input
```
int(exp(n*acoth(a*x))*(-a*c*x+c)^(3/2),x)
```

output
```
sqrt(c)*c*( - int(e**(acoth(a*x)*n)*sqrt( - a*x + 1)*x,x)*a + int(e**(acot
h(a*x)*n)*sqrt( - a*x + 1),x))
```

3.293 $\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2}{3}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-1+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-1+n),-\frac{1}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

output
```
2/3*((a-1/x)/(a+1/x))^(-1/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(1/2)*
hypergeom([-3/2, -1/2+1/2*n],[-1/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1}{2}(-1+n)}(1+ax)\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-1+n),-\frac{1}{2},\frac{1}{\cdots}\right)}{3a}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
(2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)*Sqrt[
c - a*c*x]*Hypergeometric2F1[-3/2, (-1 + n)/2, -1/2, 2/(1 + a*x)])/(3*a*(1
 - 1/(a*x))^(n/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6727, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6727}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 142}$$

$$\frac{2}{3}x\sqrt{c-acx}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}-\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{n-1}{2},-\frac{1}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

input
```
Int[E^(n*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
(2*((a - x^(-1))/(a + x^(-1)))^((-1 + n)/2)*(1 - 1/(a*x))^(-1/2 + (1 - n)/
2)*(1 + 1/(a*x))^((2 + n)/2)*x*Sqrt[c - a*c*x]*Hypergeometric2F1[-3/2, (-1
 + n)/2, -1/2, 2/((a + x^(-1))*x)])/3
```

Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] && !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[p]
```

Maple [F]

$$\int e^{n\ \text{arccoth}(ax)}\sqrt{-acx+c}dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\sqrt{-c\,(ax-1)}e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\sqrt{c-a\,c\,x}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(1/2),x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(1/2), x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\sqrt{c}\left(\int e^{acoth(ax)n}\sqrt{-ax+1}dx\right)$$

input
```
int(exp(n*acoth(a*x))*(-a*c*x+c)^(1/2),x)
```

output
```
sqrt(c)*int(e**(acoth(a*x)*n)*sqrt( - a*x + 1),x)
```

3.294 $\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+n}{2},\frac{1}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{\sqrt{c-acx}}$$

output
```
2*((a-1/x)/(a+1/x))^(1/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*hypergeom([-1/2, 1/2
+1/2*n],[1/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))/(-a*c*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1+n}{2}}(1+ax)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+n}{2},\frac{1}{2},\frac{2}{1+ax}\right)}{a\sqrt{c-acx}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

output
```
(2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((1 + n)/2)*(1 + a*x)*Hyperg
eometric2F1[-1/2, (1 + n)/2, 1/2, 2/(1 + a*x)])/(a*(1 - 1/(a*x))^(n/2)*Sqr
t[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6727, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \; 6727$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{n/2}}{\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$\Big\downarrow \; 142$$

$$\frac{2x\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{n+1}{2},\frac{1}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{\sqrt{c-acx}}$$

input
```
Int[E^(n*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

output
```
(2*((a - x^(-1))/(a + x^(-1)))^((1 + n)/2)*(1 - 1/(a*x))^(1/2 + (-1 - n)/2
)*(1 + 1/(a*x))^((2 + n)/2)*x*Hypergeometric2F1[-1/2, (1 + n)/2, 1/2, 2/((
a + x^(-1))*x)])/Sqrt[c - a*c*x]
```

Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{\sqrt{-acx+c}}dx$$

input
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x)
```

output
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{-acx+c}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\sqrt{-c\,(ax-1)}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(1/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/sqrt(-c*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{-acx+c}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a*c*x + c), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{-acx+c}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a*c*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-a\,c\,x}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - a*c*x)^(1/2),x)
```

output
```
int(exp(n*acoth(a*x))/(c - a*c*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx=\frac{\int \frac{e^{acoth(ax)n}}{\sqrt{-ax+1}}dx}{\sqrt{c}}$$

input
```
int(exp(n*acoth(a*x))/(-a*c*x+c)^(1/2),x)
```

output
```
int(e**(acoth(a*x)*n)/sqrt( - a*x + 1),x)/sqrt(c)
```

3.295 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(c-acx)^{3/2}}$$

output
```
-2*((a-1/x)/(a+1/x))^(3/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*hypergeom([1/2, 3/2
+1/2*n],[3/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))/(-a*c*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1+n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{3}{2},\frac{2}{1+ax}\right)}{ac\sqrt{c-acx}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

output
```
(2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((1 + n)/2)*Hypergeometric2F
1[1/2, (3 + n)/2, 3/2, 2/(1 + a*x)])/(a*c*(1 - 1/(a*x))^(n/2)*Sqrt[c - a*c
*x])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6727, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{n/2}}{\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$\Big\downarrow \text{142}$$

$$-\frac{2x\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)+\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{3}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(c-acx)^{3/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

output
```
(-2*((a - x^(-1))/(a + x^(-1)))^((3 + n)/2)*(1 - 1/(a*x))^(3/2 + (-3 - n)/
2)*(1 + 1/(a*x))^((2 + n)/2)*x*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/((
a + x^(-1))*x)])/(c - a*c*x)^(3/2)
```

Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] && !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[p]
```

Maple [F]

$$\int \frac{\mathrm{e}^{n \operatorname{arccoth}(ax)}}{(-acx+c)^{\frac{3}{2}}}dx$$

input
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x)
```

output
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*x^2 - 2*a
*c^2*x + c^2), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{e^{n\operatorname{acoth}(ax)}}{(-c(ax-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(3/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(-c*(a*x - 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{(c-a\,c\,x)^{3/2}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - a*c*x)^(3/2),x)
```

output
```
int(exp(n*acoth(a*x))/(c - a*c*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{\int \frac{e^{acoth(ax)n}}{\sqrt{-ax+1}\,ax-\sqrt{-ax+1}}dx}{\sqrt{c}\,c}$$

input
```
int(exp(n*acoth(a*x))/(-a*c*x+c)^(3/2),x)
```

output
```
( - int(e**(acoth(a*x)*n)/(sqrt( - a*x + 1)*a*x - sqrt( - a*x + 1)),x))/(s
qrt(c)*c)
```

3.296 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = -\frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{5+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5+n}{2},\frac{5}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{3(c-acx)^{5/2}}$$

output
```
-2/3*((a-1/x)/(a+1/x))^(5/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*hypergeom([3/2, 5
/2+1/2*n],[5/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))/(-a*c*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.19

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(-1-ax+(-1+ax)\left(\frac{-1+ax}{1+ax}\right)^{\frac{1+n}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{ac^2(3+n)(-1+ax)\sqrt{c-acx}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

output
```
((1 + 1/(a*x))^(n/2)*(-1 - a*x + (-1 + a*x)*((-1 + a*x)/(1 + a*x))^((1 + n
)/2)*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/(1 + a*x)]))/(a*c^2*(3 + n)*
(1 - 1/(a*x))^(n/2)*(-1 + a*x)*Sqrt[c - a*c*x])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6727, 105, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(1+\frac{1}{ax}\right)^{n/2}\sqrt{\frac{1}{x}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{a\sqrt{\frac{1}{x}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+3}-\frac{a\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{n/2}}{\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2(n+3)}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$\Big\downarrow \text{142}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{a\sqrt{\frac{1}{x}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+3}-\frac{a\sqrt{\frac{1}{x}}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{3}{2},\frac{}{(a\cdots}\right.}{n+3}\right.}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

output
```
-(((1 - 1/(a*x))^(5/2)*((a*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((2 +
n)/2)*Sqrt[x^(-1)])/(3 + n) - (a*((a - x^(-1))/(a + x^(-1)))^((3 + n)/2)*(
1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*Sqrt[x^(-1)]*Hypergeom
etric2F1[1/2, (3 + n)/2, 3/2, 2/((a + x^(-1))*x)])/(3 + n)))/((x^(-1))^(5/
2)*(c - a*c*x)^(5/2)))
```

Defintions of rubi rules used

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{(-acx+c)^{\frac{5}{2}}}dx$$

input
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x)
```

output
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^3*c^3*x^3 - 3*
a^2*c^3*x^2 + 3*a*c^3*x - c^3), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(5/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(-c*(a*x - 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{(c-a\,c\,x)^{5/2}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - a*c*x)^(5/2),x)
```

output
```
int(exp(n*acoth(a*x))/(c - a*c*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx=\frac{\int \frac{e^{acoth(ax)n}}{\sqrt{-ax+1}\,a^2x^2-2\sqrt{-ax+1}\,ax+\sqrt{-ax+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(exp(n*acoth(a*x))/(-a*c*x+c)^(5/2),x)
```

output
```
int(e**(acoth(a*x)*n)/(sqrt( - a*x + 1)*a**2*x**2 - 2*sqrt( - a*x + 1)*a*x
 + sqrt( - a*x + 1)),x)/(sqrt(c)*c**2)
```

3.297 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{7+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{7+n}{2},\frac{7}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{5(c-acx)^{7/2}}$$

output
```
-2/5*((a-1/x)/(a+1/x))^(7/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*hypergeom([5/2, 7
/2+1/2*n],[7/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))/(-a*c*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.41

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left((9+2n-3ax)(1+ax)+3(-1+ax)^2\left(\frac{-1+ax}{1+ax}\right)^{\frac{1+n}{2}}\text{Hyperg}\right.}{2ac^3(3+n)(5+n)(-1+ax)^2\sqrt{c-acx}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

output
```
((1 + 1/(a*x))^(n/2)*((9 + 2*n - 3*a*x)*(1 + a*x) + 3*(-1 + a*x)^2*((-1 +
a*x)/(1 + a*x))^((1 + n)/2)*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/(1 +
a*x)]))/(2*a*c^3*(3 + n)*(5 + n)*(1 - 1/(a*x))^(n/2)*(-1 + a*x)^2*Sqrt[c -
 a*c*x])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 249 vs. $2(98) = 196$.

Time = 0.61 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.54, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6727, 105, 105, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-7)}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{3/2}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{a\left(\frac{1}{x}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+5}-\frac{3a\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(1+\frac{1}{ax}\right)^{n/2}\sqrt{\frac{1}{x}}d\frac{1}{x}}{2(n+5)}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{a\left(\frac{1}{x}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+5}-\frac{3a\left(\frac{a\sqrt{\frac{1}{x}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+3}-\frac{a\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{n/2}}{\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2(n+3)}\right)}{2(n+5)}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$\Big\downarrow \text{142}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{a\left(\frac{1}{x}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+5}-\frac{3a\left(\frac{a\sqrt{\frac{1}{x}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+3}-\frac{a\sqrt{\frac{1}{x}}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(\frac{1}{ax}+\cdots\right.}{\cdots}\right.}{2(n+5)}\right.}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

output
```
-(((1 - 1/(a*x))^(7/2)*((a*(1 - 1/(a*x))^((-5 - n)/2)*(1 + 1/(a*x))^((2 +
n)/2)*(x^(-1))^(3/2))/(5 + n) - (3*a*((a*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1
/(a*x))^((2 + n)/2)*Sqrt[x^(-1)])/(3 + n) - (a*((a - x^(-1))/(a + x^(-1)))
^((3 + n)/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*Sqrt[x^(
-1)]*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/((a + x^(-1))*x)])/(3 + n)))
/(2*(5 + n))))/((x^(-1))^(7/2)*(c - a*c*x)^(7/2)))
```

Defintions of rubi rules used

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{(-acx+c)^{\frac{7}{2}}}dx$$

input
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x)
```

output
```
int(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^4*x^4 - 4*a
^3*c^4*x^3 + 6*a^2*c^4*x^2 - 4*a*c^4*x + c^4), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(7/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx=\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{{(c-a\,c\,x)}^{7/2}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - a*c*x)^(7/2),x)
```

output
```
int(exp(n*acoth(a*x))/(c - a*c*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = -\frac{\int \frac{e^{acoth(ax)n}}{\sqrt{-ax+1}\,a^3x^3-3\sqrt{-ax+1}\,a^2x^2+3\sqrt{-ax+1}\,ax-\sqrt{-ax+1}}dx}{\sqrt{c}\,c^3}$$

input
```
int(exp(n*acoth(a*x))/(-a*c*x+c)^(7/2),x)
```

output
```
( - int(e**(acoth(a*x)*n)/(sqrt( - a*x + 1)*a**3*x**3 - 3*sqrt( - a*x + 1)
*a**2*x**2 + 3*sqrt( - a*x + 1)*a*x - sqrt( - a*x + 1)),x))/(sqrt(c)*c**3)
```

3.298 $\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 278

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx &= -\frac{(56+14n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{4+n}{2}}}{a(4+n)(6+n)}\\&\quad+\frac{2(56+14n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{4+n}{2}}}{a^2(6+n)(8+6n+n^2)\,x}\\&\quad+\frac{(8+n)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{6+n}\\&\quad-\frac{\left(a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{a}\end{aligned}$$

output
```
-(n^2+14*n+56)*(1-1/a/x)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)*(-a*c*x+c)^(2+1/2*
n)/a/(4+n)/(6+n)+2*(n^2+14*n+56)*(1-1/a/x)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)*
(-a*c*x+c)^(2+1/2*n)/a^2/(6+n)/(n^2+6*n+8)/x+(8+n)*(1-1/a/x)^(-2-1/2*n)*(1
+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(2+1/2*n)/(6+n)-(a-1/x)*(1-1/a/x)^(-2-1/2*n
)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(2+1/2*n)/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.42

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx = \frac{2c^2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(1+ax)(c-acx)^{n/2}\left(n^2(-1+ax)^2+8(7-4ax+a^2x^2)+2n(7-10ax+\right.}{a(2+n)(4+n)(6+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(2 + n/2),x]
```

output
```
(2*c^2*(1 + 1/(a*x))^(n/2)*(1 + a*x)*(c - a*c*x)^(n/2)*(n^2*(-1 + a*x)^2 +
 8*(7 - 4*a*x + a^2*x^2) + 2*n*(7 - 10*a*x + 3*a^2*x^2)))/(a*(2 + n)*(4 +
n)*(6 + n)*(1 - 1/(a*x))^(n/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6727, 27, 101, 27, 88, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{\frac{n}{2}+2}e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$\left(\frac{1}{x}\right)^{\frac{n+4}{2}}\left(-\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\right)(c-acx)^{\frac{n+4}{2}}\int \frac{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-4}}{a^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+4}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}\int \left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-4}d\frac{1}{x}}{a^2}$$

$$\Big\downarrow \text{101}$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+4}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}\left(a\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}-a\int -\frac{1}{2}\left(1+\frac{1}{ax}\right)^{n/2}\left(a(n+8)-\frac{n+4}{x}\right)\left(\frac{1}{x}\right.\right.}{a^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+4}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}\left(\frac{1}{2}a\int\left(1+\frac{1}{ax}\right)^{n/2}\left(a(n+8)-\frac{n+4}{x}\right)\left(\frac{1}{x}\right)^{-\frac{n}{2}-4}d\frac{1}{x}+a\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}\right.\right.}{a^2}$$

$$\Big\downarrow \; 88$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+4}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}\left(\frac{1}{2}a\left(-\frac{(n^2+14n+56)\int\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-3}d\frac{1}{x}}{n+6}-\frac{2a(n+8)\left(\frac{1}{x}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+6}\right)+a\right.}{a^2}$$

$$\Big\downarrow \; 55$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+4}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}\left(\frac{1}{2}a\left(-\frac{(n^2+14n+56)\left(-\frac{2\int\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-2}d\frac{1}{x}}{a(n+4)}-\frac{2\left(\frac{1}{x}\right)^{-\frac{n}{2}-2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+4}\right)}{n+6}-\frac{2a(n\right.\right.}{a^2}$$

$$\Big\downarrow \; 48$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+4}{2}}\left(\frac{1}{2}a\left(-\frac{(n^2+14n+56)\left(\frac{4\left(\frac{1}{x}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{a(n+2)(n+4)}-\frac{2\left(\frac{1}{x}\right)^{-\frac{n}{2}-2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+4}\right)}{n+6}-\frac{2a(n+8)\left(\frac{1}{x}\right)^{-\frac{n}{2}-3}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+6}\right)+a\left(a-\right.\right.}{a^2}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(2 + n/2),x]
```

output
```
-((((a*(-(((56 + 14*n + n^2)*((-2*(1 + 1/(a*x))^((2 + n)/2)*(x^(-1))^(-2 -
 n/2))/(4 + n) + (4*(1 + 1/(a*x))^((2 + n)/2)*(x^(-1))^(-1 - n/2))/(a*(2 +
 n)*(4 + n))))/(6 + n)) - (2*a*(8 + n)*(1 + 1/(a*x))^((2 + n)/2)*(x^(-1))^
(-3 - n/2))/(6 + n)))/2 + a*(a - x^(-1))*(1 + 1/(a*x))^((2 + n)/2)*(x^(-1)
)^(-3 - n/2))*(1 - 1/(a*x))^(-2 - n/2)*(x^(-1))^((4 + n)/2)*(c - a*c*x)^((
4 + n)/2))/a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.37

method	result	size
gosper	$\frac{2(ax+1)\left(a^{2}n^{2}x^{2}+6n\,x^{2}a^{2}+8a^{2}x^{2}-2n^{2}xa-20nax-32ax+n^{2}+14n+56\right)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}(-acx+c)^{2+\frac{n}{2}}}{(ax-1)^{2}a\left(n^{3}+12n^{2}+44n+48\right)}$	104
orering	$\frac{2(ax+1)\left(a^{2}n^{2}x^{2}+6n\,x^{2}a^{2}+8a^{2}x^{2}-2n^{2}xa-20nax-32ax+n^{2}+14n+56\right)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}(-acx+c)^{2+\frac{n}{2}}}{(ax-1)^{2}a\left(n^{3}+12n^{2}+44n+48\right)}$	104

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x,method=_RETURNVERBOSE)
```

output
```
2*(a*x+1)*(a^2*n^2*x^2+6*a^2*n*x^2+8*a^2*x^2-2*a*n^2*x-20*a*n*x-32*a*x+n^2
+14*n+56)*exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n)/(a*x-1)^2/a/(n^3+12*n^2
+44*n+48)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.67

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx$$
$$=\frac{2\left((a^3n^2+6\,a^3n+8\,a^3)x^3-(a^2n^2+14\,a^2n+24\,a^2)x^2+n^2-(an^2+6\,an-24\,a)x+14\,n+56\right)(-acx}{an^3+12\,an^2+(a^3n^3+12\,a^3n^2+44\,a^3n+48\,a^3)x^2+44\,an-2\,(a^2n^3+12\,a^2n^2+44\,a^2n+48}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="fricas")
```

output
```
2*((a^3*n^2 + 6*a^3*n + 8*a^3)*x^3 - (a^2*n^2 + 14*a^2*n + 24*a^2)*x^2 + n
^2 - (a*n^2 + 6*a*n - 24*a)*x + 14*n + 56)*(-a*c*x + c)^(1/2*n + 2)*((a*x
+ 1)/(a*x - 1))^(1/2*n)/(a*n^3 + 12*a*n^2 + (a^3*n^3 + 12*a^3*n^2 + 44*a^3
*n + 48*a^3)*x^2 + 44*a*n - 2*(a^2*n^3 + 12*a^2*n^2 + 44*a^2*n + 48*a^2)*x
 + 48*a)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx=\int(-c(ax-1))^{\frac{n}{2}+2}e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(2+1/2*n),x)
```

output
```
Integral((-c*(a*x - 1))**(n/2 + 2)*exp(n*acoth(a*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.44

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx$$
$$=\frac{2\left((n^2+6n+8)a^3(-c)^{\frac{1}{2}n}c^2x^3-(n^2+14n+24)a^2(-c)^{\frac{1}{2}n}c^2x^2-(n^2+6n-24)a(-c)^{\frac{1}{2}n}c^2x+(n^2+\right.}{(n^3+12n^2+44n+48)a}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="maxima")
```

output
```
2*((n^2 + 6*n + 8)*a^3*(-c)^(1/2*n)*c^2*x^3 - (n^2 + 14*n + 24)*a^2*(-c)^(
1/2*n)*c^2*x^2 - (n^2 + 6*n - 24)*a*(-c)^(1/2*n)*c^2*x + (n^2 + 14*n + 56)
*(-c)^(1/2*n)*c^2)*(a*x + 1)^(1/2*n)/((n^3 + 12*n^2 + 44*n + 48)*a)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx=\int(-acx+c)^{\frac{1}{2}n+2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^(1/2*n + 2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.80

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx$$
$$=\frac{\left(\frac{ax+1}{ax}\right)^{n/2}\left(\frac{x^3\,(c-a\,c\,x)^{\frac{n}{2}+2}\,(2\,n^2+12\,n+16)}{n^3+12\,n^2+44\,n+48}+\frac{(c-a\,c\,x)^{\frac{n}{2}+2}\,(2\,n^2+28\,n+112)}{a^3\,(n^3+12\,n^2+44\,n+48)}-\frac{2\,x\,(c-a\,c\,x)^{\frac{n}{2}+2}\,(n^2+6\,n-24)}{a^2\,(n^3+12\,n^2+44\,n+48)}-\frac{x^2\,(c-a\,c\,x)^{\frac{n}{2}+2}}{a\,(n^3+12\,n}\right.}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\,\left(\frac{1}{a^2}-\frac{2\,x}{a}+x^2\right)}$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 + 2),x)
```

output
```
(((a*x + 1)/(a*x))^(n/2)*((x^3*(c - a*c*x)^(n/2 + 2)*(12*n + 2*n^2 + 16))/
(44*n + 12*n^2 + n^3 + 48) + ((c - a*c*x)^(n/2 + 2)*(28*n + 2*n^2 + 112))/
(a^3*(44*n + 12*n^2 + n^3 + 48)) - (2*x*(c - a*c*x)^(n/2 + 2)*(6*n + n^2 -
 24))/(a^2*(44*n + 12*n^2 + n^3 + 48)) - (x^2*(c - a*c*x)^(n/2 + 2)*(28*n
+ 2*n^2 + 48))/(a*(44*n + 12*n^2 + n^3 + 48))))/(((a*x - 1)/(a*x))^(n/2)*(
1/a^2 - (2*x)/a + x^2))
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx=c^2\left(\left(\int e^{acoth(ax)n}(-acx+c)^{\frac{n}{2}}\,x^2dx\right)a^2-2\left(\int e^{acoth(ax)n}(-acx+c)^{\frac{n}{2}}\,xdx\right)a+\int e^{acoth(ax)n}(-acx+c)^{\frac{n}{2}}\,dx\right)$$

input
```
int(exp(n*acoth(a*x))*(-a*c*x+c)^(2+1/2*n),x)
```

output
```
c**2*(int(e**(acoth(a*x)*n)*( - a*c*x + c)**(n/2)*x**2,x)*a**2 - 2*int(e**
(acoth(a*x)*n)*( - a*c*x + c)**(n/2)*x,x)*a + int(e**(acoth(a*x)*n)*( - a*
c*x + c)**(n/2),x))
```

3.299 $\int e^{n \coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 127

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx = -\frac{2(6+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{2+n}{2}}}{a(2+n)(4+n)} + \frac{2\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{2+n}{2}}}{4+n}$$

output
```
-2*(6+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)*(-a*c*x+c)^(1+1/2*n)/a/(
2+n)/(4+n)+2*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(1+1/2*
n)/(4+n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.61

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx = -\frac{2c\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(1+ax)(c-acx)^{n/2}(-6+2ax+n(-1+ax))}{a(2+n)(4+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(1 + n/2),x]
```

output
```
(-2*c*(1 + 1/(a*x))^(n/2)*(1 + a*x)*(c - a*c*x)^(n/2)*(-6 + 2*a*x + n*(-1
+ a*x)))/(a*(2 + n)*(4 + n)*(1 - 1/(a*x))^(n/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6727, 27, 88, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{\frac{n}{2}+1}e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6727}$$

$$\left(\frac{1}{x}\right)^{\frac{n+2}{2}}\left(-\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\right)(c-acx)^{\frac{n+2}{2}}\int \frac{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-3}}{a}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}(c-acx)^{\frac{n+2}{2}}\int \left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-3}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 88}$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}(c-acx)^{\frac{n+2}{2}}\left(-\frac{(n+6)\int \left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-2}d\frac{1}{x}}{n+4}-\frac{2a\left(\frac{1}{x}\right)^{-\frac{n}{2}-2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+4}\right)}{a}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{\left(\frac{1}{x}\right)^{\frac{n+2}{2}}\left(\frac{2(n+6)\left(\frac{1}{x}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{(n+2)(n+4)}-\frac{2a\left(\frac{1}{x}\right)^{-\frac{n}{2}-2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+4}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}(c-acx)^{\frac{n+2}{2}}}{a}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(1 + n/2),x]
```

output
```
-((((-2*a*(1 + 1/(a*x))^((2 + n)/2)*(x^(-1))^(-2 - n/2))/(4 + n) + (2*(6 +
 n)*(1 + 1/(a*x))^((2 + n)/2)*(x^(-1))^(-1 - n/2))/((2 + n)*(4 + n)))*(1 -
 1/(a*x))^(-1 - n/2)*(x^(-1))^((2 + n)/2)*(c - a*c*x)^((2 + n)/2))/a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.48

method	result
gosper	$\frac{2(-acx+c)^{1+\frac{n}{2}} \mathrm{e}^{n \,\mathrm{arccoth}(ax)} (nax+2ax-n-6)(ax+1)}{(ax-1)a(n^2+6n+8)}$
orering	$\frac{2(-acx+c)^{1+\frac{n}{2}} \mathrm{e}^{n \,\mathrm{arccoth}(ax)} (nax+2ax-n-6)(ax+1)}{(ax-1)a(n^2+6n+8)}$
parallelrisch	$-\frac{12 \,\mathrm{e}^{n \,\mathrm{arccoth}(ax)} (-acx+c)^{1+\frac{n}{2}} -2x^2(-acx+c)^{1+\frac{n}{2}} \mathrm{e}^{n \,\mathrm{arccoth}(ax)} a^2 n-4x^2(-acx+c)^{1+\frac{n}{2}} \mathrm{e}^{n \,\mathrm{arccoth}(ax)} a^2+8 \,\mathrm{e}^{n \,\mathrm{arccoth}(a}}{(ax-1)a(n^2+6n+8)}$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x,method=_RETURNVERBOSE)
```

output
```
2*(-a*c*x+c)^(1+1/2*n)*exp(n*arccoth(a*x))*(a*n*x+2*a*x-n-6)*(a*x+1)/(a*x-
1)/a/(n^2+6*n+8)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.73

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx$$
$$= -\frac{2\,((a^2n+2\,a^2)x^2-4\,ax-n-6)(-acx+c)^{\frac{1}{2}\,n+1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{an^2+6\,an-(a^2n^2+6\,a^2n+8\,a^2)x+8\,a}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="fricas")
```

output
```
-2*((a^2*n + 2*a^2)*x^2 - 4*a*x - n - 6)*(-a*c*x + c)^(1/2*n + 1)*((a*x +
1)/(a*x - 1))^(1/2*n)/(a*n^2 + 6*a*n - (a^2*n^2 + 6*a^2*n + 8*a^2)*x + 8*a
)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx$$
$$= \begin{cases} c^{\frac{n}{2}+1}xe^{\frac{i\pi n}{2}} \\ 0^{\frac{n}{2}+1}xe^{\infty n} \\ -\frac{\int \frac{1}{axe^{4\operatorname{acoth}(ax)}-e^{4\operatorname{acoth}(ax)}}\,dx}{c} \\ \int e^{-2\operatorname{acoth}(ax)}\,dx \\ \frac{2a^2nx^2(-acx+c)^{\frac{n}{2}+1}e^{n\operatorname{acoth}(ax)}}{a^2n^2x+6a^2nx+8a^2x-an^2-6an-8a} + \frac{4a^2x^2(-acx+c)^{\frac{n}{2}+1}e^{n\operatorname{acoth}(ax)}}{a^2n^2x+6a^2nx+8a^2x-an^2-6an-8a} - \frac{8ax(-acx+c)^{\frac{n}{2}+1}e^{n\operatorname{acoth}(ax)}}{a^2n^2x+6a^2nx+8a^2x-an^2-6an-8a} - \frac{2n(-acx+c}{a^2n^2x+6a^2nx-} \end{cases}$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1+1/2*n),x)
```

output
```
Piecewise((c**(n/2 + 1)*x*exp(I*pi*n/2), Eq(a, 0)), (0**(n/2 + 1)*x*exp(oo
*n), Eq(a, 1/x)), (-Integral(1/(a*x*exp(4*acoth(a*x)) - exp(4*acoth(a*x)))
, x)/c, Eq(n, -4)), (Integral(exp(-2*acoth(a*x)), x), Eq(n, -2)), (2*a**2*
n*x**2*(-a*c*x + c)**(n/2 + 1)*exp(n*acoth(a*x))/(a**2*n**2*x + 6*a**2*n*x
 + 8*a**2*x - a*n**2 - 6*a*n - 8*a) + 4*a**2*x**2*(-a*c*x + c)**(n/2 + 1)*
exp(n*acoth(a*x))/(a**2*n**2*x + 6*a**2*n*x + 8*a**2*x - a*n**2 - 6*a*n -
8*a) - 8*a*x*(-a*c*x + c)**(n/2 + 1)*exp(n*acoth(a*x))/(a**2*n**2*x + 6*a*
*2*n*x + 8*a**2*x - a*n**2 - 6*a*n - 8*a) - 2*n*(-a*c*x + c)**(n/2 + 1)*ex
p(n*acoth(a*x))/(a**2*n**2*x + 6*a**2*n*x + 8*a**2*x - a*n**2 - 6*a*n - 8*
a) - 12*(-a*c*x + c)**(n/2 + 1)*exp(n*acoth(a*x))/(a**2*n**2*x + 6*a**2*n*
x + 8*a**2*x - a*n**2 - 6*a*n - 8*a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx = -\frac{2\left(a^2(-c)^{\frac{1}{2}n}c(n+2)x^2-4\,a(-c)^{\frac{1}{2}n}cx-(-c)^{\frac{1}{2}n}c(n+6)\right)(ax+1)^{\frac{1}{2}n}}{(n^2+6\,n+8)a}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="maxima")
```

output
```
-2*(a^2*(-c)^(1/2*n)*c*(n + 2)*x^2 - 4*a*(-c)^(1/2*n)*c*x - (-c)^(1/2*n)*c
*(n + 6))*(a*x + 1)^(1/2*n)/((n^2 + 6*n + 8)*a)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx = \int(-acx+c)^{\frac{1}{2}n+1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^(1/2*n + 1)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.10

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx$$
$$=-\frac{\left(\frac{(2\,n+12)\,(c-a\,c\,x)^{\frac{n}{2}+1}}{a^2\,(n^2+6\,n+8)}-\frac{x^2\,(2\,n+4)\,(c-a\,c\,x)^{\frac{n}{2}+1}}{n^2+6\,n+8}+\frac{8\,x\,(c-a\,c\,x)^{\frac{n}{2}+1}}{a\,(n^2+6\,n+8)}\right)\,\left(\frac{a\,x+1}{a\,x}\right)^{n/2}}{\left(x-\frac{1}{a}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 + 1),x)
```

output
```
-((((2*n + 12)*(c - a*c*x)^(n/2 + 1))/(a^2*(6*n + n^2 + 8)) - (x^2*(2*n +
4)*(c - a*c*x)^(n/2 + 1))/(6*n + n^2 + 8) + (8*x*(c - a*c*x)^(n/2 + 1))/(a
*(6*n + n^2 + 8)))*((a*x + 1)/(a*x))^(n/2))/((x - 1/a)*((a*x - 1)/(a*x))^(
n/2))
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx=c\left(-\left(\int e^{acoth(ax)n}(-acx+c)^{\frac{n}{2}}\,xdx\right)a+\int e^{acoth(ax)n}(-acx+c)^{\frac{n}{2}}\,dx\right)$$

input
```
int(exp(n*acoth(a*x))*(-a*c*x+c)^(1+1/2*n),x)
```

output
```
c*( - int(e**(acoth(a*x)*n)*( - a*c*x + c)**(n/2)*x,x)*a + int(e**(acoth(a
*x)*n)*( - a*c*x + c)**(n/2),x))
```

3.300 $\int e^{n \coth^{-1}(ax)}(c-acx)^{n/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 36

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\frac{2e^{n \coth^{-1}(ax)}(1+ax)(c-acx)^{n/2}}{a(2+n)}$$

output
```
2*exp(n*arccoth(a*x))*(a*x+1)*(-a*c*x+c)^(1/2*n)/a/(2+n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{n/2}\,dx=-\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}}x(c-acx)^{n/2}}{-1-\frac{n}{2}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(n/2),x]
```

output
```
-(((1 + 1/(a*x))^(1 + n/2)*x*(c - a*c*x)^(n/2))/((-1 - n/2)*(1 - 1/(a*x))^
(n/2)))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{n/2}e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \ 6726$$

$$\frac{2(ax+1)(c-acx)^{n/2}e^{n\coth^{-1}(ax)}}{a(n+2)}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(n/2),x]
```

output
```
(2*E^(n*ArcCoth[a*x])*(1 + a*x)*(c - a*c*x)^(n/2))/(a*(2 + n))
```

Defintions of rubi rules used

rule 6726
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] &&  !IntegerQ[p] && EqQ[p, n/2]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{2\,\mathrm{e}^{n\ \operatorname{arccoth}(ax)}(ax+1)(-acx+c)^{\frac{n}{2}}}{a(2+n)}$
orering	$\frac{2\,\mathrm{e}^{n\ \operatorname{arccoth}(ax)}(ax+1)(-acx+c)^{\frac{n}{2}}}{a(2+n)}$
parallelrisch	$-\frac{-2\,\mathrm{e}^{n\ \operatorname{arccoth}(ax)}x(-acx+c)^{\frac{n}{2}}a-2\,\mathrm{e}^{n\ \operatorname{arccoth}(ax)}(-acx+c)^{\frac{n}{2}}}{a(2+n)}$
risch	$\frac{2(ax+1)(ax+1)^{\frac{n}{2}}(ax-1)^{-\frac{n}{2}}(ax-1)^{\frac{n}{2}}c^{\frac{n}{2}}\mathrm{e}^{-\frac{in\pi\left(-\operatorname{csgn}(ic(ax-1))^{3}-\operatorname{csgn}(ic(ax-1))^{2}\operatorname{csgn}(ic)-\operatorname{csgn}(i(ax-1))\operatorname{csgn}(ic(ax-1))^{2}+\operatorname{csg}}{4}}}{a(2+n)}$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x,method=_RETURNVERBOSE)
```

output
```
2*exp(n*arccoth(a*x))*(a*x+1)*(-a*c*x+c)^(1/2*n)/a/(2+n)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\frac{2\,(ax+1)(-acx+c)^{\frac{1}{2}\,n}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{an+2\,a}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="fricas")
```

output
```
2*(a*x + 1)*(-a*c*x + c)^(1/2*n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*n + 2*a)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx = \begin{cases} -\frac{x}{c} & \text{for } a=0\wedge n=-2 \\ c^{\frac{n}{2}}xe^{\frac{i\pi n}{2}} & \text{for } a=0 \\ -\frac{\int \frac{1}{axe^{2\operatorname{acoth}(ax)}-e^{2\operatorname{acoth}(ax)}}\,dx}{c} & \text{for } n=-2 \\ \frac{2ax(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an+2a}+\frac{2(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an+2a} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1/2*n),x)
```

output
```
Piecewise((-x/c, Eq(a, 0) & Eq(n, -2)), (c**(n/2)*x*exp(I*pi*n/2), Eq(a, 0
)), (-Integral(1/(a*x*exp(2*acoth(a*x)) - exp(2*acoth(a*x))), x)/c, Eq(n,
-2)), (2*a*x*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))/(a*n + 2*a) + 2*(-a*c*x
 + c)**(n/2)*exp(n*acoth(a*x))/(a*n + 2*a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx = \frac{2\left(a(-c)^{\frac{1}{2}n}x+(-c)^{\frac{1}{2}n}\right)(ax+1)^{\frac{1}{2}n}}{a(n+2)}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="maxima")
```

output
```
2*(a*(-c)^(1/2*n)*x + (-c)^(1/2*n))*(a*x + 1)^(1/2*n)/(a*(n + 2))
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\int(-acx+c)^{\frac{1}{2}n}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^(1/2*n)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [B] (verification not implemented)

Time = 13.65 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.53

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\frac{2\left(\frac{1}{a\,x}+1\right)^{n/2}(c-a\,c\,x)^{n/2}\,(a\,x+1)}{a\left(1-\frac{1}{a\,x}\right)^{n/2}\,(n+2)}$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2),x)
```

output
```
(2*(1/(a*x) + 1)^(n/2)*(c - a*c*x)^(n/2)*(a*x + 1))/(a*(1 - 1/(a*x))^(n/2)
*(n + 2))
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\int e^{acoth(ax)n}(-acx+c)^{\frac{n}{2}}\,dx$$

input
```
int(exp(n*acoth(a*x))*(-a*c*x+c)^(1/2*n),x)
```

output
```
int(e**(acoth(a*x)*n)*( - a*c*x + c)**(n/2),x)
```

3.301 $\int e^{n \coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx = \frac{2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x(c-acx)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{n}$$

output
```
2*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1/2*n)*x*(-a*c*x+c)^(-1+1/2*n)*hypergeom(
[1, -1/2*n],[1-1/2*n],2/(a+1/x)/x)/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx = -\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(c-acx)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{2}{1+ax}\right)}{acn}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-1 + n/2),x]
```

output
```
(-2*(1 + 1/(a*x))^(n/2)*(c - a*c*x)^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 -
 n/2, 2/(1 + a*x)])/(a*c*n*(1 - 1/(a*x))^(n/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6727, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{\frac{n}{2}-1}e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6727}$$

$$\left(\frac{1}{x}\right)^{\frac{n-2}{2}}\left(-\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\right)(c-acx)^{\frac{n-2}{2}}\int \frac{a\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-1}}{a-\frac{1}{x}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-a\left(\frac{1}{x}\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}(c-acx)^{\frac{n-2}{2}}\int \frac{\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-\frac{n}{2}-1}}{a-\frac{1}{x}}d\frac{1}{x}$$

$$\Big\downarrow \text{141}$$

$$\frac{2\left(\frac{1}{x}\right)^{\frac{n-2}{2}-\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{n/2}(c-acx)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{n}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-1 + n/2),x]
```

output
```
(2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^(n/2)*(x^(-1))^((-2 + n)/2 - n/2)
*(c - a*c*x)^((-2 + n)/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, 2/((a + x^
(-1))*x)])/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \mathrm{e}^{n\ \mathrm{arccoth}(ax)} (-acx+c)^{-1+\frac{n}{2}}\, dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\, dx = \int (-acx+c)^{\frac{1}{2}n-1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="fricas")
```

output
```
integral((-a*c*x + c)^(1/2*n - 1)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx=\int(-c(ax-1))^{\frac{n}{2}-1}\,e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(-1+1/2*n),x)
```

output
```
Integral((-c*(a*x - 1))**(n/2 - 1)*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx=\int(-acx+c)^{\frac{1}{2}\,n-1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^(1/2*n - 1)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx=\int(-acx+c)^{\frac{1}{2}\,n-1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^(1/2*n - 1)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^{\frac{n}{2}-1}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 1),x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 1), x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx=-\frac{\int \frac{e^{acoth(ax)n}(-acx+c)^{\frac{n}{2}}}{ax-1}dx}{c}$$

input
```
int(exp(n*acoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x)
```

output
```
( - int((e**(acoth(a*x)*n)*( - a*c*x + c)**(n/2))/(a*x - 1),x))/c
```

3.302 $\int e^{n \coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx = -\frac{2\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x(c-acx)^{\frac{1}{2}(-4+n)}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2-n}$$

output
```
-2*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(-1+1/2*n)*x*(-a*c*x+c)^(-2+1/2*n)*hyperg
eom([2, 1-1/2*n],[2-1/2*n],2/(a+1/x)/x)/(2-n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

$$\int e^{n \coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx = \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(c-acx)^{n/2}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{2}{1+ax}\right)}{ac^2(-2+n)(1+ax)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-2 + n/2),x]
```

output
```
(2*(1 + 1/(a*x))^(n/2)*(c - a*c*x)^(n/2)*Hypergeometric2F1[2, 1 - n/2, 2 -
 n/2, 2/(1 + a*x)])/(a*c^2*(-2 + n)*(1 - 1/(a*x))^(n/2)*(1 + a*x))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6727, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{\frac{n}{2}-2}e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$\left(\frac{1}{x}\right)^{\frac{n-4}{2}}\left(-\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\right)(c-acx)^{\frac{n-4}{2}}\int \frac{a^2\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-n/2}}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-a^2\left(\frac{1}{x}\right)^{\frac{n-4}{2}}\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}(c-acx)^{\frac{n-4}{2}}\int \frac{\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{1}{x}\right)^{-n/2}}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{141}$$

$$-\frac{2\left(\frac{1}{x}\right)^{\frac{n-4}{2}-\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}(c-acx)^{\frac{n-4}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2-n}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-2 + n/2),x]
```

output
```
(-2*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*(x^(-1))^(1 + (-4 +
 n)/2 - n/2)*(c - a*c*x)^((-4 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n/
2, 2/((a + x^(-1))*x)])/(2 - n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \mathrm{e}^{n \ \mathrm{arccoth}(ax)}(-acx+c)^{-2+\frac{n}{2}}\,dx$$

input
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx=\int(-acx+c)^{\frac{1}{2}n-2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="fricas")
```

output
```
integral((-a*c*x + c)^(1/2*n - 2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx=\int\left(-c(ax-1)\right)^{\frac{n}{2}-2}e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(-2+1/2*n),x)
```

output
```
Integral((-c*(a*x - 1))**(n/2 - 2)*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx=\int(-acx+c)^{\frac{1}{2}\,n-2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="maxima")
```

output
```
integrate((-a*c*x + c)^(1/2*n - 2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx=\int(-acx+c)^{\frac{1}{2}\,n-2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="giac")
```

output
```
integrate((-a*c*x + c)^(1/2*n - 2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx=\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^{\frac{n}{2}-2}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 2),x)
```

output
```
int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 2), x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx=\frac{\int \frac{e^{acoth(ax)n}(-acx+c)^{\frac{n}{2}}}{a^2x^2-2ax+1}dx}{c^2}$$

input
```
int(exp(n*acoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x)
```

output
```
int((e**(acoth(a*x)*n)*( - a*c*x + c)**(n/2))/(a**2*x**2 - 2*a*x + 1),x)/c
**2
```

3.303 $\int e^{\coth^{-1}(x)} x(1+x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 62

$$\int e^{\coth^{-1}(x)} x(1+x)\,dx = \frac{5}{3}\sqrt{1-\frac{1}{x^2}}x+\sqrt{1-\frac{1}{x^2}}x^2 + \frac{1}{3}\sqrt{1-\frac{1}{x^2}}x^3+\operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
5/3*(1-1/x^2)^(1/2)*x+(1-1/x^2)^(1/2)*x^2+1/3*(1-1/x^2)^(1/2)*x^3+arctanh(
(1-1/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

$$\int e^{\coth^{-1}(x)} x(1+x)\,dx = \frac{1}{3}\sqrt{1-\frac{1}{x^2}}x(5+3x+x^2)+\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCoth[x]*x*(1 + x),x]
```

output
```
(Sqrt[1 - x^(-2)]*x*(5 + 3*x + x^2))/3 + Log[(1 + Sqrt[1 - x^(-2)])*x]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6729, 107, 105, 105, 103, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(x+1)e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6729}$$

$$-\int \frac{\left(1+\frac{1}{x}\right)^{3/2}x^4}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3-\frac{2}{3}\int \frac{\left(1+\frac{1}{x}\right)^{3/2}x^3}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3-\frac{2}{3}\left(\frac{3}{2}\int \frac{\sqrt{1+\frac{1}{x}}x^2}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3-\frac{2}{3}\left(\frac{3}{2}\left(\int \frac{x}{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}}d\frac{1}{x}-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)$$

$$\Big\downarrow \text{103}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3-\frac{2}{3}\left(\frac{3}{2}\left(-\int \frac{1}{1-\frac{1}{x^2}}d\left(\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\right)-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3-$$
$$\frac{2}{3}\left(\frac{3}{2}\left(-\operatorname{arctanh}\left(\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\right)-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1x}\right)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)$$

input
```
Int[E^ArcCoth[x]*x*(1 + x),x]
```

output
```
(Sqrt[1 - x^(-1)]*(1 + x^(-1))^(5/2)*x^3)/3 - (2*(-1/2*(Sqrt[1 - x^(-1)]*(
1 + x^(-1))^(3/2)*x^2) + (3*(-(Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x) - ArcT
anh[Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]]))/2))/3
```

Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6729
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-d^p)*(e*x)^m*(1/x)^m   Subst[Int[((1 + c*(x/d))^p*
((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[
{a, c, d, e, m, n}, x] && EqQ[a^2*c^2 - d^2, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{(x^2+3x+5)(x-1)}{3\sqrt{\frac{x-1}{1+x}}}+\frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	62
default	$\frac{(x-1)\left(((x-1)(1+x))^{\frac{3}{2}}+3x\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)+6\sqrt{x^2-1}\right)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	67
trager	$\frac{(1+x)(x^2+3x+5)\sqrt{-\frac{1-x}{1+x}}}{3}-\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)$	67

input
```
int(1/((x-1)/(1+x))^(1/2)*x*(1+x),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^2+3*x+5)*(x-1)/((x-1)/(1+x))^(1/2)+ln(x+(x^2-1)^(1/2))/((x-1)/(1+x)
)^(1/2)/(1+x)*((x-1)*(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx=\frac{1}{3}\left(x^3+4\,x^2+8\,x+5\right)\sqrt{\frac{x-1}{x+1}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x*(1+x),x, algorithm="fricas")
```

output
```
1/3*(x^3 + 4*x^2 + 8*x + 5)*sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x +
1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx = \int \frac{x(x+1)}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*x*(1+x),x)
```

output
```
Integral(x*(x + 1)/sqrt((x - 1)/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(50) = 100$.

Time $= 0.03$ (sec) , antiderivative size $= 110$, normalized size of antiderivative $= 1.77$

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx = -\frac{2\left(3\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}} - 8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}} + 9\sqrt{\frac{x-1}{x+1}}\right)}{3\left(\frac{3(x-1)}{x+1} - \frac{3(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1\right)} + \log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x*(1+x),x, algorithm="maxima")
```

output
```
-2/3*(3*((x - 1)/(x + 1))^(5/2) - 8*((x - 1)/(x + 1))^(3/2) + 9*sqrt((x -
1)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1
)^3 - 1) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx = \frac{1}{3}\sqrt{x^2-1}\left(x\left(\frac{x}{\operatorname{sgn}(x+1)}+\frac{3}{\operatorname{sgn}(x+1)}\right)+\frac{5}{\operatorname{sgn}(x+1)}\right) - \frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x*(1+x),x, algorithm="giac")
```

output
```
1/3*sqrt(x^2 - 1)*(x*(x/sgn(x + 1) + 3/sgn(x + 1)) + 5/sgn(x + 1)) - log(a
bs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.52

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right) - \frac{6\sqrt{\frac{x-1}{x+1}} - \frac{16\left(\frac{x-1}{x+1}\right)^{3/2}}{3} + 2\left(\frac{x-1}{x+1}\right)^{5/2}}{\frac{3(x-1)}{x+1} - \frac{3(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1}$$

input
```
int((x*(x + 1))/((x - 1)/(x + 1))^(1/2),x)
```

output
```
2*atanh(((x - 1)/(x + 1))^(1/2)) - (6*((x - 1)/(x + 1))^(1/2) - (16*((x -
1)/(x + 1))^(3/2))/3 + 2*((x - 1)/(x + 1))^(5/2))/((3*(x - 1))/(x + 1) - (
3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx = \frac{\sqrt{x+1}\,\sqrt{x-1}\,x^2}{3}+\sqrt{x+1}\,\sqrt{x-1}\,x+\frac{5\sqrt{x+1}\,\sqrt{x-1}}{3}+2\log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right)$$

input
```
int(1/((x-1)/(1+x))^(1/2)*x*(1+x),x)
```

output
```
(sqrt(x + 1)*sqrt(x - 1)*x**2 + 3*sqrt(x + 1)*sqrt(x - 1)*x + 5*sqrt(x + 1
)*sqrt(x - 1) + 6*log((sqrt(x - 1) + sqrt(x + 1))/sqrt(2)))/3
```

3.304 $\int e^{\coth^{-1}(x)}(1+x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 49

$$\int e^{\coth^{-1}(x)}(1+x)\,dx=2\sqrt{1-\frac{1}{x^2}}x+\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2+\frac{3}{2}\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
2*(1-1/x^2)^(1/2)*x+1/2*(1-1/x^2)^(1/2)*x^2+3/2*arctanh((1-1/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int e^{\coth^{-1}(x)}(1+x)\,dx=\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x(4+x)+\frac{3}{2}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCoth[x]*(1 + x),x]
```

output
```
(Sqrt[1 - x^(-2)]*x*(4 + x))/2 + (3*Log[(1 + Sqrt[1 - x^(-2)])*x])/2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6725, 105, 105, 103, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6725}$$

$$-\int \frac{\left(1+\frac{1}{x}\right)^{3/2}x^3}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2-\frac{3}{2}\int \frac{\sqrt{1+\frac{1}{x}}x^2}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2-\frac{3}{2}\left(\int \frac{x}{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}}d\frac{1}{x}-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)$$

$$\Big\downarrow \text{103}$$

$$\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2-\frac{3}{2}\left(-\int \frac{1}{1-\frac{1}{x^2}}d\left(\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\right)-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2-\frac{3}{2}\left(-\text{arctanh}\left(\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\right)-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)$$

input
```
Int[E^ArcCoth[x]*(1 + x),x]
```

output
```
(Sqrt[1 - x^(-1)]*(1 + x^(-1))^(3/2)*x^2)/2 - (3*(-(Sqrt[1 - x^(-1)]*Sqrt[
1 + x^(-1)]*x) - ArcTanh[Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]]))/2
```

Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

method	result	size
default	$\frac{(x-1)\left(x\sqrt{x^{2}-1}+3\ln\left(x+\sqrt{x^{2}-1}\right)+4\sqrt{x^{2}-1}\right)}{2\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	57
risch	$\frac{(4+x)(x-1)}{2\sqrt{\frac{x-1}{1+x}}}+\frac{3\ln\left(x+\sqrt{x^{2}-1}\right)\sqrt{(x-1)(1+x)}}{2\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	58
trager	$\frac{(1+x)(4+x)\sqrt{-\frac{1-x}{1+x}}}{2}+\frac{3\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)}{2}$	59

input
```
int(1/((x-1)/(1+x))^(1/2)*(1+x),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x-1)*(x*(x^2-1)^(1/2)+3*ln(x+(x^2-1)^(1/2))+4*(x^2-1)^(1/2))/((x-1)/(
1+x))^(1/2)/((x-1)*(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int e^{\coth^{-1}(x)}(1+x)\,dx=\frac{1}{2}\left(x^{2}+5\,x+4\right)\sqrt{\frac{x-1}{x+1}}+\frac{3}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1+x),x, algorithm="fricas")
```

output
```
1/2*(x^2 + 5*x + 4)*sqrt((x - 1)/(x + 1)) + 3/2*log(sqrt((x - 1)/(x + 1))
+ 1) - 3/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}(1+x)\,dx = \int \frac{x+1}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*(1+x),x)
```

output
```
Integral((x + 1)/sqrt((x - 1)/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(39) = 78$.

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.78

$$\int e^{\coth^{-1}(x)}(1+x)\,dx = \frac{3\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}} - 5\sqrt{\frac{x-1}{x+1}}}{\frac{2(x-1)}{x+1} - \frac{(x-1)^2}{(x+1)^2} - 1} + \frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}} + 1\right) - \frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}} - 1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1+x),x, algorithm="maxima")
```

output
```
(3*((x - 1)/(x + 1))^(3/2) - 5*sqrt((x - 1)/(x + 1)))/(2*(x - 1)/(x + 1) -
 (x - 1)^2/(x + 1)^2 - 1) + 3/2*log(sqrt((x - 1)/(x + 1)) + 1) - 3/2*log(s
qrt((x - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int e^{\coth^{-1}(x)}(1+x)\,dx$$
$$=\frac{1}{2}\sqrt{x^2-1}\left(\frac{x}{\operatorname{sgn}(x+1)}+\frac{4}{\operatorname{sgn}(x+1)}\right)-\frac{3\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{2\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1+x),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 - 1)*(x/sgn(x + 1) + 4/sgn(x + 1)) - 3/2*log(abs(-x + sqrt(x^
2 - 1)))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.66 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.39

$$\int e^{\coth^{-1}(x)}(1+x)\,dx=3\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)+\frac{5\sqrt{\frac{x-1}{x+1}}-3\left(\frac{x-1}{x+1}\right)^{3/2}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2(x-1)}{x+1}+1}$$

input
```
int((x + 1)/((x - 1)/(x + 1))^(1/2),x)
```

output
```
3*atanh(((x - 1)/(x + 1))^(1/2)) + (5*((x - 1)/(x + 1))^(1/2) - 3*((x - 1)
/(x + 1))^(3/2))/((x - 1)^2/(x + 1)^2 - (2*(x - 1))/(x + 1) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int e^{\coth^{-1}(x)}(1+x)\,dx=\frac{\sqrt{x+1}\,\sqrt{x-1}\,x}{2}+2\sqrt{x+1}\,\sqrt{x-1}+3\log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right)$$

input
```
int(1/((x-1)/(1+x))^(1/2)*(1+x),x)
```

output

```
(sqrt(x + 1)*sqrt(x - 1)*x + 4*sqrt(x + 1)*sqrt(x - 1) + 6*log((sqrt(x - 1
) + sqrt(x + 1))/sqrt(2)))/2
```

3.305 $\int e^{\coth^{-1}(x)}(1-x)x\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = -\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

output
```
-1/3*(1-1/x^2)^(3/2)*x^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = -\frac{1}{3}\sqrt{1-\frac{1}{x^2}}x(-1+x^2)$$

input
```
Integrate[E^ArcCoth[x]*(1 - x)*x,x]
```

output
```
-1/3*(Sqrt[1 - x^(-2)]*x*(-1 + x^2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6728, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)xe^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \ 6728$$

$$\int \sqrt{1-\frac{1}{x^2}}x^4d\frac{1}{x}$$

$$\Big\downarrow \ 242$$

$$-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

input
```
Int[E^ArcCoth[x]*(1 - x)*x,x]
```

output
```
-1/3*((1 - x^(-2))^(3/2)*x^3)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6728
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_))^(p_.), x_S
ymbol] :> Simp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + p + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && In
tegerQ[p] && IntegerQ[n] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

method	result	size
gosper	$-\frac{(x-1)^{2}(1+x)}{3\sqrt{\frac{x-1}{1+x}}}$	22
default	$-\frac{(x-1)^{2}(1+x)}{3\sqrt{\frac{x-1}{1+x}}}$	22
risch	$-\frac{(x^{2}-1)(x-1)}{3\sqrt{\frac{x-1}{1+x}}}$	22
trager	$-\frac{(1+x)(x^{2}-1)\sqrt{-\frac{1-x}{1+x}}}{3}$	25
orering	$\frac{(x-1)(1+x)(1-x)}{3\sqrt{\frac{x-1}{1+x}}}$	25

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)*x,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(x-1)^2*(1+x)/((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx=-\frac{1}{3}\left(x^{3}+x^{2}-x-1\right)\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)*x,x, algorithm="fricas")
```

output
```
-1/3*(x^3 + x^2 - x - 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = -\int \left(-\frac{x}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\right)dx - \int \frac{x^2}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*(1-x)*x,x)
```

output
```
-Integral(-x/sqrt(x/(x + 1) - 1/(x + 1)), x) - Integral(x**2/sqrt(x/(x + 1
) - 1/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = \frac{8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}}{3\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)*x,x, algorithm="maxima")
```

output
```
8/3*((x - 1)/(x + 1))^(3/2)/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (
x - 1)^3/(x + 1)^3 - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = -\frac{(x^2-1)^{\frac{3}{2}}}{3\,\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)*x,x, algorithm="giac")
```

output
```
-1/3*(x^2 - 1)^(3/2)/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = -\frac{\left(\frac{x-1}{x+1}\right)^{3/2}(x+1)^3}{3}$$

input
```
int(-(x*(x - 1))/((x - 1)/(x + 1))^(1/2),x)
```

output
```
-(((x - 1)/(x + 1))^(3/2)*(x + 1)^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = \frac{\sqrt{x+1}\,\sqrt{x-1}\,(-x^2+1)}{3}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)*x,x)
```

output
```
(sqrt(x + 1)*sqrt(x - 1)*( - x**2 + 1))/3
```

3.306 $\int e^{\coth^{-1}(x)}(1-x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 35

$$\int e^{\coth^{-1}(x)}(1-x)\,dx=-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2+\frac{1}{2}\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
-1/2*(1-1/x^2)^(1/2)*x^2+1/2*arctanh((1-1/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int e^{\coth^{-1}(x)}(1-x)\,dx=-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2+\frac{1}{2}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCoth[x]*(1 - x),x]
```

output
```
-1/2*(Sqrt[1 - x^(-2)]*x^2) + Log[(1 + Sqrt[1 - x^(-2)])*x]/2
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6724, 243, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \; 6724$$

$$\int \sqrt{1-\frac{1}{x^2}}x^3d\frac{1}{x}$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \sqrt{1-\frac{1}{x^2}}x^2d\frac{1}{x^2}$$

$$\Big\downarrow \; 51$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x^2}-\sqrt{1-\frac{1}{x^2}}x\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\int \frac{1}{1-\sqrt{1-\frac{1}{x^2}}}d\sqrt{1-\frac{1}{x^2}}-\sqrt{1-\frac{1}{x^2}}x\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)-\sqrt{1-\frac{1}{x^2}}x\right)$$

input
```
Int[E^ArcCoth[x]*(1 - x),x]
```

output
```
(-(Sqrt[1 - x^(-2)]*x) + ArcTanh[Sqrt[1 - x^(-2)]])/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

method	result	size
default	$-\frac{(x-1)\left(x\sqrt{x^{2}-1}-\ln\left(x+\sqrt{x^{2}-1}\right)\right)}{2\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	48
risch	$-\frac{x(x-1)}{2\sqrt{\frac{x-1}{1+x}}}+\frac{\ln\left(x+\sqrt{x^{2}-1}\right)\sqrt{(x-1)(1+x)}}{2\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	56
trager	$-\frac{(1+x)\sqrt{-\frac{1-x}{1+x}}\,x}{2}-\frac{\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)}{2}$	60

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x-1)*(x*(x^2-1)^(1/2)-ln(x+(x^2-1)^(1/2)))/((x-1)/(1+x))^(1/2)/((x-1
)*(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int e^{\coth^{-1}(x)}(1-x)\,dx=-\frac{1}{2}\left(x^{2}+x\right)\sqrt{\frac{x-1}{x+1}}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x),x, algorithm="fricas")
```

output
```
-1/2*(x^2 + x)*sqrt((x - 1)/(x + 1)) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1)
- 1/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}(1-x)\,dx = -\int \frac{x}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx - \int \left(-\frac{1}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\right)dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*(1-x),x)
```

output
```
-Integral(x/sqrt(x/(x + 1) - 1/(x + 1)), x) - Integral(-1/sqrt(x/(x + 1) -
 1/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.37

$$\int e^{\coth^{-1}(x)}(1-x)\,dx = \frac{\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}} + \sqrt{\frac{x-1}{x+1}}}{\frac{2(x-1)}{x+1} - \frac{(x-1)^2}{(x+1)^2} - 1} + \frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x),x, algorithm="maxima")
```

output
```
(((x - 1)/(x + 1))^(3/2) + sqrt((x - 1)/(x + 1)))/(2*(x - 1)/(x + 1) - (x
- 1)^2/(x + 1)^2 - 1) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1) - 1/2*log(sqrt(
(x - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int e^{\coth^{-1}(x)}(1-x)\,dx=-\frac{\sqrt{x^2-1}x}{2\,\mathrm{sgn}\,(x+1)}-\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{2\,\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x),x, algorithm="giac")
```

output
```
-1/2*sqrt(x^2 - 1)*x/sgn(x + 1) - 1/2*log(abs(-x + sqrt(x^2 - 1)))/sgn(x +
 1)
```

Mupad [B] (verification not implemented)

Time = 13.55 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int e^{\coth^{-1}(x)}(1-x)\,dx=\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{\sqrt{\frac{x-1}{x+1}}+\left(\frac{x-1}{x+1}\right)^{3/2}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2\,(x-1)}{x+1}+1}$$

input
```
int(-(x - 1)/((x - 1)/(x + 1))^(1/2),x)
```

output
```
atanh(((x - 1)/(x + 1))^(1/2)) - (((x - 1)/(x + 1))^(1/2) + ((x - 1)/(x +
1))^(3/2))/((x - 1)^2/(x + 1)^2 - (2*(x - 1))/(x + 1) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int e^{\coth^{-1}(x)}(1-x)\,dx=-\frac{\sqrt{x+1}\,\sqrt{x-1}\,x}{2}+\log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right)$$

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x),x)
```

output
```
( - sqrt(x + 1)*sqrt(x - 1)*x + 2*log((sqrt(x - 1) + sqrt(x + 1))/sqrt(2))
)/2
```

3.307 $\int e^{\coth^{-1}(x)} x(1+x)^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 82

$$\int e^{\coth^{-1}(x)} x(1+x)^2 \, dx = 3\sqrt{1-\frac{1}{x^2}}x + \frac{15}{8}\sqrt{1-\frac{1}{x^2}}x^2 + \sqrt{1-\frac{1}{x^2}}x^3 + \frac{1}{4}\sqrt{1-\frac{1}{x^2}}x^4 + \frac{15}{8}\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
3*(1-1/x^2)^(1/2)*x+15/8*(1-1/x^2)^(1/2)*x^2+(1-1/x^2)^(1/2)*x^3+1/4*(1-1/
x^2)^(1/2)*x^4+15/8*arctanh((1-1/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.63

$$\int e^{\coth^{-1}(x)} x(1+x)^2 \, dx = \frac{1}{8}\sqrt{1-\frac{1}{x^2}}x(24+15x+8x^2+2x^3) + \frac{15}{8}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCoth[x]*x*(1 + x)^2,x]
```

output
```
(Sqrt[1 - x^(-2)]*x*(24 + 15*x + 8*x^2 + 2*x^3))/8 + (15*Log[(1 + Sqrt[1 -
 x^(-2)])*x])/8
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.76, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6729, 107, 105, 105, 105, 103, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(x+1)^2 e^{\coth^{-1}(x)} \, dx$$

$$\Big\downarrow \text{6729}$$

$$-\int \frac{\left(1+\frac{1}{x}\right)^{5/2} x^5}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{4}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{7/2}x^4-\frac{3}{4}\int \frac{\left(1+\frac{1}{x}\right)^{5/2} x^4}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{4}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{7/2}x^4-\frac{3}{4}\left(\frac{5}{3}\int \frac{\left(1+\frac{1}{x}\right)^{3/2} x^3}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}-\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3\right)$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{4}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{7/2}x^4-\frac{3}{4}\left(\frac{5}{3}\left(\frac{3}{2}\int \frac{\sqrt{1+\frac{1}{x}}x^2}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)-\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3\right)$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{4}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{7/2}x^4-$$
$$\frac{3}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(\int\frac{x}{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}}d\frac{1}{x}-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)-\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}$$

$\big\downarrow$ 103

$$\frac{1}{4}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{7/2}x^4-$$
$$\frac{3}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-\int\frac{1}{1-\frac{1}{x^2}}d\left(\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\right)-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)-\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right.$$

$\big\downarrow$ 219

$$\frac{1}{4}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{7/2}x^4-$$
$$\frac{3}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-\text{arctanh}\left(\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\right)-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)-\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+\right.$$

input
```
Int[E^ArcCoth[x]*x*(1 + x)^2,x]
```

output
```
(Sqrt[1 - x^(-1)]*(1 + x^(-1))^(7/2)*x^4)/4 - (3*(-1/3*(Sqrt[1 - x^(-1)]*(
1 + x^(-1))^(5/2)*x^3) + (5*(-1/2*(Sqrt[1 - x^(-1)]*(1 + x^(-1))^(3/2)*x^2
) + (3*(-(Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x) - ArcTanh[Sqrt[1 - x^(-1)]*
Sqrt[1 + x^(-1)]]))/2))/3))/4
```

Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6729
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-d^p)*(e*x)^m*(1/x)^m   Subst[Int[((1 + c*(x/d))^p*
((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[
{a, c, d, e, m, n}, x] && EqQ[a^2*c^2 - d^2, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{\left(2 x^{3}+8 x^{2}+15 x +24\right) \left(x -1\right)}{8 \sqrt{\frac{x -1}{1+x}}}+\frac{15 \ln \left(x +\sqrt{x^{2}-1}\right) \sqrt{\left(x -1\right) \left(1+x \right)}}{8 \sqrt{\frac{x -1}{1+x}}\, \left(1+x \right)}$	70
trager	$\frac{\left(1+x \right) \left(2 x^{3}+8 x^{2}+15 x +24\right) \sqrt{-\frac{1-x}{1+x}}}{8}-\frac{15 \ln \left(-\sqrt{-\frac{1-x}{1+x}}\, x -\sqrt{-\frac{1-x}{1+x}}+x \right)}{8}$	74
default	$\frac{\left(x -1\right) \left(2 x \left(x^{2}-1\right)^{\frac{3}{2}}+8 \left(\left(x -1\right) \left(1+x \right)\right)^{\frac{3}{2}}+17 x \sqrt{x^{2}-1}+32 \sqrt{x^{2}-1}+15 \ln \left(x +\sqrt{x^{2}-1}\right)\right)}{8 \sqrt{\frac{x -1}{1+x}}\, \sqrt{\left(x -1\right) \left(1+x \right)}}$	79

input
```
int(1/((x-1)/(1+x))^(1/2)*x*(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*x^3+8*x^2+15*x+24)*(x-1)/((x-1)/(1+x))^(1/2)+15/8*ln(x+(x^2-1)^(1/2
))/((x-1)/(1+x))^(1/2)/(1+x)*((x-1)*(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx = \frac{1}{8}\left(2\,x^4+10\,x^3+23\,x^2+39\,x+24\right)\sqrt{\frac{x-1}{x+1}} + \frac{15}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{15}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="fricas")
```

output
```
1/8*(2*x^4 + 10*x^3 + 23*x^2 + 39*x + 24)*sqrt((x - 1)/(x + 1)) + 15/8*log
(sqrt((x - 1)/(x + 1)) + 1) - 15/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx = \int \frac{x(x+1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*x*(1+x)**2,x)
```

output
```
Integral(x*(x + 1)**2/sqrt((x - 1)/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(66) = 132.

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.68

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx = \frac{15\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}} - 55\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}} + 73\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}} - 49\sqrt{\frac{x-1}{x+1}}}{4\left(\frac{4(x-1)}{x+1} - \frac{6(x-1)^2}{(x+1)^2} + \frac{4(x-1)^3}{(x+1)^3} - \frac{(x-1)^4}{(x+1)^4} - 1\right)} + \frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="maxima")
```

output
```
1/4*(15*((x - 1)/(x + 1))^(7/2) - 55*((x - 1)/(x + 1))^(5/2) + 73*((x - 1)
/(x + 1))^(3/2) - 49*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)
^2/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) + 15/8*log
(sqrt((x - 1)/(x + 1)) + 1) - 15/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx = \frac{1}{8}\left(\left(2x\left(\frac{x}{\operatorname{sgn}(x+1)} + \frac{4}{\operatorname{sgn}(x+1)}\right) + \frac{15}{\operatorname{sgn}(x+1)}\right)x + \frac{24}{\operatorname{sgn}(x+1)}\right)\sqrt{x^2-1} - \frac{15\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{8\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="giac")
```

output
```
1/8*((2*x*(x/sgn(x + 1) + 4/sgn(x + 1)) + 15/sgn(x + 1))*x + 24/sgn(x + 1)
)*sqrt(x^2 - 1) - 15/8*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.44

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx = \frac{15\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4} + \frac{\frac{49\sqrt{\frac{x-1}{x+1}}}{4} - \frac{73\left(\frac{x-1}{x+1}\right)^{3/2}}{4} + \frac{55\left(\frac{x-1}{x+1}\right)^{5/2}}{4} - \frac{15\left(\frac{x-1}{x+1}\right)^{7/2}}{4}}{\frac{6\,(x-1)^2}{(x+1)^2} - \frac{4\,(x-1)}{x+1} - \frac{4\,(x-1)^3}{(x+1)^3} + \frac{(x-1)^4}{(x+1)^4} + 1}$$

input
```
int((x*(x + 1)^2)/((x - 1)/(x + 1))^(1/2),x)
```

output
```
(15*atanh(((x - 1)/(x + 1))^(1/2)))/4 + ((49*((x - 1)/(x + 1))^(1/2))/4 -
(73*((x - 1)/(x + 1))^(3/2))/4 + (55*((x - 1)/(x + 1))^(5/2))/4 - (15*((x
- 1)/(x + 1))^(7/2))/4)/((6*(x - 1)^2)/(x + 1)^2 - (4*(x - 1))/(x + 1) - (
4*(x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx = \frac{\sqrt{x+1}\,\sqrt{x-1}\,x^3}{4} + \sqrt{x+1}\,\sqrt{x-1}\,x^2 + \frac{15\sqrt{x+1}\,\sqrt{x-1}\,x}{8} + 3\sqrt{x+1}\,\sqrt{x-1} + \frac{15\log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right)}{4}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*x*(1+x)^2,x)
```

output
```
(2*sqrt(x + 1)*sqrt(x - 1)*x**3 + 8*sqrt(x + 1)*sqrt(x - 1)*x**2 + 15*sqrt
(x + 1)*sqrt(x - 1)*x + 24*sqrt(x + 1)*sqrt(x - 1) + 30*log((sqrt(x - 1) +
 sqrt(x + 1))/sqrt(2)))/8
```

3.308 $\int e^{\coth^{-1}(x)}(1+x)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 69

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx=\frac{11}{3}\sqrt{1-\frac{1}{x^2}}x+\frac{3}{2}\sqrt{1-\frac{1}{x^2}}x^2+\frac{1}{3}\sqrt{1-\frac{1}{x^2}}x^3+\frac{5}{2}\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
11/3*(1-1/x^2)^(1/2)*x+3/2*(1-1/x^2)^(1/2)*x^2+1/3*(1-1/x^2)^(1/2)*x^3+5/2
*arctanh((1-1/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.68

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx=\frac{1}{6}\sqrt{1-\frac{1}{x^2}}x\left(22+9x+2x^2\right)+\frac{5}{2}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCoth[x]*(1 + x)^2,x]
```

output
```
(Sqrt[1 - x^(-2)]*x*(22 + 9*x + 2*x^2))/6 + (5*Log[(1 + Sqrt[1 - x^(-2)])*
x])/2
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.62, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6725, 105, 105, 105, 103, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)^2 e^{\coth^{-1}(x)} \, dx$$

$$\Big\downarrow \text{ 6725}$$

$$-\int \frac{\left(1+\frac{1}{x}\right)^{5/2} x^4}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 105}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3-\frac{5}{3}\int \frac{\left(1+\frac{1}{x}\right)^{3/2} x^3}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 105}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3-\frac{5}{3}\left(\frac{3}{2}\int \frac{\sqrt{1+\frac{1}{x}}x^2}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)$$

$$\Big\downarrow \text{ 105}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3-\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{x}{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}}d\frac{1}{x}-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)$$

$$\Big\downarrow \text{ 103}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3-$$
$$\frac{5}{3}\left(\frac{3}{2}\left(-\int\frac{1}{1-\frac{1}{x^2}}d\left(\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\right)-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{5/2}x^3-$$
$$\frac{5}{3}\left(\frac{3}{2}\left(-\text{arctanh}\left(\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\right)-\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x\right)-\frac{1}{2}\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2\right)$$

input
```
Int[E^ArcCoth[x]*(1 + x)^2,x]
```

output
```
(Sqrt[1 - x^(-1)]*(1 + x^(-1))^(5/2)*x^3)/3 - (5*(-1/2*(Sqrt[1 - x^(-1)]*(
1 + x^(-1))^(3/2)*x^2) + (3*(-(Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x) - ArcT
anh[Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]]))/2))/3
```

Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{(2x^2+9x+22)(x-1)}{6\sqrt{\frac{x-1}{1+x}}}+\frac{5\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{2\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	65
trager	$\frac{(1+x)(2x^2+9x+22)\sqrt{-\frac{1-x}{1+x}}}{6}+\frac{5\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)}{2}$	66
default	$\frac{(x-1)\left(2((x-1)(1+x))^{\frac{3}{2}}+9x\sqrt{x^2-1}+15\ln\left(x+\sqrt{x^2-1}\right)+24\sqrt{x^2-1}\right)}{6\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	69

input
```
int(1/((x-1)/(1+x))^(1/2)*(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*x^2+9*x+22)*(x-1)/((x-1)/(1+x))^(1/2)+5/2*ln(x+(x^2-1)^(1/2))/((x-1
)/(1+x))^(1/2)/(1+x)*((x-1)*(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx=\frac{1}{6}\left(2\,x^3+11\,x^2+31\,x+22\right)\sqrt{\frac{x-1}{x+1}}+\frac{5}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{5}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1+x)^2,x, algorithm="fricas")
```

output
```
1/6*(2*x^3 + 11*x^2 + 31*x + 22)*sqrt((x - 1)/(x + 1)) + 5/2*log(sqrt((x -
 1)/(x + 1)) + 1) - 5/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx = \int \frac{(x+1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*(1+x)**2,x)
```

output
```
Integral((x + 1)**2/sqrt((x - 1)/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(53) = 106$.

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.62

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx = -\frac{15\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}} - 40\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}} + 33\sqrt{\frac{x-1}{x+1}}}{3\left(\frac{3(x-1)}{x+1} - \frac{3(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1\right)} + \frac{5}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{5}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1+x)^2,x, algorithm="maxima")
```

output
```
-1/3*(15*((x - 1)/(x + 1))^(5/2) - 40*((x - 1)/(x + 1))^(3/2) + 33*sqrt((x
 - 1)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x
+ 1)^3 - 1) + 5/2*log(sqrt((x - 1)/(x + 1)) + 1) - 5/2*log(sqrt((x - 1)/(x
 + 1)) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx = \frac{1}{6}\sqrt{x^2-1}\left(x\left(\frac{2\,x}{\operatorname{sgn}(x+1)}+\frac{9}{\operatorname{sgn}(x+1)}\right)+\frac{22}{\operatorname{sgn}(x+1)}\right) - \frac{5\,\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{2\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1+x)^2,x, algorithm="giac")
```

output
```
1/6*sqrt(x^2 - 1)*(x*(2*x/sgn(x + 1) + 9/sgn(x + 1)) + 22/sgn(x + 1)) - 5/
2*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.60 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.36

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx = 5\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right) - \frac{11\sqrt{\frac{x-1}{x+1}} - \frac{40\left(\frac{x-1}{x+1}\right)^{3/2}}{3} + 5\left(\frac{x-1}{x+1}\right)^{5/2}}{\frac{3\,(x-1)}{x+1} - \frac{3\,(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1}$$

input
```
int((x + 1)^2/((x - 1)/(x + 1))^(1/2),x)
```

output
```
5*atanh(((x - 1)/(x + 1))^(1/2)) - (11*((x - 1)/(x + 1))^(1/2) - (40*((x -
 1)/(x + 1))^(3/2))/3 + 5*((x - 1)/(x + 1))^(5/2))/((3*(x - 1))/(x + 1) -
(3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx=\frac{\sqrt{x+1}\,\sqrt{x-1}\,x^2}{3}+\frac{3\sqrt{x+1}\,\sqrt{x-1}\,x}{2}+\frac{11\sqrt{x+1}\,\sqrt{x-1}}{3}+5\log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right)$$

input
```
int(1/((x-1)/(1+x))^(1/2)*(1+x)^2,x)
```

output
```
(2*sqrt(x + 1)*sqrt(x - 1)*x**2 + 9*sqrt(x + 1)*sqrt(x - 1)*x + 22*sqrt(x
+ 1)*sqrt(x - 1) + 30*log((sqrt(x - 1) + sqrt(x + 1))/sqrt(2)))/6
```

3.309 $\int e^{\coth^{-1}(x)}(1-x)^2 x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 71

$$\int e^{\coth^{-1}(x)}(1-x)^2 x\,dx = -\frac{1}{8}\sqrt{1-\frac{1}{x^2}}x^2-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3 +\frac{1}{4}\sqrt{1-\frac{1}{x^2}}x^4-\frac{1}{8}\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
-1/8*(1-1/x^2)^(1/2)*x^2-1/3*(1-1/x^2)^(3/2)*x^3+1/4*(1-1/x^2)^(1/2)*x^4-1
/8*arctanh((1-1/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int e^{\coth^{-1}(x)}(1-x)^2 x\,dx = \frac{1}{24}\sqrt{1-\frac{1}{x^2}}x(8-3x-8x^2+6x^3)-\frac{1}{8}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCoth[x]*(1 - x)^2*x,x]
```

output
```
(Sqrt[1 - x^(-2)]*x*(8 - 3*x - 8*x^2 + 6*x^3))/24 - Log[(1 + Sqrt[1 - x^(-2)])*x]/8
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {6728, 25, 539, 534, 243, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^2 x e^{\coth^{-1}(x)} \, dx$$

$$\Big\downarrow \text{6728}$$

$$\int -\sqrt{1-\frac{1}{x^2}}\left(1-\frac{1}{x}\right)x^5d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-\int \sqrt{1-\frac{1}{x^2}}\left(1-\frac{1}{x}\right)x^5d\frac{1}{x}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{4}\int \sqrt{1-\frac{1}{x^2}}\left(4-\frac{1}{x}\right)x^4d\frac{1}{x}+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{4}\left(-\int \sqrt{1-\frac{1}{x^2}}x^3d\frac{1}{x}-\frac{4}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3\right)+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \sqrt{1-\frac{1}{x^2}}x^2d\frac{1}{x^2}-\frac{4}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3\right)+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x^2}+\sqrt{1-\frac{1}{x^2}}x\right)-\frac{4}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3\right)+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4$$

$$\Big\downarrow 73$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\sqrt{1-\frac{1}{x^2}}x-\int \frac{1}{1-\sqrt{1-\frac{1}{x^2}}}d\sqrt{1-\frac{1}{x^2}}\right)-\frac{4}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3\right)+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4$$

$$\Big\downarrow 219$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\sqrt{1-\frac{1}{x^2}}x-\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)\right)-\frac{4}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3\right)+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4$$

input
```
Int[E^ArcCoth[x]*(1 - x)^2*x,x]
```

output
```
((1 - x^(-2))^(3/2)*x^4)/4 + ((-4*(1 - x^(-2))^(3/2)*x^3)/3 + (Sqrt[1 - x^
(-2)]*x - ArcTanh[Sqrt[1 - x^(-2)]])/2)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6728
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_))^(p_.), x_S
ymbol] :> Simp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + p + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && In
tegerQ[p] && IntegerQ[n] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

method	result	size
default	$-\frac{(x-1)\left(-6x(x^2-1)^{\frac{3}{2}}+8((x-1)(1+x))^{\frac{3}{2}}-3x\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)\right)}{24\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	70
risch	$\frac{(6x^3-8x^2-3x+8)(x-1)}{24\sqrt{\frac{x-1}{1+x}}}-\frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{8\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	70
trager	$\frac{(1+x)(6x^3-8x^2-3x+8)\sqrt{-\frac{1-x}{1+x}}}{24}+\frac{\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)}{8}$	74

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^2*x,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(x-1)*(-6*x*(x^2-1)^(3/2)+8*((x-1)*(1+x))^(3/2)-3*x*(x^2-1)^(1/2)+3*
ln(x+(x^2-1)^(1/2)))/((x-1)/(1+x))^(1/2)/((x-1)*(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx = \frac{1}{24}\left(6\,x^4-2\,x^3-11\,x^2+5\,x+8\right)\sqrt{\frac{x-1}{x+1}} - \frac{1}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+\frac{1}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="fricas")
```

output
```
1/24*(6*x^4 - 2*x^3 - 11*x^2 + 5*x + 8)*sqrt((x - 1)/(x + 1)) - 1/8*log(sq
rt((x - 1)/(x + 1)) + 1) + 1/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx = \int \frac{x(x-1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*(1-x)**2*x,x)
```

output
```
Integral(x*(x - 1)**2/sqrt((x - 1)/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(55) = 110$.

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.94

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx = -\frac{3\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}} + 53\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}} - 11\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}} + 3\sqrt{\frac{x-1}{x+1}}}{12\left(\frac{4(x-1)}{x+1} - \frac{6(x-1)^2}{(x+1)^2} + \frac{4(x-1)^3}{(x+1)^3} - \frac{(x-1)^4}{(x+1)^4} - 1\right)} - \frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}} + 1\right) + \frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}} - 1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="maxima")
```

output
```
-1/12*(3*((x - 1)/(x + 1))^(7/2) + 53*((x - 1)/(x + 1))^(5/2) - 11*((x - 1
)/(x + 1))^(3/2) + 3*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)
^2/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) - 1/8*log(
sqrt((x - 1)/(x + 1)) + 1) + 1/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx = \frac{1}{24}\left(\left(2x\left(\frac{3x}{\operatorname{sgn}(x+1)} - \frac{4}{\operatorname{sgn}(x+1)}\right) - \frac{3}{\operatorname{sgn}(x+1)}\right)x + \frac{8}{\operatorname{sgn}(x+1)}\right)\sqrt{x^2-1} + \frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{8\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="giac")
```

output
```
1/24*((2*x*(3*x/sgn(x + 1) - 4/sgn(x + 1)) - 3/sgn(x + 1))*x + 8/sgn(x + 1
))*sqrt(x^2 - 1) + 1/8*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.71 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.66

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx=\frac{\frac{\sqrt{\frac{x-1}{x+1}}}{4}-\frac{11\left(\frac{x-1}{x+1}\right)^{3/2}}{12}+\frac{53\left(\frac{x-1}{x+1}\right)^{5/2}}{12}+\frac{\left(\frac{x-1}{x+1}\right)^{7/2}}{4}}{\frac{6(x-1)^2}{(x+1)^2}-\frac{4(x-1)}{x+1}-\frac{4(x-1)^3}{(x+1)^3}+\frac{(x-1)^4}{(x+1)^4}+1}-\frac{\operatorname{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4}$$

input
```
int((x*(x - 1)^2)/((x - 1)/(x + 1))^(1/2),x)
```

output
```
(((x - 1)/(x + 1))^(1/2)/4 - (11*((x - 1)/(x + 1))^(3/2))/12 + (53*((x - 1
)/(x + 1))^(5/2))/12 + ((x - 1)/(x + 1))^(7/2)/4)/((6*(x - 1)^2)/(x + 1)^2
 - (4*(x - 1))/(x + 1) - (4*(x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1
) - atanh(((x - 1)/(x + 1))^(1/2))/4
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx=\frac{\sqrt{x+1}\,\sqrt{x-1}\,x^3}{4}-\frac{\sqrt{x+1}\,\sqrt{x-1}\,x^2}{3}-\frac{\sqrt{x+1}\,\sqrt{x-1}\,x}{8}+\frac{\sqrt{x+1}\,\sqrt{x-1}}{3}-\frac{\log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right)}{4}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^2*x,x)
```

output
```
(6*sqrt(x + 1)*sqrt(x - 1)*x**3 - 8*sqrt(x + 1)*sqrt(x - 1)*x**2 - 3*sqrt(
x + 1)*sqrt(x - 1)*x + 8*sqrt(x + 1)*sqrt(x - 1) - 6*log((sqrt(x - 1) + sq
rt(x + 1))/sqrt(2)))/24
```

3.310 $\int e^{\coth^{-1}(x)}(1-x)^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 53

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx=-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}x^2+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{2}\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
-1/2*(1-1/x^2)^(1/2)*x^2+1/3*(1-1/x^2)^(3/2)*x^3+1/2*arctanh((1-1/x^2)^(1/
2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx=\frac{1}{6}\sqrt{1-\frac{1}{x^2}}x\left(-2-3x+2x^2\right)+\frac{1}{2}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCoth[x]*(1 - x)^2,x]
```

output
```
(Sqrt[1 - x^(-2)]*x*(-2 - 3*x + 2*x^2))/6 + Log[(1 + Sqrt[1 - x^(-2)])*x]/
2
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6724, 25, 534, 243, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^2 e^{\coth^{-1}(x)} \, dx$$

$$\Big\downarrow \text{6724}$$

$$\int -\sqrt{1-\frac{1}{x^2}}\left(1-\frac{1}{x}\right) x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-\int \sqrt{1-\frac{1}{x^2}}\left(1-\frac{1}{x}\right) x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{534}$$

$$\int \sqrt{1-\frac{1}{x^2}} x^3 d\frac{1}{x}+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \sqrt{1-\frac{1}{x^2}} x^2 d\frac{1}{x^2}+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x^2}-\sqrt{1-\frac{1}{x^2}} x\right)+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\int \frac{1}{1-\sqrt{1-\frac{1}{x^2}}}d\sqrt{1-\frac{1}{x^2}}-\sqrt{1-\frac{1}{x^2}} x\right)+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)-\sqrt{1-\frac{1}{x^2}} x\right)+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3$$

input
```
Int[E^ArcCoth[x]*(1 - x)^2,x]
```

output
```
((1 - x^(-2))^(3/2)*x^3)/3 + (-(Sqrt[1 - x^(-2)]*x) + ArcTanh[Sqrt[1 - x^(
-2)]])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{(x-1)\left(2((x-1)(1+x))^{\frac{3}{2}}-3x\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)\right)}{6\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	60
risch	$\frac{(2x^2-3x-2)(x-1)}{6\sqrt{\frac{x-1}{1+x}}}+\frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{2\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	65
trager	$\frac{(1+x)(2x^2-3x-2)\sqrt{-\frac{1-x}{1+x}}}{6}-\frac{\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)}{2}$	69

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(x-1)*(2*((x-1)*(1+x))^(3/2)-3*x*(x^2-1)^(1/2)+3*ln(x+(x^2-1)^(1/2)))/
((x-1)/(1+x))^(1/2)/((x-1)*(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx=\frac{1}{6}\left(2\,x^3-x^2-5\,x-2\right)\sqrt{\frac{x-1}{x+1}}+\frac{1}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^2,x, algorithm="fricas")
```

output
```
1/6*(2*x^3 - x^2 - 5*x - 2)*sqrt((x - 1)/(x + 1)) + 1/2*log(sqrt((x - 1)/(
x + 1)) + 1) - 1/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx=\int\frac{(x-1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*(1-x)**2,x)
```

output
```
Integral((x - 1)**2/sqrt((x - 1)/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(41) = 82.

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.11

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx=-\frac{3\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}+8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-3\sqrt{\frac{x-1}{x+1}}}{3\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^2,x, algorithm="maxima")
```

output
```
-1/3*(3*((x - 1)/(x + 1))^(5/2) + 8*((x - 1)/(x + 1))^(3/2) - 3*sqrt((x -
1)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1
)^3 - 1) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1) - 1/2*log(sqrt((x - 1)/(x +
1)) - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx = \frac{1}{6}\sqrt{x^2-1}\left(x\left(\frac{2x}{\operatorname{sgn}(x+1)} - \frac{3}{\operatorname{sgn}(x+1)}\right) - \frac{2}{\operatorname{sgn}(x+1)}\right) - \frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{2\operatorname{sgn}(x+1)}$$

input

```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^2,x, algorithm="giac")
```

output

```
1/6*sqrt(x^2 - 1)*(x*(2*x/sgn(x + 1) - 3/sgn(x + 1)) - 2/sgn(x + 1)) - 1/2
*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx = \operatorname{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right) - \frac{\frac{8\left(\frac{x-1}{x+1}\right)^{3/2}}{3} - \sqrt{\frac{x-1}{x+1}} + \left(\frac{x-1}{x+1}\right)^{5/2}}{\frac{3(x-1)}{x+1} - \frac{3(x-1)^2}{(x+1)^2} + \frac{(x-1)^3}{(x+1)^3} - 1}$$

input

```
int((x - 1)^2/((x - 1)/(x + 1))^(1/2),x)
```

output

```
atanh(((x - 1)/(x + 1))^(1/2)) - ((8*((x - 1)/(x + 1))^(3/2))/3 - ((x - 1)
/(x + 1))^(1/2) + ((x - 1)/(x + 1))^(5/2))/((3*(x - 1))/(x + 1) - (3*(x -
1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int e^{\coth^{-1}(x)}(1-x)^2\,dx = \frac{\sqrt{x+1}\,\sqrt{x-1}\,x^2}{3} - \frac{\sqrt{x+1}\,\sqrt{x-1}\,x}{2} - \frac{\sqrt{x+1}\,\sqrt{x-1}}{3} + \log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right)$$

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^2,x)
```

output
```
(2*sqrt(x + 1)*sqrt(x - 1)*x**2 - 3*sqrt(x + 1)*sqrt(x - 1)*x - 2*sqrt(x +
 1)*sqrt(x - 1) + 6*log((sqrt(x - 1) + sqrt(x + 1))/sqrt(2)))/6
```

3.311 $\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = \sqrt{1-\frac{1}{x^2}}x$$

output
```
(1-1/x^2)^(1/2)*x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = x\sqrt{\frac{-1+x^2}{x^2}}$$

input
```
Integrate[(E^ArcCoth[x]*x)/(1 + x),x]
```

output
```
x*Sqrt[(-1 + x^2)/x^2]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6729, 106}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(x)}}{x+1} dx$$
$$\Big\downarrow \text{6729}$$
$$-\int \frac{x^2}{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}}d\frac{1}{x}$$
$$\Big\downarrow \text{106}$$
$$\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x$$

input
```
Int[(E^ArcCoth[x]*x)/(1 + x),x]
```

output
```
Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x
```

Defintions of rubi rules used

rule 106
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f, m, n,
p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

rule 6729
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-d^p)*(e*x)^m*(1/x)^m   Subst[Int[((1 + c*(x/d))^p*
((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[
{a, c, d, e, m, n}, x] && EqQ[a^2*c^2 - d^2, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

method	result	size
gosper	$\frac{x-1}{\sqrt{\frac{x-1}{1+x}}}$	16
risch	$\frac{x-1}{\sqrt{\frac{x-1}{1+x}}}$	16
orering	$\frac{x-1}{\sqrt{\frac{x-1}{1+x}}}$	16
trager	$(1+x)\sqrt{-\frac{1-x}{1+x}}$	19
default	$\frac{(x-1)\sqrt{x^2-1}}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	32

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1+x),x,method=_RETURNVERBOSE)
```

output
```
(x-1)/((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = (x+1)\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1+x),x, algorithm="fricas")
```

output
```
(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*x/(1+x),x)
```

output
```
Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1+x),x, algorithm="maxima")
```

output
```
-2*sqrt((x - 1)/(x + 1))/((x - 1)/(x + 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = \frac{\sqrt{x^2-1}}{\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1+x),x, algorithm="giac")
```

output
```
sqrt(x^2 - 1)/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

input
```
int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)),x)
```

output
```
-(2*((x - 1)/(x + 1))^(1/2))/((x - 1)/(x + 1) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = \sqrt{x+1}\,\sqrt{x-1}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1+x),x)
```

output
```
sqrt(x + 1)*sqrt(x - 1)
```

3.312 $\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 12

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = \operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
arctanh((1-1/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = \log\left(x\left(1+\sqrt{\frac{-1+x^2}{x^2}}\right)\right)$$

input
```
Integrate[E^ArcCoth[x]/(1 + x),x]
```

output
```
Log[x*(1 + Sqrt[(-1 + x^2)/x^2])]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6725, 103, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(x)}}{x+1} dx$$

$$\Big\downarrow \text{6725}$$

$$-\int \frac{x}{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{103}$$

$$\int \frac{1}{1-\frac{1}{x^2}}d\left(\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\right)$$

$$\Big\downarrow \text{219}$$

$$\text{arctanh}\left(\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\right)$$

input
```
Int[E^ArcCoth[x]/(1 + x),x]
```

output
```
ArcTanh[Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]]
```

Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(10) = 20.

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.92

method	result	size
default	$\frac{(x-1)\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}}$	35
trager	$-\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)$	39

input
```
int(1/((x-1)/(1+x))^(1/2)/(1+x),x,method=_RETURNVERBOSE)
```

output
```
1/((x-1)/(1+x))^(1/2)*(x-1)/((x-1)*(1+x))^(1/2)*ln(x+(x^2-1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(10) = 20.

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.58

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx=\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1+x),x, algorithm="fricas")
```

output
```
log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 2.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = -\log\left(\sqrt{1-\frac{2}{x+1}}-1\right)+\log\left(\sqrt{1-\frac{2}{x+1}}+1\right)$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)/(1+x),x)
```

output
```
-log(sqrt(1 - 2/(x + 1)) - 1) + log(sqrt(1 - 2/(x + 1)) + 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(10) = 20$.

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.58

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = \log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1+x),x, algorithm="maxima")
```

output
```
log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = -\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1+x),x, algorithm="giac")
```

output
```
-log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)$$

input
```
int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)),x)
```

output
```
2*atanh(((x - 1)/(x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx = 2\log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right)$$

input
```
int(1/((x-1)/(1+x))^(1/2)/(1+x),x)
```

output
```
2*log((sqrt(x - 1) + sqrt(x + 1))/sqrt(2))
```

3.313 $\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx=\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}-\sqrt{1-\frac{1}{x^2}}x-2\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
2*(1-1/x^2)^(1/2)/(1-1/x)-(1-1/x^2)^(1/2)*x-2*arctanh((1-1/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx=-\frac{\sqrt{1-\frac{1}{x^2}}(-3+x)x}{-1+x}-2\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[(E^ArcCoth[x]*x)/(1 - x),x]
```

output
```
-((Sqrt[1 - x^(-2)]*(-3 + x)*x)/(-1 + x)) - 2*Log[(1 + Sqrt[1 - x^(-2)])*x
]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6728, 564, 534, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(x)}}{1-x} \, dx$$

$$\Big\downarrow \text{6728}$$

$$\int \frac{\sqrt{1-\frac{1}{x^2}}x^2}{\left(1-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{564}$$

$$\int \frac{\left(1+\frac{2}{x}\right)x^2}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}$$

$$\Big\downarrow \text{534}$$

$$2\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}-\sqrt{1-\frac{1}{x^2}}x+\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}$$

$$\Big\downarrow \text{243}$$

$$\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x^2}-\sqrt{1-\frac{1}{x^2}}x+\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}$$

$$\Big\downarrow \text{73}$$

$$-2\int \frac{1}{1-\sqrt{1-\frac{1}{x^2}}}d\sqrt{1-\frac{1}{x^2}}-\sqrt{1-\frac{1}{x^2}}x+\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}$$

$$\Big\downarrow \text{219}$$

$$-2\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)-\sqrt{1-\frac{1}{x^2}}x+\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}$$

input
```
Int[(E^ArcCoth[x]*x)/(1 - x),x]
```

output
```
(2*Sqrt[1 - x^(-2)])/(1 - x^(-1)) - Sqrt[1 - x^(-2)]*x - 2*ArcTanh[Sqrt[1
- x^(-2)]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 6728
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_))^(p_.), x_S
ymbol] :> Simp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + p + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && In
tegerQ[p] && IntegerQ[n] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

method	result	size
risch	$-\frac{x^{2}-2x-3}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}-\frac{2\ln\left(x+\sqrt{x^{2}-1}\right)\sqrt{(x-1)(1+x)}}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	65
trager	$-\frac{(1+x)(-3+x)\sqrt{-\frac{1-x}{1+x}}}{x-1}+2\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)$	67
default	$\frac{(x^{2}-1)^{\frac{3}{2}}-2x^{2}\sqrt{x^{2}-1}-2\ln\left(x+\sqrt{x^{2}-1}\right)x^{2}+4x\sqrt{x^{2}-1}+4\ln\left(x+\sqrt{x^{2}-1}\right)x-2\sqrt{x^{2}-1}-2\ln\left(x+\sqrt{x^{2}-1}\right)}{(x-1)\sqrt{(x-1)(1+x)}\,\sqrt{\frac{x-1}{1+x}}}$	106

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1-x),x,method=_RETURNVERBOSE)
```

output
```
-(x^2-2*x-3)/((x-1)/(1+x))^(1/2)/(1+x)-2*ln(x+(x^2-1)^(1/2))/((x-1)/(1+x))
^(1/2)/(1+x)*((x-1)*(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = -\frac{2\,(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-2\,(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)+(x^{2}-2\,x-3)\sqrt{\frac{x-1}{x+1}}}{x-1}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1-x),x, algorithm="fricas")
```

output
```
-(2*(x - 1)*log(sqrt((x - 1)/(x + 1)) + 1) - 2*(x - 1)*log(sqrt((x - 1)/(x
 + 1)) - 1) + (x^2 - 2*x - 3)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = -\int \frac{x}{x\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}-\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*x/(1-x),x)
```

output
```
-Integral(x/(x*sqrt(x/(x + 1) - 1/(x + 1)) - sqrt(x/(x + 1) - 1/(x + 1))),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = \frac{2\left(\frac{2(x-1)}{x+1}-1\right)}{\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-\sqrt{\frac{x-1}{x+1}}} - 2\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) + 2\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1-x),x, algorithm="maxima")
```

output
```
2*(2*(x - 1)/(x + 1) - 1)/(((x - 1)/(x + 1))^(3/2) - sqrt((x - 1)/(x + 1))
) - 2*log(sqrt((x - 1)/(x + 1)) + 1) + 2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = \frac{2\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)} - \frac{\sqrt{x^2-1}}{\operatorname{sgn}(x+1)} - \frac{4}{\left(x-\sqrt{x^2-1}-1\right)\operatorname{sgn}(x+1)} - 2\operatorname{sgn}(x+1)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1-x),x, algorithm="giac")
```

output
```
2*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1) - sqrt(x^2 - 1)/sgn(x + 1) - 4/(
(x - sqrt(x^2 - 1) - 1)*sgn(x + 1)) - 2*sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.64 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = -\frac{2\,x+8\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)\,\sqrt{\frac{x-1}{x+1}}-6}{2\,\sqrt{\frac{x-1}{x+1}}}$$

input
```
int(-x/(((x - 1)/(x + 1))^(1/2)*(x - 1)),x)
```

output
```
-(2*x + 8*atanh(((x - 1)/(x + 1))^(1/2))*((x - 1)/(x + 1))^(1/2) - 6)/(2*(
(x - 1)/(x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx = \frac{-8\sqrt{x-1}\,\log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right)+5\sqrt{x-1}-2\sqrt{x+1}\,x+6\sqrt{x+1}}{2\sqrt{x-1}}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1-x),x)
```

output
```
( - 8*sqrt(x - 1)*log((sqrt(x - 1) + sqrt(x + 1))/sqrt(2)) + 5*sqrt(x - 1)
 - 2*sqrt(x + 1)*x + 6*sqrt(x + 1))/(2*sqrt(x - 1))
```

3.314 $\int \frac{e^{\coth^{-1}(x)}}{1-x} dx$

Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{e^{\coth^{-1}(x)}}{1-x} dx = \frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}} - \operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
2*(1-1/x^2)^(1/2)/(1-1/x)-arctanh((1-1/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{e^{\coth^{-1}(x)}}{1-x} dx = \frac{2\sqrt{1-\frac{1}{x^2}}x}{-1+x} - \log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCoth[x]/(1 - x),x]
```

output
```
(2*Sqrt[1 - x^(-2)]*x)/(-1 + x) - Log[(1 + Sqrt[1 - x^(-2)])*x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6724, 564, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(x)}}{1-x} dx$$

$$\Big\downarrow \text{6724}$$

$$\int \frac{\sqrt{1-\frac{1}{x^2}}x}{\left(1-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{564}$$

$$\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x^2}+\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}-\int \frac{1}{1-\sqrt{1-\frac{1}{x^2}}}d\sqrt{1-\frac{1}{x^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{1-\frac{1}{x^2}}}{1-\frac{1}{x}}-\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

input
```
Int[E^ArcCoth[x]/(1 - x),x]
```

output
```
(2*Sqrt[1 - x^(-2)])/(1 - x^(-1)) - ArcTanh[Sqrt[1 - x^(-2)]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

method	result	size
risch	$\frac{2}{\sqrt{\frac{x-1}{1+x}}}-\frac{\ln\left(x+\sqrt{x^{2}-1}\right)\sqrt{(x-1)(1+x)}}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	52
trager	$\frac{2\sqrt{-\frac{1-x}{1+x}}\,(1+x)}{x-1}-\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)$	61
default	$\frac{(x^{2}-1)^{\frac{3}{2}}-x^{2}\sqrt{x^{2}-1}-\ln\left(x+\sqrt{x^{2}-1}\right)x^{2}+2x\sqrt{x^{2}-1}+2\ln\left(x+\sqrt{x^{2}-1}\right)x-\sqrt{x^{2}-1}-\ln\left(x+\sqrt{x^{2}-1}\right)}{(x-1)\sqrt{(x-1)(1+x)}\,\sqrt{\frac{x-1}{1+x}}}$	106

input
```
int(1/((x-1)/(1+x))^(1/2)/(1-x),x,method=_RETURNVERBOSE)
```

output
```
2/((x-1)/(1+x))^(1/2)-ln(x+(x^2-1)^(1/2))/((x-1)/(1+x))^(1/2)/(1+x)*((x-1)
*(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx=-\frac{(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)-2\,(x+1)\sqrt{\frac{x-1}{x+1}}}{x-1}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1-x),x, algorithm="fricas")
```

output
```
-((x - 1)*log(sqrt((x - 1)/(x + 1)) + 1) - (x - 1)*log(sqrt((x - 1)/(x + 1
)) - 1) - 2*(x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx = -\int \frac{1}{x\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}-\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)/(1-x),x)
```

output
```
-Integral(1/(x*sqrt(x/(x + 1) - 1/(x + 1)) - sqrt(x/(x + 1) - 1/(x + 1))),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx = \frac{2}{\sqrt{\frac{x-1}{x+1}}} - \log\left(\sqrt{\frac{x-1}{x+1}}+1\right) + \log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1-x),x, algorithm="maxima")
```

output
```
2/sqrt((x - 1)/(x + 1)) - log(sqrt((x - 1)/(x + 1)) + 1) + log(sqrt((x - 1
)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx = \frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\mathrm{sgn}\,(x+1)} - \frac{4}{\left(x-\sqrt{x^2-1}-1\right)\mathrm{sgn}\,(x+1)} - 2\,\mathrm{sgn}(x+1)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1-x),x, algorithm="giac")
```

output
```
log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1) - 4/((x - sqrt(x^2 - 1) - 1)*sgn(x
 + 1)) - 2*sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.55 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx = \frac{2}{\sqrt{\frac{x-1}{x+1}}} - 2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)$$

input
```
int(-1/(((x - 1)/(x + 1))^(1/2)*(x - 1)),x)
```

output
```
2/((x - 1)/(x + 1))^(1/2) - 2*atanh(((x - 1)/(x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx = \frac{-2\sqrt{x-1}\,\log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right) + 2\sqrt{x-1} + 2\sqrt{x+1}}{\sqrt{x-1}}$$

input
```
int(1/((x-1)/(1+x))^(1/2)/(1-x),x)
```

output
```
(2*( - sqrt(x - 1)*log((sqrt(x - 1) + sqrt(x + 1))/sqrt(2)) + sqrt(x - 1)
+ sqrt(x + 1)))/sqrt(x - 1)
```

3.315 $\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = -\frac{\sqrt{1-\frac{1}{x^2}}}{1+\frac{1}{x}}+\operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
-(1-1/x^2)^(1/2)/(1+1/x)+arctanh((1-1/x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = -\frac{\sqrt{1-\frac{1}{x^2}}x}{1+x}+\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[(E^ArcCoth[x]*x)/(1 + x)^2,x]
```

output
```
-((Sqrt[1 - x^(-2)]*x)/(1 + x)) + Log[(1 + Sqrt[1 - x^(-2)])*x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6729, 107, 103, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(x)}}{(x+1)^2}dx$$

$$\Big\downarrow \text{ 6729}$$

$$-\int \frac{x}{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 107}$$

$$-\int \frac{x}{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}}d\frac{1}{x}-\frac{\sqrt{1-\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

$$\Big\downarrow \text{ 103}$$

$$\int \frac{1}{1-\frac{1}{x^2}}d\left(\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\right)-\frac{\sqrt{1-\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

$$\Big\downarrow \text{ 219}$$

$$\text{arctanh}\left(\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\right)-\frac{\sqrt{1-\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

input
```
Int[(E^ArcCoth[x]*x)/(1 + x)^2,x]
```

output
```
-(Sqrt[1 - x^(-1)]/Sqrt[1 + x^(-1)]) + ArcTanh[Sqrt[1 - x^(-1)]*Sqrt[1 + x
^(-1)]]
```

Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6729
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-d^p)*(e*x)^m*(1/x)^m   Subst[Int[((1 + c*(x/d))^p*
((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[
{a, c, d, e, m, n}, x] && EqQ[a^2*c^2 - d^2, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

method	result	size
trager	$-\sqrt{-\frac{1-x}{1+x}}-\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)$	56
risch	$-\frac{x-1}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}+\frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(1+x)}}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	59
default	$\frac{(x-1)\left((x^2-1)^{\frac{3}{2}}-x^2\sqrt{x^2-1}+2\ln\left(x+\sqrt{x^2-1}\right)x^2-2x\sqrt{x^2-1}+4\ln\left(x+\sqrt{x^2-1}\right)x-\sqrt{x^2-1}+2\ln\left(x+\sqrt{x^2-1}\right)\right)}{2\sqrt{\frac{x-1}{1+x}}\,\sqrt{(x-1)(1+x)}\,(1+x)^2}$	110

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(1-x)/(1+x))^(1/2)-ln(-(-(1-x)/(1+x))^(1/2)*x-(-(1-x)/(1+x))^(1/2)+x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = -\sqrt{\frac{x-1}{x+1}} + \log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="fricas")
```

output
```
-sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)
/(x + 1)) - 1)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*x/(1+x)**2,x)
```

output
```
Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = -\sqrt{\frac{x-1}{x+1}} + \log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="maxima")
```

output
```
-sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)
/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = -\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)} - \frac{2}{\left(x-\sqrt{x^2-1}+1\right)\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="giac")
```

output
```
-log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1) - 2/((x - sqrt(x^2 - 1) + 1)*sgn(
x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right) - \sqrt{\frac{x-1}{x+1}}$$

input
```
int(x/((((x - 1)/(x + 1))^(1/2)*(x + 1)^2),x)
```

output
```
2*atanh(((x - 1)/(x + 1))^(1/2)) - ((x - 1)/(x + 1))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx = \frac{-\sqrt{x+1}\,\sqrt{x-1}+2\log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right)x+2\log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right)-x-1}{x+1}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1+x)^2,x)
```

output
```
( - sqrt(x + 1)*sqrt(x - 1) + 2*log((sqrt(x - 1) + sqrt(x + 1))/sqrt(2))*x
 + 2*log((sqrt(x - 1) + sqrt(x + 1))/sqrt(2)) - x - 1)/(x + 1)
```

3.316 $\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2} \, dx = \frac{\sqrt{1-\frac{1}{x^2}}}{1+\frac{1}{x}}$$

output
```
(1-1/x^2)^(1/2)/(1+1/x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2} \, dx = \frac{\sqrt{1-\frac{1}{x^2}}x}{1+x}$$

input
```
Integrate[E^ArcCoth[x]/(1 + x)^2,x]
```

output
```
(Sqrt[1 - x^(-2)]*x)/(1 + x)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6725, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(x)}}{(x+1)^2}dx$$

$$\Big\downarrow \text{ 6725}$$

$$-\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{\sqrt{1-\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

input
```
Int[E^ArcCoth[x]/(1 + x)^2,x]
```

output
```
Sqrt[1 - x^(-1)]/Sqrt[1 + x^(-1)]
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6725
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^p   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a
)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2,
 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$\sqrt{\frac{x-1}{1+x}}$	12
trager	$\sqrt{-\frac{1-x}{1+x}}$	15
gosper	$\frac{x-1}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	21
risch	$\frac{x-1}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	21
orering	$\frac{x-1}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$	21
default	$\frac{\sqrt{x^2-1}\,(x-1)}{(1+x)\sqrt{(x-1)(1+x)}\,\sqrt{\frac{x-1}{1+x}}}$	37

input
```
int(1/((x-1)/(1+x))^(1/2)/(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1+x)^2,x, algorithm="fricas")
```

output
```
sqrt((x - 1)/(x + 1))
```

Sympy [A] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)/(1+x)**2,x)
```

output
```
sqrt((x - 1)/(x + 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1+x)^2,x, algorithm="maxima")
```

output
```
sqrt((x - 1)/(x + 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \frac{2}{\left(x-\sqrt{x^2-1}+1\right)\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1+x)^2,x, algorithm="giac")
```

output
```
2/((x - sqrt(x^2 - 1) + 1)*sgn(x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \sqrt{1-\frac{2}{x+1}}$$

input
```
int(1/((((x - 1)/(x + 1))^(1/2)*(x + 1)^2),x)
```

output
```
(1 - 2/(x + 1))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx = \frac{\sqrt{x+1}\,\sqrt{x-1}+x+1}{x+1}$$

input
```
int(1/((x-1)/(1+x))^(1/2)/(1+x)^2,x)
```

output
```
(sqrt(x + 1)*sqrt(x - 1) + x + 1)/(x + 1)
```

3.317 $\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx=-\frac{2\sqrt{1-\frac{1}{x^2}}}{3\left(1-\frac{1}{x}\right)^2}-\frac{5\sqrt{1-\frac{1}{x^2}}}{3\left(1-\frac{1}{x}\right)}+\operatorname{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

output
```
-2/3*(1-1/x^2)^(1/2)/(1-1/x)^2-5*(1-1/x^2)^(1/2)/(3-3/x)+arctanh((1-1/x^2)
^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx=\frac{\sqrt{1-\frac{1}{x^2}}(5-7x)x}{3(-1+x)^2}+\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

input
```
Integrate[(E^ArcCoth[x]*x)/(1 - x)^2,x]
```

output
```
(Sqrt[1 - x^(-2)]*(5 - 7*x)*x)/(3*(-1 + x)^2) + Log[(1 + Sqrt[1 - x^(-2)])
*x]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {6728, 25, 570, 532, 25, 532, 27, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(x)}}{(1-x)^2} \, dx$$

$$\Big\downarrow \text{6728}$$

$$\int -\frac{\sqrt{1-\frac{1}{x^2}}x}{\left(1-\frac{1}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{1-\frac{1}{x^2}}x}{\left(1-\frac{1}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{570}$$

$$-\int \frac{\left(1+\frac{1}{x}\right)^3 x}{\left(1-\frac{1}{x^2}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{532}$$

$$\frac{1}{3}\int -\frac{\left(3+\frac{5}{x}\right)x}{\left(1-\frac{1}{x^2}\right)^{3/2}}d\frac{1}{x}-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}\int \frac{\left(3+\frac{5}{x}\right)x}{\left(1-\frac{1}{x^2}\right)^{3/2}}d\frac{1}{x}-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{1}{3}\left(\int -\frac{3x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}-\frac{\frac{5}{x}+3}{\sqrt{1-\frac{1}{x^2}}}\right)-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-3\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}-\frac{\frac{5}{x}+3}{\sqrt{1-\frac{1}{x^2}}}\right)-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{x}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x^2}-\frac{\frac{5}{x}+3}{\sqrt{1-\frac{1}{x^2}}}\right)-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(3\int \frac{1}{1-\sqrt{1-\frac{1}{x^2}}}d\sqrt{1-\frac{1}{x^2}}-\frac{\frac{5}{x}+3}{\sqrt{1-\frac{1}{x^2}}}\right)-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(3\text{arctanh}\left(\sqrt{1-\frac{1}{x^2}}\right)-\frac{\frac{5}{x}+3}{\sqrt{1-\frac{1}{x^2}}}\right)-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}$$

input
```
Int[(E^ArcCoth[x]*x)/(1 - x)^2,x]
```

output
```
(-4*(1 + x^(-1)))/(3*(1 - x^(-2))^(3/2)) + (-((3 + 5/x)/Sqrt[1 - x^(-2)])
+ 3*ArcTanh[Sqrt[1 - x^(-2)]])/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6728
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_))^(p_.), x_S
ymbol] :> Simp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + p + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && In
tegerQ[p] && IntegerQ[n] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

method	result
trager	$-\frac{(1+x)(7x-5)\sqrt{-\frac{1-x}{1+x}}}{3(x-1)^{2}}-\ln\left(-\sqrt{-\frac{1-x}{1+x}}\, x-\sqrt{-\frac{1-x}{1+x}}+x\right)$
risch	$-\frac{7x^{2}+2x-5}{3(x-1)\sqrt{\frac{x-1}{1+x}}\,(1+x)}+\frac{\ln\left(x+\sqrt{x^{2}-1}\right)\sqrt{(x-1)(1+x)}}{\sqrt{\frac{x-1}{1+x}}\,(1+x)}$
default	$-\frac{3x\left(x^{2}-1\right)^{\frac{3}{2}}-3\sqrt{x^{2}-1}\,x^{3}-3\ln\left(x+\sqrt{x^{2}-1}\right)x^{3}-2\left(x^{2}-1\right)^{\frac{3}{2}}+9x^{2}\sqrt{x^{2}-1}+9\ln\left(x+\sqrt{x^{2}-1}\right)x^{2}-9x\sqrt{x^{2}-1}-9\ln\left(x+\sqrt{x^{2}-1}\right)x+3}{3(x-1)^{2}\sqrt{(x-1)(1+x)}\,\sqrt{\frac{x-1}{1+x}}}$

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1-x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(1+x)*(7*x-5)/(x-1)^2*(-(1-x)/(1+x))^(1/2)-ln(-(-(1-x)/(1+x))^(1/2)*x
-(-(1-x)/(1+x))^(1/2)+x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.38

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = \frac{3\left(x^{2}-2\,x+1\right)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-3\left(x^{2}-2\,x+1\right)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)-\left(7\,x^{2}+2\,x-5\right)\sqrt{\frac{x-1}{x+1}}}{3\left(x^{2}-2\,x+1\right)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="fricas")
```

output
```
1/3*(3*(x^2 - 2*x + 1)*log(sqrt((x - 1)/(x + 1)) + 1) - 3*(x^2 - 2*x + 1)*
log(sqrt((x - 1)/(x + 1)) - 1) - (7*x^2 + 2*x - 5)*sqrt((x - 1)/(x + 1)))/
(x^2 - 2*x + 1)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x-1)^2}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*x/(1-x)**2,x)
```

output
```
Integral(x/(sqrt((x - 1)/(x + 1))*(x - 1)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = -\frac{\frac{6\,(x-1)}{x+1}+1}{3\,\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}} + \log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="maxima")
```

output
```
-1/3*(6*(x - 1)/(x + 1) + 1)/((x - 1)/(x + 1))^(3/2) + log(sqrt((x - 1)/(x
 + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.30

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = -\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)} + \frac{2\left(9\,\left(x-\sqrt{x^2-1}\right)^2 - 12\,x + 12\,\sqrt{x^2-1} + 7\right)}{3\,\left(x-\sqrt{x^2-1}-1\right)^3\operatorname{sgn}(x+1)} + \frac{7}{3}\operatorname{sgn}(x+1)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="giac")
```

output
```
-log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1) + 2/3*(9*(x - sqrt(x^2 - 1))^2 -
12*x + 12*sqrt(x^2 - 1) + 7)/((x - sqrt(x^2 - 1) - 1)^3*sgn(x + 1)) + 7/3*
sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right) - \frac{\frac{2\,(x-1)}{x+1}+\frac{1}{3}}{\left(\frac{x-1}{x+1}\right)^{3/2}}$$

input
```
int(x/((((x - 1)/(x + 1))^(1/2)*(x - 1)^2),x)
```

output
```
2*atanh(((x - 1)/(x + 1))^(1/2)) - ((2*(x - 1))/(x + 1) + 1/3)/((x - 1)/(x
 + 1))^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx$$
$$= \frac{6\sqrt{x-1}\log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right)x - 6\sqrt{x-1}\log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right) + \sqrt{x-1}\,x - \sqrt{x-1} - 7\sqrt{x+1}\,x + 5\sqrt{x+1}}{3\sqrt{x-1}\,(x-1)}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1-x)^2,x)
```

output
```
(6*sqrt(x - 1)*log((sqrt(x - 1) + sqrt(x + 1))/sqrt(2))*x - 6*sqrt(x - 1)*
log((sqrt(x - 1) + sqrt(x + 1))/sqrt(2)) + sqrt(x - 1)*x - sqrt(x - 1) - 7
*sqrt(x + 1)*x + 5*sqrt(x + 1))/(3*sqrt(x - 1)*(x - 1))
```

3.318 $\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 24

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = -\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{3\left(1-\frac{1}{x}\right)^3}$$

output
```
-1/3*(1-1/x^2)^(3/2)/(1-1/x)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = -\frac{\sqrt{1-\frac{1}{x^2}}x(1+x)}{3(-1+x)^2}$$

input
```
Integrate[E^ArcCoth[x]/(1 - x)^2,x]
```

output
```
-1/3*(Sqrt[1 - x^(-2)]*x*(1 + x))/(-1 + x)^2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6724, 25, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2} \, dx$$

$$\Big\downarrow \; 6724$$

$$\int -\frac{\sqrt{1-\frac{1}{x^2}}}{\left(1-\frac{1}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\sqrt{1-\frac{1}{x^2}}}{\left(1-\frac{1}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \; 460$$

$$-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{3\left(1-\frac{1}{x}\right)^3}$$

input
```
Int[E^ArcCoth[x]/(1 - x)^2,x]
```

output
```
-1/3*(1 - x^(-2))^(3/2)/(1 - x^(-1))^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 6724
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[-d^n   Subst[Int[(d + c*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(p + 2)), x],
 x, 1/x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0] && IntegerQ[p] && In
tegerQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{1+x}{3(x-1)\sqrt{\frac{x-1}{1+x}}}$	22
trager	$-\frac{(1+x)^{2}\sqrt{-\frac{1-x}{1+x}}}{3(x-1)^{2}}$	27
orering	$-\frac{(x-1)(1+x)}{3\sqrt{\frac{x-1}{1+x}}\,(1-x)^{2}}$	27
risch	$-\frac{x^{2}+2x+1}{3\sqrt{\frac{x-1}{1+x}}\,(1+x)(x-1)}$	32
default	$-\frac{(x^{2}-1)^{\frac{3}{2}}}{3\sqrt{\frac{x-1}{1+x}}\,(x-1)^{2}\sqrt{(x-1)(1+x)}}$	35

input
```
int(1/((x-1)/(1+x))^(1/2)/(1-x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(1+x)/(x-1)/((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = -\frac{(x^2+2x+1)\sqrt{\frac{x-1}{x+1}}}{3(x^2-2x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1-x)^2,x, algorithm="fricas")
```

output
```
-1/3*(x^2 + 2*x + 1)*sqrt((x - 1)/(x + 1))/(x^2 - 2*x + 1)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = \int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,(x-1)^2}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)/(1-x)**2,x)
```

output
```
Integral(1/(sqrt((x - 1)/(x + 1))*(x - 1)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = -\frac{1}{3\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1-x)^2,x, algorithm="maxima")
```

output
```
-1/3/((x - 1)/(x + 1))^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(18) = 36$.

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = \frac{2\left(3\left(x-\sqrt{x^2-1}\right)^2+1\right)}{3\left(x-\sqrt{x^2-1}-1\right)^3\operatorname{sgn}(x+1)} + \frac{1}{3}\operatorname{sgn}(x+1)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1-x)^2,x, algorithm="giac")
```

output
```
2/3*(3*(x - sqrt(x^2 - 1))^2 + 1)/((x - sqrt(x^2 - 1) - 1)^3*sgn(x + 1)) +
 1/3*sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = -\frac{1}{3\left(\frac{x-1}{x+1}\right)^{3/2}}$$

input
```
int(1/(((x - 1)/(x + 1))^(1/2)*(x - 1)^2),x)
```

output
```
-1/(3*((x - 1)/(x + 1))^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = \frac{-\sqrt{x-1}\,x+\sqrt{x-1}-\sqrt{x+1}\,x-\sqrt{x+1}}{3\sqrt{x-1}\,(x-1)}$$

input
```
int(1/((x-1)/(1+x))^(1/2)/(1-x)^2,x)
```

output
```
( - sqrt(x - 1)*x + sqrt(x - 1) - sqrt(x + 1)*x - sqrt(x + 1))/(3*sqrt(x -
 1)*(x - 1))
```

3.319 $\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{16\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}-\frac{8\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}$$

output
```
16/105*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)-8/35*(1+1/a/
x)^(3/2)*x^2*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/7*(1+1/a/x)^(3/2)*x^3*(-
a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(8-4ax+3a^2x^2+15a^3x^3)}{105a^3\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*x^2*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(8 - 4*a*x + 3*a^2*x^2 + 15*a^3*x^3))
/(105*a^3*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6730, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-acx}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \ 6730$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 55$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{4\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{7a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 55$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{4\left(-\frac{2\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{5a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 48$$

$$-\frac{\sqrt{\frac{1}{x}}\left(-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}-\frac{4\left(\frac{4\left(\frac{1}{ax}+1\right)^{3/2}}{15a\left(\frac{1}{x}\right)^{3/2}}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^ArcCoth[a*x]*x^2*Sqrt[c - a*c*x],x]
```

output
```
-((((-4*((-2*(1 + 1/(a*x))^(3/2))/(5*(x^(-1))^(5/2)) + (4*(1 + 1/(a*x))^(3
/2))/(15*a*(x^(-1))^(3/2))))/(7*a) - (2*(1 + 1/(a*x))^(3/2))/(7*(x^(-1))^(
7/2)))*Sqrt[x^(-1)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{2(ax+1)(15a^2x^2-12ax+8)\sqrt{-acx+c}}{105a^3\sqrt{\frac{ax-1}{ax+1}}}$	49
orering	$\frac{2(ax+1)(15a^2x^2-12ax+8)\sqrt{-acx+c}}{105a^3\sqrt{\frac{ax-1}{ax+1}}}$	49
default	$\frac{2\sqrt{-c(ax-1)}\,(ax+1)(15a^2x^2-12ax+8)}{105\sqrt{\frac{ax-1}{ax+1}}\,a^3}$	50
risch	$-\frac{2c(ax-1)(15a^3x^3+3a^2x^2-4ax+8)}{105\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a^3}$	59

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
2/105*(a*x+1)*(15*a^2*x^2-12*a*x+8)*(-a*c*x+c)^(1/2)/a^3/((a*x-1)/(a*x+1))
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.49

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\,(15\,a^4x^4+18\,a^3x^3-a^2x^2+4\,ax+8)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^4x-a^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fri
cas")
```

output
```
2/105*(15*a^4*x^4 + 18*a^3*x^3 - a^2*x^2 + 4*a*x + 8)*sqrt(-a*c*x + c)*sqr
t((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \int \frac{x^2\sqrt{-c\,(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\left(15\,a^3\sqrt{-c}x^3+3\,a^2\sqrt{-c}x^2-4\,a\sqrt{-c}x+8\,\sqrt{-c}\right)\sqrt{ax+1}}{105\,a^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="max
ima")
```

output
```
2/105*(15*a^3*sqrt(-c)*x^3 + 3*a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x + 8*sqrt(
-c))*sqrt(a*x + 1)/a^3
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.72 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.41

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,(a\,x+1)^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,(15\,a^2\,x^2-12\,a\,x+8)}{105\,a^3\,(a\,x-1)}$$

input
```
int((x^2*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2)*(15*a^2*x^2 -
 12*a*x + 8))/(105*a^3*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.26

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c}\,\sqrt{ax+1}\,i(-15a^3x^3-3a^2x^2+4ax-8)}{105a^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a*x + 1)*i*( - 15*a**3*x**3 - 3*a**2*x**2 + 4*a*x - 8))/(1
05*a**3)
```

3.320 $\int e^{\coth^{-1}(ax)} x\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 92

$$\int e^{\coth^{-1}(ax)} x\sqrt{c-acx}\,dx = -\frac{4\left(1+\frac{1}{ax}\right)^{3/2} x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2} x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}$$

output
```
-4/15*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/5*(1+1/a/x)^(
3/2)*x^2*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(ax)} x\sqrt{c-acx}\,dx = \frac{2\sqrt{1+\frac{1}{ax}}(1+ax)(-2+3ax)\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*x*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[1 + 1/(a*x)]*(1 + a*x)*(-2 + 3*a*x)*Sqrt[c - a*c*x])/(15*a^2*Sqrt[
1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6730, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-acx}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{2\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{5a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{4\left(\frac{1}{ax}+1\right)^{3/2}}{15a\left(\frac{1}{x}\right)^{3/2}}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^ArcCoth[a*x]*x*Sqrt[c - a*c*x],x]
```

output
```
-((((-2*(1 + 1/(a*x))^(3/2))/(5*(x^(-1))^(5/2)) + (4*(1 + 1/(a*x))^(3/2))/
(15*a*(x^(-1))^(3/2)))*Sqrt[x^(-1)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.45

method	result	size
gosper	$\frac{2(ax+1)(3ax-2)\sqrt{-acx+c}}{15a^{2}\sqrt{\frac{ax-1}{ax+1}}}$	41
orering	$\frac{2(ax+1)(3ax-2)\sqrt{-acx+c}}{15a^{2}\sqrt{\frac{ax-1}{ax+1}}}$	41
default	$\frac{2\sqrt{-c(ax-1)}\,(ax+1)(3ax-2)}{15\sqrt{\frac{ax-1}{ax+1}}\,a^{2}}$	42
risch	$-\frac{2c(ax-1)\left(3a^{2}x^{2}+ax-2\right)}{15\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a^{2}}$	50

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(a*x+1)*(3*a*x-2)*(-a*c*x+c)^(1/2)/a^2/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\left(3a^3x^3+4a^2x^2-ax-2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^3x-a^2\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="frica
s")
```

output
```
2/15*(3*a^3*x^3 + 4*a^2*x^2 - a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*
x + 1))/(a^3*x - a^2)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\int\frac{x\sqrt{-c\left(ax-1\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(x*sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.45

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\left(3\,a^2\sqrt{-c}x^2+a\sqrt{-c}x-2\,\sqrt{-c}\right)\sqrt{ax+1}}{15\,a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="maxim
a")
```

output
```
2/15*(3*a^2*sqrt(-c)*x^2 + a*sqrt(-c)*x - 2*sqrt(-c))*sqrt(a*x + 1)/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\,c^2\left(\frac{2\,\sqrt{2}\sqrt{-c}}{ac}-\frac{3\,(acx+c)^2\sqrt{-acx-c}+5\,(-acx-c)^{\frac{3}{2}}c}{ac^3}\right)}{15\,a|c|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="giac"
)
```

output
```
2/15*c^2*(2*sqrt(2)*sqrt(-c)/(a*c) - (3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 5
*(-a*c*x - c)^(3/2)*c)/(a*c^3))/(a*abs(c)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.71 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\left(3\,a\,x-2\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,a^2\,\left(a\,x-1\right)}$$

input
```
int((x*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*(a*x + 1)^2*(3*a*x - 2)*((a*x - 1)/(a*x + 1))^(1/2))/
(15*a^2*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.30

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\sqrt{c}\,\sqrt{ax+1}\,i(-3a^2x^2-ax+2)}{15a^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a*x + 1)*i*( - 3*a**2*x**2 - a*x + 2))/(15*a**2)
```

3.321 $\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2e^{\coth^{-1}(ax)}(1+ax)\sqrt{c-acx}}{3a}$$

output
```
2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

output
```
(2*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {6726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6726}$$

$$\frac{2(ax+1)\sqrt{c-acx}e^{\coth^{-1}(ax)}}{3a}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

output
```
(2*E^ArcCoth[a*x]*(1 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Defintions of rubi rules used

rule 6726
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] &&  !IntegerQ[p] && EqQ[p, n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

method	result	size
gosper	$\frac{2(ax+1)\sqrt{-acx+c}}{3\sqrt{\frac{ax-1}{ax+1}}\,a}$	35
orering	$\frac{2(ax+1)\sqrt{-acx+c}}{3\sqrt{\frac{ax-1}{ax+1}}\,a}$	35
default	$\frac{2\sqrt{-c(ax-1)}\,(ax+1)}{3\sqrt{\frac{ax-1}{ax+1}}\,a}$	36
risch	$-\frac{2c(ax+1)(ax-1)}{3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a}$	42

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,(a^2x^2+2\,ax+1)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^2x-a)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
2/3*(a^2*x^2 + 2*a*x + 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*
x - a)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\frac{\sqrt{-c(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\left(a\sqrt{-c}x+\sqrt{-c}\right)\sqrt{ax+1}}{3a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
2/3*(a*sqrt(-c)*x + sqrt(-c))*sqrt(a*x + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\,c^2\left(\frac{2\sqrt{2}\sqrt{-c}}{c}+\frac{(-acx-c)^{\frac{3}{2}}}{c^2}\right)}{3\,a|c|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
2/3*c^2*(2*sqrt(2)*sqrt(-c)/c + (-a*c*x - c)^(3/2)/c^2)/(a*abs(c)*sgn(a*x
+ 1))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,(a\,x+1)^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,(a\,x-1)}$$

input
```
int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x -
1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{2\sqrt{c}\,\sqrt{ax+1}\,i(ax+1)}{3a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x)
```

output
```
( - 2*sqrt(c)*sqrt(a*x + 1)*i*(a*x + 1))/(3*a)
```

3.322 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}-\frac{2\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\text{arcsinh}\left(\sqrt{\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-2*(1/a/x)^(1/2)*(-a*c*x
+c)^(1/2)*arcsinh((1/a/x)^(1/2))/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}-\sqrt{\frac{1}{x}}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x,x]
```

output
```
(2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)] - Sqrt[x^(-1)]*ArcSinh[Sqrt[
x^(-1)]/Sqrt[a]]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6730, 57, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{57}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{63}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{2\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x,x]
```

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*Sqrt[1 + 1/(a*x)])/Sqrt[x^(-1)] + (2*
ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[a]))/Sqrt[1 - 1/(a*x)])
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{2\sqrt{-c(ax-1)}\left(-\sqrt{c}\,\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)+\sqrt{-c(ax+1)}\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}}$	69

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(-c*(a*x-1))^(1/2)*(-c^(1/2)*arctan((-c*(a*x+1))^(1/2)/c^(1/2))+(-c*(a*x
+1))^(1/2))/((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.33

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$$
$$= \left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1},\right.$$
$$\left.-\frac{2\left((ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}\right)}{ax-1}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="frica
s")
```

output
```
[((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c)*(a*x + 1
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x +
 c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*((a*x - 1)*sqrt(c)*
arctan(sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x
 - c)) - sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{-c\,(ax-1)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{-acx+c}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\,c^3\left(\frac{\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{c\arctan\left(\frac{\sqrt{2}\sqrt{-c}}{\sqrt{c}}\right)-\sqrt{2}\sqrt{-c}\sqrt{c}}{c^{\frac{5}{2}}} - \frac{\sqrt{-acx-c}}{c^2}\right)}{|c|\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac"
)
```

output
```
2*c^3*(arctan(sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) - (c*arctan(sqrt(2)*sqrt(-
c)/sqrt(c)) - sqrt(2)*sqrt(-c)*sqrt(c))/c^(5/2) - sqrt(-a*c*x - c)/c^2)/(a
bs(c)*sgn(a*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \sqrt{c}\,i\left(-2\sqrt{ax+1} - \log\left(\frac{2\sqrt{ax+1}-2}{\sqrt{2}}\right) + \log\left(\frac{2\sqrt{ax+1}+2}{\sqrt{2}}\right)\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x)
```

output
```
sqrt(c)*i*( - 2*sqrt(a*x + 1) - log((2*sqrt(a*x + 1) - 2)/sqrt(2)) + log((
2*sqrt(a*x + 1) + 2)/sqrt(2)))
```

3.323 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx=-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x}-\frac{a\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\text{arcsinh}\left(\sqrt{\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
-(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/x-a*(1/a/x)^(1/2)*(-a*c*
x+c)^(1/2)*arcsinh((1/a/x)^(1/2))/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.80

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx=-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}+\sqrt{a}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x^2,x]
```

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + Sqrt[a]*
ArcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6730, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{60}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}+\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{63}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}+\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)+\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x^2,x]
```

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + Sqrt[a]*
ArcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/Sqrt[1 - 1/(a*x)])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\left(\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)acx+\sqrt{-c(ax+1)}\,\sqrt{c}\right)\sqrt{-c(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}\,x\sqrt{c}}$	78
risch	$\frac{c(ax-1)}{x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}-\frac{a\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}$	106

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE
)
```

output
```
-(arctan((-c*(a*x+1))^(1/2)/c^(1/2))*a*c*x+(-c*(a*x+1))^(1/2)*c^(1/2))*(-c
*(a*x-1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x+1))^(1/2)/x/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.46

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$$
$$=\left[\frac{(a^2x^2-ax)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)-2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2(ax^2-x)},\right.$$
$$\left.-\frac{(a^2x^2-ax)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)+\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax^2-x}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fri
cas")
```

output
```
[1/2*((a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c
)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) - 2*sqr
t(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), -((a^2*x^2
 - a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(
a*x + 1))/(a*c*x - c)) + sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x +
1)))/(a*x^2 - x)]
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-acx+c}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a*c*x + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\left(\frac{a^2\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{a^2c\arctan\left(\frac{\sqrt{2}\sqrt{-c}}{\sqrt{c}}\right)+\sqrt{2}a^2\sqrt{-c}\sqrt{c}}{c^{\frac{3}{2}}} + \frac{\sqrt{-acx-c}a}{cx}\right)c^2}{a|c|\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="gia
c")
```

output
```
(a^2*arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - (a^2*c*arctan(sqrt(2)*sqrt
(-c)/sqrt(c)) + sqrt(2)*a^2*sqrt(-c)*sqrt(c))/c^(3/2) + sqrt(-a*c*x - c)*a
/(c*x))*c^2/(a*abs(c)*sgn(a*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.59

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c}\,i\left(2\sqrt{ax+1}-\log\left(\frac{2\sqrt{ax+1}-2}{\sqrt{2}}\right)ax+\log\left(\frac{2\sqrt{ax+1}+2}{\sqrt{2}}\right)ax\right)}{2x}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x)
```

output
```
(sqrt(c)*i*(2*sqrt(a*x + 1) - log((2*sqrt(a*x + 1) - 2)/sqrt(2))*a*x + log
((2*sqrt(a*x + 1) + 2)/sqrt(2))*a*x))/(2*x)
```

3.324 $\int e^{2\coth^{-1}(ax)} x^3 \sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 101

$$\int e^{2\coth^{-1}(ax)} x^3 \sqrt{c-acx}\,dx = \frac{4\sqrt{c-acx}}{a^4} - \frac{14(c-acx)^{3/2}}{3a^4c} + \frac{18(c-acx)^{5/2}}{5a^4c^2} - \frac{10(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-acx)^{9/2}}{9a^4c^4}$$

output
```
4*(-a*c*x+c)^(1/2)/a^4-14/3*(-a*c*x+c)^(3/2)/a^4/c+18/5*(-a*c*x+c)^(5/2)/a
^4/c^2-10/7*(-a*c*x+c)^(7/2)/a^4/c^3+2/9*(-a*c*x+c)^(9/2)/a^4/c^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.48

$$\int e^{2\coth^{-1}(ax)} x^3 \sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(272+136ax+102a^2x^2+85a^3x^3+35a^4x^4)}{315a^4}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[c - a*c*x]*(272 + 136*a*x + 102*a^2*x^2 + 85*a^3*x^3 + 35*a^4*x^4)
)/(315*a^4)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6717, 6680, 35, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-acx}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}x^3\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{x^3(ax+1)\sqrt{c-acx}}{1-ax}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{x^3(ax+1)}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{86}$$

$$-c\int \left(\frac{(c-acx)^{7/2}}{a^3c^4}-\frac{5(c-acx)^{5/2}}{a^3c^3}+\frac{9(c-acx)^{3/2}}{a^3c^2}-\frac{7\sqrt{c-acx}}{a^3c}+\frac{2}{a^3\sqrt{c-acx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(-\frac{2(c-acx)^{9/2}}{9a^4c^5}+\frac{10(c-acx)^{7/2}}{7a^4c^4}-\frac{18(c-acx)^{5/2}}{5a^4c^3}+\frac{14(c-acx)^{3/2}}{3a^4c^2}-\frac{4\sqrt{c-acx}}{a^4c}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

output
```
-(c*((-4*Sqrt[c - a*c*x])/(a^4*c) + (14*(c - a*c*x)^(3/2))/(3*a^4*c^2) - (
18*(c - a*c*x)^(5/2))/(5*a^4*c^3) + (10*(c - a*c*x)^(7/2))/(7*a^4*c^4) - (
2*(c - a*c*x)^(9/2))/(9*a^4*c^5)))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.45

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)}{315a^4}$	45
trager	$\frac{2\sqrt{-acx+c}\,(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)}{315a^4}$	45
orering	$\frac{2\sqrt{-acx+c}\,(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)}{315a^4}$	45
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)}{315a^4}$	46
risch	$-\frac{2c(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)(ax-1)}{315a^4\sqrt{-c(ax-1)}}$	52
derivativedivides	$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{10c(-acx+c)^{\frac{7}{2}}}{7}+\frac{18c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{14c^3(-acx+c)^{\frac{3}{2}}}{3}+4\sqrt{-acx+c}\,c^4}{a^4c^4}$	75
default	$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{10c(-acx+c)^{\frac{7}{2}}}{7}+\frac{18c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{14c^3(-acx+c)^{\frac{3}{2}}}{3}+4\sqrt{-acx+c}\,c^4}{a^4c^4}$	75

input
```
int(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(-a*c*x+c)^(1/2)*(35*a^4*x^4+85*a^3*x^3+102*a^2*x^2+136*a*x+272)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.44

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\frac{2\left(35\,a^4x^4+85\,a^3x^3+102\,a^2x^2+136\,ax+272\right)\sqrt{-acx+c}}{315\,a^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*a^4*x^4 + 85*a^3*x^3 + 102*a^2*x^2 + 136*a*x + 272)*sqrt(-a*c*x
+ c)/a^4
```

Sympy [A] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\begin{cases}\frac{2\cdot\left(2c^4\sqrt{-acx+c}-\frac{7c^3(-acx+c)^{\frac{3}{2}}}{3}+\frac{9c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{5c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{a^4c^4} & \text{for } ac\neq 0\\ \sqrt{c}\left(\frac{x^4}{4}+\frac{2x^3}{3a}+\frac{x^2}{a^2}+\frac{2x}{a^3}+\frac{2\left(\begin{cases}-x & \text{for } a=0\\ \frac{\log(ax-1)}{a} & \text{otherwise}\end{cases}\right)}{a^3}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**3*(-a*c*x+c)**(1/2),x)
```

output
```
Piecewise((2*(2*c**4*sqrt(-a*c*x + c) - 7*c**3*(-a*c*x + c)**(3/2)/3 + 9*c
**2*(-a*c*x + c)**(5/2)/5 - 5*c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2
)/9)/(a**4*c**4), Ne(a*c, 0)), (sqrt(c)*(x**4/4 + 2*x**3/(3*a) + x**2/a**2
 + 2*x/a**3 + 2*Piecewise((-x, Eq(a, 0)), (log(a*x - 1)/a, True))/a**3), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\frac{2\left(35\,(-acx+c)^{\frac{9}{2}}-225\,(-acx+c)^{\frac{7}{2}}c+567\,(-acx+c)^{\frac{5}{2}}c^2-735\,(-acx+c)^{\frac{3}{2}}c^3+630\,\sqrt{-acx+c}c^4\right)}{315\,a^4c^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*(-a*c*x + c)^(9/2) - 225*(-a*c*x + c)^(7/2)*c + 567*(-a*c*x + c)
^(5/2)*c^2 - 735*(-a*c*x + c)^(3/2)*c^3 + 630*sqrt(-a*c*x + c)*c^4)/(a^4*c
^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(83) = 166$.

Time = 0.14 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.87

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$= \frac{2\left(\frac{9\left(5\,(acx-c)^3\sqrt{-acx+c}c+21\,(acx-c)^2\sqrt{-acx+c}c-35\,(-acx+c)^{\frac{3}{2}}c^2+35\,\sqrt{-acx+c}c^3\right)}{a^3c^3}+\frac{35\,(acx-c)^4\sqrt{-acx+c}+180\,(acx-c)^3\sqrt{-acx+}}{}\right.}{315\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
2/315*(9*(5*(a*c*x - c)^3*sqrt(-a*c*x + c) + 21*(a*c*x - c)^2*sqrt(-a*c*x
+ c)*c - 35*(-a*c*x + c)^(3/2)*c^2 + 35*sqrt(-a*c*x + c)*c^3)/(a^3*c^3) +
(35*(a*c*x - c)^4*sqrt(-a*c*x + c) + 180*(a*c*x - c)^3*sqrt(-a*c*x + c)*c
+ 378*(a*c*x - c)^2*sqrt(-a*c*x + c)*c^2 - 420*(-a*c*x + c)^(3/2)*c^3 + 31
5*sqrt(-a*c*x + c)*c^4)/(a^3*c^4))/a
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{4\sqrt{c-a\,c\,x}}{a^4}-\frac{14\,(c-a\,c\,x)^{3/2}}{3\,a^4\,c}+\frac{18\,(c-a\,c\,x)^{5/2}}{5\,a^4\,c^2}-\frac{10\,(c-a\,c\,x)^{7/2}}{7\,a^4\,c^3}+\frac{2\,(c-a\,c\,x)^{9/2}}{9\,a^4\,c^4}$$

input
```
int((x^3*(c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
(4*(c - a*c*x)^(1/2))/a^4 - (14*(c - a*c*x)^(3/2))/(3*a^4*c) + (18*(c - a*
c*x)^(5/2))/(5*a^4*c^2) - (10*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c
*x)^(9/2))/(9*a^4*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.44

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\frac{2\sqrt{c}\,\sqrt{-ax+1}\,(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)}{315a^4}$$

input
```
int(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt( - a*x + 1)*(35*a**4*x**4 + 85*a**3*x**3 + 102*a**2*x**2 +
 136*a*x + 272))/(315*a**4)
```

3.325 $\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{4\sqrt{c-acx}}{a^3}-\frac{10(c-acx)^{3/2}}{3a^3c}+\frac{8(c-acx)^{5/2}}{5a^3c^2}-\frac{2(c-acx)^{7/2}}{7a^3c^3}$$

output
```
4*(-a*c*x+c)^(1/2)/a^3-10/3*(-a*c*x+c)^(3/2)/a^3/c+8/5*(-a*c*x+c)^(5/2)/a^
3/c^2-2/7*(-a*c*x+c)^(7/2)/a^3/c^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.50

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(104+52ax+39a^2x^2+15a^3x^3)}{105a^3}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[c - a*c*x]*(104 + 52*a*x + 39*a^2*x^2 + 15*a^3*x^3))/(105*a^3)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6717, 6680, 35, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-acx}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}x^2\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{x^2(ax+1)\sqrt{c-acx}}{1-ax}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{x^2(ax+1)}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{86}$$

$$-c\int \left(-\frac{(c-acx)^{5/2}}{a^2c^3}+\frac{4(c-acx)^{3/2}}{a^2c^2}-\frac{5\sqrt{c-acx}}{a^2c}+\frac{2}{a^2\sqrt{c-acx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{2(c-acx)^{7/2}}{7a^3c^4}-\frac{8(c-acx)^{5/2}}{5a^3c^3}+\frac{10(c-acx)^{3/2}}{3a^3c^2}-\frac{4\sqrt{c-acx}}{a^3c}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

output
```
-(c*((-4*Sqrt[c - a*c*x])/(a^3*c) + (10*(c - a*c*x)^(3/2))/(3*a^3*c^2) - (
8*(c - a*c*x)^(5/2))/(5*a^3*c^3) + (2*(c - a*c*x)^(7/2))/(7*a^3*c^4)))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(15a^3x^3+39a^2x^2+52ax+104)}{105a^3}$	37
trager	$\frac{2\sqrt{-acx+c}\,(15a^3x^3+39a^2x^2+52ax+104)}{105a^3}$	37
orering	$\frac{2\sqrt{-acx+c}\,(15a^3x^3+39a^2x^2+52ax+104)}{105a^3}$	37
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(15a^3x^3+39a^2x^2+52ax+104)}{105a^3}$	38
risch	$-\frac{2c(15a^3x^3+39a^2x^2+52ax+104)(ax-1)}{105a^3\sqrt{-c(ax-1)}}$	44
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}-\frac{4(-acx+c)^{\frac{5}{2}}c}{5}+\frac{5c^2(-acx+c)^{\frac{3}{2}}}{3}-2\sqrt{-acx+c}\,c^3\right)}{c^3a^3}$	61
default	$\frac{-\frac{2(-acx+c)^{\frac{7}{2}}}{7}+\frac{8(-acx+c)^{\frac{5}{2}}c}{5}-\frac{10c^2(-acx+c)^{\frac{3}{2}}}{3}+4\sqrt{-acx+c}\,c^3}{a^3c^3}$	61

input
```
int((-a*c*x+c)^(1/2)*(a*x+1)*x^2/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
2/105*(-a*c*x+c)^(1/2)*(15*a^3*x^3+39*a^2*x^2+52*a*x+104)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.45

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{2\,(15\,a^3x^3+39\,a^2x^2+52\,ax+104)\sqrt{-acx+c}}{105\,a^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*a^3*x^3 + 39*a^2*x^2 + 52*a*x + 104)*sqrt(-a*c*x + c)/a^3
```

Sympy [A] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \begin{cases} -\frac{2\left(-2c^3\sqrt{-acx+c}+\frac{5c^2(-acx+c)^{\frac{3}{2}}}{3}-\frac{4c(-acx+c)^{\frac{5}{2}}}{5}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a^3c^3} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\frac{x^3}{3}+\frac{x^2}{a}+\frac{2x}{a^2}+\frac{2\left(\begin{cases} -x & \text{for } a=0 \\ \frac{\log(ax-1)}{a} & \text{otherwise} \end{cases}\right)}{a^2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**2*(-a*c*x+c)**(1/2),x)
```

output
```
Piecewise((-2*(-2*c**3*sqrt(-a*c*x + c) + 5*c**2*(-a*c*x + c)**(3/2)/3 - 4
*c*(-a*c*x + c)**(5/2)/5 + (-a*c*x + c)**(7/2)/7)/(a**3*c**3), Ne(a*c, 0))
, (sqrt(c)*(x**3/3 + x**2/a + 2*x/a**2 + 2*Piecewise((-x, Eq(a, 0)), (log(
a*x - 1)/a, True))/a**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\frac{2\left(15\left(-acx+c\right)^{\frac{7}{2}}-84\left(-acx+c\right)^{\frac{5}{2}}c+175\left(-acx+c\right)^{\frac{3}{2}}c^2-210\sqrt{-acx+c}c^3\right)}{105\,a^3c^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-2/105*(15*(-a*c*x + c)^(7/2) - 84*(-a*c*x + c)^(5/2)*c + 175*(-a*c*x + c)
^(3/2)*c^2 - 210*sqrt(-a*c*x + c)*c^3)/(a^3*c^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(66) = 132$.

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.78

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$
$$= \frac{2\left(\frac{7\left(3\,(acx-c)^2\sqrt{-acx+c}-10\,(-acx+c)^{\frac{3}{2}}c+15\,\sqrt{-acx+c}c^2\right)}{a^2c^2}+\frac{3\left(5\,(acx-c)^3\sqrt{-acx+c}+21\,(acx-c)^2\sqrt{-acx+c}c-35\,(-acx+c)^{\frac{3}{2}}c^2+35\right.}{a^2c^3}\right.}{105\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
2/105*(7*(3*(a*c*x - c)^2*sqrt(-a*c*x + c) - 10*(-a*c*x + c)^(3/2)*c + 15*
sqrt(-a*c*x + c)*c^2)/(a^2*c^2) + 3*(5*(a*c*x - c)^3*sqrt(-a*c*x + c) + 21
*(a*c*x - c)^2*sqrt(-a*c*x + c)*c - 35*(-a*c*x + c)^(3/2)*c^2 + 35*sqrt(-a
*c*x + c)*c^3)/(a^2*c^3))/a
```

Mupad [B] (verification not implemented)

Time = 13.63 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{4\sqrt{c-a\,c\,x}}{a^3}-\frac{10\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a^3\,c}+\frac{8\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a^3\,c^2}-\frac{2\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a^3\,c^3}$$

input
```
int((x^2*(c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
(4*(c - a*c*x)^(1/2))/a^3 - (10*(c - a*c*x)^(3/2))/(3*a^3*c) + (8*(c - a*c
*x)^(5/2))/(5*a^3*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^3*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.45

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{2\sqrt{c}\,\sqrt{-ax+1}\,(15a^3x^3+39a^2x^2+52ax+104)}{105a^3}$$

input
```
int(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt( - a*x + 1)*(15*a**3*x**3 + 39*a**2*x**2 + 52*a*x + 104))/
(105*a**3)
```

3.326 $\int e^{2\coth^{-1}(ax)} x\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int e^{2\coth^{-1}(ax)} x\sqrt{c-acx}\,dx = \frac{4\sqrt{c-acx}}{a^2} - \frac{2(c-acx)^{3/2}}{a^2c} + \frac{2(c-acx)^{5/2}}{5a^2c^2}$$

output
```
4*(-a*c*x+c)^(1/2)/a^2-2*(-a*c*x+c)^(3/2)/a^2/c+2/5*(-a*c*x+c)^(5/2)/a^2/c
^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.54

$$\int e^{2\coth^{-1}(ax)} x\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(6+3ax+a^2x^2)}{5a^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[c - a*c*x]*(6 + 3*a*x + a^2*x^2))/(5*a^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6717, 6680, 35, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-acx}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}x\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{x(ax+1)\sqrt{c-acx}}{1-ax}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{x(ax+1)}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{86}$$

$$-c\int \left(\frac{(c-acx)^{3/2}}{ac^2}-\frac{3\sqrt{c-acx}}{ac}+\frac{2}{a\sqrt{c-acx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(-\frac{2(c-acx)^{5/2}}{5a^2c^3}+\frac{2(c-acx)^{3/2}}{a^2c^2}-\frac{4\sqrt{c-acx}}{a^2c}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

output
```
-(c*((-4*Sqrt[c - a*c*x])/(a^2*c) + (2*(c - a*c*x)^(3/2))/(a^2*c^2) - (2*(
c - a*c*x)^(5/2))/(5*a^2*c^3)))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(a^2x^2+3ax+6)}{5a^2}$	28
trager	$\frac{2\sqrt{-acx+c}\,(a^2x^2+3ax+6)}{5a^2}$	28
orering	$\frac{2\sqrt{-acx+c}\,(a^2x^2+3ax+6)}{5a^2}$	28
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(a^2x^2+3ax+6)}{5a^2}$	29
risch	$-\frac{2c(a^2x^2+3ax+6)(ax-1)}{5a^2\sqrt{-c(ax-1)}}$	35
derivativedivides	$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}-2(-acx+c)^{\frac{3}{2}}c+4\sqrt{-acx+c}\,c^2}{a^2c^2}$	47
default	$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}-2(-acx+c)^{\frac{3}{2}}c+4\sqrt{-acx+c}\,c^2}{a^2c^2}$	47

input
```
int((-a*c*x+c)^(1/2)*x*(a*x+1)/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
2/5*(-a*c*x+c)^(1/2)*(a^2*x^2+3*a*x+6)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.47

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\,(a^2x^2+3\,ax+6)\sqrt{-acx+c}}{5\,a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(a^2*x^2 + 3*a*x + 6)*sqrt(-a*c*x + c)/a^2
```

Sympy [A] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\begin{cases}\frac{2\cdot\left(2c^2\sqrt{-acx+c}-c(-acx+c)^{\frac{3}{2}}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a^2c^2} & \text{for } ac\neq 0\\ \sqrt{c}\left(\frac{x^2}{2}+\frac{2x}{a}+\frac{2\left(\begin{cases}-x & \text{for } a=0\\ \frac{\log(ax-1)}{a} & \text{otherwise}\end{cases}\right)}{a}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)**(1/2),x)
```

output
```
Piecewise((2*(2*c**2*sqrt(-a*c*x + c) - c*(-a*c*x + c)**(3/2) + (-a*c*x +
c)**(5/2)/5)/(a**2*c**2), Ne(a*c, 0)), (sqrt(c)*(x**2/2 + 2*x/a + 2*Piecew
ise((-x, Eq(a, 0)), (log(a*x - 1)/a, True))/a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\left((-acx+c)^{\frac{5}{2}}-5(-acx+c)^{\frac{3}{2}}c+10\sqrt{-acx+c}c^2\right)}{5a^2c^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
2/5*((-a*c*x + c)^(5/2) - 5*(-a*c*x + c)^(3/2)*c + 10*sqrt(-a*c*x + c)*c^2
)/(a^2*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$$
$$= -\frac{2\left(\frac{5\left((-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}c\right)}{ac}-\frac{3(acx-c)^2\sqrt{-acx+c}-10(-acx+c)^{\frac{3}{2}}c+15\sqrt{-acx+c}c^2}{ac^2}\right)}{15a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
-2/15*(5*((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/(a*c) - (3*(a*c*x - c
)^2*sqrt(-a*c*x + c) - 10*(-a*c*x + c)^(3/2)*c + 15*sqrt(-a*c*x + c)*c^2)/
(a*c^2))/a
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\,(c-a\,c\,x)^{5/2}-10\,c\,(c-a\,c\,x)^{3/2}+20\,c^2\,\sqrt{c-a\,c\,x}}{5\,a^2\,c^2}$$

input
```
int((x*(c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
(2*(c - a*c*x)^(5/2) - 10*c*(c - a*c*x)^(3/2) + 20*c^2*(c - a*c*x)^(1/2))/
(5*a^2*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.47

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\sqrt{c}\,\sqrt{-ax+1}\,(a^2x^2+3ax+6)}{5a^2}$$

input
```
int(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt( - a*x + 1)*(a**2*x**2 + 3*a*x + 6))/(5*a**2)
```

3.327 $\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

output
```
4*(-a*c*x+c)^(1/2)/a-2/3*(-a*c*x+c)^(3/2)/a/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2(5+ax)\sqrt{c-acx}}{3a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
(2*(5 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6680, 35, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(ax+1)\sqrt{c-acx}}{1-ax}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{ax+1}{\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{53}$$

$$-c\int \left(\frac{2}{\sqrt{c-acx}}-\frac{\sqrt{c-acx}}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c\left(\frac{2(c-acx)^{3/2}}{3ac^2}-\frac{4\sqrt{c-acx}}{ac}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
-(c*((-4*Sqrt[c - a*c*x])/(a*c) + (2*(c - a*c*x)^(3/2))/(3*a*c^2)))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
trager	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
orering	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
pseudoelliptic	$\frac{2\sqrt{-c(ax-1)}\,(ax+5)}{3a}$	21
risch	$-\frac{2c(ax+5)(ax-1)}{3a\sqrt{-c(ax-1)}}$	27
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}-2\sqrt{-acx+c}\,c\right)}{ca}$	33
default	$\frac{-\frac{2(-acx+c)^{\frac{3}{2}}}{3}+4\sqrt{-acx+c}\,c}{ac}$	33

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-a*c*x+c)^(1/2)*(a*x+5)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.50

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{-acx+c}(ax+5)}{3\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(-a*c*x + c)*(a*x + 5)/a
```

Sympy [A] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\begin{cases}-\frac{2\left(-2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac} & \text{for } ac\neq 0\\ \sqrt{c}\left(\begin{cases}-x & \text{for } a=0\\ \frac{ax+2\log(ax-1)-1}{a} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2),x)
```

output
```
Piecewise((-2*(-2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c), Ne(a*
c, 0)), (sqrt(c)*Piecewise((-x, Eq(a, 0)), ((a*x + 2*log(a*x - 1) - 1)/a,
True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=-\frac{2\left((-acx+c)^{\frac{3}{2}}-6\sqrt{-acx+c}c\right)}{3\,ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\left(3\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}c}{c}\right)}{3a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*sqrt(-a*c*x + c) - ((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/c)/a
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{4\sqrt{c-a\,c\,x}}{a}-\frac{2\,(c-a\,c\,x)^{3/2}}{3\,a\,c}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
(4*(c - a*c*x)^(1/2))/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.50

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{c}\,\sqrt{-ax+1}\,(ax+5)}{3a}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt( - a*x + 1)*(a*x + 5))/(3*a)
```

3.328 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c-acx}+2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

output
```
2*(-a*c*x+c)^(1/2)+2*c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c-acx}+2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

output
```
2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6717, 6680, 35, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-acx}}{x}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(ax+1)\sqrt{c-acx}}{x(1-ax)}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{ax+1}{x\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{90}$$

$$-c\left(\int \frac{1}{x\sqrt{c-acx}}dx-\frac{2\sqrt{c-acx}}{c}\right)$$

$$\Big\downarrow \text{73}$$

$$-c\left(-\frac{2\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}-\frac{2\sqrt{c-acx}}{c}\right)$$

$$\Big\downarrow \text{221}$$

$$-c\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\sqrt{c-acx}}{c}\right)$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

output
```
-(c*((-2*Sqrt[c - a*c*x])/c - (2*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/Sqrt[c]
))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$2\sqrt{-acx+c}+2\sqrt{c}\, \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)$	32
default	$2\sqrt{-acx+c}+2\sqrt{c}\, \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)$	32
pseudoelliptic	$2\sqrt{-c\left(ax-1\right)}+2\sqrt{c}\, \operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)$	34

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(-a*c*x+c)^(1/2)+2*c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.28

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \left[\sqrt{c}\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right) + 2\sqrt{-acx+c},\, 2\sqrt{-c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{acx-c}\right) + 2\sqrt{-acx+c}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="fricas")
```

output
```
[sqrt(c)*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2*sqrt(-a*c*x
 + c), 2*sqrt(-c)*arctan(sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x - c)) + 2*sqrt(-
a*c*x + c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(34) = 68$.

Time = 4.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.05

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \begin{cases} -\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2\sqrt{-acx+c} & \text{for } ac \neq 0 \\ \sqrt{c}\left(-\frac{3a\left(\frac{\log\left(\frac{2}{x}\right)}{a} - \frac{\log\left(2a-\frac{2}{x}\right)}{a}\right)}{2} + \frac{\log\left(\frac{a}{x}-\frac{1}{x^2}\right)}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x,x)
```

output
```
Piecewise((-2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*sqrt(-a*c*x +
 c), Ne(a*c, 0)), (sqrt(c)*(-3*a*(log(2/x)/a - log(2*a - 2/x)/a)/2 + log(a
/x - 1/x**2)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = -\sqrt{c}\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right) + 2\sqrt{-acx+c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxima")
```

output
```
-sqrt(c)*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c))) +
2*sqrt(-a*c*x + c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = -2\,c\left(\frac{\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{\sqrt{-acx+c}}{c}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")
```

output
```
-2*c*(arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)/c)
```

Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right) + 2\,\sqrt{c-a\,c\,x}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
```

output
```
2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) + 2*(c - a*c*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \sqrt{c}\,\left(2\sqrt{-ax+1} - \log\left(\sqrt{-ax+1}-1\right) + \log\left(\sqrt{-ax+1}+1\right)\right)$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x)
```

output
```
sqrt(c)*(2*sqrt( - a*x + 1) - log(sqrt( - a*x + 1) - 1) + log(sqrt( - a*x
+ 1) + 1))
```

3.329 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}}{x} + 3a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

output
```
(-a*c*x+c)^(1/2)/x+3*a*c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}}{x} + 3a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

output
```
Sqrt[c - a*c*x]/x + 3*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6717, 6680, 35, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \ 6717$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-acx}}{x^2}dx$$

$$\Big\downarrow \ 6680$$

$$-\int \frac{(ax+1)\sqrt{c-acx}}{x^2(1-ax)}dx$$

$$\Big\downarrow \ 35$$

$$-c\int \frac{ax+1}{x^2\sqrt{c-acx}}dx$$

$$\Big\downarrow \ 87$$

$$-c\left(\frac{3}{2}a\int \frac{1}{x\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{cx}\right)$$

$$\Big\downarrow \ 73$$

$$-c\left(-\frac{3\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{c}-\frac{\sqrt{c-acx}}{cx}\right)$$

$$\Big\downarrow \ 221$$

$$-c\left(-\frac{3a\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{c-acx}}{cx}\right)$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

output
```
-(c*(-(Sqrt[c - a*c*x]/(c*x)) - (3*a*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/Sqr
t[c]))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

method	result	size
risch	$-\frac{(ax-1)c}{x\sqrt{-c(ax-1)}}+3a\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)$	43
pseudoelliptic	$\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)acx+\sqrt{-c(ax-1)}\,\sqrt{c}}{x\sqrt{c}}$	43
derivativedivides	$-2ca\left(-\frac{\sqrt{-acx+c}}{2acx}-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$	45
default	$2ca\left(\frac{\sqrt{-acx+c}}{2acx}+\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$	45

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)/x/(-c*(a*x-1))^(1/2)*c+3*a*c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.40

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \left[\frac{3\,a\sqrt{c}x\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\sqrt{-acx+c}}{2\,x}, \frac{3\,a\sqrt{-c}x\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{acx-c}\right)+\sqrt{-acx+c}}{x}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(3*a*sqrt(c)*x*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2*
sqrt(-a*c*x + c))/x, (3*a*sqrt(-c)*x*arctan(sqrt(-a*c*x + c)*sqrt(-c)/(a*c
*x - c)) + sqrt(-a*c*x + c))/x]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax+1)}{x^2\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/(x**2*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{1}{2}\,ac\left(\frac{3\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\,\sqrt{-acx+c}}{acx}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-1/2*a*c*(3*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))
/sqrt(c) - 2*sqrt(-a*c*x + c)/(a*c*x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{\frac{3\,a^2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{-acx+c}a}{x}}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-(3*a^2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)*a/
x)/a
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-a\,c\,x}}{x} + 3\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)$$

input
```
int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
```

output
```
(c - a*c*x)^(1/2)/x + 3*a*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c}\left(2\sqrt{-ax+1}-3\log\left(\sqrt{-ax+1}-1\right)ax+3\log\left(\sqrt{-ax+1}+1\right)ax\right)}{2x}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x)
```

output
```
(sqrt(c)*(2*sqrt( - a*x + 1) - 3*log(sqrt( - a*x + 1) - 1)*a*x + 3*log(sqr
t( - a*x + 1) + 1)*a*x))/(2*x)
```

3.330 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx=\frac{\sqrt{c-acx}}{2x^2}+\frac{7a\sqrt{c-acx}}{4x}+\frac{7}{4}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

output
```
1/2*(-a*c*x+c)^(1/2)/x^2+7/4*a*(-a*c*x+c)^(1/2)/x+7/4*a^2*c^(1/2)*arctanh(
(-a*c*x+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx=\frac{(2+7ax)\sqrt{c-acx}}{4x^2}+\frac{7}{4}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

output
```
((2 + 7*a*x)*Sqrt[c - a*c*x])/(4*x^2) + (7*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*
c*x]/Sqrt[c]])/4
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6717, 6680, 35, 87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-acx}}{x^3}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(ax+1)\sqrt{c-acx}}{x^3(1-ax)}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{ax+1}{x^3\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{87}$$

$$-c\left(\frac{7}{4}a\int \frac{1}{x^2\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{2cx^2}\right)$$

$$\Big\downarrow \text{52}$$

$$-c\left(\frac{7}{4}a\left(\frac{1}{2}a\int \frac{1}{x\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)$$

$$\Big\downarrow \text{73}$$

$$-c\left(\frac{7}{4}a\left(-\frac{\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{c}-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)$$

$$\Big\downarrow \text{221}$$

$$-c\left(\frac{7}{4}a\left(-\frac{a\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

output
```
-(c*(-1/2*Sqrt[c - a*c*x]/(c*x^2) + (7*a*(-(Sqrt[c - a*c*x]/(c*x)) - (a*Ar
cTanh[Sqrt[c - a*c*x]/Sqrt[c]])/Sqrt[c]))/4))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\frac{7 \operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right) a^2 c\, x^2}{4}+\frac{7 \sqrt{-c(ax-1)}\,\sqrt{c}\,\left(ax+\frac{2}{7}\right)}{4}}{\sqrt{c}\, x^2}$	52
risch	$-\frac{(7a^2x^2-5ax-2)c}{4x^2\sqrt{-c(ax-1)}}+\frac{7a^2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{4}$	54
derivativedivides	$2c^2a^2\left(\frac{-\frac{7(-acx+c)^{\frac{3}{2}}}{8c}+\frac{9\sqrt{-acx+c}}{8}}{a^2c^2x^2}+\frac{7 \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}\right)$	65
default	$2c^2a^2\left(\frac{-\frac{7(-acx+c)^{\frac{3}{2}}}{8c}+\frac{9\sqrt{-acx+c}}{8}}{a^2c^2x^2}+\frac{7 \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}\right)$	65

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
7/4*(arctanh((-c*(a*x-1))^(1/2)/c^(1/2))*a^2*c*x^2+(-c*(a*x-1))^(1/2)*c^(1
/2)*(a*x+2/7))/c^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.81

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \left[\frac{7\,a^2\sqrt{c}x^2\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\,\sqrt{-acx+c}(7\,ax+2)}{8\,x^2}, \frac{7\,a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{acx-c}\right)+\sqrt{-a}}{4\,x^2}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(7*a^2*sqrt(c)*x^2*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x)
+ 2*sqrt(-a*c*x + c)*(7*a*x + 2))/x^2, 1/4*(7*a^2*sqrt(-c)*x^2*arctan(sqrt
(-a*c*x + c)*sqrt(-c)/(a*c*x - c)) + sqrt(-a*c*x + c)*(7*a*x + 2))/x^2]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax+1)}{x^3\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/(x**3*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.51

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = -\frac{1}{8}\,a^2c^2\left(\frac{2\left(7\,(-acx+c)^{\frac{3}{2}}-9\,\sqrt{-acx+c}c\right)}{(acx-c)^2c+2\,(acx-c)c^2+c^3}+\frac{7\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-1/8*a^2*c^2*(2*(7*(-a*c*x + c)^(3/2) - 9*sqrt(-a*c*x + c)*c)/((a*c*x - c)
^2*c + 2*(a*c*x - c)*c^2 + c^3) + 7*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt
(-a*c*x + c) + sqrt(c)))/c^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = -\frac{\frac{7\,a^3c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{7\,(-acx+c)^{\frac{3}{2}}a^3c-9\,\sqrt{-acx+c}a^3c^2}{a^2c^2x^2}}{4\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/4*(7*a^3*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + (7*(-a*c*x + c)
^(3/2)*a^3*c - 9*sqrt(-a*c*x + c)*a^3*c^2)/(a^2*c^2*x^2))/a
```

Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{9\sqrt{c-a\,c\,x}}{4\,x^2} + \frac{7\,a^2\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)}{4} - \frac{7\,(c-a\,c\,x)^{3/2}}{4\,c\,x^2}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

output
```
(9*(c - a*c*x)^(1/2))/(4*x^2) + (7*a^2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(
1/2)))/4 - (7*(c - a*c*x)^(3/2))/(4*c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$$
$$=\frac{\sqrt{c}\left(14\sqrt{-ax+1}\,ax+4\sqrt{-ax+1}-7\log\left(\sqrt{-ax+1}-1\right)a^2x^2+7\log\left(\sqrt{-ax+1}+1\right)a^2x^2\right)}{8x^2}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x)
```

output
```
(sqrt(c)*(14*sqrt( - a*x + 1)*a*x + 4*sqrt( - a*x + 1) - 7*log(sqrt( - a*x
 + 1) - 1)*a**2*x**2 + 7*log(sqrt( - a*x + 1) + 1)*a**2*x**2))/(8*x**2)
```

3.331 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}}{3x^3}+\frac{11a\sqrt{c-acx}}{12x^2}+\frac{11a^2\sqrt{c-acx}}{8x} + \frac{11}{8}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

output
```
1/3*(-a*c*x+c)^(1/2)/x^3+11/12*a*(-a*c*x+c)^(1/2)/x^2+11/8*a^2*(-a*c*x+c)^
(1/2)/x+11/8*a^3*c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}(8+22ax+33a^2x^2)}{24x^3}+\frac{11}{8}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

output
```
(Sqrt[c - a*c*x]*(8 + 22*a*x + 33*a^2*x^2))/(24*x^3) + (11*a^3*Sqrt[c]*Arc
Tanh[Sqrt[c - a*c*x]/Sqrt[c]])/8
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6717, 6680, 35, 87, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-acx}}{x^4}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(ax+1)\sqrt{c-acx}}{x^4(1-ax)}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{ax+1}{x^4\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{87}$$

$$-c\left(\frac{11}{6}a\int \frac{1}{x^3\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{3cx^3}\right)$$

$$\Big\downarrow \text{52}$$

$$-c\left(\frac{11}{6}a\left(\frac{3}{4}a\int \frac{1}{x^2\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{2cx^2}\right)-\frac{\sqrt{c-acx}}{3cx^3}\right)$$

$$\Big\downarrow \text{52}$$

$$-c\left(\frac{11}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{x\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)-\frac{\sqrt{c-acx}}{3cx^3}\right)$$

$$\Big\downarrow \text{73}$$

$$-c\left(\frac{11}{6}a\left(\frac{3}{4}a\left(-\frac{\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{c}-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)-\frac{\sqrt{c-acx}}{3cx^3}\right)$$

$$\Big\downarrow \text{221}$$

$$-c\left(\frac{11}{6}a\left(\frac{3}{4}a\left(-\frac{a\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)-\frac{\sqrt{c-acx}}{3cx^3}\right)$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

output
```
-(c*(-1/3*Sqrt[c - a*c*x]/(c*x^3) + (11*a*(-1/2*Sqrt[c - a*c*x]/(c*x^2) +
(3*a*(-(Sqrt[c - a*c*x]/(c*x)) - (a*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/Sqrt
[c]))/4))/6))
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{(33a^3x^3-11a^2x^2-14ax-8)c}{24x^3\sqrt{-c(ax-1)}}+\frac{11a^3\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8}$	62
pseudoelliptic	$\frac{\frac{\sqrt{-c(ax-1)}\left(33a^2x^2+22ax+8\right)\sqrt{c}}{24}+\frac{11\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)a^3c\,x^3}{8}}{\sqrt{c}\,x^3}$	62
default	$2c^3a^3\left(\frac{\frac{11(-acx+c)^{\frac{5}{2}}}{16c^2}-\frac{11(-acx+c)^{\frac{3}{2}}}{6c}+\frac{21\sqrt{-acx+c}}{16}}{a^3c^3x^3}+\frac{11\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{16c^{\frac{5}{2}}}\right)$	79
derivativedivides	$-2c^3a^3\left(-\frac{\frac{11(-acx+c)^{\frac{5}{2}}}{16c^2}-\frac{11(-acx+c)^{\frac{3}{2}}}{6c}+\frac{21\sqrt{-acx+c}}{16}}{a^3c^3x^3}-\frac{11\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{16c^{\frac{5}{2}}}\right)$	80

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(33*a^3*x^3-11*a^2*x^2-14*a*x-8)/x^3/(-c*(a*x-1))^(1/2)*c+11/8*a^3*c
^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.56

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \left[\frac{33\,a^3\sqrt{c}x^3\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\,(33\,a^2x^2+22\,ax+8)\sqrt{-acx+c}}{48\,x^3}, \frac{33\,a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-acx+}}{acx-}\right.}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(33*a^3*sqrt(c)*x^3*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x
) + 2*(33*a^2*x^2 + 22*a*x + 8)*sqrt(-a*c*x + c))/x^3, 1/24*(33*a^3*sqrt(-
c)*x^3*arctan(sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x - c)) + (33*a^2*x^2 + 22*a*
x + 8)*sqrt(-a*c*x + c))/x^3]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax+1)}{x^4\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/(x**4*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.51

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$
$$=\frac{1}{48}\,a^3c^3\left(\frac{2\left(33\,(-acx+c)^{\frac{5}{2}}-88\,(-acx+c)^{\frac{3}{2}}c+63\,\sqrt{-acx+c}c^2\right)}{(acx-c)^3c^2+3\,(acx-c)^2c^3+3\,(acx-c)c^4+c^5}-\frac{33\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/48*a^3*c^3*(2*(33*(-a*c*x + c)^(5/2) - 88*(-a*c*x + c)^(3/2)*c + 63*sqrt
(-a*c*x + c)*c^2)/((a*c*x - c)^3*c^2 + 3*(a*c*x - c)^2*c^3 + 3*(a*c*x - c)
*c^4 + c^5) - 33*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt
(c)))/c^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$
$$=-\frac{\frac{33\,a^4c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{33\,(acx-c)^2\sqrt{-acx+c}a^4c-88\,(-acx+c)^{\frac{3}{2}}a^4c^2+63\,\sqrt{-acx+c}a^4c^3}{a^3c^3x^3}}{24\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/24*(33*a^4*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - (33*(a*c*x -
c)^2*sqrt(-a*c*x + c)*a^4*c - 88*(-a*c*x + c)^(3/2)*a^4*c^2 + 63*sqrt(-a*c
*x + c)*a^4*c^3)/(a^3*c^3*x^3))/a
```

Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{21\sqrt{c-a\,c\,x}}{8\,x^3}-\frac{11\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,c\,x^3}+\frac{11\,{\left(c-a\,c\,x\right)}^{5/2}}{8\,c^2\,x^3}-\frac{a^3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,11{}\mathrm{i}}{8}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
```

output
```
(21*(c - a*c*x)^(1/2))/(8*x^3) - (a^3*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)/
c^(1/2))*11i)/8 - (11*(c - a*c*x)^(3/2))/(3*c*x^3) + (11*(c - a*c*x)^(5/2)
)/(8*c^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$
$$= \frac{\sqrt{c}\left(66\sqrt{-ax+1}\,a^2x^2+44\sqrt{-ax+1}\,ax+16\sqrt{-ax+1}-33\log\left(\sqrt{-ax+1}-1\right)a^3x^3+33\log\left(\sqrt{-ax\right.\right.}{48x^3}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x)
```

output
```
(sqrt(c)*(66*sqrt( - a*x + 1)*a**2*x**2 + 44*sqrt( - a*x + 1)*a*x + 16*sqr
t( - a*x + 1) - 33*log(sqrt( - a*x + 1) - 1)*a**3*x**3 + 33*log(sqrt( - a*
x + 1) + 1)*a**3*x**3))/(48*x**3)
```

3.332 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}+\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{75a^3\sqrt{c-acx}}{64x}+\frac{75}{64}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

output
```
1/4*(-a*c*x+c)^(1/2)/x^4+5/8*a*(-a*c*x+c)^(1/2)/x^3+25/32*a^2*(-a*c*x+c)^(
1/2)/x^2+75/64*a^3*(-a*c*x+c)^(1/2)/x+75/64*a^4*c^(1/2)*arctanh((-a*c*x+c)
^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.65

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}(16+40ax+50a^2x^2+75a^3x^3)}{64x^4}+\frac{75}{64}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

output
```
(Sqrt[c - a*c*x]*(16 + 40*a*x + 50*a^2*x^2 + 75*a^3*x^3))/(64*x^4) + (75*a
^4*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6717, 6680, 35, 87, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{2\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-acx}}{x^5}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(ax+1)\sqrt{c-acx}}{x^5(1-ax)}dx$$

$$\Big\downarrow \text{35}$$

$$-c\int \frac{ax+1}{x^5\sqrt{c-acx}}dx$$

$$\Big\downarrow \text{87}$$

$$-c\left(\frac{15}{8}a\int \frac{1}{x^4\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{4cx^4}\right)$$

$$\Big\downarrow \text{52}$$

$$-c\left(\frac{15}{8}a\left(\frac{5}{6}a\int \frac{1}{x^3\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{3cx^3}\right)-\frac{\sqrt{c-acx}}{4cx^4}\right)$$

$$\Big\downarrow \text{52}$$

$$-c\left(\frac{15}{8}a\left(\frac{5}{6}a\left(\frac{3}{4}a\int \frac{1}{x^2\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{2cx^2}\right)-\frac{\sqrt{c-acx}}{3cx^3}\right)-\frac{\sqrt{c-acx}}{4cx^4}\right)$$

$$\Big\downarrow \text{52}$$

$$-c\left(\frac{15}{8}a\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{x\sqrt{c-acx}}dx-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)-\frac{\sqrt{c-acx}}{3cx^3}\right)-\frac{\sqrt{c-acx}}{4cx^4}\right)$$

$$\Big\downarrow \text{73}$$

$$-c\left(\frac{15}{8}a\left(\frac{5}{6}a\left(\frac{3}{4}a\left(-\frac{\int\frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{c}-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)-\frac{\sqrt{c-acx}}{3cx^3}\right)-\frac{\sqrt{c-acx}}{4cx^4}\right)$$

$$\Big\downarrow \text{221}$$

$$-c\left(\frac{15}{8}a\left(\frac{5}{6}a\left(\frac{3}{4}a\left(-\frac{a\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{c-acx}}{cx}\right)-\frac{\sqrt{c-acx}}{2cx^2}\right)-\frac{\sqrt{c-acx}}{3cx^3}\right)-\frac{\sqrt{c-acx}}{4cx^4}\right)$$

input

```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

output

```
-(c*(-1/4*Sqrt[c - a*c*x]/(c*x^4) + (15*a*(-1/3*Sqrt[c - a*c*x]/(c*x^3) +
(5*a*(-1/2*Sqrt[c - a*c*x]/(c*x^2) + (3*a*(-(Sqrt[c - a*c*x]/(c*x)) - (a*A
rcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/Sqrt[c]))/4))/6))/8))
```

Defintions of rubi rules used

rule 35

```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

method	result	size
risch	$-\frac{(75a^4x^4-25a^3x^3-10a^2x^2-24ax-16)c}{64x^4\sqrt{-c(ax-1)}}+\frac{75a^4\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{64}$	70
pseudoelliptic	$\frac{\frac{\sqrt{-c(ax-1)}\,(75a^3x^3+50a^2x^2+40ax+16)\sqrt{c}}{64}+\frac{75\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)c\,a^4x^4}{64}}{\sqrt{c}\,x^4}$	70
derivativedivides	$2c^4a^4\left(\frac{-\frac{75(-acx+c)^{\frac{7}{2}}}{128c^3}+\frac{275(-acx+c)^{\frac{5}{2}}}{128c^2}-\frac{365(-acx+c)^{\frac{3}{2}}}{128c}+\frac{181\sqrt{-acx+c}}{128}}{a^4c^4x^4}+\frac{75\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{128c^{\frac{7}{2}}}\right)$	93
default	$2c^4a^4\left(\frac{-\frac{75(-acx+c)^{\frac{7}{2}}}{128c^3}+\frac{275(-acx+c)^{\frac{5}{2}}}{128c^2}-\frac{365(-acx+c)^{\frac{3}{2}}}{128c}+\frac{181\sqrt{-acx+c}}{128}}{a^4c^4x^4}+\frac{75\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{128c^{\frac{7}{2}}}\right)$	93

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/64*(75*a^4*x^4-25*a^3*x^3-10*a^2*x^2-24*a*x-16)/x^4/(-c*(a*x-1))^(1/2)*
c+75/64*a^4*c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.41

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$
$$=\left[\frac{75\,a^4\sqrt{c}x^4\log\left(\frac{acx-2\,\sqrt{-acx+c}\sqrt{c}-2\,c}{x}\right)+2\,(75\,a^3x^3+50\,a^2x^2+40\,ax+16)\sqrt{-acx+c}}{128\,x^4},\ \frac{75\,a^4\sqrt{-c}x^4\,\mathrm{arct}\cdots}{\cdots}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[1/128*(75*a^4*sqrt(c)*x^4*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/
x) + 2*(75*a^3*x^3 + 50*a^2*x^2 + 40*a*x + 16)*sqrt(-a*c*x + c))/x^4, 1/64
*(75*a^4*sqrt(-c)*x^4*arctan(sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x - c)) + (75*
a^3*x^3 + 50*a^2*x^2 + 40*a*x + 16)*sqrt(-a*c*x + c))/x^4]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax+1)}{x^5\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/(x**5*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.48

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = -\frac{1}{128}\,a^4c^4\left(\frac{2\left(75\,(-acx+c)^{\frac{7}{2}}-275\,(-acx+c)^{\frac{5}{2}}c+365\,(-acx+c)^{\frac{3}{2}}c^2-181\,\sqrt{-acx+c}c^3\right)}{(acx-c)^4c^3+4\,(acx-c)^3c^4+6\,(acx-c)^2c^5+4\,(acx-c)c^6+c^7}+\frac{75\log}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/128*a^4*c^4*(2*(75*(-a*c*x + c)^(7/2) - 275*(-a*c*x + c)^(5/2)*c + 365*
(-a*c*x + c)^(3/2)*c^2 - 181*sqrt(-a*c*x + c)*c^3)/((a*c*x - c)^4*c^3 + 4*
(a*c*x - c)^3*c^4 + 6*(a*c*x - c)^2*c^5 + 4*(a*c*x - c)*c^6 + c^7) + 75*lo
g((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/c^(7/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.19

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = -\frac{\frac{75\,a^5c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{75\,(acx-c)^3\sqrt{-acx+c}a^5c+275\,(acx-c)^2\sqrt{-acx+c}a^5c^2-365\,(-acx+c)^{\frac{3}{2}}a^5c^3+181\,\sqrt{-acx+c}a^5c^4}{a^4c^4x^4}}{64\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/64*(75*a^5*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - (75*(a*c*x -
c)^3*sqrt(-a*c*x + c)*a^5*c + 275*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^5*c^2 -
 365*(-a*c*x + c)^(3/2)*a^5*c^3 + 181*sqrt(-a*c*x + c)*a^5*c^4)/(a^4*c^4*x
^4))/a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{181\,\sqrt{c-a\,c\,x}}{64\,x^4} - \frac{365\,(c-a\,c\,x)^{3/2}}{64\,c\,x^4} + \frac{275\,(c-a\,c\,x)^{5/2}}{64\,c^2\,x^4} - \frac{75\,(c-a\,c\,x)^{7/2}}{64\,c^3\,x^4} - \frac{a^4\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,75\mathrm{i}}{64}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
```

output
```
(181*(c - a*c*x)^(1/2))/(64*x^4) - (a^4*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i
)/c^(1/2))*75i)/64 - (365*(c - a*c*x)^(3/2))/(64*c*x^4) + (275*(c - a*c*x)
^(5/2))/(64*c^2*x^4) - (75*(c - a*c*x)^(7/2))/(64*c^3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$
$$= \frac{\sqrt{c}\left(150\sqrt{-ax+1}\,a^3x^3+100\sqrt{-ax+1}\,a^2x^2+80\sqrt{-ax+1}\,ax+32\sqrt{-ax+1}-75\log\left(\sqrt{-ax+1}-\right.\right.}{128x^4}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x)
```

output
```
(sqrt(c)*(150*sqrt( - a*x + 1)*a**3*x**3 + 100*sqrt( - a*x + 1)*a**2*x**2
+ 80*sqrt( - a*x + 1)*a*x + 32*sqrt( - a*x + 1) - 75*log(sqrt( - a*x + 1)
- 1)*a**4*x**4 + 75*log(sqrt( - a*x + 1) + 1)*a**4*x**4))/(128*x**4)
```

3.333 $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 310

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}}+\frac{472\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}}+\frac{92\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{38\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

output
```
1576/315*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^4/(1-1/a/x)^(1/2)+472/315*(1+1
/a/x)^(1/2)*x*(-a*c*x+c)^(1/2)/a^3/(1-1/a/x)^(1/2)+92/105*(1+1/a/x)^(1/2)*
x^2*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)+38/63*(1+1/a/x)^(1/2)*x^3*(-a*c*x
+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/9*(1+1/a/x)^(1/2)*x^4*(-a*c*x+c)^(1/2)/(1-1/
a/x)^(1/2)-4*2^(1/2)*(1/a/x)^(1/2)*(-a*c*x+c)^(1/2)*arctanh(2^(1/2)*(1/a/x
)^(1/2)/(1+1/a/x)^(1/2))/a^4/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.42

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}(788+236ax+138a^2x^2+95a^3x^3+35a^4x^4)-630\sqrt{2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{315a^{9/2}\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(788 + 236*a*x + 138*a^2*x^2
 + 95*a^3*x^3 + 35*a^4*x^4) - 630*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sq
rt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(315*a^(9/2)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.75, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {6730, 27, 109, 27, 169, 27, 169, 25, 169, 27, 169, 27, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-acx}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int\frac{a\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{2\int -\frac{19a+\frac{17}{x}}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{9a}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\int \frac{19a+\frac{17}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{169}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{-\frac{2\int -\frac{3\left(23a+\frac{19}{x}\right)}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{6\int \frac{23a+\frac{19}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{169}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{6\left(-\frac{2\int -\frac{59a+\frac{46}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{5a}-\frac{46\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 25

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{6\left(\frac{2\int \frac{59a+\frac{46}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{5a}-\frac{46\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 169

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{6\left(\frac{2\left(-\frac{2\int -\frac{197a+\frac{118}{x}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{3a}-\frac{118\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{46\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 27

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{6\left(\frac{2\left(\frac{\int \frac{197a+\frac{118}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{3a}-\frac{118\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{46\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 169

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{6\left(\frac{2\left(\frac{-\frac{2\int -\frac{315a}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{394\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}}{3a}-\frac{118\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{46\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 27

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{6\left(\frac{2\left(\frac{315\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{394\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}}{3a}-\frac{118\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{46\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 104

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{6\left(\frac{2\left(\frac{630\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-\frac{394\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}}{3a}-\frac{118\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{46\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 219

$$-\frac{a\sqrt{\frac{1}{x}}\left(\frac{\frac{6\left(\frac{2\left(\frac{\frac{315\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}}-\frac{394\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}}{3a}-\frac{118\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{46\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{7a}-\frac{38\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}}{9a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{9a\left(\frac{1}{x}\right)^{9/2}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

output

```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*Sqrt[1 + 1/(a*x)])/(9*a*(x^(-1))^(9
/2)) + ((-38*Sqrt[1 + 1/(a*x)])/(7*(x^(-1))^(7/2)) + (6*((-46*Sqrt[1 + 1/(
a*x)])/(5*(x^(-1))^(5/2)) + (2*((-118*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)
) + ((-394*Sqrt[1 + 1/(a*x)])/Sqrt[x^(-1)] + (315*Sqrt[2]*ArcTanh[(Sqrt[2]
*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[a])/(3*a)))/(5*a)))/(7*a
))/(9*a^2)))/Sqrt[1 - 1/(a*x)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.47

method	result
risch	$-\frac{2(35a^4x^4+95a^3x^3+138a^2x^2+236ax+788)c(ax-1)}{315a^4\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}(ax-1)}{a^4\sqrt{\frac{ax-1}{ax+1}}(ax+1)\sqrt{-c(ax-1)}}$
default	$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(-35a^4x^4\sqrt{-c(ax+1)}-95a^3x^3\sqrt{-c(ax+1)}-138a^2x^2\sqrt{-c(ax+1)}+630\sqrt{c}\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)\right.}{315\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}a^4}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-2/315*(35*a^4*x^4+95*a^3*x^3+138*a^2*x^2+236*a*x+788)/a^4*c/((a*x-1)/(a*x
+1))^(1/2)/(-c*(a*x-1))^(1/2)*(a*x-1)-4/a^4*2^(1/2)*c^(1/2)*arctan(1/2*(-a
*c*x-c)^(1/2)*2^(1/2)/c^(1/2))/((a*x-1)/(a*x+1))^(1/2)/(a*x+1)*(-c*(a*x+1)
)^(1/2)/(-c*(a*x-1))^(1/2)*(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$= \left[\frac{2\left(315\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(35\,a^5x^5+130\,a^4x^4+233\right.}{315\,(a^5x-a^4)}\right.$$
$$-\frac{2\left(630\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2\,(acx-c)}\right)-(35\,a^5x^5+130\,a^4x^4+233\,a^3x^3+374\,a^2x\right.}{315\,(a^5x-a^4)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="fri
cas")
```

output
```
[2/315*(315*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(
2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a
^2*x^2 - 2*a*x + 1)) + (35*a^5*x^5 + 130*a^4*x^4 + 233*a^3*x^3 + 374*a^2*x
^2 + 1024*a*x + 788)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*x -
a^4), -2/315*(630*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x
 + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (35*a^5*x
^5 + 130*a^4*x^4 + 233*a^3*x^3 + 374*a^2*x^2 + 1024*a*x + 788)*sqrt(-a*c*x
 + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*x - a^4)]
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\int\frac{x^3\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(x**3*sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\int \frac{\sqrt{-acx+c}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a*c*x + c)*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\int \frac{x^3\,\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((x^3*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^3*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\frac{2\sqrt{c}\,i\left(-35\sqrt{ax+1}\,a^4x^4-95\sqrt{ax+1}\,a^3x^3-138\sqrt{ax+1}\,a^2x^2-236\sqrt{ax+1}\,ax-788\sqrt{ax+1}-630\right.}{315a^4}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x)
```

output
```
(2*sqrt(c)*i*( - 35*sqrt(a*x + 1)*a**4*x**4 - 95*sqrt(a*x + 1)*a**3*x**3 -
 138*sqrt(a*x + 1)*a**2*x**2 - 236*sqrt(a*x + 1)*a*x - 788*sqrt(a*x + 1) -
 630*sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2))/2)) + 1292*sqrt(2)))/(
315*a**4)
```

3.334 $\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 262

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}}+\frac{32\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}}+\frac{6\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

output
```
104/21*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^3/(1-1/a/x)^(1/2)+32/21*(1+1/a/x
)^(1/2)*x*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)+6/7*(1+1/a/x)^(1/2)*x^2*(-a
*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/7*(1+1/a/x)^(1/2)*x^3*(-a*c*x+c)^(1/2)/(
1-1/a/x)^(1/2)-4*2^(1/2)*(1/a/x)^(1/2)*(-a*c*x+c)^(1/2)*arctanh(2^(1/2)*(1
/a/x)^(1/2)/(1+1/a/x)^(1/2))/a^3/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.47

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}(52+16ax+9a^2x^2+3a^3x^3)-42\sqrt{2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{21a^{7/2}\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(52 + 16*a*x + 9*a^2*x^2 + 3
*a^3*x^3) - 42*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a
]*Sqrt[1 + 1/(a*x)])]))/(21*a^(7/2)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.75, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {6730, 27, 109, 27, 169, 27, 169, 27, 169, 27, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-acx}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{2\int -\frac{15a+\frac{13}{x}}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{7a}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\int \frac{15a+\frac{13}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{169}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{-\frac{2\int -\frac{10\left(4a+\frac{3}{x}\right)}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{5a}-\frac{6\sqrt{\frac{1}{ax}+1}}{\left(\frac{1}{x}\right)^{5/2}}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{4\int \frac{4a+\frac{3}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{a}-\frac{6\sqrt{\frac{1}{ax}+1}}{\left(\frac{1}{x}\right)^{5/2}}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{169}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{4\left(-\frac{2\int -\frac{13a+\frac{8}{x}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{3a}-\frac{8\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{a}-\frac{6\sqrt{\frac{1}{ax}+1}}{\left(\frac{1}{x}\right)^{5/2}}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 27

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{4\left(\frac{\int \frac{13a+\frac{8}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{3a}-\frac{8\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{a}-\frac{6\sqrt{\frac{1}{ax}+1}}{\left(\frac{1}{x}\right)^{5/2}}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 169

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{4\left(\frac{-\frac{2\int -\frac{21a}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{26\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}}{3a}-\frac{8\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{a}-\frac{6\sqrt{\frac{1}{ax}+1}}{\left(\frac{1}{x}\right)^{5/2}}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 27

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{4\left(\frac{21\int\frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{26\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}}{3a}-\frac{8\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{a}-\frac{6\sqrt{\frac{1}{ax}+1}}{\left(\frac{1}{x}\right)^{5/2}}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textbf{104}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{4\left(\frac{42\int\frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-\frac{26\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}}{3a}-\frac{8\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{a}-\frac{6\sqrt{\frac{1}{ax}+1}}{\left(\frac{1}{x}\right)^{5/2}}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{a\sqrt{\frac{1}{x}}\left(\frac{\frac{4\left(\frac{\frac{21\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}}-\frac{26\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}}{3a}-\frac{8\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{a}-\frac{6\sqrt{\frac{1}{ax}+1}}{\left(\frac{1}{x}\right)^{5/2}}}{7a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{7a\left(\frac{1}{x}\right)^{7/2}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

output
```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*Sqrt[1 + 1/(a*x)])/(7*a*(x^(-1))^(7
/2)) + ((-6*Sqrt[1 + 1/(a*x)])/(x^(-1))^(5/2) + (4*((-8*Sqrt[1 + 1/(a*x)])
/(3*(x^(-1))^(3/2)) + ((-26*Sqrt[1 + 1/(a*x)])/Sqrt[x^(-1)] + (21*Sqrt[2]*
ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[a])/(3*a
)))/a)/(7*a^2)))/Sqrt[1 - 1/(a*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.53

method	result
risch	$-\frac{2\left(3 a^{3} x^{3}+9 a^{2} x^{2}+16 a x +52\right) c \left(a x -1\right)}{21 a^{3} \sqrt{\frac{a x -1}{a x +1}}\, \sqrt{-c \left(a x -1\right)}}-\frac{4 \sqrt{2}\, \sqrt{c}\, \arctan\left(\frac{\sqrt{-a c x -c}\, \sqrt{2}}{2 \sqrt{c}}\right) \sqrt{-c \left(a x +1\right)}\, \left(a x -1\right)}{a^{3} \sqrt{\frac{a x -1}{a x +1}}\, \left(a x +1\right) \sqrt{-c \left(a x -1\right)}}$
default	$\frac{2 \left(a x -1\right) \sqrt{-c \left(a x -1\right)}\, \left(3 a^{3} x^{3} \sqrt{-c \left(a x +1\right)}+9 a^{2} x^{2} \sqrt{-c \left(a x +1\right)}+16 a x \sqrt{-c \left(a x +1\right)}-42 \sqrt{c}\, \sqrt{2}\, \arctan\left(\frac{\sqrt{-c \left(a x +1\right)}\, \sqrt{2}}{2 \sqrt{c}}\right)+52 \sqrt{-c(a \cdots}\right.}{21 \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(a x +1\right) \sqrt{-c \left(a x +1\right)}\, a^{3}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-2/21*(3*a^3*x^3+9*a^2*x^2+16*a*x+52)/a^3*c/((a*x-1)/(a*x+1))^(1/2)/(-c*(a
*x-1))^(1/2)*(a*x-1)-4/a^3*2^(1/2)*c^(1/2)*arctan(1/2*(-a*c*x-c)^(1/2)*2^(
1/2)/c^(1/2))/((a*x-1)/(a*x+1))^(1/2)/(a*x+1)*(-c*(a*x+1))^(1/2)/(-c*(a*x-
1))^(1/2)*(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.12

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \left[\frac{2\left(21\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(3\,a^4x^4+12\,a^3x^3+25\,a^2x^2\right.}{21\,(a^4x-a^3)}\right.$$

$$-\frac{2\left(42\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2\,(acx-c)}\right)-(3\,a^4x^4+12\,a^3x^3+25\,a^2x^2+68\,ax+52)\right.}{21\,(a^4x-a^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fri
cas")
```

output
```
[2/21*(21*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)
*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2
*x^2 - 2*a*x + 1)) + (3*a^4*x^4 + 12*a^3*x^3 + 25*a^2*x^2 + 68*a*x + 52)*s
qrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3), -2/21*(42*sqrt(2
)*(a*x - 1)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*
sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (3*a^4*x^4 + 12*a^3*x^3 + 25*a^2*
x^2 + 68*a*x + 52)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^
3)]
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \int \frac{x^2\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\int \frac{\sqrt{-acx+c}\,x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a*c*x + c)*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.55

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\frac{2\,c^2\left(\frac{2\,\sqrt{2}\left(21\,\sqrt{c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{c}}\right)-40\,\sqrt{-c}\right)}{a^2c}-\frac{42\,\sqrt{2}c^{\frac{7}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\,\sqrt{c}}\right)-3\,(acx+c)^3\sqrt{-acx-c}+7\,(-acx-c)^{\frac{3}{2}}c^2-42\,\sqrt{-acx-c}c^3}{a^2c^4}\right)}{21\,a|c|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="gia
c")
```

output
```
-2/21*c^2*(2*sqrt(2)*(21*sqrt(c)*arctan(sqrt(-c)/sqrt(c)) - 40*sqrt(-c))/(
a^2*c) - (42*sqrt(2)*c^(7/2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))
- 3*(a*c*x + c)^3*sqrt(-a*c*x - c) + 7*(-a*c*x - c)^(3/2)*c^2 - 42*sqrt(-a
*c*x - c)*c^3)/(a^2*c^4))/(a*abs(c)*sgn(a*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\int\frac{x^2\,\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((x^2*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^2*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$
$$=\frac{2\sqrt{c}\,i\left(-3\sqrt{ax+1}\,a^3x^3-9\sqrt{ax+1}\,a^2x^2-16\sqrt{ax+1}\,ax-52\sqrt{ax+1}-42\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right.\right.\right.}{21a^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x)
```

output
```
(2*sqrt(c)*i*( - 3*sqrt(a*x + 1)*a**3*x**3 - 9*sqrt(a*x + 1)*a**2*x**2 - 1
6*sqrt(a*x + 1)*a*x - 52*sqrt(a*x + 1) - 42*sqrt(2)*log(tan(asin(sqrt( - a
*x + 1)/sqrt(2))/2)) + 80*sqrt(2)))/(21*a**3)
```

3.335 $\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 214

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{76\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}+\frac{22\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{ax}}\sqrt{c-acx}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

output
```
76/15*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)+22/15*(1+1/a/x)
^(1/2)*x*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/5*(1+1/a/x)^(1/2)*x^2*(-a*c*
x+c)^(1/2)/(1-1/a/x)^(1/2)-4*2^(1/2)*(1/a/x)^(1/2)*(-a*c*x+c)^(1/2)*arctan
h(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))/a^2/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.53

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$$
$$=\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}(38+11ax+3a^2x^2)-30\sqrt{2}\sqrt{\frac{1}{x}}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{15a^{5/2}\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(38 + 11*a*x + 3*a^2*x^2) -
30*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1
/(a*x)])]))/(15*a^(5/2)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6730, 27, 107, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-acx}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6730$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{107}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}}{5a\left(\frac{1}{x}\right)^{5/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{2\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}}{5a\left(\frac{1}{x}\right)^{5/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{2\left(\frac{2\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}}{5a\left(\frac{1}{x}\right)^{5/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\frac{2\left(\frac{4\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}}{5a\left(\frac{1}{x}\right)^{5/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\downarrow \text{219}$$

$$-\frac{a\sqrt{\frac{1}{x}}\left(\frac{\frac{2\left(\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}}{5a\left(\frac{1}{x}\right)^{5/2}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

output
```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*(1 + 1/(a*x))^(5/2))/(5*a*(x^(-1))^
(5/2)) + ((-2*(1 + 1/(a*x))^(3/2))/(3*a*(x^(-1))^(3/2)) + (2*((-2*Sqrt[1 +
 1/(a*x)])/(a*Sqrt[x^(-1)]) + (2*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(S
qrt[a]*Sqrt[1 + 1/(a*x)])])/a^(3/2)))/a)/a))/Sqrt[1 - 1/(a*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.58

method	result	size
default	$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(-3a^{2}x^{2}\sqrt{-c(ax+1)}+30\sqrt{c}\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)-11ax\sqrt{-c(ax+1)}-38\sqrt{-c(ax+1)}\right)}{15\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a^{2}}$	125
risch	$-\frac{2(3a^{2}x^{2}+11ax+38)c(ax-1)}{15a^{2}\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{a^{2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}$	130

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*a^2*x
^2*(-c*(a*x+1))^(1/2)+30*c^(1/2)*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(
1/2)/c^(1/2))-11*a*x*(-c*(a*x+1))^(1/2)-38*(-c*(a*x+1))^(1/2))/(-c*(a*x+1)
)^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.30

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \left[\frac{2\left(15\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(3\,a^3x^3+14\,a^2x^2+49\,ax+\right.}{15\,(a^3x-a^2)}\right.$$

$$-\frac{2\left(30\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2\,(acx-c)}\right)-(3\,a^3x^3+14\,a^2x^2+49\,ax+38)\sqrt{-acx+c}\right.}{15\,(a^3x-a^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="frica
s")
```

output
```
[2/15*(15*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)
*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2
*x^2 - 2*a*x + 1)) + (3*a^3*x^3 + 14*a^2*x^2 + 49*a*x + 38)*sqrt(-a*c*x +
c)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*x - a^2), -2/15*(30*sqrt(2)*(a*x - 1)*s
qrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1
)/(a*x + 1))/(a*c*x - c)) - (3*a^3*x^3 + 14*a^2*x^2 + 49*a*x + 38)*sqrt(-a
*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*x - a^2)]
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\int \frac{x\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(x*sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\int \frac{\sqrt{-acx+c}x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\int \frac{x\,\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((x*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.31

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$$
$$=\frac{2\sqrt{c}\,i\left(-3\sqrt{ax+1}\,a^2x^2-11\sqrt{ax+1}\,ax-38\sqrt{ax+1}-30\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)+52\sqrt{2}\right)}{15a^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x)
```

output
```
(2*sqrt(c)*i*( - 3*sqrt(a*x + 1)*a**2*x**2 - 11*sqrt(a*x + 1)*a*x - 38*sqr
t(a*x + 1) - 30*sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2))/2)) + 52*sq
rt(2)))/(15*a**2)
```

3.336 $\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 164

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

output
```
4*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/3*(1+1/a/x)^(3/2)*x
*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*2^(1/2)*(1/a/x)^(1/2)*(-a*c*x+c)^(1/2)
*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))/a/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.64

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}(7+ax)-6\sqrt{2}\sqrt{\frac{1}{x}}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{3a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 + a*x) - 6*Sqrt[2]*Sqrt[x
^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(3*a^
(3/2)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6727, 27, 105, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2\int \frac{\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2\left(\frac{2\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 104

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2\left(\frac{4\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 219

$$-\frac{a\sqrt{\frac{1}{x}}\left(\frac{2\left(\frac{2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{a}-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}}{3a\left(\frac{1}{x}\right)^{3/2}}\right)\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

output
```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*(1 + 1/(a*x))^(3/2))/(3*a*(x^(-1))^
(3/2)) + (2*((-2*Sqrt[1 + 1/(a*x)])/(a*Sqrt[x^(-1)]) + (2*Sqrt[2]*ArcTanh[
(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/a^(3/2)))/a))/Sqrt[1
- 1/(a*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(6\sqrt{c}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)-ax\sqrt{-c(ax+1)}-7\sqrt{-c(ax+1)}\right)}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a}$	107
risch	$-\frac{2(ax+7)c(ax-1)}{3a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\,\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}\,(ax-1)}{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}$	121

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(6*c^(1/2)
*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-a*x*(-c*(a*x+1))^(
1/2)-7*(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.56

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$$

$$=\left[\frac{2\left(3\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(a^2x^2+8\,ax+7)\sqrt{-acx+c}\right.}{3\,(a^2x-a)}\right.$$

$$\left.-\frac{2\left(6\,\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2\,(acx-c)}\right)-(a^2x^2+8\,ax+7)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}\right)}{3\,(a^2x-a)}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
[2/3*(3*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*s
qrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x
^2 - 2*a*x + 1)) + (a^2*x^2 + 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(
a*x + 1)))/(a^2*x - a), -2/3*(6*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(1/2*sqrt(
2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c
)) - (a^2*x^2 + 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^
2*x - a)]
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\frac{\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\int\frac{\sqrt{-acx+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a*c*x + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.32

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{c}\,i\left(-\sqrt{ax+1}\,ax-7\sqrt{ax+1}-6\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)+8\sqrt{2}\right)}{3a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x)
```

output
```
(2*sqrt(c)*i*( - sqrt(a*x + 1)*a*x - 7*sqrt(a*x + 1) - 6*sqrt(2)*log(tan(a
sin(sqrt( - a*x + 1)/sqrt(2))/2)) + 8*sqrt(2)))/(3*a)
```

3.337 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\text{arcsinh}\left(\sqrt{\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+2*(1/a/x)^(1/2)*(-a*c*x
+c)^(1/2)*arcsinh((1/a/x)^(1/2))/(1-1/a/x)^(1/2)-4*2^(1/2)*(1/a/x)^(1/2)*(
-a*c*x+c)^(1/2)*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1-1/a/x)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.73

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}+\sqrt{\frac{1}{x}}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-2\sqrt{2}\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

output
```
(2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[x^(-1)]*ArcSinh[Sqrt[
x^(-1)]/Sqrt[a]] - 2*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(
Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.79, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6730, 27, 109, 27, 175, 63, 104, 219, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{3\coth^{-1}(ax)}}{x}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{2\int -\frac{3a+\frac{1}{x}}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\int \frac{3a+\frac{1}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{175}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{4a\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{63}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{4a\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-2\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}}{a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-2\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}}{a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{4\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-2\int\frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}}{a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{4\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{a^2}-\frac{2\sqrt{\frac{1}{ax}+1}}{a\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

output
```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*Sqrt[1 + 1/(a*x)])/(a*Sqrt[x^(-1)])
 + (-2*Sqrt[a]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]] + 4*Sqrt[2]*Sqrt[a]*ArcTanh[(
Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/a^2))/Sqrt[1 - 1/(a*x)
])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(2\sqrt{c}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)-\sqrt{c}\,\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)-\sqrt{-c(ax+1)}\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}}$	110

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(2*c^(1/2)*2
^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-c^(1/2)*arctan((-c*(
a*x+1))^(1/2)/c^(1/2))-(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.20

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \left[\frac{2\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2}{}\right.}{ax-1}\right.$$

$$-\frac{2\left(2\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2\,(acx-c)}\right)-(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)\right.}{ax-1}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="frica
s")
```

output
```
[(2*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sqrt(
-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 -
 2*a*x + 1)) + (a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x
+ c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*
sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*(2*sqr
t(2)*(a*x - 1)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(
c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (a*x - 1)*sqrt(c)*arctan(sqrt(
-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqr
t(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{-c\,(ax-1)}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))/(x*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{-acx+c}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\int \frac{\sqrt{c-a\,c\,x}}{x\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.79

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\sqrt{c}\,i\left(-2\sqrt{ax+1}-4\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)+2\sqrt{2}-\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)-1\right)+\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)+1\right)+\log\left(\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)-1\right)-\log\left(\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)+1\right)\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x)
```

output
```
sqrt(c)*i*( - 2*sqrt(a*x + 1) - 4*sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sq
rt(2))/2)) + 2*sqrt(2) - log( - sqrt(2) + tan(asin(sqrt( - a*x + 1)/sqrt(2
))/2) - 1) + log( - sqrt(2) + tan(asin(sqrt( - a*x + 1)/sqrt(2))/2) + 1) +
 log(sqrt(2) + tan(asin(sqrt( - a*x + 1)/sqrt(2))/2) - 1) - log(sqrt(2) +
tan(asin(sqrt( - a*x + 1)/sqrt(2))/2) + 1))
```

3.338 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 169

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x}+\frac{5a\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\text{arcsinh}\left(\sqrt{\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}a\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/x+5*a*(1/a/x)^(1/2)*(-a*c
*x+c)^(1/2)*arcsinh((1/a/x)^(1/2))/(1-1/a/x)^(1/2)-4*2^(1/2)*a*(1/a/x)^(1/
2)*(-a*c*x+c)^(1/2)*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1-1/a/
x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.71

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}+5\sqrt{a}\,\mathrm{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-4\sqrt{2}\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

output
```
(Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + 5*Sqrt[a]*
ArcSinh[Sqrt[x^(-1)]/Sqrt[a]] - 4*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[x^
(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/Sqrt[1 - 1/(a*x)]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.79, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6730, 27, 113, 27, 175, 63, 104, 219, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ \textbf{113}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\int -\frac{3a+\frac{5}{x}}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ \textbf{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{\int \frac{3a+\frac{5}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2a}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ \textbf{175}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-5\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{2a}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ \textbf{63}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-10\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}}{2a}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ \textbf{104}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{16a\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-10\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}}{2a}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ \textbf{219}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-10\int\frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}}{2a}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-10\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{2a}-\frac{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

output
```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(-((Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)])/a) +
 (-10*Sqrt[a]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]] + 8*Sqrt[2]*Sqrt[a]*ArcTanh[(S
qrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(2*a)))/Sqrt[1 - 1/(a*x
)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{(ax-1)\left(4\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx-5\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)acx-\sqrt{-c(ax+1)}\,\sqrt{c}\right)\sqrt{-c(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,\sqrt{c}\,x}$	119
risch	$-\frac{c(ax-1)}{x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}}-\frac{\left(\frac{4a\sqrt{2}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{5a\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)c\sqrt{-c(ax+1)}\,(ax-1)}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}$	140

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE
)
```

output
```
-(a*x-1)*(4*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-5
*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*a*c*x-(-c*(a*x+1))^(1/2)*c^(1/2))*(-c*
(a*x-1))^(1/2)/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/(-c*(a*x+1))^(1/2)/c^(1/2)/
x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.37

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$$

$$= \left[\frac{4\sqrt{2}(a^2x^2-ax)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+5\,(a^2x^2-ax)\sqrt{-c}\log\left(-\frac{a^2c}{}\right.}{2\,(ax^2-x)}\right.$$

$$-\frac{4\sqrt{2}(a^2x^2-ax)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2\,(acx-c)}\right)-5\,(a^2x^2-ax)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{a}{a}}}{acx-c}\right.}{ax^2-x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fri
cas")
```

output
```
[1/2*(4*sqrt(2)*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqr
t(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/
(a^2*x^2 - 2*a*x + 1)) + 5*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*
x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)
/(a*x^2 - x)) + 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a
*x^2 - x), -(4*sqrt(2)*(a^2*x^2 - a*x)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*
c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 5*(a^2
*x^2 - a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x -
1)/(a*x + 1))/(a*c*x - c)) - sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*
x + 1)))/(a*x^2 - x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-acx+c}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a*c*x + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.12

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c}\,i\left(-2\sqrt{ax+1}-8\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)ax+2\sqrt{2}\,ax-7\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)-\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x)
```

output
```
(sqrt(c)*i*( - 2*sqrt(a*x + 1) - 8*sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/s
qrt(2))/2))*a*x + 2*sqrt(2)*a*x - 7*log( - sqrt(2) + tan(asin(sqrt( - a*x
+ 1)/sqrt(2))/2) - 1)*a*x + 7*log( - sqrt(2) + tan(asin(sqrt( - a*x + 1)/s
qrt(2))/2) + 1)*a*x + 7*log(sqrt(2) + tan(asin(sqrt( - a*x + 1)/sqrt(2))/2
) - 1)*a*x - 7*log(sqrt(2) + tan(asin(sqrt( - a*x + 1)/sqrt(2))/2) + 1)*a*
x - 2*log((2*sqrt(a*x + 1) - 2)/sqrt(2))*a*x + 2*log((2*sqrt(a*x + 1) + 2)
/sqrt(2))*a*x))/(2*x)
```

3.339 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 225

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x}+\frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x}+\frac{23a^2\sqrt{\frac{1}{ax}}\sqrt{c-acx}\operatorname{arcsinh}\left(\sqrt{\frac{1}{ax}}\right)}{4\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}a^2\sqrt{\frac{1}{ax}}\sqrt{c-acx}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
7/4*a*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/x+1/2*a*(1+1/a/x)^(
3/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/x+23/4*a^2*(1/a/x)^(1/2)*(-a*c*x+c)^
(1/2)*arcsinh((1/a/x)^(1/2))/(1-1/a/x)^(1/2)-4*2^(1/2)*a^2*(1/a/x)^(1/2)*(
-a*c*x+c)^(1/2)*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1-1/a/x)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.59

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}(2+9ax)+\frac{23a^{3/2}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{3/2}}-\frac{16\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}}\right)}{4\sqrt{1-\frac{1}{ax}}x^2}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

output
```
(Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(2 + 9*a*x) + (23*a^(3/2)*ArcSinh[Sqrt
[x^(-1)]/Sqrt[a]])/(x^(-1))^(3/2) - (16*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*S
qrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(x^(-1))^(3/2)))/(4*Sqrt[1 - 1/
(a*x)]*x^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.70, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {6730, 27, 112, 27, 171, 27, 175, 63, 104, 219, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{3\coth^{-1}(ax)}}{x^3}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int\frac{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{112}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\int\frac{\left(a+\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\int\frac{\left(a+\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{171}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\left(-\int-\frac{9a+\frac{23}{x}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-7\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\left(\frac{1}{2}\int\frac{9a+\frac{23}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-7\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{175}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\left(\frac{1}{2}\left(32a\int\frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-23\int\frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)-7\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{63}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\left(\frac{1}{2}\left(32a\int\frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-46\int\frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\right)-7\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\left(\frac{1}{2}\left(64a\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-46\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\right)-7\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\left(\frac{1}{2}\left(32\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-46\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\right)-7\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right.\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\left(\frac{1}{2}\left(32\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-46\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)-7\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+\right.\right.}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

output
```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(-1/2*((1 + 1/(a*x))^(3/2)*Sqrt[x^(-1)])
 + (-7*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + (-46*Sqrt[a]*ArcSinh[Sqrt[x^(-1)]/
Sqrt[a]] + 32*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt
[1 + 1/(a*x)])])/2)/4))/Sqrt[1 - 1/(a*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 112
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*(m + n +
p + 1))), x] - Simp[1/(f*(m + n + p + 1))   Int[(a + b*x)^(m - 1)*(c + d*x)
^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a
*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] &&
GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p
] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.64

method	result
default	$\frac{(ax-1)\sqrt{-c(ax-1)}\left(-16\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^2c\,x^2+23c\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^2x^2+9ax\sqrt{-c(ax+1)}\,\sqrt{c}+2\sqrt{-c(ax+1)}}{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{c}\,\sqrt{-c(ax+1)}\,x^2}$
risch	$-\frac{(9a^2x^2+11ax+2)c(ax-1)}{4x^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}-\frac{\left(\frac{4a^2\sqrt{2}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{23a^2\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{4\sqrt{c}}\right)c\sqrt{-c(ax+1)}\,(ax-1)}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x,method=_RETURNVERBOSE
)
```

output
```
1/4*(a*x-1)*(-c*(a*x-1))^(1/2)*(-16*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*
2^(1/2)/c^(1/2))*a^2*c*x^2+23*c*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*a^2*x^2
+9*a*x*(-c*(a*x+1))^(1/2)*c^(1/2)+2*(-c*(a*x+1))^(1/2)*c^(1/2))/((a*x-1)/(
a*x+1))^(3/2)/(a*x+1)/c^(1/2)/(-c*(a*x+1))^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.95

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$$
$$= \left[\frac{16\sqrt{2}(a^3x^3-a^2x^2)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+23\,(a^3x^3-a^2x^2)\sqrt{-c}\log}{8\,(ax^3-x^2)}\right.$$
$$-\frac{16\sqrt{2}(a^3x^3-a^2x^2)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2\,(acx-c)}\right)-23\,(a^3x^3-a^2x^2)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}(ax+1}{acx-}\right.}{4\,(ax^3-x^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="fri
cas")
```

output
```
[1/8*(16*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x +
2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) -
3*c)/(a^2*x^2 - 2*a*x + 1)) + 23*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*
x^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x +
1)) - 2*c)/(a*x^2 - x)) + 2*(9*a^2*x^2 + 11*a*x + 2)*sqrt(-a*c*x + c)*sqrt
((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2), -1/4*(16*sqrt(2)*(a^3*x^3 - a^2*x^2)
*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x -
 1)/(a*x + 1))/(a*c*x - c)) - 23*(a^3*x^3 - a^2*x^2)*sqrt(c)*arctan(sqrt(-
a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (9*a
^2*x^2 + 11*a*x + 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 -
x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{\sqrt{-acx+c}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a*c*x + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx=\int \frac{\sqrt{c-a\,c\,x}}{x^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a*c*x)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.82

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{\sqrt{c}\,i\left(-18\sqrt{ax+1}\,ax-4\sqrt{ax+1}-32\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)a^2x^2+22\sqrt{2}\,a^2x^2-23\log\left(-\sqrt{2}\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x)
```

output
```
(sqrt(c)*i*( - 18*sqrt(a*x + 1)*a*x - 4*sqrt(a*x + 1) - 32*sqrt(2)*log(tan
(asin(sqrt( - a*x + 1)/sqrt(2))/2))*a**2*x**2 + 22*sqrt(2)*a**2*x**2 - 23*
log( - sqrt(2) + tan(asin(sqrt( - a*x + 1)/sqrt(2))/2) - 1)*a**2*x**2 + 23
*log( - sqrt(2) + tan(asin(sqrt( - a*x + 1)/sqrt(2))/2) + 1)*a**2*x**2 + 2
3*log(sqrt(2) + tan(asin(sqrt( - a*x + 1)/sqrt(2))/2) - 1)*a**2*x**2 - 23*
log(sqrt(2) + tan(asin(sqrt( - a*x + 1)/sqrt(2))/2) + 1)*a**2*x**2))/(8*x*
*2)
```

3.340 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 275

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^2}+\frac{13a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}x}+\frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x}+\frac{45a^3\sqrt{\frac{1}{ax}}\sqrt{c-acx}\operatorname{arcsinh}\left(\sqrt{\frac{1}{ax}}\right)}{8\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}a^3\sqrt{\frac{1}{ax}}\sqrt{c-acx}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
1/3*a*(1+1/a/x)^(3/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/x^2+13/8*a^2*(1+1/a
/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/x+3/4*a^2*(1+1/a/x)^(3/2)*(-a*c
*x+c)^(1/2)/(1-1/a/x)^(1/2)/x+45/8*a^3*(1/a/x)^(1/2)*(-a*c*x+c)^(1/2)*arcs
inh((1/a/x)^(1/2))/(1-1/a/x)^(1/2)-4*2^(1/2)*a^3*(1/a/x)^(1/2)*(-a*c*x+c)^
(1/2)*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.51

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}(8+26ax+57a^2x^2)+\frac{135a^{5/2}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{5/2}}-\frac{96\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{5/2}}\right)}{24\sqrt{1-\frac{1}{ax}}x^3}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

output
```
(Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(8 + 26*a*x + 57*a^2*x^2) + (135*a^(5/
2)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(5/2) - (96*Sqrt[2]*a^(5/2)*Arc
Tanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(x^(-1))^(5/2)))
/(24*Sqrt[1 - 1/(a*x)]*x^3)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.68, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {6730, 27, 112, 27, 171, 27, 171, 27, 175, 63, 104, 219, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{3\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \text{ 6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$\big\downarrow$ 112

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{3}\int \frac{3\left(a+\frac{3}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}{2\left(a-\frac{1}{x}\right)}d\frac{1}{x}-\frac{1}{3}\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\big\downarrow$ 27

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\int \frac{\left(a+\frac{3}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}{a-\frac{1}{x}}d\frac{1}{x}-\frac{1}{3}\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\big\downarrow$ 171

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\left(-\frac{1}{2}a\int -\frac{\left(3a+\frac{13}{x}\right)\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{3}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{1}{3}\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\big\downarrow$ 27

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\left(\frac{1}{4}a\int \frac{\left(3a+\frac{13}{x}\right)\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{3}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{1}{3}\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$\big\downarrow$ 171

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\left(\frac{1}{4}a\left(-\int -\frac{19a+\frac{45}{x}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-13\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{3}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{1}{3}\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+\right.\right.}{\sqrt{1-\frac{1}{ax}}}$$

$\big\downarrow$ 27

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\left(\frac{1}{4}a\left(\frac{1}{2}\int \frac{19a+\frac{45}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-13\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{3}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{1}{3}\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3}\right.}{\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 175

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\left(\frac{1}{4}a\left(\frac{1}{2}\left(64a\int\frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-45\int\frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)-13\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{3}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+\right.}{\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 63

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\left(\frac{1}{4}a\left(\frac{1}{2}\left(64a\int\frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-90\int\frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\right)-13\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{3}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+\right.}{\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 104

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\left(\frac{1}{4}a\left(\frac{1}{2}\left(128a\int\frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-90\int\frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\right)-13\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{3}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/}\right.}{\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 219

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\left(\frac{1}{4}a\left(\frac{1}{2}\left(64\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-90\int\frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\right)-13\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{3}{2}a\sqrt{\frac{1}{x}}\left(\right.\right.}{\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 222

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{2}\left(\frac{1}{4}a\left(\frac{1}{2}\left(64\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-90\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)-13\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{3}{2}a\sqrt{\right.\right.}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

output
```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(-1/3*((1 + 1/(a*x))^(3/2)*(x^(-1))^(3/2
)) + ((-3*a*(1 + 1/(a*x))^(3/2)*Sqrt[x^(-1)])/2 + (a*(-13*Sqrt[1 + 1/(a*x)
]*Sqrt[x^(-1)] + (-90*Sqrt[a]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]] + 64*Sqrt[2]*S
qrt[a]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/2))/4)
/2))/Sqrt[1 - 1/(a*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 112
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*(m + n +
p + 1))), x] - Simp[1/(f*(m + n + p + 1))   Int[(a + b*x)^(m - 1)*(c + d*x)
^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a
*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] &&
GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p
] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.60

method	result
default	$\frac{(ax-1)\sqrt{-c(ax-1)}\left(-96\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^3cx^3+135c\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^3x^3+57a^2x^2\sqrt{-c(ax+1)}\,\sqrt{c}+26ax\sqrt{-}\right.}{24\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{c}\,\sqrt{-c(ax+1)}\,x^3}$
risch	$-\frac{(57a^3x^3+83a^2x^2+34ax+8)c(ax-1)}{24x^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}-\frac{\left(\frac{4a^3\sqrt{2}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{45a^3\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{8\sqrt{c}}\right)c\sqrt{-c(ax+1)}\,(ax-1)}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x,method=_RETURNVERBOSE
)
```

output
```
1/24*(a*x-1)*(-c*(a*x-1))^(1/2)*(-96*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)
*2^(1/2)/c^(1/2))*a^3*c*x^3+135*c*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*a^3*x
^3+57*a^2*x^2*(-c*(a*x+1))^(1/2)*c^(1/2)+26*a*x*(-c*(a*x+1))^(1/2)*c^(1/2)
+8*(-c*(a*x+1))^(1/2)*c^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/c^(1/2)/(-c
*(a*x+1))^(1/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.65

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$

$$= \left[\frac{96\sqrt{2}(a^4x^4-a^3x^3)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+135\,(a^4x^4-a^3x^3)\sqrt{-c}\log}{48\,(ax^4-}\right.$$

$$-\frac{96\sqrt{2}(a^4x^4-a^3x^3)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2\,(acx-c)}\right)-135\,(a^4x^4-a^3x^3)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}(ax+}{acx}\right.}{24\,(ax^4-x^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="fri
cas")
```

output
```
[1/48*(96*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x +
 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) -
 3*c)/(a^2*x^2 - 2*a*x + 1)) + 135*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*
c*x^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x
+ 1)) - 2*c)/(a*x^2 - x)) + 2*(57*a^3*x^3 + 83*a^2*x^2 + 34*a*x + 8)*sqrt(
-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^4 - x^3), -1/24*(96*sqrt(2)*(a
^4*x^4 - a^3*x^3)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sq
rt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 135*(a^4*x^4 - a^3*x^3)*sqr
t(c)*arctan(sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(
a*c*x - c)) - (57*a^3*x^3 + 83*a^2*x^2 + 34*a*x + 8)*sqrt(-a*c*x + c)*sqrt
((a*x - 1)/(a*x + 1)))/(a*x^4 - x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{\sqrt{-acx+c}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a*c*x + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a*c*x)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.72

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$

$$= \frac{\sqrt{c}\,i\left(-114\sqrt{ax+1}\,a^2x^2-52\sqrt{ax+1}\,ax-16\sqrt{ax+1}-192\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right)}{2}\right)\right)a^3x^3+182\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x)
```

output
```
(sqrt(c)*i*( - 114*sqrt(a*x + 1)*a**2*x**2 - 52*sqrt(a*x + 1)*a*x - 16*sqr
t(a*x + 1) - 192*sqrt(2)*log(tan(asin(sqrt( - a*x + 1)/sqrt(2))/2))*a**3*x
**3 + 182*sqrt(2)*a**3*x**3 - 135*log( - sqrt(2) + tan(asin(sqrt( - a*x +
1)/sqrt(2))/2) - 1)*a**3*x**3 + 135*log( - sqrt(2) + tan(asin(sqrt( - a*x
+ 1)/sqrt(2))/2) + 1)*a**3*x**3 + 135*log(sqrt(2) + tan(asin(sqrt( - a*x +
 1)/sqrt(2))/2) - 1)*a**3*x**3 - 135*log(sqrt(2) + tan(asin(sqrt( - a*x +
1)/sqrt(2))/2) + 1)*a**3*x**3))/(48*x**3)
```

3.341 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 323

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^2} + \frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}x} + \frac{21a^3\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{32\sqrt{1-\frac{1}{ax}}x} + \frac{363a^4\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\text{arcsinh}\left(\sqrt{\frac{1}{ax}}\right)}{64\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}a^4\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
1/4*a*(1+1/a/x)^(3/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/x^3+11/24*a^2*(1+1/
a/x)^(3/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/x^2+107/64*a^3*(1+1/a/x)^(1/2)
*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/x+21/32*a^3*(1+1/a/x)^(3/2)*(-a*c*x+c)^(
1/2)/(1-1/a/x)^(1/2)/x+363/64*a^4*(1/a/x)^(1/2)*(-a*c*x+c)^(1/2)*arcsinh((
1/a/x)^(1/2))/(1-1/a/x)^(1/2)-4*2^(1/2)*a^4*(1/a/x)^(1/2)*(-a*c*x+c)^(1/2)
*arctanh(2^(1/2)*(1/a/x)^(1/2)/(1+1/a/x)^(1/2))/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.46

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}(48+136ax+214a^2x^2+447a^3x^3)+\frac{1089a^{7/2}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{7/2}}-\frac{768\sqrt{2}a^{7/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\ldots}}{\sqrt{a}\sqrt{1\ldots}}\right)}{\left(\frac{1}{x}\right)^{7/2}}\ldots\right.}{192\sqrt{1-\frac{1}{ax}}x^4}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

output
```
(Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(48 + 136*a*x + 214*a^2*x^2 + 447*a^3*
x^3) + (1089*a^(7/2)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(7/2) - (768*
Sqrt[2]*a^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])
])/(x^(-1))^(7/2)))/(192*Sqrt[1 - 1/(a*x)]*x^4)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.68, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {6730, 27, 112, 27, 171, 27, 171, 27, 171, 27, 175, 63, 104, 219, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{3\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 112$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{4}\int \frac{\left(5a+\frac{11}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)}d\frac{1}{x}-\frac{1}{4}\left(\frac{1}{x}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\int \frac{\left(5a+\frac{11}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}{a-\frac{1}{x}}d\frac{1}{x}-\frac{1}{4}\left(\frac{1}{x}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 171$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\left(-\frac{1}{3}a\int -\frac{3\left(11a+\frac{21}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}{2\left(a-\frac{1}{x}\right)}d\frac{1}{x}-\frac{11}{3}a\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{1}{4}\left(\frac{1}{x}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\left(\frac{1}{2}a\int \frac{\left(11a+\frac{21}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}{a-\frac{1}{x}}d\frac{1}{x}-\frac{11}{3}a\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{1}{4}\left(\frac{1}{x}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 171$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\left(\frac{1}{2}a\left(-\frac{1}{2}a\int -\frac{\left(21a+\frac{107}{x}\right)\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{21}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{11}{3}a\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{1}{4}\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\left(\frac{1}{2}a\left(\frac{1}{4}a\int \frac{\left(21a+\frac{107}{x}\right)\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{21}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{11}{3}a\left(\frac{1}{x}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{1}{4}\left(\frac{1}{x}\right)^{5/2}}{\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 171

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\left(\frac{1}{2}a\left(\frac{1}{4}a\left(-\int -\frac{149a+\frac{363}{x}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-107\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{21}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{11}{3}a\left(\frac{1}{x}\right)^{3}}{\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 27

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{1}{2}\int \frac{149a+\frac{363}{x}}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-107\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{21}{2}a\sqrt{\frac{1}{x}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{11}{3}a\left(\frac{1}{x}\right)^{3/2}(}{\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 175

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{1}{2}\left(512a\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-363\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)-107\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{21}{2}a\sqrt{}}{\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 63

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{1}{2}\left(512a\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-726\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\right)-107\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{21}{2}a\sqrt{}}{\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 104

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{1}{2}\left(1024a\int \frac{1}{a-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}-726\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\right)-107\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-\frac{21}{2}a\sqrt{\frac{1}{x}}(}{\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 219

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{1}{2}\left(512\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-726\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\right)-107\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)-}{\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 222

$$-\frac{a\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{8}\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{1}{2}\left(512\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-726\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)-107\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)\right.\right.\right.}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

output
```
-((a*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(-1/4*((1 + 1/(a*x))^(3/2)*(x^(-1))^(5/2
)) + ((-11*a*(1 + 1/(a*x))^(3/2)*(x^(-1))^(3/2))/3 + (a*((-21*a*(1 + 1/(a*
x))^(3/2)*Sqrt[x^(-1)])/2 + (a*(-107*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + (-72
6*Sqrt[a]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]] + 512*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqr
t[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/2))/4))/2)/8))/Sqrt[1 - 1
/(a*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 112
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*(m + n +
p + 1))), x] - Simp[1/(f*(m + n + p + 1))   Int[(a + b*x)^(m - 1)*(c + d*x)
^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a
*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] &&
GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p
] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.56

method	result
risch	$-\frac{(447a^4x^4+661a^3x^3+350a^2x^2+184ax+48)c(ax-1)}{192x^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}-\frac{\left(\frac{4a^4\sqrt{2}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{363a^4\,\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{64\sqrt{c}}\right)c\sqrt{-c(ax+}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}$
default	$\frac{(ax-1)\sqrt{-c(ax-1)}\left(-768\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^4c\,x^4+1089c\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^4x^4+447a^3x^3\sqrt{-c(ax+1)}\,\sqrt{c}+214a}{192\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{c}\,\sqrt{-c(ax+1)}\,x^4}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x,method=_RETURNVERBOSE
)
```

output
```
-1/192*(447*a^4*x^4+661*a^3*x^3+350*a^2*x^2+184*a*x+48)/x^4*c/((a*x-1)/(a*
x+1))^(1/2)/(a*x+1)/(-c*(a*x-1))^(1/2)*(a*x-1)-(4*a^4*2^(1/2)/c^(1/2)*arct
an(1/2*(-a*c*x-c)^(1/2)*2^(1/2)/c^(1/2))-363/64*a^4/c^(1/2)*arctan((-a*c*x
-c)^(1/2)/c^(1/2)))*c/((a*x-1)/(a*x+1))^(1/2)/(a*x+1)*(-c*(a*x+1))^(1/2)/(
-c*(a*x-1))^(1/2)*(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.46

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$

$$= \left[\frac{768\sqrt{2}(a^5x^5-a^4x^4)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+1089\,(a^5x^5-a^4x^4)\sqrt{-c}}{3}\right.$$

$$-\frac{768\sqrt{2}(a^5x^5-a^4x^4)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{2\,(acx-c)}\right)-1089\,(a^5x^5-a^4x^4)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}(}{}\right.}{192\,(ax^5-x^4)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="fri
cas")
```

output
```
[1/384*(768*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x
 + 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))
 - 3*c)/(a^2*x^2 - 2*a*x + 1)) + 1089*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a
^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a
*x + 1)) - 2*c)/(a*x^2 - x)) + 2*(447*a^4*x^4 + 661*a^3*x^3 + 350*a^2*x^2
+ 184*a*x + 48)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4),
 -1/192*(768*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-
a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 1089
*(a^5*x^5 - a^4*x^4)*sqrt(c)*arctan(sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqr
t((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (447*a^4*x^4 + 661*a^3*x^3 + 350*a^2
*x^2 + 184*a*x + 48)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 -
x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{-acx+c}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a*c*x + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{c-a\,c\,x}}{x^5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a*c*x)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a*c*x)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.66

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$

$$= \frac{\sqrt{c}\,i\left(-894\sqrt{ax+1}\,a^3x^3-428\sqrt{ax+1}\,a^2x^2-272\sqrt{ax+1}\,ax-96\sqrt{ax+1}-1536\sqrt{2}\log\left(\tan\left(\frac{asin\left(\right.}{}\right.\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x)
```

output
```
(sqrt(c)*i*( - 894*sqrt(a*x + 1)*a**3*x**3 - 428*sqrt(a*x + 1)*a**2*x**2 -
 272*sqrt(a*x + 1)*a*x - 96*sqrt(a*x + 1) - 1536*sqrt(2)*log(tan(asin(sqrt
( - a*x + 1)/sqrt(2))/2))*a**4*x**4 + 1690*sqrt(2)*a**4*x**4 - 1089*log( -
 sqrt(2) + tan(asin(sqrt( - a*x + 1)/sqrt(2))/2) - 1)*a**4*x**4 + 1089*log
( - sqrt(2) + tan(asin(sqrt( - a*x + 1)/sqrt(2))/2) + 1)*a**4*x**4 + 1089*
log(sqrt(2) + tan(asin(sqrt( - a*x + 1)/sqrt(2))/2) - 1)*a**4*x**4 - 1089*
log(sqrt(2) + tan(asin(sqrt( - a*x + 1)/sqrt(2))/2) + 1)*a**4*x**4))/(384*
x**4)
```

3.342 $\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 132

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx=\frac{46\sqrt{1-\frac{1}{x}}(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}}+\frac{92\sqrt{1-\frac{1}{x}}(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}x}+\frac{8\sqrt{1-\frac{1}{x}}x(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}+\frac{2\sqrt{1-\frac{1}{x}}x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}$$

output
```
46/21*(1-1/x)^(1/2)*(1+x)^(3/2)/(1+1/x)^(3/2)+92/21*(1-1/x)^(1/2)*(1+x)^(3
/2)/(1+1/x)^(3/2)/x+8/7*(1-1/x)^(1/2)*x*(1+x)^(3/2)/(1+1/x)^(3/2)+2/7*(1-1
/x)^(1/2)*x^2*(1+x)^(3/2)/(1+1/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.35

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx=\frac{2\sqrt{\frac{-1+x}{x}}\sqrt{1+x}(46+23x+12x^2+3x^3)}{21\sqrt{1+\frac{1}{x}}}$$

input
```
Integrate[E^ArcCoth[x]*x*(1 + x)^(3/2),x]
```

output
```
(2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(46 + 23*x + 12*x^2 + 3*x^3))/(21*Sqrt[1 +
 x^(-1)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6730, 100, 27, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(x+1)^{3/2}e^{\coth^{-1}(x)}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}\int \frac{\left(1+\frac{1}{x}\right)^2}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}\left(\frac{2}{7}\int \frac{20+\frac{7}{x}}{2\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}\left(\frac{1}{7}\int \frac{20+\frac{7}{x}}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}\left(\frac{1}{7}\left(23\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{8\sqrt{1-\frac{1}{x}}}{\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{1-\frac{1}{x}}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}\left(\frac{1}{7}\left(23\left(\frac{2}{3}\int\frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{8\sqrt{1-\frac{1}{x}}}{\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{1-\frac{1}{x}}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow 48$$

$$-\frac{\left(\frac{1}{7}\left(23\left(-\frac{4\sqrt{1-\frac{1}{x}}}{3\sqrt{\frac{1}{x}}}-\frac{2\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{8\sqrt{1-\frac{1}{x}}}{\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\sqrt{1-\frac{1}{x}}}{7\left(\frac{1}{x}\right)^{7/2}}\right)\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}{\left(\frac{1}{x}+1\right)^{3/2}}$$

input
```
Int[E^ArcCoth[x]*x*(1 + x)^(3/2),x]
```

output
```
-((((23*((-2*Sqrt[1 - x^(-1)])/(3*(x^(-1))^(3/2)) - (4*Sqrt[1 - x^(-1)])/(
3*Sqrt[x^(-1)])) - (8*Sqrt[1 - x^(-1)])/(x^(-1))^(5/2))/7 - (2*Sqrt[1 - x^
(-1)])/(7*(x^(-1))^(7/2)))*(x^(-1))^(3/2)*(1 + x)^(3/2))/(1 + x^(-1))^(3/2
))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.28

method	result	size
gosper	$\frac{2(x-1)\left(3x^3+12x^2+23x+46\right)}{21\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	37
default	$\frac{2(x-1)\left(3x^3+12x^2+23x+46\right)}{21\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	37
risch	$\frac{2(x-1)\left(3x^3+12x^2+23x+46\right)}{21\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	37
orering	$\frac{2(x-1)\left(3x^3+12x^2+23x+46\right)}{21\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	37

input
```
int(1/((x-1)/(1+x))^(1/2)*x*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*(x-1)*(3*x^3+12*x^2+23*x+46)/(1+x)^(1/2)/((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.25

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx = \frac{2}{21}\left(3\,x^3+12\,x^2+23\,x+46\right)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="fricas")
```

output
```
2/21*(3*x^3 + 12*x^2 + 23*x + 46)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx = \int \frac{x(x+1)^{\frac{3}{2}}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*x*(1+x)**(3/2),x)
```

output
```
Integral(x*(x + 1)**(3/2)/sqrt((x - 1)/(x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.20

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx = \frac{2\left(3\,x^4+9\,x^3+11\,x^2+23\,x-46\right)}{21\,\sqrt{x-1}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="maxima")
```

output
```
2/21*(3*x^4 + 9*x^3 + 11*x^2 + 23*x - 46)/sqrt(x - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.36

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx = -\frac{64}{21}i\sqrt{2}\mathrm{sgn}(x+1) + \frac{2\left(3\,(x-1)^{\frac{7}{2}} + 21\,(x-1)^{\frac{5}{2}} + 56\,(x-1)^{\frac{3}{2}} + 84\,\sqrt{x-1}\right)}{21\,\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="giac")
```

output
```
-64/21*I*sqrt(2)*sgn(x + 1) + 2/21*(3*(x - 1)^(7/2) + 21*(x - 1)^(5/2) + 5
6*(x - 1)^(3/2) + 84*sqrt(x - 1))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.36

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx = \sqrt{\frac{x-1}{x+1}}\left(\frac{46\,x\,\sqrt{x+1}}{21} + \frac{92\,\sqrt{x+1}}{21} + \frac{8\,x^2\,\sqrt{x+1}}{7} + \frac{2\,x^3\,\sqrt{x+1}}{7}\right)$$

input
```
int((x*(x + 1)^(3/2))/((x - 1)/(x + 1))^(1/2),x)
```

output
```
((x - 1)/(x + 1))^(1/2)*((46*x*(x + 1)^(1/2))/21 + (92*(x + 1)^(1/2))/21 +
 (8*x^2*(x + 1)^(1/2))/7 + (2*x^3*(x + 1)^(1/2))/7)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.16

$$\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx=\frac{2\sqrt{x-1}\,(3x^3+12x^2+23x+46)}{21}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*x*(1+x)^(3/2),x)
```

output
```
(2*sqrt(x - 1)*(3*x**3 + 12*x**2 + 23*x + 46))/21
```

3.343 $\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 98

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx=\frac{28\sqrt{1-\frac{1}{x}}(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}}+\frac{86\sqrt{1-\frac{1}{x}}(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}x}+\frac{2\sqrt{1-\frac{1}{x}}x(1+x)^{3/2}}{5\left(1+\frac{1}{x}\right)^{3/2}}$$

output
```
28/15*(1-1/x)^(1/2)*(1+x)^(3/2)/(1+1/x)^(3/2)+86/15*(1-1/x)^(1/2)*(1+x)^(3
/2)/(1+1/x)^(3/2)/x+2/5*(1-1/x)^(1/2)*x*(1+x)^(3/2)/(1+1/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.42

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx=\frac{2\sqrt{\frac{-1+x}{x}}\sqrt{1+x}(43+14x+3x^2)}{15\sqrt{1+\frac{1}{x}}}$$

input
```
Integrate[E^ArcCoth[x]*(1 + x)^(3/2),x]
```

output
```
(2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(43 + 14*x + 3*x^2))/(15*Sqrt[1 + x^(-1)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6727, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)^{3/2} e^{\coth^{-1}(x)} \, dx$$

$$\Big\downarrow \ 6727$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2} (x+1)^{3/2} \int \frac{\left(1+\frac{1}{x}\right)^2}{\sqrt{1-\frac{1}{x}} \left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \ 100$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2} (x+1)^{3/2} \left(\frac{2}{5} \int \frac{14+\frac{5}{x}}{2\sqrt{1-\frac{1}{x}} \left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2} (x+1)^{3/2} \left(\frac{1}{5} \int \frac{14+\frac{5}{x}}{\sqrt{1-\frac{1}{x}} \left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \ 87$$

$$-\frac{\left(\frac{1}{x}\right)^{3/2} (x+1)^{3/2} \left(\frac{1}{5} \left(\frac{43}{3} \int \frac{1}{\sqrt{1-\frac{1}{x}} \left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{28\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2\sqrt{1-\frac{1}{x}}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{\left(\frac{1}{x}+1\right)^{3/2}}$$

$$\Big\downarrow \ 48$$

$$-\frac{\left(\frac{1}{5}\left(-\frac{86\sqrt{1-\frac{1}{x}}}{3\sqrt{\frac{1}{x}}}-\frac{28\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2\sqrt{1-\frac{1}{x}}}{5\left(\frac{1}{x}\right)^{5/2}}\right)\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}{\left(\frac{1}{x}+1\right)^{3/2}}$$

input
```
Int[E^ArcCoth[x]*(1 + x)^(3/2),x]
```

output
```
-(((((-28*Sqrt[1 - x^(-1)])/(3*(x^(-1))^(3/2)) - (86*Sqrt[1 - x^(-1)])/(3*
Sqrt[x^(-1)]))/5 - (2*Sqrt[1 - x^(-1)])/(5*(x^(-1))^(5/2)))*(x^(-1))^(3/2)
*(1 + x)^(3/2))/(1 + x^(-1))^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.33

method	result	size
gosper	$\frac{2(x-1)\left(3x^{2}+14x+43\right)}{15\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	32
default	$\frac{2(x-1)\left(3x^{2}+14x+43\right)}{15\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	32
risch	$\frac{2(x-1)\left(3x^{2}+14x+43\right)}{15\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	32
orering	$\frac{2(x-1)\left(3x^{2}+14x+43\right)}{15\sqrt{1+x}\,\sqrt{\frac{x-1}{1+x}}}$	32

input
```
int(1/((x-1)/(1+x))^(1/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(x-1)*(3*x^2+14*x+43)/(1+x)^(1/2)/((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.29

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx=\frac{2}{15}\left(3\,x^{2}+14\,x+43\right)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*x^2 + 14*x + 43)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [A] (verification not implemented)

Time = 39.53 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx = 2\left(\begin{cases} 4\sqrt{2}\left(\frac{\sqrt{2}(x-1)^{\frac{5}{2}}}{40}+\frac{\sqrt{2}(x-1)^{\frac{3}{2}}}{6}+\frac{\sqrt{2}\sqrt{x-1}}{2}\right) & \text{for } \sqrt{x+1}>-\sqrt{2}\wedge\sqrt{x+1}<\sqrt{2} \end{cases}\right)$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*(1+x)**(3/2),x)
```

output
```
2*Piecewise((4*sqrt(2)*(sqrt(2)*(x - 1)**(5/2)/40 + sqrt(2)*(x - 1)**(3/2)
/6 + sqrt(2)*sqrt(x - 1)/2), (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -sqr
t(2))))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.22

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx = \frac{2\,(3\,x^3+11\,x^2+29\,x-43)}{15\,\sqrt{x-1}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="maxima")
```

output
```
2/15*(3*x^3 + 11*x^2 + 29*x - 43)/sqrt(x - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.41

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx = -\frac{64}{15}i\,\sqrt{2}\text{sgn}(x+1)+\frac{2\left(3\,(x-1)^{\frac{5}{2}}+20\,(x-1)^{\frac{3}{2}}+60\,\sqrt{x-1}\right)}{15\,\text{sgn}\,(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="giac")
```

output
```
-64/15*I*sqrt(2)*sgn(x + 1) + 2/15*(3*(x - 1)^(5/2) + 20*(x - 1)^(3/2) + 6
0*sqrt(x - 1))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.62 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx=\sqrt{\frac{x-1}{x+1}}\left(\frac{28\,x\,\sqrt{x+1}}{15}+\frac{86\,\sqrt{x+1}}{15}+\frac{2\,x^2\,\sqrt{x+1}}{5}\right)$$

input
```
int((x + 1)^(3/2)/((x - 1)/(x + 1))^(1/2),x)
```

output
```
((x - 1)/(x + 1))^(1/2)*((28*x*(x + 1)^(1/2))/15 + (86*(x + 1)^(1/2))/15 +
 (2*x^2*(x + 1)^(1/2))/5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.16

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx=\frac{2\sqrt{x-1}\,(3x^2+14x+43)}{15}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*(1+x)^(3/2),x)
```

output
```
(2*sqrt(x - 1)*(3*x**2 + 14*x + 43))/15
```

3.344 $\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx=\frac{44\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}}{105\left(1-\frac{1}{x}\right)^{3/2}}-\frac{22\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}}$$

output
```
44/105*(1+1/x)^(3/2)*(1-x)^(3/2)/(1-1/x)^(3/2)-22/35*(1+1/x)^(3/2)*(1-x)^(
3/2)*x/(1-1/x)^(3/2)+2/7*(1+1/x)^(3/2)*(1-x)^(3/2)*x^2/(1-1/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.46

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx=-\frac{2\sqrt{1+\frac{1}{x}}\sqrt{1-x}(22-11x-18x^2+15x^3)}{105\sqrt{\frac{-1+x}{x}}}$$

input
```
Integrate[E^ArcCoth[x]*(1 - x)^(3/2)*x,x]
```

output
```
(-2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(22 - 11*x - 18*x^2 + 15*x^3))/(105*Sqrt[
(-1 + x)/x])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6730, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{3/2} x e^{\coth^{-1}(x)} \, dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{(1-x)^{3/2} \left(\frac{1}{x}\right)^{3/2} \int \frac{\left(1-\frac{1}{x}\right)\sqrt{1+\frac{1}{x}}}{\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\left(1-\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{(1-x)^{3/2} \left(\frac{1}{x}\right)^{3/2} \left(-\frac{11}{7}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{\left(1-\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(1-x)^{3/2} \left(\frac{1}{x}\right)^{3/2} \left(-\frac{11}{7}\left(-\frac{2}{5}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{\left(1-\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{7\left(\frac{1}{x}\right)^{7/2}}-\frac{11}{7}\left(\frac{4\left(\frac{1}{x}+1\right)^{3/2}}{15\left(\frac{1}{x}\right)^{3/2}}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)\right)(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}{\left(1-\frac{1}{x}\right)^{3/2}}$$

input
```
Int[E^ArcCoth[x]*(1 - x)^(3/2)*x,x]
```

output
```
-((((-11*((-2*(1 + x^(-1))^(3/2))/(5*(x^(-1))^(5/2)) + (4*(1 + x^(-1))^(3/
2))/(15*(x^(-1))^(3/2))))/7 - (2*(1 + x^(-1))^(3/2))/(7*(x^(-1))^(7/2)))*(
1 - x)^(3/2)*(x^(-1))^(3/2))/(1 - x^(-1))^(3/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.33

method	result	size
gosper	$-\frac{2(1+x)\sqrt{1-x}\,(15x^2-33x+22)}{105\sqrt{\frac{x-1}{1+x}}}$	34
default	$\frac{2(x-1)(1+x)(15x^2-33x+22)}{105\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}}$	37
orering	$\frac{2(1+x)(15x^2-33x+22)(1-x)^{\frac{3}{2}}}{105(x-1)\sqrt{\frac{x-1}{1+x}}}$	39
risch	$\frac{2\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)(15x^3-18x^2-11x+22)}{105\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}\,\sqrt{-x-1}}$	62

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^(3/2)*x,x,method=_RETURNVERBOSE)
```

output
```
-2/105*(1+x)*(1-x)^(1/2)*(15*x^2-33*x+22)/((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.43

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = -\frac{2\,(15\,x^4-3\,x^3-29\,x^2+11\,x+22)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{105\,(x-1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="fricas")
```

output
```
-2/105*(15*x^4 - 3*x^3 - 29*x^2 + 11*x + 22)*sqrt(-x + 1)*sqrt((x - 1)/(x
+ 1))/(x - 1)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = \text{Timed out}$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*(1-x)**(3/2)*x,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.21

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = \frac{2}{105}\left(-15i\,x^3+18i\,x^2+11i\,x-22i\right)\sqrt{x+1}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="maxima")
```

output
```
2/105*(-15*I*x^3 + 18*I*x^2 + 11*I*x - 22*I)*sqrt(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.44

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = \frac{2\left(15\,(x+1)^3\sqrt{-x-1}-63\,(x+1)^2\sqrt{-x-1}-70\,(-x-1)^{\frac{3}{2}}\right)}{105\,\mathrm{sgn}\left(x+1\right)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="giac")
```

output
```
2/105*(15*(x + 1)^3*sqrt(-x - 1) - 63*(x + 1)^2*sqrt(-x - 1) - 70*(-x - 1)
^(3/2))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.80 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.34

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = \frac{2\sqrt{\frac{x-1}{x+1}}\,(x+1)^2\,(15\,x^2-33\,x+22)}{105\,\sqrt{1-x}}$$

input
```
int((x*(1 - x)^(3/2))/((x - 1)/(x + 1))^(1/2),x)
```

output
```
(2*((x - 1)/(x + 1))^(1/2)*(x + 1)^2*(15*x^2 - 33*x + 22))/(105*(1 - x)^(1
/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.21

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = \frac{2\sqrt{x+1}\,i(15x^3-18x^2-11x+22)}{105}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^(3/2)*x,x)
```

output
```
(2*sqrt(x + 1)*i*(15*x**3 - 18*x**2 - 11*x + 22))/105
```

3.345 $\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 68

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx=-\frac{14\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}}{15\left(1-\frac{1}{x}\right)^{3/2}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{5\left(1-\frac{1}{x}\right)^{3/2}}$$

output
```
-14/15*(1+1/x)^(3/2)*(1-x)^(3/2)/(1-1/x)^(3/2)+2/5*(1+1/x)^(3/2)*(1-x)^(3/
2)*x/(1-1/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.63

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx=-\frac{2\sqrt{1+\frac{1}{x}}\sqrt{1-x}(-7-4x+3x^2)}{15\sqrt{\frac{-1+x}{x}}}$$

input
```
Integrate[E^ArcCoth[x]*(1 - x)^(3/2),x]
```

output
```
(-2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(-7 - 4*x + 3*x^2))/(15*Sqrt[(-1 + x)/x])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6727, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{3/2} e^{\coth^{-1}(x)} \, dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{(1-x)^{3/2} \left(\frac{1}{x}\right)^{3/2} \int \frac{\left(1-\frac{1}{x}\right)\sqrt{1+\frac{1}{x}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\left(1-\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{(1-x)^{3/2} \left(\frac{1}{x}\right)^{3/2} \left(-\frac{7}{5}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{\left(1-\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{14\left(\frac{1}{x}+1\right)^{3/2}}{15\left(\frac{1}{x}\right)^{3/2}}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)(1-x)^{3/2} \left(\frac{1}{x}\right)^{3/2}}{\left(1-\frac{1}{x}\right)^{3/2}}$$

input
```
Int[E^ArcCoth[x]*(1 - x)^(3/2),x]
```

output
```
-((((-2*(1 + x^(-1))^(3/2))/(5*(x^(-1))^(5/2)) + (14*(1 + x^(-1))^(3/2))/(
15*(x^(-1))^(3/2)))*(1 - x)^(3/2)*(x^(-1))^(3/2))/(1 - x^(-1))^(3/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.43

method	result	size
gosper	$-\frac{2(1+x)(3x-7)\sqrt{1-x}}{15\sqrt{\frac{x-1}{1+x}}}$	29
default	$\frac{2(x-1)(1+x)(3x-7)}{15\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}}$	32
orering	$\frac{2(1+x)(3x-7)(1-x)^{\frac{3}{2}}}{15(x-1)\sqrt{\frac{x-1}{1+x}}}$	34
risch	$\frac{2\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)\left(3x^2-4x-7\right)}{15\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}\,\sqrt{-x-1}}$	57

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(1+x)*(3*x-7)*(1-x)^(1/2)/((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx = -\frac{2\,(3\,x^3-x^2-11\,x-7)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{15\,(x-1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(3*x^3 - x^2 - 11*x - 7)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx = \int \frac{(1-x)^{\frac{3}{2}}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*(1-x)**(3/2),x)
```

output
```
Integral((1 - x)**(3/2)/sqrt((x - 1)/(x + 1)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.25

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx = \frac{2}{15}\left(-3i\,x^2+4i\,x+7i\right)\sqrt{x+1}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="maxima")
```

output
```
2/15*(-3*I*x^2 + 4*I*x + 7*I)*sqrt(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.47

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx = \frac{2\left(3\,(x+1)^2\sqrt{-x-1}+10\,(-x-1)^{\frac{3}{2}}\right)}{15\,\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="giac")
```

output
```
2/15*(3*(x + 1)^2*sqrt(-x - 1) + 10*(-x - 1)^(3/2))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.85 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.44

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx = \frac{2\,(3\,x-7)\,\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}{15\,\sqrt{1-x}}$$

input
```
int((1 - x)^(3/2)/((x - 1)/(x + 1))^(1/2),x)
```

output
```
(2*(3*x - 7)*((x - 1)/(x + 1))^(1/2)*(x + 1)^2)/(15*(1 - x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.25

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx = \frac{2\sqrt{x+1}\,i(3x^2-4x-7)}{15}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^(3/2),x)
```

output
```
(2*sqrt(x + 1)*i*(3*x**2 - 4*x - 7))/15
```

3.346 $\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 98

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx=\frac{12\sqrt{1-\frac{1}{x}}\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}+\frac{6\sqrt{1-\frac{1}{x}}x\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}+\frac{2\sqrt{1-\frac{1}{x}}x^2\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}$$

output
```
12/5*(1-1/x)^(1/2)*(1+x)^(1/2)/(1+1/x)^(1/2)+6/5*(1-1/x)^(1/2)*x*(1+x)^(1/
2)/(1+1/x)^(1/2)+2/5*(1-1/x)^(1/2)*x^2*(1+x)^(1/2)/(1+1/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.40

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx=\frac{2\sqrt{\frac{-1+x}{x}}\sqrt{1+x}(6+3x+x^2)}{5\sqrt{1+\frac{1}{x}}}$$

input
```
Integrate[E^ArcCoth[x]*x*Sqrt[1 + x],x]
```

output
```
(2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(6 + 3*x + x^2))/(5*Sqrt[1 + x^(-1)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6730, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{x+1}e^{\coth^{-1}(x)}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{x+1}\int \frac{1+\frac{1}{x}}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}+1}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{x+1}\left(\frac{9}{5}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{\sqrt{\frac{1}{x}+1}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{x+1}\left(\frac{9}{5}\left(\frac{2}{3}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2\sqrt{1-\frac{1}{x}}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{\sqrt{\frac{1}{x}+1}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{9}{5}\left(-\frac{4\sqrt{1-\frac{1}{x}}}{3\sqrt{\frac{1}{x}}}-\frac{2\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2\sqrt{1-\frac{1}{x}}}{5\left(\frac{1}{x}\right)^{5/2}}\right)\sqrt{\frac{1}{x}}\sqrt{x+1}}{\sqrt{\frac{1}{x}+1}}$$

input
```
Int[E^ArcCoth[x]*x*Sqrt[1 + x],x]
```

output
```
-((((9*((-2*Sqrt[1 - x^(-1)])/(3*(x^(-1))^(3/2)) - (4*Sqrt[1 - x^(-1)])/(3
*Sqrt[x^(-1)])))/5 - (2*Sqrt[1 - x^(-1)])/(5*(x^(-1))^(5/2)))*Sqrt[x^(-1)]
*Sqrt[1 + x])/Sqrt[1 + x^(-1)])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.31

method	result	size
gosper	$\frac{2(x-1)(x^2+3x+6)}{5\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	30
default	$\frac{2(x-1)(x^2+3x+6)}{5\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	30
risch	$\frac{2(x-1)(x^2+3x+6)}{5\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	30
orering	$\frac{2(x-1)(x^2+3x+6)}{5\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	30

input
```
int(1/((x-1)/(1+x))^(1/2)*x*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(x-1)*(x^2+3*x+6)/((x-1)/(1+x))^(1/2)/(1+x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.27

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx = \frac{2}{5}\left(x^2+3\,x+6\right)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(x^2 + 3*x + 6)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx = \int \frac{x\sqrt{x+1}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*x*(1+x)**(1/2),x)
```

output
```
Integral(x*sqrt(x + 1)/sqrt((x - 1)/(x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.20

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx = \frac{2\,(x^3+2\,x^2+3\,x-6)}{5\,\sqrt{x-1}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="maxima")
```

output
```
2/5*(x^3 + 2*x^2 + 3*x - 6)/sqrt(x - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx = -\frac{8}{5}i\,\sqrt{2}\mathrm{sgn}(x+1) + \frac{2\left((x-1)^{\frac{5}{2}}+5\,(x-1)^{\frac{3}{2}}+10\,\sqrt{x-1}\right)}{5\,\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="giac")
```

output
```
-8/5*I*sqrt(2)*sgn(x + 1) + 2/5*((x - 1)^(5/2) + 5*(x - 1)^(3/2) + 10*sqrt
(x - 1))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx=\sqrt{\frac{x-1}{x+1}}\left(\frac{6\,x\,\sqrt{x+1}}{5}+\frac{12\,\sqrt{x+1}}{5}+\frac{2\,x^2\,\sqrt{x+1}}{5}\right)$$

input
```
int((x*(x + 1)^(1/2))/((x - 1)/(x + 1))^(1/2),x)
```

output
```
((x - 1)/(x + 1))^(1/2)*((6*x*(x + 1)^(1/2))/5 + (12*(x + 1)^(1/2))/5 + (2
*x^2*(x + 1)^(1/2))/5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.14

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx=\frac{2\sqrt{x-1}\,(x^2+3x+6)}{5}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*x*(1+x)^(1/2),x)
```

output
```
(2*sqrt(x - 1)*(x**2 + 3*x + 6))/5
```

3.347 $\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 64

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx=\frac{10\sqrt{1-\frac{1}{x}}\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}}+\frac{2\sqrt{1-\frac{1}{x}}x\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}}$$

output
```
10/3*(1-1/x)^(1/2)*(1+x)^(1/2)/(1+1/x)^(1/2)+2/3*(1-1/x)^(1/2)*x*(1+x)^(1/
2)/(1+1/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.53

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx=\frac{2\sqrt{\frac{-1+x}{x}}\sqrt{1+x}(5+x)}{3\sqrt{1+\frac{1}{x}}}$$

input
```
Integrate[E^ArcCoth[x]*Sqrt[1 + x],x]
```

output
```
(2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(5 + x))/(3*Sqrt[1 + x^(-1)])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6727, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x+1}e^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{ 6727}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{x+1}\int \frac{1+\frac{1}{x}}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}+1}}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{x+1}\left(\frac{5}{3}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{\frac{1}{x}+1}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{\left(-\frac{10\sqrt{1-\frac{1}{x}}}{3\sqrt{\frac{1}{x}}}-\frac{2\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)\sqrt{\frac{1}{x}}\sqrt{x+1}}{\sqrt{\frac{1}{x}+1}}$$

input
```
Int[E^ArcCoth[x]*Sqrt[1 + x],x]
```

output
```
-((((-2*Sqrt[1 - x^(-1)])/(3*(x^(-1))^(3/2)) - (10*Sqrt[1 - x^(-1)])/(3*Sq
rt[x^(-1)]))*Sqrt[x^(-1)]*Sqrt[1 + x])/Sqrt[1 + x^(-1)])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.39

method	result	size
gosper	$\frac{2(x-1)(x+5)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25
default	$\frac{2(x-1)(x+5)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25
risch	$\frac{2(x-1)(x+5)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25
orering	$\frac{2(x-1)(x+5)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25

input
```
int(1/((x-1)/(1+x))^(1/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x-1)*(x+5)/((x-1)/(1+x))^(1/2)/(1+x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.33

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx = \frac{2}{3}\,(x+5)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x + 5)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [A] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx = 2\left(\begin{cases} 2\sqrt{2}\left(\frac{\sqrt{2}(x-1)^{\frac{3}{2}}}{12}+\frac{\sqrt{2}\sqrt{x-1}}{2}\right) & \text{for}\ \sqrt{x+1}>-\sqrt{2}\wedge\sqrt{x+1}<\sqrt{2} \end{cases}\right)$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*(1+x)**(1/2),x)
```

output
```
2*Piecewise((2*sqrt(2)*(sqrt(2)*(x - 1)**(3/2)/12 + sqrt(2)*sqrt(x - 1)/2)
, (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -sqrt(2))))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.23

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx = \frac{2\,(x^2+4\,x-5)}{3\,\sqrt{x-1}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(x^2 + 4*x - 5)/sqrt(x - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.48

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx = -\frac{8}{3}i\sqrt{2}\mathrm{sgn}(x+1) + \frac{2\left((x-1)^{\frac{3}{2}} + 6\sqrt{x-1}\right)}{3\,\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="giac")
```

output
```
-8/3*I*sqrt(2)*sgn(x + 1) + 2/3*((x - 1)^(3/2) + 6*sqrt(x - 1))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.33

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx = \frac{2\sqrt{\frac{x-1}{x+1}}\,\sqrt{x+1}\,(x+5)}{3}$$

input
```
int((x + 1)^(1/2)/((x - 1)/(x + 1))^(1/2),x)
```

output
```
(2*((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)*(x + 5))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.14

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx = \frac{2\sqrt{x-1}\,(x+5)}{3}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*(1+x)^(1/2),x)
```

output
```
(2*sqrt(x - 1)*(x + 5))/3
```

3.348 $\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx=-\frac{4\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}x}{15\sqrt{1-\frac{1}{x}}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}x^2}{5\sqrt{1-\frac{1}{x}}}$$

output
```
-4/15*(1+1/x)^(3/2)*(1-x)^(1/2)*x/(1-1/x)^(1/2)+2/5*(1+1/x)^(3/2)*(1-x)^(1
/2)*x^2/(1-1/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx=\frac{2\sqrt{1+\frac{1}{x}}\sqrt{1-x}(-2+x+3x^2)}{15\sqrt{\frac{-1+x}{x}}}$$

input
```
Integrate[E^ArcCoth[x]*Sqrt[1 - x]*x,x]
```

output
```
(2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(-2 + x + 3*x^2))/(15*Sqrt[(-1 + x)/x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6730, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x}xe^{\coth^{-1}(x)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{1-x}\sqrt{\frac{1}{x}}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{x}}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\sqrt{1-x}\sqrt{\frac{1}{x}}\left(-\frac{2}{5}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{\sqrt{1-\frac{1}{x}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{4\left(\frac{1}{x}+1\right)^{3/2}}{15\left(\frac{1}{x}\right)^{3/2}}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{5\left(\frac{1}{x}\right)^{5/2}}\right)\sqrt{1-x}\sqrt{\frac{1}{x}}}{\sqrt{1-\frac{1}{x}}}$$

input
```
Int[E^ArcCoth[x]*Sqrt[1 - x]*x,x]
```

output
```
-((((-2*(1 + x^(-1))^(3/2))/(5*(x^(-1))^(5/2)) + (4*(1 + x^(-1))^(3/2))/(1
5*(x^(-1))^(3/2)))*Sqrt[1 - x]*Sqrt[x^(-1)])/Sqrt[1 - x^(-1)])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

method	result	size
gosper	$\frac{2(1+x)(3x-2)\sqrt{1-x}}{15\sqrt{\frac{x-1}{1+x}}}$	29
orering	$\frac{2(1+x)(3x-2)\sqrt{1-x}}{15\sqrt{\frac{x-1}{1+x}}}$	29
default	$-\frac{2(x-1)(1+x)(3x-2)}{15\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}}$	32
risch	$-\frac{2\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)(3x^2+x-2)}{15\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}\,\sqrt{-x-1}}$	55

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
2/15*(1+x)*(3*x-2)*(1-x)^(1/2)/((x-1)/(1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx = \frac{2\,(3\,x^3+4\,x^2-x-2)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{15\,(x-1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^(1/2)*x,x, algorithm="fricas")
```

output
```
2/15*(3*x^3 + 4*x^2 - x - 2)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx = \int \frac{x\sqrt{1-x}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*(1-x)**(1/2)*x,x)
```

output
```
Integral(x*sqrt(1 - x)/sqrt((x - 1)/(x + 1)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.24

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx = -\frac{2}{15}\left(-3i\,x^2-i\,x+2i\right)\sqrt{x+1}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^(1/2)*x,x, algorithm="maxima")
```

output
```
-2/15*(-3*I*x^2 - I*x + 2*I)*sqrt(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.45

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx = -\frac{2\left(3\,(x+1)^2\sqrt{-x-1}+5\,(-x-1)^{\frac{3}{2}}\right)}{15\,\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^(1/2)*x,x, algorithm="giac")
```

output
```
-2/15*(3*(x + 1)^2*sqrt(-x - 1) + 5*(-x - 1)^(3/2))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.71 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.42

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx = -\frac{2\,(3\,x-2)\,\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}{15\,\sqrt{1-x}}$$

input
```
int((x*(1 - x)^(1/2))/((x - 1)/(x + 1))^(1/2),x)
```

output
```
-(2*(3*x - 2)*((x - 1)/(x + 1))^(1/2)*(x + 1)^2)/(15*(1 - x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.24

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}x\,dx = \frac{2\sqrt{x+1}\,i(-3x^2-x+2)}{15}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^(1/2)*x,x)
```

output
```
(2*sqrt(x + 1)*i*( - 3*x**2 - x + 2))/15
```

3.349 $\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 20

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx = \frac{2}{3}e^{\coth^{-1}(x)}\sqrt{1-x}(1+x)$$

output
```
2/3/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2)*(1+x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx = \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}x}{3\sqrt{1-\frac{1}{x}}}$$

input
```
Integrate[E^ArcCoth[x]*Sqrt[1 - x],x]
```

output
```
(2*(1 + x^(-1))^(3/2)*Sqrt[1 - x]*x)/(3*Sqrt[1 - x^(-1)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {6726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x}e^{\coth^{-1}(x)} \, dx$$

$$\Big\downarrow \text{6726}$$

$$\frac{2}{3}\sqrt{1-x}(x+1)e^{\coth^{-1}(x)}$$

input
```
Int[E^ArcCoth[x]*Sqrt[1 - x],x]
```

output
```
(2*E^ArcCoth[x]*Sqrt[1 - x]*(1 + x))/3
```

Defintions of rubi rules used

rule 6726
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] &&  !IntegerQ[p] && EqQ[p, n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

method	result	size
gosper	$\frac{2\sqrt{1-x}\,(1+x)}{3\sqrt{\frac{x-1}{1+x}}}$	24
orering	$\frac{\left(\frac{2}{3}+\frac{2x}{3}\right)\sqrt{1-x}}{\sqrt{\frac{x-1}{1+x}}}$	25
default	$-\frac{2(x-1)(1+x)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}}$	27
risch	$-\frac{2(1+x)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}\,\sqrt{-x-1}}$	50

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/((x-1)/(1+x))^(1/2)*(1-x)^(1/2)*(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx = \frac{2\left(x^2+2\,x+1\right)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{3\,(x-1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x^2 + 2*x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

Sympy [F]

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx = \int \frac{\sqrt{1-x}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*(1-x)**(1/2),x)
```

output
```
Integral(sqrt(1 - x)/sqrt((x - 1)/(x + 1)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx = -\frac{2}{3}\sqrt{x+1}(-i\,x-i)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*sqrt(x + 1)*(-I*x - I)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx = \frac{2\,(-x-1)^{\frac{3}{2}}}{3\,\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="giac")
```

output
```
2/3*(-x - 1)^(3/2)/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.82 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx = -\frac{2\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}{3\sqrt{1-x}}$$

input
```
int((1 - x)^(1/2)/((x - 1)/(x + 1))^(1/2),x)
```

output
```
-(2*((x - 1)/(x + 1))^(1/2)*(x + 1)^2)/(3*(1 - x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.50

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx = -\frac{2\sqrt{x+1}\,i(x+1)}{3}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*(1-x)^(1/2),x)
```

output
```
( - 2*sqrt(x + 1)*i*(x + 1))/3
```

3.350 $\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = \frac{4\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{3\sqrt{1+x}} + \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x^2}{3\sqrt{1+x}}$$

output
```
4/3*(1-1/x)^(1/2)*(1+1/x)^(1/2)*x/(1+x)^(1/2)+2/3*(1-1/x)^(1/2)*(1+1/x)^(1
/2)*x^2/(1+x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.39

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = \frac{2\sqrt{1-\frac{1}{x^2}}x(2+x)}{3\sqrt{1+x}}$$

input
```
Integrate[(E^ArcCoth[x]*x)/Sqrt[1 + x],x]
```

output
```
(2*Sqrt[1 - x^(-2)]*x*(2 + x))/(3*Sqrt[1 + x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6730, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(x)}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}+1}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}}\sqrt{x+1}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\sqrt{\frac{1}{x}+1}\left(\frac{2}{3}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{\frac{1}{x}}\sqrt{x+1}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(-\frac{4\sqrt{1-\frac{1}{x}}}{3\sqrt{\frac{1}{x}}}-\frac{2\sqrt{1-\frac{1}{x}}}{3\left(\frac{1}{x}\right)^{3/2}}\right)\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}\sqrt{x+1}}$$

input
```
Int[(E^ArcCoth[x]*x)/Sqrt[1 + x],x]
```

output
```
-((((-2*Sqrt[1 - x^(-1)])/(3*(x^(-1))^(3/2)) - (4*Sqrt[1 - x^(-1)])/(3*Sqr
t[x^(-1)]))*Sqrt[1 + x^(-1)])/(Sqrt[x^(-1)]*Sqrt[1 + x]))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.37

method	result	size
gosper	$\frac{2(x-1)(x+2)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25
default	$\frac{2(x-1)(x+2)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25
risch	$\frac{2(x-1)(x+2)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25
orering	$\frac{2(x-1)(x+2)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	25

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x-1)*(x+2)/((x-1)/(1+x))^(1/2)/(1+x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.31

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = \frac{2}{3}\,(x+2)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x + 2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = \begin{cases} \frac{2x\sqrt{x-1}}{3} + \frac{4\sqrt{x-1}}{3} & \text{for}\ |x| > 1 \\ \frac{2ix\sqrt{1-x}}{3} + \frac{4i\sqrt{1-x}}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*x/(1+x)**(1/2),x)
```

output
```
Piecewise((2*x*sqrt(x - 1)/3 + 4*sqrt(x - 1)/3, Abs(x) > 1), (2*I*x*sqrt(1
 - x)/3 + 4*I*sqrt(1 - x)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.19

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = \frac{2\,(x^2+x-2)}{3\,\sqrt{x-1}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(x^2 + x - 2)/sqrt(x - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = -\frac{2}{3}i\,\sqrt{2}\mathrm{sgn}(x+1) + \frac{2\left((x-1)^{\frac{3}{2}}+3\,\sqrt{x-1}\right)}{3\,\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-2/3*I*sqrt(2)*sgn(x + 1) + 2/3*((x - 1)^(3/2) + 3*sqrt(x - 1))/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.79 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.31

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = \frac{2\,\sqrt{\frac{x-1}{x+1}}\,\sqrt{x+1}\,(x+2)}{3}$$

input
```
int(x/((((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)),x)
```

output
```
(2*((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)*(x + 2))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.13

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = \frac{2\sqrt{x-1}\,(x+2)}{3}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1+x)^(1/2),x)
```

output
```
(2*sqrt(x - 1)*(x + 2))/3
```

3.351 $\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}} dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}} dx = \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1+x}}$$

output
```
2*(1-1/x)^(1/2)*(1+1/x)^(1/2)*x/(1+x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}} dx = \frac{2\sqrt{1-\frac{1}{x^2}}x}{\sqrt{1+x}}$$

input
```
Integrate[E^ArcCoth[x]/Sqrt[1 + x],x]
```

output
```
(2*Sqrt[1 - x^(-2)]*x)/Sqrt[1 + x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6727, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{\frac{1}{x}+1}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{\frac{1}{x}}\sqrt{x+1}}$$

$$\Big\downarrow \text{48}$$

$$\frac{2\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}x}{\sqrt{x+1}}$$

input
```
Int[E^ArcCoth[x]/Sqrt[1 + x],x]
```

output
```
(2*Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/Sqrt[1 + x]
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{2x-2}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	22
default	$\frac{2x-2}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	22
risch	$\frac{2x-2}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	22
orering	$\frac{2x-2}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	23

input
```
int(1/((x-1)/(1+x))^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(x-1)/((x-1)/(1+x))^(1/2)/(1+x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = 2\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = \begin{cases} 2\sqrt{x-1} & \text{for } |x|>1 \\ 2i\sqrt{1-x} & \text{otherwise} \end{cases}$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((2*sqrt(x - 1), Abs(x) > 1), (2*I*sqrt(1 - x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.23

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = 2\sqrt{x-1}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = -2i\sqrt{2}\operatorname{sgn}(x+1) + \frac{2\sqrt{x-1}}{\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-2*I*sqrt(2)*sgn(x + 1) + 2*sqrt(x - 1)/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = 2\sqrt{\frac{x-1}{x+1}}\,\sqrt{x+1}$$

input
```
int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)),x)
```

output
```
2*((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.20

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = 2\sqrt{x-1}$$

input
```
int(1/((x-1)/(1+x))^(1/2)/(1+x)^(1/2),x)
```

output
```
2*sqrt(x - 1)
```

3.352 $\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx=\frac{8\sqrt{1-\frac{1}{x^2}}x}{3\sqrt{1-x}}+\frac{2\sqrt{1-\frac{1}{x^2}}x^2}{3\sqrt{1-x}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

output
```
8/3*(1-1/x^2)^(1/2)*x/(1-x)^(1/2)+2/3*(1-1/x^2)^(1/2)*x^2/(1-x)^(1/2)-2*2^
(1/2)*(1-1/x)^(1/2)*arctanh(2^(1/2)*(1/x)^(1/2)/(1+1/x)^(1/2))/(1-x)^(1/2)
/(1/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx=\frac{2\sqrt{\frac{-1+x}{x}}x\left(\sqrt{1+\frac{1}{x}}(4+x)-3\sqrt{2}\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\sqrt{2}\sqrt{\frac{1}{1+x}}\right)\right)}{3\sqrt{1-x}}$$

input
```
Integrate[(E^ArcCoth[x]*x)/Sqrt[1 - x],x]
```

output
```
(2*Sqrt[(-1 + x)/x]*x*(Sqrt[1 + x^(-1)]*(4 + x) - 3*Sqrt[2]*Sqrt[x^(-1)]*A
rcTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]]))/(3*Sqrt[1 - x])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6730, 107, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(x)}}{\sqrt{1-x}}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(1-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$\Big\downarrow \text{107}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\left(\int \frac{\sqrt{1+\frac{1}{x}}}{\left(1-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\left(2\int \frac{1}{\left(1-\frac{1}{x}\right)\sqrt{1+\frac{1}{x}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{3\left(\frac{1}{x}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\left(4\int \frac{1}{1-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{3\left(\frac{1}{x}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\left(2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)-\frac{2\left(\frac{1}{x}+1\right)^{3/2}}{3\left(\frac{1}{x}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

input
```
Int[(E^ArcCoth[x]*x)/Sqrt[1 - x],x]
```

output
```
-((Sqrt[1 - x^(-1)]*((-2*(1 + x^(-1))^(3/2))/(3*(x^(-1))^(3/2)) - (2*Sqrt[
1 + x^(-1)])/Sqrt[x^(-1)] + 2*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[
1 + x^(-1)]]))/(Sqrt[1 - x]*Sqrt[x^(-1)]))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{2(x-1)\left(-3\sqrt{2}\,\arctan\left(\frac{\sqrt{-x-1}\,\sqrt{2}}{2}\right)+\sqrt{-x-1}\,x+4\sqrt{-x-1}\right)}{3\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}\,\sqrt{-x-1}}$	68
risch	$\frac{2(4+x)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{3\sqrt{-x-1}\,\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}}+\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{-x-1}\,\sqrt{2}}{2}\right)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{\sqrt{\frac{x-1}{1+x}}\,(1+x)\sqrt{1-x}}$	111

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x-1)*(-3*2^(1/2)*arctan(1/2*(-x-1)^(1/2)*2^(1/2))+(-x-1)^(1/2)*x+4*(-
x-1)^(1/2))/((x-1)/(1+x))^(1/2)/(1-x)^(1/2)/(-x-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.69

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \frac{2\left(3\sqrt{2}(x-1)\arctan\left(\frac{\sqrt{2}(x+1)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{2\,(x-1)}\right)-(x^2+5\,x+4)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}\right)}{3\,(x-1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(3*sqrt(2)*(x - 1)*arctan(1/2*sqrt(2)*(x + 1)*sqrt(-x + 1)*sqrt((x - 1
)/(x + 1))/(x - 1)) - (x^2 + 5*x + 4)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/
(x - 1)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\sqrt{1-x}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*x/(1-x)**(1/2),x)
```

output
```
Integral(x/(sqrt((x - 1)/(x + 1))*sqrt(1 - x)), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \int \frac{x}{\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-x + 1)*sqrt((x - 1)/(x + 1))), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = -\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x-1}\right)}{\operatorname{sgn}(x+1)} - \frac{2\left((-x-1)^{\frac{3}{2}} - 3\sqrt{-x-1}\right)}{3\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x - 1))/sgn(x + 1) - 2/3*((-x - 1)^(3/
2) - 3*sqrt(-x - 1))/sgn(x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,\sqrt{1-x}}\,dx$$

input
```
int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)),x)
```

output
```
int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.36

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \frac{2i\left(\sqrt{x+1}\,x + 4\sqrt{x+1} + 3\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)\right) - 5\sqrt{2}\right)}{3}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1-x)^(1/2),x)
```

output
```
(2*i*(sqrt(x + 1)*x + 4*sqrt(x + 1) + 3*sqrt(2)*log(tan(asin(sqrt( - x + 1
)/sqrt(2))/2)) - 5*sqrt(2)))/3
```

3.353 $\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 90

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx=\frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{\sqrt{1-x}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

output
```
2*(1-1/x)^(1/2)*(1+1/x)^(1/2)*x/(1-x)^(1/2)-2*2^(1/2)*(1-1/x)^(1/2)*arctan
h(2^(1/2)*(1/x)^(1/2)/(1+1/x)^(1/2))/(1-x)^(1/2)/(1/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx=\frac{2\sqrt{\frac{-1+x}{x}}x\left(\sqrt{1+\frac{1}{x}}-\sqrt{2}\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\sqrt{2}\sqrt{\frac{1}{1+x}}\right)\right)}{\sqrt{1-x}}$$

input
```
Integrate[E^ArcCoth[x]/Sqrt[1 - x],x]
```

output
```
(2*Sqrt[(-1 + x)/x]*x*(Sqrt[1 + x^(-1)] - Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[Sqr
t[2]*Sqrt[(1 + x)^(-1)]]))/Sqrt[1 - x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6727, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(1-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\left(2\int \frac{1}{\left(1-\frac{1}{x}\right)\sqrt{1+\frac{1}{x}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\left(4\int \frac{1}{1-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{1-\frac{1}{x}}\left(2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

input
```
Int[E^ArcCoth[x]/Sqrt[1 - x],x]
```

output
```
-((Sqrt[1 - x^(-1)]*((-2*Sqrt[1 + x^(-1)])/Sqrt[x^(-1)] + 2*Sqrt[2]*ArcTan
h[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]]))/(Sqrt[1 - x]*Sqrt[x^(-1)]))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

method	result	size
default	$-\frac{2(x-1)\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-x-1}\,\sqrt{2}}{2}\right)-\sqrt{-x-1}\right)}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}\,\sqrt{-x-1}}$	58
risch	$\frac{2\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{\sqrt{-x-1}\,\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}}+\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{-x-1}\,\sqrt{2}}{2}\right)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{\sqrt{\frac{x-1}{1+x}}\,(1+x)\sqrt{1-x}}$	108

input
```
int(1/((x-1)/(1+x))^(1/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/((x-1)/(1+x))^(1/2)*(x-1)*(2^(1/2)*arctan(1/2*(-x-1)^(1/2)*2^(1/2))-(-x
-1)^(1/2))/(1-x)^(1/2)/(-x-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx=\frac{2\left(\sqrt{2}(x-1)\arctan\left(\frac{\sqrt{2}(x+1)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{2\,(x-1)}\right)-(x+1)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}\right)}{x-1}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(2)*(x - 1)*arctan(1/2*sqrt(2)*(x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x
 + 1))/(x - 1)) - (x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

Sympy [A] (verification not implemented)

Time = 14.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx$$
$$= -2\left(\left\{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-x-1}}{2} - \operatorname{acos}\left(\frac{\sqrt{2}}{\sqrt{1-x}}\right)\right) \quad \text{for } \sqrt{1-x} > -\sqrt{2} \wedge \sqrt{1-x} < \sqrt{2}\right)\right.$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)/(1-x)**(1/2),x)
```

output
```
-2*Piecewise((sqrt(2)*(sqrt(2)*sqrt(-x - 1)/2 - acos(sqrt(2)/sqrt(1 - x)))
, (sqrt(1 - x) < sqrt(2)) & (sqrt(1 - x) > -sqrt(2))))
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \int \frac{1}{\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x + 1)*sqrt((x - 1)/(x + 1))), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.39

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = -\frac{2\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x-1}\right) - \sqrt{-x-1}\right)}{\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="giac")
```

output
```
-2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x - 1)) - sqrt(-x - 1))/sgn(x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx=\int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,\sqrt{1-x}}\,dx$$

input
```
int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)),x)
```

output
```
int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.34

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx=2i\left(\sqrt{x+1}+\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)\right)-\sqrt{2}\right)$$

input
```
int(1/((x-1)/(1+x))^(1/2)/(1-x)^(1/2),x)
```

output
```
2*i*(sqrt(x + 1) + sqrt(2)*log(tan(asin(sqrt( - x + 1)/sqrt(2))/2)) - sqrt
(2))
```

3.354 $\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \frac{2\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{(1+x)^{3/2}} + \frac{\sqrt{2}\left(1+\frac{1}{x}\right)^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1-\frac{1}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

output
```
2*(1-1/x)^(1/2)*(1+1/x)^(3/2)*x^2/(1+x)^(3/2)+2^(1/2)*(1+1/x)^(3/2)*arctan
(2^(1/2)*(1/x)^(1/2)/(1-1/x)^(1/2))/(1/x)^(3/2)/(1+x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.75

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \frac{\sqrt{1+\frac{1}{x}}x\left(2\sqrt{\frac{-1+x}{x}}-\sqrt{2}\sqrt{\frac{1}{x}}\arctan\left(\frac{\sqrt{\frac{-1+x}{x^2}}x}{\sqrt{2}}\right)\right)}{\sqrt{1+x}}$$

input
```
Integrate[(E^ArcCoth[x]*x)/(1 + x)^(3/2),x]
```

output
```
(Sqrt[1 + x^(-1)]*x*(2*Sqrt[(-1 + x)/x] - Sqrt[2]*Sqrt[x^(-1)]*ArcTan[(Sqr
t[(-1 + x)/x^2]*x)/Sqrt[2]]))/Sqrt[1 + x]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6730, 107, 104, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(x)}}{(x+1)^{3/2}}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\left(\frac{1}{x}+1\right)^{3/2}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

$$\Big\downarrow \text{107}$$

$$-\frac{\left(\frac{1}{x}+1\right)^{3/2}\left(-\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2\sqrt{1-\frac{1}{x}}}{\sqrt{\frac{1}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{\left(\frac{1}{x}+1\right)^{3/2}\left(-2\int \frac{1}{1+\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1-\frac{1}{x}}}-\frac{2\sqrt{1-\frac{1}{x}}}{\sqrt{\frac{1}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\left(\frac{1}{x}+1\right)^{3/2}\left(-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1-\frac{1}{x}}}\right)-\frac{2\sqrt{1-\frac{1}{x}}}{\sqrt{\frac{1}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

input
```
Int[(E^ArcCoth[x]*x)/(1 + x)^(3/2),x]
```

output
```
-(((1 + x^(-1))^(3/2)*((-2*Sqrt[1 - x^(-1)])/Sqrt[x^(-1)] - Sqrt[2]*ArcTan
[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 - x^(-1)]]))/((x^(-1))^(3/2)*(1 + x)^(3/2))
)
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{\sqrt{x-1}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{x-1}\,\sqrt{2}}{2}\right)-2\sqrt{x-1}\right)}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	47
risch	$\frac{2x-2}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{x-1}\,\sqrt{2}}{2}\right)\sqrt{x-1}}{\sqrt{\frac{x-1}{1+x}}\,\sqrt{1+x}}$	60

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(x-1)^(1/2)*(2^(1/2)*arctan(1/2*(x-1)^(1/2)*2^(1/2))-2*(x-1)^(1/2))/((x-1
)/(1+x))^(1/2)/(1+x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.53

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx=-\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}\right)+2\,\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))) + 2*sqrt(x
+ 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*x/(1+x)**(3/2),x)
```

output
```
Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \int \frac{x}{(x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/((x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: (2*a
tan(i)-4*i)/sqrt(2)*sign(sageVARx+1)+2*(sqrt(sageVARx-1)/sign(sageVARx+1)-
atan(sqrt(sageVARx-1)/sqrt(2))/sqrt(2)/sign(sageVARx+1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{3/2}}\,dx$$

input
```
int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)),x)
```

output
```
int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.24

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = -\sqrt{2}\,atan\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)+2\sqrt{x-1}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1+x)^(3/2),x)
```

output
```
- sqrt(2)*atan(sqrt(x - 1)/sqrt(2)) + 2*sqrt(x - 1)
```

3.355 $\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 55

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx=-\frac{\sqrt{2}\left(1+\frac{1}{x}\right)^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1-\frac{1}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

output
```
-2^(1/2)*(1+1/x)^(3/2)*arctan(2^(1/2)*(1/x)^(1/2)/(1-1/x)^(1/2))/(1/x)^(3/
2)/(1+x)^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx=\sqrt{2}\sqrt{\frac{1}{1+x}}\sqrt{1+x}\arctan\left(\frac{\sqrt{\frac{-1+x}{x^2}}x}{\sqrt{2}}\right)$$

input
```
Integrate[E^ArcCoth[x]/(1 + x)^(3/2),x]
```

output
```
Sqrt[2]*Sqrt[(1 + x)^(-1)]*Sqrt[1 + x]*ArcTan[(Sqrt[(-1 + x)/x^2]*x)/Sqrt[
2]]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6727, 104, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(x)}}{(x+1)^{3/2}}\,dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(\frac{1}{x}+1\right)^{3/2}\int \frac{1}{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{2\left(\frac{1}{x}+1\right)^{3/2}\int \frac{1}{1+\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1-\frac{1}{x}}}}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\sqrt{2}\left(\frac{1}{x}+1\right)^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1-\frac{1}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

input
```
Int[E^ArcCoth[x]/(1 + x)^(3/2),x]
```

output
```
-((Sqrt[2]*(1 + x^(-1))^(3/2)*ArcTan[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 - x^(-1
)]])/((x^(-1))^(3/2)*(1 + x)^(3/2)))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\sqrt{2}\, \arctan\left(\frac{\sqrt{x-1}\, \sqrt{2}}{2}\right) \sqrt{x-1}}{\sqrt{\frac{x-1}{1+x}}\, \sqrt{1+x}}$	37

input
```
int(1/((x-1)/(1+x))^(1/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*arctan(1/2*(x-1)^(1/2)*2^(1/2))/((x-1)/(1+x))^(1/2)/(1+x)^(1/2)*(x
-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.47

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}\right)$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1)))
```

Sympy [A] (verification not implemented)

Time = 36.90 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = 2\left(\begin{cases} \frac{\sqrt{2}\,\mathrm{acos}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{2} & \text{for}\ \sqrt{x+1} > -\sqrt{2} \wedge \sqrt{x+1} < \sqrt{2} \end{cases}\right)$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)/(1+x)**(3/2),x)
```

output
```
2*Piecewise((sqrt(2)*acos(sqrt(2)/sqrt(x + 1))/2, (sqrt(x + 1) < sqrt(2))
& (sqrt(x + 1) > -sqrt(2))))
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = \int \frac{1}{(x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: -sqr
t(2)*atan(i)*sign(sageVARx+1)+sqrt(2)*atan(sqrt(sageVARx-1)/sqrt(2))/sign(
sageVARx+1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = \int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{3/2}}\,dx$$

input
```
int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)),x)
```

output
```
int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.24

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = \sqrt{2}\,atan\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)$$

input
```
int(1/((x-1)/(1+x))^(1/2)/(1+x)^(3/2),x)
```

output
```
sqrt(2)*atan(sqrt(x - 1)/sqrt(2))
```

3.356 $\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = -\frac{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x^2}{(1-x)^{3/2}} + \frac{3\left(1-\frac{1}{x}\right)^{3/2}\sqrt{1+\frac{1}{x}}x^2}{(1-x)^{3/2}} - \frac{5\left(1-\frac{1}{x}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

output
```
-(1-1/x)^(1/2)*(1+1/x)^(1/2)*x^2/(1-x)^(3/2)+3*(1-1/x)^(3/2)*(1+1/x)^(1/2)
*x^2/(1-x)^(3/2)-5/2*(1-1/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/(1+1/x)^(1/
2))*2^(1/2)/(1-x)^(3/2)/(1/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = -\frac{\sqrt{\frac{-1+x}{x}}x\left(2\sqrt{1+\frac{1}{x}}(3-2x)+5\sqrt{2}(-1+x)\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\sqrt{2}\sqrt{\frac{1}{1+x}}\right)\right)}{2(1-x)^{3/2}}$$

input
```
Integrate[(E^ArcCoth[x]*x)/(1 - x)^(3/2),x]
```

output
```
-1/2*(Sqrt[(-1 + x)/x]*x*(2*Sqrt[1 + x^(-1)]*(3 - 2*x) + 5*Sqrt[2]*(-1 + x
)*Sqrt[x^(-1)]*ArcTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]]))/(1 - x)^(3/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6730, 107, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(x)}}{(1-x)^{3/2}}dx$$

$$\Big\downarrow \text{ 6730}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(1-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{ 107}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\left(\frac{5}{4}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(1-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}+\frac{\left(\frac{1}{x}+1\right)^{3/2}}{2\left(1-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\left(\frac{5}{4}\left(2\int \frac{1}{\left(1-\frac{1}{x}\right)\sqrt{1+\frac{1}{x}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)+\frac{\left(\frac{1}{x}+1\right)^{3/2}}{2\left(1-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{ 104}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\left(\frac{5}{4}\left(4\int \frac{1}{1-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)+\frac{\left(\frac{1}{x}+1\right)^{3/2}}{2\left(1-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\left(\frac{5}{4}\left(2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)-\frac{2\sqrt{\frac{1}{x}+1}}{\sqrt{\frac{1}{x}}}\right)+\frac{\left(\frac{1}{x}+1\right)^{3/2}}{2\left(1-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

input
```
Int[(E^ArcCoth[x]*x)/(1 - x)^(3/2),x]
```

output
```
-(((1 - x^(-1))^(3/2)*((1 + x^(-1))^(3/2)/(2*(1 - x^(-1))*Sqrt[x^(-1)]) +
(5*((-2*Sqrt[1 + x^(-1)])/Sqrt[x^(-1)] + 2*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x
^(-1)])/Sqrt[1 + x^(-1)]]))/4))/((1 - x)^(3/2)*(x^(-1))^(3/2)))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{(x-1)\left(-5\sqrt{2}\,\arctan\left(\frac{\sqrt{-x-1}\,\sqrt{2}}{2}\right)x+4\sqrt{-x-1}\,x+5\sqrt{2}\,\arctan\left(\frac{\sqrt{-x-1}\,\sqrt{2}}{2}\right)-6\sqrt{-x-1}\right)}{2\sqrt{\frac{x-1}{1+x}}\,(1-x)^{\frac{3}{2}}\sqrt{-x-1}}$	88
risch	$-\frac{(2x^2-x-3)\sqrt{\frac{(1+x)(1-x)}{x-1}}}{\sqrt{-x-1}\,\sqrt{\frac{x-1}{1+x}}\,(1+x)\sqrt{1-x}}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{-x-1}\,\sqrt{2}}{2}\right)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{2\sqrt{\frac{x-1}{1+x}}\,(1+x)\sqrt{1-x}}$	120

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x-1)*(-5*2^(1/2)*arctan(1/2*(-x-1)^(1/2)*2^(1/2))*x+4*(-x-1)^(1/2)*x+
5*2^(1/2)*arctan(1/2*(-x-1)^(1/2)*2^(1/2))-6*(-x-1)^(1/2))/((x-1)/(1+x))^(
1/2)/(1-x)^(3/2)/(-x-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = -\frac{5\,\sqrt{2}(x^2-2\,x+1)\arctan\left(\frac{\sqrt{2}(x+1)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{2\,(x-1)}\right)-2\,(2\,x^2-x-3)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{2\,(x^2-2\,x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(5*sqrt(2)*(x^2 - 2*x + 1)*arctan(1/2*sqrt(2)*(x + 1)*sqrt(-x + 1)*sq
rt((x - 1)/(x + 1))/(x - 1)) - 2*(2*x^2 - x - 3)*sqrt(-x + 1)*sqrt((x - 1)
/(x + 1)))/(x^2 - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 87.89 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = 2\left(\begin{cases}\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-x-1}}{2}-\operatorname{acos}\left(\frac{\sqrt{2}}{\sqrt{1-x}}\right)\right) & \text{for } \sqrt{1-x}>-\sqrt{2}\wedge\sqrt{1-x}<\sqrt{2}\end{cases}\right) - 2\left(\begin{cases}\frac{\sqrt{2}\left(\frac{\operatorname{acos}\left(\frac{\sqrt{2}}{\sqrt{1-x}}\right)}{2}-\frac{\sqrt{2}\sqrt{1-\frac{2}{1-x}}}{2\sqrt{1-x}}\right)}{2} & \text{for } \sqrt{1-x}>-\sqrt{2}\wedge\sqrt{1-x}<\sqrt{2}\end{cases}\right)$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)*x/(1-x)**(3/2),x)
```

output
```
2*Piecewise((sqrt(2)*(sqrt(2)*sqrt(-x - 1)/2 - acos(sqrt(2)/sqrt(1 - x))),
 (sqrt(1 - x) < sqrt(2)) & (sqrt(1 - x) > -sqrt(2)))) - 2*Piecewise((sqrt(
2)*(acos(sqrt(2)/sqrt(1 - x))/2 - sqrt(2)*sqrt(1 - 2/(1 - x))/(2*sqrt(1 -
x)))/2, (sqrt(1 - x) < sqrt(2)) & (sqrt(1 - x) > -sqrt(2))))
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = \int \frac{x}{(-x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/((-x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.40

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = \frac{5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x-1}\right) - 4\sqrt{-x-1} + \frac{2\sqrt{-x-1}}{x-1}}{2\,\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="giac")
```

output
```
1/2*(5*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x - 1)) - 4*sqrt(-x - 1) + 2*sqrt(
-x - 1)/(x - 1))/sgn(x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = \int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(1-x)^{3/2}}\,dx$$

input
```
int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)),x)
```

output
```
int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = \frac{i\left(-16\sqrt{x+1}\,x + 24\sqrt{x+1} - 20\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)\right)x + 20\sqrt{2}\log\left(\tan\left(\frac{asin(\cdots}{}\right.\right.\right.}{8x-8}$$

input
```
int(1/((x-1)/(1+x))^(1/2)*x/(1-x)^(3/2),x)
```

output
```
(i*( - 16*sqrt(x + 1)*x + 24*sqrt(x + 1) - 20*sqrt(2)*log(tan(asin(sqrt( -
 x + 1)/sqrt(2))/2))*x + 20*sqrt(2)*log(tan(asin(sqrt( - x + 1)/sqrt(2))/2
)) + 17*sqrt(2)*x - 17*sqrt(2)))/(8*(x - 1))
```

3.357 $\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 90

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = -\frac{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}x}{(1-x)^{3/2}} - \frac{\left(1-\frac{1}{x}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

output
```
-(1-1/x)^(1/2)*(1+1/x)^(1/2)*x/(1-x)^(3/2)-1/2*(1-1/x)^(3/2)*arctanh(2^(1/
2)*(1/x)^(1/2)/(1+1/x)^(1/2))*2^(1/2)/(1-x)^(3/2)/(1/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = -\frac{\sqrt{\frac{-1+x}{x}}x\left(2\sqrt{1+\frac{1}{x}}+\sqrt{2}(-1+x)\sqrt{\frac{1}{x}}\operatorname{arctanh}\left(\sqrt{2}\sqrt{\frac{1}{1+x}}\right)\right)}{2(1-x)^{3/2}}$$

input
```
Integrate[E^ArcCoth[x]/(1 - x)^(3/2),x]
```

output
```
-1/2*(Sqrt[(-1 + x)/x]*x*(2*Sqrt[1 + x^(-1)] + Sqrt[2]*(-1 + x)*Sqrt[x^(-1
)]*ArcTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]]))/(1 - x)^(3/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6727, 105, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\int \frac{\sqrt{1+\frac{1}{x}}}{\left(1-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}d\frac{1}{x}}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\left(\frac{1}{2}\int \frac{1}{\left(1-\frac{1}{x}\right)\sqrt{1+\frac{1}{x}}\sqrt{\frac{1}{x}}}d\frac{1}{x}+\frac{\sqrt{\frac{1}{x}+1}\sqrt{\frac{1}{x}}}{1-\frac{1}{x}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\left(\int \frac{1}{1-\frac{2}{x^2}}d\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}+\frac{\sqrt{\frac{1}{x}+1}\sqrt{\frac{1}{x}}}{1-\frac{1}{x}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(1-\frac{1}{x}\right)^{3/2}\left(\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{2}}+\frac{\sqrt{\frac{1}{x}+1}\sqrt{\frac{1}{x}}}{1-\frac{1}{x}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

input
```
Int[E^ArcCoth[x]/(1 - x)^(3/2),x]
```

output
```
-(((1 - x^(-1))^(3/2)*((Sqrt[1 + x^(-1)]*Sqrt[x^(-1)])/(1 - x^(-1)) + ArcT
anh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]]/Sqrt[2]))/((1 - x)^(3/2)*(x^(
-1))^(3/2)))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{(x-1)\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-x-1}\,\sqrt{2}}{2}\right)x-\sqrt{2}\,\arctan\left(\frac{\sqrt{-x-1}\,\sqrt{2}}{2}\right)+2\sqrt{-x-1}\right)}{2\sqrt{\frac{x-1}{1+x}}\,(1-x)^{\frac{3}{2}}\sqrt{-x-1}}$	77
risch	$\frac{\sqrt{\frac{(1+x)(1-x)}{x-1}}}{\sqrt{-x-1}\,\sqrt{\frac{x-1}{1+x}}\,\sqrt{1-x}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{-x-1}\,\sqrt{2}}{2}\right)\sqrt{\frac{(1+x)(1-x)}{x-1}}\,(x-1)}{2\sqrt{\frac{x-1}{1+x}}\,(1+x)\sqrt{1-x}}$	104

input
```
int(1/((x-1)/(1+x))^(1/2)/(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((x-1)/(1+x))^(1/2)*(x-1)*(2^(1/2)*arctan(1/2*(-x-1)^(1/2)*2^(1/2))*x
-2^(1/2)*arctan(1/2*(-x-1)^(1/2)*2^(1/2))+2*(-x-1)^(1/2))/(1-x)^(3/2)/(-x-
1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = -\frac{\sqrt{2}(x^2-2\,x+1)\arctan\left(\frac{\sqrt{2}(x+1)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{2\,(x-1)}\right)+2\,(x+1)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{2\,(x^2-2\,x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*(x^2 - 2*x + 1)*arctan(1/2*sqrt(2)*(x + 1)*sqrt(-x + 1)*sqrt
((x - 1)/(x + 1))/(x - 1)) + 2*(x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))
/(x^2 - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 85.63 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = -2\left(\begin{cases} \frac{\sqrt{2}\left(\frac{\operatorname{acos}\left(\frac{\sqrt{2}}{\sqrt{1-x}}\right)}{2}-\frac{\sqrt{2}\sqrt{1-\frac{2}{1-x}}}{2\sqrt{1-x}}\right)}{2} & \text{for}\ \sqrt{1-x}>-\sqrt{2}\wedge\sqrt{1-x}<\sqrt{2} \end{cases}\right)$$

input
```
integrate(1/((x-1)/(1+x))**(1/2)/(1-x)**(3/2),x)
```

output
```
-2*Piecewise((sqrt(2)*(acos(sqrt(2)/sqrt(1 - x))/2 - sqrt(2)*sqrt(1 - 2/(1
 - x))/(2*sqrt(1 - x)))/2, (sqrt(1 - x) < sqrt(2)) & (sqrt(1 - x) > -sqrt(
2))))
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = \int \frac{1}{(-x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.44

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x-1}\right)+\frac{2\sqrt{-x-1}}{x-1}}{2\operatorname{sgn}(x+1)}$$

input
```
integrate(1/((x-1)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x - 1)) + 2*sqrt(-x - 1)/(x - 1))/sg
n(x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = \int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,(1-x)^{3/2}}\,dx$$

input
```
int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)),x)
```

output
```
int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = \frac{i\left(2\sqrt{x+1}-\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)\right)x+\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{2}\right)\right)\right)}{2x-2}$$

input
```
int(1/((x-1)/(1+x))^(1/2)/(1-x)^(3/2),x)
```

output
```
(i*(2*sqrt(x + 1) - sqrt(2)*log(tan(asin(sqrt( - x + 1)/sqrt(2))/2))*x + s
qrt(2)*log(tan(asin(sqrt( - x + 1)/sqrt(2))/2))))/(2*(x - 1))
```

3.358 $\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 142

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{152c\sqrt{1-\frac{1}{a^2x^2}}x}{105a^2\sqrt{c-acx}}+\frac{38\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-acx}}{105a^2}+\frac{6\sqrt{1-\frac{1}{a^2x^2}}x(c-acx)^{3/2}}{35a^2c}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}x^2(c-acx)^{3/2}}{7ac}$$

output
```
152/105*c*(1-1/a^2/x^2)^(1/2)*x/a^2/(-a*c*x+c)^(1/2)+38/105*(1-1/a^2/x^2)^
(1/2)*x*(-a*c*x+c)^(1/2)/a^2+6/35*(1-1/a^2/x^2)^(1/2)*x*(-a*c*x+c)^(3/2)/a
^2/c-2/7*(1-1/a^2/x^2)^(1/2)*x^2*(-a*c*x+c)^(3/2)/a/c
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.47

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(-104+52ax-39a^2x^2+15a^3x^3)}{105a^3\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(x^2*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

output
```
(2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-104 + 52*a*x - 39*a^2*x^2 + 15*a^3*
x^3))/(105*a^3*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6730, 27, 87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-acx}e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{a\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{13}{7}\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{13}{7}\left(-\frac{4\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{5a}-\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2a\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{13}{7}\left(-\frac{4\left(-\frac{2\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{3a}-\frac{2\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)-\frac{2a\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(-\frac{2a\sqrt{\frac{1}{ax}+1}}{7\left(\frac{1}{x}\right)^{7/2}}-\frac{13}{7}\left(-\frac{4\left(\frac{4\sqrt{\frac{1}{ax}+1}}{3a\sqrt{\frac{1}{x}}}-\frac{2\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)\right)\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(x^2*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

output
```
-((((-13*((-4*((-2*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) + (4*Sqrt[1 + 1/(
a*x)])/(3*a*Sqrt[x^(-1)])))/(5*a) - (2*Sqrt[1 + 1/(a*x)])/(5*(x^(-1))^(5/2
))))/7 - (2*a*Sqrt[1 + 1/(a*x)])/(7*(x^(-1))^(7/2)))*Sqrt[x^(-1)]*Sqrt[c -
 a*c*x])/(a*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.42

method	result	size
risch	$-\frac{2c\sqrt{\frac{ax-1}{ax+1}}\,(15a^3x^3-39a^2x^2+52ax-104)(ax+1)}{105\sqrt{-c(ax-1)}\,a^3}$	59
gosper	$\frac{2(ax+1)(15a^3x^3-39a^2x^2+52ax-104)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{105a^3(ax-1)}$	64
orering	$\frac{2(ax+1)(15a^3x^3-39a^2x^2+52ax-104)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{105a^3(ax-1)}$	64
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,(15a^3x^3-39a^2x^2+52ax-104)}{105(ax-1)a^3}$	65

input
```
int(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*c*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(15*a^3*x^3-39*a^2*x^2
+52*a*x-104)/a^3*(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.49

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\,(15\,a^4x^4-24\,a^3x^3+13\,a^2x^2-52\,ax-104)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^4x-a^3)}$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

output
```
2/105*(15*a^4*x^4 - 24*a^3*x^3 + 13*a^2*x^2 - 52*a*x - 104)*sqrt(-a*c*x +
c)*sqrt((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \int x^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}\,dx$$

input
```
integrate(x**2*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x**2*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\left(15\,a^4\sqrt{-c}x^4-24\,a^3\sqrt{-c}x^3+13\,a^2\sqrt{-c}x^2-52\,a\sqrt{-c}x-104\,\sqrt{-c}\right)(ax-1)}{105\,(a^4x-a^3)\sqrt{ax+1}}$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

output
```
2/105*(15*a^4*sqrt(-c)*x^4 - 24*a^3*sqrt(-c)*x^3 + 13*a^2*sqrt(-c)*x^2 - 5
2*a*sqrt(-c)*x - 104*sqrt(-c))*(a*x - 1)/((a^4*x - a^3)*sqrt(a*x + 1))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.71 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.62

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,(15\,a^3\,x^3-9\,a^2\,x^2+4\,a\,x-48)}{105\,a^3} - \frac{304\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{105\,a^3\,(a\,x-1)}$$

input
```
int(x^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(4*a*x - 9*a^2*x^2 + 15*a
^3*x^3 - 48))/(105*a^3) - (304*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/
2))/(105*a^3*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.25

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c}\,\sqrt{ax+1}\,i(-15a^3x^3+39a^2x^2-52ax+104)}{105a^3}$$

input
```
int(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a*x + 1)*i*( - 15*a**3*x**3 + 39*a**2*x**2 - 52*a*x + 104)
)/(105*a**3)
```

3.359 $\int e^{-\coth^{-1}(ax)} x\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 104

$$\int e^{-\coth^{-1}(ax)} x\sqrt{c-acx}\,dx = -\frac{8c\sqrt{1-\frac{1}{a^2x^2}}\,x}{5a\sqrt{c-acx}} - \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-acx}}{5a} - \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{3/2}}{5ac}$$

output
```
-8/5*c*(1-1/a^2/x^2)^(1/2)*x/a/(-a*c*x+c)^(1/2)-2/5*(1-1/a^2/x^2)^(1/2)*x*
(-a*c*x+c)^(1/2)/a-2/5*(1-1/a^2/x^2)^(1/2)*x*(-a*c*x+c)^(3/2)/a/c
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.56

$$\int e^{-\coth^{-1}(ax)} x\sqrt{c-acx}\,dx = \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}(6-3ax+a^2x^2)}{5a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(x*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

output
```
(2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(6 - 3*a*x + a^2*x^2))/(5*a^2*Sqrt[1
- 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6730, 27, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-acx}e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{a\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{9}{5}\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{9}{5}\left(-\frac{2\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{3a}-\frac{2\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)-\frac{2a\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(-\frac{2a\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}-\frac{9}{5}\left(\frac{4\sqrt{\frac{1}{ax}+1}}{3a\sqrt{\frac{1}{x}}}-\frac{2\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)\right)\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(x*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

output
```
-((((-9*((-2*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) + (4*Sqrt[1 + 1/(a*x)])
/(3*a*Sqrt[x^(-1)])))/5 - (2*a*Sqrt[1 + 1/(a*x)])/(5*(x^(-1))^(5/2)))*Sqrt
[x^(-1)]*Sqrt[c - a*c*x])/(a*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.48

method	result	size
risch	$-\frac{2c\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-3ax+6)(ax+1)}{5\sqrt{-c(ax-1)}\,a^2}$	50
gosper	$\frac{2(ax+1)(a^2x^2-3ax+6)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{5a^2(ax-1)}$	55
orering	$\frac{2(ax+1)(a^2x^2-3ax+6)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{5a^2(ax-1)}$	55
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,(a^2x^2-3ax+6)}{5(ax-1)a^2}$	56

input
```
int(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*c*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(a^2*x^2-3*a*x+6)/a^2*(a
*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\,(a^3x^3-2\,a^2x^2+3\,ax+6)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{5\,(a^3x-a^2)}$$

input
```
integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

output
```
2/5*(a^3*x^3 - 2*a^2*x^2 + 3*a*x + 6)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
 + 1))/(a^3*x - a^2)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\int x\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}\,dx$$

input
```
integrate(x*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.66

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\left(a^3\sqrt{-c}x^3-2\,a^2\sqrt{-c}x^2+3\,a\sqrt{-c}x+6\,\sqrt{-c}\right)(ax-1)}{5\,(a^3x-a^2)\sqrt{ax+1}}$$

input
```
integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

output
```
2/5*(a^3*sqrt(-c)*x^3 - 2*a^2*sqrt(-c)*x^2 + 3*a*sqrt(-c)*x + 6*sqrt(-c))*
(a*x - 1)/((a^3*x - a^2)*sqrt(a*x + 1))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.66

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = -\frac{4\sqrt{-acx-c}|c|}{a^2c} - \frac{2\left((acx+c)^2\sqrt{-acx-c}|c|+5(-acx-c)^{\frac{3}{2}}c|c|\right)}{5a^2c^3}$$

input
```
integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
-4*sqrt(-a*c*x - c)*abs(c)/(a^2*c) - 2/5*((a*c*x + c)^2*sqrt(-a*c*x - c)*a
bs(c) + 5*(-a*c*x - c)^(3/2)*c*abs(c))/(a^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.55

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,(a^3\,x^3-2\,a^2\,x^2+3\,a\,x+6)}{5\,a^2\,(a\,x-1)}$$

input
```
int(x*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3*a*x - 2*a^2*x^2 + a^3*
x^3 + 6))/(5*a^2*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.27

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\sqrt{c}\,\sqrt{ax+1}\,i(-a^2x^2+3ax-6)}{5a^2}$$

input
```
int(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output

```
(2*sqrt(c)*sqrt(a*x + 1)*i*( - a**2*x**2 + 3*a*x - 6))/(5*a**2)
```

3.360 $\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{8c\sqrt{1-\frac{1}{a^2x^2}}x}{3\sqrt{c-acx}}+\frac{2}{3}\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-acx}$$

output
```
8/3*c*(1-1/a^2/x^2)^(1/2)*x/(-a*c*x+c)^(1/2)+2/3*(1-1/a^2/x^2)^(1/2)*x*(-a
*c*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}(-5+ax)\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

output
```
(2*Sqrt[1 + 1/(a*x)]*(-5 + a*x)*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6727, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{a\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{5}{3}\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{10\sqrt{\frac{1}{ax}+1}}{3\sqrt{\frac{1}{x}}}-\frac{2a\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

output
```
-((((-2*a*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) + (10*Sqrt[1 + 1/(a*x)])/(
3*Sqrt[x^(-1)]))*Sqrt[x^(-1)]*Sqrt[c - a*c*x])/(a*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{2c\sqrt{\frac{ax-1}{ax+1}}\,(ax-5)(ax+1)}{3\sqrt{-c(ax-1)}\,a}$	42
gosper	$\frac{2(ax+1)(ax-5)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3(ax-1)a}$	47
orering	$\frac{2(ax+1)(ax-5)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3(ax-1)a}$	47
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,(ax-5)}{3(ax-1)a}$	48

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*c*((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x-1))^(1/2)*(a*x-5)/a*(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\,(a^2x^2-4\,ax-5)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^2x-a)}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(a^2*x^2 - 4*a*x - 5)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*
x - a)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \int \sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\,(a^2\sqrt{-c}x^2 - 4\,a\sqrt{-c}x - 5\,\sqrt{-c})(ax-1)}{3\,(a^2x-a)\sqrt{ax+1}}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x - 5*sqrt(-c))*(a*x - 1)/((a^2*x - a
)*sqrt(a*x + 1))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.69

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\,(-acx-c)^{\frac{3}{2}}\,|c|}{3\,ac^2} + \frac{4\,\sqrt{-acx-c}|c|}{ac}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
2/3*(-a*c*x - c)^(3/2)*abs(c)/(a*c^2) + 4*sqrt(-a*c*x - c)*abs(c)/(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\,\sqrt{c-a\,c\,x}\,(a\,x-3)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a} - \frac{16\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,(a\,x-1)}$$

input
```
int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*(a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (16*(c
 - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.32

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{c}\,\sqrt{ax+1}\,i(-ax+5)}{3a}$$

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a*x + 1)*i*( - a*x + 5))/(3*a)
```

3.361 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\text{arcsinh}\left(\sqrt{\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+2*(1/a/x)^(1/2)*(-a*c*x
+c)^(1/2)*arcsinh((1/a/x)^(1/2))/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.81

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}+\sqrt{\frac{1}{x}}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x),x]
```

output
```
(2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[x^(-1)]*ArcSinh[Sqrt[
x^(-1)]/Sqrt[a]]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6730, 27, 87, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{ 6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{a\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2a\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 63}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-2\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}-\frac{2a\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 222}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(-2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-\frac{2a\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}\right)\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x),x]
```

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*a*Sqrt[1 + 1/(a*x)])/Sqrt[x^(-1)] - 2
*Sqrt[a]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/(a*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\left(\sqrt{c}\,\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)+\sqrt{-c(ax+1)}\right)}{(ax-1)\sqrt{-c(ax+1)}}$	80

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(c^(1/2)*arctan((-c*(
a*x+1))^(1/2)/c^(1/2))+(-c*(a*x+1))^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.32

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$$
$$= \left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}, \frac{2\left((ax-\right.}{}\right.$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas"
)
```

output
```
[((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x +
 c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), 2*((a*x - 1)*sqrt(c)*a
rctan(sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x
- c)) + sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}}{x}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1))/x, x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx=\int \frac{\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.44

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = -2\left(\frac{\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}} + \frac{\sqrt{-acx-c}}{c}\right)|c|$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
```

output
```
-2*(arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) + sqrt(-a*c*x - c)/c)*abs(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

input
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
```

output
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \sqrt{c}\,i\left(-2\sqrt{ax+1} + \log\left(\frac{2\sqrt{ax+1}-2}{\sqrt{2}}\right) - \log\left(\frac{2\sqrt{ax+1}+2}{\sqrt{2}}\right)\right)$$

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x)
```

output
```
sqrt(c)*i*( - 2*sqrt(a*x + 1) + log((2*sqrt(a*x + 1) - 2)/sqrt(2)) - log((
2*sqrt(a*x + 1) + 2)/sqrt(2)))
```

3.362 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}x}-\frac{3a\sqrt{\frac{1}{ax}}\sqrt{c-acx}\operatorname{arcsinh}\left(\sqrt{\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/x-3*a*(1/a/x)^(1/2)*(-a*c
*x+c)^(1/2)*arcsinh((1/a/x)^(1/2))/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.81

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx=\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}-3\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x^2),x]
```

output
```
(Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] - 3*Sqrt[a]*
ArcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/Sqrt[1 - 1/(a*x)]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6730, 27, 90, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{a\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{a-\frac{1}{x}}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{90}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{3}{2}a\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{63}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(3a\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}-a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(3a^{3/2}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x^2),x]
```

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(-(a*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + 3*a
^(3/2)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/(a*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\left(-3\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)acx+\sqrt{-c(ax+1)}\,\sqrt{c}\right)}{(ax-1)\sqrt{-c(ax+1)}\,x\sqrt{c}}$	90
risch	$-\frac{(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{x\sqrt{-c(ax-1)}}-\frac{3a\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}}{\sqrt{-c(ax-1)}}$	95

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*arctan((-c*(a*x+1))
^(1/2)/c^(1/2))*a*c*x+(-c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/
2)/x/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.52

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$$
$$=\left[\frac{3\,(a^2x^2-ax)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)+2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\,(ax^2-x)},\right.$$
$$\left.-\frac{3\,(a^2x^2-ax)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax^2-x}\right]$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="frica
s")
```

output
```
[1/2*(3*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x +
 c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*s
qrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), -(3*(a^2
*x^2 - a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x -
1)/(a*x + 1))/(a*c*x - c)) - sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*
x + 1)))/(a*x^2 - x)]
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)}}{x^2}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1))/x**2, x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.51

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = a\left(\frac{3\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{\sqrt{-acx-c}}{acx}\right)|c|$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac"
)
```

output
```
a*(3*arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - sqrt(-a*c*x - c)/(a*c*x))*
abs(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x^2}\,dx$$

input
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
```

output
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c}\,i\left(-2\sqrt{ax+1}-3\log\left(\frac{2\sqrt{ax+1}-2}{\sqrt{2}}\right)ax+3\log\left(\frac{2\sqrt{ax+1}+2}{\sqrt{2}}\right)ax\right)}{2x}$$

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

output
```
(sqrt(c)*i*( - 2*sqrt(a*x + 1) - 3*log((2*sqrt(a*x + 1) - 2)/sqrt(2))*a*x
+ 3*log((2*sqrt(a*x + 1) + 2)/sqrt(2))*a*x))/(2*x)
```

3.363 $\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 139

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{4\sqrt{c-acx}}{a^4}+\frac{2(c-acx)^{3/2}}{3a^4c}+\frac{2(c-acx)^{5/2}}{5a^4c^2}-\frac{2(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

output
```
4*(-a*c*x+c)^(1/2)/a^4+2/3*(-a*c*x+c)^(3/2)/a^4/c+2/5*(-a*c*x+c)^(5/2)/a^4
/c^2-2/7*(-a*c*x+c)^(7/2)/a^4/c^3+2/9*(-a*c*x+c)^(9/2)/a^4/c^4-4*2^(1/2)*c
^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.61

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{2\left(\sqrt{c-acx}(788-236ax+138a^2x^2-95a^3x^3+35a^4x^4)-630\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{315a^4}$$

input
```
Integrate[(x^3*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*(Sqrt[c - a*c*x]*(788 - 236*a*x + 138*a^2*x^2 - 95*a^3*x^3 + 35*a^4*x^4
) - 630*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]))/(315*
a^4)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6717, 6680, 35, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-acx}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x^3\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{x^3(1-ax)\sqrt{c-acx}}{ax+1}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{x^3(c-acx)^{3/2}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(\frac{(c-acx)^{7/2}}{a^3c^2}-\frac{(c-acx)^{5/2}}{a^3c}-\frac{(c-acx)^{3/2}}{a^3(ax+1)}+\frac{(c-acx)^{3/2}}{a^3}\right)dx}{c}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{4\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}-\frac{2(c-acx)^{9/2}}{9a^4c^3}+\frac{2(c-acx)^{7/2}}{7a^4c^2}-\frac{2(c-acx)^{5/2}}{5a^4c}-\frac{2(c-acx)^{3/2}}{3a^4}-\frac{4c\sqrt{c-acx}}{a^4}}{c}$$

input
```
Int[(x^3*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((-4*c*Sqrt[c - a*c*x])/a^4 - (2*(c - a*c*x)^(3/2))/(3*a^4) - (2*(c - a*
c*x)^(5/2))/(5*a^4*c) + (2*(c - a*c*x)^(7/2))/(7*a^4*c^2) - (2*(c - a*c*x)
^(9/2))/(9*a^4*c^3) + (4*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*
Sqrt[c])])/a^4)/c)
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

method	result	size
pseudoelliptic	$\frac{-4\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\frac{2\sqrt{-c(ax-1)}\,\left(35a^4x^4-95a^3x^3+138a^2x^2-236ax+788\right)}{315}}{a^4}$	75
risch	$-\frac{2(35a^4x^4-95a^3x^3+138a^2x^2-236ax+788)(ax-1)c}{315a^4\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\,\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^4}$	82
derivativedivides	$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{2c^2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c^3(-acx+c)^{\frac{3}{2}}}{3}+4\sqrt{-acx+c}\,c^4-4c^{\frac{9}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^4c^4}$	101
default	$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{2c^2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c^3(-acx+c)^{\frac{3}{2}}}{3}+4\sqrt{-acx+c}\,c^4-4c^{\frac{9}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^4c^4}$	101

input
```
int(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
2/315*(-630*c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2)
)+(-c*(a*x-1))^(1/2)*(35*a^4*x^4-95*a^3*x^3+138*a^2*x^2-236*a*x+788))/a^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.26

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$

$$=\left[\frac{2\left(315\,\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\,\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+(35\,a^4x^4-95\,a^3x^3+138\,a^2x^2-236\,ax+788)\sqrt{-acx+c}\right)}{315\,a^4}\right.$$

$$-\frac{2\left(630\,\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{acx-c}\right)-(35\,a^4x^4-95\,a^3x^3+138\,a^2x^2-236\,ax+788)\sqrt{-acx+c}\right)}{315\,a^4}$$

input
```
integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[2/315*(315*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c
) - 3*c)/(a*x + 1)) + (35*a^4*x^4 - 95*a^3*x^3 + 138*a^2*x^2 - 236*a*x + 7
88)*sqrt(-a*c*x + c))/a^4, -2/315*(630*sqrt(2)*sqrt(-c)*arctan(sqrt(2)*sqr
t(-a*c*x + c)*sqrt(-c)/(a*c*x - c)) - (35*a^4*x^4 - 95*a^3*x^3 + 138*a^2*x
^2 - 236*a*x + 788)*sqrt(-a*c*x + c))/a^4]
```

Sympy [A] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.26

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$= \begin{cases} \frac{2\cdot\left(\frac{2\sqrt{2}c^5\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^4\sqrt{-acx+c}+\frac{c^3(-acx+c)^{\frac{3}{2}}}{3}+\frac{c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{a^4c^4} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\frac{x^4}{4}-\frac{2x^3}{3a}+\frac{x^2}{a^2}-\frac{2x}{a^3}+\frac{2\left(\begin{cases} x & \text{for } a=0 \\ \frac{\log(ax+1)}{a} & \text{otherwise}\end{cases}\right)}{a^3}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(x**3*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise((2*(2*sqrt(2)*c**5*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/s
qrt(-c) + 2*c**4*sqrt(-a*c*x + c) + c**3*(-a*c*x + c)**(3/2)/3 + c**2*(-a*
c*x + c)**(5/2)/5 - c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/(a**4
*c**4), Ne(a*c, 0)), (sqrt(c)*(x**4/4 - 2*x**3/(3*a) + x**2/a**2 - 2*x/a**
3 + 2*Piecewise((x, Eq(a, 0)), (log(a*x + 1)/a, True))/a**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{2\left(315\sqrt{2}c^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+35(-acx+c)^{\frac{9}{2}}-45(-acx+c)^{\frac{7}{2}}c+63(-acx+c)^{\frac{5}{2}}c^2+105(-ac\right.}{315a^4c^4}$$

input
```
integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
2/315*(315*sqrt(2)*c^(9/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt
(2)*sqrt(c) + sqrt(-a*c*x + c))) + 35*(-a*c*x + c)^(9/2) - 45*(-a*c*x + c)
^(7/2)*c + 63*(-a*c*x + c)^(5/2)*c^2 + 105*(-a*c*x + c)^(3/2)*c^3 + 630*sq
rt(-a*c*x + c)*c^4)/(a^4*c^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.14

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^4\sqrt{-c}} + \frac{2\left(35(acx-c)^4\sqrt{-acx+c}a^{32}c^{32}+45(acx-c)^3\sqrt{-acx+c}a^{32}c^{33}+63(acx-c)^2\sqrt{-acx+c}a^{32}c^{34}\right.}{315a^{36}c^{36}}$$

input
```
integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^4*sqrt(-c)) +
 2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c)*a^32*c^32 + 45*(a*c*x - c)^3*sqr
t(-a*c*x + c)*a^32*c^33 + 63*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^32*c^34 + 10
5*(-a*c*x + c)^(3/2)*a^32*c^35 + 630*sqrt(-a*c*x + c)*a^32*c^36)/(a^36*c^3
6)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{4\sqrt{c-a\,c\,x}}{a^4}+\frac{2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a^4\,c}+\frac{2\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a^4\,c^2}-\frac{2\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a^4\,c^3}+\frac{2\,{\left(c-a\,c\,x\right)}^{9/2}}{9\,a^4\,c^4}+\frac{\sqrt{2}\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right)\,4{}\mathrm{i}}{a^4}$$

input
```
int((x^3*(c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
(4*(c - a*c*x)^(1/2))/a^4 + (2*(c - a*c*x)^(3/2))/(3*a^4*c) + (2*(c - a*c*
x)^(5/2))/(5*a^4*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x)
^(9/2))/(9*a^4*c^4) + (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)
/(2*c^(1/2)))*4i)/a^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.76

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{2\sqrt{c}\,\big(35\sqrt{-ax+1}\,a^4x^4-95\sqrt{-ax+1}\,a^3x^3+138\sqrt{-ax+1}\,a^2x^2-236\sqrt{-ax+1}\,ax+788\sqrt{-ax+1}}{315a^4}$$

input
```
int(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
(2*sqrt(c)*(35*sqrt( - a*x + 1)*a**4*x**4 - 95*sqrt( - a*x + 1)*a**3*x**3
+ 138*sqrt( - a*x + 1)*a**2*x**2 - 236*sqrt( - a*x + 1)*a*x + 788*sqrt( -
a*x + 1) + 315*sqrt(2)*log(sqrt( - a*x + 1) - sqrt(2)) - 315*sqrt(2)*log(s
qrt( - a*x + 1) + sqrt(2))))/(315*a**4)
```

3.364 $\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\frac{4\sqrt{c-acx}}{a^3}-\frac{2(c-acx)^{3/2}}{3a^3c} - \frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{4\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

output
```
-4*(-a*c*x+c)^(1/2)/a^3-2/3*(-a*c*x+c)^(3/2)/a^3/c-2/7*(-a*c*x+c)^(7/2)/a^
3/c^3+4*2^(1/2)*c^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(-52+16ax-9a^2x^2+3a^3x^3)+84\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{21a^3}$$

input
```
Integrate[(x^2*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*Sqrt[c - a*c*x]*(-52 + 16*a*x - 9*a^2*x^2 + 3*a^3*x^3) + 84*Sqrt[2]*Sqr
t[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(21*a^3)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6717, 6680, 35, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-acx}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x^2\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{x^2(1-ax)\sqrt{c-acx}}{ax+1}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{x^2(c-acx)^{3/2}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(\frac{(c-acx)^{3/2}}{a^2(ax+1)}-\frac{(c-acx)^{5/2}}{a^2c}\right)dx}{c}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{4\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}+\frac{2(c-acx)^{7/2}}{7a^3c^2}+\frac{2(c-acx)^{3/2}}{3a^3}+\frac{4c\sqrt{c-acx}}{a^3}}{c}$$

input
```
Int[(x^2*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((4*c*Sqrt[c - a*c*x])/a^3 + (2*(c - a*c*x)^(3/2))/(3*a^3) + (2*(c - a*c
*x)^(7/2))/(7*a^3*c^2) - (4*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[
2]*Sqrt[c])])/a^3)/c)
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.70

method	result	size
pseudoelliptic	$\frac{84\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+2\left(3a^{3}x^{3}-9a^{2}x^{2}+16ax-52\right)\sqrt{-c(ax-1)}}{21a^{3}}$	68
risch	$-\frac{2\left(3a^{3}x^{3}-9a^{2}x^{2}+16ax-52\right)(ax-1)c}{21a^{3}\sqrt{-c(ax-1)}}+\frac{4\sqrt{2}\,\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^{3}}$	74
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}+\frac{c^{2}(-acx+c)^{\frac{3}{2}}}{3}+2\sqrt{-acx+c}\,c^{3}-2c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{c^{3}a^{3}}$	75
default	$\frac{-\frac{2(-acx+c)^{\frac{7}{2}}}{7}-\frac{2c^{2}(-acx+c)^{\frac{3}{2}}}{3}-4\sqrt{-acx+c}\,c^{3}+4c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^{3}c^{3}}$	75

input
```
int(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/21*(84*c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))+2
*(3*a^3*x^3-9*a^2*x^2+16*a*x-52)*(-c*(a*x-1))^(1/2))/a^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.63

$$\int e^{-2\coth^{-1}(ax)}x^{2}\sqrt{c-acx}\,dx$$
$$=\left[\frac{2\left(21\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\left(3a^{3}x^{3}-9a^{2}x^{2}+16ax-52\right)\sqrt{-acx+c}\right)}{21a^{3}},\ \frac{2\left(42\sqrt{2}\sqrt{-c}\,a\right.}{}\right.$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[2/21*(21*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
- 3*c)/(a*x + 1)) + (3*a^3*x^3 - 9*a^2*x^2 + 16*a*x - 52)*sqrt(-a*c*x + c)
)/a^3, 2/21*(42*sqrt(2)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/
(a*c*x - c)) + (3*a^3*x^3 - 9*a^2*x^2 + 16*a*x - 52)*sqrt(-a*c*x + c))/a^3
]
```

Sympy [A] (verification not implemented)

Time = 3.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.38

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2\sqrt{2}c^4\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^3\sqrt{-acx+c}+\frac{c^2(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a^3c^3} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\frac{x^3}{3}-\frac{x^2}{a}+\frac{2x}{a^2}-\frac{2\left(\begin{cases} x & \text{for } a=0 \\ \frac{\log(ax+1)}{a} & \text{otherwise}\end{cases}\right)}{a^2}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise((-2*(2*sqrt(2)*c**4*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/
sqrt(-c) + 2*c**3*sqrt(-a*c*x + c) + c**2*(-a*c*x + c)**(3/2)/3 + (-a*c*x
+ c)**(7/2)/7)/(a**3*c**3), Ne(a*c, 0)), (sqrt(c)*(x**3/3 - x**2/a + 2*x/a
**2 - 2*Piecewise((x, Eq(a, 0)), (log(a*x + 1)/a, True))/a**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\frac{2\left(21\sqrt{2}c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+3(-acx+c)^{\frac{7}{2}}+7(-acx+c)^{\frac{3}{2}}c^2+42\sqrt{-acx+c}c^3\right)}{21\,a^3c^3}$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-2/21*(21*sqrt(2)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(
2)*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(7/2) + 7*(-a*c*x + c)^(3
/2)*c^2 + 42*sqrt(-a*c*x + c)*c^3)/(a^3*c^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$
$$= -\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^3\sqrt{-c}}$$
$$+ \frac{2\left(3\,(acx-c)^3\sqrt{-acx+c}a^{18}c^{18}-7\,(-acx+c)^{\frac{3}{2}}a^{18}c^{20}-42\,\sqrt{-acx+c}a^{18}c^{21}\right)}{21\,a^{21}c^{21}}$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^3*sqrt(-c))
+ 2/21*(3*(a*c*x - c)^3*sqrt(-a*c*x + c)*a^18*c^18 - 7*(-a*c*x + c)^(3/2)*
a^18*c^20 - 42*sqrt(-a*c*x + c)*a^18*c^21)/(a^21*c^21)
```

Mupad [B] (verification not implemented)

Time = 13.62 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\frac{4\sqrt{c-a\,c\,x}}{a^3}-\frac{2\,(c-a\,c\,x)^{3/2}}{3\,a^3\,c}$$
$$-\frac{2\,(c-a\,c\,x)^{7/2}}{7\,a^3\,c^3}-\frac{\sqrt{2}\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}}{a^3}$$

input
```
int((x^2*(c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
- (4*(c - a*c*x)^(1/2))/a^3 - (2*(c - a*c*x)^(3/2))/(3*a^3*c) - (2*(c - a*
c*x)^(7/2))/(7*a^3*c^3) - (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)
*1i)/(2*c^(1/2)))*4i)/a^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$
$$= \frac{2\sqrt{c}\left(3\sqrt{-ax+1}\,a^3x^3-9\sqrt{-ax+1}\,a^2x^2+16\sqrt{-ax+1}\,ax-52\sqrt{-ax+1}-21\sqrt{2}\log\left(\sqrt{-ax+1}-\right.\right.}{21a^3}$$

input
```
int(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
(2*sqrt(c)*(3*sqrt( - a*x + 1)*a**3*x**3 - 9*sqrt( - a*x + 1)*a**2*x**2 +
16*sqrt( - a*x + 1)*a*x - 52*sqrt( - a*x + 1) - 21*sqrt(2)*log(sqrt( - a*x
 + 1) - sqrt(2)) + 21*sqrt(2)*log(sqrt( - a*x + 1) + sqrt(2))))/(21*a**3)
```

3.365 $\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{4\sqrt{c-acx}}{a^2}+\frac{2(c-acx)^{3/2}}{3a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

output
```
4*(-a*c*x+c)^(1/2)/a^2+2/3*(-a*c*x+c)^(3/2)/a^2/c+2/5*(-a*c*x+c)^(5/2)/a^2
/c^2-4*2^(1/2)*c^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}(38-11ax+3a^2x^2)-60\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{15a^2}$$

input
```
Integrate[(x*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*Sqrt[c - a*c*x]*(38 - 11*a*x + 3*a^2*x^2) - 60*Sqrt[2]*Sqrt[c]*ArcTanh[
Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(15*a^2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6717, 6680, 35, 90, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-acx}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{x(1-ax)\sqrt{c-acx}}{ax+1}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{x(c-acx)^{3/2}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{90}$$

$$-\frac{-\frac{\int \frac{(c-acx)^{3/2}}{ax+1}dx}{a}-\frac{2(c-acx)^{5/2}}{5a^2c}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{-\frac{2c\int \frac{\sqrt{c-acx}}{ax+1}dx+\frac{2(c-acx)^{3/2}}{3a}}{a}-\frac{2(c-acx)^{5/2}}{5a^2c}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{-\frac{2c\left(2c\int \frac{1}{(ax+1)\sqrt{c-acx}}dx+\frac{2\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{a}-\frac{2(c-acx)^{5/2}}{5a^2c}}{c}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{-\frac{2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{4\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{a}-\frac{2(c-acx)^{5/2}}{5a^2c}}{c}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{-\frac{2(c-acx)^{5/2}}{5a^2c}-\frac{2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{2\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{a}}{c}$$

input
```
Int[(x*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((-2*(c - a*c*x)^(5/2))/(5*a^2*c) - ((2*(c - a*c*x)^(3/2))/(3*a) + 2*c*(
(2*Sqrt[c - a*c*x])/a - (2*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2
]*Sqrt[c])])/a))/a)/c)
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$\frac{-60\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+(6a^2x^2-22ax+76)\sqrt{-c(ax-1)}}{15a^2}$	59
risch	$-\frac{2(3a^2x^2-11ax+38)(ax-1)c}{15a^2\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\,\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^2}$	66
derivativedivides	$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2(-acx+c)^{\frac{3}{2}}c}{3}+4\sqrt{-acx+c}\,c^2-4c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^2c^2}$	73
default	$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2(-acx+c)^{\frac{3}{2}}c}{3}+4\sqrt{-acx+c}\,c^2-4c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a^2c^2}$	73

input
```
int(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-60*c^(1/2)*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))+
(6*a^2*x^2-22*a*x+76)*(-c*(a*x-1))^(1/2))/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.47

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \left[\frac{2\left(15\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+(3a^2x^2-11ax+38)\sqrt{-acx+c}\right)}{15a^2},\right.$$
$$\left.-\frac{2\left(30\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{acx-c}\right)-(3a^2x^2-11ax+38)\sqrt{-acx+c}\right)}{15a^2}\right]$$

input
```
integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[2/15*(15*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
- 3*c)/(a*x + 1)) + (3*a^2*x^2 - 11*a*x + 38)*sqrt(-a*c*x + c))/a^2, -2/15
*(30*sqrt(2)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x - c)
) - (3*a^2*x^2 - 11*a*x + 38)*sqrt(-a*c*x + c))/a^2]
```

Sympy [A] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \begin{cases} \frac{2\cdot\left(\frac{2\sqrt{2}c^3\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^2\sqrt{-acx+c}+\frac{c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a^2c^2} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\frac{x^2}{2}-\frac{2x}{a}+\frac{2\left(\begin{cases} x & \text{for } a=0 \\ \frac{\log(ax+1)}{a} & \text{otherwise} \end{cases}\right)}{a}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise((2*(2*sqrt(2)*c**3*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/s
qrt(-c) + 2*c**2*sqrt(-a*c*x + c) + c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)
**(5/2)/5)/(a**2*c**2), Ne(a*c, 0)), (sqrt(c)*(x**2/2 - 2*x/a + 2*Piecewis
e((x, Eq(a, 0)), (log(a*x + 1)/a, True))/a), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\left(15\sqrt{2}c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+3(-acx+c)^{\frac{5}{2}}+5(-acx+c)^{\frac{3}{2}}c+30\sqrt{-acx+c}c^2\right)}{15a^2c^2}$$

input
```
integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
2/15*(15*sqrt(2)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2
)*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(5/2) + 5*(-a*c*x + c)^(3/
2)*c + 30*sqrt(-a*c*x + c)*c^2)/(a^2*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}} + \frac{2\left(3\,(acx-c)^2\sqrt{-acx+c}a^8c^8+5\,(-acx+c)^{\frac{3}{2}}a^8c^9+30\sqrt{-acx+c}a^8c^{10}\right)}{15\,a^{10}c^{10}}$$

input
```
integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^2*sqrt(-c)) +
 2/15*(3*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^8*c^8 + 5*(-a*c*x + c)^(3/2)*a^8
*c^9 + 30*sqrt(-a*c*x + c)*a^8*c^10)/(a^10*c^10)
```

Mupad [B] (verification not implemented)

Time = 13.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{4\sqrt{c-a\,c\,x}}{a^2} + \frac{2\,(c-a\,c\,x)^{3/2}}{3\,a^2\,c} + \frac{2\,(c-a\,c\,x)^{5/2}}{5\,a^2\,c^2} + \frac{\sqrt{2}\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}}{a^2}$$

input
```
int((x*(c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
(4*(c - a*c*x)^(1/2))/a^2 + (2*(c - a*c*x)^(3/2))/(3*a^2*c) + (2*(c - a*c*
x)^(5/2))/(5*a^2*c^2) + (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1
i)/(2*c^(1/2)))*4i)/a^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$$
$$= \frac{2\sqrt{c}\left(3\sqrt{-ax+1}\,a^2x^2-11\sqrt{-ax+1}\,ax+38\sqrt{-ax+1}+15\sqrt{2}\log\left(\sqrt{-ax+1}-\sqrt{2}\right)-15\sqrt{2}\log\left(\sqrt{-\right.\right.}{15a^2}$$

input
```
int(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
(2*sqrt(c)*(3*sqrt( - a*x + 1)*a**2*x**2 - 11*sqrt( - a*x + 1)*a*x + 38*sq
rt( - a*x + 1) + 15*sqrt(2)*log(sqrt( - a*x + 1) - sqrt(2)) - 15*sqrt(2)*l
og(sqrt( - a*x + 1) + sqrt(2))))/(15*a**2)
```

3.366 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=-\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
-4*(-a*c*x+c)^(1/2)/a-2/3*(-a*c*x+c)^(3/2)/a/c+4*2^(1/2)*c^(1/2)*arctanh(1
/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2(-7+ax)\sqrt{c-acx}+12\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{3a}$$

input
```
Integrate[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

output
```
(2*(-7 + a*x)*Sqrt[c - a*c*x] + 12*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]
/(Sqrt[2]*Sqrt[c])])/(3*a)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6717, 6680, 35, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-acx}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(1-ax)\sqrt{c-acx}}{ax+1}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{(c-acx)^{3/2}}{ax+1}dx}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\int \frac{\sqrt{c-acx}}{ax+1}dx+\frac{2(c-acx)^{3/2}}{3a}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{2c\left(2c\int \frac{1}{(ax+1)\sqrt{c-acx}}dx+\frac{2\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{4\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2c\left(\frac{2\sqrt{c-acx}}{a}-\frac{2\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}\right)+\frac{2(c-acx)^{3/2}}{3a}}{c}$$

input
```
Int[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((2*(c - a*c*x)^(3/2))/(3*a) + 2*c*((2*Sqrt[c - a*c*x])/a - (2*Sqrt[2]*S
qrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a))/c)
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{2(ax-7)(ax-1)c}{3a\sqrt{-c(ax-1)}}+\frac{4\sqrt{2}\,\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a}$	57
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}+2\sqrt{-acx+c}\,c-2c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	59
default	$\frac{-\frac{2(-acx+c)^{\frac{3}{2}}}{3}-4\sqrt{-acx+c}\,c+4c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$	59
pseudoelliptic	$\frac{4\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\frac{2ax\sqrt{-c(ax-1)}}{3}-\frac{14\sqrt{-c(ax-1)}}{3}}{a}$	59

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(a*x-7)*(a*x-1)/a/(-c*(a*x-1))^(1/2)*c+4*2^(1/2)*c^(1/2)*arctanh(1/2*
(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.63

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$$
$$=\left[\frac{2\left(3\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{-acx+c}(ax-7)\right)}{3a},\ \frac{2\left(6\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{acx-c}\right)-\right.}{3a}\right.$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[2/3*(3*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
3*c)/(a*x + 1)) + sqrt(-a*c*x + c)*(a*x - 7))/a, 2/3*(6*sqrt(2)*sqrt(-c)*a
rctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x - c)) + sqrt(-a*c*x + c)*(a
*x - 7))/a]
```

Sympy [A] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \begin{cases} -\frac{2\cdot\left(\frac{2\sqrt{2}c^2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac} & \text{for } ac\neq 0 \\ \sqrt{c}\left(\begin{cases} -x & \text{for } a=0 \\ \frac{ax-2\log(ax+1)+1}{a} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise((-2*(2*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/
sqrt(-c) + 2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c), Ne(a*c, 0)
), (sqrt(c)*Piecewise((-x, Eq(a, 0)), ((a*x - 2*log(a*x + 1) + 1)/a, True)
), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{2\left(3\sqrt{2}c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+(-acx+c)^{\frac{3}{2}}+6\sqrt{-acx+c}c\right)}{3\,ac}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-2/3*(3*sqrt(2)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)
*sqrt(c) + sqrt(-a*c*x + c))) + (-a*c*x + c)^(3/2) + 6*sqrt(-a*c*x + c)*c)
/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2\left((-acx+c)^{\frac{3}{2}}a^2c^2+6\sqrt{-acx+c}a^2c^3\right)}{3a^3c^3}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) -
2/3*((-a*c*x + c)^(3/2)*a^2*c^2 + 6*sqrt(-a*c*x + c)*a^2*c^3)/(a^3*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{4\sqrt{2}\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)}{a} - \frac{2\,(c-a\,c\,x)^{3/2}}{3\,a\,c} - \frac{4\sqrt{c-a\,c\,x}}{a}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
(4*2^(1/2)*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/a - (2*
(c - a*c*x)^(3/2))/(3*a*c) - (4*(c - a*c*x)^(1/2))/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$$
$$=\frac{2\sqrt{c}\left(\sqrt{-ax+1}\,ax-7\sqrt{-ax+1}-3\sqrt{2}\log\left(\sqrt{-ax+1}-\sqrt{2}\right)+3\sqrt{2}\log\left(\sqrt{-ax+1}+\sqrt{2}\right)\right)}{3a}$$

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
(2*sqrt(c)*(sqrt( - a*x + 1)*a*x - 7*sqrt( - a*x + 1) - 3*sqrt(2)*log(sqrt
( - a*x + 1) - sqrt(2)) + 3*sqrt(2)*log(sqrt( - a*x + 1) + sqrt(2))))/(3*a
)
```

3.367 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c-acx}+2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

output
```
2*(-a*c*x+c)^(1/2)+2*c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))-4*2^(1/2)*c
^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c-acx}+2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] - 4*Sqrt[2]
*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6717, 6680, 35, 95, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-acx}}{x}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(1-ax)\sqrt{c-acx}}{x(ax+1)}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{(c-acx)^{3/2}}{x(ax+1)}dx}{c}$$

$$\Big\downarrow \text{95}$$

$$-\frac{\frac{\int \frac{ac^2(1-3ax)}{x(ax+1)\sqrt{c-acx}}dx}{a}-2c\sqrt{c-acx}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\int \frac{1-3ax}{x(ax+1)\sqrt{c-acx}}dx-2c\sqrt{c-acx}}{c}$$

$$\Big\downarrow \text{174}$$

$$-\frac{c^2\left(\int \frac{1}{x\sqrt{c-acx}}dx-4a\int \frac{1}{(ax+1)\sqrt{c-acx}}dx\right)-2c\sqrt{c-acx}}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c^2\left(\frac{8\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{c}-\frac{2\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-2c\sqrt{c-acx}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{c^2\left(\frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-2c\sqrt{c-acx}}{c}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c^2\left(\frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-2c\sqrt{c-acx}}{c}$$

input
```
Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
-((-2*c*Sqrt[c - a*c*x] + c^2*((-2*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/Sqrt[
c] + (4*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/Sqrt[c]))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$2\sqrt{-acx+c}+2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)-4\sqrt{2}\,\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)$	58
default	$2\sqrt{-acx+c}+2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)-4\sqrt{2}\,\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)$	58
pseudoelliptic	$2\sqrt{-c\,(ax-1)}+2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)-4\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)$	61

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(-a*c*x+c)^(1/2)+2*c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))-4*2^(1/2)*c
^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.30

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \left[2\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{c}\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\sqrt{-acx+c},\ -4\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{acx-c}\right)+2\sqrt{-c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{acx-c}\right)+2\sqrt{-acx+c}\right]$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

output
```
[2*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/
(a*x + 1)) + sqrt(c)*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2
*sqrt(-a*c*x + c), -4*sqrt(2)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqr
t(-c)/(a*c*x - c)) + 2*sqrt(-c)*arctan(sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x -
c)) + 2*sqrt(-a*c*x + c)]
```

Sympy [A] (verification not implemented)

Time = 4.59 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.65

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \begin{cases} -\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{4\sqrt{2}c\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} + 2\sqrt{-acx+c} & \text{for } ac \neq 0 \\ \sqrt{c}\left(-\frac{3a\left(\frac{\log\left(-\frac{2}{x}\right)}{a} - \frac{\log\left(2a+\frac{2}{x}\right)}{a}\right)}{2} + \frac{\log\left(\frac{a}{x}+\frac{1}{x^2}\right)}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

output
```
Piecewise((-2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 4*sqrt(2)*c*ata
n(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2*sqrt(-a*c*x + c), Ne
(a*c, 0)), (sqrt(c)*(-3*a*(log(-2/x)/a - log(2*a + 2/x)/a)/2 + log(a/x + x
**(-2))/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{2}\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right) - \sqrt{c}\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right) + 2\sqrt{-acx+c}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

output
```
2*sqrt(2)*sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(
c) + sqrt(-a*c*x + c))) - sqrt(c)*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-
a*c*x + c) + sqrt(c))) + 2*sqrt(-a*c*x + c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{2\,c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2\sqrt{-acx+c}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

output
```
4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 2*c*a
rctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*sqrt(-a*c*x + c)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = 2\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right) + 2\sqrt{c-a\,c\,x} - 4\sqrt{2}\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)$$

input
```
int(((c - a*c*x)^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
```

output
```
2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) + 2*(c - a*c*x)^(1/2) - 4*2^(1/
2)*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \sqrt{c}\left(2\sqrt{-ax+1}+2\sqrt{2}\log\left(\sqrt{-ax+1}-\sqrt{2}\right)-2\sqrt{2}\log\left(\sqrt{-ax+1}+\sqrt{2}\right)-\log\left(\sqrt{-ax+1}-1\right)+\log\left(\sqrt{-ax+1}+1\right)\right)$$

input

```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x)
```

output

```
sqrt(c)*(2*sqrt( - a*x + 1) + 2*sqrt(2)*log(sqrt( - a*x + 1) - sqrt(2)) -
2*sqrt(2)*log(sqrt( - a*x + 1) + sqrt(2)) - log(sqrt( - a*x + 1) - 1) + lo
g(sqrt( - a*x + 1) + 1))
```

3.368 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}}{x} - 5a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

output
```
(-a*c*x+c)^(1/2)/x-5*a*c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))+4*2^(1/2)
*a*c^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}}{x} - 5a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
Sqrt[c - a*c*x]/x - 5*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] + 4*Sqrt[
2]*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6717, 6680, 35, 109, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-acx}}{x^2}dx$$

$$\Big\downarrow \text{ 6680}$$

$$-\int \frac{(1-ax)\sqrt{c-acx}}{x^2(ax+1)}dx$$

$$\Big\downarrow \text{ 35}$$

$$-\frac{\int \frac{(c-acx)^{3/2}}{x^2(ax+1)}dx}{c}$$

$$\Big\downarrow \text{ 109}$$

$$-\frac{-\int \frac{ac^2(5-3ax)}{2x(ax+1)\sqrt{c-acx}}dx-\frac{c\sqrt{c-acx}}{x}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{-\frac{1}{2}ac^2\int \frac{5-3ax}{x(ax+1)\sqrt{c-acx}}dx-\frac{c\sqrt{c-acx}}{x}}{c}$$

$$\Big\downarrow \text{ 174}$$

$$-\frac{-\frac{1}{2}ac^2\left(5\int \frac{1}{x\sqrt{c-acx}}dx-8a\int \frac{1}{(ax+1)\sqrt{c-acx}}dx\right)-\frac{c\sqrt{c-acx}}{x}}{c}$$

$$\Big\downarrow \ 73$$

$$-\frac{-\frac{1}{2}ac^2\left(\frac{16\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{c}-\frac{10\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-\frac{c\sqrt{c-acx}}{x}}{c}$$

$$\Big\downarrow \ 219$$

$$-\frac{-\frac{1}{2}ac^2\left(\frac{8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{10\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-\frac{c\sqrt{c-acx}}{x}}{c}$$

$$\Big\downarrow \ 221$$

$$-\frac{-\frac{1}{2}ac^2\left(\frac{8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{10\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{c\sqrt{c-acx}}{x}}{c}$$

input
```
Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
-((-((c*Sqrt[c - a*c*x])/x) - (a*c^2*((-10*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]
])/Sqrt[c] + (8*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/Sqrt[c
]))/2)/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

method	result	size
pseudoelliptic	$\frac{4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx-5\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)acx+\sqrt{-c(ax-1)}\,\sqrt{c}}{\sqrt{c}\,x}$	70
derivativedivides	$-2ca\left(-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{-acx+c}}{2acx}+\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$	71
default	$2ca\left(\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{-acx+c}}{2acx}-\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$	71
risch	$-\frac{(ax-1)c}{x\sqrt{-c(ax-1)}}+\frac{a\left(\frac{8\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{10\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)c}{2}$	73

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(4*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-5*arctanh
((-c*(a*x-1))^(1/2)/c^(1/2))*a*c*x+(-c*(a*x-1))^(1/2)*c^(1/2))/c^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.38

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$$
$$=\left[\frac{4\sqrt{2}a\sqrt{c}x\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+5\,a\sqrt{c}x\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\sqrt{-acx+c}}{2x},\ \frac{4\sqrt{2}a\sqrt{-cx}}{}\right.$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(4*sqrt(2)*a*sqrt(c)*x*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c
) - 3*c)/(a*x + 1)) + 5*a*sqrt(c)*x*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c
) - 2*c)/x) + 2*sqrt(-a*c*x + c))/x, (4*sqrt(2)*a*sqrt(-c)*x*arctan(sqrt(2
)*sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x - c)) - 5*a*sqrt(-c)*x*arctan(sqrt(-a*c
*x + c)*sqrt(-c)/(a*c*x - c)) + sqrt(-a*c*x + c))/x]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax-1)}{x^2\,(ax+1)}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*(a*x - 1)/(x**2*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.42

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{1}{2}\,ac\left(\frac{4\,\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{\sqrt{c}} - \frac{5\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{\sqrt{c}} - \frac{2\,\sqrt{-acx+c}}{acx}\right)$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

output
```
-1/2*a*c*(4*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqr
t(c) + sqrt(-a*c*x + c)))/sqrt(c) - 5*log((sqrt(-a*c*x + c) - sqrt(c))/(sq
rt(-a*c*x + c) + sqrt(c)))/sqrt(c) - 2*sqrt(-a*c*x + c)/(a*c*x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{4\sqrt{2}ac\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{5\,ac\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{\sqrt{-acx+c}}{x}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

output
```
-4*sqrt(2)*a*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 5*
a*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + sqrt(-a*c*x + c)/x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-a\,c\,x}}{x} - 5\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right) + 4\,\sqrt{2}\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)$$

input
```
int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
```

output
```
(c - a*c*x)^(1/2)/x - 5*a*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) + 4*2^(
1/2)*a*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$$
$$= \frac{\sqrt{c}\left(2\sqrt{-ax+1}-4\sqrt{2}\log\left(\sqrt{-ax+1}-\sqrt{2}\right)ax+4\sqrt{2}\log\left(\sqrt{-ax+1}+\sqrt{2}\right)ax+5\log\left(\sqrt{-ax+1}-\right.\right.}{2x}$$

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x)
```

output
```
(sqrt(c)*(2*sqrt( - a*x + 1) - 4*sqrt(2)*log(sqrt( - a*x + 1) - sqrt(2))*a
*x + 4*sqrt(2)*log(sqrt( - a*x + 1) + sqrt(2))*a*x + 5*log(sqrt( - a*x + 1
) - 1)*a*x - 5*log(sqrt( - a*x + 1) + 1)*a*x))/(2*x)
```

3.369 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{\sqrt{c-acx}}{2x^2}-\frac{9a\sqrt{c-acx}}{4x}+\frac{23}{4}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

output
```
1/2*(-a*c*x+c)^(1/2)/x^2-9/4*a*(-a*c*x+c)^(1/2)/x+23/4*a^2*c^(1/2)*arctanh
((-a*c*x+c)^(1/2)/c^(1/2))-4*2^(1/2)*a^2*c^(1/2)*arctanh(1/2*(-a*c*x+c)^(1
/2)*2^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{(2-9ax)\sqrt{c-acx}}{4x^2}+\frac{23}{4}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
((2 - 9*a*x)*Sqrt[c - a*c*x])/(4*x^2) + (23*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a
*c*x]/Sqrt[c]])/4 - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]
*Sqrt[c])]
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {6717, 6680, 35, 109, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-acx}}{x^3}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(1-ax)\sqrt{c-acx}}{x^3(ax+1)}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{(c-acx)^{3/2}}{x^3(ax+1)}dx}{c}$$

$$\Big\downarrow \text{109}$$

$$-\frac{-\frac{1}{2}\int \frac{ac^2(9-7ax)}{2x^2(ax+1)\sqrt{c-acx}}dx-\frac{c\sqrt{c-acx}}{2x^2}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{1}{4}ac^2\int \frac{9-7ax}{x^2(ax+1)\sqrt{c-acx}}dx-\frac{c\sqrt{c-acx}}{2x^2}}{c}$$

$$\Big\downarrow \text{168}$$

$$-\frac{-\frac{1}{4}ac^2\left(-\frac{\int \frac{ac(23-9ax)}{2x(ax+1)\sqrt{c-acx}}dx}{c}-\frac{9\sqrt{c-acx}}{cx}\right)-\frac{c\sqrt{c-acx}}{2x^2}}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{1}{4}ac^2\left(-\frac{1}{2}a\int \frac{23-9ax}{x(ax+1)\sqrt{c-acx}}dx-\frac{9\sqrt{c-acx}}{cx}\right)-\frac{c\sqrt{c-acx}}{2x^2}}{c}$$

$$\Big\downarrow \; 174$$

$$-\frac{-\frac{1}{4}ac^2\left(-\frac{1}{2}a\left(23\int \frac{1}{x\sqrt{c-acx}}dx-32a\int \frac{1}{(ax+1)\sqrt{c-acx}}dx\right)-\frac{9\sqrt{c-acx}}{cx}\right)-\frac{c\sqrt{c-acx}}{2x^2}}{c}$$

$$\Big\downarrow \; 73$$

$$-\frac{-\frac{1}{4}ac^2\left(-\frac{1}{2}a\left(\frac{64\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{c}-\frac{46\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-\frac{9\sqrt{c-acx}}{cx}\right)-\frac{c\sqrt{c-acx}}{2x^2}}{c}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{1}{4}ac^2\left(-\frac{1}{2}a\left(\frac{32\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{46\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-\frac{9\sqrt{c-acx}}{cx}\right)-\frac{c\sqrt{c-acx}}{2x^2}}{c}$$

$$\Big\downarrow \; 221$$

$$-\frac{-\frac{1}{4}ac^2\left(-\frac{1}{2}a\left(\frac{32\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{46\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{9\sqrt{c-acx}}{cx}\right)-\frac{c\sqrt{c-acx}}{2x^2}}{c}$$

input
```
Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
-((-1/2*(c*Sqrt[c - a*c*x])/x^2 - (a*c^2*((-9*Sqrt[c - a*c*x])/(c*x) - (a*
((-46*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/Sqrt[c] + (32*Sqrt[2]*ArcTanh[Sqrt
[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/Sqrt[c]))/2))/4)/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$-\frac{\sqrt{-c(ax-1)}\,(9ax-2)\sqrt{c}+a^2 c\,x^2\left(16\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)-23\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)\right)}{4\sqrt{c}\,x^2}$	80
risch	$\frac{(9a^2x^2-11ax+2)c}{4x^2\sqrt{-c(ax-1)}}-\frac{a^2\left(\frac{32\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{46\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)c}{8}$	84
derivativedivides	$2c^2a^2\left(-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{\frac{\frac{9(-acx+c)^{\frac{3}{2}}}{8}-\frac{7\sqrt{-acx+c}\,c}{8}}{a^2c^2x^2}+\frac{23\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8\sqrt{c}}}{c}\right)$	94
default	$2c^2a^2\left(-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{\frac{\frac{9(-acx+c)^{\frac{3}{2}}}{8}-\frac{7\sqrt{-acx+c}\,c}{8}}{a^2c^2x^2}+\frac{23\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8\sqrt{c}}}{c}\right)$	94

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/c^(1/2)*((-c*(a*x-1))^(1/2)*(9*a*x-2)*c^(1/2)+a^2*c*x^2*(16*2^(1/2)*a
rctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))-23*arctanh((-c*(a*x-1))^(1/
2)/c^(1/2))))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.04

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$$

$$= \left[\frac{16\sqrt{2}a^2\sqrt{c}x^2\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+23\,a^2\sqrt{c}x^2\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)-2\sqrt{-acx+c}(9\,ax-}{8\,x^2}\right.$$

$$-\frac{16\sqrt{2}a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{acx-c}\right)-23\,a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{acx-c}\right)+\sqrt{-acx+c}(9\,ax-2)}{4\,x^2}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(16*sqrt(2)*a^2*sqrt(c)*x^2*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*s
qrt(c) - 3*c)/(a*x + 1)) + 23*a^2*sqrt(c)*x^2*log((a*c*x - 2*sqrt(-a*c*x +
 c)*sqrt(c) - 2*c)/x) - 2*sqrt(-a*c*x + c)*(9*a*x - 2))/x^2, -1/4*(16*sqrt
(2)*a^2*sqrt(-c)*x^2*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x - c))
 - 23*a^2*sqrt(-c)*x^2*arctan(sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x - c)) + sqr
t(-a*c*x + c)*(9*a*x - 2))/x^2]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx=\int \frac{\sqrt{-c\,(ax-1)}(ax-1)}{x^3\,(ax+1)}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*(a*x - 1)/(x**3*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.43

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx =\frac{1}{8}\,a^2c^2\left(\frac{2\left(9\,(-acx+c)^{\frac{3}{2}}-7\,\sqrt{-acx+c}c\right)}{(acx-c)^2c+2\,(acx-c)c^2+c^3}+\frac{16\,\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{3}{2}}}-\frac{23\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

output
```
1/8*a^2*c^2*(2*(9*(-a*c*x + c)^(3/2) - 7*sqrt(-a*c*x + c)*c)/((a*c*x - c)^
2*c + 2*(a*c*x - c)*c^2 + c^3) + 16*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-
a*c*x + c))/(sqrt(2)*sqrt(c) + sqrt(-a*c*x + c)))/c^(3/2) - 23*log((sqrt(-
a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/c^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{4\sqrt{2}a^2c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{23\,a^2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}} + \frac{9\,(-acx+c)^{\frac{3}{2}}a^2c - 7\sqrt{-acx+c}a^2c^2}{4\,a^2c^2x^2}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

output
```
4*sqrt(2)*a^2*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 2
3/4*a^2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/4*(9*(-a*c*x + c)
^(3/2)*a^2*c - 7*sqrt(-a*c*x + c)*a^2*c^2)/(a^2*c^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{9\,(c-a\,c\,x)^{3/2}}{4\,c\,x^2} - \frac{a^2\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,23\mathrm{i}}{4} - \frac{7\sqrt{c-a\,c\,x}}{4\,x^2} + \sqrt{2}\,a^2\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

output
```
(9*(c - a*c*x)^(3/2))/(4*c*x^2) - (a^2*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)
/c^(1/2))*23i)/4 - (7*(c - a*c*x)^(1/2))/(4*x^2) + 2^(1/2)*a^2*c^(1/2)*ata
n((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*4i
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$$
$$=\frac{\sqrt{c}\left(-18\sqrt{-ax+1}\,ax+4\sqrt{-ax+1}+16\sqrt{2}\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^2x^2-16\sqrt{2}\log\left(\sqrt{-ax+1}+\sqrt{2}\right)\right.}{8x^2}$$

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x)
```

output
```
(sqrt(c)*( - 18*sqrt( - a*x + 1)*a*x + 4*sqrt( - a*x + 1) + 16*sqrt(2)*log
(sqrt( - a*x + 1) - sqrt(2))*a**2*x**2 - 16*sqrt(2)*log(sqrt( - a*x + 1) +
 sqrt(2))*a**2*x**2 - 23*log(sqrt( - a*x + 1) - 1)*a**2*x**2 + 23*log(sqrt
( - a*x + 1) + 1)*a**2*x**2))/(8*x**2)
```

3.370 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}}{3x^3}-\frac{13a\sqrt{c-acx}}{12x^2}+\frac{19a^2\sqrt{c-acx}}{8x} - \frac{45}{8}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

output
```
1/3*(-a*c*x+c)^(1/2)/x^3-13/12*a*(-a*c*x+c)^(1/2)/x^2+19/8*a^2*(-a*c*x+c)^
(1/2)/x-45/8*a^3*c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))+4*2^(1/2)*a^3*c
^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}(8-26ax+57a^2x^2)}{24x^3} - \frac{45}{8}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
(Sqrt[c - a*c*x]*(8 - 26*a*x + 57*a^2*x^2))/(24*x^3) - (45*a^3*Sqrt[c]*Arc
Tanh[Sqrt[c - a*c*x]/Sqrt[c]])/8 + 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c -
a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {6717, 6680, 35, 109, 27, 168, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-acx}}{x^4}dx$$

$$\Big\downarrow \text{6680}$$

$$-\int \frac{(1-ax)\sqrt{c-acx}}{x^4(ax+1)}dx$$

$$\Big\downarrow \text{35}$$

$$-\frac{\int \frac{(c-acx)^{3/2}}{x^4(ax+1)}dx}{c}$$

$$\Big\downarrow \text{109}$$

$$-\frac{-\frac{1}{3}\int \frac{ac^2(13-11ax)}{2x^3(ax+1)\sqrt{c-acx}}dx-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{1}{6}ac^2\int \frac{13-11ax}{x^3(ax+1)\sqrt{c-acx}}dx-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \text{168}$$

$$-\frac{-\frac{1}{6}ac^2\left(-\frac{\int \frac{3ac(19-13ax)}{2x^2(ax+1)\sqrt{c-acx}}dx}{2c}-\frac{13\sqrt{c-acx}}{2cx^2}\right)-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{1}{6}ac^2\left(-\frac{3}{4}a\int \frac{19-13ax}{x^2(ax+1)\sqrt{c-acx}}dx-\frac{13\sqrt{c-acx}}{2cx^2}\right)-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \text{168}$$

$$-\frac{-\frac{1}{6}ac^2\left(-\frac{3}{4}a\left(-\frac{\int \frac{ac(45-19ax)}{2x(ax+1)\sqrt{c-acx}}dx}{c}-\frac{19\sqrt{c-acx}}{cx}\right)-\frac{13\sqrt{c-acx}}{2cx^2}\right)-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{1}{6}ac^2\left(-\frac{3}{4}a\left(-\frac{1}{2}a\int \frac{45-19ax}{x(ax+1)\sqrt{c-acx}}dx-\frac{19\sqrt{c-acx}}{cx}\right)-\frac{13\sqrt{c-acx}}{2cx^2}\right)-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \text{174}$$

$$-\frac{-\frac{1}{6}ac^2\left(-\frac{3}{4}a\left(-\frac{1}{2}a\left(45\int \frac{1}{x\sqrt{c-acx}}dx-64a\int \frac{1}{(ax+1)\sqrt{c-acx}}dx\right)-\frac{19\sqrt{c-acx}}{cx}\right)-\frac{13\sqrt{c-acx}}{2cx^2}\right)-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{-\frac{1}{6}ac^2\left(-\frac{3}{4}a\left(-\frac{1}{2}a\left(\frac{128\int \frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{c}-\frac{90\int \frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-\frac{19\sqrt{c-acx}}{cx}\right)-\frac{13\sqrt{c-acx}}{2cx^2}\right)-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{1}{6}ac^2\left(-\frac{3}{4}a\left(-\frac{1}{2}a\left(\frac{64\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{90\int\frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-\frac{19\sqrt{c-acx}}{cx}\right)-\frac{13\sqrt{c-acx}}{2cx^2}\right)-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

$$\Big\downarrow \text{221}$$

$$-\frac{-\frac{1}{6}ac^2\left(-\frac{3}{4}a\left(-\frac{1}{2}a\left(\frac{64\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{90\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{19\sqrt{c-acx}}{cx}\right)-\frac{13\sqrt{c-acx}}{2cx^2}\right)-\frac{c\sqrt{c-acx}}{3x^3}}{c}$$

input
```
Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
-((-1/3*(c*Sqrt[c - a*c*x])/x^3 - (a*c^2*((-13*Sqrt[c - a*c*x])/(2*c*x^2)
- (3*a*((-19*Sqrt[c - a*c*x])/(c*x) - (a*((-90*ArcTanh[Sqrt[c - a*c*x]/Sqr
t[c]])/Sqrt[c] + (64*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/S
qrt[c]))/2))/4))/6)/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{\frac{\sqrt{-c(ax-1)}\left(57a^2x^2-26ax+8\right)\sqrt{c}}{3}+a^3cx^3\left(32\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)-45\,\operatorname{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right)\right)}{8\sqrt{c}\,x^3}$
risch	$-\frac{(57a^3x^3-83a^2x^2+34ax-8)c}{24x^3\sqrt{-c(ax-1)}}+\frac{a^3\left(\frac{64\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{90\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)c}{16}$
derivativedivides	$-2c^3a^3\left(-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{\frac{-\frac{19(-acx+c)^{\frac{5}{2}}}{16}+\frac{11(-acx+c)^{\frac{3}{2}}c}{6}-\frac{13\sqrt{-acx+c}\,c^2}{16}}{a^3c^3x^3}+\frac{45\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+}}{\sqrt{c}}\right.}{16\sqrt{c}}}{c^2}\right.$
default	$2c^3a^3\left(\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}-\frac{\frac{-\frac{19(-acx+c)^{\frac{5}{2}}}{16}+\frac{11(-acx+c)^{\frac{3}{2}}c}{6}-\frac{13\sqrt{-acx+c}\,c^2}{16}}{a^3c^3x^3}+\frac{45\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{16\sqrt{c}}}{c^2}\right)$

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/c^(1/2)*(1/3*(-c*(a*x-1))^(1/2)*(57*a^2*x^2-26*a*x+8)*c^(1/2)+a^3*c*x^
3*(32*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(1/2))-45*arctanh((
-c*(a*x-1))^(1/2)/c^(1/2))))/x^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.83

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$

$$=\left[\frac{96\sqrt{2}a^3\sqrt{c}x^3\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+135\,a^3\sqrt{c}x^3\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\left(57\,a^2x^2-26\,ax\right.}{48\,x^3}\right.$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(96*sqrt(2)*a^3*sqrt(c)*x^3*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*
sqrt(c) - 3*c)/(a*x + 1)) + 135*a^3*sqrt(c)*x^3*log((a*c*x + 2*sqrt(-a*c*x
 + c)*sqrt(c) - 2*c)/x) + 2*(57*a^2*x^2 - 26*a*x + 8)*sqrt(-a*c*x + c))/x^
3, 1/24*(96*sqrt(2)*a^3*sqrt(-c)*x^3*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(
-c)/(a*c*x - c)) - 135*a^3*sqrt(-c)*x^3*arctan(sqrt(-a*c*x + c)*sqrt(-c)/(
a*c*x - c)) + (57*a^2*x^2 - 26*a*x + 8)*sqrt(-a*c*x + c))/x^3]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax-1)}{x^4\,(ax+1)}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*(a*x - 1)/(x**4*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.44

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$
$$= \frac{1}{48}\,a^3c^3\left(\frac{2\left(57\,(-acx+c)^{\frac{5}{2}}-88\,(-acx+c)^{\frac{3}{2}}c+39\,\sqrt{-acx+c}c^2\right)}{(acx-c)^3c^2+3\,(acx-c)^2c^3+3\,(acx-c)c^4+c^5}-\frac{96\,\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{5}{2}}}+\cdots\right.$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

output
```
1/48*a^3*c^3*(2*(57*(-a*c*x + c)^(5/2) - 88*(-a*c*x + c)^(3/2)*c + 39*sqrt
(-a*c*x + c)*c^2)/((a*c*x - c)^3*c^2 + 3*(a*c*x - c)^2*c^3 + 3*(a*c*x - c)
*c^4 + c^5) - 96*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2
)*sqrt(c) + sqrt(-a*c*x + c)))/c^(5/2) + 135*log((sqrt(-a*c*x + c) - sqrt(
c))/(sqrt(-a*c*x + c) + sqrt(c)))/c^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = -\frac{4\sqrt{2}a^3c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{45\,a^3c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{8\sqrt{-c}} + \frac{57\,(acx-c)^2\sqrt{-acx+c}a^3c - 88\,(-acx+c)^{\frac{3}{2}}a^3c^2 + 39\sqrt{-acx+c}a^3c^3}{24\,a^3c^3x^3}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

output
```
-4*sqrt(2)*a^3*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) +
45/8*a^3*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/24*(57*(a*c*x -
c)^2*sqrt(-a*c*x + c)*a^3*c - 88*(-a*c*x + c)^(3/2)*a^3*c^2 + 39*sqrt(-a*c
*x + c)*a^3*c^3)/(a^3*c^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{13\sqrt{c-a\,c\,x}}{8\,x^3} + \frac{a^3\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,45\mathrm{i}}{8} - \frac{11\,(c-a\,c\,x)^{3/2}}{3\,c\,x^3} + \frac{19\,(c-a\,c\,x)^{5/2}}{8\,c^2\,x^3} - \sqrt{2}\,a^3\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\sqrt{c}}\right)\,4\mathrm{i}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
```

output
```
(13*(c - a*c*x)^(1/2))/(8*x^3) + (a^3*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)/
c^(1/2))*45i)/8 - (11*(c - a*c*x)^(3/2))/(3*c*x^3) + (19*(c - a*c*x)^(5/2)
)/(8*c^2*x^3) - 2^(1/2)*a^3*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2
*c^(1/2)))*4i
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$
$$= \frac{\sqrt{c}\left(114\sqrt{-ax+1}\,a^2x^2-52\sqrt{-ax+1}\,ax+16\sqrt{-ax+1}-96\sqrt{2}\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^3x^3+96\sqrt{2}\,\mathrm{l}\right.}{48x^3}$$

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x)
```

output
```
(sqrt(c)*(114*sqrt( - a*x + 1)*a**2*x**2 - 52*sqrt( - a*x + 1)*a*x + 16*sq
rt( - a*x + 1) - 96*sqrt(2)*log(sqrt( - a*x + 1) - sqrt(2))*a**3*x**3 + 96
*sqrt(2)*log(sqrt( - a*x + 1) + sqrt(2))*a**3*x**3 + 135*log(sqrt( - a*x +
 1) - 1)*a**3*x**3 - 135*log(sqrt( - a*x + 1) + 1)*a**3*x**3))/(48*x**3)
```

3.371 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} - \frac{149a^3\sqrt{c-acx}}{64x} + \frac{363}{64}a^4\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^4\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

output

```
1/4*(-a*c*x+c)^(1/2)/x^4-17/24*a*(-a*c*x+c)^(1/2)/x^3+107/96*a^2*(-a*c*x+c
)^(1/2)/x^2-149/64*a^3*(-a*c*x+c)^(1/2)/x+363/64*a^4*c^(1/2)*arctanh((-a*c
*x+c)^(1/2)/c^(1/2))-4*2^(1/2)*a^4*c^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^
(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.74

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}(48-136ax+214a^2x^2-447a^3x^3)}{192x^4} + \frac{363}{64}a^4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^5),x]
```

output
```
(Sqrt[c - a*c*x]*(48 - 136*a*x + 214*a^2*x^2 - 447*a^3*x^3))/(192*x^4) + (
363*a^4*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64 - 4*Sqrt[2]*a^4*Sqrt[
c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.17, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {6717, 6680, 35, 109, 27, 168, 27, 168, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-2\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-acx}}{x^5}dx$$

$$\Big\downarrow \text{ 6680}$$

$$-\int \frac{(1-ax)\sqrt{c-acx}}{x^5(ax+1)}dx$$

$$\Big\downarrow \text{ 35}$$

$$-\frac{\int \frac{(c-acx)^{3/2}}{x^5(ax+1)}dx}{c}$$

$\Big\downarrow$ 109

$$-\frac{-\frac{1}{4}\int \frac{ac^2(17-15ax)}{2x^4(ax+1)\sqrt{c-acx}}dx-\frac{c\sqrt{c-acx}}{4x^4}}{c}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{1}{8}ac^2\int \frac{17-15ax}{x^4(ax+1)\sqrt{c-acx}}dx-\frac{c\sqrt{c-acx}}{4x^4}}{c}$$

$\Big\downarrow$ 168

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{\int \frac{ac(107-85ax)}{2x^3(ax+1)\sqrt{c-acx}}dx}{3c}-\frac{17\sqrt{c-acx}}{3cx^3}\right)-\frac{c\sqrt{c-acx}}{4x^4}}{c}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\int \frac{107-85ax}{x^3(ax+1)\sqrt{c-acx}}dx-\frac{17\sqrt{c-acx}}{3cx^3}\right)-\frac{c\sqrt{c-acx}}{4x^4}}{c}$$

$\Big\downarrow$ 168

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\left(-\frac{\int \frac{3ac(149-107ax)}{2x^2(ax+1)\sqrt{c-acx}}dx}{2c}-\frac{107\sqrt{c-acx}}{2cx^2}\right)-\frac{17\sqrt{c-acx}}{3cx^3}\right)-\frac{c\sqrt{c-acx}}{4x^4}}{c}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\left(-\frac{3}{4}a\int \frac{149-107ax}{x^2(ax+1)\sqrt{c-acx}}dx-\frac{107\sqrt{c-acx}}{2cx^2}\right)-\frac{17\sqrt{c-acx}}{3cx^3}\right)-\frac{c\sqrt{c-acx}}{4x^4}}{c}$$

$\Big\downarrow$ 168

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\left(-\frac{3}{4}a\left(-\frac{\int \frac{ac(363-149ax)}{2x(ax+1)\sqrt{c-acx}}dx}{c}-\frac{149\sqrt{c-acx}}{cx}\right)-\frac{107\sqrt{c-acx}}{2cx^2}\right)-\frac{17\sqrt{c-acx}}{3cx^3}\right)-\frac{c\sqrt{c-acx}}{4x^4}}{c}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\left(-\frac{3}{4}a\left(-\frac{1}{2}a\int \frac{363-149ax}{x(ax+1)\sqrt{c-acx}}dx-\frac{149\sqrt{c-acx}}{cx}\right)-\frac{107\sqrt{c-acx}}{2cx^2}\right)-\frac{17\sqrt{c-acx}}{3cx^3}\right)-\frac{c\sqrt{c-acx}}{4x^4}}{c}$$

$\Big\downarrow$ 174

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\left(-\frac{3}{4}a\left(-\frac{1}{2}a\left(363\int\frac{1}{x\sqrt{c-acx}}dx-512a\int\frac{1}{(ax+1)\sqrt{c-acx}}dx\right)-\frac{149\sqrt{c-acx}}{cx}\right)-\frac{107\sqrt{c-acx}}{2cx^2}\right)-\frac{17\sqrt{c-acx}}{3cx^3}\right)}{c}$$

$\big\downarrow$ 73

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\left(-\frac{3}{4}a\left(-\frac{1}{2}a\left(\frac{1024\int\frac{1}{2-\frac{c-acx}{c}}d\sqrt{c-acx}}{c}-\frac{726\int\frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-\frac{149\sqrt{c-acx}}{cx}\right)-\frac{107\sqrt{c-acx}}{2cx^2}\right)-\frac{17\sqrt{c}}{3c}\right.}{c}$$

$\big\downarrow$ 219

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\left(-\frac{3}{4}a\left(-\frac{1}{2}a\left(\frac{512\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{726\int\frac{1}{\frac{1}{a}-\frac{c-acx}{ac}}d\sqrt{c-acx}}{ac}\right)-\frac{149\sqrt{c-acx}}{cx}\right)-\frac{107\sqrt{c-acx}}{2cx^2}\right)-\frac{17\sqrt{}}{3}\right.}{c}$$

$\big\downarrow$ 221

$$-\frac{-\frac{1}{8}ac^2\left(-\frac{1}{6}a\left(-\frac{3}{4}a\left(-\frac{1}{2}a\left(\frac{512\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{726\text{arctanh}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{149\sqrt{c-acx}}{cx}\right)-\frac{107\sqrt{c-acx}}{2cx^2}\right)-\frac{17\sqrt{c}}{3c}\right.}{c}$$

input
```
Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^5),x]
```

output
```
-((-1/4*(c*Sqrt[c - a*c*x])/x^4 - (a*c^2*((-17*Sqrt[c - a*c*x])/(3*c*x^3)
- (a*((-107*Sqrt[c - a*c*x])/(2*c*x^2) - (3*a*((-149*Sqrt[c - a*c*x])/(c*x
) - (a*((-726*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/Sqrt[c] + (512*Sqrt[2]*Arc
Tanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/Sqrt[c]))/2))/4))/6))/8)/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6680
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{\frac{\sqrt{-c(ax-1)}\left(447a^3x^3-214a^2x^2+136ax-48\right)\sqrt{c}}{3}+a^4cx^4\left(256\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-c(ax-1)}\,\sqrt{2}}{2\sqrt{c}}\right)-363\,\text{arctanh}\left(\frac{\sqrt{-c(ax-1)}}{\sqrt{c}}\right.\right.}{64\sqrt{c}\,x^4}$
risch	$\frac{(447a^4x^4-661a^3x^3+350a^2x^2-184ax+48)c}{192x^4\sqrt{-c(ax-1)}}-\frac{a^4\left(\frac{512\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{726\,\text{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)c}{128}$
derivativedivides	$2c^4a^4\left(-\frac{2\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{7}{2}}}+\frac{\frac{\frac{149(-acx+c)^{\frac{7}{2}}}{128}-\frac{1127(-acx+c)^{\frac{5}{2}}c}{384}+\frac{1049c^2(-acx+c)^{\frac{3}{2}}}{384}-\frac{107\sqrt{-acx+c}\,c^3}{128}}{a^4c^4x^4}+}{c^3}\right.$
default	$2c^4a^4\left(-\frac{2\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{7}{2}}}+\frac{\frac{\frac{149(-acx+c)^{\frac{7}{2}}}{128}-\frac{1127(-acx+c)^{\frac{5}{2}}c}{384}+\frac{1049c^2(-acx+c)^{\frac{3}{2}}}{384}-\frac{107\sqrt{-acx+c}\,c^3}{128}}{a^4c^4x^4}+}{c^3}\right.$

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/64/c^(1/2)*(1/3*(-c*(a*x-1))^(1/2)*(447*a^3*x^3-214*a^2*x^2+136*a*x-48)
*c^(1/2)+a^4*c*x^4*(256*2^(1/2)*arctanh(1/2*(-c*(a*x-1))^(1/2)*2^(1/2)/c^(
1/2))-363*arctanh((-c*(a*x-1))^(1/2)/c^(1/2))))/x^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.68

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$
$$=\left[\frac{768\sqrt{2}a^4\sqrt{c}x^4\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+1089\,a^4\sqrt{c}x^4\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)-2\left(447\,a^3x^3-21\right.}{384\,x^4}\right.$$
$$-\frac{768\sqrt{2}a^4\sqrt{-c}x^4\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{acx-c}\right)-1089\,a^4\sqrt{-c}x^4\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{acx-c}\right)+\left(447\,a^3x^3-214\,a^2\right.}{192\,x^4}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

output
```
[1/384*(768*sqrt(2)*a^4*sqrt(c)*x^4*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c
)*sqrt(c) - 3*c)/(a*x + 1)) + 1089*a^4*sqrt(c)*x^4*log((a*c*x - 2*sqrt(-a*
c*x + c)*sqrt(c) - 2*c)/x) - 2*(447*a^3*x^3 - 214*a^2*x^2 + 136*a*x - 48)*
sqrt(-a*c*x + c))/x^4, -1/192*(768*sqrt(2)*a^4*sqrt(-c)*x^4*arctan(sqrt(2)
*sqrt(-a*c*x + c)*sqrt(-c)/(a*c*x - c)) - 1089*a^4*sqrt(-c)*x^4*arctan(sqr
t(-a*c*x + c)*sqrt(-c)/(a*c*x - c)) + (447*a^3*x^3 - 214*a^2*x^2 + 136*a*x
 - 48)*sqrt(-a*c*x + c))/x^4]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{-c\,(ax-1)}(ax-1)}{x^5\,(ax+1)}\,dx$$

input
```
integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

output
```
Integral(sqrt(-c*(a*x - 1))*(a*x - 1)/(x**5*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.43

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$
$$= \frac{1}{384}\,a^4c^4\left(\frac{2\left(447\,(-acx+c)^{\frac{7}{2}} - 1127\,(-acx+c)^{\frac{5}{2}}c + 1049\,(-acx+c)^{\frac{3}{2}}c^2 - 321\,\sqrt{-acx+c}c^3\right)}{(acx-c)^4c^3 + 4\,(acx-c)^3c^4 + 6\,(acx-c)^2c^5 + 4\,(acx-c)c^6 + c^7} + \frac{768}{}\right.$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

output
```
1/384*a^4*c^4*(2*(447*(-a*c*x + c)^(7/2) - 1127*(-a*c*x + c)^(5/2)*c + 104
9*(-a*c*x + c)^(3/2)*c^2 - 321*sqrt(-a*c*x + c)*c^3)/((a*c*x - c)^4*c^3 +
4*(a*c*x - c)^3*c^4 + 6*(a*c*x - c)^2*c^5 + 4*(a*c*x - c)*c^6 + c^7) + 768
*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) + sqrt
(-a*c*x + c)))/c^(7/2) - 1089*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*
x + c) + sqrt(c)))/c^(7/2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.08

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{4\sqrt{2}a^4c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{363\,a^4c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{64\sqrt{-c}} - \frac{447\,(acx-c)^3\sqrt{-acx+c}a^4c + 1127\,(acx-c)^2\sqrt{-acx+c}a^4c^2 - 1049\,(-acx+c)^{\frac{3}{2}}a^4c^3 + 321\,\sqrt{-acx}}{192\,a^4c^4x^4}$$

input
```
integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

output
```
4*sqrt(2)*a^4*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 3
63/64*a^4*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 1/192*(447*(a*c*x
 - c)^3*sqrt(-a*c*x + c)*a^4*c + 1127*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c
^2 - 1049*(-a*c*x + c)^(3/2)*a^4*c^3 + 321*sqrt(-a*c*x + c)*a^4*c^4)/(a^4*
c^4*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{1049\,{(c-a\,c\,x)}^{3/2}}{192\,c\,x^4} - \frac{a^4\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,363{}\mathrm{i}}{64} - \frac{107\,\sqrt{c-a\,c\,x}}{64\,x^4} - \frac{1127\,{(c-a\,c\,x)}^{5/2}}{192\,c^2\,x^4} + \frac{149\,{(c-a\,c\,x)}^{7/2}}{64\,c^3\,x^4} + \sqrt{2}\,a^4\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right)\,4{}\mathrm{i}$$

input
```
int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
```

output
```
(1049*(c - a*c*x)^(3/2))/(192*c*x^4) - (a^4*c^(1/2)*atan(((c - a*c*x)^(1/2
)*1i)/c^(1/2))*363i)/64 - (107*(c - a*c*x)^(1/2))/(64*x^4) - (1127*(c - a*
c*x)^(5/2))/(192*c^2*x^4) + (149*(c - a*c*x)^(7/2))/(64*c^3*x^4) + 2^(1/2)
*a^4*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*4i
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$
$$= \frac{\sqrt{c}\left(-894\sqrt{-ax+1}\,a^3x^3+428\sqrt{-ax+1}\,a^2x^2-272\sqrt{-ax+1}\,ax+96\sqrt{-ax+1}+768\sqrt{2}\log\left(\sqrt{-ax\right.}\right.}{}$$

input
```
int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x)
```

output
```
(sqrt(c)*( - 894*sqrt( - a*x + 1)*a**3*x**3 + 428*sqrt( - a*x + 1)*a**2*x*
*2 - 272*sqrt( - a*x + 1)*a*x + 96*sqrt( - a*x + 1) + 768*sqrt(2)*log(sqrt
( - a*x + 1) - sqrt(2))*a**4*x**4 - 768*sqrt(2)*log(sqrt( - a*x + 1) + sqr
t(2))*a**4*x**4 - 1089*log(sqrt( - a*x + 1) - 1)*a**4*x**4 + 1089*log(sqrt
( - a*x + 1) + 1)*a**4*x**4))/(384*x**4)
```

3.372 $\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 281

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{1312\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{45a^4\sqrt{1-\frac{1}{ax}}}-\frac{656\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}}-\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{164\sqrt{1+\frac{1}{ax}}x^2\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

output

```
1312/45*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^4/(1-1/a/x)^(1/2)-656/45*(1+1/a
/x)^(1/2)*x*(-a*c*x+c)^(1/2)/a^3/(1-1/a/x)^(1/2)-82/9*x^2*(-a*c*x+c)^(1/2)
/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+164/15*(1+1/a/x)^(1/2)*x^2*(-a*c*x+c)
^(1/2)/a^2/(1-1/a/x)^(1/2)-8/9*x^3*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1
/a/x)^(1/2)+2/9*x^4*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.26

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(656+328ax-82a^2x^2+41a^3x^3-20a^4x^4+5a^5x^5)}{45a^5\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(x^3*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

output
```
(2*Sqrt[c - a*c*x]*(656 + 328*a*x - 82*a^2*x^2 + 41*a^3*x^3 - 20*a^4*x^4 +
 5*a^5*x^5))/(45*a^5*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.73, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6730, 27, 100, 27, 87, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-acx}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{11/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2}{9}\int -\frac{28a-\frac{9}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2a^2}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{1}{9}\int \frac{28a-\frac{9}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}-\frac{2a^2}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{9}\left(41\int \frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}+\frac{8a}{\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{9}\left(41\left(6\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}+\frac{2}{\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{9}\left(41\left(6\left(-\frac{4\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{5a}-\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)+\frac{2}{\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{9}\left(41\left(6\left(-\frac{4\left(-\frac{2\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{3a}-\frac{2\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)+\frac{2}{\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)-\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{9}\left(\frac{8a}{\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+41\left(6\left(-\frac{4\left(\frac{4\sqrt{\frac{1}{ax}+1}}{3a\sqrt{\frac{1}{x}}}-\frac{2\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)}{5a}-\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(\frac{1}{x}\right)^{5/2}}\right)+\frac{2}{\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)\right)-\frac{2a^2}{9\left(\frac{1}{x}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(x^3*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

output
```
-((((41*(6*((-4*((-2*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) + (4*Sqrt[1 + 1
/(a*x)])/(3*a*Sqrt[x^(-1)])))/(5*a) - (2*Sqrt[1 + 1/(a*x)])/(5*(x^(-1))^(5
/2))) + 2/(Sqrt[1 + 1/(a*x)]*(x^(-1))^(5/2))) + (8*a)/(Sqrt[1 + 1/(a*x)]*(
x^(-1))^(7/2)))/9 - (2*a^2)/(9*Sqrt[1 + 1/(a*x)]*(x^(-1))^(9/2)))*Sqrt[x^(
-1)]*Sqrt[c - a*c*x])/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6730

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.28

method	result	size
gosper	$\frac{2(ax+1)\left(5a^{5}x^{5}-20a^{4}x^{4}+41a^{3}x^{3}-82a^{2}x^{2}+328ax+656\right)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{45a^{4}\left(ax-1\right)^{2}}$	80
orering	$\frac{2(ax+1)\left(5a^{5}x^{5}-20a^{4}x^{4}+41a^{3}x^{3}-82a^{2}x^{2}+328ax+656\right)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{45a^{4}\left(ax-1\right)^{2}}$	80
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(ax+1\right)\sqrt{-c\left(ax-1\right)}\,\left(5a^{5}x^{5}-20a^{4}x^{4}+41a^{3}x^{3}-82a^{2}x^{2}+328ax+656\right)}{45\left(ax-1\right)^{2}a^{4}}$	81
risch	$-\frac{2\left(5a^{4}x^{4}-25a^{3}x^{3}+66a^{2}x^{2}-148ax+476\right)\left(ax+1\right)c\sqrt{\frac{ax-1}{ax+1}}}{45a^{4}\sqrt{-c\left(ax-1\right)}}-\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a^{4}\sqrt{-c\left(ax-1\right)}}$	99

input
```
int(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*(a*x+1)*(5*a^5*x^5-20*a^4*x^4+41*a^3*x^3-82*a^2*x^2+328*a*x+656)*(-a*
c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/a^4/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.27

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{2\,(5\,a^5x^5-20\,a^4x^4+41\,a^3x^3-82\,a^2x^2+328\,ax+656)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{45\,(a^5x-a^4)}$$

input
```
integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

output
```
2/45*(5*a^5*x^5 - 20*a^4*x^4 + 41*a^3*x^3 - 82*a^2*x^2 + 328*a*x + 656)*sq
rt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^5*x - a^4)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.42

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$=\frac{2\left(5\,a^6\sqrt{-c}x^6-15\,a^5\sqrt{-c}x^5+21\,a^4\sqrt{-c}x^4-41\,a^3\sqrt{-c}x^3+246\,a^2\sqrt{-c}x^2+984\,a\sqrt{-c}x+656\,\sqrt{-c}\right)(}{45\,(a^6x^2-2\,a^5x+a^4)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

output
```
2/45*(5*a^6*sqrt(-c)*x^6 - 15*a^5*sqrt(-c)*x^5 + 21*a^4*sqrt(-c)*x^4 - 41*
a^3*sqrt(-c)*x^3 + 246*a^2*sqrt(-c)*x^2 + 984*a*sqrt(-c)*x + 656*sqrt(-c))
*(a*x - 1)^2/((a^6*x^2 - 2*a^5*x + a^4)*(a*x + 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.72 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.26

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^5\,x^5-20\,a^4\,x^4+41\,a^3\,x^3-82\,a^2\,x^2+328\,a\,x+656\right)}{45\,a^4\,\left(a\,x-1\right)}$$

input
```
int(x^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(328*a*x - 82*a^2*x^2 + 4
1*a^3*x^3 - 20*a^4*x^4 + 5*a^5*x^5 + 656))/(45*a^4*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{2\sqrt{c}\,i(-5a^5x^5+20a^4x^4-41a^3x^3+82a^2x^2-328ax-656)}{45\sqrt{ax+1}\,a^4}$$

input
```
int(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(2*sqrt(c)*i*( - 5*a**5*x**5 + 20*a**4*x**4 - 41*a**3*x**3 + 82*a**2*x**2
- 328*a*x - 656))/(45*sqrt(a*x + 1)*a**4)
```

3.373 $\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 231

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=-\frac{2672\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}-\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{1336\sqrt{1+\frac{1}{ax}}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}-\frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

output
```
-2672/105*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^3/(1-1/a/x)^(1/2)-334/35*x*(-
a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+1336/105*(1+1/a/x)^(1/2
)*x*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)-44/35*x^2*(-a*c*x+c)^(1/2)/a/(1-1
/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/7*x^3*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/(1+1/
a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.28

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c-acx}(-1336-668ax+167a^2x^2-66a^3x^3+15a^4x^4)}{105a^4\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(x^2*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

output
```
(2*Sqrt[c - a*c*x]*(-1336 - 668*a*x + 167*a^2*x^2 - 66*a^3*x^3 + 15*a^4*x^
4))/(105*a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.76, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6730, 27, 100, 27, 87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-acx}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{9/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2}{7}\int -\frac{22a-\frac{7}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a^2}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{1}{7}\int \frac{22a-\frac{7}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}-\frac{2a^2}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 87$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{7}\left(\frac{167}{5}\int \frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}+\frac{44a}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 55$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{7}\left(\frac{167}{5}\left(4\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}+\frac{2}{\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{44a}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 55$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{7}\left(\frac{167}{5}\left(4\left(-\frac{2\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{3a}-\frac{2\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)+\frac{2}{\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{44a}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 48$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{7}\left(\frac{44a}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{167}{5}\left(4\left(\frac{4\sqrt{\frac{1}{ax}+1}}{3a\sqrt{\frac{1}{x}}}-\frac{2\sqrt{\frac{1}{ax}+1}}{3\left(\frac{1}{x}\right)^{3/2}}\right)+\frac{2}{\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)\right)-\frac{2a^2}{7\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(x^2*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

output
```
-(((((167*(4*((-2*Sqrt[1 + 1/(a*x)])/(3*(x^(-1))^(3/2)) + (4*Sqrt[1 + 1/(a
*x)])/(3*a*Sqrt[x^(-1)])) + 2/(Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2))))/5 + (44
*a)/(5*Sqrt[1 + 1/(a*x)]*(x^(-1))^(5/2)))/7 - (2*a^2)/(7*Sqrt[1 + 1/(a*x)]
*(x^(-1))^(7/2)))*Sqrt[x^(-1)]*Sqrt[c - a*c*x])/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.31

method	result	size
gosper	$\frac{2(ax+1)(15a^4x^4-66a^3x^3+167a^2x^2-668ax-1336)\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{105a^3(ax-1)^2}$	72
orering	$\frac{2(ax+1)(15a^4x^4-66a^3x^3+167a^2x^2-668ax-1336)\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{105a^3(ax-1)^2}$	72
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,(15a^4x^4-66a^3x^3+167a^2x^2-668ax-1336)}{105(ax-1)^2a^3}$	73
risch	$-\frac{2(15a^3x^3-81a^2x^2+248ax-916)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{105a^3\sqrt{-c(ax-1)}}+\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a^3\sqrt{-c(ax-1)}}$	91

input
```
int(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(a*x+1)*(15*a^4*x^4-66*a^3*x^3+167*a^2*x^2-668*a*x-1336)*(-a*c*x+c)^
(1/2)*((a*x-1)/(a*x+1))^(3/2)/a^3/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\,(15\,a^4x^4-66\,a^3x^3+167\,a^2x^2-668\,ax-1336)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^4x-a^3)}$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

output
```
2/105*(15*a^4*x^4 - 66*a^3*x^3 + 167*a^2*x^2 - 668*a*x - 1336)*sqrt(-a*c*x
 + c)*sqrt((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.45

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\left(15\,a^5\sqrt{-c}x^5-51\,a^4\sqrt{-c}x^4+101\,a^3\sqrt{-c}x^3-501\,a^2\sqrt{-c}x^2-2004\,a\sqrt{-c}x-1336\,\sqrt{-c}\right)(ax-1)^2}{105\,(a^5x^2-2\,a^4x+a^3)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

output
```
2/105*(15*a^5*sqrt(-c)*x^5 - 51*a^4*sqrt(-c)*x^4 + 101*a^3*sqrt(-c)*x^3 -
501*a^2*sqrt(-c)*x^2 - 2004*a*sqrt(-c)*x - 1336*sqrt(-c))*(a*x - 1)^2/((a^
5*x^2 - 2*a^4*x + a^3)*(a*x + 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.71 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.38

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^3\,x^3-51\,a^2\,x^2+116\,a\,x-552\right)}{105\,a^3} - \frac{3776\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{105\,a^3\,\left(a\,x-1\right)}$$

input
```
int(x^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(116*a*x - 51*a^2*x^2 + 1
5*a^3*x^3 - 552))/(105*a^3) - (3776*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1)
)^(1/2))/(105*a^3*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.20

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{2\sqrt{c}\,i(-15a^4x^4+66a^3x^3-167a^2x^2+668ax+1336)}{105\sqrt{ax+1}\,a^3}$$

input
```
int(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(2*sqrt(c)*i*( - 15*a**4*x**4 + 66*a**3*x**3 - 167*a**2*x**2 + 668*a*x + 1
336))/(105*sqrt(a*x + 1)*a**3)
```

3.374 $\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 182

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=-\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{316\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

output
```
-158/15*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+316/15*(1+1/a
/x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)-32/15*x*(-a*c*x+c)^(1/2)/a/
(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/5*x^2*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/(
1+1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.31

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}(158+79ax-16a^2x^2+3a^3x^3)}{15a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(x*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

output
```
(2*Sqrt[c - a*c*x]*(158 + 79*a*x - 16*a^2*x^2 + 3*a^3*x^3))/(15*a^3*Sqrt[1
 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6730, 27, 100, 27, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-acx}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{7/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2}{5}\int -\frac{16a-\frac{5}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{1}{5}\int \frac{16a-\frac{5}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{5}\left(\frac{79}{3}\int \frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}+\frac{32a}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{5}\left(\frac{79}{3}\left(2\int \frac{1}{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}+\frac{2}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{32a}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{5}\left(\frac{32a}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{79}{3}\left(\frac{2}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}-\frac{4\sqrt{\frac{1}{ax}+1}}{\sqrt{\frac{1}{x}}}\right)\right)-\frac{2a^2}{5\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(x*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

output
```
-(((((79*(2/(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) - (4*Sqrt[1 + 1/(a*x)])/Sqrt[
x^(-1)]))/3 + (32*a)/(3*Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2)))/5 - (2*a^2)/(5*
Sqrt[1 + 1/(a*x)]*(x^(-1))^(5/2)))*Sqrt[x^(-1)]*Sqrt[c - a*c*x])/(a^2*Sqrt
[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{2(ax+1)(3a^3x^3-16a^2x^2+79ax+158)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{15a^2(ax-1)^2}$	64
orering	$\frac{2(ax+1)(3a^3x^3-16a^2x^2+79ax+158)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{15a^2(ax-1)^2}$	64
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,(3a^3x^3-16a^2x^2+79ax+158)}{15(ax-1)^2a^2}$	65
risch	$-\frac{2(3a^2x^2-19ax+98)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{15a^2\sqrt{-c(ax-1)}}-\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a^2\sqrt{-c(ax-1)}}$	83

input

```
int(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/15*(a*x+1)*(3*a^3*x^3-16*a^2*x^2+79*a*x+158)*(-a*c*x+c)^(1/2)*((a*x-1)/(
a*x+1))^(3/2)/a^2/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.34

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{2\,(3\,a^3x^3-16\,a^2x^2+79\,ax+158)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^3x-a^2)}$$

input

```
integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

output

```
2/15*(3*a^3*x^3 - 16*a^2*x^2 + 79*a*x + 158)*sqrt(-a*c*x + c)*sqrt((a*x -
1)/(a*x + 1))/(a^3*x - a^2)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \text{Timed out}$$

input
```
integrate(x*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.50

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\left(3a^4\sqrt{-c}x^4-13a^3\sqrt{-c}x^3+63a^2\sqrt{-c}x^2+237a\sqrt{-c}x+158\sqrt{-c}\right)(ax-1)^2}{15\left(a^4x^2-2a^3x+a^2\right)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

output
```
2/15*(3*a^4*sqrt(-c)*x^4 - 13*a^3*sqrt(-c)*x^3 + 63*a^2*sqrt(-c)*x^2 + 237
*a*sqrt(-c)*x + 158*sqrt(-c))*(a*x - 1)^2/((a^4*x^2 - 2*a^3*x + a^2)*(a*x
+ 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.64 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.32

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,(3\,a^3\,x^3-16\,a^2\,x^2+79\,a\,x+158)}{15\,a^2\,(a\,x-1)}$$

input
```
int(x*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(79*a*x - 16*a^2*x^2 + 3*
a^3*x^3 + 158))/(15*a^2*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.21

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{2\sqrt{c}\,i(-3a^3x^3+16a^2x^2-79ax-158)}{15\sqrt{ax+1}\,a^2}$$

input
```
int(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(2*sqrt(c)*i*( - 3*a**3*x**3 + 16*a**2*x**2 - 79*a*x - 158))/(15*sqrt(a*x
+ 1)*a**2)
```

3.375 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=-\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{46\sqrt{c-acx}}{3a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

output
```
-20/3*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-46/3*(-a*c*x+c)^(
1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)/x+2/3*x*(-a*c*x+c)^(1/2)/(1-1/a/x
)^(1/2)/(1+1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.35

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{c-acx}(-23-10ax+a^2x^2)}{3a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

output
```
(2*Sqrt[c - a*c*x]*(-23 - 10*a*x + a^2*x^2))/(3*a^2*Sqrt[1 - 1/(a^2*x^2)]*
x)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6727, 27, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6727}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{5/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{2}{3}\int -\frac{10a-\frac{3}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\frac{1}{3}\int \frac{10a-\frac{3}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{1}{3}\left(23\int \frac{1}{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}+\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{1}{3}\left(\frac{20a}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}+\frac{46\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2a^2}{3\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

output
```
-(((((20*a)/(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + (46*Sqrt[x^(-1)])/Sqrt[1 +
1/(a*x)])/3 - (2*a^2)/(3*Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2)))*Sqrt[x^(-1)]*S
qrt[c - a*c*x])/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6727
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_), x_Symbol] :> Si
mp[(-(1/x)^p)*((c + d*x)^p/(1 + c/(d*x))^p)   Subst[Int[((1 + c*(x/d))^p*((
1 + x/a)^(n/2)/x^(p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.40

method	result	size
gosper	$\frac{2(ax+1)\left(a^{2}x^{2}-10ax-23\right)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3a\left(ax-1\right)^{2}}$	55
orering	$\frac{2(ax+1)\left(a^{2}x^{2}-10ax-23\right)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3a\left(ax-1\right)^{2}}$	55
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,\left(a^{2}x^{2}-10ax-23\right)}{3\left(ax-1\right)^{2}a}$	56
risch	$-\frac{2(ax-11)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{3a\sqrt{-c(ax-1)}}+\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	74

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a*x+1)*(a^2*x^2-10*a*x-23)*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/a
/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.36

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\left(a^2x^2-10\,ax-23\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(a^2*x^2 - 10*a*x - 23)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^
2*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.55

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\left(a^3\sqrt{-c}x^3-9\,a^2\sqrt{-c}x^2-33\,a\sqrt{-c}x-23\,\sqrt{-c}\right)(ax-1)^2}{3\left(a^3x^2-2\,a^2x+a\right)(ax+1)^{\frac{3}{2}}}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
2/3*(a^3*sqrt(-c)*x^3 - 9*a^2*sqrt(-c)*x^2 - 33*a*sqrt(-c)*x - 23*sqrt(-c)
)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.52

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2\sqrt{c-a\,c\,x}\,(a\,x-9)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a} - \frac{64\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,(a\,x-1)}$$

input
```
int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*(c - a*c*x)^(1/2)*(a*x - 9)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (64*(c
 - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.22

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2\sqrt{c}\,i(-a^2x^2+10ax+23)}{3\sqrt{ax+1}\,a}$$

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(2*sqrt(c)*i*( - a**2*x**2 + 10*a*x + 23))/(3*sqrt(a*x + 1)*a)
```

3.376 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{10\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}-\frac{2\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\text{arcsinh}\left(\sqrt{\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
2*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+10*(-a*c*x+c)^(1/2)/a/(
1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)/x-2*(1/a/x)^(1/2)*(-a*c*x+c)^(1/2)*arcsinh(
(1/a/x)^(1/2))/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{2\sqrt{c-acx}\left(a+\frac{5}{x}-\sqrt{a}\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}\,\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
(2*Sqrt[c - a*c*x]*(a + 5/x - Sqrt[a]*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]*ArcSi
nh[Sqrt[x^(-1)]/Sqrt[a]]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6730, 27, 100, 27, 87, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(2\int -\frac{4a-\frac{1}{x}}{2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2a^2}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\int \frac{4a-\frac{1}{x}}{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2a^2}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(a\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-\frac{2a^2}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}-\frac{10a\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{63}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(2a\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}-\frac{2a^2}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}-\frac{10a\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(2a^{3/2}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-\frac{2a^2}{\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}}-\frac{10a\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((-2*a^2)/(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)])
 - (10*a*Sqrt[x^(-1)])/Sqrt[1 + 1/(a*x)] + 2*a^(3/2)*ArcSinh[Sqrt[x^(-1)]/
Sqrt[a]]))/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(\sqrt{c}\,\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)\sqrt{-c(ax+1)}+acx+5c\right)}{(ax-1)^{2}c}$	80

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*(c^(1/2)*ar
ctan((-c*(a*x+1))^(1/2)/c^(1/2))*(-c*(a*x+1))^(1/2)+a*c*x+5*c)/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.55

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}(ax+5)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}, -\frac{2\left((ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+c}(ax+5)\sqrt{\frac{ax-1}{ax+1}}\right)}{ax-1}\right]$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas"
)
```

output
```
[((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c)*(a*x + 1
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x +
 c)*(a*x + 5)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*((a*x - 1)*sqrt(c)*
arctan(sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x
 - c)) - sqrt(-a*c*x + c)*(a*x + 5)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x}\,dx$$

input
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
```

output
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{\sqrt{c}\,i\left(-\sqrt{ax+1}\log\left(\frac{2\sqrt{ax+1}-2}{\sqrt{2}}\right)+\sqrt{ax+1}\log\left(\frac{2\sqrt{ax+1}+2}{\sqrt{2}}\right)-2ax-10\right)}{\sqrt{ax+1}}$$

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x)
```

output
```
(sqrt(c)*i*( - sqrt(a*x + 1)*log((2*sqrt(a*x + 1) - 2)/sqrt(2)) + sqrt(a*x
 + 1)*log((2*sqrt(a*x + 1) + 2)/sqrt(2)) - 2*a*x - 10))/sqrt(a*x + 1)
```

3.377 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{7a\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\operatorname{arcsinh}\left(\sqrt{\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

output
```
-8*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)/x-(1+1/a/x)^(1/2)*(-a*
c*x+c)^(1/2)/(1-1/a/x)^(1/2)/x+7*a*(1/a/x)^(1/2)*(-a*c*x+c)^(1/2)*arcsinh(
(1/a/x)^(1/2))/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.57

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}\left(-1-9ax+\frac{7a^{3/2}\sqrt{1+\frac{1}{ax}}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{3/2}}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
(Sqrt[c - a*c*x]*(-1 - 9*a*x + (7*a^(3/2)*Sqrt[1 + 1/(a*x)]*ArcSinh[Sqrt[x
^(-1)]/Sqrt[a]])/(x^(-1))^(3/2)))/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6730, 27, 100, 27, 90, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2}{\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{x}}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}-2a^2\int \frac{3a-\frac{1}{x}}{2a\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}-a\int\frac{3a-\frac{1}{x}}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 90}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{7}{2}a\int\frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}-a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 63}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}-a\left(7a\int\frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}-a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 222}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{8a^2\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{ax}+1}}-a\left(7a^{3/2}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)\right)\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((8*a^2*Sqrt[x^(-1)])/Sqrt[1 + 1/(a*x)] -
a*(-(a*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + 7*a^(3/2)*ArcSinh[Sqrt[x^(-1)]/Sq
rt[a]])))/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.62

method	result	size
default	$-\frac{\left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}} (a x+1) \left(7 \arctan\left(\frac{\sqrt{-c (a x+1)}}{\sqrt{c}}\right) a x \sqrt{-c (a x+1)}+9 \sqrt{c}\, a x+\sqrt{c}\right) \sqrt{-c (a x-1)}}{(a x-1)^{2} \sqrt{c}\, x}$	86
risch	$\frac{(a x+1) c \sqrt{\frac{a x-1}{a x+1}}}{x \sqrt{-c (a x-1)}}-\frac{\left(-\frac{8 a}{\sqrt{-a c x-c}}-\frac{7 a \arctan\left(\frac{\sqrt{-a c x-c}}{\sqrt{c}}\right)}{\sqrt{c}}\right) c \sqrt{\frac{a x-1}{a x+1}}\, \sqrt{-c (a x+1)}}{\sqrt{-c (a x-1)}}$	112

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(7*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*a*x
*(-c*(a*x+1))^(1/2)+9*c^(1/2)*a*x+c^(1/2))*(-c*(a*x-1))^(1/2)/(a*x-1)^2/c^
(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.72

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$$
$$= \left[\frac{7\,(a^2x^2-ax)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\,\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)-2\,\sqrt{-acx+c}(9\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\,(ax^2-x)}, \frac{7\,(a\right.$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="frica
s")
```

output
```
[1/2*(7*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x +
 c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) - 2*s
qrt(-a*c*x + c)*(9*a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), (7*(a^
2*x^2 - a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x -
 1)/(a*x + 1))/(a*c*x - c)) - sqrt(-a*c*x + c)*(9*a*x + 1)*sqrt((a*x - 1)/
(a*x + 1)))/(a*x^2 - x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^2}\,dx$$

input
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
```

output
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c}\,i\left(7\sqrt{ax+1}\log\left(\frac{2\sqrt{ax+1}-2}{\sqrt{2}}\right)ax-7\sqrt{ax+1}\log\left(\frac{2\sqrt{ax+1}+2}{\sqrt{2}}\right)ax+18ax+2\right)}{2\sqrt{ax+1}\,x}$$

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

output
```
(sqrt(c)*i*(7*sqrt(a*x + 1)*log((2*sqrt(a*x + 1) - 2)/sqrt(2))*a*x - 7*sqr
t(a*x + 1)*log((2*sqrt(a*x + 1) + 2)/sqrt(2))*a*x + 18*a*x + 2))/(2*sqrt(a
*x + 1)*x)
```

3.378 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 190

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}x^2} + \frac{47a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x} - \frac{47a^2\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\mathrm{arcsinh}\left(\sqrt{\frac{1}{ax}}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

output
```
-8*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)/x^2-1/2*(1+1/a/x)^(1/2
)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/x^2+47/4*a*(1+1/a/x)^(1/2)*(-a*c*x+c)^(
1/2)/(1-1/a/x)^(1/2)/x-47/4*a^2*(1/a/x)^(1/2)*(-a*c*x+c)^(1/2)*arcsinh((1/
a/x)^(1/2))/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = -\frac{\sqrt{c-acx}\left(2-13ax-47a^2x^2+\frac{47a^{5/2}\sqrt{1+\frac{1}{ax}}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{5/2}}\right)}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^3),x]
```

output
```
-1/4*(Sqrt[c - a*c*x]*(2 - 13*a*x - 47*a^2*x^2 + (47*a^(5/2)*Sqrt[1 + 1/(a
*x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(5/2)))/(a*Sqrt[1 - 1/(a^2*x^
2)]*x^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.74, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6730, 27, 100, 27, 90, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}\,e^{-3\coth^{-1}(ax)}}{x^3}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{x}}}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{3/2}}{\sqrt{\frac{1}{ax}+1}}-2a^2\int \frac{\left(11a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}{2a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{3/2}}{\sqrt{\frac{1}{ax}+1}}-a\int \frac{\left(11a-\frac{1}{x}\right)\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{90}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{3/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{47}{4}a\int \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{60}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{3/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{47}{4}a\left(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-\frac{1}{2}a\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)-\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{63}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{3/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{47}{4}a\left(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-a\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\right)-\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{8a^2\left(\frac{1}{x}\right)^{3/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{47}{4}a\left(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-a^{3/2}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)-\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}\right)\right)\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^3),x]
```

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((8*a^2*(x^(-1))^(3/2))/Sqrt[1 + 1/(a*x)]
- a*(-1/2*(a*Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2)) + (47*a*(a*Sqrt[1 + 1/(a*x)
]*Sqrt[x^(-1)] - a^(3/2)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/4)))/(a^2*Sqrt[1
- 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(47\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^{2}x^{2}\sqrt{-c(ax+1)}+47\sqrt{c}\,a^{2}x^{2}+13\sqrt{c}\,ax-2\sqrt{c}\right)}{4(ax-1)^{2}\sqrt{c}\,x^{2}}$	103
risch	$-\frac{(15a^{2}x^{2}+13ax-2)c\sqrt{\frac{ax-1}{ax+1}}}{4x^{2}\sqrt{-c(ax-1)}}-\frac{\left(\frac{8a^{2}}{\sqrt{-acx-c}}+\frac{47a^{2}\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{4\sqrt{c}}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}}{\sqrt{-c(ax-1)}}$	126

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*(47*arcta
n((-c*(a*x+1))^(1/2)/c^(1/2))*a^2*x^2*(-c*(a*x+1))^(1/2)+47*c^(1/2)*a^2*x^
2+13*c^(1/2)*a*x-2*c^(1/2))/c^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$$

$$= \left[\frac{47\,(a^3x^3-a^2x^2)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\,\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)+2\,(47\,a^2x^2+13\,ax-2)\sqrt{-acx-}}{8\,(ax^3-x^2)}\right.$$

$$\left.-\frac{47\,(a^3x^3-a^2x^2)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(47\,a^2x^2+13\,ax-2)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{4\,(ax^3-x^2)}\right]$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="frica
s")
```

output
```
[1/8*(47*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*
c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x))
+ 2*(47*a^2*x^2 + 13*a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/
(a*x^3 - x^2), -1/4*(47*(a^3*x^3 - a^2*x^2)*sqrt(c)*arctan(sqrt(-a*c*x + c
)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (47*a^2*x^2 +
 13*a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^3}\,dx$$

input
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
```

output
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$$
$$= \frac{\sqrt{c}\,i\left(-47\sqrt{ax+1}\log\left(\frac{2\sqrt{ax+1}-2}{\sqrt{2}}\right)a^2x^2+47\sqrt{ax+1}\log\left(\frac{2\sqrt{ax+1}+2}{\sqrt{2}}\right)a^2x^2-94a^2x^2-26ax+4\right)}{8\sqrt{ax+1}\,x^2}$$

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

output
```
(sqrt(c)*i*( - 47*sqrt(a*x + 1)*log((2*sqrt(a*x + 1) - 2)/sqrt(2))*a**2*x*
*2 + 47*sqrt(a*x + 1)*log((2*sqrt(a*x + 1) + 2)/sqrt(2))*a**2*x**2 - 94*a*
*2*x**2 - 26*a*x + 4))/(8*sqrt(a*x + 1)*x**2)
```

3.379 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 238

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^3}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}x^3}+\frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{12\sqrt{1-\frac{1}{ax}}x^2}-\frac{119a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}x}+\frac{119a^3\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\text{arcsinh}\left(\sqrt{\frac{1}{ax}}\right)}{8\sqrt{1-\frac{1}{ax}}}$$

output
```
-8*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)/x^3-1/3*(1+1/a/x)^(1/2
)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/x^3+119/12*a*(1+1/a/x)^(1/2)*(-a*c*x+c)
^(1/2)/(1-1/a/x)^(1/2)/x^2-119/8*a^2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1
/a/x)^(1/2)/x+119/8*a^3*(1/a/x)^(1/2)*(-a*c*x+c)^(1/2)*arcsinh((1/a/x)^(1/
2))/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}\left(-8+38ax-119a^2x^2-357a^3x^3+\frac{357a^{7/2}\sqrt{1+\frac{1}{ax}}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{7/2}}\right)}{24a\sqrt{1-\frac{1}{a^2x^2}}x^4}$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
(Sqrt[c - a*c*x]*(-8 + 38*a*x - 119*a^2*x^2 - 357*a^3*x^3 + (357*a^(7/2)*S
qrt[1 + 1/(a*x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(7/2)))/(24*a*Sqr
t[1 - 1/(a^2*x^2)]*x^4)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.72, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6730, 27, 100, 27, 90, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-3\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \ 6730$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{3/2}}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{3/2}}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 100$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}}-2a^2\int \frac{\left(19a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}{2a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}}-a\int \frac{\left(19a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{3/2}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 90$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{119}{6}a\int \frac{\left(\frac{1}{x}\right)^{3/2}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 60$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{119}{6}a\left(\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{3}{4}a\int \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 60$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{119}{6}a\left(\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{3}{4}a\left(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-\frac{1}{2}a\int \frac{1}{\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}}d\frac{1}{x}\right)\right)-\frac{1}{3}a\left(\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 63$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{119}{6}a\left(\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{3}{4}a\left(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-a\int \frac{1}{\sqrt{1+\frac{1}{x^2a}}}d\sqrt{\frac{1}{x}}\right)\right)-\frac{1}{3}a\left(\frac{1}{x}\right)^{5}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 222

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{8a^2\left(\frac{1}{x}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{119}{6}a\left(\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{3}{4}a\left(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-a^{3/2}\text{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)\right)-\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}\cdots}\right.\right.}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((8*a^2*(x^(-1))^(5/2))/Sqrt[1 + 1/(a*x)]
- a*(-1/3*(a*Sqrt[1 + 1/(a*x)]*(x^(-1))^(5/2)) + (119*a*((a*Sqrt[1 + 1/(a*
x)]*(x^(-1))^(3/2))/2 - (3*a*(a*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] - a^(3/2)*A
rcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/4))/6)))/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.48

method	result	siz
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(357\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^{3}x^{3}\sqrt{-c(ax+1)}+357a^{3}x^{3}\sqrt{c}+119\sqrt{c}\,a^{2}x^{2}-38\sqrt{c}\,ax+8\sqrt{c}\right)}{24(ax-1)^{2}\sqrt{c}\,x^{3}}$	11
risch	$\frac{(165a^{3}x^{3}+119a^{2}x^{2}-38ax+8)c\sqrt{\frac{ax-1}{ax+1}}}{24x^{3}\sqrt{-c(ax-1)}}-\frac{\left(-\frac{8a^{3}}{\sqrt{-acx-c}}-\frac{119a^{3}\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{8\sqrt{c}}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}}{\sqrt{-c(ax-1)}}$	13

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*(357*ar
ctan((-c*(a*x+1))^(1/2)/c^(1/2))*a^3*x^3*(-c*(a*x+1))^(1/2)+357*a^3*x^3*c^
(1/2)+119*c^(1/2)*a^2*x^2-38*c^(1/2)*a*x+8*c^(1/2))/c^(1/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.19

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$
$$= \left[\frac{357\,(a^4x^4-a^3x^3)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\,\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)-2\,(357\,a^3x^3+119\,a^2x^2-38\,ax}{48\,(ax^4-x^3)}\right.$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="frica
s")
```

output
```
[1/48*(357*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-
a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)
) - 2*(357*a^3*x^3 + 119*a^2*x^2 - 38*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x
- 1)/(a*x + 1)))/(a*x^4 - x^3), 1/24*(357*(a^4*x^4 - a^3*x^3)*sqrt(c)*arct
an(sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c
)) - (357*a^3*x^3 + 119*a^2*x^2 - 38*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x -
 1)/(a*x + 1)))/(a*x^4 - x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^4}\,dx$$

input
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

output
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$
$$= \frac{\sqrt{c}\,i\Big(357\sqrt{ax+1}\log\Big(\frac{2\sqrt{ax+1}-2}{\sqrt{2}}\Big)a^3x^3 - 357\sqrt{ax+1}\log\Big(\frac{2\sqrt{ax+1}+2}{\sqrt{2}}\Big)a^3x^3 + 714a^3x^3 + 238a^2x^2 - 76ax}{48\sqrt{ax+1}\,x^3}$$

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

output
```
(sqrt(c)*i*(357*sqrt(a*x + 1)*log((2*sqrt(a*x + 1) - 2)/sqrt(2))*a**3*x**3
 - 357*sqrt(a*x + 1)*log((2*sqrt(a*x + 1) + 2)/sqrt(2))*a**3*x**3 + 714*a*
*3*x**3 + 238*a**2*x**2 - 76*a*x + 16))/(48*sqrt(a*x + 1)*x**3)
```

3.380 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 286

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx =& -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x^4}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}x^4}\\&+\frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}x^3}-\frac{1115a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{96\sqrt{1-\frac{1}{ax}}x^2}\\&+\frac{1115a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}x}\\&-\frac{1115a^4\sqrt{\frac{1}{ax}}\sqrt{c-acx}\,\text{arcsinh}\left(\sqrt{\frac{1}{ax}}\right)}{64\sqrt{1-\frac{1}{ax}}}\end{aligned}$$

output
```
-8*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)/x^4-1/4*(1+1/a/x)^(1/2
)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/x^4+223/24*a*(1+1/a/x)^(1/2)*(-a*c*x+c)
^(1/2)/(1-1/a/x)^(1/2)/x^3-1115/96*a^2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1
-1/a/x)^(1/2)/x^2+1115/64*a^3*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(
1/2)/x-1115/64*a^4*(1/a/x)^(1/2)*(-a*c*x+c)^(1/2)*arcsinh((1/a/x)^(1/2))/(
1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.37

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = -\frac{\sqrt{c-acx}\left(48-200ax+446a^2x^2-1115a^3x^3-3345a^4x^4+\frac{3345a^{9/2}\sqrt{1+\frac{1}{ax}}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{9/2}}\right)}{192a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}$$

input
```
Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
-1/192*(Sqrt[c - a*c*x]*(48 - 200*a*x + 446*a^2*x^2 - 1115*a^3*x^3 - 3345*
a^4*x^4 + (3345*a^(9/2)*Sqrt[1 + 1/(a*x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(
x^(-1))^(9/2)))/(a*Sqrt[1 - 1/(a^2*x^2)]*x^5)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.71, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6730, 27, 100, 27, 90, 60, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-acx}e^{-3\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{ 6730}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{5/2}}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\int \frac{\left(a-\frac{1}{x}\right)^2\left(\frac{1}{x}\right)^{5/2}}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 100

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{7/2}}{\sqrt{\frac{1}{ax}+1}}-2a^2\int \frac{\left(27a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{5/2}}{2a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 27

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{7/2}}{\sqrt{\frac{1}{ax}+1}}-a\int \frac{\left(27a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{5/2}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 90

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{7/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{223}{8}a\int \frac{\left(\frac{1}{x}\right)^{5/2}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{4}a\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 60

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{7/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{223}{8}a\left(\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}-\frac{5}{6}a\int \frac{\left(\frac{1}{x}\right)^{3/2}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{1}{4}a\left(\frac{1}{x}\right)^{7/2}\sqrt{\frac{1}{ax}+1}\right)\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 60

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{7/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{223}{8}a\left(\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}-\frac{5}{6}a\left(\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{3}{4}a\int \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)\right)-\frac{1}{4}a\left(\frac{1}{x}\right.\right.\right.}{a^2\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 60

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{7/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{223}{8}a\left(\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}-\frac{5}{6}a\left(\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{3}{4}a\left(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-\frac{1}{2}a\int\right.\right.\right.\right.\right.}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 63$$

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\frac{8a^2\left(\frac{1}{x}\right)^{7/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{223}{8}a\left(\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}-\frac{5}{6}a\left(\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{3}{4}a\left(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-a\int -\right.\right.\right.\right.\right.}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 222$$

$$-\frac{\sqrt{\frac{1}{x}}\left(\frac{8a^2\left(\frac{1}{x}\right)^{7/2}}{\sqrt{\frac{1}{ax}+1}}-a\left(\frac{223}{8}a\left(\frac{1}{3}a\left(\frac{1}{x}\right)^{5/2}\sqrt{\frac{1}{ax}+1}-\frac{5}{6}a\left(\frac{1}{2}a\left(\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{3}{4}a\left(a\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}-a^{3/2}\text{arcsinh}\left(\right.\right.\right.\right.\right.\right.}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
-((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*((8*a^2*(x^(-1))^(7/2))/Sqrt[1 + 1/(a*x)]
- a*(-1/4*(a*Sqrt[1 + 1/(a*x)]*(x^(-1))^(7/2)) + (223*a*((a*Sqrt[1 + 1/(a*
x)]*(x^(-1))^(5/2))/3 - (5*a*((a*Sqrt[1 + 1/(a*x)]*(x^(-1))^(3/2))/2 - (3*
a*(a*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] - a^(3/2)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]
]))/4))/6))/8)))/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.44

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(3345\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^4x^4\sqrt{-c(ax+1)}+3345a^4x^4\sqrt{c}+1115a^3x^3\sqrt{c}-446\sqrt{c}\,a^2x^2+200\sqrt{c}\,ax\right.}{192(ax-1)^2\sqrt{c}\,x^4}$
risch	$-\frac{(1809a^4x^4+1115a^3x^3-446a^2x^2+200ax-48)c\sqrt{\frac{ax-1}{ax+1}}}{192x^4\sqrt{-c(ax-1)}}-\frac{\left(\frac{8a^4}{\sqrt{-acx-c}}+\frac{1115a^4\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{64\sqrt{c}}\right)c\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax+1)}}{\sqrt{-c(ax-1)}}$

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/192*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*(3345*a
rctan((-c*(a*x+1))^(1/2)/c^(1/2))*a^4*x^4*(-c*(a*x+1))^(1/2)+3345*a^4*x^4*
c^(1/2)+1115*a^3*x^3*c^(1/2)-446*c^(1/2)*a^2*x^2+200*c^(1/2)*a*x-48*c^(1/2
))/c^(1/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.05

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$

$$= \left[\frac{3345\left(a^5x^5-a^4x^4\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\left(3345a^4x^4+1115a^3x^3-44\right.}{384\left(ax^5-x^4\right)}\right.$$

$$-\frac{3345\left(a^5x^5-a^4x^4\right)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}(ax+1)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\left(3345a^4x^4+1115a^3x^3-446a^2x^2+200ax\right.}{192\left(ax^5-x^4\right)}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="frica
s")
```

output
```
[1/384*(3345*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt
(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 -
x)) + 2*(3345*a^4*x^4 + 1115*a^3*x^3 - 446*a^2*x^2 + 200*a*x - 48)*sqrt(-a
*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4), -1/192*(3345*(a^5*x^5
- a^4*x^4)*sqrt(c)*arctan(sqrt(-a*c*x + c)*(a*x + 1)*sqrt(c)*sqrt((a*x - 1
)/(a*x + 1))/(a*c*x - c)) - (3345*a^4*x^4 + 1115*a^3*x^3 - 446*a^2*x^2 + 2
00*a*x - 48)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^5}\,dx$$

input
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
```

output
```
int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.37

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c}\,i\left(-3345\sqrt{ax+1}\log\left(\frac{2\sqrt{ax+1}-2}{\sqrt{2}}\right)a^4x^4+3345\sqrt{ax+1}\log\left(\frac{2\sqrt{ax+1}+2}{\sqrt{2}}\right)a^4x^4-6690a^4x^4-2230a^3x^3+\right.}{384\sqrt{ax+1}\,x^4}$$

input
```
int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x)
```

output
```
(sqrt(c)*i*( - 3345*sqrt(a*x + 1)*log((2*sqrt(a*x + 1) - 2)/sqrt(2))*a**4*
x**4 + 3345*sqrt(a*x + 1)*log((2*sqrt(a*x + 1) + 2)/sqrt(2))*a**4*x**4 - 6
690*a**4*x**4 - 2230*a**3*x**3 + 892*a**2*x**2 - 400*a*x + 96))/(384*sqrt(
a*x + 1)*x**4)
```

3.381 $\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 189

$$\begin{aligned}\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^4\,dx &= \frac{a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4(ex)^m}{1+m}-\frac{3a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5(ex)^m}{2+m}\\&\quad+\frac{a^4c^4(17+4m)x^5(ex)^m\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-5-m),\frac{1}{2}(-3-m),\frac{1}{a^2x^2}\right)}{(2+m)(5+m)}\\&\quad-\frac{a^3c^4(7+4m)x^4(ex)^m\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-4-m),\frac{1}{2}(-2-m),\frac{1}{a^2x^2}\right)}{(1+m)(4+m)}\end{aligned}$$

output
```
a^3*c^4*(1-1/a^2/x^2)^(3/2)*x^4*(e*x)^m/(1+m)-3*a^4*c^4*(1-1/a^2/x^2)^(3/2
)*x^5*(e*x)^m/(2+m)+a^4*c^4*(17+4*m)*x^5*(e*x)^m*hypergeom([-1/2, -5/2-1/2
*m],[-3/2-1/2*m],1/a^2/x^2)/(2+m)/(5+m)-a^3*c^4*(7+4*m)*x^4*(e*x)^m*hyperg
eom([-1/2, -2-1/2*m],[-1-1/2*m],1/a^2/x^2)/(1+m)/(4+m)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.85

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^4\,dx$$
$$=ac^4x^2(ex)^m\left(\frac{a^3x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{5}{2}-\frac{m}{2},-\frac{3}{2}-\frac{m}{2},\frac{1}{a^2x^2}\right)}{5+m}-\frac{3a^2x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-2-\frac{m}{2},-1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{4+m}+\frac{3ax\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{2}-\frac{m}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{a^2x^2}\right)}{3+m}-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},-1-\frac{m}{2},-\frac{m}{2},\frac{1}{a^2x^2}\right)}{2+m}\right)$$

input
```
Integrate[E^ArcCoth[a*x]*(e*x)^m*(c - a*c*x)^4,x]
```

output
```
a*c^4*x^2*(e*x)^m*((a^3*x^3*Hypergeometric2F1[-1/2, -5/2 - m/2, -3/2 - m/2
, 1/(a^2*x^2)])/(5 + m) - (3*a^2*x^2*Hypergeometric2F1[-1/2, -2 - m/2, -1
- m/2, 1/(a^2*x^2)])/(4 + m) + (3*a*x*Hypergeometric2F1[-1/2, -3/2 - m/2,
-1/2 - m/2, 1/(a^2*x^2)])/(3 + m) - Hypergeometric2F1[-1/2, -1 - m/2, -1/2
*m, 1/(a^2*x^2)]/(2 + m))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6729, 147, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^4e^{\coth^{-1}(ax)}(ex)^m\,dx$$
$$\Big\downarrow \text{ 6729}$$

$$-a^4c^4\left(\frac{1}{x}\right)^m(ex)^m\int\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-6}d\frac{1}{x}$$

$$\Big\downarrow \text{147}$$

$$-a^4c^4\left(\frac{1}{x}\right)^m(ex)^m\int\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-6}-\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-5}}{a}+\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)}{a^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-a^4c^4\left(\frac{1}{x}\right)^m(ex)^m\left(-\frac{\left(\frac{1}{x}\right)^{-m-5}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-m-5),\frac{1}{2}(-m-3),\frac{1}{a^2x^2}\right)}{m+5}+\frac{3\left(\frac{1}{x}\right)^{-m-4}\text{Hyperge}}{}\right.$$

input

```
Int[E^ArcCoth[a*x]*(e*x)^m*(c - a*c*x)^4,x]
```

output

```
-(a^4*c^4*(x^(-1))^m*(e*x)^m*(-(((x^(-1))^(-5 - m)*Hypergeometric2F1[-1/2,
 (-5 - m)/2, (-3 - m)/2, 1/(a^2*x^2)])/(5 + m)) + (3*(x^(-1))^(-4 - m)*Hyp
ergeometric2F1[-1/2, (-4 - m)/2, (-2 - m)/2, 1/(a^2*x^2)])/(a*(4 + m)) - (
3*(x^(-1))^(-3 - m)*Hypergeometric2F1[-1/2, (-3 - m)/2, (-1 - m)/2, 1/(a^2
*x^2)])/(a^2*(3 + m)) + ((x^(-1))^(-2 - m)*Hypergeometric2F1[-1/2, (-2 - m
)/2, -1/2*m, 1/(a^2*x^2)])/(a^3*(2 + m))))
```

Defintions of rubi rules used

rule 147

```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Int[ExpandIntegrand[(a + b*x)^n*(c + d*x)^n*(f*x)^p, (a + b*x)^(m -
n), x], x] /; FreeQ[{a, b, c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && IG
tQ[m - n, 0] && NeQ[m + n + p + 2, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6729
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-d^p)*(e*x)^m*(1/x)^m   Subst[Int[((1 + c*(x/d))^p*
((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[
{a, c, d, e, m, n}, x] && EqQ[a^2*c^2 - d^2, 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(ex)^m (-acx+c)^4}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^4,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^4,x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^4\,dx = \int \frac{(acx-c)^4(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^4,x, algorithm="fri
cas")
```

output
```
integral((a^4*c^4*x^4 - 2*a^3*c^4*x^3 + 2*a*c^4*x - c^4)*(e*x)^m*sqrt((a*x
 - 1)/(a*x + 1)), x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^4\,dx = c^4\left(\int \frac{(ex)^m}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \left(-\frac{4ax(ex)^m}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx + \int \frac{6a^2x^2(ex)^m}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx + \int \left(-\frac{4a^3x^3(ex)^m}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx + \int \frac{a^4x^4(ex)^m}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(e*x)**m*(-a*c*x+c)**4,x)
```

output
```
c**4*(Integral((e*x)**m/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-
4*a*x*(e*x)**m/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(6*a**2*x**
2*(e*x)**m/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-4*a**3*x**3*(
e*x)**m/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**4*x**4*(e*x)**
m/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^4\,dx = \int \frac{(acx-c)^4(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^4,x, algorithm="max
ima")
```

output
```
integrate((a*c*x - c)^4*(e*x)^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^4\,dx=\int\frac{(acx-c)^4(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^4,x, algorithm="gia
c")
```

output
```
integrate((a*c*x - c)^4*(e*x)^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^4\,dx=\int\frac{(e\,x)^m\,(c-a\,c\,x)^4}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(((e*x)^m*(c - a*c*x)^4)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(((e*x)^m*(c - a*c*x)^4)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^4\,dx=e^m c^4\left(\left(\int\frac{x^m\sqrt{ax+1}\,x^4}{\sqrt{ax-1}}dx\right)a^4-4\left(\int\frac{x^m\sqrt{ax+1}\,x^3}{\sqrt{ax-1}}dx\right)a^3+6\left(\int\frac{x^m\sqrt{ax+1}\,x^2}{\sqrt{ax-1}}dx\right)a^2-4\left(\int\frac{x^m\sqrt{ax+1}\,x}{\sqrt{ax-1}}dx\right)a+\int\frac{x^m\sqrt{ax+1}}{\sqrt{ax-1}}dx\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^4,x)
```

output
```
e**m*c**4*(int((x**m*sqrt(a*x + 1)*x**4)/sqrt(a*x - 1),x)*a**4 - 4*int((x*
*m*sqrt(a*x + 1)*x**3)/sqrt(a*x - 1),x)*a**3 + 6*int((x**m*sqrt(a*x + 1)*x
**2)/sqrt(a*x - 1),x)*a**2 - 4*int((x**m*sqrt(a*x + 1)*x)/sqrt(a*x - 1),x)
*a + int((x**m*sqrt(a*x + 1))/sqrt(a*x - 1),x))
```

3.382 $\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 145

$$\begin{aligned}\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^3\,dx &= \frac{a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4(ex)^m}{1+m}\\ &\quad -\frac{a^3c^3(5+2m)x^4(ex)^m\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-4-m),\frac{1}{2}(-2-m),\frac{1}{a^2x^2}\right)}{(1+m)(4+m)}\\ &\quad +\frac{2a^2c^3x^3(ex)^m\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-3-m),\frac{1}{2}(-1-m),\frac{1}{a^2x^2}\right)}{3+m}\end{aligned}$$

output
```
a^3*c^3*(1-1/a^2/x^2)^(3/2)*x^4*(e*x)^m/(1+m)-a^3*c^3*(5+2*m)*x^4*(e*x)^m*
hypergeom([-1/2, -2-1/2*m],[-1-1/2*m],1/a^2/x^2)/(1+m)/(4+m)+2*a^2*c^3*x^3
*(e*x)^m*hypergeom([-1/2, -3/2-1/2*m],[-1/2-1/2*m],1/a^2/x^2)/(3+m)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.83

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^3\,dx$$
$$=ac^3x^2(ex)^m\left(-\frac{a^2x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-2-\frac{m}{2},-1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{4+m}+\frac{2ax\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{2}-\frac{m}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{a^2x^2}\right)}{3+m}-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},-1-\frac{m}{2},-\frac{m}{2},\frac{1}{a^2x^2}\right)}{2+m}\right)$$

input
```
Integrate[E^ArcCoth[a*x]*(e*x)^m*(c - a*c*x)^3,x]
```

output
```
a*c^3*x^2*(e*x)^m*(-((a^2*x^2*Hypergeometric2F1[-1/2, -2 - m/2, -1 - m/2,
1/(a^2*x^2)])/(4 + m)) + (2*a*x*Hypergeometric2F1[-1/2, -3/2 - m/2, -1/2 -
 m/2, 1/(a^2*x^2)])/(3 + m) - Hypergeometric2F1[-1/2, -1 - m/2, -1/2*m, 1/
(a^2*x^2)]/(2 + m))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6729, 147, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^3e^{\coth^{-1}(ax)}(ex)^m\,dx$$
$$\Big\downarrow \text{ 6729}$$
$$a^3c^3\left(\frac{1}{x}\right)^m(ex)^m\int \left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-5}d\frac{1}{x}$$
$$\Big\downarrow \text{ 147}$$

$$a^3c^3\left(\frac{1}{x}\right)^m(ex)^m\int\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-5}-\frac{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-4}}{a}+\frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m}}{a^2}\right.$$

$\downarrow$ 2009

$$a^3c^3\left(\frac{1}{x}\right)^m(ex)^m\left(-\frac{\left(\frac{1}{x}\right)^{-m-4}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-m-4),\frac{1}{2}(-m-2),\frac{1}{a^2x^2}\right)}{m+4}+\frac{2\left(\frac{1}{x}\right)^{-m-3}\text{Hypergeor}}{}\right.$$

input
```
Int[E^ArcCoth[a*x]*(e*x)^m*(c - a*c*x)^3,x]
```

output
```
a^3*c^3*(x^(-1))^m*(e*x)^m*(-(((x^(-1))^(-4 - m)*Hypergeometric2F1[-1/2, (
-4 - m)/2, (-2 - m)/2, 1/(a^2*x^2)])/(4 + m)) + (2*(x^(-1))^(-3 - m)*Hyper
geometric2F1[-1/2, (-3 - m)/2, (-1 - m)/2, 1/(a^2*x^2)])/(a*(3 + m)) - ((x
^(-1))^(-2 - m)*Hypergeometric2F1[-1/2, (-2 - m)/2, -1/2*m, 1/(a^2*x^2)])/
(a^2*(2 + m)))
```

Defintions of rubi rules used

rule 147
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Int[ExpandIntegrand[(a + b*x)^n*(c + d*x)^n*(f*x)^p, (a + b*x)^(m -
n), x], x] /; FreeQ[{a, b, c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && IG
tQ[m - n, 0] && NeQ[m + n + p + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6729
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-d^p)*(e*x)^m*(1/x)^m   Subst[Int[((1 + c*(x/d))^p*
((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[
{a, c, d, e, m, n}, x] && EqQ[a^2*c^2 - d^2, 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(ex)^m (-acx+c)^3}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^3,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^3,x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^3\,dx = \int -\frac{(acx-c)^3(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^3,x, algorithm="fri
cas")
```

output
```
integral(-(a^3*c^3*x^3 - a^2*c^3*x^2 - a*c^3*x + c^3)*(e*x)^m*sqrt((a*x -
1)/(a*x + 1)), x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^3\,dx = -c^3\left(\int\left(-\frac{(ex)^m}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx + \int\frac{3ax(ex)^m}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx + \int\left(-\frac{3a^2x^2(ex)^m}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx + \int\frac{a^3x^3(ex)^m}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(e*x)**m*(-a*c*x+c)**3,x)
```

output
```
-c**3*(Integral(-(e*x)**m/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral
(3*a*x*(e*x)**m/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-3*a**2*x
**2*(e*x)**m/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**3*x**3*(e
*x)**m/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^3\,dx=\int -\frac{(acx-c)^3(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^3,x, algorithm="max
ima")
```

output
```
-integrate((a*c*x - c)^3*(e*x)^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^3,x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^3\,dx=\int\frac{(e\,x)^m\,(c-a\,c\,x)^3}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(((e*x)^m*(c - a*c*x)^3)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(((e*x)^m*(c - a*c*x)^3)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^3\,dx=e^mc^3\left(-\left(\int\frac{x^m\sqrt{ax+1}\,x^3}{\sqrt{ax-1}}dx\right)a^3+3\left(\int\frac{x^m\sqrt{ax+1}\,x^2}{\sqrt{ax-1}}dx\right)a^2-3\left(\int\frac{x^m\sqrt{ax+1}\,x}{\sqrt{ax-1}}dx\right)a+\int\frac{x^m\sqrt{ax+1}}{\sqrt{ax-1}}dx\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^3,x)
```

output
```
e**m*c**3*( - int((x**m*sqrt(a*x + 1)*x**3)/sqrt(a*x - 1),x)*a**3 + 3*int(
(x**m*sqrt(a*x + 1)*x**2)/sqrt(a*x - 1),x)*a**2 - 3*int((x**m*sqrt(a*x + 1
)*x)/sqrt(a*x - 1),x)*a + int((x**m*sqrt(a*x + 1))/sqrt(a*x - 1),x))
```

3.383 $\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^2\,dx = \frac{a^2c^2x^3(ex)^m \operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-3-m),\frac{1}{2}(-1-m),\frac{1}{a^2x^2}\right)}{3+m} - \frac{ac^2x^2(ex)^m \operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-2-m),-\frac{m}{2},\frac{1}{a^2x^2}\right)}{2+m}$$

output
```
a^2*c^2*x^3*(e*x)^m*hypergeom([-1/2, -3/2-1/2*m],[-1/2-1/2*m],1/a^2/x^2)/(
3+m)-a*c^2*x^2*(e*x)^m*hypergeom([-1/2, -1-1/2*m],[-1/2*m],1/a^2/x^2)/(2+m
)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^2\,dx = \frac{ac^2x^2(ex)^m \left(a(2+m)x \operatorname{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{2}-\frac{m}{2},-\frac{1}{2}-\frac{m}{2},\frac{1}{a^2x^2}\right)-(3+m)\operatorname{Hypergeometric2F\right.}{(2+m)(3+m)}$$

input
```
Integrate[E^ArcCoth[a*x]*(e*x)^m*(c - a*c*x)^2,x]
```

output
```
(a*c^2*x^2*(e*x)^m*(a*(2 + m)*x*Hypergeometric2F1[-1/2, -3/2 - m/2, -1/2 -
 m/2, 1/(a^2*x^2)] - (3 + m)*Hypergeometric2F1[-1/2, -1 - m/2, -1/2*m, 1/(
a^2*x^2)]))/((2 + m)*(3 + m))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6729, 92, 135, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^2 e^{\coth^{-1}(ax)}(ex)^m\,dx$$

$$\Big\downarrow \text{6729}$$

$$-a^2c^2\left(\frac{1}{x}\right)^m(ex)^m\int \left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-4}d\frac{1}{x}$$

$$\Big\downarrow \text{92}$$

$$-a^2c^2\left(\frac{1}{x}\right)^m(ex)^m\left(\int \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-4}d\frac{1}{x}-\frac{\int \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-3}d\frac{1}{x}}{a}\right)$$

$$\Big\downarrow \text{135}$$

$$-a^2c^2\left(\frac{1}{x}\right)^m(ex)^m\left(\int \sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{x}\right)^{-m-4}d\frac{1}{x}-\frac{\int \sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{x}\right)^{-m-3}d\frac{1}{x}}{a}\right)$$

$$\Big\downarrow \text{278}$$

$$-a^2c^2\left(\frac{1}{x}\right)^m(ex)^m\left(\frac{\left(\frac{1}{x}\right)^{-m-2}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-m-2),-\frac{m}{2},\frac{1}{a^2x^2}\right)}{a(m+2)}-\frac{\left(\frac{1}{x}\right)^{-m-3}\text{Hypergeometric2F}$$

input
```
Int[E^ArcCoth[a*x]*(e*x)^m*(c - a*c*x)^2,x]
```

output
```
-(a^2*c^2*(x^(-1))^m*(e*x)^m*(-(((x^(-1))^(-3 - m)*Hypergeometric2F1[-1/2,
 (-3 - m)/2, (-1 - m)/2, 1/(a^2*x^2)])/(3 + m)) + ((x^(-1))^(-2 - m)*Hyper
geometric2F1[-1/2, (-2 - m)/2, -1/2*m, 1/(a^2*x^2)])/(a*(2 + m))))
```

Defintions of rubi rules used

rule 92
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Simp[a   Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Simp[b/f   In
t[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, f, m,
 n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[p] &&
!IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

rule 135
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f, m, n, p},
 x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && GtQ[a, 0] && GtQ[c, 0]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6729
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-d^p)*(e*x)^m*(1/x)^m   Subst[Int[((1 + c*(x/d))^p*
((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[
{a, c, d, e, m, n}, x] && EqQ[a^2*c^2 - d^2, 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(ex)^m (-acx+c)^2}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^2,x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^2\,dx = \int \frac{(acx-c)^2(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^2,x, algorithm="fri
cas")
```

output
```
integral((a^2*c^2*x^2 - c^2)*(e*x)^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^2\,dx = c^2\left(\int \frac{(ex)^m}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx + \int \left(-\frac{2ax(ex)^m}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx + \int \frac{a^2x^2(ex)^m}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(e*x)**m*(-a*c*x+c)**2,x)
```

output
```
c**2*(Integral((e*x)**m/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-
2*a*x*(e*x)**m/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**2*x**2*
(e*x)**m/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^2\,dx=\int\frac{(acx-c)^2(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^2,x, algorithm="max
ima")
```

output
```
integrate((a*c*x - c)^2*(e*x)^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^2\,dx=\int\frac{(acx-c)^2(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^2,x, algorithm="gia
c")
```

output
```
integrate((a*c*x - c)^2*(e*x)^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^2\,dx=\int\frac{(e\,x)^m\,(c-a\,c\,x)^2}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(((e*x)^m*(c - a*c*x)^2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(((e*x)^m*(c - a*c*x)^2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^2\,dx=e^m c^2\left(\left(\int\frac{x^m\sqrt{ax+1}\,x^2}{\sqrt{ax-1}}dx\right)a^2-2\left(\int\frac{x^m\sqrt{ax+1}\,x}{\sqrt{ax-1}}dx\right)a+\int\frac{x^m\sqrt{ax+1}}{\sqrt{ax-1}}dx\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^2,x)
```

output
```
e**m*c**2*(int((x**m*sqrt(a*x + 1)*x**2)/sqrt(a*x - 1),x)*a**2 - 2*int((x*
*m*sqrt(a*x + 1)*x)/sqrt(a*x - 1),x)*a + int((x**m*sqrt(a*x + 1))/sqrt(a*x
 - 1),x))
```

3.384 $\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 42

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)\,dx = -\frac{acx^2(ex)^m \operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-2-m),-\frac{m}{2},\frac{1}{a^2x^2}\right)}{2+m}$$

output
```
-a*c*x^2*(e*x)^m*hypergeom([-1/2, -1-1/2*m],[-1/2*m],1/a^2/x^2)/(2+m)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)\,dx = -\frac{acx^2(ex)^m \operatorname{Hypergeometric2F1}\left(-\frac{1}{2},-1-\frac{m}{2},-\frac{m}{2},\frac{1}{a^2x^2}\right)}{2+m}$$

input
```
Integrate[E^ArcCoth[a*x]*(e*x)^m*(c - a*c*x),x]
```

output
```
-((a*c*x^2*(e*x)^m*Hypergeometric2F1[-1/2, -1 - m/2, -1/2*m, 1/(a^2*x^2)])
/(2 + m))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6729, 135, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)e^{\coth^{-1}(ax)}(ex)^m\,dx$$

$$\Big\downarrow \text{6729}$$

$$ac\left(\frac{1}{x}\right)^m(ex)^m\int \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-3}d\frac{1}{x}$$

$$\Big\downarrow \text{135}$$

$$ac\left(\frac{1}{x}\right)^m(ex)^m\int \sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{x}\right)^{-m-3}d\frac{1}{x}$$

$$\Big\downarrow \text{278}$$

$$-\frac{acx^2(ex)^m\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-m-2),-\frac{m}{2},\frac{1}{a^2x^2}\right)}{m+2}$$

input
```
Int[E^ArcCoth[a*x]*(e*x)^m*(c - a*c*x),x]
```

output
```
-((a*c*x^2*(e*x)^m*Hypergeometric2F1[-1/2, (-2 - m)/2, -1/2*m, 1/(a^2*x^2)
])/(2 + m))
```

Defintions of rubi rules used

rule 135
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f, m, n, p},
 x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && GtQ[a, 0] && GtQ[c, 0]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6729
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-d^p)*(e*x)^m*(1/x)^m   Subst[Int[((1 + c*(x/d))^p*
((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[
{a, c, d, e, m, n}, x] && EqQ[a^2*c^2 - d^2, 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(ex)^m (-acx+c)}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c),x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)\,dx=\int -\frac{(acx-c)(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c),x, algorithm="frica
s")
```

output
```
integral(-(a*c*x + c)*(e*x)^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)\,dx=-c\left(\int\left(-\frac{(ex)^m}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{ax(ex)^m}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(e*x)**m*(-a*c*x+c),x)
```

output
```
-c*(Integral(-(e*x)**m/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a*
x*(e*x)**m/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)\,dx=\int-\frac{(acx-c)(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c),x, algorithm="maxim
a")
```

output
```
-integrate((a*c*x - c)*(e*x)^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)\,dx = \int \frac{(e\,x)^m\,(c-a\,c\,x)}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(((e*x)^m*(c - a*c*x))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(((e*x)^m*(c - a*c*x))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)\,dx = e^m c\left(-\left(\int \frac{x^m\sqrt{ax+1}\,x}{\sqrt{ax-1}}dx\right)a + \int \frac{x^m\sqrt{ax+1}}{\sqrt{ax-1}}dx\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c),x)
```

output
```
e**m*c*( - int((x**m*sqrt(a*x + 1)*x)/sqrt(a*x - 1),x)*a + int((x**m*sqrt(
a*x + 1))/sqrt(a*x - 1),x))
```

3.385 $\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{c-acx} dx$

Optimal result

Integrand size = 21, antiderivative size = 133

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{c-acx} dx = \frac{(ex)^m}{ac(1+m)\sqrt{1-\frac{1}{a^2x^2}}} + \frac{2(ex)^m \text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-m}{2},\frac{3-m}{2},\frac{1}{a^2x^2}\right)}{a^2c(1-m)x} - \frac{(1+2m)(ex)^m \text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{a^2x^2}\right)}{acm(1+m)}$$

output
```
(e*x)^m/a/c/(1+m)/(1-1/a^2/x^2)^(1/2)+2*(e*x)^m*hypergeom([3/2, 1/2-1/2*m]
,[3/2-1/2*m],1/a^2/x^2)/a^2/c/(1-m)/x-(1+2*m)*(e*x)^m*hypergeom([3/2, -1/2
*m],[1-1/2*m],1/a^2/x^2)/a/c/m/(1+m)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{c-acx}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}x(ex)^m\sqrt{1-ax}\sqrt{\frac{1+ax}{a^2}}\left(\text{AppellF1}\left(m,-\frac{1}{2},\frac{1}{2},1+m,-ax,ax\right)-\text{AppellF1}\left(m,-\frac{1}{2},\frac{3}{2},1+m\right.\right.}{cm\sqrt{-1+ax}\sqrt{1+ax}\sqrt{-\frac{1}{a^2}+x^2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*(e*x)^m)/(c - a*c*x),x]
```

output
```
-((Sqrt[1 - 1/(a^2*x^2)]*x*(e*x)^m*Sqrt[1 - a*x]*Sqrt[(1 + a*x)/a^2]*(Appe
llF1[m, -1/2, 1/2, 1 + m, -(a*x), a*x] - AppellF1[m, -1/2, 3/2, 1 + m, -(a
*x), a*x]))/(c*m*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[-a^(-2) + x^2]))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6729, 147, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{c-acx}\,dx$$

$$\Big\downarrow \text{ 6729}$$

$$\frac{\left(\frac{1}{x}\right)^m(ex)^m\int \frac{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-1}}{\left(1-\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{ 147}$$

$$\frac{\left(\frac{1}{x}\right)^m (ex)^m \int \left(\frac{\left(\frac{1}{x}\right)^{-m-1}}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{\left(\frac{1}{x}\right)^{1-m}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{2\left(\frac{1}{x}\right)^{-m}}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}\right)d\frac{1}{x}}{ac}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{1}{x}\right)^m (ex)^m \left(\frac{2\left(\frac{1}{x}\right)^{1-m}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-m}{2},\frac{3-m}{2},\frac{1}{a^2x^2}\right)}{a(1-m)}+\frac{\left(\frac{1}{x}\right)^{2-m}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2-m}{2},\frac{4-m}{2},\frac{1}{a^2x^2}\right)}{a^2(2-m)}-\frac{\left(\frac{1}{x}\right)^{-m}\text{Hyper}}{}\right.}{ac}$$

input
```
Int[(E^ArcCoth[a*x]*(e*x)^m)/(c - a*c*x),x]
```

output
```
((x^(-1))^m*(e*x)^m*((2*(x^(-1))^(1 - m)*Hypergeometric2F1[3/2, (1 - m)/2,
 (3 - m)/2, 1/(a^2*x^2)])/(a*(1 - m)) + ((x^(-1))^(2 - m)*Hypergeometric2F
1[3/2, (2 - m)/2, (4 - m)/2, 1/(a^2*x^2)])/(a^2*(2 - m)) - Hypergeometric2
F1[3/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)]/(m*(x^(-1))^m)))/(a*c)
```

Defintions of rubi rules used

rule 147
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Int[ExpandIntegrand[(a + b*x)^n*(c + d*x)^n*(f*x)^p, (a + b*x)^(m -
n), x], x] /; FreeQ[{a, b, c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && IG
tQ[m - n, 0] && NeQ[m + n + p + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6729
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-d^p)*(e*x)^m*(1/x)^m   Subst[Int[((1 + c*(x/d))^p*
((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[
{a, c, d, e, m, n}, x] && EqQ[a^2*c^2 - d^2, 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}\,(-acx+c)}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c),x)
```

Fricas [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{c-acx}\,dx = \int -\frac{(ex)^m}{(acx-c)\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c),x, algorithm="frica
s")
```

output
```
integral(-(a*x + 1)*(e*x)^m*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x^2 - 2*a*c*x
 + c), x)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{c-acx}\,dx = -\frac{\int \frac{(ex)^m}{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(e*x)**m/(-a*c*x+c),x)
```

output
```
-Integral((e*x)**m/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))), x)/c
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{c-acx}\,dx=\int -\frac{(ex)^m}{(acx-c)\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c),x, algorithm="maxim
a")
```

output
```
-integrate((e*x)^m/((a*c*x - c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{c-acx}\,dx=\int -\frac{(ex)^m}{(acx-c)\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c),x, algorithm="giac"
)
```

output
```
integrate(-(e*x)^m/((a*c*x - c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{c-acx}\,dx=\int \frac{(e\,x)^m}{(c-a\,c\,x)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((e*x)^m/((c - a*c*x)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((e*x)^m/((c - a*c*x)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{c-acx}\,dx = -\frac{e^m\left(\int \frac{x^m\sqrt{ax+1}}{\sqrt{ax-1}\,ax-\sqrt{ax-1}}dx\right)}{c}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c),x)
```

output
```
( - e**m*int((x**m*sqrt(a*x + 1))/(sqrt(a*x - 1)*a*x - sqrt(a*x - 1)),x))/
c
```

3.386 $\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^2}\,dx = -\frac{4\left(a+\frac{1}{x}\right)(ex)^m}{3a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x} + \frac{(1-4m)(ex)^m \operatorname{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-m}{2},\frac{3-m}{2},\frac{1}{a^2x^2}\right)}{3a^2c^2(1-m)x} - \frac{(1+4m)(ex)^m \operatorname{Hypergeometric2F1}\left(\frac{3}{2},\frac{2-m}{2},\frac{4-m}{2},\frac{1}{a^2x^2}\right)}{3a^3c^2(2-m)x^2}$$

output
```
-4/3*(a+1/x)*(e*x)^m/a^3/c^2/(1-1/a^2/x^2)^(3/2)/x+1/3*(1-4*m)*(e*x)^m*hyp
ergeom([3/2, 1/2-1/2*m],[3/2-1/2*m],1/a^2/x^2)/a^2/c^2/(1-m)/x-1/3*(1+4*m)
*(e*x)^m*hypergeom([3/2, 1-1/2*m],[2-1/2*m],1/a^2/x^2)/a^3/c^2/(2-m)/x^2
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.78

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^2}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}x(ex)^m\sqrt{1-ax}\sqrt{\frac{1+ax}{a^2}}\left(\text{AppellF1}\left(m,-\frac{1}{2},\frac{3}{2},1+m,-ax,ax\right)-\text{AppellF1}\left(m,-\frac{1}{2},\frac{5}{2},1+m\right.\right.}{c^2m\sqrt{-1+ax}\sqrt{1+ax}\sqrt{-\frac{1}{a^2}+x^2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*(e*x)^m)/(c - a*c*x)^2,x]
```

output
```
-((Sqrt[1 - 1/(a^2*x^2)]*x*(e*x)^m*Sqrt[1 - a*x]*Sqrt[(1 + a*x)/a^2]*(Appe
llF1[m, -1/2, 3/2, 1 + m, -(a*x), a*x] - AppellF1[m, -1/2, 5/2, 1 + m, -(a
*x), a*x]))/(c^2*m*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[-a^(-2) + x^2]))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.37, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6729, 147, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^2}\,dx$$

$$\Big\downarrow \text{6729}$$

$$-\frac{\left(\frac{1}{x}\right)^m(ex)^m\int \frac{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m}}{\left(1-\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{a^2c^2}$$

$$\Big\downarrow \text{147}$$

$$-\frac{\left(\frac{1}{x}\right)^m (ex)^m \int \left(\frac{3\left(\frac{1}{x}\right)^{1-m}}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{3\left(\frac{1}{x}\right)^{2-m}}{a^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{\left(\frac{1}{x}\right)^{3-m}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{\left(\frac{1}{x}\right)^{-m}}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}\right)d\frac{1}{x}}{a^2c^2}$$

$\Big\downarrow$ 2009

$$-\frac{\left(\frac{1}{x}\right)^m (ex)^m \left(\frac{\left(\frac{1}{x}\right)^{1-m}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-m}{2},\frac{3-m}{2},\frac{1}{a^2x^2}\right)}{1-m}+\frac{3\left(\frac{1}{x}\right)^{2-m}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{2-m}{2},\frac{4-m}{2},\frac{1}{a^2x^2}\right)}{a(2-m)}+\frac{3\left(\frac{1}{x}\right)^{3-m}\text{H}\dots}{\dots}\right.}{a^2c^2}$$

input
```
Int[(E^ArcCoth[a*x]*(e*x)^m)/(c - a*c*x)^2,x]
```

output
```
-(((x^(-1))^m*(e*x)^m*(((x^(-1))^(1 - m)*Hypergeometric2F1[5/2, (1 - m)/2,
 (3 - m)/2, 1/(a^2*x^2)])/(1 - m) + (3*(x^(-1))^(2 - m)*Hypergeometric2F1[
5/2, (2 - m)/2, (4 - m)/2, 1/(a^2*x^2)])/(a*(2 - m)) + (3*(x^(-1))^(3 - m)
*Hypergeometric2F1[5/2, (3 - m)/2, (5 - m)/2, 1/(a^2*x^2)])/(a^2*(3 - m))
+ ((x^(-1))^(4 - m)*Hypergeometric2F1[5/2, (4 - m)/2, (6 - m)/2, 1/(a^2*x^
2)])/(a^3*(4 - m))))/(a^2*c^2))
```

Defintions of rubi rules used

rule 147
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Int[ExpandIntegrand[(a + b*x)^n*(c + d*x)^n*(f*x)^p, (a + b*x)^(m -
n), x], x] /; FreeQ[{a, b, c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && IG
tQ[m - n, 0] && NeQ[m + n + p + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6729
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-d^p)*(e*x)^m*(1/x)^m   Subst[Int[((1 + c*(x/d))^p*
((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[
{a, c, d, e, m, n}, x] && EqQ[a^2*c^2 - d^2, 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}\,(-acx+c)^2}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^2,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^2,x)
```

Fricas [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^2}\,dx = \int \frac{(ex)^m}{(acx-c)^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^2,x, algorithm="fri
cas")
```

output
```
integral((a*x + 1)*(e*x)^m*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^3 - 3*a^2*
c^2*x^2 + 3*a*c^2*x - c^2), x)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^2}\,dx = \frac{\int \frac{(ex)^m}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(e*x)**m/(-a*c*x+c)**2,x)
```

output
```
Integral((e*x)**m/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a*x*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c*
*2
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^2}\,dx = \int \frac{(ex)^m}{(acx-c)^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^2,x, algorithm="max
ima")
```

output
```
integrate((e*x)^m/((a*c*x - c)^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^2}\,dx = \int \frac{(ex)^m}{(acx-c)^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^2,x, algorithm="gia
c")
```

output
```
integrate((e*x)^m/((a*c*x - c)^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^2}\,dx = \int \frac{(e\,x)^m}{(c-a\,c\,x)^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((e*x)^m/((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((e*x)^m/((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^2}\,dx = \frac{e^m\left(\int \frac{x^m\sqrt{ax+1}}{\sqrt{ax-1}\,a^2x^2-2\sqrt{ax-1}\,ax+\sqrt{ax-1}}dx\right)}{c^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^2,x)
```

output
```
(e**m*int((x**m*sqrt(a*x + 1))/(sqrt(a*x - 1)*a**2*x**2 - 2*sqrt(a*x - 1)*
a*x + sqrt(a*x - 1)),x))/c**2
```

3.387 $\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 201

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^3}\,dx &= -\frac{(ex)^m}{a^3c^3(1+m)\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}+\frac{8\left(a+\frac{1}{x}\right)(ex)^m}{5a^4c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^2}\\&\quad-\frac{(1+8m-8m^2)(ex)^m\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{2-m}{2},\frac{4-m}{2},\frac{1}{a^2x^2}\right)}{5a^3c^3(2-m)(1+m)x^2}\\&\quad-\frac{4(1-2m)(ex)^m\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{3-m}{2},\frac{5-m}{2},\frac{1}{a^2x^2}\right)}{5a^4c^3(3-m)x^3}\end{aligned}$$

output
```
-(e*x)^m/a^3/c^3/(1+m)/(1-1/a^2/x^2)^(3/2)/x^2+8/5*(a+1/x)*(e*x)^m/a^4/c^3
/(1-1/a^2/x^2)^(5/2)/x^2-1/5*(-8*m^2+8*m+1)*(e*x)^m*hypergeom([5/2, 1-1/2*
m],[2-1/2*m],1/a^2/x^2)/a^3/c^3/(2-m)/(1+m)/x^2-4/5*(1-2*m)*(e*x)^m*hyperg
eom([5/2, 3/2-1/2*m],[5/2-1/2*m],1/a^2/x^2)/a^4/c^3/(3-m)/x^3
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.35 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.60

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^3}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}x(ex)^m\sqrt{1-ax}\sqrt{\frac{1+ax}{a^2}}\left(\text{AppellF1}\left(m,-\frac{1}{2},\frac{5}{2},1+m,-ax,ax\right)-\text{AppellF1}\left(m,-\frac{1}{2},\frac{7}{2},1+m\right.\right.}{c^3m\sqrt{-1+ax}\sqrt{1+ax}\sqrt{-\frac{1}{a^2}+x^2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*(e*x)^m)/(c - a*c*x)^3,x]
```

output
```
-((Sqrt[1 - 1/(a^2*x^2)]*x*(e*x)^m*Sqrt[1 - a*x]*Sqrt[(1 + a*x)/a^2]*(Appe
llF1[m, -1/2, 5/2, 1 + m, -(a*x), a*x] - AppellF1[m, -1/2, 7/2, 1 + m, -(a
*x), a*x]))/(c^3*m*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[-a^(-2) + x^2]))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6729, 147, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^3}dx$$

$$\Big\downarrow \text{6729}$$

$$\frac{\left(\frac{1}{x}\right)^m(ex)^m\int \frac{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{1-m}}{\left(1-\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{a^3c^3}$$

$$\Big\downarrow \text{147}$$

$$\frac{\left(\frac{1}{x}\right)^m (ex)^m \int \left(\frac{\left(\frac{1}{x}\right)^{1-m}}{\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{4\left(\frac{1}{x}\right)^{2-m}}{a\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{6\left(\frac{1}{x}\right)^{3-m}}{a^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{4\left(\frac{1}{x}\right)^{4-m}}{a^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{(}{a^4\left(1-\frac{1}{ax}\right.}\right.}{a^3c^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{\left(\frac{1}{x}\right)^m (ex)^m \left(\frac{\left(\frac{1}{x}\right)^{2-m} \text{Hypergeometric2F1}\left(\frac{7}{2},\frac{2-m}{2},\frac{4-m}{2},\frac{1}{a^2x^2}\right)}{2-m} + \frac{4\left(\frac{1}{x}\right)^{3-m} \text{Hypergeometric2F1}\left(\frac{7}{2},\frac{3-m}{2},\frac{5-m}{2},\frac{1}{a^2x^2}\right)}{a(3-m)} + \frac{6\left(\frac{1}{x}\right)^{4-m} \text{Hyp}}{}\right.}{}$$

input
```
Int[(E^ArcCoth[a*x]*(e*x)^m)/(c - a*c*x)^3,x]
```

output
```
((x^(-1))^m*(e*x)^m*(((x^(-1))^(2 - m)*Hypergeometric2F1[7/2, (2 - m)/2, (
4 - m)/2, 1/(a^2*x^2)])/(2 - m) + (4*(x^(-1))^(3 - m)*Hypergeometric2F1[7/
2, (3 - m)/2, (5 - m)/2, 1/(a^2*x^2)])/(a*(3 - m)) + (6*(x^(-1))^(4 - m)*H
ypergeometric2F1[7/2, (4 - m)/2, (6 - m)/2, 1/(a^2*x^2)])/(a^2*(4 - m)) +
(4*(x^(-1))^(5 - m)*Hypergeometric2F1[7/2, (5 - m)/2, (7 - m)/2, 1/(a^2*x^
2)])/(a^3*(5 - m)) + ((x^(-1))^(6 - m)*Hypergeometric2F1[7/2, (6 - m)/2, (
8 - m)/2, 1/(a^2*x^2)])/(a^4*(6 - m))))/(a^3*c^3)
```

Defintions of rubi rules used

rule 147
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Int[ExpandIntegrand[(a + b*x)^n*(c + d*x)^n*(f*x)^p, (a + b*x)^(m -
n), x], x] /; FreeQ[{a, b, c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && IG
tQ[m - n, 0] && NeQ[m + n + p + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6729
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-d^p)*(e*x)^m*(1/x)^m   Subst[Int[((1 + c*(x/d))^p*
((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(n/2), x], x, 1/x], x] /; FreeQ[
{a, c, d, e, m, n}, x] && EqQ[a^2*c^2 - d^2, 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}\,(-acx+c)^3}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^3,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^3,x)
```

Fricas [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^3}\,dx = \int -\frac{(ex)^m}{(acx-c)^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^3,x, algorithm="fri
cas")
```

output
```
integral(-(a*x + 1)*(e*x)^m*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^3*x^4 - 4*a^3
*c^3*x^3 + 6*a^2*c^3*x^2 - 4*a*c^3*x + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(e*x)**m/(-a*c*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^3}\,dx = \int -\frac{(ex)^m}{(acx-c)^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^3,x, algorithm="max
ima")
```

output
```
-integrate((e*x)^m/((a*c*x - c)^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^3}\,dx = \int -\frac{(ex)^m}{(acx-c)^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^3,x, algorithm="gia
c")
```

output
```
integrate(-(e*x)^m/((a*c*x - c)^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^3}\,dx = \int \frac{(e\,x)^m}{(c-a\,c\,x)^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((e*x)^m/((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((e*x)^m/((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^3}\,dx = -\frac{e^m\left(\int \frac{x^m\sqrt{ax+1}}{\sqrt{ax-1}\,a^3x^3-3\sqrt{ax-1}\,a^2x^2+3\sqrt{ax-1}\,ax-\sqrt{ax-1}}dx\right)}{c^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^3,x)
```

output
```
( - e**m*int((x**m*sqrt(a*x + 1))/(sqrt(a*x - 1)*a**3*x**3 - 3*sqrt(a*x -
1)*a**2*x**2 + 3*sqrt(a*x - 1)*a*x - sqrt(a*x - 1)),x))/c**3
```

3.388 $\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 214

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{5/2}\,dx=\frac{(9+4m)\left(1+\frac{1}{ax}\right)^{3/2}x(ex)^m(c-acx)^{5/2}}{(1+m)(7+2m)\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x(ex)^m(c-acx)^{5/2}}{a(1+m)\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{(71+72m+16m^2)(ex)^m(c-acx)^{5/2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{5}{2}-m,-\frac{3}{2}-m,-\frac{1}{ax}\right)}{a(1+m)(5+2m)(7+2m)\left(1-\frac{1}{ax}\right)^{5/2}}$$

output
```
(9+4*m)*(1+1/a/x)^(3/2)*x*(e*x)^m*(-a*c*x+c)^(5/2)/(1+m)/(7+2*m)/(1-1/a/x)
^(5/2)-(a-1/x)*(1+1/a/x)^(3/2)*x*(e*x)^m*(-a*c*x+c)^(5/2)/a/(1+m)/(1-1/a/x
)^(5/2)-(16*m^2+72*m+71)*(e*x)^m*(-a*c*x+c)^(5/2)*hypergeom([-1/2, -5/2-m]
,[-3/2-m],-1/a/x)/a/(1+m)/(5+2*m)/(7+2*m)/(1-1/a/x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.62

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{5/2}\,dx = \frac{c^2x(ex)^m\sqrt{c-acx}\left((5+2m)\sqrt{1+\frac{1}{ax}}(1+ax)(7+2ax+2m(1+ax))-a(71+72m+16\right.}{(1+m)(5+2m)(7+2m)\sqrt{1-\frac{1}{a}}}$$

input
```
Integrate[E^ArcCoth[a*x]*(e*x)^m*(c - a*c*x)^(5/2),x]
```

output
```
(c^2*x*(e*x)^m*Sqrt[c - a*c*x]*((5 + 2*m)*Sqrt[1 + 1/(a*x)]*(1 + a*x)*(7 +
 2*a*x + 2*m*(1 + a*x)) - a*(71 + 72*m + 16*m^2)*x*Hypergeometric2F1[-1/2,
 -5/2 - m, -3/2 - m, -(1/(a*x))]))/((1 + m)*(5 + 2*m)*(7 + 2*m)*Sqrt[1 - 1
/(a*x)])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6730, 27, 101, 27, 88, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{5/2}e^{\coth^{-1}(ax)}(ex)^m\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{5}{2}}(c-acx)^{5/2}(ex)^m\int \frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-\frac{9}{2}}}{a^2}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{5}{2}}(c-acx)^{5/2}(ex)^m\int \left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-\frac{9}{2}}d\frac{1}{x}}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{101}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{5}{2}}(c-acx)^{5/2}(ex)^m\left(\frac{a\left(a-\frac{1}{x}\right)\left(\frac{1}{ax}+1\right)^{3/2}\left(\frac{1}{x}\right)^{-m-\frac{7}{2}}}{m+1}-\frac{a\int -\frac{1}{2}\sqrt{1+\frac{1}{ax}}\left(a(4m+9)-\frac{4m+5}{x}\right)\left(\frac{1}{x}\right)^{-m-\frac{9}{2}}d\frac{1}{x}}{m+1}\right)}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{5}{2}}(c-acx)^{5/2}(ex)^m\left(\frac{a\int \sqrt{1+\frac{1}{ax}}\left(a(4m+9)-\frac{4m+5}{x}\right)\left(\frac{1}{x}\right)^{-m-\frac{9}{2}}d\frac{1}{x}}{2(m+1)}+\frac{a\left(a-\frac{1}{x}\right)\left(\frac{1}{ax}+1\right)^{3/2}\left(\frac{1}{x}\right)^{-m-\frac{7}{2}}}{m+1}\right)}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{88}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{5}{2}}(c-acx)^{5/2}(ex)^m\left(\frac{a\left(-\frac{\left(16m^2+72m+71\right)\int \sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-\frac{7}{2}}d\frac{1}{x}}{2m+7}-\frac{2a(4m+9)\left(\frac{1}{ax}+1\right)^{3/2}\left(\frac{1}{x}\right)^{-m-\frac{7}{2}}}{2m+7}\right)}{2(m+1)}+\frac{a\left(a-\frac{1}{x}\right)\left(\frac{1}{ax}+1\right)^{3/}}{m+1}\right.}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$\Big\downarrow \text{74}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{5}{2}}(c-acx)^{5/2}(ex)^m\left(\frac{a\left(\frac{2\left(16m^2+72m+71\right)\left(\frac{1}{x}\right)^{-m-\frac{5}{2}}\text{Hypergeometric2F1}\left(-\frac{1}{2},-m-\frac{5}{2},-m-\frac{3}{2},-\frac{1}{ax}\right)}{(2m+5)(2m+7)}-\frac{2a(4m+9)\left(\frac{1}{ax}+1\right)^{3/2}\left(\frac{1}{x}\right)^{-}}{2m+7}\right.}{2(m+1)}\right.}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}$$

input
```
Int[E^ArcCoth[a*x]*(e*x)^m*(c - a*c*x)^(5/2),x]
```

output
```
-(((x^(-1))^(5/2 + m)*(e*x)^m*(c - a*c*x)^(5/2)*((a*(a - x^(-1))*(1 + 1/(a
*x))^(3/2)*(x^(-1))^(-7/2 - m))/(1 + m) + (a*((-2*a*(9 + 4*m)*(1 + 1/(a*x)
)^(3/2)*(x^(-1))^(-7/2 - m))/(7 + 2*m) + (2*(71 + 72*m + 16*m^2)*(x^(-1))^
(-5/2 - m)*Hypergeometric2F1[-1/2, -5/2 - m, -3/2 - m, -(1/(a*x))])/((5 +
2*m)*(7 + 2*m))))/(2*(1 + m))))/(a^2*(1 - 1/(a*x))^(5/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{(ex)^m (-acx+c)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(5/2),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(5/2),x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{5/2}\,dx = \int \frac{(-acx+c)^{\frac{5}{2}}(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(5/2),x, algorithm=
"fricas")
```

output
```
integral((a^2*c^2*x^2 - c^2)*sqrt(-a*c*x + c)*(e*x)^m*sqrt((a*x - 1)/(a*x
+ 1)), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(e*x)**m*(-a*c*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{5/2}\,dx=\int\frac{(-acx+c)^{\frac{5}{2}}(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((-a*c*x + c)^(5/2)*(e*x)^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{5/2}\,dx=\int\frac{(-acx+c)^{\frac{5}{2}}(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((-a*c*x + c)^(5/2)*(e*x)^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{5/2}\,dx=\int\frac{(e\,x)^m\,(c-a\,c\,x)^{5/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(((e*x)^m*(c - a*c*x)^(5/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(((e*x)^m*(c - a*c*x)^(5/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{5/2}\,dx = e^m\sqrt{c}\,c^2\left(\left(\int \frac{x^m\sqrt{ax+1}\,\sqrt{-ax+1}\,x^2}{\sqrt{ax-1}}dx\right)a^2 - 2\left(\int \frac{x^m\sqrt{ax+1}\,\sqrt{-ax+1}\,x}{\sqrt{ax-1}}dx\right)a + \int \frac{x^m\sqrt{ax+1}\,\sqrt{-ax+1}}{\sqrt{ax-1}}dx\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(5/2),x)
```

output
```
e**m*sqrt(c)*c**2*(int((x**m*sqrt(a*x + 1)*sqrt( - a*x + 1)*x**2)/sqrt(a*x
 - 1),x)*a**2 - 2*int((x**m*sqrt(a*x + 1)*sqrt( - a*x + 1)*x)/sqrt(a*x - 1
),x)*a + int((x**m*sqrt(a*x + 1)*sqrt( - a*x + 1))/sqrt(a*x - 1),x))
```

3.389 $\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 134

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{3/2}\,dx=\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(ex)^m(c-acx)^{3/2}}{(5+2m)\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{2(7+4m)(ex)^m(c-acx)^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{2}-m,-\frac{1}{2}-m,-\frac{1}{ax}\right)}{a(3+2m)(5+2m)\left(1-\frac{1}{ax}\right)^{3/2}}$$

output
```
2*(1+1/a/x)^(3/2)*x*(e*x)^m*(-a*c*x+c)^(3/2)/(5+2*m)/(1-1/a/x)^(3/2)-2*(7+
4*m)*(e*x)^m*(-a*c*x+c)^(3/2)*hypergeom([-1/2, -3/2-m],[-1/2-m],-1/a/x)/a/
(3+2*m)/(5+2*m)/(1-1/a/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{3/2}\,dx=-\frac{2cx(ex)^m\sqrt{c-acx}\left((3+2m)\sqrt{1+\frac{1}{ax}}(1+ax)-(7+4m)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{2}-m,-\frac{1}{2}-m,\right.\right.}{(3+2m)(5+2m)\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*(e*x)^m*(c - a*c*x)^(3/2),x]
```

output
```
(-2*c*x*(e*x)^m*Sqrt[c - a*c*x]*((3 + 2*m)*Sqrt[1 + 1/(a*x)]*(1 + a*x) - (
7 + 4*m)*Hypergeometric2F1[-1/2, -3/2 - m, -1/2 - m, -(1/(a*x))]))/((3 + 2
*m)*(5 + 2*m)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6730, 27, 88, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-acx)^{3/2}e^{\coth^{-1}(ax)}(ex)^m\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{3}{2}}(c-acx)^{3/2}(ex)^m\int \frac{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-\frac{7}{2}}}{a}d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{3}{2}}(c-acx)^{3/2}(ex)^m\int \left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-\frac{7}{2}}d\frac{1}{x}}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{88}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{3}{2}}(c-acx)^{3/2}(ex)^m\left(-\frac{(4m+7)\int \sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-\frac{5}{2}}d\frac{1}{x}}{2m+5}-\frac{2a\left(\frac{1}{ax}+1\right)^{3/2}\left(\frac{1}{x}\right)^{-m-\frac{5}{2}}}{2m+5}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{74}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{3}{2}}(c-acx)^{3/2}(ex)^m\left(\frac{2(4m+7)\left(\frac{1}{x}\right)^{-m-\frac{3}{2}}\text{Hypergeometric2F1}\left(-\frac{1}{2},-m-\frac{3}{2},-m-\frac{1}{2},-\frac{1}{ax}\right)}{(2m+3)(2m+5)}-\frac{2a\left(\frac{1}{ax}+1\right)^{3/2}\left(\frac{1}{x}\right)^{-m-\frac{5}{2}}}{2m+5}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

input
```
Int[E^ArcCoth[a*x]*(e*x)^m*(c - a*c*x)^(3/2),x]
```

output
```
-(((x^(-1))^(3/2 + m)*(e*x)^m*(c - a*c*x)^(3/2)*((-2*a*(1 + 1/(a*x))^(3/2)
*(x^(-1))^(-5/2 - m))/(5 + 2*m) + (2*(7 + 4*m)*(x^(-1))^(-3/2 - m)*Hyperge
ometric2F1[-1/2, -3/2 - m, -1/2 - m, -(1/(a*x))])/((3 + 2*m)*(5 + 2*m))))/
(a*(1 - 1/(a*x))^(3/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{(ex)^m (-acx+c)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(3/2),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(3/2),x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{3/2}\,dx = \int \frac{(-acx+c)^{\frac{3}{2}}(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(3/2),x, algorithm=
"fricas")
```

output
```
integral(-(a*c*x + c)*sqrt(-a*c*x + c)*(e*x)^m*sqrt((a*x - 1)/(a*x + 1)),
x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(e*x)**m*(-a*c*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{3/2}\,dx=\int \frac{(-acx+c)^{\frac{3}{2}}(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((-a*c*x + c)^(3/2)*(e*x)^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(3/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{3/2}\,dx=\int \frac{(e\,x)^m\,(c-a\,c\,x)^{3/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(((e*x)^m*(c - a*c*x)^(3/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(((e*x)^m*(c - a*c*x)^(3/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m(c-acx)^{3/2}\,dx = e^m\sqrt{c}\,c\left(-\left(\int \frac{x^m\sqrt{ax+1}\,\sqrt{-ax+1}\,x}{\sqrt{ax-1}}dx\right)a + \int \frac{x^m\sqrt{ax+1}\,\sqrt{-ax+1}}{\sqrt{ax-1}}dx\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(3/2),x)
```

output
```
e**m*sqrt(c)*c*( - int((x**m*sqrt(a*x + 1)*sqrt( - a*x + 1)*x)/sqrt(a*x -
1),x)*a + int((x**m*sqrt(a*x + 1)*sqrt( - a*x + 1))/sqrt(a*x - 1),x))
```

3.390 $\int e^{\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int e^{\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx = \frac{2x(ex)^m\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{2}-m,-\frac{1}{2}-m,-\frac{1}{ax}\right)}{(3+2m)\sqrt{1-\frac{1}{ax}}}$$

output
```
2*x*(e*x)^m*(-a*c*x+c)^(1/2)*hypergeom([-1/2, -3/2-m],[-1/2-m],-1/a/x)/(3+
2*m)/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int e^{\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx = -\frac{x(ex)^m\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{2}-m,-\frac{1}{2}-m,-\frac{1}{ax}\right)}{\left(-\frac{3}{2}-m\right)\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*(e*x)^m*Sqrt[c - a*c*x],x]
```

output
```
-((x*(e*x)^m*Sqrt[c - a*c*x]*Hypergeometric2F1[-1/2, -3/2 - m, -1/2 - m, -
(1/(a*x))])/((-3/2 - m)*Sqrt[1 - 1/(a*x)]))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {6730, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{\coth^{-1}(ax)}(ex)^m\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{1}{2}}\sqrt{c-acx}(ex)^m\int \sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-\frac{5}{2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{74}$$

$$\frac{2x\sqrt{c-acx}(ex)^m\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-m-\frac{3}{2},-m-\frac{1}{2},-\frac{1}{ax}\right)}{(2m+3)\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^ArcCoth[a*x]*(e*x)^m*Sqrt[c - a*c*x],x]
```

output
```
(2*x*(e*x)^m*Sqrt[c - a*c*x]*Hypergeometric2F1[-1/2, -3/2 - m, -1/2 - m, -
(1/(a*x))])/((3 + 2*m)*Sqrt[1 - 1/(a*x)])
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] && !IntegerQ[p]
```

Maple [F]

$$\int \frac{(ex)^m \sqrt{-acx+c}}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(1/2),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(1/2),x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx = \int \frac{\sqrt{-acx+c}(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(sqrt(-a*c*x + c)*(a*x + 1)*(e*x)^m*sqrt((a*x - 1)/(a*x + 1))/(a*x
 - 1), x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx=\int\frac{(ex)^m\sqrt{-c(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(e*x)**m*(-a*c*x+c)**(1/2),x)
```

output
```
Integral((e*x)**m*sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx=\int\frac{\sqrt{-acx+c}(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a*c*x + c)*(e*x)^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx=\int\frac{\sqrt{-acx+c}(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(-a*c*x + c)*(e*x)^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx=\int \frac{(e\,x)^m\,\sqrt{c-a\,c\,x}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(((e*x)^m*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(((e*x)^m*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx=e^m\sqrt{c}\left(\int \frac{x^m\sqrt{ax+1}\,\sqrt{-ax+1}}{\sqrt{ax-1}}dx\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m*(-a*c*x+c)^(1/2),x)
```

output
```
e**m*sqrt(c)*int((x**m*sqrt(a*x + 1)*sqrt( - a*x + 1))/sqrt(a*x - 1),x)
```

3.391 $\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-acx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-acx}}\,dx = \frac{2\sqrt{1-\frac{1}{ax}}\,x(ex)^m \operatorname{AppellF1}\left(-\frac{1}{2}-m,1,-\frac{1}{2},\frac{1}{2}-m,\frac{1}{ax},-\frac{1}{ax}\right)}{(1+2m)\sqrt{c-acx}}$$

output
```
2*(1-1/a/x)^(1/2)*x*(e*x)^m*AppellF1(-1/2-m,1,-1/2,1/2-m,1/a/x,-1/a/x)/(1+
2*m)/(-a*c*x+c)^(1/2)
```

Mathematica [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-acx}}\,dx = \int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-acx}}\,dx$$

input
```
Integrate[(E^ArcCoth[a*x]*(e*x)^m)/Sqrt[c - a*c*x],x]
```

output
```
Integrate[(E^ArcCoth[a*x]*(e*x)^m)/Sqrt[c - a*c*x], x]
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6730, 27, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-acx}}\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{x}\right)^{m-\frac{1}{2}}(ex)^m\int \frac{a\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-\frac{3}{2}}}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-acx}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{x}\right)^{m-\frac{1}{2}}(ex)^m\int \frac{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-\frac{3}{2}}}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-acx}}$$

$$\Big\downarrow \text{150}$$

$$\frac{2x\sqrt{1-\frac{1}{ax}}(ex)^m \operatorname{AppellF1}\left(-m-\frac{1}{2},-\frac{1}{2},1,\frac{1}{2}-m,-\frac{1}{ax},\frac{1}{ax}\right)}{(2m+1)\sqrt{c-acx}}$$

input
```
Int[(E^ArcCoth[a*x]*(e*x)^m)/Sqrt[c - a*c*x],x]
```

output
```
(2*Sqrt[1 - 1/(a*x)]*x*(e*x)^m*AppellF1[-1/2 - m, -1/2, 1, 1/2 - m, -(1/(a
*x)), 1/(a*x)])/((1 + 2*m)*Sqrt[c - a*c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-acx+c}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^(1/2),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-acx}}\,dx = \int \frac{(ex)^m}{\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-sqrt(-a*c*x + c)*(a*x + 1)*(e*x)^m*sqrt((a*x - 1)/(a*x + 1))/(a^
2*c*x^2 - 2*a*c*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-acx}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(e*x)**m/(-a*c*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-acx}}\,dx = \int \frac{(ex)^m}{\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((e*x)^m/(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-acx}}\,dx = \int \frac{(ex)^m}{\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((e*x)^m/(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-acx}}\,dx = \int \frac{(e\,x)^m}{\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((e*x)^m/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((e*x)^m/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-acx}}\,dx = \frac{e^m\left(\int \frac{x^m\sqrt{ax+1}}{\sqrt{ax-1}\,\sqrt{-ax+1}}dx\right)}{\sqrt{c}}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^(1/2),x)
```

output
```
(e**m*int((x**m*sqrt(a*x + 1))/(sqrt(a*x - 1)*sqrt( - a*x + 1)),x))/sqrt(c
)
```

3.392 $\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^{3/2}}\,dx = -\frac{2\left(1-\frac{1}{ax}\right)^{3/2} x(ex)^m \operatorname{AppellF1}\left(\frac{1}{2}-m,2,-\frac{1}{2},\frac{3}{2}-m,\frac{1}{ax},-\frac{1}{ax}\right)}{(1-2m)(c-acx)^{3/2}}$$

output
```
-2*(1-1/a/x)^(3/2)*x*(e*x)^m*AppellF1(1/2-m,2,-1/2,3/2-m,1/a/x,-1/a/x)/(1-
2*m)/(-a*c*x+c)^(3/2)
```

Mathematica [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^{3/2}}\,dx = \int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^{3/2}}\,dx$$

input
```
Integrate[(E^ArcCoth[a*x]*(e*x)^m)/(c - a*c*x)^(3/2),x]
```

output
```
Integrate[(E^ArcCoth[a*x]*(e*x)^m)/(c - a*c*x)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6730, 27, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^{3/2}}dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{m-\frac{3}{2}}(ex)^m\int \frac{a^2\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-\frac{1}{2}}}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{(c-acx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{x}\right)^{m-\frac{3}{2}}(ex)^m\int \frac{\sqrt{1+\frac{1}{ax}}\left(\frac{1}{x}\right)^{-m-\frac{1}{2}}}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{(c-acx)^{3/2}}$$

$$\Big\downarrow \text{150}$$

$$-\frac{2x\left(1-\frac{1}{ax}\right)^{3/2}(ex)^m \text{AppellF1}\left(\frac{1}{2}-m,-\frac{1}{2},2,\frac{3}{2}-m,-\frac{1}{ax},\frac{1}{ax}\right)}{(1-2m)(c-acx)^{3/2}}$$

input
```
Int[(E^ArcCoth[a*x]*(e*x)^m)/(c - a*c*x)^(3/2),x]
```

output
```
(-2*(1 - 1/(a*x))^(3/2)*x*(e*x)^m*AppellF1[1/2 - m, -1/2, 2, 3/2 - m, -(1/
(a*x)), 1/(a*x)])/((1 - 2*m)*(c - a*c*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}\,(-acx+c)^{\frac{3}{2}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^(3/2),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^{3/2}}\,dx = \int \frac{(ex)^m}{(-acx+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^(3/2),x, algorithm=
"fricas")
```

output
```
integral(sqrt(-a*c*x + c)*(a*x + 1)*(e*x)^m*sqrt((a*x - 1)/(a*x + 1))/(a^3
*c^2*x^3 - 3*a^2*c^2*x^2 + 3*a*c^2*x - c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(e*x)**m/(-a*c*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^{3/2}}\,dx = \int \frac{(ex)^m}{(-acx+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((e*x)^m/((-a*c*x + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^{3/2}}\,dx = \int \frac{(ex)^m}{(-acx+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((e*x)^m/((-a*c*x + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^{3/2}}\,dx=\int \frac{(e\,x)^m}{(c-a\,c\,x)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((e*x)^m/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((e*x)^m/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{(c-acx)^{3/2}}\,dx=-\frac{e^m\left(\int \frac{x^m\sqrt{ax+1}}{\sqrt{ax-1}\,\sqrt{-ax+1}\,ax-\sqrt{ax-1}\,\sqrt{-ax+1}}dx\right)}{\sqrt{c}\,c}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(-a*c*x+c)^(3/2),x)
```

output
```
( - e**m*int((x**m*sqrt(a*x + 1))/(sqrt(a*x - 1)*sqrt( - a*x + 1)*a*x - sq
rt(a*x - 1)*sqrt( - a*x + 1)),x))/(sqrt(c)*c)
```

3.393 $\int e^{-\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int e^{-\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx = \frac{2\sqrt{1+\frac{1}{ax}}x(ex)^m\sqrt{c-acx}}{(3+2m)\sqrt{1-\frac{1}{ax}}} - \frac{2(5+4m)(ex)^m\sqrt{c-acx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2}-m,\frac{1}{2}-m,-\frac{1}{ax}\right)}{a\left(3+8m+4m^2\right)\sqrt{1-\frac{1}{ax}}}$$

output
```
2*(1+1/a/x)^(1/2)*x*(e*x)^m*(-a*c*x+c)^(1/2)/(3+2*m)/(1-1/a/x)^(1/2)-2*(5+
4*m)*(e*x)^m*(-a*c*x+c)^(1/2)*hypergeom([1/2, -1/2-m],[1/2-m],-1/a/x)/a/(4
*m^2+8*m+3)/(1-1/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.79

$$\int e^{-\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx = \frac{2(ex)^m\sqrt{c-acx}\left(a(1+2m)\sqrt{1+\frac{1}{ax}}x-(5+4m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2}-m,\frac{1}{2}-m,-\frac{1}{ax}\right)\right)}{a(1+2m)(3+2m)\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[((e*x)^m*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

output
```
(2*(e*x)^m*Sqrt[c - a*c*x]*(a*(1 + 2*m)*Sqrt[1 + 1/(a*x)]*x - (5 + 4*m)*Hy
pergeometric2F1[1/2, -1/2 - m, 1/2 - m, -(1/(a*x))]))/(a*(1 + 2*m)*(3 + 2*
m)*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6730, 27, 88, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-acx}e^{-\coth^{-1}(ax)}(ex)^m\,dx$$

$$\Big\downarrow \text{6730}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{1}{2}}\sqrt{c-acx}(ex)^m\int\frac{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-\frac{5}{2}}}{a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{1}{2}}\sqrt{c-acx}(ex)^m\int\frac{\left(a-\frac{1}{x}\right)\left(\frac{1}{x}\right)^{-m-\frac{5}{2}}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{88}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{1}{2}}\sqrt{c-acx}(ex)^m\left(-\frac{(4m+5)\int\frac{\left(\frac{1}{x}\right)^{-m-\frac{3}{2}}}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2m+3}-\frac{2a\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)^{-m-\frac{3}{2}}}{2m+3}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{74}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+\frac{1}{2}}\sqrt{c-acx}(ex)^m\left(\frac{2(4m+5)\left(\frac{1}{x}\right)^{-m-\frac{1}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},-m-\frac{1}{2},\frac{1}{2}-m,-\frac{1}{ax}\right)}{(2m+1)(2m+3)}-\frac{2a\sqrt{\frac{1}{ax}+1}\left(\frac{1}{x}\right)^{-m-\frac{3}{2}}}{2m+3}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[((e*x)^m*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

output
```
-(((x^(-1))^(1/2 + m)*(e*x)^m*Sqrt[c - a*c*x]*((-2*a*Sqrt[1 + 1/(a*x)]*(x^
(-1))^(-3/2 - m))/(3 + 2*m) + (2*(5 + 4*m)*(x^(-1))^(-1/2 - m)*Hypergeomet
ric2F1[1/2, -1/2 - m, 1/2 - m, -(1/(a*x))])/((1 + 2*m)*(3 + 2*m))))/(a*Sqr
t[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 6730
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(p
_), x_Symbol] :> Simp[(-(e*x)^m)*(1/x)^(m + p)*((c + d*x)^p/(1 + c/(d*x))^p
)   Subst[Int[((1 + c*(x/d))^p*((1 + x/a)^(n/2)/x^(m + p + 2)))/(1 - x/a)^(
n/2), x], x, 1/x], x] /; FreeQ[{a, c, d, e, m, n, p}, x] && EqQ[a^2*c^2 - d
^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int (ex)^m \sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}dx$$

input
```
int((e*x)^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
int((e*x)^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

Fricas [F]

$$\int e^{-\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx = \int \sqrt{-acx+c}(ex)^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((e*x)^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="f
ricas")
```

output
```
integral(sqrt(-a*c*x + c)*(e*x)^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx = \int \sqrt{-acx+c}(ex)^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((e*x)^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a*c*x + c)*(e*x)^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx = \int \sqrt{-acx+c}(ex)^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((e*x)^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(-a*c*x + c)*(e*x)^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx=\int (e\,x)^m\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((e*x)^m*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((e*x)^m*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{-\coth^{-1}(ax)}(ex)^m\sqrt{c-acx}\,dx=e^m\sqrt{c}\left(\int\frac{x^m\sqrt{ax-1}\,\sqrt{-ax+1}}{\sqrt{ax+1}}dx\right)$$

input
```
int((e*x)^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
e**m*sqrt(c)*int((x**m*sqrt(a*x - 1)*sqrt( - a*x + 1))/sqrt(a*x + 1),x)
```

3.394 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal result

Integrand size = 20, antiderivative size = 114

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2}+c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x-\frac{c^4\csc^{-1}(ax)}{2a}-\frac{3c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-1/3*c^4*(1-1/a^2/x^2)^(3/2)/a+1/2*c^4*(1-1/a^2/x^2)^(1/2)*(6*a-1/x)/a^2+c
^4*(1-1/a^2/x^2)^(3/2)*x-1/2*c^4*arccsc(a*x)/a-3*c^4*arctanh((1-1/a^2/x^2)
^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.54

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{c^4\left(-2+9ax-14a^2x^2-15a^3x^3+16a^4x^4+6a^5x^5+24a^4\sqrt{1-\frac{1}{a^2x^2}}x^4\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+9a^4\sqrt{1-\frac{1}{a^2}}\right.}{6a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^4,x]
```

output
```
(c^4*(-2 + 9*a*x - 14*a^2*x^2 - 15*a^3*x^3 + 16*a^4*x^4 + 6*a^5*x^5 + 24*a
^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 9*a^4*Sqr
t[1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] - 18*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*
ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(6*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {6731, 27, 540, 2340, 27, 535, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^4 e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c\int \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3x^2}{a^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^4\int \sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3x^2d\frac{1}{x}}{a^3}$$

$$\Big\downarrow \text{540}$$

$$-\frac{c^4\left(a^3x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\int\sqrt{1-\frac{1}{a^2x^2}}\left(3a^2-\frac{a}{x}+\frac{1}{x^2}\right)xd\frac{1}{x}\right)}{a^3}$$

$$\Big\downarrow \text{2340}$$

$$-\frac{c^4\left(\frac{1}{3}a^2\int-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)x}{a}d\frac{1}{x}+\frac{1}{3}a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)\right)}{a^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\left(-a\int\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)xd\frac{1}{x}+\frac{1}{3}a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)\right)}{a^3}$$

$$\Big\downarrow \text{535}$$

$$-\frac{c^4\left(-a\left(\frac{1}{2}\int\frac{\left(6a-\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)\right)+\frac{1}{3}a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)\right)}{a^3}$$

$$\Big\downarrow \text{538}$$

$$-\frac{c^4\left(-a\left(\frac{1}{2}\left(6a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)\right)+\frac{1}{3}a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3x\left(-\left(1-\frac{1}{a^2x^2}\right.\right.}{a^3}$$

$$\Big\downarrow \text{223}$$

$$-\frac{c^4\left(-a\left(\frac{1}{2}\left(6a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-a\arcsin\left(\frac{1}{ax}\right)\right)+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)\right)+\frac{1}{3}a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3x\left(-\left(1-\frac{1}{a^2x^2}\right.\right.}{a^3}$$

$$\Big\downarrow \text{243}$$

$$-\frac{c^4\left(-a\left(\frac{1}{2}\left(3a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-a\arcsin\left(\frac{1}{ax}\right)\right)+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)\right)+\frac{1}{3}a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3x\left(-\left(1-\frac{1}{a^2x^2}\right.\right.}{a^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c^4\left(-a\left(\frac{1}{2}\left(-6a^3\int\frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-a\arcsin\left(\frac{1}{ax}\right)\right)+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)\right)+\frac{1}{3}a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right.}{a^3}$$

$\downarrow$ 221

$$-\frac{c^4\left(-a\left(\frac{1}{2}\left(-6a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-a\arcsin\left(\frac{1}{ax}\right)\right)+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)\right)+\frac{1}{3}a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3x\left(-(\right.\right.}{a^3}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a*x))^4,x]
```

output
```
-((c^4*((a^2*(1 - 1/(a^2*x^2))^(3/2))/3 - a^3*(1 - 1/(a^2*x^2))^(3/2)*x -
a*((Sqrt[1 - 1/(a^2*x^2)]*(6*a - x^(-1)))/2 + (-(a*ArcSin[1/(a*x)]) - 6*a*
ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/2)))/a^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.38

method	result
risch	$\frac{(ax-1)(16a^2x^2-9ax+2)c^4}{6x^3a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(-\frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}-\frac{3a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a^3\sqrt{(ax-1)(ax+1)}\right)c^4\sqrt{(ax-1)(ax+1)}}{a^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)c^4\left(-18\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^4x^4+18\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^2x^2+3\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^3x^3+3a^3\sqrt{a^2}\,x^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+18\ln\left(a^2\cdots\right.\right.}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^4x^3\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x-1)*(16*a^2*x^2-9*a*x+2)/x^3*c^4/a^4/((a*x-1)/(a*x+1))^(1/2)+(-1/2
*a^3*arctan(1/(a^2*x^2-1)^(1/2))-3*a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1
/2))/(a^2)^(1/2)+a^3*((a*x-1)*(a*x+1))^(1/2))*c^4/a^4/((a*x-1)/(a*x+1))^(1
/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.37

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{6\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-18\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+18\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(6\,a^4c^4x^4+22\,a\cdots}{6\,a^4x^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/6*(6*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 18*a^3*c^4*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) + 18*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x +
1)) - 1) + (6*a^4*c^4*x^4 + 22*a^3*c^4*x^3 + 7*a^2*c^4*x^2 - 7*a*c^4*x + 2
*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$=\frac{c^4\left(\int \frac{a^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \frac{1}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \left(-\frac{4a}{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int \frac{6a^2}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \left(-\frac{4}{x\sqrt{\frac{ax}{ax-}}}\right.\right.}{a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**4,x)
```

output
```
c**4*(Integral(a**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x*
*4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-4*a/(x**3*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))), x) + Integral(6*a**2/(x**2*sqrt(a*x/(a*x + 1) -
1/(a*x + 1))), x) + Integral(-4*a**3/(x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)))
, x))/a**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(100) = 200$.

Time = 0.12 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.96

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$=\frac{1}{3}\left(\frac{3\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{9\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{9\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{21\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-17\,c^4\left(\frac{ax-1}{ax+1}\right.}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(a}{(a}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
1/3*(3*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 9*c^4*log(sqrt((a*x - 1
)/(a*x + 1)) + 1)/a^2 + 9*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (21
*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 17*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 37
*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a
*x - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a
*x + 1)^4 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(100) = 200$.

Time = 0.14 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.18

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4 \arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\text{sgn}(ax+1)} + \frac{3c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\text{sgn}(ax+1)} + \frac{\sqrt{a^2x^2-1}c^4}{a\text{sgn}(ax+1)} + \frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^4|a|+12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^4+36\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^4-9\left(x|a|-\sqrt{a}\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|\text{sgn}(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="giac")
```

output
```
c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) + 3*c^4*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)
*c^4/(a*sgn(a*x + 1)) + 1/3*(9*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)
 + 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4 + 36*(x*abs(a) - sqrt(a^2*x^2
 - 1))^2*a*c^4 - 9*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*abs(a) + 16*a*c^4)/(
((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.85 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.61

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{5c^4\sqrt{\frac{ax-1}{ax+1}}+\frac{37c^4\left(\frac{ax-1}{ax+1}\right)^{3/2}}{3}+\frac{17c^4\left(\frac{ax-1}{ax+1}\right)^{5/2}}{3}-7c^4\left(\frac{ax-1}{ax+1}\right)^{7/2}}{a+\frac{2a(ax-1)}{ax+1}-\frac{2a(ax-1)^3}{(ax+1)^3}-\frac{a(ax-1)^4}{(ax+1)^4}} + \frac{c^4\,\text{atan}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a} - \frac{6c^4\,\text{atanh}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}$$

input
```
int((c - c/(a*x))^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(5*c^4*((a*x - 1)/(a*x + 1))^(1/2) + (37*c^4*((a*x - 1)/(a*x + 1))^(3/2))/
3 + (17*c^4*((a*x - 1)/(a*x + 1))^(5/2))/3 - 7*c^4*((a*x - 1)/(a*x + 1))^(
7/2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*
(a*x - 1)^4)/(a*x + 1)^4) + (c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6
*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.43

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$=\frac{c^4\left(6atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^3x^3-6atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^3x^3+6\sqrt{ax+1}\,\sqrt{ax-1}\,a\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x)
```

output
```
(c**4*(6*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**3*x**3 - 6*atan(sqrt(a
*x - 1) + sqrt(a*x + 1) + 1)*a**3*x**3 + 6*sqrt(a*x + 1)*sqrt(a*x - 1)*a**
3*x**3 + 16*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 - 9*sqrt(a*x + 1)*sqrt(a
*x - 1)*a*x + 2*sqrt(a*x + 1)*sqrt(a*x - 1) - 36*log((sqrt(a*x - 1) + sqrt
(a*x + 1))/sqrt(2))*a**3*x**3 - 8*a**3*x**3))/(6*a**4*x**3)
```

3.395 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{c^3\csc^{-1}(ax)}{2a}-\frac{2c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
1/2*c^3*(1-1/a^2/x^2)^(1/2)*(4*a+1/x)/a^2+c^3*(1-1/a^2/x^2)^(3/2)*x+1/2*c^
3*arccsc(a*x)/a-2*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.90

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$$
$$=\frac{c^3\left(1-4ax-3a^2x^2+4a^3x^3+2a^4x^4+2a^3\sqrt{1-\frac{1}{a^2x^2}}x^3\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+2a^3\sqrt{1-\frac{1}{a^2x^2}}x^3\arcsin\left(\frac{1}{ax}\right)\right.}{2a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^3,x]
```

output
```
(c^3*(1 - 4*a*x - 3*a^2*x^2 + 4*a^3*x^3 + 2*a^4*x^4 + 2*a^3*Sqrt[1 - 1/(a^
2*x^2)]*x^3*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 2*a^3*Sqrt[1 - 1/(a^2*x^2)
]*x^3*ArcSin[1/(a*x)] - 4*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^3*ArcTanh[Sqrt[1 - 1
/(a^2*x^2)]]))/(2*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6731, 27, 540, 535, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^3 e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \; 6731$$

$$-c\int \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2 x^2}{a^2}d\frac{1}{x}$$

$$\Big\downarrow \; 27$$

$$-\frac{c^3\int \sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2 x^2 d\frac{1}{x}}{a^2}$$

$$\Big\downarrow \; 540$$

$$-\frac{c^3\left(a^2x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\int \sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right)xd\frac{1}{x}\right)}{a^2}$$

$$\Big\downarrow \; 535$$

$$-\frac{c^3\left(-\frac{1}{2}\int \frac{\left(4a+\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a^2x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)\right)}{a^2}$$

$$\Big\downarrow \; 538$$

$$-\frac{c^3\left(\frac{1}{2}\left(-\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-4a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+a^2x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)\right)}{a^2}$$

$$\Big\downarrow \ 223$$

$$-\frac{c^3\left(\frac{1}{2}\left(-4a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-a\arcsin\left(\frac{1}{ax}\right)\right)+a^2x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)\right)}{a^2}$$

$$\Big\downarrow \ 243$$

$$-\frac{c^3\left(\frac{1}{2}\left(-2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-a\arcsin\left(\frac{1}{ax}\right)\right)+a^2x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)\right)}{a^2}$$

$$\Big\downarrow \ 73$$

$$-\frac{c^3\left(\frac{1}{2}\left(4a^3\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-a\arcsin\left(\frac{1}{ax}\right)\right)+a^2x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)\right)}{a^2}$$

$$\Big\downarrow \ 221$$

$$-\frac{c^3\left(\frac{1}{2}\left(4a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-a\arcsin\left(\frac{1}{ax}\right)\right)+a^2x\left(-\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)-\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)\right)}{a^2}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a*x))^3,x]
```

output
```
-((c^3*(-1/2*(Sqrt[1 - 1/(a^2*x^2)]*(4*a + x^(-1))) - a^2*(1 - 1/(a^2*x^2)
)^(3/2)*x + (-(a*ArcSin[1/(a*x)]) + 4*a*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/2)
)/a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.59

method	result
risch	$\frac{(ax-1)\left(2a^2x^2+4ax-1\right)c^3}{2x^2a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}-\frac{2a^3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}\right)c^3\sqrt{(ax-1)(ax+1)}}{a^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)c^3\left(-4\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^3x^3+4\sqrt{a^2}\,\left(a^2x^2-1\right)^{\frac{3}{2}}ax-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^2x^2-a^2\sqrt{a^2}\,x^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+4\ln\left(\frac{a^2x+\sqrt{a^2x}}{\sqrt{a}}\right.\right.}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^3x^2\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x-1)*(2*a^2*x^2+4*a*x-1)/x^2*c^3/a^3/((a*x-1)/(a*x+1))^(1/2)+(1/2*a
^2*arctan(1/(a^2*x^2-1)^(1/2))-2*a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2
))/(a^2)^(1/2))*c^3/a^3/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a
*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.66

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = -\frac{2\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+4\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-4\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(2\,a^3c^3x^3+6\,a^2}{2\,a^3x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + 4*a^2*c^3*x^2*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) - 4*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x + 1
)) - 1) - (2*a^3*c^3*x^3 + 6*a^2*c^3*x^2 + 3*a*c^3*x - c^3)*sqrt((a*x - 1)
/(a*x + 1)))/(a^3*x^2)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = \frac{c^3\left(\int\frac{a^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{3a}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^2}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**3,x)
```

output
```
c**3*(Integral(a**3/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x
**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(3*a/(x**2*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))), x) + Integral(-3*a**2/(x*sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(78) = 156.

Time = 0.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.28

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx =$$
$$-\left(\frac{c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{2\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{2\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{3\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-6\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(ax}{(a}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
-(c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 2*c^3*log(sqrt((a*x - 1)/(a*
x + 1)) + 1)/a^2 - 2*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (3*c^3*(
(a*x - 1)/(a*x + 1))^(5/2) - 6*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*sqr
t((a*x - 1)/(a*x + 1)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x +
1)^2 - (a*x - 1)^3*a^2/(a*x + 1)^3 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(78) = 156.

Time = 0.14 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.51

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$$
$$= -\frac{c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}+\frac{2\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^3}{a\mathrm{sgn}\left(ax+1\right)}$$
$$+\frac{\left(x|a|-\sqrt{a^2x^2-1}\right)^3c^3|a|+4\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^3-\left(x|a|-\sqrt{a^2x^2-1}\right)c^3|a|+4\,ac^3}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^2a|a|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="giac")
```

output
```
-c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) + 2*c^3*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1
)*c^3/(a*sgn(a*x + 1)) + ((x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a) + 4*
(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3 - (x*abs(a) - sqrt(a^2*x^2 - 1))*c^
3*abs(a) + 4*a*c^3)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^2*a*abs(a)*sgn
(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.62 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.85

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}+6\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-3\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a+\frac{a\,(a\,x-1)}{a\,x+1}-\frac{a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a\,(a\,x-1)^3}{(a\,x+1)^3}}-\frac{c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{4\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a*x))^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(5*c^3*((a*x - 1)/(a*x + 1))^(1/2) + 6*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 3
*c^3*((a*x - 1)/(a*x + 1))^(5/2))/(a + (a*(a*x - 1))/(a*x + 1) - (a*(a*x -
 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (c^3*atan(((a*x - 1)/(
a*x + 1))^(1/2)))/a - (4*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.53

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\left(-2atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^2x^2+2atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^2x^2+2\sqrt{ax+1}\,\sqrt{ax-}\right.}{2a^3x^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x)
```

output
```
(c**3*( - 2*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**2*x**2 + 2*atan(sqr
t(a*x - 1) + sqrt(a*x + 1) + 1)*a**2*x**2 + 2*sqrt(a*x + 1)*sqrt(a*x - 1)*
a**2*x**2 + 4*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x - sqrt(a*x + 1)*sqrt(a*x - 1
) - 8*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**2*x**2))/(2*a**3*x**
2)
```

3.396 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a}+\frac{c^2\csc^{-1}(ax)}{a}-\frac{c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
c^2*(1-1/a^2/x^2)^(1/2)*(a+1/x)*x/a+c^2*arccsc(a*x)/a-c^2*arctanh((1-1/a^2
/x^2)^(1/2))/a
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 158 vs. 2(62) = 124.

Time = 0.23 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.55

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$$
$$=\frac{c^2\left(-2-2ax+2a^2x^2+2a^3x^3-2a^2\sqrt{1-\frac{1}{a^2x^2}}x^2\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}x^2\arcsin\left(\frac{1}{ax}\right)-2a^2\right.}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^2,x]
```

output
```
(c^2*(-2 - 2*a*x + 2*a^2*x^2 + 2*a^3*x^3 - 2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2
*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcSin[
1/(a*x)] - 2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])
)/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6731, 27, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^2 e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c\int \frac{c\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2}{a}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^2\int \sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 536}$$

$$-\frac{c^2\left(\int \frac{\left(-1-\frac{1}{ax}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-x\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)\right)}{a}$$

$$\Big\downarrow \text{ 538}$$

$$-\frac{c^2\left(-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\left(a+\frac{1}{x}\right)\right)}{a}$$

$$\Big\downarrow \text{223}$$

$$-\frac{c^2\left(-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\left(a+\frac{1}{x}\right)-\arcsin\left(\frac{1}{ax}\right)\right)}{a}$$

$$\Big\downarrow \text{243}$$

$$-\frac{c^2\left(-\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\left(a+\frac{1}{x}\right)-\arcsin\left(\frac{1}{ax}\right)\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c^2\left(a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-x\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)-\arcsin\left(\frac{1}{ax}\right)\right)}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c^2\left(\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\left(a+\frac{1}{x}\right)-\arcsin\left(\frac{1}{ax}\right)\right)}{a}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a*x))^2,x]
```

output
```
-((c^2*(-(Sqrt[1 - 1/(a^2*x^2)]*(a + x^(-1))*x) - ArcSin[1/(a*x)] + ArcTan
h[Sqrt[1 - 1/(a^2*x^2)]]))/a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(58) = 116$.

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.11

method	result
risch	$\frac{(ax-1)c^2}{x\,a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\sqrt{(ax-1)(ax+1)}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-\frac{a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}\right)c^2\sqrt{(ax-1)(ax+1)}}{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)c^2\left(-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax-a\sqrt{a^2}\,x\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right.}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)/x*c^2/a^2/((a*x-1)/(a*x+1))^(1/2)+1/a*(((a*x-1)*(a*x+1))^(1/2)+arc
tan(1/(a^2*x^2-1)^(1/2))-a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(
1/2))*c^2/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(58) = 116$.

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.92

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx =$$
$$-\frac{2\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2c^2x^2+2\,ac^2x+c^2)\sqrt{}}{a^2x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
-(2*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + a*c^2*x*log(sqrt((a*x - 1)
/(a*x + 1)) + 1) - a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c^2*x
^2 + 2*a*c^2*x + c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$$
$$=\frac{c^2\left(\int\frac{a^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{2a}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**2,x)
```

output
```
c**2*(Integral(a**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-2*a/(x*sqrt(a*x/(a*x
 + 1) - 1/(a*x + 1))), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(58) = 116$.

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.02

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx =$$
$$-\left(\frac{4\,c^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
-(4*c^2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*
c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c^2*log(sqrt((a*x - 1)/(a*x +
1)) + 1)/a^2 - c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(58) = 116$.

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.21

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = -\frac{2\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)} + \frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)} + \frac{\sqrt{a^2x^2-1}c^2}{a\mathrm{sgn}\left(ax+1\right)} + \frac{2\,c^2}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="giac")
```

output
```
-2*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) + c^2*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1
)*c^2/(a*sgn(a*x + 1)) + 2*c^2/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs
(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.45

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = \frac{4\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}} - \frac{2\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a} - \frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a*x))^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(4*c^2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (2
*c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c^2*atanh(((a*x - 1)/(a*x +
 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.66

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$$
$$= \frac{c^2\left(-2atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)ax+2atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)ax+\sqrt{ax+1}\,\sqrt{ax-1}\,ax\right.}{a^2x}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x)
```

output
```
(c**2*( - 2*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a*x + 2*atan(sqrt(a*x
- 1) + sqrt(a*x + 1) + 1)*a*x + sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + sqrt(a*x
 + 1)*sqrt(a*x - 1) - 2*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x +
 a*x))/(a**2*x)
```

3.397 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=c\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{c\csc^{-1}(ax)}{a}$$

output
```
c*(1-1/a^2/x^2)^(1/2)*x+c*arccsc(a*x)/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x+\arcsin\left(\frac{1}{ax}\right)\right)}{a}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x)),x]
```

output
```
(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x + ArcSin[1/(a*x)]))/a
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6731, 247, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right) e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \sqrt{1-\frac{1}{a^2x^2}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{247}$$

$$-c\left(x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^2}\right)$$

$$\Big\downarrow \text{223}$$

$$-c\left(x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{\arcsin\left(\frac{1}{ax}\right)}{a}\right)$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a*x)),x]
```

output
```
-(c*(-(Sqrt[1 - 1/(a^2*x^2)]*x) - ArcSin[1/(a*x)]/a))
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(25) = 50$.

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

method	result	size
default	$\frac{(ax-1)c\left(\sqrt{a^2x^2-1}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a}$	63

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
1/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)/((a*x-1)*(a*x+1))^(1/2)*c/a*((a^2*x^2-1)
^(1/2)+arctan(1/(a^2*x^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = -\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="fricas")
```

output
```
-(2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*c*x + c)*sqrt((a*x - 1)/(a*x
+ 1)))/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c\left(\int\frac{a}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x),x)
```

output
```
c*(Integral(a/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(25) = 50$.

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=-2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="maxima")
```

output
```
-2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + c*arct
an(sqrt((a*x - 1)/(a*x + 1)))/a^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=-\frac{c\arctan\left(\sqrt{a^2x^2-1}\right)-\sqrt{a^2x^2-1}c}{a\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="giac")
```

output
```
-(c*arctan(sqrt(a^2*x^2 - 1)) - sqrt(a^2*x^2 - 1)*c)/(a*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a*x))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) - (2*c*ata
n(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.96

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c\left(-2atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)+2atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)+\sqrt{ax+1}\,\sqrt{ax-1}\right)}{a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x)
```

output
```
(c*( - 2*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1) + 2*atan(sqrt(a*x - 1) +
sqrt(a*x + 1) + 1) + sqrt(a*x + 1)*sqrt(a*x - 1)))/a
```

3.398 $\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 71

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{c\left(a-\frac{1}{x}\right)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}+\frac{2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
-2*(1-1/a^2/x^2)^(1/2)/c/(a-1/x)+(1-1/a^2/x^2)^(1/2)*x/c+2*arctanh((1-1/a^
2/x^2)^(1/2))/a/c
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-3+ax)+2(-1+ax)\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac(-1+ax)}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a*x)),x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-3 + a*x) + 2*(-1 + a*x)*Log[(1 + Sqrt[1 - 1/(
a^2*x^2)])*x])/(a*c*(-1 + a*x))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6731, 27, 564, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}x^2}{c^2\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{564}$$

$$-\frac{a^2\left(\frac{\int \frac{\left(a+\frac{2}{x}\right)x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^2}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\left(\frac{\int \frac{\left(a+\frac{2}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^3}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}\right)}{c}$$

$$\Big\downarrow \text{534}$$

$$-\frac{a^2\left(\frac{2\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}}{a^3}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}\right)}{c}$$

$$\Big\downarrow \text{243}$$

$$-\frac{a^2\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}}{a^3}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}\right)}{c}$$

$$\Big\downarrow \; 73$$

$$-\frac{a^2\left(\frac{-2a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}}{a^3}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}\right)}{c}$$

$$\Big\downarrow \; 221$$

$$-\frac{a^2\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{-2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}}{a^3}\right)}{c}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a*x)),x]
```

output
```
-((a^2*((2*Sqrt[1 - 1/(a^2*x^2)])/(a^2*(a - x^(-1))) + (-(a*Sqrt[1 - 1/(a^
2*x^2)]*x) - 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a^3))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(65) = 130$.

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.01

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{2\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a\sqrt{a^{2}}}-\frac{2\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2a\left(x-\frac{1}{a}\right)}}{a^{3}\left(x-\frac{1}{a}\right)}\right)a\sqrt{(ax-1)(ax+1)}}{c\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-2\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,a^{2}x^{2}-2\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{3}x^{2}+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^{2}}+4\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,ax+4\ln}{a\sqrt{a^{2}}\,(ax-1)c\sqrt{(ax-1)(ax+1)}\,\sqrt{\frac{ax-}{ax-}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c/((a*x-1)/(a*x+1))^(1/2)+(2/a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1
)^(1/2))/(a^2)^(1/2)-2/a^3/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2))*a/c/
((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{2\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2x^2-2\,ax-3\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="fricas")
```

output
```
(2*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 2*(a*x - 1)*log(sqrt((a*
x - 1)/(a*x + 1)) - 1) + (a^2*x^2 - 2*a*x - 3)*sqrt((a*x - 1)/(a*x + 1)))/
(a^2*c*x - a*c)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{a\int \frac{x}{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x),x)
```

output
```
a*Integral(x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))), x)/c
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.63

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -2\,a\left(\frac{\frac{2\,(ax-1)}{ax+1}-1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="maxima")
```

output
```
-2*a*((2*(a*x - 1)/(a*x + 1) - 1)/(a^2*c*((a*x - 1)/(a*x + 1))^(3/2) - a^2
*c*sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c)
 + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c))
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \int \frac{1}{\left(c-\frac{c}{ax}\right)\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{2\,a\,x+8\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-6}{2\,a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(1/((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(2*a*x + 8*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2)
- 6)/(2*a*c*((a*x - 1)/(a*x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{8\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)-5\sqrt{ax-1}+2\sqrt{ax+1}\,ax-6\sqrt{ax+1}}{2\sqrt{ax-1}\,ac}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x)
```

output
```
(8*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) - 5*sqrt(a*x
 - 1) + 2*sqrt(a*x + 1)*a*x - 6*sqrt(a*x + 1))/(2*sqrt(a*x - 1)*a*c)
```

3.399 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 120

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=-\frac{14\sqrt{1-\frac{1}{a^2x^2}}}{3c^2\left(a-\frac{1}{x}\right)}-\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^2\left(a-\frac{1}{x}\right)^2}+\frac{5a\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^2\left(a-\frac{1}{x}\right)}+\frac{3\,\mathrm{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

output
```
-14/3*(1-1/a^2/x^2)^(1/2)/c^2/(a-1/x)-2/3*a^2*(1-1/a^2/x^2)^(1/2)*x/c^2/(a
-1/x)^2+5/3*a*(1-1/a^2/x^2)^(1/2)*x/c^2/(a-1/x)+3*arctanh((1-1/a^2/x^2)^(1
/2))/a/c^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{14-5ax-16a^2x^2+3a^3x^3+9a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)\,\mathrm{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^2,x]
```

output
```
(14 - 5*a*x - 16*a^2*x^2 + 3*a^3*x^3 + 9*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a
*x)*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x*(-1
 + a*x))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {6731, 27, 570, 532, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}{c^3\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^3x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^3c^2}$$

$$\Big\downarrow \text{532}$$

$$-\frac{\frac{4a\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(3a^3+\frac{9a^2}{x}+\frac{8a}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^3c^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{3}\int \frac{\left(3a^3+\frac{9a^2}{x}+\frac{8a}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{4a\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^3c^2}$$

$\Big\downarrow$ 2336

$$-\frac{\frac{1}{3}\left(\frac{a\left(9a+\frac{11}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{3a^2\left(a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^3c^2}$$

$\Big\downarrow$ 27

$$-\frac{\frac{1}{3}\left(3a^2\int \frac{\left(a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a\left(9a+\frac{11}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^3c^2}$$

$\Big\downarrow$ 534

$$-\frac{\frac{1}{3}\left(3a^2\left(3\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a\left(9a+\frac{11}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^3c^2}$$

$\Big\downarrow$ 243

$$-\frac{\frac{1}{3}\left(3a^2\left(\frac{3}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a\left(9a+\frac{11}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^3c^2}$$

$\Big\downarrow$ 73

$$-\frac{\frac{1}{3}\left(3a^2\left(-3a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a\left(9a+\frac{11}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^3c^2}$$

$\Big\downarrow$ 221

$$-\frac{\frac{1}{3}\left(3a^2\left(-3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a\left(9a+\frac{11}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^3c^2}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a*x))^2,x]
```

output
```
-(((4*a*(a + x^(-1)))/(3*(1 - 1/(a^2*x^2))^(3/2)) + ((a*(9*a + 11/x))/Sqrt
[1 - 1/(a^2*x^2)] + 3*a^2*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x) - 3*ArcTanh[Sqrt[1
 - 1/(a^2*x^2)]]))/3)/(a^3*c^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.54

method	result
risch	$\frac{ax-1}{a\,c^{2}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{2}\sqrt{a^{2}}}-\frac{13\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2a\left(x-\frac{1}{a}\right)}}{3a^{4}\left(x-\frac{1}{a}\right)}-\frac{2\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2a\left(x-\frac{1}{a}\right)}}{3a^{5}\left(x-\frac{1}{a}\right)^{2}}\right)a^{2}\sqrt{(ax-1)(ax+1)}}{c^{2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{9\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{4}x^{3}+9\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{3}x^{3}-27\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{3}x^{2}-6\sqrt{a^{2}}\,((ax-1)(ax+1}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)+(3/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-13/3/a^4/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2
)-2/3/a^5/(x-1/a)^2*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2))*a^2/c^2/((a*x-1)/(a
*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.12

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{9\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-16\,a^2x^2-5\,ax\right.}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
1/3*(9*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a^2*x
^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 16*a^2*x
^2 - 5*a*x + 14)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a
*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{x^2}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**2,x)
```

output
```
a**2*Integral(x**2/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a*x*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c
**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.14

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{1}{3}\,a\left(\frac{\frac{11\,(ax-1)}{ax+1}-\frac{18\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
1/3*a*((11*(a*x - 1)/(a*x + 1) - 18*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^2*
((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)) + 9*log
(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 9*log(sqrt((a*x - 1)/(a*x + 1)
) - 1)/(a^2*c^2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2} - \frac{\frac{11\,(a\,x-1)}{3\,(a\,x+1)} - \frac{6\,(a\,x-1)^2}{(a\,x+1)^2} + \frac{1}{3}}{a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} - a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(1/((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^2) - ((11*(a*x - 1))/(3*(a*x +
 1)) - (6*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(a*c^2*((a*x - 1)/(a*x + 1))^(3/
2) - a*c^2*((a*x - 1)/(a*x + 1))^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.08

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$
$$= \frac{18\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax - 18\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right) + 4\sqrt{ax-1}\,ax - 4\sqrt{ax-1} + 3\sqrt{ax}}{3\sqrt{ax-1}\,a\,c^2\,(ax-1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x)
```

output
```
(18*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x - 18*sq
rt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 4*sqrt(a*x - 1)
*a*x - 4*sqrt(a*x - 1) + 3*sqrt(a*x + 1)*a**2*x**2 - 19*sqrt(a*x + 1)*a*x
+ 14*sqrt(a*x + 1))/(3*sqrt(a*x - 1)*a*c**2*(a*x - 1))
```

3.400 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 154

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{94\sqrt{1-\frac{1}{a^2x^2}}}{15c^3\left(a-\frac{1}{x}\right)}-\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x}{5c^3\left(a-\frac{1}{x}\right)^3}-\frac{13a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}{15c^3\left(a-\frac{1}{x}\right)^2}+\frac{34a\sqrt{1-\frac{1}{a^2x^2}}\,x}{15c^3\left(a-\frac{1}{x}\right)}+\frac{4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

output

```
-94/15*(1-1/a^2/x^2)^(1/2)/c^3/(a-1/x)-2/5*a^3*(1-1/a^2/x^2)^(1/2)*x/c^3/(
a-1/x)^3-13/15*a^2*(1-1/a^2/x^2)^(1/2)*x/c^3/(a-1/x)^2+34/15*a*(1-1/a^2/x^
2)^(1/2)*x/c^3/(a-1/x)+4*arctanh((1-1/a^2/x^2)^(1/2))/a/c^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.68

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{-94+128ax+73a^2x^2-134a^3x^3+15a^4x^4+60a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^3,x]
```

output
```
(-94 + 128*a*x + 73*a^2*x^2 - 134*a^3*x^3 + 15*a^4*x^4 + 60*a*Sqrt[1 - 1/(
a^2*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^3*Sqrt[
1 - 1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {6731, 27, 570, 532, 25, 2336, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{a^4\sqrt{1-\frac{1}{a^2x^2}}x^2}{c^4\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^4x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}}{a^4c^3}$$

$$\Big\downarrow \text{532}$$

$$-\frac{\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}\int -\frac{\left(5a^4+\frac{20a^3}{x}+\frac{27a^2}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^4c^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{5}\int \frac{\left(5a^4+\frac{20a^3}{x}+\frac{27a^2}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}+\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^4c^3}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{1}{5}\left(\frac{4a^2\left(5a+\frac{8}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(15a^4+\frac{60a^3}{x}+\frac{64a^2}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^4c^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{\left(15a^4+\frac{60a^3}{x}+\frac{64a^2}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{4a^2\left(5a+\frac{8}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^4c^3}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{a^2\left(60a+\frac{79}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{15a^3\left(a+\frac{4}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{4a^2\left(5a+\frac{8}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^4c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a^3\int \frac{\left(a+\frac{4}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^2\left(60a+\frac{79}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a^2\left(5a+\frac{8}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^4c^3}$$

$$\Big\downarrow \text{534}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a^3\left(4\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^2\left(60a+\frac{79}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a^2\left(5a+\frac{8}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^4c^3}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a^3\left(2\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^2\left(60a+\frac{79}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a^2\left(5a+\frac{8}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^4c^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a^3\left(-4a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^2\left(60a+\frac{79}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a^2\left(5a+\frac{8}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^4c^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{8a^2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{1}{5}\left(\frac{4a^2\left(5a+\frac{8}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{1}{3}\left(\frac{a^2\left(60a+\frac{79}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+15a^3\left(-4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)\right)\right)}{a^4c^3}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a*x))^3,x]
```

output
```
-(((8*a^2*(a + x^(-1)))/(5*(1 - 1/(a^2*x^2))^(5/2)) + ((4*a^2*(5*a + 8/x))
/(3*(1 - 1/(a^2*x^2))^(3/2)) + ((a^2*(60*a + 79/x))/Sqrt[1 - 1/(a^2*x^2)]
+ 15*a^3*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x) - 4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])
)/3)/5)/(a^4*c^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.46

method	result
risch	$\frac{ax-1}{a\,c^{3}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{4\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{3}\sqrt{a^{2}}}-\frac{104\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2a\left(x-\frac{1}{a}\right)}}{15a^{5}\left(x-\frac{1}{a}\right)}-\frac{2\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2a\left(x-\frac{1}{a}\right)}}{5a^{7}\left(x-\frac{1}{a}\right)^{3}}-\frac{31\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2a\left(x-\frac{1}{a}\right)}}{15a^{6}\left(x-\frac{1}{a}\right)^{2}}\right)a^{3}\sqrt{(}}{c^{3}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-60\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,a^{4}x^{4}-60\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{5}x^{4}+45\sqrt{a^{2}}\,((ax-1)(ax+1))^{\frac{3}{2}}a^{2}x^{2}+240\sqrt{a^{2}}\,\sqrt{(ax-1)(ax}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^3/((a*x-1)/(a*x+1))^(1/2)+(4/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-104/15/a^5/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1
/2)-2/5/a^7/(x-1/a)^3*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-31/15/a^6/(x-1/a)^
2*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2))*a^3/c^3/((a*x-1)/(a*x+1))^(1/2)*((a*x
-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3}}\,dx = \frac{60\,(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-60\,(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(15}{15\,(a^{4}c^{3}x^{3}-3\,a^{3}c^{3}x^{2}+3\,a^{2}c^{3}x-ac^{3})}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
1/15*(60*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
 1) - 60*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) -
 1) + (15*a^4*x^4 - 134*a^3*x^3 + 73*a^2*x^2 + 128*a*x - 94)*sqrt((a*x - 1
)/(a*x + 1)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{a^3 \int \frac{x^3}{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**3,x)
```

output
```
a**3*Integral(x**3/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**2*x
**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.99

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{1}{30}\,a\left(\frac{\frac{22\,(ax-1)}{ax+1}+\frac{155\,(ax-1)^2}{(ax+1)^2}-\frac{240\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{120\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{120\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
1/30*a*((22*(a*x - 1)/(a*x + 1) + 155*(a*x - 1)^2/(a*x + 1)^2 - 240*(a*x -
 1)^3/(a*x + 1)^3 + 3)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^3*((a*
x - 1)/(a*x + 1))^(5/2)) + 120*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3
) - 120*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.41

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c^3|a|\text{sgn}\,(ax+1)}+\frac{\sqrt{a^2x^2-1}}{ac^3\text{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="giac")
```

output
```
-4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c^3*abs(a)*sgn(a*x + 1)) + sqr
t(a^2*x^2 - 1)/(a*c^3*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{8\,\text{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3}-\frac{\frac{31\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}-\frac{16\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{22\,\left(a\,x-1\right)}{15\,\left(a\,x+1\right)}+\frac{1}{5}}{2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

input
```
int(1/((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(8*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^3) - ((31*(a*x - 1)^2)/(3*(a*x
 + 1)^2) - (16*(a*x - 1)^3)/(a*x + 1)^3 + (22*(a*x - 1))/(15*(a*x + 1)) +
1/5)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(5/2) - 2*a*c^3*((a*x - 1)/(a*x + 1))^
(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.29

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$

$$= \frac{240\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^2x^2-480\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax+240\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax}}{\sqrt{2}}\right.}{30\sqrt{}}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x)
```

output
```
(240*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**2*x**2
- 480*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x + 240
*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 127*sqrt(a*x
 - 1)*a**2*x**2 - 254*sqrt(a*x - 1)*a*x + 127*sqrt(a*x - 1) + 30*sqrt(a*x
+ 1)*a**3*x**3 - 298*sqrt(a*x + 1)*a**2*x**2 + 444*sqrt(a*x + 1)*a*x - 188
*sqrt(a*x + 1))/(30*sqrt(a*x - 1)*a*c**3*(a**2*x**2 - 2*a*x + 1))
```

3.401 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 188

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx = -\frac{824\sqrt{1-\frac{1}{a^2x^2}}}{105c^4\left(a-\frac{1}{x}\right)} - \frac{2a^4\sqrt{1-\frac{1}{a^2x^2}}x}{7c^4\left(a-\frac{1}{x}\right)^4} - \frac{17a^3\sqrt{1-\frac{1}{a^2x^2}}x}{35c^4\left(a-\frac{1}{x}\right)^3} - \frac{113a^2\sqrt{1-\frac{1}{a^2x^2}}x}{105c^4\left(a-\frac{1}{x}\right)^2} + \frac{299a\sqrt{1-\frac{1}{a^2x^2}}x}{105c^4\left(a-\frac{1}{x}\right)} + \frac{5\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

output
```
-824/105*(1-1/a^2/x^2)^(1/2)/c^4/(a-1/x)-2/7*a^4*(1-1/a^2/x^2)^(1/2)*x/c^4
/(a-1/x)^4-17/35*a^3*(1-1/a^2/x^2)^(1/2)*x/c^4/(a-1/x)^3-113/105*a^2*(1-1/
a^2/x^2)^(1/2)*x/c^4/(a-1/x)^2+299/105*a*(1-1/a^2/x^2)^(1/2)*x/c^4/(a-1/x)
+5*arctanh((1-1/a^2/x^2)^(1/2))/a/c^4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.60

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} \, dx = \frac{824-1947ax+485a^2x^2+1812a^3x^3-1339a^4x^4+105a^5x^5+525a\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)^3\text{arctanh}\left(\sqrt{1\right.}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)^3}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^4,x]
```

output
```
(824 - 1947*a*x + 485*a^2*x^2 + 1812*a^3*x^3 - 1339*a^4*x^4 + 105*a^5*x^5
+ 525*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]
])/(105*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3)
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6731, 27, 570, 532, 25, 2336, 25, 2336, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} \, dx$$

$$\Big\downarrow \ 6731$$

$$-c\int \frac{a^5\sqrt{1-\frac{1}{a^2x^2}}x^2}{c^5\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^5\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \textbf{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^5 x^2}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}d\frac{1}{x}}{a^5c^4}$$

$$\Big\downarrow \textbf{532}$$

$$-\frac{\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{1}{7}\int -\frac{\left(7a^5+\frac{35a^4}{x}+\frac{61a^3}{x^2}-\frac{7a^2}{x^3}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}}{a^5c^4}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{\frac{1}{7}\int \frac{\left(7a^5+\frac{35a^4}{x}+\frac{61a^3}{x^2}-\frac{7a^2}{x^3}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}+\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^5c^4}$$

$$\Big\downarrow \textbf{2336}$$

$$-\frac{\frac{1}{7}\left(\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}\int -\frac{\left(35a^5+\frac{175a^4}{x}+\frac{272a^3}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^5c^4}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\int \frac{\left(35a^5+\frac{175a^4}{x}+\frac{272a^3}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}+\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^5c^4}$$

$$\Big\downarrow \textbf{2336}$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{a^3\left(175a+\frac{307}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(105a^5+\frac{525a^4}{x}+\frac{614a^3}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)+\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^5c^4}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{\left(105a^5+\frac{525a^4}{x}+\frac{614a^3}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{a^3\left(175a+\frac{307}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^5c^4}$$

$$\Big\downarrow \textbf{2336}$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{a^3\left(525a+\frac{719}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{105a^4\left(a+\frac{5}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{a^3\left(175a+\frac{307}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^5c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(105a^4\int \frac{\left(a+\frac{5}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^3\left(525a+\frac{719}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{a^3\left(175a+\frac{307}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^5c^4}$$

$$\Big\downarrow \text{534}$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(105a^4\left(5\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^3\left(525a+\frac{719}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{a^3\left(175a+\frac{307}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{16a^3 \ldots}{7(1- \ldots}}{a^5c^4}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(105a^4\left(\frac{5}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^3\left(525a+\frac{719}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{a^3\left(175a+\frac{307}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{16a \ldots}{7(1- \ldots}}{a^5c^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(105a^4\left(-5a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^3\left(525a+\frac{719}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{a^3\left(175a+\frac{307}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{4a^3 \ldots}{5(1- \ldots}}{a^5c^4}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{16a^3\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{1}{7}\left(\frac{4a^3\left(7a+\frac{17}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{1}{5}\left(\frac{a^3\left(175a+\frac{307}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{1}{3}\left(105a^4\left(-5\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^3 \ldots}{\ldots}\right.\right.\right.}{a^5c^4}$$

input

```
Int[E^ArcCoth[a*x]/(c - c/(a*x))^4,x]
```

output

```
-(((16*a^3*(a + x^(-1)))/(7*(1 - 1/(a^2*x^2))^(7/2)) + ((4*a^3*(7*a + 17/x
))/(5*(1 - 1/(a^2*x^2))^(5/2)) + ((a^3*(175*a + 307/x))/(3*(1 - 1/(a^2*x^2
))^(3/2)) + ((a^3*(525*a + 719/x))/Sqrt[1 - 1/(a^2*x^2)] + 105*a^4*(-(a*Sq
rt[1 - 1/(a^2*x^2)]*x) - 5*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/3)/5)/7)/(a^5*
c^4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.41

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{1024\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{105a^6\left(x-\frac{1}{a}\right)}-\frac{2\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{7a^9\left(x-\frac{1}{a}\right)^4}-\frac{57\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{35a^8\left(x-\frac{1}{a}\right)^3}-\frac{446\sqrt{}}{}\right.}{c^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{525\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^6x^5+525\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^5x^5-2625\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^5x^4-420\sqrt{a^2}\,((ax-}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^4/((a*x-1)/(a*x+1))^(1/2)+(5/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-1024/105/a^6/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^
(1/2)-2/7/a^9/(x-1/a)^4*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-57/35/a^8/(x-1/a
)^3*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-446/105/a^7/(x-1/a)^2*((x-1/a)^2*a^2
+2*a*(x-1/a))^(1/2))*a^4/c^4/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/
2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.09

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{525\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-525\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log}{105\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/105*(525*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1
)/(a*x + 1)) + 1) - 525*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(
sqrt((a*x - 1)/(a*x + 1)) - 1) + (105*a^5*x^5 - 1339*a^4*x^4 + 1812*a^3*x^
3 + 485*a^2*x^2 - 1947*a*x + 824)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^4*x^4
- 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\int \frac{x^4}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**4,x)
```

output
```
a**4*Integral(x**4/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**3*x
**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**2*x**2*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1)) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))), x)/c**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.90

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{1}{420}\,a\left(\frac{\frac{111\,(ax-1)}{ax+1}+\frac{469\,(ax-1)^2}{(ax+1)^2}+\frac{2765\,(ax-1)^3}{(ax+1)^3}-\frac{4200\,(ax-1)^4}{(ax+1)^4}+15}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{2100\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{2100\,\log\left(\sqrt{\right.}}{a^2c^4}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
1/420*a*((111*(a*x - 1)/(a*x + 1) + 469*(a*x - 1)^2/(a*x + 1)^2 + 2765*(a*
x - 1)^3/(a*x + 1)^3 - 4200*(a*x - 1)^4/(a*x + 1)^4 + 15)/(a^2*c^4*((a*x -
 1)/(a*x + 1))^(9/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2)) + 2100*log(sqr
t((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 2100*log(sqrt((a*x - 1)/(a*x + 1))
 - 1)/(a^2*c^4))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.73

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{10\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4} - \frac{\frac{67\,(a\,x-1)^2}{15\,(a\,x+1)^2}+\frac{79\,(a\,x-1)^3}{3\,(a\,x+1)^3}-\frac{40\,(a\,x-1)^4}{(a\,x+1)^4}+\frac{37\,(a\,x-1)}{35\,(a\,x+1)}+\frac{1}{7}}{4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}-4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

input
```
int(1/((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(10*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^4) - ((67*(a*x - 1)^2)/(15*(a
*x + 1)^2) + (79*(a*x - 1)^3)/(3*(a*x + 1)^3) - (40*(a*x - 1)^4)/(a*x + 1)
^4 + (37*(a*x - 1))/(35*(a*x + 1)) + 1/7)/(4*a*c^4*((a*x - 1)/(a*x + 1))^(
7/2) - 4*a*c^4*((a*x - 1)/(a*x + 1))^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.43

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$

$$= \frac{1050\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^3x^3-3150\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^2x^2+3150\sqrt{ax-1}\log\left(\frac{\sqrt{ax-}}{}\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x)
```

output
```
(1050*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**3*x**3
 - 3150*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**2*x*
*2 + 3150*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x -
 1050*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 754*sqr
t(a*x - 1)*a**3*x**3 - 2262*sqrt(a*x - 1)*a**2*x**2 + 2262*sqrt(a*x - 1)*a
*x - 754*sqrt(a*x - 1) + 105*sqrt(a*x + 1)*a**4*x**4 - 1444*sqrt(a*x + 1)*
a**3*x**3 + 3256*sqrt(a*x + 1)*a**2*x**2 - 2771*sqrt(a*x + 1)*a*x + 824*sq
rt(a*x + 1))/(105*sqrt(a*x - 1)*a*c**4*(a**3*x**3 - 3*a**2*x**2 + 3*a*x -
1))
```

3.402 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=-\frac{c^5}{4a^5x^4}+\frac{c^5}{a^4x^3}-\frac{c^5}{a^3x^2}-\frac{2c^5}{a^2x}+c^5x-\frac{3c^5\log(x)}{a}$$

output
```
-1/4*c^5/a^5/x^4+c^5/a^4/x^3-c^5/a^3/x^2-2*c^5/a^2/x+c^5*x-3*c^5*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=-\frac{c^5\left(\frac{5a^4}{4}+\frac{1}{4x^4}-\frac{a}{x^3}+\frac{a^2}{x^2}+\frac{2a^3}{x}-a^5x+3a^4\log(x)\right)}{a^5}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

output
```
-((c^5*((5*a^4)/4 + 1/(4*x^4) - a/x^3 + a^2/x^2 + (2*a^3)/x - a^5*x + 3*a^
4*Log[x]))/a^5)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^5 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{c^5 e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^5}{a^5}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^5\int e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^5dx}{a^5}$$

$$\Big\downarrow \text{ 6681}$$

$$\frac{c^5\int \frac{e^{2\text{arctanh}(ax)}(1-ax)^5}{x^5}dx}{a^5}$$

$$\Big\downarrow \text{ 6679}$$

$$\frac{c^5\int \frac{(1-ax)^4(ax+1)}{x^5}dx}{a^5}$$

$$\Big\downarrow \text{ 84}$$

$$\frac{c^5\int \left(a^5-\frac{3a^4}{x}+\frac{2a^3}{x^2}+\frac{2a^2}{x^3}-\frac{3a}{x^4}+\frac{1}{x^5}\right)dx}{a^5}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^5\left(a^5x-3a^4\log(x)-\frac{2a^3}{x}-\frac{a^2}{x^2}+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{a^5}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

output
```
(c^5*(-1/4*1/x^4 + a/x^3 - a^2/x^2 - (2*a^3)/x + a^5*x - 3*a^4*Log[x]))/a^
5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result
default	$\frac{c^5\left(x\,a^5-\frac{a^2}{x^2}-\frac{2a^3}{x}-\frac{1}{4x^4}-3a^4\ln(x)+\frac{a}{x^3}\right)}{a^5}$
risch	$c^5x+\frac{-2a^3c^5x^3-a^2c^5x^2+a\,c^5x-\frac{1}{4}c^5}{a^5x^4}-\frac{3c^5\ln(x)}{a}$
norman	$\frac{c^5x+a^4c^5x^5-\frac{c^5}{4a}-2a^2c^5x^3-c^5a\,x^2}{a^4x^4}-\frac{3c^5\ln(x)}{a}$
parallelrisch	$-\frac{-4a^5c^5x^5+12c^5\ln(x)a^4x^4+8a^3c^5x^3+4a^2c^5x^2-4a\,c^5x+c^5}{4a^5x^4}$
meijerg	$-\frac{c^5(-ax-\ln(-ax+1))}{a}-\frac{4c^5\ln(-ax+1)}{a}-\frac{5c^5(\ln(x)+\ln(-a)-\ln(-ax+1))}{a}+\frac{5c^5\left(-\frac{1}{2a^2x^2}-\frac{1}{ax}+\ln(x)+\ln(-a)-\ln(\right.}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x,method=_RETURNVERBOSE)
```

output
```
c^5/a^5*(x*a^5-a^2/x^2-2*a^3/x-1/4/x^4-3*a^4*ln(x)+a/x^3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx = \frac{4\,a^5c^5x^5-12\,a^4c^5x^4\log\left(x\right)-8\,a^3c^5x^3-4\,a^2c^5x^2+4\,ac^5x-c^5}{4\,a^5x^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="fricas")
```

output
```
1/4*(4*a^5*c^5*x^5 - 12*a^4*c^5*x^4*log(x) - 8*a^3*c^5*x^3 - 4*a^2*c^5*x^2
 + 4*a*c^5*x - c^5)/(a^5*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=\frac{a^5c^5x-3a^4c^5\log\left(x\right)+\frac{-8a^3c^5x^3-4a^2c^5x^2+4ac^5x-c^5}{4x^4}}{a^5}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**5,x)
```

output
```
(a**5*c**5*x - 3*a**4*c**5*log(x) + (-8*a**3*c**5*x**3 - 4*a**2*c**5*x**2
+ 4*a*c**5*x - c**5)/(4*x**4))/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=c^5x-\frac{3\,c^5\log\left(x\right)}{a}-\frac{8\,a^3c^5x^3+4\,a^2c^5x^2-4\,ac^5x+c^5}{4\,a^5x^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="maxima")
```

output
```
c^5*x - 3*c^5*log(x)/a - 1/4*(8*a^3*c^5*x^3 + 4*a^2*c^5*x^2 - 4*a*c^5*x +
c^5)/(a^5*x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=c^5x-\frac{3\,c^5\log\left(|x|\right)}{a}-\frac{8\,a^3c^5x^3+4\,a^2c^5x^2-4\,ac^5x+c^5}{4\,a^5x^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="giac")
```

output
```
c^5*x - 3*c^5*log(abs(x))/a - 1/4*(8*a^3*c^5*x^3 + 4*a^2*c^5*x^2 - 4*a*c^5
*x + c^5)/(a^5*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx$$
$$= -\frac{c^5 \left(4 a^2 x^2 - 4 a x + 8 a^3 x^3 - 4 a^5 x^5 + 12 a^4 x^4 \ln(x) + 1\right)}{4 a^5 x^4}$$

input
```
int(((c - c/(a*x))^5*(a*x + 1))/(a*x - 1),x)
```

output
```
-(c^5*(4*a^2*x^2 - 4*a*x + 8*a^3*x^3 - 4*a^5*x^5 + 12*a^4*x^4*log(x) + 1))
/(4*a^5*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx = \frac{c^5(-12\log(x)\,a^4x^4 + 4a^5x^5 - 8a^3x^3 - 4a^2x^2 + 4ax - 1)}{4a^5x^4}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x)
```

output
```
(c**5*( - 12*log(x)*a**4*x**4 + 4*a**5*x**5 - 8*a**3*x**3 - 4*a**2*x**2 +
4*a*x - 1))/(4*a**5*x**4)
```

3.403 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 40

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{c^4}{3a^4x^3}-\frac{c^4}{a^3x^2}+c^4x-\frac{2c^4\log(x)}{a}$$

output
```
1/3*c^4/a^4/x^3-c^4/a^3/x^2+c^4*x-2*c^4*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{c^4\left(\frac{4a^3}{3}-\frac{1}{3x^3}+\frac{a}{x^2}-a^4x+2a^3\log(x)\right)}{a^4}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

output
```
-((c^4*((4*a^3)/3 - 1/(3*x^3) + a/x^2 - a^4*x + 2*a^3*Log[x]))/a^4)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6681, 6679, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^4 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^4 e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\int e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^4dx}{a^4}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{c^4\int \frac{e^{2\text{arctanh}(ax)}(1-ax)^4}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{c^4\int \frac{(1-ax)^3(ax+1)}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{84}$$

$$-\frac{c^4\int \left(-a^4+\frac{2a^3}{x}-\frac{2a}{x^3}+\frac{1}{x^4}\right)dx}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^4\left(a^4(-x)+2a^3\log(x)+\frac{a}{x^2}-\frac{1}{3x^3}\right)}{a^4}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

output
```
-((c^4*(-1/3*1/x^3 + a/x^2 - a^4*x + 2*a^3*Log[x]))/a^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result
default	$\frac{c^{4}\left(x\,a^{4}-\frac{a}{x^{2}}-2a^{3}\ln\left(x\right)+\frac{1}{3x^{3}}\right)}{a^{4}}$
risch	$c^{4}x+\frac{-a\,c^{4}x+\frac{1}{3}c^{4}}{a^{4}x^{3}}-\frac{2c^{4}\ln\left(x\right)}{a}$
norman	$\frac{a^{3}c^{4}x^{4}+\frac{c^{4}}{3a}-c^{4}x}{a^{3}x^{3}}-\frac{2c^{4}\ln\left(x\right)}{a}$
parallelrisch	$-\frac{-3a^{4}c^{4}x^{4}+6c^{4}\ln\left(x\right)a^{3}x^{3}+3a\,c^{4}x-c^{4}}{3a^{4}x^{3}}$
meijerg	$-\frac{c^{4}\left(-ax-\ln\left(-ax+1\right)\right)}{a}-\frac{3c^{4}\ln\left(-ax+1\right)}{a}-\frac{2c^{4}\left(\ln\left(x\right)+\ln\left(-a\right)-\ln\left(-ax+1\right)\right)}{a}+\frac{2c^{4}\left(\frac{1}{ax}-\ln\left(x\right)-\ln\left(-a\right)+\ln\left(-ax+1\right)\right)}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
c^4/a^4*(x*a^4-a/x^2-2*a^3*ln(x)+1/3/x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{3\,a^4c^4x^4-6\,a^3c^4x^3\log\left(x\right)-3\,ac^4x+c^4}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*c^4*x^4 - 6*a^3*c^4*x^3*log(x) - 3*a*c^4*x + c^4)/(a^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{a^4c^4x-2a^3c^4\log(x)+\frac{-3ac^4x+c^4}{3x^3}}{a^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**4,x)
```

output
```
(a**4*c**4*x - 2*a**3*c**4*log(x) + (-3*a*c**4*x + c**4)/(3*x**3))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = c^4x-\frac{2\,c^4\log(x)}{a}-\frac{3\,ac^4x-c^4}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
c^4*x - 2*c^4*log(x)/a - 1/3*(3*a*c^4*x - c^4)/(a^4*x^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = c^4x-\frac{2\,c^4\log(|x|)}{a}-\frac{3\,ac^4x-c^4}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="giac")
```

output
```
c^4*x - 2*c^4*log(abs(x))/a - 1/3*(3*a*c^4*x - c^4)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 13.66 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx = -\frac{c^4\,(3\,a\,x-3\,a^4\,x^4+6\,a^3\,x^3\,\ln(x)-1)}{3\,a^4\,x^3}$$

input
```
int(((c - c/(a*x))^4*(a*x + 1))/(a*x - 1),x)
```

output
```
-(c^4*(3*a*x - 3*a^4*x^4 + 6*a^3*x^3*log(x) - 1))/(3*a^4*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx = \frac{c^4(-6\log(x)\,a^3x^3+3a^4x^4-3ax+1)}{3a^4x^3}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x)
```

output
```
(c**4*( - 6*log(x)*a**3*x**3 + 3*a**4*x**4 - 3*a*x + 1))/(3*a**4*x**3)
```

3.404 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 39

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{c^3\log(x)}{a}$$

output
```
-1/2*c^3/a^3/x^2+c^3/a^2/x+c^3*x-c^3*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{c^3\left(3a^2+\frac{1}{x^2}-\frac{2a}{x}-2a^3x+2a^2\log(x)\right)}{2a^3}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

output
```
-1/2*(c^3*(3*a^2 + x^(-2) - (2*a)/x - 2*a^3*x + 2*a^2*Log[x]))/a^3
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^3 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^3 e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^3}{a^3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\int e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^3dx}{a^3}$$

$$\Big\downarrow \text{6681}$$

$$\frac{c^3\int \frac{e^{2\text{arctanh}(ax)}(1-ax)^3}{x^3}dx}{a^3}$$

$$\Big\downarrow \text{6679}$$

$$\frac{c^3\int \frac{(1-ax)^2(ax+1)}{x^3}dx}{a^3}$$

$$\Big\downarrow \text{84}$$

$$\frac{c^3\int \left(a^3-\frac{a^2}{x}-\frac{a}{x^2}+\frac{1}{x^3}\right)dx}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3\left(a^3x-a^2\log(x)+\frac{a}{x}-\frac{1}{2x^2}\right)}{a^3}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

output
```
(c^3*(-1/2*1/x^2 + a/x + a^3*x - a^2*Log[x]))/a^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result
default	$\frac{c^3\left(a^3x-\frac{1}{2x^2}+\frac{a}{x}-a^2\ln(x)\right)}{a^3}$
risch	$c^3x+\frac{a\,c^3x-\frac{1}{2}c^3}{a^3x^2}-\frac{c^3\ln(x)}{a}$
norman	$\frac{c^3x+a^2c^3x^3-\frac{c^3}{2a}}{a^2x^2}-\frac{c^3\ln(x)}{a}$
parallelrisch	$-\frac{-2a^3c^3x^3+2c^3\ln(x)a^2x^2-2a\,c^3x+c^3}{2a^3x^2}$
meijerg	$-\frac{c^3(-ax-\ln(-ax+1))}{a}-\frac{2c^3\ln(-ax+1)}{a}+\frac{2c^3\left(\frac{1}{ax}-\ln(x)-\ln(-a)+\ln(-ax+1)\right)}{a}+\frac{c^3\left(-\frac{1}{2a^2x^2}-\frac{1}{ax}+\ln(x)+\ln(-a)-\right.}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
c^3/a^3*(a^3*x-1/2/x^2+a/x-a^2*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{2\,a^3c^3x^3-2\,a^2c^3x^2\log(x)+2\,ac^3x-c^3}{2\,a^3x^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
1/2*(2*a^3*c^3*x^3 - 2*a^2*c^3*x^2*log(x) + 2*a*c^3*x - c^3)/(a^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{a^3c^3x-a^2c^3\log{(x)}+\frac{2ac^3x-c^3}{2x^2}}{a^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**3,x)
```

output
```
(a**3*c**3*x - a**2*c**3*log(x) + (2*a*c**3*x - c**3)/(2*x**2))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=c^3x-\frac{c^3\log{(x)}}{a}+\frac{2\,ac^3x-c^3}{2\,a^3x^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
c^3*x - c^3*log(x)/a + 1/2*(2*a*c^3*x - c^3)/(a^3*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=c^3x-\frac{c^3\log{(|x|)}}{a}+\frac{2\,ac^3x-c^3}{2\,a^3x^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="giac")
```

output
```
c^3*x - c^3*log(abs(x))/a + 1/2*(2*a*c^3*x - c^3)/(a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 13.83 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\,\left(2\,a\,x+2\,a^3\,x^3-2\,a^2\,x^2\,\ln\left(x\right)-1\right)}{2\,a^3\,x^2}$$

input
```
int(((c - c/(a*x))^3*(a*x + 1))/(a*x - 1),x)
```

output
```
(c^3*(2*a*x + 2*a^3*x^3 - 2*a^2*x^2*log(x) - 1))/(2*a^3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3(-2\log(x)\,a^2x^2+2a^3x^3+2ax-1)}{2a^3x^2}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x)
```

output
```
(c**3*( - 2*log(x)*a**2*x**2 + 2*a**3*x**3 + 2*a*x - 1))/(2*a**3*x**2)
```

3.405 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 16

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2}{a^2x}+c^2x$$

output
```
c^2/a^2/x+c^2*x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2}{a^2x}+c^2x$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

output
```
c^2/(a^2*x) + c^2*x
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6717, 27, 6681, 6679, 82, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^2 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^2 e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\int e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^2dx}{a^2}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{c^2\int \frac{e^{2\text{arctanh}(ax)}(1-ax)^2}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{c^2\int \frac{(1-ax)(ax+1)}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{82}$$

$$-\frac{c^2\int \frac{1-a^2x^2}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{244}$$

$$-\frac{c^2\int \left(\frac{1}{x^2}-a^2\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2\left(a^2(-x)-\frac{1}{x}\right)}{a^2}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

output
```
-((c^2*(-x^(-1) - a^2*x))/a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	s
default	$\frac{c^{2}\left(a^{2} x+\frac{1}{x}\right)}{a^{2}}$	1
risch	$\frac{c^{2}}{a^{2} x}+c^{2} x$	1
gosper	$\frac{c^{2}\left(a^{2} x^{2}+1\right)}{x\, a^{2}}$	2
parallelrisch	$\frac{c^{2} a^{2} x^{2}+c^{2}}{a^{2} x}$	2
norman	$\frac{\frac{c^{2}}{a}+a\, c^{2} x^{2}}{a x}$	2
orering	$\frac{x\left(a^{2} x^{2}+1\right)\left(c-\frac{c}{a x}\right)^{2}}{\left(a x-1\right)^{2}}$	3
meijerg	$-\frac{c^{2}\left(-a x-\ln\left(-a x+1\right)\right)}{a}-\frac{c^{2} \ln\left(-a x+1\right)}{a}+\frac{c^{2}\left(\ln\left(x\right)+\ln\left(-a\right)-\ln\left(-a x+1\right)\right)}{a}+\frac{c^{2}\left(\frac{1}{a x}-\ln\left(x\right)-\ln\left(-a\right)+\ln\left(-a x+1\right)\right)}{a}$	9

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2/a^2*(a^2*x+1/x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{a^2c^2x^2+c^2}{a^2x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
(a^2*c^2*x^2 + c^2)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{a^2c^2x+\frac{c^2}{x}}{a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**2,x)
```

output
```
(a**2*c**2*x + c**2/x)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=c^2x+\frac{c^2}{a^2x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
c^2*x + c^2/(a^2*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=c^2x+\frac{c^2}{a^2x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="giac")
```

output
```
c^2*x + c^2/(a^2*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\left(a^2\,x^2+1\right)}{a^2\,x}$$

input
```
int(((c - c/(a*x))^2*(a*x + 1))/(a*x - 1),x)
```

output
```
(c^2*(a^2*x^2 + 1))/(a^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2(a^2x^2+1)}{a^2x}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x)
```

output
```
(c**2*(a**2*x**2 + 1))/(a**2*x)
```

3.406 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 11

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\log(x)}{a}$$

output
```
c*x+c*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\log(x)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x)),x]
```

output
```
c*x + (c*Log[x])/a
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6717, 27, 6681, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right) e^{2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{ce^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)}{a}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c\int e^{2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)dx}{a}$$

$$\Big\downarrow \text{ 6681}$$

$$\frac{c\int \frac{e^{2\text{arctanh}(ax)}(1-ax)}{x}dx}{a}$$

$$\Big\downarrow \text{ 6679}$$

$$\frac{c\int \frac{ax+1}{x}dx}{a}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{c\int \left(a+\frac{1}{x}\right)dx}{a}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c(ax+\log(x))}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x)),x]
```

output
```
(c*(a*x + Log[x]))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{c(ax+\ln(x))}{a}$	12
norman	$xc+\frac{c\ln(x)}{a}$	12
risch	$xc+\frac{c\ln(x)}{a}$	12
parallelrisch	$\frac{acx+c\ln(x)}{a}$	14
meijerg	$-\frac{c(-ax-\ln(-ax+1))}{a}+\frac{c(\ln(x)+\ln(-a)-\ln(-ax+1))}{a}$	43

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
c/a*(a*x+ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{acx+c\log(x)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="fricas")
```

output
```
(a*c*x + c*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{acx+c\log(x)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x)
```

output
```
(a*c*x + c*log(x))/a
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\log(x)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="maxima")
```

output
```
c*x + c*log(x)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\log(|x|)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="giac")
```

output
```
c*x + c*log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c\,(\ln(x)+a\,x)}{a}$$

input
```
int(((c - c/(a*x))*(a*x + 1))/(a*x - 1),x)
```

output
```
(c*(log(x) + a*x))/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c(\log(x)+ax)}{a}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x),x)
```

output
```
(c*(log(x) + a*x))/a
```

3.407 $\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 37

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{x}{c}+\frac{2}{ac(1-ax)}+\frac{3\log(1-ax)}{ac}$$

output
```
x/c+2/a/c/(-a*x+1)+3*ln(-a*x+1)/a/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{ax+\frac{2}{1-ax}+3\log(1-ax)}{ac}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x)),x]
```

output
```
(a*x + 2/(1 - a*x) + 3*Log[1 - a*x])/(a*c)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{ae^{2\text{arctanh}(ax)}}{c\left(a-\frac{1}{x}\right)}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{e^{2\text{arctanh}(ax)}}{a-\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{6681}$$

$$\frac{a\int \frac{e^{2\text{arctanh}(ax)}x}{1-ax}dx}{c}$$

$$\Big\downarrow \text{6679}$$

$$\frac{a\int \frac{x(ax+1)}{(1-ax)^2}dx}{c}$$

$$\Big\downarrow \text{86}$$

$$\frac{a\int \left(\frac{3}{(ax-1)a}+\frac{2}{(ax-1)^2a}+\frac{1}{a}\right)dx}{c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(\frac{2}{a^2(1-ax)}+\frac{3\log(1-ax)}{a^2}+\frac{x}{a}\right)}{c}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a*x)),x]
```

output
```
(a*(x/a + 2/(a^2*(1 - a*x)) + (3*Log[1 - a*x])/a^2))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{a\left(\frac{x}{a}-\frac{2}{(ax-1)a^{2}}+\frac{3\ln(ax-1)}{a^{2}}\right)}{c}$	35
risch	$\frac{x}{c}-\frac{2}{ac(ax-1)}+\frac{3\ln(ax-1)}{ac}$	36
norman	$\frac{\frac{a\,x^{2}}{c}-\frac{3x}{c}}{ax-1}+\frac{3\ln(ax-1)}{ac}$	39
parallelrisch	$\frac{a^{2}x^{2}+3a\ln(ax-1)x-3ax-3\ln(ax-1)}{c(ax-1)a}$	45

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
a/c*(x/a-2/(a*x-1)/a^2+3/a^2*ln(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a^{2}x^{2}-ax+3\,(ax-1)\log{(ax-1)}-2}{a^{2}cx-ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="fricas")
```

output
```
(a^2*x^2 - a*x + 3*(a*x - 1)*log(a*x - 1) - 2)/(a^2*c*x - a*c)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{2}{a^2cx-ac}+\frac{x}{c}+\frac{3\log(ax-1)}{ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x)
```

output
```
-2/(a**2*c*x - a*c) + x/c + 3*log(a*x - 1)/(a*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{x}{c}-\frac{2}{a^2cx-ac}+\frac{3\log(ax-1)}{ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="maxima")
```

output
```
x/c - 2/(a^2*c*x - a*c) + 3*log(a*x - 1)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{x}{c}+\frac{3\log(|ax-1|)}{ac}-\frac{2}{(ax-1)ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="giac")
```

output
```
x/c + 3*log(abs(a*x - 1))/(a*c) - 2/((a*x - 1)*a*c)
```

Mupad [B] (verification not implemented)

Time = 13.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{x}{c}+\frac{2}{a\,\left(c-a\,c\,x\right)}+\frac{3\,\ln\left(a\,x-1\right)}{a\,c}$$

input
```
int((a*x + 1)/((c - c/(a*x))*(a*x - 1)),x)
```

output
```
x/c + 2/(a*(c - a*c*x)) + (3*log(a*x - 1))/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{3\log(ax-1)\,ax-3\log(ax-1)+a^2x^2-3ax}{ac\,(ax-1)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x),x)
```

output
```
(3*log(a*x - 1)*a*x - 3*log(a*x - 1) + a**2*x**2 - 3*a*x)/(a*c*(a*x - 1))
```

3.408 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{x}{c^2}-\frac{1}{ac^2(1-ax)^2}+\frac{5}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}$$

output
```
x/c^2-1/a/c^2/(-a*x+1)^2+5/a/c^2/(-a*x+1)+4*ln(-a*x+1)/a/c^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=-\frac{a^2\left(-\frac{x}{a^2}+\frac{1}{a^3(1-ax)^2}-\frac{5}{a^3(1-ax)}-\frac{4\log(1-ax)}{a^3}\right)}{c^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

output
```
-((a^2*(-(x/a^2) + 1/(a^3*(1 - a*x)^2) - 5/(a^3*(1 - a*x)) - (4*Log[1 - a*
x])/a^3))/c^2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^2e^{2\text{arctanh}(ax)}}{c^2\left(a-\frac{1}{x}\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\int \frac{e^{2\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^2}dx}{c^2}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{a^2\int \frac{e^{2\text{arctanh}(ax)}x^2}{(1-ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{a^2\int \frac{x^2(ax+1)}{(1-ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{86}$$

$$-\frac{a^2\int \left(-\frac{1}{a^2}-\frac{4}{a^2(ax-1)}-\frac{5}{a^2(ax-1)^2}-\frac{2}{a^2(ax-1)^3}\right)dx}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\left(-\frac{5}{a^3(1-ax)}+\frac{1}{a^3(1-ax)^2}-\frac{4\log(1-ax)}{a^3}-\frac{x}{a^2}\right)}{c^2}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

output
```
-((a^2*(-(x/a^2) + 1/(a^3*(1 - a*x)^2) - 5/(a^3*(1 - a*x)) - (4*Log[1 - a*
x])/a^3))/c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x}{c^2}+\frac{-5c^2x+\frac{4c^2}{a}}{c^4(ax-1)^2}+\frac{4\ln(ax-1)}{a\,c^2}$	47
default	$\frac{a^2\left(\frac{x}{a^2}-\frac{5}{(ax-1)a^3}+\frac{4\ln(ax-1)}{a^3}-\frac{1}{a^3(ax-1)^2}\right)}{c^2}$	49
norman	$\frac{\frac{a^2x^3}{c}-\frac{6a\,x^2}{c}+\frac{4x}{c}}{(ax-1)^2c}+\frac{4\ln(ax-1)}{a\,c^2}$	53
parallelrisch	$\frac{a^3x^3+4a^2\ln(ax-1)x^2-6a^2x^2-8a\ln(ax-1)x+4ax+4\ln(ax-1)}{c^2(ax-1)^2a}$	67

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
x/c^2+(-5*c^2*x+4*c^2/a)/c^4/(a*x-1)^2+4/a/c^2*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int\frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{a^3x^3-2\,a^2x^2-4\,ax+4\,(a^2x^2-2\,ax+1)\log\left(ax-1\right)+4}{a^3c^2x^2-2\,a^2c^2x+ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
(a^3*x^3 - 2*a^2*x^2 - 4*a*x + 4*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 4)/(
a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{-5ax+4}{a^3c^2x^2-2a^2c^2x+ac^2}+\frac{x}{c^2}+\frac{4\log\left(ax-1\right)}{ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**2,x)
```

output
```
(-5*a*x + 4)/(a**3*c**2*x**2 - 2*a**2*c**2*x + a*c**2) + x/c**2 + 4*log(a*
x - 1)/(a*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=-\frac{5\,ax-4}{a^3c^2x^2-2\,a^2c^2x+ac^2}+\frac{x}{c^2}+\frac{4\,\log\left(ax-1\right)}{ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
-(5*a*x - 4)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) + x/c^2 + 4*log(a*x - 1)/
(a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{x}{c^2}+\frac{4\,\log\left(|ax-1|\right)}{ac^2}-\frac{5\,ax-4}{(ax-1)^2ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="giac")
```

output
```
x/c^2 + 4*log(abs(a*x - 1))/(a*c^2) - (5*a*x - 4)/((a*x - 1)^2*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{x}{c^2}-\frac{5\,x-\frac{4}{a}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2}+\frac{4\,\ln\left(a\,x-1\right)}{a\,c^2}$$

input
```
int((a*x + 1)/((c - c/(a*x))^2*(a*x - 1)),x)
```

output
```
x/c^2 - (5*x - 4/a)/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) + (4*log(a*x - 1))/(a*
c^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{4\log(ax-1)\,a^2x^2-8\log(ax-1)\,ax+4\log(ax-1)+a^3x^3-4a^2x^2+2}{a\,c^2\,(a^2x^2-2ax+1)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x)
```

output
```
(4*log(a*x - 1)*a**2*x**2 - 8*log(a*x - 1)*a*x + 4*log(a*x - 1) + a**3*x**
3 - 4*a**2*x**2 + 2)/(a*c**2*(a**2*x**2 - 2*a*x + 1))
```

3.409 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{x}{c^3}+\frac{2}{3ac^3(1-ax)^3}-\frac{7}{2ac^3(1-ax)^2}+\frac{9}{ac^3(1-ax)}+\frac{5\log(1-ax)}{ac^3}$$

output
```
x/c^3+2/3/a/c^3/(-a*x+1)^3-7/2/a/c^3/(-a*x+1)^2+9/a/c^3/(-a*x+1)+5*ln(-a*x
+1)/a/c^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{-37+81ax-36a^2x^2-18a^3x^3+6a^4x^4+30(-1+ax)^3\log(1-ax)}{6ac^3(-1+ax)^3}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

output
```
(-37 + 81*a*x - 36*a^2*x^2 - 18*a^3*x^3 + 6*a^4*x^4 + 30*(-1 + a*x)^3*Log[
1 - a*x])/(6*a*c^3*(-1 + a*x)^3)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^3e^{2\text{arctanh}(ax)}}{c^3\left(a-\frac{1}{x}\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\int \frac{e^{2\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^3}dx}{c^3}$$

$$\Big\downarrow \text{6681}$$

$$\frac{a^3\int \frac{e^{2\text{arctanh}(ax)}x^3}{(1-ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{6679}$$

$$\frac{a^3\int \frac{x^3(ax+1)}{(1-ax)^4}dx}{c^3}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^3\int \left(\frac{1}{a^3}+\frac{5}{a^3(ax-1)}+\frac{9}{a^3(ax-1)^2}+\frac{7}{a^3(ax-1)^3}+\frac{2}{a^3(ax-1)^4}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\left(\frac{9}{a^4(1-ax)}-\frac{7}{2a^4(1-ax)^2}+\frac{2}{3a^4(1-ax)^3}+\frac{5\log(1-ax)}{a^4}+\frac{x}{a^3}\right)}{c^3}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

output
```
(a^3*(x/a^3 + 2/(3*a^4*(1 - a*x)^3) - 7/(2*a^4*(1 - a*x)^2) + 9/(a^4*(1 -
a*x)) + (5*Log[1 - a*x])/a^4))/c^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{x}{c^{3}}+\frac{-9 a \,c^{3} x^{2}+\frac{29 c^{3} x}{2}-\frac{37 c^{3}}{6 a}}{c^{6} \left(a x -1\right)^{3}}+\frac{5 \ln \left(a x -1\right)}{a \,c^{3}}$	56
default	$\frac{a^{3} \left(\frac{x}{a^{3}}-\frac{9}{a^{4} \left(a x -1\right)}+\frac{5 \ln \left(a x -1\right)}{a^{4}}-\frac{7}{2 a^{4} \left(a x -1\right)^{2}}-\frac{2}{3 a^{4} \left(a x -1\right)^{3}}\right)}{c^{3}}$	61
norman	$\frac{\frac{a^{3} x^{4}}{c}-\frac{5 x}{c}+\frac{25 a \,x^{2}}{2 c}-\frac{55 a^{2} x^{3}}{6 c}}{\left(a x -1\right)^{3} c^{2}}+\frac{5 \ln \left(a x -1\right)}{a \,c^{3}}$	64
parallelrisch	$\frac{6 a^{4} x^{4}+30 a^{3} \ln \left(a x -1\right) x^{3}-55 a^{3} x^{3}-90 a^{2} \ln \left(a x -1\right) x^{2}+75 a^{2} x^{2}+90 a \ln \left(a x -1\right) x -30 a x -30 \ln \left(a x -1\right)}{6 c^{3} \left(a x -1\right)^{3} a}$	91

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
x/c^3+(-9*a*c^3*x^2+29/2*c^3*x-37/6*c^3/a)/c^6/(a*x-1)^3+5/a/c^3*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.37

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{6\,a^4x^4 - 18\,a^3x^3 - 36\,a^2x^2 + 81\,ax + 30\,(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1)\log\left(ax-1\right) - 37}{6\,(a^4c^3x^3 - 3\,a^3c^3x^2 + 3\,a^2c^3x - ac^3)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
1/6*(6*a^4*x^4 - 18*a^3*x^3 - 36*a^2*x^2 + 81*a*x + 30*(a^3*x^3 - 3*a^2*x^
2 + 3*a*x - 1)*log(a*x - 1) - 37)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3
*x - a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{-54a^2x^2+87ax-37}{6a^4c^3x^3-18a^3c^3x^2+18a^2c^3x-6ac^3}+\frac{x}{c^3}+\frac{5\log(ax-1)}{ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**3,x)
```

output
```
(-54*a**2*x**2 + 87*a*x - 37)/(6*a**4*c**3*x**3 - 18*a**3*c**3*x**2 + 18*a
**2*c**3*x - 6*a*c**3) + x/c**3 + 5*log(a*x - 1)/(a*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{54\,a^2x^2-87\,ax+37}{6\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}+\frac{x}{c^3}+\frac{5\,\log(ax-1)}{ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
-1/6*(54*a^2*x^2 - 87*a*x + 37)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x
 - a*c^3) + x/c^3 + 5*log(a*x - 1)/(a*c^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3}+\frac{5\,\log(|ax-1|)}{ac^3}-\frac{54\,a^2x^2-87\,ax+37}{6\,(ax-1)^3ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="giac")
```

output
```
x/c^3 + 5*log(abs(a*x - 1))/(a*c^3) - 1/6*(54*a^2*x^2 - 87*a*x + 37)/((a*x
 - 1)^3*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 13.67 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{9\,a\,x^2-\frac{29\,x}{2}+\frac{37}{6\,a}}{-a^3\,c^3\,x^3+3\,a^2\,c^3\,x^2-3\,a\,c^3\,x+c^3}+\frac{x}{c^3}+\frac{5\,\ln\left(a\,x-1\right)}{a\,c^3}$$

input
```
int((a*x + 1)/((c - c/(a*x))^3*(a*x - 1)),x)
```

output
```
(9*a*x^2 - (29*x)/2 + 37/(6*a))/(c^3 + 3*a^2*c^3*x^2 - a^3*c^3*x^3 - 3*a*c
^3*x) + x/c^3 + (5*log(a*x - 1))/(a*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$
$$= \frac{30\log(ax-1)\,a^3x^3-90\log(ax-1)\,a^2x^2+90\log(ax-1)\,ax-30\log(ax-1)+6a^4x^4-30a^3x^3+45a}{6a\,c^3\left(a^3x^3-3a^2x^2+3ax-1\right)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x)
```

output
```
(30*log(a*x - 1)*a**3*x**3 - 90*log(a*x - 1)*a**2*x**2 + 90*log(a*x - 1)*a
*x - 30*log(a*x - 1) + 6*a**4*x**4 - 30*a**3*x**3 + 45*a*x - 25)/(6*a*c**3
*(a**3*x**3 - 3*a**2*x**2 + 3*a*x - 1))
```

3.410 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\frac{x}{c^4}-\frac{1}{2ac^4(1-ax)^4}+\frac{3}{ac^4(1-ax)^3}-\frac{8}{ac^4(1-ax)^2}+\frac{14}{ac^4(1-ax)}+\frac{6\log(1-ax)}{ac^4}$$

output
```
x/c^4-1/2/a/c^4/(-a*x+1)^4+3/a/c^4/(-a*x+1)^3-8/a/c^4/(-a*x+1)^2+14/a/c^4/
(-a*x+1)+6*ln(-a*x+1)/a/c^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\frac{17-56ax+60a^2x^2-16a^3x^3-8a^4x^4+2a^5x^5+12(-1+ax)^4\log(1-ax)}{2ac^4(-1+ax)^4}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

output
```
(17 - 56*a*x + 60*a^2*x^2 - 16*a^3*x^3 - 8*a^4*x^4 + 2*a^5*x^5 + 12*(-1 +
a*x)^4*Log[1 - a*x])/(2*a*c^4*(-1 + a*x)^4)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6681, 6679, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^4 e^{2\text{arctanh}(ax)}}{c^4\left(a-\frac{1}{x}\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\int \frac{e^{2\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^4}dx}{c^4}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{a^4\int \frac{e^{2\text{arctanh}(ax)}x^4}{(1-ax)^4}dx}{c^4}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{a^4\int \frac{x^4(ax+1)}{(1-ax)^5}dx}{c^4}$$

$$\Big\downarrow \text{86}$$

$$-\frac{a^4\int \left(-\frac{1}{a^4}-\frac{6}{a^4(ax-1)}-\frac{14}{a^4(ax-1)^2}-\frac{16}{a^4(ax-1)^3}-\frac{9}{a^4(ax-1)^4}-\frac{2}{a^4(ax-1)^5}\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4\left(-\frac{14}{a^5(1-ax)}+\frac{8}{a^5(1-ax)^2}-\frac{3}{a^5(1-ax)^3}+\frac{1}{2a^5(1-ax)^4}-\frac{6\log(1-ax)}{a^5}-\frac{x}{a^4}\right)}{c^4}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

output
```
-((a^4*(-(x/a^4) + 1/(2*a^5*(1 - a*x)^4) - 3/(a^5*(1 - a*x)^3) + 8/(a^5*(1
 - a*x)^2) - 14/(a^5*(1 - a*x)) - (6*Log[1 - a*x])/a^5))/c^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

method	result
risch	$\frac{x}{c^4}+\frac{-14a^2c^4x^3+34a\,c^4x^2-29c^4x+\frac{17c^4}{2a}}{c^8(ax-1)^4}+\frac{6\ln(ax-1)}{a\,c^4}$
default	$\frac{a^4\left(\frac{x}{a^4}-\frac{14}{a^5(ax-1)}+\frac{6\ln(ax-1)}{a^5}-\frac{1}{2a^5(ax-1)^4}-\frac{8}{a^5(ax-1)^2}-\frac{3}{a^5(ax-1)^3}\right)}{c^4}$
norman	$\frac{\frac{a^4x^5}{c}+\frac{6x}{c}-\frac{21a\,x^2}{c}+\frac{26a^2x^3}{c}-\frac{25a^3x^4}{2c}}{(ax-1)^4c^3}+\frac{6\ln(ax-1)}{a\,c^4}$
parallelrisch	$\frac{2a^5x^5+12\ln(ax-1)x^4a^4-25a^4x^4-48a^3\ln(ax-1)x^3+52a^3x^3+72a^2\ln(ax-1)x^2-42a^2x^2-48a\ln(ax-1)x+12ax+12\ln(ax-}{2c^4(ax-1)^4a}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
x/c^4+(-14*a^2*c^4*x^3+34*a*c^4*x^2-29*c^4*x+17/2*c^4/a)/c^8/(a*x-1)^4+6/a
/c^4*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int\frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$=\frac{2\,a^5x^5-8\,a^4x^4-16\,a^3x^3+60\,a^2x^2-56\,ax+12\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log{(ax-1)}+17}{2\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/2*(2*a^5*x^5 - 8*a^4*x^4 - 16*a^3*x^3 + 60*a^2*x^2 - 56*a*x + 12*(a^4*x^
4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 17)/(a^5*c^4*x^4 - 4
*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\frac{-28a^3x^3+68a^2x^2-58ax+17}{2a^5c^4x^4-8a^4c^4x^3+12a^3c^4x^2-8a^2c^4x+2ac^4}+\frac{x}{c^4}+\frac{6\log{(ax-1)}}{ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**4,x)
```

output
```
(-28*a**3*x**3 + 68*a**2*x**2 - 58*a*x + 17)/(2*a**5*c**4*x**4 - 8*a**4*c*
*4*x**3 + 12*a**3*c**4*x**2 - 8*a**2*c**4*x + 2*a*c**4) + x/c**4 + 6*log(a
*x - 1)/(a*c**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=-\frac{28\,a^3x^3-68\,a^2x^2+58\,ax-17}{2\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}+\frac{x}{c^4}+\frac{6\,\log\left(ax-1\right)}{ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
-1/2*(28*a^3*x^3 - 68*a^2*x^2 + 58*a*x - 17)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3
+ 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4) + x/c^4 + 6*log(a*x - 1)/(a*c^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4}+\frac{6\log\left(|ax-1|\right)}{ac^4}-\frac{28\,a^3x^3-68\,a^2x^2+58\,ax-17}{2\,(ax-1)^4ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="giac")
```

output
```
x/c^4 + 6*log(abs(a*x - 1))/(a*c^4) - 1/2*(28*a^3*x^3 - 68*a^2*x^2 + 58*a*
x - 17)/((a*x - 1)^4*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 13.65 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4}-\frac{29\,x-34\,a\,x^2-\frac{17}{2\,a}+14\,a^2\,x^3}{a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3+6\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}+\frac{6\,\ln\left(a\,x-1\right)}{a\,c^4}$$

input
```
int((a*x + 1)/((c - c/(a*x))^4*(a*x - 1)),x)
```

output
```
x/c^4 - (29*x - 34*a*x^2 - 17/(2*a) + 14*a^2*x^3)/(c^4 + 6*a^2*c^4*x^2 - 4
*a^3*c^4*x^3 + a^4*c^4*x^4 - 4*a*c^4*x) + (6*log(a*x - 1))/(a*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.48

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$

$$= \frac{12\log(ax-1)\,a^4x^4-48\log(ax-1)\,a^3x^3+72\log(ax-1)\,a^2x^2-48\log(ax-1)\,ax+12\log(ax-1)+}{2a\,c^4\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x)
```

output
```
(12*log(a*x - 1)*a**4*x**4 - 48*log(a*x - 1)*a**3*x**3 + 72*log(a*x - 1)*a
**2*x**2 - 48*log(a*x - 1)*a*x + 12*log(a*x - 1) + 2*a**5*x**5 - 12*a**4*x
**4 + 36*a**2*x**2 - 40*a*x + 13)/(2*a*c**4*(a**4*x**4 - 4*a**3*x**3 + 6*a
**2*x**2 - 4*a*x + 1))
```

3.411 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a}+\frac{3c^4\csc^{-1}(ax)}{2a}-\frac{c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
1/2*c^4*(1-1/a^2/x^2)^(1/2)*(2*a+3/x)/a^2+1/3*c^4*(1-1/a^2/x^2)^(3/2)*(3*a
+1/x)*x/a+3/2*c^4*arccsc(a*x)/a-c^4*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.70

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{c^4\left(-8+12ax+40a^2x^2+12a^3x^3-32a^4x^4-24a^5x^5+42a^4\sqrt{1-\frac{1}{a^2x^2}}x^4\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-15a^4\sqrt{}\right.}{24a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

output
```
-1/24*(c^4*(-8 + 12*a*x + 40*a^2*x^2 + 12*a^3*x^3 - 32*a^4*x^4 - 24*a^5*x^
5 + 42*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 1
5*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] + 24*a^4*Sqrt[1 - 1/(a^2*x
^2)]*x^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6731, 27, 536, 535, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^4 e^{3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3 \int \frac{c\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)x^2}{a}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4 \int \left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)x^2d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{536}$$

$$-\frac{c^4\left(\int \sqrt{1-\frac{1}{a^2x^2}}\left(-1-\frac{3}{ax}\right)xd\frac{1}{x}-\frac{1}{3}x\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)\right)}{a}$$

$$\Big\downarrow \text{535}$$

$$-\frac{c^4\left(\frac{1}{2}\int -\frac{\left(2a+\frac{3}{x}\right)x}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{3}x\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{\left(2a+\frac{3}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}{2a}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^4\left(-\frac{1}{2}\int \frac{\left(2a+\frac{3}{x}\right)x}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{1}{3}x\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{\left(2a+\frac{3}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}{2a}\right)}{a}$$

$$\Big\downarrow \; 27$$

$$-\frac{c^4\left(-\frac{\int \frac{\left(2a+\frac{3}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{1}{3}x\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{\left(2a+\frac{3}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}{2a}\right)}{a}$$

$$\Big\downarrow \; 538$$

$$-\frac{c^4\left(-\frac{3\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{1}{3}x\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{\left(2a+\frac{3}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}{2a}\right)}{a}$$

$$\Big\downarrow \; 223$$

$$-\frac{c^4\left(-\frac{2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+3a\arcsin\left(\frac{1}{ax}\right)}{2a}-\frac{1}{3}x\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{\left(2a+\frac{3}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}{2a}\right)}{a}$$

$$\Big\downarrow \; 243$$

$$-\frac{c^4\left(-\frac{a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+3a\arcsin\left(\frac{1}{ax}\right)}{2a}-\frac{1}{3}x\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{\left(2a+\frac{3}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}{2a}\right)}{a}$$

$$\Big\downarrow \; 73$$

$$-\frac{c^4\left(-\frac{3a\arcsin\left(\frac{1}{ax}\right)-2a^3\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}}{2a}-\frac{1}{3}x\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{\left(2a+\frac{3}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}{2a}\right)}{a}$$

$$\Big\downarrow \; 221$$

$$-\frac{c^4\left(-\frac{3a\arcsin\left(\frac{1}{ax}\right)-2a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}-\frac{1}{3}x\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{\left(2a+\frac{3}{x}\right)\sqrt{1-\frac{1}{a^2x^2}}}{2a}\right)}{a}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

output
```
-((c^4*(-1/2*(Sqrt[1 - 1/(a^2*x^2)]*(2*a + 3/x))/a - ((1 - 1/(a^2*x^2))^(3
/2)*(3*a + x^(-1))*x)/3 - (3*a*ArcSin[1/(a*x)] - 2*a*ArcTanh[Sqrt[1 - 1/(a
^2*x^2)]])/(2*a)))/a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.52

method	result
risch	$\frac{(ax-1)\left(8a^2x^2+3ax-2\right)c^4}{6x^3a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}-\frac{a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a^3\sqrt{(ax-1)(ax+1)}\right)c^4\sqrt{(ax-1)(ax+1)}}{a^4(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{(ax-1)^2c^4\left(-6\sqrt{a^2}\sqrt{a^2x^2-1}\,a^4x^4+6\sqrt{a^2}\left(a^2x^2-1\right)^{\frac{3}{2}}a^2x^2-9\sqrt{a^2}\sqrt{a^2x^2-1}\,a^3x^3-9a^3\sqrt{a^2}\,x^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+6\ln(\frac{a^2x+}{}\right.}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a^4x^3\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x-1)*(8*a^2*x^2+3*a*x-2)/x^3*c^4/a^4/((a*x-1)/(a*x+1))^(1/2)+(3/2*a
^3*arctan(1/(a^2*x^2-1)^(1/2))-a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))
/(a^2)^(1/2)+a^3*((a*x-1)*(a*x+1))^(1/2))*c^4/a^4/(a*x+1)/((a*x-1)/(a*x+1)
)^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.51

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\frac{18\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+6\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(6\,a^4c^4x^4+14}{6\,a^4x^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
-1/6*(18*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 6*a^3*c^4*x^3*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x +
1)) - 1) - (6*a^4*c^4*x^4 + 14*a^3*c^4*x^3 + 11*a^2*c^4*x^2 + a*c^4*x - 2*
c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$

$$= \frac{c^4\left(\int\left(-\frac{4a}{\frac{ax^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{6a^2}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}dx+\int\left(-\frac{4a^3}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-x\right.\right.}{a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**4,x)
```

output
```
c**4*(Integral(-4*a/(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**2/(a
*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-4*a**3/(a*x**2*sqrt(a*x/(a*x +
 1) - 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
1)), x) + Integral(a**4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x**5*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**4*sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))/(a*x + 1)), x))/a**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(91) = 182$.

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.17

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx =$$
$$-\frac{1}{3}\left(\frac{9\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{3\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+c^4\left(\frac{ax-1}{ax+1}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(a}{(a}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
-1/3*(9*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^4*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/a^2 - 3*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (3
*c^4*((a*x - 1)/(a*x + 1))^(7/2) + c^4*((a*x - 1)/(a*x + 1))^(5/2) + 29*c^
4*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x
- 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x
+ 1)^4 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(91) = 182$.

Time = 0.15 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.41

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$= -\frac{3\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}+\frac{c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^4}{a\mathrm{sgn}\left(ax+1\right)}$$
$$-\frac{3\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^4|a|-12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^4-12\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^4-3\left(x|a|-\sqrt{a^2x^2-1}\right)\cdots}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="giac")
```

output
```
-3*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) + c^4*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1
)*c^4/(a*sgn(a*x + 1)) - 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a
) - 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4 - 12*(x*abs(a) - sqrt(a^2*x^
2 - 1))^2*a*c^4 - 3*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*abs(a) - 8*a*c^4)/(
((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.78

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx=\frac{5\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{29\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}+c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{3\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a*x))^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(5*c^4*((a*x - 1)/(a*x + 1))^(1/2) + (29*c^4*((a*x - 1)/(a*x + 1))^(3/2))/
3 + (c^4*((a*x - 1)/(a*x + 1))^(5/2))/3 + c^4*((a*x - 1)/(a*x + 1))^(7/2))
/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x
- 1)^4)/(a*x + 1)^4) - (3*c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c^
4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.50

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$= \frac{c^4\left(-18atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^3x^3+18atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^3x^3+6\sqrt{ax+1}\,\sqrt{ax}\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x)
```

output
```
(c**4*( - 18*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**3*x**3 + 18*atan(s
qrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**3*x**3 + 6*sqrt(a*x + 1)*sqrt(a*x - 1
)*a**3*x**3 + 8*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 + 3*sqrt(a*x + 1)*sq
rt(a*x - 1)*a*x - 2*sqrt(a*x + 1)*sqrt(a*x - 1) - 12*log((sqrt(a*x - 1) +
sqrt(a*x + 1))/sqrt(2))*a**3*x**3))/(6*a**4*x**3)
```

3.412 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{3c^3\csc^{-1}(ax)}{2a}$$

output
```
3/2*c^3*(1-1/a^2/x^2)^(1/2)/a^2/x+c^3*(1-1/a^2/x^2)^(3/2)*x+3/2*c^3*arccsc
(a*x)/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}(1+2a^2x^2)+3ax\arcsin\left(\frac{1}{ax}\right)\right)}{2a^2x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

output
```
(c^3*(Sqrt[1 - 1/(a^2*x^2)]*(1 + 2*a^2*x^2) + 3*a*x*ArcSin[1/(a*x)]))/(2*a
^2*x)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6731, 247, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^3 e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{247}$$

$$-c^3\left(-\frac{3\int \sqrt{1-\frac{1}{a^2x^2}}d\frac{1}{x}}{a^2}-x\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{211}$$

$$-c^3\left(-\frac{3\left(\frac{1}{2}\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)}{a^2}-x\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{223}$$

$$-c^3\left(-\frac{3\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2x}+\frac{1}{2}a\arcsin\left(\frac{1}{ax}\right)\right)}{a^2}-x\left(1-\frac{1}{a^2x^2}\right)^{3/2}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

output
```
-(c^3*(-((1 - 1/(a^2*x^2))^(3/2)*x) - (3*(Sqrt[1 - 1/(a^2*x^2)]/(2*x) + (a
*ArcSin[1/(a*x)])/2))/a^2))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.72

method	result	size
default	$-\frac{(ax-1)^{2} c^{3} \left(-3 a^{2} x^{2} \sqrt{a^{2} x^{2}-1}-3 a^{2} x^{2} \arctan\left(\frac{1}{\sqrt{a^{2} x^{2}-1}}\right)+\left(a^{2} x^{2}-1\right)^{\frac{3}{2}}\right)}{2 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} (ax+1) \sqrt{(ax-1)(ax+1)}\, a^{3} x^{2}}$	105
risch	$\frac{(ax-1) c^{3}}{2 x^{2} a^{3} \sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(a^{2} \sqrt{(ax-1)(ax+1)}+\frac{3 a^{2} \arctan\left(\frac{1}{\sqrt{a^{2} x^{2}-1}}\right)}{2}\right) c^{3} \sqrt{(ax-1)(ax+1)}}{a^{3} \sqrt{\frac{ax-1}{ax+1}}\, (ax+1)}$	110

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x-1)^2*c^3*(-3*a^2*x^2*(a^2*x^2-1)^(1/2)-3*a^2*x^2*arctan(1/(a^2*x
^2-1)^(1/2))+(a^2*x^2-1)^(3/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(
a*x+1))^(1/2)/a^3/x^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{6\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(2\,a^3c^3x^3+2\,a^2c^3x^2+ac^3x+c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a^3x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
-1/2*(6*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (2*a^3*c^3*x^3 + 2
*a^2*c^3*x^2 + a*c^3*x + c^3)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*x^2)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$$
$$= \frac{c^3\left(\int \frac{3a}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{3a^2}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int \frac{a^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-}}{ax+}}}{a^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**3,x)
```

output
```
c**3*(Integral(3*a/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x
**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**2/(a
*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**3/(a*x*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + I
ntegral(-1/(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**3*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(53) = 106$.

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.48

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = -\left(\frac{3\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2} - \frac{3\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} + 2\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 3\,c^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1} - \frac{(ax-1)^2a^2}{(ax+1)^2} - \frac{(ax-1)^3a^2}{(ax+1)^3} + a^2}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
-(3*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - (3*c^3*((a*x - 1)/(a*x + 1
))^(5/2) + 2*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^3*sqrt((a*x - 1)/(a*x +
 1)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - (a*x - 1)^3
*a^2/(a*x + 1)^3 + a^2))*a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = -\frac{3\,c^3\arctan\left(\sqrt{a^2x^2-1}\right) - 2\,\sqrt{a^2x^2-1}c^3 - \frac{\sqrt{a^2x^2-1}c^3}{a^2x^2}}{2\,a\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="giac")
```

output
```
-1/2*(3*c^3*arctan(sqrt(a^2*x^2 - 1)) - 2*sqrt(a^2*x^2 - 1)*c^3 - sqrt(a^2
*x^2 - 1)*c^3/(a^2*x^2))/(a*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.74 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.95

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{3\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+c^3\,x\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{c^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a^2\,x}+\frac{c^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a^3\,x^2}$$

input
```
int((c - c/(a*x))^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(c^3*((a*x - 1)/(a*x + 1))^(1/2))/a - (3*c^3*atan(((a*x - 1)/(a*x + 1))^(1
/2)))/a + c^3*x*((a*x - 1)/(a*x + 1))^(1/2) + (c^3*((a*x - 1)/(a*x + 1))^(
1/2))/(2*a^2*x) + (c^3*((a*x - 1)/(a*x + 1))^(1/2))/(2*a^3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.49

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$$
$$=\frac{c^3\left(-6atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^2x^2+6atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^2x^2+2\sqrt{ax+1}\,\sqrt{ax-1}\right.}{2a^3x^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x)
```

output
```
(c**3*( - 6*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**2*x**2 + 6*atan(sqr
t(a*x - 1) + sqrt(a*x + 1) + 1)*a**2*x**2 + 2*sqrt(a*x + 1)*sqrt(a*x - 1)*
a**2*x**2 + sqrt(a*x + 1)*sqrt(a*x - 1)))/(2*a**3*x**2)
```

3.413 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a}+\frac{c^2\csc^{-1}(ax)}{a}+\frac{c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
c^2*(1-1/a^2/x^2)^(1/2)*(a-1/x)*x/a+c^2*arccsc(a*x)/a+c^2*arctanh((1-1/a^2
/x^2)^(1/2))/a
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(63) = 126.

Time = 0.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.44

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=-\frac{c^2\left(-1+ax+a^2x^2-a^3x^3+4a^2\sqrt{1-\frac{1}{a^2x^2}}x^2\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}x^2\arcsin\left(\frac{1}{ax}\right)-a^2\sqrt{1\cdots}\right.}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

output
```
-((c^2*(-1 + a*x + a^2*x^2 - a^3*x^3 + 4*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*Arc
Sin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcSin[1/(a
*x)] - a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(a^3
*Sqrt[1 - 1/(a^2*x^2)]*x^2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {6731, 27, 566, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^2 e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c^3\int \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{c\left(a-\frac{1}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-ac^2\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{a-\frac{1}{x}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 566}$$

$$-ac^2\int \sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{a}+\frac{1}{xa^2}\right)x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 536}$$

$$-ac^2\left(\int \frac{\left(\frac{1}{a^2}-\frac{1}{a^3x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a^2}\right)$$

$$\Big\downarrow \text{ 538}$$

$$-ac^2\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^2}-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^3}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a^2}\right)$$

$$\Big\downarrow \text{223}$$

$$-ac^2\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^2}-\frac{\arcsin\left(\frac{1}{ax}\right)}{a^2}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a^2}\right)$$

$$\Big\downarrow \text{243}$$

$$-ac^2\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}-\frac{\arcsin\left(\frac{1}{ax}\right)}{a^2}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a^2}\right)$$

$$\Big\downarrow \text{73}$$

$$-ac^2\left(-\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-\frac{\arcsin\left(\frac{1}{ax}\right)}{a^2}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a^2}\right)$$

$$\Big\downarrow \text{221}$$

$$-ac^2\left(-\frac{\arcsin\left(\frac{1}{ax}\right)}{a^2}-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a^2}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

output
```
-(a*c^2*(-((Sqrt[1 - 1/(a^2*x^2)]*(a - x^(-1))*x)/a^2) - ArcSin[1/(a*x)]/a
^2 - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536

```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538

```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 566

```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Int[x^m*(a/c + b*(x/d))*(a + b*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(59) = 118$.

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.08

method	result
risch	$-\frac{(ax-1)c^2}{x\,a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\sqrt{(ax-1)(ax+1)}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c^2\sqrt{(ax-1)(ax+1)}}{a(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)^2c^2\left(-\sqrt{a^2}\sqrt{a^2x^2-1}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+\sqrt{a^2}\sqrt{a^2x^2-1}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x+a\sqrt{a^2}\,x\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right.\right.}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)/x*c^2/a^2/((a*x-1)/(a*x+1))^(1/2)+1/a*(a*ln(a^2*x/(a^2)^(1/2)+(a^
2*x^2-1)^(1/2))/(a^2)^(1/2)+((a*x-1)*(a*x+1))^(1/2)+arctan(1/(a^2*x^2-1)^(
1/2)))*c^2/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.81

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = -\frac{2\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2c^2x^2-c^2)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
-(2*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - a*c^2*x*log(sqrt((a*x - 1)
/(a*x + 1)) + 1) + a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c^2*x
^2 - c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$$
$$=\frac{c^2\left(\int\left(-\frac{2a}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-}}{ax+1}}}{a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**2,x)
```

output
```
c**2*(Integral(-2*a/(a*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**2/(a*x*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x +
1))/(a*x + 1)), x) + Integral(1/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/
(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(59) = 118$.

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.98

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=$$
$$-\left(\frac{4\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
-(4*c^2*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) +
2*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c^2*log(sqrt((a*x - 1)/(a*x
+ 1)) + 1)/a^2 + c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(59) = 118.

Time = 0.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.19

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=-\frac{2\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\,(ax+1)}-\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\,(ax+1)}+\frac{\sqrt{a^2x^2-1}c^2}{a\mathrm{sgn}\,(ax+1)}-\frac{2\,c^2}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="giac")
```

output
```
-2*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c^2*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1
)*c^2/(a*sgn(a*x + 1)) - 2*c^2/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs
(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.73 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{4\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a-\frac{a\,\left(a\,x-1\right)^2}{\left(a\,x+1\right)^2}}-\frac{2\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a*x))^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(4*c^2*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (2
*c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (2*c^2*atanh(((a*x - 1)/(a*x +
 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.67

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$$
$$= \frac{c^2\left(-2atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)ax+2atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)ax+\sqrt{ax+1}\,\sqrt{ax-1}\,ax\right.}{a^2x}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x)
```

output
```
(c**2*( - 2*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a*x + 2*atan(sqrt(a*x
- 1) + sqrt(a*x + 1) + 1)*a*x + sqrt(a*x + 1)*sqrt(a*x - 1)*a*x - sqrt(a*x
 + 1)*sqrt(a*x - 1) + 2*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x -
 a*x))/(a**2*x)
```

3.414 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{c\csc^{-1}(ax)}{a}+\frac{2\text{carctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
c*(1-1/a^2/x^2)^(1/2)*x-c*arccsc(a*x)/a+2*c*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x-2\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-2\arcsin\left(\frac{1}{ax}\right)+2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x)),x]
```

output
```
(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x - 2*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*Ar
cSin[1/(a*x)] + 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/a
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6731, 27, 570, 540, 25, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right) e^{3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3 \int \frac{a^2 \left(1-\frac{1}{a^2x^2}\right)^{3/2} x^2}{c^2 \left(a-\frac{1}{x}\right)^2} d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-a^2 c \int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2} x^2}{\left(a-\frac{1}{x}\right)^2} d\frac{1}{x}$$

$$\Big\downarrow \text{570}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)^2 x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^2}$$

$$\Big\downarrow \text{540}$$

$$-\frac{c\left(a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int -\frac{\left(2a+\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c\left(\int \frac{\left(2a+\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-a^2x\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}$$

$$\Big\downarrow \text{538}$$

$$-\frac{c\left(2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^2}$$

$$\Big\downarrow \text{223}$$

$$-\frac{c\left(2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)+a\arcsin\left(\frac{1}{ax}\right)\right)}{a^2}$$

$$\Big\downarrow \text{243}$$

$$-\frac{c\left(a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)+a\arcsin\left(\frac{1}{ax}\right)\right)}{a^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c\left(-2a^3\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-a^2x\sqrt{1-\frac{1}{a^2x^2}}+a\arcsin\left(\frac{1}{ax}\right)\right)}{a^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c\left(-2a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)+a\arcsin\left(\frac{1}{ax}\right)\right)}{a^2}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a*x)),x]
```

output
```
-((c*(-(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + a*ArcSin[1/(a*x)] - 2*a*ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]]))/a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] && !(IGtQ[m, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(45) = 90$.

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.96

method	result	size
default	$-\frac{(ax-1)^2 c\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}-2a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)-2\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$	145

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)^2*c*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arctan(1/(a^2*x^2-1)^(1/2))*(a
^2)^(1/2)-2*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))-
2*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a
*x-1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.80

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="fricas")
```

output
```
(2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) + 2*c*log(sqrt((a*x - 1)/(a*x + 1))
 + 1) - 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$$
$$=\frac{c\left(\int\frac{a}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)}{a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x),x)
```

output
```
c*(Integral(a/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x**2*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
 + 1)), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(45) = 90$.

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.33

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=$$
$$-2a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="maxima")
```

output
```
-2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - c*arct
an(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a
^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(45) = 90$.

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.86

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\text{sgn}\left(ax+1\right)} - \frac{2\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\text{sgn}\left(ax+1\right)} + \frac{\sqrt{a^2x^2-1}c}{a\text{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="giac")
```

output
```
2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 2*c*log(abs(-
x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*c
/(a*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.55 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a} + \frac{4\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a} + \frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}$$

input
```
int((c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (4*c*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a + (2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x +
 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$$
$$=\frac{c\Big(2atan\big(\sqrt{ax-1}+\sqrt{ax+1}-1\big)-2atan\big(\sqrt{ax-1}+\sqrt{ax+1}+1\big)+\sqrt{ax+1}\,\sqrt{ax-1}+4\log\Big(\frac{\sqrt{ax}}{}$$
$$a$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x)
```

output
```
(c*(2*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1) - 2*atan(sqrt(a*x - 1) + sqr
t(a*x + 1) + 1) + sqrt(a*x + 1)*sqrt(a*x - 1) + 4*log((sqrt(a*x - 1) + sqr
t(a*x + 1))/sqrt(2))))/a
```

3.415 $\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 116

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{7a\sqrt{1-\frac{1}{a^2x^2}}}{3c\left(a-\frac{1}{x}\right)^2}-\frac{19\sqrt{1-\frac{1}{a^2x^2}}}{3c\left(a-\frac{1}{x}\right)}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\left(a-\frac{1}{x}\right)^2}+\frac{4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
-7/3*a*(1-1/a^2/x^2)^(1/2)/c/(a-1/x)^2-19/3*(1-1/a^2/x^2)^(1/2)/c/(a-1/x)+
a^2*(1-1/a^2/x^2)^(1/2)*x/c/(a-1/x)^2+4*arctanh((1-1/a^2/x^2)^(1/2))/a/c
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(19-26ax+3a^2x^2)}{(-1+ax)^2}+12\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{3ac}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x)),x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(19 - 26*a*x + 3*a^2*x^2))/(-1 + a*x)^2 + 12*L
og[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(3*a*c)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6731, 27, 570, 532, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{c^4\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^4x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^4c}$$

$$\Big\downarrow \text{532}$$

$$-\frac{\frac{8a^2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(3a^4+\frac{12a^3}{x}+\frac{13a^2}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^4c}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{3}\int \frac{\left(3a^4+\frac{12a^3}{x}+\frac{13a^2}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{8a^2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^4c}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{1}{3}\left(\frac{4a^2\left(3a+\frac{4}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{3a^3\left(a+\frac{4}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^4c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{1}{3}\left(3a^3\int \frac{\left(a+\frac{4}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{4a^2\left(3a+\frac{4}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^4c}$$

$$\Big\downarrow \text{534}$$

$$-\frac{\frac{1}{3}\left(3a^3\left(4\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{4a^2\left(3a+\frac{4}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^4c}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\frac{1}{3}\left(3a^3\left(2\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{4a^2\left(3a+\frac{4}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^4c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{1}{3}\left(3a^3\left(-4a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{4a^2\left(3a+\frac{4}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{8a^2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^4c}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{8a^2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{1}{3}\left(\frac{4a^2\left(3a+\frac{4}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+3a^3\left(-4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4c}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a*x)),x]
```

output
```
-(((8*a^2*(a + x^(-1)))/(3*(1 - 1/(a^2*x^2))^(3/2)) + ((4*a^2*(3*a + 4/x))
/Sqrt[1 - 1/(a^2*x^2)] + 3*a^3*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x) - 4*ArcTanh[S
qrt[1 - 1/(a^2*x^2)]]))/3)/(a^4*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.58

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{4\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a\sqrt{a^{2}}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2a\left(x-\frac{1}{a}\right)}}{3a^{4}\left(x-\frac{1}{a}\right)^{2}}-\frac{20\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2a\left(x-\frac{1}{a}\right)}}{3a^{3}\left(x-\frac{1}{a}\right)}\right)a\sqrt{(ax-1)(ax+1)}}{c(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{-12\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{3}x^{3}-12\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{4}x^{3}+9\sqrt{a^{2}}\,((ax-1)(ax+1))^{\frac{3}{2}}ax+36\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,a}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c/((a*x-1)/(a*x+1))^(1/2)+(4/a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1
)^(1/2))/(a^2)^(1/2)-4/3/a^4/(x-1/a)^2*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-2
0/3/a^3/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2))*a/c/(a*x+1)/((a*x-1)/(a
*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{12\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-12\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(3\,a^3x^3-23\,a^2x^2-7}{3\,(a^3cx^2-2\,a^2cx+ac)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="fricas")
```

output
```
1/3*(12*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 12*(a^2
*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 23*a^2
*x^2 - 7*a*x + 19)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c
)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{a\int \frac{x}{\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x),x)
```

output
```
a*Integral(x/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a*
x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))/(a*x + 1)), x)/c
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{2}{3}a\left(\frac{\frac{8(ax-1)}{ax+1}-\frac{12(ax-1)^2}{(ax+1)^2}+1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{6\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{6\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="maxima")
```

output
```
2/3*a*((8*(a*x - 1)/(a*x + 1) - 12*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c*((a
*x - 1)/(a*x + 1))^(5/2) - a^2*c*((a*x - 1)/(a*x + 1))^(3/2)) + 6*log(sqrt
((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 6*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
(a^2*c))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.54

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c|a|\mathrm{sgn}\,(ax+1)}+\frac{\sqrt{a^2x^2-1}}{ac\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="giac")
```

output
```
-4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c*abs(a)*sgn(a*x + 1)) + sqrt(
a^2*x^2 - 1)/(a*c*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{8\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}-\frac{\frac{16\,(a\,x-1)}{3\,(a\,x+1)}-\frac{8\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{2}{3}}{a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(1/((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
(8*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c) - ((16*(a*x - 1))/(3*(a*x + 1
)) - (8*(a*x - 1)^2)/(a*x + 1)^2 + 2/3)/(a*c*((a*x - 1)/(a*x + 1))^(3/2) -
 a*c*((a*x - 1)/(a*x + 1))^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$$
$$=\frac{24\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax-24\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)+5\sqrt{ax-1}\,ax-5\sqrt{ax-1}+3\sqrt{ax}}{3\sqrt{ax-1}\,ac\,(ax-1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x)
```

output
```
(24*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x - 24*sq
rt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 5*sqrt(a*x - 1)
*a*x - 5*sqrt(a*x - 1) + 3*sqrt(a*x + 1)*a**2*x**2 - 26*sqrt(a*x + 1)*a*x
+ 19*sqrt(a*x + 1))/(3*sqrt(a*x - 1)*a*c*(a*x - 1))
```

3.416 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{9a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^2\left(a-\frac{1}{x}\right)^3}-\frac{43a\sqrt{1-\frac{1}{a^2x^2}}}{15c^2\left(a-\frac{1}{x}\right)^2}-\frac{118\sqrt{1-\frac{1}{a^2x^2}}}{15c^2\left(a-\frac{1}{x}\right)}+\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)^3}+\frac{5\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

output
```
-9/5*a^2*(1-1/a^2/x^2)^(1/2)/c^2/(a-1/x)^3-43/15*a*(1-1/a^2/x^2)^(1/2)/c^2
/(a-1/x)^2-118/15*(1-1/a^2/x^2)^(1/2)/c^2/(a-1/x)+a^3*(1-1/a^2/x^2)^(1/2)*
x/c^2/(a-1/x)^3+5*arctanh((1-1/a^2/x^2)^(1/2))/a/c^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.70

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{-118+161ax+91a^2x^2-173a^3x^3+15a^4x^4+75a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

output
```
(-118 + 161*a*x + 91*a^2*x^2 - 173*a^3*x^3 + 15*a^4*x^4 + 75*a*Sqrt[1 - 1/
(a^2*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^2*Sqrt
[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6731, 27, 570, 532, 25, 2336, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{c^5\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^5\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^5}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^5x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}}{a^5c^2}$$

$$\Big\downarrow \text{532}$$

$$-\frac{\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}\int -\frac{\left(5a^5+\frac{25a^4}{x}+\frac{39a^3}{x^2}-\frac{5a^2}{x^3}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^5c^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{5}\int \frac{\left(5a^5+\frac{25a^4}{x}+\frac{39a^3}{x^2}-\frac{5a^2}{x^3}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}+\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^5c^2}$$

$\downarrow$ 2336

$$-\frac{\frac{1}{5}\left(\frac{4a^3\left(5a+\frac{11}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(15a^5+\frac{75a^4}{x}+\frac{88a^3}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^5c^2}$$

$\downarrow$ 25

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{\left(15a^5+\frac{75a^4}{x}+\frac{88a^3}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{4a^3\left(5a+\frac{11}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^5c^2}$$

$\downarrow$ 2336

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{a^3\left(75a+\frac{103}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{15a^4\left(a+\frac{5}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{4a^3\left(5a+\frac{11}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^5c^2}$$

$\downarrow$ 27

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a^4\int \frac{\left(a+\frac{5}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^3\left(75a+\frac{103}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a^3\left(5a+\frac{11}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^5c^2}$$

$\downarrow$ 534

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a^4\left(5\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^3\left(75a+\frac{103}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a^3\left(5a+\frac{11}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^5c^2}$$

$\downarrow$ 243

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a^4\left(\frac{5}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^3\left(75a+\frac{103}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a^3\left(5a+\frac{11}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^5c^2}$$

$\downarrow$ 73

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a^4\left(-5a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^3\left(75a+\frac{103}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a^3\left(5a+\frac{11}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)}}{a^5c^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{16a^3\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{1}{5}\left(\frac{4a^3\left(5a+\frac{11}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{1}{3}\left(15a^4\left(-5\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^3\left(75a+\frac{103}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)\right)}{a^5c^2}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

output
```
-(((16*a^3*(a + x^(-1)))/(5*(1 - 1/(a^2*x^2))^(5/2)) + ((4*a^3*(5*a + 11/x
))/(3*(1 - 1/(a^2*x^2))^(3/2)) + ((a^3*(75*a + 103/x))/Sqrt[1 - 1/(a^2*x^2
)] + 15*a^4*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x) - 5*ArcTanh[Sqrt[1 - 1/(a^2*x^2)
]]))/3)/5)/(a^5*c^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.51

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}-\frac{143\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{15a^4\left(x-\frac{1}{a}\right)}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{5a^6\left(x-\frac{1}{a}\right)^3}-\frac{52\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{15a^5\left(x-\frac{1}{a}\right)^2}\right)a^2\sqrt{(}}{c^2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{-75\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^4x^4-75\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^5x^4+60\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2+300\sqrt{a^2}\,\sqrt{(ax-1)(ax}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)+(5/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-143/15/a^4/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1
/2)-4/5/a^6/(x-1/a)^3*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-52/15/a^5/(x-1/a)^
2*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2))*a^2/c^2/(a*x+1)/((a*x-1)/(a*x+1))^(1/
2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.14

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$
$$=\frac{75\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-75\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(15}{15\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
1/15*(75*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
 1) - 75*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) -
 1) + (15*a^4*x^4 - 173*a^3*x^3 + 91*a^2*x^2 + 161*a*x - 118)*sqrt((a*x -
1)/(a*x + 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2 \int \frac{x^2}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**2,x)
```

output
```
a**2*Integral(x**2/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a*x*sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a
*x + 1)), x)/c**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{1}{15}a\left(\frac{\frac{17(ax-1)}{ax+1}+\frac{100(ax-1)^2}{(ax+1)^2}-\frac{150(ax-1)^3}{(ax+1)^3}+3}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{75\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{75\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
1/15*a*((17*(a*x - 1)/(a*x + 1) + 100*(a*x - 1)^2/(a*x + 1)^2 - 150*(a*x -
 1)^3/(a*x + 1)^3 + 3)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^2*((a*
x - 1)/(a*x + 1))^(5/2)) + 75*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2)
 - 75*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{5\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c^2|a|\mathrm{sgn}\left(ax+1\right)} + \frac{\sqrt{a^2x^2-1}}{ac^2\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="giac")
```

output
```
-5*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c^2*abs(a)*sgn(a*x + 1)) + sqr
t(a^2*x^2 - 1)/(a*c^2*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.65 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{10\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2} - \frac{\frac{20\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}-\frac{10\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{17\,\left(a\,x-1\right)}{15\,\left(a\,x+1\right)}+\frac{1}{5}}{a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

input
```
int(1/((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
(10*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^2) - ((20*(a*x - 1)^2)/(3*(a*
x + 1)^2) - (10*(a*x - 1)^3)/(a*x + 1)^3 + (17*(a*x - 1))/(15*(a*x + 1)) +
 1/5)/(a*c^2*((a*x - 1)/(a*x + 1))^(5/2) - a*c^2*((a*x - 1)/(a*x + 1))^(7/
2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.34

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

$$= \frac{300\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^2x^2 - 600\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax + 300\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax}}{\sqrt{2}}\right.}{30\sqrt{}}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x)
```

output
```
(300*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**2*x**2
- 600*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x + 300
*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 157*sqrt(a*x
 - 1)*a**2*x**2 - 314*sqrt(a*x - 1)*a*x + 157*sqrt(a*x - 1) + 30*sqrt(a*x
+ 1)*a**3*x**3 - 376*sqrt(a*x + 1)*a**2*x**2 + 558*sqrt(a*x + 1)*a*x - 236
*sqrt(a*x + 1))/(30*sqrt(a*x - 1)*a*c**2*(a**2*x**2 - 2*a*x + 1))
```

3.417 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 182

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{11a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^3\left(a-\frac{1}{x}\right)^4}-\frac{15a^2\sqrt{1-\frac{1}{a^2x^2}}}{7c^3\left(a-\frac{1}{x}\right)^3}-\frac{24a\sqrt{1-\frac{1}{a^2x^2}}}{7c^3\left(a-\frac{1}{x}\right)^2}-\frac{66\sqrt{1-\frac{1}{a^2x^2}}}{7c^3\left(a-\frac{1}{x}\right)}+\frac{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\left(a-\frac{1}{x}\right)^4}+\frac{6\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

output
```
-11/7*a^3*(1-1/a^2/x^2)^(1/2)/c^3/(a-1/x)^4-15/7*a^2*(1-1/a^2/x^2)^(1/2)/c
^3/(a-1/x)^3-24/7*a*(1-1/a^2/x^2)^(1/2)/c^3/(a-1/x)^2-66/7*(1-1/a^2/x^2)^(
1/2)/c^3/(a-1/x)+a^4*(1-1/a^2/x^2)^(1/2)*x/c^3/(a-1/x)^4+6*arctanh((1-1/a^
2/x^2)^(1/2))/a/c^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.62

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{66-156ax+39a^2x^2+145a^3x^3-109a^4x^4+7a^5x^5+42a\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)^3}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

output
```
(66 - 156*a*x + 39*a^2*x^2 + 145*a^3*x^3 - 109*a^4*x^4 + 7*a^5*x^5 + 42*a*
Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(7*a^
2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3)
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.87, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6731, 27, 570, 532, 25, 2336, 27, 2336, 27, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$

$$\Big\downarrow \; 6731$$

$$-c^3\int \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{c^6\left(a-\frac{1}{x}\right)^6}d\frac{1}{x}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^6\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^6}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^6 x^2}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}d\frac{1}{x}}{a^6c^3}$$

$$\Big\downarrow \text{532}$$

$$-\frac{\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{1}{7}\int -\frac{\left(7a^6+\frac{42a^5}{x}+\frac{80a^4}{x^2}-\frac{42a^3}{x^3}-\frac{7a^2}{x^4}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}}{a^6c^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{7}\int \frac{\left(7a^6+\frac{42a^5}{x}+\frac{80a^4}{x^2}-\frac{42a^3}{x^3}-\frac{7a^2}{x^4}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{1}{7}\left(\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}\int -\frac{5\left(7a^6+\frac{42a^5}{x}+\frac{71a^4}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}\right)+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{1}{7}\left(\int \frac{\left(7a^6+\frac{42a^5}{x}+\frac{71a^4}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{1}{7}\left(-\frac{1}{3}\int -\frac{3\left(7a^6+\frac{42a^5}{x}+\frac{52a^4}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{2a^4\left(7a+\frac{13}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{1}{7}\left(\int \frac{\left(7a^6+\frac{42a^5}{x}+\frac{52a^4}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{2a^4\left(7a+\frac{13}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{1}{7}\left(-\int -\frac{7a^5\left(a+\frac{6}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2a^4\left(7a+\frac{13}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a^4\left(42a+\frac{59}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{1}{7}\left(7a^5\int \frac{\left(a+\frac{6}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2a^4\left(7a+\frac{13}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a^4\left(42a+\frac{59}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \; 534$$

$$-\frac{\frac{1}{7}\left(7a^5\left(6\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{2a^4\left(7a+\frac{13}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a^4\left(42a+\frac{59}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \; 243$$

$$-\frac{\frac{1}{7}\left(7a^5\left(3\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{2a^4\left(7a+\frac{13}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a^4\left(42a+\frac{59}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}}{a^6c^3}$$

$$\Big\downarrow \; 73$$

$$-\frac{\frac{1}{7}\left(7a^5\left(-6a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{2a^4\left(7a+\frac{13}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a^4\left(42a+\frac{59}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)+\frac{3}{7(}}{a^6c^3}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{32a^4\left(a+\frac{1}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{1}{7}\left(7a^5\left(-6\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{2a^4\left(7a+\frac{13}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a^4\left(42a+\frac{59}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{16a^4}{x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)}{a^6c^3}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

output
```
-(((32*a^4*(a + x^(-1)))/(7*(1 - 1/(a^2*x^2))^(7/2)) + ((2*a^4*(7*a + 13/x
))/(1 - 1/(a^2*x^2))^(3/2) + (a^4*(42*a + 59/x))/Sqrt[1 - 1/(a^2*x^2)] + (
16*a^4)/((1 - 1/(a^2*x^2))^(5/2)*x) + 7*a^5*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x)
- 6*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/7)/(a^6*c^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.46

method	result
risch	$\frac{ax-1}{a\,c^{3}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{6\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{3}\sqrt{a^{2}}}-\frac{88\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2a\left(x-\frac{1}{a}\right)}}{7a^{5}\left(x-\frac{1}{a}\right)}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2a\left(x-\frac{1}{a}\right)}}{7a^{8}\left(x-\frac{1}{a}\right)^{4}}-\frac{20\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2a\left(x-\frac{1}{a}\right)}}{7a^{7}\left(x-\frac{1}{a}\right)^{3}}-\frac{45\sqrt{\left(x\right.}}{}\right.}{c^{3}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-42\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,a^{5}x^{5}-42\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{6}x^{5}+35\sqrt{a^{2}}\,((ax-1)(ax+1))^{\frac{3}{2}}a^{3}x^{3}+210\sqrt{(ax-1)(ax+1)}\,\sqrt{}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^3/((a*x-1)/(a*x+1))^(1/2)+(6/a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-88/7/a^5/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2
)-4/7/a^8/(x-1/a)^4*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-20/7/a^7/(x-1/a)^3*(
(x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-45/7/a^6/(x-1/a)^2*((x-1/a)^2*a^2+2*a*(x-
1/a))^(1/2))*a^3/c^3/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+
1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.12

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{42\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-42\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log\Big(\sqrt{}}{7\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
1/7*(42*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1)/(
a*x + 1)) + 1) - 42*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt
((a*x - 1)/(a*x + 1)) - 1) + (7*a^5*x^5 - 109*a^4*x^4 + 145*a^3*x^3 + 39*a
^2*x^2 - 156*a*x + 66)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 4*a^4*c^3
*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{a^3\int \frac{x^3}{\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**3,x)
```

output
```
a**3*Integral(x**3/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- 4*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 6*a**2*x**2*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**
3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.93

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{1}{14}a\left(\frac{\frac{6(ax-1)}{ax+1}+\frac{21(ax-1)^2}{(ax+1)^2}+\frac{112(ax-1)^3}{(ax+1)^3}-\frac{168(ax-1)^4}{(ax+1)^4}+1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{84\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{84\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
1/14*a*((6*(a*x - 1)/(a*x + 1) + 21*(a*x - 1)^2/(a*x + 1)^2 + 112*(a*x - 1
)^3/(a*x + 1)^3 - 168*(a*x - 1)^4/(a*x + 1)^4 + 1)/(a^2*c^3*((a*x - 1)/(a*
x + 1))^(9/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2)) + 84*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/(a^2*c^3) - 84*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*
c^3))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.75

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{12\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3} - \frac{\frac{3\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{16\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{24\,(a\,x-1)^4}{(a\,x+1)^4}+\frac{6\,(a\,x-1)}{7\,(a\,x+1)}+\frac{1}{7}}{2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}-2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

input
```
int(1/((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
(12*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^3) - ((3*(a*x - 1)^2)/(a*x +
1)^2 + (16*(a*x - 1)^3)/(a*x + 1)^3 - (24*(a*x - 1)^4)/(a*x + 1)^4 + (6*(a
*x - 1))/(7*(a*x + 1)) + 1/7)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 2*a*c
^3*((a*x - 1)/(a*x + 1))^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.47

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{84\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^3x^3-252\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^2x^2+252\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{a}}{\sqrt{2}}\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x)
```

output
```
(84*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**3*x**3 -
 252*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**2*x**2
+ 252*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x - 84*
sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 60*sqrt(a*x -
 1)*a**3*x**3 - 180*sqrt(a*x - 1)*a**2*x**2 + 180*sqrt(a*x - 1)*a*x - 60*s
qrt(a*x - 1) + 7*sqrt(a*x + 1)*a**4*x**4 - 116*sqrt(a*x + 1)*a**3*x**3 + 2
61*sqrt(a*x + 1)*a**2*x**2 - 222*sqrt(a*x + 1)*a*x + 66*sqrt(a*x + 1))/(7*
sqrt(a*x - 1)*a*c**3*(a**3*x**3 - 3*a**2*x**2 + 3*a*x - 1))
```

3.418 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 215

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{13a^4\sqrt{1-\frac{1}{a^2x^2}}}{9c^4\left(a-\frac{1}{x}\right)^5}-\frac{115a^3\sqrt{1-\frac{1}{a^2x^2}}}{63c^4\left(a-\frac{1}{x}\right)^4}-\frac{262a^2\sqrt{1-\frac{1}{a^2x^2}}}{105c^4\left(a-\frac{1}{x}\right)^3}-\frac{1259a\sqrt{1-\frac{1}{a^2x^2}}}{315c^4\left(a-\frac{1}{x}\right)^2}-\frac{3464\sqrt{1-\frac{1}{a^2x^2}}}{315c^4\left(a-\frac{1}{x}\right)}+\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4\left(a-\frac{1}{x}\right)^5}+\frac{7\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

output
```
-13/9*a^4*(1-1/a^2/x^2)^(1/2)/c^4/(a-1/x)^5-115/63*a^3*(1-1/a^2/x^2)^(1/2)
/c^4/(a-1/x)^4-262/105*a^2*(1-1/a^2/x^2)^(1/2)/c^4/(a-1/x)^3-1259/315*a*(1
-1/a^2/x^2)^(1/2)/c^4/(a-1/x)^2-3464/315*(1-1/a^2/x^2)^(1/2)/c^4/(a-1/x)+a
^5*(1-1/a^2/x^2)^(1/2)*x/c^4/(a-1/x)^5+7*arctanh((1-1/a^2/x^2)^(1/2))/a/c^
4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.56

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{-3464+11651ax-10232a^2x^2-5567a^3x^3+13241a^4x^4-6224a^5x^5+315a^6x^6+2205a\sqrt{1-\frac{1}{a^2x^2}}\,x(-}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^4}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

output
```
(-3464 + 11651*a*x - 10232*a^2*x^2 - 5567*a^3*x^3 + 13241*a^4*x^4 - 6224*a
^5*x^5 + 315*a^6*x^6 + 2205*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^4*ArcTanh
[Sqrt[1 - 1/(a^2*x^2)]])/(315*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^4
)
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.99, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {6731, 27, 570, 532, 25, 2336, 27, 2336, 25, 2336, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$

$$\Big\downarrow \; 6731$$

$$-c^3\int \frac{a^7\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{c^7\left(a-\frac{1}{x}\right)^7}d\frac{1}{x}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^7\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^7}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^7x^2}{\left(1-\frac{1}{a^2x^2}\right)^{11/2}}d\frac{1}{x}}{a^7c^4}$$

$$\Big\downarrow \text{532}$$

$$-\frac{\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{1}{9}\int -\frac{\left(9a^7+\frac{63a^6}{x}+\frac{134a^5}{x^2}-\frac{198a^4}{x^3}-\frac{63a^3}{x^4}-\frac{9a^2}{x^5}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}d\frac{1}{x}}{a^7c^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{9}\int \frac{\left(9a^7+\frac{63a^6}{x}+\frac{134a^5}{x^2}-\frac{198a^4}{x^3}-\frac{63a^3}{x^4}-\frac{9a^2}{x^5}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}d\frac{1}{x}+\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}}{a^7c^4}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{1}{9}\left(-\frac{1}{7}\int -\frac{3\left(21a^7+\frac{147a^6}{x}+\frac{307a^5}{x^2}+\frac{21a^4}{x^3}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}-\frac{16a^5\left(9a-\frac{5}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\right)+\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}}{a^7c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{1}{9}\left(\frac{3}{7}\int \frac{\left(21a^7+\frac{147a^6}{x}+\frac{307a^5}{x^2}+\frac{21a^4}{x^3}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}-\frac{16a^5\left(9a-\frac{5}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\right)+\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}}{a^7c^4}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}\int -\frac{\left(105a^7+\frac{735a^6}{x}+\frac{1312a^5}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}\right)-\frac{16a^5\left(9a-\frac{5}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\right)+\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}}{a^7c^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\int \frac{\left(105a^7+\frac{735a^6}{x}+\frac{1312a^5}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}+\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)-\frac{16a^5\left(9a-\frac{5}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\right)+\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}}{a^7c^4}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{a^5\left(735a+\frac{1417}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(315a^7+\frac{2205a^6}{x}+\frac{2834a^5}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)+\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)-\frac{16a^5\left(9a-\frac{5}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\right)+\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}}{a^7c^4}$$

$\downarrow$ 25

$$-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{\left(315a^7+\frac{2205a^6}{x}+\frac{2834a^5}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{a^5\left(735a+\frac{1417}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)-\frac{16a^5\left(9a-\frac{5}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\right)+\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}}{a^7c^4}$$

$\downarrow$ 2336

$$-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{a^5\left(2205a+\frac{3149}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{315a^6\left(a+\frac{7}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{a^5\left(735a+\frac{1417}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)-\frac{16a^5\left(9a-\frac{5}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\right)+\frac{64a}{9(1-}}{a^7c^4}$$

$\downarrow$ 27

$$-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(315a^6\int \frac{\left(a+\frac{7}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^5\left(2205a+\frac{3149}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{a^5\left(735a+\frac{1417}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)-\frac{16a^5\left(9a-\frac{5}{x}\right)}{7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\right)+\frac{64}{9(1-}}{a^7c^4}$$

$\downarrow$ 534

$$-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(315a^6\left(7\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^5\left(2205a+\frac{3149}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{a^5\left(735a+\frac{1417}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)-}{a^7c^4}$$

$\downarrow$ 243

$$-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(315a^6\left(\frac{7}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^5\left(2205a+\frac{3149}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{a^5\left(735a+\frac{1417}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\right)-}{a^7c^4}$$

$\downarrow$ 73

$$-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(315a^6\left(-7a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^5\left(2205a+\frac{3149}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{a^5\left(735a+\frac{1417}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+}{a^7c^4}$$

$\downarrow$ 221

$$-\frac{\frac{64a^5\left(a+\frac{1}{x}\right)}{9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{1}{9}\left(\frac{3}{7}\left(\frac{8a^5\left(21a+\frac{41}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{1}{5}\left(\frac{a^5\left(735a+\frac{1417}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{1}{3}\left(315a^6\left(-7\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)\right.\right.\right.\right.}{a^7c^4}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

output
```
-(((64*a^5*(a + x^(-1)))/(9*(1 - 1/(a^2*x^2))^(9/2)) + ((-16*a^5*(9*a - 5/
x))/(7*(1 - 1/(a^2*x^2))^(7/2)) + (3*((8*a^5*(21*a + 41/x))/(5*(1 - 1/(a^2
*x^2))^(5/2)) + ((a^5*(735*a + 1417/x))/(3*(1 - 1/(a^2*x^2))^(3/2)) + ((a^
5*(2205*a + 3149/x))/Sqrt[1 - 1/(a^2*x^2)] + 315*a^6*(-(a*Sqrt[1 - 1/(a^2*
x^2)]*x) - 7*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/3)/5))/7)/9)/(a^7*c^4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.42

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{7\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{4964\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{315a^6\left(x-\frac{1}{a}\right)}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{9a^{10}\left(x-\frac{1}{a}\right)^5}-\frac{164\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{63a^9\left(x-\frac{1}{a}\right)^4}-\frac{697}{}\right.}{c^4(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{-2205\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^6x^6-2205\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^7x^6+1890\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^4x^4+13230\sqrt{(ax-1}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^4/((a*x-1)/(a*x+1))^(1/2)+(7/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-4964/315/a^6/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^
(1/2)-4/9/a^10/(x-1/a)^5*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-164/63/a^9/(x-1
/a)^4*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-697/105/a^8/(x-1/a)^3*((x-1/a)^2*a
^2+2*a*(x-1/a))^(1/2)-3226/315/a^7/(x-1/a)^2*((x-1/a)^2*a^2+2*a*(x-1/a))^(
1/2))*a^4/c^4/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$=\frac{2205\left(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+5\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2205\left(a^5x^5-5\,a^4x^4+10\,a^3x^3\right.}{315\left(a^6c^4x^5-5\,a^5\right.}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/315*(2205*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*lo
g(sqrt((a*x - 1)/(a*x + 1)) + 1) - 2205*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3
- 10*a^2*x^2 + 5*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (315*a^6*x^
6 - 6224*a^5*x^5 + 13241*a^4*x^4 - 5567*a^3*x^3 - 10232*a^2*x^2 + 11651*a*
x - 3464)*sqrt((a*x - 1)/(a*x + 1)))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4
*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$=\frac{a^4\int \frac{x^4}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{5a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{10a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{10a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{5ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**4,x)
```

output
```
a**4*Integral(x**4/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- 5*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 10*a**3*x**3*s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 10*a**2*x**2*sqrt(a*x/(a*x +
1) - 1/(a*x + 1))/(a*x + 1) + 5*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
 + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.86

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$=\frac{1}{1260}a\left(\frac{\frac{235(ax-1)}{ax+1}+\frac{801(ax-1)^2}{(ax+1)^2}+\frac{2289(ax-1)^3}{(ax+1)^3}+\frac{11760(ax-1)^4}{(ax+1)^4}-\frac{17640(ax-1)^5}{(ax+1)^5}+35}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}+\frac{8820\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
1/1260*a*((235*(a*x - 1)/(a*x + 1) + 801*(a*x - 1)^2/(a*x + 1)^2 + 2289*(a
*x - 1)^3/(a*x + 1)^3 + 11760*(a*x - 1)^4/(a*x + 1)^4 - 17640*(a*x - 1)^5/
(a*x + 1)^5 + 35)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(11/2) - a^2*c^4*((a*x -
1)/(a*x + 1))^(9/2)) + 8820*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) -
 8820*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.29

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=-\frac{7\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c^4|a|\text{sgn}\,(ax+1)}+\frac{\sqrt{a^2x^2-1}}{ac^4\text{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="giac")
```

output
```
-7*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c^4*abs(a)*sgn(a*x + 1)) + sqr
t(a^2*x^2 - 1)/(a*c^4*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.74 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.71

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\frac{14\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4}-\frac{\frac{89\,{\left(a\,x-1\right)}^2}{35\,{\left(a\,x+1\right)}^2}+\frac{109\,{\left(a\,x-1\right)}^3}{15\,{\left(a\,x+1\right)}^3}+\frac{112\,{\left(a\,x-1\right)}^4}{3\,{\left(a\,x+1\right)}^4}-\frac{56\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{47\,\left(a\,x-1\right)}{63\,\left(a\,x+1\right)}+\frac{1}{9}}{4\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}-4\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}$$

input
```
int(1/((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
(14*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^4) - ((89*(a*x - 1)^2)/(35*(a
*x + 1)^2) + (109*(a*x - 1)^3)/(15*(a*x + 1)^3) + (112*(a*x - 1)^4)/(3*(a*
x + 1)^4) - (56*(a*x - 1)^5)/(a*x + 1)^5 + (47*(a*x - 1))/(63*(a*x + 1)) +
 1/9)/(4*a*c^4*((a*x - 1)/(a*x + 1))^(9/2) - 4*a*c^4*((a*x - 1)/(a*x + 1))
^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.57

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$= \frac{8820\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^4x^4 - 35280\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^3x^3 + 52920\sqrt{ax-1}\log\left(\frac{\sqrt{a}}{}\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x)
```

output
```
(8820*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**4*x**4
 - 35280*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**3*x
**3 + 52920*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**
2*x**2 - 35280*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*
a*x + 8820*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 75
13*sqrt(a*x - 1)*a**4*x**4 - 30052*sqrt(a*x - 1)*a**3*x**3 + 45078*sqrt(a*
x - 1)*a**2*x**2 - 30052*sqrt(a*x - 1)*a*x + 7513*sqrt(a*x - 1) + 630*sqrt
(a*x + 1)*a**5*x**5 - 13078*sqrt(a*x + 1)*a**4*x**4 + 39560*sqrt(a*x + 1)*
a**3*x**3 - 50694*sqrt(a*x + 1)*a**2*x**2 + 30230*sqrt(a*x + 1)*a*x - 6928
*sqrt(a*x + 1))/(630*sqrt(a*x - 1)*a*c**4*(a**4*x**4 - 4*a**3*x**3 + 6*a**
2*x**2 - 4*a*x + 1))
```

3.419 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=\frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}+c^5x-\frac{c^5\log(x)}{a}$$

output
```
1/4*c^5/a^5/x^4-1/3*c^5/a^4/x^3-c^5/a^3/x^2+2*c^5/a^2/x+c^5*x-c^5*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=\frac{c^5\left(\frac{1}{4x^4}-\frac{a}{3x^3}-\frac{a^2}{x^2}+\frac{2a^3}{x}+a^5x-a^4\log(x)\right)}{a^5}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

output
```
(c^5*(1/(4*x^4) - a/(3*x^3) - a^2/x^2 + (2*a^3)/x + a^5*x - a^4*Log[x]))/a
^5
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^5 e^{4\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{c^5 \left(a-\frac{1}{x}\right)^5 e^{4\text{arctanh}(ax)}}{a^5}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c^5 \int e^{4\text{arctanh}(ax)} \left(a-\frac{1}{x}\right)^5 dx}{a^5}$$

$$\Big\downarrow \text{ 6681}$$

$$-\frac{c^5 \int \frac{e^{4\text{arctanh}(ax)}(1-ax)^5}{x^5}dx}{a^5}$$

$$\Big\downarrow \text{ 6679}$$

$$-\frac{c^5 \int \frac{(1-ax)^3(ax+1)^2}{x^5}dx}{a^5}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{c^5 \int \left(-a^5+\frac{a^4}{x}+\frac{2a^3}{x^2}-\frac{2a^2}{x^3}-\frac{a}{x^4}+\frac{1}{x^5}\right)dx}{a^5}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{c^5 \left(a^5(-x)+a^4\log(x)-\frac{2a^3}{x}+\frac{a^2}{x^2}+\frac{a}{3x^3}-\frac{1}{4x^4}\right)}{a^5}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

output
```
-((c^5*(-1/4*1/x^4 + a/(3*x^3) + a^2/x^2 - (2*a^3)/x - a^5*x + a^4*Log[x])
)/a^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

method	result
default	$\frac{c^{5}\left(x\,a^{5}-\frac{a^{2}}{x^{2}}+\frac{2a^{3}}{x}+\frac{1}{4x^{4}}-a^{4}\ln\left(x\right)-\frac{a}{3x^{3}}\right)}{a^{5}}$
risch	$c^{5}x+\frac{2a^{3}c^{5}x^{3}-a^{2}c^{5}x^{2}-\frac{1}{3}a\,c^{5}x+\frac{1}{4}c^{5}}{a^{5}x^{4}}-\frac{c^{5}\ln\left(x\right)}{a}$
parallelrisch	$-\frac{-12a^{5}c^{5}x^{5}+12c^{5}\ln\left(x\right)a^{4}x^{4}-24a^{3}c^{5}x^{3}+12a^{2}c^{5}x^{2}+4a\,c^{5}x-3c^{5}}{12a^{5}x^{4}}$
norman	$\frac{a^{4}c^{5}x^{5}+a^{5}c^{5}x^{6}-\frac{c^{5}}{4a}+\frac{7c^{5}x}{12}-3a^{2}c^{5}x^{3}+\frac{2c^{5}a\,x^{2}}{3}}{(ax-1)a^{4}x^{4}}-\frac{c^{5}\ln\left(x\right)}{a}$
meijerg	$-\frac{c^{5}\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln\left(-ax+1\right)\right)}{a}-\frac{3c^{5}\left(\frac{ax}{-ax+1}+\ln\left(-ax+1\right)\right)}{a}+\frac{c^{5}x}{-ax+1}+\frac{5c^{5}\left(1+\ln\left(x\right)+\ln\left(-a\right)+\frac{2ax}{-2ax+2}-\ln(-a\right.}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x,method=_RETURNVERBOSE)
```

output
```
c^5/a^5*(x*a^5-a^2/x^2+2*a^3/x+1/4/x^4-a^4*ln(x)-1/3*a/x^3)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5}\,dx = \frac{12\,a^{5}c^{5}x^{5}-12\,a^{4}c^{5}x^{4}\log\left(x\right)+24\,a^{3}c^{5}x^{3}-12\,a^{2}c^{5}x^{2}-4\,ac^{5}x+3\,c^{5}}{12\,a^{5}x^{4}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="fricas")
```

output
```
1/12*(12*a^5*c^5*x^5 - 12*a^4*c^5*x^4*log(x) + 24*a^3*c^5*x^3 - 12*a^2*c^5
*x^2 - 4*a*c^5*x + 3*c^5)/(a^5*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=\frac{a^5c^5x-a^4c^5\log(x)+\frac{24a^3c^5x^3-12a^2c^5x^2-4ac^5x+3c^5}{12x^4}}{a^5}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**5,x)
```

output
```
(a**5*c**5*x - a**4*c**5*log(x) + (24*a**3*c**5*x**3 - 12*a**2*c**5*x**2 -
 4*a*c**5*x + 3*c**5)/(12*x**4))/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=c^5x-\frac{c^5\log(x)}{a}+\frac{24\,a^3c^5x^3-12\,a^2c^5x^2-4\,ac^5x+3\,c^5}{12\,a^5x^4}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="maxima")
```

output
```
c^5*x - c^5*log(x)/a + 1/12*(24*a^3*c^5*x^3 - 12*a^2*c^5*x^2 - 4*a*c^5*x +
 3*c^5)/(a^5*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(60) = 120.

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.92

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx=\frac{c^5\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{c^5\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(12\,c^5+\frac{37\,c^5}{ax-1}+\frac{52\,c^5}{(ax-1)^2}+\frac{42\,c^5}{(ax-1)^3}+\frac{12\,c^5}{(ax-1)^4}\right)(ax-1)}{12\,a\left(\frac{1}{ax-1}+1\right)^4}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="giac")
```

output
```
c^5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - c^5*log(abs(-1/(a*x - 1) -
1))/a + 1/12*(12*c^5 + 37*c^5/(a*x - 1) + 52*c^5/(a*x - 1)^2 + 42*c^5/(a*x
 - 1)^3 + 12*c^5/(a*x - 1)^4)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^4)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx = -\frac{c^5\,\left(4\,a\,x+12\,a^2\,x^2-24\,a^3\,x^3-12\,a^5\,x^5+12\,a^4\,x^4\,\ln\left(x\right)-3\right)}{12\,a^5\,x^4}$$

input
```
int(((c - c/(a*x))^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
-(c^5*(4*a*x + 12*a^2*x^2 - 24*a^3*x^3 - 12*a^5*x^5 + 12*a^4*x^4*log(x) -
3))/(12*a^5*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx = \frac{c^5(-12\log(x)\,a^4x^4+12a^5x^5+24a^3x^3-12a^2x^2-4ax+3)}{12a^5x^4}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x)
```

output
```
(c**5*( - 12*log(x)*a**4*x**4 + 12*a**5*x**5 + 24*a**3*x**3 - 12*a**2*x**2
 - 4*a*x + 3))/(12*a**5*x**4)
```

3.420 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 30

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

output
```
-1/3*c^4/a^4/x^3+2*c^4/a^2/x+c^4*x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{c^4\left(-\frac{1}{3x^3}+\frac{2a^2}{x}+a^4x\right)}{a^4}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

output
```
(c^4*(-1/3*1/x^3 + (2*a^2)/x + a^4*x))/a^4
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6717, 27, 6681, 6679, 82, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^4 e^{4\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{c^4 \left(a-\frac{1}{x}\right)^4 e^{4\text{arctanh}(ax)}}{a^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c^4 \int e^{4\text{arctanh}(ax)} \left(a-\frac{1}{x}\right)^4 dx}{a^4}$$

$$\Big\downarrow \text{ 6681}$$

$$\frac{c^4 \int \frac{e^{4\text{arctanh}(ax)} (1-ax)^4}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{ 6679}$$

$$\frac{c^4 \int \frac{(1-ax)^2 (ax+1)^2}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{ 82}$$

$$\frac{c^4 \int \frac{\left(1-a^2x^2\right)^2}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{c^4 \int \left(a^4-\frac{2a^2}{x^2}+\frac{1}{x^4}\right) dx}{a^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^4 \left(a^4x+\frac{2a^2}{x}-\frac{1}{3x^3}\right)}{a^4}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

output
```
(c^4*(-1/3*1/x^3 + (2*a^2)/x + a^4*x))/a^4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

method	result
default	$\frac{c^{4}\left(x\,a^{4}+\frac{2a^{2}}{x}-\frac{1}{3x^{3}}\right)}{a^{4}}$
gosper	$\frac{c^{4}\left(3a^{4}x^{4}+6a^{2}x^{2}-1\right)}{3x^{3}a^{4}}$
risch	$c^{4}x+\frac{2a^{2}c^{4}x^{2}-\frac{1}{3}c^{4}}{a^{4}x^{3}}$
parallelrisch	$\frac{3a^{4}c^{4}x^{4}+6a^{2}c^{4}x^{2}-c^{4}}{3a^{4}x^{3}}$
orering	$\frac{\left(3a^{4}x^{4}+6a^{2}x^{2}-1\right)x\left(c-\frac{c}{ax}\right)^{4}}{3\left(ax-1\right)^{4}}$
norman	$\frac{a^{3}c^{4}x^{4}+a^{4}c^{4}x^{5}+\frac{c^{4}}{3a}-\frac{c^{4}x}{3}-2a\,c^{4}x^{2}}{\left(ax-1\right)a^{3}x^{3}}$
meijerg	$-\frac{c^{4}\left(-\frac{ax\left(-3ax+6\right)}{3\left(-ax+1\right)}-2\ln\left(-ax+1\right)\right)}{a}-\frac{2c^{4}\left(\frac{ax}{-ax+1}+\ln\left(-ax+1\right)\right)}{a}-\frac{c^{4}x}{-ax+1}+\frac{4c^{4}\left(1+\ln\left(x\right)+\ln\left(-a\right)+\frac{2ax}{-2ax+2}-\ln(-a\right.}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
c^4/a^4*(x*a^4+2*a^2/x-1/3/x^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{4}\,dx=\frac{3\,a^{4}c^{4}x^{4}+6\,a^{2}c^{4}x^{2}-c^{4}}{3\,a^{4}x^{3}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*c^4*x^4 + 6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{a^4c^4x+\frac{6a^2c^4x^2-c^4}{3x^3}}{a^4}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**4,x)
```

output
```
(a**4*c**4*x + (6*a**2*c**4*x**2 - c**4)/(3*x**3))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = c^4x+\frac{6\,a^2c^4x^2-c^4}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
c^4*x + 1/3*(6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(28) = 56.

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.97

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{(ax-1)c^4}{a} - \frac{5\,c^4+\frac{9\,c^4}{ax-1}+\frac{3\,c^4}{(ax-1)^2}}{3\,a\left(\frac{1}{ax-1}+1\right)^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="giac")
```

output
```
(a*x - 1)*c^4/a - 1/3*(5*c^4 + 9*c^4/(a*x - 1) + 3*c^4/(a*x - 1)^2)/(a*(1/
(a*x - 1) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 13.84 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\left(a^4x^4+2a^2x^2-\frac{1}{3}\right)}{a^4x^3}$$

input
```
int(((c - c/(a*x))^4*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(c^4*(2*a^2*x^2 + a^4*x^4 - 1/3))/(a^4*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4(3a^4x^4+6a^2x^2-1)}{3a^4x^3}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x)
```

output
```
(c**4*(3*a**4*x**4 + 6*a**2*x**2 - 1))/(3*a**4*x**3)
```

3.421 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 38

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{c^3\log(x)}{a}$$

output
```
1/2*c^3/a^3/x^2+c^3/a^2/x+c^3*x+c^3*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\left(\frac{3a^2}{2}+\frac{1}{2x^2}+\frac{a}{x}+a^3x+a^2\log(x)\right)}{a^3}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

output
```
(c^3*((3*a^2)/2 + 1/(2*x^2) + a/x + a^3*x + a^2*Log[x]))/a^3
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^3 e^{4\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{c^3\left(a-\frac{1}{x}\right)^3 e^{4\text{arctanh}(ax)}}{a^3}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c^3\int e^{4\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^3dx}{a^3}$$

$$\Big\downarrow \text{ 6681}$$

$$-\frac{c^3\int \frac{e^{4\text{arctanh}(ax)}(1-ax)^3}{x^3}dx}{a^3}$$

$$\Big\downarrow \text{ 6679}$$

$$-\frac{c^3\int \frac{(1-ax)(ax+1)^2}{x^3}dx}{a^3}$$

$$\Big\downarrow \text{ 84}$$

$$-\frac{c^3\int \left(-a^3-\frac{a^2}{x}+\frac{a}{x^2}+\frac{1}{x^3}\right)dx}{a^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{c^3\left(a^3(-x)-a^2\log(x)-\frac{a}{x}-\frac{1}{2x^2}\right)}{a^3}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

output
```
-((c^3*(-1/2*1/x^2 - a/x - a^3*x - a^2*Log[x]))/a^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

method	result
default	$\frac{c^{3}\left(a^{3}x+\frac{1}{2x^{2}}+\frac{a}{x}+a^{2}\ln\left(x\right)\right)}{a^{3}}$
risch	$c^{3}x+\frac{a\,c^{3}x+\frac{1}{2}c^{3}}{a^{3}x^{2}}+\frac{c^{3}\ln\left(x\right)}{a}$
parallelrisch	$\frac{2a^{3}c^{3}x^{3}+2c^{3}\ln\left(x\right)a^{2}x^{2}+2a\,c^{3}x+c^{3}}{2a^{3}x^{2}}$
norman	$\frac{a^{3}c^{3}x^{4}-\frac{c^{3}}{2a}-\frac{c^{3}x}{2}}{\left(ax-1\right)a^{2}x^{2}}+\frac{c^{3}\ln\left(x\right)}{a}$
meijerg	$-\frac{c^{3}\left(-\frac{ax\left(-3ax+6\right)}{3\left(-ax+1\right)}-2\ln\left(-ax+1\right)\right)}{a}-\frac{c^{3}\left(\frac{ax}{-ax+1}+\ln\left(-ax+1\right)\right)}{a}-\frac{2c^{3}x}{-ax+1}+\frac{2c^{3}\left(1+\ln\left(x\right)+\ln\left(-a\right)+\frac{2ax}{-2ax+2}-\ln(-ax\right.}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
c^3/a^3*(a^3*x+1/2/x^2+a/x+a^2*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3}\,dx=\frac{2\,a^{3}c^{3}x^{3}+2\,a^{2}c^{3}x^{2}\log\left(x\right)+2\,ac^{3}x+c^{3}}{2\,a^{3}x^{2}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
1/2*(2*a^3*c^3*x^3 + 2*a^2*c^3*x^2*log(x) + 2*a*c^3*x + c^3)/(a^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{a^3c^3x+a^2c^3\log{(x)}+\frac{2ac^3x+c^3}{2x^2}}{a^3}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**3,x)
```

output
```
(a**3*c**3*x + a**2*c**3*log(x) + (2*a*c**3*x + c**3)/(2*x**2))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=c^3x+\frac{c^3\log{(x)}}{a}+\frac{2\,ac^3x+c^3}{2\,a^3x^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
c^3*x + c^3*log(x)/a + 1/2*(2*a*c^3*x + c^3)/(a^3*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(36) = 72.

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.58

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{c^3\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{c^3\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(2\,c^3+\frac{c^3}{ax-1}-\frac{2\,c^3}{(ax-1)^2}\right)(ax-1)}{2\,a\left(\frac{1}{ax-1}+1\right)^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="giac")
```

output
```
-c^3*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + c^3*log(abs(-1/(a*x - 1) -
 1))/a + 1/2*(2*c^3 + c^3/(a*x - 1) - 2*c^3/(a*x - 1)^2)*(a*x - 1)/(a*(1/(
a*x - 1) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3\,\left(a\,x+a^3\,x^3+a^2\,x^2\,\ln\left(x\right)+\frac{1}{2}\right)}{a^3\,x^2}$$

input
```
int(((c - c/(a*x))^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(c^3*(a*x + a^3*x^3 + a^2*x^2*log(x) + 1/2))/(a^3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{c^3(2\log(x)\,a^2x^2+2a^3x^3+2ax+1)}{2a^3x^2}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x)
```

output
```
(c**3*(2*log(x)*a**2*x**2 + 2*a**3*x**3 + 2*a*x + 1))/(2*a**3*x**2)
```

3.422 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 27

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=-\frac{c^2}{a^2x}+c^2x+\frac{2c^2\log(x)}{a}$$

output
```
-c^2/a^2/x+c^2*x+2*c^2*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\left(-\frac{1}{x}+a^2x+2a\log(x)\right)}{a^2}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

output
```
(c^2*(-x^(-1) + a^2*x + 2*a*Log[x]))/a^2
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6681, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^2 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{c^2\left(a-\frac{1}{x}\right)^2 e^{4\text{arctanh}(ax)}}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c^2\int e^{4\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^2dx}{a^2}$$

$$\Big\downarrow \text{ 6681}$$

$$\frac{c^2\int \frac{e^{4\text{arctanh}(ax)}(1-ax)^2}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{ 6679}$$

$$\frac{c^2\int \frac{(ax+1)^2}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{c^2\int \left(a^2+\frac{2a}{x}+\frac{1}{x^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^2\left(a^2x+2a\log(x)-\frac{1}{x}\right)}{a^2}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

output
```
(c^2*(-x^(-1) + a^2*x + 2*a*Log[x]))/a^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{c^2(a^2x-\frac{1}{x}+2\ln(x)a)}{a^2}$	24
risch	$-\frac{c^2}{a^2x}+c^2x+\frac{2c^2\ln(x)}{a}$	28
parallelrisch	$\frac{c^2a^2x^2+2c^2\ln(x)ax-c^2}{a^2x}$	33
norman	$\frac{\frac{c^2}{a}-2a\,c^2x^2+a^2c^2x^3}{(ax-1)ax}+\frac{2c^2\ln(x)}{a}$	53
meijerg	$-\frac{c^2\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{2c^2x}{-ax+1}-\frac{c^2\left(\frac{1}{ax}-1-2\ln(x)-2\ln(-a)-\frac{3ax}{-3ax+3}+2\ln(-ax+1)\right)}{a}$	100

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2/a^2*(a^2*x-1/x+2*ln(x)*a)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{a^2c^2x^2+2\,ac^2x\log(x)-c^2}{a^2x}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
(a^2*c^2*x^2 + 2*a*c^2*x*log(x) - c^2)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{a^2c^2x+2ac^2\log(x)-\frac{c^2}{x}}{a^2}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**2,x)
```

output
```
(a**2*c**2*x + 2*a*c**2*log(x) - c**2/x)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=c^2x+\frac{2\,c^2\log(x)}{a}-\frac{c^2}{a^2x}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
c^2*x + 2*c^2*log(x)/a - c^2/(a^2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(27) = 54.

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.48

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=-\frac{2\,c^2\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{2\,c^2\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{c^2+\frac{2\,c^2}{ax-1}}{a^2\left(\frac{1}{(ax-1)a}+\frac{1}{(ax-1)^2a}\right)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="giac")
```

output
```
-2*c^2*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 2*c^2*log(abs(-1/(a*x -
1) - 1))/a + (c^2 + 2*c^2/(a*x - 1))/(a^2*(1/((a*x - 1)*a) + 1/((a*x - 1)^
2*a)))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\,\left(a^2\,x^2+2\,a\,x\,\ln\left(x\right)-1\right)}{a^2\,x}$$

input
```
int(((c - c/(a*x))^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(c^2*(a^2*x^2 + 2*a*x*log(x) - 1))/(a^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2(2\log(x)\,ax+a^2x^2-1)}{a^2x}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x)
```

output
```
(c**2*(2*log(x)*a*x + a**2*x**2 - 1))/(a**2*x)
```

3.423 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 25

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}$$

output
```
c*x-c*ln(x)/a+4*c*ln(-a*x+1)/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c(ax-\log(x)+4\log(1-ax))}{a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x)),x]
```

output
```
(c*(a*x - Log[x] + 4*Log[1 - a*x]))/a
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6717, 27, 6681, 6679, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right) e^{4\coth^{-1}(ax)} \, dx$$
$$\Big\downarrow \text{6717}$$
$$\int \frac{c\left(a-\frac{1}{x}\right) e^{4\text{arctanh}(ax)}}{a}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{c\int e^{4\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)dx}{a}$$
$$\Big\downarrow \text{6681}$$
$$-\frac{c\int \frac{e^{4\text{arctanh}(ax)}(1-ax)}{x}dx}{a}$$
$$\Big\downarrow \text{6679}$$
$$-\frac{c\int \frac{(ax+1)^2}{x(1-ax)}dx}{a}$$
$$\Big\downarrow \text{93}$$
$$-\frac{c\int \left(-\frac{4a}{ax-1}-a+\frac{1}{x}\right)dx}{a}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{c(-ax-4\log(1-ax)+\log(x))}{a}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a*x)),x]
```

output
```
-((c*(-(a*x) + Log[x] - 4*Log[1 - a*x]))/a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result
default	$\frac{c(ax+4\ln(ax-1)-\ln(x))}{a}$
parallelrisch	$-\frac{-acx+c\ln(x)-4c\ln(ax-1)}{a}$
risch	$xc-\frac{c\ln(x)}{a}+\frac{4c\ln(-ax+1)}{a}$
norman	$\frac{ac\,x^2-xc}{ax-1}-\frac{c\ln(x)}{a}+\frac{4c\ln(ax-1)}{a}$
meijerg	$-\frac{c\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}+\frac{c\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}-\frac{cx}{-ax+1}-\frac{c\left(1+\ln(x)+\ln(-a)+\frac{2ax}{-2ax+2}-\ln(-ax+1)\right)}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
c/a*(a*x+4*ln(a*x-1)-ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{acx+4\,c\log\left(ax-1\right)-c\log\left(x\right)}{a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="fricas")
```

output
```
(a*c*x + 4*c*log(a*x - 1) - c*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\left(-\log{(x)}+4\log{\left(x-\frac{1}{a}\right)}\right)}{a}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x),x)
```

output
```
c*x + c*(-log(x) + 4*log(x - 1/a))/a
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{4\,c\log{(ax-1)}}{a}-\frac{c\log{(x)}}{a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="maxima")
```

output
```
c*x + 4*c*log(a*x - 1)/a - c*log(x)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(25) = 50.

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{(ax-1)c}{a}-\frac{3\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{c\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="giac")
```

output
```
(a*x - 1)*c/a - 3*c*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - c*log(abs(-
1/(a*x - 1) - 1))/a
```

Mupad [B] (verification not implemented)

Time = 13.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=c\,x-\frac{c\,\ln{(x)}}{a}+\frac{4\,c\,\ln{(a\,x-1)}}{a}$$

input
```
int(((c - c/(a*x))*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
c*x - (c*log(x))/a + (4*c*log(a*x - 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c(4\log(ax-1)-\log(x)+ax)}{a}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x)
```

output
```
(c*(4*log(a*x - 1) - log(x) + a*x))/a
```

3.424 $\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{x}{c}-\frac{2}{ac(1-ax)^2}+\frac{8}{ac(1-ax)}+\frac{5\log(1-ax)}{ac}$$

output
```
x/c-2/a/c/(-a*x+1)^2+8/a/c/(-a*x+1)+5*ln(-a*x+1)/a/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=-\frac{a\left(-\frac{x}{a}+\frac{2}{a^2(1-ax)^2}-\frac{8}{a^2(1-ax)}-\frac{5\log(1-ax)}{a^2}\right)}{c}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x)),x]
```

output
```
-((a*(-(x/a) + 2/(a^2*(1 - a*x)^2) - 8/(a^2*(1 - a*x)) - (5*Log[1 - a*x])/
a^2))/c)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6681, 6679, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{ae^{4\text{arctanh}(ax)}}{c\left(a-\frac{1}{x}\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{e^{4\text{arctanh}(ax)}}{a-\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{a\int \frac{e^{4\text{arctanh}(ax)}x}{1-ax}dx}{c}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{a\int \frac{x(ax+1)^2}{(1-ax)^3}dx}{c}$$

$$\Big\downarrow \text{86}$$

$$-\frac{a\int \left(-\frac{5}{(ax-1)a}-\frac{8}{(ax-1)^2a}-\frac{4}{(ax-1)^3a}-\frac{1}{a}\right)dx}{c}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(-\frac{8}{a^2(1-ax)}+\frac{2}{a^2(1-ax)^2}-\frac{5\log(1-ax)}{a^2}-\frac{x}{a}\right)}{c}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - c/(a*x)),x]
```

output
```
-((a*(-(x/a) + 2/(a^2*(1 - a*x)^2) - 8/(a^2*(1 - a*x)) - (5*Log[1 - a*x])/
a^2))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{x}{c}+\frac{-8xc+\frac{6c}{a}}{c^{2}\left(ax-1\right)^{2}}+\frac{5\ln\left(ax-1\right)}{ac}$	43
default	$\frac{a\left(\frac{x}{a}-\frac{8}{\left(ax-1\right)a^{2}}+\frac{5\ln\left(ax-1\right)}{a^{2}}-\frac{2}{a^{2}\left(ax-1\right)^{2}}\right)}{c}$	47
norman	$\frac{\frac{a^{2}x^{3}}{c}-\frac{8a\,x^{2}}{c}+\frac{5x}{c}}{\left(ax-1\right)^{2}}+\frac{5\ln\left(ax-1\right)}{ac}$	50
parallelrisch	$\frac{a^{3}x^{3}+5a^{2}\ln\left(ax-1\right)x^{2}-8a^{2}x^{2}-10a\ln\left(ax-1\right)x+5ax+5\ln\left(ax-1\right)}{\left(ax-1\right)^{2}ca}$	67

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
x/c+(-8*x*c+6*c/a)/c^2/(a*x-1)^2+5/a/c*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int\frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a^{3}x^{3}-2\,a^{2}x^{2}-7\,ax+5\,(a^{2}x^{2}-2\,ax+1)\log\left(ax-1\right)+6}{a^{3}cx^{2}-2\,a^{2}cx+ac}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="fricas")
```

output
```
(a^3*x^3 - 2*a^2*x^2 - 7*a*x + 5*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 6)/(
a^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{-8ax+6}{a^3cx^2-2a^2cx+ac}+\frac{x}{c}+\frac{5\log(ax-1)}{ac}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x),x)
```

output
```
(-8*a*x + 6)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + x/c + 5*log(a*x - 1)/(a*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{2\,(4\,ax-3)}{a^3cx^2-2\,a^2cx+ac}+\frac{x}{c}+\frac{5\,\log(ax-1)}{ac}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="maxima")
```

output
```
-2*(4*a*x - 3)/(a^3*c*x^2 - 2*a^2*c*x + a*c) + x/c + 5*log(a*x - 1)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{ax-1}{ac}-\frac{5\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac}-\frac{2\left(\frac{4\,a^3c}{ax-1}+\frac{a^3c}{(ax-1)^2}\right)}{a^4c^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="giac")
```

output
```
(a*x - 1)/(a*c) - 5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) - 2*(4*a^
3*c/(a*x - 1) + a^3*c/(a*x - 1)^2)/(a^4*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{x}{c} - \frac{8\,x-\frac{6}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c} + \frac{5\,\ln\left(a\,x-1\right)}{a\,c}$$

input
```
int((a*x + 1)^2/((c - c/(a*x))*(a*x - 1)^2),x)
```

output
```
x/c - (8*x - 6/a)/(c + a^2*c*x^2 - 2*a*c*x) + (5*log(a*x - 1))/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{10\log(ax-1)\,a^2x^2-20\log(ax-1)\,ax+10\log(ax-1)+2a^3x^3-11a^2x^2+5}{2ac\,(a^2x^2-2ax+1)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x)
```

output
```
(10*log(a*x - 1)*a**2*x**2 - 20*log(a*x - 1)*a*x + 10*log(a*x - 1) + 2*a**
3*x**3 - 11*a**2*x**2 + 5)/(2*a*c*(a**2*x**2 - 2*a*x + 1))
```

3.425 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{x}{c^2}+\frac{4}{3ac^2(1-ax)^3}-\frac{6}{ac^2(1-ax)^2}+\frac{13}{ac^2(1-ax)}+\frac{6\log(1-ax)}{ac^2}$$

output
```
x/c^2+4/3/a/c^2/(-a*x+1)^3-6/a/c^2/(-a*x+1)^2+13/a/c^2/(-a*x+1)+6*ln(-a*x+
1)/a/c^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{-25+57ax-30a^2x^2-9a^3x^3+3a^4x^4+18(-1+ax)^3\log(1-ax)}{3ac^2(-1+ax)^3}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

output
```
(-25 + 57*a*x - 30*a^2*x^2 - 9*a^3*x^3 + 3*a^4*x^4 + 18*(-1 + a*x)^3*Log[1
 - a*x])/(3*a*c^2*(-1 + a*x)^3)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx$$

$$\Big\downarrow \ 6717$$

$$\int \frac{a^2e^{4\text{arctanh}(ax)}}{c^2\left(a-\frac{1}{x}\right)^2}dx$$

$$\Big\downarrow \ 27$$

$$\frac{a^2\int \frac{e^{4\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^2}dx}{c^2}$$

$$\Big\downarrow \ 6681$$

$$\frac{a^2\int \frac{e^{4\text{arctanh}(ax)}x^2}{(1-ax)^2}dx}{c^2}$$

$$\Big\downarrow \ 6679$$

$$\frac{a^2\int \frac{x^2(ax+1)^2}{(1-ax)^4}dx}{c^2}$$

$$\Big\downarrow \ 99$$

$$\frac{a^2\int \left(\frac{1}{a^2}+\frac{6}{a^2(ax-1)}+\frac{13}{a^2(ax-1)^2}+\frac{12}{a^2(ax-1)^3}+\frac{4}{a^2(ax-1)^4}\right)dx}{c^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{a^2\left(\frac{13}{a^3(1-ax)}-\frac{6}{a^3(1-ax)^2}+\frac{4}{3a^3(1-ax)^3}+\frac{6\log(1-ax)}{a^3}+\frac{x}{a^2}\right)}{c^2}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

output
```
(a^2*(x/a^2 + 4/(3*a^3*(1 - a*x)^3) - 6/(a^3*(1 - a*x)^2) + 13/(a^3*(1 - a
*x)) + (6*Log[1 - a*x])/a^3))/c^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x}{c^{2}}+\frac{-13a\,c^{2}x^{2}+20c^{2}x-\frac{25c^{2}}{3a}}{c^{4}\left(ax-1\right)^{3}}+\frac{6\ln\left(ax-1\right)}{a\,c^{2}}$	56
default	$\frac{a^{2}\left(\frac{x}{a^{2}}-\frac{13}{\left(ax-1\right)a^{3}}+\frac{6\ln\left(ax-1\right)}{a^{3}}-\frac{6}{a^{3}\left(ax-1\right)^{2}}-\frac{4}{3a^{3}\left(ax-1\right)^{3}}\right)}{c^{2}}$	61
norman	$\frac{\frac{a^{3}x^{4}}{c}-\frac{6x}{c}+\frac{15a\,x^{2}}{c}-\frac{34a^{2}x^{3}}{3c}}{\left(ax-1\right)^{3}c}+\frac{6\ln\left(ax-1\right)}{a\,c^{2}}$	64
parallelrisch	$\frac{3a^{4}x^{4}+18a^{3}\ln\left(ax-1\right)x^{3}-34a^{3}x^{3}-54a^{2}\ln\left(ax-1\right)x^{2}+45a^{2}x^{2}+54a\ln\left(ax-1\right)x-18ax-18\ln\left(ax-1\right)}{3\left(ax-1\right)^{3}c^{2}a}$	91

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
x/c^2+(-13*a*c^2*x^2+20*c^2*x-25/3*c^2/a)/c^4/(a*x-1)^3+6/a/c^2*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int\frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{2}}\,dx=\frac{3\,a^{4}x^{4}-9\,a^{3}x^{3}-30\,a^{2}x^{2}+57\,ax+18\,(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1)\log\left(ax-1\right)-25}{3\,(a^{4}c^{2}x^{3}-3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x-ac^{2})}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*x^4 - 9*a^3*x^3 - 30*a^2*x^2 + 57*a*x + 18*(a^3*x^3 - 3*a^2*x^2
 + 3*a*x - 1)*log(a*x - 1) - 25)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*
x - a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{-39a^2x^2+60ax-25}{3a^4c^2x^3-9a^3c^2x^2+9a^2c^2x-3ac^2}+\frac{x}{c^2}+\frac{6\log{(ax-1)}}{ac^2}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**2,x)
```

output
```
(-39*a**2*x**2 + 60*a*x - 25)/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**
2*c**2*x - 3*a*c**2) + x/c**2 + 6*log(a*x - 1)/(a*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{39\,a^2x^2-60\,ax+25}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}+\frac{x}{c^2}+\frac{6\,\log{(ax-1)}}{ac^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
-1/3*(39*a^2*x^2 - 60*a*x + 25)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
 - a*c^2) + x/c^2 + 6*log(a*x - 1)/(a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{ax-1}{ac^2}-\frac{6\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^2}-\frac{\frac{39\,a^5c^4}{ax-1}+\frac{18\,a^5c^4}{(ax-1)^2}+\frac{4\,a^5c^4}{(ax-1)^3}}{3\,a^6c^6}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="giac")
```

output
```
(a*x - 1)/(a*c^2) - 6*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^2) - 1/3
*(39*a^5*c^4/(a*x - 1) + 18*a^5*c^4/(a*x - 1)^2 + 4*a^5*c^4/(a*x - 1)^3)/(
a^6*c^6)
```

Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{13\,a\,x^2-20\,x+\frac{25}{3\,a}}{-a^3\,c^2\,x^3+3\,a^2\,c^2\,x^2-3\,a\,c^2\,x+c^2}+\frac{x}{c^2}+\frac{6\,\ln\left(a\,x-1\right)}{a\,c^2}$$

input
```
int((a*x + 1)^2/((c - c/(a*x))^2*(a*x - 1)^2),x)
```

output
```
(13*a*x^2 - 20*x + 25/(3*a))/(c^2 + 3*a^2*c^2*x^2 - a^3*c^2*x^3 - 3*a*c^2*
x) + x/c^2 + (6*log(a*x - 1))/(a*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$
$$= \frac{18\log(ax-1)\,a^3x^3-54\log(ax-1)\,a^2x^2+54\log(ax-1)\,ax-18\log(ax-1)+3a^4x^4-19a^3x^3+27a}{3a\,c^2\left(a^3x^3-3a^2x^2+3ax-1\right)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x)
```

output
```
(18*log(a*x - 1)*a**3*x**3 - 54*log(a*x - 1)*a**2*x**2 + 54*log(a*x - 1)*a
*x - 18*log(a*x - 1) + 3*a**4*x**4 - 19*a**3*x**3 + 27*a*x - 15)/(3*a*c**2
*(a**3*x**3 - 3*a**2*x**2 + 3*a*x - 1))
```

3.426 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3}-\frac{1}{ac^3(1-ax)^4}+\frac{16}{3ac^3(1-ax)^3}-\frac{25}{2ac^3(1-ax)^2}+\frac{19}{ac^3(1-ax)}+\frac{7\log(1-ax)}{ac^3}$$

output
```
x/c^3-1/a/c^3/(-a*x+1)^4+16/3/a/c^3/(-a*x+1)^3-25/2/a/c^3/(-a*x+1)^2+19/a/
c^3/(-a*x+1)+7*ln(-a*x+1)/a/c^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{65-218ax+243a^2x^2-78a^3x^3-24a^4x^4+6a^5x^5+42(-1+ax)^4\log(1-ax)}{6ac^3(-1+ax)^4}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

output
```
(65 - 218*a*x + 243*a^2*x^2 - 78*a^3*x^3 - 24*a^4*x^4 + 6*a^5*x^5 + 42*(-1
 + a*x)^4*Log[1 - a*x])/(6*a*c^3*(-1 + a*x)^4)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{a^3e^{4\text{arctanh}(ax)}}{c^3\left(a-\frac{1}{x}\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3\int \frac{e^{4\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^3}dx}{c^3}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{a^3\int \frac{e^{4\text{arctanh}(ax)}x^3}{(1-ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{a^3\int \frac{x^3(ax+1)^2}{(1-ax)^5}dx}{c^3}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^3\int \left(-\frac{1}{a^3}-\frac{7}{a^3(ax-1)}-\frac{19}{a^3(ax-1)^2}-\frac{25}{a^3(ax-1)^3}-\frac{16}{a^3(ax-1)^4}-\frac{4}{a^3(ax-1)^5}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\left(-\frac{19}{a^4(1-ax)}+\frac{25}{2a^4(1-ax)^2}-\frac{16}{3a^4(1-ax)^3}+\frac{1}{a^4(1-ax)^4}-\frac{7\log(1-ax)}{a^4}-\frac{x}{a^3}\right)}{c^3}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

output
```
-((a^3*(-(x/a^3) + 1/(a^4*(1 - a*x)^4) - 16/(3*a^4*(1 - a*x)^3) + 25/(2*a^
4*(1 - a*x)^2) - 19/(a^4*(1 - a*x)) - (7*Log[1 - a*x])/a^4))/c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.75

method	result
risch	$\frac{x}{c^{3}}+\frac{-19a^{2}c^{3}x^{3}+\frac{89a\,c^{3}x^{2}}{2}-\frac{112c^{3}x}{3}+\frac{65c^{3}}{6a}}{c^{6}\left(ax-1\right)^{4}}+\frac{7\ln\left(ax-1\right)}{a\,c^{3}}$
default	$\frac{a^{3}\left(\frac{x}{a^{3}}-\frac{19}{a^{4}\left(ax-1\right)}+\frac{7\ln\left(ax-1\right)}{a^{4}}-\frac{1}{a^{4}\left(ax-1\right)^{4}}-\frac{25}{2a^{4}\left(ax-1\right)^{2}}-\frac{16}{3a^{4}\left(ax-1\right)^{3}}\right)}{c^{3}}$
norman	$\frac{\frac{a^{4}x^{5}}{c}+\frac{7x}{c}-\frac{49a\,x^{2}}{2c}+\frac{91a^{2}x^{3}}{3c}-\frac{89a^{3}x^{4}}{6c}}{\left(ax-1\right)^{4}c^{2}}+\frac{7\ln\left(ax-1\right)}{a\,c^{3}}$
parallelrisch	$\frac{6a^{5}x^{5}+42\ln\left(ax-1\right)x^{4}a^{4}-89a^{4}x^{4}-168a^{3}\ln\left(ax-1\right)x^{3}+182a^{3}x^{3}+252a^{2}\ln\left(ax-1\right)x^{2}-147a^{2}x^{2}-168a\ln\left(ax-1\right)x+42ax+42}{6\left(ax-1\right)^{4}c^{3}a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
x/c^3+(-19*a^2*c^3*x^3+89/2*a*c^3*x^2-112/3*c^3*x+65/6*c^3/a)/c^6/(a*x-1)^
4+7/a/c^3*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.42

$$\int\frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3}}\,dx=\frac{6\,a^{5}x^{5}-24\,a^{4}x^{4}-78\,a^{3}x^{3}+243\,a^{2}x^{2}-218\,ax+42\,(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1)\log\left(ax-1\right)+}{6\,(a^{5}c^{3}x^{4}-4\,a^{4}c^{3}x^{3}+6\,a^{3}c^{3}x^{2}-4\,a^{2}c^{3}x+ac^{3})}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
1/6*(6*a^5*x^5 - 24*a^4*x^4 - 78*a^3*x^3 + 243*a^2*x^2 - 218*a*x + 42*(a^4
*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 65)/(a^5*c^3*x^4
- 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{-114a^3x^3+267a^2x^2-224ax+65}{6a^5c^3x^4-24a^4c^3x^3+36a^3c^3x^2-24a^2c^3x+6ac^3}+\frac{x}{c^3}+\frac{7\log(ax-1)}{ac^3}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**3,x)
```

output
```
(-114*a**3*x**3 + 267*a**2*x**2 - 224*a*x + 65)/(6*a**5*c**3*x**4 - 24*a**
4*c**3*x**3 + 36*a**3*c**3*x**2 - 24*a**2*c**3*x + 6*a*c**3) + x/c**3 + 7*
log(a*x - 1)/(a*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{114\,a^3x^3-267\,a^2x^2+224\,ax-65}{6\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}+\frac{x}{c^3}+\frac{7\,\log(ax-1)}{ac^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
-1/6*(114*a^3*x^3 - 267*a^2*x^2 + 224*a*x - 65)/(a^5*c^3*x^4 - 4*a^4*c^3*x
^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + x/c^3 + 7*log(a*x - 1)/(a*c^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.22

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{ax-1}{ac^3}-\frac{7\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^3}-\frac{\frac{114\,a^7c^9}{ax-1}+\frac{75\,a^7c^9}{(ax-1)^2}+\frac{32\,a^7c^9}{(ax-1)^3}+\frac{6\,a^7c^9}{(ax-1)^4}}{6\,a^8c^{12}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="giac")
```

output
```
(a*x - 1)/(a*c^3) - 7*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^3) - 1/6
*(114*a^7*c^9/(a*x - 1) + 75*a^7*c^9/(a*x - 1)^2 + 32*a^7*c^9/(a*x - 1)^3
+ 6*a^7*c^9/(a*x - 1)^4)/(a^8*c^12)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{x}{c^3}-\frac{\frac{112\,x}{3}-\frac{89\,a\,x^2}{2}-\frac{65}{6\,a}+19\,a^2\,x^3}{a^4\,c^3\,x^4-4\,a^3\,c^3\,x^3+6\,a^2\,c^3\,x^2-4\,a\,c^3\,x+c^3}+\frac{7\,\ln(a\,x-1)}{a\,c^3}$$

input
```
int((a*x + 1)^2/((c - c/(a*x))^3*(a*x - 1)^2),x)
```

output
```
x/c^3 - ((112*x)/3 - (89*a*x^2)/2 - 65/(6*a) + 19*a^2*x^3)/(c^3 + 6*a^2*c^
3*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x) + (7*log(a*x - 1))/(a*c^3
)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.45

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$
$$=\frac{84\log(ax-1)\,a^4x^4-336\log(ax-1)\,a^3x^3+504\log(ax-1)\,a^2x^2-336\log(ax-1)\,ax+84\log(ax-1}{12a\,c^3\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x)
```

output
```
(84*log(a*x - 1)*a**4*x**4 - 336*log(a*x - 1)*a**3*x**3 + 504*log(a*x - 1)
*a**2*x**2 - 336*log(a*x - 1)*a*x + 84*log(a*x - 1) + 12*a**5*x**5 - 87*a*
*4*x**4 + 252*a**2*x**2 - 280*a*x + 91)/(12*a*c**3*(a**4*x**4 - 4*a**3*x**
3 + 6*a**2*x**2 - 4*a*x + 1))
```

3.427 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4}+\frac{4}{5ac^4(1-ax)^5}-\frac{5}{ac^4(1-ax)^4}+\frac{41}{3ac^4(1-ax)^3}-\frac{22}{ac^4(1-ax)^2}+\frac{26}{ac^4(1-ax)}+\frac{8\log(1-ax)}{ac^4}$$

output
```
x/c^4+4/5/a/c^4/(-a*x+1)^5-5/a/c^4/(-a*x+1)^4+41/3/a/c^4/(-a*x+1)^3-22/a/c
^4/(-a*x+1)^2+26/a/c^4/(-a*x+1)+8*ln(-a*x+1)/a/c^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.75

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{-202+890ax-1480a^2x^2+1080a^3x^3-240a^4x^4-75a^5x^5+15a^6x^6+120(-1+ax)^5\log(1-ax)}{15ac^4(-1+ax)^5}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

output
```
(-202 + 890*a*x - 1480*a^2*x^2 + 1080*a^3*x^3 - 240*a^4*x^4 - 75*a^5*x^5 +
 15*a^6*x^6 + 120*(-1 + a*x)^5*Log[1 - a*x])/(15*a*c^4*(-1 + a*x)^5)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{a^4e^{4\text{arctanh}(ax)}}{c^4\left(a-\frac{1}{x}\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{a^4\int \frac{e^{4\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^4}dx}{c^4}$$

$$\Big\downarrow \text{6681}$$

$$\frac{a^4\int \frac{e^{4\text{arctanh}(ax)}x^4}{(1-ax)^4}dx}{c^4}$$

$$\Big\downarrow \text{6679}$$

$$\frac{a^4\int \frac{x^4(ax+1)^2}{(1-ax)^6}dx}{c^4}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^4\int \left(\frac{1}{a^4}+\frac{8}{a^4(ax-1)}+\frac{26}{a^4(ax-1)^2}+\frac{44}{a^4(ax-1)^3}+\frac{41}{a^4(ax-1)^4}+\frac{20}{a^4(ax-1)^5}+\frac{4}{a^4(ax-1)^6}\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4\left(\frac{26}{a^5(1-ax)}-\frac{22}{a^5(1-ax)^2}+\frac{41}{3a^5(1-ax)^3}-\frac{5}{a^5(1-ax)^4}+\frac{4}{5a^5(1-ax)^5}+\frac{8\log(1-ax)}{a^5}+\frac{x}{a^4}\right)}{c^4}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

output
```
(a^4*(x/a^4 + 4/(5*a^5*(1 - a*x)^5) - 5/(a^5*(1 - a*x)^4) + 41/(3*a^5*(1 -
 a*x)^3) - 22/(a^5*(1 - a*x)^2) + 26/(a^5*(1 - a*x)) + (8*Log[1 - a*x])/a^
5))/c^4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

method	result
risch	$\frac{x}{c^4}+\frac{-26a^3c^4x^4+82a^2c^4x^3-\frac{311a\,c^4x^2}{3}+\frac{181c^4x}{3}-\frac{202c^4}{15a}}{c^8(ax-1)^5}+\frac{8\ln(ax-1)}{a\,c^4}$
default	$\frac{a^4\left(\frac{x}{a^4}-\frac{26}{a^5(ax-1)}+\frac{8\ln(ax-1)}{a^5}-\frac{5}{a^5(ax-1)^4}-\frac{4}{5a^5(ax-1)^5}-\frac{22}{a^5(ax-1)^2}-\frac{41}{3a^5(ax-1)^3}\right)}{c^4}$
norman	$\frac{\frac{a^5x^6}{c}-\frac{8x}{c}+\frac{36a\,x^2}{c}-\frac{188a^2x^3}{3c}+\frac{154a^3x^4}{3c}-\frac{277a^4x^5}{15c}}{(ax-1)^5c^3}+\frac{8\ln(ax-1)}{a\,c^4}$
parallelrisch	$\frac{15x^6a^6+120\ln(ax-1)x^5a^5-277a^5x^5-600\ln(ax-1)x^4a^4+770a^4x^4+1200a^3\ln(ax-1)x^3-940a^3x^3-1200a^2\ln(ax-1)x^2+5}{15(ax-1)^5c^4a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
x/c^4+(-26*a^3*c^4*x^4+82*a^2*c^4*x^3-311/3*a*c^4*x^2+181/3*c^4*x-202/15*c
^4/a)/c^8/(a*x-1)^5+8/a/c^4*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.47

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$=\frac{15\,a^6x^6-75\,a^5x^5-240\,a^4x^4+1080\,a^3x^3-1480\,a^2x^2+890\,ax+120\,(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x}{15\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/15*(15*a^6*x^6 - 75*a^5*x^5 - 240*a^4*x^4 + 1080*a^3*x^3 - 1480*a^2*x^2
+ 890*a*x + 120*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1
)*log(a*x - 1) - 202)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a
^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{-390a^4x^4+1230a^3x^3-1555a^2x^2+905ax-202}{15a^6c^4x^5-75a^5c^4x^4+150a^4c^4x^3-150a^3c^4x^2+75a^2c^4x-15ac^4} + \frac{x}{c^4} + \frac{8\log(ax-1)}{ac^4}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**4,x)
```

output
```
(-390*a**4*x**4 + 1230*a**3*x**3 - 1555*a**2*x**2 + 905*a*x - 202)/(15*a**
6*c**4*x**5 - 75*a**5*c**4*x**4 + 150*a**4*c**4*x**3 - 150*a**3*c**4*x**2
+ 75*a**2*c**4*x - 15*a*c**4) + x/c**4 + 8*log(a*x - 1)/(a*c**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{390\,a^4x^4-1230\,a^3x^3+1555\,a^2x^2-905\,ax+202}{15\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)} + \frac{x}{c^4} + \frac{8\,\log(ax-1)}{ac^4}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
-1/15*(390*a^4*x^4 - 1230*a^3*x^3 + 1555*a^2*x^2 - 905*a*x + 202)/(a^6*c^4
*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c
^4) + x/c^4 + 8*log(a*x - 1)/(a*c^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.18

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{ax-1}{ac^4} - \frac{8\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^4} - \frac{\frac{390\,a^9c^{16}}{ax-1} + \frac{330\,a^9c^{16}}{(ax-1)^2} + \frac{205\,a^9c^{16}}{(ax-1)^3} + \frac{75\,a^9c^{16}}{(ax-1)^4} + \frac{12\,a^9c^{16}}{(ax-1)^5}}{15\,a^{10}c^{20}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="giac")
```

output
```
(a*x - 1)/(a*c^4) - 8*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^4) - 1/1
5*(390*a^9*c^16/(a*x - 1) + 330*a^9*c^16/(a*x - 1)^2 + 205*a^9*c^16/(a*x -
 1)^3 + 75*a^9*c^16/(a*x - 1)^4 + 12*a^9*c^16/(a*x - 1)^5)/(a^10*c^20)
```

Mupad [B] (verification not implemented)

Time = 13.67 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4} + \frac{\frac{311\,a\,x^2}{3} - \frac{181\,x}{3} + \frac{202}{15\,a} - 82\,a^2\,x^3 + 26\,a^3\,x^4}{-a^5\,c^4\,x^5 + 5\,a^4\,c^4\,x^4 - 10\,a^3\,c^4\,x^3 + 10\,a^2\,c^4\,x^2 - 5\,a\,c^4\,x + c^4} + \frac{8\,\ln(a\,x-1)}{a\,c^4}$$

input
```
int((a*x + 1)^2/((c - c/(a*x))^4*(a*x - 1)^2),x)
```

output
```
x/c^4 + ((311*a*x^2)/3 - (181*x)/3 + 202/(15*a) - 82*a^2*x^3 + 26*a^3*x^4)
/(c^4 + 10*a^2*c^4*x^2 - 10*a^3*c^4*x^3 + 5*a^4*c^4*x^4 - a^5*c^4*x^5 - 5*
a*c^4*x) + (8*log(a*x - 1))/(a*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.51

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{120\log(ax-1)\,a^5x^5-600\log(ax-1)\,a^4x^4+1200\log(ax-1)\,a^3x^3-1200\log(ax-1)\,a^2x^2+600\log(}{15a\,c^4\left(a^5x^5-5a^4x^4+10a^3x^3-\right.}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x)
```

output
```
(120*log(a*x - 1)*a**5*x**5 - 600*log(a*x - 1)*a**4*x**4 + 1200*log(a*x -
1)*a**3*x**3 - 1200*log(a*x - 1)*a**2*x**2 + 600*log(a*x - 1)*a*x - 120*lo
g(a*x - 1) + 15*a**6*x**6 - 123*a**5*x**5 + 600*a**3*x**3 - 1000*a**2*x**2
 + 650*a*x - 154)/(15*a*c**4*(a**5*x**5 - 5*a**4*x**4 + 10*a**3*x**3 - 10*
a**2*x**2 + 5*a*x - 1))
```

3.428 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 130

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=-\frac{11c^4\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}+\frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^4\sqrt{1-\frac{1}{a^2x^2}}x-\frac{25c^4\csc^{-1}(ax)}{2a}-\frac{5c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-11*c^4*(1-1/a^2/x^2)^(1/2)/a+1/3*c^4*(1-1/a^2/x^2)^(3/2)/a+5/2*c^4*(1-1/a
^2/x^2)^(1/2)/a^2/x+c^4*(1-1/a^2/x^2)^(1/2)*x-25/2*c^4*arccsc(a*x)/a-5*c^4
*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.35

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{c^4\left(2-15ax+62a^2x^2+9a^3x^3-64a^4x^4+6a^5x^5+90a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^4\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-30a^4\sqrt{1-\frac{1}{a^2x}}\right.}{6a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}$$

input
```
Integrate[(c - c/(a*x))^4/E^ArcCoth[a*x],x]
```

output
```
(c^4*(2 - 15*a*x + 62*a^2*x^2 + 9*a^3*x^3 - 64*a^4*x^4 + 6*a^5*x^5 + 90*a^
4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 30*a^4*Sqr
t[1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] - 30*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*
ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(6*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {6731, 27, 540, 2340, 25, 2340, 25, 2340, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^4 e^{-\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{ 6731}$$
$$-\frac{\int \frac{c^5\left(a-\frac{1}{x}\right)^5x^2}{a^5\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$
$$\Big\downarrow \text{ 27}$$
$$-\frac{c^4\int \frac{\left(a-\frac{1}{x}\right)^5x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^5}$$

$$\Big\downarrow \ 540$$

$$-\frac{c^4\left(a^5 x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{\left(5a^4-\frac{10a^3}{x}+\frac{10a^2}{x^2}-\frac{5a}{x^3}+\frac{1}{x^4}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a^5}$$

$$\Big\downarrow \ 2340$$

$$-\frac{c^4\left(\frac{1}{3}a^2\int -\frac{\left(15a^2-\frac{30a}{x}+\frac{32}{x^2}-\frac{15}{x^3a}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^5}$$

$$\Big\downarrow \ 25$$

$$-\frac{c^4\left(-\frac{1}{3}a^2\int \frac{\left(15a^2-\frac{30a}{x}+\frac{32}{x^2}-\frac{15}{x^3a}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^5}$$

$$\Big\downarrow \ 2340$$

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^2\int -\frac{\left(30-\frac{75}{ax}+\frac{64}{a^2x^2}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^5}$$

$$\Big\downarrow \ 25$$

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\int \frac{\left(30-\frac{75}{ax}+\frac{64}{a^2x^2}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^5}$$

$$\Big\downarrow \ 2340$$

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(a^2\left(-\int -\frac{15\left(2a-\frac{5}{x}\right)x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-64\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^5}$$

$$\Big\downarrow \ 27$$

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{15\int \frac{\left(2a-\frac{5}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-64\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^5}$$

$$\Big\downarrow \ 538$$

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{15\left(2a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-5\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a}-64\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\Big(-\cdots}{a^5}$$

$\Big\downarrow$ 223

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{15\left(2a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-5a\arcsin\left(\frac{1}{ax}\right)\right)}{a}-64\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\Big(-\sqrt{\cdots}}{a^5}$$

$\Big\downarrow$ 243

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{15\left(a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-5a\arcsin\left(\frac{1}{ax}\right)\right)}{a}-64\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\Big(-\sqrt{\cdots}}{a^5}$$

$\Big\downarrow$ 73

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{15\left(-2a^3\int\frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-5a\arcsin\left(\frac{1}{ax}\right)\right)}{a}-64\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}\cdots}{a^5}$$

$\Big\downarrow$ 221

$$-\frac{c^4\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{15\left(-2a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-5a\arcsin\left(\frac{1}{ax}\right)\right)}{a}-64\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{15a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\Big(\cdots}{a^5}$$

input

```
Int[(c - c/(a*x))^4/E^ArcCoth[a*x],x]
```

output
```
-((c^4*((a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2) - a^5*Sqrt[1 - 1/(a^2*x^2)]*x
- (a^2*((15*a*Sqrt[1 - 1/(a^2*x^2)])/(2*x) + (a^2*(-64*Sqrt[1 - 1/(a^2*x^2
)] + (15*(-5*a*ArcSin[1/(a*x)] - 2*a*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/a))/
2))/3))/a^5)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{(ax+1)(64a^2x^2-15ax+2)c^4\sqrt{\frac{ax-1}{ax+1}}}{6x^3a^4}+\frac{\left(-\frac{25a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}-\frac{5a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a^3\sqrt{(ax-1)(ax+1)}\right)c^4\sqrt{\frac{ax-}{ax+}}}{a^4(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^4\left(-66\sqrt{a^2}\sqrt{a^2x^2-1}\,a^4x^4+66\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^2x^2-75\sqrt{a^2}\sqrt{a^2x^2-1}\,a^3x^3-75a^3\sqrt{a^2}\,x^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+6\right.}{6\sqrt{(a}}$

input
```
int((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(a*x+1)*(64*a^2*x^2-15*a*x+2)/x^3*c^4/a^4*((a*x-1)/(a*x+1))^(1/2)+(-2
5/2*a^3*arctan(1/(a^2*x^2-1)^(1/2))-5*a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)
^(1/2))/(a^2)^(1/2)+a^3*((a*x-1)*(a*x+1))^(1/2))*c^4/a^4/(a*x-1)*((a*x-1)/
(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.20

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{150\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-30\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+30\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(6\,a^4c^4x^4-5}{6\,a^4x^3}$$

input
```
integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
1/6*(150*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 30*a^3*c^4*x^3*lo
g(sqrt((a*x - 1)/(a*x + 1)) + 1) + 30*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) + (6*a^4*c^4*x^4 - 58*a^3*c^4*x^3 - 49*a^2*c^4*x^2 + 13*a*c^4*x
 - 2*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\left(\int a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\frac{4a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^3}\right)dx+\int\frac{6a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx+\int\left(-\frac{4}{}\right.}{a^4}$$

input
```
integrate((c-c/a/x)**4*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c**4*(Integral(a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/x**4, x) + Integral(-4*a*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/x**3, x) + Integral(6*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))
/x**2, x) + Integral(-4*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x, x))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.72

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$=\frac{1}{3}\left(\frac{75\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{87\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+61\,c^4\left(\cdots\right.}{\frac{2\,(ax-1)a^2}{ax+1}-\cdots}\right.$$

input
```
integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
1/3*(75*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 15*c^4*log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^2 + 15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 +
(87*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 61*c^4*((a*x - 1)/(a*x + 1))^(5/2) -
 55*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 45*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2
*(a*x - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2
/(a*x + 1)^4 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(114) = 228.

Time = 0.17 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.04

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx=\frac{25\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}(ax+1)}{a}$$
$$+\frac{5\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c^4\mathrm{sgn}(ax+1)}{a}$$
$$-\frac{15\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^4|a|\mathrm{sgn}(ax+1)+60\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^4\mathrm{sgn}(ax+1)+132\left(x|a|-\sqrt{a^2x\cdots\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2\cdots\right.}$$

input
```
integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
25*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 5*c^4*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*
c^4*sgn(a*x + 1)/a - 1/3*(15*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*s
gn(a*x + 1) + 60*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) + 132
*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^4*sgn(a*x + 1) - 15*(x*abs(a) - sqrt
(a^2*x^2 - 1))*c^4*abs(a)*sgn(a*x + 1) + 64*a*c^4*sgn(a*x + 1))/(((x*abs(a
) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 13.66 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.42

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{25\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a} - \frac{15\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{55\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-\frac{61\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-29\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}} - \frac{10\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(25*c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (15*c^4*((a*x - 1)/(a*x + 1
))^(1/2) + (55*c^4*((a*x - 1)/(a*x + 1))^(3/2))/3 - (61*c^4*((a*x - 1)/(a*
x + 1))^(5/2))/3 - 29*c^4*((a*x - 1)/(a*x + 1))^(7/2))/(a + (2*a*(a*x - 1)
)/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x - 1)^4)/(a*x + 1)^4)
 - (10*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.25

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$= \frac{c^4\left(150atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^3x^3-150atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^3x^3+6\sqrt{ax+1}\,\sqrt{ax}\right.}{}$$

input
```
int((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(c**4*(150*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**3*x**3 - 150*atan(sq
rt(a*x - 1) + sqrt(a*x + 1) + 1)*a**3*x**3 + 6*sqrt(a*x + 1)*sqrt(a*x - 1)
*a**3*x**3 - 64*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 + 15*sqrt(a*x + 1)*s
qrt(a*x - 1)*a*x - 2*sqrt(a*x + 1)*sqrt(a*x - 1) - 60*log((sqrt(a*x - 1) +
 sqrt(a*x + 1))/sqrt(2))*a**3*x**3 + 24*a**3*x**3))/(6*a**4*x**3)
```

3.429 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 106

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{13c^3\csc^{-1}(ax)}{2a}-\frac{4c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-4*c^3*(1-1/a^2/x^2)^(1/2)/a+1/2*c^3*(1-1/a^2/x^2)^(1/2)/a^2/x+c^3*(1-1/a^
2/x^2)^(1/2)*x-13/2*c^3*arccsc(a*x)/a-4*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.58

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$$
$$=\frac{c^3\left(-1+8ax-a^2x^2-8a^3x^3+2a^4x^4+10a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-8a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3\arcsin\left(\frac{1}{a\right.}\right.}{2a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}$$

input
```
Integrate[(c - c/(a*x))^3/E^ArcCoth[a*x],x]
```

output
```
(c^3*(-1 + 8*a*x - a^2*x^2 - 8*a^3*x^3 + 2*a^4*x^4 + 10*a^3*Sqrt[1 - 1/(a^
2*x^2)]*x^3*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 8*a^3*Sqrt[1 - 1/(a^2*x^2)
]*x^3*ArcSin[1/(a*x)] - 8*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^3*ArcTanh[Sqrt[1 - 1
/(a^2*x^2)]]))/(2*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6731, 27, 540, 2340, 25, 2340, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^3 e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{c^4\left(a-\frac{1}{x}\right)^4x^2}{a^4\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\int \frac{\left(a-\frac{1}{x}\right)^4x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^4}$$

$$\Big\downarrow \text{540}$$

$$-\frac{c^3\left(a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{\left(4a^3-\frac{6a^2}{x}+\frac{4a}{x^2}-\frac{1}{x^3}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a^4}$$

$$\Big\downarrow \text{2340}$$

$$-\frac{c^3\left(\frac{1}{2}a^2\int -\frac{\left(8a-\frac{13}{x}+\frac{8}{x^2a}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\int \frac{\left(8a-\frac{13}{x}+\frac{8}{x^2a}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{2340}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\left(a^2\left(-\int -\frac{\left(8a-\frac{13}{x}\right)x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-8a\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\left(a^2\int \frac{\left(8a-\frac{13}{x}\right)x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8a\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\left(\int \frac{\left(8a-\frac{13}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8a\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{538}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\left(8a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-13\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8a\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{223}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\left(8a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8a\sqrt{1-\frac{1}{a^2x^2}}-13a\arcsin\left(\frac{1}{ax}\right)\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{243}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\left(4a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-8a\sqrt{1-\frac{1}{a^2x^2}}-13a\arcsin\left(\frac{1}{ax}\right)\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\left(-8a^3\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-8a\sqrt{1-\frac{1}{a^2x^2}}-13a\arcsin\left(\frac{1}{ax}\right)\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c^3\left(-\frac{1}{2}a^2\left(-8a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-8a\sqrt{1-\frac{1}{a^2x^2}}-13a\arcsin\left(\frac{1}{ax}\right)\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^4}$$

input
```
Int[(c - c/(a*x))^3/E^ArcCoth[a*x],x]
```

output
```
-((c^3*(-1/2*(a^2*Sqrt[1 - 1/(a^2*x^2)])/x - a^4*Sqrt[1 - 1/(a^2*x^2)]*x -
 (a^2*(-8*a*Sqrt[1 - 1/(a^2*x^2)] - 13*a*ArcSin[1/(a*x)] - 8*a*ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]]))/2))/a^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{(ax+1)(8ax-1)c^3\sqrt{\frac{ax-1}{ax+1}}}{2x^2a^3}+\frac{\left(-\frac{13a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}-\frac{4a^3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a^2\sqrt{(ax-1)(ax+1)}\right)c^3\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1}}{a^3(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)c^3\left(-8\sqrt{a^2}\sqrt{a^2x^2-1}a^3x^3+16\sqrt{(ax-1)(ax+1)}\sqrt{a^2}a^2x^2+8\sqrt{a^2}\left(a^2x^2-1\right)^{\frac{3}{2}}ax-13\sqrt{a^2}\sqrt{a^2x^2-1}a^2x^2-13a^2\sqrt{a}}{2\sqrt{(ax-1)(ax+1)}a^3}$

input
```
int((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x+1)*(8*a*x-1)/x^2*c^3/a^3*((a*x-1)/(a*x+1))^(1/2)+(-13/2*a^2*arct
an(1/(a^2*x^2-1)^(1/2))-4*a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2
)^(1/2)+a^2*((a*x-1)*(a*x+1))^(1/2))*c^3/a^3/(a*x-1)*((a*x-1)/(a*x+1))^(1/
2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.35

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$$
$$=\frac{26\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-8\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+8\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^3c^3x^3-6\,a^2c\right.}{2\,a^3x^2}$$

input
```
integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(26*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - 8*a^2*c^3*x^2*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x + 1
)) - 1) + (2*a^3*c^3*x^3 - 6*a^2*c^3*x^2 - 7*a*c^3*x + c^3)*sqrt((a*x - 1)
/(a*x + 1)))/(a^3*x^2)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = \frac{c^3\left(\int a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^3}\right)dx+\int\frac{3a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx+\int\left(-\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right)dx\right)}{a^3}$$

input
```
integrate((c-c/a/x)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c**3*(Integral(a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/x**3, x) + Integral(3*a*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/x**2, x) + Integral(-3*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
)/x, x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(94) = 188.

Time = 0.13 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.90

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = \left(\frac{13\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{4\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{4\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(a}{(}}\right.$$

input
```
integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
(13*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 4*c^3*log(sqrt((a*x - 1)/(
a*x + 1)) + 1)/a^2 + 4*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (11*c^
3*((a*x - 1)/(a*x + 1))^(5/2) + 2*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*
sqrt((a*x - 1)/(a*x + 1)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x
 + 1)^2 - (a*x - 1)^3*a^2/(a*x + 1)^3 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(94) = 188$.

Time = 0.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.19

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = \frac{13\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}(ax+1)}{a} + \frac{4\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c^3\mathrm{sgn}(ax+1)}{a} - \frac{\left(x|a|-\sqrt{a^2x^2-1}\right)^3 c^3|a|\mathrm{sgn}(ax+1)+8\left(x|a|-\sqrt{a^2x^2-1}\right)^2 ac^3\mathrm{sgn}(ax+1)-\left(x|a|-\sqrt{a^2x^2-1}\right)c}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^2 a|a|}$$

input
```
integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
13*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 4*c^3*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*
c^3*sgn(a*x + 1)/a - ((x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a)*sgn(a*x
+ 1) + 8*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x + 1) - (x*abs(a) -
 sqrt(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) + 8*a*c^3*sgn(a*x + 1))/(((x*a
bs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^2*a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.54

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = \frac{2\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-5\,c^3\sqrt{\frac{a\,x-1}{a\,x+1}}+11\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a+\frac{a\,(a\,x-1)}{a\,x+1}-\frac{a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a\,(a\,x-1)^3}{(a\,x+1)^3}} + \frac{13\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a} - \frac{8\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*((a*x - 1)/(a*x + 1))^(1/2) + 1
1*c^3*((a*x - 1)/(a*x + 1))^(5/2))/(a + (a*(a*x - 1))/(a*x + 1) - (a*(a*x
- 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (13*c^3*atan(((a*x -
1)/(a*x + 1))^(1/2)))/a - (8*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$$
$$=\frac{c^3\left(26atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^2x^2-26atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^2x^2+2\sqrt{ax+1}\,\sqrt{ax-\right.}{2a^3x^2}$$

input
```
int((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(c**3*(26*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**2*x**2 - 26*atan(sqrt
(a*x - 1) + sqrt(a*x + 1) + 1)*a**2*x**2 + 2*sqrt(a*x + 1)*sqrt(a*x - 1)*a
**2*x**2 - 8*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + sqrt(a*x + 1)*sqrt(a*x - 1)
 - 16*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**2*x**2))/(2*a**3*x**
2)
```

3.430 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=-\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}+c^2\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{3c^2\csc^{-1}(ax)}{a}-\frac{3c^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-c^2*(1-1/a^2/x^2)^(1/2)/a+c^2*(1-1/a^2/x^2)^(1/2)*x-3*c^2*arccsc(a*x)/a-3
*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}(-1+ax)-3\arcsin\left(\frac{1}{ax}\right)-3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a}$$

input
```
Integrate[(c - c/(a*x))^2/E^ArcCoth[a*x],x]
```

output
```
(c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) - 3*ArcSin[1/(a*x)] - 3*ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]]))/a
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {6731, 27, 540, 2340, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^2 e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{c^3\left(a-\frac{1}{x}\right)^3x^2}{a^3\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\int \frac{\left(a-\frac{1}{x}\right)^3x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^3}$$

$$\Big\downarrow \text{540}$$

$$-\frac{c^2\left(a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{\left(3a^2-\frac{3a}{x}+\frac{1}{x^2}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a^3}$$

$$\Big\downarrow \text{2340}$$

$$-\frac{c^2\left(a^2\int -\frac{3\left(a-\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\left(-3a\int \frac{\left(a-\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^3}$$

$$\Big\downarrow \text{538}$$

$$-\frac{c^2\left(-3a\left(a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^3}$$

$$\Big\downarrow \text{223}$$

$$-\frac{c^2\left(-3a\left(a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-a\arcsin\left(\frac{1}{ax}\right)\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^3}$$

$$\Big\downarrow \text{243}$$

$$-\frac{c^2\left(-3a\left(\frac{1}{2}a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-a\arcsin\left(\frac{1}{ax}\right)\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c^2\left(-3a\left(a^3\left(-\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-a\arcsin\left(\frac{1}{ax}\right)\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c^2\left(-3a\left(-a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-a\arcsin\left(\frac{1}{ax}\right)\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^3}$$

input
```
Int[(c - c/(a*x))^2/E^ArcCoth[a*x],x]
```

output
```
-((c^2*(a^2*Sqrt[1 - 1/(a^2*x^2)] - a^3*Sqrt[1 - 1/(a^2*x^2)]*x - 3*a*(-(a
*ArcSin[1/(a*x)]) - a*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])))/a^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.74

method	result
risch	$-\frac{(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{x\,a^2}+\frac{\left(\sqrt{(ax-1)(ax+1)}-3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-\frac{3a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{a(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^2\left(-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^2x^2+4\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,ax+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-3\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{}}{\sqrt{a^2}}\right.\right.}{\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

input
```
int((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*x+1)/x*c^2/a^2*((a*x-1)/(a*x+1))^(1/2)+1/a*(((a*x-1)*(a*x+1))^(1/2)-3*
arctan(1/(a^2*x^2-1)^(1/2))-3*a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a
^2)^(1/2))*c^2/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = \frac{6\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-3\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2c^2x^2-c^2)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

input
```
integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
(6*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - 3*a*c^2*x*log(sqrt((a*x - 1
)/(a*x + 1)) + 1) + 3*a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c^
2*x^2 - c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = \frac{c^2\left(\int a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx+\int\left(-\frac{2a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right)dx\right)}{a^2}$$

input
```
integrate((c-c/a/x)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c**2*(Integral(a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/x**2, x) + Integral(-2*a*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/x, x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.64

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = -\left(\frac{4\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

input
```
integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-(4*c^2*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) -
6*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^2*log(sqrt((a*x - 1)/(a*
x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.69

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx = \frac{6\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a} + \frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c^2\operatorname{sgn}(ax+1)}{a} - \frac{2\,c^2\operatorname{sgn}(ax+1)}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|}$$

input
```
integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
6*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^2*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c
^2*sgn(a*x + 1)/a - 2*c^2*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2
+ 1)*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{4\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a-\frac{a\,(a\,x-1)^2}{(a\,x+1)^2}}+\frac{6\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(4*c^2*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) + (6
*c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6*c^2*atanh(((a*x - 1)/(a*x +
 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.36

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$$
$$=\frac{c^2\Big(6atan\big(\sqrt{ax-1}+\sqrt{ax+1}-1\big)\,ax-6atan\big(\sqrt{ax-1}+\sqrt{ax+1}+1\big)\,ax+\sqrt{ax+1}\,\sqrt{ax-1}\,ax-}{a^2x}$$

input
```
int((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(c**2*(6*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a*x - 6*atan(sqrt(a*x - 1
) + sqrt(a*x + 1) + 1)*a*x + sqrt(a*x + 1)*sqrt(a*x - 1)*a*x - sqrt(a*x +
1)*sqrt(a*x - 1) - 6*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x - a*
x))/(a**2*x)
```

3.431 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{c\csc^{-1}(ax)}{a}-\frac{2c\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
c*(1-1/a^2/x^2)^(1/2)*x-c*arccsc(a*x)/a-2*c*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}x-2\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-2\arcsin\left(\frac{1}{ax}\right)-2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a}$$

input
```
Integrate[(c - c/(a*x))/E^ArcCoth[a*x],x]
```

output
```
(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x - 2*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*Ar
cSin[1/(a*x)] - 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/a
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6731, 27, 540, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right) e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \ 6731$$

$$-\frac{\int \frac{c^2\left(a-\frac{1}{x}\right)^2x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \ 27$$

$$-\frac{c\int \frac{\left(a-\frac{1}{x}\right)^2x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^2}$$

$$\Big\downarrow \ 540$$

$$-\frac{c\left(a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{\left(2a-\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a^2}$$

$$\Big\downarrow \ 538$$

$$-\frac{c\left(-2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^2}$$

$$\Big\downarrow \ 223$$

$$-\frac{c\left(-2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)+a\arcsin\left(\frac{1}{ax}\right)\right)}{a^2}$$

$$\Big\downarrow \ 243$$

$$-\frac{c\left(-a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)+a\arcsin\left(\frac{1}{ax}\right)\right)}{a^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c\left(2a^3\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-a^2x\sqrt{1-\frac{1}{a^2x^2}}+a\arcsin\left(\frac{1}{ax}\right)\right)}{a^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c\left(2a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)+a\arcsin\left(\frac{1}{ax}\right)\right)}{a^2}$$

input
```
Int[(c - c/(a*x))/E^ArcCoth[a*x],x]
```

output
```
-((c*(-(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + a*ArcSin[1/(a*x)] + 2*a*ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]]))/a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(45) = 90$.

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.80

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c\left(2\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}-\sqrt{a^2x^2-1}\,\sqrt{a^2}-\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}-2a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$	137

input
```
int((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c*(2*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)-(
a^2*x^2-1)^(1/2)*(a^2)^(1/2)-arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)-2*a*l
n((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x-1)*(a*x+
1))^(1/2)/a/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.80

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = \frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

input
```
integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
(2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) - 2*c*log(sqrt((a*x - 1)/(a*x + 1))
 + 1) + 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = \frac{c\left(\int a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right)dx\right)}{a}$$

input
```
integrate((c-c/a/x)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c*(Integral(a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/x, x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(45) = 90$.

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.33

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = -2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - c*arct
an(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a
^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.73

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a} + \frac{2\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c\operatorname{sgn}(ax+1)}{a}$$

input
```
integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 2*c*log(abs(-x*
abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn
(a*x + 1)/a
```

Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{4\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}$$

input
```
int((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (4*c*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a + (2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x +
 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$$
$$=\frac{c\left(2atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)-2atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)+\sqrt{ax+1}\,\sqrt{ax-1}-4\log\left(\frac{\sqrt{ax}}{}\right.\right.}{a}$$

input
```
int((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(c*(2*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1) - 2*atan(sqrt(a*x - 1) + sqr
t(a*x + 1) + 1) + sqrt(a*x + 1)*sqrt(a*x - 1) - 4*log((sqrt(a*x - 1) + sqr
t(a*x + 1))/sqrt(2))))/a
```

3.432 $\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}} dx$

Optimal result

Integrand size = 22, antiderivative size = 19

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}} dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/c
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}} dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x)/c
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6731, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{242}$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x)/c
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

method	result	size
gosper	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}$	28
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}$	28
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}$	28
trager	$\frac{(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{ac}$	30
orering	$\frac{(ax-1)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2x\left(c-\frac{c}{ax}\right)}$	46

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
1/a*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="fricas")
```

output
```
(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a*c)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{a\int \frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax-1}\,dx}{c}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x),x)
```

output
```
a*Integral(x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x - 1), x)/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{2\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="maxima")
```

output
```
-2*a*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="giac")
```

output
```
sqrt(a^2*x^2 - 1)*sgn(a*x + 1)/(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x)),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{\sqrt{ax+1}\,\sqrt{ax-1}}{ac}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x)
```

output
```
(sqrt(a*x + 1)*sqrt(a*x - 1))/(a*c)
```

3.433 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

output
```
-(1-1/a^2/x^2)^(1/2)/c^2/(a-1/x)+(1-1/a^2/x^2)^(1/2)*x/c^2+arctanh((1-1/a^
2/x^2)^(1/2))/a/c^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{-2-ax+a^2x^2+a\sqrt{1-\frac{1}{a^2x^2}}\,x\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^2),x]
```

output
```
(-2 - a*x + a^2*x^2 + a*Sqrt[1 - 1/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]])/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {6731, 27, 564, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx$$

$$\Big\downarrow \; 6731$$

$$-\frac{\int \frac{ax^2}{c\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\int \frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \; 564$$

$$-\frac{a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}-\int -\frac{\left(a+\frac{1}{x}\right)x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{c^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{a\left(\int \frac{\left(a+\frac{1}{x}\right)x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}\right)}{c^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(\frac{\int \frac{\left(a+\frac{1}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}\right)}{c^2}$$

$$\Big\downarrow \; 534$$

$$-\frac{a\left(\frac{\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}}{a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}\right)}{c^2}$$

$$\Big\downarrow \; 243$$

$$-\frac{a\left(\frac{\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}}{a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}\right)}{c^2}$$

$$\Big\downarrow \; 73$$

$$-\frac{a\left(\frac{a^2\left(-\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}}{a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}\right)}{c^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{a\left(\frac{-\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}}{a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}\right)}{c^2}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^2),x]
```

output
```
-((a*(Sqrt[1 - 1/(a^2*x^2)]/(a*(a - x^(-1))) + (-(a*Sqrt[1 - 1/(a^2*x^2)]*
x) - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a^2))/c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(64) = 128$.

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.06

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{a^4\left(x-\frac{1}{a}\right)}\right)a^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c^2(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(2\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2+3\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^2x^2-4\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x-((ax-}{2a\sqrt{(ax-1)(ax+1)}\,c^2(ax-1)}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^2*((a*x-1)/(a*x+1))^(1/2)+(1/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-1/a^4/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2))*
a^2/c^2*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.39

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2x^2-ax-2)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2x-ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x -
1)/(a*x + 1)) - 1) + (a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c
^2*x - a*c^2)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2 \int \frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-2ax+1}\,dx}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**2,x)
```

output
```
a**2*Integral(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 2*a*x +
1), x)/c**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.71

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -a\left(\frac{\frac{3\,(ax-1)}{ax+1}-1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^2\sqrt{\frac{ax-1}{ax+1}}} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
-a*((3*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2) - a^2
*c^2*sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*
c^2) + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{2\,a\,x+4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-4}{2\,a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^2,x)
```

output
```
(2*a*x + 4*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2)
- 4)/(2*a*c^2*((a*x - 1)/(a*x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{4\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)-3\sqrt{ax-1}+2\sqrt{ax+1}\,ax-4\sqrt{ax+1}}{2\sqrt{ax-1}\,a\,c^2}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x)
```

output
```
(4*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) - 3*sqrt(a*x
 - 1) + 2*sqrt(a*x + 1)*a*x - 4*sqrt(a*x + 1))/(2*sqrt(a*x - 1)*a*c**2)
```

3.434 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=-\frac{2}{3c^3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}+\frac{10\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^3}-\frac{\left(7a+\frac{6}{x}\right)x}{3ac^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

output
```
-2/3/c^3/(1-1/a^2/x^2)^(1/2)/(a-1/x)+10/3*(1-1/a^2/x^2)^(1/2)*x/c^3-1/3*(7
*a+6/x)*x/a/c^3/(1-1/a^2/x^2)^(1/2)+2*arctanh((1-1/a^2/x^2)^(1/2))/a/c^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{10-4ax-11a^2x^2+3a^3x^3+6a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^3),x]
```

output
```
(10 - 4*a*x - 11*a^2*x^2 + 3*a^3*x^3 + 6*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a
*x)*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x*(-1
 + a*x))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6731, 27, 570, 532, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{a^2x^2}{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^2\int \frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{ 570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^2x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^2c^3}$$

$$\Big\downarrow \text{ 532}$$

$$-\frac{\frac{2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(3a^2+\frac{6a}{x}+\frac{4}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^2c^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{1}{3}\int \frac{\left(3a^2+\frac{6a}{x}+\frac{4}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^2c^3}$$

$$\Big\downarrow \text{ 2336}$$

$$-\frac{\frac{1}{3}\left(\frac{6a+\frac{7}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{3a\left(a+\frac{2}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^2c^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{1}{3}\left(3a\int \frac{\left(a+\frac{2}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{6a+\frac{7}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^2c^3}$$

$$\Big\downarrow \text{ 534}$$

$$-\frac{\frac{1}{3}\left(3a\left(2\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{6a+\frac{7}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^2c^3}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{\frac{1}{3}\left(3a\left(\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{6a+\frac{7}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^2c^3}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\frac{1}{3}\left(3a\left(-2a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{6a+\frac{7}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^2c^3}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\frac{1}{3}\left(3a\left(-2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{6a+\frac{7}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}}{a^2c^3}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^3),x]
```

output
```
-(((2*(a + x^(-1)))/(3*(1 - 1/(a^2*x^2))^(3/2)) + ((6*a + 7/x)/Sqrt[1 - 1/
(a^2*x^2)] + 3*a*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x) - 2*ArcTanh[Sqrt[1 - 1/(a^2
*x^2)]]))/3)/(a^2*c^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.68

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3}+\frac{\left(-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2 a^2+2a\left(x-\frac{1}{a}\right)}}{3a^6\left(x-\frac{1}{a}\right)^2}-\frac{8\sqrt{\left(x-\frac{1}{a}\right)^2 a^2+2a\left(x-\frac{1}{a}\right)}}{3a^5\left(x-\frac{1}{a}\right)}+\frac{2\ln\left(\frac{a^2 x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^3\sqrt{a^2}}\right)a^3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c^3(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-27\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^3x^3-24\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^4x^3+15\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}ax+81\sqrt{(a}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^3*((a*x-1)/(a*x+1))^(1/2)+(-1/3/a^6/(x-1/a)^2*((x-1/a)^2*a^2
+2*a*(x-1/a))^(1/2)-8/3/a^5/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)+2/a^
3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2))*a^3/c^3*((a*x-1)/(a
*x+1))^(1/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.22

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{6\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-11\,a^2x^2-4\,ax\right.}{3\left(a^3c^3x^2-2\,a^2c^3x+ac^3\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
1/3*(6*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*(a^2*x
^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 11*a^2*x
^2 - 4*a*x + 10)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a
*c^3)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{a^3\int \frac{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-3a^2x^2+3ax-1}\,dx}{c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**3,x)
```

output
```
a**3*Integral(x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - 3*a**2*x
**2 + 3*a*x - 1), x)/c**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{1}{6}\,a\left(\frac{\frac{14\,(ax-1)}{ax+1}-\frac{27\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
1/6*a*((14*(a*x - 1)/(a*x + 1) - 27*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^3*
((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)) + 12*lo
g(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 12*log(sqrt((a*x - 1)/(a*x +
1)) - 1)/(a^2*c^3))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3} - \frac{\frac{14\,(a\,x-1)}{3\,(a\,x+1)} - \frac{9\,(a\,x-1)^2}{(a\,x+1)^2} + \frac{1}{3}}{2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} - 2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^3,x)
```

output
```
(4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^3) - ((14*(a*x - 1))/(3*(a*x +
 1)) - (9*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(
3/2) - 2*a*c^3*((a*x - 1)/(a*x + 1))^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.18

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$
$$= \frac{12\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax - 12\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right) + 3\sqrt{ax-1}\,ax - 3\sqrt{ax-1} + 3\sqrt{ax}}{3\sqrt{ax-1}\,a\,c^3\,(ax-1)}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x)
```

output
```
(12*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x - 12*sq
rt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 3*sqrt(a*x - 1)
*a*x - 3*sqrt(a*x - 1) + 3*sqrt(a*x + 1)*a**2*x**2 - 14*sqrt(a*x + 1)*a*x
+ 10*sqrt(a*x + 1))/(3*sqrt(a*x - 1)*a*c**3*(a*x - 1))
```

3.435 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 143

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx = -\frac{4}{5c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)} - \frac{3\left(5a+\frac{4}{x}\right)}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{12\sqrt{1-\frac{1}{a^2x^2}}x}{5c^4} - \frac{\left(7a+\frac{5}{x}\right)x}{5ac^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

output
```
-4/5/c^4/(1-1/a^2/x^2)^(3/2)/(a-1/x)-3/5*(5*a+4/x)/a^2/c^4/(1-1/a^2/x^2)^(
1/2)+12/5*(1-1/a^2/x^2)^(1/2)*x/c^4-1/5*(7*a+5/x)*x/a/c^4/(1-1/a^2/x^2)^(3
/2)+3*arctanh((1-1/a^2/x^2)^(1/2))/a/c^4
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.73

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx = \frac{-24+33ax+18a^2x^2-34a^3x^3+5a^4x^4+15a\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}x(-1+ax)^2}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^4),x]
```

output
```
(-24 + 33*a*x + 18*a^2*x^2 - 34*a^3*x^3 + 5*a^4*x^4 + 15*a*Sqrt[1 - 1/(a^2
*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(5*a^2*c^4*Sqrt[1 -
1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6731, 27, 570, 532, 25, 2336, 27, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{a^3x^2}{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^3\int \frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{ 570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^3x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}}{a^3c^4}$$

$$\Big\downarrow \text{ 532}$$

$$-\frac{\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}\int -\frac{\left(5a^3+\frac{15a^2}{x}+\frac{16a}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^3c^4}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{1}{5}\int \frac{\left(5a^3+\frac{15a^2}{x}+\frac{16a}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

$$\Big\downarrow \text{ 2336}$$

$$-\frac{\frac{1}{5}\left(\frac{a\left(5a+\frac{7}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{3\left(5a^3+\frac{15a^2}{x}+\frac{14a}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{1}{5}\left(\int \frac{\left(5a^3+\frac{15a^2}{x}+\frac{14a}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{a\left(5a+\frac{7}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

$$\Big\downarrow \text{ 2336}$$

$$-\frac{\frac{1}{5}\left(-\int -\frac{5a^2\left(a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a\left(5a+\frac{7}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a\left(15a+\frac{19}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{1}{5}\left(5a^2\int \frac{\left(a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a\left(5a+\frac{7}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a\left(15a+\frac{19}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

$$\Big\downarrow \text{ 534}$$

$$-\frac{\frac{1}{5}\left(5a^2\left(3\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a\left(5a+\frac{7}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a\left(15a+\frac{19}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{\frac{1}{5}\left(5a^2\left(\frac{3}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a\left(5a+\frac{7}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a\left(15a+\frac{19}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\frac{1}{5}\left(5a^2\left(-3a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a\left(5a+\frac{7}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a\left(15a+\frac{19}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\frac{1}{5}\left(5a^2\left(-3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a\left(5a+\frac{7}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a\left(15a+\frac{19}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{4a\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^3c^4}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^4),x]
```

output
```
-(((4*a*(a + x^(-1)))/(5*(1 - 1/(a^2*x^2))^(5/2)) + ((a*(5*a + 7/x))/(1 -
1/(a^2*x^2))^(3/2) + (a*(15*a + 19/x))/Sqrt[1 - 1/(a^2*x^2)] + 5*a^2*(-(a*
Sqrt[1 - 1/(a^2*x^2)]*x) - 3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/5)/(a^3*c^4)
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.57

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^4}+\frac{\left(-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{5a^8\left(x-\frac{1}{a}\right)^3}-\frac{6\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{5a^7\left(x-\frac{1}{a}\right)^2}-\frac{24\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{5a^6\left(x-\frac{1}{a}\right)}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}\right)a^4 \sqrt{}}{c^4(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-125\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^4x^4-120\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^5x^4+85\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2+50\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^4*((a*x-1)/(a*x+1))^(1/2)+(-1/5/a^8/(x-1/a)^3*((x-1/a)^2*a^2
+2*a*(x-1/a))^(1/2)-6/5/a^7/(x-1/a)^2*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-24
/5/a^6/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)+3/a^4*ln(a^2*x/(a^2)^(1/2
)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2))*a^4/c^4*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*
((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.19

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$=\frac{15\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(5\,a}{5\,(a^4c^4x^3-3\,a^3c^4x^2+3\,a^2c^4x-ac^4)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/5*(15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) -
1) + (5*a^4*x^4 - 34*a^3*x^3 + 18*a^2*x^2 + 33*a*x - 24)*sqrt((a*x - 1)/(a
*x + 1)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4 \int \frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-4a^3x^3+6a^2x^2-4ax+1}\,dx}{c^4}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**4,x)
```

output
```
a**4*Integral(x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 4*a**3*x
**3 + 6*a**2*x**2 - 4*a*x + 1), x)/c**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.07

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{1}{20}\,a\left(\frac{\frac{9\,(ax-1)}{ax+1}+\frac{75\,(ax-1)^2}{(ax+1)^2}-\frac{125\,(ax-1)^3}{(ax+1)^3}+1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
1/20*a*((9*(a*x - 1)/(a*x + 1) + 75*(a*x - 1)^2/(a*x + 1)^2 - 125*(a*x - 1
)^3/(a*x + 1)^3 + 1)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^4*((a*x
- 1)/(a*x + 1))^(5/2)) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) -
 60*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.41

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{c^4|a|} + \frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{ac^4}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="giac")
```

output
```
-3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c^4*abs(a)) + sqr
t(a^2*x^2 - 1)*sgn(a*x + 1)/(a*c^4)
```

Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4} - \frac{\frac{15\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{25\,(a\,x-1)^3}{(a\,x+1)^3} + \frac{9\,(a\,x-1)}{5\,(a\,x+1)} + \frac{1}{5}}{4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2} - 4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^4,x)
```

output
```
(6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^4) - ((15*(a*x - 1)^2)/(a*x +
1)^2 - (25*(a*x - 1)^3)/(a*x + 1)^3 + (9*(a*x - 1))/(5*(a*x + 1)) + 1/5)/(
4*a*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 4*a*c^4*((a*x - 1)/(a*x + 1))^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.39

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{60\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^2x^2 - 120\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax + 60\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right.}{10\sqrt{ax}}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x)
```

output
```
(60*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**2*x**2 -
 120*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x + 60*s
qrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 33*sqrt(a*x -
1)*a**2*x**2 - 66*sqrt(a*x - 1)*a*x + 33*sqrt(a*x - 1) + 10*sqrt(a*x + 1)*
a**3*x**3 - 78*sqrt(a*x + 1)*a**2*x**2 + 114*sqrt(a*x + 1)*a*x - 48*sqrt(a
*x + 1))/(10*sqrt(a*x - 1)*a*c**4*(a**2*x**2 - 2*a*x + 1))
```

3.436 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\frac{c^4}{3a^4x^3}-\frac{3c^4}{a^3x^2}+\frac{16c^4}{a^2x}+c^4x+\frac{26c^4\log(x)}{a}-\frac{32c^4\log(1+ax)}{a}$$

output
```
1/3*c^4/a^4/x^3-3*c^4/a^3/x^2+16*c^4/a^2/x+c^4*x+26*c^4*ln(x)/a-32*c^4*ln(
a*x+1)/a
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx =-\frac{c^4\left(-\frac{1}{3x^3}+\frac{3a}{x^2}-\frac{16a^2}{x}-a^4x-26a^3\log(x)+32a^3\log(1+ax)\right)}{a^4}$$

input
```
Integrate[(c - c/(a*x))^4/E^(2*ArcCoth[a*x]),x]
```

output
```
-((c^4*(-1/3*1/x^3 + (3*a)/x^2 - (16*a^2)/x - a^4*x - 26*a^3*Log[x] + 32*a
^3*Log[1 + a*x]))/a^4)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^4 e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^4 e^{-2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^4}{a^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\int e^{-2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^4dx}{a^4}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{c^4\int \frac{e^{-2\text{arctanh}(ax)}(1-ax)^4}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{c^4\int \frac{(1-ax)^5}{x^4(ax+1)}dx}{a^4}$$

$$\Big\downarrow \text{99}$$

$$-\frac{c^4\int \left(\frac{32a^4}{ax+1}-a^4-\frac{26a^3}{x}+\frac{16a^2}{x^2}-\frac{6a}{x^3}+\frac{1}{x^4}\right)dx}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^4\left(a^4(-x)-26a^3\log(x)+32a^3\log(ax+1)-\frac{16a^2}{x}+\frac{3a}{x^2}-\frac{1}{3x^3}\right)}{a^4}$$

input
```
Int[(c - c/(a*x))^4/E^(2*ArcCoth[a*x]),x]
```

output
```
-((c^4*(-1/3*1/x^3 + (3*a)/x^2 - (16*a^2)/x - a^4*x - 26*a^3*Log[x] + 32*a
^3*Log[1 + a*x]))/a^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

method	result
default	$\frac{c^4\left(x\,a^4-32a^3\ln(ax+1)+\frac{1}{3x^3}-\frac{3a}{x^2}+\frac{16a^2}{x}+26a^3\ln(x)\right)}{a^4}$
risch	$c^4x+\frac{16a^2c^4x^2-3a\,c^4x+\frac{1}{3}c^4}{a^4x^3}+\frac{26c^4\ln(-x)}{a}-\frac{32c^4\ln(ax+1)}{a}$
norman	$\frac{a^3c^4x^4+\frac{c^4}{3a}-3c^4x+16a\,c^4x^2}{a^3x^3}+\frac{26c^4\ln(x)}{a}-\frac{32c^4\ln(ax+1)}{a}$
parallelrisch	$\frac{3a^4c^4x^4+78c^4\ln(x)a^3x^3-96c^4\ln(ax+1)a^3x^3+48a^2c^4x^2-9a\,c^4x+c^4}{3a^4x^3}$
meijerg	$\frac{c^4(ax-\ln(ax+1))}{a}-\frac{5c^4\ln(ax+1)}{a}+\frac{10c^4(\ln(x)+\ln(a)-\ln(ax+1))}{a}-\frac{10c^4\left(-\frac{1}{ax}-\ln(x)-\ln(a)+\ln(ax+1)\right)}{a}+\frac{5c^4\left(-\right.}{}$

input
```
int((c-c/a/x)^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^4/a^4*(x*a^4-32*a^3*ln(a*x+1)+1/3/x^3-3*a/x^2+16*a^2/x+26*a^3*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{3\,a^4c^4x^4-96\,a^3c^4x^3\log\left(ax+1\right)+78\,a^3c^4x^3\log\left(x\right)+48\,a^2c^4x^2-9\,ac^4x+c^4}{3\,a^4x^3}$$

input
```
integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/3*(3*a^4*c^4*x^4 - 96*a^3*c^4*x^3*log(a*x + 1) + 78*a^3*c^4*x^3*log(x) +
 48*a^2*c^4*x^2 - 9*a*c^4*x + c^4)/(a^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = c^4x+\frac{2c^4\cdot\left(13\log\left(x\right)-16\log\left(x+\frac{1}{a}\right)\right)}{a}+\frac{48a^2c^4x^2-9ac^4x+c^4}{3a^4x^3}$$

input
```
integrate((c-c/a/x)**4*(a*x-1)/(a*x+1),x)
```

output
```
c**4*x + 2*c**4*(13*log(x) - 16*log(x + 1/a))/a + (48*a**2*c**4*x**2 - 9*a
*c**4*x + c**4)/(3*a**4*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = c^4x-\frac{32\,c^4\log\left(ax+1\right)}{a}+\frac{26\,c^4\log\left(x\right)}{a}+\frac{48\,a^2c^4x^2-9\,ac^4x+c^4}{3\,a^4x^3}$$

input
```
integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
c^4*x - 32*c^4*log(a*x + 1)/a + 26*c^4*log(x)/a + 1/3*(48*a^2*c^4*x^2 - 9*
a*c^4*x + c^4)/(a^4*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = c^4x - \frac{32\,c^4\log\left(|ax+1|\right)}{a} + \frac{26\,c^4\log\left(|x|\right)}{a} + \frac{48\,a^2c^4x^2 - 9\,ac^4x + c^4}{3\,a^4x^3}$$

input
```
integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
c^4*x - 32*c^4*log(abs(a*x + 1))/a + 26*c^4*log(abs(x))/a + 1/3*(48*a^2*c^
4*x^2 - 9*a*c^4*x + c^4)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = c^4\,x + \frac{16\,a^2\,c^4\,x^2 - 3\,a\,c^4\,x + \frac{c^4}{3}}{a^4\,x^3} + \frac{26\,c^4\,\ln\left(x\right)}{a} - \frac{32\,c^4\,\ln\left(a\,x+1\right)}{a}$$

input
```
int(((c - c/(a*x))^4*(a*x - 1))/(a*x + 1),x)
```

output
```
c^4*x + (c^4/3 + 16*a^2*c^4*x^2 - 3*a*c^4*x)/(a^4*x^3) + (26*c^4*log(x))/a
 - (32*c^4*log(a*x + 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$= \frac{c^4(-96\log(ax+1)\,a^3x^3+78\log(x)\,a^3x^3+3a^4x^4+48a^2x^2-9ax+1)}{3a^4x^3}$$

input
```
int((c-c/a/x)^4*(a*x-1)/(a*x+1),x)
```

output
```
(c**4*( - 96*log(a*x + 1)*a**3*x**3 + 78*log(x)*a**3*x**3 + 3*a**4*x**4 +
48*a**2*x**2 - 9*a*x + 1))/(3*a**4*x**3)
```

3.437 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+c^3x+\frac{11c^3\log(x)}{a}-\frac{16c^3\log(1+ax)}{a}$$

output
```
-1/2*c^3/a^3/x^2+5*c^3/a^2/x+c^3*x+11*c^3*ln(x)/a-16*c^3*ln(a*x+1)/a
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{c^3\left(\frac{1}{2x^2}-\frac{5a}{x}-a^3x-11a^2\log(x)+16a^2\log(1+ax)\right)}{a^3}$$

input
```
Integrate[(c - c/(a*x))^3/E^(2*ArcCoth[a*x]),x]
```

output
```
-((c^3*(1/(2*x^2) - (5*a)/x - a^3*x - 11*a^2*Log[x] + 16*a^2*Log[1 + a*x])
)/a^3)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^3 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^3 e^{-2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^3}{a^3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\int e^{-2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^3dx}{a^3}$$

$$\Big\downarrow \text{6681}$$

$$\frac{c^3\int \frac{e^{-2\text{arctanh}(ax)}(1-ax)^3}{x^3}dx}{a^3}$$

$$\Big\downarrow \text{6679}$$

$$\frac{c^3\int \frac{(1-ax)^4}{x^3(ax+1)}dx}{a^3}$$

$$\Big\downarrow \text{99}$$

$$\frac{c^3\int \left(-\frac{16a^3}{ax+1}+a^3+\frac{11a^2}{x}-\frac{5a}{x^2}+\frac{1}{x^3}\right)dx}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3\left(a^3x+11a^2\log(x)-16a^2\log(ax+1)+\frac{5a}{x}-\frac{1}{2x^2}\right)}{a^3}$$

input
```
Int[(c - c/(a*x))^3/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^3*(-1/2*1/x^2 + (5*a)/x + a^3*x + 11*a^2*Log[x] - 16*a^2*Log[1 + a*x]))
/a^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

method	result
default	$\frac{c^3\left(a^3x-16a^2\ln(ax+1)-\frac{1}{2x^2}+\frac{5a}{x}+11a^2\ln(x)\right)}{a^3}$
risch	$c^3x+\frac{5a\,c^3x-\frac{1}{2}c^3}{a^3x^2}-\frac{16c^3\ln(ax+1)}{a}+\frac{11c^3\ln(-x)}{a}$
norman	$\frac{a^2c^3x^3-\frac{c^3}{2a}+5c^3x}{a^2x^2}+\frac{11c^3\ln(x)}{a}-\frac{16c^3\ln(ax+1)}{a}$
parallelrisch	$\frac{2a^3c^3x^3+22c^3\ln(x)a^2x^2-32c^3\ln(ax+1)a^2x^2+10a\,c^3x-c^3}{2a^3x^2}$
meijerg	$\frac{c^3(ax-\ln(ax+1))}{a}-\frac{4c^3\ln(ax+1)}{a}+\frac{6c^3(\ln(x)+\ln(a)-\ln(ax+1))}{a}-\frac{4c^3\left(-\frac{1}{ax}-\ln(x)-\ln(a)+\ln(ax+1)\right)}{a}+\frac{c^3\left(-\frac{1}{2a^2x}\right.}{}$

input
```
int((c-c/a/x)^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^3/a^3*(a^3*x-16*a^2*ln(a*x+1)-1/2/x^2+5*a/x+11*a^2*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$$
$$=\frac{2\,a^3c^3x^3-32\,a^2c^3x^2\log(ax+1)+22\,a^2c^3x^2\log(x)+10\,ac^3x-c^3}{2\,a^3x^2}$$

input
```
integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/2*(2*a^3*c^3*x^3 - 32*a^2*c^3*x^2*log(a*x + 1) + 22*a^2*c^3*x^2*log(x) +
 10*a*c^3*x - c^3)/(a^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = c^3x + \frac{c^3\cdot\left(11\log{(x)} - 16\log{\left(x+\frac{1}{a}\right)}\right)}{a} + \frac{10ac^3x - c^3}{2a^3x^2}$$

input
```
integrate((c-c/a/x)**3*(a*x-1)/(a*x+1),x)
```

output
```
c**3*x + c**3*(11*log(x) - 16*log(x + 1/a))/a + (10*a*c**3*x - c**3)/(2*a*
*3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = c^3x - \frac{16\,c^3\log{(ax+1)}}{a} + \frac{11\,c^3\log{(x)}}{a} + \frac{10\,ac^3x - c^3}{2\,a^3x^2}$$

input
```
integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
c^3*x - 16*c^3*log(a*x + 1)/a + 11*c^3*log(x)/a + 1/2*(10*a*c^3*x - c^3)/(
a^3*x^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = c^3x - \frac{16\,c^3\log{(|ax+1|)}}{a} + \frac{11\,c^3\log{(|x|)}}{a} + \frac{10\,ac^3x - c^3}{2\,a^3x^2}$$

input
```
integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
c^3*x - 16*c^3*log(abs(a*x + 1))/a + 11*c^3*log(abs(x))/a + 1/2*(10*a*c^3*
x - c^3)/(a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = c^3\,x-\frac{\frac{c^3}{2}-5\,a\,c^3\,x}{a^3\,x^2}+\frac{11\,c^3\,\ln\left(x\right)}{a}-\frac{16\,c^3\,\ln\left(a\,x+1\right)}{a}$$

input
```
int(((c - c/(a*x))^3*(a*x - 1))/(a*x + 1),x)
```

output
```
c^3*x - (c^3/2 - 5*a*c^3*x)/(a^3*x^2) + (11*c^3*log(x))/a - (16*c^3*log(a*
x + 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = \frac{c^3(-32\log(ax+1)\,a^2x^2+22\log(x)\,a^2x^2+2a^3x^3+10ax-1)}{2a^3x^2}$$

input
```
int((c-c/a/x)^3*(a*x-1)/(a*x+1),x)
```

output
```
(c**3*( - 32*log(a*x + 1)*a**2*x**2 + 22*log(x)*a**2*x**2 + 2*a**3*x**3 +
10*a*x - 1))/(2*a**3*x**2)
```

3.438 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 40

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}-\frac{8c^2\log(1+ax)}{a}$$

output
```
c^2/a^2/x+c^2*x+4*c^2*ln(x)/a-8*c^2*ln(a*x+1)/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=-\frac{c^2\left(-\frac{1}{x}-a^2x-4a\log(x)+8a\log(1+ax)\right)}{a^2}$$

input
```
Integrate[(c - c/(a*x))^2/E^(2*ArcCoth[a*x]),x]
```

output
```
-((c^2*(-x^(-1) - a^2*x - 4*a*Log[x] + 8*a*Log[1 + a*x]))/a^2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^2 e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^2 e^{-2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^2}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\int e^{-2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)^2dx}{a^2}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{c^2\int \frac{e^{-2\text{arctanh}(ax)}(1-ax)^2}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{c^2\int \frac{(1-ax)^3}{x^2(ax+1)}dx}{a^2}$$

$$\Big\downarrow \text{99}$$

$$-\frac{c^2\int \left(\frac{8a^2}{ax+1}-a^2-\frac{4a}{x}+\frac{1}{x^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2\left(a^2(-x)-4a\log(x)+8a\log(ax+1)-\frac{1}{x}\right)}{a^2}$$

input
```
Int[(c - c/(a*x))^2/E^(2*ArcCoth[a*x]),x]
```

output
```
-((c^2*(-x^(-1) - a^2*x - 4*a*Log[x] + 8*a*Log[1 + a*x]))/a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{c^2\left(a^2x-8a\ln(ax+1)+\frac{1}{x}+4\ln(x)a\right)}{a^2}$	31
risch	$\frac{c^2}{a^2x}+c^2x-\frac{8c^2\ln(ax+1)}{a}+\frac{4c^2\ln(-x)}{a}$	43
parallelrisch	$\frac{c^2a^2x^2+4c^2\ln(x)ax-8c^2\ln(ax+1)ax+c^2}{a^2x}$	44
norman	$\frac{\frac{c^2}{a}+a\,c^2x^2}{ax}+\frac{4c^2\ln(x)}{a}-\frac{8c^2\ln(ax+1)}{a}$	49
meijerg	$\frac{c^2(ax-\ln(ax+1))}{a}-\frac{3c^2\ln(ax+1)}{a}+\frac{3c^2(\ln(x)+\ln(a)-\ln(ax+1))}{a}-\frac{c^2\left(-\frac{1}{ax}-\ln(x)-\ln(a)+\ln(ax+1)\right)}{a}$	87

input
```
int((c-c/a/x)^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^2/a^2*(a^2*x-8*a*ln(a*x+1)+1/x+4*ln(x)*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{a^2c^2x^2-8\,ac^2x\log\left(ax+1\right)+4\,ac^2x\log\left(x\right)+c^2}{a^2x}$$

input
```
integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
(a^2*c^2*x^2 - 8*a*c^2*x*log(a*x + 1) + 4*a*c^2*x*log(x) + c^2)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = c^2x + \frac{4c^2\left(\log(x) - 2\log\left(x + \frac{1}{a}\right)\right)}{a} + \frac{c^2}{a^2x}$$

input
```
integrate((c-c/a/x)**2*(a*x-1)/(a*x+1),x)
```

output
```
c**2*x + 4*c**2*(log(x) - 2*log(x + 1/a))/a + c**2/(a**2*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = c^2x - \frac{8\,c^2\log(ax+1)}{a} + \frac{4\,c^2\log(x)}{a} + \frac{c^2}{a^2x}$$

input
```
integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
c^2*x - 8*c^2*log(a*x + 1)/a + 4*c^2*log(x)/a + c^2/(a^2*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = c^2x - \frac{8\,c^2\log(|ax+1|)}{a} + \frac{4\,c^2\log(|x|)}{a} + \frac{c^2}{a^2x}$$

input
```
integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
c^2*x - 8*c^2*log(abs(a*x + 1))/a + 4*c^2*log(abs(x))/a + c^2/(a^2*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = c^2 x + \frac{c^2}{a^2 x} + \frac{4 c^2 \ln(x)}{a} - \frac{8 c^2 \ln(a x+1)}{a}$$

input
```
int(((c - c/(a*x))^2*(a*x - 1))/(a*x + 1),x)
```

output
```
c^2*x + c^2/(a^2*x) + (4*c^2*log(x))/a - (8*c^2*log(a*x + 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \frac{c^2(-8\log(ax+1)\,ax+4\log(x)\,ax+a^2x^2+1)}{a^2x}$$

input
```
int((c-c/a/x)^2*(a*x-1)/(a*x+1),x)
```

output
```
(c**2*( - 8*log(a*x + 1)*a*x + 4*log(x)*a*x + a**2*x**2 + 1))/(a**2*x)
```

3.439 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\log(x)}{a}-\frac{4c\log(1+ax)}{a}$$

output
```
c*x+c*ln(x)/a-4*c*ln(a*x+1)/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\log(x)}{a}-\frac{4c\log(1+ax)}{a}$$

input
```
Integrate[(c - c/(a*x))/E^(2*ArcCoth[a*x]),x]
```

output
```
c*x + (c*Log[x])/a - (4*c*Log[1 + a*x])/a
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6717, 27, 6681, 6679, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right) e^{-2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{ce^{-2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)}{a}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c\int e^{-2\text{arctanh}(ax)}\left(a-\frac{1}{x}\right)dx}{a}$$

$$\Big\downarrow \text{ 6681}$$

$$\frac{c\int \frac{e^{-2\text{arctanh}(ax)}(1-ax)}{x}dx}{a}$$

$$\Big\downarrow \text{ 6679}$$

$$\frac{c\int \frac{(1-ax)^2}{x(ax+1)}dx}{a}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{c\int \left(-\frac{4a}{ax+1}+a+\frac{1}{x}\right)dx}{a}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c(ax-4\log(ax+1)+\log(x))}{a}$$

input
```
Int[(c - c/(a*x))/E^(2*ArcCoth[a*x]),x]
```

output
```
(c*(a*x + Log[x] - 4*Log[1 + a*x]))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{c(ax-4\ln(ax+1)+\ln(x))}{a}$	20
parallelrisch	$\frac{acx+c\ln(x)-4c\ln(ax+1)}{a}$	23
norman	$xc+\frac{c\ln(x)}{a}-\frac{4c\ln(ax+1)}{a}$	24
risch	$xc+\frac{c\ln(-x)}{a}-\frac{4c\ln(ax+1)}{a}$	26
meijerg	$\frac{c(ax-\ln(ax+1))}{a}-\frac{2c\ln(ax+1)}{a}+\frac{c(\ln(x)+\ln(a)-\ln(ax+1))}{a}$	49

input
```
int((c-c/a/x)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c/a*(a*x-4*ln(a*x+1)+ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{acx-4\,c\log\left(ax+1\right)+c\log\left(x\right)}{a}$$

input
```
integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
(a*c*x - 4*c*log(a*x + 1) + c*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx+\frac{c\left(\log{(x)}-4\log{\left(x+\frac{1}{a}\right)}\right)}{a}$$

input
```
integrate((c-c/a/x)*(a*x-1)/(a*x+1),x)
```

output
```
c*x + c*(log(x) - 4*log(x + 1/a))/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx-\frac{4\,c\log{(ax+1)}}{a}+\frac{c\log{(x)}}{a}$$

input
```
integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
c*x - 4*c*log(a*x + 1)/a + c*log(x)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=cx-\frac{4\,c\log{(|ax+1|)}}{a}+\frac{c\log{(|x|)}}{a}$$

input
```
integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
c*x - 4*c*log(abs(a*x + 1))/a + c*log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = c\,x+\frac{c\,\ln(x)}{a}-\frac{4\,c\,\ln(a\,x+1)}{a}$$

input
```
int(((c - c/(a*x))*(a*x - 1))/(a*x + 1),x)
```

output
```
c*x + (c*log(x))/a - (4*c*log(a*x + 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = \frac{c(-4\log(ax+1)+\log(x)+ax)}{a}$$

input
```
int((c-c/a/x)*(a*x-1)/(a*x+1),x)
```

output
```
(c*( - 4*log(a*x + 1) + log(x) + a*x))/a
```

3.440 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}} dx$

Optimal result

Integrand size = 22, antiderivative size = 20

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}} dx = \frac{x}{c} - \frac{\log(1+ax)}{ac}$$

output
```
x/c-ln(a*x+1)/a/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}} dx = \frac{a\left(\frac{x}{a} - \frac{\log(1+ax)}{a^2}\right)}{c}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))),x]
```

output
```
(a*(x/a - Log[1 + a*x]/a^2))/c
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{ae^{-2\text{arctanh}(ax)}}{c\left(a-\frac{1}{x}\right)}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\int \frac{e^{-2\text{arctanh}(ax)}}{a-\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{ 6681}$$

$$\frac{a\int \frac{e^{-2\text{arctanh}(ax)}x}{1-ax}dx}{c}$$

$$\Big\downarrow \text{ 6679}$$

$$\frac{a\int \frac{x}{ax+1}dx}{c}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{a\int \left(\frac{1}{a}-\frac{1}{a(ax+1)}\right)dx}{c}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a\left(\frac{x}{a}-\frac{\log(ax+1)}{a^2}\right)}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))),x]
```

output
```
(a*(x/a - Log[1 + a*x]/a^2))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{ax-\ln(ax+1)}{ac}$	20
norman	$\frac{x}{c}-\frac{\ln(ax+1)}{ac}$	21
risch	$\frac{x}{c}-\frac{\ln(ax+1)}{ac}$	21
default	$\frac{a\left(\frac{x}{a}-\frac{\ln(ax+1)}{a^2}\right)}{c}$	23

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
(a*x-ln(a*x+1))/a/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{ax-\log(ax+1)}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="fricas")
```

output
```
(a*x - log(a*x + 1))/(a*c)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = a\left(\frac{x}{ac}-\frac{\log(ax+1)}{a^2c}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x),x)
```

output
```
a*(x/(a*c) - log(a*x + 1)/(a**2*c))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{x}{c} - \frac{\log(ax+1)}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="maxima")
```

output
```
x/c - log(a*x + 1)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{x}{c} - \frac{\log(|ax+1|)}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="giac")
```

output
```
x/c - log(abs(a*x + 1))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{\ln(a\,x+1)-a\,x}{a\,c}$$

input
```
int((a*x - 1)/((c - c/(a*x))*(a*x + 1)),x)
```

output
```
-(log(a*x + 1) - a*x)/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{-\log(ax+1)+ax}{ac}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x),x)
```

output
```
( - log(a*x + 1) + a*x)/(a*c)
```

3.441 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{x}{c^2}-\frac{\text{arctanh}(ax)}{ac^2}$$

output
```
x/c^2-arctanh(a*x)/a/c^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{x}{c^2}-\frac{\text{arctanh}(ax)}{ac^2}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

output
```
x/c^2 - ArcTanh[a*x]/(a*c^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6717, 27, 6681, 6679, 82, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^2e^{-2\text{arctanh}(ax)}}{c^2\left(a-\frac{1}{x}\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\int \frac{e^{-2\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^2}dx}{c^2}$$

$$\Big\downarrow \text{6681}$$

$$-\frac{a^2\int \frac{e^{-2\text{arctanh}(ax)}x^2}{(1-ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{6679}$$

$$-\frac{a^2\int \frac{x^2}{(1-ax)(ax+1)}dx}{c^2}$$

$$\Big\downarrow \text{82}$$

$$-\frac{a^2\int \frac{x^2}{1-a^2x^2}dx}{c^2}$$

$$\Big\downarrow \text{262}$$

$$-\frac{a^2\left(\frac{\int \frac{1}{1-a^2x^2}dx}{a^2}-\frac{x}{a^2}\right)}{c^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a^2\left(\frac{\text{arctanh}(ax)}{a^3}-\frac{x}{a^2}\right)}{c^2}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

output
```
-((a^2*(-(x/a^2) + ArcTanh[a*x]/a^3))/c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

method	result	size
parallelrisch	$-\frac{-2ax+\ln(ax+1)-\ln(ax-1)}{2a\,c^{2}}$	28
default	$\frac{a^{2}\left(\frac{x}{a^{2}}-\frac{\ln(ax+1)}{2a^{3}}+\frac{\ln(ax-1)}{2a^{3}}\right)}{c^{2}}$	36
risch	$\frac{x}{c^{2}}-\frac{\ln(ax+1)}{2a\,c^{2}}+\frac{\ln(-ax+1)}{2a\,c^{2}}$	36
norman	$\frac{\frac{a\,x^{2}}{c}-\frac{x}{c}}{c(ax-1)}+\frac{\ln(ax-1)}{2a\,c^{2}}-\frac{\ln(ax+1)}{2a\,c^{2}}$	56

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*a*x+ln(a*x+1)-ln(a*x-1))/a/c^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{2}}\,dx=\frac{2\,ax-\log\left(ax+1\right)+\log\left(ax-1\right)}{2\,ac^{2}}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
1/2*(2*a*x - log(a*x + 1) + log(a*x - 1))/(a*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = a^2\left(\frac{x}{a^2c^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{2}-\frac{\log\left(x+\frac{1}{a}\right)}{2}}{a^3c^2}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)**2,x)
```

output
```
a**2*(x/(a**2*c**2) + (log(x - 1/a)/2 - log(x + 1/a)/2)/(a**3*c**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{x}{c^2}-\frac{\log(ax+1)}{2\,ac^2}+\frac{\log(ax-1)}{2\,ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
x/c^2 - 1/2*log(a*x + 1)/(a*c^2) + 1/2*log(a*x - 1)/(a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{x}{c^2}-\frac{\log(|ax+1|)}{2\,ac^2}+\frac{\log(|ax-1|)}{2\,ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="giac")
```

output
```
x/c^2 - 1/2*log(abs(a*x + 1))/(a*c^2) + 1/2*log(abs(a*x - 1))/(a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{\operatorname{atanh}(a\,x)-a\,x}{a\,c^2}$$

input
```
int((a*x - 1)/((c - c/(a*x))^2*(a*x + 1)),x)
```

output
```
-(atanh(a*x) - a*x)/(a*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.83

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{\log(a^2x-a)-\log(a^2x+a)+2ax}{2a\,c^2}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^2,x)
```

output
```
(log(a**2*x - a) - log(a**2*x + a) + 2*a*x)/(2*a*c**2)
```

3.442 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx = \frac{x}{c^3} + \frac{1}{2ac^3(1-ax)} + \frac{5\log(1-ax)}{4ac^3} - \frac{\log(1+ax)}{4ac^3}$$

output
```
x/c^3+1/2/a/c^3/(-a*x+1)+5/4*ln(-a*x+1)/a/c^3-1/4*ln(a*x+1)/a/c^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx = \frac{x}{c^3} + \frac{1}{2ac^3(1-ax)} + \frac{5\log(1-ax)}{4ac^3} - \frac{\log(1+ax)}{4ac^3}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

output
```
x/c^3 + 1/(2*a*c^3*(1 - a*x)) + (5*Log[1 - a*x])/(4*a*c^3) - Log[1 + a*x]/
(4*a*c^3)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{a^3e^{-2\text{arctanh}(ax)}}{c^3\left(a-\frac{1}{x}\right)^3}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^3\int \frac{e^{-2\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^3}dx}{c^3}$$

$$\Big\downarrow \text{ 6681}$$

$$\frac{a^3\int \frac{e^{-2\text{arctanh}(ax)}x^3}{(1-ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{ 6679}$$

$$\frac{a^3\int \frac{x^3}{(1-ax)^2(ax+1)}dx}{c^3}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{a^3\int \left(-\frac{1}{4a^3(ax+1)}+\frac{1}{a^3}+\frac{5}{4a^3(ax-1)}+\frac{1}{2a^3(ax-1)^2}\right)dx}{c^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3\left(\frac{1}{2a^4(1-ax)}+\frac{5\log(1-ax)}{4a^4}-\frac{\log(ax+1)}{4a^4}+\frac{x}{a^3}\right)}{c^3}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

output
```
(a^3*(x/a^3 + 1/(2*a^4*(1 - a*x)) + (5*Log[1 - a*x])/(4*a^4) - Log[1 + a*x
]/(4*a^4)))/c^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{a^{3}\left(\frac{x}{a^{3}}-\frac{\ln(ax+1)}{4a^{4}}-\frac{1}{2a^{4}(ax-1)}+\frac{5\ln(ax-1)}{4a^{4}}\right)}{c^{3}}$	48
risch	$\frac{x}{c^{3}}-\frac{1}{2a(ax-1)c^{3}}+\frac{5\ln(-ax+1)}{4a\,c^{3}}-\frac{\ln(ax+1)}{4a\,c^{3}}$	51
parallelrisch	$\frac{4a^{2}x^{2}+5a\ln(ax-1)x-\ln(ax+1)xa-6ax-5\ln(ax-1)+\ln(ax+1)}{4c^{3}(ax-1)a}$	63
norman	$\frac{\frac{a^{2}x^{3}}{c}+\frac{3x}{2c}-\frac{5a\,x^{2}}{2c}}{c^{2}(ax-1)^{2}}+\frac{5\ln(ax-1)}{4a\,c^{3}}-\frac{\ln(ax+1)}{4a\,c^{3}}$	67

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3/c^3*(x/a^3-1/4*ln(a*x+1)/a^4-1/2/a^4/(a*x-1)+5/4/a^4*ln(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3}}\,dx=\frac{4\,a^{2}x^{2}-4\,ax-(ax-1)\log(ax+1)+5\,(ax-1)\log(ax-1)-2}{4\,(a^{2}c^{3}x-ac^{3})}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
 - 2)/(a^2*c^3*x - a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = a^3\left(-\frac{1}{2a^5c^3x-2a^4c^3}+\frac{x}{a^3c^3}+\frac{\frac{5\log\left(x-\frac{1}{a}\right)}{4}-\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^4c^3}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)**3,x)
```

output
```
a**3*(-1/(2*a**5*c**3*x - 2*a**4*c**3) + x/(a**3*c**3) + (5*log(x - 1/a)/4
 - log(x + 1/a)/4)/(a**4*c**3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{1}{2\left(a^2c^3x-ac^3\right)}+\frac{x}{c^3}-\frac{\log\left(ax+1\right)}{4\,ac^3}+\frac{5\,\log\left(ax-1\right)}{4\,ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
-1/2/(a^2*c^3*x - a*c^3) + x/c^3 - 1/4*log(a*x + 1)/(a*c^3) + 5/4*log(a*x
- 1)/(a*c^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3}-\frac{\log\left(|ax+1|\right)}{4\,ac^3}+\frac{5\,\log\left(|ax-1|\right)}{4\,ac^3}-\frac{1}{2\,(ax-1)ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="giac")
```

output
```
x/c^3 - 1/4*log(abs(a*x + 1))/(a*c^3) + 5/4*log(abs(a*x - 1))/(a*c^3) - 1/
2/((a*x - 1)*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{x}{c^3}+\frac{1}{2\,a\,\left(c^3-a\,c^3\,x\right)}+\frac{5\,\ln\left(a\,x-1\right)}{4\,a\,c^3}-\frac{\ln\left(a\,x+1\right)}{4\,a\,c^3}$$

input
```
int((a*x - 1)/((c - c/(a*x))^3*(a*x + 1)),x)
```

output
```
x/c^3 + 1/(2*a*(c^3 - a*c^3*x)) + (5*log(a*x - 1))/(4*a*c^3) - log(a*x + 1
)/(4*a*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{5\log(ax-1)\,ax-5\log(ax-1)-\log(ax+1)\,ax+\log(ax+1)+4a^2x^2-6ax}{4a\,c^3\,(ax-1)}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^3,x)
```

output
```
(5*log(a*x - 1)*a*x - 5*log(a*x - 1) - log(a*x + 1)*a*x + log(a*x + 1) + 4
*a**2*x**2 - 6*a*x)/(4*a*c**3*(a*x - 1))
```

3.443 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\frac{x}{c^4}-\frac{1}{4ac^4(1-ax)^2}+\frac{7}{4ac^4(1-ax)}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(1+ax)}{8ac^4}$$

output
```
x/c^4-1/4/a/c^4/(-a*x+1)^2+7/4/a/c^4/(-a*x+1)+17/8*ln(-a*x+1)/a/c^4-1/8*ln
(a*x+1)/a/c^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=-\frac{a^4\left(-\frac{x}{a^4}+\frac{1}{4a^5(1-ax)^2}-\frac{7}{4a^5(1-ax)}-\frac{17\log(1-ax)}{8a^5}+\frac{\log(1+ax)}{8a^5}\right)}{c^4}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

output
```
-((a^4*(-(x/a^4) + 1/(4*a^5*(1 - a*x)^2) - 7/(4*a^5*(1 - a*x)) - (17*Log[1
 - a*x])/(8*a^5) + Log[1 + a*x]/(8*a^5)))/c^4)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6681, 6679, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{a^4e^{-2\text{arctanh}(ax)}}{c^4\left(a-\frac{1}{x}\right)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^4\int \frac{e^{-2\text{arctanh}(ax)}}{\left(a-\frac{1}{x}\right)^4}dx}{c^4}$$

$$\Big\downarrow \text{ 6681}$$

$$-\frac{a^4\int \frac{e^{-2\text{arctanh}(ax)}x^4}{(1-ax)^4}dx}{c^4}$$

$$\Big\downarrow \text{ 6679}$$

$$-\frac{a^4\int \frac{x^4}{(1-ax)^3(ax+1)}dx}{c^4}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{a^4\int \left(\frac{1}{8a^4(ax+1)}-\frac{1}{a^4}-\frac{17}{8a^4(ax-1)}-\frac{7}{4a^4(ax-1)^2}-\frac{1}{2a^4(ax-1)^3}\right)dx}{c^4}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^4\left(-\frac{7}{4a^5(1-ax)}+\frac{1}{4a^5(1-ax)^2}-\frac{17\log(1-ax)}{8a^5}+\frac{\log(ax+1)}{8a^5}-\frac{x}{a^4}\right)}{c^4}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

output
```
-((a^4*(-(x/a^4) + 1/(4*a^5*(1 - a*x)^2) - 7/(4*a^5*(1 - a*x)) - (17*Log[1
 - a*x])/(8*a^5) + Log[1 + a*x]/(8*a^5)))/c^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6679
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Simp[c^p   Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x]
, x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p]
|| GtQ[c, 0])
```

rule 6681
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Simp[d^p   Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; F
reeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{a^{4}\left(\frac{x}{a^{4}}-\frac{\ln(ax+1)}{8a^{5}}-\frac{1}{4a^{5}(ax-1)^{2}}-\frac{7}{4a^{5}(ax-1)}+\frac{17\ln(ax-1)}{8a^{5}}\right)}{c^{4}}$	60
risch	$\frac{x}{c^{4}}+\frac{-\frac{7c^{4}x}{4}+\frac{3c^{4}}{2a}}{c^{8}(ax-1)^{2}}+\frac{17\ln(-ax+1)}{8a\,c^{4}}-\frac{\ln(ax+1)}{8a\,c^{4}}$	62
norman	$\frac{\frac{a^{3}x^{4}}{c}-\frac{9x}{4c}+\frac{23a\,x^{2}}{4c}-\frac{9a^{2}x^{3}}{2c}}{c^{3}(ax-1)^{3}}+\frac{17\ln(ax-1)}{8a\,c^{4}}-\frac{\ln(ax+1)}{8a\,c^{4}}$	78
parallelrisch	$\frac{8a^{3}x^{3}+17a^{2}\ln(ax-1)x^{2}-\ln(ax+1)x^{2}a^{2}-28a^{2}x^{2}-34a\ln(ax-1)x+2\ln(ax+1)xa+18ax+17\ln(ax-1)-\ln(ax+1)}{8c^{4}(ax-1)^{2}a}$	101

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
a^4/c^4*(x/a^4-1/8*ln(a*x+1)/a^5-1/4/a^5/(a*x-1)^2-7/4/a^5/(a*x-1)+17/8/a^
5*ln(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.24

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{4}}\,dx = \frac{8\,a^{3}x^{3}-16\,a^{2}x^{2}-6\,ax-(a^{2}x^{2}-2\,ax+1)\log(ax+1)+17\,(a^{2}x^{2}-2\,ax+1)\log(ax-1)+12}{8\,(a^{3}c^{4}x^{2}-2\,a^{2}c^{4}x+ac^{4})}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
 17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^3*c^4*x^2 - 2*a^2*c^4*x +
a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = a^4\left(\frac{-7ax+6}{4a^7c^4x^2-8a^6c^4x+4a^5c^4}+\frac{x}{a^4c^4}+\frac{\frac{17\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^4}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)**4,x)
```

output
```
a**4*((-7*a*x + 6)/(4*a**7*c**4*x**2 - 8*a**6*c**4*x + 4*a**5*c**4) + x/(a
**4*c**4) + (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{7\,ax-6}{4\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}+\frac{x}{c^4}-\frac{\log\left(ax+1\right)}{8\,ac^4}+\frac{17\,\log\left(ax-1\right)}{8\,ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
-1/4*(7*a*x - 6)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4) + x/c^4 - 1/8*log(a*x
 + 1)/(a*c^4) + 17/8*log(a*x - 1)/(a*c^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{x}{c^4}-\frac{\log\left(|ax+1|\right)}{8\,ac^4}+\frac{17\,\log\left(|ax-1|\right)}{8\,ac^4}-\frac{7\,ax-6}{4\left(ax-1\right)^2ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="giac")
```

output
```
x/c^4 - 1/8*log(abs(a*x + 1))/(a*c^4) + 17/8*log(abs(a*x - 1))/(a*c^4) - 1
/4*(7*a*x - 6)/((a*x - 1)^2*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\frac{x}{c^4}-\frac{\frac{7\,x}{4}-\frac{3}{2\,a}}{a^2\,c^4\,x^2-2\,a\,c^4\,x+c^4}+\frac{17\,\ln\left(a\,x-1\right)}{8\,a\,c^4}-\frac{\ln\left(a\,x+1\right)}{8\,a\,c^4}$$

input
```
int((a*x - 1)/((c - c/(a*x))^4*(a*x + 1)),x)
```

output
```
x/c^4 - ((7*x)/4 - 3/(2*a))/(c^4 + a^2*c^4*x^2 - 2*a*c^4*x) + (17*log(a*x
- 1))/(8*a*c^4) - log(a*x + 1)/(8*a*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.40

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$=\frac{17\log(ax-1)\,a^2x^2-34\log(ax-1)\,ax+17\log(ax-1)-\log(ax+1)\,a^2x^2+2\log(ax+1)\,ax-\log(a}{8a\,c^4\,(a^2x^2-2ax+1)}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^4,x)
```

output
```
(17*log(a*x - 1)*a**2*x**2 - 34*log(a*x - 1)*a*x + 17*log(a*x - 1) - log(a
*x + 1)*a**2*x**2 + 2*log(a*x + 1)*a*x - log(a*x + 1) + 8*a**3*x**3 - 19*a
**2*x**2 + 9)/(8*a*c**4*(a**2*x**2 - 2*a*x + 1))
```

3.444 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$

Optimal result

Integrand size = 22, antiderivative size = 159

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{23c^4\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}+\frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^4\sqrt{1-\frac{1}{a^2x^2}}x+\frac{91c^4\csc^{-1}(ax)}{2a}-\frac{7c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
23*c^4*(1-1/a^2/x^2)^(1/2)/a-1/3*c^4*(1-1/a^2/x^2)^(3/2)/a+64*c^4*(a-1/x)/
a^2/(1-1/a^2/x^2)^(1/2)-7/2*c^4*(1-1/a^2/x^2)^(1/2)/a^2/x+c^4*(1-1/a^2/x^2
)^(1/2)*x+91/2*c^4*arccsc(a*x)/a-7*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.39 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.57

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$= \frac{c^4\left(2772\sqrt{2}a^3x^3(-1+ax)^3(1+ax)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{2},\frac{7}{2},\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)+1980\sqrt{2}a^2x^2(-1+ax)^4(\right.}{}$$

input
```
Integrate[(c - c/(a*x))^4/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^4*(2772*Sqrt[2]*a^3*x^3*(-1 + a*x)^3*(1 + a*x)*Hypergeometric2F1[3/2, 5
/2, 7/2, (1 - 1/(a*x))/2] + 1980*Sqrt[2]*a^2*x^2*(-1 + a*x)^4*(1 + a*x)*Hy
pergeometric2F1[3/2, 7/2, 9/2, (1 - 1/(a*x))/2] + 35*(-198*a^2*Sqrt[1 + 1/
(a*x)]*x^2 + 1716*a^3*Sqrt[1 + 1/(a*x)]*x^3 - 7425*a^4*Sqrt[1 + 1/(a*x)]*x
^4 + 26268*a^5*Sqrt[1 + 1/(a*x)]*x^5 + 29403*a^6*Sqrt[1 + 1/(a*x)]*x^6 - 5
0160*a^7*Sqrt[1 + 1/(a*x)]*x^7 + 396*a^8*Sqrt[1 + 1/(a*x)]*x^8 + 66726*a^6
*Sqrt[1 - 1/(a*x)]*x^6*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 66726*a^7*Sqrt[
1 - 1/(a*x)]*x^7*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 1980*a^6*Sqrt[1 - 1/(
a*x)]*x^6*ArcSin[1/(a*x)] - 1980*a^7*Sqrt[1 - 1/(a*x)]*x^7*ArcSin[1/(a*x)]
 - 2772*a^7*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^7*ArcTanh[Sqrt[1 - 1
/(a^2*x^2)]] + 44*Sqrt[2]*a*x*(-1 + a*x)^5*(1 + a*x)*Hypergeometric2F1[3/2
, 9/2, 11/2, (1 - 1/(a*x))/2] + 36*Sqrt[2]*(-1 + a*x)^6*(1 + a*x)*Hypergeo
metric2F1[3/2, 11/2, 13/2, (1 - 1/(a*x))/2])))/(13860*a^7*Sqrt[1 - 1/(a*x)
]*x^6*(1 + a*x))
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6731, 27, 528, 2338, 2340, 25, 2340, 25, 2340, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^4 e^{-3\coth^{-1}(ax)}\,dx$$

$\Big\downarrow$ 6731

$$-\frac{\int \frac{c^7\left(a-\frac{1}{x}\right)^7 x^2}{a^7\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$\Big\downarrow$ 27

$$-\frac{c^4\int \frac{\left(a-\frac{1}{x}\right)^7 x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^7}$$

$\Big\downarrow$ 528

$$-\frac{c^4\left(a^2\int \frac{\left(a^5-\frac{7a^4}{x}-\frac{42a^3}{x^2}+\frac{22a^2}{x^3}-\frac{7a}{x^4}+\frac{1}{x^5}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{64a^5\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^7}$$

$\Big\downarrow$ 2338

$$-\frac{c^4\left(a^2\left(a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{\left(7a^4+\frac{42a^3}{x}-\frac{22a^2}{x^2}+\frac{7a}{x^3}-\frac{1}{x^4}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-\frac{64a^5\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^7}$$

$\Big\downarrow$ 2340

$$-\frac{c^4\left(a^2\left(\frac{1}{3}a^2\int -\frac{\left(21a^2+\frac{126a}{x}-\frac{68}{x^2}+\frac{21}{x^3a}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{64a^5\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^7}$$

$\Big\downarrow$ 25

$$-\frac{c^4\left(a^2\left(-\frac{1}{3}a^2\int \frac{\left(21a^2+\frac{126a}{x}-\frac{68}{x^2}+\frac{21}{x^3a}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{64a^5\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^7}$$

$\Big\downarrow$ 2340

$$-\frac{c^4\left(a^2\left(-\frac{1}{3}a^2\left(-\frac{1}{2}a^2\int -\frac{\left(42+\frac{273}{ax}-\frac{136}{a^2x^2}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{64a^5\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^7}$$

$\Big\downarrow$ 25

$$-\frac{c^4\left(a^2\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\int\frac{\left(42+\frac{273}{ax}-\frac{136}{a^2x^2}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{64a^5\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^7}$$

$$\Big\downarrow \text{2340}$$

$$-\frac{c^4\left(a^2\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(136\sqrt{1-\frac{1}{a^2x^2}}-a^2\int-\frac{21\left(2a+\frac{13}{x}\right)x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-\frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right.\right.\right.}{a^7}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\left(a^2\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{21\int\frac{\left(2a+\frac{13}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}+136\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)\right.}{a^7}$$

$$\Big\downarrow \text{538}$$

$$-\frac{c^4\left(a^2\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{21\left(13\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+2a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)}{a}+136\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a\right.\right.}{a^7}$$

$$\Big\downarrow \text{223}$$

$$-\frac{c^4\left(a^2\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{21\left(2a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+13a\arcsin\left(\frac{1}{ax}\right)\right)}{a}+136\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\right.\right.}{a^7}$$

$$\Big\downarrow \text{243}$$

$$-\frac{c^4\left(a^2\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{21\left(a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+13a\arcsin\left(\frac{1}{ax}\right)\right)}{a}+136\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+a^5x\right.\right.}{a^7}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c^4\left(a^2\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{21\left(13a\arcsin\left(\frac{1}{ax}\right)-2a^3\int\frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+136\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}\right.\right.}{a^7}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c^4\left(a^2\left(-\frac{1}{3}a^2\left(\frac{1}{2}a^2\left(\frac{21\left(13a\arcsin\left(\frac{1}{ax}\right)-2a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a}+136\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{21a\sqrt{1-\frac{1}{a^2x^2}}}{2x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\right.\right.}{a^7}$$

input
```
Int[(c - c/(a*x))^4/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c^4*((-64*a^5*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] + a^2*(-1/3*(a^2*Sqrt
[1 - 1/(a^2*x^2)])/x^2 - a^5*Sqrt[1 - 1/(a^2*x^2)]*x - (a^2*((-21*a*Sqrt[1
 - 1/(a^2*x^2)])/(2*x) + (a^2*(136*Sqrt[1 - 1/(a^2*x^2)] + (21*(13*a*ArcSi
n[1/(a*x)] - 2*a*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/a))/2))/3)))/a^7)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.20

method	result
risch	$\frac{(ax+1)(136a^2x^2-21ax+2)c^4\sqrt{\frac{ax-1}{ax+1}}}{6x^3a^4}+\frac{\left(\frac{91a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}-\frac{7a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{64a^2\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}+a^3\sqrt{\right.}{a^4(ax-1)}$
default	$-\frac{\left(-138\sqrt{a^2x^2-1}\sqrt{a^2}a^6x^6+138(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}a^4x^4-549\sqrt{a^2}\sqrt{a^2x^2-1}a^5x^5-273\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}a^5x^5+138\ln\left(\frac{a^2x}{\right.}\right.}{}$

input
```
int((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x+1)*(136*a^2*x^2-21*a*x+2)/x^3*c^4/a^4*((a*x-1)/(a*x+1))^(1/2)+(91
/2*a^3*arctan(1/(a^2*x^2-1)^(1/2))-7*a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^
(1/2))/(a^2)^(1/2)+64*a^2/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+a^3*((
a*x-1)*(a*x+1))^(1/2))*c^4/a^4/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a
*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\frac{546a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+42a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-42a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(6a^4c^4x^4+}{6a^4x^3}$$

input
```
integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(546*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 42*a^3*c^4*x^3*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1) - 42*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) - (6*a^4*c^4*x^4 + 526*a^3*c^4*x^3 + 115*a^2*c^4*x^2 - 19*a*c^
4*x + 2*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^5+x^4}\right)dx+\int\frac{5a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}dx+\int\left(-\frac{10a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\right)dx+\int\frac{10a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}dx+\int\right.}{a^4}$$

input
```
integrate((c-c/a/x)**4*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c**4*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + In
tegral(5*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integra
l(-10*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integra
l(10*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-5
*a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**5*x*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.55

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\frac{1}{3}\left(\frac{273\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{21\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{21\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{192\,c^4\sqrt{\frac{ax-1}{ax+1}}}{a^2}+\frac{15}{}\right.$$

input
```
integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-1/3*(273*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 21*c^4*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 - 21*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2
- 192*c^4*sqrt((a*x - 1)/(a*x + 1))/a^2 + (153*c^4*((a*x - 1)/(a*x + 1))^(
7/2) + 91*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 169*c^4*((a*x - 1)/(a*x + 1))^
(3/2) - 123*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)*a^2/(a*x + 1) - 2*
(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 + a^2))*a
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\int\left(c-\frac{c}{ax}\right)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.33

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx &= \frac{41\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{169\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-\frac{91\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-51\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}} \\ &\quad+\frac{64\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{91\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,14{}\mathrm{i}}{a}\end{aligned}$$

input
```
int((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(41*c^4*((a*x - 1)/(a*x + 1))^(1/2) + (169*c^4*((a*x - 1)/(a*x + 1))^(3/2)
)/3 - (91*c^4*((a*x - 1)/(a*x + 1))^(5/2))/3 - 51*c^4*((a*x - 1)/(a*x + 1)
)^(7/2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 -
(a*(a*x - 1)^4)/(a*x + 1)^4) + (64*c^4*((a*x - 1)/(a*x + 1))^(1/2))/a - (9
1*c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (c^4*atan(((a*x - 1)/(a*x + 1
))^(1/2)*1i)*14i)/a
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.70

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$= \frac{c^4\left(-546atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^4x^4-546atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^3x^3+546atan\left(\sqrt{ax}\right.\right.}{}$$

input
```
int((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(c**4*( - 546*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**4*x**4 - 546*atan
(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**3*x**3 + 546*atan(sqrt(a*x - 1) + s
qrt(a*x + 1) + 1)*a**4*x**4 + 546*atan(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*
a**3*x**3 + 6*sqrt(a*x + 1)*sqrt(a*x - 1)*a**4*x**4 + 526*sqrt(a*x + 1)*sq
rt(a*x - 1)*a**3*x**3 + 115*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 - 19*sqr
t(a*x + 1)*sqrt(a*x - 1)*a*x + 2*sqrt(a*x + 1)*sqrt(a*x - 1) - 84*log((sqr
t(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**4*x**4 - 84*log((sqrt(a*x - 1) + s
qrt(a*x + 1))/sqrt(2))*a**3*x**3 + 284*a**4*x**4 + 284*a**3*x**3))/(6*a**4
*x**3*(a*x + 1))
```

3.445 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=\frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\sqrt{1-\frac{1}{a^2x^2}}x+\frac{33c^3\csc^{-1}(ax)}{2a}-\frac{6c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
6*c^3*(1-1/a^2/x^2)^(1/2)/a+32*c^3*(a-1/x)/a^2/(1-1/a^2/x^2)^(1/2)-1/2*c^3
*(1-1/a^2/x^2)^(1/2)/a^2/x+c^3*(1-1/a^2/x^2)^(1/2)*x+33/2*c^3*arccsc(a*x)/
a-6*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.54 (sec) , antiderivative size = 663, normalized size of antiderivative = 4.91

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$$

$$= \frac{c^3\left(420a^2\sqrt{1+\frac{1}{ax}}x^2 - 3465a^3\sqrt{1+\frac{1}{ax}}x^3 + 16800a^4\sqrt{1+\frac{1}{ax}}x^4 + 17955a^5\sqrt{1+\frac{1}{ax}}x^5 - 32340a^6\sqrt{1+\right.}{}$$

input
```
Integrate[(c - c/(a*x))^3/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^3*(420*a^2*Sqrt[1 + 1/(a*x)]*x^2 - 3465*a^3*Sqrt[1 + 1/(a*x)]*x^3 + 168
00*a^4*Sqrt[1 + 1/(a*x)]*x^4 + 17955*a^5*Sqrt[1 + 1/(a*x)]*x^5 - 32340*a^6
*Sqrt[1 + 1/(a*x)]*x^6 + 630*a^7*Sqrt[1 + 1/(a*x)]*x^7 + 44730*a^5*Sqrt[1
 - 1/(a*x)]*x^5*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 44730*a^6*Sqrt[1 - 1/(a
*x)]*x^6*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2520*a^5*Sqrt[1 - 1/(a*x)]*x^
5*ArcSin[1/(a*x)] - 2520*a^6*Sqrt[1 - 1/(a*x)]*x^6*ArcSin[1/(a*x)] - 3780*
a^6*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^6*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]] + 126*Sqrt[2]*a^2*x^2*(-1 + a*x)^3*(1 + a*x)*Hypergeometric2F1[3/2, 5
/2, 7/2, (1 - 1/(a*x))/2] + 90*Sqrt[2]*a*x*(-1 + a*x)^4*(1 + a*x)*Hypergeo
metric2F1[3/2, 7/2, 9/2, (1 - 1/(a*x))/2] - 70*Sqrt[2]*Hypergeometric2F1[3
/2, 9/2, 11/2, (1 - 1/(a*x))/2] + 280*Sqrt[2]*a*x*Hypergeometric2F1[3/2, 9
/2, 11/2, (1 - 1/(a*x))/2] - 350*Sqrt[2]*a^2*x^2*Hypergeometric2F1[3/2, 9/
2, 11/2, (1 - 1/(a*x))/2] + 350*Sqrt[2]*a^4*x^4*Hypergeometric2F1[3/2, 9/2
, 11/2, (1 - 1/(a*x))/2] - 280*Sqrt[2]*a^5*x^5*Hypergeometric2F1[3/2, 9/2,
 11/2, (1 - 1/(a*x))/2] + 70*Sqrt[2]*a^6*x^6*Hypergeometric2F1[3/2, 9/2, 1
1/2, (1 - 1/(a*x))/2]))/(630*a^6*Sqrt[1 - 1/(a*x)]*x^5*(1 + a*x))
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6731, 27, 528, 2338, 2340, 27, 2340, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^3 e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{c^6\left(a-\frac{1}{x}\right)^6x^2}{a^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^3\int \frac{\left(a-\frac{1}{x}\right)^6x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^6}$$

$$\Big\downarrow \text{ 528}$$

$$-\frac{c^3\left(a^2\int \frac{\left(a^4-\frac{6a^3}{x}-\frac{16a^2}{x^2}+\frac{6a}{x^3}-\frac{1}{x^4}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{32a^4\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^6}$$

$$\Big\downarrow \text{ 2338}$$

$$-\frac{c^3\left(a^2\left(a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int \frac{\left(6a^3+\frac{16a^2}{x}-\frac{6a}{x^2}+\frac{1}{x^3}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-\frac{32a^4\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^6}$$

$$\Big\downarrow \text{ 2340}$$

$$-\frac{c^3\left(a^2\left(\frac{1}{2}a^2\int -\frac{3\left(4a+\frac{11}{x}-\frac{4}{x^2a}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{32a^4\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^6}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^3\left(a^2\left(-\frac{3}{2}a^2\int \frac{\left(4a+\frac{11}{x}-\frac{4}{x^2a}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{32a^4\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^6}$$

$\downarrow$ 2340

$$-\frac{c^3\left(a^2\left(-\frac{3}{2}a^2\left(4a\sqrt{1-\frac{1}{a^2x^2}}-a^2\int -\frac{\left(4a+\frac{11}{x}\right)x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{32a^4\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^6}$$

$\downarrow$ 25

$$-\frac{c^3\left(a^2\left(-\frac{3}{2}a^2\left(a^2\int \frac{\left(4a+\frac{11}{x}\right)x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+4a\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{32a^4\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^6}$$

$\downarrow$ 27

$$-\frac{c^3\left(a^2\left(-\frac{3}{2}a^2\left(\int \frac{\left(4a+\frac{11}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+4a\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{32a^4\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^6}$$

$\downarrow$ 538

$$-\frac{c^3\left(a^2\left(-\frac{3}{2}a^2\left(4a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+11\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+4a\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\cdots\right)}{a^6}$$

$\downarrow$ 223

$$-\frac{c^3\left(a^2\left(-\frac{3}{2}a^2\left(4a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+4a\sqrt{1-\frac{1}{a^2x^2}}+11a\arcsin\left(\frac{1}{ax}\right)\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\cdots\right)}{a^6}$$

$\downarrow$ 243

$$-\frac{c^3\left(a^2\left(-\frac{3}{2}a^2\left(2a\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+4a\sqrt{1-\frac{1}{a^2x^2}}+11a\arcsin\left(\frac{1}{ax}\right)\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\cdots\right)}{a^6}$$

$\downarrow$ 73

$$-\frac{c^3\left(a^2\left(-\frac{3}{2}a^2\left(-4a^3\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}+4a\sqrt{1-\frac{1}{a^2x^2}}+11a\arcsin\left(\frac{1}{ax}\right)\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{\cdots}\right.\cdots\right.\right.}{a^6}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c^3\left(a^2\left(-\frac{3}{2}a^2\left(-4a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+4a\sqrt{1-\frac{1}{a^2x^2}}+11a\arcsin\left(\frac{1}{ax}\right)\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+a^4x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right.\right.\right.}{a^6}$$

input
```
Int[(c - c/(a*x))^3/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c^3*((-32*a^4*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] + a^2*((a^2*Sqrt[1 -
1/(a^2*x^2)])/(2*x) - a^4*Sqrt[1 - 1/(a^2*x^2)]*x - (3*a^2*(4*a*Sqrt[1 - 1
/(a^2*x^2)] + 11*a*ArcSin[1/(a*x)] - 4*a*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/
2))/a^6)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.34

method	result
risch	$\frac{(ax+1)(12ax-1)c^3\sqrt{\frac{ax-1}{ax+1}}}{2x^2a^3}+\frac{\left(\frac{33a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}-\frac{6a^3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{32a\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}+a^2\sqrt{(ax-1)(ax\cdots}\right.}{a^3(ax-1)}$
default	$-\frac{\left(-12\sqrt{a^2}\sqrt{a^2x^2-1}\,a^5x^5+12\sqrt{a^2}\left(a^2x^2-1\right)^{\frac{3}{2}}a^3x^3-57\sqrt{a^2}\sqrt{a^2x^2-1}\,a^4x^4-33a^4\sqrt{a^2}\,x^4\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+12\ln\left(\frac{a^2x+\sqrt{a^2\cdots}}{\sqrt{\cdots}}\right.\right.}{\cdots}$

input
```
int((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x+1)*(12*a*x-1)/x^2*c^3/a^3*((a*x-1)/(a*x+1))^(1/2)+(33/2*a^2*arcta
n(1/(a^2*x^2-1)^(1/2))-6*a^3*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)
^(1/2)+32*a/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+a^2*((a*x-1)*(a*x+1)
)^(1/2))*c^3/a^3/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{66\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+12\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-12\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(2\,a^3c^3x^3+\cdots}{2\,a^3x^2}$$

input
```
integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(66*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + 12*a^2*c^3*x^2*lo
g(sqrt((a*x - 1)/(a*x + 1)) + 1) - 12*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) - (2*a^3*c^3*x^3 + 78*a^2*c^3*x^2 + 11*a*c^3*x - c^3)*sqrt((a*x
 - 1)/(a*x + 1)))/(a^3*x^2)
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = \frac{c^3\left(\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\left(-\frac{4a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\right)dx+\int\frac{6a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(-\frac{4a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^4}{}\right.}{a^3}$$

input
```
integrate((c-c/a/x)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Int
egral(-4*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integra
l(6*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-4*
a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**4*x*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.67

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\left(\frac{33\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{6\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{6\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{32\,c^3\sqrt{\frac{ax-1}{ax+1}}}{a^2}+\frac{11\,c^3\left(\frac{ax-}{ax+}\right.}{\frac{(ax-1)}{ax+1}}\right.$$

input
```
integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-(33*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 6*c^3*log(sqrt((a*x - 1)/
(a*x + 1)) + 1)/a^2 - 6*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 32*c^
3*sqrt((a*x - 1)/(a*x + 1))/a^2 + (11*c^3*((a*x - 1)/(a*x + 1))^(5/2) - 6*
c^3*((a*x - 1)/(a*x + 1))^(3/2) - 13*c^3*sqrt((a*x - 1)/(a*x + 1)))/((a*x
- 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - (a*x - 1)^3*a^2/(a*x +
1)^3 + a^2))*a
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx=\int \left(c-\frac{c}{ax}\right)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

input
```
integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 13.51 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.41

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx=\frac{13\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}+6\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-11\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a+\frac{a\,(a\,x-1)}{a\,x+1}-\frac{a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}-\frac{a\,{(a\,x-1)}^3}{{(a\,x+1)}^3}}+\frac{32\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{33\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,12{}\mathrm{i}}{a}$$

input
```
int((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(13*c^3*((a*x - 1)/(a*x + 1))^(1/2) + 6*c^3*((a*x - 1)/(a*x + 1))^(3/2) -
11*c^3*((a*x - 1)/(a*x + 1))^(5/2))/(a + (a*(a*x - 1))/(a*x + 1) - (a*(a*x
 - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (32*c^3*((a*x - 1)/(
a*x + 1))^(1/2))/a - (33*c^3*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (c^3*a
tan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*12i)/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.86

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$$

$$= \frac{c^3\left(-66atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^3x^3-66atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^2x^2+66atan\left(\sqrt{ax-\right.\right.}{}$$

input
```
int((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(c**3*( - 66*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**3*x**3 - 66*atan(s
qrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**2*x**2 + 66*atan(sqrt(a*x - 1) + sqrt
(a*x + 1) + 1)*a**3*x**3 + 66*atan(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**2
*x**2 + 2*sqrt(a*x + 1)*sqrt(a*x - 1)*a**3*x**3 + 78*sqrt(a*x + 1)*sqrt(a*
x - 1)*a**2*x**2 + 11*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x - sqrt(a*x + 1)*sqrt
(a*x - 1) - 24*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**3*x**3 - 24
*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**2*x**2 + 53*a**3*x**3 + 5
3*a**2*x**2))/(2*a**3*x**2*(a*x + 1))
```

3.446 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^2\sqrt{1-\frac{1}{a^2x^2}}x+\frac{5c^2\csc^{-1}(ax)}{a}-\frac{5c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
c^2*(1-1/a^2/x^2)^(1/2)/a+16*c^2*(a-1/x)/a^2/(1-1/a^2/x^2)^(1/2)+c^2*(1-1/
a^2/x^2)^(1/2)*x+5*c^2*arccsc(a*x)/a-5*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 424, normalized size of antiderivative = 4.04

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$$

$$=\frac{c^2\left(-35a^2\sqrt{1+\frac{1}{ax}}x^2+315a^3\sqrt{1+\frac{1}{ax}}x^3+280a^4\sqrt{1+\frac{1}{ax}}x^4-595a^5\sqrt{1+\frac{1}{ax}}x^5+35a^6\sqrt{1+\frac{1}{ax}}x^6+9\right.}{}$$

input
```
Integrate[(c - c/(a*x))^2/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^2*(-35*a^2*Sqrt[1 + 1/(a*x)]*x^2 + 315*a^3*Sqrt[1 + 1/(a*x)]*x^3 + 280*
a^4*Sqrt[1 + 1/(a*x)]*x^4 - 595*a^5*Sqrt[1 + 1/(a*x)]*x^5 + 35*a^6*Sqrt[1
+ 1/(a*x)]*x^6 + 910*a^4*Sqrt[1 - 1/(a*x)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sq
rt[2]] + 910*a^5*Sqrt[1 - 1/(a*x)]*x^5*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] -
 105*a^4*Sqrt[1 - 1/(a*x)]*x^4*ArcSin[1/(a*x)] - 105*a^5*Sqrt[1 - 1/(a*x)]
*x^5*ArcSin[1/(a*x)] - 175*a^5*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^5
*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]] + 7*Sqrt[2]*a*x*(-1 + a*x)^3*(1 + a*x)*Hyp
ergeometric2F1[3/2, 5/2, 7/2, (1 - 1/(a*x))/2] + 5*Sqrt[2]*(-1 + a*x)^4*(1
 + a*x)*Hypergeometric2F1[3/2, 7/2, 9/2, (1 - 1/(a*x))/2]))/(35*a^5*Sqrt[1
 - 1/(a*x)]*x^4*(1 + a*x))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6731, 27, 528, 2338, 2340, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^2 e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{c^5\left(a-\frac{1}{x}\right)^5x^2}{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\int \frac{\left(a-\frac{1}{x}\right)^5x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^5}$$

$$\Big\downarrow \text{528}$$

$$-\frac{c^2\left(a^2\int\frac{\left(a^3-\frac{5a^2}{x}-\frac{5a}{x^2}+\frac{1}{x^3}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{16a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^5}$$

$\Big\downarrow$ 2338

$$-\frac{c^2\left(a^2\left(a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int\frac{\left(5a^2+\frac{5a}{x}-\frac{1}{x^2}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-\frac{16a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^5}$$

$\Big\downarrow$ 2340

$$-\frac{c^2\left(a^2\left(a^2\int-\frac{5\left(a+\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{16a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^5}$$

$\Big\downarrow$ 27

$$-\frac{c^2\left(a^2\left(-5a\int\frac{\left(a+\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{16a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^5}$$

$\Big\downarrow$ 538

$$-\frac{c^2\left(a^2\left(-5a\left(\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{16a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^5}$$

$\Big\downarrow$ 223

$$-\frac{c^2\left(a^2\left(-5a\left(a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a\arcsin\left(\frac{1}{ax}\right)\right)-a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{16a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^5}$$

$\Big\downarrow$ 243

$$-\frac{c^2\left(a^2\left(-5a\left(\frac{1}{2}a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+a\arcsin\left(\frac{1}{ax}\right)\right)-a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{16a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^5}$$

$\Big\downarrow$ 73

$$-\frac{c^2\left(a^2\left(-5a\left(a\arcsin\left(\frac{1}{ax}\right)-a^3\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}\right)-a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{16a^3(a}{\sqrt{1-}}\right.}{a^5}$$

$$\Big\downarrow \; 221$$

$$-\frac{c^2\left(a^2\left(-5a\left(a\arcsin\left(\frac{1}{ax}\right)-a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-a^2\sqrt{1-\frac{1}{a^2x^2}}+a^3x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{16a^3\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^5}$$

input
```
Int[(c - c/(a*x))^2/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c^2*((-16*a^3*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] + a^2*(-(a^2*Sqrt[1 -
 1/(a^2*x^2)]) - a^3*Sqrt[1 - 1/(a^2*x^2)]*x - 5*a*(a*ArcSin[1/(a*x)] - a*
ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))))/a^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.61

method	result
risch	$\frac{(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{x\,a^2}+\frac{\left(a\sqrt{(ax-1)(ax+1)}+5a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-\frac{5a^2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{16\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}\right)c^2\sqrt{\frac{ax-}{ax}}$ ...
default	$-\frac{\left(-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^4x^4-4\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^3x^3+\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^2x^2-7\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^3x^3+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right.}{}$ ...

input
```
int((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x+1)/x*c^2/a^2*((a*x-1)/(a*x+1))^(1/2)+(a*((a*x-1)*(a*x+1))^(1/2)+5*a*a
rctan(1/(a^2*x^2-1)^(1/2))-5*a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(
a^2)^(1/2)+16/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*c^2/a^2/(a*x-1)*(
(a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\frac{10\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+5\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-5\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2c^2x^2+18\,ac^2x+\cdots}{a^2x}$$

input
```
integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-(10*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + 5*a*c^2*x*log(sqrt((a*x -
 1)/(a*x + 1)) + 1) - 5*a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*
c^2*x^2 + 18*a*c^2*x + c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$$
$$=\frac{c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\right)dx+\int\frac{3a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(-\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^3x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a^2}$$

input
```
integrate((c-c/a/x)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + In
tegral(3*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-
3*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x*s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.42

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=$$
$$-\left(\frac{4\,c^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{10\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{5\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{5\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{16\,c^2\sqrt{}}{a^2}\right.$$

input
```
integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-(4*c^2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 10
*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 5*c^2*log(sqrt((a*x - 1)/(a*x
 + 1)) + 1)/a^2 - 5*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 16*c^2*sq
rt((a*x - 1)/(a*x + 1))/a^2)*a
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\int\left(c-\frac{c}{ax}\right)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{16\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}+\frac{4\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)^2}{(a\,x+1)^2}}-\frac{10\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,10\mathrm{i}}{a}$$

input
```
int((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(16*c^2*((a*x - 1)/(a*x + 1))^(1/2))/a + (4*c^2*((a*x - 1)/(a*x + 1))^(1/2
))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (10*c^2*atan(((a*x - 1)/(a*x + 1))^
(1/2)))/a + (c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*10i)/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.02

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$$
$$=\frac{c^2\left(-10atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^2x^2-10atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)ax+10atan\left(\sqrt{ax-1}\right.\right.}{}$$

input
```
int((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(c**2*( - 10*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**2*x**2 - 10*atan(s
qrt(a*x - 1) + sqrt(a*x + 1) - 1)*a*x + 10*atan(sqrt(a*x - 1) + sqrt(a*x +
 1) + 1)*a**2*x**2 + 10*atan(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a*x + sqrt
(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 + 18*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + s
qrt(a*x + 1)*sqrt(a*x - 1) - 10*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2
))*a**2*x**2 - 10*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x + 15*a*
*2*x**2 + 15*a*x))/(a**2*x*(a*x + 1))
```

3.447 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 75

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c\sqrt{1-\frac{1}{a^2x^2}}x+\frac{c\csc^{-1}(ax)}{a}-\frac{4c\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
8*c*(a-1/x)/a^2/(1-1/a^2/x^2)^(1/2)+c*(1-1/a^2/x^2)^(1/2)*x+c*arccsc(a*x)/
a-4*c*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.56 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.12

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$$
$$=\frac{5a^2cx^2\left((1+ax)\left(\sqrt{1+\frac{1}{ax}}(2-3ax+a^2x^2)+6a\sqrt{1-\frac{1}{ax}}x\arcsin\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-2a\sqrt{1-\frac{1}{ax}}x\arcsin\left(\frac{1}{ax}\right.\right.\right.}{5a}$$

input
```
Integrate[(c - c/(a*x))/E^(3*ArcCoth[a*x]),x]
```

output
```
(5*a^2*c*x^2*((1 + a*x)*(Sqrt[1 + 1/(a*x)]*(2 - 3*a*x + a^2*x^2) + 6*a*Sqr
t[1 - 1/(a*x)]*x*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*a*Sqrt[1 - 1/(a*x)]
*x*ArcSin[1/(a*x)]) - 4*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^2*Ar
cTanh[Sqrt[1 - 1/(a^2*x^2)]]) + Sqrt[2]*c*(-1 + a*x)^3*(1 + a*x)*Hypergeom
etric2F1[3/2, 5/2, 7/2, (1 - 1/(a*x))/2])/(5*a^4*Sqrt[1 - 1/(a*x)]*x^3*(1
+ a*x))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6731, 27, 528, 2338, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right) e^{-3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{c^4\left(a-\frac{1}{x}\right)^4 x^2}{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\int \frac{\left(a-\frac{1}{x}\right)^4 x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^4}$$

$$\Big\downarrow \text{528}$$

$$-\frac{c\left(a^2\int \frac{\left(a^2-\frac{4a}{x}-\frac{1}{x^2}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{8a^2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^4}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{c\left(a^2\left(a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-\int\frac{\left(4a+\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)-\frac{8a^2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^4}$$

$$\Big\downarrow \text{538}$$

$$-\frac{c\left(a^2\left(-4a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)\right)-\frac{8a^2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^4}$$

$$\Big\downarrow \text{223}$$

$$-\frac{c\left(a^2\left(-4a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-a\arcsin\left(\frac{1}{ax}\right)\right)-\frac{8a^2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^4}$$

$$\Big\downarrow \text{243}$$

$$-\frac{c\left(a^2\left(-2a\int\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-a\arcsin\left(\frac{1}{ax}\right)\right)-\frac{8a^2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c\left(a^2\left(4a^3\int\frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-a^2x\sqrt{1-\frac{1}{a^2x^2}}-a\arcsin\left(\frac{1}{ax}\right)\right)-\frac{8a^2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^4}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c\left(a^2\left(4a\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+a^2x\left(-\sqrt{1-\frac{1}{a^2x^2}}\right)-a\arcsin\left(\frac{1}{ax}\right)\right)-\frac{8a^2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{a^4}$$

input
```
Int[(c - c/(a*x))/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c*((-8*a^2*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] + a^2*(-(a^2*Sqrt[1 - 1/
(a^2*x^2)]*x) - a*ArcSin[1/(a*x)] + 4*a*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])))/
a^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(69) = 138$.

Time = 0.10 (sec) , antiderivative size = 376, normalized size of antiderivative = 5.01

method	result
default	$-\frac{\left(-4\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^2x^2-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^2x^2-a^2\sqrt{a^2}\,x^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+4\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2+4\right.}{}$

input
```
int((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-4*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*a^2*x^2-(a^2)^(1/2)*(a^2*x^2-1)^(
1/2)*a^2*x^2-a^2*(a^2)^(1/2)*x^2*arctan(1/(a^2*x^2-1)^(1/2))+4*ln((a^2*x+(
(a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^3*x^2+4*((a*x-1)*(a*x+1
))^(3/2)*(a^2)^(1/2)-8*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*a*x-2*(a^2)^(1/
2)*(a^2*x^2-1)^(1/2)*a*x-2*a*(a^2)^(1/2)*x*arctan(1/(a^2*x^2-1)^(1/2))+8*l
n((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x-4*((a*x-1
)*(a*x+1))^(1/2)*(a^2)^(1/2)-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)-arctan(1/(a^2*x
^2-1)^(1/2))*(a^2)^(1/2)+4*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)
)/(a^2)^(1/2)))/a*c*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x-1)*(a*x+1))^
(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = -\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+4\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-4\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(acx+9\,c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

input
```
integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-(2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) + 4*c*log(sqrt((a*x - 1)/(a*x + 1)
) + 1) - 4*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a*c*x + 9*c)*sqrt((a*x
- 1)/(a*x + 1)))/a
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = \frac{c\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(-\frac{2a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^2x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a}$$

input
```
integrate((c-c/a/x)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(
-2*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**2*x*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.80

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx =$$
$$-2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

input
```
integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + c*arct
an(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 2*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)
/a^2 - 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 4*c*sqrt((a*x - 1)/(a*
x + 1))/a^2)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\int\left(c-\frac{c}{ax}\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{8\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}+\frac{c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,8{}\mathrm{i}}{a}$$

input
```
int((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) - (2*c*ata
n(((a*x - 1)/(a*x + 1))^(1/2)))/a + (c*atan(((a*x - 1)/(a*x + 1))^(1/2)*1i
)*8i)/a + (8*c*((a*x - 1)/(a*x + 1))^(1/2))/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.20

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$$
$$=\frac{c\left(-4atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)ax-4atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)+4atan\left(\sqrt{ax-1}+\sqrt{ax+\right.\right.}{}$$

input
```
int((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(c*( - 4*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a*x - 4*atan(sqrt(a*x - 1
) + sqrt(a*x + 1) - 1) + 4*atan(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a*x + 4
*atan(sqrt(a*x - 1) + sqrt(a*x + 1) + 1) + 2*sqrt(a*x + 1)*sqrt(a*x - 1)*a
*x + 18*sqrt(a*x + 1)*sqrt(a*x - 1) - 16*log((sqrt(a*x - 1) + sqrt(a*x + 1
))/sqrt(2))*a*x - 16*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 17*a*x
 + 17))/(2*a*(a*x + 1))
```

3.448 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}-\frac{2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
2*(a-1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)+(1-1/a^2/x^2)^(1/2)*x/c-2*arctanh((1-1
/a^2/x^2)^(1/2))/a/c
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(3+ax)-2(1+ax)\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a(c+acx)}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))),x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(3 + a*x) - 2*(1 + a*x)*Log[(1 + Sqrt[1 - 1/(a^
2*x^2)])*x]])/(a*(c + a*c*x))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6731, 27, 528, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{c^2\left(a-\frac{1}{x}\right)^2x^2}{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(a-\frac{1}{x}\right)^2x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^2c}$$

$$\Big\downarrow \text{528}$$

$$-\frac{a^2\int \frac{\left(a-\frac{2}{x}\right)x^2}{a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}}{a^2c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{\left(a-\frac{2}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}}{a^2c}$$

$$\Big\downarrow \text{534}$$

$$-\frac{a\left(-2\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}}{a^2c}$$

$$\Big\downarrow \text{243}$$

$$-\frac{a\left(-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}}{a^2c}$$

$$\Big\downarrow \; 73$$

$$-\frac{a\left(2a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}}{a^2c}$$

$$\Big\downarrow \; 221$$

$$-\frac{a\left(2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{2\left(a-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}}{a^2c}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))),x]
```

output
```
-(((-2*(a - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)] + a*(-(a*Sqrt[1 - 1/(a^2*x^2)]*
x) + 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(a^2*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(66) = 132$.

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.90

method	result
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}+\frac{\left(-\frac{2\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a\sqrt{a^{2}}}+\frac{2\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{a^{3}\left(x+\frac{1}{a}\right)}\right)a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c(ax-1)}$
default	$-\frac{\left(2\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{3}x^{2}-2\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,a^{2}x^{2}+4\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{2}x+((ax-1)(ax+1))^{\frac{3}{2}}\cdots}{a\sqrt{a^{2}}\,c\sqrt{(ax-1)(ax+1)}\,(ax\cdots}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

output
```
1/a*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/c+(-2/a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-
1)^(1/2))/(a^2)^(1/2)+2/a^3/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a/c
*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{(ax+3)\sqrt{\frac{ax-1}{ax+1}} - 2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + 2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="fricas")
```

output
```
((a*x + 3)*sqrt((a*x - 1)/(a*x + 1)) - 2*log(sqrt((a*x - 1)/(a*x + 1)) + 1
) + 2*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{a\left(\int \left(-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\right)dx + \int \frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx\right)}{c}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x),x)
```

output
```
a*(Integral(-x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x) + Int
egral(a*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x))/c
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.67

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -2a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c} - \frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="maxima")
```

output
```
-2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) + log(
sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x + 1)) - 1
)/(a^2*c) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{c-\frac{c}{ax}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}}+\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}-\frac{4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x)),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1)) + (2*((a
*x - 1)/(a*x + 1))^(1/2))/(a*c) - (4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(
a*c)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.32

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$$
$$=\frac{2\sqrt{ax+1}\,\sqrt{ax-1}\,ax+6\sqrt{ax+1}\,\sqrt{ax-1}-8\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax-8\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)+5ax+5}{2ac\,(ax+1)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x)
```

output
```
(2*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 6*sqrt(a*x + 1)*sqrt(a*x - 1) - 8*log
((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x - 8*log((sqrt(a*x - 1) + sqr
t(a*x + 1))/sqrt(2)) + 5*a*x + 5)/(2*a*c*(a*x + 1))
```

3.449 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

output
```
(a-1/x)/a^2/c^2/(1-1/a^2/x^2)^(1/2)+(1-1/a^2/x^2)^(1/2)*x/c^2-arctanh((1-1
/a^2/x^2)^(1/2))/a/c^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{-2+ax+a^2x^2-a\sqrt{1-\frac{1}{a^2x^2}}\,x\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

output
```
(-2 + a*x + a^2*x^2 - a*Sqrt[1 - 1/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]])/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6731, 27, 528, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{c\left(a-\frac{1}{x}\right)x^2}{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(a-\frac{1}{x}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{ac^2}$$

$$\Big\downarrow \text{528}$$

$$-\frac{a^2\int \frac{\left(a-\frac{1}{x}\right)x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a-\frac{1}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}}{ac^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(a-\frac{1}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a-\frac{1}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}}{ac^2}$$

$$\Big\downarrow \text{534}$$

$$-\frac{-\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a-\frac{1}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}-ax\sqrt{1-\frac{1}{a^2x^2}}}{ac^2}$$

$$\Big\downarrow \text{243}$$

$$-\frac{-\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-\frac{a-\frac{1}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}-ax\sqrt{1-\frac{1}{a^2x^2}}}{ac^2}$$

$$\Big\downarrow \; 73$$

$$-\frac{a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}-\frac{a-\frac{1}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}}{ac^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-\frac{a-\frac{1}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}-ax\sqrt{1-\frac{1}{a^2x^2}}}{ac^2}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

output
```
-((-((a - x^(-1))/(a*Sqrt[1 - 1/(a^2*x^2)])) - a*Sqrt[1 - 1/(a^2*x^2)]*x +
 ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(65) = 130$.

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.94

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2}+\frac{\left(-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}+\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^4\left(x+\frac{1}{a}\right)}\right)a^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c^2(ax-1)}$
default	$-\frac{\left(-3\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^2x^2+2\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-6\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,ax+4}{2a\sqrt{a^2}\,c^2\sqrt{(ax-1)(ax+1)}}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^2*((a*x-1)/(a*x+1))^(1/2)+(-1/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)+1/a^4/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))
*a^2/c^2*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{(ax+2)\sqrt{\frac{ax-1}{ax+1}} - \log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + \log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
((a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)
+ log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c^2)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\left(\int\left(-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\right)dx + \int\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\,dx\right)}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**2,x)
```

output
```
a**2*(Integral(-x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**2*x
**2 - a*x + 1), x) + Integral(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*
*3*x**3 - a**2*x**2 - a*x + 1), x))/c**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.76

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c^2}{ax+1}-a^2c^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
-a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c^2/(a*x + 1) - a^2*c^2) +
log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - log(sqrt((a*x - 1)/(a*x + 1
)) - 1)/(a^2*c^2) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 13.64 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2-\frac{a\,c^2\,(a\,x-1)}{a\,x+1}}+\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^2,x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c^2 - (a*c^2*(a*x - 1))/(a*x + 1)) + ((
a*x - 1)/(a*x + 1))^(1/2)/(a*c^2) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))
/(a*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.34

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{2\sqrt{ax+1}\,\sqrt{ax-1}\,ax+4\sqrt{ax+1}\,\sqrt{ax-1}-4\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax-4\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)+3ax+3}{2a\,c^2\,(ax+1)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x)
```

output
```
(2*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 4*sqrt(a*x + 1)*sqrt(a*x - 1) - 4*log
((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x - 4*log((sqrt(a*x - 1) + sqr
t(a*x + 1))/sqrt(2)) + 3*a*x + 3)/(2*a*c**2*(a*x + 1))
```

3.450 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 41

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=-\frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3}$$

output
```
-x/c^3/(1-1/a^2/x^2)^(1/2)+2*(1-1/a^2/x^2)^(1/2)*x/c^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=\frac{-2+a^2x^2}{a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

output
```
(-2 + a^2*x^2)/(a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6731, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{245}$$

$$-\frac{\frac{2\int \frac{1}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{208}$$

$$-\frac{\frac{2}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

output
```
-((2/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) - x/Sqrt[1 - 1/(a^2*x^2)])/c^3)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

method	result	size
trager	$\frac{(a^2x^2-2)\sqrt{-\frac{-ax+1}{ax+1}}}{a\,c^3(ax-1)}$	41
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(a^2x^2-2)(ax+1)}{a(ax-1)^2c^3}$	44
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(a^3x^3+a^2x^2-2ax-2)}{a(ax-1)^2c^3}$	50
orering	$\frac{(a^2x^2-2)(ax-1)(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{a^4x^3\left(c-\frac{c}{ax}\right)^3}$	55
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3(ax-1)}$	59

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a/c^3*(a^2*x^2-2)/(a*x-1)*(-(-a*x+1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{(a^2x^2-2)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3x-ac^3}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

output
```
(a^2*x^2 - 2)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3*x - a*c^3)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{a^3\left(\int \left(-\frac{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\right)dx + \int \frac{ax^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\,dx\right)}{c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**3,x)
```

output
```
a**3*(Integral(-x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**3
*x**3 + 2*a*x - 1), x) + Integral(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))
/(a**4*x**4 - 2*a**3*x**3 + 2*a*x - 1), x))/c**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(37) = 74.

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.24

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{1}{2}a\left(\frac{\frac{5(ax-1)}{ax+1}-1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^3\sqrt{\frac{ax-1}{ax+1}}} - \frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

output
```
-1/2*a*((5*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2) -
 a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{\left(\frac{\sqrt{a^2x^2-1}}{c^3}-\frac{1}{\sqrt{a^2x^2-1}c^3}\right)\mathrm{sgn}(ax+1)}{a}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
```

output
```
(sqrt(a^2*x^2 - 1)/c^3 - 1/(sqrt(a^2*x^2 - 1)*c^3))*sgn(a*x + 1)/a
```

Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{a^2\,x^2-2}{\left(x\,a^2\,c^3+a\,c^3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^3,x)
```

output
```
(a^2*x^2 - 2)/((a*c^3 + a^2*c^3*x)*((a*x - 1)/(a*x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \frac{\sqrt{ax+1}\,(a^2x^2-2)}{\sqrt{ax-1}\,a\,c^3\,(ax+1)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x)
```

output
```
(sqrt(a*x + 1)*(a**2*x**2 - 2))/(sqrt(a*x - 1)*a*c**3*(a*x + 1))
```

3.451 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=-\frac{3a+\frac{4}{x}}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4}-\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

output
```
-1/3*(3*a+4/x)/a^2/c^4/(1-1/a^2/x^2)^(1/2)+4/3*(1-1/a^2/x^2)^(1/2)*x/c^4-1
/3*a*x/c^4/(1-1/a^2/x^2)^(1/2)/(a-1/x)+arctanh((1-1/a^2/x^2)^(1/2))/a/c^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\frac{8-5ax-7a^2x^2+3a^3x^3+3a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

output
```
(8 - 5*a*x - 7*a^2*x^2 + 3*a^3*x^3 + 3*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x
)*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 +
 a*x))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6731, 27, 569, 25, 532, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{ax^2}{c\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\int \frac{x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{ 569}$$

$$-\frac{a\left(\frac{x}{3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}-\frac{\int -\frac{\left(4a+\frac{3}{x}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{3a^2}\right)}{c^4}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a\left(\frac{\int \frac{\left(4a+\frac{3}{x}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{3a^2}+\frac{x}{3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)}{c^4}$$

$$\Big\downarrow \ 532$$

$$-\frac{a\left(\frac{\frac{3a+\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{\left(4a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{3a^2}+\frac{x}{3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)}{c^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{a\left(\frac{\int \frac{\left(4a+\frac{3}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{3a+\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}}{3a^2}+\frac{x}{3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)}{c^4}$$

$$\Big\downarrow \ 534$$

$$-\frac{a\left(\frac{3\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{3a+\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{x}{3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)}{c^4}$$

$$\Big\downarrow \ 243$$

$$-\frac{a\left(\frac{\frac{3}{2}\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}+\frac{3a+\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{x}{3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)}{c^4}$$

$$\Big\downarrow \ 73$$

$$-\frac{a\left(\frac{-3a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-4ax\sqrt{1-\frac{1}{a^2x^2}}+\frac{3a+\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}}{3a^2}+\frac{x}{3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)}{c^4}$$

$$\Big\downarrow \ 221$$

$$-\frac{a\left(\frac{-3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{3a+\frac{4}{x}}{a\sqrt{1-\frac{1}{a^2x^2}}}-4ax\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{x}{3\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}\right)}{c^4}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

output
```
-((a*(x/(3*Sqrt[1 - 1/(a^2*x^2)]*(a - x^(-1))) + ((3*a + 4/x)/(a*Sqrt[1 -
1/(a^2*x^2)]) - 4*a*Sqrt[1 - 1/(a^2*x^2)]*x - 3*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]])/(3*a^2)))/c^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 569
```
Int[((x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol] :
> Simp[(-x^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*p*(c + d*x))), x] + Simp[1/(2
*c^2*p)   Int[x^m*(a + b*x^2)^p*(c*(m + 2*p + 1) - d*(m + 2*p + 2)*x), x],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[m + 2*p, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(96) = 192$.

Time = 0.16 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.98

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^4}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{19\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{12a^6\left(x-\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{6a^7\left(x-\frac{1}{a}\right)^2}+\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{4a^6\left(x+\frac{1}{a}\right)}\right)a^4\sqrt{\frac{ax}{ax}}}{c^4(ax-1)}$
default	$\frac{\left(24\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^6x^5+45\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^5x^5-24\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^5x^4-21\sqrt{a^2}\,((ax-1)(}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^4*((a*x-1)/(a*x+1))^(1/2)+(1/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-19/12/a^6/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/
2)-1/6/a^7/(x-1/a)^2*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)+1/4/a^6/(x+1/a)*(a^
2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^4/c^4*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*((
a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.22

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-7\,a^2x^2-5\,ax\right.}{3\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

output
```
1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x
^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 7*a^2*x^
2 - 5*a*x + 8)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c
^4)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\left(\int\left(-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\right)dx+\int\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\,dx\right)}{c^4}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**4,x)
```

output
```
a**4*(Integral(-x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 3*a**4
*x**4 + 2*a**3*x**3 + 2*a**2*x**2 - 3*a*x + 1), x) + Integral(a*x**5*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 3*a**4*x**4 + 2*a**3*x**3 + 2*a*
*2*x**2 - 3*a*x + 1), x))/c**4
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.45

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$=\frac{1}{12}a\left(\frac{\frac{17\,(ax-1)}{ax+1}-\frac{42\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{12\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}+\frac{3\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

output
```
1/12*a*((17*(a*x - 1)/(a*x + 1) - 42*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^4
*((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2)) + 12*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 12*log(sqrt((a*x - 1)/(a*x +
 1)) - 1)/(a^2*c^4) + 3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^4))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx=\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^4}-\frac{\frac{17\,(a\,x-1)}{3\,(a\,x+1)}-\frac{14\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}+\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^4,x)
```

output
```
((a*x - 1)/(a*x + 1))^(1/2)/(4*a*c^4) - ((17*(a*x - 1))/(3*(a*x + 1)) - (1
4*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(4*a*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 4
*a*c^4*((a*x - 1)/(a*x + 1))^(5/2)) + (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)
))/(a*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.42

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx =\frac{24\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^2x^2-24\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)-11\sqrt{ax-1}\,a^2x^2+11\sqrt{ax-1}+}{12\sqrt{ax-1}\,a\,c^4\left(a^2x^2-1\right)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x)
```

output
```
(24*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**2*x**2 -
 24*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) - 11*sqrt(a
*x - 1)*a**2*x**2 + 11*sqrt(a*x - 1) + 12*sqrt(a*x + 1)*a**3*x**3 - 28*sqr
t(a*x + 1)*a**2*x**2 - 20*sqrt(a*x + 1)*a*x + 32*sqrt(a*x + 1))/(12*sqrt(a
*x - 1)*a*c**4*(a**2*x**2 - 1))
```

3.452 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 143

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = -\frac{2}{5c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)} - \frac{2\left(15a+\frac{14}{x}\right)}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{28\sqrt{1-\frac{1}{a^2x^2}}\,x}{15c^5} - \frac{\left(13a+\frac{10}{x}\right)x}{15ac^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^5}$$

output
```
-2/5/c^5/(1-1/a^2/x^2)^(3/2)/(a-1/x)-2/15*(15*a+14/x)/a^2/c^5/(1-1/a^2/x^2
)^(1/2)+28/15*(1-1/a^2/x^2)^(1/2)*x/c^5-1/15*(13*a+10/x)*x/a/c^5/(1-1/a^2/
x^2)^(3/2)+2*arctanh((1-1/a^2/x^2)^(1/2))/a/c^5
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.73

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = \frac{-56+82ax+32a^2x^2-76a^3x^3+15a^4x^4+30a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^5),x]
```

output
```
(-56 + 82*a*x + 32*a^2*x^2 - 76*a^3*x^3 + 15*a^4*x^4 + 30*a*Sqrt[1 - 1/(a^
2*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^5*Sqrt[1
- 1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6731, 27, 570, 532, 25, 2336, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{a^2x^2}{c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^2\int \frac{x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{c^5}$$

$$\Big\downarrow \text{ 570}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)^2x^2}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}d\frac{1}{x}}{a^2c^5}$$

$$\Big\downarrow \text{ 532}$$

$$-\frac{\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{1}{5}\int -\frac{\left(5a^2+\frac{10a}{x}+\frac{8}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}}{a^2c^5}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{1}{5}\int \frac{\left(5a^2+\frac{10a}{x}+\frac{8}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}d\frac{1}{x}+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

$\big\downarrow$ 2336

$$-\frac{\frac{1}{5}\left(\frac{10a+\frac{13}{x}}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{1}{3}\int -\frac{\left(15a^2+\frac{30a}{x}+\frac{26}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}\right)+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

$\big\downarrow$ 25

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{\left(15a^2+\frac{30a}{x}+\frac{26}{x^2}\right)x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{10a+\frac{13}{x}}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

$\big\downarrow$ 2336

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{30a+\frac{41}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}-\int -\frac{15a\left(a+\frac{2}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}\right)+\frac{10a+\frac{13}{x}}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

$\big\downarrow$ 27

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a\int \frac{\left(a+\frac{2}{x}\right)x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{30a+\frac{41}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{10a+\frac{13}{x}}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

$\big\downarrow$ 534

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a\left(2\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{30a+\frac{41}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{10a+\frac{13}{x}}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

$\big\downarrow$ 243

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a\left(\int \frac{x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x^2}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{30a+\frac{41}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{10a+\frac{13}{x}}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

$\big\downarrow$ 73

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a\left(-2a^2\int \frac{1}{a^2-a^2\sqrt{1-\frac{1}{a^2x^2}}}d\sqrt{1-\frac{1}{a^2x^2}}-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{30a+\frac{41}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{10a+\frac{13}{x}}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

$\big\downarrow$ 221

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(15a\left(-2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\right)+\frac{30a+\frac{41}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{10a+\frac{13}{x}}{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\right)+\frac{2\left(a+\frac{1}{x}\right)}{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}}{a^2c^5}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^5),x]
```

output
```
-(((2*(a + x^(-1)))/(5*(1 - 1/(a^2*x^2))^(5/2)) + ((10*a + 13/x)/(3*(1 - 1
/(a^2*x^2))^(3/2)) + ((30*a + 41/x)/Sqrt[1 - 1/(a^2*x^2)] + 15*a*(-(a*Sqrt
[1 - 1/(a^2*x^2)]*x) - 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/3)/5)/(a^2*c^5))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(125) = 250.

Time = 0.17 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.81

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^5}+\frac{\left(\frac{2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^5\sqrt{a^2}}-\frac{383\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{120a^7\left(x-\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{10a^9\left(x-\frac{1}{a}\right)^3}-\frac{41\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{60a^8\left(x-\frac{1}{a}\right)^2}+\sqrt{a}\right.}{c^5(ax-1)}$
default	$-\frac{\left(-75\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^6x^6-60\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^7x^6+45\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^4x^4+150\sqrt{(ax-1)(ax+1)}\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^5*((a*x-1)/(a*x+1))^(1/2)+(2/a^5*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-383/120/a^7/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(
1/2)-1/10/a^9/(x-1/a)^3*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-41/60/a^8/(x-1/a
)^2*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)+1/8/a^7/(x+1/a)*(a^2*(x+1/a)^2-2*a*(
x+1/a))^(1/2))*a^5/c^5/(a*x-1)*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.19

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx$$
$$=\frac{30\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-30\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(15}{15\left(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="fricas")
```

output
```
1/15*(30*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
 1) - 30*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) -
 1) + (15*a^4*x^4 - 76*a^3*x^3 + 32*a^2*x^2 + 82*a*x - 56)*sqrt((a*x - 1)/
(a*x + 1)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.23

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx$$
$$= \frac{1}{120}a\left(\frac{\frac{32\,(ax-1)}{ax+1}+\frac{310\,(ax-1)^2}{(ax+1)^2}-\frac{585\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{240\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^5}-\frac{240\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^5}+\frac{15\sqrt{}}{a^2c}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="maxima")
```

output
```
1/120*a*((32*(a*x - 1)/(a*x + 1) + 310*(a*x - 1)^2/(a*x + 1)^2 - 585*(a*x
- 1)^3/(a*x + 1)^3 + 3)/(a^2*c^5*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^5*((a
*x - 1)/(a*x + 1))^(5/2)) + 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^
5) - 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^5) + 15*sqrt((a*x - 1)/
(a*x + 1))/(a^2*c^5))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.46 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{8\,a\,c^5} - \frac{\frac{62\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2} - \frac{39\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3} + \frac{32\,\left(a\,x-1\right)}{15\,\left(a\,x+1\right)} + \frac{1}{5}}{8\,a\,c^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2} - 8\,a\,c^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}} + \frac{4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^5}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^5,x)
```

output
```
((a*x - 1)/(a*x + 1))^(1/2)/(8*a*c^5) - ((62*(a*x - 1)^2)/(3*(a*x + 1)^2)
- (39*(a*x - 1)^3)/(a*x + 1)^3 + (32*(a*x - 1))/(15*(a*x + 1)) + 1/5)/(8*a
*c^5*((a*x - 1)/(a*x + 1))^(5/2) - 8*a*c^5*((a*x - 1)/(a*x + 1))^(7/2)) +
(4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.87

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx$$
$$= \frac{120\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^3x^3 - 120\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^2x^2 - 120\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)}{}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x)
```

output
```
(120*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**3*x**3
- 120*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**2*x**2
 - 120*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x + 12
0*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 47*sqrt(a*x
 - 1)*a**3*x**3 - 47*sqrt(a*x - 1)*a**2*x**2 - 47*sqrt(a*x - 1)*a*x + 47*s
qrt(a*x - 1) + 30*sqrt(a*x + 1)*a**4*x**4 - 152*sqrt(a*x + 1)*a**3*x**3 +
64*sqrt(a*x + 1)*a**2*x**2 + 164*sqrt(a*x + 1)*a*x - 112*sqrt(a*x + 1))/(3
0*sqrt(a*x - 1)*a*c**5*(a**3*x**3 - a**2*x**2 - a*x + 1))
```

3.453 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2} dx$

Optimal result

Integrand size = 22, antiderivative size = 235

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2} dx = \frac{173c^5\sqrt{1-\frac{1}{a^2x^2}}}{105a\sqrt{c-\frac{c}{ax}}}+\frac{227c^4\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{105a}+\frac{59c^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35a}+\frac{9c^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7a}+c\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{7c^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
173/105*c^5*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+227/105*c^4*(1-1/a^2/x^2
)^(1/2)*(c-c/a/x)^(1/2)/a+59/35*c^3*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(3/2)/a+
9/7*c^2*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(5/2)/a+c*(1-1/a^2/x^2)^(1/2)*(c-c/a
/x)^(7/2)*x-7*c^(9/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))
/a
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.46

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{c^4\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(-30+162ax-356a^2x^2+292a^3x^3+105a^4x^4)-735a^3x^3\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{105a^4\sqrt{1-\frac{1}{ax}}x^3}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(9/2),x]
```

output
```
(c^4*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-30 + 162*a*x - 356*a^2*x^2 + 2
92*a^3*x^3 + 105*a^4*x^4) - 735*a^3*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(105*
a^4*Sqrt[1 - 1/(a*x)]*x^3)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.78, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {6731, 585, 27, 108, 27, 170, 27, 170, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{9/2}e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{7/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}x^2}{a^4}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\int \left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}x^2d\frac{1}{x}}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{108}$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(\int -\frac{\left(a-\frac{1}{x}\right)^3\left(7a+\frac{9}{x}\right)x}{2a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-x\left(a-\frac{1}{x}\right)^4\sqrt{\frac{1}{ax}+1}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^4-\frac{\int \frac{\left(a-\frac{1}{x}\right)^3\left(7a+\frac{9}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{170}$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^4-\frac{\frac{2}{7}a\int \frac{\left(a-\frac{1}{x}\right)^2\left(49a+\frac{59}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{18}{7}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{2a}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^4-\frac{\frac{1}{7}a\int \frac{\left(a-\frac{1}{x}\right)^2\left(49a+\frac{59}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{18}{7}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{2a}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{170}$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^4-\frac{\frac{1}{7}a\left(\frac{2}{5}a\int \frac{\left(a-\frac{1}{x}\right)\left(245a+\frac{227}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{118}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)+\frac{18}{7}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{2a}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^4-\frac{\frac{1}{7}a\left(\frac{1}{5}a\int\frac{\left(a-\frac{1}{x}\right)\left(245a+\frac{227}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{118}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)+\frac{18}{7}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3}{2a}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 164

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^4-\frac{\frac{1}{7}a\left(\frac{1}{5}a\left(245a^2\int\frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{3}a\left(400a-\frac{227}{x}\right)\sqrt{\frac{1}{ax}+1}\right)+\frac{118}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)+\frac{18}{7}a\sqrt{}}{2a}}{a^4\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 73

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^4-\frac{\frac{1}{7}a\left(\frac{1}{5}a\left(490a^3\int\frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{3}a\left(400a-\frac{227}{x}\right)\sqrt{\frac{1}{ax}+1}\right)+\frac{118}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)+\frac{18}{7}}{2a}}{a^4\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 221

$$-\frac{c^4\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^4-\frac{\frac{1}{7}a\left(\frac{1}{5}a\left(\frac{2}{3}a\left(400a-\frac{227}{x}\right)\sqrt{\frac{1}{ax}+1}-490a^2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{118}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)+\frac{18}{7}a\sqrt{\frac{1}{ax}+1}}{2a}}{a^4\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[E^ArcCoth[a*x]*(c - c/(a*x))^(9/2),x]
```

output

```
-((c^4*Sqrt[c - c/(a*x)]*(-((a - x^(-1))^4*Sqrt[1 + 1/(a*x)]*x) - ((18*a*(
a - x^(-1))^3*Sqrt[1 + 1/(a*x)])/7 + (a*((118*a*(a - x^(-1))^2*Sqrt[1 + 1/
(a*x)])/5 + (a*((2*a*(400*a - 227/x)*Sqrt[1 + 1/(a*x)])/3 - 490*a^2*ArcTan
h[Sqrt[1 + 1/(a*x)]]))/5))/7)/(2*a)))/(a^4*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.71

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^4 \left(210 a^{\frac{9}{2}} \sqrt{x(ax+1)}\, x^4+584 a^{\frac{7}{2}} x^3 \sqrt{x(ax+1)}-735 \ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right) a^4 x^4-712 a^{\frac{5}{2}} x^2 \sqrt{x(ax+1)}+324 a^{\frac{3}{2}} x \sqrt{\cdots}\right.}{210\sqrt{\frac{ax-1}{ax+1}}\, x^3 a^{\frac{9}{2}} \sqrt{x(ax+1)}}$
risch	$\frac{\left(105 a^5 x^5+397 a^4 x^4-64 a^3 x^3-194 a^2 x^2+132 a x-30\right) c^4 \sqrt{\frac{c(ax-1)}{ax}}}{105 x^3 a^4 \sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{7 \ln\left(\frac{\frac{1}{2} ac+a^2 cx}{\sqrt{a^2 c}}+\sqrt{a^2 c\, x^2+acx}\right) c^4 \sqrt{\frac{c(ax-1)}{ax}}\, \sqrt{(ax+1)acx}}{2\sqrt{a^2 c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/210/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c^4*(210*a^(9/2)*(x*(a
*x+1))^(1/2)*x^4+584*a^(7/2)*x^3*(x*(a*x+1))^(1/2)-735*ln(1/2*(2*(x*(a*x+1
))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^4*x^4-712*a^(5/2)*x^2*(x*(a*x+1))^(1/
2)+324*a^(3/2)*x*(x*(a*x+1))^(1/2)-60*(x*(a*x+1))^(1/2)*a^(1/2))/x^3/a^(9/
2)/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.86

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx = \left[\frac{735\left(a^4c^4x^4-a^3c^4x^3\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(105\,a\right.}{420\left(a^5x^4-a\right.}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

output
```
[1/420*(735*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*
x - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt
((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(105*a^5*c^4*x^5 + 397*a^4*c^4*x^4
 - 64*a^3*c^4*x^3 - 194*a^2*c^4*x^2 + 132*a*c^4*x - 30*c^4)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/210*(735*(a^4*
c^4*x^4 - a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(
105*a^5*c^4*x^5 + 397*a^4*c^4*x^4 - 64*a^3*c^4*x^3 - 194*a^2*c^4*x^2 + 132
*a*c^4*x - 30*c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5
*x^4 - a^4*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{9/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.52

$$\int e^{\coth^{-1}(ax)}\left(c - \frac{c}{ax}\right)^{9/2} dx = \frac{\sqrt{c}\,c^4\left(105\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^4x^4 + 292\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^3x^3 - 356\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2 + 16\right.}{105a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x)
```

output
```
(sqrt(c)*c**4*(105*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**4*x**4 + 292*sqrt(x)*s
qrt(a)*sqrt(a*x + 1)*a**3*x**3 - 356*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**2*x*
*2 + 162*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x - 30*sqrt(x)*sqrt(a)*sqrt(a*x +
 1) - 735*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a**4*x**4 - 337*a**4*x**4))
/(105*a**5*x**4)
```

3.454 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2} dx$

Optimal result

Integrand size = 22, antiderivative size = 196

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2} dx = \frac{49c^4\sqrt{1-\frac{1}{a^2x^2}}}{15a\sqrt{c-\frac{c}{ax}}} + \frac{31c^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{15a} + \frac{7c^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5a} + c\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}x - \frac{5c^{7/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
49/15*c^4*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+31/15*c^3*(1-1/a^2/x^2)^(1
/2)*(c-c/a/x)^(1/2)/a+7/5*c^2*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(3/2)/a+c*(1-1
/a^2/x^2)^(1/2)*(c-c/a/x)^(5/2)*x-5*c^(7/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^
(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.52

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(6-28ax+56a^2x^2+15a^3x^3)-75a^2x^2\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{15a^3\sqrt{1-\frac{1}{ax}}x^2}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(7/2),x]
```

output
```
(c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(6 - 28*a*x + 56*a^2*x^2 + 15*a^
3*x^3) - 75*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(15*a^3*Sqrt[1 - 1/(a*x)]
*x^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {6731, 585, 27, 108, 27, 170, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{7/2}e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}x^2}{a^3}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\int\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}x^2d\frac{1}{x}}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{108}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\int-\frac{\left(a-\frac{1}{x}\right)^2\left(5a+\frac{7}{x}\right)x}{2a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-x\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{ax}+1}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^3-\frac{\int\frac{\left(a-\frac{1}{x}\right)^2\left(5a+\frac{7}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{170}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^3-\frac{\frac{2}{5}a\int\frac{\left(a-\frac{1}{x}\right)\left(25a+\frac{31}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{14}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^3-\frac{\frac{1}{5}a\int\frac{\left(a-\frac{1}{x}\right)\left(25a+\frac{31}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{14}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{164}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^3-\frac{\frac{1}{5}a\left(25a^2\int\frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{3}a\left(80a-\frac{31}{x}\right)\sqrt{\frac{1}{ax}+1}\right)+\frac{14}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^3-\frac{\frac{1}{5}a\left(50a^3\int\frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{3}a\left(80a-\frac{31}{x}\right)\sqrt{\frac{1}{ax}+1}\right)+\frac{14}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c^3\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(a-\frac{1}{x}\right)^3-\frac{\frac{1}{5}a\left(\frac{2}{3}a\left(80a-\frac{31}{x}\right)\sqrt{\frac{1}{ax}+1}-50a^2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{14}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2}{2a}\right)\sqrt{c-\frac{c}{ax}}}{a^3\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a*x))^(7/2),x]
```

output
```
-((c^3*Sqrt[c - c/(a*x)]*(-((a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*x) - ((14*a*(
a - x^(-1))^2*Sqrt[1 + 1/(a*x)])/5 + (a*((2*a*(80*a - 31/x)*Sqrt[1 + 1/(a*
x)])/3 - 50*a^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/5)/(2*a)))/(a^3*Sqrt[1 - 1/(a
*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^{3} \left(-30 a^{\frac{7}{2}} x^{3} \sqrt{x(ax+1)}+75 \ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right) a^{3} x^{3}-112 a^{\frac{5}{2}} x^{2} \sqrt{x(ax+1)}+56 a^{\frac{3}{2}} x \sqrt{x(ax+1)}-12\sqrt{x(ax+}\right.}{30\sqrt{\frac{ax-1}{ax+1}}\, x^{2} a^{\frac{7}{2}} \sqrt{x(ax+1)}}$
risch	$\frac{(15a^{4}x^{4}+71a^{3}x^{3}+28a^{2}x^{2}-22ax+6)c^{3}\sqrt{\frac{c(ax-1)}{ax}}}{15x^{2}a^{3}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{5\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)c^{3}\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^{2}c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)/x^2*c^3/a^(7/2)*(-30*a
^(7/2)*x^3*(x*(a*x+1))^(1/2)+75*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+
1)/a^(1/2))*a^3*x^3-112*a^(5/2)*x^2*(x*(a*x+1))^(1/2)+56*a^(3/2)*x*(x*(a*x
+1))^(1/2)-12*(x*(a*x+1))^(1/2)*a^(1/2))/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.12

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2} dx = \left[\frac{75\left(a^{3}c^{3}x^{3}-a^{2}c^{3}x^{2}\right)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\left(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(15\,a^{4}c\right.}{60\left(a^{4}x^{3}-a^{3}x^{2}\right)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

output
```
[1/60*(75*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
- 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 71*a^3*c^3*x^3 + 2
8*a^2*c^3*x^2 - 22*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(75*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqr
t(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 71*a^3*c^
3*x^3 + 28*a^2*c^3*x^2 - 22*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqr
t((a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{7/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.54

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\frac{\sqrt{c}\,c^3\left(60\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^3x^3+224\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2-112\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax+24\sqrt{x}\cdots\right.}{60a^4x^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x)
```

output
```
(sqrt(c)*c**3*(60*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**3*x**3 + 224*sqrt(x)*sq
rt(a)*sqrt(a*x + 1)*a**2*x**2 - 112*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x + 24
*sqrt(x)*sqrt(a)*sqrt(a*x + 1) - 300*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*
a**3*x**3 - 197*a**3*x**3))/(60*a**4*x**3)
```

3.455 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{11c^3\sqrt{1-\frac{1}{a^2x^2}}}{3a\sqrt{c-\frac{c}{ax}}}+\frac{5c^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{3a}+c\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{3c^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
11/3*c^3*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+5/3*c^2*(1-1/a^2/x^2)^(1/2)
*(c-c/a/x)^(1/2)/a+c*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(3/2)*x-3*c^(5/2)*arcta
nh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.57

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(-2+10ax+3a^2x^2)-9ax\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{3a^2\sqrt{1-\frac{1}{ax}}x}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(5/2),x]
```

output
```
(c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + 10*a*x + 3*a^2*x^2) - 9*a*
x*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.73, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6731, 585, 27, 100, 27, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{5/2} e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c\int \sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}x^2}{a^2}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\int \left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}x^2d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 100}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\int -\frac{1}{2}\left(3a-\frac{2}{x}\right)\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}-a^2x\left(\frac{1}{ax}+1\right)^{3/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(a^2x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{1}{2}\int\left(3a-\frac{2}{x}\right)\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{90}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{4}{3}a\left(\frac{1}{ax}+1\right)^{3/2}-3a\int\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}\right)-a^2x\left(\frac{1}{ax}+1\right)^{3/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{60}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{4}{3}a\left(\frac{1}{ax}+1\right)^{3/2}-3a\left(\int\frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+2\sqrt{\frac{1}{ax}+1}\right)\right)-a^2x\left(\frac{1}{ax}+1\right)^{3/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{4}{3}a\left(\frac{1}{ax}+1\right)^{3/2}-3a\left(2a\int\frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+2\sqrt{\frac{1}{ax}+1}\right)\right)-a^2x\left(\frac{1}{ax}+1\right)^{3/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c^2\left(\frac{1}{2}\left(\frac{4}{3}a\left(\frac{1}{ax}+1\right)^{3/2}-3a\left(2\sqrt{\frac{1}{ax}+1}-2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)\right)-a^2x\left(\frac{1}{ax}+1\right)^{3/2}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a*x))^(5/2),x]
```

output
```
-((c^2*Sqrt[c - c/(a*x)]*(-(a^2*(1 + 1/(a*x))^(3/2)*x) + ((4*a*(1 + 1/(a*x
))^(3/2))/3 - 3*a*(2*Sqrt[1 + 1/(a*x)] - 2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/2)
)/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^{2} \left(-6 a^{\frac{5}{2}} x^{2} \sqrt{x(ax+1)}+9 \ln\left(\frac{2 \sqrt{x(ax+1)}\, \sqrt{a}+2 a x+1}{2 \sqrt{a}}\right) a^{2} x^{2}-20 a^{\frac{3}{2}} x \sqrt{x(ax+1)}+4 \sqrt{x(ax+1)}\, \sqrt{a}\right)}{6 \sqrt{\frac{ax-1}{ax+1}}\, x\, a^{\frac{5}{2}} \sqrt{x(ax+1)}}$	132
risch	$\frac{\left(3 a^{3} x^{3}+13 a^{2} x^{2}+8 a x-2\right) c^{2} \sqrt{\frac{c(ax-1)}{ax}}}{3 x\, a^{2} \sqrt{\frac{ax-1}{ax+1}}\, (ax+1)}-\frac{3 \ln\left(\frac{\frac{1}{2} a c+a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c\, x^{2}+a c x}\right) c^{2} \sqrt{\frac{c(ax-1)}{ax}}\, \sqrt{(ax+1) a c x}}{2 \sqrt{a^{2} c}\, \sqrt{\frac{ax-1}{ax+1}}\, (ax+1)}$	168

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)/x*c^2/a^(5/2)*(-6*a^(5/
2)*x^2*(x*(a*x+1))^(1/2)+9*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^
(1/2))*a^2*x^2-20*a^(3/2)*x*(x*(a*x+1))^(1/2)+4*(x*(a*x+1))^(1/2)*a^(1/2))
/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.43

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \left[\frac{9\,(a^2c^2x^2-ac^2x)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(3\,a^3c^2x^3}{12\,(a^3x^2-a^2x)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(9*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(
2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 13*a^2*c^2*x^2 + 8*a*c^2
*x - 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 -
a^2*x), 1/6*(9*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a
*c*x - c)) + 2*(3*a^3*c^2*x^3 + 13*a^2*c^2*x^2 + 8*a*c^2*x - 2*c^2)*sqrt((
a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{5/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.55

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{\sqrt{c}\,c^2\left(6\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2+20\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax-4\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}-18\log\left(\sqrt{ax+\right.}{6a^3x^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x)
```

output
```
(sqrt(c)*c**2*(6*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**2*x**2 + 20*sqrt(x)*sqrt
(a)*sqrt(a*x + 1)*a*x - 4*sqrt(x)*sqrt(a)*sqrt(a*x + 1) - 18*log(sqrt(a*x
+ 1) + sqrt(x)*sqrt(a))*a**2*x**2 - 11*a**2*x**2))/(6*a**3*x**2)
```

3.456 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
c^2*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+c^3*(1-1/a^2/x^2)^(3/2)*x/(c-c/a
/x)^(3/2)-c^(3/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\frac{c\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(2+ax)-\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(3/2),x]
```

output
```
(c*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 + a*x) - ArcTanh[Sqrt[1 + 1/(a*
x)]]))/(a*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6731, 580, 576, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{3/2} e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c\int \sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 580}$$

$$-c\left(-\frac{c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 576}$$

$$-c\left(-\frac{c\left(\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 573}$$

$$-c\left(-\frac{c\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-2\int\frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\downarrow 219$$

$$-c\left(-\frac{c\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{\sqrt{c}}\right)}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a*x))^(3/2),x]
```

output
```
-(c*(-((c^2*(1 - 1/(a^2*x^2))^(3/2)*x)/(c - c/(a*x))^(3/2)) - (c*((2*Sqrt[
1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] - (2*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*
x^2)])/Sqrt[c - c/(a*x)]])/Sqrt[c]))/(2*a)))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 576
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-(e*x)^(m + 1))*(c + d*x)^n*((a + b*x^2)^p/(e*(n - m - 1
))), x] - Simp[b*c*(n/(d^2*(n - m - 1)))   Int[(e*x)^m*(c + d*x)^(n + 1)*(a
 + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c^2 + a*
d^2, 0] && EqQ[n + p, 0] && GtQ[p, 0] && NeQ[m - n + 1, 0] &&  !IGtQ[m, 0]
&&  !(IntegerQ[m + p] && LtQ[m + p + 2, 0]) && RationalQ[m]
```

rule 580
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 2)*((a + b*x^2)^(p +
1)/(b*e*(m + 1))), x] + Simp[d*((2*m + p + 3)/(e*(m + 1)))   Int[(e*x)^(m +
 1)*(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p},
 x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p - 1, 0] && LtQ[m, -1] && IntegerQ
[p + 1/2]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c\left(2a^{\frac{3}{2}} x\sqrt{x(ax+1)}-\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+4\sqrt{x(ax+1)}\,\sqrt{a}\right)}{2\sqrt{\frac{ax-1}{ax+1}}\, a^{\frac{3}{2}}\sqrt{x(ax+1)}}$	106
risch	$\frac{(a^2x^2+3ax+2)c\sqrt{\frac{c(ax-1)}{ax}}}{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	151

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c*(2*a^(3/2)*x*(x*(a*x+1
))^(1/2)-ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x+4*(x*(a
*x+1))^(1/2)*a^(1/2))/a^(3/2)/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.68

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\left[\frac{(acx-c)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2cx^2+3\,acx+}{4\,(a^2x-a)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*c*x^2 + 3*a*c*x + 2*c)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*((a*c*x - c)*sqrt(-c)*arctan(2*(a^
2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2
*a^2*c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 + 3*a*c*x + 2*c)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int \frac{\left(c-\frac{c}{a\,x}\right)^{3/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{\sqrt{c}\,c\left(4\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax+8\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}-4\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)ax+7ax\right)}{4a^2x}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x)
```

output
```
(sqrt(c)*c*(4*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x + 8*sqrt(x)*sqrt(a)*sqrt(a
*x + 1) - 4*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a*x + 7*a*x))/(4*a**2*x)
```

3.457 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 78

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
c*(1-1/a^2/x^2)^(1/2)*x/(c-c/a/x)^(1/2)+c^(1/2)*arctanh(c^(1/2)*(1-1/a^2/x
^2)^(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(1+ax+\sqrt{1+\frac{1}{ax}}\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

output
```
(Sqrt[c - c/(a*x)]*(1 + a*x + Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]]
))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6731, 575, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{575}$$

$$-c\left(\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2ac}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{573}$$

$$-c\left(-\frac{\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{219}$$

$$-c\left(-\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

output
```
-(c*(-((Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)]) - ArcTanh[(Sqrt[c]*Sqr
t[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]]/(a*Sqrt[c])))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 575
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(e*x)^(m + 1)*(c + d*x)^n*((a + b*x^2)^p/(e*(m + 1))), x]
 + Simp[b*(n/(d*e*(m + 1)))   Int[(e*x)^(m + 1)*(c + d*x)^(n + 1)*(a + b*x^
2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[b*c^2 + a*d^2, 0] &&
 EqQ[n + p, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m + p] && LeQ[m +
p + 2, 0])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\left(2\sqrt{x(ax+1)}\,\sqrt{a}+\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{x(ax+1)}\,\sqrt{a}}$	87
risch	$\frac{x\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	127

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*(x*(a*x+1))^(1/2)*a
^(1/2)+ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(x*(a*x+1))^
(1/2)/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(66) = 132.

Time = 0.12 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.78

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)}, -\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(a^2x-a)}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)
/(a*x)))/(a^2*x - a), -1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sq
rt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*
c*x - c)) - 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x)))/(a^2*x - a)]
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.40

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)\right)}{a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x)
```

output
```
(sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x + 1) + log(sqrt(a*x + 1) + sqrt(x)*sqrt
(a))))/a
```

3.458 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 134

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{3\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}-\frac{2\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/(c-c/a/x)^(1/2)+3*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2
)/(c-c/a/x)^(1/2))/a/c^(1/2)-2*2^(1/2)*arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(
1/2)*2^(1/2)/(c-c/a/x)^(1/2))/a/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.71

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(\sqrt{1+\frac{1}{ax}}\,x+\frac{3\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}-\frac{2\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]/Sqrt[c - c/(a*x)],x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(Sqrt[1 + 1/(a*x)]*x + (3*ArcTanh[Sqrt[1 + 1/(a*x)]])/a
 - (2*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a))/Sqrt[c - c/(a*x)]
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6731, 585, 27, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a\sqrt{1+\frac{1}{ax}}x^2}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\int \frac{\sqrt{1+\frac{1}{ax}}x^2}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 110}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(3a+\frac{1}{x}\right)x}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(3a+\frac{1}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 174$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{4\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+3\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 73$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{8a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+6a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 221$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-6\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^ArcCoth[a*x]/Sqrt[c - c/(a*x)],x]
```

output
```
-((a*Sqrt[1 - 1/(a*x)]*(-((Sqrt[1 + 1/(a*x)]*x)/a) + (-6*ArcTanh[Sqrt[1 +
1/(a*x)]] + 4*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(2*a^2)))/Sqrt[c
 - c/(a*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.13

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-2\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)\sqrt{a}+3\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,a^{\frac{3}{2}}\,c\sqrt{x(ax+1)}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{3\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a\sqrt{a^{2}c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}c+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{a^{2}\sqrt{c}}\right)\sqrt{(ax+1}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*(x*(a*x+1))^(1/2)*a
^(3/2)*(1/a)^(1/2)-2*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a
+3*a*x+1)/(a*x-1))*a^(1/2)+3*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/
a^(1/2))*a*(1/a)^(1/2))/a^(3/2)/c/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(111) = 222.

Time = 0.15 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.86

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$
$$= \left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}+\cdots}{4\,(a^2cx-ac)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x)) + 2*sqrt(2)*(a*c*x - c)*log(-(17*a^3*x^3 - 3*a^2*x^2 - 13*a*x -
4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x))/sqrt(c) - 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1))/sqrt(c))/(
a^2*c*x - a*c), 1/2*(2*sqrt(2)*(a*c*x - c)*sqrt(-1/c)*arctan(2*sqrt(2)*(a^
2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/
(3*a^2*x^2 - 2*a*x - 1)) - 3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a
*c*x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^2*c*x - a*c)]
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(1/2),x)
```

output
```
Integral(1/(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x)))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{1}{\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.88

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}-1\right)-\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}+1\right)-\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x)
```

output
```
(sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x + 1) + sqrt(2)*log(sqrt(a*x + 1) + sqrt
(x)*sqrt(a) - sqrt(2) - 1) - sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) -
 sqrt(2) + 1) - sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) - 1)
 + sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1) + 3*log(sqrt
(a*x + 1) + sqrt(x)*sqrt(a))))/(a*c)
```

3.459 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 170

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{ax}}}+\frac{5\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}-\frac{7\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{\sqrt{2}ac^{3/2}}$$

output
```
-(1-1/a^2/x^2)^(1/2)*x/(c-c/a/x)^(3/2)+2*(1-1/a^2/x^2)^(1/2)*x/c/(c-c/a/x)
^(1/2)+5*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(3/2)-7/
2*arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))*2^(1/2)
/a/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.72

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(-2+ax)+10(-1+ax)\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-7\sqrt{2}(-1+ax\right.}{2ac\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^(3/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-2 + a*x) + 10*(-1 + a*x)*Arc
Tanh[Sqrt[1 + 1/(a*x)]] - 7*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]/S
qrt[2]]))/(2*a*c*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.81, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {6731, 585, 27, 110, 27, 168, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^2\sqrt{1+\frac{1}{ax}}x^2}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\int \frac{\sqrt{1+\frac{1}{ax}}x^2}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 110$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}-\frac{\int -\frac{\left(4a+\frac{3}{x}\right)x^2}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(4a+\frac{3}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 168$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{-\frac{\int -\frac{\left(5a+\frac{2}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-4x\sqrt{\frac{1}{ax}+1}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\int \frac{\left(5a+\frac{2}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-4x\sqrt{\frac{1}{ax}+1}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 174$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{7\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-4x\sqrt{\frac{1}{ax}+1}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 73$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{14a\int\frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+10a\int\frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{a}-4x\sqrt{\frac{1}{ax}+1}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{7\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-10\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}-4x\sqrt{\frac{1}{ax}+1}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a*x))^(3/2),x]
```

output
```
-((a^2*Sqrt[1 - 1/(a*x)]*((Sqrt[1 + 1/(a*x)]*x)/(a*(a - x^(-1))) + (-4*Sqr
t[1 + 1/(a*x)]*x + (-10*ArcTanh[Sqrt[1 + 1/(a*x)]] + 7*Sqrt[2]*ArcTanh[Sqr
t[1 + 1/(a*x)]/Sqrt[2]])/a)/(2*a^2)))/(c*Sqrt[c - c/(a*x)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.52

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\left(4a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x-7a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)x+10\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^{2}\sqrt{\frac{1}{a}}\,x-8\sqrt{x(ax}\right.}{4\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)a^{\frac{3}{2}}c^{2}\sqrt{x(ax+1)}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a^{2}\sqrt{a^{2}c}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}c+3\left(x-\frac{1}{a}\right)ac+2c}}{a^{4}c\left(x-\frac{1}{a}\right)}-\frac{7\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}\right.}{4a^{3}\sqrt{c}}\right.}{c\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x\sqrt{\frac{c(ax-1)}{ax}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*x*(4*a^(5/2)*(1/
a)^(1/2)*(x*(a*x+1))^(1/2)*x-7*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(
x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x+10*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1
/2)+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*x-8*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^
(1/2)-10*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/
2)+7*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1
))*a^(1/2))/a^(3/2)/c^2/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.49

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \left[\frac{7\sqrt{2}(a^2x^2-2ax+1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx}{a}}}{a^3x^3-3a^2x^2+3ax-1}\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(7*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c
*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*
x - 1)) + 10*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4
*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(a^3*x^3 - a^2*x^2 - 2*a*x)*sqrt((a*x -
 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2
), 1/4*(7*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2
 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*
c*x^2 - 2*a*c*x - c)) - 10*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^
2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2
*c*x^2 - a*c*x - c)) + 4*(a^3*x^3 - a^2*x^2 - 2*a*x)*sqrt((a*x - 1)/(a*x +
 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a\,x}\right)^{3/2}\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.48

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\sqrt{c}\left(4\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax-8\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+7\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}-1\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x)
```

output
```
(sqrt(c)*(4*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x - 8*sqrt(x)*sqrt(a)*sqrt(a*x
 + 1) + 7*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1)*a*x -
 7*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1) - 7*sqrt(2)*
log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) + 1)*a*x + 7*sqrt(2)*log(sqr
t(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) + 1) - 7*sqrt(2)*log(sqrt(a*x + 1)
+ sqrt(x)*sqrt(a) + sqrt(2) - 1)*a*x + 7*sqrt(2)*log(sqrt(a*x + 1) + sqrt(
x)*sqrt(a) + sqrt(2) - 1) + 7*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)
+ sqrt(2) + 1)*a*x - 7*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(
2) + 1) + 20*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a*x - 20*log(sqrt(a*x +
1) + sqrt(x)*sqrt(a))))/(4*a*c**2*(a*x - 1))
```

3.460 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 213

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11\sqrt{1-\frac{1}{a^2x^2}}\,x}{8c\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{23\sqrt{1-\frac{1}{a^2x^2}}\,x}{8c^2\sqrt{c-\frac{c}{ax}}}+\frac{7\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{5/2}}-\frac{79\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{8\sqrt{2}ac^{5/2}}$$

output
```
-1/2*(1-1/a^2/x^2)^(1/2)*x/(c-c/a/x)^(5/2)-11/8*(1-1/a^2/x^2)^(1/2)*x/c/(c
-c/a/x)^(3/2)+23/8*(1-1/a^2/x^2)^(1/2)*x/c^2/(c-c/a/x)^(1/2)+7*arctanh(c^(
1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(5/2)-79/16*arctanh(1/2*c^(1
/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))*2^(1/2)/a/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.63

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(23-35ax+8a^2x^2)+112(-1+ax)^2\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-79\right.}{16ac^2\sqrt{c-\frac{c}{ax}}(-1+ax)^2}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^(5/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(23 - 35*a*x + 8*a^2*x^2) + 11
2*(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 79*Sqrt[2]*(-1 + a*x)^2*ArcTan
h[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(16*a*c^2*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.82, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {6731, 585, 27, 110, 27, 168, 27, 168, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\left(c-\frac{c}{ax}\right)^{7/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^3\sqrt{1+\frac{1}{ax}}x^2}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\int \frac{\sqrt{1+\frac{1}{ax}}x^2}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{110}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}-\frac{\int -\frac{\left(6a+\frac{5}{x}\right)x^2}{2a\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(6a+\frac{5}{x}\right)x^2}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{11x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{\int -\frac{\left(46a+\frac{33}{x}\right)x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\int \frac{\left(46a+\frac{33}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}+\frac{11x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{-\frac{\int -\frac{\left(56a+\frac{23}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-46x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{11x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{\int \frac{\left(56a+\frac{23}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-46x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{11x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{174}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{79\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+56\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-46x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{11x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{158a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+112a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{a}-46x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{11x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{79\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-112\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}-46x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{11x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)^2}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

input

```
Int[E^ArcCoth[a*x]/(c - c/(a*x))^(5/2),x]
```

output
```
-((a^3*Sqrt[1 - 1/(a*x)]*((Sqrt[1 + 1/(a*x)]*x)/(2*a*(a - x^(-1))^2) + ((1
1*Sqrt[1 + 1/(a*x)]*x)/(2*(a - x^(-1))) + (-46*Sqrt[1 + 1/(a*x)]*x + (-112
*ArcTanh[Sqrt[1 + 1/(a*x)]] + 79*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]
])/a)/(4*a))/(4*a^2)))/(c^2*Sqrt[c - c/(a*x)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.54

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{7\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^3\sqrt{a^2c}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+3\left(x-\frac{1}{a}\right)ac+2c}}{2a^6c\left(x-\frac{1}{a}\right)^2}-\frac{19\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+3\left(x-\frac{1}{a}\right)ac+2c}}{8a^5c\left(x-\frac{1}{a}\right)}-\frac{79\sqrt{2}\cdots}{\cdots}\right.}{c^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x\sqrt{\frac{c(a\cdots}{\cdots}}}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(32a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x^2-79a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)x^2-140a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x+112\ln\left(\frac{2\sqrt{x(ax+1}\cdots}{2\cdots}\right.\right.}{\cdots}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/c^2/((a*x-1)/(a*x+1))^(1/2)/(c*(a*x-1)/a/x)^(1/2)*(a*x-1)+(7/2/a^3*ln(
(1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2+a*c*x)^(1/2))/(a^2*c)^(1/2)-1/2
/a^6/c/(x-1/a)^2*((x-1/a)^2*a^2*c+3*(x-1/a)*a*c+2*c)^(1/2)-19/8/a^5/c/(x-1
/a)*((x-1/a)^2*a^2*c+3*(x-1/a)*a*c+2*c)^(1/2)-79/32/a^4/c^(1/2)*2^(1/2)*ln
((4*c+3*(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*((x-1/a)^2*a^2*c+3*(x-1/a)*a*c+2*c)^
(1/2))/(x-1/a)))*a^2/c^2/((a*x-1)/(a*x+1))^(1/2)/(a*x+1)/x/(c*(a*x-1)/a/x)
^(1/2)*((a*x+1)*a*c*x)^(1/2)*(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 668, normalized size of antiderivative = 3.14

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \left[\frac{79\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{c}}{a^3x^3-3a^2x^2+3ax-1}\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(79*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3*c
*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sq
rt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*
a^2*x^2 + 3*a*x - 1)) + 112*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(
-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(8*a^4*x^4 -
 27*a^3*x^3 - 12*a^2*x^2 + 23*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3), 1/32*(79*
sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x
^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^
2*c*x^2 - 2*a*c*x - c)) - 112*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*a
rctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 4*(8*a^4*x^4 - 27*a^3*x^3 - 12*a^2*x
^2 + 23*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x
^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a\,x}\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.97

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\frac{\sqrt{c}\,\left(96\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2-420\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax+276\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+237\sqrt{2}\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x)
```

output
```
(sqrt(c)*(96*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**2*x**2 - 420*sqrt(x)*sqrt(a)
*sqrt(a*x + 1)*a*x + 276*sqrt(x)*sqrt(a)*sqrt(a*x + 1) + 237*sqrt(2)*log(s
qrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1)*a**2*x**2 - 474*sqrt(2)*log(
sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1)*a*x + 237*sqrt(2)*log(sqrt(
a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1) - 237*sqrt(2)*log(sqrt(a*x + 1)
+ sqrt(x)*sqrt(a) - sqrt(2) + 1)*a**2*x**2 + 474*sqrt(2)*log(sqrt(a*x + 1)
 + sqrt(x)*sqrt(a) - sqrt(2) + 1)*a*x - 237*sqrt(2)*log(sqrt(a*x + 1) + sq
rt(x)*sqrt(a) - sqrt(2) + 1) - 237*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqr
t(a) + sqrt(2) - 1)*a**2*x**2 + 474*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sq
rt(a) + sqrt(2) - 1)*a*x - 237*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)
 + sqrt(2) - 1) + 237*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2
) + 1)*a**2*x**2 - 474*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(
2) + 1)*a*x + 237*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) +
1) + 672*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a**2*x**2 - 1344*log(sqrt(a*
x + 1) + sqrt(x)*sqrt(a))*a*x + 672*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)) +
 172*a**2*x**2 - 344*a*x + 172))/(96*a*c**3*(a**2*x**2 - 2*a*x + 1))
```

3.461 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 143

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{5c^{9/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

output
```
5*c^4*(c-c/a/x)^(1/2)/a+5/3*c^3*(c-c/a/x)^(3/2)/a+c^2*(c-c/a/x)^(5/2)/a+5/
7*c*(c-c/a/x)^(7/2)/a+(c-c/a/x)^(9/2)*x-5*c^(9/2)*arctanh((c-c/a/x)^(1/2)/
c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.64

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\frac{c^4\sqrt{c-\frac{c}{ax}}\left(6-18ax+4a^2x^2+92a^3x^3+21a^4x^4\right)}{21a^4x^3}-\frac{5c^{9/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

output
```
(c^4*Sqrt[c - c/(a*x)]*(6 - 18*a*x + 4*a^2*x^2 + 92*a^3*x^3 + 21*a^4*x^4))
/(21*a^4*x^3) - (5*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6717, 6683, 1035, 281, 899, 87, 60, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{9/2} e^{2\coth^{-1}(ax)} dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^{9/2}(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{9/2}}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}dx}{a}$$

$$\Big\downarrow \text{899}$$

$$-\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}x^2d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{87}$$

$$-\frac{c\left(-\frac{5}{2}\int \left(c-\frac{c}{ax}\right)^{7/2}xd\frac{1}{x}-\frac{ax\left(c-\frac{c}{ax}\right)^{9/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{c\left(-\frac{5}{2}\left(c\int \left(c-\frac{c}{ax}\right)^{5/2}xd\frac{1}{x}+\frac{2}{7}\left(c-\frac{c}{ax}\right)^{7/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{9/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{c\left(-\frac{5}{2}\left(c\left(c\int \left(c-\frac{c}{ax}\right)^{3/2}xd\frac{1}{x}+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)+\frac{2}{7}\left(c-\frac{c}{ax}\right)^{7/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{9/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{c\left(-\frac{5}{2}\left(c\left(c\left(c\int \sqrt{c-\frac{c}{ax}}xd\frac{1}{x}+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)+\frac{2}{7}\left(c-\frac{c}{ax}\right)^{7/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{9/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{c\left(-\frac{5}{2}\left(c\left(c\left(c\left(c\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+2\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)+\frac{2}{7}\left(c-\frac{c}{ax}\right)^{7/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{9/}}{c}\right.}{a}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{c\left(-\frac{5}{2}\left(c\left(c\left(c\left(2\sqrt{c-\frac{c}{ax}}-2a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)+\frac{2}{7}\left(c-\frac{c}{ax}\right)^{7/2}\right)-\frac{ax(c-}{c}\right.}{a}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{c\left(-\frac{5}{2}\left(c\left(c\left(c\left(2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)+\frac{2}{7}\left(c-\frac{c}{ax}\right)^{7/2}\right)-\right.}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

output
```
-((c*(-((a*(c - c/(a*x))^(9/2)*x)/c) - (5*((2*(c - c/(a*x))^(7/2))/7 + c*(
(2*(c - c/(a*x))^(5/2))/5 + c*((2*(c - c/(a*x))^(3/2))/3 + c*(2*Sqrt[c - c
/(a*x)] - 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])))))/2))/a)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.08

method	result
risch	$\frac{\left(21 a^{5} x^{5}+71 a^{4} x^{4}-88 a^{3} x^{3}-22 a^{2} x^{2}+24 a x -6\right) c^{4} \sqrt{\frac{c \left(a x -1\right)}{a x}}}{21 x^{3} a^{4} \left(a x -1\right)}-\frac{5 \ln \left(\frac{-\frac{1}{2} a c +a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}-a c x}\right) c^{4} \sqrt{\frac{c \left(a x -1\right)}{a x}}\, \sqrt{c \left(a x -1\right) a x}}{2 \sqrt{a^{2} c}\, \left(a x -1\right)}$
default	$-\frac{\sqrt{\frac{c \left(a x -1\right)}{a x}}\, c^{4} \left(-210 \sqrt{a \,x^{2}-x}\, a^{\frac{9}{2}} x^{5}+168 \left(a \,x^{2}-x \right)^{\frac{3}{2}} a^{\frac{7}{2}} x^{3}+105 \ln \left(\frac{2 \sqrt{a \,x^{2}-x}\, \sqrt{a}+2 a x -1}{2 \sqrt{a}}\right) a^{4} x^{5}-16 a^{\frac{5}{2}} \left(a \,x^{2}-x \right)^{\frac{3}{2}} x^{2}-24 a^{\frac{3}{2}} \left(a \,x^{2}\right.}{42 x^{4} \sqrt{x \left(a x -1\right)}\, a^{\frac{9}{2}}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/21*(21*a^5*x^5+71*a^4*x^4-88*a^3*x^3-22*a^2*x^2+24*a*x-6)/x^3*c^4/a^4/(a
*x-1)*(c*(a*x-1)/a/x)^(1/2)-5/2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c
*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)*c^4/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x
-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.70

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx = \left[\frac{105\,a^3c^{\frac{9}{2}}x^3\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(21\,a^4c^4x^4+92\,a^3c^4x^3+4\,a^2c^4x^2-18\right.}{42\,a^4x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

output
```
[1/42*(105*a^3*c^(9/2)*x^3*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(
a*x)) + c) + 2*(21*a^4*c^4*x^4 + 92*a^3*c^4*x^3 + 4*a^2*c^4*x^2 - 18*a*c^4
*x + 6*c^4)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3), 1/21*(105*a^3*sqrt(-c)*c^4
*x^3*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) + (21*a^4*c^
4*x^4 + 92*a^3*c^4*x^3 + 4*a^2*c^4*x^2 - 18*a*c^4*x + 6*c^4)*sqrt((a*c*x -
 c)/(a*x)))/(a^4*x^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.49 (sec) , antiderivative size = 2222, normalized size of antiderivative = 15.54

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(9/2),x)
```

output
```
c**4*Piecewise((-sqrt(c)*acosh(sqrt(a)*sqrt(x))/a + sqrt(c)*sqrt(x)*sqrt(a
*x - 1)/sqrt(a), Abs(a*x) > 1), (-I*sqrt(a)*sqrt(c)*x**(3/2)/sqrt(-a*x + 1
) + I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(-a
*x + 1)), True)) + 2*c**4*Piecewise((2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/
sqrt(-c) + 2*sqrt(c - c/(a*x)), Ne(c/a, 0)), (-sqrt(c)*log(x), True))/a +
2*c**4*Piecewise((-4*a**(11/2)*sqrt(c)*x**(7/2)/(15*a**(7/2)*x**(7/2) - 15
*a**(5/2)*x**(5/2)) + 4*a**(9/2)*sqrt(c)*x**(5/2)/(15*a**(7/2)*x**(7/2) -
15*a**(5/2)*x**(5/2)) + 4*a**5*sqrt(c)*x**3*sqrt(a*x - 1)/(15*a**(7/2)*x**
(7/2) - 15*a**(5/2)*x**(5/2)) - 2*a**4*sqrt(c)*x**2*sqrt(a*x - 1)/(15*a**(
7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) - 8*a**3*sqrt(c)*x*sqrt(a*x - 1)/(15
*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 6*a**2*sqrt(c)*sqrt(a*x - 1)/
(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)), Abs(a*x) > 1), (-4*a**(11/2
)*sqrt(c)*x**(7/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*a**(9
/2)*sqrt(c)*x**(5/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*I*a
**5*sqrt(c)*x**3*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/
2)) - 2*I*a**4*sqrt(c)*x**2*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(
5/2)*x**(5/2)) - 8*I*a**3*sqrt(c)*x*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) -
 15*a**(5/2)*x**(5/2)) + 6*I*a**2*sqrt(c)*sqrt(-a*x + 1)/(15*a**(7/2)*x**(
7/2) - 15*a**(5/2)*x**(5/2)), True))/a**3 - c**4*Piecewise((-16*a**(19/2)*
sqrt(c)*x**(13/2)/(105*a**(13/2)*x**(13/2) - 315*a**(11/2)*x**(11/2) + ...
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c - c/(a*x))^(9/2)/(a*x - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{9/2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a*x))^(9/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a*x))^(9/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\frac{\sqrt{c}\,c^4\left(21\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^4x^4+92\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^3x^3+4\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^2x^2-18\sqrt{x}\cdots\right.}{21a^5x^4}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x)
```

output
```
(sqrt(c)*c**4*(21*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**4*x**4 + 92*sqrt(x)*sqr
t(a)*sqrt(a*x - 1)*a**3*x**3 + 4*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**2*x**2 -
 18*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a*x + 6*sqrt(x)*sqrt(a)*sqrt(a*x - 1) -
105*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))*a**4*x**4 - 83*a**4*x**4))/(21*a*
*5*x**4)
```

3.462 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$

Optimal result

Integrand size = 24, antiderivative size = 118

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{3c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

output
```
3*c^3*(c-c/a/x)^(1/2)/a+c^2*(c-c/a/x)^(3/2)/a+3/5*c*(c-c/a/x)^(5/2)/a+(c-c
/a/x)^(7/2)*x-3*c^(7/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{c^3\sqrt{c-\frac{c}{ax}}(-2+4ax+8a^2x^2+5a^3x^3)}{5a^3x^2}-\frac{3c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

output
```
(c^3*Sqrt[c - c/(a*x)]*(-2 + 4*a*x + 8*a^2*x^2 + 5*a^3*x^3))/(5*a^3*x^2) -
 (3*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6717, 6683, 1035, 281, 899, 87, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{7/2} e^{2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)} \left(c-\frac{c}{ax}\right)^{7/2} dx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^{7/2} (ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{ 1035}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}dx}{a}$$

$$\Big\downarrow \text{ 899}$$

$$-\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}x^2d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{c\left(-\frac{3}{2}\int \left(c-\frac{c}{ax}\right)^{5/2}xd\frac{1}{x}-\frac{ax\left(c-\frac{c}{ax}\right)^{7/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{60}$$

$$-\frac{c\left(-\frac{3}{2}\left(c\int\left(c-\frac{c}{ax}\right)^{3/2}xd\frac{1}{x}+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{7/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{60}$$

$$-\frac{c\left(-\frac{3}{2}\left(c\left(c\int\sqrt{c-\frac{c}{ax}}xd\frac{1}{x}+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{7/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{60}$$

$$-\frac{c\left(-\frac{3}{2}\left(c\left(c\left(c\int\frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+2\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{7/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c\left(-\frac{3}{2}\left(c\left(c\left(2\sqrt{c-\frac{c}{ax}}-2a\int\frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{7/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c\left(-\frac{3}{2}\left(c\left(c\left(2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{2}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{7/2}}{c}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

output
```
-((c*(-((a*(c - c/(a*x))^(7/2)*x)/c) - (3*((2*(c - c/(a*x))^(5/2))/5 + c*(
(2*(c - c/(a*x))^(3/2))/3 + c*(2*Sqrt[c - c/(a*x)] - 2*Sqrt[c]*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]]))))/2))/a)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.22

method	result
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^{3} \left(-30\sqrt{a\,x^{2}-x}\, a^{\frac{7}{2}} x^{4}+20 a^{\frac{5}{2}} \left(a\,x^{2}-x\right)^{\frac{3}{2}} x^{2}+15 \ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right) a^{3} x^{4}+4 a^{\frac{3}{2}} \left(a\,x^{2}-x\right)^{\frac{3}{2}} x-4 \left(a\,x^{2}-x\right)^{\frac{3}{2}} \sqrt{a}\right.}{10 x^{3} \sqrt{x(ax-1)}\, a^{\frac{7}{2}}}$
risch	$\frac{\left(5a^{4}x^{4}+3a^{3}x^{3}-4a^{2}x^{2}-6ax+2\right)c^{3}\sqrt{\frac{c(ax-1)}{ax}}}{5x^{2}a^{3}(ax-1)}-\frac{3\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)c^{3}\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{2\sqrt{a^{2}c}\,(ax-1)}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*(c*(a*x-1)/a/x)^(1/2)/x^3*c^3*(-30*(a*x^2-x)^(1/2)*a^(7/2)*x^4+20*a^
(5/2)*(a*x^2-x)^(3/2)*x^2+15*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^
(1/2))*a^3*x^4+4*a^(3/2)*(a*x^2-x)^(3/2)*x-4*(a*x^2-x)^(3/2)*a^(1/2))/(x*(
a*x-1))^(1/2)/a^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.87

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\left[\frac{15\,a^2c^{\frac{7}{2}}x^2\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(5\,a^3c^3x^3+8\,a^2c^3x^2+4\,ac^3x-2\,c^3\right)\sqrt{\frac{c}{}}}{10\,a^3x^2}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

output
```
[1/10*(15*a^2*c^(7/2)*x^2*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) + c) + 2*(5*a^3*c^3*x^3 + 8*a^2*c^3*x^2 + 4*a*c^3*x - 2*c^3)*sqrt((a*
c*x - c)/(a*x)))/(a^3*x^2), 1/5*(15*a^2*sqrt(-c)*c^3*x^2*arctan(a*sqrt(-c)
*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) + (5*a^3*c^3*x^3 + 8*a^2*c^3*x^2 +
 4*a*c^3*x - 2*c^3)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.91 (sec) , antiderivative size = 740, normalized size of antiderivative = 6.27

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\text{Too large to display}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(7/2),x)
```

output
```
c**3*Piecewise((-sqrt(c)*acosh(sqrt(a)*sqrt(x))/a + sqrt(c)*sqrt(x)*sqrt(a
*x - 1)/sqrt(a), Abs(a*x) > 1), (-I*sqrt(a)*sqrt(c)*x**(3/2)/sqrt(-a*x + 1
) + I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(-a
*x + 1)), True)) + c**3*Piecewise((2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sq
rt(-c) + 2*sqrt(c - c/(a*x)), Ne(c/a, 0)), (-sqrt(c)*log(x), True))/a - c*
*3*Piecewise((0, Eq(c, 0)), (2*a*(c - c/(a*x))**(3/2)/(3*c), True))/a**2 +
 c**3*Piecewise((-4*a**(11/2)*sqrt(c)*x**(7/2)/(15*a**(7/2)*x**(7/2) - 15*
a**(5/2)*x**(5/2)) + 4*a**(9/2)*sqrt(c)*x**(5/2)/(15*a**(7/2)*x**(7/2) - 1
5*a**(5/2)*x**(5/2)) + 4*a**5*sqrt(c)*x**3*sqrt(a*x - 1)/(15*a**(7/2)*x**(
7/2) - 15*a**(5/2)*x**(5/2)) - 2*a**4*sqrt(c)*x**2*sqrt(a*x - 1)/(15*a**(7
/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) - 8*a**3*sqrt(c)*x*sqrt(a*x - 1)/(15*
a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 6*a**2*sqrt(c)*sqrt(a*x - 1)/(
15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)), Abs(a*x) > 1), (-4*a**(11/2)
*sqrt(c)*x**(7/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*a**(9/
2)*sqrt(c)*x**(5/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*I*a*
*5*sqrt(c)*x**3*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2
)) - 2*I*a**4*sqrt(c)*x**2*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5
/2)*x**(5/2)) - 8*I*a**3*sqrt(c)*x*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) -
15*a**(5/2)*x**(5/2)) + 6*I*a**2*sqrt(c)*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7
/2) - 15*a**(5/2)*x**(5/2)), True))/a**3
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c - c/(a*x))^(7/2)/(a*x - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{7/2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a*x))^(7/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a*x))^(7/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.89

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\frac{\sqrt{c}\,c^3\left(20\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^3x^3+32\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^2x^2+16\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax-8\sqrt{x}\,\sqrt{\right.}{20a^4x^3}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x)
```

output
```
(sqrt(c)*c**3*(20*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**3*x**3 + 32*sqrt(x)*sqr
t(a)*sqrt(a*x - 1)*a**2*x**2 + 16*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a*x - 8*sq
rt(x)*sqrt(a)*sqrt(a*x - 1) - 60*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))*a**3
*x**3 - 41*a**3*x**3))/(20*a**4*x**3)
```

3.463 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

output
```
c^2*(c-c/a/x)^(1/2)/a+1/3*c*(c-c/a/x)^(3/2)/a+(c-c/a/x)^(5/2)*x-c^(5/2)*ar
ctanh((c-c/a/x)^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\frac{c^2\sqrt{c-\frac{c}{ax}}(2-2ax+3a^2x^2)-3ac^{5/2}x\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{3a^2x}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

output
```
(c^2*Sqrt[c - c/(a*x)]*(2 - 2*a*x + 3*a^2*x^2) - 3*a*c^(5/2)*x*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]])/(3*a^2*x)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6717, 6683, 1035, 281, 899, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{5/2} e^{2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)} \left(c-\frac{c}{ax}\right)^{5/2} dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^{5/2} (ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}dx}{a}$$

$$\Big\downarrow \text{899}$$

$$-\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}x^2d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{87}$$

$$-\frac{c\left(-\frac{1}{2}\int \left(c-\frac{c}{ax}\right)^{3/2}xd\frac{1}{x}-\frac{ax\left(c-\frac{c}{ax}\right)^{5/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{60}$$

$$-\frac{c\left(\frac{1}{2}\left(-c\int \sqrt{c-\frac{c}{ax}}xd\frac{1}{x}-\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{5/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{60}$$

$$-\frac{c\left(\frac{1}{2}\left(-c\left(c\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+2\sqrt{c-\frac{c}{ax}}\right)-\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{5/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c\left(\frac{1}{2}\left(-c\left(2\sqrt{c-\frac{c}{ax}}-2a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}\right)-\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{5/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c\left(\frac{1}{2}\left(-c\left(2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)\right)-\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{5/2}}{c}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

output
```
-((c*(-((a*(c - c/(a*x))^(5/2)*x)/c) + ((-2*(c - c/(a*x))^(3/2))/3 - c*(2*
Sqrt[c - c/(a*x)] - 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]))/2))/a)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^{2} \left(-6\sqrt{a\,x^{2}-x}\, a^{\frac{5}{2}} x^{3}+3\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right) a^{2} x^{3}+4\left(a\,x^{2}-x\right)^{\frac{3}{2}}\sqrt{a}\right)}{6x^{2}\sqrt{x(ax-1)}\, a^{\frac{5}{2}}}$	108
risch	$\frac{(3a^{3}x^{3}-5a^{2}x^{2}+4ax-2)c^{2}\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^{2}(ax-1)}-\frac{\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)c^{2}\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{2\sqrt{a^{2}c}\,(ax-1)}$	139

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(c*(a*x-1)/a/x)^(1/2)/x^2*c^2*(-6*(a*x^2-x)^(1/2)*a^(5/2)*x^3+3*ln(1/
2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a^2*x^3+4*(a*x^2-x)^(3/2)*a
^(1/2))/(x*(a*x-1))^(1/2)/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.01

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2} dx = \left[\frac{3\,ac^{\frac{5}{2}}x\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(3\,a^{2}c^{2}x^{2}-2\,ac^{2}x+2\,c^{2}\right)\sqrt{\frac{acx-c}{ax}}}{6\,a^{2}x}, \frac{3\,a\sqrt{-c}\cdots}{\cdots}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*a*c^(5/2)*x*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) +
 c) + 2*(3*a^2*c^2*x^2 - 2*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*
x), 1/3*(3*a*sqrt(-c)*c^2*x*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a
*c*x - c)) + (3*a^2*c^2*x^2 - 2*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/
(a^2*x)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.70 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.52

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = c^2\left(\begin{cases}-\frac{\sqrt{c}\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{\sqrt{c}\sqrt{x}\sqrt{ax-1}}{\sqrt{a}} & \text{for } |ax|>1\\ -\frac{i\sqrt{a}\sqrt{c}x^{\frac{3}{2}}}{\sqrt{-ax+1}}+\frac{i\sqrt{c}\,\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{i\sqrt{c}\sqrt{x}}{\sqrt{a}\sqrt{-ax+1}} & \text{otherwise}\end{cases}\right) - \frac{c^2\left(\begin{cases}0 & \text{for } c=0\\ \frac{2a\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)}{a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(5/2),x)
```

output
```
c**2*Piecewise((-sqrt(c)*acosh(sqrt(a)*sqrt(x))/a + sqrt(c)*sqrt(x)*sqrt(a
*x - 1)/sqrt(a), Abs(a*x) > 1), (-I*sqrt(a)*sqrt(c)*x**(3/2)/sqrt(-a*x + 1
) + I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(-a
*x + 1)), True)) - c**2*Piecewise((0, Eq(c, 0)), (2*a*(c - c/(a*x))**(3/2)
/(3*c), True))/a**2
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c - c/(a*x))^(5/2)/(a*x - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionLimit: Max order reached or unable to make series expansio
n Error:
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{5/2}(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a*x))^(5/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a*x))^(5/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{\sqrt{c}\,c^2\left(6\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^2x^2-4\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax+4\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}-6\log\left(\sqrt{ax-1}\right.\right.}{6a^3x^2}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x)
```

output
```
(sqrt(c)*c**2*(6*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**2*x**2 - 4*sqrt(x)*sqrt(
a)*sqrt(a*x - 1)*a*x + 4*sqrt(x)*sqrt(a)*sqrt(a*x - 1) - 6*log(sqrt(a*x -
1) + sqrt(x)*sqrt(a))*a**2*x**2 - 5*a**2*x**2))/(6*a**3*x**2)
```

3.464 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 70

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2} dx=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

output
```
-c*(c-c/a/x)^(1/2)/a+(c-c/a/x)^(3/2)*x+c^(3/2)*arctanh((c-c/a/x)^(1/2)/c^(
1/2))/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2} dx=\frac{c\sqrt{c-\frac{c}{ax}}(-2+ax)+c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

output
```
(c*Sqrt[c - c/(a*x)]*(-2 + a*x) + c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c
]])/a
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6717, 6683, 1035, 281, 899, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{3/2} e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{ 1035}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{c\int \left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}dx}{a}$$

$$\Big\downarrow \text{ 899}$$

$$-\frac{c\int \left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}x^2d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{c\left(\frac{1}{2}\int \sqrt{c-\frac{c}{ax}}xd\frac{1}{x}-\frac{ax\left(c-\frac{c}{ax}\right)^{3/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{c\left(\frac{1}{2}\left(c\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+2\sqrt{c-\frac{c}{ax}}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{3/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c\left(\frac{1}{2}\left(2\sqrt{c-\frac{c}{ax}}-2a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{3/2}}{c}\right)}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c\left(\frac{1}{2}\left(2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)\right)-\frac{ax\left(c-\frac{c}{ax}\right)^{3/2}}{c}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

output
```
-((c*(-((a*(c - c/(a*x))^(3/2)*x)/c) + (2*Sqrt[c - c/(a*x)] - 2*Sqrt[c]*Ar
cTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/2))/a)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.47

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c\left(-2\sqrt{a\,x^{2}-x}\, a^{\frac{3}{2}} x^{2}+4\left(a\,x^{2}-x\right)^{\frac{3}{2}} \sqrt{a}+\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right) a\,x^{2}\right)}{2x\sqrt{x(ax-1)}\, a^{\frac{3}{2}}}$	103
risch	$\frac{(a^{2}x^{2}-3ax+2)c\sqrt{\frac{c(ax-1)}{ax}}}{a(ax-1)}+\frac{\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right) c\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{2\sqrt{a^{2}c}\,(ax-1)}$	122

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*(a*x-1)/a/x)^(1/2)/x*c*(-2*(a*x^2-x)^(1/2)*a^(3/2)*x^2+4*(a*x^2-x)^
(3/2)*a^(1/2)+ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*x^2)/(
x*(a*x-1))^(1/2)/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.09

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\left[\frac{c^{\frac{3}{2}}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{2\,a},\right.$$
$$\left.-\frac{\sqrt{-c}c\arctan\left(\frac{a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)-(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{a}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(c^(3/2)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) +
2*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a, -(sqrt(-c)*c*arctan(a*sqrt(-c)
*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) - (a*c*x - 2*c)*sqrt((a*c*x - c)/(
a*x)))/a]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.56 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.47

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = c\left(\begin{cases}-\frac{\sqrt{c}\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{\sqrt{c}\sqrt{x}\sqrt{ax-1}}{\sqrt{a}} & \text{for } |ax|>1\\ -\frac{i\sqrt{a}\sqrt{c}x^{\frac{3}{2}}}{\sqrt{-ax+1}}+\frac{i\sqrt{c}\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{i\sqrt{c}\sqrt{x}}{\sqrt{a}\sqrt{-ax+1}} & \text{otherwise}\end{cases}\right) - \frac{c\left(\begin{cases}\frac{2c\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{c-\frac{c}{ax}} & \text{for } \frac{c}{a}\neq 0\\ -\sqrt{c}\log(x) & \text{otherwise}\end{cases}\right)}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(3/2),x)
```

output
```
c*Piecewise((-sqrt(c)*acosh(sqrt(a)*sqrt(x))/a + sqrt(c)*sqrt(x)*sqrt(a*x
- 1)/sqrt(a), Abs(a*x) > 1), (-I*sqrt(a)*sqrt(c)*x**(3/2)/sqrt(-a*x + 1) +
 I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(-a*x
+ 1)), True)) - c*Piecewise((2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sqrt(-c)
 + 2*sqrt(c - c/(a*x)), Ne(c/a, 0)), (-sqrt(c)*log(x), True))/a
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c - c/(a*x))^(3/2)/(a*x - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int \frac{\left(c-\frac{c}{a\,x}\right)^{3/2}(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a*x))^(3/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a*x))^(3/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\frac{\sqrt{c}\,c\left(4\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax-8\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}+4\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}\right)ax-7ax\right)}{4a^2x}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x)
```

output
```
(sqrt(c)*c*(4*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a*x - 8*sqrt(x)*sqrt(a)*sqrt(a
*x - 1) + 4*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))*a*x - 7*a*x))/(4*a**2*x)
```

3.465 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 50

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x+\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

output
```
(c-c/a/x)^(1/2)*x+3*c^(1/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x+\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6683, 1035, 281, 899, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}dx$$
$$\Big\downarrow \text{6717}$$
$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}dx$$
$$\Big\downarrow \text{6683}$$
$$-\int \frac{\sqrt{c-\frac{c}{ax}}(ax+1)}{1-ax}dx$$
$$\Big\downarrow \text{1035}$$
$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}{\frac{1}{x}-a}dx$$
$$\Big\downarrow \text{281}$$
$$\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}dx}{a}$$
$$\Big\downarrow \text{899}$$
$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}$$
$$\Big\downarrow \text{87}$$
$$-\frac{c\left(\frac{3}{2}\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{ax\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$
$$\Big\downarrow \text{73}$$

$$-\frac{c\left(-\frac{3a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{ax\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

$$\Big\downarrow \; 221$$

$$-\frac{c\left(-\frac{3\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{ax\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
-((c*(-((a*Sqrt[c - c/(a*x)]*x)/c) - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
)/Sqrt[c]))/a)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(42) = 84$.

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.96

method	result	size
risch	$x\sqrt{\frac{c(ax-1)}{ax}}+\frac{3\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{2\sqrt{a^{2}c}\,(ax-1)}$	98
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{a\,x^{2}-x}\,\sqrt{a}-4\sqrt{x(ax-1)}\,\sqrt{a}-\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-2\ln\left(\frac{2\sqrt{x(ax-1)}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\right)}{2\sqrt{x(ax-1)}\,\sqrt{a}}$	120

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(c*(a*x-1)/a/x)^(1/2)+3/2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2
-a*c*x)^(1/2))/(a^2*c)^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.66

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+3\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a}, \frac{ax\sqrt{\frac{acx-c}{ax}}-3\,\sqrt{-c}\arctan\left(\frac{a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)}{a}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 3*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)
*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(
-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)))/a]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))/(a*x - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(42) = 84$.

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{3\sqrt{c}\log(|a||c|)\operatorname{sgn}(x)}{2a}-\frac{3\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}|a|+ac\right|\right)}{2a\operatorname{sgn}(x)}+\frac{\sqrt{a^2cx^2-acx}|a|}{a^2\operatorname{sgn}(x)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
3/2*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a - 3/2*sqrt(c)*log(abs(-2*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a*sgn(x)) + sqrt(
a^2*c*x^2 - a*c*x)*abs(a)/(a^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}+3\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}\right)\right)}{a}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x)
```

output
```
(sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 3*log(sqrt(a*x - 1) + sqrt(x)*sq
rt(a))))/a
```

3.466 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=-\frac{5}{a\sqrt{c-\frac{c}{ax}}}+\frac{x}{\sqrt{c-\frac{c}{ax}}}+\frac{5\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

output
```
-5/a/(c-c/a/x)^(1/2)+x/(c-c/a/x)^(1/2)+5*arctanh((c-c/a/x)^(1/2)/c^(1/2))/
a/c^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{ax-5\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\frac{1}{ax}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

output
```
(a*x - 5*Hypergeometric2F1[-1/2, 1, 1/2, 1 - 1/(a*x)])/(a*Sqrt[c - c/(a*x)
])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6717, 6683, 1035, 281, 899, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{ax+1}{\sqrt{c-\frac{c}{ax}}(1-ax)}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{a+\frac{1}{x}}{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx}{a}$$

$$\Big\downarrow \text{899}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{87}$$

$$-\frac{c\left(\frac{5}{2}\int \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{ax}{c\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

$\Big\downarrow$ 61

$$-\frac{c\left(\frac{5}{2}\left(\frac{\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}+\frac{2}{c\sqrt{c-\frac{c}{ax}}}\right)-\frac{ax}{c\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

$\Big\downarrow$ 73

$$-\frac{c\left(\frac{5}{2}\left(\frac{2}{c\sqrt{c-\frac{c}{ax}}}-\frac{2a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c^2}\right)-\frac{ax}{c\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

$\Big\downarrow$ 221

$$-\frac{c\left(\frac{5}{2}\left(\frac{2}{c\sqrt{c-\frac{c}{ax}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{c^{3/2}}\right)-\frac{ax}{c\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

output
```
-((c*(-((a*x)/(c*Sqrt[c - c/(a*x)])) + (5*(2/(c*Sqrt[c - c/(a*x)]) - (2*Ar
cTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/c^(3/2)))/2))/a)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(60) = 120$.

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.14

method	result
risch	$\frac{ax-1}{a\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{5\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{2a\sqrt{a^{2}c}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}c+\left(x-\frac{1}{a}\right)ac}}{a^{3}c\left(x-\frac{1}{a}\right)}\right)\sqrt{c(ax-1)ax}}{\sqrt{\frac{c(ax-1)}{ax}}\,x}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(10\sqrt{x(ax-1)}\,a^{\frac{5}{2}}x^{2}+5\ln\left(\frac{2\sqrt{x(ax-1)}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^{2}x^{2}-8a^{\frac{3}{2}}\left(x(ax-1)\right)^{\frac{3}{2}}-20\sqrt{x(ax-1)}\,a^{\frac{3}{2}}x-10\ln\left(\frac{2\sqrt{x(ax-1)}\,\sqrt{a}}{2\sqrt{a}}\right.}{2\sqrt{x(ax-1)}\,c(ax-1)^{2}\sqrt{a}}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/(c*(a*x-1)/a/x)^(1/2)+(5/2/a*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/
2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-4/a^3/c/(x-1/a)*((x-1/a)^2*a^2*c
+(x-1/a)*a*c)^(1/2))/(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.64

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \left[\frac{5\,(ax-1)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(a^2x^2-5\,ax)\sqrt{\frac{acx-c}{ax}}}{2\,(a^2cx-ac)},\right.$$
$$\left.-\frac{5\,(ax-1)\sqrt{-c}\arctan\left(\frac{a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)-(a^2x^2-5\,ax)\sqrt{\frac{acx-c}{ax}}}{a^2cx-ac}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(5*(a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) + c) + 2*(a^2*x^2 - 5*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c),
-(5*(a*x - 1)*sqrt(-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x
- c)) - (a^2*x^2 - 5*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{ax+1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(1/2),x)
```

output
```
Integral((a*x + 1)/(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{ax+1}{(ax-1)\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)/((a*x - 1)*sqrt(c - c/(a*x))), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(60) = 120.

Time = 0.16 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.43

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{5\log\left(c^2|a|\right)\operatorname{sgn}(x)}{6\,a\sqrt{c}} - \frac{5\log\left(\left|2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^3\sqrt{c}|a|-5\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^2ac+4\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-\right.\right.\right.}{6\,a\sqrt{c}} + \frac{\sqrt{a^2cx^2-acx}|a|\operatorname{sgn}(x)}{a^2c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
5/6*log(c^2*abs(a))*sgn(x)/(a*sqrt(c)) - 5/6*log(abs(2*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - a*c*x))^3*sqrt(c)*abs(a) - 5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - a*c*x))^2*a*c + 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*c^(3/2)*ab
s(a) - a*c^2))*sgn(x)/(a*sqrt(c)) + sqrt(a^2*c*x^2 - a*c*x)*abs(a)*sgn(x)/
(a^2*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{a\,x+1}{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}\,dx$$

input
```
int((a*x + 1)/((c - c/(a*x))^(1/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - c/(a*x))^(1/2)*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{c}\,\left(20\sqrt{ax-1}\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}\right)-17\sqrt{ax-1}+4\sqrt{x}\,\sqrt{a}\,ax-20\sqrt{x}\,\sqrt{a}\right)}{4\sqrt{ax-1}\,ac}$$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x)
```

output
```
(sqrt(c)*(20*sqrt(a*x - 1)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a)) - 17*sqrt(
a*x - 1) + 4*sqrt(x)*sqrt(a)*a*x - 20*sqrt(x)*sqrt(a)))/(4*sqrt(a*x - 1)*a
*c)
```

3.467 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=-\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{ac\sqrt{c-\frac{c}{ax}}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{7\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}$$

output
```
-7/3/a/(c-c/a/x)^(3/2)-7/a/c/(c-c/a/x)^(1/2)+x/(c-c/a/x)^(3/2)+7*arctanh((
c-c/a/x)^(1/2)/c^(1/2))/a/c^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{x\left(3ax-7\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\frac{1}{ax}\right)\right)}{3c\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

output
```
(x*(3*a*x - 7*Hypergeometric2F1[-3/2, 1, -1/2, 1 - 1/(a*x)]))/(3*c*Sqrt[c
- c/(a*x)]*(-1 + a*x))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6717, 6683, 1035, 281, 899, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \ 6717$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \ 6683$$

$$-\int \frac{ax+1}{\left(c-\frac{c}{ax}\right)^{3/2}(1-ax)}dx$$

$$\Big\downarrow \ 1035$$

$$-\int \frac{a+\frac{1}{x}}{\left(\frac{1}{x}-a\right)\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \ 281$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx}{a}$$

$$\Big\downarrow \ 899$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \ 87$$

$$-\frac{c\left(\frac{7}{2}\int \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{a}$$

$\Big\downarrow$ 61

$$-\frac{c\left(\frac{7}{2}\left(\frac{\int \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{a}$$

$\Big\downarrow$ 61

$$-\frac{c\left(\frac{7}{2}\left(\frac{\frac{\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}+\frac{2}{c\sqrt{c-\frac{c}{ax}}}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{a}$$

$\Big\downarrow$ 73

$$-\frac{c\left(\frac{7}{2}\left(\frac{\frac{2}{c\sqrt{c-\frac{c}{ax}}}-\frac{2a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c^2}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{a}$$

$\Big\downarrow$ 221

$$-\frac{c\left(\frac{7}{2}\left(\frac{\frac{2}{c\sqrt{c-\frac{c}{ax}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{c^{3/2}}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

output
```
-((c*(-((a*x)/(c*(c - c/(a*x))^(3/2))) + (7*(2/(3*c*(c - c/(a*x))^(3/2)) +
 (2/(c*Sqrt[c - c/(a*x)]) - (2*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/c^(3/2)
)/c))/2))/a)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(81) = 162$.

Time = 0.17 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.12

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{7\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^2\sqrt{a^2c}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+\left(x-\frac{1}{a}\right)ac}}{3a^5c\left(x-\frac{1}{a}\right)^2}-\frac{22\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+\left(x-\frac{1}{a}\right)ac}}{3a^4c\left(x-\frac{1}{a}\right)}\right)a\sqrt{c(ax-1)ax}}{cx\sqrt{\frac{c(ax-1)}{ax}}}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(42\sqrt{x(ax-1)}\,a^{\frac{7}{2}}x^3+21\ln\left(\frac{2\sqrt{x(ax-1)}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^3x^3-36(x(ax-1))^{\frac{3}{2}}a^{\frac{5}{2}}x-126\sqrt{x(ax-1)}\,a^{\frac{5}{2}}x^2-63\ln\left(\frac{2\sqrt{x(ax-}}{}\right.\right.}{6\sqrt{x}}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c/(c*(a*x-1)/a/x)^(1/2)+(7/2/a^2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)
^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-4/3/a^5/c/(x-1/a)^2*((x-1/a)
^2*a^2*c+(x-1/a)*a*c)^(1/2)-22/3/a^4/c/(x-1/a)*((x-1/a)^2*a^2*c+(x-1/a)*a*
c)^(1/2))/c*a/x/(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.60

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \left[\frac{21\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(3\,a^3x^3-28\,a^2x^2+2\right.}{6\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}\right.$$

$$\left.-\frac{21\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\arctan\left(\frac{a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)-\left(3\,a^3x^3-28\,a^2x^2+21\,ax\right)\sqrt{\frac{acx-c}{ax}}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(21*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((
a*c*x - c)/(a*x)) + c) + 2*(3*a^3*x^3 - 28*a^2*x^2 + 21*a*x)*sqrt((a*c*x -
 c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/3*(21*(a^2*x^2 - 2*a*x
 + 1)*sqrt(-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) -
(3*a^3*x^3 - 28*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 -
2*a^2*c^2*x + a*c^2)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(3/2),x)
```

output
```
Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(3/2)*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(81) = 162$.

Time = 0.18 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.58

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{7\log\left(c^2|a||c|\right)\operatorname{sgn}(x)}{10\,ac^{\frac{3}{2}}}-\frac{7\log\left(\left|2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^5\sqrt{c}|a|-9\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^4ac+16\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-\cdots}\right.\right.\right.}{10\,ac^{\cdots}}+\frac{\sqrt{a^2cx^2-acx}|a|\operatorname{sgn}(x)}{a^2c^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
7/10*log(c^2*abs(a)*abs(c))*sgn(x)/(a*c^(3/2)) - 7/10*log(abs(2*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*sqrt(c)*abs(a) - 9*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - a*c*x))^4*a*c + 16*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^
3*c^(3/2)*abs(a) - 14*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a*c^2 +
6*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*c^(5/2)*abs(a) - a*c^3))*sgn(x
)/(a*c^(3/2)) + sqrt(a^2*c*x^2 - a*c*x)*abs(a)*sgn(x)/(a^2*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{a\,x+1}{\left(c-\frac{c}{a\,x}\right)^{3/2}\,\left(a\,x-1\right)}\,dx$$

input
```
int((a*x + 1)/((c - c/(a*x))^(3/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - c/(a*x))^(3/2)*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.20

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\sqrt{c}\left(42\sqrt{ax-1}\log\left(\sqrt{ax-1}+\sqrt{x}\sqrt{a}\right)ax-42\sqrt{ax-1}\log\left(\sqrt{ax-1}+\sqrt{x}\sqrt{a}\right)+5\cdots\right.}{6\sqrt{ax-1}\,a\,c^{2}\left(ax-\cdots\right.}$$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x)
```

output
```
(sqrt(c)*(42*sqrt(a*x - 1)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))*a*x - 42*s
qrt(a*x - 1)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a)) + 5*sqrt(a*x - 1)*a*x -
5*sqrt(a*x - 1) + 6*sqrt(x)*sqrt(a)*a**2*x**2 - 56*sqrt(x)*sqrt(a)*a*x + 4
2*sqrt(x)*sqrt(a)))/(6*sqrt(a*x - 1)*a*c**2*(a*x - 1))
```

3.468 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{9}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{9\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}$$

output
```
-9/5/a/(c-c/a/x)^(5/2)-3/a/c/(c-c/a/x)^(3/2)-9/a/c^2/(c-c/a/x)^(1/2)+x/(c-
c/a/x)^(5/2)+9*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.49

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{9\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-\frac{1}{ax}\right)}{5a\left(c-\frac{c}{ax}\right)^{5/2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

output
```
x/(c - c/(a*x))^(5/2) - (9*Hypergeometric2F1[-5/2, 1, -3/2, 1 - 1/(a*x)])/
(5*a*(c - c/(a*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6717, 6683, 1035, 281, 899, 87, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{ax+1}{\left(c-\frac{c}{ax}\right)^{5/2}(1-ax)}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{a+\frac{1}{x}}{\left(\frac{1}{x}-a\right)\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{7/2}}dx}{a}$$

$$\Big\downarrow \text{899}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\left(c-\frac{c}{ax}\right)^{7/2}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \; 87$$

$$-\frac{c\left(\frac{9}{2}\int \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{a}$$

$$\Big\downarrow \; 61$$

$$-\frac{c\left(\frac{9}{2}\left(\frac{\int \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{a}$$

$$\Big\downarrow \; 61$$

$$-\frac{c\left(\frac{9}{2}\left(\frac{\frac{\int \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{a}$$

$$\Big\downarrow \; 61$$

$$-\frac{c\left(\frac{9}{2}\left(\frac{\frac{\frac{\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}+\frac{2}{c\sqrt{c-\frac{c}{ax}}}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{a}$$

$$\Big\downarrow \; 73$$

$$-\frac{c\left(\frac{9}{2}\left(\frac{\frac{\frac{2}{c\sqrt{c-\frac{c}{ax}}}-\frac{2a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c^2}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{a}$$

$$\Big\downarrow \; 221$$

$$-\frac{c\left(\frac{9}{2}\left(\frac{\frac{\frac{2}{c\sqrt{c-\frac{c}{ax}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{c^{3/2}}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

output
```
-((c*(-((a*x)/(c*(c - c/(a*x))^(5/2))) + (9*(2/(5*c*(c - c/(a*x))^(5/2)) +
 (2/(3*c*(c - c/(a*x))^(3/2)) + (2/(c*Sqrt[c - c/(a*x)]) - (2*ArcTanh[Sqrt
[c - c/(a*x)]/Sqrt[c]])/c^(3/2))/c)/c))/2))/a)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(102) = 204.

Time = 0.18 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.09

method	result
risch	$\frac{ax-1}{a\,c^{2}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{9\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{2a^{3}\sqrt{a^{2}c}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}c+\left(x-\frac{1}{a}\right)ac}}{5a^{7}c\left(x-\frac{1}{a}\right)^{3}}-\frac{18\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}c+\left(x-\frac{1}{a}\right)ac}}{5a^{6}c\left(x-\frac{1}{a}\right)^{2}}-\frac{54\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}c+(x}}{5a^{5}c\left(x-\frac{1}{a}\right)}}{c^{2}x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-90\sqrt{x(ax-1)}\,a^{\frac{9}{2}}x^{4}+80(x(ax-1))^{\frac{3}{2}}a^{\frac{7}{2}}x^{2}-45\ln\left(\frac{2\sqrt{x(ax-1)}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^{4}x^{4}+360\sqrt{x(ax-1)}\,a^{\frac{7}{2}}x^{3}-132(x(ax\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^2/(c*(a*x-1)/a/x)^(1/2)+(9/2/a^3*ln((-1/2*a*c+a^2*c*x)/(a^2*
c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-4/5/a^7/c/(x-1/a)^3*((x-1/
a)^2*a^2*c+(x-1/a)*a*c)^(1/2)-18/5/a^6/c/(x-1/a)^2*((x-1/a)^2*a^2*c+(x-1/a
)*a*c)^(1/2)-54/5/a^5/c/(x-1/a)*((x-1/a)^2*a^2*c+(x-1/a)*a*c)^(1/2))*a^2/c
^2/x/(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.57

$$\int\frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\left[\frac{45\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(5\,a^{4}x^{4}-69\right.}{10\left(a^{4}c^{3}x^{3}-3\,a^{3}c^{3}x^{2}+3\,a^{2}c^{3}x-ac^{3}\right)}\right.$$

$$-\frac{45\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\sqrt{-c}\arctan\left(\frac{a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)-\left(5\,a^{4}x^{4}-69\,a^{3}x^{3}+105\,a^{2}x^{2}-45\,ax\right)\sqrt{\frac{acx-}{ax}}}{5\left(a^{4}c^{3}x^{3}-3\,a^{3}c^{3}x^{2}+3\,a^{2}c^{3}x-ac^{3}\right)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/10*(45*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqr
t(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(5*a^4*x^4 - 69*a^3*x^3 + 105*a^2*
x^2 - 45*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^
2*c^3*x - a*c^3), -1/5*(45*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arct
an(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) - (5*a^4*x^4 - 69*a^3
*x^3 + 105*a^2*x^2 - 45*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3
*c^3*x^2 + 3*a^2*c^3*x - a*c^3)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(5/2),x)
```

output
```
Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(5/2)*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(102) = 204.

Time = 0.24 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{9\log\left(c^4|a|\right)\operatorname{sgn}(x)}{14\,ac^{\frac{5}{2}}} - \frac{9\log\left(\left|2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^7\sqrt{c}|a|-13\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^6ac+36\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-\cdots}\right.\right.\right.}{\cdots} + \frac{\sqrt{a^2cx^2-acx}|a|\operatorname{sgn}(x)}{a^2c^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
9/14*log(c^4*abs(a))*sgn(x)/(a*c^(5/2)) - 9/14*log(abs(2*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - a*c*x))^7*sqrt(c)*abs(a) - 13*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - a*c*x))^6*a*c + 36*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*c^(3
/2)*abs(a) - 55*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a*c^2 + 50*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*c^(5/2)*abs(a) - 27*(sqrt(a^2*c)*
x - sqrt(a^2*c*x^2 - a*c*x))^2*a*c^3 + 8*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 a*c*x))*c^(7/2)*abs(a) - a*c^4))*sgn(x)/(a*c^(5/2)) + sqrt(a^2*c*x^2 - a*
c*x)*abs(a)*sgn(x)/(a^2*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{a\,x+1}{\left(c-\frac{c}{a\,x}\right)^{5/2}\,(a\,x-1)}\,dx$$

input
```
int((a*x + 1)/((c - c/(a*x))^(5/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - c/(a*x))^(5/2)*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.48

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\sqrt{c}\left(180\sqrt{ax-1}\log\left(\sqrt{ax-1}+\sqrt{x}\sqrt{a}\right)a^2x^2-360\sqrt{ax-1}\log\left(\sqrt{ax-1}+\sqrt{x}\sqrt{a}\right)\right.}{}$$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x)
```

output
```
(sqrt(c)*(180*sqrt(a*x - 1)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))*a**2*x**2
 - 360*sqrt(a*x - 1)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))*a*x + 180*sqrt(a
*x - 1)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a)) + 81*sqrt(a*x - 1)*a**2*x**2
- 162*sqrt(a*x - 1)*a*x + 81*sqrt(a*x - 1) + 20*sqrt(x)*sqrt(a)*a**3*x**3
- 276*sqrt(x)*sqrt(a)*a**2*x**2 + 420*sqrt(x)*sqrt(a)*a*x - 180*sqrt(x)*sq
rt(a)))/(20*sqrt(a*x - 1)*a*c**3*(a**2*x**2 - 2*a*x + 1))
```

3.469 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=-\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{11}{ac^3\sqrt{c-\frac{c}{ax}}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{11\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}$$

output
```
-11/7/a/(c-c/a/x)^(7/2)-11/5/a/c/(c-c/a/x)^(5/2)-11/3/a/c^2/(c-c/a/x)^(3/2
)-11/a/c^3/(c-c/a/x)^(1/2)+x/(c-c/a/x)^(7/2)+11*arctanh((c-c/a/x)^(1/2)/c^
(1/2))/a/c^(7/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.32

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\frac{7x-\frac{11\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},1-\frac{1}{ax}\right)}{a}}{7\left(c-\frac{c}{ax}\right)^{7/2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(7/2),x]
```

output
```
(7*x - (11*Hypergeometric2F1[-7/2, 1, -5/2, 1 - 1/(a*x)])/a)/(7*(c - c/(a*
x))^(7/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6717, 6683, 1035, 281, 899, 87, 61, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{ax+1}{\left(c-\frac{c}{ax}\right)^{7/2}(1-ax)}dx$$

$$\Big\downarrow \text{ 1035}$$

$$-\int \frac{a+\frac{1}{x}}{\left(\frac{1}{x}-a\right)\left(c-\frac{c}{ax}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{9/2}}dx}{a}$$

$$\Big\downarrow \text{ 899}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\left(c-\frac{c}{ax}\right)^{9/2}}d\frac{1}{x}}{a}$$

$\downarrow$ 87

$$-\frac{c\left(\frac{11}{2}\int \frac{x}{\left(c-\frac{c}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{7/2}}\right)}{a}$$

$\downarrow$ 61

$$-\frac{c\left(\frac{11}{2}\left(\frac{\int \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}d\frac{1}{x}}{c}+\frac{2}{7c\left(c-\frac{c}{ax}\right)^{7/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{7/2}}\right)}{a}$$

$\downarrow$ 61

$$-\frac{c\left(\frac{11}{2}\left(\frac{\frac{\int \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{c}+\frac{2}{7c\left(c-\frac{c}{ax}\right)^{7/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{7/2}}\right)}{a}$$

$\downarrow$ 61

$$-\frac{c\left(\frac{11}{2}\left(\frac{\frac{\frac{\int \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{c}+\frac{2}{7c\left(c-\frac{c}{ax}\right)^{7/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{7/2}}\right)}{a}$$

$\downarrow$ 61

$$-\frac{c\left(\frac{11}{2}\left(\frac{\frac{\frac{\frac{\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}+\frac{2}{c\sqrt{c-\frac{c}{ax}}}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{c}+\frac{2}{7c\left(c-\frac{c}{ax}\right)^{7/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{7/2}}\right)}{a}$$

$\downarrow$ 73

$$-\frac{c\left(\frac{11}{2}\left(\frac{\frac{\frac{\frac{2}{c\sqrt{c-\frac{c}{ax}}}-\frac{2a\int\frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c^2}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{c}+\frac{2}{7c\left(c-\frac{c}{ax}\right)^{7/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{7/2}}\right)}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c\left(\frac{11}{2}\left(\frac{\frac{\frac{\frac{2}{c\sqrt{c-\frac{c}{ax}}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{c^{3/2}}}{c}+\frac{2}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{2}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{c}+\frac{2}{7c\left(c-\frac{c}{ax}\right)^{7/2}}\right)-\frac{ax}{c\left(c-\frac{c}{ax}\right)^{7/2}}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(7/2),x]
```

output
```
-((c*(-((a*x)/(c*(c - c/(a*x))^(7/2))) + (11*(2/(7*c*(c - c/(a*x))^(7/2))
+ (2/(5*c*(c - c/(a*x))^(5/2)) + (2/(3*c*(c - c/(a*x))^(3/2)) + (2/(c*Sqrt
[c - c/(a*x)]) - (2*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/c^(3/2))/c)/c)/c))
/2))/a)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(123) = 246$.

Time = 0.18 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.01

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{11\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^4\sqrt{a^2c}}-\frac{4\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+\left(x-\frac{1}{a}\right)ac}}{7a^9c\left(x-\frac{1}{a}\right)^4}-\frac{102\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+\left(x-\frac{1}{a}\right)ac}}{35a^8c\left(x-\frac{1}{a}\right)^3}-\frac{712\sqrt{\left(x-\frac{1}{a}\right)^2a^2c-}}{105a^7c\left(x-\right.}\right.}{c^3x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2310\sqrt{x(ax-1)}\,a^{\frac{11}{2}}x^5+2100(x(ax-1))^{\frac{3}{2}}a^{\frac{9}{2}}x^3-1155\ln\left(\frac{2\sqrt{x(ax-1)}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^5x^5+11550\sqrt{x(ax-1)}\,a^{\frac{9}{2}}x^4\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^3/(c*(a*x-1)/a/x)^(1/2)+(11/2/a^4*ln((-1/2*a*c+a^2*c*x)/(a^2
*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-4/7/a^9/c/(x-1/a)^4*((x-1
/a)^2*a^2*c+(x-1/a)*a*c)^(1/2)-102/35/a^8/c/(x-1/a)^3*((x-1/a)^2*a^2*c+(x-
1/a)*a*c)^(1/2)-712/105/a^7/c/(x-1/a)^2*((x-1/a)^2*a^2*c+(x-1/a)*a*c)^(1/2
)-1516/105/a^6/c/(x-1/a)*((x-1/a)^2*a^2*c+(x-1/a)*a*c)^(1/2))*a^3/c^3/x/(c
*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.45

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \left[\frac{1155\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(\ldots}{210\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-\ldots}\right.$$

$$-\frac{1155\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\sqrt{-c}\arctan\left(\frac{a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)-(105\,a^5x^5-1936\,a^4x^4+4466\,a^3\ldots}{105\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

output
```
[1/210*(1155*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*log(-2*
a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(105*a^5*x^5 - 1936
*a^4*x^4 + 4466*a^3*x^3 - 3850*a^2*x^2 + 1155*a*x)*sqrt((a*c*x - c)/(a*x))
)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4), -1/
105*(1155*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*arctan(a*
sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) - (105*a^5*x^5 - 1936*a^4*
x^4 + 4466*a^3*x^3 - 3850*a^2*x^2 + 1155*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^
5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(7/2),x)
```

output
```
Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(7/2)*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{ax+1}{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(7/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(123) = 246.

Time = 0.34 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.70

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{11\log\left(c^4|a||c|\right)\text{sgn}(x)}{18\,ac^{\frac{7}{2}}} - \frac{11\log\left(\left|2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^9\sqrt{c}|a|-17\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^8ac+64\left(\sqrt{a^2c}x-\sqrt{a^2cx^2}\right.\right.\right.}{} + \frac{\sqrt{a^2cx^2-acx}|a|\text{sgn}(x)}{a^2c^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

output
```
11/18*log(c^4*abs(a)*abs(c))*sgn(x)/(a*c^(7/2)) - 11/18*log(abs(2*(sqrt(a^
2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^9*sqrt(c)*abs(a) - 17*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - a*c*x))^8*a*c + 64*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x
))^7*c^(3/2)*abs(a) - 140*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^6*a*c^
2 + 196*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*c^(5/2)*abs(a) - 182*(
sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a*c^3 + 112*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - a*c*x))^3*c^(7/2)*abs(a) - 44*(sqrt(a^2*c)*x - sqrt(a^2*c*x
^2 - a*c*x))^2*a*c^4 + 10*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*c^(9/2
)*abs(a) - a*c^5))*sgn(x)/(a*c^(7/2)) + sqrt(a^2*c*x^2 - a*c*x)*abs(a)*sgn
(x)/(a^2*c^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{a\,x+1}{\left(c-\frac{c}{a\,x}\right)^{7/2}\,(a\,x-1)}\,dx$$

input
```
int((a*x + 1)/((c - c/(a*x))^(7/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - c/(a*x))^(7/2)*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.63

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\frac{\sqrt{c}\left(1155\sqrt{ax-1}\log\left(\sqrt{ax-1}+\sqrt{x}\sqrt{a}\right)a^3x^3-3465\sqrt{ax-1}\log\left(\sqrt{ax-1}+\sqrt{x}\sqrt{\right.$$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x)
```

output
```
(sqrt(c)*(1155*sqrt(a*x - 1)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))*a**3*x**
3 - 3465*sqrt(a*x - 1)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))*a**2*x**2 + 34
65*sqrt(a*x - 1)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))*a*x - 1155*sqrt(a*x
- 1)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a)) + 751*sqrt(a*x - 1)*a**3*x**3 -
2253*sqrt(a*x - 1)*a**2*x**2 + 2253*sqrt(a*x - 1)*a*x - 751*sqrt(a*x - 1)
+ 105*sqrt(x)*sqrt(a)*a**4*x**4 - 1936*sqrt(x)*sqrt(a)*a**3*x**3 + 4466*sq
rt(x)*sqrt(a)*a**2*x**2 - 3850*sqrt(x)*sqrt(a)*a*x + 1155*sqrt(x)*sqrt(a))
)/(105*sqrt(a*x - 1)*a*c**4*(a**3*x**3 - 3*a**2*x**2 + 3*a*x - 1))
```

3.470 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 235

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\frac{33c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35a\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3c^5\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{51c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35a\sqrt{c-\frac{c}{ax}}}+\frac{9c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{7a}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{3c^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
33/35*c^6*(1-1/a^2/x^2)^(3/2)/a/(c-c/a/x)^(3/2)+3*c^5*(1-1/a^2/x^2)^(1/2)/
a/(c-c/a/x)^(1/2)+51/35*c^5*(1-1/a^2/x^2)^(3/2)/a/(c-c/a/x)^(1/2)+9/7*c^4*
(1-1/a^2/x^2)^(3/2)*(c-c/a/x)^(1/2)/a+c^3*(1-1/a^2/x^2)^(3/2)*(c-c/a/x)^(3
/2)*x-3*c^(9/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.46

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{c^4\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(10-26ax-12a^2x^2+164a^3x^3+35a^4x^4)-105a^3x^3\text{arctanh}\left(\sqrt{1+\right.\right.}{35a^4\sqrt{1-\frac{1}{ax}}x^3}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

output
```
(c^4*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(10 - 26*a*x - 12*a^2*x^2 + 164*
a^3*x^3 + 35*a^4*x^4) - 105*a^3*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(35*a^4*S
qrt[1 - 1/(a*x)]*x^3)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.71, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6731, 585, 27, 108, 27, 170, 27, 164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{9/2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}x^2}{a^3}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\int \left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}x^2d\frac{1}{x}}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 108$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(\int -\frac{3\left(a-\frac{1}{x}\right)^2\left(a+\frac{3}{x}\right)\sqrt{1+\frac{1}{ax}}x}{2a}d\frac{1}{x}-x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(a-\frac{1}{x}\right)^3-\frac{3\int \left(a-\frac{1}{x}\right)^2\left(a+\frac{3}{x}\right)\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 170$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(a-\frac{1}{x}\right)^3-\frac{3\left(\frac{2}{7}a\int \frac{1}{2}\left(a-\frac{1}{x}\right)\left(7a+\frac{17}{x}\right)\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}+\frac{6}{7}a\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^2\right)}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 27$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(a-\frac{1}{x}\right)^3-\frac{3\left(\frac{1}{7}a\int \left(a-\frac{1}{x}\right)\left(7a+\frac{17}{x}\right)\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}+\frac{6}{7}a\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^2\right)}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 164$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(a-\frac{1}{x}\right)^3-\frac{3\left(\frac{1}{7}a\left(7a^2\int \sqrt{1+\frac{1}{ax}}xd\frac{1}{x}+\frac{2}{5}a\left(28a-\frac{17}{x}\right)\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{6}{7}a\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)^2\right)}{2a}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow 60$$

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(a-\frac{1}{x}\right)^3-\frac{3\left(\frac{1}{7}a\left(7a^2\left(\int\frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+2\sqrt{\frac{1}{ax}+1}\right)+\frac{2}{5}a\left(28a-\frac{17}{x}\right)\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{6}{7}a\left(\frac{1}{ax}+1\right)^{3/2}(a-\cdots}{2a}}{a^3\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 73

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(a-\frac{1}{x}\right)^3-\frac{3\left(\frac{1}{7}a\left(7a^2\left(2a\int\frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+2\sqrt{\frac{1}{ax}+1}\right)+\frac{2}{5}a\left(28a-\frac{17}{x}\right)\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{6}{7}a\left(\frac{1}{ax}+1\cdots}{2a}}{a^3\sqrt{1-\frac{1}{ax}}}$$

$\downarrow$ 221

$$-\frac{c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(a-\frac{1}{x}\right)^3-\frac{3\left(\frac{1}{7}a\left(7a^2\left(2\sqrt{\frac{1}{ax}+1}-2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{2}{5}a\left(28a-\frac{17}{x}\right)\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{6}{7}a\left(\frac{1}{ax}+1\right)^{3/2}\left(a-\frac{1}{x}\right)\cdots}{2a}}{a^3\sqrt{1-\frac{1}{ax}}}$$

input

```
Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

output

```
-((c^4*Sqrt[c - c/(a*x)]*(-((a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*x) - (3*((6
*a*(a - x^(-1))^2*(1 + 1/(a*x))^(3/2))/7 + (a*((2*a*(28*a - 17/x)*(1 + 1/(
a*x))^(3/2))/5 + 7*a^2*(2*Sqrt[1 + 1/(a*x)] - 2*ArcTanh[Sqrt[1 + 1/(a*x)]]
)))/7))/(2*a)))/(a^3*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.76

method	result
default	$-\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,c^4\left(-70a^{\frac{9}{2}}\sqrt{x(ax+1)}\,x^4+105\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^4x^4-328a^{\frac{7}{2}}x^3\sqrt{x(ax+1)}+24a^{\frac{5}{2}}x^2\sqrt{x(ax+1)}+5}{70\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^3a^{\frac{9}{2}}\sqrt{x(ax+1)}}$
risch	$\frac{(35a^5x^5+199a^4x^4+152a^3x^3-38a^2x^2-16ax+10)c^4\sqrt{\frac{c(ax-1)}{ax}}}{35x^3a^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^4\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/70/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x^3*c^
4/a^(9/2)*(-70*a^(9/2)*(x*(a*x+1))^(1/2)*x^4+105*ln(1/2*(2*(x*(a*x+1))^(1/
2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^4*x^4-328*a^(7/2)*x^3*(x*(a*x+1))^(1/2)+24*
a^(5/2)*x^2*(x*(a*x+1))^(1/2)+52*a^(3/2)*x*(x*(a*x+1))^(1/2)-20*(x*(a*x+1)
)^(1/2)*a^(1/2))/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.86

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \left[\frac{105\left(a^4c^4x^4-a^3c^4x^3\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(35a^5\right.}{140\left(a^5x^4-a^4\right.}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

output
```
[1/140*(105*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*
x - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt
((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(35*a^5*c^4*x^5 + 199*a^4*c^4*x^4
+ 152*a^3*c^4*x^3 - 38*a^2*c^4*x^2 - 16*a*c^4*x + 10*c^4)*sqrt((a*x - 1)/(
a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/70*(105*(a^4*c^4
*x^4 - a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x -
 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(35*
a^5*c^4*x^5 + 199*a^4*c^4*x^4 + 152*a^3*c^4*x^3 - 38*a^2*c^4*x^2 - 16*a*c^
4*x + 10*c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4
- a^4*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{9/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.52

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{\sqrt{c}\,c^4\left(35\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^4x^4+164\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^3x^3-12\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2-26\sqrt{}\right.}{35a^5x^4}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x)
```

output
```
(sqrt(c)*c**4*(35*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**4*x**4 + 164*sqrt(x)*sq
rt(a)*sqrt(a*x + 1)*a**3*x**3 - 12*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**2*x**2
 - 26*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x + 10*sqrt(x)*sqrt(a)*sqrt(a*x + 1)
 - 105*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a**4*x**4 - 99*a**4*x**4))/(35
*a**5*x**4)
```

3.471 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 195

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{23c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15a\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{7c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5a\sqrt{c-\frac{c}{ax}}}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}\,x-\frac{c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
23/15*c^5*(1-1/a^2/x^2)^(3/2)/a/(c-c/a/x)^(3/2)+c^4*(1-1/a^2/x^2)^(1/2)/a/
(c-c/a/x)^(1/2)+7/5*c^4*(1-1/a^2/x^2)^(3/2)/a/(c-c/a/x)^(1/2)+c^3*(1-1/a^2
/x^2)^(3/2)*(c-c/a/x)^(1/2)*x-c^(7/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/
(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.52

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(-6+8ax+44a^2x^2+15a^3x^3)-15a^2x^2\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{15a^3\sqrt{1-\frac{1}{ax}}x^2}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

output
```
(c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-6 + 8*a*x + 44*a^2*x^2 + 15*a^
3*x^3) - 15*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(15*a^3*Sqrt[1 - 1/(a*x)]
*x^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.68, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6731, 585, 27, 100, 27, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{7/2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c^3\int \left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}x^2}{a^2}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\int \left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}x^2d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\int -\frac{1}{2}\left(a-\frac{2}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}-a^2x\left(\frac{1}{ax}+1\right)^{5/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{2}\int \left(a-\frac{2}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}-a^2x\left(\frac{1}{ax}+1\right)^{5/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{90}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{4}{5}a\left(\frac{1}{ax}+1\right)^{5/2}-a\int \left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}\right)-a^2x\left(\frac{1}{ax}+1\right)^{5/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{60}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{4}{5}a\left(\frac{1}{ax}+1\right)^{5/2}-a\left(\int \sqrt{1+\frac{1}{ax}}xd\frac{1}{x}+\frac{2}{3}\left(\frac{1}{ax}+1\right)^{3/2}\right)\right)-a^2x\left(\frac{1}{ax}+1\right)^{5/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{60}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{4}{5}a\left(\frac{1}{ax}+1\right)^{5/2}-a\left(\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{3}\left(\frac{1}{ax}+1\right)^{3/2}+2\sqrt{\frac{1}{ax}+1}\right)\right)-a^2x\left(\frac{1}{ax}+1\right)^{5/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{4}{5}a\left(\frac{1}{ax}+1\right)^{5/2}-a\left(2a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{3}\left(\frac{1}{ax}+1\right)^{3/2}+2\sqrt{\frac{1}{ax}+1}\right)\right)-a^2x\left(\frac{1}{ax}+1\right)^{5/2}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c^3\left(\frac{1}{2}\left(\frac{4}{5}a\left(\frac{1}{ax}+1\right)^{5/2}-a\left(-2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)+\frac{2}{3}\left(\frac{1}{ax}+1\right)^{3/2}+2\sqrt{\frac{1}{ax}+1}\right)\right)-a^2x\left(\frac{1}{ax}+1\right)^{5/2}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

output
```
-((c^3*Sqrt[c - c/(a*x)]*(-(a^2*(1 + 1/(a*x))^(5/2)*x) + ((4*a*(1 + 1/(a*x
))^(5/2))/5 - a*(2*Sqrt[1 + 1/(a*x)] + (2*(1 + 1/(a*x))^(3/2))/3 - 2*ArcTa
nh[Sqrt[1 + 1/(a*x)]]))/2))/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.83

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\, c^3 \left(30a^{\frac{7}{2}}x^3\sqrt{x(ax+1)}+88a^{\frac{5}{2}}x^2\sqrt{x(ax+1)}-15\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^3x^3+16a^{\frac{3}{2}}x\sqrt{x(ax+1)}-12\sqrt{x(ax}\right.}{30\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^2a^{\frac{7}{2}}\sqrt{x(ax+1)}}$
risch	$\frac{(15a^4x^4+59a^3x^3+52a^2x^2+2ax-6)c^3\sqrt{\frac{c(ax-1)}{ax}}}{15x^2a^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^3\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/30/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^3*(30
*a^(7/2)*x^3*(x*(a*x+1))^(1/2)+88*a^(5/2)*x^2*(x*(a*x+1))^(1/2)-15*ln(1/2*
(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3*x^3+16*a^(3/2)*x*(x*(a*
x+1))^(1/2)-12*(x*(a*x+1))^(1/2)*a^(1/2))/x^2/a^(7/2)/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.13

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\left[\frac{15\,(a^3c^3x^3-a^2c^3x^2)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(15\,a^4c}{60\,(a^4x^3-a^3x^2)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

output
```
[1/60*(15*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
- 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 59*a^3*c^3*x^3 + 5
2*a^2*c^3*x^2 + 2*a*c^3*x - 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(15*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt
(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 59*a^3*c^3
*x^3 + 52*a^2*c^3*x^2 + 2*a*c^3*x - 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\int \frac{\left(c-\frac{c}{a\,x}\right)^{7/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.54

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{\sqrt{c}\,c^3\left(60\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^3x^3+176\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2+32\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax-24\sqrt{x}\right.}{60a^4x^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x)
```

output
```
(sqrt(c)*c**3*(60*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**3*x**3 + 176*sqrt(x)*sq
rt(a)*sqrt(a*x + 1)*a**2*x**2 + 32*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x - 24*
sqrt(x)*sqrt(a)*sqrt(a*x + 1) - 60*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a*
*3*x**3 - 53*a**3*x**3))/(60*a**4*x**3)
```

3.472 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{c^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
-1/3*c^4*(1-1/a^2/x^2)^(3/2)/a/(c-c/a/x)^(3/2)-c^3*(1-1/a^2/x^2)^(1/2)/a/(
c-c/a/x)^(1/2)+c^5*(1-1/a^2/x^2)^(5/2)*x/(c-c/a/x)^(5/2)+c^(5/2)*arctanh(c
^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.57

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(2+2ax+3a^2x^2)+3ax\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{3a^2\sqrt{1-\frac{1}{ax}}x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

output
```
(c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 + 2*a*x + 3*a^2*x^2) + 3*a*x*
ArcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6731, 580, 576, 576, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{5/2} e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{580}$$

$$-c^3\left(\frac{c\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

$$\Big\downarrow \text{576}$$

$$-c^3\left(\frac{c\left(\frac{\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}+\frac{2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

$$\Big\downarrow \text{576}$$

$$-c^3\left(\frac{c\left(\frac{\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}+\frac{2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

$$\Big\downarrow \; 573$$

$$-c^3\left(\frac{c\left(\frac{\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-2\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}+\frac{2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

$$\Big\downarrow \; 219$$

$$-c^3\left(\frac{c\left(\frac{\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{\sqrt{c}}}{c}+\frac{2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{2a}-\frac{c^2x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

output
```
-(c^3*(-((c^2*(1 - 1/(a^2*x^2))^(5/2)*x)/(c - c/(a*x))^(5/2)) + (c*((2*(1
- 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) + ((2*Sqrt[1 - 1/(a^2*x^2)])
/Sqrt[c - c/(a*x)] - (2*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c
/(a*x)]])/Sqrt[c])/c))/(2*a)))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 576
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-(e*x)^(m + 1))*(c + d*x)^n*((a + b*x^2)^p/(e*(n - m - 1
))), x] - Simp[b*c*(n/(d^2*(n - m - 1)))   Int[(e*x)^m*(c + d*x)^(n + 1)*(a
 + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c^2 + a*
d^2, 0] && EqQ[n + p, 0] && GtQ[p, 0] && NeQ[m - n + 1, 0] &&  !IGtQ[m, 0]
&&  !(IntegerQ[m + p] && LtQ[m + p + 2, 0]) && RationalQ[m]
```

rule 580
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 2)*((a + b*x^2)^(p +
1)/(b*e*(m + 1))), x] + Simp[d*((2*m + p + 3)/(e*(m + 1)))   Int[(e*x)^(m +
 1)*(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p},
 x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p - 1, 0] && LtQ[m, -1] && IntegerQ
[p + 1/2]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\, c^2\left(6a^{\frac{5}{2}}x^2\sqrt{x(ax+1)}+3\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^2x^2+4a^{\frac{3}{2}}x\sqrt{x(ax+1)}+4\sqrt{x(ax+1)}\,\sqrt{a}\right)}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x\,a^{\frac{5}{2}}\sqrt{x(ax+1)}}$	144
risch	$\frac{(3a^3x^3+5a^2x^2+4ax+2)c^2\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}+\frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^2\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	168

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x*c^2/a^
(5/2)*(6*a^(5/2)*x^2*(x*(a*x+1))^(1/2)+3*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/
2)+2*a*x+1)/a^(1/2))*a^2*x^2+4*a^(3/2)*x*(x*(a*x+1))^(1/2)+4*(x*(a*x+1))^(
1/2)*a^(1/2))/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.44

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\left[\frac{3\left(a^2c^2x^2-ac^2x\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3a^3c^2x^3\right.}{12\left(a^3x^2-a^2x\right)}\right.$$

$$-\frac{3\left(a^2c^2x^2-ac^2x\right)\sqrt{-c}\arctan\left(\frac{2\left(a^2x^2+ax\right)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)-2\left(3a^3c^2x^3+5a^2c^2x^2+4ac^2x+2c^2\right)\sqrt{\frac{ax}{ax}}}{6\left(a^3x^2-a^2x\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(
2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + 4*a*c^2*
x + 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a
^2*x), -1/6*(3*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a
*c*x - c)) - 2*(3*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + 4*a*c^2*x + 2*c^2)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{5/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.56

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{\sqrt{c}\,c^2\left(6\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2+4\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax+4\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+6\log\left(\sqrt{ax+1}\right.\right.}{6a^3x^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x)
```

output
```
(sqrt(c)*c**2*(6*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**2*x**2 + 4*sqrt(x)*sqrt(
a)*sqrt(a*x + 1)*a*x + 4*sqrt(x)*sqrt(a)*sqrt(a*x + 1) + 6*log(sqrt(a*x +
1) + sqrt(x)*sqrt(a))*a**2*x**2 + 5*a**2*x**2))/(6*a**3*x**2)
```

3.473 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 118

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=-\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
-3*c^2*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+c^3*(1-1/a^2/x^2)^(3/2)*x/(c-
c/a/x)^(3/2)+3*c^(3/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
)/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.56

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},1+\frac{1}{ax}\right)}{5a\left(1-\frac{1}{ax}\right)^{3/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

output
```
(-2*(1 + 1/(a*x))^(5/2)*(c - c/(a*x))^(3/2)*Hypergeometric2F1[2, 5/2, 7/2,
 1 + 1/(a*x)])/(5*a*(1 - 1/(a*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6731, 575, 576, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{3/2} e^{3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3 \int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2} x^2}{\left(c-\frac{c}{ax}\right)^{3/2}} d\frac{1}{x}$$

$$\Big\downarrow \text{575}$$

$$-c^3 \left(\frac{3\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2ac}-\frac{x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{576}$$

$$-c^3 \left(\frac{3\left(\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{2ac}-\frac{x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{573}$$

$$-c^3\left(\frac{3\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-2\int\frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{2ac}-\frac{x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{219}$$

$$-c^3\left(\frac{3\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{\sqrt{c}}\right)}{2ac}-\frac{x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

output
```
-(c^3*(-(((1 - 1/(a^2*x^2))^(3/2)*x)/(c - c/(a*x))^(3/2)) + (3*((2*Sqrt[1
- 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] - (2*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^
2)])/Sqrt[c - c/(a*x)]])/Sqrt[c]))/(2*a*c)))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 575
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(e*x)^(m + 1)*(c + d*x)^n*((a + b*x^2)^p/(e*(m + 1))), x]
 + Simp[b*(n/(d*e*(m + 1)))   Int[(e*x)^(m + 1)*(c + d*x)^(n + 1)*(a + b*x^
2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[b*c^2 + a*d^2, 0] &&
 EqQ[n + p, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m + p] && LeQ[m +
p + 2, 0])
```

rule 576
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-(e*x)^(m + 1))*(c + d*x)^n*((a + b*x^2)^p/(e*(n - m - 1
))), x] - Simp[b*c*(n/(d^2*(n - m - 1)))   Int[(e*x)^m*(c + d*x)^(n + 1)*(a
 + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c^2 + a*
d^2, 0] && EqQ[n + p, 0] && GtQ[p, 0] && NeQ[m - n + 1, 0] &&  !IGtQ[m, 0]
&&  !(IntegerQ[m + p] && LtQ[m + p + 2, 0]) && RationalQ[m]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\, c\left(2a^{\frac{3}{2}} x\sqrt{x(ax+1)}+3\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-4\sqrt{x(ax+1)}\,\sqrt{a}\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{3}{2}}\sqrt{x(ax+1)}}$	118
risch	$\frac{(a^2x^2-ax-2)c\sqrt{\frac{c(ax-1)}{ax}}}{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}+\frac{3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	151

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2
)*(2*a^(3/2)*x*(x*(a*x+1))^(1/2)+3*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a
*x+1)/a^(1/2))*a*x-4*(x*(a*x+1))^(1/2)*a^(1/2))/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.67

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\left[\frac{3\,(acx-c)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2cx^2-acx-}{4\,(a^2x-a)},\ -\frac{3\,(acx-c)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(a^2cx^2-acx-2\,c)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(a^2x-a)}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3
*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
- c)/(a*x - 1)) + 4*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x)))/(a^2*x - a), -1/2*(3*(a*c*x - c)*sqrt(-c)*arctan(2*
(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
/(2*a^2*c*x^2 - a*c*x - c)) - 2*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(
a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^{3/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\frac{\sqrt{c}\,c\left(4\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax-8\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+12\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)ax-9ax\right)}{4a^2x}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x)
```

output
```
(sqrt(c)*c*(4*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x - 8*sqrt(x)*sqrt(a)*sqrt(a
*x + 1) + 12*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a*x - 9*a*x))/(4*a**2*x)
```

3.474 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 135

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{5\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
c*(1-1/a^2/x^2)^(1/2)*x/(c-c/a/x)^(1/2)+5*c^(1/2)*arctanh(c^(1/2)*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2))/a-4*2^(1/2)*c^(1/2)*arctanh(1/2*c^(1/2)*(1-1/
a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}x+\frac{5\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}-\frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

input

```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output

```
(Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x + (5*ArcTanh[Sqrt[1 + 1/(a*x)]])/a
 - (4*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a))/Sqrt[1 - 1/(a*x)]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.77, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6731, 585, 27, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c\sqrt{1-\frac{1}{ax}}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}x^2}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x^2}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 109$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(-\frac{\int -\frac{\left(5a+\frac{3}{x}\right)x}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(5a+\frac{3}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 174$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{8\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 73$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{16a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+10a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 221$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-10\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input

```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
-((a*c*Sqrt[1 - 1/(a*x)]*(-((Sqrt[1 + 1/(a*x)]*x)/a) + (-10*ArcTanh[Sqrt[1
 + 1/(a*x)]] + 8*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(2*a^2)))/Sqr
t[c - c/(a*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.19

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-4\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)\sqrt{a}+5\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$
risch	$\frac{x\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2cx^2+acx}\right)}{2\sqrt{a^2c}}-\frac{2\sqrt{2}\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{a\sqrt{c}}\right)\sqrt{\frac{c(ax-1)}{ax}}\sqrt{(a}}{\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*(x*
(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2)-4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*
(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+5*ln(1/2*(2*(x*(a*x+1))^(1/2)*a
^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/(x*(a*x+1))^(1/2)/a^(3/2)/(1/a)^(1
/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(112) = 224$.

Time = 0.17 (sec) , antiderivative size = 512, normalized size of antiderivative = 3.79

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$= \left[\frac{4\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+5(ax-1)\sqrt{c}\log}{4(a^2x-a)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*
c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 5*
(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 +
 a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x
- 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^2*x - a), 1/2*(4*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^
2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2
*c*x^2 - 2*a*c*x - c)) - 5*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c
*x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a^2*x - a)]
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=\frac{\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+2\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}-1\right)-2\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}+1\right)\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x)
```

output
```
(sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x + 1) + 2*sqrt(2)*log(sqrt(a*x + 1) + sq
rt(x)*sqrt(a) - sqrt(2) - 1) - 2*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(
a) - sqrt(2) + 1) - 2*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2
) - 1) + 2*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1) + 5*
log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))))/a
```

3.475 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 170

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = -\frac{3c\sqrt{1-\frac{1}{a^2x^2}}}{a\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}} - \frac{5\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

output
```
-3*c*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(3/2)+c*(1-1/a^2/x^2)^(1/2)*x/(c-c/a/
x)^(3/2)+7*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(1/2)-
5*2^(1/2)*arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))
/a/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.68

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(a\sqrt{1+\frac{1}{ax}}x(-3+ax)+7(-1+ax)\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-5\sqrt{2}(-1+ax)\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{a\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x*(-3 + a*x) + 7*(-1 + a*x)*ArcTan
h[Sqrt[1 + 1/(a*x)]] - 5*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt
[2]]))/(a*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.79, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6731, 585, 27, 109, 25, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-\frac{c}{ax}\right)^{7/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^2\left(1+\frac{1}{ax}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^2}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 109$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{2x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}-\frac{\int -\frac{\left(3a+\frac{2}{x}\right)x^2}{a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(3a+\frac{2}{x}\right)x^2}{a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}+\frac{2x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(3a+\frac{2}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 168$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{-\frac{\int -\frac{\left(7a+\frac{3}{x}\right)x}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-3x\sqrt{\frac{1}{ax}+1}}{a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\int \frac{\left(7a+\frac{3}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-3x\sqrt{\frac{1}{ax}+1}}{a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 174$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{10\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+7\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-3x\sqrt{\frac{1}{ax}+1}}{a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 73$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{20a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+14a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{2a}-3x\sqrt{\frac{1}{ax}+1}}{a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 221$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{10\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-14\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{2a}-3x\sqrt{\frac{1}{ax}+1}}{a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

output
```
-((a^2*Sqrt[1 - 1/(a*x)]*((2*Sqrt[1 + 1/(a*x)]*x)/(a*(a - x^(-1))) + (-3*S
qrt[1 + 1/(a*x)]*x + (-14*ArcTanh[Sqrt[1 + 1/(a*x)]] + 10*Sqrt[2]*ArcTanh[
Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(2*a))/a^2))/Sqrt[c - c/(a*x)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.52

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x-5a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)x+7\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^{2}\sqrt{\frac{1}{a}}\,x-6\sqrt{x(ax+}\right.}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{3}{2}}c\sqrt{x(ax+1)}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{7\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a\sqrt{a^{2}c}}-\frac{2\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}c+3\left(x-\frac{1}{a}\right)ac+2c}}{a^{3}c\left(x-\frac{1}{a}\right)}-\frac{5\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}\right.}{2a^{2}\sqrt{c}}\right.}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^(5/2)*(1/
a)^(1/2)*(x*(a*x+1))^(1/2)*x-5*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(
x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x+7*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/
2)+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*x-6*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^(
1/2)-7*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)
+5*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))
*a^(1/2))/a^(3/2)/c/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 581, normalized size of antiderivative = 3.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

$$= \left[\frac{7\,(a^2x^2-2\,ax+1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^3x^3-2\,a^2x^2-3\,ax}{4\,(a^3cx^2-2\,a^2cx} \right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(7*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a
^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x)) - c)/(a*x - 1)) + 4*(a^3*x^3 - 2*a^2*x^2 - 3*a*x)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) + 5*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*
log(-(17*a^3*x^3 - 3*a^2*x^2 - 13*a*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 +
 a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/sqrt(c) - 1)/(a^3*
x^3 - 3*a^2*x^2 + 3*a*x - 1))/sqrt(c))/(a^3*c*x^2 - 2*a^2*c*x + a*c), 1/2*
(5*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*sqrt(-1/c)*arctan(2*sqrt(2)*(a^2*x^2
+ a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(3*a^2
*x^2 - 2*a*x - 1)) - 7*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^2 +
a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x
^2 - a*c*x - c)) + 2*(a^3*x^3 - 2*a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{1}{\sqrt{c-\frac{c}{a\,x}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.48

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{c}\left(2\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax-6\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+5\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}-1\right)ax-5\sqrt{2}\log(\sqrt{}\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x)
```

output
```
(sqrt(c)*(2*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x - 6*sqrt(x)*sqrt(a)*sqrt(a*x
 + 1) + 5*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1)*a*x -
 5*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1) - 5*sqrt(2)*
log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) + 1)*a*x + 5*sqrt(2)*log(sqr
t(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) + 1) - 5*sqrt(2)*log(sqrt(a*x + 1)
+ sqrt(x)*sqrt(a) + sqrt(2) - 1)*a*x + 5*sqrt(2)*log(sqrt(a*x + 1) + sqrt(
x)*sqrt(a) + sqrt(2) - 1) + 5*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)
+ sqrt(2) + 1)*a*x - 5*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(
2) + 1) + 14*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a*x - 14*log(sqrt(a*x +
1) + sqrt(x)*sqrt(a))))/(2*a*c*(a*x - 1))
```

3.476 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal result

Integrand size $=24$, antiderivative size $=208$

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=-\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{15\sqrt{1-\frac{1}{a^2x^2}}}{4a\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{9\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}-\frac{51\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{4\sqrt{2}ac^{3/2}}$$

output

```
-2*c*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(5/2)-15/4*(1-1/a^2/x^2)^(1/2)/a/(c-c
/a/x)^(3/2)+c*(1-1/a^2/x^2)^(1/2)*x/(c-c/a/x)^(5/2)+9*arctanh(c^(1/2)*(1-1
/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(3/2)-51/8*arctanh(1/2*c^(1/2)*(1-1/a
^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))*2^(1/2)/a/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.65

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(15-23ax+4a^2x^2)+72(-1+ax)^2\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-51\right.}{8ac\sqrt{c-\frac{c}{ax}}(-1+ax)^2}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(15 - 23*a*x + 4*a^2*x^2) + 72
*(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 51*Sqrt[2]*(-1 + a*x)^2*ArcTanh
[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(8*a*c*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.83, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {6731, 585, 27, 109, 25, 27, 168, 27, 168, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-\frac{c}{ax}\right)^{9/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^3\left(1+\frac{1}{ax}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\int\frac{\left(1+\frac{1}{ax}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^3}d\frac{1}{x}}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}-\frac{\int-\frac{\left(4a+\frac{3}{x}\right)x^2}{a\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\int\frac{\left(4a+\frac{3}{x}\right)x^2}{a\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\int\frac{\left(4a+\frac{3}{x}\right)x^2}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{7x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{\int-\frac{3\left(10a+\frac{7}{x}\right)x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\int\frac{\left(10a+\frac{7}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}+\frac{7x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(-\frac{\int -\frac{\left(12a+\frac{5}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-10x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{7x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$\Big\downarrow$ 25

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{\int \frac{\left(12a+\frac{5}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-10x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{7x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$\Big\downarrow$ 174

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{17\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+12\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-10x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{7x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$\Big\downarrow$ 73

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{34a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+24a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{a}-10x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{7x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$\Big\downarrow$ 221

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{17\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-24\,\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}-10x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{7x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{2a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{a\left(a-\frac{1}{x}\right)^2}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

output
```
-((a^3*Sqrt[1 - 1/(a*x)]*((Sqrt[1 + 1/(a*x)]*x)/(a*(a - x^(-1))^2) + ((7*S
qrt[1 + 1/(a*x)]*x)/(2*(a - x^(-1))) + (3*(-10*Sqrt[1 + 1/(a*x)]*x + (-24*
ArcTanh[Sqrt[1 + 1/(a*x)]] + 17*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]
)/a))/(4*a))/(2*a^2)))/(c*Sqrt[c - c/(a*x)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.57

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{9\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^2\sqrt{a^2c}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+3\left(x-\frac{1}{a}\right)ac+2c}}{a^5c\left(x-\frac{1}{a}\right)^2}-\frac{15\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+3\left(x-\frac{1}{a}\right)ac+2c}}{4a^4c\left(x-\frac{1}{a}\right)}-\frac{51\sqrt{2}\ \cdots}{\cdots}\right.}{c\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x\sqrt{\frac{c(ax-\cdots}{ax}}}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x^2-51a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)x^2-92a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x+72\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{\cdots}}{2\sqrt{a}\cdots}\right.\right.}{\cdots}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/c/((a*x-1)/(a*x+1))^(1/2)/(c*(a*x-1)/a/x)^(1/2)*(a*x-1)+(9/2/a^2*ln((1
/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2+a*c*x)^(1/2))/(a^2*c)^(1/2)-1/a^5
/c/(x-1/a)^2*((x-1/a)^2*a^2*c+3*(x-1/a)*a*c+2*c)^(1/2)-15/4/a^4/c/(x-1/a)*
((x-1/a)^2*a^2*c+3*(x-1/a)*a*c+2*c)^(1/2)-51/16/a^3/c^(1/2)*2^(1/2)*ln((4*
c+3*(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*((x-1/a)^2*a^2*c+3*(x-1/a)*a*c+2*c)^(1/2
))/(x-1/a)))/c*a/((a*x-1)/(a*x+1))^(1/2)/(a*x+1)/x/(c*(a*x-1)/a/x)^(1/2)*(
(a*x+1)*a*c*x)^(1/2)*(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 668, normalized size of antiderivative = 3.21

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\left[\frac{51\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{\cdots}}{a^3x^3-3a^2x^2+3ax-1}\right.}{\cdots}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(51*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3*c
*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sq
rt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*
a^2*x^2 + 3*a*x - 1)) + 72*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-
(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(4*a^4*x^4 -
19*a^3*x^3 - 8*a^2*x^2 + 15*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2), 1/16*(51*sq
rt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2
 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*
c*x^2 - 2*a*c*x - c)) - 72*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arct
an(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 4*(4*a^4*x^4 - 19*a^3*x^3 - 8*a^2*x^2 +
 15*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*c^2*x^3 -
 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a\,x}\right)^{3/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.02

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\sqrt{c}\left(16\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2-92\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax+60\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+51\sqrt{2}\log\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x)
```

output
```
(sqrt(c)*(16*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**2*x**2 - 92*sqrt(x)*sqrt(a)*
sqrt(a*x + 1)*a*x + 60*sqrt(x)*sqrt(a)*sqrt(a*x + 1) + 51*sqrt(2)*log(sqrt
(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1)*a**2*x**2 - 102*sqrt(2)*log(sqr
t(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1)*a*x + 51*sqrt(2)*log(sqrt(a*x
+ 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1) - 51*sqrt(2)*log(sqrt(a*x + 1) + sqr
t(x)*sqrt(a) - sqrt(2) + 1)*a**2*x**2 + 102*sqrt(2)*log(sqrt(a*x + 1) + sq
rt(x)*sqrt(a) - sqrt(2) + 1)*a*x - 51*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*
sqrt(a) - sqrt(2) + 1) - 51*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) +
sqrt(2) - 1)*a**2*x**2 + 102*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) +
 sqrt(2) - 1)*a*x - 51*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(
2) - 1) + 51*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1)*a*
*2*x**2 - 102*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1)*a
*x + 51*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1) + 144*l
og(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a**2*x**2 - 288*log(sqrt(a*x + 1) + sq
rt(x)*sqrt(a))*a*x + 144*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)) + 37*a**2*x*
*2 - 74*a*x + 37))/(16*a*c**2*(a**2*x**2 - 2*a*x + 1))
```

3.477 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 249

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = -\frac{5c\sqrt{1-\frac{1}{a^2x^2}}}{3a\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{29\sqrt{1-\frac{1}{a^2x^2}}}{12a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{73\sqrt{1-\frac{1}{a^2x^2}}}{16ac\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{11\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{5/2}}-\frac{249\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{16\sqrt{2}ac^{5/2}}$$

output
```
-5/3*c*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(7/2)-29/12*(1-1/a^2/x^2)^(1/2)/a/(
c-c/a/x)^(5/2)-73/16*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a/x)^(3/2)+c*(1-1/a^2/x^
2)^(1/2)*x/(c-c/a/x)^(7/2)+11*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x
)^(1/2))/a/c^(5/2)-249/32*arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/
(c-c/a/x)^(1/2))*2^(1/2)/a/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.57

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(-219+554ax-415a^2x^2+48a^3x^3)+1056(-1+ax)^3\text{arctan}\right.}{96ac^2\sqrt{c-\frac{c}{ax}}(-1+ax)^3}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-219 + 554*a*x - 415*a^2*x^2
+ 48*a^3*x^3) + 1056*(-1 + a*x)^3*ArcTanh[Sqrt[1 + 1/(a*x)]] - 747*Sqrt[2]
*(-1 + a*x)^3*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(96*a*c^2*Sqrt[c - c/(a
*x)]*(-1 + a*x)^3)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.84, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6731, 585, 27, 109, 25, 27, 168, 27, 168, 27, 168, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-\frac{c}{ax}\right)^{11/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^4\left(1+\frac{1}{ax}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x^2}{\left(a-\frac{1}{x}\right)^4}d\frac{1}{x}}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}-\frac{\int -\frac{\left(5a+\frac{4}{x}\right)x^2}{a\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{3a}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(5a+\frac{4}{x}\right)x^2}{a\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{3a}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(5a+\frac{4}{x}\right)x^2}{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}-\frac{\int -\frac{\left(58a+\frac{45}{x}\right)x^2}{2\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\int \frac{\left(58a+\frac{45}{x}\right)x^2}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{8a}+\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{103x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{\int -\frac{3\left(146a+\frac{103}{x}\right)x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}}{8a}+\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\int \frac{\left(146a+\frac{103}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}+\frac{103x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{8a}+\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 168$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(-\frac{\int -\frac{\left(176a+\frac{73}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-146x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{103x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{8a}+\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(\frac{\int \frac{\left(176a+\frac{73}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-146x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{103x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{8a}+\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 174$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(\frac{249\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+176\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-146x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{103x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{8a}+\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 73$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(\frac{498a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+352a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{a}-146x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{103x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{8a}+\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 221$$

$$-\frac{a^4\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(\frac{249\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-352\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}-146x\sqrt{\frac{1}{ax}+1}\right)}{4a}+\frac{103x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}}{8a}+\frac{9x\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)^2}}{3a^2}+\frac{2x\sqrt{\frac{1}{ax}+1}}{3a\left(a-\frac{1}{x}\right)^3}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

output
```
-((a^4*Sqrt[1 - 1/(a*x)]*((2*Sqrt[1 + 1/(a*x)]*x)/(3*a*(a - x^(-1))^3) + (
(9*Sqrt[1 + 1/(a*x)]*x)/(4*(a - x^(-1))^2) + ((103*Sqrt[1 + 1/(a*x)]*x)/(2
*(a - x^(-1))) + (3*(-146*Sqrt[1 + 1/(a*x)]*x + (-352*ArcTanh[Sqrt[1 + 1/(
a*x)]] + 249*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a))/(4*a))/(8*a))
/(3*a^2)))/(c^2*Sqrt[c - c/(a*x)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.51

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{11\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^3\sqrt{a^2c}}-\frac{2\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+3\left(x-\frac{1}{a}\right)ac+2c}}{3a^7c\left(x-\frac{1}{a}\right)^3}-\frac{11\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+3\left(x-\frac{1}{a}\right)ac+2c}}{4a^6c\left(x-\frac{1}{a}\right)^2}-27\cdots\right.}{c^2\sqrt{\cdots}}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(192\sqrt{x(ax+1)}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^3-747a^{\frac{7}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)x^3-1660a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x^2+1056\ln\left(2\sqrt{\cdots}\right.\right.}{\cdots}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/c^2/((a*x-1)/(a*x+1))^(1/2)/(c*(a*x-1)/a/x)^(1/2)*(a*x-1)+(11/2/a^3*ln
((1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2+a*c*x)^(1/2))/(a^2*c)^(1/2)-2/
3/a^7/c/(x-1/a)^3*((x-1/a)^2*a^2*c+3*(x-1/a)*a*c+2*c)^(1/2)-11/4/a^6/c/(x-
1/a)^2*((x-1/a)^2*a^2*c+3*(x-1/a)*a*c+2*c)^(1/2)-271/48/a^5/c/(x-1/a)*((x-
1/a)^2*a^2*c+3*(x-1/a)*a*c+2*c)^(1/2)-249/64/a^4/c^(1/2)*2^(1/2)*ln((4*c+3
*(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*((x-1/a)^2*a^2*c+3*(x-1/a)*a*c+2*c)^(1/2))/
(x-1/a)))*a^2/c^2/((a*x-1)/(a*x+1))^(1/2)/(a*x+1)/x/(c*(a*x-1)/a/x)^(1/2)*
((a*x+1)*a*c*x)^(1/2)*(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 736, normalized size of antiderivative = 2.96

$$\int\frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\left[\frac{747\sqrt{2}(a^4x^4-4a^3x^3+6a^2x^2-4ax+1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+\cdots}{a^3x^3-3a^2x^2+\cdots}\right.}{\cdots}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/384*(747*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*
log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 1056*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^
2 - 4*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/
(a*x - 1)) + 8*(48*a^5*x^5 - 367*a^4*x^4 + 139*a^3*x^3 + 335*a^2*x^2 - 219
*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*c^3*x^4 - 4*
a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3), 1/192*(747*sqrt(2)*(a^
4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*
x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a
^2*c*x^2 - 2*a*c*x - c)) - 1056*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x +
 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 4*(48*a^5*x^5 - 367*a^
4*x^4 + 139*a^3*x^3 + 335*a^2*x^2 - 219*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqr
t((a*c*x - c)/(a*x)))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2
*c^3*x + a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a\,x}\right)^{5/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 583, normalized size of antiderivative = 2.34

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x)
```

output
```
(sqrt(c)*(576*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**3*x**3 - 4980*sqrt(x)*sqrt(
a)*sqrt(a*x + 1)*a**2*x**2 + 6648*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x - 2628
*sqrt(x)*sqrt(a)*sqrt(a*x + 1) + 2241*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*
sqrt(a) - sqrt(2) - 1)*a**3*x**3 - 6723*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x
)*sqrt(a) - sqrt(2) - 1)*a**2*x**2 + 6723*sqrt(2)*log(sqrt(a*x + 1) + sqrt
(x)*sqrt(a) - sqrt(2) - 1)*a*x - 2241*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*
sqrt(a) - sqrt(2) - 1) - 2241*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)
- sqrt(2) + 1)*a**3*x**3 + 6723*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a
) - sqrt(2) + 1)*a**2*x**2 - 6723*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt
(a) - sqrt(2) + 1)*a*x + 2241*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)
- sqrt(2) + 1) - 2241*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2
) - 1)*a**3*x**3 + 6723*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt
(2) - 1)*a**2*x**2 - 6723*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sq
rt(2) - 1)*a*x + 2241*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2
) - 1) + 2241*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1)*a
**3*x**3 - 6723*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1)
*a**2*x**2 + 6723*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) +
1)*a*x - 2241*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1) +
 6336*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a**3*x**3 - 19008*log(sqrt(a*x
+ 1) + sqrt(x)*sqrt(a))*a**2*x**2 + 19008*log(sqrt(a*x + 1) + sqrt(x)*s...
```

3.478 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 196

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=-\frac{73c^4\sqrt{1-\frac{1}{a^2x^2}}}{5a\sqrt{c-\frac{c}{ax}}}-\frac{7c^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{5a}+\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5a}+c\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{9c^{7/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
-73/5*c^4*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)-7/5*c^3*(1-1/a^2/x^2)^(1/2
)*(c-c/a/x)^(1/2)/a+3/5*c^2*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(3/2)/a+c*(1-1/a
^2/x^2)^(1/2)*(c-c/a/x)^(5/2)*x-9*c^(7/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1
/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.52

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(-2+16ax-92a^2x^2+5a^3x^3)-45a^2x^2\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{5a^3\sqrt{1-\frac{1}{ax}}x^2}$$

input
```
Integrate[(c - c/(a*x))^(7/2)/E^ArcCoth[a*x],x]
```

output
```
(c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + 16*a*x - 92*a^2*x^2 + 5*a^
3*x^3) - 45*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(5*a^3*Sqrt[1 - 1/(a*x)]*
x^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.76, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.417$, Rules used = {6731, 585, 27, 109, 27, 170, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int\left(c-\frac{c}{ax}\right)^{7/2}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int\frac{\left(c-\frac{c}{ax}\right)^{9/2}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\int\frac{\left(a-\frac{1}{x}\right)^4x^2}{a^4\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^4 x^2}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 109$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\int \frac{3\left(a-\frac{1}{x}\right)^2\left(3a+\frac{1}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-ax\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\frac{3}{2}\int \frac{\left(a-\frac{1}{x}\right)^2\left(3a+\frac{1}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-ax\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 170$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\frac{3}{2}\left(\frac{2}{5}a\int \frac{\left(15a-\frac{7}{x}\right)\left(a-\frac{1}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)-ax\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\frac{3}{2}\left(\frac{1}{5}a\int \frac{\left(15a-\frac{7}{x}\right)\left(a-\frac{1}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)-ax\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 164$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\frac{3}{2}\left(\frac{1}{5}a\left(15a^2\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{2}{3}a\left(80a-\frac{7}{x}\right)\sqrt{\frac{1}{ax}+1}\right)+\frac{2}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)-ax\sqrt{\frac{1}{ax}+1}\left(a-\right.\right.}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 73$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\frac{3}{2}\left(\frac{1}{5}a\left(30a^3\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}-\frac{2}{3}a\left(80a-\frac{7}{x}\right)\sqrt{\frac{1}{ax}+1}\right)+\frac{2}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)-ax\sqrt{\frac{1}{ax}+1}\left(\right.\right.}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c^3\left(-\frac{3}{2}\left(\frac{1}{5}a\left(-30a^2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)-\frac{2}{3}a\left(80a-\frac{7}{x}\right)\sqrt{\frac{1}{ax}+1}\right)+\frac{2}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)-ax\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right.\right.}{a^4\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(c - c/(a*x))^(7/2)/E^ArcCoth[a*x],x]
```

output
```
-((c^3*Sqrt[c - c/(a*x)]*(-(a*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*x) - (3*((2
*a*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)])/5 + (a*((-2*a*(80*a - 7/x)*Sqrt[1 + 1
/(a*x)])/3 - 30*a^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/5))/2))/(a^4*Sqrt[1 - 1/(
a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.82

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(10a^{\frac{7}{2}}x^3\sqrt{x(ax+1)}-184a^{\frac{5}{2}}x^2\sqrt{x(ax+1)}-45\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^3x^3+32a^{\frac{3}{2}}x\sqrt{x(ax+1)}-\right.}{10x^2a^{\frac{7}{2}}(ax-1)\sqrt{x(ax+1)}}$
risch	$\frac{(5a^4x^4-87a^3x^3-76a^2x^2+14ax-2)c^3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{5x^2a^3(ax-1)}-\frac{9\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,(ax-1)}$

input
```
int((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^3*(10*a^(7/2)
*x^3*(x*(a*x+1))^(1/2)-184*a^(5/2)*x^2*(x*(a*x+1))^(1/2)-45*ln(1/2*(2*(x*(
a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3*x^3+32*a^(3/2)*x*(x*(a*x+1))^(
1/2)-4*(x*(a*x+1))^(1/2)*a^(1/2))/x^2/a^(7/2)/(a*x-1)/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.12

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\left[\frac{45\,(a^3c^3x^3-a^2c^3x^2)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(5\,a^4c^3}{20\,(a^4x^3-a^3x^2)}\right.$$

input
```
integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
[1/20*(45*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
- 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(5*a^4*c^3*x^4 - 87*a^3*c^3*x^3 - 76
*a^2*c^3*x^2 + 14*a*c^3*x - 2*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^4*x^3 - a^3*x^2), 1/10*(45*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt
(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(5*a^4*c^3*x^4 - 87*a^3*c^3*
x^3 - 76*a^2*c^3*x^2 + 14*a*c^3*x - 2*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\int\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \int \left(c-\frac{c}{a\,x}\right)^{7/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.54

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \frac{\sqrt{c}\,c^3\left(20\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^3x^3-368\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2+64\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax-8\sqrt{x}\,\sqrt{\right.}{20a^4x^3}$$

input
```
int((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*c**3*(20*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**3*x**3 - 368*sqrt(x)*sq
rt(a)*sqrt(a*x + 1)*a**2*x**2 + 64*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x - 8*s
qrt(x)*sqrt(a)*sqrt(a*x + 1) - 180*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a*
*3*x**3 + 281*a**3*x**3))/(20*a**4*x**3)
```

3.479 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 157

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=-\frac{17c^3\sqrt{1-\frac{1}{a^2x^2}}}{3a\sqrt{c-\frac{c}{ax}}}+\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{3a}+c\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{7c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
-17/3*c^3*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+1/3*c^2*(1-1/a^2/x^2)^(1/2
)*(c-c/a/x)^(1/2)/a+c*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(3/2)*x-7*c^(5/2)*arct
anh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.57

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(2-22ax+3a^2x^2)-21ax\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{3a^2\sqrt{1-\frac{1}{ax}}x}$$

input
```
Integrate[(c - c/(a*x))^(5/2)/E^ArcCoth[a*x],x]
```

output
```
(c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 - 22*a*x + 3*a^2*x^2) - 21*a*
x*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.72, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6731, 585, 27, 109, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{5/2} e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2} x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^3 x^2}{a^3\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^3 x^2}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(-\int \frac{\left(a-\frac{1}{x}\right)\left(7a+\frac{1}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-ax\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{2}\int \frac{\left(a-\frac{1}{x}\right)\left(7a+\frac{1}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-ax\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 164$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{2}{3}a\left(16a+\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}-7a^2\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)-ax\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 73$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(\frac{2}{3}a\left(16a+\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}-14a^3\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}\right)-ax\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 221$$

$$-\frac{c^2\left(\frac{1}{2}\left(14a^2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)+\frac{2}{3}a\left(16a+\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\right)-ax\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-\frac{c}{ax}}}{a^3\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(c - c/(a*x))^(5/2)/E^ArcCoth[a*x],x]
```

output
```
-((c^2*Sqrt[c - c/(a*x)]*(-(a*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*x) + ((2*a*
(16*a + x^(-1))*Sqrt[1 + 1/(a*x)])/3 + 14*a^2*ArcTanh[Sqrt[1 + 1/(a*x)]])/
2))/(a^3*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

method	result	si
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^2\left(6a^{\frac{5}{2}}x^2\sqrt{x(ax+1)}-44a^{\frac{3}{2}}x\sqrt{x(ax+1)}-21\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^2x^2+4\sqrt{x(ax+1)}\,\sqrt{a}\right)}{6x\,a^{\frac{5}{2}}(ax-1)\sqrt{x(ax+1)}}$	14
risch	$\frac{(3a^3x^3-19a^2x^2-20ax+2)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^2(ax-1)}-\frac{7\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,(ax-1)}$	10

input
```
int((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^2*(6*a^(5/2)*x
^2*(x*(a*x+1))^(1/2)-44*a^(3/2)*x*(x*(a*x+1))^(1/2)-21*ln(1/2*(2*(x*(a*x+1
))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*x^2+4*(x*(a*x+1))^(1/2)*a^(1/2))/x/
a^(5/2)/(a*x-1)/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.43

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \left[\frac{21\,(a^2c^2x^2-ac^2x)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(3\,a^3c^2x^3}{12\,(a^3x^2-a^2x)}\right.$$

input

```
integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output

```
[1/12*(21*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*
(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*
x - c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 - 19*a^2*c^2*x^2 - 20*a*c
^2*x + 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2
- a^2*x), 1/6*(21*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
- a*c*x - c)) + 2*(3*a^3*c^2*x^3 - 19*a^2*c^2*x^2 - 20*a*c^2*x + 2*c^2)*sq
rt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \text{Timed out}$$

input

```
integrate((c-c/a/x)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int \left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int \left(c-\frac{c}{a\,x}\right)^{5/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.55

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2} dx = \frac{\sqrt{c}\,c^2\left(6\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2 - 44\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax + 4\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1} - 42\log\left(\sqrt{ax+\right.}{6a^3x^2}$$

input
```
int((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*c**2*(6*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**2*x**2 - 44*sqrt(x)*sqrt
(a)*sqrt(a*x + 1)*a*x + 4*sqrt(x)*sqrt(a)*sqrt(a*x + 1) - 42*log(sqrt(a*x
+ 1) + sqrt(x)*sqrt(a))*a**2*x**2 + 21*a**2*x**2))/(6*a**3*x**2)
```

3.480 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=-\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+c\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x-\frac{5c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
-c^2*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+c*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)
^(1/2)*x-5*c^(3/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\frac{c\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(-2+ax)-5\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(c - c/(a*x))^(3/2)/E^ArcCoth[a*x],x]
```

output
```
(c*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + a*x) - 5*ArcTanh[Sqrt[1 + 1/
(a*x)]]))/(a*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6731, 585, 27, 100, 27, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{3/2} e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^2}{a^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^2}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left(\int -\frac{\left(5a-\frac{2}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-a^2x\sqrt{\frac{1}{ax}+1}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left(a^2x\left(-\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}\int\frac{\left(5a-\frac{2}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 90$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(4a\sqrt{\frac{1}{ax}+1}-5a\int\frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)-a^2x\sqrt{\frac{1}{ax}+1}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 73$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(4a\sqrt{\frac{1}{ax}+1}-10a^2\int\frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}\right)-a^2x\sqrt{\frac{1}{ax}+1}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 221$$

$$-\frac{c\left(\frac{1}{2}\left(10a\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)+4a\sqrt{\frac{1}{ax}+1}\right)-a^2x\sqrt{\frac{1}{ax}+1}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(c - c/(a*x))^(3/2)/E^ArcCoth[a*x],x]
```

output
```
-((c*Sqrt[c - c/(a*x)]*(-(a^2*Sqrt[1 + 1/(a*x)]*x) + (4*a*Sqrt[1 + 1/(a*x)
] + 10*a*ArcTanh[Sqrt[1 + 1/(a*x)]])/2))/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c\left(2a^{\frac{3}{2}}x\sqrt{x(ax+1)}-5\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-4\sqrt{x(ax+1)}\,\sqrt{a}\right)}{2a^{\frac{3}{2}}\,(ax-1)\sqrt{x(ax+1)}}$	118
risch	$\frac{(a^2x^2-ax-2)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{a(ax-1)}-\frac{5\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,(ax-1)}$	151

input
```
int((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c*(2*a^(3/2)*x*(
x*(a*x+1))^(1/2)-5*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a
*x-4*(x*(a*x+1))^(1/2)*a^(1/2))/a^(3/2)/(a*x-1)/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.72

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\left[\frac{5\,(acx-c)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2cx^2-acx-}{4\,(a^2x-a)}\right.$$

input
```
integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(5*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3
*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
- c)/(a*x - 1)) + 4*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(5*(a*c*x - c)*sqrt(-c)*arctan(2*(
a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/
(2*a^2*c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \int \left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int \left(c-\frac{c}{a\,x}\right)^{3/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.51

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{\sqrt{c}\,c\left(4\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax-8\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}-20\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)ax-9ax\right)}{4a^2x}$$

input
```
int((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*c*(4*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x - 8*sqrt(x)*sqrt(a)*sqrt(a
*x + 1) - 20*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a*x - 9*a*x))/(4*a**2*x)
```

3.481 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
c*(1-1/a^2/x^2)^(1/2)*x/(c-c/a/x)^(1/2)-3*c^(1/2)*arctanh(c^(1/2)*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}x-\frac{3\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x - (3*ArcTanh[Sqrt[1 + 1/(a*x)]])/a
))/Sqrt[1 - 1/(a*x)]
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6731, 580, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 580}$$

$$-\frac{-\frac{3c\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{ 573}$$

$$-\frac{\frac{3c^2\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\frac{3c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

output
```
-((-((c^2*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)]) + (3*c^(3/2)*ArcTanh
[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a)/c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 580
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 2)*((a + b*x^2)^(p +
1)/(b*e*(m + 1))), x] + Simp[d*((2*m + p + 3)/(e*(m + 1)))   Int[(e*x)^(m +
 1)*(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p},
 x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p - 1, 0] && LtQ[m, -1] && IntegerQ
[p + 1/2]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{x(ax+1)}\,\sqrt{a}-3\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2(ax-1)\sqrt{x(ax+1)}\,\sqrt{a}}$	101
risch	$\frac{(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}-\frac{3\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^{2}c}\,(ax-1)}$	139

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*(x*(a*x+1))
^(1/2)*a^(1/2)-3*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a
*x-1)/(x*(a*x+1))^(1/2)/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(67) = 134$.

Time = 0.12 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.76

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$= \left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)}, \ldots\right.$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x)))/(a^2*x - a), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 -
 a*c*x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*x - a)]
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))), x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.42

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}-3\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)\right)}{a}$$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x + 1) - 3*log(sqrt(a*x + 1) + sqrt(x)*sq
rt(a))))/a
```

3.482 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}} dx$

Optimal result

Integrand size = 24, antiderivative size = 78

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}} dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{ax}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/(c-c/a/x)^(1/2)-arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/
(c-c/a/x)^(1/2))/a/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}} dx = \frac{\sqrt{1-\frac{1}{ax}}\left(\sqrt{1+\frac{1}{ax}}x - \frac{\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(Sqrt[1 + 1/(a*x)]*x - ArcTanh[Sqrt[1 + 1/(a*x)]]/a))/S
qrt[c - c/(a*x)]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6731, 579, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 579}$$

$$-\frac{-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{ 573}$$

$$-\frac{\frac{c\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

input
```
Int[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]),x]
```

output
```
-((-((c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sq
rt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a)/c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 579
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 1)*((a + b*x^2)^(p +
1)/(b*c*e*(m + 1))), x] - Simp[d*((n - m - 2)/(c*e*(m + 1)))   Int[(e*x)^(m
 + 1)*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0] && LtQ[m, -1] && (IntegerQ[2*p] |
| IntegerQ[m])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.31

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2\sqrt{x(ax+1)}\,\sqrt{a}+\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2\sqrt{a}\,c(ax-1)\sqrt{x(ax+1)}}$	102
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{\frac{c(ax-1)}{ax}}}-\frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)acx}}{2a\sqrt{a^2c}\,\sqrt{\frac{c(ax-1)}{ax}}\,x}$	133

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(1/2)/c*(-2
*(x*(a*x+1))^(1/2)*a^(1/2)+ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^
(1/2)))/(a*x-1)/(x*(a*x+1))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(66) = 132$.

Time = 0.13 (sec) , antiderivative size = 299, normalized size of antiderivative = 3.83

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$
$$=\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2cx-ac\right)}, \ldots (ax\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)
/(a*x)))/(a^2*c*x - a*c), 1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 -
 a*c*x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*c*x - a*c)]
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(-1 + 1/(a*x))), x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a*x)), x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(1/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}-\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)\right)}{ac}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x)
```

output
```
(sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x + 1) - log(sqrt(a*x + 1) + sqrt(x)*sqrt
(a))))/(a*c)
```

3.483 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 136

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{ax}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}-\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}$$

output

```
(1-1/a^2/x^2)^(1/2)*x/c/(c-c/a/x)^(1/2)+arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2
)/(c-c/a/x)^(1/2))/a/c^(3/2)-arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/
2)/(c-c/a/x)^(1/2))*2^(1/2)/a/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.67

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\left(1-\frac{1}{ax}\right)^{3/2}\left(a\sqrt{1+\frac{1}{ax}}\,x+\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

input

```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(3/2)),x]
```

output
```
((1 - 1/(a*x))^(3/2)*(a*Sqrt[1 + 1/(a*x)]*x + ArcTanh[Sqrt[1 + 1/(a*x)]] -
 Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(a*(c - c/(a*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6731, 585, 27, 114, 27, 35, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{ax^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\int \frac{x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 114}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(-\frac{\int -\frac{\left(a+\frac{1}{x}\right)x}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(a+\frac{1}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 35$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\sqrt{1+\frac{1}{ax}}x}{a-\frac{1}{x}}d\frac{1}{x}}{2a}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 94$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{2\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}+\frac{\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}}{2a}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 73$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{4\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+2\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{2a}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 221$$

$$-\frac{a\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a}-\frac{2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}}{2a}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{c\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(3/2)),x]
```

output
```
-((a*Sqrt[1 - 1/(a*x)]*(-((Sqrt[1 + 1/(a*x)]*x)/a) + ((-2*ArcTanh[Sqrt[1 +
 1/(a*x)]])/a + (2*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a)/(2*a))/
(c*Sqrt[c - c/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)\sqrt{a}\right)}{2a^{\frac{3}{2}}c^{2}(ax-1)\sqrt{x(ax+1)}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ac\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a^{2}\sqrt{a^{2}c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}c+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{2a^{3}\sqrt{c}}\right)a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(a}}{cx\sqrt{\frac{c(ax-1)}{ax}}}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)/c^2*(2
*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2)+ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)
+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x
+1))^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/(a*x-1)/(x*(a*x+1))^(1/2)/(1/a)^(1
/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(113) = 226$.

Time = 0.15 (sec) , antiderivative size = 522, normalized size of antiderivative = 3.84

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4(2a^3x^3+3a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4(a^2x^2+ax)\sqrt{}}{4(a^2c^2x-}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)
/(a*x)) + sqrt(2)*(a*c*x - c)*log(-(17*a^3*x^3 - 3*a^2*x^2 - 13*a*x - 4*sq
rt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x))/sqrt(c) - 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1))/sqrt(c))/(a^2*
c^2*x - a*c^2), 1/2*(sqrt(2)*(a*c*x - c)*sqrt(-1/c)*arctan(2*sqrt(2)*(a^2*
x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(3
*a^2*x^2 - 2*a*x - 1)) - (a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(
-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x
 - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^2*c^2*x - a*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(3/2), x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a\,x}\right)^{3/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(3/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\sqrt{c}\left(2\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}-1\right)-\sqrt{2}\log\left(\sqrt{ax+1}+\right.\right.}{}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x)
```

output
```
(sqrt(c)*(2*sqrt(x)*sqrt(a)*sqrt(a*x + 1) + sqrt(2)*log(sqrt(a*x + 1) + sq
rt(x)*sqrt(a) - sqrt(2) - 1) - sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)
 - sqrt(2) + 1) - sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) -
1) + sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1) + 2*log(sq
rt(a*x + 1) + sqrt(x)*sqrt(a))))/(2*a*c**2)
```

3.484 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 179

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}} \, dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{2c\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{2c^2\sqrt{c-\frac{c}{ax}}}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{5/2}}-\frac{9\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{2\sqrt{2}ac^{5/2}}$$

output

```
-1/2*(1-1/a^2/x^2)^(1/2)*x/c/(c-c/a/x)^(3/2)+3/2*(1-1/a^2/x^2)^(1/2)*x/c^2
/(c-c/a/x)^(1/2)+3*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/
c^(5/2)-9/4*arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2
))*2^(1/2)/a/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.69

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(-3+2ax)+12(-1+ax)\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-9\sqrt{2}(-1+a\right.}{4ac^2\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(5/2)),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-3 + 2*a*x) + 12*(-1 + a*x)*A
rcTanh[Sqrt[1 + 1/(a*x)]] - 9*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]
/Sqrt[2]]))/(4*a*c^2*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.78, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6731, 585, 27, 114, 27, 168, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^2x^2}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\int \frac{x^2}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{114}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)}-\frac{\int -\frac{3\left(2a+\frac{1}{x}\right)x^2}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\int \frac{\left(2a+\frac{1}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\left(-\frac{\int -\frac{\left(2a+\frac{1}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-2x\sqrt{\frac{1}{ax}+1}\right)}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\left(\frac{\int \frac{\left(2a+\frac{1}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-2x\sqrt{\frac{1}{ax}+1}\right)}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{174}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\left(\frac{3\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+2\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-2x\sqrt{\frac{1}{ax}+1}\right)}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 73$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\left(\frac{6a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+4a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{a}-2x\sqrt{\frac{1}{ax}+1}\right)}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 221$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{3\left(\frac{3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-4\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}-2x\sqrt{\frac{1}{ax}+1}\right)}{4a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{2a\left(a-\frac{1}{x}\right)}\right)}{c^2\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(5/2)),x]
```

output
```
-((a^2*Sqrt[1 - 1/(a*x)]*((Sqrt[1 + 1/(a*x)]*x)/(2*a*(a - x^(-1))) + (3*(-
2*Sqrt[1 + 1/(a*x)]*x + (-4*ArcTanh[Sqrt[1 + 1/(a*x)]] + 3*Sqrt[2]*ArcTanh
[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a))/(4*a^2)))/(c^2*Sqrt[c - c/(a*x)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.47

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(8a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x-9a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)x-12\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+12\ln\left(\frac{2\sqrt{}}{}\right.\right.}{8a^{\frac{3}{2}}c^{3}(ax-1)^{2}\sqrt{x(ax+1}}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^{2}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{3\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a^{3}\sqrt{a^{2}c}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}c+3\left(x-\frac{1}{a}\right)ac+2c}}{2a^{5}c\left(x-\frac{1}{a}\right)}-\frac{9\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}c+}}{x-\frac{1}{a}}\right.}{8a^{4}\sqrt{c}}\right.}{c^{2}x\sqrt{\frac{c(ax-1)}{ax}}}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(8*a^(5/2)*(1/
a)^(1/2)*(x*(a*x+1))^(1/2)*x-9*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(
x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x-12*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^
(1/2)+12*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*(1/a)^(
1/2)*x-12*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1
/2)+9*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-
1))*a^(1/2))/a^(3/2)/c^3/(a*x-1)^2/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 596, normalized size of antiderivative = 3.33

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \left[\frac{9\sqrt{2}(a^2x^2-2ax+1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx}{a}}}{a^3x^3-3a^2x^2+3ax-1}\right.}{}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/16*(9*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*
c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a
*x - 1)) + 12*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x +
4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(2*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*
c^3), 1/8*(9*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*
x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a
^2*c*x^2 - 2*a*c*x - c)) - 12*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2
*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*
a^2*c*x^2 - a*c*x - c)) + 4*(2*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/(
a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a\,x}\right)^{5/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(5/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.41

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\sqrt{c}\,\big(8\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax-12\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+9\sqrt{2}\log\big(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}-}{}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x)
```

output
```
(sqrt(c)*(8*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x - 12*sqrt(x)*sqrt(a)*sqrt(a*
x + 1) + 9*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1)*a*x
- 9*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1) - 9*sqrt(2)
*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) + 1)*a*x + 9*sqrt(2)*log(sq
rt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) + 1) - 9*sqrt(2)*log(sqrt(a*x + 1)
 + sqrt(x)*sqrt(a) + sqrt(2) - 1)*a*x + 9*sqrt(2)*log(sqrt(a*x + 1) + sqrt
(x)*sqrt(a) + sqrt(2) - 1) + 9*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)
 + sqrt(2) + 1)*a*x - 9*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt
(2) + 1) + 24*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a*x - 24*log(sqrt(a*x +
 1) + sqrt(x)*sqrt(a))))/(8*a*c**3*(a*x - 1))
```

3.485 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 216

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{4c\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{15\sqrt{1-\frac{1}{a^2x^2}}\,x}{16c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{35\sqrt{1-\frac{1}{a^2x^2}}\,x}{16c^3\sqrt{c-\frac{c}{ax}}} + \frac{5\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{7/2}} - \frac{115\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{16\sqrt{2}ac^{7/2}}$$

output

```
-1/4*(1-1/a^2/x^2)^(1/2)*x/c/(c-c/a/x)^(5/2)-15/16*(1-1/a^2/x^2)^(1/2)*x/c
^2/(c-c/a/x)^(3/2)+35/16*(1-1/a^2/x^2)^(1/2)*x/c^3/(c-c/a/x)^(1/2)+5*arcta
nh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(7/2)-115/32*arctanh(1
/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))*2^(1/2)/a/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.62

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}x(35-55ax+16a^2x^2)+160(-1+ax)^2\operatorname{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)-\right.}{32ac^3\sqrt{c-\frac{c}{ax}}(-1+ax)^2}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(7/2)),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(35 - 55*a*x + 16*a^2*x^2) + 1
60*(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 115*Sqrt[2]*(-1 + a*x)^2*ArcT
anh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(32*a*c^3*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.81, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6731, 585, 27, 114, 27, 168, 27, 168, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^3x^2}{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\int \frac{x^2}{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{114}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}-\frac{\int -\frac{5\left(2a+\frac{1}{x}\right)x^2}{2a\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{5\int \frac{\left(2a+\frac{1}{x}\right)x^2}{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{8a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{5\left(\frac{3x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}-\frac{\int -\frac{\left(14a+\frac{9}{x}\right)x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}\right)}{8a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{5\left(\frac{\int \frac{\left(14a+\frac{9}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}+\frac{3x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}\right)}{8a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{5\left(\frac{-\frac{\int -\frac{\left(16a+\frac{7}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-14x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{3x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}\right)}{8a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{5\left(\frac{\frac{\int \frac{\left(16a+\frac{7}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-14x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{3x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}\right)}{8a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 174$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{5\left(\frac{\frac{23\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+16\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-14x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{3x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}\right)}{8a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 73$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{5\left(\frac{\frac{46a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+32a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{a}-14x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{3x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}\right)}{8a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 221$$

$$-\frac{a^3\sqrt{1-\frac{1}{ax}}\left(\frac{5\left(\frac{\frac{23\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-32\,\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}-14x\sqrt{\frac{1}{ax}+1}}{4a}+\frac{3x\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)}\right)}{8a^2}+\frac{x\sqrt{\frac{1}{ax}+1}}{4a\left(a-\frac{1}{x}\right)^2}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(7/2)),x]
```

output
```
-((a^3*Sqrt[1 - 1/(a*x)]*((Sqrt[1 + 1/(a*x)]*x)/(4*a*(a - x^(-1))^2) + (5*
((3*Sqrt[1 + 1/(a*x)]*x)/(2*(a - x^(-1))) + (-14*Sqrt[1 + 1/(a*x)]*x + (-3
2*ArcTanh[Sqrt[1 + 1/(a*x)]] + 23*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2
]])/a)/(4*a)))/(8*a^2)))/(c^3*Sqrt[c - c/(a*x)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.46

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^4\sqrt{a^2c}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+3\left(x-\frac{1}{a}\right)ac+2c}}{4a^7c\left(x-\frac{1}{a}\right)^2}-\frac{23\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+3\left(x-\frac{1}{a}\right)ac+2c}}{16a^6c\left(x-\frac{1}{a}\right)}-\frac{115\sqrt{2}\ln\left(\frac{4}{}\right.}{}\right.}{c^3x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(64a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x^2-115a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)x^2-220a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x+1\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^3*((a*x-1)/(a*x+1))^(1/2)/(c*(a*x-1)/a/x)^(1/2)+(5/2/a^4*ln(
(1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2+a*c*x)^(1/2))/(a^2*c)^(1/2)-1/4
/a^7/c/(x-1/a)^2*((x-1/a)^2*a^2*c+3*(x-1/a)*a*c+2*c)^(1/2)-23/16/a^6/c/(x-
1/a)*((x-1/a)^2*a^2*c+3*(x-1/a)*a*c+2*c)^(1/2)-115/64/a^5/c^(1/2)*2^(1/2)*
ln((4*c+3*(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*((x-1/a)^2*a^2*c+3*(x-1/a)*a*c+2*c
)^(1/2))/(x-1/a)))/c^3*a^3*((a*x-1)/(a*x+1))^(1/2)/x/(c*(a*x-1)/a/x)^(1/2)
*((a*x+1)*a*c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 668, normalized size of antiderivative = 3.09

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \left[\frac{115\sqrt{2}(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)}{a^3x^3-3\,a^2x^2+3\,ax-1}\right.}{}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

output
```
[1/128*(115*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3
*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 -
3*a^2*x^2 + 3*a*x - 1)) + 160*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*lo
g(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((
a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(16*a^4*x^
4 - 39*a^3*x^3 - 20*a^2*x^2 + 35*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*
x - c)/(a*x)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4), 1/64*(
115*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a
^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(
3*a^2*c*x^2 - 2*a*c*x - c)) - 160*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-
c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 4*(16*a^4*x^4 - 39*a^3*x^3 - 20*
a^2*x^2 + 35*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*
c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(7/2), x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a\,x}\right)^{7/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(7/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.94

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\frac{\sqrt{c}\left(64\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2-220\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax+140\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+115\sqrt{2}\right.}{}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x)
```

output
```
(sqrt(c)*(64*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**2*x**2 - 220*sqrt(x)*sqrt(a)
*sqrt(a*x + 1)*a*x + 140*sqrt(x)*sqrt(a)*sqrt(a*x + 1) + 115*sqrt(2)*log(s
qrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1)*a**2*x**2 - 230*sqrt(2)*log(
sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1)*a*x + 115*sqrt(2)*log(sqrt(
a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1) - 115*sqrt(2)*log(sqrt(a*x + 1)
+ sqrt(x)*sqrt(a) - sqrt(2) + 1)*a**2*x**2 + 230*sqrt(2)*log(sqrt(a*x + 1)
 + sqrt(x)*sqrt(a) - sqrt(2) + 1)*a*x - 115*sqrt(2)*log(sqrt(a*x + 1) + sq
rt(x)*sqrt(a) - sqrt(2) + 1) - 115*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqr
t(a) + sqrt(2) - 1)*a**2*x**2 + 230*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sq
rt(a) + sqrt(2) - 1)*a*x - 115*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)
 + sqrt(2) - 1) + 115*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2
) + 1)*a**2*x**2 - 230*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(
2) + 1)*a*x + 115*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) +
1) + 320*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a**2*x**2 - 640*log(sqrt(a*x
 + 1) + sqrt(x)*sqrt(a))*a*x + 320*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)) +
90*a**2*x**2 - 180*a*x + 90))/(64*a*c**4*(a**2*x**2 - 2*a*x + 1))
```

3.486 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{11c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{32\sqrt{2}c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
-21*c^3*(c-c/a/x)^(1/2)/a-5/3*c^2*(c-c/a/x)^(3/2)/a+3/5*c*(c-c/a/x)^(5/2)/
a+(c-c/a/x)^(7/2)*x-11*c^(7/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+32*2^(1/
2)*c^(7/2)*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.77

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-6+52ax-376a^2x^2+15a^3x^3\right)}{15a^3x^2}-\frac{11c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{32\sqrt{2}c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

input
```
Integrate[(c - c/(a*x))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^3*Sqrt[c - c/(a*x)]*(-6 + 52*a*x - 376*a^2*x^2 + 15*a^3*x^3))/(15*a^3*x
^2) - (11*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (32*Sqrt[2]*c^(7
/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6717, 6683, 1035, 281, 899, 109, 27, 171, 27, 171, 27, 171, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{7/2} e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\left(c-\frac{c}{ax}\right)^{7/2}}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{9/2}}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{9/2}x^2}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a\left(-\frac{\int \frac{c^2\left(11a+\frac{3}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}x}{2a\left(a+\frac{1}{x}\right)}d\frac{1}{x}}{a}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(-\frac{c^2\int \frac{\left(11a+\frac{3}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}x}{a+\frac{1}{x}}d\frac{1}{x}}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$$\Big\downarrow \text{171}$$

$$-\frac{a\left(-\frac{c^2\left(\frac{2}{5}\int \frac{5c\left(11a-\frac{5}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}x}{2\left(a+\frac{1}{x}\right)}d\frac{1}{x}+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(-\frac{c^2\left(c\int \frac{\left(11a-\frac{5}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}x}{a+\frac{1}{x}}d\frac{1}{x}+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$$\Big\downarrow \text{171}$$

$$-\frac{a\left(-\frac{c^2\left(c\left(\frac{2}{3}\int \frac{3c\left(11a-\frac{21}{x}\right)\sqrt{c-\frac{c}{ax}}\,x}{2\left(a+\frac{1}{x}\right)}d\frac{1}{x}-\frac{10}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(-\frac{c^2\left(c\left(c\int \frac{\left(11a-\frac{21}{x}\right)\sqrt{c-\frac{c}{ax}}\,x}{a+\frac{1}{x}}d\frac{1}{x}-\frac{10}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$$\Big\downarrow \text{171}$$

$$-\frac{a\left(-\frac{c^2\left(c\left(c\left(2\int \frac{c\left(11a-\frac{53}{x}\right)x}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-42\sqrt{c-\frac{c}{ax}}\right)-\frac{10}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$\Big\downarrow$ 27

$$-\frac{a\left(-\frac{c^2\left(c\left(c\left(c\int \frac{\left(11a-\frac{53}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-42\sqrt{c-\frac{c}{ax}}\right)-\frac{10}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$\Big\downarrow$ 174

$$-\frac{a\left(-\frac{c^2\left(c\left(c\left(c\left(11\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-64\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}\right)-42\sqrt{c-\frac{c}{ax}}\right)-\frac{10}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$\Big\downarrow$ 73

$$-\frac{a\left(-\frac{c^2\left(c\left(c\left(c\left(\frac{128a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{22a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}\right)-42\sqrt{c-\frac{c}{ax}}\right)-\frac{10}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

$\Big\downarrow$ 221

$$-\frac{a\left(-\frac{c^2\left(c\left(c\left(c\left(\frac{64\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{22\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-42\sqrt{c-\frac{c}{ax}}\right)-\frac{10}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)+\frac{6}{5}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{7/2}}{a}\right)}{c}$$

input
```
Int[(c - c/(a*x))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-((a*(-((c*(c - c/(a*x))^(7/2)*x)/a) - (c^2*((6*(c - c/(a*x))^(5/2))/5 + c
*((-10*(c - c/(a*x))^(3/2))/3 + c*(-42*Sqrt[c - c/(a*x)] + c*((-22*ArcTanh
[Sqrt[c - c/(a*x)]/Sqrt[c]])/Sqrt[c] + (64*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x
)]/(Sqrt[2]*Sqrt[c])])/Sqrt[c])))))/(2*a^2)))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.37

method	result
risch	$\frac{(15a^4x^4-391a^3x^3+428a^2x^2-58ax+6)c^3\sqrt{\frac{c(ax-1)}{ax}}}{15x^2a^3(ax-1)}+\frac{\left(-\frac{11a^3\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}}-\frac{16a^2\sqrt{2}\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}}{}\right.}{}\right.}{a^3(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(-1110\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^4+480a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax-1)}\,x^4+660\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x^2+555\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\right.}{30x}$

input
```
int((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/15*(15*a^4*x^4-391*a^3*x^3+428*a^2*x^2-58*a*x+6)/x^2*c^3/a^3/(a*x-1)*(c*
(a*x-1)/a/x)^(1/2)+(-11/2*a^3*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x
^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-16*a^2*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*
c+2*2^(1/2)*c^(1/2)*((x+1/a)^2*a^2*c-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))*c
^3/a^3/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.09

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\left[\frac{480\sqrt{2}a^2c^{\frac{7}{2}}x^2\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+165\,a^2c^{\frac{7}{2}}x^2\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}\right.}{30\,a^3x^2}\right.$$

$$-\frac{480\sqrt{2}a^2\sqrt{-c}c^3x^2\arctan\left(\frac{\sqrt{2}a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)-165\,a^2\sqrt{-c}c^3x^2\arctan\left(\frac{a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)-(15\,a^3c^3x^3-37}{15\,a^3x^2}$$

input
```
integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/30*(480*sqrt(2)*a^2*c^(7/2)*x^2*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
 - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 165*a^2*c^(7/2)*x^2*log(-2*a*c*x
+ 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(15*a^3*c^3*x^3 - 376*a^2
*c^3*x^2 + 52*a*c^3*x - 6*c^3)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2), -1/15*(
480*sqrt(2)*a^2*sqrt(-c)*c^3*x^2*arctan(sqrt(2)*a*sqrt(-c)*x*sqrt((a*c*x -
 c)/(a*x))/(a*c*x - c)) - 165*a^2*sqrt(-c)*c^3*x^2*arctan(a*sqrt(-c)*x*sqr
t((a*c*x - c)/(a*x))/(a*c*x - c)) - (15*a^3*c^3*x^3 - 376*a^2*c^3*x^2 + 52
*a*c^3*x - 6*c^3)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2)]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\int\frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)**(7/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral((-c*(-1 + 1/(a*x)))**(7/2)*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\int\frac{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(c - c/(a*x))^(7/2)/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \int \frac{\left(c-\frac{c}{a\,x}\right)^{7/2}(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a*x))^(7/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a*x))^(7/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.21

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \frac{\sqrt{c}\,c^3\left(60\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^3x^3-1504\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^2x^2+208\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax-24\sqrt{\ldots}\right.}{\ldots}$$

input
```
int((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*c**3*(60*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**3*x**3 - 1504*sqrt(x)*s
qrt(a)*sqrt(a*x - 1)*a**2*x**2 + 208*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a*x - 2
4*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 960*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*
sqrt(a) - sqrt(2)*i + i)*a**3*x**3 + 960*sqrt(2)*log(sqrt(a*x - 1) + sqrt(
x)*sqrt(a) + sqrt(2)*i - i)*a**3*x**3 - 960*sqrt(2)*log(2*sqrt(x)*sqrt(a)*
sqrt(a*x - 1) + 2*sqrt(2) + 2*a*x + 2)*a**3*x**3 - 660*log(sqrt(a*x - 1) +
 sqrt(x)*sqrt(a))*a**3*x**3 + 709*a**3*x**3))/(60*a**4*x**3)
```

3.487 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 138

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{9c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{16\sqrt{2}c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
-7*c^2*(c-c/a/x)^(1/2)/a+1/3*c*(c-c/a/x)^(3/2)/a+(c-c/a/x)^(5/2)*x-9*c^(5/
2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+16*2^(1/2)*c^(5/2)*arctanh(1/2*(c-c/
a/x)^(1/2)*2^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.84

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\frac{c^2\sqrt{c-\frac{c}{ax}}(2-26ax+3a^2x^2)-27ac^{5/2}x\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+48\sqrt{2}ac^{5/2}x\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{3a^2x}$$

input
```
Integrate[(c - c/(a*x))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^2*Sqrt[c - c/(a*x)]*(2 - 26*a*x + 3*a^2*x^2) - 27*a*c^(5/2)*x*ArcTanh[S
qrt[c - c/(a*x)]/Sqrt[c]] + 48*Sqrt[2]*a*c^(5/2)*x*ArcTanh[Sqrt[c - c/(a*x
)]/(Sqrt[2]*Sqrt[c])])/(3*a^2*x)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6717, 6683, 1035, 281, 899, 109, 27, 171, 27, 171, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{5/2} e^{-2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\left(c-\frac{c}{ax}\right)^{5/2}}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x^2}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a\left(-\frac{\int \frac{c^2\left(9a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}x}{2a\left(a+\frac{1}{x}\right)}d\frac{1}{x}}{a}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(-\frac{c^2\int \frac{\left(9a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}x}{a+\frac{1}{x}}d\frac{1}{x}}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

$$\Big\downarrow \text{171}$$

$$-\frac{a\left(-\frac{c^2\left(\frac{2}{3}\int \frac{3c\left(9a-\frac{7}{x}\right)\sqrt{c-\frac{c}{ax}}x}{2\left(a+\frac{1}{x}\right)}d\frac{1}{x}+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(-\frac{c^2\left(c\int \frac{\left(9a-\frac{7}{x}\right)\sqrt{c-\frac{c}{ax}}x}{a+\frac{1}{x}}d\frac{1}{x}+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

$$\Big\downarrow \text{171}$$

$$-\frac{a\left(-\frac{c^2\left(c\left(2\int \frac{c\left(9a-\frac{23}{x}\right)x}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-14\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(-\frac{c^2\left(c\left(c\int \frac{\left(9a-\frac{23}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-14\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

$$\Big\downarrow \text{174}$$

$$-\frac{a\left(-\frac{c^2\left(c\left(c\left(9\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-32\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}\right)-14\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a\left(-\frac{c^2\left(c\left(c\left(\frac{64a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{18a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}\right)-14\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

$$\Big\downarrow \text{221}$$

$$-\frac{a\left(-\frac{c^2\left(c\left(c\left(\frac{32\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{18\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-14\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{5/2}}{a}\right)}{c}$$

input
```
Int[(c - c/(a*x))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-((a*(-((c*(c - c/(a*x))^(5/2)*x)/a) - (c^2*((2*(c - c/(a*x))^(3/2))/3 + c
*(-14*Sqrt[c - c/(a*x)] + c*((-18*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/Sqrt
[c] + (32*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/Sqrt[c])))
)/(2*a^2)))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.54

method	result
risch	$\frac{(3a^3x^3-29a^2x^2+28ax-2)c^2\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^2(ax-1)}+\frac{\left(-\frac{9a^2\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}}-\frac{8a\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x+\frac{1}{a}\right)^2a^2c}}{x+\frac{1}{a}}\right.}{\sqrt{c}}\right.}{a^2(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^2\left(48\sqrt{x(ax-1)}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x^3-90\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x^3+48a^{\frac{3}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+45\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a^2x^3-4\right.}{6x^2a^{\frac{5}{2}}\sqrt{x(ax-1)}\sqrt{\frac{1}{a}}}$

input
```
int((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*a^3*x^3-29*a^2*x^2+28*a*x-2)/x*c^2/a^2/(a*x-1)*(c*(a*x-1)/a/x)^(1/2
)+(-9/2*a^2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(
a^2*c)^(1/2)-8*a*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(
(x+1/a)^2*a^2*c-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))*c^2/a^2/(a*x-1)*(c*(a*
x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.19

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\left[\frac{48\sqrt{2}ac^{\frac{5}{2}}x\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+27\,ac^{\frac{5}{2}}x\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+}{6\,a^2x}\right.$$

$$-\frac{48\sqrt{2}a\sqrt{-c}c^2x\arctan\left(\frac{\sqrt{2}a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)-27\,a\sqrt{-c}c^2x\arctan\left(\frac{a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)-(3\,a^2c^2x^2-26\,ac^2x+}{3\,a^2x}$$

input
```
integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/6*(48*sqrt(2)*a*c^(5/2)*x*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/
(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 27*a*c^(5/2)*x*log(-2*a*c*x + 2*a*sqrt(
c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(3*a^2*c^2*x^2 - 26*a*c^2*x + 2*c^2)
*sqrt((a*c*x - c)/(a*x)))/(a^2*x), -1/3*(48*sqrt(2)*a*sqrt(-c)*c^2*x*arcta
n(sqrt(2)*a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) - 27*a*sqrt(-c
)*c^2*x*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) - (3*a^2*
c^2*x^2 - 26*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*x)]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)**(5/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral((-c*(-1 + 1/(a*x)))**(5/2)*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\frac{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(c - c/(a*x))^(5/2)/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \int \frac{\left(c-\frac{c}{a\,x}\right)^{5/2}(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a*x))^(5/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a*x))^(5/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.30

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \frac{\sqrt{c}\,c^2\left(6\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^2x^2-52\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax+4\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}+48\sqrt{2}\log\left(\sqrt{ax}\right.\right.}{}$$

input
```
int((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*c**2*(6*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**2*x**2 - 52*sqrt(x)*sqrt
(a)*sqrt(a*x - 1)*a*x + 4*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 48*sqrt(2)*log(s
qrt(a*x - 1) + sqrt(x)*sqrt(a) - sqrt(2)*i + i)*a**2*x**2 + 48*sqrt(2)*log
(sqrt(a*x - 1) + sqrt(x)*sqrt(a) + sqrt(2)*i - i)*a**2*x**2 - 48*sqrt(2)*l
og(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 2*sqrt(2) + 2*a*x + 2)*a**2*x**2 - 54
*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))*a**2*x**2 + 11*a**2*x**2))/(6*a**3*x
**2)
```

3.488 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 113

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{7c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{8\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
-c*(c-c/a/x)^(1/2)/a+(c-c/a/x)^(3/2)*x-7*c^(3/2)*arctanh((c-c/a/x)^(1/2)/c
^(1/2))/a+8*2^(1/2)*c^(3/2)*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\frac{c\sqrt{c-\frac{c}{ax}}(-2+ax)-7c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+8\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

input
```
Integrate[(c - c/(a*x))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c*Sqrt[c - c/(a*x)]*(-2 + a*x) - 7*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt
[c]] + 8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6717, 6683, 1035, 281, 899, 109, 27, 171, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{3/2} e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}x^2}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a\left(-\frac{\int \frac{c^2\left(7a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}x}{2a\left(a+\frac{1}{x}\right)}d\frac{1}{x}}{a}-\frac{cx\left(c-\frac{c}{ax}\right)^{3/2}}{a}\right)}{c}$$

$\Big\downarrow$ 27

$$-\frac{a\left(-\frac{c^2\int \frac{\left(7a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}x}{a+\frac{1}{x}}d\frac{1}{x}}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{3/2}}{a}\right)}{c}$$

$\Big\downarrow$ 171

$$-\frac{a\left(-\frac{c^2\left(2\int \frac{c\left(7a-\frac{9}{x}\right)x}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-2\sqrt{c-\frac{c}{ax}}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{3/2}}{a}\right)}{c}$$

$\Big\downarrow$ 27

$$-\frac{a\left(-\frac{c^2\left(c\int \frac{\left(7a-\frac{9}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-2\sqrt{c-\frac{c}{ax}}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{3/2}}{a}\right)}{c}$$

$\Big\downarrow$ 174

$$-\frac{a\left(-\frac{c^2\left(c\left(7\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-16\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}\right)-2\sqrt{c-\frac{c}{ax}}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{3/2}}{a}\right)}{c}$$

$\Big\downarrow$ 73

$$-\frac{a\left(-\frac{c^2\left(c\left(\frac{32a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{14a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}\right)-2\sqrt{c-\frac{c}{ax}}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{3/2}}{a}\right)}{c}$$

$\Big\downarrow$ 221

$$-\frac{a\left(-\frac{c^2\left(c\left(\frac{16\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{14\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-2\sqrt{c-\frac{c}{ax}}\right)}{2a^2}-\frac{cx\left(c-\frac{c}{ax}\right)^{3/2}}{a}\right)}{c}$$

input
```
Int[(c - c/(a*x))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-((a*(-((c*(c - c/(a*x))^(3/2)*x)/a) - (c^2*(-2*Sqrt[c - c/(a*x)] + c*((-1
4*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/Sqrt[c] + (16*Sqrt[2]*ArcTanh[Sqrt[c
 - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/Sqrt[c])))/(2*a^2)))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(94) = 188$.

Time = 0.19 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.71

method	result
risch	$\frac{(a^2x^2-3ax+2)c\sqrt{\frac{c(ax-1)}{ax}}}{a(ax-1)}+\frac{\left(-\frac{7a\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}}-\frac{4\sqrt{2}\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{\sqrt{c}}}{a(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c\left(8\sqrt{x(ax-1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,x^2-10\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,x^2+4\left(a\,x^2-x\right)^{\frac{3}{2}}\sqrt{a}\,\sqrt{\frac{1}{a}}+5\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a\,x^2-8\sqrt{a}\,\sqrt{2}}{2x\,a^{\frac{3}{2}}\sqrt{x(ax-1)}\,\sqrt{\frac{1}{a}}}$

input
```
int((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
(a^2*x^2-3*a*x+2)*c/a*(c*(a*x-1)/a/x)^(1/2)/(a*x-1)+(-7/2*a*ln((-1/2*a*c+a
^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-4*2^(1/2)/c^(
1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*((x+1/a)^2*a^2*c-3*(x+1/a)*a*
c+2*c)^(1/2))/(x+1/a)))*c/a*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)/(a
*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.23

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\left[\frac{8\sqrt{2}c^{\frac{3}{2}}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+7\,c^{\frac{3}{2}}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(acx}{2\,a}\right.$$

$$\left.-\frac{8\sqrt{2}\sqrt{-c}c\arctan\left(\frac{\sqrt{2}a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)-7\sqrt{-c}c\arctan\left(\frac{a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)-(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{a}\right]$$

input
```
integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/2*(8*sqrt(2)*c^(3/2)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)
) + 3*a*c*x - c)/(a*x + 1)) + 7*c^(3/2)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt(
(a*c*x - c)/(a*x)) + c) + 2*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a, -(8*
sqrt(2)*sqrt(-c)*c*arctan(sqrt(2)*a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*
c*x - c)) - 7*sqrt(-c)*c*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*
x - c)) - (a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int\frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)**(3/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral((-c*(-1 + 1/(a*x)))**(3/2)*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int \frac{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(c - c/(a*x))^(3/2)/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int \frac{\left(c-\frac{c}{a\,x}\right)^{3/2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a*x))^(3/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a*x))^(3/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.23

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{\sqrt{c}\,c\left(4\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax-8\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}+16\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}\,i+i\right)a\right.}{}$$

input
```
int((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*c*(4*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a*x - 8*sqrt(x)*sqrt(a)*sqrt(a
*x - 1) + 16*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a) - sqrt(2)*i + i)*
a*x + 16*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a) + sqrt(2)*i - i)*a*x
- 16*sqrt(2)*log(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 2*sqrt(2) + 2*a*x + 2)*
a*x - 28*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))*a*x - 7*a*x))/(4*a**2*x)
```

3.489 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x-\frac{5\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
(c-c/a/x)^(1/2)*x-5*c^(1/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+4*2^(1/2)*c
^(1/2)*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x-\frac{5\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

output
```
Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (
4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6717, 6683, 1035, 281, 899, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a\left(-\frac{\int \frac{c^2\left(5a-\frac{3}{x}\right)x}{2a\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(-\frac{c^2\int \frac{\left(5a-\frac{3}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a^2}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \text{174}$$

$$-\frac{a\left(-\frac{c^2\left(5\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-8\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}\right)}{2a^2}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a\left(-\frac{c^2\left(\frac{16a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{10a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}\right)}{2a^2}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \text{221}$$

$$-\frac{a\left(-\frac{c^2\left(\frac{8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{10\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{2a^2}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

output
```
-((a*(-((c*Sqrt[c - c/(a*x)]*x)/a) - (c^2*((-10*ArcTanh[Sqrt[c - c/(a*x)]/
Sqrt[c]])/Sqrt[c] + (8*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])
])/Sqrt[c]))/(2*a^2)))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(75) = 150$.

Time = 0.18 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.83

method	result
risch	$x\sqrt{\frac{c(ax-1)}{ax}}+\frac{\left(-\frac{5\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{a\sqrt{c}}\right)\sqrt{c(ax-1)}}{ax-1}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{x(ax-1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-2\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax-1)}\,a-3ax+1}{ax+1}\right)\sqrt{a}}{2\sqrt{x(ax-1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$

input

```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output

```
x*(c*(a*x-1)/a/x)^(1/2)+(-5/2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x
^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-2/a*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2
*2^(1/2)*c^(1/2)*((x+1/a)^2*a^2*c-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))*(c*(
a*x-1)*a*x)^(1/2)*(c*(a*x-1)/a/x)^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.57

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+4\,\sqrt{2}\sqrt{c}\log\left(-\frac{2\,\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+5\,\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\ ax\sqrt{\vphantom{x}}\right.$$

input

```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output

```
[1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 4*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*
sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 5*sqrt(c)*lo
g(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c
*x - c)/(a*x)) - 4*sqrt(2)*sqrt(-c)*arctan(sqrt(2)*a*sqrt(-c)*x*sqrt((a*c*
x - c)/(a*x))/(a*c*x - c)) + 5*sqrt(-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x -
c)/(a*x))/(a*c*x - c)))/a]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=\frac{\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}+2\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}\,i+i\right)+2\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}+\sqrt{2}\,i-\right.\right.}{a}$$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 2*sqrt(2)*log(sqrt(a*x - 1) + sq
rt(x)*sqrt(a) - sqrt(2)*i + i) + 2*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqr
t(a) + sqrt(2)*i - i) - 2*sqrt(2)*log(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 2*
sqrt(2) + 2*a*x + 2) - 5*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))))/a
```

3.490 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\,x}{c}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

output
```
(c-c/a/x)^(1/2)*x/c-3*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(1/2)+2*2^(1/2)
*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))/a/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\,x}{c}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

output
```
(Sqrt[c - c/(a*x)]*x)/c - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*Sqrt[c
]) + (2*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6717, 6683, 1035, 281, 899, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{1-ax}{\sqrt{c-\frac{c}{ax}}(ax+1)}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\frac{1}{x}-a}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\sqrt{c-\frac{c}{ax}}}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{110}$$

$$-\frac{a\left(\frac{\int -\frac{c\left(3a-\frac{1}{x}\right)x}{2a\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}-\frac{x\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$\Big\downarrow$ 27

$$-\frac{a\left(-\frac{c\int \frac{\left(3a-\frac{1}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a^2}-\frac{x\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$\Big\downarrow$ 174

$$-\frac{a\left(-\frac{c\left(3\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-4\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}\right)}{2a^2}-\frac{x\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$\Big\downarrow$ 73

$$-\frac{a\left(-\frac{c\left(\frac{8a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{6a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}\right)}{2a^2}-\frac{x\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$\Big\downarrow$ 221

$$-\frac{a\left(-\frac{c\left(\frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{6\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{2a^2}-\frac{x\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

output
```
-((a*(-((Sqrt[c - c/(a*x)]*x)/a) - (c*((-6*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[
c]])/Sqrt[c] + (4*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/Sq
rt[c]))/(2*a^2)))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.43

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(2\sqrt{x(ax-1)}\, a^{\frac{3}{2}} \sqrt{\frac{1}{a}}-3\ln\left(\frac{2\sqrt{x(ax-1)}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right) a\sqrt{\frac{1}{a}}-2\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax-1)}\, a-3ax+1}{ax+1}\right)\sqrt{a}\right)}{2\sqrt{x(ax-1)}\, c\, a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{a\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{3\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{2a\sqrt{a^{2}c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x+\frac{1}{a}\right)^{2}a^{2}c-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{a^{2}\sqrt{c}}\right)\sqrt{c(ax-1)ax}}{\sqrt{\frac{c(ax-1)}{ax}}\, x}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*(x*(a*x-1))^(1/2)*a^(3/2)*(1/a)^(1/2)-3*ln(
1/2*(2*(x*(a*x-1))^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)-2*2^(1/2)
*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x-1))^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))/
(x*(a*x-1))^(1/2)/c/a^(3/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.47

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+2\,\sqrt{2}\sqrt{c}\log\left(-\frac{\frac{2\,\sqrt{2}ax\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}+3\,ax-1}{ax+1}\right)+3\,\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,ac},\right.$$
$$\left.-\frac{2\,\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{1}{2}\,\sqrt{2}\sqrt{-\frac{1}{c}}\sqrt{\frac{acx-c}{ax}}\right)-ax\sqrt{\frac{acx-c}{ax}}-3\,\sqrt{-c}\arctan\left(\frac{a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)}{ac}\right]$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 2*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*
x*sqrt((a*c*x - c)/(a*x))/sqrt(c) + 3*a*x - 1)/(a*x + 1)) + 3*sqrt(c)*log(
-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/(a*c), -(2*sqrt(2)*
c*sqrt(-1/c)*arctan(1/2*sqrt(2)*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))) - a*x*
sqrt((a*c*x - c)/(a*x)) - 3*sqrt(-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/
(a*x))/(a*c*x - c)))/(a*c)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{ax-1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(1/2),x)
```

output
```
Integral((a*x - 1)/(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{ax-1}{(ax+1)\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*sqrt(c - c/(a*x))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{a\,x-1}{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a*x))^(1/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a*x))^(1/2)*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$
$$=\frac{\sqrt{c}\,\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}+\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}\,i+i\right)+\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}+\sqrt{2}\,i-i\right)\right.}{ac}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x)
```

output
```
(sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x - 1) + sqrt(2)*log(sqrt(a*x - 1) + sqrt
(x)*sqrt(a) - sqrt(2)*i + i) + sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a)
 + sqrt(2)*i - i) - sqrt(2)*log(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 2*sqrt(2
) + 2*a*x + 2) - 3*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))))/(a*c)
```

3.491 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

output
```
(c-c/a/x)^(1/2)*x/c^2-arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(3/2)+2^(1/2)*a
rctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))/a/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

output
```
(Sqrt[c - c/(a*x)]*x)/c^2 - ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*c^(3/2))
 + (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6717, 6683, 1035, 281, 899, 114, 27, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{3/2}(ax+1)}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\frac{1}{x}-a}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{x^2}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{114}$$

$$-\frac{a\left(-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{2\left(a+\frac{1}{x}\right)}d\frac{1}{x}}{ac}-\frac{x\sqrt{c-\frac{c}{ax}}}{ac}\right)}{c}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{a\left(-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{a+\frac{1}{x}}d\frac{1}{x}}{2ac}-\frac{x\sqrt{c-\frac{c}{ax}}}{ac}\right)}{c}$$

$$\Big\downarrow \textbf{94}$$

$$-\frac{a\left(-\frac{\frac{c\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}-\frac{2c\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}}{2ac}-\frac{x\sqrt{c-\frac{c}{ax}}}{ac}\right)}{c}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{a\left(-\frac{4\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}-2\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{2ac}-\frac{x\sqrt{c-\frac{c}{ax}}}{ac}\right)}{c}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{a\left(-\frac{\frac{2\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}}{2ac}-\frac{x\sqrt{c-\frac{c}{ax}}}{ac}\right)}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

output
```
-((a*(-((Sqrt[c - c/(a*x)]*x)/(a*c)) - ((-2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*
x)]/Sqrt[c]])/a + (2*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sq
rt[c])])/a)/(2*a*c)))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.45

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(2\sqrt{x(ax-1)}\, a^{\frac{3}{2}} \sqrt{\frac{1}{a}}-\ln\left(\frac{2\sqrt{x(ax-1)}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right) a\sqrt{\frac{1}{a}}-\sqrt{2}\, \ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax-1)}\, a-3ax+1}{ax+1}\right)\sqrt{a}\right)}{2a^{\frac{3}{2}}\sqrt{x(ax-1)}\, c^{2}\sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{ac\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{2a^{2}\sqrt{a^{2}c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x+\frac{1}{a}\right)^{2}a^{2}c-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{2a^{3}\sqrt{c}}\right)a\sqrt{c(ax-1)a}}{cx\sqrt{\frac{c(ax-1)}{ax}}}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(2*(x*(a*x-1))^(1/2)*a^(3/2)*(1/a)^(1/
2)-ln(1/2*(2*(x*(a*x-1))^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)-2^(
1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x-1))^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/
2))/(x*(a*x-1))^(1/2)/c^2/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.47

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+\sqrt{2}\sqrt{c}\log\left(-\frac{\frac{2\sqrt{2}ax\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}+3\,ax-1}{ax+1}\right)+\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}\right.}{2\,ac^2}\right.$$

$$\left.-\frac{\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{c}}\sqrt{\frac{acx-c}{ax}}\right)-ax\sqrt{\frac{acx-c}{ax}}-\sqrt{-c}\arctan\left(\frac{a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)}{ac^2}\right]$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*x*
sqrt((a*c*x - c)/(a*x))/sqrt(c) + 3*a*x - 1)/(a*x + 1)) + sqrt(c)*log(-2*a
*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/(a*c^2), -(sqrt(2)*c*sq
rt(-1/c)*arctan(1/2*sqrt(2)*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))) - a*x*sqrt
((a*c*x - c)/(a*x)) - sqrt(-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))
/(a*c*x - c)))/(a*c^2)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(3/2),x)
```

output
```
Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(3/2)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{a\,x-1}{\left(c-\frac{c}{a\,x}\right)^{3/2}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a*x))^(3/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a*x))^(3/2)*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.17

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\sqrt{c}\left(2\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}+\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}\,i+i\right)+\sqrt{2}\log\left(\sqrt{ax-1}+\right.\right.}{}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x)
```

output
```
(sqrt(c)*(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + sqrt(2)*log(sqrt(a*x - 1) + sq
rt(x)*sqrt(a) - sqrt(2)*i + i) + sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(
a) + sqrt(2)*i - i) - sqrt(2)*log(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 2*sqrt
(2) + 2*a*x + 2) - 2*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))))/(2*a*c**2)
```

3.492 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=-\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}+\frac{\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}+\frac{\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}ac^{5/2}}$$

output
```
-2/a/c^2/(c-c/a/x)^(1/2)+x/c^2/(c-c/a/x)^(1/2)+arctanh((c-c/a/x)^(1/2)/c^(
1/2))/a/c^(5/2)+1/2*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a
/c^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\frac{ax-\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a-\frac{1}{x}}{2a}\right)-\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\frac{1}{ax}\right)}{ac^2\sqrt{c-\frac{c}{ax}}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

output
```
(a*x - Hypergeometric2F1[-1/2, 1, 1/2, (a - x^(-1))/(2*a)] - Hypergeometri
c2F1[-1/2, 1, 1/2, 1 - 1/(a*x)])/(a*c^2*Sqrt[c - c/(a*x)])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {6717, 6683, 1035, 281, 899, 114, 27, 169, 25, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{5/2}(ax+1)}dx$$

$$\Big\downarrow \text{ 1035}$$

$$-\int \frac{\frac{1}{x}-a}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}dx}{c}$$

$$\Big\downarrow \text{ 899}$$

$$-\frac{a\int \frac{x^2}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 114}$$

$$-\frac{a\left(-\frac{\int -\frac{c\left(a+\frac{3}{x}\right)x}{2a\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{ac}-\frac{x}{ac\sqrt{c-\frac{c}{ax}}}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(\frac{\int \frac{\left(a+\frac{3}{x}\right)x}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{2a^2}-\frac{x}{ac\sqrt{c-\frac{c}{ax}}}\right)}{c}$$

$$\Big\downarrow \text{169}$$

$$-\frac{a\left(\frac{\frac{4}{c\sqrt{c-\frac{c}{ax}}}-\frac{\int -\frac{c\left(a+\frac{2}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c^2}}{2a^2}-\frac{x}{ac\sqrt{c-\frac{c}{ax}}}\right)}{c}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\left(\frac{\frac{\int \frac{c\left(a+\frac{2}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c^2}+\frac{4}{c\sqrt{c-\frac{c}{ax}}}}{2a^2}-\frac{x}{ac\sqrt{c-\frac{c}{ax}}}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(\frac{\frac{\int \frac{\left(a+\frac{2}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}+\frac{4}{c\sqrt{c-\frac{c}{ax}}}}{2a^2}-\frac{x}{ac\sqrt{c-\frac{c}{ax}}}\right)}{c}$$

$$\Big\downarrow \text{174}$$

$$-\frac{a\left(\frac{\frac{\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}+\frac{4}{c\sqrt{c-\frac{c}{ax}}}}{2a^2}-\frac{x}{ac\sqrt{c-\frac{c}{ax}}}\right)}{c}$$

$$\Big\downarrow 73$$

$$-\frac{a\left(\frac{-\frac{\frac{2a\int\frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}}{c}-\frac{2a\int\frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}+\frac{4}{c\sqrt{c-\frac{c}{ax}}}}{2a^2}-\frac{x}{ac\sqrt{c-\frac{c}{ax}}}\right)}{c}$$

$$\Big\downarrow 221$$

$$-\frac{a\left(\frac{-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}}{c}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}+\frac{4}{c\sqrt{c-\frac{c}{ax}}}}{2a^2}-\frac{x}{ac\sqrt{c-\frac{c}{ax}}}\right)}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

output
```
-((a*(-(x/(a*c*Sqrt[c - c/(a*x)])) + (4/(c*Sqrt[c - c/(a*x)]) + ((-2*ArcTa
nh[Sqrt[c - c/(a*x)]/Sqrt[c]])/Sqrt[c] - (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)
]/(Sqrt[2]*Sqrt[c])])/Sqrt[c])/c)/(2*a^2)))/c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(98) = 196$.

Time = 0.23 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.96

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^3\sqrt{a^2c}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+\left(x-\frac{1}{a}\right)ac}}{a^5c\left(x-\frac{1}{a}\right)}-\frac{\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-3\left(x+\frac{1}{a}\right.}}{x+\frac{1}{a}}\right.}{4a^4\sqrt{c}}\right.}{c^2x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-8\sqrt{x(ax-1)}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^2+4(x(ax-1))^{\frac{3}{2}}a^{\frac{5}{2}}\sqrt{\frac{1}{a}}-2\ln\left(\frac{2\sqrt{x(ax-1)}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a^3x^2+a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(a}}{ax}\right.\right.}{}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^2/(c*(a*x-1)/a/x)^(1/2)+(1/2/a^3*ln((-1/2*a*c+a^2*c*x)/(a^2*
c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-1/a^5/c/(x-1/a)*((x-1/a)^2
*a^2*c+(x-1/a)*a*c)^(1/2)-1/4/a^4*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*
2^(1/2)*c^(1/2)*((x+1/a)^2*a^2*c-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))*a^2/c
^2/x/(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.62

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \left[\frac{\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+2\,(ax-1)\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\right.}{4\,(a^2c^3x-ac^3)}\right.$$

$$-\frac{\sqrt{2}(ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)+2\,(ax-1)\sqrt{-c}\arctan\left(\frac{a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)-2\,(a^2x^2-2\,ax)\sqrt{}}{2\,(a^2c^3x-ac^3)}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(2)*(a*x - 1)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 2*(a*x - 1)*sqrt(c)*log(-2*a*c*x - 2
*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 4*(a^2*x^2 - 2*a*x)*sqrt((a*c*
x - c)/(a*x)))/(a^2*c^3*x - a*c^3), -1/2*(sqrt(2)*(a*x - 1)*sqrt(-c)*arcta
n(sqrt(2)*a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) + 2*(a*x - 1)*
sqrt(-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) - 2*(a^2
*x^2 - 2*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c^3*x - a*c^3)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(5/2),x)
```

output
```
Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(5/2)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\int \frac{a\,x-1}{\left(c-\frac{c}{a\,x}\right)^{5/2}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a*x))^(5/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a*x))^(5/2)*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.31

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\frac{\sqrt{c}\left(\sqrt{ax-1}\,\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}\,i+i\right)+\sqrt{ax-1}\,\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{}\right.\right.}{}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x)
```

output
```
(sqrt(c)*(sqrt(a*x - 1)*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a) - sqrt
(2)*i + i) + sqrt(a*x - 1)*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a) + s
qrt(2)*i - i) - sqrt(a*x - 1)*sqrt(2)*log(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1)
+ 2*sqrt(2) + 2*a*x + 2) + 4*sqrt(a*x - 1)*log(sqrt(a*x - 1) + sqrt(x)*sqr
t(a)) - 5*sqrt(a*x - 1) + 4*sqrt(x)*sqrt(a)*a*x - 8*sqrt(x)*sqrt(a)))/(4*s
qrt(a*x - 1)*a*c**3)
```

3.493 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 147

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=-\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}ac^{7/2}}$$

output
```
-4/3/a/c^2/(c-c/a/x)^(3/2)-7/2/a/c^3/(c-c/a/x)^(1/2)+x/c^2/(c-c/a/x)^(3/2)
+3*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(7/2)+1/4*arctanh(1/2*(c-c/a/x)^(1
/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(7/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.54

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\frac{x\left(3ax-\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a-\frac{1}{x}}{2a}\right)-3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\right.\right.}{3c^3\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

output
```
(x*(3*a*x - Hypergeometric2F1[-3/2, 1, -1/2, (a - x^(-1))/(2*a)] - 3*Hyper
geometric2F1[-3/2, 1, -1/2, 1 - 1/(a*x)]))/(3*c^3*Sqrt[c - c/(a*x)]*(-1 +
a*x))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6717, 6683, 1035, 281, 899, 114, 27, 169, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{7/2}(ax+1)}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\frac{1}{x}-a}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{x^2}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \; 114$$

$$-\frac{a\left(-\frac{\int -\frac{c\left(3a+\frac{5}{x}\right)x}{2a\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{ac}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(\frac{\int \frac{\left(3a+\frac{5}{x}\right)x}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{c}$$

$$\Big\downarrow \; 169$$

$$-\frac{a\left(\frac{\frac{8}{3c\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\int -\frac{3c\left(3a+\frac{4}{x}\right)x}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{3c^2}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(\frac{\frac{\int \frac{\left(3a+\frac{4}{x}\right)x}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}+\frac{8}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{c}$$

$$\Big\downarrow \; 169$$

$$-\frac{a\left(\frac{\frac{\frac{7}{c\sqrt{c-\frac{c}{ax}}}-\frac{\int -\frac{c\left(6a+\frac{7}{x}\right)x}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c^2}}{c}+\frac{8}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(\frac{\frac{\frac{\int \frac{\left(6a+\frac{7}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2c}+\frac{7}{c\sqrt{c-\frac{c}{ax}}}}{c}+\frac{8}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{c}$$

$\Big\downarrow$ 174

$$-\frac{a\left(\frac{\frac{\frac{\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+6\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2c}+\frac{7}{c\sqrt{c-\frac{c}{ax}}}}{c}+\frac{8}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{c}$$

$\Big\downarrow$ 73

$$-\frac{a\left(\frac{\frac{\frac{-\frac{12a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{2a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}}{2c}+\frac{7}{c\sqrt{c-\frac{c}{ax}}}}{c}+\frac{8}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{c}$$

$\Big\downarrow$ 221

$$-\frac{a\left(\frac{\frac{\frac{-\frac{12\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}}{2c}+\frac{7}{c\sqrt{c-\frac{c}{ax}}}}{c}+\frac{8}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

output
```
-((a*(-(x/(a*c*(c - c/(a*x))^(3/2))) + (8/(3*c*(c - c/(a*x))^(3/2)) + (7/(
c*Sqrt[c - c/(a*x)]) + ((-12*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/Sqrt[c] -
 (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/Sqrt[c])/(2*c))/c)
/(2*a^2)))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(120) = 240$.

Time = 0.23 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.84

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2 a^2 c+\left(x-\frac{1}{a}\right)ac}}{3a^7 c\left(x-\frac{1}{a}\right)^2}-\frac{17\sqrt{\left(x-\frac{1}{a}\right)^2 a^2 c+\left(x-\frac{1}{a}\right)ac}}{6a^6 c\left(x-\frac{1}{a}\right)}+\frac{3\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^4\sqrt{a^2c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)\cdots}{\cdots}\right.}{\cdots}\cdots\right.}{c^3 x\sqrt{\frac{c(ax-1)}{ax}}\cdots}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-84\sqrt{x(ax-1)}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^3+60(x(ax-1))^{\frac{3}{2}}a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x-36\ln\left(\frac{2\sqrt{x(ax-1)}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a^4x^3+3a^{\frac{7}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\cdots}}{\cdots}\right.\cdots\right.}{\cdots}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^3/(c*(a*x-1)/a/x)^(1/2)+(-1/3/a^7/c/(x-1/a)^2*((x-1/a)^2*a^2
*c+(x-1/a)*a*c)^(1/2)-17/6/a^6/c/(x-1/a)*((x-1/a)^2*a^2*c+(x-1/a)*a*c)^(1/
2)+3/2/a^4*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a
^2*c)^(1/2)-1/8/a^5*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2
)*((x+1/a)^2*a^2*c-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))*a^3/c^3/x/(c*(a*x-1
)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.56

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \left[\frac{3\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{c}\log\left(-\frac{2\,\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+36\,(a^2x^2-2\,ax+1)\sqrt{c}\,\cdots}{24\,(a^3c^4x^2-2\,a^2c^4\cdots}\right.$$

$$-\frac{3\,\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)+36\,(a^2x^2-2\,ax+1)\sqrt{-c}\arctan\left(\frac{a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)\cdots}{12\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x
*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 36*(a^2*x^2 - 2*a*x +
 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 4*
(6*a^3*x^3 - 29*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 -
2*a^2*c^4*x + a*c^4), -1/12*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arct
an(sqrt(2)*a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) + 36*(a^2*x^2
 - 2*a*x + 1)*sqrt(-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x
- c)) - 2*(6*a^3*x^3 - 29*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*
c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(7/2),x)
```

output
```
Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(7/2)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(7/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{a\,x-1}{\left(c-\frac{c}{a\,x}\right)^{7/2}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a*x))^(7/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a*x))^(7/2)*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.07

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{\sqrt{c}\left(3\sqrt{ax-1}\,\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}\,i+i\right)ax-3\sqrt{ax-1}\,\sqrt{2}\log\left(\sqrt{ax-\right.\right.}{}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x)
```

output
```
(sqrt(c)*(3*sqrt(a*x - 1)*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a) - sq
rt(2)*i + i)*a*x - 3*sqrt(a*x - 1)*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqr
t(a) - sqrt(2)*i + i) + 3*sqrt(a*x - 1)*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x
)*sqrt(a) + sqrt(2)*i - i)*a*x - 3*sqrt(a*x - 1)*sqrt(2)*log(sqrt(a*x - 1)
 + sqrt(x)*sqrt(a) + sqrt(2)*i - i) - 3*sqrt(a*x - 1)*sqrt(2)*log(2*sqrt(x
)*sqrt(a)*sqrt(a*x - 1) + 2*sqrt(2) + 2*a*x + 2)*a*x + 3*sqrt(a*x - 1)*sqr
t(2)*log(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 2*sqrt(2) + 2*a*x + 2) + 72*sqr
t(a*x - 1)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))*a*x - 72*sqrt(a*x - 1)*log
(sqrt(a*x - 1) + sqrt(x)*sqrt(a)) + 16*sqrt(a*x - 1)*a*x - 16*sqrt(a*x - 1
) + 24*sqrt(x)*sqrt(a)*a**2*x**2 - 116*sqrt(x)*sqrt(a)*a*x + 84*sqrt(x)*sq
rt(a)))/(24*sqrt(a*x - 1)*a*c**4*(a*x - 1))
```

3.494 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx = -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{5\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}}+\frac{\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}ac^{9/2}}$$

output
```
-6/5/a/c^2/(c-c/a/x)^(5/2)-11/6/a/c^3/(c-c/a/x)^(3/2)-21/4/a/c^4/(c-c/a/x)
^(1/2)+x/c^2/(c-c/a/x)^(5/2)+5*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(9/2)+
1/8*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(9/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.48

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx = \frac{ax^2\left(5ax-\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{a-\frac{1}{x}}{2a}\right)-5\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\right.\right.}{5c^4\sqrt{c-\frac{c}{ax}}(-1+ax)^2}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2)),x]
```

output
```
(a*x^2*(5*a*x - Hypergeometric2F1[-5/2, 1, -3/2, (a - x^(-1))/(2*a)] - 5*H
ypergeometric2F1[-5/2, 1, -3/2, 1 - 1/(a*x)]))/(5*c^4*Sqrt[c - c/(a*x)]*(-
1 + a*x)^2)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.12, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6717, 6683, 1035, 281, 899, 114, 27, 169, 27, 169, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{9/2}(ax+1)}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\frac{1}{x}-a}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{9/2}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{x^2}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \textbf{114}$$

$$-\frac{a\left(-\frac{\int -\frac{c\left(5a+\frac{7}{x}\right)x}{2a\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}d\frac{1}{x}}{ac}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{a\left(\frac{\int \frac{\left(5a+\frac{7}{x}\right)x}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}d\frac{1}{x}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \textbf{169}$$

$$-\frac{a\left(\frac{\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\int -\frac{5c\left(5a+\frac{6}{x}\right)x}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{5c^2}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{a\left(\frac{\frac{\int \frac{\left(5a+\frac{6}{x}\right)x}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{c}+\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \textbf{169}$$

$$-\frac{a\left(\frac{\frac{\frac{11}{3c\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\int -\frac{3c\left(10a+\frac{11}{x}\right)x}{2\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{3c^2}}{c}+\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{a\left(\frac{\frac{\frac{\int \frac{\left(10a+\frac{11}{x}\right)x}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{2c}+\frac{11}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \; 169$$

$$-\frac{a\left(\frac{\frac{\frac{\frac{21}{c\sqrt{c-\frac{c}{ax}}}-\frac{\int -\frac{c\left(20a+\frac{21}{x}\right)x}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c^2}}{2c}+\frac{11}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(\frac{\frac{\frac{\frac{\int \frac{\left(20a+\frac{21}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2c}+\frac{21}{c\sqrt{c-\frac{c}{ax}}}}{2c}+\frac{11}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \; 174$$

$$-\frac{a\left(\frac{\frac{\frac{\frac{\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+20\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2c}+\frac{21}{c\sqrt{c-\frac{c}{ax}}}}{2c}+\frac{11}{3c\left(c-\frac{c}{ax}\right)^{3/2}}}{c}+\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{a\left(\frac{\frac{\frac{-\frac{40a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}}{2c}-\frac{2a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}}{2c}+\frac{21}{c\sqrt{c-\frac{c}{ax}}}}{c}+\frac{11}{3c\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{a\left(\frac{\frac{\frac{-\frac{\frac{40\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}}{2c}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}}{2c}+\frac{21}{c\sqrt{c-\frac{c}{ax}}}}{c}+\frac{11}{3c\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{12}{5c\left(c-\frac{c}{ax}\right)^{5/2}}}{2a^2}-\frac{x}{ac\left(c-\frac{c}{ax}\right)^{5/2}}\right)}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2)),x]
```

output
```
-((a*(-(x/(a*c*(c - c/(a*x))^(5/2))) + (12/(5*c*(c - c/(a*x))^(5/2)) + (11
/(3*c*(c - c/(a*x))^(3/2)) + (21/(c*Sqrt[c - c/(a*x)]) + ((-40*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]])/Sqrt[c] - (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqr
t[2]*Sqrt[c])])/Sqrt[c])/(2*c))/(2*c))/c)/(2*a^2)))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(141) = 282$.

Time = 0.24 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.83

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+\left(x-\frac{1}{a}\right)ac}}{5a^9c\left(x-\frac{1}{a}\right)^3}-\frac{37\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+\left(x-\frac{1}{a}\right)ac}}{30a^8c\left(x-\frac{1}{a}\right)^2}-\frac{317\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+\left(x-\frac{1}{a}\right)ac}}{60a^7c\left(x-\frac{1}{a}\right)}+\frac{5\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c}\right.}{2a^5\sqrt{a^2c}}\right.}{c^4x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(1260a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\sqrt{x(ax-1)}\,x^4+600\ln\left(\frac{2\sqrt{x(ax-1)}\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a^5x^4-15a^{\frac{9}{2}}\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{x(ax-1)}\,a-3ax+1}{ax+1}\right)x^4\right.}{}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^4/(c*(a*x-1)/a/x)^(1/2)+(-1/5/a^9/c/(x-1/a)^3*((x-1/a)^2*a^2
*c+(x-1/a)*a*c)^(1/2)-37/30/a^8/c/(x-1/a)^2*((x-1/a)^2*a^2*c+(x-1/a)*a*c)^
(1/2)-317/60/a^7/c/(x-1/a)*((x-1/a)^2*a^2*c+(x-1/a)*a*c)^(1/2)+5/2/a^5*ln(
(-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-1/
16/a^6*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*((x+1/a)^2*
a^2*c-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))*a^4/c^4/x/(c*(a*x-1)/a/x)^(1/2)*
(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.60

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx=\left[\frac{15\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3acx-c}{ax+1}\right)+600(a^3x^3-3}{240(a^4}\right.$$

$$-\frac{15\sqrt{2}(a^3x^3-3a^2x^2+3ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)+600(a^3x^3-3a^2x^2+3ax-1)\sqrt{-c}\,\mathrm{ar}}{120(a^4c^5x^3-3a^3c^5x^2+3a^2c^5x-}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="fricas")
```

output
```
[1/240*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(2*sqrt(
2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 600*(a^
3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((
a*c*x - c)/(a*x)) + c) + 4*(60*a^4*x^4 - 497*a^3*x^3 + 740*a^2*x^2 - 315*a
*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x -
a*c^5), -1/120*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arct
an(sqrt(2)*a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) + 600*(a^3*x^
3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(
a*x))/(a*c*x - c)) - 2*(60*a^4*x^4 - 497*a^3*x^3 + 740*a^2*x^2 - 315*a*x)*
sqrt((a*c*x - c)/(a*x)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^
5)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx=\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{9}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(9/2),x)
```

output
```
Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(9/2)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx=\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(9/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx = \int \frac{a\,x-1}{\left(c-\frac{c}{a\,x}\right)^{9/2}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a*x))^(9/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a*x))^(9/2)*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.76

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx = \frac{\sqrt{c}\,\left(15\sqrt{ax-1}\,\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}\,i+i\right)a^2x^2-30\sqrt{ax-1}\,\sqrt{2}\log\left(\sqrt{a}\right.\right.}{}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x)
```

output
```
(sqrt(c)*(15*sqrt(a*x - 1)*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a) - s
qrt(2)*i + i)*a**2*x**2 - 30*sqrt(a*x - 1)*sqrt(2)*log(sqrt(a*x - 1) + sqr
t(x)*sqrt(a) - sqrt(2)*i + i)*a*x + 15*sqrt(a*x - 1)*sqrt(2)*log(sqrt(a*x
- 1) + sqrt(x)*sqrt(a) - sqrt(2)*i + i) + 15*sqrt(a*x - 1)*sqrt(2)*log(sqr
t(a*x - 1) + sqrt(x)*sqrt(a) + sqrt(2)*i - i)*a**2*x**2 - 30*sqrt(a*x - 1)
*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a) + sqrt(2)*i - i)*a*x + 15*sqr
t(a*x - 1)*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a) + sqrt(2)*i - i) -
15*sqrt(a*x - 1)*sqrt(2)*log(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 2*sqrt(2) +
 2*a*x + 2)*a**2*x**2 + 30*sqrt(a*x - 1)*sqrt(2)*log(2*sqrt(x)*sqrt(a)*sqr
t(a*x - 1) + 2*sqrt(2) + 2*a*x + 2)*a*x - 15*sqrt(a*x - 1)*sqrt(2)*log(2*s
qrt(x)*sqrt(a)*sqrt(a*x - 1) + 2*sqrt(2) + 2*a*x + 2) + 1200*sqrt(a*x - 1)
*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))*a**2*x**2 - 2400*sqrt(a*x - 1)*log(s
qrt(a*x - 1) + sqrt(x)*sqrt(a))*a*x + 1200*sqrt(a*x - 1)*log(sqrt(a*x - 1)
 + sqrt(x)*sqrt(a)) + 632*sqrt(a*x - 1)*a**2*x**2 - 1264*sqrt(a*x - 1)*a*x
 + 632*sqrt(a*x - 1) + 240*sqrt(x)*sqrt(a)*a**3*x**3 - 1988*sqrt(x)*sqrt(a
)*a**2*x**2 + 2960*sqrt(x)*sqrt(a)*a*x - 1260*sqrt(x)*sqrt(a)))/(240*sqrt(
a*x - 1)*a*c**5*(a**2*x**2 - 2*a*x + 1))
```

3.495 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 232

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\frac{1049c^4\sqrt{1-\frac{1}{a^2x^2}}}{15a\sqrt{c-\frac{c}{ax}}}+\frac{311c^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{15a}+\frac{47c^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5a}+\frac{10\left(c-\frac{c}{ax}\right)^{7/2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\left(c-\frac{c}{ax}\right)^{9/2}x}{c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{13c^{7/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
1049/15*c^4*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+311/15*c^3*(1-1/a^2/x^2)
^(1/2)*(c-c/a/x)^(1/2)/a+47/5*c^2*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(3/2)/a+10
*(c-c/a/x)^(7/2)/a/(1-1/a^2/x^2)^(1/2)+(c-c/a/x)^(9/2)*x/c/(1-1/a^2/x^2)^(
1/2)-13*c^(7/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.57

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{c^3\sqrt{c-\frac{c}{ax}}\left(6-62ax+548a^2x^2+1441a^3x^3+15a^4x^4-45a^3\sqrt{1+\frac{1}{ax}}x^3\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\dots\right)}{15a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}$$

input
```
Integrate[(c - c/(a*x))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^3*Sqrt[c - c/(a*x)]*(6 - 62*a*x + 548*a^2*x^2 + 1441*a^3*x^3 + 15*a^4*x
^4 - 45*a^3*Sqrt[1 + 1/(a*x)]*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]] + 150*a^3*x^3
*Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(15*a^4*Sqrt[1 - 1/(a^2*x^
2)]*x^3)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.76, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6731, 585, 27, 109, 27, 167, 27, 170, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{7/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{13/2}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^5 x^2}{a^5\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^5 x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^5\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\int \frac{\left(a-\frac{1}{x}\right)^3\left(13a+\frac{3}{x}\right)x}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{ax\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}\right)}{a^5\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{2}\int \frac{\left(a-\frac{1}{x}\right)^3\left(13a+\frac{3}{x}\right)x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{ax\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}\right)}{a^5\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{167}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(2a\int -\frac{\left(a-\frac{1}{x}\right)^2\left(13a+\frac{47}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{20a\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}\right)}{a^5\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-a\int \frac{\left(a-\frac{1}{x}\right)^2\left(13a+\frac{47}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{20a\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}\right)}{a^5\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{170}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-a\left(\frac{2}{5}a\int \frac{\left(a-\frac{1}{x}\right)\left(65a+\frac{311}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{94}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)-\frac{20a\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}\right)}{a^5\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-a\left(\frac{1}{5}a\int\frac{\left(a-\frac{1}{x}\right)\left(65a+\frac{311}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{94}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)-\frac{20a\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^4}{\sqrt{\frac{1}{ax}+1}}\right)}{a^5\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 164

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-a\left(\frac{1}{5}a\left(65a^2\int\frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{3}a\left(1360a-\frac{311}{x}\right)\sqrt{\frac{1}{ax}+1}\right)+\frac{94}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)-\frac{20a\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)\cdots}{a^5\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 73

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-a\left(\frac{1}{5}a\left(130a^3\int\frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{3}a\left(1360a-\frac{311}{x}\right)\sqrt{\frac{1}{ax}+1}\right)+\frac{94}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)-\frac{20a\cdots}{\sqrt{\cdots}}\cdots}{a^5\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 221

$$-\frac{c^3\left(\frac{1}{2}\left(-a\left(\frac{1}{5}a\left(\frac{2}{3}a\left(1360a-\frac{311}{x}\right)\sqrt{\frac{1}{ax}+1}-130a^2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{94}{5}a\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\right)-\frac{20a\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\cdots}{a^5\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(c - c/(a*x))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c^3*Sqrt[c - c/(a*x)]*(-((a*(a - x^(-1))^4*x)/Sqrt[1 + 1/(a*x)]) + ((-2
0*a*(a - x^(-1))^3)/Sqrt[1 + 1/(a*x)] - a*((94*a*(a - x^(-1))^2*Sqrt[1 + 1
/(a*x)])/5 + (a*((2*a*(1360*a - 311/x)*Sqrt[1 + 1/(a*x)])/3 - 130*a^2*ArcT
anh[Sqrt[1 + 1/(a*x)]]))/5))/2))/(a^5*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.91

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(30a^{\frac{9}{2}}\sqrt{x(ax+1)}\,x^4+3182a^{\frac{7}{2}}x^3\sqrt{x(ax+1)}-195\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^4x^4+1096a^{\frac{5}{2}}x^2\sqrt{x(}\right.}{30(ax-1)^2x^2a^{\frac{7}{2}}\sqrt{x(ax+1)}}$
risch	$\frac{(15a^4x^4+631a^3x^3+548a^2x^2-62ax+6)c^3\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{15x^2a^3(ax-1)}+\frac{\left(-\frac{13a^3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2\sqrt{a^2c}}+\frac{64a\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-\left(x+\frac{1}{a}\right)}}{c\left(x+\frac{1}{a}\right)}\right.}{a^3(ax-1)}$

input
```
int((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^3*(
30*a^(9/2)*(x*(a*x+1))^(1/2)*x^4+3182*a^(7/2)*x^3*(x*(a*x+1))^(1/2)-195*ln
(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^4*x^4+1096*a^(5/2)*x
^2*(x*(a*x+1))^(1/2)-195*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1
/2))*a^3*x^3-124*a^(3/2)*x*(x*(a*x+1))^(1/2)+12*(x*(a*x+1))^(1/2)*a^(1/2))
/x^2/a^(7/2)/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.79

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\left[\frac{195\left(a^3c^3x^3-a^2c^3x^2\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(15a^4\right.}{60\left(a^4x^3-a^3x^2\right)}\right.$$

input
```
integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
[1/60*(195*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 1591*a^3*c^3*x^3
+ 548*a^2*c^3*x^2 - 62*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(195*(a^3*c^3*x^3 - a^2*c^3*x^2
)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqr
t((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 1591
*a^3*c^3*x^3 + 548*a^2*c^3*x^2 - 62*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x +
 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \int \left(c-\frac{c}{ax}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

input
```
integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \int \left(c-\frac{c}{a\,x}\right)^{7/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}dx$$

input
```
int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.49

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \frac{\sqrt{c}\,c^3\left(-780\sqrt{ax+1}\log\left(\sqrt{ax+1}+\sqrt{x}\sqrt{a}\right)a^3x^3-7249\sqrt{ax+1}\,a^3x^3+60\sqrt{x}\sqrt{a}\,a^4x^4+\right.}{60\sqrt{ax+1}\,a^4x^3}$$

input
```
int((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*c**3*( - 780*sqrt(a*x + 1)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a
**3*x**3 - 7249*sqrt(a*x + 1)*a**3*x**3 + 60*sqrt(x)*sqrt(a)*a**4*x**4 + 6
364*sqrt(x)*sqrt(a)*a**3*x**3 + 2192*sqrt(x)*sqrt(a)*a**2*x**2 - 248*sqrt(
x)*sqrt(a)*a*x + 24*sqrt(x)*sqrt(a)))/(60*sqrt(a*x + 1)*a**4*x**3)
```

3.496 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 193

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{83c^3\sqrt{1-\frac{1}{a^2x^2}}}{3a\sqrt{c-\frac{c}{ax}}}+\frac{29c^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{3a}+\frac{10\left(c-\frac{c}{ax}\right)^{5/2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\left(c-\frac{c}{ax}\right)^{7/2}x}{c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{11c^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
83/3*c^3*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+29/3*c^2*(1-1/a^2/x^2)^(1/2
)*(c-c/a/x)^(1/2)/a+10*(c-c/a/x)^(5/2)/a/(1-1/a^2/x^2)^(1/2)+(c-c/a/x)^(7/
2)*x/c/(1-1/a^2/x^2)^(1/2)-11*c^(5/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/
(c-c/a/x)^(1/2))/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.64

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{c^2\sqrt{c-\frac{c}{ax}}\left(-2+32ax+103a^2x^2+3a^3x^3-3a^2\sqrt{1+\frac{1}{ax}}x^2\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)+30a^2x^2\text{ F}\right.}{3a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

input
```
Integrate[(c - c/(a*x))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^2*Sqrt[c - c/(a*x)]*(-2 + 32*a*x + 103*a^2*x^2 + 3*a^3*x^3 - 3*a^2*Sqrt
[1 + 1/(a*x)]*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]] + 30*a^2*x^2*Hypergeometric2F
1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(3*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.75, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6731, 585, 27, 109, 27, 167, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{5/2} e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{11/2}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^4x^2}{a^4\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^4x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 109$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(-\int \frac{\left(a-\frac{1}{x}\right)^2\left(11a+\frac{1}{x}\right)x}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{ax\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{2}\int \frac{\left(a-\frac{1}{x}\right)^2\left(11a+\frac{1}{x}\right)x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{ax\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 167$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(2a\int -\frac{\left(a-\frac{1}{x}\right)\left(11a+\frac{29}{x}\right)x}{2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{20a\left(a-\frac{1}{x}\right)^2}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-a\int \frac{\left(a-\frac{1}{x}\right)\left(11a+\frac{29}{x}\right)x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{20a\left(a-\frac{1}{x}\right)^2}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 164$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-a\left(11a^2\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{3}a\left(112a-\frac{29}{x}\right)\sqrt{\frac{1}{ax}+1}\right)-\frac{20a\left(a-\frac{1}{x}\right)^2}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 73$$

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-a\left(22a^3\int\frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{3}a\left(112a-\frac{29}{x}\right)\sqrt{\frac{1}{ax}+1}\right)-\frac{20a\left(a-\frac{1}{x}\right)^2}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c^2\left(\frac{1}{2}\left(-a\left(\frac{2}{3}a\left(112a-\frac{29}{x}\right)\sqrt{\frac{1}{ax}+1}-22a^2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)-\frac{20a\left(a-\frac{1}{x}\right)^2}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax\left(a-\frac{1}{x}\right)^3}{\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{a^4\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(c - c/(a*x))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c^2*Sqrt[c - c/(a*x)]*(-((a*(a - x^(-1))^3*x)/Sqrt[1 + 1/(a*x)]) + ((-2
0*a*(a - x^(-1))^2)/Sqrt[1 + 1/(a*x)] - a*((2*a*(112*a - 29/x)*Sqrt[1 + 1/
(a*x)])/3 - 22*a^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/2))/(a^4*Sqrt[1 - 1/(a*x)]
))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.01

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^2\left(6a^{\frac{7}{2}}x^3\sqrt{x(ax+1)}+266a^{\frac{5}{2}}x^2\sqrt{x(ax+1)}-33\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^3x^3+64a^{\frac{3}{2}}x\sqrt{x(ax+1)}-\right.}{6(ax-1)^2x\,a^{\frac{5}{2}}\sqrt{x(ax+1)}}$
risch	$\frac{(3a^3x^3+37a^2x^2+32ax-2)c^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^2(ax-1)}+\frac{\left(-\frac{11a^2\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2\sqrt{a^2c}}+\frac{32\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-\left(x+\frac{1}{a}\right)ac}}{c\left(x+\frac{1}{a}\right)}\right)c^2\sqrt{\frac{ax-}{ax+}}}{a^2(ax-1)}$

input
```
int((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^2*(6
*a^(7/2)*x^3*(x*(a*x+1))^(1/2)+266*a^(5/2)*x^2*(x*(a*x+1))^(1/2)-33*ln(1/2
*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3*x^3+64*a^(3/2)*x*(x*(a
*x+1))^(1/2)-33*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*
x^2-4*(x*(a*x+1))^(1/2)*a^(1/2))/x/a^(5/2)/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.97

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\left[\frac{33\left(a^2c^2x^2-ac^2x\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3a^3c^2x^3\right.}{12\left(a^3x^2-a^2x\right)}\right.$$

input
```
integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
[1/12*(33*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*
(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*
x - c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 133*a^2*c^2*x^2 + 32*a*
c^2*x - 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2
 - a^2*x), 1/6*(33*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
 - a*c*x - c)) + 2*(3*a^3*c^2*x^3 + 133*a^2*c^2*x^2 + 32*a*c^2*x - 2*c^2)*
sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\int\left(c-\frac{c}{a\,x}\right)^{5/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.52

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\frac{\sqrt{c}\,c^2\left(-264\sqrt{ax+1}\log\left(\sqrt{ax+1}+\sqrt{x}\sqrt{a}\right)a^2x^2-1571\sqrt{ax+1}\,a^2x^2+24\sqrt{x}\sqrt{a}\,a^3x^3+\right.}{24\sqrt{ax+1}\,a^3x^2}$$

input
```
int((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*c**2*( - 264*sqrt(a*x + 1)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a
**2*x**2 - 1571*sqrt(a*x + 1)*a**2*x**2 + 24*sqrt(x)*sqrt(a)*a**3*x**3 + 1
064*sqrt(x)*sqrt(a)*a**2*x**2 + 256*sqrt(x)*sqrt(a)*a*x - 16*sqrt(x)*sqrt(
a)))/(24*sqrt(a*x + 1)*a**3*x**2)
```

3.497 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 152

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{11c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{10\left(c-\frac{c}{ax}\right)^{3/2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\left(c-\frac{c}{ax}\right)^{5/2}x}{c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{9c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
11*c^2*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+10*(c-c/a/x)^(3/2)/a/(1-1/a^2
/x^2)^(1/2)+(c-c/a/x)^(5/2)*x/c/(1-1/a^2/x^2)^(1/2)-9*c^(3/2)*arctanh(c^(1
/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.47

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{c\sqrt{c-\frac{c}{ax}}\left(2+10ax+a^2x^2+9ax\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{1}{ax}\right)\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - c/(a*x))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c*Sqrt[c - c/(a*x)]*(2 + 10*a*x + a^2*x^2 + 9*a*x*Hypergeometric2F1[-1/2,
 1, 1/2, 1 + 1/(a*x)]))/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.72, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6731, 585, 27, 109, 27, 163, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{3/2} e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{9/2}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^3x^2}{a^3\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^3x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left(-\int \frac{\left(a-\frac{1}{x}\right)\left(9a-\frac{1}{x}\right)x}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{ax\left(a-\frac{1}{x}\right)^2}{\sqrt{\frac{1}{ax}+1}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{2}\int \frac{(a-\frac{1}{x})(9a-\frac{1}{x})x}{(1+\frac{1}{ax})^{3/2}}d\frac{1}{x}-\frac{ax(a-\frac{1}{x})^2}{\sqrt{\frac{1}{ax}+1}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 163$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-9a^2\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{2a(21a+\frac{1}{x})}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax(a-\frac{1}{x})^2}{\sqrt{\frac{1}{ax}+1}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 73$$

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-18a^3\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}-\frac{2a(21a+\frac{1}{x})}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax(a-\frac{1}{x})^2}{\sqrt{\frac{1}{ax}+1}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 221$$

$$-\frac{c\left(\frac{1}{2}\left(18a^2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)-\frac{2a(21a+\frac{1}{x})}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{ax(a-\frac{1}{x})^2}{\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{a^3\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(c - c/(a*x))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c*Sqrt[c - c/(a*x)]*(-((a*(a - x^(-1))^2*x)/Sqrt[1 + 1/(a*x)]) + ((-2*a
*(21*a + x^(-1)))/Sqrt[1 + 1/(a*x)] + 18*a^2*ArcTanh[Sqrt[1 + 1/(a*x)]])/2
))/(a^3*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.11

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c\left(2a^{\frac{5}{2}}x^2\sqrt{x(ax+1)}+38a^{\frac{3}{2}}x\sqrt{x(ax+1)}-9\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^2x^2-9\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax}{2\sqrt{a}}\right.\right.}{2(ax-1)^2a^{\frac{3}{2}}\sqrt{x(ax+1)}}$
risch	$\frac{(a^2x^2+3ax+2)c\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{a(ax-1)}+\frac{\left(-\frac{9a\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2\sqrt{a^2c}}+\frac{16\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-\left(x+\frac{1}{a}\right)ac}}{ac\left(x+\frac{1}{a}\right)}\right)c\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{(}}{a(ax-1)}$

input
```
int((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c*(2*a
^(5/2)*x^2*(x*(a*x+1))^(1/2)+38*a^(3/2)*x*(x*(a*x+1))^(1/2)-9*ln(1/2*(2*(x
*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*x^2-9*ln(1/2*(2*(x*(a*x+1))^
(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x+4*(x*(a*x+1))^(1/2)*a^(1/2))/a^(3/2)/(
x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.07

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\left[\frac{9\,(acx-c)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2cx^2+19\,acx}{4\,(a^2x-a)}\right.$$

input
```
integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(9*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3
*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
- c)/(a*x - 1)) + 4*(a^2*c*x^2 + 19*a*c*x + 2*c)*sqrt((a*x - 1)/(a*x + 1))
*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(9*(a*c*x - c)*sqrt(-c)*arctan(
2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 + 19*a*c*x + 2*c)*sqrt((a*x -
 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int \left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \int \left(c-\frac{c}{a\,x}\right)^{3/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}dx$$

input
```
int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.51

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \frac{\sqrt{c}\,c\left(-9\sqrt{ax+1}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)ax+18\sqrt{ax+1}\,ax+\sqrt{x}\,\sqrt{a}\,a^2x^2+19\sqrt{x}\,\sqrt{a}\,a\right.}{\sqrt{ax+1}\,a^2x}$$

input
```
int((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*c*( - 9*sqrt(a*x + 1)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a*x +
18*sqrt(a*x + 1)*a*x + sqrt(x)*sqrt(a)*a**2*x**2 + 19*sqrt(x)*sqrt(a)*a*x
+ 2*sqrt(x)*sqrt(a)))/(sqrt(a*x + 1)*a**2*x)
```

3.498 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{10\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{7\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
10*(c-c/a/x)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+(c-c/a/x)^(3/2)*x/c/(1-1/a^2/x^2)
^(1/2)-7*c^(1/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.58

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(9+ax-7\sqrt{1+\frac{1}{ax}}\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a*x)]*(9 + a*x - 7*Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)
]]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6731, 585, 27, 100, 27, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\int -\frac{\left(7a-\frac{2}{x}\right)x}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{2}\int \frac{\left(7a-\frac{2}{x}\right)x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-7a\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{18a}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-14a^2\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}-\frac{18a}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\left(\frac{1}{2}\left(14a\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)-\frac{18a}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a*x)]*(-((a^2*x)/Sqrt[1 + 1/(a*x)]) + ((-18*a)/Sqrt[1 + 1/(
a*x)] + 14*a*ArcTanh[Sqrt[1 + 1/(a*x)]])/2))/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.27

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2a^{\frac{3}{2}}x\sqrt{x(ax+1)}-7\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+18\sqrt{x(ax+1)}\sqrt{a}-7\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2(ax-1)^2\sqrt{a}\sqrt{x(ax+1)}}$
risch	$\frac{(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}+\frac{\left(-\frac{7\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2\sqrt{a^2c}}+\frac{8\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-\left(x+\frac{1}{a}\right)ac}}{a^2c\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{(ax+1)acx}}{ax-1}$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*a
^(3/2)*x*(x*(a*x+1))^(1/2)-7*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/
a^(1/2))*a*x+18*(x*(a*x+1))^(1/2)*a^(1/2)-7*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^
(1/2)+2*a*x+1)/a^(1/2)))/a^(1/2)/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.60

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=\left[\frac{7\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+9\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)},\right.$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)))/(a^2*x - a), 1/2*(7*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
 - a*c*x - c)) + 2*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \sqrt{c-\frac{c}{a\,x}}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} dx$$

input
```
int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.51

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\sqrt{c}\left(-28\sqrt{ax+1}\log\left(\sqrt{ax+1}+\sqrt{x}\sqrt{a}\right)+33\sqrt{ax+1}+4\sqrt{x}\sqrt{a}\,ax+36\sqrt{x}\sqrt{a}\right)}{4\sqrt{ax+1}\,a}$$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*( - 28*sqrt(a*x + 1)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)) + 33*sq
rt(a*x + 1) + 4*sqrt(x)*sqrt(a)*a*x + 36*sqrt(x)*sqrt(a)))/(4*sqrt(a*x + 1
)*a)
```

3.499 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

output
```
5*(c-c/a/x)^(1/2)/a/c/(1-1/a^2/x^2)^(1/2)+(c-c/a/x)^(1/2)*x/c/(1-1/a^2/x^2
)^(1/2)-5*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.58

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{1-\frac{1}{ax}}\left(ax+5\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{1}{ax}\right)\right)}{a\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(a*x + 5*Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))
/(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6731, 580, 578, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \text{ 6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{ 580}$$

$$-\frac{-\frac{5c\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{2a}-\frac{c^2x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{ 578}$$

$$-\frac{-\frac{5c\left(c\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2c\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{2a}-\frac{c^2x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{ 573}$$

$$-\frac{-\frac{5c\left(\frac{2c\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}-2c^2\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{2a}-\frac{c^2x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{-\frac{5c\left(\frac{2c\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}-2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)\right)}{2a}-\frac{c^2x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

output
```
-((-((c^2*Sqrt[c - c/(a*x)]*x)/Sqrt[1 - 1/(a^2*x^2)]) - (5*c*((2*c*Sqrt[c
- c/(a*x)])/Sqrt[1 - 1/(a^2*x^2)] - 2*c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/
(a^2*x^2)])/Sqrt[c - c/(a*x)]]))/(2*a))/c^3)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 578
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-c)*(e*x)^(m + 1)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)
/(a*e*(p + 1))), x] + Simp[c*((m - n + 2)/(a*(p + 1)))   Int[(e*x)^m*(c + d
*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] &&
EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0] && LtQ[p, -1] && RationalQ[m]
```

rule 580
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 2)*((a + b*x^2)^(p +
1)/(b*e*(m + 1))), x] + Simp[d*((2*m + p + 3)/(e*(m + 1)))   Int[(e*x)^(m +
 1)*(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p},
 x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p - 1, 0] && LtQ[m, -1] && IntegerQ
[p + 1/2]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.26

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2a^{\frac{3}{2}}x\sqrt{x(ax+1)}-5\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+10\sqrt{x(ax+1)}\,\sqrt{a}-5\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right.\right.}{2(ax-1)^{2}\sqrt{a}\,c\sqrt{x(ax+1)}}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{5\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a\sqrt{a^{2}c}}+\frac{4\sqrt{\left(x+\frac{1}{a}\right)^{2}a^{2}c-\left(x+\frac{1}{a}\right)ac}}{a^{3}c\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)acx}}{\sqrt{\frac{c(ax-1)}{ax}}\,x}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*a
^(3/2)*x*(x*(a*x+1))^(1/2)-5*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/
a^(1/2))*a*x+10*(x*(a*x+1))^(1/2)*a^(1/2)-5*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^
(1/2)+2*a*x+1)/a^(1/2)))/a^(1/2)/c/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.57

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \left[\frac{5\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^{2}x^{2}+5\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^{2}cx-ac)},\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(5*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*x^2 + 5*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)))/(a^2*c*x - a*c), 1/2*(5*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2
+ a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c
*x^2 - a*c*x - c)) + 2*(a^2*x^2 + 5*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a
*c*x - c)/(a*x)))/(a^2*c*x - a*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(c - c/(a*x)), x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(c - c/(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(1/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.53

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{c}\left(-20\sqrt{ax+1}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)+17\sqrt{ax+1}+4\sqrt{x}\,\sqrt{a}\,ax+20\sqrt{x}\,\sqrt{a}\right)}{4\sqrt{ax+1}\,ac}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x)
```

output
```
(sqrt(c)*( - 20*sqrt(a*x + 1)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)) + 17*sq
rt(a*x + 1) + 4*sqrt(x)*sqrt(a)*a*x + 20*sqrt(x)*sqrt(a)))/(4*sqrt(a*x + 1
)*a*c)
```

3.500 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{ax}}}-\frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}$$

output
```
3*(1-1/a^2/x^2)^(1/2)*x/c/(c-c/a/x)^(1/2)-2*(c-c/a/x)^(1/2)*x/c^2/(1-1/a^2
/x^2)^(1/2)-3*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(3/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.55

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},1+\frac{1}{ax}\right)}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

output
```
(2*(1 - 1/(a*x))^(3/2)*Hypergeometric2F1[-1/2, 2, 1/2, 1 + 1/(a*x)])/(a*Sq
rt[1 + 1/(a*x)]*(c - c/(a*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6731, 578, 579, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{578}$$

$$-\frac{3c\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2cx\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{579}$$

$$-\frac{3c\left(-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)+\frac{2cx\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{573}$$

$$-\frac{3c\left(\frac{c\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)+\frac{2cx\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{3c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)+\frac{2cx\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

output
```
-(((2*c*Sqrt[c - c/(a*x)]*x)/Sqrt[1 - 1/(a^2*x^2)] + 3*c*(-((c*Sqrt[1 - 1/
(a^2*x^2)]*x)/Sqrt[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a
^2*x^2)])/Sqrt[c - c/(a*x)]])/a))/c^3)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 578
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-c)*(e*x)^(m + 1)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)
/(a*e*(p + 1))), x] + Simp[c*((m - n + 2)/(a*(p + 1)))   Int[(e*x)^m*(c + d
*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] &&
EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0] && LtQ[p, -1] && RationalQ[m]
```

rule 579
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 1)*((a + b*x^2)^(p +
1)/(b*c*e*(m + 1))), x] - Simp[d*((n - m - 2)/(c*e*(m + 1)))   Int[(e*x)^(m
 + 1)*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0] && LtQ[m, -1] && (IntegerQ[2*p] |
| IntegerQ[m])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.27

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2a^{\frac{3}{2}}x\sqrt{x(ax+1)}+3\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-6\sqrt{x(ax+1)}\,\sqrt{a}+3\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax\cdots}{2\sqrt{a}}\right.\right.}{2(ax-1)^2\sqrt{a}\,c^2\sqrt{x(ax+1)}}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ac\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^2\sqrt{a^2c}}+\frac{2\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-\left(x+\frac{1}{a}\right)ac}}{a^4c\left(x+\frac{1}{a}\right)}\right)a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)acx}}{cx\sqrt{\frac{c(ax-1)}{ax}}}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(
1/2)/c^2*(-2*a^(3/2)*x*(x*(a*x+1))^(1/2)+3*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(
1/2)+2*a*x+1)/a^(1/2))*a*x-6*(x*(a*x+1))^(1/2)*a^(1/2)+3*ln(1/2*(2*(x*(a*x
+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.66

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+3\,a\cdots}{4\,(a^2c^2x-ac^2)}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*x^2 + 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)))/(a^2*c^2*x - a*c^2), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*
x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a
^2*c*x^2 - a*c*x - c)) + 2*(a^2*x^2 + 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqr
t((a*c*x - c)/(a*x)))/(a^2*c^2*x - a*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2), x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a\,x}\right)^{3/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.53

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\frac{\sqrt{c}\left(-12\sqrt{ax+1}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)+9\sqrt{ax+1}+4\sqrt{x}\,\sqrt{a}\,ax+12\sqrt{x}\,\sqrt{a}\right)}{4\sqrt{ax+1}\,a\,c^{2}}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x)
```

output
```
(sqrt(c)*( - 12*sqrt(a*x + 1)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)) + 9*sqr
t(a*x + 1) + 4*sqrt(x)*sqrt(a)*a*x + 12*sqrt(x)*sqrt(a)))/(4*sqrt(a*x + 1)
*a*c**2)
```

3.501 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 174

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\frac{2\sqrt{c-\frac{c}{ax}}}{ac^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{5/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{\sqrt{2}ac^{5/2}}$$

output
```
2*(c-c/a/x)^(1/2)/a/c^3/(1-1/a^2/x^2)^(1/2)+(c-c/a/x)^(1/2)*x/c^3/(1-1/a^2
/x^2)^(1/2)-arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(5/2)
-1/2*arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))*2^(1
/2)/a/c^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.52

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(ax+\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+\frac{1}{x}}{2a}\right)+\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2}\right.\right.}{ac^2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(a*x + Hypergeometric2F1[-1/2, 1, 1/2, (a + x^(-1))/(2*
a)] + Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(a*c^2*Sqrt[1 + 1/(a*
x)]*Sqrt[c - c/(a*x)])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.70, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6731, 585, 27, 114, 27, 169, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{ax^2}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{c^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\sqrt{c-\frac{c}{ax}}\int \frac{x^2}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{c^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 114$$

$$-\frac{a\sqrt{c-\frac{c}{ax}}\left(-\frac{\int \frac{\left(a-\frac{3}{x}\right)x}{2a\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{x}{a\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\sqrt{c-\frac{c}{ax}}\left(-\frac{\int \frac{\left(a-\frac{3}{x}\right)x}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{2a^2}-\frac{x}{a\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 169$$

$$-\frac{a\sqrt{c-\frac{c}{ax}}\left(-\frac{\int \frac{\left(a-\frac{2}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4}{\sqrt{\frac{1}{ax}+1}}}{2a^2}-\frac{x}{a\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 174$$

$$-\frac{a\sqrt{c-\frac{c}{ax}}\left(-\frac{-\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4}{\sqrt{\frac{1}{ax}+1}}}{2a^2}-\frac{x}{a\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 73$$

$$-\frac{a\sqrt{c-\frac{c}{ax}}\left(-\frac{-2a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+2a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{4}{\sqrt{\frac{1}{ax}+1}}}{2a^2}-\frac{x}{a\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 221$$

$$-\frac{a\left(-\frac{-2\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)-\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)+\frac{4}{\sqrt{\frac{1}{ax}+1}}}{2a^2}-\frac{x}{a\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{c^3\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

output
```
-((a*Sqrt[c - c/(a*x)]*(-(x/(a*Sqrt[1 + 1/(a*x)])) - (4/Sqrt[1 + 1/(a*x)]
- 2*ArcTanh[Sqrt[1 + 1/(a*x)]] - Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]
])/(2*a^2)))/(c^3*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.48

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^{2}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a^{3}\sqrt{a^{2}c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}c+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{4a^{4}\sqrt{c}}+\frac{\sqrt{\left(x+\frac{1}{a}\right)^{2}a^{2}\cdots}}{a^{5}c\left(x\cdots\right.}\right.}{c^{2}x\sqrt{\frac{c(ax-1)}{ax}}}$
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-4a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x+a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)x+2\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right.\cdots\right.}{4(ax-1)^{2}a^{\frac{3}{2}}c^{3}\sqrt{\frac{1}{a}}\,\sqrt{x\cdots}}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^2*((a*x-1)/(a*x+1))^(1/2)/(c*(a*x-1)/a/x)^(1/2)+(-1/2/a^3*ln
((1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2+a*c*x)^(1/2))/(a^2*c)^(1/2)-1/
4/a^4/c^(1/2)*2^(1/2)*ln((4*c+3*(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*((x-1/a)^2*a
^2*c+3*(x-1/a)*a*c+2*c)^(1/2))/(x-1/a))+1/a^5/c/(x+1/a)*((x+1/a)^2*a^2*c-(
x+1/a)*a*c)^(1/2))*a^2/c^2*((a*x-1)/(a*x+1))^(1/2)/x/(c*(a*x-1)/a/x)^(1/2)
*((a*x+1)*a*c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.01

$$\int\frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx=\left[\frac{\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^{3}cx^{3}-3\,a^{2}cx^{2}-13\,acx-4\,\sqrt{2}(3\,a^{3}x^{3}+4\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1}\right)+\cdots}{\cdots}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*
x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 2*(a
*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2*x^2 + a
*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
1)) + 8*(a^2*x^2 + 2*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^2*c^3*x - a*c^3), 1/4*(sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(
a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/
(3*a^2*c*x^2 - 2*a*c*x - c)) + 2*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
 - a*c*x - c)) + 4*(a^2*x^2 + 2*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x)))/(a^2*c^3*x - a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a\,x}\right)^{5/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(5/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.97

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\sqrt{c}\left(\sqrt{ax+1}\,\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}-1\right)-\sqrt{ax+1}\,\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{\right.}\right.}{}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x)
```

output
```
(sqrt(c)*(sqrt(a*x + 1)*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt
(2) - 1) - sqrt(a*x + 1)*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqr
t(2) + 1) - sqrt(a*x + 1)*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sq
rt(2) - 1) + sqrt(a*x + 1)*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + s
qrt(2) + 1) - 4*sqrt(a*x + 1)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)) + 5*sqr
t(a*x + 1) + 4*sqrt(x)*sqrt(a)*a*x + 8*sqrt(x)*sqrt(a)))/(4*sqrt(a*x + 1)*
a*c**3)
```

3.502 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 215

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = -\frac{x}{2c^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{1-\frac{1}{a^2x^2}}\,x}{4c^3\sqrt{c-\frac{c}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\,x}{4c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{7/2}}-\frac{11\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{4\sqrt{2}ac^{7/2}}$$

output
```
-1/2*x/c^3/(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)+7/4*(1-1/a^2/x^2)^(1/2)*x/c
^3/(c-c/a/x)^(1/2)-1/4*(c-c/a/x)^(1/2)*x/c^4/(1-1/a^2/x^2)^(1/2)+arctanh(c
^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(7/2)-11/8*arctanh(1/2*c^(
1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))*2^(1/2)/a/c^(7/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.56

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\left(2ax(-3+2ax)+11(-1+ax)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+\frac{1}{x}}{2a}\right)+(4-\right.}{4ac^3\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}(-1+ax)}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

output
```
(Sqrt[1 - 1/(a*x)]*(2*a*x*(-3 + 2*a*x) + 11*(-1 + a*x)*Hypergeometric2F1[-
1/2, 1, 1/2, (a + x^(-1))/(2*a)] + (4 - 4*a*x)*Hypergeometric2F1[-1/2, 1,
1/2, 1 + 1/(a*x)]))/(4*a*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*(-1 + a*x
))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.74, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6731, 585, 27, 114, 27, 168, 25, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \frac{a^2x^2}{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\int \frac{x^2}{\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 114$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}-\frac{\int -\frac{\left(6a+\frac{5}{x}\right)x^2}{2a\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{2a}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(6a+\frac{5}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{4a^2}+\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 168$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{-\frac{\int -\frac{\left(2a+\frac{9}{x}\right)x}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{6x}{\sqrt{\frac{1}{ax}+1}}}{4a^2}+\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 25$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\int \frac{\left(2a+\frac{9}{x}\right)x}{\left(a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{6x}{\sqrt{\frac{1}{ax}+1}}}{4a^2}+\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \ 169$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\int \frac{\left(4a+\frac{7}{x}\right)x}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{7}{\sqrt{\frac{1}{ax}+1}}}{a}-\frac{6x}{\sqrt{\frac{1}{ax}+1}}}{4a^2}+\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{1}{2}\int \frac{\left(4a+\frac{7}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{7}{\sqrt{\frac{1}{ax}+1}}}{a}-\frac{6x}{\sqrt{\frac{1}{ax}+1}}}{4a^2}+\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{174}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{1}{2}\left(11\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+4\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{7}{\sqrt{\frac{1}{ax}+1}}}{a}-\frac{6x}{\sqrt{\frac{1}{ax}+1}}}{4a^2}+\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{1}{2}\left(22a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+8a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}\right)-\frac{7}{\sqrt{\frac{1}{ax}+1}}}{a}-\frac{6x}{\sqrt{\frac{1}{ax}+1}}}{4a^2}+\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{a^2\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{1}{2}\left(11\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-8\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)-\frac{7}{\sqrt{\frac{1}{ax}+1}}}{a}-\frac{6x}{\sqrt{\frac{1}{ax}+1}}}{4a^2}+\frac{x}{2a\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}}\right)}{c^3\sqrt{c-\frac{c}{ax}}}$$

input

```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

output
```
-((a^2*Sqrt[1 - 1/(a*x)]*(x/(2*a*(a - x^(-1))*Sqrt[1 + 1/(a*x)]) + ((-6*x)
/Sqrt[1 + 1/(a*x)] + (-7/Sqrt[1 + 1/(a*x)] + (-8*ArcTanh[Sqrt[1 + 1/(a*x)]
] + 11*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/2)/a)/(4*a^2)))/(c^3*Sq
rt[c - c/(a*x)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.35

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x^2-11a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)x^2+4a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x+8\ln\right.}{16(ax-1)^3}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^4\sqrt{a^2c}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+3\left(x-\frac{1}{a}\right)ac+2c}}{4a^6c\left(x-\frac{1}{a}\right)}-\frac{11\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+}}{x-\frac{1}{a}}\right.}{16a^5\sqrt{c}}\right.}{c^3x\sqrt{\frac{c(ax-1)}{ax}}}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16
*a^(7/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*x^2-11*a^(5/2)*2^(1/2)*ln((2*2^(1/2
)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x^2+4*a^(5/2)*(1/a)^(1
/2)*(x*(a*x+1))^(1/2)*x+8*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(
1/2))*a^3*(1/a)^(1/2)*x^2-28*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2)-8*ln(1/
2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+11*2^(1/2)*
ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/a
^(3/2)/c^4/(1/a)^(1/2)/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 594, normalized size of antiderivative = 2.76

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\left[\frac{11\sqrt{2}(a^2x^2-2\,ax+1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{}}{a^3x^3-3\,a^2x^2+3\,ax-1}\right.}{}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

output
```
[1/32*(11*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2
*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((
a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*
a*x - 1)) + 8*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x +
4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(4*a^3*x^3 + a^2*x^2 - 7*a*x)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*
c^4), 1/16*(11*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^
2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3
*a^2*c*x^2 - 2*a*c*x - c)) - 8*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^
2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2
*a^2*c*x^2 - a*c*x - c)) + 4*(4*a^3*x^3 + a^2*x^2 - 7*a*x)*sqrt((a*x - 1)/
(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2), x)
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a\,x}\right)^{7/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.57

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx=\frac{\sqrt{c}\left(55\sqrt{ax+1}\,\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}-1\right)ax-55\sqrt{ax+1}\,\sqrt{2}\log\left(\sqrt{ax+\right.\right.}{}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x)
```

output
```
(sqrt(c)*(55*sqrt(a*x + 1)*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - s
qrt(2) - 1)*a*x - 55*sqrt(a*x + 1)*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqr
t(a) - sqrt(2) - 1) - 55*sqrt(a*x + 1)*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)
*sqrt(a) - sqrt(2) + 1)*a*x + 55*sqrt(a*x + 1)*sqrt(2)*log(sqrt(a*x + 1) +
 sqrt(x)*sqrt(a) - sqrt(2) + 1) - 55*sqrt(a*x + 1)*sqrt(2)*log(sqrt(a*x +
1) + sqrt(x)*sqrt(a) + sqrt(2) - 1)*a*x + 55*sqrt(a*x + 1)*sqrt(2)*log(sqr
t(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) - 1) + 55*sqrt(a*x + 1)*sqrt(2)*log
(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1)*a*x - 55*sqrt(a*x + 1)*sqr
t(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1) + 80*sqrt(a*x + 1)
*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a*x - 80*sqrt(a*x + 1)*log(sqrt(a*x
+ 1) + sqrt(x)*sqrt(a)) - 84*sqrt(a*x + 1)*a*x + 84*sqrt(a*x + 1) + 80*sqr
t(x)*sqrt(a)*a**2*x**2 + 20*sqrt(x)*sqrt(a)*a*x - 140*sqrt(x)*sqrt(a)))/(8
0*sqrt(a*x + 1)*a*c**4*(a*x - 1))
```

3.503 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 164

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = -\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

output
```
-1/8*c*(1-1/a^2/x^2)^(1/2)*x/a^2/(c-c/a/x)^(1/2)+1/12*c*(1-1/a^2/x^2)^(1/2
)*x^2/a/(c-c/a/x)^(1/2)+1/3*c*(1-1/a^2/x^2)^(1/2)*x^3/(c-c/a/x)^(1/2)+1/8*
c^(1/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$

$$= \frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(-3+2ax+8a^2x^2)}{-1+ax}-3\sqrt{c}\log(1-ax)+3\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-1-a\right.}{48a^3}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^2,x]
```

output
```
((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-3 + 2*a*x + 8*a^2*x^
2))/(-1 + a*x) - 3*Sqrt[c]*Log[1 - a*x] + 3*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt
[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(48*a
^3)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6733, 575, 579, 579, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6733}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^4}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{575}$$

$$-c\left(\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{6ac}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{579}$$

$$-c\left(\frac{-\frac{3\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{4a}-\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}}{6ac}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \; 579$$

$$-c\left(\frac{-\frac{3\left(-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}}{6ac}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \; 573$$

$$-c\left(\frac{-\frac{3\left(\frac{c\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}}{6ac}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \; 219$$

$$-c\left(\frac{-\frac{3\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}}{6ac}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}\right)$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^2,x]
```

output
```
-(c*(-1/3*(Sqrt[1 - 1/(a^2*x^2)]*x^3)/Sqrt[c - c/(a*x)] + (-1/2*(c*Sqrt[1
- 1/(a^2*x^2)]*x^2)/Sqrt[c - c/(a*x)] - (3*(-((c*Sqrt[1 - 1/(a^2*x^2)]*x)/
Sqrt[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt
[c - c/(a*x)]])/a))/(4*a))/(6*a*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 575
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(e*x)^(m + 1)*(c + d*x)^n*((a + b*x^2)^p/(e*(m + 1))), x]
 + Simp[b*(n/(d*e*(m + 1)))   Int[(e*x)^(m + 1)*(c + d*x)^(n + 1)*(a + b*x^
2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[b*c^2 + a*d^2, 0] &&
 EqQ[n + p, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m + p] && LeQ[m +
p + 2, 0])
```

rule 579
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 1)*((a + b*x^2)^(p +
1)/(b*c*e*(m + 1))), x] - Simp[d*((n - m - 2)/(c*e*(m + 1)))   Int[(e*x)^(m
 + 1)*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0] && LtQ[m, -1] && (IntegerQ[2*p] |
| IntegerQ[m])
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\left(16a^{\frac{5}{2}}x^{2}\sqrt{x(ax+1)}+4a^{\frac{3}{2}}x\sqrt{x(ax+1)}-6\sqrt{x(ax+1)}\,\sqrt{a}+3\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{48\sqrt{\frac{ax-1}{ax+1}}\, a^{\frac{5}{2}}\sqrt{x(ax+1)}}$	121
risch	$\frac{(8a^{2}x^{2}+2ax-3)x\sqrt{\frac{c(ax-1)}{ax}}}{24a^{2}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{16a^{2}\sqrt{a^{2}c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	148

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/48/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x/a^(5/2)*(16*a^(5/2)*x
^2*(x*(a*x+1))^(1/2)+4*a^(3/2)*x*(x*(a*x+1))^(1/2)-6*(x*(a*x+1))^(1/2)*a^(
1/2)+3*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(x*(a*x+1))^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.05

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$= \left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(8\,a^4x^4+10\,a^3x^3-a^2x^2-3\,a}{96\,(a^4x-a^3)}\right.$$
$$-\frac{3\,(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(8\,a^4x^4+10\,a^3x^3-a^2x^2-3\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx}{ax}}}{48\,(a^4x-a^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)*x^2,x, algorithm="fric
as")
```

output
```
[1/96*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
 c)/(a*x - 1)) + 4*(8*a^4*x^4 + 10*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*x -
1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), -1/48*(3*(a*x - 1)*s
qrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(8*a^4*x^4 + 10*a^3*x^3 -
 a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*
x - a^3)]
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)*x**2,x)
```

output
```
Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)*x^2,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)*x^2,x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a*x))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{x^2\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.41

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{\sqrt{c}\left(8\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2+2\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax-3\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+3\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)\right)}{24a^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)*x^2,x)
```

output
```
(sqrt(c)*(8*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**2*x**2 + 2*sqrt(x)*sqrt(a)*sq
rt(a*x + 1)*a*x - 3*sqrt(x)*sqrt(a)*sqrt(a*x + 1) + 3*log(sqrt(a*x + 1) +
sqrt(x)*sqrt(a))))/(24*a**3)
```

3.504 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

output
```
1/4*c*(1-1/a^2/x^2)^(1/2)*x/a/(c-c/a/x)^(1/2)+1/2*c*(1-1/a^2/x^2)^(1/2)*x^
2/(c-c/a/x)^(1/2)-1/4*c^(1/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x
)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.19

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$=\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(1+2ax)+\sqrt{c}(-1+ax)\log(1-ax)+\sqrt{c}(1-ax)\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c\right.}{8a^2(-1+ax)}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x,x]
```

output
```
(2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(1 + 2*a*x) + Sqrt[c]*(
-1 + a*x)*Log[1 - a*x] + Sqrt[c]*(1 - a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a
^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(8*a^2*(-1 + a
*x))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6733, 575, 579, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6733}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^3}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{575}$$

$$-c\left(\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{4ac}-\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{579}$$

$$-c\left(\frac{-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{4ac}-\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{573}$$

$$-c\left(\frac{\frac{c\int\frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{4ac}-\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{219}$$

$$-c\left(\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{4ac}-\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}\right)$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x,x]
```

output
```
-(c*(-1/2*(Sqrt[1 - 1/(a^2*x^2)]*x^2)/Sqrt[c - c/(a*x)] + (-((c*Sqrt[1 - 1
/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(
a^2*x^2)])/Sqrt[c - c/(a*x)]])/a)/(4*a*c)))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 575
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(e*x)^(m + 1)*(c + d*x)^n*((a + b*x^2)^p/(e*(m + 1))), x]
 + Simp[b*(n/(d*e*(m + 1)))   Int[(e*x)^(m + 1)*(c + d*x)^(n + 1)*(a + b*x^
2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[b*c^2 + a*d^2, 0] &&
 EqQ[n + p, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m + p] && LeQ[m +
p + 2, 0])
```

rule 579
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 1)*((a + b*x^2)^(p +
1)/(b*c*e*(m + 1))), x] - Simp[d*((n - m - 2)/(c*e*(m + 1)))   Int[(e*x)^(m
 + 1)*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0] && LtQ[m, -1] && (IntegerQ[2*p] |
| IntegerQ[m])
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\left(4a^{\frac{3}{2}}x\sqrt{x(ax+1)}+2\sqrt{x(ax+1)}\,\sqrt{a}-\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{8\sqrt{\frac{ax-1}{ax+1}}\, a^{\frac{3}{2}}\sqrt{x(ax+1)}}$	104
risch	$\frac{(2ax+1)x\sqrt{\frac{c(ax-1)}{ax}}}{4a\sqrt{\frac{ax-1}{ax+1}}}-\frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{8a\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	140

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/8/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(4*a^(3/2)*x*(x*(a*x+1
))^(1/2)+2*(x*(a*x+1))^(1/2)*a^(1/2)-ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2
*a*x+1)/a^(1/2)))/a^(3/2)/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.56

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4(2a^3x^3+3a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4(2a^3x^3+3a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{}}{16(a^3x-a^2)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)*x,x, algorithm="fricas
")
```

output
```
[1/16*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c
)/(a*x - 1)) + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x)))/(a^3*x - a^2), 1/8*((a*x - 1)*sqrt(-c)*arctan(2*(a
^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(
2*a^2*c*x^2 - a*c*x - c)) + 2*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = \int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)*x,x)
```

output
```
Integral(x*sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)*x,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)*x,x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{x\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$=\frac{\sqrt{c}\left(2\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax+\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}-\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)\right)}{4a^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)*x,x)
```

output
```
(sqrt(c)*(2*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x + sqrt(x)*sqrt(a)*sqrt(a*x +
 1) - log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))))/(4*a**2)
```

3.505 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 78

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
c*(1-1/a^2/x^2)^(1/2)*x/(c-c/a/x)^(1/2)+c^(1/2)*arctanh(c^(1/2)*(1-1/a^2/x
^2)^(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(1+ax+\sqrt{1+\frac{1}{ax}}\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

output
```
(Sqrt[c - c/(a*x)]*(1 + a*x + Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]]
))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6731, 575, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x^2}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{575}$$

$$-c\left(\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2ac}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{573}$$

$$-c\left(-\frac{\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{219}$$

$$-c\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

output
```
-(c*(-((Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)]) - ArcTanh[(Sqrt[c]*Sqr
t[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]]/(a*Sqrt[c])))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 575
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(e*x)^(m + 1)*(c + d*x)^n*((a + b*x^2)^p/(e*(m + 1))), x]
 + Simp[b*(n/(d*e*(m + 1)))   Int[(e*x)^(m + 1)*(c + d*x)^(n + 1)*(a + b*x^
2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[b*c^2 + a*d^2, 0] &&
 EqQ[n + p, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m + p] && LeQ[m +
p + 2, 0])
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\left(2\sqrt{x(ax+1)}\,\sqrt{a}+\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{x(ax+1)}\,\sqrt{a}}$	87
risch	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	127

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*(x*(a*x+1))^(1/2)*a
^(1/2)+ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(x*(a*x+1))^
(1/2)/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(66) = 132.

Time = 0.11 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.78

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)}, -\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(a^2x-a)}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)
/(a*x)))/(a^2*x - a), -1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sq
rt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*
c*x - c)) - 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x)))/(a^2*x - a)]
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.40

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)\right)}{a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x)
```

output
```
(sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x + 1) + log(sqrt(a*x + 1) + sqrt(x)*sqrt
(a))))/a
```

3.506 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 76

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=-\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

output
```
-2*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)+2*c^(1/2)*arctanh(c^(1/2)*(1-1/a^
2/x^2)^(1/2)/(c-c/a/x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.74

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$$
$$=\frac{-2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x+\sqrt{c}(1-ax)\log(1-ax)+\sqrt{c}(-1+ax)\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+\right.}{-1+ax}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x,x]
```

output
```
(-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x + Sqrt[c]*(1 - a*x)*Log[1
- a*x] + Sqrt[c]*(-1 + a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c
 - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(-1 + a*x)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6733, 576, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6733}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{576}$$

$$-c\left(\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{573}$$

$$-c\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-2\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{219}$$

$$-c\left(\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{\sqrt{c}}\right)$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x,x]
```

output
```
-(c*((2*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] - (2*ArcTanh[(Sqrt[c]*Sqr
t[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/Sqrt[c]))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 576
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-(e*x)^(m + 1))*(c + d*x)^n*((a + b*x^2)^p/(e*(n - m - 1
))), x] - Simp[b*c*(n/(d^2*(n - m - 1)))   Int[(e*x)^m*(c + d*x)^(n + 1)*(a
 + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c^2 + a*
d^2, 0] && EqQ[n + p, 0] && GtQ[p, 0] && NeQ[m - n + 1, 0] &&  !IGtQ[m, 0]
&&  !(IntegerQ[m + p] && LtQ[m + p + 2, 0]) && RationalQ[m]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-2\sqrt{x(ax+1)}\,\sqrt{a}\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{x(ax+1)}\,\sqrt{a}}$	88
risch	$-\frac{2\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{a\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	127

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*(ln(1/2*(2*(x*(a*x+1))^(1/
2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x-2*(x*(a*x+1))^(1/2)*a^(1/2))/(x*(a*x+1))^
(1/2)/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(64) = 128.

Time = 0.11 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.62

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(ax-1)}, -\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{ax-1}\right]$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas
")
```

output
```
[1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) - 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) +
2*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\frac{2\sqrt{c}\left(-\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)ax-ax\right)}{ax}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x)
```

output
```
(2*sqrt(c)*( - sqrt(x)*sqrt(a)*sqrt(a*x + 1) + log(sqrt(a*x + 1) + sqrt(x)
*sqrt(a))*a*x - a*x))/(a*x)
```

3.507 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}$$

output
```
-2/3*a*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(1+ax)}{-3+3ax}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^2,x]
```

output
```
(-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 + a*x))/(-3 + 3*a*x)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6733, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}}{x^2}\,dx$$

$$\Big\downarrow \text{6733}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{458}$$

$$-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^2,x]
```

output
```
(-2*a*c^2*(1 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2))
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

method	result	size
orering	$-\frac{2(ax+1)\sqrt{c-\frac{c}{ax}}}{3x\sqrt{\frac{ax-1}{ax+1}}}$	39
gosper	$-\frac{2(ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{3x\sqrt{\frac{ax-1}{ax+1}}}$	41
default	$-\frac{2(ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{3x\sqrt{\frac{ax-1}{ax+1}}}$	41
risch	$-\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(a^2x^2+2ax+1)}{3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x}$	56

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3/x*(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2\left(a^2x^2+2\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fric
as")
```

output
```
-2/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
))/(a*x^2 - x)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 13.67 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2\sqrt{c-\frac{c}{a\,x}}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,x\,\left(a\,x-1\right)}$$

input
```
int((c - c/(a*x))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
-(2*(c - c/(a*x))^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2))/(3*x*(a*x
 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax+\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+a^2x^2\right)}{3a\,x^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x + sqrt(x)*sqrt(a)*sqrt(a*
x + 1) + a**2*x**2))/(3*a*x**2)
```

3.508 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=-\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}$$

output
```
-2/15*a^2*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)+2/5*a^2*c*(1-1/a^2/x^2)^
(3/2)/(c-c/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\left(-3-ax+2a^2x^2\right)}{15x(-1+ax)}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^3,x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-3 - a*x + 2*a^2*x^2))/(15*x
*(-1 + a*x))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6733, 572, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6733}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}x}d\frac{1}{x}$$

$$\Big\downarrow \text{572}$$

$$-c\left(\frac{1}{5}a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{2a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{458}$$

$$-c\left(\frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}\right)$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^3,x]
```

output
```
-(c*((2*a^2*c*(1 - 1/(a^2*x^2))^(3/2))/(15*(c - c/(a*x))^(3/2)) - (2*a^2*(
1 - 1/(a^2*x^2))^(3/2))/(5*Sqrt[c - c/(a*x)])))
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 572
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 2))), x] + Simp[c*(n/(d
*(n + 2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && NeQ[n + 2*p + 2, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.58

method	result	size
orering	$\frac{2(2ax-3)(ax+1)\sqrt{c-\frac{c}{ax}}}{15x^2\sqrt{\frac{ax-1}{ax+1}}}$	45
gosper	$\frac{2(ax+1)(2ax-3)\sqrt{\frac{c(ax-1)}{ax}}}{15x^2\sqrt{\frac{ax-1}{ax+1}}}$	47
default	$\frac{2(ax+1)(2ax-3)\sqrt{\frac{c(ax-1)}{ax}}}{15x^2\sqrt{\frac{ax-1}{ax+1}}}$	47
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(2a^3x^3+a^2x^2-4ax-3)}{15\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x^2}$	64

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/15*(2*a*x-3)/x^2*(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\,(2\,a^3x^3+a^2x^2-4\,ax-3)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{15\,(ax^3-x^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="fric
as")
```

output
```
2/15*(2*a^3*x^3 + a^2*x^2 - 4*a*x - 3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x))/(a*x^3 - x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.69

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)^2\,(2\,a\,x-3)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,x^2\,(a\,x-1)}$$

input
```
int((c - c/(a*x))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(2*(c - c/(a*x))^(1/2)*(a*x + 1)^2*(2*a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2)
)/(15*x^2*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\sqrt{c}\left(2\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2-\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax-3\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}-2a^3x^3\right)}{15a\,x^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x)
```

output
```
(2*sqrt(c)*(2*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**2*x**2 - sqrt(x)*sqrt(a)*sq
rt(a*x + 1)*a*x - 3*sqrt(x)*sqrt(a)*sqrt(a*x + 1) - 2*a**3*x**3))/(15*a*x*
*3)
```

3.509 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{22a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{12a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}} -\frac{2}{7}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}$$

output
```
-22/105*a^3*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)+12/35*a^3*c*(1-1/a^2/x
^2)^(3/2)/(c-c/a/x)^(1/2)-2/7*a^3*(1-1/a^2/x^2)^(3/2)*(c-c/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(15+3ax-4a^2x^2+8a^3x^3)}{105x^2(-1+ax)}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^4,x]
```

output
```
(-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(15 + 3*a*x - 4*a^2*x^2 + 8*
a^3*x^3))/(105*x^2*(-1 + a*x))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6733, 581, 27, 672, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{ 6733}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 581}$$

$$-c\left(\frac{2a^2\int \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{6}{x}\right)}{2a\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{7c^2}+\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{7c}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-c\left(\frac{1}{7}a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{6}{x}\right)}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{7c}\right)$$

$$\Big\downarrow \text{ 672}$$

$$-c\left(\frac{1}{7}a\left(\frac{11}{5}a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{12a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}\right)+\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{7c}\right)$$

$$\Big\downarrow \text{ 458}$$

$$-c\left(\frac{1}{7}a\left(\frac{22a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{12a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}\right)+\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{7c}\right)$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^4,x]
```

output
```
-(c*((a*((22*a^2*c*(1 - 1/(a^2*x^2))^(3/2))/(15*(c - c/(a*x))^(3/2)) - (12
*a^2*(1 - 1/(a^2*x^2))^(3/2))/(5*Sqrt[c - c/(a*x)])))/7 + (2*a^3*(1 - 1/(a
^2*x^2))^(3/2)*Sqrt[c - c/(a*x)])/(7*c)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 672
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Simp[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2))   Int[(d + e*x
)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^
2 + a*e^2, 0] && NeQ[m + 2*p + 2, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.47

method	result	size
orering	$-\frac{2\left(8a^{2}x^{2}-12ax+15\right)\left(ax+1\right)\sqrt{c-\frac{c}{ax}}}{105x^{3}\sqrt{\frac{ax-1}{ax+1}}}$	53
gosper	$-\frac{2\left(ax+1\right)\left(8a^{2}x^{2}-12ax+15\right)\sqrt{\frac{c\left(ax-1\right)}{ax}}}{105x^{3}\sqrt{\frac{ax-1}{ax+1}}}$	55
default	$-\frac{2\left(ax+1\right)\left(8a^{2}x^{2}-12ax+15\right)\sqrt{\frac{c\left(ax-1\right)}{ax}}}{105x^{3}\sqrt{\frac{ax-1}{ax+1}}}$	55
risch	$-\frac{2\sqrt{\frac{c\left(ax-1\right)}{ax}}\,\left(8a^{4}x^{4}+4a^{3}x^{3}-a^{2}x^{2}+18ax+15\right)}{105\sqrt{\frac{ax-1}{ax+1}}\,\left(ax+1\right)x^{3}}$	73

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/105*(8*a^2*x^2-12*a*x+15)/x^3*(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{2\left(8\,a^4x^4+4\,a^3x^3-a^2x^2+18\,ax+15\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{105\left(ax^4-x^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="fric
as")
```

output
```
-2/105*(8*a^4*x^4 + 4*a^3*x^3 - a^2*x^2 + 18*a*x + 15)*sqrt((a*x - 1)/(a*x
 + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(8\,a^3\,x^3+12\,a^2\,x^2+11\,a\,x+29\right)\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{105\,x^3} - \frac{88\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{105\,x^3\,\left(a\,x-1\right)}$$

input
```
int((c - c/(a*x))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
- (2*((a*x - 1)/(a*x + 1))^(1/2)*(11*a*x + 12*a^2*x^2 + 8*a^3*x^3 + 29)*((
c*(a*x - 1))/(a*x))^(1/2))/(105*x^3) - (88*((a*x - 1)/(a*x + 1))^(1/2)*((c
*(a*x - 1))/(a*x))^(1/2))/(105*x^3*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2\sqrt{c}\,\left(-8\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^3x^3+4\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2-3\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax-15\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+8\right.}{105a\,x^4}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x)
```

output
```
(2*sqrt(c)*( - 8*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**3*x**3 + 4*sqrt(x)*sqrt(
a)*sqrt(a*x + 1)*a**2*x**2 - 3*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x - 15*sqrt
(x)*sqrt(a)*sqrt(a*x + 1) + 8*a**4*x**4))/(105*a*x**4)
```

3.510 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 152

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\frac{26a^4c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{46a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}} - \frac{10}{21}a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}+\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{9c}$$

output
```
-26/315*a^4*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)+46/105*a^4*c*(1-1/a^2/
x^2)^(3/2)/(c-c/a/x)^(1/2)-10/21*a^4*(1-1/a^2/x^2)^(3/2)*(c-c/a/x)^(1/2)+2
/9*a^4*(1-1/a^2/x^2)^(3/2)*(c-c/a/x)^(3/2)/c
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.49

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\left(-35-5ax+6a^2x^2-8a^3x^3+16a^4x^4\right)}{315x^3(-1+ax)}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^5,x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-35 - 5*a*x + 6*a^2*x^2 - 8*
a^3*x^3 + 16*a^4*x^4))/(315*x^3*(-1 + a*x))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6733, 581, 27, 2170, 27, 672, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6733}$$

$$-c\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{581}$$

$$-c\left(-\frac{2a^3\int \frac{3\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{c^3}{ax}-\frac{5c^3}{a^2x^2}+c^3\right)}{2\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{9c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{9c^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(-\frac{a^3\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{c^3}{ax}-\frac{5c^3}{a^2x^2}+c^3\right)}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{3c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{9c^2}\right)$$

$$\Big\downarrow \text{2170}$$

$$-c\left(-\frac{a^3\left(-\frac{2a^4\int -\frac{c^5\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{23}{x}\right)}{2a^5\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{7c^2}-\frac{10}{7}ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}\right)}{3c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{9c^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(-\frac{a^3\left(\frac{c^3\int \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{23}{x}\right)}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{7a}-\frac{10}{7}ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}\right)}{3c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{9c^2}\right)$$

$$\Big\downarrow \text{672}$$

$$-c\left(-\frac{a^3\left(\frac{c^3\left(\frac{46a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}-\frac{13}{5}a\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}\right)}{7a}-\frac{10}{7}ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}\right)}{3c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/}}{9c^2}\right.$$

$$\Big\downarrow \text{458}$$

$$-c\left(-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{9c^2}-\frac{a^3\left(\frac{c^3\left(\frac{46a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}-\frac{26a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}}\right)}{7a}-\frac{10}{7}ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-\frac{c}{ax}}\right)}{3c^3}\right)$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^5,x]
```

output
```
-(c*(-1/3*(a^3*((c^3*((-26*a^2*c*(1 - 1/(a^2*x^2))^(3/2))/(15*(c - c/(a*x)
)^(3/2)) + (46*a^2*(1 - 1/(a^2*x^2))^(3/2))/(5*Sqrt[c - c/(a*x)])))/(7*a)
- (10*a*c^2*(1 - 1/(a^2*x^2))^(3/2)*Sqrt[c - c/(a*x)])/7))/c^3 - (2*a^4*(1
 - 1/(a^2*x^2))^(3/2)*(c - c/(a*x))^(3/2))/(9*c^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 672
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Simp[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2))   Int[(d + e*x
)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^
2 + a*e^2, 0] && NeQ[m + 2*p + 2, 0]
```

rule 2170
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.40

method	result	size
orering	$\frac{2(16a^3x^3-24a^2x^2+30ax-35)(ax+1)\sqrt{c-\frac{c}{ax}}}{315x^4\sqrt{\frac{ax-1}{ax+1}}}$	61
gosper	$\frac{2(ax+1)(16a^3x^3-24a^2x^2+30ax-35)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4\sqrt{\frac{ax-1}{ax+1}}}$	63
default	$\frac{2(ax+1)(16a^3x^3-24a^2x^2+30ax-35)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4\sqrt{\frac{ax-1}{ax+1}}}$	63
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(16a^5x^5+8a^4x^4-2a^3x^3+a^2x^2-40ax-35)}{315\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x^4}$	80

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2/315*(16*a^3*x^3-24*a^2*x^2+30*a*x-35)/x^4*(a*x+1)/((a*x-1)/(a*x+1))^(1/2
)*(c-c/a/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.55

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2\,(16\,a^5x^5+8\,a^4x^4-2\,a^3x^3+a^2x^2-40\,ax-35)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{315\,(ax^5-x^4)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="fric
as")
```

output
```
2/315*(16*a^5*x^5 + 8*a^4*x^4 - 2*a^3*x^3 + a^2*x^2 - 40*a*x - 35)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^5 - x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^5\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^5*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^5\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^5*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 13.71 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.71

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}\,\left(16\,a^4\,x^4+24\,a^3\,x^3+22\,a^2\,x^2+23\,a\,x-17\right)}{315\,x^4} - \frac{104\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{315\,x^4\,\left(a\,x-1\right)}$$

input
```
int((c - c/(a*x))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2)*(23*a*x + 22*a^
2*x^2 + 24*a^3*x^3 + 16*a^4*x^4 - 17))/(315*x^4) - (104*((a*x - 1)/(a*x +
1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2))/(315*x^4*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.65

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$
$$= \frac{2\sqrt{c}\,\big(16\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^4x^4 - 8\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^3x^3 + 6\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2 - 5\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax -}{315a\,x^5}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x)
```

output
```
(2*sqrt(c)*(16*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**4*x**4 - 8*sqrt(x)*sqrt(a)
*sqrt(a*x + 1)*a**3*x**3 + 6*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**2*x**2 - 5*s
qrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x - 35*sqrt(x)*sqrt(a)*sqrt(a*x + 1) - 16*a
**5*x**5))/(315*a*x**5)
```

3.511 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 130

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\frac{75\sqrt{c-\frac{c}{ax}}x}{64a^3}+\frac{25\sqrt{c-\frac{c}{ax}}x^2}{32a^2}+\frac{5\sqrt{c-\frac{c}{ax}}x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4+\frac{75\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}$$

output
```
75/64*(c-c/a/x)^(1/2)*x/a^3+25/32*(c-c/a/x)^(1/2)*x^2/a^2+5/8*(c-c/a/x)^(1
/2)*x^3/a+1/4*(c-c/a/x)^(1/2)*x^4+75/64*c^(1/2)*arctanh((c-c/a/x)^(1/2)/c^
(1/2))/a^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.38

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(a^4x^4+15\,\text{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},1-\frac{1}{ax}\right)\right)}{4a^4}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

output
```
(Sqrt[c - c/(a*x)]*(a^4*x^4 + 15*Hypergeometric2F1[1/2, 4, 3/2, 1 - 1/(a*x
)]))/(4*a^4)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6717, 6683, 1070, 281, 948, 87, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}x^3dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}x^3(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}x^3}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^3}{\sqrt{c-\frac{c}{ax}}}dx}{a}$$

$$\Big\downarrow \text{948}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^5}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{87}$$

$$-\frac{c\left(\frac{15}{8}\int \frac{x^4}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{ax^4\sqrt{c-\frac{c}{ax}}}{4c}\right)}{a}$$

$$\Big\downarrow \; 52$$

$$-\frac{c\left(\frac{15}{8}\left(\frac{5\int \frac{x^3}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{6a}-\frac{x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)-\frac{ax^4\sqrt{c-\frac{c}{ax}}}{4c}\right)}{a}$$

$$\Big\downarrow \; 52$$

$$-\frac{c\left(\frac{15}{8}\left(\frac{5\left(\frac{3\int \frac{x^2}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{4a}-\frac{x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a}-\frac{x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)-\frac{ax^4\sqrt{c-\frac{c}{ax}}}{4c}\right)}{a}$$

$$\Big\downarrow \; 52$$

$$-\frac{c\left(\frac{15}{8}\left(\frac{5\left(\frac{3\left(\frac{\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a}-\frac{x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)-\frac{ax^4\sqrt{c-\frac{c}{ax}}}{4c}\right)}{a}$$

$$\Big\downarrow \; 73$$

$$-\frac{c\left(\frac{15}{8}\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a}-\frac{x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)-\frac{ax^4\sqrt{c-\frac{c}{ax}}}{4c}\right)}{a}$$

$$\Big\downarrow \; 221$$

$$-\frac{c\left(\frac{15}{8}\left(\frac{5\left(\frac{3\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a}-\frac{x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)-\frac{ax^4\sqrt{c-\frac{c}{ax}}}{4c}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

output
```
-((c*(-1/4*(a*Sqrt[c - c/(a*x)]*x^4)/c + (15*(-1/3*(Sqrt[c - c/(a*x)]*x^3)
/c + (5*(-1/2*(Sqrt[c - c/(a*x)]*x^2)/c + (3*(-((Sqrt[c - c/(a*x)]*x)/c) -
 ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*Sqrt[c])))/(4*a)))/(6*a)))/8))/a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

method	result
risch	$\frac{(16a^3x^3+40a^2x^2+50ax+75)x\sqrt{\frac{c(ax-1)}{ax}}}{64a^3}+\frac{75\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{128a^3\sqrt{a^2c}\,(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(32x(a\,x^2-x)^{\frac{3}{2}}a^{\frac{7}{2}}+112(a\,x^2-x)^{\frac{3}{2}}a^{\frac{5}{2}}+212\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x+256\sqrt{x(ax-1)}\,a^{\frac{3}{2}}-106\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}+128a\ln\left(\frac{2\sqrt{x(ax-1}}{2}\right.\right.}{128\sqrt{x(ax-1)}\,a^{\frac{9}{2}}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/64*(16*a^3*x^3+40*a^2*x^2+50*a*x+75)/a^3*x*(c*(a*x-1)/a/x)^(1/2)+75/128/
a^3*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(
1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.45

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$
$$=\left[\frac{2\,(16\,a^4x^4+40\,a^3x^3+50\,a^2x^2+75\,ax)\sqrt{\frac{acx-c}{ax}}+75\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{128\,a^4},\ \frac{(16\,a^4x^4\,-}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)*x^3,x, algorithm="fricas")
```

output
```
[1/128*(2*(16*a^4*x^4 + 40*a^3*x^3 + 50*a^2*x^2 + 75*a*x)*sqrt((a*c*x - c)
/(a*x)) + 75*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))
+ c))/a^4, 1/64*((16*a^4*x^4 + 40*a^3*x^3 + 50*a^2*x^2 + 75*a*x)*sqrt((a*c
*x - c)/(a*x)) - 75*sqrt(-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(
a*c*x - c)))/a^4]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int \frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)*x**3,x)
```

output
```
Integral(x**3*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}x^3}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)*x^3,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))*x^3/(a*x - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx =\frac{1}{64}\sqrt{a^2cx^2-acx}\left(2\left(4\,x\left(\frac{2\,x|a|}{a^2\text{sgn}\left(x\right)}+\frac{5\,|a|}{a^3\text{sgn}\left(x\right)}\right)+\frac{25\,|a|}{a^4\text{sgn}\left(x\right)}\right)x+\frac{75\,|a|}{a^5\text{sgn}\left(x\right)}\right) +\frac{75\,\sqrt{c}\log\left(|a||c|\right)\text{sgn}(x)}{128\,a^4}-\frac{75\,\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2}cx-\sqrt{a^2cx^2-acx}\right)\sqrt{c}|a|+ac\right|\right)}{128\,a^4\text{sgn}\left(x\right)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)*x^3,x, algorithm="giac")
```

output
```
1/64*sqrt(a^2*c*x^2 - a*c*x)*(2*(4*x*(2*x*abs(a)/(a^2*sgn(x)) + 5*abs(a)/(
a^3*sgn(x))) + 25*abs(a)/(a^4*sgn(x)))*x + 75*abs(a)/(a^5*sgn(x))) + 75/12
8*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a^4 - 75/128*sqrt(c)*log(abs(-2*(sqrt(
a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a^4*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int \frac{x^3\,\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

input
```
int((x^3*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x^3*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.65

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$
$$=\frac{\sqrt{c}\,(16\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^3x^3+40\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^2x^2+50\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax+75\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}+7}{64a^4}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)*x^3,x)
```

output
```
(sqrt(c)*(16*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**3*x**3 + 40*sqrt(x)*sqrt(a)*
sqrt(a*x - 1)*a**2*x**2 + 50*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a*x + 75*sqrt(x
)*sqrt(a)*sqrt(a*x - 1) + 75*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))))/(64*a*
*4)
```

3.512 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 105

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{11\sqrt{c-\frac{c}{ax}}x}{8a^2}+\frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3+\frac{11\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}$$

output
```
11/8*(c-c/a/x)^(1/2)*x/a^2+11/12*(c-c/a/x)^(1/2)*x^2/a+1/3*(c-c/a/x)^(1/2)
*x^3+11/8*c^(1/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.48

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(a^3x^3+11\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\frac{1}{ax}\right)\right)}{3a^3}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

output
```
(Sqrt[c - c/(a*x)]*(a^3*x^3 + 11*Hypergeometric2F1[1/2, 3, 3/2, 1 - 1/(a*x
)]))/(3*a^3)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6717, 6683, 1070, 281, 948, 87, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{c-\frac{c}{ax}} e^{2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)} \sqrt{c-\frac{c}{ax}} x^2 dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}} x^2 (ax+1)}{1-ax} dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(a+\frac{1}{x}\right) \sqrt{c-\frac{c}{ax}} x^2}{\frac{1}{x}-a} dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c \int \frac{\left(a+\frac{1}{x}\right) x^2}{\sqrt{c-\frac{c}{ax}}} dx}{a}$$

$$\Big\downarrow \text{948}$$

$$-\frac{c \int \frac{\left(a+\frac{1}{x}\right) x^4}{\sqrt{c-\frac{c}{ax}}} d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{87}$$

$$-\frac{c\left(\frac{11}{6}\int \frac{x^3}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{ax^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{a}$$

$$\Big\downarrow \; 52$$

$$-\frac{c\left(\frac{11}{6}\left(\frac{3\int \frac{x^2}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{4a}-\frac{x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)-\frac{ax^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{a}$$

$$\Big\downarrow \; 52$$

$$-\frac{c\left(\frac{11}{6}\left(\frac{3\left(\frac{\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)-\frac{ax^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{a}$$

$$\Big\downarrow \; 73$$

$$-\frac{c\left(\frac{11}{6}\left(\frac{3\left(-\frac{\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)-\frac{ax^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{a}$$

$$\Big\downarrow \; 221$$

$$-\frac{c\left(\frac{11}{6}\left(\frac{3\left(-\frac{\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)-\frac{ax^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

output
```
-((c*(-1/3*(a*Sqrt[c - c/(a*x)]*x^3)/c + (11*(-1/2*(Sqrt[c - c/(a*x)]*x^2)
/c + (3*(-((Sqrt[c - c/(a*x)]*x)/c) - ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(
a*Sqrt[c])))/(4*a)))/6))/a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.13

method	result
risch	$\frac{(8a^2x^2+22ax+33)x\sqrt{\frac{c(ax-1)}{ax}}}{24a^2}+\frac{11\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{16a^2\sqrt{a^2c}\,(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16(a\,x^2-x)^{\frac{3}{2}}a^{\frac{5}{2}}+60\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x+96\sqrt{x(ax-1)}\,a^{\frac{3}{2}}-30\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}+48a\ln\left(\frac{2\sqrt{x(ax-1)}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-15\ln\left(2\sqrt{a}\cdots\right)\right)}{48\sqrt{x(ax-1)}\,a^{\frac{7}{2}}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*a^2*x^2+22*a*x+33)/a^2*x*(c*(a*x-1)/a/x)^(1/2)+11/16/a^2*ln((-1/2*
a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)/(a*x-1)*
(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.64

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$= \left[\frac{2\,(8\,a^3x^3+22\,a^2x^2+33\,ax)\sqrt{\frac{acx-c}{ax}}+33\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{48\,a^3},\ \frac{(8\,a^3x^3+22\,a^2x^2+3}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)*x^2,x, algorithm="fricas")
```

output
```
[1/48*(2*(8*a^3*x^3 + 22*a^2*x^2 + 33*a*x)*sqrt((a*c*x - c)/(a*x)) + 33*sq
rt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^3, 1/24
*((8*a^3*x^3 + 22*a^2*x^2 + 33*a*x)*sqrt((a*c*x - c)/(a*x)) - 33*sqrt(-c)*
arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)))/a^3]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)*x**2,x)
```

output
```
Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}x^2}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)*x^2,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))*x^2/(a*x - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.21

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\frac{1}{24}\sqrt{a^2cx^2-acx}\left(2x\left(\frac{4x|a|}{a^2\mathrm{sgn}(x)}+\frac{11|a|}{a^3\mathrm{sgn}(x)}\right)+\frac{33|a|}{a^4\mathrm{sgn}(x)}\right)$$
$$+\frac{11\sqrt{c}\log(|a||c|)\,\mathrm{sgn}(x)}{16a^3}-\frac{11\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}|a|+ac\right|\right)}{16a^3\mathrm{sgn}(x)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)*x^2,x, algorithm="giac")
```

output
```
1/24*sqrt(a^2*c*x^2 - a*c*x)*(2*x*(4*x*abs(a)/(a^2*sgn(x)) + 11*abs(a)/(a^
3*sgn(x))) + 33*abs(a)/(a^4*sgn(x))) + 11/16*sqrt(c)*log(abs(a)*abs(c))*sg
n(x)/a^3 - 11/16*sqrt(c)*log(abs(-2*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*
x))*sqrt(c)*abs(a) + a*c))/(a^3*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{x^2\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x^2*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x^2*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \frac{\sqrt{c}\,\left(8\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^2x^2+22\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax+33\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}+33\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}\right)\right)}{24a^3}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)*x^2,x)
```

output
```
(sqrt(c)*(8*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**2*x**2 + 22*sqrt(x)*sqrt(a)*s
qrt(a*x - 1)*a*x + 33*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 33*log(sqrt(a*x - 1)
 + sqrt(x)*sqrt(a))))/(24*a**3)
```

3.513 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 80

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{7\sqrt{c-\frac{c}{ax}}x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2+\frac{7\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}$$

output
```
7/4*(c-c/a/x)^(1/2)*x/a+1/2*(c-c/a/x)^(1/2)*x^2+7/4*c^(1/2)*arctanh((c-c/a
/x)^(1/2)/c^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(a\sqrt{1-\frac{1}{ax}}x(7+2ax)+7\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\right)\right)}{4a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

output
```
(Sqrt[c - c/(a*x)]*(a*Sqrt[1 - 1/(a*x)]*x*(7 + 2*a*x) + 7*ArcTanh[Sqrt[1 -
 1/(a*x)]]))/(4*a^2*Sqrt[1 - 1/(a*x)])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {6717, 6683, 1070, 281, 948, 87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}xdx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}x(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{ 1070}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}x}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{c\int \frac{\left(a+\frac{1}{x}\right)x}{\sqrt{c-\frac{c}{ax}}}dx}{a}$$

$$\Big\downarrow \text{ 948}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^3}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{c\left(\frac{7}{4}\int \frac{x^2}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{ax^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{a}$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{c\left(\frac{7}{4}\left(\frac{\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)-\frac{ax^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c\left(\frac{7}{4}\left(-\frac{\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)-\frac{ax^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c\left(\frac{7}{4}\left(-\frac{\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}-\frac{x\sqrt{c-\frac{c}{ax}}}{c}\right)-\frac{ax^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

output
```
-((c*(-1/2*(a*Sqrt[c - c/(a*x)]*x^2)/c + (7*(-((Sqrt[c - c/(a*x)]*x)/c) -
ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*Sqrt[c])))/4))/a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.39

method	result
risch	$\frac{(2ax+7)x\sqrt{\frac{c(ax-1)}{ax}}}{4a}+\frac{7\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{8a\sqrt{a^2c}\,(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x+16\sqrt{x(ax-1)}\,a^{\frac{3}{2}}-2\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}+8a\ln\left(\frac{2\sqrt{x(ax-1)}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\right)}{8\sqrt{x(ax-1)}\,a^{\frac{5}{2}}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*a*x+7)/a*x*(c*(a*x-1)/a/x)^(1/2)+7/8/a*ln((-1/2*a*c+a^2*c*x)/(a^2*c
)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/
2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.95

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$=\left[\frac{2\left(2\,a^2x^2+7\,ax\right)\sqrt{\frac{acx-c}{ax}}+7\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{8\,a^2},\ \frac{\left(2\,a^2x^2+7\,ax\right)\sqrt{\frac{acx-c}{ax}}-7\,\sqrt{-c}\,a}{4\,a^2}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)*x,x, algorithm="fricas")
```

output
```
[1/8*(2*(2*a^2*x^2 + 7*a*x)*sqrt((a*c*x - c)/(a*x)) + 7*sqrt(c)*log(-2*a*c
*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^2, 1/4*((2*a^2*x^2 + 7*
a*x)*sqrt((a*c*x - c)/(a*x)) - 7*sqrt(-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x
- c)/(a*x))/(a*c*x - c)))/a^2]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)*x,x)
```

output
```
Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}x}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)*x,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))*x/(a*x - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.40

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx&=\frac{1}{4}\sqrt{a^2cx^2-acx}\left(\frac{2x|a|}{a^2\mathrm{sgn}(x)}+\frac{7|a|}{a^3\mathrm{sgn}(x)}\right)\\&\quad+\frac{7\sqrt{c}\log(|a||c|)\,\mathrm{sgn}(x)}{8a^2}\\&\quad-\frac{7\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}|a|+ac\right|\right)}{8a^2\mathrm{sgn}(x)}\end{aligned}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)*x,x, algorithm="giac")
```

output
```
1/4*sqrt(a^2*c*x^2 - a*c*x)*(2*x*abs(a)/(a^2*sgn(x)) + 7*abs(a)/(a^3*sgn(x
))) + 7/8*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a^2 - 7/8*sqrt(c)*log(abs(-2*(
sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a^2*sgn(x
))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int \frac{x\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.61

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{\sqrt{c}\,\left(2\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax+7\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}+7\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}\right)\right)}{4a^2}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)*x,x)
```

output
```
(sqrt(c)*(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a*x + 7*sqrt(x)*sqrt(a)*sqrt(a*x
 - 1) + 7*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))))/(4*a**2)
```

3.514 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 50

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x+\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

output
```
(c-c/a/x)^(1/2)*x+3*c^(1/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x+\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6683, 1035, 281, 899, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}dx}{a}$$

$$\Big\downarrow \text{899}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{87}$$

$$-\frac{c\left(\frac{3}{2}\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{ax\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c\left(-\frac{3a\int\frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{ax\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c\left(-\frac{3\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{ax\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
-((c*(-((a*Sqrt[c - c/(a*x)]*x)/c) - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
)/Sqrt[c]))/a)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(42) = 84$.

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.96

method	result	size
risch	$x\sqrt{\frac{c(ax-1)}{ax}}+\frac{3\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{2\sqrt{a^{2}c}\,(ax-1)}$	98
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{x(ax-1)}\,\sqrt{a}-2\sqrt{a\,x^{2}-x}\,\sqrt{a}+\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)+2\ln\left(\frac{2\sqrt{x(ax-1)}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\right)}{2\sqrt{x(ax-1)}\,\sqrt{a}}$	118

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(c*(a*x-1)/a/x)^(1/2)+3/2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2
-a*c*x)^(1/2))/(a^2*c)^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.66

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+3\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a}, \frac{ax\sqrt{\frac{acx-c}{ax}}-3\,\sqrt{-c}\arctan\left(\frac{a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)}{a}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 3*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)
*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(
-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)))/a]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))/(a*x - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(42) = 84$.

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{3\sqrt{c}\log\left(|a||c|\right)\mathrm{sgn}(x)}{2a}-\frac{3\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}|a|+ac\right|\right)}{2a\mathrm{sgn}\left(x\right)}+\frac{\sqrt{a^2cx^2-acx}|a|}{a^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
3/2*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a - 3/2*sqrt(c)*log(abs(-2*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a*sgn(x)) + sqrt(
a^2*c*x^2 - a*c*x)*abs(a)/(a^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}+3\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}\right)\right)}{a}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x)
```

output
```
(sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 3*log(sqrt(a*x - 1) + sqrt(x)*sq
rt(a))))/a
```

3.515 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 47

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = 2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

output
```
2*(c-c/a/x)^(1/2)+2*c^(1/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = 2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

output
```
2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6717, 6683, 1070, 281, 948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x}dx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(ax+1)}{x(1-ax)}dx$$

$$\Big\downarrow \text{ 1070}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}{\left(\frac{1}{x}-a\right)x}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x}dx}{a}$$

$$\Big\downarrow \text{ 948}$$

$$-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 90}$$

$$-\frac{c\left(a\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{2a\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{c\left(-\frac{2a^2\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{2a\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c\left(-\frac{2a\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{2a\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

output
```
-((c*((-2*a*Sqrt[c - c/(a*x)])/c - (2*a*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
)/Sqrt[c]))/a)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(39) = 78$.

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.09

method	result	size
risch	$2\sqrt{\frac{c(ax-1)}{ax}}+\frac{a\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{\sqrt{a^{2}c}\,(ax-1)}$	98
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-2\sqrt{x(ax-1)}\,a^{\frac{3}{2}}x^{2}+2\left(a\,x^{2}-x\right)^{\frac{3}{2}}\sqrt{a}-\ln\left(\frac{2\sqrt{x(ax-1)}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\,x^{2}\right)}{x\sqrt{x(ax-1)}\,\sqrt{a}}$	99

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(c*(a*x-1)/a/x)^(1/2)+a*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a
*c*x)^(1/2))/(a^2*c)^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.55

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \left[\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,\sqrt{\frac{acx-c}{ax}},\ -2\,\sqrt{-c}\arctan\left(\frac{a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)+2\,\sqrt{\frac{acx-c}{ax}}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas")
```

output
```
[sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*sqr
t((a*c*x - c)/(a*x)), -2*sqrt(-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*
x))/(a*c*x - c)) + 2*sqrt((a*c*x - c)/(a*x))]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(34) = 68$.

Time = 4.01 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.02

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \begin{cases} -\frac{2a\left(\frac{c^2\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{c\sqrt{c-\frac{c}{ax}}}{a}\right)}{c} & \text{for}\ \frac{c}{a}\neq 0 \\ -\frac{3a\sqrt{c}\left(\frac{\log\left(\frac{2}{x}\right)}{a}-\frac{\log\left(2a-\frac{2}{x}\right)}{a}\right)}{2}+\frac{\sqrt{c}\log\left(\frac{a}{x}-\frac{1}{x^2}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x,x)
```

output
```
Piecewise((-2*a*(c**2*atan(sqrt(c - c/(a*x))/sqrt(-c))/(a*sqrt(-c)) - c*sq
rt(c - c/(a*x))/a)/c, Ne(c/a, 0)), (-3*a*sqrt(c)*(log(2/x)/a - log(2*a - 2
/x)/a)/2 + sqrt(c)*log(a/x - 1/x**2)/2, True))
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{x\,(a\,x-1)}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\frac{2\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}+\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}\right)ax+ax\right)}{ax}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x)
```

output
```
(2*sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x - 1) + log(sqrt(a*x - 1) + sqrt(x)*sq
rt(a))*a*x + a*x))/(a*x)
```

3.516 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 42

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = 4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}$$

output
```
4*a*(c-c/a/x)^(1/2)-2/3*a*(c-c/a/x)^(3/2)/c
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2\sqrt{c-\frac{c}{ax}}(1+5ax)}{3x}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

output
```
(2*Sqrt[c - c/(a*x)]*(1 + 5*a*x))/(3*x)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6717, 6683, 1070, 281, 946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(ax+1)}{x^2(1-ax)}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}{\left(\frac{1}{x}-a\right)x^2}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^2}dx}{a}$$

$$\Big\downarrow \text{946}$$

$$-\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{53}$$

$$-\frac{c\int \left(\frac{2a}{\sqrt{c-\frac{c}{ax}}}-\frac{a\sqrt{c-\frac{c}{ax}}}{c}\right)d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\left(\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c^2}-\frac{4a^2\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

output
```
-((c*((-4*a^2*Sqrt[c - c/(a*x)])/c + (2*a^2*(c - c/(a*x))^(3/2))/(3*c^2)))
/a)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

method	result
orering	$\frac{2(5ax+1)\sqrt{c-\frac{c}{ax}}}{3x}$
gosper	$\frac{2(5ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{3x}$
trager	$\frac{2(5ax+1)\sqrt{-\frac{-acx+c}{ax}}}{3x}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(5a^2x^2-4ax-1)}{3(ax-1)x}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(6\sqrt{x(ax-1)}\,a^{\frac{5}{2}}x^3+6\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x^3-12a^{\frac{3}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x-3\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^2x^3+3\ln\left(\frac{2\sqrt{x(ax-1)}\,\sqrt{a}+2ax-}{2\sqrt{a}}\right.\right.}{3x^2\sqrt{x(ax-1)}\,\sqrt{a}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(5*a*x+1)/x*(c-c/a/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2\,(5\,ax+1)\sqrt{\frac{acx-c}{ax}}}{3\,x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
2/3*(5*a*x + 1)*sqrt((a*c*x - c)/(a*x))/x
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{x^2\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**2*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^2}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(36) = 72.

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.88

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2\left(3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^2a^2c+3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)ac^{\frac{3}{2}}|a|-a^2c^2\right)}{3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^3|a|\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^2*c + 3*(sqrt(a^2*c)*
x - sqrt(a^2*c*x^2 - a*c*x))*a*c^(3/2)*abs(a) - a^2*c^2)/((sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - a*c*x))^3*abs(a)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 13.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2\sqrt{c-\frac{c}{a\,x}}\,(5\,a\,x+1)}{3\,x}$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
```

output
```
(2*(c - c/(a*x))^(1/2)*(5*a*x + 1))/(3*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2\sqrt{c}\left(5\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax+\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}-3a^2x^2\right)}{3a\,x^2}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x)
```

output
```
(2*sqrt(c)*(5*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a*x + sqrt(x)*sqrt(a)*sqrt(a*x
 - 1) - 3*a**2*x**2))/(3*a*x**2)
```

3.517 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 69

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = 4a^2\sqrt{c-\frac{c}{ax}} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}$$

output
```
4*a^2*(c-c/a/x)^(1/2)-2*a^2*(c-c/a/x)^(3/2)/c+2/5*a^2*(c-c/a/x)^(5/2)/c^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.52

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\sqrt{c-\frac{c}{ax}}(1+3ax+6a^2x^2)}{5x^2}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

output
```
(2*Sqrt[c - c/(a*x)]*(1 + 3*a*x + 6*a^2*x^2))/(5*x^2)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6717, 6683, 1070, 281, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(ax+1)}{x^3(1-ax)}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}{\left(\frac{1}{x}-a\right)x^3}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^3}dx}{a}$$

$$\Big\downarrow \text{948}$$

$$-\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{86}$$

$$-\frac{c\int \left(\frac{\left(c-\frac{c}{ax}\right)^{3/2}a^2}{c^2}-\frac{3\sqrt{c-\frac{c}{ax}}a^2}{c}+\frac{2a^2}{\sqrt{c-\frac{c}{ax}}}\right)d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\left(-\frac{2a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^3}+\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{c^2}-\frac{4a^3\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

output
```
-((c*((-4*a^3*Sqrt[c - c/(a*x)])/c + (2*a^3*(c - c/(a*x))^(3/2))/c^2 - (2*
a^3*(c - c/(a*x))^(5/2))/(5*c^3)))/a)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.48

method	result
orering	$\frac{2(6a^2x^2+3ax+1)\sqrt{c-\frac{c}{ax}}}{5x^2}$
gosper	$\frac{2(6a^2x^2+3ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{5x^2}$
trager	$\frac{2(6a^2x^2+3ax+1)\sqrt{-\frac{-acx+c}{ax}}}{5x^2}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(6a^3x^3-3a^2x^2-2ax-1)}{5(ax-1)x^2}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-10\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}x^4-10a^{\frac{7}{2}}\sqrt{x(ax-1)}\,x^4+20a^{\frac{5}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x^2+5\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^3x^4-5\ln\left(\frac{2\sqrt{x(ax-1)}}{2\sqrt{}}\right.\right.}{5x^3\sqrt{x(ax-1)}\,\sqrt{a}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*(6*a^2*x^2+3*a*x+1)/x^2*(c-c/a/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.52

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\left(6\,a^2x^2+3\,ax+1\right)\sqrt{\frac{acx-c}{ax}}}{5\,x^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
2/5*(6*a^2*x^2 + 3*a*x + 1)*sqrt((a*c*x - c)/(a*x))/x^2
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{x^3\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**3*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^3}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(61) = 122$.

Time = 0.19 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.38

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\left(5\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^3 a^2c^{\frac{3}{2}}|a| + 5\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^2 a^3c^2 - 5\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-ac\right.}\right.}{5\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^5|a|\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
2/5*(5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*a^2*c^(3/2)*abs(a) + 5*
(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^3*c^2 - 5*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - a*c*x))*a^2*c^(5/2)*abs(a) + a^3*c^3)/((sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - a*c*x))^5*abs(a)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 13.65 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\sqrt{c-\frac{c}{a\,x}}\,\left(6\,a^2\,x^2+3\,a\,x+1\right)}{5\,x^2}$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

output
```
(2*(c - c/(a*x))^(1/2)*(3*a*x + 6*a^2*x^2 + 1))/(5*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\sqrt{c}\left(6\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^2x^2+3\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax+\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}-6a^3x^3\right)}{5a\,x^3}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x)
```

output
```
(2*sqrt(c)*(6*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**2*x**2 + 3*sqrt(x)*sqrt(a)*
sqrt(a*x - 1)*a*x + sqrt(x)*sqrt(a)*sqrt(a*x - 1) - 6*a**3*x**3))/(5*a*x**
3)
```

3.518 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 96

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = 4a^3\sqrt{c-\frac{c}{ax}}-\frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}$$

output
```
4*a^3*(c-c/a/x)^(1/2)-10/3*a^3*(c-c/a/x)^(3/2)/c+8/5*a^3*(c-c/a/x)^(5/2)/c
^2-2/7*a^3*(c-c/a/x)^(7/2)/c^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.46

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2\sqrt{c-\frac{c}{ax}}(15+39ax+52a^2x^2+104a^3x^3)}{105x^3}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

output
```
(2*Sqrt[c - c/(a*x)]*(15 + 39*a*x + 52*a^2*x^2 + 104*a^3*x^3))/(105*x^3)
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6717, 6683, 1070, 281, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(ax+1)}{x^4(1-ax)}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}{\left(\frac{1}{x}-a\right)x^4}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^4}dx}{a}$$

$$\Big\downarrow \text{948}$$

$$-\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{86}$$

$$-\frac{c\int \left(-\frac{\left(c-\frac{c}{ax}\right)^{5/2}a^3}{c^3}+\frac{4\left(c-\frac{c}{ax}\right)^{3/2}a^3}{c^2}-\frac{5\sqrt{c-\frac{c}{ax}}a^3}{c}+\frac{2a^3}{\sqrt{c-\frac{c}{ax}}}\right)d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\left(\frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^4}-\frac{8a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^3}+\frac{10a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c^2}-\frac{4a^4\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

input

```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

output

```
-((c*((-4*a^4*Sqrt[c - c/(a*x)])/c + (10*a^4*(c - c/(a*x))^(3/2))/(3*c^2)
- (8*a^4*(c - c/(a*x))^(5/2))/(5*c^3) + (2*a^4*(c - c/(a*x))^(7/2))/(7*c^4
)))/a)
```

Defintions of rubi rules used

rule 86

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 281

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.43

method	result
orering	$\frac{2(104a^3x^3+52a^2x^2+39ax+15)\sqrt{c-\frac{c}{ax}}}{105x^3}$
gosper	$\frac{2(104a^3x^3+52a^2x^2+39ax+15)\sqrt{\frac{c(ax-1)}{ax}}}{105x^3}$
trager	$\frac{2(104a^3x^3+52a^2x^2+39ax+15)\sqrt{-\frac{-acx+c}{ax}}}{105x^3}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(104a^4x^4-52a^3x^3-13a^2x^2-24ax-15)}{105(ax-1)x^3}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(210\sqrt{x(ax-1)}\,a^{\frac{9}{2}}x^5+210\sqrt{a\,x^2-x}\,a^{\frac{9}{2}}x^5-420(a\,x^2-x)^{\frac{3}{2}}a^{\frac{7}{2}}x^3-105\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^4x^5+105\ln\left(\frac{2\sqrt{x(ax}}{}\right.\right.}{105x^4\sqrt{x(ax-1)}\,\sqrt{a}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/105*(104*a^3*x^3+52*a^2*x^2+39*a*x+15)/x^3*(c-c/a/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.46

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2\left(104\,a^3x^3+52\,a^2x^2+39\,ax+15\right)\sqrt{\frac{acx-c}{ax}}}{105\,x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
2/105*(104*a^3*x^3 + 52*a^2*x^2 + 39*a*x + 15)*sqrt((a*c*x - c)/(a*x))/x^3
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{x^4\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**4*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^4}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(82) = 164$.

Time = 0.24 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.11

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$$
$$= \frac{2\left(140\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^4a^4c^2+105\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^3a^3c^{\frac{5}{2}}|a|-231\left(\sqrt{a^2c}x-\sqrt{a^2c\right.}\right.}{105\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^7|a|\mathrm{sgn}\left(\right.}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
2/105*(140*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a^4*c^2 + 105*(sqrt
(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*a^3*c^(5/2)*abs(a) - 231*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^4*c^3 + 105*(sqrt(a^2*c)*x - sqrt(a^2*
c*x^2 - a*c*x))*a^3*c^(7/2)*abs(a) - 15*a^4*c^4)/((sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - a*c*x))^7*abs(a)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{208\,a^3\,\sqrt{c-\frac{c}{a\,x}}}{105}+\frac{2\,\sqrt{c-\frac{c}{a\,x}}}{7\,x^3}+\frac{26\,a\,\sqrt{c-\frac{c}{a\,x}}}{35\,x^2}+\frac{104\,a^2\,\sqrt{c-\frac{c}{a\,x}}}{105\,x}$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
```

output
```
(208*a^3*(c - c/(a*x))^(1/2))/105 + (2*(c - c/(a*x))^(1/2))/(7*x^3) + (26*
a*(c - c/(a*x))^(1/2))/(35*x^2) + (104*a^2*(c - c/(a*x))^(1/2))/(105*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$$
$$= \frac{2\sqrt{c}\,\left(104\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^3x^3+52\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^2x^2+39\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax+15\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,-\right.}{105a\,x^4}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x)
```

output
```
(2*sqrt(c)*(104*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**3*x**3 + 52*sqrt(x)*sqrt(
a)*sqrt(a*x - 1)*a**2*x**2 + 39*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a*x + 15*sqr
t(x)*sqrt(a)*sqrt(a*x - 1) - 104*a**4*x**4))/(105*a*x**4)
```

3.519 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = 4a^4\sqrt{c-\frac{c}{ax}} - \frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}$$

output
```
4*a^4*(c-c/a/x)^(1/2)-14/3*a^4*(c-c/a/x)^(3/2)/c+18/5*a^4*(c-c/a/x)^(5/2)/
c^2-10/7*a^4*(c-c/a/x)^(7/2)/c^3+2/9*a^4*(c-c/a/x)^(9/2)/c^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2\sqrt{c-\frac{c}{ax}}(35+85ax+102a^2x^2+136a^3x^3+272a^4x^4)}{315x^4}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

output
```
(2*Sqrt[c - c/(a*x)]*(35 + 85*a*x + 102*a^2*x^2 + 136*a^3*x^3 + 272*a^4*x^
4))/(315*x^4)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6717, 6683, 1070, 281, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{2\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(ax+1)}{x^5(1-ax)}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}{\left(\frac{1}{x}-a\right)x^5}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^5}dx}{a}$$

$$\Big\downarrow \text{948}$$

$$-\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}x^3}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{86}$$

$$-\frac{c\int\left(\frac{\left(c-\frac{c}{ax}\right)^{7/2}a^4}{c^4}-\frac{5\left(c-\frac{c}{ax}\right)^{5/2}a^4}{c^3}+\frac{9\left(c-\frac{c}{ax}\right)^{3/2}a^4}{c^2}-\frac{7\sqrt{c-\frac{c}{ax}}a^4}{c}+\frac{2a^4}{\sqrt{c-\frac{c}{ax}}}\right)d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\left(-\frac{2a^5\left(c-\frac{c}{ax}\right)^{9/2}}{9c^5}+\frac{10a^5\left(c-\frac{c}{ax}\right)^{7/2}}{7c^4}-\frac{18a^5\left(c-\frac{c}{ax}\right)^{5/2}}{5c^3}+\frac{14a^5\left(c-\frac{c}{ax}\right)^{3/2}}{3c^2}-\frac{4a^5\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

output
```
-((c*((-4*a^5*Sqrt[c - c/(a*x)])/c + (14*a^5*(c - c/(a*x))^(3/2))/(3*c^2)
- (18*a^5*(c - c/(a*x))^(5/2))/(5*c^3) + (10*a^5*(c - c/(a*x))^(7/2))/(7*c
^4) - (2*a^5*(c - c/(a*x))^(9/2))/(9*c^5)))/a)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.40

method	result
orering	$\frac{2\left(272 a^{4} x^{4}+136 a^{3} x^{3}+102 a^{2} x^{2}+85 a x +35\right) \sqrt{c -\frac{c}{a x}}}{315 x^{4}}$
gosper	$\frac{2\left(272 a^{4} x^{4}+136 a^{3} x^{3}+102 a^{2} x^{2}+85 a x +35\right) \sqrt{\frac{c \left(a x -1\right)}{a x}}}{315 x^{4}}$
trager	$\frac{2\left(272 a^{4} x^{4}+136 a^{3} x^{3}+102 a^{2} x^{2}+85 a x +35\right) \sqrt{-\frac{-a c x +c}{a x}}}{315 x^{4}}$
risch	$\frac{2 \sqrt{\frac{c \left(a x -1\right)}{a x}}\, \left(272 a^{5} x^{5}-136 a^{4} x^{4}-34 a^{3} x^{3}-17 a^{2} x^{2}-50 a x -35\right)}{315 \left(a x -1\right) x^{4}}$
default	$-\frac{\sqrt{\frac{c \left(a x -1\right)}{a x}}\, \left(-630 \sqrt{a \,x^{2}-x}\, a^{\frac{11}{2}} x^{6}-630 a^{\frac{11}{2}} \sqrt{x \left(a x -1\right)}\, x^{6}+1260 \left(a \,x^{2}-x \right)^{\frac{3}{2}} a^{\frac{9}{2}} x^{4}+315 \ln\left(\frac{2 \sqrt{a \,x^{2}-x}\, \sqrt{a}+2 a x -1}{2 \sqrt{a}}\right) a^{5} x^{6}-315 \ln\left(\right.}{315 x^{5} \sqrt{x \left(a x -1\right)}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2/315*(272*a^4*x^4+136*a^3*x^3+102*a^2*x^2+85*a*x+35)/x^4*(c-c/a/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2\,(272\,a^4x^4+136\,a^3x^3+102\,a^2x^2+85\,ax+35)\sqrt{\frac{acx-c}{ax}}}{315\,x^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
2/315*(272*a^4*x^4 + 136*a^3*x^3 + 102*a^2*x^2 + 85*a*x + 35)*sqrt((a*c*x
- c)/(a*x))/x^4
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax+1)}{x^5\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**5*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^5}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(103) = 206$.

Time = 0.29 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.01

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2\left(630\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^5 a^4c^{\frac{5}{2}}|a|+252\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^4 a^5c^3-1365\left(\sqrt{a^2c}x-\sqrt{a^2\right.}{315\left(\sqrt{a^2c}x-\sqrt{\right.}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac")
```

output
```
2/315*(630*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*a^4*c^(5/2)*abs(a)
+ 252*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a^5*c^3 - 1365*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*a^4*c^(7/2)*abs(a) + 1035*(sqrt(a^2*c)*
x - sqrt(a^2*c*x^2 - a*c*x))^2*a^5*c^4 - 315*(sqrt(a^2*c)*x - sqrt(a^2*c*x
^2 - a*c*x))*a^4*c^(9/2)*abs(a) + 35*a^5*c^5)/((sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - a*c*x))^9*abs(a)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 13.71 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{544\,a^4\,\sqrt{c-\frac{c}{a\,x}}}{315}+\frac{2\,\sqrt{c-\frac{c}{a\,x}}}{9\,x^4}+\frac{34\,a\,\sqrt{c-\frac{c}{a\,x}}}{63\,x^3} + \frac{68\,a^2\,\sqrt{c-\frac{c}{a\,x}}}{105\,x^2}+\frac{272\,a^3\,\sqrt{c-\frac{c}{a\,x}}}{315\,x}$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
```

output
```
(544*a^4*(c - c/(a*x))^(1/2))/315 + (2*(c - c/(a*x))^(1/2))/(9*x^4) + (34*
a*(c - c/(a*x))^(1/2))/(63*x^3) + (68*a^2*(c - c/(a*x))^(1/2))/(105*x^2) +
 (272*a^3*(c - c/(a*x))^(1/2))/(315*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$
$$= \frac{2\sqrt{c}\left(272\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^4x^4+136\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^3x^3+102\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^2x^2+85\sqrt{x}\,\sqrt{a}\,\sqrt{ax-\right.}{315a\,x^5}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x)
```

output
```
(2*sqrt(c)*(272*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**4*x**4 + 136*sqrt(x)*sqrt
(a)*sqrt(a*x - 1)*a**3*x**3 + 102*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**2*x**2
+ 85*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a*x + 35*sqrt(x)*sqrt(a)*sqrt(a*x - 1)
- 272*a**5*x**5))/(315*a*x**5)
```

3.520 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 260

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\frac{149c\sqrt{1-\frac{1}{a^2x^2}}\,x}{64a^3\sqrt{c-\frac{c}{ax}}}+\frac{107c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{96a^2\sqrt{c-\frac{c}{ax}}}+\frac{17c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{24a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^4}{4\sqrt{c-\frac{c}{ax}}}+\frac{363\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{64a^4}-\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{a^4}$$

output
```
149/64*c*(1-1/a^2/x^2)^(1/2)*x/a^3/(c-c/a/x)^(1/2)+107/96*c*(1-1/a^2/x^2)^
(1/2)*x^2/a^2/(c-c/a/x)^(1/2)+17/24*c*(1-1/a^2/x^2)^(1/2)*x^3/a/(c-c/a/x)^
(1/2)+1/4*c*(1-1/a^2/x^2)^(1/2)*x^4/(c-c/a/x)^(1/2)+363/64*c^(1/2)*arctanh
(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a^4-4*2^(1/2)*c^(1/2)*arctan
h(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.97

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(447+214ax+136a^2x^2+48a^3x^3)}{-1+ax} - 1089\sqrt{c}\log(1-ax)+768\sqrt{2}\sqrt{c}\log\left((-1+ax)^2\right)+1089\sqrt{}}{}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

output
```
((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(447 + 214*a*x + 136*a
^2*x^2 + 48*a^3*x^3))/(-1 + a*x) - 1089*Sqrt[c]*Log[1 - a*x] + 768*Sqrt[2]
*Sqrt[c]*Log[(-1 + a*x)^2] + 1089*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^
2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 768*Sqrt[2]*Sq
rt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^
2 + c*(-1 - 2*a*x + 3*a^2*x^2)])/(384*a^4)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.72, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.519, Rules used = {6733, 585, 27, 109, 27, 168, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6733}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{c\sqrt{1-\frac{1}{ax}}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}x^5}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x^5}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(-\frac{\int -\frac{\left(17a+\frac{15}{x}\right)x^4}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(17a+\frac{15}{x}\right)x^4}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{-\frac{\int -\frac{\left(107a+\frac{85}{x}\right)x^3}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{3a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\int \frac{\left(107a+\frac{85}{x}\right)x^3}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{6a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{-\frac{\int -\frac{3\left(149a+\frac{107}{x}\right)x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-\frac{107}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$\Big\downarrow$ 27

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\int \frac{\left(149a+\frac{107}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{107}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$\Big\downarrow$ 168

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(-\frac{\int -\frac{\left(363a+\frac{149}{x}\right)x}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-149x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{107}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$\Big\downarrow$ 27

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{\int \frac{\left(363a+\frac{149}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-149x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{107}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$\Big\downarrow$ 174

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(\frac{512\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+363\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-149x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{107}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 73$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(\frac{1024a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+726a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{2a}-149x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{107}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 221$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\frac{3\left(\frac{512\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-726\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{2a}-149x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{107}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a}-\frac{17}{3}x^3\sqrt{\frac{1}{ax}+1}}{8a^2}-\frac{x^4\sqrt{\frac{1}{ax}+1}}{4a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

output
```
-((a*c*Sqrt[1 - 1/(a*x)]*(-1/4*(Sqrt[1 + 1/(a*x)]*x^4)/a + ((-17*Sqrt[1 +
1/(a*x)]*x^3)/3 + ((-107*Sqrt[1 + 1/(a*x)]*x^2)/2 + (3*(-149*Sqrt[1 + 1/(a
*x)]*x + (-726*ArcTanh[Sqrt[1 + 1/(a*x)]] + 512*Sqrt[2]*ArcTanh[Sqrt[1 + 1
/(a*x)]/Sqrt[2]])/(2*a)))/(4*a))/(6*a))/(8*a^2)))/Sqrt[c - c/(a*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.86

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(96a^{\frac{9}{2}}x^{3}\sqrt{x(ax+1)}\,\sqrt{\frac{1}{a}}+272a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x^{2}+428a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x+894\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-768\sqrt{2}\right.}{384\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{9}{2}}\sqrt{x(ax+1)}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{(48a^{3}x^{3}+136a^{2}x^{2}+214ax+447)x\sqrt{\frac{c(ax-1)}{ax}}}{192a^{3}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{363\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{128a^{3}\sqrt{a^{2}c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}c}}{x-\frac{1}{a}}\right.}{a^{4}\sqrt{c}}\right.}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/384/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(96*
a^(9/2)*x^3*(x*(a*x+1))^(1/2)*(1/a)^(1/2)+272*a^(7/2)*(1/a)^(1/2)*(x*(a*x+
1))^(1/2)*x^2+428*a^(5/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*x+894*(x*(a*x+1))^
(1/2)*a^(3/2)*(1/a)^(1/2)-768*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1)
)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+1089*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/
2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/a^(9/2)/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.18

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$
$$= \left[\frac{768\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+1089\,(ax-1)\sqrt{\ldots}}{\ldots}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)*x^3,x, algorithm="fric
as")
```

output
```
[1/768*(768*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 1
3*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(
a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1))
+ 1089*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c
)/(a*x - 1)) + 4*(48*a^5*x^5 + 184*a^4*x^4 + 350*a^3*x^3 + 661*a^2*x^2 + 4
47*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4),
1/384*(768*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a
*c*x - c)) - 1089*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqr
t((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c))
+ 2*(48*a^5*x^5 + 184*a^4*x^4 + 350*a^3*x^3 + 661*a^2*x^2 + 447*a*x)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)*x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)*x^3,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)*x^3,x, algorithm="giac
")
```

output
```
integrate(sqrt(c - c/(a*x))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int \frac{x^3\,\sqrt{c-\frac{c}{a\,x}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((x^3*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^3*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.65

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$
$$=\frac{\sqrt{c}\left(48\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^3x^3+136\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2+214\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax+447\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)*x^3,x)
```

output
```
(sqrt(c)*(48*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**3*x**3 + 136*sqrt(x)*sqrt(a)
*sqrt(a*x + 1)*a**2*x**2 + 214*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x + 447*sqr
t(x)*sqrt(a)*sqrt(a*x + 1) + 384*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(
a) - sqrt(2) - 1) - 384*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt
(2) + 1) - 384*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) - 1)
+ 384*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1) + 1089*lo
g(sqrt(a*x + 1) + sqrt(x)*sqrt(a))))/(192*a**4)
```

3.521 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 220

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{19c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{13c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}}+\frac{45\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}-\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

output
```
19/8*c*(1-1/a^2/x^2)^(1/2)*x/a^2/(c-c/a/x)^(1/2)+13/12*c*(1-1/a^2/x^2)^(1/
2)*x^2/a/(c-c/a/x)^(1/2)+1/3*c*(1-1/a^2/x^2)^(1/2)*x^3/(c-c/a/x)^(1/2)+45/
8*c^(1/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a^3-4*2^(1/
2)*c^(1/2)*arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2)
)/a^3
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.11

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$= \frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(57+26ax+8a^2x^2)}{-1+ax} - 135\sqrt{c}\log(1-ax) + 96\sqrt{2}\sqrt{c}\log\left((-1+ax)^2\right) + 135\sqrt{c}\log\left(2a^2\sqrt{c}\right.}{}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

output
```
((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(57 + 26*a*x + 8*a^2*x
^2))/(-1 + a*x) - 135*Sqrt[c]*Log[1 - a*x] + 96*Sqrt[2]*Sqrt[c]*Log[(-1 +
a*x)^2] + 135*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(
a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 96*Sqrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a
^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3
*a^2*x^2)])/(48*a^3)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.73, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6733, 585, 27, 109, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \ 6733$$
$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}$$
$$\Big\downarrow \ 585$$

$$-\frac{c\sqrt{1-\frac{1}{ax}}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}x^4}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x^4}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(-\frac{\int -\frac{\left(13a+\frac{11}{x}\right)x^3}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{3a}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(13a+\frac{11}{x}\right)x^3}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{6a^2}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{-\frac{\int -\frac{3\left(19a+\frac{13}{x}\right)x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-\frac{13}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a^2}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\int \frac{\left(19a+\frac{13}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a}-\frac{13}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a^2}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(-\frac{\int -\frac{\left(45a+\frac{19}{x}\right)x}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-19x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{13}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a^2}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$\Big\downarrow$ 27

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{\int \frac{\left(45a+\frac{19}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-19x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{13}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a^2}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$\Big\downarrow$ 174

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{64\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+45\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-19x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{13}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a^2}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$\Big\downarrow$ 73

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{128a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+90a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{2a}-19x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{13}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a^2}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$\Big\downarrow$ 221

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{3\left(\frac{64\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-90\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{2a}-19x\sqrt{\frac{1}{ax}+1}\right)}{4a}-\frac{13}{2}x^2\sqrt{\frac{1}{ax}+1}}{6a^2}-\frac{x^3\sqrt{\frac{1}{ax}+1}}{3a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

output
```
-((a*c*Sqrt[1 - 1/(a*x)]*(-1/3*(Sqrt[1 + 1/(a*x)]*x^3)/a + ((-13*Sqrt[1 +
1/(a*x)]*x^2)/2 + (3*(-19*Sqrt[1 + 1/(a*x)]*x + (-90*ArcTanh[Sqrt[1 + 1/(a
*x)]] + 64*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(2*a)))/(4*a))/(6*a
^2)))/Sqrt[c - c/(a*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.92

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x^2+52a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x+114\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-96\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{x(ax+1)}\,a+3ax}{ax-1}\right.\right.}{48\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{7}{2}}\sqrt{x(ax+1)}\sqrt{\frac{1}{a}}}$
risch	$\frac{(8a^2x^2+26ax+57)x\sqrt{\frac{c(ax-1)}{ax}}}{24a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{45\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{16a^2\sqrt{a^2c}}-\frac{2\sqrt{2}\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right.}{a^3\sqrt{c}}\right.}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/48/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*a
^(7/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*x^2+52*a^(5/2)*(1/a)^(1/2)*(x*(a*x+1)
)^(1/2)*x+114*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2)-96*2^(1/2)*ln((2*2^(1/
2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+135*ln(1/2*(2
*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/a^(7/2)/(x*(a*
x+1))^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.51

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\left[\frac{96\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+135(ax-1)\sqrt{c}\,\mathrm{l}}{96}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)*x^2,x, algorithm="fric
as")
```

output
```
[1/96*(96*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*
a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*
x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) +
135*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x
^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(8*a^4*x^4 + 34*a^3*x^3 + 83*a^2*x^2 + 57*a*x)*sqrt((a*x - 1
)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(96*sqrt(2)*(a*x
 - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - 135*(a*x -
 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 + 34*a^3*
x^3 + 83*a^2*x^2 + 57*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^4*x - a^3)]
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)*x**2,x)
```

output
```
Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)*x^2,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)*x^2,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \int \frac{x^2\sqrt{c-\frac{c}{a\,x}}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.69

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \frac{\sqrt{c}\left(8\sqrt{x}\sqrt{a}\sqrt{ax+1}\,a^2x^2+26\sqrt{x}\sqrt{a}\sqrt{ax+1}\,ax+57\sqrt{x}\sqrt{a}\sqrt{ax+1}+48\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\sqrt{a}\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)*x^2,x)
```

output
```
(sqrt(c)*(8*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**2*x**2 + 26*sqrt(x)*sqrt(a)*s
qrt(a*x + 1)*a*x + 57*sqrt(x)*sqrt(a)*sqrt(a*x + 1) + 48*sqrt(2)*log(sqrt(
a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1) - 48*sqrt(2)*log(sqrt(a*x + 1) +
 sqrt(x)*sqrt(a) - sqrt(2) + 1) - 48*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*s
qrt(a) + sqrt(2) - 1) + 48*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + s
qrt(2) + 1) + 135*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))))/(24*a**3)
```

3.522 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 180

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{9c\sqrt{1-\frac{1}{a^2x^2}}x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}}+\frac{23\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}-\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{a^2}$$

output
```
9/4*c*(1-1/a^2/x^2)^(1/2)*x/a/(c-c/a/x)^(1/2)+1/2*c*(1-1/a^2/x^2)^(1/2)*x^
2/(c-c/a/x)^(1/2)+23/4*c^(1/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/
x)^(1/2))/a^2-4*2^(1/2)*c^(1/2)*arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^
(1/2)/(c-c/a/x)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.31

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$=\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(9+2ax)}{-1+ax}-23\sqrt{c}\log(1-ax)+16\sqrt{2}\sqrt{c}\log\left((-1+ax)^2\right)+23\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\right.}{}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

output
```
((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(9 + 2*a*x))/(-1 + a*x
) - 23*Sqrt[c]*Log[1 - a*x] + 16*Sqrt[2]*Sqrt[c]*Log[(-1 + a*x)^2] + 23*Sq
rt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-
1 - a*x + 2*a^2*x^2)] - 16*Sqrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[
1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)])/(8*a
^2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.73, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6733, 585, 27, 109, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6733}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c\sqrt{1-\frac{1}{ax}}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}x^3}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x^3}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 109$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(-\frac{\int -\frac{\left(9a+\frac{7}{x}\right)x^2}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-\frac{x^2\sqrt{\frac{1}{ax}+1}}{2a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(9a+\frac{7}{x}\right)x^2}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{4a^2}-\frac{x^2\sqrt{\frac{1}{ax}+1}}{2a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 168$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{-\frac{\int -\frac{\left(23a+\frac{9}{x}\right)x}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-9x\sqrt{\frac{1}{ax}+1}}{4a^2}-\frac{x^2\sqrt{\frac{1}{ax}+1}}{2a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{\int \frac{\left(23a+\frac{9}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-9x\sqrt{\frac{1}{ax}+1}}{4a^2}-\frac{x^2\sqrt{\frac{1}{ax}+1}}{2a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 174$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{32\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+23\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a}-9x\sqrt{\frac{1}{ax}+1}}{4a^2}-\frac{x^2\sqrt{\frac{1}{ax}+1}}{2a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{64a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+46a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{2a}-9x\sqrt{\frac{1}{ax}+1}}{4a^2}-\frac{x^2\sqrt{\frac{1}{ax}+1}}{2a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\frac{32\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-46\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{2a}-9x\sqrt{\frac{1}{ax}+1}}{4a^2}-\frac{x^2\sqrt{\frac{1}{ax}+1}}{2a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input

```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

output

```
-((a*c*Sqrt[1 - 1/(a*x)]*(-1/2*(Sqrt[1 + 1/(a*x)]*x^2)/a + (-9*Sqrt[1 + 1/
(a*x)]*x + (-46*ArcTanh[Sqrt[1 + 1/(a*x)]] + 32*Sqrt[2]*ArcTanh[Sqrt[1 + 1
/(a*x)]/Sqrt[2]])/(2*a))/(4*a^2)))/Sqrt[c - c/(a*x)])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,x+18\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-16\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)\sqrt{a}+23\ln\left(\frac{2\sqrt{x(ax+1)}}{2\sqrt{}}\right.\right.}{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{5}{2}}\sqrt{x(ax+1)}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{(2ax+9)x\sqrt{\frac{c(ax-1)}{ax}}}{4a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{23\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{8a\sqrt{a^{2}c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}c+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{a^{2}\sqrt{c}}\right)\sqrt{\frac{c(a}{a}}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/8/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(4*a^(
5/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*x+18*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1
/2)-16*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x
-1))*a^(1/2)+23*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1
/a)^(1/2))/a^(5/2)/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.98

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$

$$=\left[\frac{16\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^{3}cx^{3}-3\,a^{2}cx^{2}-13\,acx-4\,\sqrt{2}(3\,a^{3}x^{3}+4\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1}\right)+23\,(ax-1)\sqrt{c}\,\mathrm{lo}}{16\,(a^{3}x-}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)*x,x, algorithm="fricas
")
```

output
```
[1/16*(16*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*
a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*
x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) +
23*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^
2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a
*x - 1)) + 4*(2*a^3*x^3 + 11*a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x)))/(a^3*x - a^2), 1/8*(16*sqrt(2)*(a*x - 1)*sqrt(-c)*a
rctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a
*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - 23*(a*x - 1)*sqrt(-c)*arct
an(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(2*a^3*x^3 + 11*a^2*x^2 + 9*a*x)*sqrt
((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)*x,x)
```

output
```
Integral(x*sqrt(-c*(-1 + 1/(a*x)))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)*x,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)*x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = \int \frac{x\sqrt{c-\frac{c}{a\,x}}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.74

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$= \frac{\sqrt{c}\left(2\sqrt{x}\sqrt{a}\sqrt{ax+1}\,ax+9\sqrt{x}\sqrt{a}\sqrt{ax+1}+8\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\sqrt{a}-\sqrt{2}-1\right)-8\sqrt{2}\log\left(\sqrt{ax\right.}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)*x,x)
```

output
```
(sqrt(c)*(2*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x + 9*sqrt(x)*sqrt(a)*sqrt(a*x
 + 1) + 8*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1) - 8*s
qrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) + 1) - 8*sqrt(2)*log(
sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) - 1) + 8*sqrt(2)*log(sqrt(a*x +
1) + sqrt(x)*sqrt(a) + sqrt(2) + 1) + 23*log(sqrt(a*x + 1) + sqrt(x)*sqrt(
a))))/(4*a**2)
```

3.523 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 135

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{5\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output

```
c*(1-1/a^2/x^2)^(1/2)*x/(c-c/a/x)^(1/2)+5*c^(1/2)*arctanh(c^(1/2)*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2))/a-4*2^(1/2)*c^(1/2)*arctanh(1/2*c^(1/2)*(1-1/
a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}x+\frac{5\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}-\frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
(Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x + (5*ArcTanh[Sqrt[1 + 1/(a*x)]])/a
 - (4*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a))/Sqrt[1 - 1/(a*x)]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.77, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6731, 585, 27, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6731}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{585}$$

$$-\frac{c\sqrt{1-\frac{1}{ax}}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}x^2}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x^2}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 109$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(-\frac{\int -\frac{\left(5a+\frac{3}{x}\right)x}{2a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(5a+\frac{3}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 174$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{8\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 73$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{16a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+10a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \; 221$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-10\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{2a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
-((a*c*Sqrt[1 - 1/(a*x)]*(-((Sqrt[1 + 1/(a*x)]*x)/a) + (-10*ArcTanh[Sqrt[1
 + 1/(a*x)]] + 8*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(2*a^2)))/Sqr
t[c - c/(a*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.19

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)\sqrt{a}+5\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$
risch	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2\sqrt{a^{2}c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}c+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{a\sqrt{c}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(a}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*(x*
(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2)-4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*
(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+5*ln(1/2*(2*(x*(a*x+1))^(1/2)*a
^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/(x*(a*x+1))^(1/2)/a^(3/2)/(1/a)^(1
/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(112) = 224.

Time = 0.17 (sec) , antiderivative size = 512, normalized size of antiderivative = 3.79

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$= \left[\frac{4\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+5(ax-1)\sqrt{c}\log}{4(a^2x-a)} \right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*
c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 5*
(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 +
 a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x
- 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^2*x - a), 1/2*(4*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^
2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2
*c*x^2 - 2*a*c*x - c)) - 5*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c
*x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a^2*x - a)]
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=\frac{\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+2\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}-1\right)-2\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}+1\right)\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x)
```

output
```
(sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x + 1) + 2*sqrt(2)*log(sqrt(a*x + 1) + sq
rt(x)*sqrt(a) - sqrt(2) - 1) - 2*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(
a) - sqrt(2) + 1) - 2*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2
) - 1) + 2*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1) + 5*
log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))))/a
```

3.524 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 129

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + 2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right) - 4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

output
```
2*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)+2*c^(1/2)*arctanh(c^(1/2)*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2))-4*2^(1/2)*c^(1/2)*arctanh(1/2*c^(1/2)*(1-1/a^
2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.69

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x}{-1+ax} - \sqrt{c}\log(1-ax) + 2\sqrt{2}\sqrt{c}\log\left((-1+ax)^2\right) + \sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2 + c(-1-ax+2a^2x^2)\right) - 2\sqrt{2}\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2 + c(-1-2ax+3a^2x^2)\right)$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x)/(-1 + a*x) - Sqrt[c]*Log[1
 - a*x] + 2*Sqrt[2]*Sqrt[c]*Log[(-1 + a*x)^2] + Sqrt[c]*Log[2*a^2*Sqrt[c]*
Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] -
2*Sqrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c -
 c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6733, 585, 27, 95, 25, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}}{x}dx$$

$\big\downarrow$ 6733

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}$$

$\big\downarrow$ 585

$$-\frac{c\sqrt{1-\frac{1}{ax}}\int \frac{a\left(1+\frac{1}{ax}\right)^{3/2}x}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$\big\downarrow$ 27

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\int \frac{\left(1+\frac{1}{ax}\right)^{3/2}x}{a-\frac{1}{x}}d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$\big\downarrow$ 95

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(-\int -\frac{\left(a+\frac{3}{x}\right)x}{a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{2\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$\big\downarrow$ 25

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\int \frac{\left(a+\frac{3}{x}\right)x}{a\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{2\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$\big\downarrow$ 27

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{\int \frac{\left(a+\frac{3}{x}\right)x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$\big\downarrow$ 174

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{4\int \frac{1}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$\big\downarrow$ 73

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{8a\int \frac{1}{2a-\frac{a}{x^2}}d\sqrt{1+\frac{1}{ax}}+2a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{ac\sqrt{1-\frac{1}{ax}}\left(\frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)-2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}-\frac{2\sqrt{\frac{1}{ax}+1}}{a}\right)}{\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

output
```
-((a*c*Sqrt[1 - 1/(a*x)]*((-2*Sqrt[1 + 1/(a*x)])/a + (-2*ArcTanh[Sqrt[1 +
1/(a*x)]] + 4*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/a))/Sqrt[c - c/(
a*x)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.25

method	result
default	$-\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(2\sqrt{a}\,\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)x-\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,ax-2\sqrt{x(ax+1)}\,\sqrt{a}\,\sqrt{\frac{1}{a}}\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{x(ax+1)}\,\sqrt{a}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{\sqrt{a^{2}c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}c+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)}{\sqrt{c}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(a}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(2*a^(1/2
)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*
x-ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*(1/a)^(1/2)*a*x-2*
(x*(a*x+1))^(1/2)*a^(1/2)*(1/a)^(1/2))/(x*(a*x+1))^(1/2)/a^(1/2)/(1/a)^(1/
2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(106) = 212$.

Time = 0.16 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.80

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$$
$$= \left[\frac{2\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+(ax-1)\sqrt{c}\log\left(\right.}{2\,(ax-1)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas
")
```

output
```
[1/2*(2*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*
c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + (a
*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a
*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
1)) + 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x
- 1), (2*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(
-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c
*x - c)) - (a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(a*
x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{2\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}-1\right)ax-\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}+\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x)
```

output
```
(2*sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x + 1) + sqrt(2)*log(sqrt(a*x + 1) + sq
rt(x)*sqrt(a) - sqrt(2) - 1)*a*x - sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqr
t(a) - sqrt(2) + 1)*a*x - sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sq
rt(2) - 1)*a*x + sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1
)*a*x + log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a*x + a*x))/(a*x)
```

3.525 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 125

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

output
```
2/3*a*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)+4*a*c*(1-1/a^2/x^2)^(1/2)/(c
-c/a/x)^(1/2)-4*2^(1/2)*a*c^(1/2)*arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*
2^(1/2)/(c-c/a/x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.24

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$$
$$=\frac{2a\left(\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(1+7ax)+3\sqrt{2}\sqrt{c}(-1+ax)\log\left((-1+ax)^2\right)-3\sqrt{2}\sqrt{c}(-1+ax)\log\left(2\sqrt{2}a^2\right.\right.}{-3+3ax}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

output
```
(2*a*(Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 + 7*a*x) + 3*Sqrt[2]*Sqrt
[c]*(-1 + a*x)*Log[(-1 + a*x)^2] - 3*Sqrt[2]*Sqrt[c]*(-1 + a*x)*Log[2*Sqrt
[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*
x + 3*a^2*x^2)]))/(-3 + 3*a*x)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6733, 466, 466, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6733}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{466}$$

$$-c^3\left(\frac{2\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{466}$$

$$-c^3\left(\frac{2\left(\frac{2\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{471}$$

$$-c^3\left(\frac{2\left(-\frac{4\int \frac{1}{\frac{c^2}{a^2x^2}-\frac{2c}{a^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$$\Big\downarrow \; 221$$

$$-c^3\left(\frac{2\left(\frac{2\sqrt{2}a\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{c^{3/2}}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

output
```
-(c^3*((-2*a*(1 - 1/(a^2*x^2))^(3/2))/(3*c*(c - c/(a*x))^(3/2)) + (2*((-2*
a*Sqrt[1 - 1/(a^2*x^2)])/(c*Sqrt[c - c/(a*x)]) + (2*Sqrt[2]*a*ArcTanh[(Sqr
t[c]*Sqrt[1 - 1/(a^2*x^2)])/(Sqrt[2]*Sqrt[c - c/(a*x)])])/c^(3/2)))/c))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{2(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(-3a\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)x^{2}+7x\sqrt{x(ax+1)}\,a\sqrt{\frac{1}{a}}+\sqrt{x(ax+1)}\,\sqrt{\frac{1}{a}}\right)}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x\sqrt{x(ax+1)}\,\sqrt{\frac{1}{a}}}$	140
risch	$\frac{2\left(7a^{2}x^{2}+8ax+1\right)\sqrt{\frac{c(ax-1)}{ax}}}{3x\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{2a\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}c+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	180

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(-3*a*2^
(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x^2+
7*x*(x*(a*x+1))^(1/2)*a*(1/a)^(1/2)+(x*(a*x+1))^(1/2)*(1/a)^(1/2))/x/(x*(a
*x+1))^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.82

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \left[\frac{3\sqrt{2}(a^2x^2-ax)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+2\left(7a^2x^2+8a\right.}{3\left(ax^2-x\right)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fric
as")
```

output
```
[1/3*(3*sqrt(2)*(a^2*x^2 - a*x)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 -
 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)
) + 2*(7*a^2*x^2 + 8*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x)))/(a*x^2 - x), 2/3*(3*sqrt(2)*(a^2*x^2 - a*x)*sqrt(-c)*arctan(2*sqrt(
2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*
x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + (7*a^2*x^2 + 8*a*x + 1)*sqrt((a*x - 1)/
(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^2 - x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a*x))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.22

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$$
$$= \frac{2\sqrt{c}\left(7\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax+\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+3\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}-1\right)a^2x^2-3\sqrt{2}\log\left(\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x)
```

output
```
(2*sqrt(c)*(7*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x + sqrt(x)*sqrt(a)*sqrt(a*x
 + 1) + 3*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1)*a**2*
x**2 - 3*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) + 1)*a**2*x
**2 - 3*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) - 1)*a**2*x*
*2 + 3*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1)*a**2*x**
2 - a**2*x**2))/(3*a*x**2)
```

3.526 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 170

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

output
```
2/5*a^2*c^3*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(5/2)+2/3*a^2*c^2*(1-1/a^2/x^2)^
(3/2)/(c-c/a/x)^(3/2)+4*a^2*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)-4*2^(1/2
)*a^2*c^(1/2)*arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1
/2))
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.95

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(3+11ax+38a^2x^2)}{15x(-1+ax)} + 2\sqrt{2}a^2\sqrt{c}\log\left((-1+ax)^2\right) - 2\sqrt{2}a^2\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2 + c(-1-2ax+3a^2x^2)\right)$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 + 11*a*x + 38*a^2*x^2))/(1
5*x*(-1 + a*x)) + 2*Sqrt[2]*a^2*Sqrt[c]*Log[(-1 + a*x)^2] - 2*Sqrt[2]*a^2*
Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*
x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6733, 572, 466, 466, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{ 6733}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{5/2}x}d\frac{1}{x}$$

$$\Big\downarrow \ 572$$

$$-c^3\left(a\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{2a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

$$\Big\downarrow \ 466$$

$$-c^3\left(a\left(\frac{2\int \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)-\frac{2a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

$$\Big\downarrow \ 466$$

$$-c^3\left(a\left(\frac{2\left(\frac{2\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)-\frac{2a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

$$\Big\downarrow \ 471$$

$$-c^3\left(a\left(\frac{2\left(-\frac{4\int \frac{1}{\frac{c^2}{a^2x^2}-\frac{2c}{a^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)-\frac{2a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

$$\Big\downarrow \ 221$$

$$-c^3\left(a\left(\frac{2\left(\frac{2\sqrt{2}a\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{c^{3/2}}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)-\frac{2a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}\right)$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

output
```
-(c^3*((-2*a^2*(1 - 1/(a^2*x^2))^(5/2))/(5*(c - c/(a*x))^(5/2)) + a*((-2*a
*(1 - 1/(a^2*x^2))^(3/2))/(3*c*(c - c/(a*x))^(3/2)) + (2*((-2*a*Sqrt[1 - 1
/(a^2*x^2)])/(c*Sqrt[c - c/(a*x)]) + (2*Sqrt[2]*a*ArcTanh[(Sqrt[c]*Sqrt[1
- 1/(a^2*x^2)])/(Sqrt[2]*Sqrt[c - c/(a*x)])])/c^(3/2)))/c)))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 572
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 2))), x] + Simp[c*(n/(d
*(n + 2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && NeQ[n + 2*p + 2, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.97

method	result
default	$\frac{2(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(-15a^2\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)x^3+38x^2\sqrt{x(ax+1)}\,a^2\sqrt{\frac{1}{a}}+11x\sqrt{x(ax+1)}\,a\sqrt{\frac{1}{a}}+3\sqrt{x(ax+1)}\,\sqrt{}\right.}{15\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^2\sqrt{x(ax+1)}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{2(38a^3x^3+49a^2x^2+14ax+3)\sqrt{\frac{c(ax-1)}{ax}}}{15x^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{2a^2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)}}{\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/15/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(-15*a^
2*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*
x^3+38*x^2*(x*(a*x+1))^(1/2)*a^2*(1/a)^(1/2)+11*x*(x*(a*x+1))^(1/2)*a*(1/a
)^(1/2)+3*(x*(a*x+1))^(1/2)*(1/a)^(1/2))/x^2/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.24

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$$
$$=\left[\frac{15\sqrt{2}(a^3x^3-a^2x^2)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\,(38\,a^3x^3-}{15\,(ax^3-x^2)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="fric
as")
```

output
```
[1/15*(15*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c
*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*
x - 1)) + 2*(38*a^3*x^3 + 49*a^2*x^2 + 14*a*x + 3)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a*x^3 - x^2), 2/15*(15*sqrt(2)*(a^3*x^3 - a^2
*x^2)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + (38*a^3*x^
3 + 49*a^2*x^2 + 14*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a*x^3 - x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a*x))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.01

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$$
$$= \frac{2\sqrt{c}\,\left(38\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2+11\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax+3\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+15\sqrt{2}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x)
```

output
```
(2*sqrt(c)*(38*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**2*x**2 + 11*sqrt(x)*sqrt(a
)*sqrt(a*x + 1)*a*x + 3*sqrt(x)*sqrt(a)*sqrt(a*x + 1) + 15*sqrt(2)*log(sqr
t(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1)*a**3*x**3 - 15*sqrt(2)*log(sqr
t(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) + 1)*a**3*x**3 - 15*sqrt(2)*log(sqr
t(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) - 1)*a**3*x**3 + 15*sqrt(2)*log(sqr
t(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1)*a**3*x**3 - 14*a**3*x**3))/(15
*a*x**3)
```

3.527 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 209

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

output

```
4/7*a^3*c^3*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(5/2)+2/3*a^3*c^2*(1-1/a^2/x^2)^
(3/2)/(c-c/a/x)^(3/2)-2/7*a^3*c^2*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(3/2)+4*a^
3*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)-4*2^(1/2)*a^3*c^(1/2)*arctanh(1/2*
c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.81

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(3+9ax+16a^2x^2+52a^3x^3)}{21x^2(-1+ax)} + 2\sqrt{2}a^3\sqrt{c}\log\left((-1+ax)^2\right) - 2\sqrt{2}a^3\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2 + c(-1-2ax+3a^2x^2)\right)$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 + 9*a*x + 16*a^2*x^2 + 52*
a^3*x^3))/(21*x^2*(-1 + a*x)) + 2*Sqrt[2]*a^3*Sqrt[c]*Log[(-1 + a*x)^2] -
2*Sqrt[2]*a^3*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt
[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6733, 581, 27, 672, 466, 466, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{ 6733}$$

$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{5/2}x^2}d\frac{1}{x}$$

$\downarrow$ 581

$$-c^3\left(\frac{2a^2\int -\frac{c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a-\frac{10}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{7c^2}+\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c\left(c-\frac{c}{ax}\right)^{3/2}}\right)$$

$\downarrow$ 27

$$-c^3\left(\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{1}{7}a\int\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a-\frac{10}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}\right)$$

$\downarrow$ 672

$$-c^3\left(\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{1}{7}a\left(\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}-7a\int\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}\right)\right)$$

$\downarrow$ 466

$$-c^3\left(\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{1}{7}a\left(\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}-7a\left(\frac{2\int\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)\right)\right)$$

$\downarrow$ 466

$$-c^3\left(\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{1}{7}a\left(\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}-7a\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)\right)\right)$$

$\downarrow$ 471

$$-c^3\left(\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{1}{7}a\left(\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}-7a\left(\frac{2\left(-\frac{4\int\frac{1}{\frac{c^2}{a^2x^2}-\frac{2c}{a^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)\right)\right)$$

$\downarrow$ 221

$$-c^3\left(\frac{2a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{1}{7}a\left(\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}-7a\left(\frac{2\left(\frac{2\sqrt{2}a\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{c^{3/2}}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a(1-}{3c(c-}\right.\right.\right.$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

output
```
-(c^3*((2*a^3*(1 - 1/(a^2*x^2))^(5/2))/(7*c*(c - c/(a*x))^(3/2)) - (a*((4*
a^2*(1 - 1/(a^2*x^2))^(5/2))/(c - c/(a*x))^(5/2) - 7*a*((-2*a*(1 - 1/(a^2*
x^2))^(3/2))/(3*c*(c - c/(a*x))^(3/2)) + (2*((-2*a*Sqrt[1 - 1/(a^2*x^2)])/
(c*Sqrt[c - c/(a*x)]) + (2*Sqrt[2]*a*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)
])/(Sqrt[2]*Sqrt[c - c/(a*x)])])/c^(3/2)))/c)))/7))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 672
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Simp[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2))   Int[(d + e*x
)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^
2 + a*e^2, 0] && NeQ[m + 2*p + 2, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.89

method	result
default	$-\frac{2(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(21a^3\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)x^4-52a^3\sqrt{\frac{1}{a}}\,x^3\sqrt{x(ax+1)}-16x^2\sqrt{x(ax+1)}\,a^2\sqrt{\frac{1}{a}}-9x\sqrt{x(ax+}\right.}{21\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^3\sqrt{x(ax+1)}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{2\left(52a^4x^4+68a^3x^3+25a^2x^2+12ax+3\right)\sqrt{\frac{c(ax-1)}{ax}}}{21x^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{2a^3\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{}}{\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/21/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x^3*(2
1*a^3*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-
1))*x^4-52*a^3*(1/a)^(1/2)*x^3*(x*(a*x+1))^(1/2)-16*x^2*(x*(a*x+1))^(1/2)*
a^2*(1/a)^(1/2)-9*x*(x*(a*x+1))^(1/2)*a*(1/a)^(1/2)-3*(x*(a*x+1))^(1/2)*(1
/a)^(1/2))/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.90

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \left[\frac{21\sqrt{2}(a^4x^4-a^3x^3)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\,(52\,a^4x^4-}{21\,(ax^4-x^3)} \right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="fric
as")
```

output
```
[1/21*(21*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c
*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*
x - 1)) + 2*(52*a^4*x^4 + 68*a^3*x^3 + 25*a^2*x^2 + 12*a*x + 3)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^4 - x^3), 2/21*(21*sqrt(2)*(
a^4*x^4 - a^3*x^3)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt
((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c))
 + (52*a^4*x^4 + 68*a^3*x^3 + 25*a^2*x^2 + 12*a*x + 3)*sqrt((a*x - 1)/(a*x
 + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^4 - x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a*x))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.90

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$$
$$= \frac{2\sqrt{c}\left(52\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^3x^3+16\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2+9\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax+3\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+21\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x)
```

output
```
(2*sqrt(c)*(52*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**3*x**3 + 16*sqrt(x)*sqrt(a
)*sqrt(a*x + 1)*a**2*x**2 + 9*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x + 3*sqrt(x
)*sqrt(a)*sqrt(a*x + 1) + 21*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) -
 sqrt(2) - 1)*a**4*x**4 - 21*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) -
 sqrt(2) + 1)*a**4*x**4 - 21*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) +
 sqrt(2) - 1)*a**4*x**4 + 21*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) +
 sqrt(2) + 1)*a**4*x**4 - 28*a**4*x**4))/(21*a*x**4)
```

3.528 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 246

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{226a^4c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{315\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2a^4c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{38a^4c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{63\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^4c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + \frac{2a^4c\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{9\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

output

```
226/315*a^4*c^3*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(5/2)+2/3*a^4*c^2*(1-1/a^2/x
^2)^(3/2)/(c-c/a/x)^(3/2)-38/63*a^4*c^2*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(3/2
)+4*a^4*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)+2/9*a^4*c*(1-1/a^2/x^2)^(5/2
)/(c-c/a/x)^(1/2)-4*2^(1/2)*a^4*c^(1/2)*arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^
(1/2)*2^(1/2)/(c-c/a/x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.72

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$
$$=\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(35+95ax+138a^2x^2+236a^3x^3+788a^4x^4)}{315x^3(-1+ax)}$$
$$+2\sqrt{2}a^4\sqrt{c}\log\left((-1+ax)^2\right)$$
$$-2\sqrt{2}a^4\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-1-2ax+3a^2x^2)\right)$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(35 + 95*a*x + 138*a^2*x^2 +
236*a^3*x^3 + 788*a^4*x^4))/(315*x^3*(-1 + a*x)) + 2*Sqrt[2]*a^4*Sqrt[c]*L
og[(-1 + a*x)^2] - 2*Sqrt[2]*a^4*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1
- 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6733, 581, 27, 2170, 27, 672, 466, 466, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{3\coth^{-1}(ax)}}{x^5}dx$$
$$\Big\downarrow \ 6733$$
$$-c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{5/2}x^3}d\frac{1}{x}$$
$$\Big\downarrow \ 581$$

$$-c^3\left(-\frac{2a^3\int -\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(-\frac{11c^3}{ax}+\frac{19c^3}{a^2x^2}+c^3\right)}{2\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{9c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{9c^2\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \ 27$$

$$-c^3\left(\frac{a^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(-\frac{11c^3}{ax}+\frac{19c^3}{a^2x^2}+c^3\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{9c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{9c^2\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \ 2170$$

$$-c^3\left(\frac{a^3\left(\frac{38ac^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2a^4\int \frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(50a-\frac{113}{x}\right)}{2a^5\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{7c^2}\right)}{9c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{9c^2\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \ 27$$

$$-c^3\left(\frac{a^3\left(\frac{38ac^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\int \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(50a-\frac{113}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}}{7a}\right)}{9c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{9c^2\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \ 672$$

$$-c^3\left(\frac{a^3\left(\frac{38ac^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\left(\frac{226a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}-63a\int\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}d\frac{1}{x}\right)}{7a}\right)}{9c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{9c^2\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{466}$$

$$-c^3\left(\frac{a^3\left(\frac{38ac^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\left(\frac{226a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}-63a\left(\frac{2\int\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\left(c-\frac{c}{ax}\right)^{3/2}}d\frac{1}{x}}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)\right)}{7a}\right)}{9c^3}-\frac{2a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{9c^2\sqrt{c-\frac{c}{ax}}}\right)$$

$$\Big\downarrow \text{466}$$

$$-c^3\left(\frac{a^3\left(\frac{38ac^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\left(\frac{226a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}-63a\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{c}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)\right)}{7a}\right)}{9c^3}-\frac{2}{\cdots}\right.$$

$\downarrow$ 471

$$-c^3\left(\frac{a^3\left(\frac{38ac^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\left(\frac{226a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}-63a\left(\frac{2\left(-\frac{4\int\frac{1}{\frac{c^2}{a^2x^2}-\frac{2c}{a^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)\right)}{7a}\right)}{9c^3}-\right.$$

↓ 221

$$-c^3\left(\frac{a^3\left(\frac{38ac^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\left(\frac{226a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}-63a\left(\frac{2\left(\frac{2\sqrt{2}a\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)}{c^{3/2}}-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}\right)}{c}-\frac{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c\left(c-\frac{c}{ax}\right)^{3/2}}\right)\right)}{7a}\right)}{9c^3}\right.$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

output
```
-(c^3*((-2*a^4*(1 - 1/(a^2*x^2))^(5/2))/(9*c^2*Sqrt[c - c/(a*x)]) + (a^3*(
(38*a*c^2*(1 - 1/(a^2*x^2))^(5/2))/(7*(c - c/(a*x))^(3/2)) - (c^3*((226*a^
2*(1 - 1/(a^2*x^2))^(5/2))/(5*(c - c/(a*x))^(5/2)) - 63*a*((-2*a*(1 - 1/(a
^2*x^2))^(3/2))/(3*c*(c - c/(a*x))^(3/2)) + (2*((-2*a*Sqrt[1 - 1/(a^2*x^2)
])/(c*Sqrt[c - c/(a*x)]) + (2*Sqrt[2]*a*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x
^2)])/(Sqrt[2]*Sqrt[c - c/(a*x)])])/c^(3/2)))/c)))/(7*a)))/(9*c^3)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 672
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Simp[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2))   Int[(d + e*x
)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^
2 + a*e^2, 0] && NeQ[m + 2*p + 2, 0]
```

rule 2170
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.84

method	result
risch	$\frac{2\left(788a^5x^5+1024a^4x^4+374a^3x^3+233a^2x^2+130ax+35\right)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{2a^4\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+3\left(x-\frac{1}{a}\right)ac\cdots}}{x-\frac{1}{a}}\cdots\right.}{\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\cdots}$
default	$-\frac{2(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(315a^4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)x^5-788a^4\sqrt{\frac{1}{a}}\,x^4\sqrt{x(ax+1)}-236a^3\sqrt{\frac{1}{a}}\,x^3\sqrt{x(ax+1)}-138x^2\cdots\right.}{315\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^4\sqrt{x(ax+1)}\,\sqrt{\frac{1}{a}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2/315*(788*a^5*x^5+1024*a^4*x^4+374*a^3*x^3+233*a^2*x^2+130*a*x+35)/x^4/((
a*x-1)/(a*x+1))^(1/2)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)-2*a^4*2^(1/2)/c^(1/2)*
ln((4*c+3*(x-1/a)*a*c+2*2^(1/2)*c^(1/2)*((x-1/a)^2*a^2*c+3*(x-1/a)*a*c+2*c
)^(1/2))/(x-1/a))/((a*x-1)/(a*x+1))^(1/2)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*((
a*x+1)*a*c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.68

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$

$$= \left[\frac{315\sqrt{2}(a^5x^5-a^4x^4)\sqrt{c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\,(788\,a^5x}{315\,(ax^5-x^4)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="fric
as")
```

output
```
[1/315*(315*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2
*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((
a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*
a*x - 1)) + 2*(788*a^5*x^5 + 1024*a^4*x^4 + 374*a^3*x^3 + 233*a^2*x^2 + 13
0*a*x + 35)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^5 - x^
4), 2/315*(315*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*
x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a
^2*c*x^2 - 2*a*c*x - c)) + (788*a^5*x^5 + 1024*a^4*x^4 + 374*a^3*x^3 + 233
*a^2*x^2 + 130*a*x + 35)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
)/(a*x^5 - x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a*x))/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a*x))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.84

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2\sqrt{c}\left(788\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^4x^4+236\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^3x^3+138\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2+95\sqrt{x}\,\sqrt{a}\,\sqrt{ax-}\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x)
```

output
```
(2*sqrt(c)*(788*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**4*x**4 + 236*sqrt(x)*sqrt
(a)*sqrt(a*x + 1)*a**3*x**3 + 138*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**2*x**2
+ 95*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x + 35*sqrt(x)*sqrt(a)*sqrt(a*x + 1)
+ 315*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) - 1)*a**5*x**5
 - 315*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) - sqrt(2) + 1)*a**5*x**
5 - 315*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) - 1)*a**5*x*
*5 + 315*sqrt(2)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a) + sqrt(2) + 1)*a**5*x
**5 - 508*a**5*x**5))/(315*a*x**5)
```

3.529 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 164

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{11c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}}-\frac{11c\sqrt{1-\frac{1}{a^2x^2}}x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^3}{3\sqrt{c-\frac{c}{ax}}}-\frac{11\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

output
```
11/8*c*(1-1/a^2/x^2)^(1/2)*x/a^2/(c-c/a/x)^(1/2)-11/12*c*(1-1/a^2/x^2)^(1/
2)*x^2/a/(c-c/a/x)^(1/2)+1/3*c*(1-1/a^2/x^2)^(1/2)*x^3/(c-c/a/x)^(1/2)-11/
8*c^(1/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.90

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(33-22ax+8a^2x^2)}{-1+ax}+33\sqrt{c}\log(1-ax)-33\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-1-\right.}{48a^3}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^ArcCoth[a*x],x]
```

output
```
((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(33 - 22*a*x + 8*a^2*x
^2))/(-1 + a*x) + 33*Sqrt[c]*Log[1 - a*x] - 33*Sqrt[c]*Log[2*a^2*Sqrt[c]*S
qrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(4
8*a^3)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6733, 580, 579, 579, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{c-\frac{c}{ax}} e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2} x^4}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{580}$$

$$-\frac{-\frac{11c\int \frac{\sqrt{c-\frac{c}{ax}}x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{6a}-\frac{c^2x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{579}$$

$$-\frac{-\frac{11c\left(-\frac{3\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{4a}-\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}\right)}{6a}-\frac{c^2x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{579}$$

$$-\frac{-\frac{11c\left(-\frac{3\left(-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}\right)}{6a}-\frac{c^2x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \; 573$$

$$-\frac{-\frac{11c\left(-\frac{3\left(\frac{c\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}\right)}{6a}-\frac{c^2x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{11c\left(-\frac{3\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}\right)}{6a}-\frac{c^2x^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}}{c}$$

input

```
Int[(Sqrt[c - c/(a*x)]*x^2)/E^ArcCoth[a*x],x]
```

output

```
-((-1/3*(c^2*Sqrt[1 - 1/(a^2*x^2)]*x^3)/Sqrt[c - c/(a*x)] - (11*c*(-1/2*(c
*Sqrt[1 - 1/(a^2*x^2)]*x^2)/Sqrt[c - c/(a*x)] - (3*(-((c*Sqrt[1 - 1/(a^2*x
^2)]*x)/Sqrt[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2
)])/Sqrt[c - c/(a*x)]])/a))/(4*a)))/(6*a))/c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 579
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 1)*((a + b*x^2)^(p +
1)/(b*c*e*(m + 1))), x] - Simp[d*((n - m - 2)/(c*e*(m + 1)))   Int[(e*x)^(m
 + 1)*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0] && LtQ[m, -1] && (IntegerQ[2*p] |
| IntegerQ[m])
```

rule 580
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 2)*((a + b*x^2)^(p +
1)/(b*e*(m + 1))), x] + Simp[d*((2*m + p + 3)/(e*(m + 1)))   Int[(e*x)^(m +
 1)*(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p},
 x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p - 1, 0] && LtQ[m, -1] && IntegerQ
[p + 1/2]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{5}{2}}x^{2}\sqrt{x(ax+1)}-44a^{\frac{3}{2}}x\sqrt{x(ax+1)}+66\sqrt{x(ax+1)}\,\sqrt{a}-33\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{48a^{\frac{5}{2}}(ax-1)\sqrt{x(ax+1)}}$	133
risch	$\frac{(8a^{2}x^{2}-22ax+33)(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{24a^{2}(ax-1)}-\frac{11\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{16a^{2}\sqrt{a^{2}c}\,(ax-1)}$	160

input
```
int((c-c/a/x)^(1/2)*x^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*a^(5/2)*x
^2*(x*(a*x+1))^(1/2)-44*a^(3/2)*x*(x*(a*x+1))^(1/2)+66*(x*(a*x+1))^(1/2)*a
^(1/2)-33*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(5/2)/(
a*x-1)/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.05

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^{2}\,dx$$
$$=\left[\frac{33\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(8\,a^{4}x^{4}-14\,a^{3}x^{3}+11\,a^{2}x^{2}-}{96\,(a^{4}x-a^{3})}\right.$$

input
```
integrate((c-c/a/x)^(1/2)*x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

output
```
[1/96*(33*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3
*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
- c)/(a*x - 1)) + 4*(8*a^4*x^4 - 14*a^3*x^3 + 11*a^2*x^2 + 33*a*x)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(33*(a*x -
 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 - 14*a^3*
x^3 + 11*a^2*x^2 + 33*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^4*x - a^3)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*x**2*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \int \sqrt{c-\frac{c}{ax}}x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int x^2\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.41

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\frac{\sqrt{c}\left(8\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2-22\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax+33\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}-33\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)\right)}{24a^3}$$

input
```
int((c-c/a/x)^(1/2)*x^2*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*(8*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**2*x**2 - 22*sqrt(x)*sqrt(a)*s
qrt(a*x + 1)*a*x + 33*sqrt(x)*sqrt(a)*sqrt(a*x + 1) - 33*log(sqrt(a*x + 1)
 + sqrt(x)*sqrt(a))))/(24*a**3)
```

3.530 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=-\frac{7c\sqrt{1-\frac{1}{a^2x^2}}x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}x^2}{2\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

output
```
-7/4*c*(1-1/a^2/x^2)^(1/2)*x/a/(c-c/a/x)^(1/2)+1/2*c*(1-1/a^2/x^2)^(1/2)*x
^2/(c-c/a/x)^(1/2)+7/4*c^(1/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/
x)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.12

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(-7+2ax)}{-4+4ax}-\frac{7\sqrt{c}\log(1-ax)}{8a^2}+\frac{7\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-1-ax+2a^2x^2)\right)}{8a^2}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*x)/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-7 + 2*a*x))/(-4 + 4*a*x) -
(7*Sqrt[c]*Log[1 - a*x])/(8*a^2) + (7*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1
/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(8*a^2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6733, 580, 579, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{ax}}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 580}$$

$$-\frac{-\frac{7c\int \frac{\sqrt{c-\frac{c}{ax}}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{4a}-\frac{c^2x^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{ 579}$$

$$-\frac{-\frac{7c\left(-\frac{\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{c^2x^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{ 573}$$

$$-\frac{-\frac{7c\left(\frac{c\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{c^2x^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{7c\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a}-\frac{c^2x^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}}{c}$$

input
```
Int[(Sqrt[c - c/(a*x)]*x)/E^ArcCoth[a*x],x]
```

output
```
-((-1/2*(c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/Sqrt[c - c/(a*x)] - (7*c*(-((c*Sqr
t[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[
1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a))/(4*a))/c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 579
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 1)*((a + b*x^2)^(p +
1)/(b*c*e*(m + 1))), x] - Simp[d*((n - m - 2)/(c*e*(m + 1)))   Int[(e*x)^(m
 + 1)*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0] && LtQ[m, -1] && (IntegerQ[2*p] |
| IntegerQ[m])
```

rule 580
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 2)*((a + b*x^2)^(p +
1)/(b*e*(m + 1))), x] + Simp[d*((2*m + p + 3)/(e*(m + 1)))   Int[(e*x)^(m +
 1)*(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p},
 x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p - 1, 0] && LtQ[m, -1] && IntegerQ
[p + 1/2]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4a^{\frac{3}{2}}x\sqrt{x(ax+1)}-14\sqrt{x(ax+1)}\,\sqrt{a}+7\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{8a^{\frac{3}{2}}(ax-1)\sqrt{x(ax+1)}}$	116
risch	$\frac{(2ax-7)(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{4a(ax-1)}+\frac{7\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{8a\sqrt{a^2c}\,(ax-1)}$	152

input
```
int((c-c/a/x)^(1/2)*x*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(4*a^(
3/2)*x*(x*(a*x+1))^(1/2)-14*(x*(a*x+1))^(1/2)*a^(1/2)+7*ln(1/2*(2*(x*(a*x+
1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x-1)/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.59

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$= \left[\frac{7\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(2\,a^3x^3-5\,a^2x^2-7\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{16\,(a^3x-a^2)}, -\frac{7\,(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(2\,a^3x^3-5\,a^2x^2-7\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{8\,(a^3x-a^2)}\right]$$

input
```
integrate((c-c/a/x)^(1/2)*x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
 c)/(a*x - 1)) + 4*(2*a^3*x^3 - 5*a^2*x^2 - 7*a*x)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2), -1/8*(7*(a*x - 1)*sqrt(-c)*arct
an(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(2*a^3*x^3 - 5*a^2*x^2 - 7*a*x)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int x\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*x*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))), x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\sqrt{c-\frac{c}{ax}}x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\sqrt{c-\frac{c}{ax}}x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int x\sqrt{c-\frac{c}{a\,x}}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.40

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$=\frac{\sqrt{c}\left(2\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax-7\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+7\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)\right)}{4a^2}$$

input
```
int((c-c/a/x)^(1/2)*x*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*(2*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x - 7*sqrt(x)*sqrt(a)*sqrt(a*x
 + 1) + 7*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))))/(4*a**2)
```

3.531 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
c*(1-1/a^2/x^2)^(1/2)*x/(c-c/a/x)^(1/2)-3*c^(1/2)*arctanh(c^(1/2)*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\,x-\frac{3\,\text{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x - (3*ArcTanh[Sqrt[1 + 1/(a*x)]])/a
))/Sqrt[1 - 1/(a*x)]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6731, 580, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{580}$$

$$-\frac{-\frac{3c\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{573}$$

$$-\frac{\frac{3c^2\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{a}-\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{3c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}-\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

output
```
-((-((c^2*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)]) + (3*c^(3/2)*ArcTanh
[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a)/c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 580
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[(-d^2)*(e*x)^(m + 1)*(c + d*x)^(n - 2)*((a + b*x^2)^(p +
1)/(b*e*(m + 1))), x] + Simp[d*((2*m + p + 3)/(e*(m + 1)))   Int[(e*x)^(m +
 1)*(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p},
 x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p - 1, 0] && LtQ[m, -1] && IntegerQ
[p + 1/2]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{x(ax+1)}\,\sqrt{a}-3\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2(ax-1)\sqrt{x(ax+1)}\,\sqrt{a}}$	101
risch	$\frac{(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}-\frac{3\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{2\sqrt{a^2c}\,(ax-1)}$	139

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*(x*(a*x+1))
^(1/2)*a^(1/2)-3*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a
*x-1)/(x*(a*x+1))^(1/2)/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(67) = 134$.

Time = 0.12 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.76

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$= \left[\frac{3\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)}, -\right.$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x)))/(a^2*x - a), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 -
 a*c*x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*x - a)]
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))), x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.42

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}-3\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)\right)}{a}$$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x + 1) - 3*log(sqrt(a*x + 1) + sqrt(x)*sq
rt(a))))/a
```

3.532 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 76

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + 2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

output
```
2*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)+2*c^(1/2)*arctanh(c^(1/2)*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.74

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$$
$$= \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x + \sqrt{c}(1-ax)\log(1-ax) + \sqrt{c}(-1+ax)\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2 + c(\right.}{-1+ax}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x),x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x + Sqrt[c]*(1 - a*x)*Log[1 -
 a*x] + Sqrt[c]*(-1 + a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c
- c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(-1 + a*x)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6733, 574, 573, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{ 6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 574}$$

$$-\frac{c\int \frac{\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{2c^2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{ 573}$$

$$-\frac{-2c^2\int \frac{1}{1-\frac{c}{x^2}}d\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{2c^2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{-2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)-\frac{2c^2\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x),x]
```

output
```
-(((-2*c^2*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] - 2*c^(3/2)*ArcTanh[(S
qrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 573
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2*c   Subst[Int[1/(a - c*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 574
```
Int[((e_.)*(x_))^(n_)*((c_) + (d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^2*(e*x)^(n + 1)*(c + d*x)^(m - 2)*((a + b*x^2)^(p + 1)/
(b*e*(n + p + 2))), x] + Simp[c*((2*n + p + 3)/(n + p + 2))   Int[(e*x)^n*(
c + d*x)^(m - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x
] && EqQ[b*c^2 + a*d^2, 0] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && IntegerQ
[2*p]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{\sqrt{\frac{a x -1}{a x +1}}\, \left(a x +1\right) \sqrt{\frac{c \left(a x -1\right)}{a x}}\, \left(\ln \left(\frac{2 \sqrt{x \left(a x +1\right)}\, \sqrt{a}+2 a x +1}{2 \sqrt{a}}\right) a x +2 \sqrt{x \left(a x +1\right)}\, \sqrt{a}\right)}{\left(a x -1\right) \sqrt{x \left(a x +1\right)}\, \sqrt{a}}$	100
risch	$\frac{2 \left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}\, \sqrt{\frac{c \left(a x -1\right)}{a x}}}{a x -1}+\frac{a \ln \left(\frac{\frac{1}{2} a c +a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}+a c x}\right) \sqrt{\frac{a x -1}{a x +1}}\, \sqrt{\frac{c \left(a x -1\right)}{a x}}\, \sqrt{\left(a x +1\right) a c x}}{\sqrt{a^{2} c}\, \left(a x -1\right)}$	139

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(ln(1/2*(2*(x*(a*x+1
))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x+2*(x*(a*x+1))^(1/2)*a^(1/2))/(a*x-1
)/(x*(a*x+1))^(1/2)/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(64) = 128$.

Time = 0.15 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.62

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(ax-1)}, -\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{ax-1}\right]$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) -
2*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x)))/x, x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
```

output
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.54

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{2\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)ax+ax\right)}{ax}$$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x)
```

output
```
(2*sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x + 1) + log(sqrt(a*x + 1) + sqrt(x)*sq
rt(a))*a*x + a*x))/(a*x)
```

3.533 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 70

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{8ac\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}} - \frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}$$

output
```
-8/3*a*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)-2/3*a*(1-1/a^2/x^2)^(1/2)*(c-
c/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.66

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(-1+5ax)}{-3+3ax}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^2),x]
```

output
```
(-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-1 + 5*a*x))/(-3 + 3*a*x)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6733, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \ 6733$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \ 459$$

$$-\frac{\frac{4}{3}c\int \frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2}{3}ac\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{c}$$

$$\Big\downarrow \ 458$$

$$-\frac{\frac{8ac^2\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}+\frac{2}{3}ac\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^2),x]
```

output
```
-(((8*a*c^2*Sqrt[1 - 1/(a^2*x^2)])/(3*Sqrt[c - c/(a*x)]) + (2*a*c*Sqrt[1 -
 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/3)/c)
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

method	result	size
orering	$-\frac{2(5ax-1)(ax+1)\sqrt{c-\frac{c}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{3x(ax-1)}$	52
gosper	$-\frac{2(ax+1)(5ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{3x(ax-1)}$	54
default	$-\frac{2(ax+1)(5ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{3x(ax-1)}$	54
risch	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,(5a^2x^2+4ax-1)}{3(ax-1)x}$	57

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(5*a*x-1)*(a*x+1)/x/(a*x-1)*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=-\frac{2\,(5\,a^2x^2+4\,ax-1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\,(ax^2-x)}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas
")
```

output
```
-2/3*(5*a^2*x^2 + 4*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x))/(a*x^2 - x)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x^2}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x)))/x**2, x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 13.61 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{2\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,(5\,a^2\,x^2+4\,a\,x-1)}{3\,x\,(a\,x-1)}$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
```

output
```
-(2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(4*a*x + 5*a^2*x^2 - 1
))/(3*x*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2\sqrt{c}\left(-5\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax+\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}+3a^2x^2\right)}{3a\,x^2}$$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

output
```
(2*sqrt(c)*( - 5*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x + sqrt(x)*sqrt(a)*sqrt(
a*x + 1) + 3*a**2*x**2))/(3*a*x**2)
```

3.534 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\frac{8a^2c\sqrt{1-\frac{1}{a^2x^2}}}{5\sqrt{c-\frac{c}{ax}}}+\frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}$$

output
```
8/5*a^2*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)+2/5*a^2*(1-1/a^2/x^2)^(1/2)*
(c-c/a/x)^(1/2)+2/5*a^2*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(3/2)/c
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.51

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(1-3ax+6a^2x^2)}{5x(-1+ax)}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^3),x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 - 3*a*x + 6*a^2*x^2))/(5*x
*(-1 + a*x))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6733, 572, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}x}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{572}$$

$$-\frac{-\frac{3}{5}a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{c}$$

$$\Big\downarrow \text{459}$$

$$-\frac{-\frac{3}{5}a\left(\frac{4}{3}c\int \frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2}{3}ac\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\right)-\frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{c}$$

$$\Big\downarrow \text{458}$$

$$-\frac{-\frac{3}{5}a\left(\frac{8ac^2\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}+\frac{2}{3}ac\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\right)-\frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^3),x]
```

output
```
-(((-3*a*((8*a*c^2*Sqrt[1 - 1/(a^2*x^2)])/(3*Sqrt[c - c/(a*x)]) + (2*a*c*S
qrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/3))/5 - (2*a^2*Sqrt[1 - 1/(a^2*x^2
)]*(c - c/(a*x))^(3/2))/5)/c)
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

rule 572
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 2))), x] + Simp[c*(n/(d
*(n + 2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && NeQ[n + 2*p + 2, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.53

method	result	size
orering	$\frac{2(6a^{2}x^{2}-3ax+1)(ax+1)\sqrt{c-\frac{c}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{5x^{2}(ax-1)}$	60
gosper	$\frac{2(ax+1)(6a^{2}x^{2}-3ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{5x^{2}(ax-1)}$	62
default	$\frac{2(ax+1)(6a^{2}x^{2}-3ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{5x^{2}(ax-1)}$	62
risch	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,(6a^{3}x^{3}+3a^{2}x^{2}-2ax+1)}{5(ax-1)x^{2}}$	65

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*(6*a^2*x^2-3*a*x+1)*(a*x+1)/x^2/(a*x-1)*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+
1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.61

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\,(6\,a^3x^3+3\,a^2x^2-2\,ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{5\,(ax^3-x^2)}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas
")
```

output
```
2/5*(6*a^3*x^3 + 3*a^2*x^2 - 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x))/(a*x^3 - x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^3, x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(6\,a^3\,x^3+3\,a^2\,x^2-2\,a\,x+1\right)}{5\,x^2\,\left(a\,x-1\right)}$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^3,x)
```

output
```
(2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3*a^2*x^2 - 2*a*x + 6*
a^3*x^3 + 1))/(5*x^2*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\sqrt{c}\left(6\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2-3\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax+\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}-6a^3x^3\right)}{5a\,x^3}$$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x)
```

output
```
(2*sqrt(c)*(6*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**2*x**2 - 3*sqrt(x)*sqrt(a)*
sqrt(a*x + 1)*a*x + sqrt(x)*sqrt(a)*sqrt(a*x + 1) - 6*a**3*x**3))/(5*a*x**
3)
```

3.535 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 152

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{152a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}-\frac{38}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{4a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c}-\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2}$$

output
```
-152/105*a^3*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)-38/105*a^3*(1-1/a^2/x^2
)^(1/2)*(c-c/a/x)^(1/2)+4/35*a^3*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(3/2)/c-2/7
*a^3*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(5/2)/c^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(-15+39ax-52a^2x^2+104a^3x^3)}{105x^2(-1+ax)}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^4),x]
```

output
```
(-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-15 + 39*a*x - 52*a^2*x^2 +
 104*a^3*x^3))/(105*x^2*(-1 + a*x))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6733, 574, 581, 27, 672, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{574}$$

$$-\frac{\frac{13}{7}c\int \frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}d\frac{1}{x}-\frac{2c^2\sqrt{1-\frac{1}{a^2x^2}}}{7x^3\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{581}$$

$$-\frac{\frac{13}{7}c\left(\frac{2a^2\int \frac{c^2\left(3a+\frac{2}{x}\right)\sqrt{c-\frac{c}{ax}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{5c^2}+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}\right)-\frac{2c^2\sqrt{1-\frac{1}{a^2x^2}}}{7x^3\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{13}{7}c\left(\frac{1}{5}a\int \frac{\left(3a+\frac{2}{x}\right)\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}\right)-\frac{2c^2\sqrt{1-\frac{1}{a^2x^2}}}{7x^3\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{672}$$

$$-\frac{\frac{13}{7}c\left(\frac{1}{5}a\left(\frac{7}{3}a\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{4}{3}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\right)+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}\right)-\frac{2c^2\sqrt{1-\frac{1}{a^2x^2}}}{7x^3\sqrt{c-\frac{c}{ax}}}}{c}$$

$$\Big\downarrow \text{458}$$

$$-\frac{\frac{13}{7}c\left(\frac{1}{5}a\left(\frac{14a^2c\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}-\frac{4}{3}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\right)+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}\right)-\frac{2c^2\sqrt{1-\frac{1}{a^2x^2}}}{7x^3\sqrt{c-\frac{c}{ax}}}}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^4),x]
```

output
```
-(((13*c*((a*((14*a^2*c*Sqrt[1 - 1/(a^2*x^2)])/(3*Sqrt[c - c/(a*x)]) - (4*
a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/3))/5 + (2*a^3*Sqrt[1 - 1/(a^
2*x^2)]*(c - c/(a*x))^(3/2))/(5*c)))/7 - (2*c^2*Sqrt[1 - 1/(a^2*x^2)])/(7*
Sqrt[c - c/(a*x)]*x^3))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 574
```
Int[((e_.)*(x_))^(n_)*((c_) + (d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^2*(e*x)^(n + 1)*(c + d*x)^(m - 2)*((a + b*x^2)^(p + 1)/
(b*e*(n + p + 2))), x] + Simp[c*((2*n + p + 3)/(n + p + 2))   Int[(e*x)^n*(
c + d*x)^(m - 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x
] && EqQ[b*c^2 + a*d^2, 0] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && IntegerQ
[2*p]
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 672
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Simp[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2))   Int[(d + e*x
)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^
2 + a*e^2, 0] && NeQ[m + 2*p + 2, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.45

method	result	size
orering	$-\frac{2(104a^3x^3-52a^2x^2+39ax-15)(ax+1)\sqrt{c-\frac{c}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{105x^3(ax-1)}$	68
gosper	$-\frac{2(ax+1)(104a^3x^3-52a^2x^2+39ax-15)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{105(ax-1)x^3}$	70
default	$-\frac{2(ax+1)(104a^3x^3-52a^2x^2+39ax-15)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{105(ax-1)x^3}$	70
risch	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,(104a^4x^4+52a^3x^3-13a^2x^2+24ax-15)}{105(ax-1)x^3}$	73

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/105*(104*a^3*x^3-52*a^2*x^2+39*a*x-15)*(a*x+1)/x^3/(a*x-1)*(c-c/a/x)^(1
/2)*((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.51

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{2\,(104\,a^4x^4+52\,a^3x^3-13\,a^2x^2+24\,ax-15)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{105\,(ax^4-x^3)}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas
")
```

output
```
-2/105*(104*a^4*x^4 + 52*a^3*x^3 - 13*a^2*x^2 + 24*a*x - 15)*sqrt((a*x - 1
)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^4, x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.66

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx= -\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(104\,a^3\,x^3+156\,a^2\,x^2+143\,a\,x+167\right)\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{105\,x^3} -\frac{304\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{105\,x^3\,\left(a\,x-1\right)}$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^4,x)
```

output
```
- (2*((a*x - 1)/(a*x + 1))^(1/2)*(143*a*x + 156*a^2*x^2 + 104*a^3*x^3 + 16
7)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3) - (304*((a*x - 1)/(a*x + 1))^(1/
2)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.53

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2\sqrt{c}\left(-104\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^3x^3+52\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,a^2x^2-39\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}\,ax+15\sqrt{x}\,\sqrt{a}\,\sqrt{ax+1}}{105a\,x^4}$$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x)
```

output
```
(2*sqrt(c)*( - 104*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a**3*x**3 + 52*sqrt(x)*sq
rt(a)*sqrt(a*x + 1)*a**2*x**2 - 39*sqrt(x)*sqrt(a)*sqrt(a*x + 1)*a*x + 15*
sqrt(x)*sqrt(a)*sqrt(a*x + 1) + 104*a**4*x**4))/(105*a*x**4)
```

3.536 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 172

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=-\frac{149\sqrt{c-\frac{c}{ax}}x}{64a^3}+\frac{107\sqrt{c-\frac{c}{ax}}x^2}{96a^2}-\frac{17\sqrt{c-\frac{c}{ax}}x^3}{24a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}x^4+\frac{363\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}-\frac{4\sqrt{2}\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

output

```
-149/64*(c-c/a/x)^(1/2)*x/a^3+107/96*(c-c/a/x)^(1/2)*x^2/a^2-17/24*(c-c/a/
x)^(1/2)*x^3/a+1/4*(c-c/a/x)^(1/2)*x^4+363/64*c^(1/2)*arctanh((c-c/a/x)^(1
/2)/c^(1/2))/a^4-4*2^(1/2)*c^(1/2)*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(
1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.67

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$
$$=\frac{a\sqrt{c-\frac{c}{ax}}x(-447+214ax-136a^2x^2+48a^3x^3)+1089\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-768\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{192a^4}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

output
```
(a*Sqrt[c - c/(a*x)]*x*(-447 + 214*a*x - 136*a^2*x^2 + 48*a^3*x^3) + 1089*
Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 768*Sqrt[2]*Sqrt[c]*ArcTanh[S
qrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(192*a^4)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.20, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.593, Rules used = {6717, 6683, 1070, 281, 948, 109, 27, 168, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}x^3dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}x^3(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}x^3}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{948}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^5}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a\left(-\frac{\int \frac{c^2\left(17a-\frac{15}{x}\right)x^4}{2a\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{4a}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(-\frac{c^2\int \frac{\left(17a-\frac{15}{x}\right)x^4}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{\int \frac{c\left(107a-\frac{85}{x}\right)x^3}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{3ac}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{\int \frac{\left(107a-\frac{85}{x}\right)x^3}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{6a}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{-\frac{\int \frac{3c\left(149a-\frac{107}{x}\right)x^2}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2ac}-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{2c}}{6a}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{-\frac{3\int \frac{\left(149a-\frac{107}{x}\right)x^2}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{4a}-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{2c}}{6a}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{-\frac{3\left(-\frac{\int \frac{c\left(363a-\frac{149}{x}\right)x}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{ac}-\frac{149x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{2c}}{6a}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{-\frac{3\left(-\frac{\int \frac{\left(363a-\frac{149}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{149x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{2c}}{6a}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \; 174$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{-\frac{3\left(-\frac{363\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-512\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{149x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{2c}}{6a}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$$\Big\downarrow \; 73$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{-\frac{3\left(-\frac{\frac{1024a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{726a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}}{2a}-\frac{149x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{2c}}{6a}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

$\big\downarrow$ 221

$$-\frac{a\left(-\frac{c^2\left(-\frac{-\frac{3\left(-\frac{\frac{512\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{726\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a}-\frac{149x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{2c}}{6a}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{3c}\right)}{8a^2}-\frac{cx^4\sqrt{c-\frac{c}{ax}}}{4a}\right)}{c}$$

input
```
Int[(Sqrt[c - c/(a*x)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

output
```
-((a*(-1/4*(c*Sqrt[c - c/(a*x)]*x^4)/a - (c^2*((-17*Sqrt[c - c/(a*x)]*x^3)
/(3*c) - ((-107*Sqrt[c - c/(a*x)]*x^2)/(2*c) - (3*((-149*Sqrt[c - c/(a*x)]
*x)/c - ((-726*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/Sqrt[c] + (512*Sqrt[2]*
ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/Sqrt[c])/(2*a)))/(4*a))/(6*a
)))/(8*a^2)))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.15

method	result
risch	$\frac{(48a^3x^3-136a^2x^2+214ax-447)x\sqrt{\frac{c(ax-1)}{ax}}}{192a^3}+\frac{\left(\frac{363\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{128a^3\sqrt{a^2c}}+\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x+\frac{1}{a}\right)^2a^2}}{x+\frac{1}{a}}\right.}{a^4\sqrt{c}}\right.}{ax-1}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-96x\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{9}{2}}\sqrt{\frac{1}{a}}+176\sqrt{\frac{1}{a}}\,\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{7}{2}}-252\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}x+768\sqrt{x(ax-1)}\,\sqrt{\frac{1}{a}}\,a^{\frac{5}{2}}+126\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-}\right.}{384\sqrt{x(a}}$

input
```
int((c-c/a/x)^(1/2)*x^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*a^3*x^3-136*a^2*x^2+214*a*x-447)/a^3*x*(c*(a*x-1)/a/x)^(1/2)+(36
3/128/a^3*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^
2*c)^(1/2)+2/a^4*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*(
(x+1/a)^2*a^2*c-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))/(a*x-1)*(c*(a*x-1)/a/x
)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.67

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$
$$=\left[\frac{768\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+2\left(48\,a^4x^4-136\,a^3x^3+214\,a^2x^2-447\,ax\right)\sqrt{\frac{acx-c}{ax}}+1089\,\sqrt{}}{384\,a^4}\right.$$

input
```
integrate((c-c/a/x)^(1/2)*x^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/384*(768*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) - 3*a*c*x + c)/(a*x + 1)) + 2*(48*a^4*x^4 - 136*a^3*x^3 + 214*a^2*x^2
 - 447*a*x)*sqrt((a*c*x - c)/(a*x)) + 1089*sqrt(c)*log(-2*a*c*x - 2*a*sqrt
(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^4, 1/192*(768*sqrt(2)*sqrt(-c)*arcta
n(sqrt(2)*a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) + (48*a^4*x^4
- 136*a^3*x^3 + 214*a^2*x^2 - 447*a*x)*sqrt((a*c*x - c)/(a*x)) - 1089*sqrt
(-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)))/a^4]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*x**3*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x**3*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}x^3}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*x^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))*x^3/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*x^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \int \frac{x^3\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x^3*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x^3*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$
$$= \frac{\sqrt{c}\,(48\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^3x^3 - 136\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^2x^2 + 214\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax - 447\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}}{}$$

input
```
int((c-c/a/x)^(1/2)*x^3*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*(48*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**3*x**3 - 136*sqrt(x)*sqrt(a)
*sqrt(a*x - 1)*a**2*x**2 + 214*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a*x - 447*sqr
t(x)*sqrt(a)*sqrt(a*x - 1) - 384*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(
a) - sqrt(2)*i + i) - 384*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a) + sq
rt(2)*i - i) + 384*sqrt(2)*log(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 2*sqrt(2)
 + 2*a*x + 2) + 1089*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))))/(192*a**4)
```

3.537 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 147

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{19\sqrt{c-\frac{c}{ax}}x}{8a^2}-\frac{13\sqrt{c-\frac{c}{ax}}x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3-\frac{45\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}+\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

output
```
19/8*(c-c/a/x)^(1/2)*x/a^2-13/12*(c-c/a/x)^(1/2)*x^2/a+1/3*(c-c/a/x)^(1/2)
*x^3-45/8*c^(1/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a^3+4*2^(1/2)*c^(1/2)*a
rctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.73

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\frac{a\sqrt{c-\frac{c}{ax}}x(57-26ax+8a^2x^2)-135\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+96\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{24a^3}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(a*Sqrt[c - c/(a*x)]*x*(57 - 26*a*x + 8*a^2*x^2) - 135*Sqrt[c]*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]] + 96*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sq
rt[2]*Sqrt[c])])/(24*a^3)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.18, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.519, Rules used = {6717, 6683, 1070, 281, 948, 109, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}x^2dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}x^2(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}x^2}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{948}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^4}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \; 109$$

$$-\frac{a\left(-\frac{\int \frac{c^2\left(13a-\frac{11}{x}\right)x^3}{2a\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{3a}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(-\frac{c^2\int \frac{\left(13a-\frac{11}{x}\right)x^3}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{6a^2}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

$$\Big\downarrow \; 168$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{\int \frac{3c\left(19a-\frac{13}{x}\right)x^2}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2ac}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a^2}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{3\int \frac{\left(19a-\frac{13}{x}\right)x^2}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{4a}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a^2}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

$$\Big\downarrow \; 168$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{3\left(-\frac{\int \frac{c\left(45a-\frac{19}{x}\right)x}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{ac}-\frac{19x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a^2}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

$\Big\downarrow$ 27

$$-\frac{a\left(-\frac{c^2\left(-\frac{3\left(-\frac{\int \frac{\left(45a-\frac{19}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{19x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a^2}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

$\Big\downarrow$ 174

$$-\frac{a\left(-\frac{c^2\left(-\frac{3\left(-\frac{45\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-64\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{19x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a^2}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

$\Big\downarrow$ 73

$$-\frac{a\left(-\frac{c^2\left(-\frac{3\left(-\frac{-\frac{128a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{90a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}}{2a}-\frac{19x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a^2}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

$$\Big\downarrow \; 221$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{3\left(-\frac{\frac{64\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{90\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a}-\frac{19x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{2c}\right)}{6a^2}-\frac{cx^3\sqrt{c-\frac{c}{ax}}}{3a}\right)}{c}$$

input
```
Int[(Sqrt[c - c/(a*x)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-((a*(-1/3*(c*Sqrt[c - c/(a*x)]*x^3)/a - (c^2*((-13*Sqrt[c - c/(a*x)]*x^2)
/(2*c) - (3*((-19*Sqrt[c - c/(a*x)]*x)/c - ((-90*ArcTanh[Sqrt[c - c/(a*x)]
/Sqrt[c]])/Sqrt[c] + (64*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c
]])]/Sqrt[c])/(2*a)))/(4*a)))/(6*a^2)))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.29

method	result
risch	$\frac{(8a^2x^2-26ax+57)x\sqrt{\frac{c(ax-1)}{ax}}}{24a^2}+\frac{\left(-\frac{45\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{16a^2\sqrt{a^2c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-3\left(x+\frac{1}{a}\right)ac}}{x+\frac{1}{a}}\right)}{a^3\sqrt{c}}\right.}{ax-1}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16\sqrt{\frac{1}{a}}\,(a\,x^2-x)^{\frac{3}{2}}a^{\frac{7}{2}}-36\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}x+96\sqrt{x(ax-1)}\,\sqrt{\frac{1}{a}}\,a^{\frac{5}{2}}+18\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}-96\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax-1)}\,c}{ax+1}\right.\right.}{48\sqrt{x(ax-1)}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}}$

input
```
int((c-c/a/x)^(1/2)*x^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*a^2*x^2-26*a*x+57)/a^2*x*(c*(a*x-1)/a/x)^(1/2)+(-45/16/a^2*ln((-1/
2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-2/a^3*
2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*((x+1/a)^2*a^2*c-3
*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1
)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.88

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\left[\frac{96\sqrt{2}\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+2\left(8\,a^3x^3-26\,a^2x^2+57\,ax\right)\sqrt{\frac{acx-c}{ax}}+135\sqrt{c}\log\left(-2\,acx+\right.}{48\,a^3}\right.$$
$$-\frac{96\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)-\left(8\,a^3x^3-26\,a^2x^2+57\,ax\right)\sqrt{\frac{acx-c}{ax}}-135\sqrt{-c}\arctan\left(\frac{a\sqrt{-c}x\sqrt{\cdots}}{acx-\cdots}\right.}{24\,a^3}$$

input
```
integrate((c-c/a/x)^(1/2)*x^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/48*(96*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) + 3*a*c*x - c)/(a*x + 1)) + 2*(8*a^3*x^3 - 26*a^2*x^2 + 57*a*x)*sqrt((
a*c*x - c)/(a*x)) + 135*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x -
 c)/(a*x)) + c))/a^3, -1/24*(96*sqrt(2)*sqrt(-c)*arctan(sqrt(2)*a*sqrt(-c)
*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) - (8*a^3*x^3 - 26*a^2*x^2 + 57*a*x
)*sqrt((a*c*x - c)/(a*x)) - 135*sqrt(-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x -
 c)/(a*x))/(a*c*x - c)))/a^3]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*x**2*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}x^2}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*x^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))*x^2/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*x^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \int \frac{x^2\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x^2*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x^2*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.96

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$= \frac{\sqrt{c}\left(8\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^2x^2-26\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax+57\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}+48\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}\right.\right.}{}$$

input
```
int((c-c/a/x)^(1/2)*x^2*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*(8*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**2*x**2 - 26*sqrt(x)*sqrt(a)*s
qrt(a*x - 1)*a*x + 57*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 48*sqrt(2)*log(sqrt(
a*x - 1) + sqrt(x)*sqrt(a) - sqrt(2)*i + i) + 48*sqrt(2)*log(sqrt(a*x - 1)
 + sqrt(x)*sqrt(a) + sqrt(2)*i - i) - 48*sqrt(2)*log(2*sqrt(x)*sqrt(a)*sqr
t(a*x - 1) + 2*sqrt(2) + 2*a*x + 2) - 135*log(sqrt(a*x - 1) + sqrt(x)*sqrt
(a))))/(24*a**3)
```

3.538 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 122

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=-\frac{9\sqrt{c-\frac{c}{ax}}x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}x^2+\frac{23\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}-\frac{4\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

output
```
-9/4*(c-c/a/x)^(1/2)*x/a+1/2*(c-c/a/x)^(1/2)*x^2+23/4*c^(1/2)*arctanh((c-c
/a/x)^(1/2)/c^(1/2))/a^2-4*2^(1/2)*c^(1/2)*arctanh(1/2*(c-c/a/x)^(1/2)*2^(
1/2)/c^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\frac{a\sqrt{c-\frac{c}{ax}}x(-9+2ax)+23\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-16\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4a^2}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*x)/E^(2*ArcCoth[a*x]),x]
```

output
```
(a*Sqrt[c - c/(a*x)]*x*(-9 + 2*a*x) + 23*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]
/Sqrt[c]] - 16*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])
])/(4*a^2)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {6717, 6683, 1070, 281, 948, 109, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}xdx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}x(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{ 1070}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}x}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{ 948}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \ 109$$

$$-\frac{a\left(-\frac{\int \frac{c^2\left(9a-\frac{7}{x}\right)x^2}{2a\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{cx^2\sqrt{c-\frac{c}{ax}}}{2a}\right)}{c}$$

$$\Big\downarrow \ 27$$

$$-\frac{a\left(-\frac{c^2\int \frac{\left(9a-\frac{7}{x}\right)x^2}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{4a^2}-\frac{cx^2\sqrt{c-\frac{c}{ax}}}{2a}\right)}{c}$$

$$\Big\downarrow \ 168$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{\int \frac{c\left(23a-\frac{9}{x}\right)x}{2\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{ac}-\frac{9x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a^2}-\frac{cx^2\sqrt{c-\frac{c}{ax}}}{2a}\right)}{c}$$

$$\Big\downarrow \ 27$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{\int \frac{\left(23a-\frac{9}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{9x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a^2}-\frac{cx^2\sqrt{c-\frac{c}{ax}}}{2a}\right)}{c}$$

$$\Big\downarrow \ 174$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{23\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-32\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a}-\frac{9x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a^2}-\frac{cx^2\sqrt{c-\frac{c}{ax}}}{2a}\right)}{c}$$

$$\Big\downarrow \ 73$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{\frac{64a\int \frac{1}{2a-\frac{a}{\frac{cx^2}{c}}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{46a\int \frac{1}{a-\frac{a}{\frac{cx^2}{c}}}d\sqrt{c-\frac{c}{ax}}}{c}}{2a}-\frac{9x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a^2}-\frac{cx^2\sqrt{c-\frac{c}{ax}}}{2a}\right)}{c}$$

$$\Big\downarrow \text{221}$$

$$-\frac{a\left(-\frac{c^2\left(-\frac{\frac{32\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{46\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a}-\frac{9x\sqrt{c-\frac{c}{ax}}}{c}\right)}{4a^2}-\frac{cx^2\sqrt{c-\frac{c}{ax}}}{2a}\right)}{c}$$

input
```
Int[(Sqrt[c - c/(a*x)]*x)/E^(2*ArcCoth[a*x]),x]
```

output
```
-((a*(-1/2*(c*Sqrt[c - c/(a*x)]*x^2)/a - (c^2*((-9*Sqrt[c - c/(a*x)]*x)/c
- ((-46*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/Sqrt[c] + (32*Sqrt[2]*ArcTanh[
Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/Sqrt[c])/(2*a)))/(4*a^2)))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.48

method	result
risch	$\frac{(2ax-9)x\sqrt{\frac{c(ax-1)}{ax}}}{4a}+\frac{\left(\frac{23\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{8a\sqrt{a^2c}}+\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{a^2\sqrt{c}}\right)\sqrt{\frac{c(}{}}}{ax-1}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-4\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}x+16\sqrt{x(ax-1)}\,\sqrt{\frac{1}{a}}\,a^{\frac{5}{2}}+2\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}-16\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax-1)}\,a-3ax+1}{ax+1}\right)a^{\frac{3}{2}}\sqrt{2}-24\right.}{8\sqrt{x(ax-1)}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}}$

input
```
int((c-c/a/x)^(1/2)*x*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*a*x-9)/a*x*(c*(a*x-1)/a/x)^(1/2)+(23/8/a*ln((-1/2*a*c+a^2*c*x)/(a^2
*c)^(1/2)+(a^2*c*x^2-a*c*x)^(1/2))/(a^2*c)^(1/2)+2/a^2*2^(1/2)/c^(1/2)*ln(
(4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*((x+1/a)^2*a^2*c-3*(x+1/a)*a*c+2*c)^(
1/2))/(x+1/a)))/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.10

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$= \left[\frac{16\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+2\left(2\,a^2x^2-9\,ax\right)\sqrt{\frac{acx-c}{ax}}+23\,\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}\right.}{8\,a^2}\right.$$

input
```
integrate((c-c/a/x)^(1/2)*x*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/8*(16*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)
) - 3*a*c*x + c)/(a*x + 1)) + 2*(2*a^2*x^2 - 9*a*x)*sqrt((a*c*x - c)/(a*x)
) + 23*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/
a^2, 1/4*(16*sqrt(2)*sqrt(-c)*arctan(sqrt(2)*a*sqrt(-c)*x*sqrt((a*c*x - c)
/(a*x))/(a*c*x - c)) + (2*a^2*x^2 - 9*a*x)*sqrt((a*c*x - c)/(a*x)) - 23*sq
rt(-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)))/a^2]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*x*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}x}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*x*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))*x/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*x*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\frac{x\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$= \frac{\sqrt{c}\left(2\sqrt{x}\sqrt{a}\sqrt{ax-1}\,ax-9\sqrt{x}\sqrt{a}\sqrt{ax-1}-8\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{x}\sqrt{a}-\sqrt{2}\,i+i\right)-8\sqrt{2}\log\left(\sqrt{ax\right.$$

input
```
int((c-c/a/x)^(1/2)*x*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a*x - 9*sqrt(x)*sqrt(a)*sqrt(a*x
 - 1) - 8*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a) - sqrt(2)*i + i) - 8
*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a) + sqrt(2)*i - i) + 8*sqrt(2)*
log(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 2*sqrt(2) + 2*a*x + 2) + 23*log(sqrt
(a*x - 1) + sqrt(x)*sqrt(a))))/(4*a**2)
```

3.539 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x-\frac{5\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

output
```
(c-c/a/x)^(1/2)*x-5*c^(1/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+4*2^(1/2)*c
^(1/2)*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\sqrt{c-\frac{c}{ax}}x-\frac{5\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

output
```
Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (
4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6717, 6683, 1035, 281, 899, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \; 109$$

$$-\frac{a\left(-\frac{\int \frac{c^2\left(5a-\frac{3}{x}\right)x}{2a\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(-\frac{c^2\int \frac{\left(5a-\frac{3}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{2a^2}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \; 174$$

$$-\frac{a\left(-\frac{c^2\left(5\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-8\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}\right)}{2a^2}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \; 73$$

$$-\frac{a\left(-\frac{c^2\left(\frac{16a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{10a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}\right)}{2a^2}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \; 221$$

$$-\frac{a\left(-\frac{c^2\left(\frac{8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{10\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{2a^2}-\frac{cx\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

output
```
-((a*(-((c*Sqrt[c - c/(a*x)]*x)/a) - (c^2*((-10*ArcTanh[Sqrt[c - c/(a*x)]/
Sqrt[c]])/Sqrt[c] + (8*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])
])/Sqrt[c]))/(2*a^2)))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(75) = 150$.

Time = 0.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.83

method	result
risch	$x\sqrt{\frac{c(ax-1)}{ax}}+\frac{\left(-\frac{5\ln\left(\frac{-\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{a\sqrt{c}}\right)\sqrt{c(ax-1)}}{ax-1}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{x(ax-1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-2\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax-1)}\,a-3ax+1}{ax+1}\right)\sqrt{a}\right.}{2\sqrt{x(ax-1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
x*(c*(a*x-1)/a/x)^(1/2)+(-5/2*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x
^2-a*c*x)^(1/2))/(a^2*c)^(1/2)-2/a*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2
*2^(1/2)*c^(1/2)*((x+1/a)^2*a^2*c-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))*(c*(
a*x-1)*a*x)^(1/2)*(c*(a*x-1)/a/x)^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.57

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$

$$=\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+4\,\sqrt{2}\sqrt{c}\log\left(-\frac{2\,\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+5\,\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\ \frac{ax\sqrt{\vphantom{x}}}{}\right.$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 4*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*
sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 5*sqrt(c)*lo
g(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c
*x - c)/(a*x)) - 4*sqrt(2)*sqrt(-c)*arctan(sqrt(2)*a*sqrt(-c)*x*sqrt((a*c*
x - c)/(a*x))/(a*c*x - c)) + 5*sqrt(-c)*arctan(a*sqrt(-c)*x*sqrt((a*c*x -
c)/(a*x))/(a*c*x - c)))/a]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=\frac{\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}+2\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}\,i+i\right)+2\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}+\sqrt{2}\,i-\right.\right.}{a}$$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 2*sqrt(2)*log(sqrt(a*x - 1) + sq
rt(x)*sqrt(a) - sqrt(2)*i + i) + 2*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqr
t(a) + sqrt(2)*i - i) - 2*sqrt(2)*log(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 2*
sqrt(2) + 2*a*x + 2) - 5*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))))/a
```

3.540 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 86

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = 2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

output
```
2*(c-c/a/x)^(1/2)+2*c^(1/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))-4*2^(1/2)*c^(
1/2)*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = 2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 4*Sqr
t[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6717, 6683, 1070, 281, 948, 95, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x(ax+1)}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}}{\left(a+\frac{1}{x}\right)x}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x}dx}{c}$$

$$\Big\downarrow \text{948}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{95}$$

$$-\frac{a\left(\int \frac{c^2\left(a-\frac{3}{x}\right)x}{a\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-\frac{2c\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\left(\frac{c^2\int \frac{\left(a-\frac{3}{x}\right)x}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}}{a}-\frac{2c\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \; 174$$

$$-\frac{a\left(\frac{c^2\left(\int \frac{x}{\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}-4\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}\right)}{a}-\frac{2c\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \; 73$$

$$-\frac{a\left(\frac{c^2\left(\frac{8a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}-\frac{2a\int \frac{1}{a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}}{c}\right)}{a}-\frac{2c\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

$$\Big\downarrow \; 221$$

$$-\frac{a\left(\frac{c^2\left(\frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{a}-\frac{2c\sqrt{c-\frac{c}{ax}}}{a}\right)}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
-((a*((-2*c*Sqrt[c - c/(a*x)])/a + (c^2*((-2*ArcTanh[Sqrt[c - c/(a*x)]/Sqr
t[c]])/Sqrt[c] + (4*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/
Sqrt[c]))/a))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(69) = 138$.

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.92

method	result
risch	$2\sqrt{\frac{c(ax-1)}{ax}}+\frac{\left(\frac{a\ln\left(\frac{-\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{\sqrt{a^{2}c}}+\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x+\frac{1}{a}\right)^{2}a^{2}c-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{\sqrt{c}}\right)\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-4\sqrt{a\,x^{2}-x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,x^{2}+2\sqrt{x(ax-1)}\,x^{2}a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+2\left(a\,x^{2}-x\right)^{\frac{3}{2}}\sqrt{a}\,\sqrt{\frac{1}{a}}+2\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a\,x^{2}-2\sqrt{a}\right.}{x\sqrt{x(ax-1)}\,\sqrt{a}\,\sqrt{\frac{1}{a}}}$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(c*(a*x-1)/a/x)^(1/2)+(a*ln((-1/2*a*c+a^2*c*x)/(a^2*c)^(1/2)+(a^2*c*x^2-
a*c*x)^(1/2))/(a^2*c)^(1/2)+2*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1
/2)*c^(1/2)*((x+1/a)^2*a^2*c-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a)))/(a*x-1)*(
c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.56

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \left[2\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right) + \sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right) + 2\sqrt{\frac{acx-c}{ax}}, 4\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right) - 2\sqrt{-c}\arctan\left(\frac{a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right) + 2\sqrt{\frac{acx-c}{ax}}\right]$$

input

```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

output

```
[2*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) - 3*
a*c*x + c)/(a*x + 1)) + sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x -
 c)/(a*x)) + c) + 2*sqrt((a*c*x - c)/(a*x)), 4*sqrt(2)*sqrt(-c)*arctan(sqr
t(2)*a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) - 2*sqrt(-c)*arctan
(a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) + 2*sqrt((a*c*x - c)/(a
*x))]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(70) = 140$.

Time = 4.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.64

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \begin{cases} -\frac{2a\left(\frac{c^2\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\sqrt{2}c^2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c-\frac{c}{ax}}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{c\sqrt{c-\frac{c}{ax}}}{a}\right)}{c} & \text{for}\ \frac{c}{a}\neq 0 \\ -\frac{3a\sqrt{c}\left(\frac{\log\left(-\frac{2}{x}\right)}{a}-\frac{\log\left(2a+\frac{2}{x}\right)}{a}\right)}{2}+\frac{\sqrt{c}\log\left(\frac{a}{x}+\frac{1}{x^2}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

output
```
Piecewise((-2*a*(c**2*atan(sqrt(c - c/(a*x))/sqrt(-c))/(a*sqrt(-c)) - 2*sq
rt(2)*c**2*atan(sqrt(2)*sqrt(c - c/(a*x))/(2*sqrt(-c)))/(a*sqrt(-c)) - c*s
qrt(c - c/(a*x))/a)/c, Ne(c/a, 0)), (-3*a*sqrt(c)*(log(-2/x)/a - log(2*a +
 2/x)/a)/2 + sqrt(c)*log(a/x + x**(-2))/2, True))
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{x\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.40

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{2\sqrt{c}\left(\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}-\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}\,i+i\right)ax-\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}+\sqrt{2}\,i\right.}{ax}$$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x)
```

output
```
(2*sqrt(c)*(sqrt(x)*sqrt(a)*sqrt(a*x - 1) - sqrt(2)*log(sqrt(a*x - 1) + sq
rt(x)*sqrt(a) - sqrt(2)*i + i)*a*x - sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*s
qrt(a) + sqrt(2)*i - i)*a*x + sqrt(2)*log(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1)
+ 2*sqrt(2) + 2*a*x + 2)*a*x + log(sqrt(a*x - 1) + sqrt(x)*sqrt(a))*a*x +
a*x))/(a*x)
```

3.541 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4\sqrt{2}a\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

output
```
-4*a*(c-c/a/x)^(1/2)-2/3*a*(c-c/a/x)^(3/2)/c+4*2^(1/2)*a*c^(1/2)*arctanh(1
/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2\sqrt{c-\frac{c}{ax}}(1-7ax)}{3x}+4\sqrt{2}a\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
(2*Sqrt[c - c/(a*x)]*(1 - 7*a*x))/(3*x) + 4*Sqrt[2]*a*Sqrt[c]*ArcTanh[Sqrt
[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6683, 1070, 281, 946, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^2(ax+1)}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}}{\left(a+\frac{1}{x}\right)x^2}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^2}dx}{c}$$

$$\Big\downarrow \text{946}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a\left(2c\int \frac{\sqrt{c-\frac{c}{ax}}}{a+\frac{1}{x}}d\frac{1}{x}+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{c}$$

$\Big\downarrow$ 60

$$-\frac{a\left(2c\left(2c\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+2\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{c}$$

$\Big\downarrow$ 73

$$-\frac{a\left(2c\left(2\sqrt{c-\frac{c}{ax}}-4a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{c}$$

$\Big\downarrow$ 221

$$-\frac{a\left(2c\left(2\sqrt{c-\frac{c}{ax}}-2\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
-((a*((2*(c - c/(a*x))^(3/2))/3 + 2*c*(2*Sqrt[c - c/(a*x)] - 2*Sqrt[2]*Sqr
t[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])))/c)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 946

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1070

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(67) = 134$.

Time = 0.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.73

method	result
risch	$-\frac{2(7a^2x^2-8ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{3x(ax-1)}-\frac{2a\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)ax}}{\sqrt{c}\,(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-18\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x^3+6\sqrt{x(ax-1)}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x^3+12(a\,x^2-x)^{\frac{3}{2}}a^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,x+9\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a^2x^3-6a^{\frac{3}{2}}\right.}{3x^2\sqrt{x(ax-1)}\,\sqrt{a}\,\sqrt{\frac{1}{a}}}$

input

```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

output

```
-2/3*(7*a^2*x^2-8*a*x+1)/x/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)-2*a*2^(1/2)/c^(1/
2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*((x+1/a)^2*a^2*c-3*(x+1/a)*a*c+
2*c)^(1/2))/(x+1/a))/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.06

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \left[\frac{2\left(3\sqrt{2}a\sqrt{c}x\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-(7\,ax-1)\sqrt{\frac{acx-c}{ax}}\right)}{3\,x},\right.$$
$$\left.-\frac{2\left(6\sqrt{2}a\sqrt{-c}x\arctan\left(\frac{\sqrt{2}a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)+(7\,ax-1)\sqrt{\frac{acx-c}{ax}}\right)}{3\,x}\right]$$

input

```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

output
```
[2/3*(3*sqrt(2)*a*sqrt(c)*x*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(
a*x)) + 3*a*c*x - c)/(a*x + 1)) - (7*a*x - 1)*sqrt((a*c*x - c)/(a*x)))/x,
-2/3*(6*sqrt(2)*a*sqrt(-c)*x*arctan(sqrt(2)*a*sqrt(-c)*x*sqrt((a*c*x - c)/
(a*x))/(a*c*x - c)) + (7*a*x - 1)*sqrt((a*c*x - c)/(a*x)))/x]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{x^2\,(ax+1)}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**2*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^2}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{x^2\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.65

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$$
$$= \frac{2\sqrt{c}\left(-7\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax+\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}+3\sqrt{2}\log\left(\sqrt{ax-1}+\sqrt{x}\,\sqrt{a}-\sqrt{2}\,i+i\right)a^2x^2+3\sqrt{2}\,\mathrm{lo}\right.}{3a\,x}$$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x)
```

output
```
(2*sqrt(c)*( - 7*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a*x + sqrt(x)*sqrt(a)*sqrt(
a*x - 1) + 3*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a) - sqrt(2)*i + i)*
a**2*x**2 + 3*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a) + sqrt(2)*i - i)
*a**2*x**2 - 3*sqrt(2)*log(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 2*sqrt(2) + 2
*a*x + 2)*a**2*x**2 + a**2*x**2))/(3*a*x**2)
```

3.542 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=4a^2\sqrt{c-\frac{c}{ax}}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-4\sqrt{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

output
```
4*a^2*(c-c/a/x)^(1/2)+2/3*a^2*(c-c/a/x)^(3/2)/c+2/5*a^2*(c-c/a/x)^(5/2)/c^
2-4*2^(1/2)*a^2*c^(1/2)*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.70

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=\frac{2\sqrt{c-\frac{c}{ax}}(3-11ax+38a^2x^2)}{15x^2}-4\sqrt{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
(2*Sqrt[c - c/(a*x)]*(3 - 11*a*x + 38*a^2*x^2))/(15*x^2) - 4*Sqrt[2]*a^2*S
qrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6717, 6683, 1070, 281, 948, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^3(ax+1)}dx$$

$$\Big\downarrow \text{1070}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}}{\left(a+\frac{1}{x}\right)x^3}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^3}dx}{c}$$

$$\Big\downarrow \text{948}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{90}$$

$$-\frac{a\left(-a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}d\frac{1}{x}-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{5c}\right)}{c}$$

$$\Big\downarrow \ 60$$

$$-\frac{a\left(-a\left(2c\int \frac{\sqrt{c-\frac{c}{ax}}}{a+\frac{1}{x}}d\frac{1}{x}+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{5c}\right)}{c}$$

$$\Big\downarrow \ 60$$

$$-\frac{a\left(-a\left(2c\left(2c\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}d\frac{1}{x}+2\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{5c}\right)}{c}$$

$$\Big\downarrow \ 73$$

$$-\frac{a\left(-a\left(2c\left(2\sqrt{c-\frac{c}{ax}}-4a\int \frac{1}{2a-\frac{a}{cx^2}}d\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{5c}\right)}{c}$$

$$\Big\downarrow \ 221$$

$$-\frac{a\left(-a\left(2c\left(2\sqrt{c-\frac{c}{ax}}-2\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)\right)+\frac{2}{3}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{5c}\right)}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
-((a*((-2*a*(c - c/(a*x))^(5/2))/(5*c) - a*((2*(c - c/(a*x))^(3/2))/3 + 2*
c*(2*Sqrt[c - c/(a*x)] - 2*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt
[2]*Sqrt[c])]))))/c)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.35

method	result
risch	$\frac{2(38a^3x^3-49a^2x^2+14ax-3)\sqrt{\frac{c(ax-1)}{ax}}}{15x^2(ax-1)}+\frac{2a^2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c(ax-1)}}{\sqrt{c}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-90\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^4+30a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax-1)}\,x^4+60\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x^2+45\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a^3x^4}{15x^3\sqrt{}}$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/15*(38*a^3*x^3-49*a^2*x^2+14*a*x-3)/x^2/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)+2*
a^2*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*((x+1/a)^2*a^2
*c-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a))/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*
x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.67

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \left[\frac{2\left(15\sqrt{2}a^2\sqrt{c}x^2\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+\left(38\,a^2x^2-11\,ax+3\right)\sqrt{\frac{acx-c}{ax}}\right)}{15\,x^2}, \frac{2\left(30\sqrt{2}a^2\sqrt{-c}x^2\,\mathrm{ar}\right.}{}\right.$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

output
```
[2/15*(15*sqrt(2)*a^2*sqrt(c)*x^2*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
 c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + (38*a^2*x^2 - 11*a*x + 3)*sqrt((a*c
*x - c)/(a*x)))/x^2, 2/15*(30*sqrt(2)*a^2*sqrt(-c)*x^2*arctan(sqrt(2)*a*sq
rt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) + (38*a^2*x^2 - 11*a*x + 3)*
sqrt((a*c*x - c)/(a*x)))/x^2]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{x^3\,(ax+1)}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**3*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^3}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(94) = 188$.

Time = 0.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.46

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{4\sqrt{2}a^3c\arctan\left(-\frac{\sqrt{2}\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}|a|\right)}{2a\sqrt{-c}}\right)}{\sqrt{-c}|a|\text{sgn}(x)} + \frac{2\left(60\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^4a^5c-45\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^3a^4c^{\frac{3}{2}}|a|+35\left(\sqrt{a^2c}x-\sqrt{a^2cx^2}\right.}{15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^5a^2|a|\text{sgn}\left(x\right.}$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

output
```
-4*sqrt(2)*a^3*c*arctan(-1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*
c*x))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) + 2/15*(6
0*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a^5*c - 45*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - a*c*x))^3*a^4*c^(3/2)*abs(a) + 35*(sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - a*c*x))^2*a^5*c^2 - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x)
)*a^4*c^(5/2)*abs(a) + 3*a^5*c^3)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x
))^5*a^2*abs(a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{x^3\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.37

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$$
$$= \frac{2\sqrt{c}\,(38\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^2x^2 - 11\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax + 3\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1} - 15\sqrt{2}\log\big(\sqrt{ax-1} + \sqrt{x}\,\sqrt{}$$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x)
```

output
```
(2*sqrt(c)*(38*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**2*x**2 - 11*sqrt(x)*sqrt(a
)*sqrt(a*x - 1)*a*x + 3*sqrt(x)*sqrt(a)*sqrt(a*x - 1) - 15*sqrt(2)*log(sqr
t(a*x - 1) + sqrt(x)*sqrt(a) - sqrt(2)*i + i)*a**3*x**3 - 15*sqrt(2)*log(s
qrt(a*x - 1) + sqrt(x)*sqrt(a) + sqrt(2)*i - i)*a**3*x**3 + 15*sqrt(2)*log
(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 2*sqrt(2) + 2*a*x + 2)*a**3*x**3 - 14*a
**3*x**3))/(15*a*x**3)
```

3.543 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 113

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -4a^3\sqrt{c-\frac{c}{ax}}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+4\sqrt{2}a^3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

output
```
-4*a^3*(c-c/a/x)^(1/2)-2/3*a^3*(c-c/a/x)^(3/2)/c-2/7*a^3*(c-c/a/x)^(7/2)/c
^3+4*2^(1/2)*a^3*c^(1/2)*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.77

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2\sqrt{c-\frac{c}{ax}}(3-9ax+16a^2x^2-52a^3x^3)}{21x^3}+4\sqrt{2}a^3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
(2*Sqrt[c - c/(a*x)]*(3 - 9*a*x + 16*a^2*x^2 - 52*a^3*x^3))/(21*x^3) + 4*S
qrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6717, 6683, 1070, 281, 948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}dx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^4(ax+1)}dx$$

$$\Big\downarrow \text{ 1070}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}}{\left(a+\frac{1}{x}\right)x^4}dx$$

$$\Big\downarrow \text{ 281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^4}dx}{c}$$

$$\Big\downarrow \text{ 948}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{a\int\left(\frac{a^2\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}-\frac{a\left(c-\frac{c}{ax}\right)^{5/2}}{c}\right)d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(-4\sqrt{2}a^2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)+\frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}}{7c^2}+\frac{2}{3}a^2\left(c-\frac{c}{ax}\right)^{3/2}+4a^2c\sqrt{c-\frac{c}{ax}}\right)}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
-((a*(4*a^2*c*Sqrt[c - c/(a*x)] + (2*a^2*(c - c/(a*x))^(3/2))/3 + (2*a^2*(
c - c/(a*x))^(7/2))/(7*c^2) - 4*Sqrt[2]*a^2*c^(3/2)*ArcTanh[Sqrt[c - c/(a*
x)]/(Sqrt[2]*Sqrt[c])]))/c)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.42

method	result
risch	$-\frac{2\left(52a^4x^4-68a^3x^3+25a^2x^2-12ax+3\right)\sqrt{\frac{c(ax-1)}{ax}}}{21x^3(ax-1)}-\frac{2a^3\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)\sqrt{\frac{c(ax-1}{ax}}}{\sqrt{c}\,(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-126\sqrt{a\,x^2-x}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^5+42a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax-1)}\,x^5+84\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^3+63\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a^4x^5\right.}{}$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/21*(52*a^4*x^4-68*a^3*x^3+25*a^2*x^2-12*a*x+3)/x^3/(a*x-1)*(c*(a*x-1)/a
/x)^(1/2)-2*a^3*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*2^(1/2)*c^(1/2)*((
x+1/a)^2*a^2*c-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a))/(a*x-1)*(c*(a*x-1)/a/x)^
(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.85

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$$
$$= \left[\frac{2\left(21\sqrt{2}a^3\sqrt{c}x^3\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-(52\,a^3x^3-16\,a^2x^2+9\,ax-3)\sqrt{\frac{acx-c}{ax}}\right)}{21\,x^3},\right.$$
$$\left.-\frac{2\left(42\sqrt{2}a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{2}a\sqrt{-c}x\sqrt{\frac{acx-c}{ax}}}{acx-c}\right)+(52\,a^3x^3-16\,a^2x^2+9\,ax-3)\sqrt{\frac{acx-c}{ax}}\right)}{21\,x^3}\right]$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

output
```
[2/21*(21*sqrt(2)*a^3*sqrt(c)*x^3*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
- c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) - (52*a^3*x^3 - 16*a^2*x^2 + 9*a*x -
 3)*sqrt((a*c*x - c)/(a*x)))/x^3, -2/21*(42*sqrt(2)*a^3*sqrt(-c)*x^3*arcta
n(sqrt(2)*a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x - c)) + (52*a^3*x^3
- 16*a^2*x^2 + 9*a*x - 3)*sqrt((a*c*x - c)/(a*x)))/x^3]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}(ax-1)}{x^4\,(ax+1)}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**4*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^4}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(94) = 188.

Time = 0.29 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.15

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{4\sqrt{2}a^4c\arctan\left(-\frac{\sqrt{2}\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}|a|\right)}{2a\sqrt{-c}}\right)}{\sqrt{-c}|a|\mathrm{sgn}\,(x)} - \frac{2\left(84\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^6a^7c-84\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^5a^6c^{\frac{3}{2}}|a|+112\left(\sqrt{a^2c}x-\sqrt{a^2cx\right.\right.}}{2\ldots}$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

output
```
4*sqrt(2)*a^4*c*arctan(-1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c
*x))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) - 2/21*(84
*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^6*a^7*c - 84*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - a*c*x))^5*a^6*c^(3/2)*abs(a) + 112*(sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - a*c*x))^4*a^7*c^2 - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x
))^3*a^6*c^(5/2)*abs(a) + 63*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a
^7*c^3 - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^6*c^(7/2)*abs(a) +
 3*a^7*c^4)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^7*a^3*abs(a)*sgn(x)
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{x^4\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.53

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$$
$$=\frac{2\sqrt{c}\left(-52\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^3x^3+16\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^2x^2-9\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,ax+3\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}+\right.}{}$$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x)
```

output
```
(2*sqrt(c)*( - 52*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**3*x**3 + 16*sqrt(x)*sqr
t(a)*sqrt(a*x - 1)*a**2*x**2 - 9*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a*x + 3*sqr
t(x)*sqrt(a)*sqrt(a*x - 1) + 21*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a
) - sqrt(2)*i + i)*a**4*x**4 + 21*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt
(a) + sqrt(2)*i - i)*a**4*x**4 - 21*sqrt(2)*log(2*sqrt(x)*sqrt(a)*sqrt(a*x
 - 1) + 2*sqrt(2) + 2*a*x + 2)*a**4*x**4 + 28*a**4*x**4))/(21*a*x**4)
```

3.544 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 163

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = 4a^4\sqrt{c-\frac{c}{ax}}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} - 4\sqrt{2}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

output
```
4*a^4*(c-c/a/x)^(1/2)+2/3*a^4*(c-c/a/x)^(3/2)/c+2/5*a^4*(c-c/a/x)^(5/2)/c^
2-2/7*a^4*(c-c/a/x)^(7/2)/c^3+2/9*a^4*(c-c/a/x)^(9/2)/c^4-4*2^(1/2)*a^4*c^
(1/2)*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.58

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2\sqrt{c-\frac{c}{ax}}(35-95ax+138a^2x^2-236a^3x^3+788a^4x^4)}{315x^4} - 4\sqrt{2}a^4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

output
```
(2*Sqrt[c - c/(a*x)]*(35 - 95*a*x + 138*a^2*x^2 - 236*a^3*x^3 + 788*a^4*x^
4))/(315*x^4) - 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*S
qrt[c])]
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6717, 6683, 1070, 281, 948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-2\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}dx$$

$$\Big\downarrow \text{ 6683}$$

$$-\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{x^5(ax+1)}dx$$

$$\Big\downarrow \text{ 1070}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\sqrt{c-\frac{c}{ax}}}{\left(a+\frac{1}{x}\right)x^5}dx$$

$\Big\downarrow$ 281

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^5}dx}{c}$$

$\Big\downarrow$ 948

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^3}d\frac{1}{x}}{c}$$

$\Big\downarrow$ 99

$$-\frac{a\int \left(\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^2}-\frac{a^2\left(c-\frac{c}{ax}\right)^{5/2}}{c}+a^2\left(c-\frac{c}{ax}\right)^{3/2}-\frac{a^3\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}\right)d\frac{1}{x}}{c}$$

$\Big\downarrow$ 2009

$$-\frac{a\left(4\sqrt{2}a^3c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)-\frac{2a^3\left(c-\frac{c}{ax}\right)^{9/2}}{9c^3}+\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^2}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c}-\frac{2}{3}a^3\left(c-\frac{c}{ax}\right)^{3/2}-4a^3c\sqrt{c-\frac{c}{ax}}\right)}{c}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

output
```
-((a*(-4*a^3*c*Sqrt[c - c/(a*x)] - (2*a^3*(c - c/(a*x))^(3/2))/3 - (2*a^3*
(c - c/(a*x))^(5/2))/(5*c) + (2*a^3*(c - c/(a*x))^(7/2))/(7*c^2) - (2*a^3*
(c - c/(a*x))^(9/2))/(9*c^3) + 4*Sqrt[2]*a^3*c^(3/2)*ArcTanh[Sqrt[c - c/(a
*x)]/(Sqrt[2]*Sqrt[c])]))/c)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1070
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^
(p_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(m + n*(p + r))*(
b + a/x^n)^p*(c + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, m,
 n, q}, x] && EqQ[mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03

method	result
risch	$\frac{2(788a^5x^5-1024a^4x^4+374a^3x^3-233a^2x^2+130ax-35)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4(ax-1)}+\frac{2a^4\sqrt{2}\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-3\left(x+\frac{1}{a}\right)ac-}{x+\frac{1}{a}}\right)}{\sqrt{c}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-1890\sqrt{a\,x^2-x}\,a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\,x^6+630a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\,\sqrt{x(ax-1)}\,x^6+1260\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^4+945\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\right.}{}$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2/315*(788*a^5*x^5-1024*a^4*x^4+374*a^3*x^3-233*a^2*x^2+130*a*x-35)/x^4/(a
*x-1)*(c*(a*x-1)/a/x)^(1/2)+2*a^4*2^(1/2)/c^(1/2)*ln((4*c-3*(x+1/a)*a*c+2*
2^(1/2)*c^(1/2)*((x+1/a)^2*a^2*c-3*(x+1/a)*a*c+2*c)^(1/2))/(x+1/a))/(a*x-1
)*(c*(a*x-1)/a/x)^(1/2)*(c*(a*x-1)*a*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.36

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$
$$=\left[\frac{2\left(315\sqrt{2}a^4\sqrt{c}x^4\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+\left(788\,a^4x^4-236\,a^3x^3+138\,a^2x^2-95\,ax+35\right)\sqrt{\frac{acx-}{ax}}\right.}{315\,x^4}\right.$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

output
```
[2/315*(315*sqrt(2)*a^4*sqrt(c)*x^4*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
 - c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + (788*a^4*x^4 - 236*a^3*x^3 + 138*
a^2*x^2 - 95*a*x + 35)*sqrt((a*c*x - c)/(a*x)))/x^4, 2/315*(630*sqrt(2)*a^
4*sqrt(-c)*x^4*arctan(sqrt(2)*a*sqrt(-c)*x*sqrt((a*c*x - c)/(a*x))/(a*c*x
- c)) + (788*a^4*x^4 - 236*a^3*x^3 + 138*a^2*x^2 - 95*a*x + 35)*sqrt((a*c*
x - c)/(a*x)))/x^4]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{x^5\,(ax+1)}\,dx$$

input
```
integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**5*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^5}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(136) = 272$.

Time = 0.37 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.66

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\frac{4\sqrt{2}a^5c\arctan\left(-\frac{\sqrt{2}\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}|a|\right)}{2a\sqrt{-c}}\right)}{\sqrt{-c}|a|\mathrm{sgn}\left(x\right)} + \frac{2\left(1260\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^8a^9c-1260\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^7a^8c^{\frac{3}{2}}|a|+2100\left(\sqrt{a^2c}x-\right.\right.}{}$$

input
```
integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

output
```
-4*sqrt(2)*a^5*c*arctan(-1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*
c*x))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) + 2/315*(
1260*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^8*a^9*c - 1260*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 - a*c*x))^7*a^8*c^(3/2)*abs(a) + 2100*(sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - a*c*x))^6*a^9*c^2 - 3150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - a*c*x))^5*a^8*c^(5/2)*abs(a) + 3528*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a
*c*x))^4*a^9*c^3 - 2625*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*a^8*c^
(7/2)*abs(a) + 1215*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^9*c^4 -
315*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^8*c^(9/2)*abs(a) + 35*a^9*
c^5)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^9*a^4*abs(a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x-1)}{x^5\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.17

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$
$$= \frac{2\sqrt{c}\left(788\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^4x^4-236\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^3x^3+138\sqrt{x}\,\sqrt{a}\,\sqrt{ax-1}\,a^2x^2-95\sqrt{x}\,\sqrt{a}\,\sqrt{ax-}\right.}{}$$

input
```
int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x)
```

output
```
(2*sqrt(c)*(788*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**4*x**4 - 236*sqrt(x)*sqrt
(a)*sqrt(a*x - 1)*a**3*x**3 + 138*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a**2*x**2
- 95*sqrt(x)*sqrt(a)*sqrt(a*x - 1)*a*x + 35*sqrt(x)*sqrt(a)*sqrt(a*x - 1)
- 315*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a) - sqrt(2)*i + i)*a**5*x*
*5 - 315*sqrt(2)*log(sqrt(a*x - 1) + sqrt(x)*sqrt(a) + sqrt(2)*i - i)*a**5
*x**5 + 315*sqrt(2)*log(2*sqrt(x)*sqrt(a)*sqrt(a*x - 1) + 2*sqrt(2) + 2*a*
x + 2)*a**5*x**5 - 508*a**5*x**5))/(315*a*x**5)
```

3.545 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 244

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = -\frac{1115c\sqrt{1-\frac{1}{a^2x^2}}x}{64a^3\sqrt{c-\frac{c}{ax}}}+\frac{1115c\sqrt{1-\frac{1}{a^2x^2}}x^2}{96a^2\sqrt{c-\frac{c}{ax}}} -\frac{223\sqrt{c-\frac{c}{ax}}x^2}{24a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{19\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{a^2x^2}}} +\frac{\left(c-\frac{c}{ax}\right)^{3/2}x^4}{4c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{1115\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{64a^4}$$

output
```
-1115/64*c*(1-1/a^2/x^2)^(1/2)*x/a^3/(c-c/a/x)^(1/2)+1115/96*c*(1-1/a^2/x^
2)^(1/2)*x^2/a^2/(c-c/a/x)^(1/2)-223/24*(c-c/a/x)^(1/2)*x^2/a^2/(1-1/a^2/x
^2)^(1/2)-19/24*(c-c/a/x)^(1/2)*x^3/a/(1-1/a^2/x^2)^(1/2)+1/4*(c-c/a/x)^(3
/2)*x^4/c/(1-1/a^2/x^2)^(1/2)+1115/64*c^(1/2)*arctanh(c^(1/2)*(1-1/a^2/x^2
)^(1/2)/(c-c/a/x)^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.68

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(-3345-1115ax+446a^2x^2-200a^3x^3+48a^4x^4)}{-1+a^2x^2}-3345\sqrt{c}\log(1-ax)+3345\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\right.}{384a^4}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

output
```
((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-3345 - 1115*a*x + 44
6*a^2*x^2 - 200*a^3*x^3 + 48*a^4*x^4))/(-1 + a^2*x^2) - 3345*Sqrt[c]*Log[1
 - a*x] + 3345*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/
(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(384*a^4)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.70, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6733, 585, 27, 100, 27, 87, 52, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x^5}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^5}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^5}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 100$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{4}\int -\frac{\left(25a-\frac{8}{x}\right)x^4}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x^4}{4\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{8}\int \frac{\left(25a-\frac{8}{x}\right)x^4}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x^4}{4\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 87$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{8}\left(\frac{223}{6}\int \frac{x^3}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{25ax^3}{3\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^4}{4\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 52$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{8}\left(\frac{223}{6}\left(-\frac{5\int \frac{x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{4a}-\frac{x^2}{2\sqrt{\frac{1}{ax}+1}}\right)+\frac{25ax^3}{3\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^4}{4\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 52$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{8}\left(\frac{223}{6}\left(-\frac{5\left(-\frac{3\int \frac{x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{2a}-\frac{x}{\sqrt{\frac{1}{ax}+1}}\right)}{4a}-\frac{x^2}{2\sqrt{\frac{1}{ax}+1}}\right)+\frac{25ax^3}{3\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^4}{4\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \ 61$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{8}\left(\frac{223}{6}\left(-\frac{5\left(-\frac{3\left(\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{\sqrt{\frac{1}{ax}+1}}\right)}{2a}-\frac{x}{\sqrt{\frac{1}{ax}+1}}\right)}{4a}-\frac{x^2}{2\sqrt{\frac{1}{ax}+1}}\right)+\frac{25ax^3}{3\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^4}{4\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 73

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{8}\left(\frac{223}{6}\left(-\frac{5\left(-\frac{3\left(2a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{\sqrt{\frac{1}{ax}+1}}\right)}{2a}-\frac{x}{\sqrt{\frac{1}{ax}+1}}\right)}{4a}-\frac{x^2}{2\sqrt{\frac{1}{ax}+1}}\right)+\frac{25ax^3}{3\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^4}{4\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$\Big\downarrow$ 221

$$-\frac{\left(\frac{1}{8}\left(\frac{223}{6}\left(-\frac{5\left(-\frac{3\left(\frac{2}{\sqrt{\frac{1}{ax}+1}}-2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{2a}-\frac{x}{\sqrt{\frac{1}{ax}+1}}\right)}{4a}-\frac{x^2}{2\sqrt{\frac{1}{ax}+1}}\right)+\frac{25ax^3}{3\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^4}{4\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(Sqrt[c - c/(a*x)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a*x)]*(-1/4*(a^2*x^4)/Sqrt[1 + 1/(a*x)] + ((25*a*x^3)/(3*Sq
rt[1 + 1/(a*x)]) + (223*(-1/2*x^2/Sqrt[1 + 1/(a*x)] - (5*(-(x/Sqrt[1 + 1/(
a*x)]) - (3*(2/Sqrt[1 + 1/(a*x)] - 2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(2*a)))/
(4*a)))/6)/8))/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.81

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\,(96a^{\frac{9}{2}}\sqrt{x(ax+1)}\,x^4-400a^{\frac{7}{2}}x^3\sqrt{x(ax+1)}+892a^{\frac{5}{2}}x^2\sqrt{x(ax+1)}-2230a^{\frac{3}{2}}x\sqrt{x(ax+1)}+3345\ln(\frac{2\sqrt{}}{}}{384(ax-1)^2a^{\frac{7}{2}}\sqrt{x(ax+1)}}$
risch	$\frac{(48a^3x^3-248a^2x^2+694ax-1809)(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{192a^3(ax-1)}+\frac{(\frac{1115\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{128a^3\sqrt{a^2c}}-\frac{8\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-\left(x+\frac{1}{a}\right)ac}}{a^5c\left(x+\frac{1}{a}\right)}}{ax-1}$

input
```
int((c-c/a/x)^(1/2)*x^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(9
6*a^(9/2)*(x*(a*x+1))^(1/2)*x^4-400*a^(7/2)*x^3*(x*(a*x+1))^(1/2)+892*a^(5
/2)*x^2*(x*(a*x+1))^(1/2)-2230*a^(3/2)*x*(x*(a*x+1))^(1/2)+3345*ln(1/2*(2*
(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x-6690*(x*(a*x+1))^(1/2)*a^(
1/2)+3345*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(7/2)/(
x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.45

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$
$$=\left[\frac{3345\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(48\,a^5x^5-200\,a^4x^4+446\cdots}{768\,(a^5x-a^4)}\right.$$
$$-\frac{3345\,(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(48\,a^5x^5-200\,a^4x^4+446\,a^3x^3-1115\,a^2\cdots}{384\,(a^5x-a^4)}$$

input
```
integrate((c-c/a/x)^(1/2)*x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

output
```
[1/768*(3345*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3
+ 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)) - c)/(a*x - 1)) + 4*(48*a^5*x^5 - 200*a^4*x^4 + 446*a^3*x^3 - 1115*a^2*
x^2 - 3345*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x
- a^4), -1/384*(3345*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*
sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c
)) - 2*(48*a^5*x^5 - 200*a^4*x^4 + 446*a^3*x^3 - 1115*a^2*x^2 - 3345*a*x)*
sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*x**3*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \int \sqrt{c-\frac{c}{ax}}x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx=\int x^3\,\sqrt{c-\frac{c}{a\,x}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int(x^3*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x^3*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.39

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx$$
$$=\frac{\sqrt{c}\,\big(3345\sqrt{ax+1}\,\log\big(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\big)-2097\sqrt{ax+1}+48\sqrt{x}\,\sqrt{a}\,a^4x^4-200\sqrt{x}\,\sqrt{a}\,a^3x^3+446\sqrt{x}}{192\sqrt{ax+1}\,a^4}$$

input
```
int((c-c/a/x)^(1/2)*x^3*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*(3345*sqrt(a*x + 1)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)) - 2097*s
qrt(a*x + 1) + 48*sqrt(x)*sqrt(a)*a**4*x**4 - 200*sqrt(x)*sqrt(a)*a**3*x**
3 + 446*sqrt(x)*sqrt(a)*a**2*x**2 - 1115*sqrt(x)*sqrt(a)*a*x - 3345*sqrt(x
)*sqrt(a)))/(192*sqrt(a*x + 1)*a**4)
```

3.546 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 202

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx=\frac{119c\sqrt{1-\frac{1}{a^2x^2}}x}{8a^2\sqrt{c-\frac{c}{ax}}}-\frac{119\sqrt{c-\frac{c}{ax}}x}{12a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5\sqrt{c-\frac{c}{ax}}x^2}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{3c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{119\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

output
```
119/8*c*(1-1/a^2/x^2)^(1/2)*x/a^2/(c-c/a/x)^(1/2)-119/12*(c-c/a/x)^(1/2)*x
/a^2/(1-1/a^2/x^2)^(1/2)-5/4*(c-c/a/x)^(1/2)*x^2/a/(1-1/a^2/x^2)^(1/2)+1/3
*(c-c/a/x)^(3/2)*x^3/c/(1-1/a^2/x^2)^(1/2)-119/8*c^(1/2)*arctanh(c^(1/2)*(
1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(357+119ax-38a^2x^2+8a^3x^3)}{-1+a^2x^2}+357\sqrt{c}\log(1-ax)-357\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x\right.}{48a^3}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

output
```
((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(357 + 119*a*x - 38*a^
2*x^2 + 8*a^3*x^3))/(-1 + a^2*x^2) + 357*Sqrt[c]*Log[1 - a*x] - 357*Sqrt[c
]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 -
a*x + 2*a^2*x^2)])/(48*a^3)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.71, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6733, 585, 27, 100, 27, 87, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x^4}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^4}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^4}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{3}\int -\frac{\left(19a-\frac{6}{x}\right)x^3}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x^3}{3\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{6}\int \frac{\left(19a-\frac{6}{x}\right)x^3}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x^3}{3\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{6}\left(\frac{119}{4}\int \frac{x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{19ax^2}{2\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^3}{3\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{52}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{6}\left(\frac{119}{4}\left(-\frac{3\int \frac{x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{2a}-\frac{x}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{19ax^2}{2\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^3}{3\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{6}\left(\frac{119}{4}\left(-\frac{3\left(\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{\sqrt{\frac{1}{ax}+1}}\right)}{2a}-\frac{x}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{19ax^2}{2\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^3}{3\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{6}\left(\frac{119}{4}\left(-\frac{3\left(2a\int\frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{\sqrt{\frac{1}{ax}+1}}\right)}{2a}-\frac{x}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{19ax^2}{2\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^3}{3\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 221$$

$$-\frac{\left(\frac{1}{6}\left(\frac{119}{4}\left(-\frac{3\left(\frac{2}{\sqrt{\frac{1}{ax}+1}}-2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{2a}-\frac{x}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{19ax^2}{2\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^3}{3\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(Sqrt[c - c/(a*x)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a*x)]*(-1/3*(a^2*x^3)/Sqrt[1 + 1/(a*x)] + ((19*a*x^2)/(2*Sq
rt[1 + 1/(a*x)]) + (119*(-(x/Sqrt[1 + 1/(a*x)]) - (3*(2/Sqrt[1 + 1/(a*x)]
- 2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(2*a)))/4)/6))/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.89

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{7}{2}}x^3\sqrt{x(ax+1)}-76a^{\frac{5}{2}}x^2\sqrt{x(ax+1)}+238a^{\frac{3}{2}}x\sqrt{x(ax+1)}-357\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+\right.}{48(ax-1)^2a^{\frac{5}{2}}\sqrt{x(ax+1)}}$
risch	$\frac{(8a^2x^2-46ax+165)(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{24a^2(ax-1)}+\frac{\left(-\frac{119\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{16a^2\sqrt{a^2c}}+\frac{8\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-\left(x+\frac{1}{a}\right)ac}}{a^4c\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{}}{ax-1}$

input
```
int((c-c/a/x)^(1/2)*x^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16
*a^(7/2)*x^3*(x*(a*x+1))^(1/2)-76*a^(5/2)*x^2*(x*(a*x+1))^(1/2)+238*a^(3/2
)*x*(x*(a*x+1))^(1/2)-357*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(
1/2))*a*x+714*(x*(a*x+1))^(1/2)*a^(1/2)-357*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^
(1/2)+2*a*x+1)/a^(1/2)))/a^(5/2)/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.67

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$=\left[\frac{357\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(8\,a^4x^4-38\,a^3x^3+119\,a^2x}{96\,(a^4x-a^3)}\right.$$

input
```
integrate((c-c/a/x)^(1/2)*x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

output
```
[1/96*(357*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 +
3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
 - c)/(a*x - 1)) + 4*(8*a^4*x^4 - 38*a^3*x^3 + 119*a^2*x^2 + 357*a*x)*sqrt
((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(357*(a
*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 - 38*
a^3*x^3 + 119*a^2*x^2 + 357*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^4*x - a^3)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*x**2*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \int \sqrt{c-\frac{c}{ax}}x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \int x^2\,\sqrt{c-\frac{c}{a\,x}}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.41

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = \frac{\sqrt{c}\left(-2856\sqrt{ax+1}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)+1917\sqrt{ax+1}+64\sqrt{x}\,\sqrt{a}\,a^3x^3-304\sqrt{x}\,\sqrt{a}\,a^2x^2+952\sqrt{}}{192\sqrt{ax+1}\,a^3}$$

input
```
int((c-c/a/x)^(1/2)*x^2*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*( - 2856*sqrt(a*x + 1)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)) + 191
7*sqrt(a*x + 1) + 64*sqrt(x)*sqrt(a)*a**3*x**3 - 304*sqrt(x)*sqrt(a)*a**2*
x**2 + 952*sqrt(x)*sqrt(a)*a*x + 2856*sqrt(x)*sqrt(a)))/(192*sqrt(a*x + 1)
*a**3)
```

3.547 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{11\sqrt{c-\frac{c}{ax}}x}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{47\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

output
```
-47/4*(c-c/a/x)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)-11/4*(c-c/a/x)^(1/2)*x/a/(1-
1/a^2/x^2)^(1/2)+1/2*(c-c/a/x)^(3/2)*x^2/c/(1-1/a^2/x^2)^(1/2)+47/4*c^(1/2
)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 2.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$=\frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(-47-13ax+2a^2x^2)}{-4+4a^2x^2}-\frac{47\sqrt{c}\log(1-ax)}{8a^2}+\frac{47\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-1-ax+2a^2x^2)\right)}{8a^2}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*x)/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-47 - 13*a*x + 2*a^2*x^2))/(
-4 + 4*a^2*x^2) - (47*Sqrt[c]*Log[1 - a*x])/(8*a^2) + (47*Sqrt[c]*Log[2*a^
2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^
2*x^2)])/(8*a^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.72, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {6733, 585, 27, 100, 27, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x^3}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^3}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^3}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 100}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\int -\frac{\left(13a-\frac{4}{x}\right)x^2}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x^2}{2\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{4}\int \frac{\left(13a-\frac{4}{x}\right)x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x^2}{2\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{4}\left(\frac{47}{2}\int \frac{x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{13ax}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^2}{2\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{4}\left(\frac{47}{2}\left(\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{13ax}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^2}{2\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{4}\left(\frac{47}{2}\left(2a\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}+\frac{2}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{13ax}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^2}{2\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\left(\frac{1}{4}\left(\frac{47}{2}\left(\frac{2}{\sqrt{\frac{1}{ax}+1}}-2\operatorname{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{13ax}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x^2}{2\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(Sqrt[c - c/(a*x)]*x)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a*x)]*(-1/2*(a^2*x^2)/Sqrt[1 + 1/(a*x)] + ((13*a*x)/Sqrt[1
+ 1/(a*x)] + (47*(2/Sqrt[1 + 1/(a*x)] - 2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/2)/
4))/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4a^{\frac{5}{2}}x^{2}\sqrt{x(ax+1)}-26a^{\frac{3}{2}}x\sqrt{x(ax+1)}+47\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-94\sqrt{x(ax+1)}\,\sqrt{a}+47\ln\left(\right.\right.}{8(ax-1)^{2}a^{\frac{3}{2}}\sqrt{x(ax+1)}}$
risch	$\frac{(2ax-15)(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{4a(ax-1)}+\frac{\left(\frac{47\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{8a\sqrt{a^{2}c}}-\frac{8\sqrt{\left(x+\frac{1}{a}\right)^{2}a^{2}c-\left(x+\frac{1}{a}\right)ac}}{a^{3}c\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax}}{ax-1}$

input
```
int((c-c/a/x)^(1/2)*x*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(4*a
^(5/2)*x^2*(x*(a*x+1))^(1/2)-26*a^(3/2)*x*(x*(a*x+1))^(1/2)+47*ln(1/2*(2*(
x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x-94*(x*(a*x+1))^(1/2)*a^(1/2
)+47*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(3/2)/(x*(a*
x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.99

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx$$
$$=\left[\frac{47\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx+4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(2\,a^{3}x^{3}-13\,a^{2}x^{2}-47\,ax)\sqrt{}}{16\,(a^{3}x-a^{2})}\right.$$
$$\left.-\frac{47\,(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^{2}x^{2}+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)-2\,(2\,a^{3}x^{3}-13\,a^{2}x^{2}-47\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{8\,(a^{3}x-a^{2})}\right]$$

input
```
integrate((c-c/a/x)^(1/2)*x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(47*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3
*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
- c)/(a*x - 1)) + 4*(2*a^3*x^3 - 13*a^2*x^2 - 47*a*x)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2), -1/8*(47*(a*x - 1)*sqrt(-c)*
arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(2*a^3*x^3 - 13*a^2*x^2 - 47*a*x)
*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*x*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx=\int\sqrt{c-\frac{c}{ax}}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = \int x\,\sqrt{c-\frac{c}{a\,x}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.44

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x\,dx = \frac{\sqrt{c}\,\left(47\sqrt{ax+1}\,\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)-36\sqrt{ax+1}+2\sqrt{x}\,\sqrt{a}\,a^2x^2-13\sqrt{x}\,\sqrt{a}\,ax-47\sqrt{x}\,\sqrt{a}\right)}{4\sqrt{ax+1}\,a^2}$$

input
```
int((c-c/a/x)^(1/2)*x*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*(47*sqrt(a*x + 1)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)) - 36*sqrt(
a*x + 1) + 2*sqrt(x)*sqrt(a)*a**2*x**2 - 13*sqrt(x)*sqrt(a)*a*x - 47*sqrt(
x)*sqrt(a)))/(4*sqrt(a*x + 1)*a**2)
```

3.548 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{10\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{7\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

output
```
10*(c-c/a/x)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+(c-c/a/x)^(3/2)*x/c/(1-1/a^2/x^2)
^(1/2)-7*c^(1/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.58

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\left(9+ax-7\sqrt{1+\frac{1}{ax}}\,\mathrm{arctanh}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a*x)]*(9 + a*x - 7*Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)
]]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6731, 585, 27, 100, 27, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6731}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x^2}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{585}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^2}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x^2}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\int -\frac{\left(7a-\frac{2}{x}\right)x}{2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(-\frac{1}{2}\int \frac{\left(7a-\frac{2}{x}\right)x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 87$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-7a\int \frac{x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{18a}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 73$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{1}{2}\left(-14a^2\int \frac{1}{\frac{a}{x^2}-a}d\sqrt{1+\frac{1}{ax}}-\frac{18a}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \; 221$$

$$-\frac{\left(\frac{1}{2}\left(14a\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)-\frac{18a}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{a^2x}{\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a*x)]*(-((a^2*x)/Sqrt[1 + 1/(a*x)]) + ((-18*a)/Sqrt[1 + 1/(
a*x)] + 14*a*ArcTanh[Sqrt[1 + 1/(a*x)]])/2))/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 6731
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/
x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2]
&& IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.27

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2a^{\frac{3}{2}}x\sqrt{x(ax+1)}-7\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+18\sqrt{x(ax+1)}\,\sqrt{a}-7\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right.\right.}{2(ax-1)^{2}\sqrt{a}\,\sqrt{x(ax+1)}}$
risch	$\frac{(ax+1)x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}+\frac{\left(-\frac{7\ln\left(\frac{\frac{1}{2}ac+a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2\sqrt{a^{2}c}}+\frac{8\sqrt{\left(x+\frac{1}{a}\right)^{2}a^{2}c-\left(x+\frac{1}{a}\right)ac}}{a^{2}c\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{(ax+1)acx}}{ax-1}$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*a
^(3/2)*x*(x*(a*x+1))^(1/2)-7*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/
a^(1/2))*a*x+18*(x*(a*x+1))^(1/2)*a^(1/2)-7*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^
(1/2)+2*a*x+1)/a^(1/2)))/a^(1/2)/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.60

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \left[\frac{7\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^{2}x^{2}+9\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^{2}x-a)},\right.$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)))/(a^2*x - a), 1/2*(7*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
 - a*c*x - c)) + 2*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \sqrt{c-\frac{c}{a\,x}}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.51

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\sqrt{c}\,\left(-28\sqrt{ax+1}\,\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)+33\sqrt{ax+1}+4\sqrt{x}\,\sqrt{a}\,ax+36\sqrt{x}\,\sqrt{a}\right)}{4\sqrt{ax+1}\,a}$$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*( - 28*sqrt(a*x + 1)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a)) + 33*sq
rt(a*x + 1) + 4*sqrt(x)*sqrt(a)*a*x + 36*sqrt(x)*sqrt(a)))/(4*sqrt(a*x + 1
)*a)
```

3.549 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 109

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = -\frac{12\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\left(c-\frac{c}{ax}\right)^{3/2}}{c\sqrt{1-\frac{1}{a^2x^2}}}+2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

output
```
-12*(c-c/a/x)^(1/2)/(1-1/a^2/x^2)^(1/2)+2*(c-c/a/x)^(3/2)/c/(1-1/a^2/x^2)^
(1/2)+2*c^(1/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.20

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x(1+5ax)}{-1+a^2x^2}-\sqrt{c}\log(1-ax)+\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2+c\left(-1-ax+2a^2x^2\right)\right)$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
(-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x*(1 + 5*a*x))/(-1 + a^2*x^2
) - Sqrt[c]*Log[1 - a*x] + Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]
*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6733, 585, 27, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{ 6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{ 585}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x}{a^2\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \frac{\left(a-\frac{1}{x}\right)^2x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 98}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \left(\frac{xa^2}{\sqrt{1+\frac{1}{ax}}}+\frac{a}{\sqrt{1+\frac{1}{ax}}}-\frac{4a}{\left(1+\frac{1}{ax}\right)^{3/2}}\right)d\frac{1}{x}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-2a^2\text{arctanh}\left(\sqrt{\frac{1}{ax}+1}\right)+2a^2\sqrt{\frac{1}{ax}+1}+\frac{8a^2}{\sqrt{\frac{1}{ax}+1}}\right)\sqrt{c-\frac{c}{ax}}}{a^2\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
-((Sqrt[c - c/(a*x)]*((8*a^2)/Sqrt[1 + 1/(a*x)] + 2*a^2*Sqrt[1 + 1/(a*x)]
- 2*a^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(a^2*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 585
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_^2))^(p_)
, x_Symbol] :> Simp[a^p*c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^F
racPart[n])   Int[(e*x)^m*(1 - d*(x/c))^p*(1 + d*(x/c))^(n + p), x], x] /;
FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[a, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\left(10a^{\frac{3}{2}}x\sqrt{x(ax+1)}-\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^2x^2-\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+2\sqrt{x(ax+1)}\right.}{(ax-1)^2\sqrt{a}\sqrt{x(ax+1)}}$
risch	$-\frac{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}+\frac{\left(\frac{a\ln\left(\frac{\frac{1}{2}ac+a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{\sqrt{a^2c}}-\frac{8\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-\left(x+\frac{1}{a}\right)ac}}{ac\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}\sqrt{(ax+1)acx}}{ax-1}$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(10*a^(3/
2)*x*(x*(a*x+1))^(1/2)-ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2
))*a^2*x^2-ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x+2*(x*
(a*x+1))^(1/2)*a^(1/2))/a^(1/2)/(x*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.54

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$$
$$=\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\,(5\,ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(ax-1)},\right.$$
$$\left.-\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(5\,ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{ax-1}\right]$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) - 4*(5*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*
x)))/(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqr
t((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c))
+ 2*(5*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x -
1)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
```

output
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{2\sqrt{c}\left(\sqrt{ax+1}\log\left(\sqrt{ax+1}+\sqrt{x}\,\sqrt{a}\right)ax-5\sqrt{ax+1}\,ax-5\sqrt{x}\,\sqrt{a}\,ax-\sqrt{x}\,\sqrt{a}\right)}{\sqrt{ax+1}\,ax}$$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x)
```

output
```
(2*sqrt(c)*(sqrt(a*x + 1)*log(sqrt(a*x + 1) + sqrt(x)*sqrt(a))*a*x - 5*sqr
t(a*x + 1)*a*x - 5*sqrt(x)*sqrt(a)*a*x - sqrt(x)*sqrt(a)))/(sqrt(a*x + 1)*
a*x)
```

3.550 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 109

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{64a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
64/3*a*(c-c/a/x)^(1/2)/(1-1/a^2/x^2)^(1/2)-16/3*a*(c-c/a/x)^(3/2)/c/(1-1/a
^2/x^2)^(1/2)-2/3*a*(c-c/a/x)^(5/2)/c^2/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.53

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(-1+10ax+23a^2x^2)}{-3+3a^2x^2}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-1 + 10*a*x + 23*a^2*x^2))/(
-3 + 3*a^2*x^2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6733, 459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{459}$$

$$-\frac{\frac{8}{3}c\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{2ac\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{459}$$

$$-\frac{\frac{8}{3}c\left(4c\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{2ac\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2ac\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{458}$$

$$-\frac{\frac{8}{3}c\left(\frac{2ac\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8ac^2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2ac\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
-(((8*c*((-8*a*c^2*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a^2*x^2)] + (2*a*c*(c -
c/(a*x))^(3/2))/Sqrt[1 - 1/(a^2*x^2)]))/3 + (2*a*c*(c - c/(a*x))^(5/2))/(3
*Sqrt[1 - 1/(a^2*x^2)]))/c^3)
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.55

method	result	size
orering	$\frac{2(23a^2x^2+10ax-1)(ax+1)\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3x(ax-1)^2}$	60
gosper	$\frac{2(ax+1)(23a^2x^2+10ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3x(ax-1)^2}$	62
default	$\frac{2(ax+1)(23a^2x^2+10ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3x(ax-1)^2}$	62
risch	$\frac{2(11a^2x^2+10ax-1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{3x(ax-1)}+\frac{8a^2x\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$	101

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(23*a^2*x^2+10*a*x-1)*(a*x+1)/x/(a*x-1)^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*
x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.54

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{2\left(23\,a^2x^2+10\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas
")
```

output
```
2/3*(23*a^2*x^2 + 10*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x))/(a*x^2 - x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.63 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx=\frac{2\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(23\,a^2\,x^2+10\,a\,x-1\right)}{3\,x\,\left(a\,x-1\right)}$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
```

output
```
(2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(10*a*x + 23*a^2*x^2 -
1))/(3*x*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.54

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$$
$$= \frac{2\sqrt{c}\left(-26\sqrt{ax+1}\,a^2x^2+23\sqrt{x}\,\sqrt{a}\,a^2x^2+10\sqrt{x}\,\sqrt{a}\,ax-\sqrt{x}\,\sqrt{a}\right)}{3\sqrt{ax+1}\,a\,x^2}$$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

output
```
(2*sqrt(c)*( - 26*sqrt(a*x + 1)*a**2*x**2 + 23*sqrt(x)*sqrt(a)*a**2*x**2 +
 10*sqrt(x)*sqrt(a)*a*x - sqrt(x)*sqrt(a)))/(3*sqrt(a*x + 1)*a*x**2)
```

3.551 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 150

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{224a^2c\sqrt{1-\frac{1}{a^2x^2}}}{15\sqrt{c-\frac{c}{ax}}}-\frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}-\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-224/15*a^2*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)-56/15*a^2*(1-1/a^2/x^2)^
(1/2)*(c-c/a/x)^(1/2)-7/5*a^2*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(3/2)/c-a^2*(c
-c/a/x)^(7/2)/c^3/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(3-16ax+79a^2x^2+158a^3x^3)}{15x\left(-1+a^2x^2\right)}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

output
```
(-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 - 16*a*x + 79*a^2*x^2 + 1
58*a^3*x^3))/(15*x*(-1 + a^2*x^2))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6733, 572, 459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{572}$$

$$-\frac{-\frac{7}{5}a\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}-\frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}}{5\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{459}$$

$$-\frac{-\frac{7}{5}a\left(\frac{8}{3}c\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{2ac\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-\frac{1}{a^2x^2}}}\right)-\frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}}{5\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{459}$$

$$-\frac{-\frac{7}{5}a\left(\frac{8}{3}c\left(4c\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{2ac\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2ac\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-\frac{1}{a^2x^2}}}\right)-\frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}}{5\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \text{458}$$

$$-\frac{-\frac{7}{5}a\left(\frac{8}{3}c\left(\frac{2ac\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8ac^2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2ac\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-\frac{1}{a^2x^2}}}\right)-\frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}}{5\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

output
```
-(((-7*a*((8*c*((-8*a*c^2*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a^2*x^2)] + (2*a*
c*(c - c/(a*x))^(3/2))/Sqrt[1 - 1/(a^2*x^2)]))/3 + (2*a*c*(c - c/(a*x))^(5
/2))/(3*Sqrt[1 - 1/(a^2*x^2)])))/5 - (2*a^2*(c - c/(a*x))^(7/2))/(5*Sqrt[1
 - 1/(a^2*x^2)]))/c^3)
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

rule 572
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 2))), x] + Simp[c*(n/(d
*(n + 2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && NeQ[n + 2*p + 2, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.45

method	result	size
orering	$-\frac{2(158a^3x^3+79a^2x^2-16ax+3)(ax+1)\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{15x^2(ax-1)^2}$	68
gosper	$-\frac{2(ax+1)(158a^3x^3+79a^2x^2-16ax+3)\sqrt{\frac{c(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{15x^2(ax-1)^2}$	70
default	$-\frac{2(ax+1)(158a^3x^3+79a^2x^2-16ax+3)\sqrt{\frac{c(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{15x^2(ax-1)^2}$	70
risch	$-\frac{2(98a^3x^3+79a^2x^2-16ax+3)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{15x^2(ax-1)}-\frac{8a^3x\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$	109

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(158*a^3*x^3+79*a^2*x^2-16*a*x+3)*(a*x+1)/x^2/(a*x-1)^2*(c-c/a/x)^(1
/2)*((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.46

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx=-\frac{2\,(158\,a^3x^3+79\,a^2x^2-16\,ax+3)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{15\,(ax^3-x^2)}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas
")
```

output
```
-2/15*(158*a^3*x^3 + 79*a^2*x^2 - 16*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x))/(a*x^3 - x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.65 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{2\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(158\,a^3\,x^3+79\,a^2\,x^2-16\,a\,x+3\right)}{15\,x^2\,\left(a\,x-1\right)}$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
```

output
```
-(2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(79*a^2*x^2 - 16*a*x +
 158*a^3*x^3 + 3))/(15*x^2*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{2\sqrt{c}\left(158\sqrt{ax+1}\,a^3x^3-158\sqrt{x}\,\sqrt{a}\,a^3x^3-79\sqrt{x}\,\sqrt{a}\,a^2x^2+16\sqrt{x}\,\sqrt{a}\,ax-3\sqrt{x}\,\sqrt{a}\right)}{15\sqrt{ax+1}\,a\,x^3}$$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

output
```
(2*sqrt(c)*(158*sqrt(a*x + 1)*a**3*x**3 - 158*sqrt(x)*sqrt(a)*a**3*x**3 -
79*sqrt(x)*sqrt(a)*a**2*x**2 + 16*sqrt(x)*sqrt(a)*a*x - 3*sqrt(x)*sqrt(a))
)/(15*sqrt(a*x + 1)*a*x**3)
```

3.552 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 188

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx=\frac{1888a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}+\frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c}+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2}+\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}$$

output

```
1888/105*a^3*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)+472/105*a^3*(1-1/a^2/x^
2)^(1/2)*(c-c/a/x)^(1/2)+59/35*a^3*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(3/2)/c+2
/7*a^3*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(5/2)/c^2+a^3*(c-c/a/x)^(7/2)/c^3/(1-
1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\left(-15+66ax-167a^2x^2+668a^3x^3+1336a^4x^4\right)}{105x^2\left(-1+a^2x^2\right)}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
(2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-15 + 66*a*x - 167*a^2*x^2 +
 668*a^3*x^3 + 1336*a^4*x^4))/(105*x^2*(-1 + a^2*x^2))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6733, 581, 27, 669, 459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \text{6733}$$

$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{581}$$

$$-\frac{\frac{2a^2\int \frac{c^2\left(9a-\frac{2}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}{2a\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{7c^2}+\frac{2a^3\left(c-\frac{c}{ax}\right)^{9/2}}{7c\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{1}{7}a\int \frac{\left(9a-\frac{2}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}+\frac{2a^3\left(c-\frac{c}{ax}\right)^{9/2}}{7c\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \; 669$$

$$-\frac{\frac{1}{7}a\left(-\frac{59}{2}ac\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{11a^2\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2a^3\left(c-\frac{c}{ax}\right)^{9/2}}{7c\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \; 459$$

$$-\frac{\frac{1}{7}a\left(-\frac{59}{2}ac\left(\frac{8}{5}c\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2}{5}ac\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{11a^2\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2a^3\left(c-\frac{c}{ax}\right)^{9/2}}{7c\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \; 459$$

$$-\frac{\frac{1}{7}a\left(-\frac{59}{2}ac\left(\frac{8}{5}c\left(\frac{4}{3}c\int \frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2}{3}ac\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{5}ac\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{11a^2\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{c^3}$$

$$\Big\downarrow \; 458$$

$$-\frac{\frac{1}{7}a\left(-\frac{59}{2}ac\left(\frac{8}{5}c\left(\frac{8ac^2\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}+\frac{2}{3}ac\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{5}ac\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}\right)-\frac{11a^2\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)+\frac{2a}{7}}{c^3}$$

input

```
Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

output

```
-(((a*((-59*a*c*((8*c*((8*a*c^2*Sqrt[1 - 1/(a^2*x^2)])/(3*Sqrt[c - c/(a*x)
]) + (2*a*c*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/3))/5 + (2*a*c*Sqrt[1
 - 1/(a^2*x^2)]*(c - c/(a*x))^(3/2))/5))/2 - (11*a^2*(c - c/(a*x))^(7/2))/
Sqrt[1 - 1/(a^2*x^2)]))/7 + (2*a^3*(c - c/(a*x))^(9/2))/(7*c*Sqrt[1 - 1/(a
^2*x^2)]))/c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 669
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*
(p + 1))), x] - Simp[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1)))
Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.40

method	result	size
orering	$\frac{2(1336a^4x^4+668a^3x^3-167a^2x^2+66ax-15)(ax+1)\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{105x^3(ax-1)^2}$	76
gosper	$\frac{2(ax+1)(1336a^4x^4+668a^3x^3-167a^2x^2+66ax-15)\sqrt{\frac{c(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{105x^3(ax-1)^2}$	78
default	$\frac{2(ax+1)(1336a^4x^4+668a^3x^3-167a^2x^2+66ax-15)\sqrt{\frac{c(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{105x^3(ax-1)^2}$	78
risch	$\frac{2(916a^4x^4+668a^3x^3-167a^2x^2+66ax-15)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{105x^3(ax-1)}+\frac{8x\,a^4\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$	117

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/105*(1336*a^4*x^4+668*a^3*x^3-167*a^2*x^2+66*a*x-15)*(a*x+1)/x^3/(a*x-1)
^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2\,(1336\,a^4x^4+668\,a^3x^3-167\,a^2x^2+66\,ax-15)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{105\,(ax^4-x^3)}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas
")
```

output
```
2/105*(1336*a^4*x^4 + 668*a^3*x^3 - 167*a^2*x^2 + 66*a*x - 15)*sqrt((a*x -
 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.79 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.53

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(1336\,a^3\,x^3+2004\,a^2\,x^2+1837\,a\,x+1903\right)\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{105\,x^3} + \frac{3776\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{105\,x^3\,\left(a\,x-1\right)}$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2)*(1837*a*x + 2004*a^2*x^2 + 1336*a^3*x^3 + 1
903)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3) + (3776*((a*x - 1)/(a*x + 1))^
(1/2)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.44

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{2\sqrt{c}\left(-1336\sqrt{ax+1}\,a^4x^4+1336\sqrt{x}\,\sqrt{a}\,a^4x^4+668\sqrt{x}\,\sqrt{a}\,a^3x^3-167\sqrt{x}\,\sqrt{a}\,a^2x^2+66\sqrt{x}\,\sqrt{a}\,ax-15 \cdots}{105\sqrt{ax+1}\,a\,x^4}$$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

output
```
(2*sqrt(c)*( - 1336*sqrt(a*x + 1)*a**4*x**4 + 1336*sqrt(x)*sqrt(a)*a**4*x**
*4 + 668*sqrt(x)*sqrt(a)*a**3*x**3 - 167*sqrt(x)*sqrt(a)*a**2*x**2 + 66*sq
rt(x)*sqrt(a)*a*x - 15*sqrt(x)*sqrt(a)))/(105*sqrt(a*x + 1)*a*x**4)
```

3.553 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 228

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\frac{928a^4c\sqrt{1-\frac{1}{a^2x^2}}}{45\sqrt{c-\frac{c}{ax}}}-\frac{232}{45}a^4\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{29a^4\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{15c}-\frac{2a^4\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{9c^2}-\frac{a^4\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a^4\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{7/2}}{9c^3}$$

output
```
-928/45*a^4*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)-232/45*a^4*(1-1/a^2/x^2)
^(1/2)*(c-c/a/x)^(1/2)-29/15*a^4*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(3/2)/c-2/9
*a^4*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(5/2)/c^2-a^4*(c-c/a/x)^(7/2)/c^3/(1-1/
a^2/x^2)^(1/2)-2/9*a^4*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(7/2)/c^3
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.38

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$
$$= -\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}(5-20ax+41a^2x^2-82a^3x^3+328a^4x^4+656a^5x^5)}{45x^3(-1+a^2x^2)}$$

input
```
Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
(-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(5 - 20*a*x + 41*a^2*x^2 - 8
2*a^3*x^3 + 328*a^4*x^4 + 656*a^5*x^5))/(45*x^3*(-1 + a^2*x^2))
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6733, 581, 27, 2166, 27, 672, 459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{ax}}e^{-3\coth^{-1}(ax)}}{x^5}\,dx$$
$$\Big\downarrow \text{6733}$$
$$-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}d\frac{1}{x}}{c^3}$$
$$\Big\downarrow \text{581}$$
$$-\frac{-\frac{2a^3\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}\left(-\frac{13c^3}{ax}-\frac{7c^3}{a^2x^2}+11c^3\right)}{2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{9c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{11/2}}{9c^2\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{a^3\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}\left(-\frac{13c^3}{ax}-\frac{7c^3}{a^2x^2}+11c^3\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}d\frac{1}{x}}{9c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{11/2}}{9c^2\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \; 2166$$

$$-\frac{-\frac{a^3\left(-c\int \frac{c^3\left(97a+\frac{14}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{17ac^3\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{9c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{11/2}}{9c^2\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{a^3\left(-\frac{c^4\int \frac{\left(97a+\frac{14}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{2a}-\frac{17ac^3\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{9c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{11/2}}{9c^2\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \; 672$$

$$-\frac{-\frac{a^3\left(-\frac{c^4\left(87a\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-4a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a}-\frac{17ac^3\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{9c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{11/2}}{9c^2\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \; 459$$

$$-\frac{-\frac{a^3\left(-\frac{c^4\left(87a\left(\frac{8}{5}c\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2}{5}ac\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}\right)-4a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a}-\frac{17ac^3\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{9c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{11/2}}{9c^2\sqrt{1-\frac{1}{a^2x^2}}}}{c^3}$$

$$\Big\downarrow \; 459$$

$$-\frac{-\frac{a^3\left(-\frac{c^4\left(87a\left(\frac{8}{5}c\left(\frac{4}{3}c\int \frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+\frac{2}{3}ac\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{5}ac\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}\right)-4a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}\right)}{2a}-\frac{17ac^3\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}\right)}{9c^3}}{c^3}$$

$\downarrow$ 458

$$-\frac{-\frac{2a^4\left(c-\frac{c}{ax}\right)^{11/2}}{9c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{a^3\left(-\frac{17ac^3\left(c-\frac{c}{ax}\right)^{7/2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^4\left(87a\left(\frac{8}{5}c\left(\frac{8ac^2\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}+\frac{2}{3}ac\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\right)+\frac{2}{5}ac\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}\right)-4a^2\sqrt{1-\frac{1}{a^2x^2}}\right.}{2a}\right)}{9c^3}}{c^3}$$

input
```
Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
-((-1/9*(a^3*(-1/2*(c^4*(87*a*((8*c*((8*a*c^2*Sqrt[1 - 1/(a^2*x^2)])/(3*Sq
rt[c - c/(a*x)]) + (2*a*c*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/3))/5 +
 (2*a*c*Sqrt[1 - 1/(a^2*x^2)]*(c - c/(a*x))^(3/2))/5) - 4*a^2*Sqrt[1 - 1/(
a^2*x^2)]*(c - c/(a*x))^(5/2)))/a - (17*a*c^3*(c - c/(a*x))^(7/2))/Sqrt[1
- 1/(a^2*x^2)]))/c^3 - (2*a^4*(c - c/(a*x))^(11/2))/(9*c^2*Sqrt[1 - 1/(a^2
*x^2)]))/c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

rule 581
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*x^
2)^p*ExpandToSum[d^m*(m + n + 2*p + 1)*x^m - (m + n + 2*p + 1)*(c + d*x)^m
+ c*(c + d*x)^(m - 2)*(c*(m + n - 1) + c*(m + n + 2*p + 1) + 2*d*(m + n + p
)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] &
& IGtQ[m, 1] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[2*p] || ILtQ[m + n, 0]
)
```

rule 672
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Simp[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2))   Int[(d + e*x
)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^
2 + a*e^2, 0] && NeQ[m + 2*p + 2, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 6733
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Simp[-c^n   Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Int
egerQ[(n - 1)/2] && IntegerQ[m] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.37

method	result	size
orering	$-\frac{2\left(656a^{5}x^{5}+328a^{4}x^{4}-82a^{3}x^{3}+41a^{2}x^{2}-20ax+5\right)\left(ax+1\right)\sqrt{c-\frac{c}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{45x^{4}\left(ax-1\right)^{2}}$	84
gosper	$-\frac{2\left(ax+1\right)\left(656a^{5}x^{5}+328a^{4}x^{4}-82a^{3}x^{3}+41a^{2}x^{2}-20ax+5\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{45x^{4}\left(ax-1\right)^{2}}$	86
default	$-\frac{2\left(ax+1\right)\left(656a^{5}x^{5}+328a^{4}x^{4}-82a^{3}x^{3}+41a^{2}x^{2}-20ax+5\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{45x^{4}\left(ax-1\right)^{2}}$	86
risch	$-\frac{2\left(476a^{5}x^{5}+328a^{4}x^{4}-82a^{3}x^{3}+41a^{2}x^{2}-20ax+5\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{45x^{4}\left(ax-1\right)}-\frac{8x\,a^{5}\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$	125

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/45*(656*a^5*x^5+328*a^4*x^4-82*a^3*x^3+41*a^2*x^2-20*a*x+5)*(a*x+1)/x^4
/(a*x-1)^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.37

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\frac{2\left(656\,a^5x^5+328\,a^4x^4-82\,a^3x^3+41\,a^2x^2-20\,ax+5\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{45\left(ax^5-x^4\right)}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fricas
")
```

output
```
-2/45*(656*a^5*x^5 + 328*a^4*x^4 - 82*a^3*x^3 + 41*a^2*x^2 - 20*a*x + 5)*s
qrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^5 - x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.73 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}\,\left(656\,a^4\,x^4+984\,a^3\,x^3+902\,a^2\,x^2+943\,a\,x+923\right)}{45\,x^4} - \frac{1856\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{45\,x^4\,\left(a\,x-1\right)}$$

input
```
int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
```

output
```
- (2*((a*x - 1)/(a*x + 1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2)*(943*a*x + 90
2*a^2*x^2 + 984*a^3*x^3 + 656*a^4*x^4 + 923))/(45*x^4) - (1856*((a*x - 1)/
(a*x + 1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2))/(45*x^4*(a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.42

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{2\sqrt{c}\left(656\sqrt{ax+1}\,a^5x^5-656\sqrt{x}\,\sqrt{a}\,a^5x^5-328\sqrt{x}\,\sqrt{a}\,a^4x^4+82\sqrt{x}\,\sqrt{a}\,a^3x^3-41\sqrt{x}\,\sqrt{a}\,a^2x^2+20\sqrt{x}\cdots\right.}{45\sqrt{ax+1}\,a\,x^5}$$

input
```
int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x)
```

output
```
(2*sqrt(c)*(656*sqrt(a*x + 1)*a**5*x**5 - 656*sqrt(x)*sqrt(a)*a**5*x**5 -
328*sqrt(x)*sqrt(a)*a**4*x**4 + 82*sqrt(x)*sqrt(a)*a**3*x**3 - 41*sqrt(x)*
sqrt(a)*a**2*x**2 + 20*sqrt(x)*sqrt(a)*a*x - 5*sqrt(x)*sqrt(a)))/(45*sqrt(
a*x + 1)*a*x**5)
```

3.554 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx$

Optimal result

Integrand size = 27, antiderivative size = 62

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx = \frac{c\sqrt{1-\frac{1}{ax}}x(ex)^m \text{Hypergeometric2F1}\left(-\frac{1}{2},-1-m,-m,-\frac{1}{ax}\right)}{(1+m)\sqrt{c-\frac{c}{ax}}}$$

output
```
c*(1-1/a/x)^(1/2)*x*(e*x)^m*hypergeom([-1/2, -1-m],[-m],-1/a/x)/(1+m)/(c-c
/a/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx = \frac{\sqrt{c-\frac{c}{ax}}x(ex)^m \text{Hypergeometric2F1}\left(-\frac{1}{2},-1-m,-m,-\frac{1}{ax}\right)}{(1+m)\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*(e*x)^m,x]
```

output
```
(Sqrt[c - c/(a*x)]*x*(e*x)^m*Hypergeometric2F1[-1/2, -1 - m, -m, -(1/(a*x)
)])/((1 + m)*Sqrt[1 - 1/(a*x)])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{\coth^{-1}(ax)}(ex)^m\,dx$$

$$\Big\downarrow \text{ 6736}$$

$$\frac{\sqrt{c-\frac{c}{ax}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}(ex)^m dx}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 7268}$$

$$\frac{2\sqrt{c-\frac{c}{ax}}\int a^2e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x^2(ex)^m d\sqrt{1-\frac{1}{ax}}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 2044}$$

$$\frac{2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{ax}\right)^m(ex)^m\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)\left(\frac{1}{ax}\right)^{-m-2}d\sqrt{1-\frac{1}{ax}}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{ax}\right)^m(ex)^m\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)\left(\frac{1}{ax}\right)^{-m-2}d\sqrt{1-\frac{1}{ax}}}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*(e*x)^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\sqrt{c-\frac{c}{ax}}\,(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)*(e*x)^m,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)*(e*x)^m,x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx=\int\frac{(ex)^m\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)*(e*x)^m,x, algorithm="
fricas")
```

output
```
integral((a*x + 1)*(e*x)^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
))/(a*x - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)*(e*x)**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx=\int\frac{(ex)^m\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)*(e*x)^m,x, algorithm="
maxima")
```

output
```
integrate((e*x)^m*sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx=\int\frac{(ex)^m\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)*(e*x)^m,x, algorithm="
giac")
```

output
```
integrate((e*x)^m*sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx=\int\frac{\sqrt{c-\frac{c}{a\,x}}\,(e\,x)^m}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(((c - c/(a*x))^(1/2)*(e*x)^m)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(((c - c/(a*x))^(1/2)*(e*x)^m)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx=\frac{e^m\sqrt{c}\left(\int\frac{x^m\sqrt{ax+1}}{\sqrt{x}}dx\right)}{\sqrt{a}}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)*(e*x)^m,x)
```

output
```
(e**m*sqrt(c)*int((x**m*sqrt(a*x + 1))/sqrt(x),x))/sqrt(a)
```

3.555 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx$

Optimal result

Integrand size = 29, antiderivative size = 119

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx = -\frac{2c\sqrt{1-\frac{1}{a^2x^2}}x(ex)^m}{(1+2m)\sqrt{c-\frac{c}{ax}}} + \frac{(3+4m)\sqrt{c-\frac{c}{ax}}x(ex)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-1-m,-m,-\frac{1}{ax}\right)}{(1+m)(1+2m)\sqrt{1-\frac{1}{ax}}}$$

output
```
-2*c*(1-1/a^2/x^2)^(1/2)*x*(e*x)^m/(1+2*m)/(c-c/a/x)^(1/2)+(3+4*m)*(c-c/a/
x)^(1/2)*x*(e*x)^m*hypergeom([1/2, -1-m],[-m],-1/a/x)/(1+m)/(1+2*m)/(1-1/a
/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.80

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx$$
$$=\frac{\sqrt{c-\frac{c}{ax}}(ex)^m\left(2am\sqrt{1+\frac{1}{ax}}x-(3+4m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,1-m,-\frac{1}{ax}\right)\right)}{2am(1+m)\sqrt{1-\frac{1}{ax}}}$$

input
```
Integrate[(Sqrt[c - c/(a*x)]*(e*x)^m)/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a*x)]*(e*x)^m*(2*a*m*Sqrt[1 + 1/(a*x)]*x - (3 + 4*m)*Hypergeo
metric2F1[1/2, -m, 1 - m, -(1/(a*x))]))/(2*a*m*(1 + m)*Sqrt[1 - 1/(a*x)])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{-\coth^{-1}(ax)}(ex)^m\,dx$$
$$\Big\downarrow \text{6736}$$
$$\frac{\sqrt{c-\frac{c}{ax}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}(ex)^mdx}{\sqrt{1-\frac{1}{ax}}}$$
$$\Big\downarrow \text{7268}$$
$$\frac{2\sqrt{c-\frac{c}{ax}}\int a^2e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x^2(ex)^md\sqrt{1-\frac{1}{ax}}}{a\sqrt{1-\frac{1}{ax}}}$$
$$\Big\downarrow \text{2044}$$
$$\frac{2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{ax}\right)^m(ex)^m\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)\left(\frac{1}{ax}\right)^{-m-2}d\sqrt{1-\frac{1}{ax}}}{a\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{7299}$$

$$\frac{2\sqrt{c-\frac{c}{ax}}\left(\frac{1}{ax}\right)^m(ex)^m\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)\left(\frac{1}{ax}\right)^{-m-2}d\sqrt{1-\frac{1}{ax}}}{a\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[(Sqrt[c - c/(a*x)]*(e*x)^m)/E^ArcCoth[a*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \sqrt{c-\frac{c}{ax}}\,(ex)^m \sqrt{\frac{ax-1}{ax+1}}dx$$

input
```
int((c-c/a/x)^(1/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
int((c-c/a/x)^(1/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2),x)
```

Fricas [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx = \int (ex)^m \sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fr
icas")
```

output
```
integral((e*x)^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx = \text{Timed out}$$

input
```
integrate((c-c/a/x)**(1/2)*(e*x)**m*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx=\int (ex)^m\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((e*x)^m*sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx=\int (ex)^m\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^(1/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((e*x)^m*sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx=\int \sqrt{c-\frac{c}{a\,x}}\,(e\,x)^m\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a*x))^(1/2)*(e*x)^m*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^(1/2)*(e*x)^m*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}(ex)^m\,dx=\frac{e^m\sqrt{c}\left(-\left(\int\frac{x^m}{\sqrt{x}\,\sqrt{ax+1}}dx\right)+\left(\int\frac{x^m x}{\sqrt{x}\,\sqrt{ax+1}}dx\right)a\right)}{\sqrt{a}}$$

input

```
int((c-c/a/x)^(1/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2),x)
```

output

```
(e**m*sqrt(c)*( - int(x**m/(sqrt(x)*sqrt(a*x + 1)),x) + int((x**m*x)/(sqrt
(x)*sqrt(a*x + 1)),x)*a))/sqrt(a)
```

3.556 $\int e^{n \coth^{-1}(ax)} \left(c - \frac{c}{ax}\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 185

$$
\begin{aligned}
&\int e^{n \coth^{-1}(ax)} \left(c - \frac{c}{ax}\right) \, dx \\
&= c\left(1 - \frac{1}{ax}\right)^{1-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{n/2} x \\
&\quad - \frac{2c(1-n)\left(1 - \frac{1}{ax}\right)^{-n/2} \left(1 + \frac{1}{ax}\right)^{n/2} \text{Hypergeometric2F1}\left(1, \frac{n}{2}, \frac{2+n}{2}, \frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{an} \\
&\quad - \frac{2^{n/2} c\left(1 - \frac{1}{ax}\right)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1 - \frac{n}{2}, 1 - \frac{n}{2}, 2 - \frac{n}{2}, \frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}
\end{aligned}
$$

output
```
c*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1/2*n)*x-2*c*(1-n)*(1+1/a/x)^(1/2*n)*hype
rgeom([1, 1/2*n],[1+1/2*n],(a+1/x)/(a-1/x))/a/n/((1-1/a/x)^(1/2*n))-2^(1/2
*n)*c*(1-1/a/x)^(1-1/2*n)*hypergeom([1-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/
x)/a)/a/(2-n)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$$
$$=\frac{ce^{n\coth^{-1}(ax)}\left(-e^{2\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},-e^{2\coth^{-1}(ax)}\right)+e^{2\coth^{-1}(ax)}(-1+n)n\right.}{}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x)),x]
```

output
```
(c*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2
, 2 + n/2, -E^(2*ArcCoth[a*x])]) + E^(2*ArcCoth[a*x])*(-1 + n)*n*Hypergeom
etric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*n*x + Hyper
geometric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])] + (-1 + n)*Hypergeometr
ic2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*n*(2 + n))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6732, 138, 79, 107, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \; 6732$$

$$-c\int \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \; 138$$

$$-c\left(\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}x^2d\frac{1}{x}-\frac{\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}d\frac{1}{x}}{a^2}\right)$$

$$\Big\downarrow \text{79}$$

$$-c\left(\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}x^2d\frac{1}{x}+\frac{2^{n/2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}\right)$$

$$\Big\downarrow \text{107}$$

$$-c\left(-\frac{(1-n)\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}xd\frac{1}{x}}{a}+\frac{2^{n/2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right.}{a(2-n)}\right.$$

$$\Big\downarrow \text{141}$$

$$-c\left(\frac{2^{n/2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}+\frac{2(1-n)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hype}}{an}\right.$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - c/(a*x)),x]
```

output
```
-(c*(-((1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^(n/2)*x) + (2*(1 - n)*(1 + 1/
(a*x))^(n/2)*Hypergeometric2F1[1, n/2, (2 + n)/2, (a + x^(-1))/(a - x^(-1)
)])/(a*n*(1 - 1/(a*x))^(n/2)) + (2^(n/2)*(1 - 1/(a*x))^(1 - n/2)*Hypergeom
etric2F1[(2 - n)/2, 1 - n/2, 2 - n/2, (a - x^(-1))/(2*a)])/(a*(2 - n))))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 138
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_))^2, x_] :> Simp[b*(d/f^2)   Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1), x],
 x] + Simp[(b*e - a*f)*((d*e - c*f)/f^2)   Int[(a + b*x)^(m - 1)*((c + d*x)
^(n - 1)/(e + f*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ
[m + n, 0] && EqQ[2*b*d*e - f*(b*c + a*d), 0]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int e^{n \operatorname{arccoth}(ax)} \left(c - \frac{c}{ax}\right) dx$$

input
```
int(exp(n*arccoth(a*x))*(c-c/a/x),x)
```

output
```
int(exp(n*arccoth(a*x))*(c-c/a/x),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\int\left(c-\frac{c}{ax}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="fricas")
```

output
```
integral((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*x), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c\left(\int ae^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-\frac{e^{n\,\mathrm{acoth}\,(ax)}}{x}\right)\,dx\right)}{a}$$

input
```
integrate(exp(n*acoth(a*x))*(c-c/a/x),x)
```

output
```
c*(Integral(a*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x))/x, x))/a
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\int\left(c-\frac{c}{ax}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\int\left(c-\frac{c}{ax}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - c/(a*x)),x)
```

output
```
int(exp(n*acoth(a*x))*(c - c/(a*x)), x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\frac{c\left(\left(\int e^{acoth(ax)n}dx\right)a-\left(\int\frac{e^{acoth(ax)n}}{x}dx\right)\right)}{a}$$

input
```
int(exp(n*acoth(a*x))*(c-c/a/x),x)
```

output
```
(c*(int(e**(acoth(a*x)*n),x)*a - int(e**(acoth(a*x)*n)/x,x)))/a
```

3.557 $\int \frac{e^{n \coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{e^{n \coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c} - \frac{2(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

output

```
(1+1/a/x)^(1+1/2*n)*x/c/((1-1/a/x)^(1/2*n))-2*(1+n)*(1+1/a/x)^(1/2*n)*hype
rgeom([1, -1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/a/c/n/((1-1/a/x)^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int \frac{e^{n \coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{e^{n \coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n(1+n)\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+anx\right.\right.}{acn(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x)),x]
```

output
```
(E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(1 + n)*Hypergeometric2F1[1, 1 +
 n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + (1 + n)*Hyperge
ometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6732, 107, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}dx$$

$$\Big\downarrow \text{6732}$$

$$-\frac{\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{107}$$

$$-\frac{\frac{(n+1)\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}}{a}-x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c}$$

$$\Big\downarrow \text{141}$$

$$-\frac{\frac{2(n+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{an}-x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - c/(a*x)),x]
```

output
```
-((-(((1 + 1/(a*x))^((2 + n)/2)*x)/(1 - 1/(a*x))^(n/2)) + (2*(1 + n)*(1 +
1/(a*x))^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (a - x^(-1))/(a + x^(
-1))])/(a*n*(1 - 1/(a*x))^(n/2)))/c)
```

Defintions of rubi rules used

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{c - \frac{c}{ax}} dx$$

input
```
int(exp(n*arccoth(a*x))/(c-c/a/x),x)
```

output
```
int(exp(n*arccoth(a*x))/(c-c/a/x),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{c-\frac{c}{ax}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="fricas")
```

output
```
integral(a*x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\frac{a\int \frac{xe^{n\,\mathrm{acoth}\,(ax)}}{ax-1}\,dx}{c}$$

input
```
integrate(exp(n*acoth(a*x))/(c-c/a/x),x)
```

output
```
a*Integral(x*exp(n*acoth(a*x))/(a*x - 1), x)/c
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{c-\frac{c}{ax}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Valu
e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{c-\frac{c}{a\,x}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - c/(a*x)),x)
```

output
```
int(exp(n*acoth(a*x))/(c - c/(a*x)), x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \frac{\left(\int \frac{e^{acoth(ax)n}x}{ax-1}dx\right)a}{c}$$

input
```
int(exp(n*acoth(a*x))/(c-c/a/x),x)
```

output
```
(int((e**(acoth(a*x)*n)*x)/(a*x - 1),x)*a)/c
```

3.558 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 166

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)}+\frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2} - \frac{2(2+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{ac^2n}$$

output
```
-(3+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^2/(2+n)+(1-1/a/x)^(-1-
1/2*n)*(1+1/a/x)^(1+1/2*n)*x/c^2-2*(2+n)*(1+1/a/x)^(1/2*n)*hypergeom([1, -
1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/a/c^2/n/((1-1/a/x)^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(n(1+ax)(-3+2ax+n(-1+ax))-2(2+n)^2(-1+ax)\,\text{Hypergeometric2I}}{ac^2n(2+n)(-1+ax)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

output
```
((1 + 1/(a*x))^(n/2)*(n*(1 + a*x)*(-3 + 2*a*x + n*(-1 + a*x)) - 2*(2 + n)^
2*(-1 + a*x)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (-1 + a*x)/(1 + a*x)]))
/(a*c^2*n*(2 + n)*(1 - 1/(a*x))^(n/2)*(-1 + a*x))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6732, 114, 25, 27, 172, 25, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

$$\Big\downarrow \text{6732}$$

$$-\frac{\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{114}$$

$$-\frac{x\left(-\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}-\int -\frac{\left(a(n+2)+\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{n/2}x}{a^2}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\left(a(n+2)+\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{n/2}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \left(a(n+2)+\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c^2}$$

$$\Big\downarrow \text{172}$$

$$-\frac{\frac{\frac{a(n+3)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{n+2}-\frac{a\int -(n+2)^2\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}}{n+2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\frac{a\int (n+2)^2\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}}{n+2}+\frac{a(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{n+2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{a(n+2)\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{n/2}xd\frac{1}{x}+\frac{a(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{n+2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c^2}$$

$$\Big\downarrow \text{141}$$

$$-\frac{\frac{\frac{2a(n+2)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n}+\frac{a(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{n+2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c^2}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

output
```
-((-((1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x) + ((a*(3 + n)*(
1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(2 + n) + (2*a*(2 + n)*
(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (a - x^(-1))/(a
+ x^(-1))])/(n*(1 - 1/(a*x))^(n/2)))/a^2)/c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{\left(c - \frac{c}{ax}\right)^2} dx$$

input
```
int(exp(n*arccoth(a*x))/(c-c/a/x)^2,x)
```

output
```
int(exp(n*arccoth(a*x))/(c-c/a/x)^2,x)
```

Fricas [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{\left(c - \frac{c}{ax}\right)^2} dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c - \frac{c}{ax}\right)^2} dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="fricas")
```

output
```
integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*x^2 - 2*a*c^2*x +
c^2), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{x^2e^{n\,\mathrm{acoth}\,(ax)}}{a^2x^2-2ax+1}\,dx}{c^2}$$

input
```
integrate(exp(n*acoth(a*x))/(c-c/a/x)**2,x)
```

output
```
a**2*Integral(x**2*exp(n*acoth(a*x))/(a**2*x**2 - 2*a*x + 1), x)/c**2
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,2]%%%} Error: Bad Argument Valu
e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\left(c-\frac{c}{a\,x}\right)^2}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - c/(a*x))^2,x)
```

output
```
int(exp(n*acoth(a*x))/(c - c/(a*x))^2, x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx=\frac{\left(\int \frac{e^{acoth(ax)n}x^2}{a^2x^2-2ax+1}dx\right)a^2}{c^2}$$

input
```
int(exp(n*acoth(a*x))/(c-c/a/x)^2,x)
```

output
```
(int((e**(acoth(a*x)*n)*x**2)/(a**2*x**2 - 2*a*x + 1),x)*a**2)/c**2
```

3.559 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{2^{1+\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(c-\frac{c}{ax}\right)^{3/2}\operatorname{AppellF1}\left(\frac{5-n}{2},2,-\frac{n}{2},\frac{7-n}{2},1-\frac{1}{ax},\frac{a-\frac{1}{x}}{2a}\right)}{a(5-n)}$$

output
```
2^(1+1/2*n)*(1-1/a/x)^(1-1/2*n)*(c-c/a/x)^(3/2)*AppellF1(5/2-1/2*n,-1/2*n,
2,7/2-1/2*n,1/2*(a-1/x)/a,1-1/a/x)/a/(5-n)
```

Mathematica [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\$Aborted$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

output
```
$Aborted
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6736, 6732, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^{3/2} e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6736}$$

$$\frac{\left(c-\frac{c}{ax}\right)^{3/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}dx}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{6732}$$

$$-\frac{\left(c-\frac{c}{ax}\right)^{3/2}\int \left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{153}$$

$$-\frac{2^{\frac{5}{2}-\frac{n}{2}}\left(c-\frac{c}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{AppellF1}\left(\frac{n+2}{2},\frac{n-3}{2},2,\frac{n+4}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\left(1-\frac{1}{ax}\right)^{3/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

output
```
-((2^(5/2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - c/(a*x))^(3/2)*AppellF1[(2
 + n)/2, (-3 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2
 + n)*(1 - 1/(a*x))^(3/2)))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int e^{n \operatorname{arccoth}(ax)} \left(c - \frac{c}{ax}\right)^{\frac{3}{2}} dx$$

input
```
int(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x)
```

output
```
int(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))
/(a*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(exp(n*acoth(a*x))*(c-c/a/x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)^{3/2}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - c/(a*x))^(3/2),x)
```

output
```
int(exp(n*acoth(a*x))*(c - c/(a*x))^(3/2), x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=\frac{\sqrt{c}\,c\left(\left(\int\frac{e^{acoth(ax)n}\sqrt{ax-1}}{\sqrt{x}}dx\right)a-\left(\int\frac{e^{acoth(ax)n}\sqrt{ax-1}}{\sqrt{x}\,x}dx\right)\right)}{\sqrt{a}\,a}$$

input
```
int(exp(n*acoth(a*x))*(c-c/a/x)^(3/2),x)
```

output
```
(sqrt(c)*c*(int((e**(acoth(a*x)*n)*sqrt(a*x - 1))/sqrt(x),x)*a - int((e**(
acoth(a*x)*n)*sqrt(a*x - 1))/(sqrt(x)*x),x)))/(sqrt(a)*a)
```

3.560 $\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{2^{1+\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\sqrt{c-\frac{c}{ax}}\,\text{AppellF1}\left(\frac{3-n}{2},2,-\frac{n}{2},\frac{5-n}{2},1-\frac{1}{ax},\frac{a-\frac{1}{x}}{2a}\right)}{a(3-n)}$$

output
```
2^(1+1/2*n)*(1-1/a/x)^(1-1/2*n)*(c-c/a/x)^(1/2)*AppellF1(3/2-1/2*n,-1/2*n,
2,5/2-1/2*n,1/2*(a-1/x)/a,1-1/a/x)/a/(3-n)
```

Mathematica [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \text{\$Aborted}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
$Aborted
```

Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6736, 6732, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{ax}}e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6736}$$

$$\frac{\sqrt{c-\frac{c}{ax}}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}dx}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{6732}$$

$$-\frac{\sqrt{c-\frac{c}{ax}}\int \left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}}{\sqrt{1-\frac{1}{ax}}}$$

$$\Big\downarrow \text{153}$$

$$-\frac{2^{\frac{3}{2}-\frac{n}{2}}\sqrt{c-\frac{c}{ax}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{AppellF1}\left(\frac{n+2}{2},\frac{n-1}{2},2,\frac{n+4}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\sqrt{1-\frac{1}{ax}}}$$

input
```
Int[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

output
```
-((2^(3/2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*Sqrt[c - c/(a*x)]*AppellF1[(2 +
 n)/2, (-1 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
 n)*Sqrt[1 - 1/(a*x)]))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int e^{n\ \mathrm{arccoth}(ax)}\sqrt{c-\frac{c}{ax}}dx$$

input
```
int(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x)
```

output
```
int(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x)), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{-c\left(-1+\frac{1}{ax}\right)}e^{n\operatorname{acoth}(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(c-c/a/x)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x)))*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,\sqrt{c-\frac{c}{a\,x}}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - c/(a*x))^(1/2),x)
```

output
```
int(exp(n*acoth(a*x))*(c - c/(a*x))^(1/2), x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c}\left(\int\frac{e^{acoth(ax)n}\sqrt{ax-1}}{\sqrt{x}}dx\right)}{\sqrt{a}}$$

input
```
int(exp(n*acoth(a*x))*(c-c/a/x)^(1/2),x)
```

output
```
(sqrt(c)*int((e**(acoth(a*x)*n)*sqrt(a*x - 1))/sqrt(x),x))/sqrt(a)
```

3.561 $\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{2^{1+\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\operatorname{AppellF1}\left(\frac{1-n}{2},2,-\frac{n}{2},\frac{3-n}{2},1-\frac{1}{ax},\frac{a-\frac{1}{x}}{2a}\right)}{a(1-n)\sqrt{c-\frac{c}{ax}}}$$

output
```
2^(1+1/2*n)*(1-1/a/x)^(1-1/2*n)*AppellF1(1/2-1/2*n,-1/2*n,2,3/2-1/2*n,1/2*
(a-1/x)/a,1-1/a/x)/a/(1-n)/(c-c/a/x)^(1/2)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\$\text{Aborted}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

output
```
$Aborted
```

Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6736, 6732, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

$$\Big\downarrow \text{6736}$$

$$\frac{\sqrt{1-\frac{1}{ax}}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}}dx}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{6732}$$

$$-\frac{\sqrt{1-\frac{1}{ax}}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}$$

$$\Big\downarrow \text{153}$$

$$-\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{AppellF1}\left(\frac{n+2}{2},\frac{n+1}{2},2,\frac{n+4}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\sqrt{c-\frac{c}{ax}}}$$

input
```
Int[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

output
```
-((2^(1/2 - n/2)*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^((2 + n)/2)*AppellF1[(2 +
 n)/2, (1 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
n)*Sqrt[c - c/(a*x)]))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{\sqrt{c-\frac{c}{ax}}}dx$$

input
```
int(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x)
```

output
```
int(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

output
```
integral(a*x*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a*c*x
- c), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(c-c/a/x)**(1/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/sqrt(-c*(-1 + 1/(a*x))), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a*x)), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - c/(a*x))^(1/2),x)
```

output
```
int(exp(n*acoth(a*x))/(c - c/(a*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{x}\,e^{acoth(ax)n}}{\sqrt{ax-1}}dx\right)}{\sqrt{c}}$$

input
```
int(exp(n*acoth(a*x))/(c-c/a/x)^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(x)*e**(acoth(a*x)*n))/sqrt(a*x - 1),x))/sqrt(c)
```

3.562 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}} dx = -\frac{2^{1+\frac{n}{2}} \left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}} \operatorname{AppellF1}\left(\frac{1}{2}(-1-n),2,-\frac{n}{2},\frac{1-n}{2},1-\frac{1}{ax},\frac{a-\frac{1}{x}}{2a}\right)}{a(1+n)\left(c-\frac{c}{ax}\right)^{3/2}}$$

output
```
-2^(1+1/2*n)*(1-1/a/x)^(1-1/2*n)*AppellF1(-1/2-1/2*n,-1/2*n,2,1/2-1/2*n,1/
2*(a-1/x)/a,1-1/a/x)/a/(1+n)/(c-c/a/x)^(3/2)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}} dx = \$Aborted$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

output
```
$Aborted
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6736, 6732, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 6736}$$

$$\frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}dx}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{ 6732}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$\Big\downarrow \text{ 153}$$

$$-\frac{2^{-\frac{n}{2}-\frac{1}{2}}\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\text{AppellF1}\left(\frac{n+2}{2},\frac{n+3}{2},2,\frac{n+4}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\left(c-\frac{c}{ax}\right)^{3/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

output
```
-((2^(-1/2 - n/2)*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^((2 + n)/2)*AppellF1[(
2 + n)/2, (3 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2
 + n)*(c - c/(a*x))^(3/2)))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}dx$$

input
```
int(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x)
```

output
```
int(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

output
```
integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a^
2*c^2*x^2 - 2*a*c^2*x + c^2), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{e^{n\operatorname{acoth}(ax)}}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(c-c/a/x)**(3/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(-c*(-1 + 1/(a*x)))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^(3/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\left(c-\frac{c}{a\,x}\right)^{3/2}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - c/(a*x))^(3/2),x)
```

output
```
int(exp(n*acoth(a*x))/(c - c/(a*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{x}\,e^{acoth(ax)n}x}{\sqrt{ax-1}\,ax-\sqrt{ax-1}}dx\right)a}{\sqrt{c}\,c}$$

input
```
int(exp(n*acoth(a*x))/(c-c/a/x)^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(x)*e**(acoth(a*x)*n)*x)/(sqrt(a*x - 1)*a*x - sqrt(a*x -
 1)),x)*a)/(sqrt(c)*c)
```

3.563 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 101

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = \frac{2^{1+\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(c-\frac{c}{ax}\right)^p\text{AppellF1}\left(1-\frac{n}{2}+p,2,-\frac{n}{2},2-\frac{n}{2}+p,1-\frac{1}{ax},\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n+2p)}$$

output
```
2^(1+1/2*n)*(1-1/a/x)^(1-1/2*n)*(c-c/a/x)^p*AppellF1(1-1/2*n+p,-1/2*n,2,2-
1/2*n+p,1/2*(a-1/x)/a,1-1/a/x)/a/(2-n+2*p)
```

Mathematica [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = \int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

output
```
Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6736, 6732, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^p e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6736}$$

$$\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx$$

$$\Big\downarrow \text{6732}$$

$$-\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int \left(1-\frac{1}{ax}\right)^{p-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{153}$$

$$-\frac{2^{-\frac{n}{2}+p+1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p \text{AppellF1}\left(\frac{n+2}{2},\frac{1}{2}(n-2p),2,\frac{n+4}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

output
```
-((2^(1 - n/2 + p)*(1 + 1/(a*x))^((2 + n)/2)*(c - c/(a*x))^p*AppellF1[(2 +
 n)/2, (n - 2*p)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2
+ n)*(1 - 1/(a*x))^p))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int e^{n \operatorname{arccoth}(ax)} \left(c - \frac{c}{ax}\right)^p dx$$

input
```
int(exp(n*arccoth(a*x))*(c-c/a/x)^p,x)
```

output
```
int(exp(n*arccoth(a*x))*(c-c/a/x)^p,x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)*((a*c*x - c)/(a*x))^p, x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(-c\left(-1+\frac{1}{ax}\right)\right)^p e^{n\operatorname{acoth}(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(c-c/a/x)**p,x)
```

output
```
Integral((-c*(-1 + 1/(a*x)))**p*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)^p\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - c/(a*x))^p,x)
```

output
```
int(exp(n*acoth(a*x))*(c - c/(a*x))^p, x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\frac{\int\frac{e^{acoth(ax)n}(acx-c)^p}{x^p}dx}{a^p}$$

input
```
int(exp(n*acoth(a*x))*(c-c/a/x)^p,x)
```

output
```
int((e**(acoth(a*x)*n)*(a*c*x - c)**p)/x**p,x)/a**p
```

3.564 $\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = -\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+p}\left(c-\frac{c}{ax}\right)^p\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{1}{ax}\right)}{a(1+p)}$$

output
```
-(1+1/a/x)^(p+1)*(c-c/a/x)^p*hypergeom([2, p+1],[2+p],1+1/a/x)/a/(p+1)/((1
-1/a/x)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = -\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+p}\left(c-\frac{c}{ax}\right)^p\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{1}{ax}\right)}{a(1+p)}$$

input
```
Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

output
```
-(((1 + 1/(a*x))^(1 + p)*(c - c/(a*x))^p*Hypergeometric2F1[2, 1 + p, 2 + p
, 1 + 1/(a*x)])/(a*(1 + p)*(1 - 1/(a*x))^p))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6736, 6732, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^p e^{2p\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6736}$$

$$\left(1-\frac{1}{ax}\right)^{-p} \left(c-\frac{c}{ax}\right)^p \int e^{2p\coth^{-1}(ax)} \left(1-\frac{1}{ax}\right)^p dx$$

$$\Big\downarrow \text{ 6732}$$

$$-\left(1-\frac{1}{ax}\right)^{-p} \left(c-\frac{c}{ax}\right)^p \int \left(1+\frac{1}{ax}\right)^p x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 75}$$

$$-\frac{\left(1-\frac{1}{ax}\right)^{-p} \left(\frac{1}{ax}+1\right)^{p+1} \left(c-\frac{c}{ax}\right)^p \text{Hypergeometric2F1}\left(2,p+1,p+2,1+\frac{1}{ax}\right)}{a(p+1)}$$

input
```
Int[E^(2*p*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

output


```
-(((1 + 1/(a*x))^(1 + p)*(c - c/(a*x))^p*Hypergeometric2F1[2, 1 + p, 2 + p
, 1 + 1/(a*x)])/(a*(1 + p)*(1 - 1/(a*x))^p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int e^{2p\operatorname{arccoth}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

input
```
int(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x)
```

output
```
int(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x)
```

Fricas [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx=\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^p dx$$

input
```
integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^p*((a*c*x - c)/(a*x))^p, x)
```

Sympy [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(-c\left(-1+\frac{1}{ax}\right)\right)^p e^{2p\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(2*p*acoth(a*x))*(c-c/a/x)**p,x)
```

output
```
Integral((-c*(-1 + 1/(a*x)))**p*exp(2*p*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

input
```
integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^p, x)
```

Giac [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

input
```
integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int e^{2\,p\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)^p\,dx$$

input
```
int(exp(2*p*acoth(a*x))*(c - c/(a*x))^p,x)
```

output
```
int(exp(2*p*acoth(a*x))*(c - c/(a*x))^p, x)
```

Reduce [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\frac{\int\frac{e^{2acoth(ax)p}(acx-c)^p}{x^p}dx}{a^p}$$

input
```
int(exp(2*p*acoth(a*x))*(c-c/a/x)^p,x)
```

output
```
int((e**(2*acoth(a*x)*p)*(a*c*x - c)**p)/x**p,x)/a**p
```

3.565 $\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = \frac{2^{-p}\left(1-\frac{1}{ax}\right)^{1+p}\left(c-\frac{c}{ax}\right)^p\text{AppellF1}\left(1+2p,2,p,2(1+p),1-\frac{1}{ax},\frac{a-\frac{1}{x}}{2a}\right)}{a(1+2p)}$$

output
```
(1-1/a/x)^(p+1)*(c-c/a/x)^p*AppellF1(1+2*p,p,2,2*p+2,1/2*(a-1/x)/a,1-1/a/x
)/(2^p)/a/(1+2*p)
```

Mathematica [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

input
```
Integrate[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]),x]
```

output
```
Integrate[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]), x]
```

Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6736, 6732, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{ax}\right)^p e^{-2p\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6736}$$

$$\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx$$

$$\Big\downarrow \text{6732}$$

$$-\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int \left(1-\frac{1}{ax}\right)^{2p}\left(1+\frac{1}{ax}\right)^{-p}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{153}$$

$$-\frac{4^p\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{1-p}\text{AppellF1}\left(1-p,-2p,2,2-p,\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{a(1-p)}$$

input
```
Int[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]),x]
```

output
```
-((4^p*(1 + 1/(a*x))^(1 - p)*(c - c/(a*x))^p*AppellF1[1 - p, -2*p, 2, 2 -
p, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - p)*(1 - 1/(a*x))^p))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \left(c-\frac{c}{ax}\right)^{p} \mathrm{e}^{-2p\ \mathrm{arccoth}(ax)}dx$$

input
```
int((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x)
```

output
```
int((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x)
```

Fricas [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

input
```
integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")
```

output
```
integral(((a*c*x - c)/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Sympy [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\left(-c\left(-1+\frac{1}{ax}\right)\right)^p e^{-2p\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate((c-c/a/x)**p/exp(2*p*acoth(a*x)),x)
```

output
```
Integral((-c*(-1 + 1/(a*x)))**p*exp(-2*p*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

input
```
integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Giac [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

input
```
integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int e^{-2\,p\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)^p\,dx$$

input
```
int(exp(-2*p*acoth(a*x))*(c - c/(a*x))^p,x)
```

output
```
int(exp(-2*p*acoth(a*x))*(c - c/(a*x))^p, x)
```

Reduce [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\frac{\int\frac{(acx-c)^p}{x^p e^{2acoth(ax)p}}dx}{a^p}$$

input
```
int((c-c/a/x)^p/exp(2*p*acoth(a*x)),x)
```

output
```
int((a*c*x - c)**p/(x**p*e**(2*acoth(a*x)*p)),x)/a**p
```

3.566 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\left(c-\frac{c}{ax}\right)^p x+\frac{(2-p)\left(c-\frac{c}{ax}\right)^p \text{Hypergeometric2F1}\left(1,p,1+p,1-\frac{1}{ax}\right)}{ap}$$

output
```
(c-c/a/x)^p*x+(2-p)*(c-c/a/x)^p*hypergeom([1, p],[p+1],1-1/a/x)/a/p
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\frac{\left(c-\frac{c}{ax}\right)^p\left(apx-(-2+p)\,\text{Hypergeometric2F1}\left(1,p,1+p,1-\frac{1}{ax}\right)\right)}{ap}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

output
```
((c - c/(a*x))^p*(a*p*x - (-2 + p)*Hypergeometric2F1[1, p, 1 + p, 1 - 1/(a
*x)]))/(a*p)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6717, 6683, 1035, 281, 899, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^p(ax+1)}{1-ax}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^p}{\frac{1}{x}-a}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{p-1}dx}{a}$$

$$\Big\downarrow \text{899}$$

$$-\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{p-1}x^2d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{87}$$

$$-\frac{c\left((2-p)\int \left(c-\frac{c}{ax}\right)^{p-1}xd\frac{1}{x}-\frac{ax\left(c-\frac{c}{ax}\right)^p}{c}\right)}{a}$$

$$\Big\downarrow \text{75}$$

$$-\frac{c\left(-\frac{(2-p)\left(c-\frac{c}{ax}\right)^p \text{Hypergeometric2F1}\left(1,p,p+1,1-\frac{1}{ax}\right)}{cp}-\frac{ax\left(c-\frac{c}{ax}\right)^p}{c}\right)}{a}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

output
```
-((c*(-((a*(c - c/(a*x))^p*x)/c) - ((2 - p)*(c - c/(a*x))^p*Hypergeometric
2F1[1, p, 1 + p, 1 - 1/(a*x)])/(c*p)))/a)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [F]

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{p}}{ax-1}dx$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x)
```

output
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x)
```

Fricas [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{p}\,dx=\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{p}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*((a*c*x - c)/(a*x))^p/(a*x - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.10 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.72

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$
$$= a\left(\begin{cases}\frac{0^p x}{a}+\frac{0^p\log(ax-1)}{a^2}-\frac{a^{-p}c^p p x^{2-p}e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}1-p,2-p\\3-p\end{matrix}\middle|ax\right)}{\Gamma(3-p)\Gamma(p+1)} & \text{for } |ax|>1\\ \frac{0^p x}{a}+\frac{0^p\log(-ax+1)}{a^2}-\frac{a^{-p}c^p p x^{2-p}e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}1-p,2-p\\3-p\end{matrix}\middle|ax\right)}{\Gamma(3-p)\Gamma(p+1)} & \text{otherwise}\end{cases}\right)$$
$$+\begin{cases}\frac{0^p\log(ax-1)}{a}-\frac{a^{-p}c^p p x^{1-p}e^{i\pi p}\Gamma(p)\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}1-p,1-p\\2-p\end{matrix}\middle|ax\right)}{\Gamma(2-p)\Gamma(p+1)} & \text{for } |ax|>1\\ \frac{0^p\log(-ax+1)}{a}-\frac{a^{-p}c^p p x^{1-p}e^{i\pi p}\Gamma(p)\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}1-p,1-p\\2-p\end{matrix}\middle|ax\right)}{\Gamma(2-p)\Gamma(p+1)} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**p,x)
```

output
```
a*Piecewise((0**p*x/a + 0**p*log(a*x - 1)/a**2 - c**p*p*x**(2 - p)*exp(I*p
i*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 - p,), a*x)/(a**p*gamm
a(3 - p)*gamma(p + 1)), Abs(a*x) > 1), (0**p*x/a + 0**p*log(-a*x + 1)/a**2
 - c**p*p*x**(2 - p)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p
), (3 - p,), a*x)/(a**p*gamma(3 - p)*gamma(p + 1)), True)) + Piecewise((0*
*p*log(a*x - 1)/a - c**p*p*x**(1 - p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hy
per((1 - p, 1 - p), (2 - p,), a*x)/(a**p*gamma(2 - p)*gamma(p + 1)), Abs(a
*x) > 1), (0**p*log(-a*x + 1)/a - c**p*p*x**(1 - p)*exp(I*pi*p)*gamma(p)*g
amma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a*x)/(a**p*gamma(2 - p)*gamma(
p + 1)), True))
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c - c/(a*x))^p/(a*x - 1), x)
```

Giac [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="giac")
```

output
```
integrate((a*x + 1)*(c - c/(a*x))^p/(a*x - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^p\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a*x))^p*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a*x))^p*(a*x + 1))/(a*x - 1), x)
```

Reduce [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx = \frac{(acx-c)^p x - x^p\left(\int \frac{(acx-c)^p}{x^p ax - x^p}dx\right)p + 2x^p\left(\int \frac{(acx-c)^p}{x^p ax - x^p}dx\right)}{x^p a^p}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x)
```

output
```
((a*c*x - c)**p*x - x**p*int((a*c*x - c)**p/(x**p*a*x - x**p),x)*p + 2*x**
p*int((a*c*x - c)**p/(x**p*a*x - x**p),x))/(x**p*a**p)
```

3.567 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 83

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = \frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^p\operatorname{AppellF1}\left(\frac{1}{2}+p,2,-\frac{1}{2},\frac{3}{2}+p,1-\frac{1}{ax},\frac{a-\frac{1}{x}}{2a}\right)}{a(1+2p)}$$

output
```
2*2^(1/2)*(1-1/a/x)^(1/2)*(c-c/a/x)^p*AppellF1(1/2+p,-1/2,2,3/2+p,1/2*(a-1
/x)/a,1-1/a/x)/a/(1+2*p)
```

Mathematica [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = \int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^p,x]
```

output
```
Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^p, x]
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6736, 6732, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

$$\Big\downarrow \text{ 6736}$$

$$\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p\,dx$$

$$\Big\downarrow \text{ 6732}$$

$$-\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int \left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\sqrt{1+\frac{1}{ax}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 153}$$

$$-\frac{2^{p+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{-p}\text{AppellF1}\left(\frac{3}{2},\frac{1}{2}-p,2,\frac{5}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{3a}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a*x))^p,x]
```

output
```
-1/3*(2^(1/2 + p)*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^p*AppellF1[3/2, 1/2 -
p, 2, 5/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - 1/(a*x))^p)
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] && !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{\left(c-\frac{c}{ax}\right)^{p}}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*((a*c*x - c)/(a*x))^p*sqrt((a*x - 1)/(a*x + 1))/(a*x -
1), x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**p,x)
```

output
```
Integral((-c*(-1 + 1/(a*x)))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{a\,x}\right)^p}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a*x))^p/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^p/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\frac{\int\frac{\sqrt{ax+1}\,(acx-c)^p}{x^p\sqrt{ax-1}}dx}{a^p}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x)
```

output
```
int((sqrt(a*x + 1)*(a*c*x - c)**p)/(x**p*sqrt(a*x - 1)),x)/a**p
```

3.568 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = \frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^p \operatorname{AppellF1}\left(\frac{3}{2}+p,2,\frac{1}{2},\frac{5}{2}+p,1-\frac{1}{ax},\frac{a-\frac{1}{x}}{2a}\right)}{a(3+2p)}$$

output
```
2^(1/2)*(1-1/a/x)^(3/2)*(c-c/a/x)^p*AppellF1(3/2+p,1/2,2,5/2+p,1/2*(a-1/x)
/a,1-1/a/x)/a/(3+2*p)
```

Mathematica [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = \int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

input
```
Integrate[(c - c/(a*x))^p/E^ArcCoth[a*x],x]
```

output
```
Integrate[(c - c/(a*x))^p/E^ArcCoth[a*x], x]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6736, 6732, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

$$\Big\downarrow \text{6736}$$

$$\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx$$

$$\Big\downarrow \text{6732}$$

$$-\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\int \frac{\left(1-\frac{1}{ax}\right)^{p+\frac{1}{2}}x^2}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{153}$$

$$-\frac{2^{p+\frac{3}{2}}\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p-\frac{1}{2},2,\frac{3}{2},\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{a}$$

input
```
Int[(c - c/(a*x))^p/E^ArcCoth[a*x],x]
```

output
```
-((2^(3/2 + p)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^p*AppellF1[1/2, -1/2 - p, 2
, 3/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - 1/(a*x))^p))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6732
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> S
imp[-c^p   Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2))
), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6736
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Simp[(c + d/x)^p/(1 + d/(c*x))^p   Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a
*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Int
egerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \left(c-\frac{c}{ax}\right)^{p}\sqrt{\frac{ax-1}{ax+1}}dx$$

input
```
int((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
int((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

Fricas [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \left(c-\frac{c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
integral(((a*c*x - c)/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \sqrt{\frac{ax-1}{ax+1}}\left(-c\left(-1+\frac{1}{ax}\right)\right)^p\,dx$$

input
```
integrate((c-c/a/x)**p*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*(-c*(-1 + 1/(a*x)))**p, x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \left(c-\frac{c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \left(c-\frac{c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate((c - c/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \left(c-\frac{c}{a\,x}\right)^p\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\frac{\int \frac{\sqrt{ax-1}\,(acx-c)^p}{x^p\sqrt{ax+1}}dx}{a^p}$$

input
```
int((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
int((sqrt(a*x - 1)*(a*c*x - c)**p)/(x**p*sqrt(a*x + 1)),x)/a**p
```

3.569 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 114

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = \frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}+\frac{\left(c-\frac{c}{ax}\right)^{2+p}\text{Hypergeometric2F1}\left(1,2+p,3+p,\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(2+p)} - \frac{\left(c-\frac{c}{ax}\right)^{2+p}\text{Hypergeometric2F1}\left(1,2+p,3+p,1-\frac{1}{ax}\right)}{ac^2}$$

output
```
(c-c/a/x)^(2+p)*x/c^2+1/2*(c-c/a/x)^(2+p)*hypergeom([1, 2+p],[3+p],1/2*(a-
1/x)/a)/a/c^2/(2+p)-(c-c/a/x)^(2+p)*hypergeom([1, 2+p],[3+p],1-1/a/x)/a/c^
2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = \frac{\left(c-\frac{c}{ax}\right)^p(-1+ax)^2\left(\text{Hypergeometric2F1}\left(1,2+p,3+p,\frac{a-\frac{1}{x}}{2a}\right)+2(2+p)\left(ax-\text{Hypergeometric2F1}\right.\right.}{2a^3(2+p)x^2}$$

input
```
Integrate[(c - c/(a*x))^p/E^(2*ArcCoth[a*x]),x]
```

output
```
((c - c/(a*x))^p*(-1 + a*x)^2*(Hypergeometric2F1[1, 2 + p, 3 + p, (a - x^(
-1))/(2*a)] + 2*(2 + p)*(a*x - Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a
*x)])))/(2*a^3*(2 + p)*x^2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6717, 6683, 1035, 281, 899, 114, 27, 174, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

$$\Big\downarrow \text{6683}$$

$$-\int \frac{\left(c-\frac{c}{ax}\right)^p(1-ax)}{ax+1}dx$$

$$\Big\downarrow \text{1035}$$

$$-\int \frac{\left(\frac{1}{x}-a\right)\left(c-\frac{c}{ax}\right)^p}{a+\frac{1}{x}}dx$$

$$\Big\downarrow \text{281}$$

$$\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{p+1}}{a+\frac{1}{x}}dx}{c}$$

$$\Big\downarrow \text{899}$$

$$-\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{p+1}x^2}{a+\frac{1}{x}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \ 114$$

$$-\frac{a\left(-\frac{\int \frac{c\left(c-\frac{c}{ax}\right)^{p+1}\left(\frac{p+1}{x}+a(p+2)\right)x}{a\left(a+\frac{1}{x}\right)}d\frac{1}{x}}{ac}-\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{ac}\right)}{c}$$

$$\Big\downarrow \ 27$$

$$-\frac{a\left(-\frac{\int \frac{\left(c-\frac{c}{ax}\right)^{p+1}\left(\frac{p+1}{x}+a(p+2)\right)x}{a+\frac{1}{x}}d\frac{1}{x}}{a^2}-\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{ac}\right)}{c}$$

$$\Big\downarrow \ 174$$

$$-\frac{a\left(-\frac{(p+2)\int \left(c-\frac{c}{ax}\right)^{p+1}xd\frac{1}{x}-\int \frac{\left(c-\frac{c}{ax}\right)^{p+1}}{a+\frac{1}{x}}d\frac{1}{x}}{a^2}-\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{ac}\right)}{c}$$

$$\Big\downarrow \ 75$$

$$-\frac{a\left(-\frac{-\int \frac{\left(c-\frac{c}{ax}\right)^{p+1}}{a+\frac{1}{x}}d\frac{1}{x}-\frac{\left(c-\frac{c}{ax}\right)^{p+2}\text{Hypergeometric2F1}\left(1,p+2,p+3,1-\frac{1}{ax}\right)}{c}}{a^2}-\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{ac}\right)}{c}$$

$$\Big\downarrow \ 78$$

$$-\frac{a\left(-\frac{\frac{\left(c-\frac{c}{ax}\right)^{p+2}\text{Hypergeometric2F1}\left(1,p+2,p+3,\frac{a-\frac{1}{x}}{2a}\right)}{2c(p+2)}-\frac{\left(c-\frac{c}{ax}\right)^{p+2}\text{Hypergeometric2F1}\left(1,p+2,p+3,1-\frac{1}{ax}\right)}{c}}{a^2}-\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{ac}\right)}{c}$$

input
```
Int[(c - c/(a*x))^p/E^(2*ArcCoth[a*x]),x]
```

output
```
-((a*(-(((c - c/(a*x))^(2 + p)*x)/(a*c)) - (((c - c/(a*x))^(2 + p)*Hyperge
ometric2F1[1, 2 + p, 3 + p, (a - x^(-1))/(2*a)])/(2*c*(2 + p)) - ((c - c/(
a*x))^(2 + p)*Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a*x)])/c)/a^2))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1035
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((e_)
 + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[x^(n*(p + r))*(b + a/x^n)^p*(c
 + d/x^n)^q*(f + e/x^n)^r, x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && EqQ[
mn, -n] && IntegerQ[p] && IntegerQ[r]
```

rule 6683
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [F]

$$\int \frac{\left(c-\frac{c}{ax}\right)^{p} (ax-1)}{ax+1}dx$$

input
```
int((c-c/a/x)^p*(a*x-1)/(a*x+1),x)
```

output
```
int((c-c/a/x)^p*(a*x-1)/(a*x+1),x)
```

Fricas [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{(ax-1)\left(c-\frac{c}{ax}\right)^p}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
integral((a*x - 1)*((a*c*x - c)/(a*x))^p/(a*x + 1), x)
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^p(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)**p*(a*x-1)/(a*x+1),x)
```

output
```
Integral((-c*(-1 + 1/(a*x)))**p*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{(ax-1)\left(c-\frac{c}{ax}\right)^p}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(c - c/(a*x))^p/(a*x + 1), x)
```

Giac [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \frac{(ax-1)\left(c-\frac{c}{ax}\right)^p}{ax+1}\,dx$$

input
```
integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
integrate((a*x - 1)*(c - c/(a*x))^p/(a*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int \frac{\left(c-\frac{c}{a\,x}\right)^p\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a*x))^p*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a*x))^p*(a*x - 1))/(a*x + 1), x)
```

Reduce [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$
$$=\frac{(acx-c)^p\,apx-(acx-c)^p\,p+2(acx-c)^p+x^p\left(\int\frac{(acx-c)^p}{x^pa^2x^3-x^px}dx\right)p^2-2x^p\left(\int\frac{(acx-c)^p}{x^pa^2x^3-x^px}dx\right)p-x^p\left(\int\right.}{x^pa^pap}$$

input
```
int((c-c/a/x)^p*(a*x-1)/(a*x+1),x)
```

output
```
((a*c*x - c)**p*a*p*x - (a*c*x - c)**p*p + 2*(a*c*x - c)**p + x**p*int((a*
c*x - c)**p/(x**p*a**2*x**3 - x**p*x),x)*p**2 - 2*x**p*int((a*c*x - c)**p/
(x**p*a**2*x**3 - x**p*x),x)*p - x**p*int(((a*c*x - c)**p*x)/(x**p*a**2*x*
*2 - x**p),x)*a**2*p**2 - 2*x**p*int(((a*c*x - c)**p*x)/(x**p*a**2*x**2 -
x**p),x)*a**2*p)/(x**p*a**p*a*p)
```

3.570 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^5\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{16c^5(1+ax)^7}{7a}+\frac{4c^5(1+ax)^8}{a}-\frac{8c^5(1+ax)^9}{3a}+\frac{4c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}$$

output
```
-16/7*c^5*(a*x+1)^7/a+4*c^5*(a*x+1)^8/a-8/3*c^5*(a*x+1)^9/a+4/5*c^5*(a*x+1
)^10/a-1/11*c^5*(a*x+1)^11/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.56

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{c^5(1+ax)^7\left(281-812ax+938a^2x^2-504a^3x^3+105a^4x^4\right)}{1155a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

output
```
-1/1155*(c^5*(1 + a*x)^7*(281 - 812*a*x + 938*a^2*x^2 - 504*a^3*x^3 + 105*
a^4*x^4))/a
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^5 e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int c^5e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^5dx$$

$$\Big\downarrow \text{27}$$

$$-c^5\int e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^5dx$$

$$\Big\downarrow \text{6690}$$

$$-c^5\int (1-ax)^4(ax+1)^6dx$$

$$\Big\downarrow \text{49}$$

$$-c^5\int \left((ax+1)^{10}-8(ax+1)^9+24(ax+1)^8-32(ax+1)^7+16(ax+1)^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c^5\left(\frac{(ax+1)^{11}}{11a}-\frac{4(ax+1)^{10}}{5a}+\frac{8(ax+1)^9}{3a}-\frac{4(ax+1)^8}{a}+\frac{16(ax+1)^7}{7a}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

output
```
-(c^5*((16*(1 + a*x)^7)/(7*a) - (4*(1 + a*x)^8)/a + (8*(1 + a*x)^9)/(3*a)
- (4*(1 + a*x)^10)/(5*a) + (1 + a*x)^11/(11*a)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

method	result
gosper	$-\frac{c^5x(105a^{10}x^{10}+231a^9x^9-385a^8x^8-1155a^7x^7+330x^6a^6+2310a^5x^5+462a^4x^4-2310a^3x^3-1155a^2x^2+1155ax+1155)}{1155}$
default	$c^5\left(-\frac{1}{11}a^{10}x^{11}-\frac{1}{5}a^9x^{10}+\frac{1}{3}a^8x^9+a^7x^8-\frac{2}{7}a^6x^7-2a^5x^6-\frac{2}{5}x^5a^4+2a^3x^4+a^2x^3-a\,x^2-\right.$
orering	$\frac{x(105a^{10}x^{10}+231a^9x^9-385a^8x^8-1155a^7x^7+330x^6a^6+2310a^5x^5+462a^4x^4-2310a^3x^3-1155a^2x^2+1155ax+1155)(-a^2c}{1155(ax-1)^5(ax+1)^5}$
norman	$a^2c^5x^3+a^7c^5x^8-c^5x+2a^3c^5x^4-\frac{2}{5}a^4c^5x^5-2a^5c^5x^6-\frac{2}{7}a^6c^5x^7+\frac{1}{3}a^8c^5x^9-\frac{1}{5}a^9c^5x^{10}-$
risch	$a^2c^5x^3+a^7c^5x^8-c^5x+2a^3c^5x^4-\frac{2}{5}a^4c^5x^5-2a^5c^5x^6-\frac{2}{7}a^6c^5x^7+\frac{1}{3}a^8c^5x^9-\frac{1}{5}a^9c^5x^{10}-$
parallelrisch	$a^2c^5x^3+a^7c^5x^8-c^5x+2a^3c^5x^4-\frac{2}{5}a^4c^5x^5-2a^5c^5x^6-\frac{2}{7}a^6c^5x^7+\frac{1}{3}a^8c^5x^9-\frac{1}{5}a^9c^5x^{10}-$
meijerg	$\frac{c^5\left(-\frac{xa(2520a^{10}x^{10}+2772a^9x^9+3080a^8x^8+3465a^7x^7+3960x^6a^6+4620a^5x^5+5544a^4x^4+6930a^3x^3+9240a^2x^2+13860ax+27720)}{27720}-\ln(\right.}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/1155*c^5*x*(105*a^10*x^10+231*a^9*x^9-385*a^8*x^8-1155*a^7*x^7+330*a^6*
x^6+2310*a^5*x^5+462*a^4*x^4-2310*a^3*x^3-1155*a^2*x^2+1155*a*x+1155)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{1}{11}a^{10}c^5x^{11}-\frac{1}{5}a^9c^5x^{10}+\frac{1}{3}a^8c^5x^9+a^7c^5x^8-\frac{2}{7}a^6c^5x^7-2a^5c^5x^6-\frac{2}{5}a^4c^5x^5+2a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="fricas")
```

output
```
-1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2
/7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5
*x^3 - a*c^5*x^2 - c^5*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.42

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{a^{10}c^5x^{11}}{11}-\frac{a^9c^5x^{10}}{5}+\frac{a^8c^5x^9}{3}+a^7c^5x^8-\frac{2a^6c^5x^7}{7}-2a^5c^5x^6-\frac{2a^4c^5x^5}{5}+2a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**5,x)
```

output
```
-a**10*c**5*x**11/11 - a**9*c**5*x**10/5 + a**8*c**5*x**9/3 + a**7*c**5*x*
*8 - 2*a**6*c**5*x**7/7 - 2*a**5*c**5*x**6 - 2*a**4*c**5*x**5/5 + 2*a**3*c
**5*x**4 + a**2*c**5*x**3 - a*c**5*x**2 - c**5*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{1}{5}\,a^9c^5x^{10}+\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8-\frac{2}{7}\,a^6c^5x^7-2\,a^5c^5x^6-\frac{2}{5}\,a^4c^5x^5+2\,a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="maxima")
```

output
```
-1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2
/7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5
*x^3 - a*c^5*x^2 - c^5*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{1}{11}a^{10}c^5x^{11}-\frac{1}{5}a^9c^5x^{10}+\frac{1}{3}a^8c^5x^9+a^7c^5x^8-\frac{2}{7}a^6c^5x^7-2a^5c^5x^6-\frac{2}{5}a^4c^5x^5+2a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="giac")
```

output
```
-1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2
/7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5
*x^3 - a*c^5*x^2 - c^5*x
```

Mupad [B] (verification not implemented)

Time = 13.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{a^{10}\,c^5\,x^{11}}{11}-\frac{a^9\,c^5\,x^{10}}{5}+\frac{a^8\,c^5\,x^9}{3}+a^7\,c^5\,x^8-\frac{2\,a^6\,c^5\,x^7}{7}-2\,a^5\,c^5\,x^6-\frac{2\,a^4\,c^5\,x^5}{5}+2\,a^3\,c^5\,x^4+a^2\,c^5\,x^3-a\,c^5\,x^2-c^5\,x$$

input
```
int(((c - a^2*c*x^2)^5*(a*x + 1))/(a*x - 1),x)
```

output
```
a^2*c^5*x^3 - a*c^5*x^2 - c^5*x + 2*a^3*c^5*x^4 - (2*a^4*c^5*x^5)/5 - 2*a^
5*c^5*x^6 - (2*a^6*c^5*x^7)/7 + a^7*c^5*x^8 + (a^8*c^5*x^9)/3 - (a^9*c^5*x
^10)/5 - (a^10*c^5*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx$$
$$=\frac{c^5x(-105a^{10}x^{10}-231a^9x^9+385a^8x^8+1155a^7x^7-330a^6x^6-2310a^5x^5-462a^4x^4+2310a^3x^3+1155$$
$$1155$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x)
```

output
```
(c**5*x*( - 105*a**10*x**10 - 231*a**9*x**9 + 385*a**8*x**8 + 1155*a**7*x*
*7 - 330*a**6*x**6 - 2310*a**5*x**5 - 462*a**4*x**4 + 2310*a**3*x**3 + 115
5*a**2*x**2 - 1155*a*x - 1155))/1155
```

3.571 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=-\frac{4c^4(1+ax)^6}{3a}+\frac{12c^4(1+ax)^7}{7a}-\frac{3c^4(1+ax)^8}{4a}+\frac{c^4(1+ax)^9}{9a}$$

output
```
-4/3*c^4*(a*x+1)^6/a+12/7*c^4*(a*x+1)^7/a-3/4*c^4*(a*x+1)^8/a+1/9*c^4*(a*x
+1)^9/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{c^4(1+ax)^6\left(-65+138ax-105a^2x^2+28a^3x^3\right)}{252a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

output
```
(c^4*(1 + a*x)^6*(-65 + 138*a*x - 105*a^2*x^2 + 28*a^3*x^3))/(252*a)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-a^2cx^2)^4 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int c^4e^{2\text{arctanh}(ax)}(1-a^2x^2)^4\,dx$$

$$\Big\downarrow \text{27}$$

$$-c^4\int e^{2\text{arctanh}(ax)}(1-a^2x^2)^4\,dx$$

$$\Big\downarrow \text{6690}$$

$$-c^4\int (1-ax)^3(ax+1)^5dx$$

$$\Big\downarrow \text{49}$$

$$-c^4\int \left(-(ax+1)^8+6(ax+1)^7-12(ax+1)^6+8(ax+1)^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c^4\left(-\frac{(ax+1)^9}{9a}+\frac{3(ax+1)^8}{4a}-\frac{12(ax+1)^7}{7a}+\frac{4(ax+1)^6}{3a}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

output
```
-(c^4*((4*(1 + a*x)^6)/(3*a) - (12*(1 + a*x)^7)/(7*a) + (3*(1 + a*x)^8)/(4
*a) - (1 + a*x)^9/(9*a)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

method	result
gosper	$\frac{c^{4} x \left(28 a^{8} x^{8}+63 a^{7} x^{7}-72 x^{6} a^{6}-252 a^{5} x^{5}+378 a^{3} x^{3}+168 a^{2} x^{2}-252 a x -252\right)}{252}$
default	$c^{4} \left(\frac{1}{9} a^{8} x^{9}+\frac{1}{4} a^{7} x^{8}-\frac{2}{7} a^{6} x^{7}-a^{5} x^{6}+\frac{3}{2} a^{3} x^{4}+\frac{2}{3} a^{2} x^{3}-a \,x^{2}-x \right)$
norman	$-c^{4} x -a \,c^{4} x^{2}+\frac{2}{3} a^{2} c^{4} x^{3}+\frac{3}{2} a^{3} c^{4} x^{4}-a^{5} c^{4} x^{6}-\frac{2}{7} a^{6} c^{4} x^{7}+\frac{1}{4} a^{7} c^{4} x^{8}+\frac{1}{9} a^{8} c^{4} x^{9}$
risch	$-c^{4} x -a \,c^{4} x^{2}+\frac{2}{3} a^{2} c^{4} x^{3}+\frac{3}{2} a^{3} c^{4} x^{4}-a^{5} c^{4} x^{6}-\frac{2}{7} a^{6} c^{4} x^{7}+\frac{1}{4} a^{7} c^{4} x^{8}+\frac{1}{9} a^{8} c^{4} x^{9}$
parallelrisch	$-c^{4} x -a \,c^{4} x^{2}+\frac{2}{3} a^{2} c^{4} x^{3}+\frac{3}{2} a^{3} c^{4} x^{4}-a^{5} c^{4} x^{6}-\frac{2}{7} a^{6} c^{4} x^{7}+\frac{1}{4} a^{7} c^{4} x^{8}+\frac{1}{9} a^{8} c^{4} x^{9}$
orering	$\frac{x \left(28 a^{8} x^{8}+63 a^{7} x^{7}-72 x^{6} a^{6}-252 a^{5} x^{5}+378 a^{3} x^{3}+168 a^{2} x^{2}-252 a x -252\right) \left(-a^{2} c \,x^{2}+c \right)^{4}}{252 \left(a x +1\right)^{4} \left(a x -1\right)^{4}}$
meijerg	$-\frac{c^{4} \left(-\frac{a x \left(280 a^{8} x^{8}+315 a^{7} x^{7}+360 x^{6} a^{6}+420 a^{5} x^{5}+504 a^{4} x^{4}+630 a^{3} x^{3}+840 a^{2} x^{2}+1260 a x +2520\right)}{2520}-\ln \left(-a x +1\right)\right)}{a}+\frac{4 c^{4} \left(-\frac{a x \left(120\right.}{}\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/252*c^4*x*(28*a^8*x^8+63*a^7*x^7-72*a^6*x^6-252*a^5*x^5+378*a^3*x^3+168*
a^2*x^2-252*a*x-252)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9+\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7-a^5c^4x^6+\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3-ac^4x^2-c^4x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output
```
1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^
3*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8c^4x^9}{9}+\frac{a^7c^4x^8}{4}-\frac{2a^6c^4x^7}{7}-a^5c^4x^6+\frac{3a^3c^4x^4}{2}+\frac{2a^2c^4x^3}{3}-ac^4x^2-c^4x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**4,x)
```

output
```
a**8*c**4*x**9/9 + a**7*c**4*x**8/4 - 2*a**6*c**4*x**7/7 - a**5*c**4*x**6
+ 3*a**3*c**4*x**4/2 + 2*a**2*c**4*x**3/3 - a*c**4*x**2 - c**4*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9+\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7-a^5c^4x^6+\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3-ac^4x^2-c^4x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^
3*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9+\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7-a^5c^4x^6+\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3-ac^4x^2-c^4x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^
3*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8\,c^4\,x^9}{9}+\frac{a^7\,c^4\,x^8}{4}-\frac{2\,a^6\,c^4\,x^7}{7}-a^5\,c^4\,x^6+\frac{3\,a^3\,c^4\,x^4}{2}+\frac{2\,a^2\,c^4\,x^3}{3}-a\,c^4\,x^2-c^4\,x$$

input
```
int(((c - a^2*c*x^2)^4*(a*x + 1))/(a*x - 1),x)
```

output
```
(2*a^2*c^4*x^3)/3 - a*c^4*x^2 - c^4*x + (3*a^3*c^4*x^4)/2 - a^5*c^4*x^6 -
(2*a^6*c^4*x^7)/7 + (a^7*c^4*x^8)/4 + (a^8*c^4*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = \frac{c^4x(28a^8x^8+63a^7x^7-72a^6x^6-252a^5x^5+378a^3x^3+168a^2x^2-252ax-252)}{252}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x)
```

output
```
(c**4*x*(28*a**8*x**8 + 63*a**7*x**7 - 72*a**6*x**6 - 252*a**5*x**5 + 378*
a**3*x**3 + 168*a**2*x**2 - 252*a*x - 252))/252
```

3.572 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{4c^3(1+ax)^5}{5a}+\frac{2c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}$$

output
```
-4/5*c^3*(a*x+1)^5/a+2/3*c^3*(a*x+1)^6/a-1/7*c^3*(a*x+1)^7/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{c^3(1+ax)^5\left(29-40ax+15a^2x^2\right)}{105a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

output
```
-1/105*(c^3*(1 + a*x)^5*(29 - 40*a*x + 15*a^2*x^2))/a
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^3 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int c^3 e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^3dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^3\int e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^3dx$$

$$\Big\downarrow \text{ 6690}$$

$$-c^3\int (1-ax)^2(ax+1)^4dx$$

$$\Big\downarrow \text{ 49}$$

$$-c^3\int \left((ax+1)^6-4(ax+1)^5+4(ax+1)^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-c^3\left(\frac{(ax+1)^7}{7a}-\frac{2(ax+1)^6}{3a}+\frac{4(ax+1)^5}{5a}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

output
```
-(c^3*((4*(1 + a*x)^5)/(5*a) - (2*(1 + a*x)^6)/(3*a) + (1 + a*x)^7/(7*a)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

method	result
gosper	$-\frac{c^{3} x \left(15 x^{6} a^{6}+35 a^{5} x^{5}-21 a^{4} x^{4}-105 a^{3} x^{3}-35 a^{2} x^{2}+105 a x +105\right)}{105}$
default	$c^{3} \left(-\frac{1}{7} a^{6} x^{7}-\frac{1}{3} a^{5} x^{6}+\frac{1}{5} x^{5} a^{4}+a^{3} x^{4}+\frac{1}{3} a^{2} x^{3}-a \,x^{2}-x \right)$
norman	$a^{3} c^{3} x^{4}-c^{3} x -a \,c^{3} x^{2}+\frac{1}{3} a^{2} c^{3} x^{3}-\frac{1}{3} a^{5} c^{3} x^{6}-\frac{1}{7} a^{6} c^{3} x^{7}+\frac{1}{5} c^{3} a^{4} x^{5}$
risch	$a^{3} c^{3} x^{4}-c^{3} x -a \,c^{3} x^{2}+\frac{1}{3} a^{2} c^{3} x^{3}-\frac{1}{3} a^{5} c^{3} x^{6}-\frac{1}{7} a^{6} c^{3} x^{7}+\frac{1}{5} c^{3} a^{4} x^{5}$
parallelrisch	$a^{3} c^{3} x^{4}-c^{3} x -a \,c^{3} x^{2}+\frac{1}{3} a^{2} c^{3} x^{3}-\frac{1}{3} a^{5} c^{3} x^{6}-\frac{1}{7} a^{6} c^{3} x^{7}+\frac{1}{5} c^{3} a^{4} x^{5}$
orering	$\frac{x \left(15 x^{6} a^{6}+35 a^{5} x^{5}-21 a^{4} x^{4}-105 a^{3} x^{3}-35 a^{2} x^{2}+105 a x +105\right) \left(-a^{2} c \,x^{2}+c \right)^{3}}{105 \left(a x +1\right)^{3} \left(a x -1\right)^{3}}$
meijerg	$\frac{c^{3} \left(-\frac{a x \left(120 x^{6} a^{6}+140 a^{5} x^{5}+168 a^{4} x^{4}+210 a^{3} x^{3}+280 a^{2} x^{2}+420 a x +840\right)}{840}-\ln \left(-a x +1\right)\right)}{a}-\frac{3 c^{3} \left(-\frac{x a \left(12 a^{4} x^{4}+15 a^{3} x^{3}+20 a^{2} x^{2}+30\right.}{60}\right.}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/105*c^3*x*(15*a^6*x^6+35*a^5*x^5-21*a^4*x^4-105*a^3*x^3-35*a^2*x^2+105*
a*x+105)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7-\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5+a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3-ac^3x^2-c^3x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a
^2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{a^6c^3x^7}{7}-\frac{a^5c^3x^6}{3}+\frac{a^4c^3x^5}{5}+a^3c^3x^4+\frac{a^2c^3x^3}{3}-ac^3x^2-c^3x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**3,x)
```

output
```
-a**6*c**3*x**7/7 - a**5*c**3*x**6/3 + a**4*c**3*x**5/5 + a**3*c**3*x**4 +
 a**2*c**3*x**3/3 - a*c**3*x**2 - c**3*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7-\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5+a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3-ac^3x^2-c^3x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a
^2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7-\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5+a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3-ac^3x^2-c^3x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a
^2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{a^6\,c^3\,x^7}{7}-\frac{a^5\,c^3\,x^6}{3}+\frac{a^4\,c^3\,x^5}{5}+a^3\,c^3\,x^4+\frac{a^2\,c^3\,x^3}{3}-a\,c^3\,x^2-c^3\,x$$

input
```
int(((c - a^2*c*x^2)^3*(a*x + 1))/(a*x - 1),x)
```

output
```
(a^2*c^3*x^3)/3 - a*c^3*x^2 - c^3*x + a^3*c^3*x^4 + (a^4*c^3*x^5)/5 - (a^5
*c^3*x^6)/3 - (a^6*c^3*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = \frac{c^3x(-15a^6x^6-35a^5x^5+21a^4x^4+105a^3x^3+35a^2x^2-105ax-105)}{105}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x)
```

output
```
(c**3*x*( - 15*a**6*x**6 - 35*a**5*x**5 + 21*a**4*x**4 + 105*a**3*x**3 + 3
5*a**2*x**2 - 105*a*x - 105))/105
```

3.573 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 35

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx=-\frac{c^2(1+ax)^4}{2a}+\frac{c^2(1+ax)^5}{5a}$$

output
```
-1/2*c^2*(a*x+1)^4/a+1/5*c^2*(a*x+1)^5/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx=\frac{c^2(1+ax)^4(-3+2ax)}{10a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

output
```
(c^2*(1 + a*x)^4*(-3 + 2*a*x))/(10*a)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-a^2cx^2)^2 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int c^2 e^{2\text{arctanh}(ax)}(1-a^2x^2)^2\,dx$$

$$\Big\downarrow \text{27}$$

$$-c^2\int e^{2\text{arctanh}(ax)}(1-a^2x^2)^2\,dx$$

$$\Big\downarrow \text{6690}$$

$$-c^2\int (1-ax)(ax+1)^3dx$$

$$\Big\downarrow \text{49}$$

$$-c^2\int \left(2(ax+1)^3-(ax+1)^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c^2\left(\frac{(ax+1)^4}{2a}-\frac{(ax+1)^5}{5a}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

output
```
-(c^2*((1 + a*x)^4/(2*a) - (1 + a*x)^5/(5*a)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result
gosper	$\frac{c^{2} x \left(2 a^{4} x^{4}+5 a^{3} x^{3}-10 a x -10\right)}{10}$
default	$c^{2} \left(\frac{1}{5} x^{5} a^{4}+\frac{1}{2} a^{3} x^{4}-a \,x^{2}-x \right)$
norman	$-c^{2} x -a \,c^{2} x^{2}+\frac{1}{2} a^{3} c^{2} x^{4}+\frac{1}{5} a^{4} c^{2} x^{5}$
risch	$-c^{2} x -a \,c^{2} x^{2}+\frac{1}{2} a^{3} c^{2} x^{4}+\frac{1}{5} a^{4} c^{2} x^{5}$
parallelrisch	$-c^{2} x -a \,c^{2} x^{2}+\frac{1}{2} a^{3} c^{2} x^{4}+\frac{1}{5} a^{4} c^{2} x^{5}$
orering	$\frac{x \left(2 a^{4} x^{4}+5 a^{3} x^{3}-10 a x -10\right) \left(-a^{2} c \,x^{2}+c \right)^{2}}{10 \left(a x -1\right)^{2} \left(a x +1\right)^{2}}$
meijerg	$-\frac{c^{2} \left(-\frac{x a \left(12 a^{4} x^{4}+15 a^{3} x^{3}+20 a^{2} x^{2}+30 a x +60\right)}{60}-\ln \left(-a x +1\right)\right)}{a}+\frac{2 c^{2} \left(-\frac{a x \left(4 a^{2} x^{2}+6 a x +12\right)}{12}-\ln \left(-a x +1\right)\right)}{a}-\frac{c^{2} \left(-a x -\ln(\right.}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*c^2*x*(2*a^4*x^4+5*a^3*x^3-10*a*x-10)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5+\frac{1}{2}\,a^3c^2x^4-ac^2x^2-c^2x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{a^4c^2x^5}{5}+\frac{a^3c^2x^4}{2}-ac^2x^2-c^2x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**2,x)
```

output
```
a**4*c**2*x**5/5 + a**3*c**2*x**4/2 - a*c**2*x**2 - c**2*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5+\frac{1}{2}\,a^3c^2x^4-ac^2x^2-c^2x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5+\frac{1}{2}\,a^3c^2x^4-ac^2x^2-c^2x$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{a^4\,c^2\,x^5}{5}+\frac{a^3\,c^2\,x^4}{2}-a\,c^2\,x^2-c^2\,x$$

input
```
int(((c - a^2*c*x^2)^2*(a*x + 1))/(a*x - 1),x)
```

output
```
(a^3*c^2*x^4)/2 - a*c^2*x^2 - c^2*x + (a^4*c^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{c^2x(2a^4x^4+5a^3x^3-10ax-10)}{10}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x)
```

output
```
(c**2*x*(2*a**4*x**4 + 5*a**3*x**3 - 10*a*x - 10))/10
```

3.574 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 15

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{c(1+ax)^3}{3a}$$

output
```
-1/3*c*(a*x+1)^3/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-c\left(x+ax^2+\frac{a^2x^3}{3}\right)$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

output
```
-(c*(x + a*x^2 + (a^2*x^3)/3))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6717, 27, 6690, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-a^2cx^2)\, e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int ce^{2\text{arctanh}(ax)}(1-a^2x^2)\,dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{2\text{arctanh}(ax)}(1-a^2x^2)\,dx$$

$$\Big\downarrow \text{6690}$$

$$-c\int (ax+1)^2dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{c(ax+1)^3}{3a}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

output
```
-1/3*(c*(1 + a*x)^3)/a
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{c(ax+1)^3}{3a}$	14
gosper	$-\frac{cx(a^2x^2+3ax+3)}{3}$	18
norman	$-xc-ac\,x^2-\frac{1}{3}a^2c\,x^3$	22
parallelrisch	$-xc-ac\,x^2-\frac{1}{3}a^2c\,x^3$	22
risch	$-\frac{a^2c\,x^3}{3}-ac\,x^2-xc-\frac{c}{3a}$	28
orering	$\frac{x(a^2x^2+3ax+3)(-a^2c\,x^2+c)}{3(ax+1)(ax-1)}$	42
meijerg	$\frac{c\left(-\frac{ax\left(4a^2x^2+6ax+12\right)}{12}-\ln(-ax+1)\right)}{a}-\frac{c(-ax-\ln(-ax+1))}{a}-\frac{c\left(\frac{ax(3ax+6)}{6}+\ln(-ax+1)\right)}{a}+\frac{c\ln(-ax+1)}{a}$	91

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3*c*(a*x+1)^3/a
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3-acx^2-cx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
-1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{a^2cx^3}{3}-acx^2-cx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c),x)
```

output
```
-a**2*c*x**3/3 - a*c*x**2 - c*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3-acx^2-cx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
-1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3-acx^2-cx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
-1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{c\,x\,\left(a^2\,x^2+3\,a\,x+3\right)}{3}$$

input
```
int(((c - a^2*c*x^2)*(a*x + 1))/(a*x - 1),x)
```

output
```
-(c*x*(3*a*x + a^2*x^2 + 3))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\frac{cx(-a^2x^2-3ax-3)}{3}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x)
```

output
```
(c*x*( - a**2*x**2 - 3*a*x - 3))/3
```

3.575 $\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{1}{ac(1-ax)}$$

output
```
-1/a/c/(-a*x+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{e^{2\coth^{-1}(ax)}}{2ac}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

output
```
E^(2*ArcCoth[a*x])/(2*a*c)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6717, 27, 6690, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}dx$$
$$\Big\downarrow \text{ 6717}$$
$$-\int \frac{e^{2\text{arctanh}(ax)}}{c(1-a^2x^2)}dx$$
$$\Big\downarrow \text{ 27}$$
$$-\frac{\int \frac{e^{2\text{arctanh}(ax)}}{1-a^2x^2}dx}{c}$$
$$\Big\downarrow \text{ 6690}$$
$$-\frac{\int \frac{1}{(1-ax)^2}dx}{c}$$
$$\Big\downarrow \text{ 17}$$
$$-\frac{1}{ac(1-ax)}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

output
```
-(1/(a*c*(1 - a*x)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
norman	$\frac{x}{c(ax-1)}$	13
parallelrisch	$\frac{x}{c(ax-1)}$	13
gosper	$\frac{1}{ac(ax-1)}$	15
default	$\frac{1}{ac(ax-1)}$	15
risch	$\frac{1}{ac(ax-1)}$	15
orering	$-\frac{ax+1}{a(-a^2cx^2+c)}$	24

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/c*x/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{1}{a^2cx-ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/(a^2*c*x - a*c)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{1}{a^2cx-ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c),x)
```

output
```
1/(a**2*c*x - a*c)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{1}{a^2cx-ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/(a^2*c*x - a*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{(ax-1)ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
1/((a*x - 1)*a*c)
```

Mupad [B] (verification not implemented)

Time = 13.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{1}{a\,(c-a\,c\,x)}$$

input
```
int((a*x + 1)/((c - a^2*c*x^2)*(a*x - 1)),x)
```

output
```
-1/(a*(c - a*c*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{x}{c\,(ax-1)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x)
```

output
```
x/(c*(a*x - 1))
```

3.576 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{1}{4ac^2(1-ax)^2}-\frac{1}{4ac^2(1-ax)}-\frac{\text{arctanh}(ax)}{4ac^2}$$

output
```
-1/4/a/c^2/(-a*x+1)^2-1/4/a/c^2/(-a*x+1)-1/4*arctanh(a*x)/a/c^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{-2+ax-(-1+ax)^2\text{arctanh}(ax)}{4ac^2(-1+ax)^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

output
```
(-2 + a*x - (-1 + a*x)^2*ArcTanh[a*x])/(4*a*c^2*(-1 + a*x)^2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6717, 27, 6690, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$$
$$\Big\downarrow \text{6717}$$
$$-\int \frac{e^{2\text{arctanh}(ax)}}{c^2(1-a^2x^2)^2}dx$$
$$\Big\downarrow \text{27}$$
$$-\frac{\int \frac{e^{2\text{arctanh}(ax)}}{(1-a^2x^2)^2}dx}{c^2}$$
$$\Big\downarrow \text{6690}$$
$$-\frac{\int \frac{1}{(1-ax)^3(ax+1)}dx}{c^2}$$
$$\Big\downarrow \text{54}$$
$$-\frac{\int \left(\frac{1}{4(ax-1)^2}-\frac{1}{2(ax-1)^3}-\frac{1}{4(a^2x^2-1)}\right)dx}{c^2}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\frac{\text{arctanh}(ax)}{4a}+\frac{1}{4a(1-ax)}+\frac{1}{4a(1-ax)^2}}{c^2}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

output
```
-((1/(4*a*(1 - a*x)^2) + 1/(4*a*(1 - a*x)) + ArcTanh[a*x]/(4*a))/c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{\frac{x}{4}-\frac{1}{2a}}{c^{2}\left(ax-1\right)^{2}}+\frac{\ln\left(-ax+1\right)}{8a\,c^{2}}-\frac{\ln\left(ax+1\right)}{8a\,c^{2}}$	51
default	$\frac{-\frac{\ln\left(ax+1\right)}{8a}-\frac{1}{4a\left(ax-1\right)^{2}}+\frac{1}{4\left(ax-1\right)a}+\frac{\ln\left(ax-1\right)}{8a}}{c^{2}}$	52
norman	$\frac{-\frac{3}{4ac}+\frac{a\,x^{2}}{2c}-\frac{a^{2}x^{3}}{4c}}{c\left(ax+1\right)\left(ax-1\right)^{2}}+\frac{\ln\left(ax-1\right)}{8a\,c^{2}}-\frac{\ln\left(ax+1\right)}{8a\,c^{2}}$	77
parallelrisch	$\frac{a^{2}\ln\left(ax-1\right)x^{2}-\ln\left(ax+1\right)x^{2}a^{2}+4a^{2}x^{2}-2a\ln\left(ax-1\right)x+2\ln\left(ax+1\right)xa-6ax+\ln\left(ax-1\right)-\ln\left(ax+1\right)}{8c^{2}\left(ax-1\right)^{2}a}$	90

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*x-1/2/a)/c^2/(a*x-1)^2+1/8*ln(-a*x+1)/a/c^2-1/8*ln(a*x+1)/a/c^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{2\,ax-(a^2x^2-2\,ax+1)\log(ax+1)+(a^2x^2-2\,ax+1)\log(ax-1)-4}{8\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*lo
g(a*x - 1) - 4)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{ax-2}{4a^3c^2x^2-8a^2c^2x+4ac^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**2,x)
```

output
```
(a*x - 2)/(4*a**3*c**2*x**2 - 8*a**2*c**2*x + 4*a*c**2) + (log(x - 1/a)/8
- log(x + 1/a)/8)/(a*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{ax-2}{4\,(a^3c^2x^2-2\,a^2c^2x+ac^2)} - \frac{\log(ax+1)}{8\,ac^2} + \frac{\log(ax-1)}{8\,ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/4*(a*x - 2)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) - 1/8*log(a*x + 1)/(a*c^
2) + 1/8*log(a*x - 1)/(a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{\log(|ax+1|)}{8\,ac^2} + \frac{\log(|ax-1|)}{8\,ac^2} + \frac{ax-2}{4\,(ax-1)^2ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) + 1/4*(a*x
- 2)/((a*x - 1)^2*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{\frac{x}{4}-\frac{1}{2\,a}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2} - \frac{\operatorname{atanh}(a\,x)}{4\,a\,c^2}$$

input
```
int((a*x + 1)/((c - a^2*c*x^2)^2*(a*x - 1)),x)
```

output
```
(x/4 - 1/(2*a))/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) - atanh(a*x)/(4*a*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.82

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$$
$$= \frac{\log(ax-1)\,a^2x^2 - 2\log(ax-1)\,ax + \log(ax-1) - \log(ax+1)\,a^2x^2 + 2\log(ax+1)\,ax - \log(ax+1)}{8a\,c^2\,(a^2x^2-2ax+1)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x)
```

output
```
(log(a*x - 1)*a**2*x**2 - 2*log(a*x - 1)*a*x + log(a*x - 1) - log(a*x + 1)
*a**2*x**2 + 2*log(a*x + 1)*a*x - log(a*x + 1) + a**2*x**2 - 3)/(8*a*c**2*
(a**2*x**2 - 2*a*x + 1))
```

3.577 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{1}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^2}-\frac{3}{16ac^3(1-ax)}+\frac{1}{16ac^3(1+ax)}-\frac{\operatorname{arctanh}(ax)}{4ac^3}$$

output
```
-1/12/a/c^3/(-a*x+1)^3-1/8/a/c^3/(-a*x+1)^2-3/16/a/c^3/(-a*x+1)+1/16/a/c^3
/(a*x+1)-1/4*arctanh(a*x)/a/c^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{4+ax-6a^2x^2+3a^3x^3-3(-1+ax)^3(1+ax)\operatorname{arctanh}(ax)}{12ac^3(-1+ax)^3(1+ax)}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

output
```
(4 + a*x - 6*a^2*x^2 + 3*a^3*x^3 - 3*(-1 + a*x)^3*(1 + a*x)*ArcTanh[a*x])/
(12*a*c^3*(-1 + a*x)^3*(1 + a*x))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6717, 27, 6690, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{c^3(1-a^2x^2)^3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{2\text{arctanh}(ax)}}{(1-a^2x^2)^3}dx}{c^3}$$

$$\Big\downarrow \text{6690}$$

$$-\frac{\int \frac{1}{(1-ax)^4(ax+1)^2}dx}{c^3}$$

$$\Big\downarrow \text{54}$$

$$-\frac{\int \left(\frac{3}{16(ax-1)^2}+\frac{1}{16(ax+1)^2}-\frac{1}{4(ax-1)^3}+\frac{1}{4(ax-1)^4}-\frac{1}{4(a^2x^2-1)}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\text{arctanh}(ax)}{4a}+\frac{3}{16a(1-ax)}-\frac{1}{16a(ax+1)}+\frac{1}{8a(1-ax)^2}+\frac{1}{12a(1-ax)^3}}{c^3}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

output
```
-((1/(12*a*(1 - a*x)^3) + 1/(8*a*(1 - a*x)^2) + 3/(16*a*(1 - a*x)) - 1/(16
*a*(1 + a*x)) + ArcTanh[a*x]/(4*a))/c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

method	result
default	$\frac{\frac{1}{16a(ax+1)}-\frac{\ln(ax+1)}{8a}+\frac{1}{12a(ax-1)^{3}}-\frac{1}{8a(ax-1)^{2}}+\frac{3}{16(ax-1)a}+\frac{\ln(ax-1)}{8a}}{c^{3}}$
risch	$\frac{\frac{a^{2}x^{3}}{4}-\frac{a\,x^{2}}{2}+\frac{x}{12}+\frac{1}{3a}}{c^{3}(ax-1)^{2}(a^{2}x^{2}-1)}-\frac{\ln(ax+1)}{8a\,c^{3}}+\frac{\ln(-ax+1)}{8a\,c^{3}}$
norman	$\frac{\frac{3x}{4c}+\frac{a\,x^{2}}{4c}-\frac{11a^{2}x^{3}}{12c}-\frac{a^{3}x^{4}}{12c}+\frac{a^{4}x^{5}}{3c}}{c^{2}(ax+1)^{2}(ax-1)^{3}}+\frac{\ln(ax-1)}{8a\,c^{3}}-\frac{\ln(ax+1)}{8a\,c^{3}}$
parallelrisch	$\frac{3\ln(ax-1)x^{4}a^{4}-3\ln(ax+1)x^{4}a^{4}+8a^{4}x^{4}-6a^{3}\ln(ax-1)x^{3}+6\ln(ax+1)x^{3}a^{3}-10a^{3}x^{3}-12a^{2}x^{2}+6a\ln(ax-1)x-6\ln(ax+1)}{24c^{3}(ax-1)^{2}(a^{2}x^{2}-1)a}$

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/16/a/(a*x+1)-1/8*ln(a*x+1)/a+1/12/a/(a*x-1)^3-1/8/a/(a*x-1)^2+3/1
6/(a*x-1)/a+1/8/a*ln(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.41

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{6\,a^3x^3-12\,a^2x^2+2\,ax-3\,(a^4x^4-2\,a^3x^3+2\,ax-1)\log(ax+1)+3\,(a^4x^4-2\,a^3x^3+2\,ax-1)\log(a}{24\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/24*(6*a^3*x^3 - 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)
*log(a*x + 1) + 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) + 8)/(a^5
*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{-3a^3x^3+6a^2x^2-ax-4}{12a^5c^3x^4-24a^4c^3x^3+24a^2c^3x-12ac^3} - \frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**3,x)
```

output
```
-(-3*a**3*x**3 + 6*a**2*x**2 - a*x - 4)/(12*a**5*c**3*x**4 - 24*a**4*c**3*
x**3 + 24*a**2*c**3*x - 12*a*c**3) - (-log(x - 1/a)/8 + log(x + 1/a)/8)/(a
*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{3\,a^3x^3-6\,a^2x^2+ax+4}{12\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)} - \frac{\log(ax+1)}{8\,ac^3} + \frac{\log(ax-1)}{8\,ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/12*(3*a^3*x^3 - 6*a^2*x^2 + a*x + 4)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^
2*c^3*x - a*c^3) - 1/8*log(a*x + 1)/(a*c^3) + 1/8*log(a*x - 1)/(a*c^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\log(|ax+1|)}{8\,ac^3} + \frac{\log(|ax-1|)}{8\,ac^3} + \frac{3\,a^3x^3-6\,a^2x^2+ax+4}{12\,(ax+1)(ax-1)^3ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/8*log(abs(a*x + 1))/(a*c^3) + 1/8*log(abs(a*x - 1))/(a*c^3) + 1/12*(3*a
^3*x^3 - 6*a^2*x^2 + a*x + 4)/((a*x + 1)*(a*x - 1)^3*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 13.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\frac{x}{12}-\frac{a\,x^2}{2}+\frac{1}{3\,a}+\frac{a^2\,x^3}{4}}{-a^4\,c^3\,x^4+2\,a^3\,c^3\,x^3-2\,a\,c^3\,x+c^3} - \frac{\mathrm{atanh}(a\,x)}{4\,a\,c^3}$$

input
```
int((a*x + 1)/((c - a^2*c*x^2)^3*(a*x - 1)),x)
```

output
```
- (x/12 - (a*x^2)/2 + 1/(3*a) + (a^2*x^3)/4)/(c^3 + 2*a^3*c^3*x^3 - a^4*c^
3*x^4 - 2*a*c^3*x) - atanh(a*x)/(4*a*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.69

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$$
$$= \frac{3\log(ax-1)\,a^4x^4 - 6\log(ax-1)\,a^3x^3 + 6\log(ax-1)\,ax - 3\log(ax-1) - 3\log(ax+1)\,a^4x^4 + 6\log(}{24a\,c^3\,(a^4x^4 - 2a^3x^3 + 2ax -}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x)
```

output
```
(3*log(a*x - 1)*a**4*x**4 - 6*log(a*x - 1)*a**3*x**3 + 6*log(a*x - 1)*a*x
- 3*log(a*x - 1) - 3*log(a*x + 1)*a**4*x**4 + 6*log(a*x + 1)*a**3*x**3 - 6
*log(a*x + 1)*a*x + 3*log(a*x + 1) + 3*a**4*x**4 - 12*a**2*x**2 + 8*a*x +
5)/(24*a*c**3*(a**4*x**4 - 2*a**3*x**3 + 2*a*x - 1))
```

3.578 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{1}{32ac^4(1-ax)^4}-\frac{1}{16ac^4(1-ax)^3}-\frac{3}{32ac^4(1-ax)^2}-\frac{5}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+ax)^2}+\frac{5}{64ac^4(1+ax)}-\frac{15\text{arctanh}(ax)}{64ac^4}$$

output
```
-1/32/a/c^4/(-a*x+1)^4-1/16/a/c^4/(-a*x+1)^3-3/32/a/c^4/(-a*x+1)^2-5/32/a/
c^4/(-a*x+1)+1/64/a/c^4/(a*x+1)^2+5/64/a/c^4/(a*x+1)-15/64*arctanh(a*x)/a/
c^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{16+17ax-50a^2x^2+10a^3x^3+30a^4x^4-15a^5x^5+15(-1+ax)^4(1+ax)^2\text{arctanh}(ax)}{64ac^4(-1+ax)^4(1+ax)^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

output
```
-1/64*(16 + 17*a*x - 50*a^2*x^2 + 10*a^3*x^3 + 30*a^4*x^4 - 15*a^5*x^5 + 1
5*(-1 + a*x)^4*(1 + a*x)^2*ArcTanh[a*x])/(a*c^4*(-1 + a*x)^4*(1 + a*x)^2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{c^4(1-a^2x^2)^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{2\text{arctanh}(ax)}}{(1-a^2x^2)^4}dx}{c^4}$$

$$\Big\downarrow \text{6690}$$

$$-\frac{\int \frac{1}{(1-ax)^5(ax+1)^3}dx}{c^4}$$

$$\Big\downarrow \text{54}$$

$$-\frac{\int \left(\frac{5}{32(ax-1)^2}+\frac{5}{64(ax+1)^2}-\frac{3}{16(ax-1)^3}+\frac{1}{32(ax+1)^3}+\frac{3}{16(ax-1)^4}-\frac{1}{8(ax-1)^5}-\frac{15}{64(a^2x^2-1)}\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{15\text{arctanh}(ax)}{64a}+\frac{5}{32a(1-ax)}-\frac{5}{64a(ax+1)}+\frac{3}{32a(1-ax)^2}-\frac{1}{64a(ax+1)^2}+\frac{1}{16a(1-ax)^3}+\frac{1}{32a(1-ax)^4}}{c^4}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

output
```
-((1/(32*a*(1 - a*x)^4) + 1/(16*a*(1 - a*x)^3) + 3/(32*a*(1 - a*x)^2) + 5/
(32*a*(1 - a*x)) - 1/(64*a*(1 + a*x)^2) - 5/(64*a*(1 + a*x)) + (15*ArcTanh
[a*x])/(64*a))/c^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

method	result
risch	$\frac{\frac{15x^5a^4}{64}-\frac{15a^3x^4}{32}-\frac{5a^2x^3}{32}+\frac{25a\,x^2}{32}-\frac{17x}{64}-\frac{1}{4a}}{(ax-1)^2(a^2x^2-1)^2c^4}-\frac{15\ln(ax+1)}{128a\,c^4}+\frac{15\ln(-ax+1)}{128a\,c^4}$
default	$\frac{\frac{1}{64a(ax+1)^2}+\frac{5}{64a(ax+1)}-\frac{15\ln(ax+1)}{128a}-\frac{1}{32a(ax-1)^4}+\frac{1}{16a(ax-1)^3}-\frac{3}{32a(ax-1)^2}+\frac{5}{32(ax-1)a}+\frac{15\ln(ax-1)}{128a}}{c^4}$
norman	$\frac{-\frac{49x}{64c}-\frac{15a\,x^2}{64c}+\frac{11a^2x^3}{8c}+\frac{a^3x^4}{8c}-\frac{63a^4x^5}{64c}-\frac{a^5x^6}{64c}+\frac{a^6x^7}{4c}}{c^3(ax+1)^3(ax-1)^4}+\frac{15\ln(ax-1)}{128a\,c^4}-\frac{15\ln(ax+1)}{128a\,c^4}$
parallelrisch	$108a^3x^3-98ax-92a^4x^4-34a^5x^5+32x^6a^6+30\ln(ax+1)xa-15\ln(ax+1)+68a^2x^2+15\ln(ax-1)+15\ln(ax+1)x^2a^2-15\ln(a$

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/64*x^5*a^4-15/32*a^3*x^4-5/32*a^2*x^3+25/32*a*x^2-17/64*x-1/4/a)/(a*x-
1)^2/(a^2*x^2-1)^2/c^4-15/128*ln(a*x+1)/a/c^4+15/128*ln(-a*x+1)/a/c^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(103) = 206$.

Time = 0.10 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.79

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$
$$= \frac{30\,a^5x^5-60\,a^4x^4-20\,a^3x^3+100\,a^2x^2-34\,ax-15\,(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1)}{128\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output
```
1/128*(30*a^5*x^5 - 60*a^4*x^4 - 20*a^3*x^3 + 100*a^2*x^2 - 34*a*x - 15*(a
^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x +
1) + 15*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*
log(a*x - 1) - 32)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*
x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.17

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{15a^5x^5-30a^4x^4-10a^3x^3+50a^2x^2-17ax-16}{64a^7c^4x^6-128a^6c^4x^5-64a^5c^4x^4+256a^4c^4x^3-64a^3c^4x^2-128a^2c^4x+64ac^4} + \frac{\frac{15\log\left(x-\frac{1}{a}\right)}{128}-\frac{15\log\left(x+\frac{1}{a}\right)}{128}}{ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**4,x)
```

output
```
(15*a**5*x**5 - 30*a**4*x**4 - 10*a**3*x**3 + 50*a**2*x**2 - 17*a*x - 16)/
(64*a**7*c**4*x**6 - 128*a**6*c**4*x**5 - 64*a**5*c**4*x**4 + 256*a**4*c**
4*x**3 - 64*a**3*c**4*x**2 - 128*a**2*c**4*x + 64*a*c**4) + (15*log(x - 1/
a)/128 - 15*log(x + 1/a)/128)/(a*c**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{15\,a^5x^5-30\,a^4x^4-10\,a^3x^3+50\,a^2x^2-17\,ax-16}{64\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)} - \frac{15\,\log(ax+1)}{128\,ac^4} + \frac{15\,\log(ax-1)}{128\,ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
1/64*(15*a^5*x^5 - 30*a^4*x^4 - 10*a^3*x^3 + 50*a^2*x^2 - 17*a*x - 16)/(a^
7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*
a^2*c^4*x + a*c^4) - 15/128*log(a*x + 1)/(a*c^4) + 15/128*log(a*x - 1)/(a*
c^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{15\,\log\left(|ax+1|\right)}{128\,ac^4} + \frac{15\,\log\left(|ax-1|\right)}{128\,ac^4} + \frac{15\,a^5x^5 - 30\,a^4x^4 - 10\,a^3x^3 + 50\,a^2x^2 - 17\,ax - 16}{64\,(ax+1)^2(ax-1)^4ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
-15/128*log(abs(a*x + 1))/(a*c^4) + 15/128*log(abs(a*x - 1))/(a*c^4) + 1/6
4*(15*a^5*x^5 - 30*a^4*x^4 - 10*a^3*x^3 + 50*a^2*x^2 - 17*a*x - 16)/((a*x
+ 1)^2*(a*x - 1)^4*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{\frac{17\,x}{64} - \frac{25\,a\,x^2}{32} + \frac{1}{4\,a} + \frac{5\,a^2\,x^3}{32} + \frac{15\,a^3\,x^4}{32} - \frac{15\,a^4\,x^5}{64}}{-a^6\,c^4\,x^6 + 2\,a^5\,c^4\,x^5 + a^4\,c^4\,x^4 - 4\,a^3\,c^4\,x^3 + a^2\,c^4\,x^2 + 2\,a\,c^4\,x - c^4} - \frac{15\,\mathrm{atanh}(a\,x)}{64\,a\,c^4}$$

input
```
int((a*x + 1)/((c - a^2*c*x^2)^4*(a*x - 1)),x)
```

output
```
((17*x)/64 - (25*a*x^2)/32 + 1/(4*a) + (5*a^2*x^3)/32 + (15*a^3*x^4)/32 -
(15*a^4*x^5)/64)/(a^2*c^4*x^2 - c^4 - 4*a^3*c^4*x^3 + a^4*c^4*x^4 + 2*a^5*
c^4*x^5 - a^6*c^4*x^6 + 2*a*c^4*x) - (15*atanh(a*x))/(64*a*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.22

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$
$$= \frac{15\log(ax-1)\,a^6x^6 - 30\log(ax-1)\,a^5x^5 - 15\log(ax-1)\,a^4x^4 + 60\log(ax-1)\,a^3x^3 - 15\log(ax-1)}{}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x)
```

output
```
(15*log(a*x - 1)*a**6*x**6 - 30*log(a*x - 1)*a**5*x**5 - 15*log(a*x - 1)*a
**4*x**4 + 60*log(a*x - 1)*a**3*x**3 - 15*log(a*x - 1)*a**2*x**2 - 30*log(
a*x - 1)*a*x + 15*log(a*x - 1) - 15*log(a*x + 1)*a**6*x**6 + 30*log(a*x +
1)*a**5*x**5 + 15*log(a*x + 1)*a**4*x**4 - 60*log(a*x + 1)*a**3*x**3 + 15*
log(a*x + 1)*a**2*x**2 + 30*log(a*x + 1)*a*x - 15*log(a*x + 1) + 15*a**6*x
**6 - 75*a**4*x**4 + 40*a**3*x**3 + 85*a**2*x**2 - 64*a*x - 17)/(128*a*c**
4*(a**6*x**6 - 2*a**5*x**5 - a**4*x**4 + 4*a**3*x**3 - a**2*x**2 - 2*a*x +
 1))
```

3.579 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^5\,dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=\frac{c^5(1+ax)^8}{a}-\frac{4c^5(1+ax)^9}{3a}+\frac{3c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}$$

output
```
c^5*(a*x+1)^8/a-4/3*c^5*(a*x+1)^9/a+3/5*c^5*(a*x+1)^10/a-1/11*c^5*(a*x+1)^
11/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.59

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{c^5(1+ax)^8\left(-29+67ax-54a^2x^2+15a^3x^3\right)}{165a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

output
```
-1/165*(c^5*(1 + a*x)^8*(-29 + 67*a*x - 54*a^2*x^2 + 15*a^3*x^3))/a
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-a^2cx^2)^5 e^{4\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$\int c^5(1-a^2x^2)^5 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{27}$$

$$c^5\int e^{4\text{arctanh}(ax)}(1-a^2x^2)^5dx$$

$$\Big\downarrow \text{6690}$$

$$c^5\int (1-ax)^3(ax+1)^7dx$$

$$\Big\downarrow \text{49}$$

$$c^5\int \left(-(ax+1)^{10}+6(ax+1)^9-12(ax+1)^8+8(ax+1)^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c^5\left(-\frac{(ax+1)^{11}}{11a}+\frac{3(ax+1)^{10}}{5a}-\frac{4(ax+1)^9}{3a}+\frac{(ax+1)^8}{a}\right)$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

output
```
c^5*((1 + a*x)^8/a - (4*(1 + a*x)^9)/(3*a) + (3*(1 + a*x)^10)/(5*a) - (1 +
 a*x)^11/(11*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

method	result
default	$c^5\left(-\frac{1}{11}a^{10}x^{11}-\frac{2}{5}a^9x^{10}-\frac{1}{3}a^8x^9+a^7x^8+2a^6x^7-\frac{14}{5}x^5a^4-2a^3x^4+a^2x^3+2a\,x^2+x\right)$
gosper	$-\frac{c^5x(15a^{10}x^{10}+66a^9x^9+55a^8x^8-165a^7x^7-330x^6a^6+462a^4x^4+330a^3x^3-165a^2x^2-330ax-165)}{165}$
orering	$\frac{x(15a^{10}x^{10}+66a^9x^9+55a^8x^8-165a^7x^7-330x^6a^6+462a^4x^4+330a^3x^3-165a^2x^2-330ax-165)\left(-a^2c\,x^2+c\right)^5}{165(ax-1)^5(ax+1)^5}$
risch	$-\frac{1}{11}a^{10}c^5x^{11}-\frac{2}{5}a^9c^5x^{10}-\frac{1}{3}a^8c^5x^9+a^7c^5x^8+2a^6c^5x^7-\frac{14}{5}a^4c^5x^5-2a^3c^5x^4+a^2c^5x^3+2$
parallelrisch	$-\frac{1}{11}a^{10}c^5x^{11}-\frac{2}{5}a^9c^5x^{10}-\frac{1}{3}a^8c^5x^9+a^7c^5x^8+2a^6c^5x^7-\frac{14}{5}a^4c^5x^5-2a^3c^5x^4+a^2c^5x^3+2$
norman	$\frac{-c^5x+a^2c^5x^3+a^7c^5x^8+3a^3c^5x^4+\frac{4}{5}a^4c^5x^5-\frac{14}{5}a^5c^5x^6-2a^6c^5x^7+\frac{4}{3}a^8c^5x^9+\frac{1}{15}a^9c^5x^{10}-\frac{17}{55}a^{10}c^5x^{11}-\frac{1}{11}a^{11}c^5x^{12}-c^5a\,x^2}{ax-1}$
meijerg	$\frac{c^5\left(-\frac{xa\left(-2730x^{11}a^{11}-3276a^{10}x^{10}-4004a^9x^9-5005a^8x^8-6435a^7x^7-8580x^6a^6-12012a^5x^5-18018a^4x^4-30030a^3x^3-60060a^2x^2-18\right.}{30030(-ax+1)}\right.}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x,method=_RETURNVERBOSE)
```

output
```
c^5*(-1/11*a^10*x^11-2/5*a^9*x^10-1/3*a^8*x^9+a^7*x^8+2*a^6*x^7-14/5*x^5*a
^4-2*a^3*x^4+a^2*x^3+2*a*x^2+x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.53

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx = -\frac{1}{11}\,a^{10}c^5x^{11}-\frac{2}{5}\,a^9c^5x^{10}-\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8+2\,a^6c^5x^7 -\frac{14}{5}\,a^4c^5x^5-2\,a^3c^5x^4+a^2c^5x^3+2\,ac^5x^2+c^5x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="fricas")
```

output
```
-1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2
*a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^
2 + c^5*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(54) = 108.

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.65

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx = -\frac{a^{10}c^5x^{11}}{11}-\frac{2a^9c^5x^{10}}{5}-\frac{a^8c^5x^9}{3}+a^7c^5x^8+2a^6c^5x^7 -\frac{14a^4c^5x^5}{5}-2a^3c^5x^4+a^2c^5x^3+2ac^5x^2+c^5x$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**5,x)
```

output
```
-a**10*c**5*x**11/11 - 2*a**9*c**5*x**10/5 - a**8*c**5*x**9/3 + a**7*c**5*
x**8 + 2*a**6*c**5*x**7 - 14*a**4*c**5*x**5/5 - 2*a**3*c**5*x**4 + a**2*c*
*5*x**3 + 2*a*c**5*x**2 + c**5*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.53

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{2}{5}\,a^9c^5x^{10}-\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8+2\,a^6c^5x^7-\frac{14}{5}\,a^4c^5x^5-2\,a^3c^5x^4+a^2c^5x^3+2\,ac^5x^2+c^5x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="maxima")
```

output
```
-1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2
*a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^
2 + c^5*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.55

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx$$
$$=-\frac{\left(15\,c^5+\frac{231\,c^5}{ax-1}+\frac{1540\,c^5}{(ax-1)^2}+\frac{5775\,c^5}{(ax-1)^3}+\frac{13200\,c^5}{(ax-1)^4}+\frac{18480\,c^5}{(ax-1)^5}+\frac{14784\,c^5}{(ax-1)^6}+\frac{5280\,c^5}{(ax-1)^7}\right)(ax-1)^{11}}{165\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="giac")
```

output
```
-1/165*(15*c^5 + 231*c^5/(a*x - 1) + 1540*c^5/(a*x - 1)^2 + 5775*c^5/(a*x
- 1)^3 + 13200*c^5/(a*x - 1)^4 + 18480*c^5/(a*x - 1)^5 + 14784*c^5/(a*x -
1)^6 + 5280*c^5/(a*x - 1)^7)*(a*x - 1)^11/a
```

Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.53

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx=-\frac{a^{10}\,c^5\,x^{11}}{11}-\frac{2\,a^9\,c^5\,x^{10}}{5}-\frac{a^8\,c^5\,x^9}{3}+a^7\,c^5\,x^8+2\,a^6\,c^5\,x^7-\frac{14\,a^4\,c^5\,x^5}{5}-2\,a^3\,c^5\,x^4+a^2\,c^5\,x^3+2\,a\,c^5\,x^2+c^5\,x$$

input
```
int(((c - a^2*c*x^2)^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
c^5*x + 2*a*c^5*x^2 + a^2*c^5*x^3 - 2*a^3*c^5*x^4 - (14*a^4*c^5*x^5)/5 + 2
*a^6*c^5*x^7 + a^7*c^5*x^8 - (a^8*c^5*x^9)/3 - (2*a^9*c^5*x^10)/5 - (a^10*
c^5*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx$$
$$=\frac{c^5x(-15a^{10}x^{10}-66a^9x^9-55a^8x^8+165a^7x^7+330a^6x^6-462a^4x^4-330a^3x^3+165a^2x^2+330ax+16}{165}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x)
```

output
```
(c**5*x*( - 15*a**10*x**10 - 66*a**9*x**9 - 55*a**8*x**8 + 165*a**7*x**7 +
 330*a**6*x**6 - 462*a**4*x**4 - 330*a**3*x**3 + 165*a**2*x**2 + 330*a*x +
 165))/165
```

3.580 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx=\frac{4c^4(1+ax)^7}{7a}-\frac{c^4(1+ax)^8}{2a}+\frac{c^4(1+ax)^9}{9a}$$

output
```
4/7*c^4*(a*x+1)^7/a-1/2*c^4*(a*x+1)^8/a+1/9*c^4*(a*x+1)^9/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx=\frac{c^4(1+ax)^7\,(23-35ax+14a^2x^2)}{126a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

output
```
(c^4*(1 + a*x)^7*(23 - 35*a*x + 14*a^2*x^2))/(126*a)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-a^2cx^2)^4 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6717$$

$$\int c^4(1-a^2x^2)^4 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \; 27$$

$$c^4\int e^{4\text{arctanh}(ax)}(1-a^2x^2)^4\,dx$$

$$\Big\downarrow \; 6690$$

$$c^4\int (1-ax)^2(ax+1)^6dx$$

$$\Big\downarrow \; 49$$

$$c^4\int \left((ax+1)^8-4(ax+1)^7+4(ax+1)^6\right)dx$$

$$\Big\downarrow \; 2009$$

$$c^4\left(\frac{(ax+1)^9}{9a}-\frac{(ax+1)^8}{2a}+\frac{4(ax+1)^7}{7a}\right)$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

output
```
c^4*((4*(1 + a*x)^7)/(7*a) - (1 + a*x)^8/(2*a) + (1 + a*x)^9/(9*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

method	result
gosper	$\frac{c^{4} x \left(14 a^{8} x^{8}+63 a^{7} x^{7}+72 x^{6} a^{6}-84 a^{5} x^{5}-252 a^{4} x^{4}-126 a^{3} x^{3}+168 a^{2} x^{2}+252 a x +126\right)}{126}$
default	$c^{4} \left(\frac{1}{9} a^{8} x^{9}+\frac{1}{2} a^{7} x^{8}+\frac{4}{7} a^{6} x^{7}-\frac{2}{3} a^{5} x^{6}-2 x^{5} a^{4}-a^{3} x^{4}+\frac{4}{3} a^{2} x^{3}+2 a \,x^{2}+x \right)$
risch	$\frac{1}{9} a^{8} c^{4} x^{9}+\frac{1}{2} a^{7} c^{4} x^{8}+\frac{4}{7} a^{6} c^{4} x^{7}-\frac{2}{3} a^{5} c^{4} x^{6}-2 a^{4} c^{4} x^{5}-a^{3} c^{4} x^{4}+\frac{4}{3} a^{2} c^{4} x^{3}+2 a \,c^{4} x^{2}+c^{4} x$
parallelrisch	$\frac{1}{9} a^{8} c^{4} x^{9}+\frac{1}{2} a^{7} c^{4} x^{8}+\frac{4}{7} a^{6} c^{4} x^{7}-\frac{2}{3} a^{5} c^{4} x^{6}-2 a^{4} c^{4} x^{5}-a^{3} c^{4} x^{4}+\frac{4}{3} a^{2} c^{4} x^{3}+2 a \,c^{4} x^{2}+c^{4} x$
orering	$\frac{x \left(14 a^{8} x^{8}+63 a^{7} x^{7}+72 x^{6} a^{6}-84 a^{5} x^{5}-252 a^{4} x^{4}-126 a^{3} x^{3}+168 a^{2} x^{2}+252 a x +126\right) \left(-a^{2} c \,x^{2}+c \right)^{4}}{126 \left(a x +1\right)^{4} \left(a x -1\right)^{4}}$
norman	$\frac{-c^{4} x +a^{4} c^{4} x^{5}-a \,c^{4} x^{2}+\frac{2}{3} a^{2} c^{4} x^{3}+\frac{7}{3} a^{3} c^{4} x^{4}-\frac{4}{3} a^{5} c^{4} x^{6}-\frac{26}{21} a^{6} c^{4} x^{7}+\frac{1}{14} a^{7} c^{4} x^{8}+\frac{7}{18} a^{8} c^{4} x^{9}+\frac{1}{9} a^{9} c^{4} x^{10}}{a x -1}$
meijerg	$-\frac{c^{4} \left(-\frac{x a \left(-308 a^{9} x^{9}-385 a^{8} x^{8}-495 a^{7} x^{7}-660 x^{6} a^{6}-924 a^{5} x^{5}-1386 a^{4} x^{4}-2310 a^{3} x^{3}-4620 a^{2} x^{2}-13860 a x +27720\right)}{2772 \left(-a x +1\right)}-10 \ln \left(-a x +1\right)\right)}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/126*c^4*x*(14*a^8*x^8+63*a^7*x^7+72*a^6*x^6-84*a^5*x^5-252*a^4*x^4-126*a
^3*x^3+168*a^2*x^2+252*a*x+126)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9+\frac{1}{2}\,a^7c^4x^8+\frac{4}{7}\,a^6c^4x^7-\frac{2}{3}\,a^5c^4x^6-2\,a^4c^4x^5-a^3c^4x^4+\frac{4}{3}\,a^2c^4x^3+2\,ac^4x^2+c^4x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output
```
1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*
a^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(41) = 82.

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.92

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8c^4x^9}{9}+\frac{a^7c^4x^8}{2}+\frac{4a^6c^4x^7}{7}-\frac{2a^5c^4x^6}{3}-2a^4c^4x^5-a^3c^4x^4+\frac{4a^2c^4x^3}{3}+2ac^4x^2+c^4x$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**4,x)
```

output
```
a**8*c**4*x**9/9 + a**7*c**4*x**8/2 + 4*a**6*c**4*x**7/7 - 2*a**5*c**4*x**
6/3 - 2*a**4*c**4*x**5 - a**3*c**4*x**4 + 4*a**2*c**4*x**3/3 + 2*a*c**4*x*
*2 + c**4*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9+\frac{1}{2}\,a^7c^4x^8+\frac{4}{7}\,a^6c^4x^7-\frac{2}{3}\,a^5c^4x^6-2\,a^4c^4x^5-a^3c^4x^4+\frac{4}{3}\,a^2c^4x^3+2\,ac^4x^2+c^4x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*
a^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.73

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{\left(14\,c^4+\frac{189\,c^4}{ax-1}+\frac{1080\,c^4}{(ax-1)^2}+\frac{3360\,c^4}{(ax-1)^3}+\frac{6048\,c^4}{(ax-1)^4}+\frac{6048\,c^4}{(ax-1)^5}+\frac{2688\,c^4}{(ax-1)^6}\right)(ax-1)^9}{126\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
1/126*(14*c^4 + 189*c^4/(a*x - 1) + 1080*c^4/(a*x - 1)^2 + 3360*c^4/(a*x -
 1)^3 + 6048*c^4/(a*x - 1)^4 + 6048*c^4/(a*x - 1)^5 + 2688*c^4/(a*x - 1)^6
)*(a*x - 1)^9/a
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8\,c^4\,x^9}{9}+\frac{a^7\,c^4\,x^8}{2}+\frac{4\,a^6\,c^4\,x^7}{7}-\frac{2\,a^5\,c^4\,x^6}{3}-2\,a^4\,c^4\,x^5-a^3\,c^4\,x^4+\frac{4\,a^2\,c^4\,x^3}{3}+2\,a\,c^4\,x^2+c^4\,x$$

input
```
int(((c - a^2*c*x^2)^4*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
c^4*x + 2*a*c^4*x^2 + (4*a^2*c^4*x^3)/3 - a^3*c^4*x^4 - 2*a^4*c^4*x^5 - (2
*a^5*c^4*x^6)/3 + (4*a^6*c^4*x^7)/7 + (a^7*c^4*x^8)/2 + (a^8*c^4*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.31

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{c^4x(14a^8x^8+63a^7x^7+72a^6x^6-84a^5x^5-252a^4x^4-126a^3x^3+168a^2x^2+252ax+126)}{126}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x)
```

output
```
(c**4*x*(14*a**8*x**8 + 63*a**7*x**7 + 72*a**6*x**6 - 84*a**5*x**5 - 252*a
**4*x**4 - 126*a**3*x**3 + 168*a**2*x**2 + 252*a*x + 126))/126
```

3.581 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 35

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}$$

output
```
1/3*c^3*(a*x+1)^6/a-1/7*c^3*(a*x+1)^7/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{c^3(1+ax)^6(-4+3ax)}{21a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

output
```
-1/21*(c^3*(1 + a*x)^6*(-4 + 3*a*x))/a
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^3 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int c^3\left(1-a^2x^2\right)^3 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{ 27}$$

$$c^3\int e^{4\text{arctanh}(ax)}\left(1-a^2x^2\right)^3dx$$

$$\Big\downarrow \text{ 6690}$$

$$c^3\int (1-ax)(ax+1)^5dx$$

$$\Big\downarrow \text{ 49}$$

$$c^3\int \left(2(ax+1)^5-(ax+1)^6\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$c^3\left(\frac{(ax+1)^6}{3a}-\frac{(ax+1)^7}{7a}\right)$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

output
```
c^3*((1 + a*x)^6/(3*a) - (1 + a*x)^7/(7*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

method	result
gosper	$-\frac{c^{3} x \left(3 x^{6} a^{6}+14 a^{5} x^{5}+21 a^{4} x^{4}-35 a^{2} x^{2}-42 a x -21\right)}{21}$
default	$c^{3} \left(-\frac{1}{7} a^{6} x^{7}-\frac{2}{3} a^{5} x^{6}-x^{5} a^{4}+\frac{5}{3} a^{2} x^{3}+2 a \,x^{2}+x \right)$
risch	$-\frac{1}{7} a^{6} c^{3} x^{7}-\frac{2}{3} a^{5} c^{3} x^{6}-c^{3} a^{4} x^{5}+\frac{5}{3} a^{2} c^{3} x^{3}+2 a \,c^{3} x^{2}+c^{3} x$
parallelrisch	$-\frac{1}{7} a^{6} c^{3} x^{7}-\frac{2}{3} a^{5} c^{3} x^{6}-c^{3} a^{4} x^{5}+\frac{5}{3} a^{2} c^{3} x^{3}+2 a \,c^{3} x^{2}+c^{3} x$
orering	$\frac{x \left(3 x^{6} a^{6}+14 a^{5} x^{5}+21 a^{4} x^{4}-35 a^{2} x^{2}-42 a x -21\right) \left(-a^{2} c \,x^{2}+c \right)^{3}}{21 \left(a x +1\right)^{3} \left(a x -1\right)^{3}}$
norman	$\frac{-c^{3} x +c^{3} a^{4} x^{5}-a \,c^{3} x^{2}+\frac{1}{3} a^{2} c^{3} x^{3}+\frac{5}{3} a^{3} c^{3} x^{4}-\frac{1}{3} a^{5} c^{3} x^{6}-\frac{11}{21} a^{6} c^{3} x^{7}-\frac{1}{7} a^{7} c^{3} x^{8}}{a x -1}$
meijerg	$\frac{c^{3} \left(-\frac{a x \left(-45 a^{7} x^{7}-60 x^{6} a^{6}-84 a^{5} x^{5}-126 a^{4} x^{4}-210 a^{3} x^{3}-420 a^{2} x^{2}-1260 a x +2520\right)}{315 \left(-a x +1\right)}-8 \ln \left(-a x +1\right)\right)}{a}-\frac{2 c^{3} \left(-\frac{a x \left(-14 a^{5} x^{5}-21 a^{4}\right.}{}\right.}{}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/21*c^3*x*(3*a^6*x^6+14*a^5*x^5+21*a^4*x^4-35*a^2*x^2-42*a*x-21)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7-\frac{2}{3}\,a^5c^3x^6-a^4c^3x^5+\frac{5}{3}\,a^2c^3x^3+2\,ac^3x^2+c^3x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c
^3*x^2 + c^3*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(26) = 52.

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{a^6c^3x^7}{7}-\frac{2a^5c^3x^6}{3}-a^4c^3x^5+\frac{5a^2c^3x^3}{3}+2ac^3x^2+c^3x$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**3,x)
```

output
```
-a**6*c**3*x**7/7 - 2*a**5*c**3*x**6/3 - a**4*c**3*x**5 + 5*a**2*c**3*x**3
/3 + 2*a*c**3*x**2 + c**3*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7-\frac{2}{3}\,a^5c^3x^6-a^4c^3x^5+\frac{5}{3}\,a^2c^3x^3+2\,ac^3x^2+c^3x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c
^3*x^2 + c^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(31) = 62.

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.23

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$$
$$=-\frac{\left(3\,c^3+\frac{35\,c^3}{ax-1}+\frac{168\,c^3}{(ax-1)^2}+\frac{420\,c^3}{(ax-1)^3}+\frac{560\,c^3}{(ax-1)^4}+\frac{336\,c^3}{(ax-1)^5}\right)(ax-1)^7}{21\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/21*(3*c^3 + 35*c^3/(a*x - 1) + 168*c^3/(a*x - 1)^2 + 420*c^3/(a*x - 1)^
3 + 560*c^3/(a*x - 1)^4 + 336*c^3/(a*x - 1)^5)*(a*x - 1)^7/a
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{a^6\,c^3\,x^7}{7}-\frac{2\,a^5\,c^3\,x^6}{3}-a^4\,c^3\,x^5+\frac{5\,a^2\,c^3\,x^3}{3}+2\,a\,c^3\,x^2+c^3\,x$$

input
```
int(((c - a^2*c*x^2)^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
c^3*x + 2*a*c^3*x^2 + (5*a^2*c^3*x^3)/3 - a^4*c^3*x^5 - (2*a^5*c^3*x^6)/3
- (a^6*c^3*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{c^3x(-3a^6x^6-14a^5x^5-21a^4x^4+35a^2x^2+42ax+21)}{21}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x)
```

output
```
(c**3*x*( - 3*a**6*x**6 - 14*a**5*x**5 - 21*a**4*x**4 + 35*a**2*x**2 + 42*
a*x + 21))/21
```

3.582 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 17

$$\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx=\frac{c^2(1+ax)^5}{5a}$$

output
```
1/5*c^2*(a*x+1)^5/a
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx=c^2\left(x+2ax^2+2a^2x^3+a^3x^4+\frac{a^4x^5}{5}\right)$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

output
```
c^2*(x + 2*a*x^2 + 2*a^2*x^3 + a^3*x^4 + (a^4*x^5)/5)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6717, 27, 6690, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^2 e^{4\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$\int c^2\left(1-a^2x^2\right)^2 e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{27}$$

$$c^2\int e^{4\text{arctanh}(ax)}\left(1-a^2x^2\right)^2dx$$

$$\Big\downarrow \text{6690}$$

$$c^2\int (ax+1)^4dx$$

$$\Big\downarrow \text{17}$$

$$\frac{c^2(ax+1)^5}{5a}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

output
```
(c^2*(1 + a*x)^5)/(5*a)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result
default	$\frac{c^{2} (a x +1)^{5}}{5 a}$
gosper	$\frac{c^{2} x \left(a^{4} x^{4}+5 a^{3} x^{3}+10 a^{2} x^{2}+10 a x +5\right)}{5}$
parallelrisch	$\frac{1}{5} a^{4} c^{2} x^{5}+a^{3} c^{2} x^{4}+2 a^{2} c^{2} x^{3}+2 a \,c^{2} x^{2}+c^{2} x$
risch	$\frac{a^{4} c^{2} x^{5}}{5}+a^{3} c^{2} x^{4}+2 a^{2} c^{2} x^{3}+2 a \,c^{2} x^{2}+c^{2} x +\frac{c^{2}}{5 a}$
norman	$\frac{-c^{2} x +a^{3} c^{2} x^{4}-a \,c^{2} x^{2}+\frac{4}{5} a^{4} c^{2} x^{5}+\frac{1}{5} a^{5} c^{2} x^{6}}{a x -1}$
orering	$\frac{x \left(a^{4} x^{4}+5 a^{3} x^{3}+10 a^{2} x^{2}+10 a x +5\right) \left(-a^{2} c \,x^{2}+c \right)^{2}}{5 (a x +1)^{2} (a x -1)^{2}}$
meijerg	$-\frac{c^{2} \left(-\frac{a x \left(-14 a^{5} x^{5}-21 a^{4} x^{4}-35 a^{3} x^{3}-70 a^{2} x^{2}-210 a x +420\right)}{70 (-a x +1)}-6 \ln \left(-a x +1\right)\right)}{a}+\frac{c^{2} \left(-\frac{a x \left(-5 a^{3} x^{3}-10 a^{2} x^{2}-30 a x +60\right)}{15 (-a x +1)}-4 \ln \left(-\right.\right.}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*c^2*(a*x+1)^5/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(15) = 30.

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5+a^3c^2x^4+2\,a^2c^2x^3+2\,ac^2x^2+c^2x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(12) = 24.

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.82

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{a^4c^2x^5}{5}+a^3c^2x^4+2a^2c^2x^3+2ac^2x^2+c^2x$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**2,x)
```

output
```
a**4*c**2*x**5/5 + a**3*c**2*x**4 + 2*a**2*c**2*x**3 + 2*a*c**2*x**2 + c**
2*x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{1}{5}\,a^4c^2x^5 + a^3c^2x^4 + 2\,a^2c^2x^3 + 2\,ac^2x^2 + c^2x$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(15) = 30$.

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.76

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{\left(c^2 + \frac{10\,c^2}{ax-1} + \frac{40\,c^2}{(ax-1)^2} + \frac{80\,c^2}{(ax-1)^3} + \frac{80\,c^2}{(ax-1)^4}\right)(ax-1)^5}{5\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
1/5*(c^2 + 10*c^2/(a*x - 1) + 40*c^2/(a*x - 1)^2 + 80*c^2/(a*x - 1)^3 + 80
*c^2/(a*x - 1)^4)*(a*x - 1)^5/a
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = \frac{a^4\,c^2\,x^5}{5} + a^3\,c^2\,x^4 + 2\,a^2\,c^2\,x^3 + 2\,a\,c^2\,x^2 + c^2\,x$$

input
```
int(((c - a^2*c*x^2)^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
c^2*x + 2*a*c^2*x^2 + 2*a^2*c^2*x^3 + a^3*c^2*x^4 + (a^4*c^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{c^2x(a^4x^4+5a^3x^3+10a^2x^2+10ax+5)}{5}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x)
```

output
```
(c**2*x*(a**4*x**4 + 5*a**3*x**3 + 10*a**2*x**2 + 10*a*x + 5))/5
```

3.583 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-4cx-\frac{c(1+ax)^2}{a}-\frac{c(1+ax)^3}{3a}-\frac{8c\log(1-ax)}{a}$$

output
```
-4*c*x-c*(a*x+1)^2/a-1/3*c*(a*x+1)^3/a-8*c*ln(-a*x+1)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{c(4+21ax+6a^2x^2+a^3x^3+24\log(1-ax))}{3a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

output
```
-1/3*(c*(4 + 21*a*x + 6*a^2*x^2 + a^3*x^3 + 24*Log[1 - a*x]))/a
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2 c x^2\right) e^{4 \coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int c\left(1-a^2 x^2\right) e^{4\text{arctanh}(ax)}dx$$

$$\Big\downarrow \text{ 27}$$

$$c\int e^{4\text{arctanh}(ax)}\left(1-a^2 x^2\right)dx$$

$$\Big\downarrow \text{ 6690}$$

$$c\int \frac{(ax+1)^3}{1-ax}dx$$

$$\Big\downarrow \text{ 49}$$

$$c\int \left(-(ax+1)^2-2(ax+1)+\frac{8}{1-ax}-4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$c\left(-\frac{(ax+1)^3}{3a}-\frac{(ax+1)^2}{a}-\frac{8\log(1-ax)}{a}-4x\right)$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

output
```
c*(-4*x - (1 + a*x)^2/a - (1 + a*x)^3/(3*a) - (8*Log[1 - a*x])/a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

method	result
default	$c\left(-\frac{a^{2}x^{3}}{3}-2a\,x^{2}-7x-\frac{8\ln\left(ax-1\right)}{a}\right)$
risch	$-\frac{a^{2}c\,x^{3}}{3}-2ac\,x^{2}-7xc-\frac{8c\ln\left(ax-1\right)}{a}$
parallelrisch	$-\frac{a^{3}c\,x^{3}+6a^{2}c\,x^{2}+21acx+24c\ln\left(ax-1\right)}{3a}$
norman	$\frac{7xc-5ac\,x^{2}-\frac{5}{3}a^{2}c\,x^{3}-\frac{1}{3}a^{3}c\,x^{4}}{ax-1}-\frac{8c\ln\left(ax-1\right)}{a}$
meijerg	$\frac{c\left(-\frac{ax\left(-5a^{3}x^{3}-10a^{2}x^{2}-30ax+60\right)}{15(-ax+1)}-4\ln\left(-ax+1\right)\right)}{a}-\frac{2c\left(\frac{ax\left(-2a^{2}x^{2}-6ax+12\right)}{-4ax+4}+3\ln\left(-ax+1\right)\right)}{a}+\frac{2c\left(\frac{ax}{-ax+1}+\ln(-ax+\right.}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
c*(-1/3*a^2*x^3-2*a*x^2-7*x-8/a*ln(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{a^3cx^3+6\,a^2cx^2+21\,acx+24\,c\log\left(ax-1\right)}{3\,a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
-1/3*(a^3*c*x^3 + 6*a^2*c*x^2 + 21*a*c*x + 24*c*log(a*x - 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{a^2cx^3}{3}-2acx^2-7cx-\frac{8c\log\left(ax-1\right)}{a}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c),x)
```

output
```
-a**2*c*x**3/3 - 2*a*c*x**2 - 7*c*x - 8*c*log(a*x - 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3-2\,acx^2-7\,cx-\frac{8\,c\log\left(ax-1\right)}{a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
-1/3*a^2*c*x^3 - 2*a*c*x^2 - 7*c*x - 8*c*log(a*x - 1)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{(ax-1)^3\left(c+\frac{9\,c}{ax-1}+\frac{36\,c}{(ax-1)^2}\right)}{3\,a}+\frac{8\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
-1/3*(a*x - 1)^3*(c + 9*c/(a*x - 1) + 36*c/(a*x - 1)^2)/a + 8*c*log(abs(a*
x - 1)/((a*x - 1)^2*abs(a)))/a
```

Mupad [B] (verification not implemented)

Time = 13.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-7\,c\,x-\frac{a^2\,c\,x^3}{3}-\frac{8\,c\,\ln\left(a\,x-1\right)}{a}-2\,a\,c\,x^2$$

input
```
int(((c - a^2*c*x^2)*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
- 7*c*x - (a^2*c*x^3)/3 - (8*c*log(a*x - 1))/a - 2*a*c*x^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\frac{c(-24\log(ax-1)-a^3x^3-6a^2x^2-21ax)}{3a}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x)
```

output
```
(c*( - 24*log(a*x - 1) - a**3*x**3 - 6*a**2*x**2 - 21*a*x))/(3*a)
```

3.584 $\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 13

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{x}{c(1-ax)^2}$$

output
```
x/c/(-a*x+1)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{(1+ax)^2}{4ac(1-ax)^2}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

output
```
(1 + a*x)^2/(4*a*c*(1 - a*x)^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6717, 27, 6690, 38}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}dx$$
$$\Big\downarrow \text{6717}$$
$$\int \frac{e^{4\text{arctanh}(ax)}}{c(1-a^2x^2)}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{e^{4\text{arctanh}(ax)}}{1-a^2x^2}dx}{c}$$
$$\Big\downarrow \text{6690}$$
$$\frac{\int \frac{ax+1}{(1-ax)^3}dx}{c}$$
$$\Big\downarrow \text{38}$$
$$\frac{x}{c(1-ax)^2}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

output
```
x/(c*(1 - a*x)^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 38
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{x}{c(ax-1)^2}$	13
norman	$\frac{x}{c(ax-1)^2}$	13
risch	$\frac{x}{c(ax-1)^2}$	13
parallelrisch	$\frac{x}{c(ax-1)^2}$	13
default	$\frac{\frac{1}{(ax-1)a}+\frac{1}{a(ax-1)^2}}{c}$	28
orering	$-\frac{x(ax+1)}{(ax-1)(-a^2c\,x^2+c)}$	29

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
x/c/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{x}{a^2cx^2-2\,acx+c}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
x/(a^2*c*x^2 - 2*a*c*x + c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{x}{a^2cx^2-2acx+c}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c),x)
```

output
```
x/(a**2*c*x**2 - 2*a*c*x + c)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{x}{a^2cx^2-2\,acx+c}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
x/(a^2*c*x^2 - 2*a*c*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{\frac{1}{(ax-1)a}+\frac{1}{(ax-1)^2a}}{c}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
(1/((a*x - 1)*a) + 1/((a*x - 1)^2*a))/c
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{x}{c\,(a\,x-1)^2}$$

input
```
int((a*x + 1)^2/((c - a^2*c*x^2)*(a*x - 1)^2),x)
```

output
```
x/(c*(a*x - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.46

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{a^2x^2+1}{2ac\,(a^2x^2-2ax+1)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x)
```

output
```
(a**2*x**2 + 1)/(2*a*c*(a**2*x**2 - 2*a*x + 1))
```

3.585 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{1}{3ac^2(1-ax)^3}$$

output
```
1/3/a/c^2/(-a*x+1)^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{1}{3ac^2(1-ax)^3}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

output
```
1/(3*a*c^2*(1 - a*x)^3)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6717, 27, 6690, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{c^2(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{e^{4\text{arctanh}(ax)}}{(1-a^2x^2)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 6690}$$

$$\frac{\int \frac{1}{(1-ax)^4}dx}{c^2}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{1}{3ac^2(1-ax)^3}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

output
```
1/(3*a*c^2*(1 - a*x)^3)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{1}{3c^{2}a\left(ax-1\right)^{3}}$	16
default	$-\frac{1}{3c^{2}a\left(ax-1\right)^{3}}$	16
risch	$-\frac{1}{3c^{2}a\left(ax-1\right)^{3}}$	16
parallelrisch	$\frac{-a^{2}x^{3}+3a\,x^{2}-3x}{3\left(ax-1\right)^{3}c^{2}}$	31
orering	$-\frac{x\left(a^{2}x^{2}-3ax+3\right)\left(ax+1\right)^{2}}{3\left(ax-1\right)\left(-a^{2}c\,x^{2}+c\right)^{2}}$	44
norman	$\frac{-\frac{x}{c}-\frac{a^{3}x^{4}}{3c}+\frac{2a^{2}x^{3}}{3c}}{c\left(ax-1\right)^{3}\left(ax+1\right)}$	48

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/c^2/a/(a*x-1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(15) = 30.

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx=-\frac{1}{3\,\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/3/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(14) = 28.

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx=-\frac{1}{3a^4c^2x^3-9a^3c^2x^2+9a^2c^2x-3ac^2}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**2,x)
```

output
```
-1/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2*c**2*x - 3*a*c**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(15) = 30.

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx=-\frac{1}{3\,\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/3/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{1}{3\,(ax-1)^3ac^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/3/((a*x - 1)^3*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.22

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{1}{-3\,a^4\,c^2\,x^3+9\,a^3\,c^2\,x^2-9\,a^2\,c^2\,x+3\,a\,c^2}$$

input
```
int((a*x + 1)^2/((c - a^2*c*x^2)^2*(a*x - 1)^2),x)
```

output
```
1/(3*a*c^2 - 9*a^2*c^2*x + 9*a^3*c^2*x^2 - 3*a^4*c^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{1}{3a\,c^2\,(a^3x^3-3a^2x^2+3ax-1)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x)
```

output
```
( - 1)/(3*a*c**2*(a**3*x**3 - 3*a**2*x**2 + 3*a*x - 1))
```

3.586 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{1}{8ac^3(1-ax)^4}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{16ac^3(1-ax)}+\frac{\operatorname{arctanh}(ax)}{16ac^3}$$

output
```
1/8/a/c^3/(-a*x+1)^4+1/12/a/c^3/(-a*x+1)^3+1/16/a/c^3/(-a*x+1)^2+1/16/a/c^
3/(-a*x+1)+1/16*arctanh(a*x)/a/c^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{16-19ax+12a^2x^2-3a^3x^3+3(-1+ax)^4\operatorname{arctanh}(ax)}{48ac^3(-1+ax)^4}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

output
```
(16 - 19*a*x + 12*a^2*x^2 - 3*a^3*x^3 + 3*(-1 + a*x)^4*ArcTanh[a*x])/(48*a
*c^3*(-1 + a*x)^4)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{c^3(1-a^2x^2)^3}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{e^{4\text{arctanh}(ax)}}{(1-a^2x^2)^3}dx}{c^3}$$

$$\Big\downarrow \text{ 6690}$$

$$\frac{\int \frac{1}{(1-ax)^5(ax+1)}dx}{c^3}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{\int \left(\frac{1}{16(ax-1)^2}-\frac{1}{8(ax-1)^3}+\frac{1}{4(ax-1)^4}-\frac{1}{2(ax-1)^5}-\frac{1}{16(a^2x^2-1)}\right)dx}{c^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\text{arctanh}(ax)}{16a}+\frac{1}{16a(1-ax)}+\frac{1}{16a(1-ax)^2}+\frac{1}{12a(1-ax)^3}+\frac{1}{8a(1-ax)^4}}{c^3}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

output
```
(1/(8*a*(1 - a*x)^4) + 1/(12*a*(1 - a*x)^3) + 1/(16*a*(1 - a*x)^2) + 1/(16
*a*(1 - a*x)) + ArcTanh[a*x]/(16*a))/c^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.75

method	result
risch	$\frac{-\frac{a^{2} x^{3}}{16}+\frac{a\,x^{2}}{4}-\frac{19 x}{48}+\frac{1}{3 a}}{c^{3} \left(a x -1\right)^{4}}-\frac{\ln \left(a x -1\right)}{32 a\,c^{3}}+\frac{\ln \left(-a x -1\right)}{32 a\,c^{3}}$
default	$\frac{\frac{\ln \left(a x +1\right)}{32 a}+\frac{1}{8 a \left(a x -1\right)^{4}}-\frac{1}{12 a \left(a x -1\right)^{3}}+\frac{1}{16 a \left(a x -1\right)^{2}}-\frac{1}{16 \left(a x -1\right) a}-\frac{\ln \left(a x -1\right)}{32 a}}{c^{3}}$
norman	$\frac{\frac{15 x}{16 c}+\frac{a\,x^{2}}{8 c}-\frac{31 a^{2} x^{3}}{24 c}+\frac{11 a^{3} x^{4}}{24 c}+\frac{29 a^{4} x^{5}}{48 c}-\frac{a^{5} x^{6}}{3 c}}{c^{2} \left(a x +1\right)^{2} \left(a x -1\right)^{4}}-\frac{\ln \left(a x -1\right)}{32 a\,c^{3}}+\frac{\ln \left(a x +1\right)}{32 a\,c^{3}}$
parallelrisch	$\frac{-3 \ln \left(a x -1\right) x^{4} a^{4}+3 \ln \left(a x +1\right) x^{4} a^{4}-32 a^{4} x^{4}+12 a^{3} \ln \left(a x -1\right) x^{3}-12 \ln \left(a x +1\right) x^{3} a^{3}+122 a^{3} x^{3}-18 a^{2} \ln \left(a x -1\right) x^{2}+18 \ln \left(a x +\right.}{96 \left(a x -1\right)^{4} c^{3} a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/16*a^2*x^3+1/4*a*x^2-19/48*x+1/3/a)/c^3/(a*x-1)^4-1/32/a/c^3*ln(a*x-1)
+1/32/a/c^3*ln(-a*x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(73) = 146.

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.69

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{6\,a^3x^3-24\,a^2x^2+38\,ax-3\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log(ax+1)+3\,(a^4x^4-4\,a^3x^3+6\,a}{96\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/96*(6*a^3*x^3 - 24*a^2*x^2 + 38*a*x - 3*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^
2 - 4*a*x + 1)*log(a*x + 1) + 3*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x +
 1)*log(a*x - 1) - 32)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^
2*c^3*x + a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{3a^3x^3-12a^2x^2+19ax-16}{48a^5c^3x^4-192a^4c^3x^3+288a^3c^3x^2-192a^2c^3x+48ac^3} - \frac{\frac{\log\left(x-\frac{1}{a}\right)}{32}-\frac{\log\left(x+\frac{1}{a}\right)}{32}}{ac^3}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**3,x)
```

output
```
-(3*a**3*x**3 - 12*a**2*x**2 + 19*a*x - 16)/(48*a**5*c**3*x**4 - 192*a**4*
c**3*x**3 + 288*a**3*c**3*x**2 - 192*a**2*c**3*x + 48*a*c**3) - (log(x - 1
/a)/32 - log(x + 1/a)/32)/(a*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{3\,a^3x^3-12\,a^2x^2+19\,ax-16}{48\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)} + \frac{\log(ax+1)}{32\,ac^3} - \frac{\log(ax-1)}{32\,ac^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/48*(3*a^3*x^3 - 12*a^2*x^2 + 19*a*x - 16)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3
+ 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + 1/32*log(a*x + 1)/(a*c^3) - 1/32*
log(a*x - 1)/(a*c^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{32\,ac^3} - \frac{\frac{3\,a^3c^9}{ax-1}-\frac{3\,a^3c^9}{(ax-1)^2}+\frac{4\,a^3c^9}{(ax-1)^3}-\frac{6\,a^3c^9}{(ax-1)^4}}{48\,a^4c^{12}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
1/32*log(abs(-2/(a*x - 1) - 1))/(a*c^3) - 1/48*(3*a^3*c^9/(a*x - 1) - 3*a^
3*c^9/(a*x - 1)^2 + 4*a^3*c^9/(a*x - 1)^3 - 6*a^3*c^9/(a*x - 1)^4)/(a^4*c^
12)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{\operatorname{atanh}(a\,x)}{16\,a\,c^3} - \frac{\frac{19\,x}{48}-\frac{a\,x^2}{4}-\frac{1}{3\,a}+\frac{a^2\,x^3}{16}}{a^4\,c^3\,x^4-4\,a^3\,c^3\,x^3+6\,a^2\,c^3\,x^2-4\,a\,c^3\,x+c^3}$$

input
```
int((a*x + 1)^2/((c - a^2*c*x^2)^3*(a*x - 1)^2),x)
```

output
```
atanh(a*x)/(16*a*c^3) - ((19*x)/48 - (a*x^2)/4 - 1/(3*a) + (a^2*x^3)/16)/(
c^3 + 6*a^2*c^3*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.08

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{-6\log(ax-1)\,a^4x^4+24\log(ax-1)\,a^3x^3-36\log(ax-1)\,a^2x^2+24\log(ax-1)\,ax-6\log(ax-1)+}{192a\,c^3}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x)
```

output
```
( - 6*log(a*x - 1)*a**4*x**4 + 24*log(a*x - 1)*a**3*x**3 - 36*log(a*x - 1)
*a**2*x**2 + 24*log(a*x - 1)*a*x - 6*log(a*x - 1) + 6*log(a*x + 1)*a**4*x*
*4 - 24*log(a*x + 1)*a**3*x**3 + 36*log(a*x + 1)*a**2*x**2 - 24*log(a*x +
1)*a*x + 6*log(a*x + 1) - 3*a**4*x**4 + 30*a**2*x**2 - 64*a*x + 61)/(192*a
*c**3*(a**4*x**4 - 4*a**3*x**3 + 6*a**2*x**2 - 4*a*x + 1))
```

3.587 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\begin{aligned}\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx &= \frac{1}{20ac^4(1-ax)^5}+\frac{1}{16ac^4(1-ax)^4}\\&\quad+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^2}\\&\quad+\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)}+\frac{3\operatorname{arctanh}(ax)}{32ac^4}\end{aligned}$$

output
```
1/20/a/c^4/(-a*x+1)^5+1/16/a/c^4/(-a*x+1)^4+1/16/a/c^4/(-a*x+1)^3+1/16/a/c
^4/(-a*x+1)^2+5/64/a/c^4/(-a*x+1)-1/64/a/c^4/(a*x+1)+3/32*arctanh(a*x)/a/c
^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\begin{aligned}&\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx\\&=\frac{-48+47ax+20a^2x^2-80a^3x^3+60a^4x^4-15a^5x^5+15(-1+ax)^5(1+ax)\operatorname{arctanh}(ax)}{160ac^4(-1+ax)^5(1+ax)}\end{aligned}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

output
```
(-48 + 47*a*x + 20*a^2*x^2 - 80*a^3*x^3 + 60*a^4*x^4 - 15*a^5*x^5 + 15*(-1
 + a*x)^5*(1 + a*x)*ArcTanh[a*x])/(160*a*c^4*(-1 + a*x)^5*(1 + a*x))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{e^{4\text{arctanh}(ax)}}{c^4(1-a^2x^2)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{e^{4\text{arctanh}(ax)}}{(1-a^2x^2)^4}dx}{c^4}$$

$$\Big\downarrow \text{6690}$$

$$\frac{\int \frac{1}{(1-ax)^6(ax+1)^2}dx}{c^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{5}{64(ax-1)^2}+\frac{1}{64(ax+1)^2}-\frac{1}{8(ax-1)^3}+\frac{3}{16(ax-1)^4}-\frac{1}{4(ax-1)^5}+\frac{1}{4(ax-1)^6}-\frac{3}{32(a^2x^2-1)}\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3\text{arctanh}(ax)}{32a}+\frac{5}{64a(1-ax)}-\frac{1}{64a(ax+1)}+\frac{1}{16a(1-ax)^2}+\frac{1}{16a(1-ax)^3}+\frac{1}{16a(1-ax)^4}+\frac{1}{20a(1-ax)^5}}{c^4}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

output
```
(1/(20*a*(1 - a*x)^5) + 1/(16*a*(1 - a*x)^4) + 1/(16*a*(1 - a*x)^3) + 1/(1
6*a*(1 - a*x)^2) + 5/(64*a*(1 - a*x)) - 1/(64*a*(1 + a*x)) + (3*ArcTanh[a*
x])/(32*a))/c^4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.75

method	result
risch	$\frac{-\frac{3x^5a^4}{32}+\frac{3a^3x^4}{8}-\frac{a^2x^3}{2}+\frac{a\,x^2}{8}+\frac{47x}{160}-\frac{3}{10a}}{c^4(ax-1)^4(a^2x^2-1)}-\frac{3\ln(ax-1)}{64a\,c^4}+\frac{3\ln(-ax-1)}{64a\,c^4}$
default	$\frac{-\frac{1}{64a(ax+1)}+\frac{3\ln(ax+1)}{64a}-\frac{1}{20a(ax-1)^5}+\frac{1}{16a(ax-1)^4}-\frac{1}{16a(ax-1)^3}+\frac{1}{16a(ax-1)^2}-\frac{5}{64(ax-1)a}-\frac{3\ln(ax-1)}{64a}}{c^4}$
norman	$\frac{-\frac{a^3x^4}{2c}-\frac{29x}{32c}-\frac{3a\,x^2}{16c}+\frac{59a^2x^3}{32c}-\frac{263a^4x^5}{160c}+\frac{63a^5x^6}{80c}+\frac{81a^6x^7}{160c}-\frac{3a^7x^8}{10c}}{c^3(ax+1)^3(ax-1)^5}-\frac{3\ln(ax-1)}{64a\,c^4}+\frac{3\ln(ax+1)}{64a\,c^4}$
parallelrisch	$\frac{-160a^3x^3-290ax-360a^4x^4+354a^5x^5-96x^6a^6+60\ln(ax+1)xa-15\ln(ax+1)+520a^2x^2+15\ln(ax-1)-75\ln(ax+1)x^2a^2-}{3}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(-3/32*x^5*a^4+3/8*a^3*x^4-1/2*a^2*x^3+1/8*a*x^2+47/160*x-3/10/a)/c^4/(a*x
-1)^4/(a^2*x^2-1)-3/64/a/c^4*ln(a*x-1)+3/64/a/c^4*ln(-a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.57

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{30\,a^5x^5-120\,a^4x^4+160\,a^3x^3-40\,a^2x^2-94\,ax-15\,(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2+4\,ax-1)\,\mathrm{lo}}{320\,(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output
```
-1/320*(30*a^5*x^5 - 120*a^4*x^4 + 160*a^3*x^3 - 40*a^2*x^2 - 94*a*x - 15*
(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x + 1) + 1
5*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x - 1) +
 96)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*
c^4*x - a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.06

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{-15a^5x^5+60a^4x^4-80a^3x^3+20a^2x^2+47ax-48}{160a^7c^4x^6-640a^6c^4x^5+800a^5c^4x^4-800a^3c^4x^2+640a^2c^4x-160ac^4} + \frac{-\frac{3\log\left(x-\frac{1}{a}\right)}{64}+\frac{3\log\left(x+\frac{1}{a}\right)}{64}}{ac^4}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**4,x)
```

output
```
(-15*a**5*x**5 + 60*a**4*x**4 - 80*a**3*x**3 + 20*a**2*x**2 + 47*a*x - 48)
/(160*a**7*c**4*x**6 - 640*a**6*c**4*x**5 + 800*a**5*c**4*x**4 - 800*a**3*
c**4*x**2 + 640*a**2*c**4*x - 160*a*c**4) + (-3*log(x - 1/a)/64 + 3*log(x
+ 1/a)/64)/(a*c**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{15\,a^5x^5-60\,a^4x^4+80\,a^3x^3-20\,a^2x^2-47\,ax+48}{160\,(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4)} + \frac{3\,\log(ax+1)}{64\,ac^4} - \frac{3\,\log(ax-1)}{64\,ac^4}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
-1/160*(15*a^5*x^5 - 60*a^4*x^4 + 80*a^3*x^3 - 20*a^2*x^2 - 47*a*x + 48)/(
a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x
- a*c^4) + 3/64*log(a*x + 1)/(a*c^4) - 3/64*log(a*x - 1)/(a*c^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{3\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{64\,ac^4} + \frac{1}{128\,ac^4\left(\frac{2}{ax-1}+1\right)} - \frac{\frac{25\,a^9c^{16}}{ax-1} - \frac{20\,a^9c^{16}}{(ax-1)^2} + \frac{20\,a^9c^{16}}{(ax-1)^3} - \frac{20\,a^9c^{16}}{(ax-1)^4} + \frac{16\,a^9c^{16}}{(ax-1)^5}}{320\,a^{10}c^{20}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
3/64*log(abs(-2/(a*x - 1) - 1))/(a*c^4) + 1/128/(a*c^4*(2/(a*x - 1) + 1))
- 1/320*(25*a^9*c^16/(a*x - 1) - 20*a^9*c^16/(a*x - 1)^2 + 20*a^9*c^16/(a*
x - 1)^3 - 20*a^9*c^16/(a*x - 1)^4 + 16*a^9*c^16/(a*x - 1)^5)/(a^10*c^20)
```

Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{3\,\mathrm{atanh}(a\,x)}{32\,a\,c^4} - \frac{\frac{47\,x}{160} + \frac{a\,x^2}{8} - \frac{3}{10\,a} - \frac{a^2\,x^3}{2} + \frac{3\,a^3\,x^4}{8} - \frac{3\,a^4\,x^5}{32}}{-a^6\,c^4\,x^6 + 4\,a^5\,c^4\,x^5 - 5\,a^4\,c^4\,x^4 + 5\,a^2\,c^4\,x^2 - 4\,a\,c^4\,x + c^4}$$

input
```
int((a*x + 1)^2/((c - a^2*c*x^2)^4*(a*x - 1)^2),x)
```

output
```
(3*atanh(a*x))/(32*a*c^4) - ((47*x)/160 + (a*x^2)/8 - 3/(10*a) - (a^2*x^3)
/2 + (3*a^3*x^4)/8 - (3*a^4*x^5)/32)/(c^4 + 5*a^2*c^4*x^2 - 5*a^4*c^4*x^4
+ 4*a^5*c^4*x^5 - a^6*c^4*x^6 - 4*a*c^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{-30\log(ax-1)\,a^6x^6+120\log(ax-1)\,a^5x^5-150\log(ax-1)\,a^4x^4+150\log(ax-1)\,a^2x^2-120\log(a}{}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x)
```

output
```
( - 30*log(a*x - 1)*a**6*x**6 + 120*log(a*x - 1)*a**5*x**5 - 150*log(a*x -
 1)*a**4*x**4 + 150*log(a*x - 1)*a**2*x**2 - 120*log(a*x - 1)*a*x + 30*log
(a*x - 1) + 30*log(a*x + 1)*a**6*x**6 - 120*log(a*x + 1)*a**5*x**5 + 150*l
og(a*x + 1)*a**4*x**4 - 150*log(a*x + 1)*a**2*x**2 + 120*log(a*x + 1)*a*x
- 30*log(a*x + 1) - 15*a**6*x**6 + 165*a**4*x**4 - 320*a**3*x**3 + 155*a**
2*x**2 + 128*a*x - 177)/(640*a*c**4*(a**6*x**6 - 4*a**5*x**5 + 5*a**4*x**4
 - 5*a**2*x**2 + 4*a*x - 1))
```

3.588 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{4c^4(1-ax)^6}{3a}-\frac{12c^4(1-ax)^7}{7a}+\frac{3c^4(1-ax)^8}{4a}-\frac{c^4(1-ax)^9}{9a}$$

output
```
4/3*c^4*(-a*x+1)^6/a-12/7*c^4*(-a*x+1)^7/a+3/4*c^4*(-a*x+1)^8/a-1/9*c^4*(-
a*x+1)^9/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.53

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{c^4(-1+ax)^6\left(65+138ax+105a^2x^2+28a^3x^3\right)}{252a}$$

input
```
Integrate[(c - a^2*c*x^2)^4/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^4*(-1 + a*x)^6*(65 + 138*a*x + 105*a^2*x^2 + 28*a^3*x^3))/(252*a)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-a^2cx^2)^4 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int c^4 e^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)^4dx$$

$$\Big\downarrow \text{27}$$

$$-c^4\int e^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)^4dx$$

$$\Big\downarrow \text{6690}$$

$$-c^4\int (1-ax)^5(ax+1)^3dx$$

$$\Big\downarrow \text{49}$$

$$-c^4\int \left(-(1-ax)^8+6(1-ax)^7-12(1-ax)^6+8(1-ax)^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-c^4\left(\frac{(1-ax)^9}{9a}-\frac{3(1-ax)^8}{4a}+\frac{12(1-ax)^7}{7a}-\frac{4(1-ax)^6}{3a}\right)$$

input
```
Int[(c - a^2*c*x^2)^4/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c^4*((-4*(1 - a*x)^6)/(3*a) + (12*(1 - a*x)^7)/(7*a) - (3*(1 - a*x)^8)/(
4*a) + (1 - a*x)^9/(9*a)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

method	result
gosper	$\frac{c^{4} x \left(28 a^{8} x^{8}-63 a^{7} x^{7}-72 x^{6} a^{6}+252 a^{5} x^{5}-378 a^{3} x^{3}+168 a^{2} x^{2}+252 a x -252\right)}{252}$
default	$c^{4} \left(\frac{1}{9} a^{8} x^{9}-\frac{1}{4} a^{7} x^{8}-\frac{2}{7} a^{6} x^{7}+a^{5} x^{6}-\frac{3}{2} a^{3} x^{4}+\frac{2}{3} a^{2} x^{3}+a \,x^{2}-x \right)$
norman	$a \,c^{4} x^{2}+a^{5} c^{4} x^{6}-c^{4} x +\frac{2}{3} a^{2} c^{4} x^{3}-\frac{3}{2} a^{3} c^{4} x^{4}-\frac{2}{7} a^{6} c^{4} x^{7}-\frac{1}{4} a^{7} c^{4} x^{8}+\frac{1}{9} a^{8} c^{4} x^{9}$
risch	$a \,c^{4} x^{2}+a^{5} c^{4} x^{6}-c^{4} x +\frac{2}{3} a^{2} c^{4} x^{3}-\frac{3}{2} a^{3} c^{4} x^{4}-\frac{2}{7} a^{6} c^{4} x^{7}-\frac{1}{4} a^{7} c^{4} x^{8}+\frac{1}{9} a^{8} c^{4} x^{9}$
parallelrisch	$a \,c^{4} x^{2}+a^{5} c^{4} x^{6}-c^{4} x +\frac{2}{3} a^{2} c^{4} x^{3}-\frac{3}{2} a^{3} c^{4} x^{4}-\frac{2}{7} a^{6} c^{4} x^{7}-\frac{1}{4} a^{7} c^{4} x^{8}+\frac{1}{9} a^{8} c^{4} x^{9}$
orering	$\frac{x \left(28 a^{8} x^{8}-63 a^{7} x^{7}-72 x^{6} a^{6}+252 a^{5} x^{5}-378 a^{3} x^{3}+168 a^{2} x^{2}+252 a x -252\right) \left(-a^{2} c \,x^{2}+c \right)^{4}}{252 \left(a x -1\right)^{4} \left(a x +1\right)^{4}}$
meijerg	$\frac{c^{4} \left(\frac{a x \left(280 a^{8} x^{8}-315 a^{7} x^{7}+360 x^{6} a^{6}-420 a^{5} x^{5}+504 a^{4} x^{4}-630 a^{3} x^{3}+840 a^{2} x^{2}-1260 a x +2520\right)}{2520}-\ln \left(a x +1\right)\right)}{a}-\frac{c^{4} \left(-\frac{a x \left(-315 a^{7} x^{7}+\right.}{}\right.}{}$

input
```
int((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/252*c^4*x*(28*a^8*x^8-63*a^7*x^7-72*a^6*x^6+252*a^5*x^5-378*a^3*x^3+168*
a^2*x^2+252*a*x-252)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9-\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7+a^5c^4x^6-\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3+ac^4x^2-c^4x$$

input
```
integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^
3*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8c^4x^9}{9}-\frac{a^7c^4x^8}{4}-\frac{2a^6c^4x^7}{7}+a^5c^4x^6-\frac{3a^3c^4x^4}{2}+\frac{2a^2c^4x^3}{3}+ac^4x^2-c^4x$$

input
```
integrate((-a**2*c*x**2+c)**4*(a*x-1)/(a*x+1),x)
```

output
```
a**8*c**4*x**9/9 - a**7*c**4*x**8/4 - 2*a**6*c**4*x**7/7 + a**5*c**4*x**6
- 3*a**3*c**4*x**4/2 + 2*a**2*c**4*x**3/3 + a*c**4*x**2 - c**4*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9-\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7+a^5c^4x^6-\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3+ac^4x^2-c^4x$$

input
```
integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^
3*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{1}{9}\,a^8c^4x^9-\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7+a^5c^4x^6-\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3+ac^4x^2-c^4x$$

input
```
integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^
3*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\frac{a^8\,c^4\,x^9}{9}-\frac{a^7\,c^4\,x^8}{4}-\frac{2\,a^6\,c^4\,x^7}{7}+a^5\,c^4\,x^6-\frac{3\,a^3\,c^4\,x^4}{2}+\frac{2\,a^2\,c^4\,x^3}{3}+a\,c^4\,x^2-c^4\,x$$

input
```
int(((c - a^2*c*x^2)^4*(a*x - 1))/(a*x + 1),x)
```

output
```
a*c^4*x^2 - c^4*x + (2*a^2*c^4*x^3)/3 - (3*a^3*c^4*x^4)/2 + a^5*c^4*x^6 -
(2*a^6*c^4*x^7)/7 - (a^7*c^4*x^8)/4 + (a^8*c^4*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$=\frac{c^4x(28a^8x^8-63a^7x^7-72a^6x^6+252a^5x^5-378a^3x^3+168a^2x^2+252ax-252)}{252}$$

input
```
int((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x)
```

output
```
(c**4*x*(28*a**8*x**8 - 63*a**7*x**7 - 72*a**6*x**6 + 252*a**5*x**5 - 378*
a**3*x**3 + 168*a**2*x**2 + 252*a*x - 252))/252
```

3.589 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\frac{4c^3(1-ax)^5}{5a}-\frac{2c^3(1-ax)^6}{3a}+\frac{c^3(1-ax)^7}{7a}$$

output
```
4/5*c^3*(-a*x+1)^5/a-2/3*c^3*(-a*x+1)^6/a+1/7*c^3*(-a*x+1)^7/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{c^3(-1+ax)^5\,(29+40ax+15a^2x^2)}{105a}$$

input
```
Integrate[(c - a^2*c*x^2)^3/E^(2*ArcCoth[a*x]),x]
```

output
```
-1/105*(c^3*(-1 + a*x)^5*(29 + 40*a*x + 15*a^2*x^2))/a
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^3 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int c^3e^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)^3dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^3\int e^{-2\text{arctanh}(ax)}\left(1-a^2x^2\right)^3dx$$

$$\Big\downarrow \text{ 6690}$$

$$-c^3\int (1-ax)^4(ax+1)^2dx$$

$$\Big\downarrow \text{ 49}$$

$$-c^3\int \left((1-ax)^6-4(1-ax)^5+4(1-ax)^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-c^3\left(-\frac{(1-ax)^7}{7a}+\frac{2(1-ax)^6}{3a}-\frac{4(1-ax)^5}{5a}\right)$$

input

```
Int[(c - a^2*c*x^2)^3/E^(2*ArcCoth[a*x]),x]
```

output

```
-(c^3*((-4*(1 - a*x)^5)/(5*a) + (2*(1 - a*x)^6)/(3*a) - (1 - a*x)^7/(7*a))
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

method	result
gosper	$-\frac{c^{3} x \left(15 x^{6} a^{6}-35 a^{5} x^{5}-21 a^{4} x^{4}+105 a^{3} x^{3}-35 a^{2} x^{2}-105 a x +105\right)}{105}$
default	$c^{3} \left(-\frac{1}{7} a^{6} x^{7}+\frac{1}{3} a^{5} x^{6}+\frac{1}{5} x^{5} a^{4}-a^{3} x^{4}+\frac{1}{3} a^{2} x^{3}+a \,x^{2}-x \right)$
norman	$a \,c^{3} x^{2}-c^{3} x +\frac{1}{3} a^{2} c^{3} x^{3}-a^{3} c^{3} x^{4}+\frac{1}{3} a^{5} c^{3} x^{6}-\frac{1}{7} a^{6} c^{3} x^{7}+\frac{1}{5} c^{3} a^{4} x^{5}$
risch	$a \,c^{3} x^{2}-c^{3} x +\frac{1}{3} a^{2} c^{3} x^{3}-a^{3} c^{3} x^{4}+\frac{1}{3} a^{5} c^{3} x^{6}-\frac{1}{7} a^{6} c^{3} x^{7}+\frac{1}{5} c^{3} a^{4} x^{5}$
parallelrisch	$a \,c^{3} x^{2}-c^{3} x +\frac{1}{3} a^{2} c^{3} x^{3}-a^{3} c^{3} x^{4}+\frac{1}{3} a^{5} c^{3} x^{6}-\frac{1}{7} a^{6} c^{3} x^{7}+\frac{1}{5} c^{3} a^{4} x^{5}$
orering	$\frac{x \left(15 x^{6} a^{6}-35 a^{5} x^{5}-21 a^{4} x^{4}+105 a^{3} x^{3}-35 a^{2} x^{2}-105 a x +105\right) \left(-a^{2} c \,x^{2}+c \right)^{3}}{105 \left(a x -1\right)^{3} \left(a x +1\right)^{3}}$
meijerg	$-\frac{c^{3} \left(\frac{a x \left(120 x^{6} a^{6}-140 a^{5} x^{5}+168 a^{4} x^{4}-210 a^{3} x^{3}+280 a^{2} x^{2}-420 a x +840\right)}{840}-\ln \left(a x +1\right)\right)}{a}+\frac{c^{3} \left(-\frac{a x \left(-70 a^{5} x^{5}+84 a^{4} x^{4}-105 a^{3} x^{3}+1\right.}{420}\right.}{a}$

input
```
int((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/105*c^3*x*(15*a^6*x^6-35*a^5*x^5-21*a^4*x^4+105*a^3*x^3-35*a^2*x^2-105*
a*x+105)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7+\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5-a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+ac^3x^2-c^3x$$

input
```
integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
-1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a
^2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{a^6c^3x^7}{7}+\frac{a^5c^3x^6}{3}+\frac{a^4c^3x^5}{5}-a^3c^3x^4+\frac{a^2c^3x^3}{3}+ac^3x^2-c^3x$$

input
```
integrate((-a**2*c*x**2+c)**3*(a*x-1)/(a*x+1),x)
```

output
```
-a**6*c**3*x**7/7 + a**5*c**3*x**6/3 + a**4*c**3*x**5/5 - a**3*c**3*x**4 +
 a**2*c**3*x**3/3 + a*c**3*x**2 - c**3*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7+\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5-a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+ac^3x^2-c^3x$$

input
```
integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a
^2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{1}{7}\,a^6c^3x^7+\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5-a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+ac^3x^2-c^3x$$

input
```
integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a
^2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\frac{a^6\,c^3\,x^7}{7}+\frac{a^5\,c^3\,x^6}{3}+\frac{a^4\,c^3\,x^5}{5}-a^3\,c^3\,x^4+\frac{a^2\,c^3\,x^3}{3}+a\,c^3\,x^2-c^3\,x$$

input
```
int(((c - a^2*c*x^2)^3*(a*x - 1))/(a*x + 1),x)
```

output
```
a*c^3*x^2 - c^3*x + (a^2*c^3*x^3)/3 - a^3*c^3*x^4 + (a^4*c^3*x^5)/5 + (a^5
*c^3*x^6)/3 - (a^6*c^3*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = \frac{c^3x(-15a^6x^6+35a^5x^5+21a^4x^4-105a^3x^3+35a^2x^2+105ax-105)}{105}$$

input
```
int((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x)
```

output
```
(c**3*x*( - 15*a**6*x**6 + 35*a**5*x**5 + 21*a**4*x**4 - 105*a**3*x**3 + 3
5*a**2*x**2 + 105*a*x - 105))/105
```

3.590 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 37

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{c^2(1-ax)^4}{2a}-\frac{c^2(1-ax)^5}{5a}$$

output
```
1/2*c^2*(-a*x+1)^4/a-1/5*c^2*(-a*x+1)^5/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{10}c^2x\left(-10+10ax-5a^3x^3+2a^4x^4\right)$$

input
```
Integrate[(c - a^2*c*x^2)^2/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^2*x*(-10 + 10*a*x - 5*a^3*x^3 + 2*a^4*x^4))/10
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6717, 27, 6690, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-a^2cx^2)^2 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int c^2e^{-2\text{arctanh}(ax)}(1-a^2x^2)^2\,dx$$

$$\Big\downarrow \text{ 27}$$

$$-c^2\int e^{-2\text{arctanh}(ax)}(1-a^2x^2)^2\,dx$$

$$\Big\downarrow \text{ 6690}$$

$$-c^2\int (1-ax)^3(ax+1)dx$$

$$\Big\downarrow \text{ 49}$$

$$-c^2\int \left(2(1-ax)^3-(1-ax)^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-c^2\left(\frac{(1-ax)^5}{5a}-\frac{(1-ax)^4}{2a}\right)$$

input
```
Int[(c - a^2*c*x^2)^2/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c^2*(-1/2*(1 - a*x)^4/a + (1 - a*x)^5/(5*a)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result
gosper	$\frac{c^{2} x \left(2 a^{4} x^{4}-5 a^{3} x^{3}+10 a x -10\right)}{10}$
default	$c^{2} \left(\frac{1}{5} x^{5} a^{4}-\frac{1}{2} a^{3} x^{4}+a \,x^{2}-x \right)$
norman	$a \,c^{2} x^{2}-c^{2} x -\frac{1}{2} a^{3} c^{2} x^{4}+\frac{1}{5} a^{4} c^{2} x^{5}$
risch	$a \,c^{2} x^{2}-c^{2} x -\frac{1}{2} a^{3} c^{2} x^{4}+\frac{1}{5} a^{4} c^{2} x^{5}$
parallelrisch	$a \,c^{2} x^{2}-c^{2} x -\frac{1}{2} a^{3} c^{2} x^{4}+\frac{1}{5} a^{4} c^{2} x^{5}$
orering	$\frac{x \left(2 a^{4} x^{4}-5 a^{3} x^{3}+10 a x -10\right) \left(-a^{2} c \,x^{2}+c \right)^{2}}{10 \left(a x +1\right)^{2} \left(a x -1\right)^{2}}$
meijerg	$\frac{c^{2} \left(\frac{x a \left(12 a^{4} x^{4}-15 a^{3} x^{3}+20 a^{2} x^{2}-30 a x +60\right)}{60}-\ln \left(a x +1\right)\right)}{a}-\frac{c^{2} \left(-\frac{a x \left(-15 a^{3} x^{3}+20 a^{2} x^{2}-30 a x +60\right)}{60}+\ln \left(a x +1\right)\right)}{a}-\frac{2 c^{2} \left(\frac{a x (\cdots}{}\right.}{}$

input
```
int((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/10*c^2*x*(2*a^4*x^4-5*a^3*x^3+10*a*x-10)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5-\frac{1}{2}\,a^3c^2x^4+ac^2x^2-c^2x$$

input
```
integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{a^4c^2x^5}{5}-\frac{a^3c^2x^4}{2}+ac^2x^2-c^2x$$

input
```
integrate((-a**2*c*x**2+c)**2*(a*x-1)/(a*x+1),x)
```

output
```
a**4*c**2*x**5/5 - a**3*c**2*x**4/2 + a*c**2*x**2 - c**2*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5-\frac{1}{2}\,a^3c^2x^4+ac^2x^2-c^2x$$

input
```
integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{1}{5}\,a^4c^2x^5-\frac{1}{2}\,a^3c^2x^4+ac^2x^2-c^2x$$

input
```
integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{a^4\,c^2\,x^5}{5}-\frac{a^3\,c^2\,x^4}{2}+a\,c^2\,x^2-c^2\,x$$

input
```
int(((c - a^2*c*x^2)^2*(a*x - 1))/(a*x + 1),x)
```

output
```
a*c^2*x^2 - c^2*x - (a^3*c^2*x^4)/2 + (a^4*c^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\frac{c^2x(2a^4x^4-5a^3x^3+10ax-10)}{10}$$

input
```
int((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x)
```

output
```
(c**2*x*(2*a**4*x**4 - 5*a**3*x**3 + 10*a*x - 10))/10
```

3.591 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 16

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\frac{c(1-ax)^3}{3a}$$

output
```
1/3*c*(-a*x+1)^3/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-c\left(x-ax^2+\frac{a^2x^3}{3}\right)$$

input
```
Integrate[(c - a^2*c*x^2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(x - a*x^2 + (a^2*x^3)/3))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6717, 27, 6690, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-a^2cx^2)\, e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int ce^{-2\text{arctanh}(ax)}(1-a^2x^2)\,dx$$

$$\Big\downarrow \text{27}$$

$$-c\int e^{-2\text{arctanh}(ax)}(1-a^2x^2)\,dx$$

$$\Big\downarrow \text{6690}$$

$$-c\int (1-ax)^2dx$$

$$\Big\downarrow \text{17}$$

$$\frac{c(1-ax)^3}{3a}$$

input
```
Int[(c - a^2*c*x^2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c*(1 - a*x)^3)/(3*a)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{c(ax-1)^3}{3a}$	14
gosper	$-\frac{cx(a^2x^2-3ax+3)}{3}$	18
norman	$ac\,x^2-xc-\frac{1}{3}a^2c\,x^3$	21
parallelrisch	$ac\,x^2-xc-\frac{1}{3}a^2c\,x^3$	21
risch	$-\frac{a^2c\,x^3}{3}+ac\,x^2-xc+\frac{c}{3a}$	27
orering	$\frac{x(a^2x^2-3ax+3)(-a^2c\,x^2+c)}{3(ax-1)(ax+1)}$	42
meijerg	$-\frac{c\left(\frac{ax\left(4a^2x^2-6ax+12\right)}{12}-\ln(ax+1)\right)}{a}+\frac{c\left(-\frac{ax(-3ax+6)}{6}+\ln(ax+1)\right)}{a}+\frac{c(ax-\ln(ax+1))}{a}-\frac{c\ln(ax+1)}{a}$	86

input
```
int((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*c*(a*x-1)^3/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3+acx^2-cx$$

input
```
integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
-1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{a^2cx^3}{3}+acx^2-cx$$

input
```
integrate((-a**2*c*x**2+c)*(a*x-1)/(a*x+1),x)
```

output
```
-a**2*c*x**3/3 + a*c*x**2 - c*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3+acx^2-cx$$

input
```
integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{1}{3}\,a^2cx^3+acx^2-cx$$

input
```
integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\frac{c\,x\,\left(a^2\,x^2-3\,a\,x+3\right)}{3}$$

input
```
int(((c - a^2*c*x^2)*(a*x - 1))/(a*x + 1),x)
```

output
```
-(c*x*(a^2*x^2 - 3*a*x + 3))/3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\frac{cx(-a^2x^2+3ax-3)}{3}$$

input
```
int((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x)
```

output
```
(c*x*( - a**2*x**2 + 3*a*x - 3))/3
```

3.592 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 14

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{1}{ac(1+ax)}$$

output
```
1/a/c/(a*x+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=-\frac{e^{-2\coth^{-1}(ax)}}{2ac}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

output
```
-1/2*1/(a*c*E^(2*ArcCoth[a*x]))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6717, 27, 6690, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}dx$$
$$\Big\downarrow \text{6717}$$
$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c(1-a^2x^2)}dx$$
$$\Big\downarrow \text{27}$$
$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{1-a^2x^2}dx}{c}$$
$$\Big\downarrow \text{6690}$$
$$-\frac{\int \frac{1}{(ax+1)^2}dx}{c}$$
$$\Big\downarrow \text{17}$$
$$\frac{1}{ac(ax+1)}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

output
```
1/(a*c*(1 + a*x))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
norman	$-\frac{x}{c(ax+1)}$	14
parallelrisch	$-\frac{x}{c(ax+1)}$	14
gosper	$\frac{1}{ac(ax+1)}$	15
default	$\frac{1}{ac(ax+1)}$	15
risch	$\frac{1}{ac(ax+1)}$	15
orering	$-\frac{ax-1}{a(-a^2c\,x^2+c)}$	24

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/c*x/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{a^2cx+ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/(a^2*c*x + a*c)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{a^2cx+ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c),x)
```

output
```
1/(a**2*c*x + a*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{a^2cx+ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/(a^2*c*x + a*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{(ax+1)ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
1/((a*x + 1)*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{1}{a\,(c+a\,c\,x)}$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)*(a*x + 1)),x)
```

output
```
1/(a*(c + a*c*x))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{x}{c\,(ax+1)}$$

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x)
```

output
```
( - x)/(c*(a*x + 1))
```

3.593 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 49

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{1}{4ac^2(1+ax)^2}+\frac{1}{4ac^2(1+ax)}-\frac{\text{arctanh}(ax)}{4ac^2}$$

output
```
1/4/a/c^2/(a*x+1)^2+1/4/a/c^2/(a*x+1)-1/4*arctanh(a*x)/a/c^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{2+ax-(1+ax)^2\text{arctanh}(ax)}{4a(c+acx)^2}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

output
```
(2 + a*x - (1 + a*x)^2*ArcTanh[a*x])/(4*a*(c + a*c*x)^2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c^2(1-a^2x^2)^2}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{(1-a^2x^2)^2}dx}{c^2}$$

$$\Big\downarrow \text{6690}$$

$$-\frac{\int \frac{1}{(1-ax)(ax+1)^3}dx}{c^2}$$

$$\Big\downarrow \text{54}$$

$$-\frac{\int \left(\frac{1}{4(ax+1)^2}+\frac{1}{2(ax+1)^3}-\frac{1}{4(a^2x^2-1)}\right)dx}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\text{arctanh}(ax)}{4a}-\frac{1}{4a(ax+1)}-\frac{1}{4a(ax+1)^2}}{c^2}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

output
```
-((-1/4*1/(a*(1 + a*x)^2) - 1/(4*a*(1 + a*x)) + ArcTanh[a*x]/(4*a))/c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

method	result	size
risch	$\frac{\frac{x}{4}+\frac{1}{2a}}{\left(ax+1\right)^{2} c^{2}}+\frac{\ln \left(-ax+1\right)}{8a\,c^{2}}-\frac{\ln \left(ax+1\right)}{8a\,c^{2}}$	51
default	$\frac{\frac{1}{4a\left(ax+1\right)^{2}}+\frac{1}{4a\left(ax+1\right)}-\frac{\ln \left(ax+1\right)}{8a}+\frac{\ln \left(ax-1\right)}{8a}}{c^{2}}$	52
norman	$\frac{-\frac{3}{4ac}+\frac{a\,x^{2}}{2c}+\frac{a^{2} x^{3}}{4c}}{c\left(ax+1\right)^{2} \left(ax-1\right)}+\frac{\ln \left(ax-1\right)}{8a\,c^{2}}-\frac{\ln \left(ax+1\right)}{8a\,c^{2}}$	77
parallelrisch	$\frac{a^{2} \ln \left(ax-1\right) x^{2}-\ln \left(ax+1\right) x^{2} a^{2}-4a^{2} x^{2}+2a \ln \left(ax-1\right) x-2\ln \left(ax+1\right) x a-6ax+\ln \left(ax-1\right)-\ln \left(ax+1\right)}{8c^{2} \left(ax+1\right)^{2} a}$	90

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*x+1/2/a)/(a*x+1)^2/c^2+1/8*ln(-a*x+1)/a/c^2-1/8*ln(a*x+1)/a/c^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.55

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{2\,ax-(a^2x^2+2\,ax+1)\log(ax+1)+(a^2x^2+2\,ax+1)\log(ax-1)+4}{8\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/8*(2*a*x - (a^2*x^2 + 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 + 2*a*x + 1)*lo
g(a*x - 1) + 4)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{ax+2}{4a^3c^2x^2+8a^2c^2x+4ac^2} + \frac{\frac{\log\left(x-\frac{1}{a}\right)}{8} - \frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**2,x)
```

output
```
(a*x + 2)/(4*a**3*c**2*x**2 + 8*a**2*c**2*x + 4*a*c**2) + (log(x - 1/a)/8
- log(x + 1/a)/8)/(a*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{ax+2}{4\,(a^3c^2x^2+2\,a^2c^2x+ac^2)} - \frac{\log(ax+1)}{8\,ac^2} + \frac{\log(ax-1)}{8\,ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/4*(a*x + 2)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2) - 1/8*log(a*x + 1)/(a*c^
2) + 1/8*log(a*x - 1)/(a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{\log(|ax+1|)}{8\,ac^2} + \frac{\log(|ax-1|)}{8\,ac^2} + \frac{ax+2}{4\,(ax+1)^2ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) + 1/4*(a*x
+ 2)/((a*x + 1)^2*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 13.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{\frac{x}{4}+\frac{1}{2\,a}}{a^2\,c^2\,x^2+2\,a\,c^2\,x+c^2} - \frac{\mathrm{atanh}(a\,x)}{4\,a\,c^2}$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)^2*(a*x + 1)),x)
```

output
```
(x/4 + 1/(2*a))/(c^2 + a^2*c^2*x^2 + 2*a*c^2*x) - atanh(a*x)/(4*a*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.92

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$$
$$= \frac{\log(ax-1)\,a^2x^2+2\log(ax-1)\,ax+\log(ax-1)-\log(ax+1)\,a^2x^2-2\log(ax+1)\,ax-\log(ax+1)}{8a\,c^2\,(a^2x^2+2ax+1)}$$

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x)
```

output
```
(log(a*x - 1)*a**2*x**2 + 2*log(a*x - 1)*a*x + log(a*x - 1) - log(a*x + 1)
*a**2*x**2 - 2*log(a*x + 1)*a*x - log(a*x + 1) - a**2*x**2 + 3)/(8*a*c**2*
(a**2*x**2 + 2*a*x + 1))
```

3.594 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{1}{16ac^3(1-ax)}+\frac{1}{12ac^3(1+ax)^3}+\frac{1}{8ac^3(1+ax)^2}+\frac{3}{16ac^3(1+ax)}-\frac{\operatorname{arctanh}(ax)}{4ac^3}$$

output
```
-1/16/a/c^3/(-a*x+1)+1/12/a/c^3/(a*x+1)^3+1/8/a/c^3/(a*x+1)^2+3/16/a/c^3/(
a*x+1)-1/4*arctanh(a*x)/a/c^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{-4+ax+6a^2x^2+3a^3x^3-3(-1+ax)(1+ax)^3\operatorname{arctanh}(ax)}{12a(-1+ax)(c+acx)^3}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

output
```
(-4 + a*x + 6*a^2*x^2 + 3*a^3*x^3 - 3*(-1 + a*x)*(1 + a*x)^3*ArcTanh[a*x])
/(12*a*(-1 + a*x)*(c + a*c*x)^3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c^3(1-a^2x^2)^3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{(1-a^2x^2)^3}dx}{c^3}$$

$$\Big\downarrow \text{6690}$$

$$-\frac{\int \frac{1}{(1-ax)^2(ax+1)^4}dx}{c^3}$$

$$\Big\downarrow \text{54}$$

$$-\frac{\int \left(\frac{1}{16(ax-1)^2}+\frac{3}{16(ax+1)^2}+\frac{1}{4(ax+1)^3}+\frac{1}{4(ax+1)^4}-\frac{1}{4(a^2x^2-1)}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\text{arctanh}(ax)}{4a}+\frac{1}{16a(1-ax)}-\frac{3}{16a(ax+1)}-\frac{1}{8a(ax+1)^2}-\frac{1}{12a(ax+1)^3}}{c^3}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

output
```
-((1/(16*a*(1 - a*x)) - 1/(12*a*(1 + a*x)^3) - 1/(8*a*(1 + a*x)^2) - 3/(16
*a*(1 + a*x)) + ArcTanh[a*x]/(4*a))/c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

method	result
default	$\frac{\frac{1}{12a(ax+1)^{3}}+\frac{1}{8a(ax+1)^{2}}+\frac{3}{16a(ax+1)}-\frac{\ln(ax+1)}{8a}+\frac{1}{16(ax-1)a}+\frac{\ln(ax-1)}{8a}}{c^{3}}$
risch	$\frac{\frac{a^{2}x^{3}}{4}+\frac{a\,x^{2}}{2}+\frac{x}{12}-\frac{1}{3a}}{(ax+1)^{2}(a^{2}x^{2}-1)c^{3}}-\frac{\ln(ax+1)}{8a\,c^{3}}+\frac{\ln(-ax+1)}{8a\,c^{3}}$
norman	$\frac{-\frac{3x}{4c}+\frac{a\,x^{2}}{4c}+\frac{11a^{2}x^{3}}{12c}-\frac{a^{3}x^{4}}{12c}-\frac{a^{4}x^{5}}{3c}}{(ax+1)^{3}c^{2}(ax-1)^{2}}+\frac{\ln(ax-1)}{8a\,c^{3}}-\frac{\ln(ax+1)}{8a\,c^{3}}$
parallelrisch	$\frac{3\ln(ax-1)x^{4}a^{4}-3\ln(ax+1)x^{4}a^{4}-8a^{4}x^{4}+6a^{3}\ln(ax-1)x^{3}-6\ln(ax+1)x^{3}a^{3}-10a^{3}x^{3}+12a^{2}x^{2}-6a\ln(ax-1)x+6\ln(ax+1)x}{24c^{3}(ax+1)^{2}(a^{2}x^{2}-1)a}$

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/12/a/(a*x+1)^3+1/8/a/(a*x+1)^2+3/16/a/(a*x+1)-1/8*ln(a*x+1)/a+1/1
6/(a*x-1)/a+1/8/a*ln(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.44

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{6\,a^3x^3+12\,a^2x^2+2\,ax-3\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log\left(ax+1\right)+3\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log\left(a\right.}{24\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/24*(6*a^3*x^3 + 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)
*log(a*x + 1) + 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x - 1) - 8)/(a^5
*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{-3a^3x^3-6a^2x^2-ax+4}{12a^5c^3x^4+24a^4c^3x^3-24a^2c^3x-12ac^3} - \frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**3,x)
```

output
```
-(-3*a**3*x**3 - 6*a**2*x**2 - a*x + 4)/(12*a**5*c**3*x**4 + 24*a**4*c**3*
x**3 - 24*a**2*c**3*x - 12*a*c**3) - (-log(x - 1/a)/8 + log(x + 1/a)/8)/(a
*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.08

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{3\,a^3x^3+6\,a^2x^2+ax-4}{12\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)} - \frac{\log\left(ax+1\right)}{8\,ac^3} + \frac{\log\left(ax-1\right)}{8\,ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/12*(3*a^3*x^3 + 6*a^2*x^2 + a*x - 4)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^
2*c^3*x - a*c^3) - 1/8*log(a*x + 1)/(a*c^3) + 1/8*log(a*x - 1)/(a*c^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\log\left(|ax+1|\right)}{8\,ac^3} + \frac{\log\left(|ax-1|\right)}{8\,ac^3} + \frac{3\,a^3x^3+6\,a^2x^2+ax-4}{12\,(ax+1)^3(ax-1)ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/8*log(abs(a*x + 1))/(a*c^3) + 1/8*log(abs(a*x - 1))/(a*c^3) + 1/12*(3*a
^3*x^3 + 6*a^2*x^2 + a*x - 4)/((a*x + 1)^3*(a*x - 1)*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{\frac{x}{12}+\frac{a\,x^2}{2}-\frac{1}{3\,a}+\frac{a^2\,x^3}{4}}{-a^4\,c^3\,x^4-2\,a^3\,c^3\,x^3+2\,a\,c^3\,x+c^3} - \frac{\mathrm{atanh}(a\,x)}{4\,a\,c^3}$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)^3*(a*x + 1)),x)
```

output
```
- (x/12 + (a*x^2)/2 - 1/(3*a) + (a^2*x^3)/4)/(c^3 - 2*a^3*c^3*x^3 - a^4*c^
3*x^4 + 2*a*c^3*x) - atanh(a*x)/(4*a*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.73

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$$
$$= \frac{3\log(ax-1)\,a^4x^4+6\log(ax-1)\,a^3x^3-6\log(ax-1)\,ax-3\log(ax-1)-3\log(ax+1)\,a^4x^4-6\log(}{24a\,c^3\left(a^4x^4+2a^3x^3-2ax-\right.}$$

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x)
```

output
```
(3*log(a*x - 1)*a**4*x**4 + 6*log(a*x - 1)*a**3*x**3 - 6*log(a*x - 1)*a*x
- 3*log(a*x - 1) - 3*log(a*x + 1)*a**4*x**4 - 6*log(a*x + 1)*a**3*x**3 + 6
*log(a*x + 1)*a*x + 3*log(a*x + 1) - 3*a**4*x**4 + 12*a**2*x**2 + 8*a*x -
5)/(24*a*c**3*(a**4*x**4 + 2*a**3*x**3 - 2*a*x - 1))
```

3.595 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{1}{64ac^4(1-ax)^2}-\frac{5}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}+\frac{1}{16ac^4(1+ax)^3}+\frac{3}{32ac^4(1+ax)^2}+\frac{5}{32ac^4(1+ax)}-\frac{15\text{arctanh}(ax)}{64ac^4}$$

output
```
-1/64/a/c^4/(-a*x+1)^2-5/64/a/c^4/(-a*x+1)+1/32/a/c^4/(a*x+1)^4+1/16/a/c^4
/(a*x+1)^3+3/32/a/c^4/(a*x+1)^2+5/32/a/c^4/(a*x+1)-15/64*arctanh(a*x)/a/c^
4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{16-17ax-50a^2x^2-10a^3x^3+30a^4x^4+15a^5x^5-15(-1+ax)^2(1+ax)^4\text{arctanh}(ax)}{64a(-1+ax)^2(c+acx)^4}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

output
```
(16 - 17*a*x - 50*a^2*x^2 - 10*a^3*x^3 + 30*a^4*x^4 + 15*a^5*x^5 - 15*(-1
+ a*x)^2*(1 + a*x)^4*ArcTanh[a*x])/(64*a*(-1 + a*x)^2*(c + a*c*x)^4)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6717, 27, 6690, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{c^4(1-a^2x^2)^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{e^{-2\text{arctanh}(ax)}}{(1-a^2x^2)^4}dx}{c^4}$$

$$\Big\downarrow \text{6690}$$

$$-\frac{\int \frac{1}{(1-ax)^3(ax+1)^5}dx}{c^4}$$

$$\Big\downarrow \text{54}$$

$$-\frac{\int \left(\frac{5}{64(ax-1)^2}+\frac{5}{32(ax+1)^2}-\frac{1}{32(ax-1)^3}+\frac{3}{16(ax+1)^3}+\frac{3}{16(ax+1)^4}+\frac{1}{8(ax+1)^5}-\frac{15}{64(a^2x^2-1)}\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{15\text{arctanh}(ax)}{64a}+\frac{5}{64a(1-ax)}-\frac{5}{32a(ax+1)}+\frac{1}{64a(1-ax)^2}-\frac{3}{32a(ax+1)^2}-\frac{1}{16a(ax+1)^3}-\frac{1}{32a(ax+1)^4}}{c^4}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

output
```
-((1/(64*a*(1 - a*x)^2) + 5/(64*a*(1 - a*x)) - 1/(32*a*(1 + a*x)^4) - 1/(1
6*a*(1 + a*x)^3) - 3/(32*a*(1 + a*x)^2) - 5/(32*a*(1 + a*x)) + (15*ArcTanh
[a*x])/(64*a))/c^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6690
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Simp[c^p   Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a
, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.77

method	result
risch	$\frac{\frac{15x^5a^4}{64}+\frac{15a^3x^4}{32}-\frac{5a^2x^3}{32}-\frac{25a\,x^2}{32}-\frac{17x}{64}+\frac{1}{4a}}{(ax+1)^2(a^2x^2-1)^2c^4}-\frac{15\ln(ax+1)}{128a\,c^4}+\frac{15\ln(-ax+1)}{128a\,c^4}$
default	$\frac{\frac{1}{32a(ax+1)^4}+\frac{1}{16a(ax+1)^3}+\frac{3}{32a(ax+1)^2}+\frac{5}{32a(ax+1)}-\frac{15\ln(ax+1)}{128a}-\frac{1}{64a(ax-1)^2}+\frac{5}{64(ax-1)a}+\frac{15\ln(ax-1)}{128a}}{c^4}$
norman	$\frac{\frac{49x}{64c}-\frac{15a\,x^2}{64c}-\frac{11a^2x^3}{8c}+\frac{a^3x^4}{8c}+\frac{63a^4x^5}{64c}-\frac{a^5x^6}{64c}-\frac{a^6x^7}{4c}}{c^3(ax-1)^3(ax+1)^4}+\frac{15\ln(ax-1)}{128a\,c^4}-\frac{15\ln(ax+1)}{128a\,c^4}$
parallelrisch	$108a^3x^3-98ax+92a^4x^4-34a^5x^5-32x^6a^6-30\ln(ax+1)xa-15\ln(ax+1)-68a^2x^2+15\ln(ax-1)+15\ln(ax+1)x^2a^2-15\ln(a$

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/64*x^5*a^4+15/32*a^3*x^4-5/32*a^2*x^3-25/32*a*x^2-17/64*x+1/4/a)/(a*x+
1)^2/(a^2*x^2-1)^2/c^4-15/128*ln(a*x+1)/a/c^4+15/128*ln(-a*x+1)/a/c^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(103) = 206.

Time = 0.10 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.82

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$
$$=\frac{30\,a^5x^5+60\,a^4x^4-20\,a^3x^3-100\,a^2x^2-34\,ax-15\left(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1\right)}{128\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3\right.}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output
```
1/128*(30*a^5*x^5 + 60*a^4*x^4 - 20*a^3*x^3 - 100*a^2*x^2 - 34*a*x - 15*(a
^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x +
1) + 15*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*
log(a*x - 1) + 32)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*
x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.18

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{15a^5x^5+30a^4x^4-10a^3x^3-50a^2x^2-17ax+16}{64a^7c^4x^6+128a^6c^4x^5-64a^5c^4x^4-256a^4c^4x^3-64a^3c^4x^2+128a^2c^4x+64ac^4} + \frac{\frac{15\log\left(x-\frac{1}{a}\right)}{128}-\frac{15\log\left(x+\frac{1}{a}\right)}{128}}{ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**4,x)
```

output
```
(15*a**5*x**5 + 30*a**4*x**4 - 10*a**3*x**3 - 50*a**2*x**2 - 17*a*x + 16)/
(64*a**7*c**4*x**6 + 128*a**6*c**4*x**5 - 64*a**5*c**4*x**4 - 256*a**4*c**
4*x**3 - 64*a**3*c**4*x**2 + 128*a**2*c**4*x + 64*a*c**4) + (15*log(x - 1/
a)/128 - 15*log(x + 1/a)/128)/(a*c**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.18

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{15\,a^5x^5+30\,a^4x^4-10\,a^3x^3-50\,a^2x^2-17\,ax+16}{64\,(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4)} - \frac{15\,\log(ax+1)}{128\,ac^4} + \frac{15\,\log(ax-1)}{128\,ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
1/64*(15*a^5*x^5 + 30*a^4*x^4 - 10*a^3*x^3 - 50*a^2*x^2 - 17*a*x + 16)/(a^
7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*
a^2*c^4*x + a*c^4) - 15/128*log(a*x + 1)/(a*c^4) + 15/128*log(a*x - 1)/(a*
c^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{15\,\log\left(|ax+1|\right)}{128\,ac^4}+\frac{15\,\log\left(|ax-1|\right)}{128\,ac^4} + \frac{15\,a^5x^5+30\,a^4x^4-10\,a^3x^3-50\,a^2x^2-17\,ax+16}{64\,(ax+1)^4(ax-1)^2ac^4}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
-15/128*log(abs(a*x + 1))/(a*c^4) + 15/128*log(abs(a*x - 1))/(a*c^4) + 1/6
4*(15*a^5*x^5 + 30*a^4*x^4 - 10*a^3*x^3 - 50*a^2*x^2 - 17*a*x + 16)/((a*x
+ 1)^4*(a*x - 1)^2*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 13.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{\frac{17\,x}{64}+\frac{25\,a\,x^2}{32}-\frac{1}{4\,a}+\frac{5\,a^2\,x^3}{32}-\frac{15\,a^3\,x^4}{32}-\frac{15\,a^4\,x^5}{64}}{a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5-a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3-a^2\,c^4\,x^2+2\,a\,c^4\,x+c^4} - \frac{15\,\mathrm{atanh}(a\,x)}{64\,a\,c^4}$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)^4*(a*x + 1)),x)
```

output
```
- ((17*x)/64 + (25*a*x^2)/32 - 1/(4*a) + (5*a^2*x^3)/32 - (15*a^3*x^4)/32
- (15*a^4*x^5)/64)/(c^4 - a^2*c^4*x^2 - 4*a^3*c^4*x^3 - a^4*c^4*x^4 + 2*a^
5*c^4*x^5 + a^6*c^4*x^6 + 2*a*c^4*x) - (15*atanh(a*x))/(64*a*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.26

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$
$$= \frac{15\log(ax-1)\,a^6x^6+30\log(ax-1)\,a^5x^5-15\log(ax-1)\,a^4x^4-60\log(ax-1)\,a^3x^3-15\log(ax-1)}{}$$

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x)
```

output
```
(15*log(a*x - 1)*a**6*x**6 + 30*log(a*x - 1)*a**5*x**5 - 15*log(a*x - 1)*a
**4*x**4 - 60*log(a*x - 1)*a**3*x**3 - 15*log(a*x - 1)*a**2*x**2 + 30*log(
a*x - 1)*a*x + 15*log(a*x - 1) - 15*log(a*x + 1)*a**6*x**6 - 30*log(a*x +
1)*a**5*x**5 + 15*log(a*x + 1)*a**4*x**4 + 60*log(a*x + 1)*a**3*x**3 + 15*
log(a*x + 1)*a**2*x**2 - 30*log(a*x + 1)*a*x - 15*log(a*x + 1) - 15*a**6*x
**6 + 75*a**4*x**4 + 40*a**3*x**3 - 85*a**2*x**2 - 64*a*x + 17)/(128*a*c**
4*(a**6*x**6 + 2*a**5*x**5 - a**4*x**4 - 4*a**3*x**3 - a**2*x**2 + 2*a*x +
 1))
```

3.596 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 229

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{8(1+ax)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{32(1+ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1+ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{8(1+ax)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1+ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

output
```
8/3*(a*x+1)^6*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-32/7*(a*x+
1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3*(a*x+1)^8*(-a^2*c
*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-8/9*(a*x+1)^9*(-a^2*c*x^2+c)^(9
/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(a*x+1)^10*(-a^2*c*x^2+c)^(9/2)/a^10
/(1-1/a^2/x^2)^(9/2)/x^9
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{c^4(1+ax)^6\sqrt{c-a^2cx^2}(193-528ax+588a^2x^2-308a^3x^3+63a^4x^4)}{630a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(9/2),x]
```

output
```
(c^4*(1 + a*x)^6*Sqrt[c - a^2*c*x^2]*(193 - 528*a*x + 588*a^2*x^2 - 308*a^
3*x^3 + 63*a^4*x^4))/(630*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{9/2}e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{9/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9dx}{x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{9/2}\int (1-ax)^4(ax+1)^5dx}{a^9x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\left(c-a^2cx^2\right)^{9/2}\int\left((ax+1)^9-8(ax+1)^8+24(ax+1)^7-32(ax+1)^6+16(ax+1)^5\right)dx}{a^9x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{(ax+1)^{10}}{10a}-\frac{8(ax+1)^9}{9a}+\frac{3(ax+1)^8}{a}-\frac{32(ax+1)^7}{7a}+\frac{8(ax+1)^6}{3a}\right)\left(c-a^2cx^2\right)^{9/2}}{a^9x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

input
```
Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(9/2),x]
```

output
```
((c - a^2*c*x^2)^(9/2)*((8*(1 + a*x)^6)/(3*a) - (32*(1 + a*x)^7)/(7*a) + (
3*(1 + a*x)^8)/a - (8*(1 + a*x)^9)/(9*a) + (1 + a*x)^10/(10*a)))/(a^9*(1 -
 1/(a^2*x^2))^(9/2)*x^9)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{(63a^9x^9+70a^8x^8-315a^7x^7-360x^6a^6+630a^5x^5+756a^4x^4-630a^3x^3-840a^2x^2+315ax+630)x\,c^4\sqrt{-c(a^2x^2-1)}}{630(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	113
gosper	$\frac{x(63a^9x^9+70a^8x^8-315a^7x^7-360x^6a^6+630a^5x^5+756a^4x^4-630a^3x^3-840a^2x^2+315ax+630)(-a^2c\,x^2+c)^{\frac{9}{2}}}{630(ax-1)^4(ax+1)^5\sqrt{\frac{ax-1}{ax+1}}}$	116
orering	$\frac{x(63a^9x^9+70a^8x^8-315a^7x^7-360x^6a^6+630a^5x^5+756a^4x^4-630a^3x^3-840a^2x^2+315ax+630)(-a^2c\,x^2+c)^{\frac{9}{2}}}{630(ax-1)^4(ax+1)^5\sqrt{\frac{ax-1}{ax+1}}}$	116

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE
)
```

output
```
1/630*(63*a^9*x^9+70*a^8*x^8-315*a^7*x^7-360*a^6*x^6+630*a^5*x^5+756*a^4*x
^4-630*a^3*x^3-840*a^2*x^2+315*a*x+630)*x*c^4*(-c*(a^2*x^2-1))^(1/2)/(a*x+
1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.51

$$\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx = \frac{(63\,a^9c^4x^{10}+70\,a^8c^4x^9-315\,a^7c^4x^8-360\,a^6c^4x^7+630\,a^5c^4x^6+756\,a^4c^4x^5-630\,a^3c^4x}{630\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fri
cas")
```

output
```
1/630*(63*a^9*c^4*x^10 + 70*a^8*c^4*x^9 - 315*a^7*c^4*x^8 - 360*a^6*c^4*x^
7 + 630*a^5*c^4*x^6 + 756*a^4*c^4*x^5 - 630*a^3*c^4*x^4 - 840*a^2*c^4*x^3
+ 315*a*c^4*x^2 + 630*c^4*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int\frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="max
ima")
```

output
```
integrate((-a^2*c*x^2 + c)^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,9,9,4,0]%%%}+%%%{1,[0,8,8,4,0]%%%}+%%%{-4,[0,7,7,4,0]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int \frac{\left(c-a^2\,c\,x^2\right)^{9/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\sqrt{c}\,c^4i(-63a^{10}x^{10}-70a^9x^9+315a^8x^8+360a^7x^7-630a^6x^6-756a^5x^5+630a^4x^4+840}{630a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x)
```

output
```
(sqrt(c)*c**4*i*( - 63*a**10*x**10 - 70*a**9*x**9 + 315*a**8*x**8 + 360*a*
*7*x**7 - 630*a**6*x**6 - 756*a**5*x**5 + 630*a**4*x**4 + 840*a**3*x**3 -
315*a**2*x**2 - 630*a*x + 319))/(630*a)
```

3.597 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 183

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{8(1+ax)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{2(1+ax)^6\left(c-a^2cx^2\right)^{7/2}}{a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{6(1+ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1+ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

output
```
-8/5*(a*x+1)^5*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+2*(a*x+1)^
6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-6/7*(a*x+1)^7*(-a^2*c*x
^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(a*x+1)^8*(-a^2*c*x^2+c)^(7/2)
/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{c^3(1+ax)^5\sqrt{c-a^2cx^2}\left(-93+185ax-135a^2x^2+35a^3x^3\right)}{280a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2),x]
```

output
```
-1/280*(c^3*(1 + a*x)^5*Sqrt[c - a^2*c*x^2]*(-93 + 185*a*x - 135*a^2*x^2 +
 35*a^3*x^3))/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.50, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{7/2} e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2} \int e^{\coth^{-1}(ax)} \left(1-\frac{1}{a^2x^2}\right)^{7/2} x^7dx}{x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2} \int -(1-ax)^3(ax+1)^4dx}{a^7x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(c-a^2cx^2\right)^{7/2} \int (1-ax)^3(ax+1)^4dx}{a^7x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\left(c-a^2cx^2\right)^{7/2} \int \left(-(ax+1)^7+6(ax+1)^6-12(ax+1)^5+8(ax+1)^4\right)dx}{a^7x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-\frac{(ax+1)^8}{8a}+\frac{6(ax+1)^7}{7a}-\frac{2(ax+1)^6}{a}+\frac{8(ax+1)^5}{5a}\right) \left(c-a^2cx^2\right)^{7/2}}{a^7x^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

input
```
Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2),x]
```

output
```
-(((c - a^2*c*x^2)^(7/2)*((8*(1 + a*x)^5)/(5*a) - (2*(1 + a*x)^6)/a + (6*(
1 + a*x)^7)/(7*a) - (1 + a*x)^8/(8*a)))/(a^7*(1 - 1/(a^2*x^2))^(7/2)*x^7))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{\left(35a^{7}x^{7}+40x^{6}a^{6}-140a^{5}x^{5}-168a^{4}x^{4}+210a^{3}x^{3}+280a^{2}x^{2}-140ax-280\right)x\,c^{3}\sqrt{-c\left(a^{2}x^{2}-1\right)}}{280\left(ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}$	97
gosper	$\frac{x\left(35a^{7}x^{7}+40x^{6}a^{6}-140a^{5}x^{5}-168a^{4}x^{4}+210a^{3}x^{3}+280a^{2}x^{2}-140ax-280\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{7}{2}}}{280\left(ax+1\right)^{4}\left(ax-1\right)^{3}\sqrt{\frac{ax-1}{ax+1}}}$	100
orering	$\frac{x\left(35a^{7}x^{7}+40x^{6}a^{6}-140a^{5}x^{5}-168a^{4}x^{4}+210a^{3}x^{3}+280a^{2}x^{2}-140ax-280\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{7}{2}}}{280\left(ax+1\right)^{4}\left(ax-1\right)^{3}\sqrt{\frac{ax-1}{ax+1}}}$	100

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/280*(35*a^7*x^7+40*a^6*x^6-140*a^5*x^5-168*a^4*x^4+210*a^3*x^3+280*a^2*
x^2-140*a*x-280)*x*c^3*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1
/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.52

$$\int e^{\coth^{-1}(ax)}\left(c-a^{2}cx^{2}\right)^{7/2}\,dx = -\frac{\left(35\,a^{7}c^{3}x^{8}+40\,a^{6}c^{3}x^{7}-140\,a^{5}c^{3}x^{6}-168\,a^{4}c^{3}x^{5}+210\,a^{3}c^{3}x^{4}+280\,a^{2}c^{3}x^{3}-140\,ac^{3}x^{2}-280\,c^{3}x\right)\sqrt{-a}}{280\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fri
cas")
```

output
```
-1/280*(35*a^7*c^3*x^8 + 40*a^6*c^3*x^7 - 140*a^5*c^3*x^6 - 168*a^4*c^3*x^
5 + 210*a^3*c^3*x^4 + 280*a^2*c^3*x^3 - 140*a*c^3*x^2 - 280*c^3*x)*sqrt(-a
^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int\frac{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="max
ima")
```

output
```
integrate((-a^2*c*x^2 + c)^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,7,7,3,0]%%%}+%%%{-1,[0,6,6,3,0]%%%}+%%%{3,[0,5,5,3,0
]%%%}+%%%
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int \frac{\left(c-a^2\,c\,x^2\right)^{7/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.40

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{\sqrt{c}\,c^3i(35a^8x^8+40a^7x^7-140a^6x^6-168a^5x^5+210a^4x^4+280a^3x^3-140a^2x^2-280ax+163)}{280a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x)
```

output
```
(sqrt(c)*c**3*i*(35*a**8*x**8 + 40*a**7*x**7 - 140*a**6*x**6 - 168*a**5*x*
*5 + 210*a**4*x**4 + 280*a**3*x**3 - 140*a**2*x**2 - 280*a*x + 163))/(280*
a)
```

3.598 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 136

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{(1+ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}-\frac{4(1+ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1+ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

output
```
(a*x+1)^4*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5-4/5*(a*x+1)^5*(
-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(a*x+1)^6*(-a^2*c*x^2+
c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2(1+ax)^4\left(11-14ax+5a^2x^2\right)\sqrt{c-a^2cx^2}}{30a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2),x]
```

output
```
(c^2*(1 + a*x)^4*(11 - 14*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sq
rt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{5/2}e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5dx}{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2}\int (1-ax)^2(ax+1)^3dx}{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2}\int \left((ax+1)^5-4(ax+1)^4+4(ax+1)^3\right)dx}{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{(ax+1)^6}{6a}-\frac{4(ax+1)^5}{5a}+\frac{(ax+1)^4}{a}\right)\left(c-a^2cx^2\right)^{5/2}}{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

input
```
Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2),x]
```

output
```
((c - a^2*c*x^2)^(5/2)*((1 + a*x)^4/a - (4*(1 + a*x)^5)/(5*a) + (1 + a*x)^
6/(6*a)))/(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\left(5 a^{5} x^{5}+6 a^{4} x^{4}-15 a^{3} x^{3}-20 a^{2} x^{2}+15 a x +30\right) x \,c^{2} \sqrt{-c \left(a^{2} x^{2}-1\right)}}{30 \left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}}$	81
gosper	$\frac{x \left(5 a^{5} x^{5}+6 a^{4} x^{4}-15 a^{3} x^{3}-20 a^{2} x^{2}+15 a x +30\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}{30 \left(a x +1\right)^{3} \left(a x -1\right)^{2} \sqrt{\frac{a x -1}{a x +1}}}$	84
orering	$\frac{x \left(5 a^{5} x^{5}+6 a^{4} x^{4}-15 a^{3} x^{3}-20 a^{2} x^{2}+15 a x +30\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}{30 \left(a x +1\right)^{3} \left(a x -1\right)^{2} \sqrt{\frac{a x -1}{a x +1}}}$	84

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
1/30*(5*a^5*x^5+6*a^4*x^4-15*a^3*x^3-20*a^2*x^2+15*a*x+30)*x*c^2*(-c*(a^2*
x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.54

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\left(5\,a^5c^2x^6+6\,a^4c^2x^5-15\,a^3c^2x^4-20\,a^2c^2x^3+15\,ac^2x^2+30\,c^2x\right)\sqrt{-a^2c}}{30\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

output
```
1/30*(5*a^5*c^2*x^6 + 6*a^4*c^2*x^5 - 15*a^3*c^2*x^4 - 20*a^2*c^2*x^3 + 15
*a*c^2*x^2 + 30*c^2*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

output
```
integrate((-a^2*c*x^2 + c)^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,5,5,2,0]%%%}+%%%{1,[0,4,4,2,0]%%%}+%%%{-2,[0,3,3,2,0]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int \frac{\left(c-a^2\,c\,x^2\right)^{5/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.42

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\sqrt{c}\,c^2i(-5a^6x^6-6a^5x^5+15a^4x^4+20a^3x^3-15a^2x^2-30ax+21)}{30a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*c**2*i*( - 5*a**6*x**6 - 6*a**5*x**5 + 15*a**4*x**4 + 20*a**3*x**
3 - 15*a**2*x**2 - 30*a*x + 21))/(30*a)
```

3.599 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 93

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{2(1+ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}+\frac{(1+ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

output
```
-2/3*(a*x+1)^3*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3+1/4*(a*x+1
)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.57

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{c(1+ax)^3(-5+3ax)\sqrt{c-a^2cx^2}}{12a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2),x]
```

output
```
-1/12*(c*(1 + a*x)^3*(-5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^
2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{3/2}e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3dx}{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2}\int -\left((1-ax)(ax+1)^2\right)dx}{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(c-a^2cx^2\right)^{3/2}\int (1-ax)(ax+1)^2dx}{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\left(c-a^2cx^2\right)^{3/2}\int \left(2(ax+1)^2-(ax+1)^3\right)dx}{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\frac{2(ax+1)^3}{3a}-\frac{(ax+1)^4}{4a}\right)\left(c-a^2cx^2\right)^{3/2}}{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2),x]
```

output
```
-(((c - a^2*c*x^2)^(3/2)*((2*(1 + a*x)^3)/(3*a) - (1 + a*x)^4/(4*a)))/(a^3
*(1 - 1/(a^2*x^2))^(3/2)*x^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{\left(3 a^{3} x^{3}+4 a^{2} x^{2}-6 a x -12\right) x c \sqrt{-c \left(a^{2} x^{2}-1\right)}}{12 \left(a x +1\right) \sqrt{\frac{a x -1}{a x +1}}}$	63
gosper	$\frac{x \left(3 a^{3} x^{3}+4 a^{2} x^{2}-6 a x -12\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{12 \left(a x +1\right)^{2} \left(a x -1\right) \sqrt{\frac{a x -1}{a x +1}}}$	68
orering	$\frac{x \left(3 a^{3} x^{3}+4 a^{2} x^{2}-6 a x -12\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{12 \left(a x +1\right)^{2} \left(a x -1\right) \sqrt{\frac{a x -1}{a x +1}}}$	68

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/12*(3*a^3*x^3+4*a^2*x^2-6*a*x-12)*x*c*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((
a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.46

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{\left(3\,a^3cx^4+4\,a^2cx^3-6\,acx^2-12\,cx\right)\sqrt{-a^2c}}{12\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

output
```
-1/12*(3*a^3*c*x^4 + 4*a^2*c*x^3 - 6*a*c*x^2 - 12*c*x)*sqrt(-a^2*c)/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\frac{\left(-c(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

output
```
integrate((-a^2*c*x^2 + c)^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,3,3,1,0]%%%}+%%%{-1,[0,2,2,1,0]%%%}+%%%{1,[0,1,1,1,0
]%%%}+%%%
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^{3/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{\sqrt{c}\,ci(3a^4x^4+4a^3x^3-6a^2x^2-12ax+11)}{12a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*c*i*(3*a**4*x**4 + 4*a**3*x**3 - 6*a**2*x**2 - 12*a*x + 11))/(12*
a)
```

3.600 $\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(1+ax)^2\sqrt{c-a^2cx^2}}{2a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
1/2*(a*x+1)^2*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(2+ax)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

output
```
((2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6746, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int (ax+1)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(ax+1)^2\sqrt{c-a^2cx^2}}{2a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

output
```
((1 + a*x)^2*Sqrt[c - a^2*c*x^2])/(2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{x(ax+2)\sqrt{-a^{2}c\,x^{2}+c}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	44
orering	$\frac{x(ax+2)\sqrt{-a^{2}c\,x^{2}+c}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	44
default	$\frac{(ax+2)x\sqrt{-c(a^{2}x^{2}-1)}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	45

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/2*x*(a*x+2)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{-a^2c}(ax^2+2\,x)}{2\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
1/2*sqrt(-a^2*c)*(a*x^2 + 2*x)/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{1}{2}\sqrt{-c}\left(\frac{ax^2+2x}{\operatorname{sgn}(ax+1)}+\frac{\operatorname{sgn}(ax+1)}{a}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
1/2*sqrt(-c)*((a*x^2 + 2*x)/sgn(a*x + 1) + sgn(a*x + 1)/a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c}\,i(-a^2x^2-2ax+3)}{2a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*i*( - a**2*x**2 - 2*a*x + 3))/(2*a)
```

3.601 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 38

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

output
```
(1-1/a^2/x^2)^(1/2)*x*ln(-a*x+1)/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^ArcCoth[a*x]/Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6746, 6747, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\int \frac{1}{ax-1}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

input
```
Int[E^ArcCoth[a*x]/Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{\ln(ax-1)\sqrt{-c(a^2x^2-1)}}{ca(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	51

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-ln(a*x-1)*(-c*(a^2*x^2-1))^(1/2)/c/a/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{\sqrt{-a^2c}\log(ax-1)}{a^2c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
-sqrt(-a^2*c)*log(a*x - 1)/(a^2*c)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)(ax+1)}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{1}{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(1/(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{1}{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.50

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2\sqrt{c}\log\left(\sqrt{-ax+1}\right)i}{ac}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(2*sqrt(c)*log(sqrt( - a*x + 1))*i)/(a*c)
```

3.602 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\,(c-a^2cx^2)^{3/2}} + \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\operatorname{arctanh}(ax)}{2\,(c-a^2cx^2)^{3/2}}$$

output
```
1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+1/2*a^2*(1-1
/a^2/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3(-1+(-1+ax)\operatorname{arctanh}(ax))}{(-2+2ax)\,(c-a^2cx^2)^{3/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-1 + (-1 + a*x)*ArcTanh[a*x]))/((-2 + 2*
a*x)*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6746, 6747, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{1}{(1-ax)^2(ax+1)}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{54}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(\frac{1}{2(ax-1)^2}-\frac{1}{2(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\text{arctanh}(ax)}{2a}+\frac{1}{2a(1-ax)}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a)))
/(c - a^2*c*x^2)^(3/2)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(\ln \left(a x +1\right) x a -a \ln \left(a x -1\right) x -\ln \left(a x +1\right)+\ln \left(a x -1\right)-2\right)}{4 \sqrt{\frac{a x -1}{a x +1}}\, \left(a^{2} x^{2}-1\right) c^{2} a}$	84

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*x*a-a*ln(a*x
-1)*x-ln(a*x+1)+ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{(a^2x-a)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\sqrt{-a^2c}}{4\left(a^3c^2x-a^2c^2\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

output
```
-1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)
/(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^3*c^2*x - a^2*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(1/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(c-a^2\,c\,x^2)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,i\left(-\log\left(\sqrt{-ax+1}-\sqrt{2}\right)ax+\log\left(\sqrt{-ax+1}-\sqrt{2}\right)-\log\left(\sqrt{-ax+1}+\sqrt{2}\right)a\right.}{4a\,c^2\,(ax-}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

output

```
(sqrt(c)*i*( - log(sqrt( - a*x + 1) - sqrt(2))*a*x + log(sqrt( - a*x + 1)
- sqrt(2)) - log(sqrt( - a*x + 1) + sqrt(2))*a*x + log(sqrt( - a*x + 1) +
sqrt(2)) + 2*log(sqrt( - a*x + 1))*a*x - 2*log(sqrt( - a*x + 1)) + 2))/(4*
a*c**2*(a*x - 1))
```

3.603 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 184

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(c-a^2cx^2)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)(c-a^2cx^2)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)(c-a^2cx^2)^{5/2}} - \frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8(c-a^2cx^2)^{5/2}}$$

output
```
-1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/4*a^4*(
1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)
^(5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-3/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*ar
ctanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(2+3ax-3a^2x^2+3(-1+ax)^2(1+ax)\operatorname{arctanh}(ax))}{8c^2(-1+ax)^2(1+ax)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-1/8*(Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^2*(1 +
 a*x)*ArcTanh[a*x]))/(c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6746, 6747, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{1}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{1}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{54}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(\frac{1}{4(ax-1)^2}+\frac{1}{8(ax+1)^2}-\frac{1}{4(ax-1)^3}-\frac{3}{8(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{3\text{arctanh}(ax)}{8a}+\frac{1}{4a(1-ax)}-\frac{1}{8a(ax+1)}+\frac{1}{8a(1-ax)^2}\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(8*a*(1 - a*x)^2) + 1/(4*a*(1 - a*x)
) - 1/(8*a*(1 + a*x)) + (3*ArcTanh[a*x])/(8*a)))/(c - a^2*c*x^2)^(5/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(3\ln(ax+1)x^3a^3-3a^3\ln(ax-1)x^3-3\ln(ax+1)x^2a^2+3a^2\ln(ax-1)x^2-6a^2x^2-3\ln(ax+1)xa+3a\ln(ax-1)x+6ax}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x+1)*x
^3*a^3-3*a^3*ln(a*x-1)*x^3-3*ln(a*x+1)*x^2*a^2+3*a^2*ln(a*x-1)*x^2-6*a^2*x
^2-3*ln(a*x+1)*x*a+3*a*ln(a*x-1)*x+6*a*x+3*ln(a*x+1)-3*ln(a*x-1)+4)/(a^2*x
^2-1)/c^3/a/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{3\,(a^4x^3-a^3x^2-a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\,(3\,a^2x^2-3\,ax-2)\sqrt{-a^2c}}{16\,(a^5c^3x^3-a^4c^3x^2-a^3c^3x+a^2c^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

output
```
-1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(
-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 - 3*a*x - 2)*sqrt(-a
^2*c))/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(c-a^2\,c\,x^2)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.36

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,i\left(-3\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^3x^3+3\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^2x^2+3\log\left(\sqrt{-ax\right.\right.}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*i*( - 3*log(sqrt( - a*x + 1) - sqrt(2))*a**3*x**3 + 3*log(sqrt( -
 a*x + 1) - sqrt(2))*a**2*x**2 + 3*log(sqrt( - a*x + 1) - sqrt(2))*a*x - 3
*log(sqrt( - a*x + 1) - sqrt(2)) - 3*log(sqrt( - a*x + 1) + sqrt(2))*a**3*
x**3 + 3*log(sqrt( - a*x + 1) + sqrt(2))*a**2*x**2 + 3*log(sqrt( - a*x + 1
) + sqrt(2))*a*x - 3*log(sqrt( - a*x + 1) + sqrt(2)) + 6*log(sqrt( - a*x +
 1))*a**3*x**3 - 6*log(sqrt( - a*x + 1))*a**2*x**2 - 6*log(sqrt( - a*x + 1
))*a*x + 6*log(sqrt( - a*x + 1)) - 3*a**3*x**3 + 9*a**2*x**2 - 3*a*x - 7))
/(16*a*c**3*(a**3*x**3 - a**2*x**2 - a*x + 1))
```

3.604 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 277

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx &= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}\\&+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2\left(c-a^2cx^2\right)^{7/2}}\\&-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1+ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{5a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}(ax)}{16\left(c-a^2cx^2\right)^{7/2}}\end{aligned}$$

output
```
1/24*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^3/(-a^2*c*x^2+c)^(7/2)+3/32*a^6*
(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^2/(-a^2*c*x^2+c)^(7/2)+3/16*a^6*(1-1/a^2/
x^2)^(7/2)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)-1/32*a^6*(1-1/a^2/x^2)^(7/2)*
x^7/(a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)
/(-a^2*c*x^2+c)^(7/2)+5/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*
c*x^2+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.36

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}x(-8-25ax+25a^2x^2+15a^3x^3-15a^4x^4+15(-1+ax)^3(1+ax)^2\operatorname{arctanh}(ax))}{48c^3(-1+ax)^3(1+ax)^2\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

output
```
-1/48*(Sqrt[1 - 1/(a^2*x^2)]*x*(-8 - 25*a*x + 25*a^2*x^2 + 15*a^3*x^3 - 15
*a^4*x^4 + 15*(-1 + a*x)^3*(1 + a*x)^2*ArcTanh[a*x]))/(c^3*(-1 + a*x)^3*(1
 + a*x)^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6746, 6747, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int \frac{1}{(1-ax)^4(ax+1)^3}dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{54}$$

$$\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int\left(\frac{3}{16(ax-1)^2}+\frac{1}{8(ax+1)^2}-\frac{3}{16(ax-1)^3}+\frac{1}{16(ax+1)^3}+\frac{1}{8(ax-1)^4}-\frac{5}{16(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(\frac{5\text{arctanh}(ax)}{16a}+\frac{3}{16a(1-ax)}-\frac{1}{8a(ax+1)}+\frac{3}{32a(1-ax)^2}-\frac{1}{32a(ax+1)^2}+\frac{1}{24a(1-ax)^3}\right)}{(c-a^2cx^2)^{7/2}}$$

input
```
Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

output
```
(a^7*(1 - 1/(a^2*x^2))^(7/2)*x^7*(1/(24*a*(1 - a*x)^3) + 3/(32*a*(1 - a*x)
^2) + 3/(16*a*(1 - a*x)) - 1/(32*a*(1 + a*x)^2) - 1/(8*a*(1 + a*x)) + (5*A
rcTanh[a*x])/(16*a)))/(c - a^2*c*x^2)^(7/2)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.87

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5-15\ln(ax+1)x^4a^4+15\ln(ax-1)x^4a^4-30a^4x^4-30\ln(ax+1)x^3a^3+30a^3\ln(ax}{96\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE
)
```

output
```
1/96/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^2*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x+1
)*x^5*a^5-15*ln(a*x-1)*x^5*a^5-15*ln(a*x+1)*x^4*a^4+15*ln(a*x-1)*x^4*a^4-3
0*a^4*x^4-30*ln(a*x+1)*x^3*a^3+30*a^3*ln(a*x-1)*x^3+30*a^3*x^3+30*ln(a*x+1
)*x^2*a^2-30*a^2*ln(a*x-1)*x^2+50*a^2*x^2+15*ln(a*x+1)*x*a-15*a*ln(a*x-1)*
x-50*a*x-15*ln(a*x+1)+15*ln(a*x-1)-16)/(a^2*x^2-1)/c^4/a/(a*x+1)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.69

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{15\,(a^6x^5-a^5x^4-2\,a^4x^3+2\,a^3x^2+a^2x-a)\sqrt{-c}\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\,(15\,a^4x^4-15\,a^3x^3-25}{96\,(a^7c^4x^5-a^6c^4x^4-2\,a^5c^4x^3+2\,a^4c^4x^2+a^3c^4x-a^2c^4)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fri
cas")
```

output
```
-1/96*(15*(a^6*x^5 - a^5*x^4 - 2*a^4*x^3 + 2*a^3*x^2 + a^2*x - a)*sqrt(-c)
*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(15*a^
4*x^4 - 15*a^3*x^3 - 25*a^2*x^2 + 25*a*x + 8)*sqrt(-a^2*c))/(a^7*c^4*x^5 -
 a^6*c^4*x^4 - 2*a^5*c^4*x^3 + 2*a^4*c^4*x^2 + a^3*c^4*x - a^2*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="max
ima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\int \frac{1}{\left(c-a^2\,c\,x^2\right)^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.42

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\frac{\sqrt{c}\,i\left(-30\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^5x^5+30\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^4x^4+60\log\left(\sqrt{-\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x)
```

output
```
(sqrt(c)*i*( - 30*log(sqrt( - a*x + 1) - sqrt(2))*a**5*x**5 + 30*log(sqrt(
 - a*x + 1) - sqrt(2))*a**4*x**4 + 60*log(sqrt( - a*x + 1) - sqrt(2))*a**3
*x**3 - 60*log(sqrt( - a*x + 1) - sqrt(2))*a**2*x**2 - 30*log(sqrt( - a*x
+ 1) - sqrt(2))*a*x + 30*log(sqrt( - a*x + 1) - sqrt(2)) - 30*log(sqrt( -
a*x + 1) + sqrt(2))*a**5*x**5 + 30*log(sqrt( - a*x + 1) + sqrt(2))*a**4*x*
*4 + 60*log(sqrt( - a*x + 1) + sqrt(2))*a**3*x**3 - 60*log(sqrt( - a*x + 1
) + sqrt(2))*a**2*x**2 - 30*log(sqrt( - a*x + 1) + sqrt(2))*a*x + 30*log(s
qrt( - a*x + 1) + sqrt(2)) + 60*log(sqrt( - a*x + 1))*a**5*x**5 - 60*log(s
qrt( - a*x + 1))*a**4*x**4 - 120*log(sqrt( - a*x + 1))*a**3*x**3 + 120*log
(sqrt( - a*x + 1))*a**2*x**2 + 60*log(sqrt( - a*x + 1))*a*x - 60*log(sqrt(
 - a*x + 1)) - 15*a**5*x**5 + 75*a**4*x**4 - 30*a**3*x**3 - 130*a**2*x**2
+ 85*a*x + 47))/(192*a*c**4*(a**5*x**5 - a**4*x**4 - 2*a**3*x**3 + 2*a**2*
x**2 + a*x - 1))
```

3.605 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 169

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx = -\frac{77}{256}c^4x\sqrt{c-a^2cx^2} - \frac{77}{384}c^3x(c-a^2cx^2)^{3/2} - \frac{77}{480}c^2x(c-a^2cx^2)^{5/2} - \frac{11}{80}cx(c-a^2cx^2)^{7/2} + \frac{2(c-a^2cx^2)^{9/2}}{9a} + \frac{1}{10}x(c-a^2cx^2)^{9/2} - \frac{77c^{9/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{256a}$$

output
```
-77/256*c^4*x*(-a^2*c*x^2+c)^(1/2)-77/384*c^3*x*(-a^2*c*x^2+c)^(3/2)-77/48
0*c^2*x*(-a^2*c*x^2+c)^(5/2)-11/80*c*x*(-a^2*c*x^2+c)^(7/2)+2/9*(-a^2*c*x^
2+c)^(9/2)/a+1/10*x*(-a^2*c*x^2+c)^(9/2)-77/256*c^(9/2)*arctan(a*c^(1/2)*x
/(-a^2*c*x^2+c)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{c^4\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}(2560-10615ax-2185a^2x^2+16390a^3x^3+9210a^4x^4-15048a^5x^5\right.}{11520a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

output
```
(c^4*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(2560 - 10615*a*x - 2185*a^2*x^2 +
 16390*a^3*x^3 + 9210*a^4*x^4 - 15048*a^5*x^5 - 10552*a^6*x^6 + 7216*a^7*x
^7 + 5584*a^8*x^8 - 1408*a^9*x^9 - 1152*a^10*x^10) + 6930*Sqrt[1 - a*x]*Ar
cSin[Sqrt[1 - a*x]/Sqrt[2]]))/(11520*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6717, 6691, 469, 455, 211, 211, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{9/2}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^{9/2}dx$$

$$\Big\downarrow \text{ 6691}$$

$$-c\int (ax+1)^2\left(c-a^2cx^2\right)^{7/2}dx$$

$$\Big\downarrow \text{ 469}$$

$$-c\left(\frac{11}{10}\int (ax+1)\left(c-a^2cx^2\right)^{7/2}dx-\frac{(ax+1)\left(c-a^2cx^2\right)^{9/2}}{10ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{11}{10}\left(\int \left(c-a^2cx^2\right)^{7/2}dx-\frac{\left(c-a^2cx^2\right)^{9/2}}{9ac}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{9/2}}{10ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{11}{10}\left(\frac{7}{8}c\int \left(c-a^2cx^2\right)^{5/2}dx-\frac{\left(c-a^2cx^2\right)^{9/2}}{9ac}+\frac{1}{8}x\left(c-a^2cx^2\right)^{7/2}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{9/2}}{10ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{11}{10}\left(\frac{7}{8}c\left(\frac{5}{6}c\int \left(c-a^2cx^2\right)^{3/2}dx+\frac{1}{6}x\left(c-a^2cx^2\right)^{5/2}\right)-\frac{\left(c-a^2cx^2\right)^{9/2}}{9ac}+\frac{1}{8}x\left(c-a^2cx^2\right)^{7/2}\right)-\frac{(ax+1)\left(\right.}{}$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{11}{10}\left(\frac{7}{8}c\left(\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{c-a^2cx^2}dx+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)+\frac{1}{6}x\left(c-a^2cx^2\right)^{5/2}\right)-\frac{\left(c-a^2cx^2\right)^{9/2}}{9ac}+\frac{1}{8}x(c-\right.\right.$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{11}{10}\left(\frac{7}{8}c\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)+\frac{1}{6}x\left(c-a^2cx^2\right)^{5/2}\right)-\frac{(c}{}\right.\right.$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{11}{10}\left(\frac{7}{8}c\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)+\frac{1}{6}x\left(c-a^2cx^{2}\right.\right.\right.\right.$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{11}{10}\left(\frac{7}{8}c\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)+\frac{1}{6}x\left(c-a^2cx^2\right)^{5/2}\right)\right.\right.$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

output
```
-(c*(-1/10*((1 + a*x)*(c - a^2*c*x^2)^(9/2))/(a*c) + (11*((x*(c - a^2*c*x^
2)^(7/2))/8 - (c - a^2*c*x^2)^(9/2)/(9*a*c) + (7*c*((x*(c - a^2*c*x^2)^(5/
2))/6 + (5*c*((x*(c - a^2*c*x^2)^(3/2))/4 + (3*c*((x*Sqrt[c - a^2*c*x^2])/
2 + (Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)))/4))/6))/8)
)/10))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{(1152a^9x^9+2560a^8x^8-3024a^7x^7-10240x^6a^6+312a^5x^5+15360a^4x^4+6150a^3x^3-10240a^2x^2-8055ax+2560)(a^2x^2-1)c^5}{11520a\sqrt{-c(a^2x^2-1)}} - \frac{7}{}$
default	$\frac{x(-a^2c\,x^2+c)^{\frac{9}{2}}}{10} + \frac{9c\left(\frac{x(-a^2c\,x^2+c)^{\frac{7}{2}}}{8} + \frac{7c\left(\frac{x(-a^2c\,x^2+c)^{\frac{5}{2}}}{6} + \frac{5c\left(\frac{x(-a^2c\,x^2+c)^{\frac{3}{2}}}{4} + \frac{3c\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2} + \frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)}{4}\right)}{6}\right)}{8}\right)}{10}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/11520*(1152*a^9*x^9+2560*a^8*x^8-3024*a^7*x^7-10240*a^6*x^6+312*a^5*x^5
+15360*a^4*x^4+6150*a^3*x^3-10240*a^2*x^2-8055*a*x+2560)*(a^2*x^2-1)/a/(-c
*(a^2*x^2-1))^(1/2)*c^5-77/256/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*
c*x^2+c)^(1/2))*c^5
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.95

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx = \left[\frac{3465\sqrt{-c}c^4\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+2\left(1152a^9c^4x^9+2560a^8c^4x^8\right.}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
[1/23040*(3465*sqrt(-c)*c^4*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqr
t(-c)*x - c) + 2*(1152*a^9*c^4*x^9 + 2560*a^8*c^4*x^8 - 3024*a^7*c^4*x^7 -
 10240*a^6*c^4*x^6 + 312*a^5*c^4*x^5 + 15360*a^4*c^4*x^4 + 6150*a^3*c^4*x^
3 - 10240*a^2*c^4*x^2 - 8055*a*c^4*x + 2560*c^4)*sqrt(-a^2*c*x^2 + c))/a,
1/11520*(3465*c^(9/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 -
 c)) + (1152*a^9*c^4*x^9 + 2560*a^8*c^4*x^8 - 3024*a^7*c^4*x^7 - 10240*a^6
*c^4*x^6 + 312*a^5*c^4*x^5 + 15360*a^4*c^4*x^4 + 6150*a^3*c^4*x^3 - 10240*
a^2*c^4*x^2 - 8055*a*c^4*x + 2560*c^4)*sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 714 vs. 2(155) = 310.

Time = 3.43 (sec) , antiderivative size = 714, normalized size of antiderivative = 4.22

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(9/2),x)
```

output
```
Piecewise(((-2*c**4*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c),
 Ne(c, 0)), (a**2*sqrt(c)*x**2/2, True)) + 6*c**4*Piecewise((sqrt(-a**2*c*
x**2 + c)*(a**4*x**4/5 - a**2*x**2/15 - 2/15), Ne(c, 0)), (a**4*sqrt(c)*x*
*4/4, True)) - 6*c**4*Piecewise((sqrt(-a**2*c*x**2 + c)*(a**6*x**6/7 - a**
4*x**4/35 - 4*a**2*x**2/105 - 8/105), Ne(c, 0)), (a**6*sqrt(c)*x**6/6, Tru
e)) + 2*c**4*Piecewise((sqrt(-a**2*c*x**2 + c)*(a**8*x**8/9 - a**6*x**6/63
 - 2*a**4*x**4/105 - 8*a**2*x**2/315 - 16/315), Ne(c, 0)), (a**8*sqrt(c)*x
**8/8, True)) + c**4*Piecewise((7*c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*s
qrt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**
2), True))/256 + sqrt(-a**2*c*x**2 + c)*(a**9*x**9/10 - a**7*x**7/80 - 7*a
**5*x**5/480 - 7*a**3*x**3/384 - 7*a*x/256), Ne(c, 0)), (a**9*sqrt(c)*x**9
/9, True)) - 2*c**4*Piecewise((5*c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sq
rt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2
), True))/128 + sqrt(-a**2*c*x**2 + c)*(a**7*x**7/8 - a**5*x**5/48 - 5*a**
3*x**3/192 - 5*a*x/128), Ne(c, 0)), (a**7*sqrt(c)*x**7/7, True)) + 2*c**4*
Piecewise((c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/
sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/8 + (a**3*x*
*3/4 - a*x/8)*sqrt(-a**2*c*x**2 + c), Ne(c, 0)), (a**3*sqrt(c)*x**3/3, Tru
e)) - c**4*Piecewise((a*x*sqrt(-a**2*c*x**2 + c)/2 + c*Piecewise((log(-2*a
*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.14

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx &= \frac{1}{10}\left(-a^2cx^2+c\right)^{\frac{9}{2}}x \\ &- \frac{11}{80}\left(-a^2cx^2+c\right)^{\frac{7}{2}}cx - \frac{77}{480}\left(-a^2cx^2+c\right)^{\frac{5}{2}}c^2x - \frac{77}{384}\left(-a^2cx^2+c\right)^{\frac{3}{2}}c^3x \\ &- \frac{35}{64}\sqrt{a^2cx^2-4\,acx+3\,c}c^4x + \frac{63}{256}\sqrt{-a^2cx^2+c}c^4x - \frac{35\,c^6\arcsin\left(ax-2\right)}{64\,a\left(-c\right)^{\frac{3}{2}}} \\ &+ \frac{63\,c^{\frac{9}{2}}\arcsin\left(ax\right)}{256\,a} + \frac{2\left(-a^2cx^2+c\right)^{\frac{9}{2}}}{9\,a} + \frac{35\sqrt{a^2cx^2-4\,acx+3\,c}c^4}{32\,a}\end{aligned}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
1/10*(-a^2*c*x^2 + c)^(9/2)*x - 11/80*(-a^2*c*x^2 + c)^(7/2)*c*x - 77/480*
(-a^2*c*x^2 + c)^(5/2)*c^2*x - 77/384*(-a^2*c*x^2 + c)^(3/2)*c^3*x - 35/64
*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^4*x + 63/256*sqrt(-a^2*c*x^2 + c)*c^4*x
 - 35/64*c^6*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 63/256*c^(9/2)*arcsin(a*x)/a
 + 2/9*(-a^2*c*x^2 + c)^(9/2)/a + 35/32*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^
4/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.97

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{77\,c^5\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{256\,\sqrt{-c}|a|}+\frac{1}{11520}\sqrt{-a^2cx^2+c}\left(\frac{2560\,c^4}{a}-\left(8055\,c^4+2\left(5120\,ac^4-\left(3075\,a^2c^4+4\left(1920\,a^3c^4+\left(39\,a^4c^4-2\left(640\,a\right.\right.\right.\right.\right.\right.\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
77/256*c^5*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(
a)) + 1/11520*sqrt(-a^2*c*x^2 + c)*(2560*c^4/a - (8055*c^4 + 2*(5120*a*c^4
 - (3075*a^2*c^4 + 4*(1920*a^3*c^4 + (39*a^4*c^4 - 2*(640*a^5*c^4 + (189*a
^6*c^4 - 8*(9*a^8*c^4*x + 20*a^7*c^4)*x)*x)*x)*x)*x)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int \frac{{\left(c-a^2\,c\,x^2\right)}^{9/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(9/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - a^2*c*x^2)^(9/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.17

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx = \frac{\sqrt{c}\,c^4\left(-3465asin(ax)+1152\sqrt{-a^2x^2+1}\,a^9x^9+2560\sqrt{-a^2x^2+1}\,a^8x^8-3024\sqrt{-a^2x^2}\right.}{}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x)
```

output
```
(sqrt(c)*c**4*( - 3465*asin(a*x) + 1152*sqrt( - a**2*x**2 + 1)*a**9*x**9 +
 2560*sqrt( - a**2*x**2 + 1)*a**8*x**8 - 3024*sqrt( - a**2*x**2 + 1)*a**7*
x**7 - 10240*sqrt( - a**2*x**2 + 1)*a**6*x**6 + 312*sqrt( - a**2*x**2 + 1)
*a**5*x**5 + 15360*sqrt( - a**2*x**2 + 1)*a**4*x**4 + 6150*sqrt( - a**2*x*
*2 + 1)*a**3*x**3 - 10240*sqrt( - a**2*x**2 + 1)*a**2*x**2 - 8055*sqrt( -
a**2*x**2 + 1)*a*x + 2560*sqrt( - a**2*x**2 + 1) - 2560))/(11520*a)
```

3.606 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 146

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{2(c-a^2cx^2)^{7/2}}{7a}+\frac{1}{8}x\left(c-a^2cx^2\right)^{7/2}-\frac{45c^{7/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{128a}$$

output

```
-45/128*c^3*x*(-a^2*c*x^2+c)^(1/2)-15/64*c^2*x*(-a^2*c*x^2+c)^(3/2)-3/16*c
*x*(-a^2*c*x^2+c)^(5/2)+2/7*(-a^2*c*x^2+c)^(7/2)/a+1/8*x*(-a^2*c*x^2+c)^(7
/2)-45/128*c^(7/2)*arctan(a*c^(1/2)*x/(-a^2*c*x^2+c)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{c^3\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}(256-837ax-187a^2x^2+978a^3x^3+558a^4x^4-600a^5x^5-424a\right.}{896a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

output
```
(c^3*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(256 - 837*a*x - 187*a^2*x^2 + 978
*a^3*x^3 + 558*a^4*x^4 - 600*a^5*x^5 - 424*a^6*x^6 + 144*a^7*x^7 + 112*a^8
*x^8) + 630*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(896*a*Sqrt[1 -
a*x]*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6717, 6691, 469, 455, 211, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{7/2} e^{2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)} \left(c-a^2cx^2\right)^{7/2} dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int (ax+1)^2 \left(c-a^2cx^2\right)^{5/2} dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{9}{8}\int (ax+1) \left(c-a^2cx^2\right)^{5/2} dx-\frac{(ax+1) \left(c-a^2cx^2\right)^{7/2}}{8ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{9}{8}\left(\int \left(c-a^2cx^2\right)^{5/2} dx-\frac{\left(c-a^2cx^2\right)^{7/2}}{7ac}\right)-\frac{(ax+1) \left(c-a^2cx^2\right)^{7/2}}{8ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{9}{8}\left(\frac{5}{6}c\int \left(c-a^2cx^2\right)^{3/2}dx-\frac{\left(c-a^2cx^2\right)^{7/2}}{7ac}+\frac{1}{6}x\left(c-a^2cx^2\right)^{5/2}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{7/2}}{8ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{9}{8}\left(\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{c-a^2cx^2}dx+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)-\frac{\left(c-a^2cx^2\right)^{7/2}}{7ac}+\frac{1}{6}x\left(c-a^2cx^2\right)^{5/2}\right)-\frac{(ax+1)\left(c-\right.}{8ac}\right.$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{9}{8}\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)-\frac{\left(c-a^2cx^2\right)^{7/2}}{7ac}+\frac{1}{6}x\left(c-a^2cx\right.\right.\right.$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{9}{8}\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)-\frac{\left(c-a^2cx^2\right)^{7/2}}{7ac}+\frac{1}{6}\right.\right.\right.$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{9}{8}\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)-\frac{\left(c-a^2cx^2\right)^{7/2}}{7ac}+\frac{1}{6}x\left(c-\right.\right.\right.$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

output
```
-(c*(-1/8*((1 + a*x)*(c - a^2*c*x^2)^(7/2))/(a*c) + (9*((x*(c - a^2*c*x^2)
^(5/2))/6 - (c - a^2*c*x^2)^(7/2)/(7*a*c) + (5*c*((x*(c - a^2*c*x^2)^(3/2)
)/4 + (3*c*((x*Sqrt[c - a^2*c*x^2])/2 + (Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(2*a)))/4))/6))/8))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.84

method	result
risch	$\frac{(112a^7x^7+256x^6a^6-168a^5x^5-768a^4x^4-210a^3x^3+768a^2x^2+581ax-256)(a^2x^2-1)c^4}{896a\sqrt{-c(a^2x^2-1)}}-\frac{45\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c^4}{128\sqrt{a^2c}}$
default	$\frac{x(-a^2c\,x^2+c)^{\frac{7}{2}}}{8}+\frac{7c\left(\frac{x(-a^2c\,x^2+c)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x(-a^2c\,x^2+c)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)}{4}\right)}{6}\right)}{8}+\frac{2\left(-\left(x-\frac{1}{a}\right)^2 \ldots\right.}{\ldots}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/896*(112*a^7*x^7+256*a^6*x^6-168*a^5*x^5-768*a^4*x^4-210*a^3*x^3+768*a^2
*x^2+581*a*x-256)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c^4-45/128/(a^2*c)^
(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.96

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\left[\frac{315\sqrt{-c}c^3\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\left(112a^7c^3x^7+256a^6c^3x^6-16\right.}{1792}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
[1/1792*(315*sqrt(-c)*c^3*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(
-c)*x - c) - 2*(112*a^7*c^3*x^7 + 256*a^6*c^3*x^6 - 168*a^5*c^3*x^5 - 768*
a^4*c^3*x^4 - 210*a^3*c^3*x^3 + 768*a^2*c^3*x^2 + 581*a*c^3*x - 256*c^3)*s
qrt(-a^2*c*x^2 + c))/a, 1/896*(315*c^(7/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*s
qrt(c)*x/(a^2*c*x^2 - c)) - (112*a^7*c^3*x^7 + 256*a^6*c^3*x^6 - 168*a^5*c
^3*x^5 - 768*a^4*c^3*x^4 - 210*a^3*c^3*x^3 + 768*a^2*c^3*x^2 + 581*a*c^3*x
 - 256*c^3)*sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. 2(133) = 266.

Time = 3.09 (sec) , antiderivative size = 614, normalized size of antiderivative = 4.21

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\text{Too large to display}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Piecewise(((-2*c**3*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c),
 Ne(c, 0)), (a**2*sqrt(c)*x**2/2, True)) + 4*c**3*Piecewise((sqrt(-a**2*c*
x**2 + c)*(a**4*x**4/5 - a**2*x**2/15 - 2/15), Ne(c, 0)), (a**4*sqrt(c)*x*
*4/4, True)) - 2*c**3*Piecewise((sqrt(-a**2*c*x**2 + c)*(a**6*x**6/7 - a**
4*x**4/35 - 4*a**2*x**2/105 - 8/105), Ne(c, 0)), (a**6*sqrt(c)*x**6/6, Tru
e)) - c**3*Piecewise((5*c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*
c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))
/128 + sqrt(-a**2*c*x**2 + c)*(a**7*x**7/8 - a**5*x**5/48 - 5*a**3*x**3/19
2 - 5*a*x/128), Ne(c, 0)), (a**7*sqrt(c)*x**7/7, True)) + c**3*Piecewise((
c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqrt(-c), N
e(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/16 + sqrt(-a**2*c*x**2
+ c)*(a**5*x**5/6 - a**3*x**3/24 - a*x/16), Ne(c, 0)), (a**5*sqrt(c)*x**5/
5, True)) + c**3*Piecewise((c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a
**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), Tr
ue))/8 + (a**3*x**3/4 - a*x/8)*sqrt(-a**2*c*x**2 + c), Ne(c, 0)), (a**3*sq
rt(c)*x**3/3, True)) - c**3*Piecewise((a*x*sqrt(-a**2*c*x**2 + c)/2 + c*Pi
ecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c,
 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/2, Ne(c, 0)), (a*sqrt(c)*x,
 True)))/a, Ne(a, 0)), (-c**(7/2)*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.18

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{1}{8}\left(-a^2cx^2+c\right)^{\frac{7}{2}}x - \frac{3}{16}\left(-a^2cx^2+c\right)^{\frac{5}{2}}cx - \frac{15}{64}\left(-a^2cx^2+c\right)^{\frac{3}{2}}c^2x - \frac{5}{8}\sqrt{a^2cx^2-4\,acx+3\,c}c^3x + \frac{35}{128}\sqrt{-a^2cx^2+c}c^3x - \frac{5\,c^5\arcsin\left(ax-2\right)}{8\,a\left(-c\right)^{\frac{3}{2}}} + \frac{35\,c^{\frac{7}{2}}\arcsin\left(ax\right)}{128\,a} + \frac{2\left(-a^2cx^2+c\right)^{\frac{7}{2}}}{7\,a} + \frac{5\sqrt{a^2cx^2-4\,acx+3\,c}c^3}{4\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
1/8*(-a^2*c*x^2 + c)^(7/2)*x - 3/16*(-a^2*c*x^2 + c)^(5/2)*c*x - 15/64*(-a
^2*c*x^2 + c)^(3/2)*c^2*x - 5/8*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^3*x + 35
/128*sqrt(-a^2*c*x^2 + c)*c^3*x - 5/8*c^5*arcsin(a*x - 2)/(a*(-c)^(3/2)) +
 35/128*c^(7/2)*arcsin(a*x)/a + 2/7*(-a^2*c*x^2 + c)^(7/2)/a + 5/4*sqrt(a^
2*c*x^2 - 4*a*c*x + 3*c)*c^3/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{45\,c^4\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{128\,\sqrt{-c}|a|}$$
$$+\frac{1}{896}\sqrt{-a^2cx^2+c}\left(\frac{256\,c^3}{a}-\left(581\,c^3+2\left(384\,ac^3-\left(105\,a^2c^3+4\left(96\,a^3c^3+\left(21\,a^4c^3-2\left(7\,a^6c^3x+16\,a^5\right.\right.\right.\right.\right.\right.\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
45/128*c^4*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(
a)) + 1/896*sqrt(-a^2*c*x^2 + c)*(256*c^3/a - (581*c^3 + 2*(384*a*c^3 - (1
05*a^2*c^3 + 4*(96*a^3*c^3 + (21*a^4*c^3 - 2*(7*a^6*c^3*x + 16*a^5*c^3)*x)
*x)*x)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^{7/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(7/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - a^2*c*x^2)^(7/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx = \frac{\sqrt{c}\,c^3\left(-315asin(ax)-112\sqrt{-a^2x^2+1}\,a^7x^7-256\sqrt{-a^2x^2+1}\,a^6x^6+168\sqrt{-a^2x^2+1}\,a\right.}{}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x)
```

output
```
(sqrt(c)*c**3*( - 315*asin(a*x) - 112*sqrt( - a**2*x**2 + 1)*a**7*x**7 - 2
56*sqrt( - a**2*x**2 + 1)*a**6*x**6 + 168*sqrt( - a**2*x**2 + 1)*a**5*x**5
 + 768*sqrt( - a**2*x**2 + 1)*a**4*x**4 + 210*sqrt( - a**2*x**2 + 1)*a**3*
x**3 - 768*sqrt( - a**2*x**2 + 1)*a**2*x**2 - 581*sqrt( - a**2*x**2 + 1)*a
*x + 256*sqrt( - a**2*x**2 + 1) - 256))/(896*a)
```

3.607 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{2\left(c-a^2cx^2\right)^{5/2}}{5a}+\frac{1}{6}x\left(c-a^2cx^2\right)^{5/2}-\frac{7c^{5/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}$$

output
```
-7/16*c^2*x*(-a^2*c*x^2+c)^(1/2)-7/24*c*x*(-a^2*c*x^2+c)^(3/2)+2/5*(-a^2*c
*x^2+c)^(5/2)/a+1/6*x*(-a^2*c*x^2+c)^(5/2)-7/16*c^(5/2)*arctan(a*c^(1/2)*x
/(-a^2*c*x^2+c)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.10

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}(96-231ax-57a^2x^2+182a^3x^3+106a^4x^4-56a^5x^5-40a^6x^6)\right.}{240a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

output
```
(c^2*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(96 - 231*a*x - 57*a^2*x^2 + 182*a
^3*x^3 + 106*a^4*x^4 - 56*a^5*x^5 - 40*a^6*x^6) + 210*Sqrt[1 - a*x]*ArcSin
[Sqrt[1 - a*x]/Sqrt[2]]))/(240*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6691, 469, 455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{5/2}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int (ax+1)^2\left(c-a^2cx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{7}{6}\int (ax+1)\left(c-a^2cx^2\right)^{3/2}dx-\frac{(ax+1)\left(c-a^2cx^2\right)^{5/2}}{6ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{7}{6}\left(\int \left(c-a^2cx^2\right)^{3/2}dx-\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{5/2}}{6ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{7}{6}\left(\frac{3}{4}c\int \sqrt{c-a^2cx^2}dx-\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{5/2}}{6ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{7}{6}\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)-\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)-\frac{(ax+1)\left(c-a^2\right.}{6ac}\right.$$

$\big\downarrow$ 224

$$-c\left(\frac{7}{6}\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)-\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)-\frac{(ax+}{}\right.$$

$\big\downarrow$ 216

$$-c\left(\frac{7}{6}\left(\frac{3}{4}c\left(\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)-\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)-\frac{(ax+1)\left(c-\right.}{6a}\right.$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

output
```
-(c*(-1/6*((1 + a*x)*(c - a^2*c*x^2)^(5/2))/(a*c) + (7*((x*(c - a^2*c*x^2)
^(3/2))/4 - (c - a^2*c*x^2)^(5/2)/(5*a*c) + (3*c*((x*Sqrt[c - a^2*c*x^2])/
2 + (Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)))/4))/6))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\left(40 a^{5} x^{5}+96 a^{4} x^{4}-10 a^{3} x^{3}-192 a^{2} x^{2}-135 a x +96\right) \left(a^{2} x^{2}-1\right) c^{3}}{240 a \sqrt{-c \left(a^{2} x^{2}-1\right)}}-\frac{7 \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right) c^{3}}{16 \sqrt{a^{2} c}}$
default	$\frac{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}{6}+\frac{5 c \left(\frac{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{4}+\frac{3 c \left(\frac{x \sqrt{-a^{2} c \,x^{2}+c}}{2}+\frac{c \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right)}{2 \sqrt{a^{2} c}}\right)}{4}\right)}{6}+\frac{\frac{2 \left(-\left(x -\frac{1}{a}\right)^{2} a^{2} c -2 \left(x -\frac{1}{a}\right) a c \right)^{\frac{5}{2}}}{5}-2 a c}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(40*a^5*x^5+96*a^4*x^4-10*a^3*x^3-192*a^2*x^2-135*a*x+96)*(a^2*x^2-
1)/a/(-c*(a^2*x^2-1))^(1/2)*c^3-7/16/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/
(-a^2*c*x^2+c)^(1/2))*c^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.96

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx = \left[\frac{105\sqrt{-c}c^2\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+2\left(40\,a^5c^2x^5+96\,a^4c^2x^4-10\,a\right.}{480\,a}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/480*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) + 2*(40*a^5*c^2*x^5 + 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 192*a^2*
c^2*x^2 - 135*a*c^2*x + 96*c^2)*sqrt(-a^2*c*x^2 + c))/a, 1/240*(105*c^(5/2
)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (40*a^5*c^2*x
^5 + 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 192*a^2*c^2*x^2 - 135*a*c^2*x + 96*
c^2)*sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(110) = 220.

Time = 2.59 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.55

$$\int e^{2\coth^{-1}(ax)}(c - a^2cx^2)^{5/2}\,dx = \begin{cases} \frac{-2c^2\left(\begin{cases}\left(\frac{a^2x^2}{3}-\frac{1}{3}\right)\sqrt{-a^2cx^2+c} & \text{for } c\neq 0\\ \frac{a^2\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)+2c^2\left(\begin{cases}\sqrt{-a^2cx^2+c}\left(\frac{a^4x^4}{5}-\frac{a^2x^2}{15}-\frac{2}{15}\right)\\ \frac{a^4\sqrt{c}x^4}{4}\end{cases}\right.}{} \\ -c^{\frac{5}{2}}x \end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Piecewise(((-2*c**2*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c),
 Ne(c, 0)), (a**2*sqrt(c)*x**2/2, True)) + 2*c**2*Piecewise((sqrt(-a**2*c*
x**2 + c)*(a**4*x**4/5 - a**2*x**2/15 - 2/15), Ne(c, 0)), (a**4*sqrt(c)*x*
*4/4, True)) + c**2*Piecewise((c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt
(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2),
 True))/16 + sqrt(-a**2*c*x**2 + c)*(a**5*x**5/6 - a**3*x**3/24 - a*x/16),
 Ne(c, 0)), (a**5*sqrt(c)*x**5/5, True)) - c**2*Piecewise((a*x*sqrt(-a**2*
c*x**2 + c)/2 + c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 +
 c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/2, Ne(c
, 0)), (a*sqrt(c)*x, True)))/a, Ne(a, 0)), (-c**(5/2)*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.25

$$\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx = \frac{1}{6}\left(-a^2cx^2+c\right)^{\frac{5}{2}}x - \frac{7}{24}\left(-a^2cx^2+c\right)^{\frac{3}{2}}cx - \frac{3}{4}\sqrt{a^2cx^2-4\,acx+3\,c}c^2x + \frac{5}{16}\sqrt{-a^2cx^2+c}c^2x - \frac{3\,c^4\arcsin\left(ax-2\right)}{4\,a\left(-c\right)^{\frac{3}{2}}} + \frac{5\,c^{\frac{5}{2}}\arcsin\left(ax\right)}{16\,a} + \frac{2\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{5\,a} + \frac{3\sqrt{a^2cx^2-4\,acx+3\,c}c^2}{2\,a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(-a^2*c*x^2 + c)^(5/2)*x - 7/24*(-a^2*c*x^2 + c)^(3/2)*c*x - 3/4*sqrt(
a^2*c*x^2 - 4*a*c*x + 3*c)*c^2*x + 5/16*sqrt(-a^2*c*x^2 + c)*c^2*x - 3/4*c
^4*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 5/16*c^(5/2)*arcsin(a*x)/a + 2/5*(-a^2
*c*x^2 + c)^(5/2)/a + 3/2*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^2/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{7\,c^3\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{16\,\sqrt{-c}|a|}$$
$$-\frac{1}{240}\sqrt{-a^2cx^2+c}\left(\left(135\,c^2+2\left(96\,ac^2+\left(5\,a^2c^2-4\left(5\,a^4c^2x+12\,a^3c^2\right)x\right)x\right)x\right)x-\frac{96\,c^2}{a}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
7/16*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a)
) - 1/240*sqrt(-a^2*c*x^2 + c)*((135*c^2 + 2*(96*a*c^2 + (5*a^2*c^2 - 4*(5
*a^4*c^2*x + 12*a^3*c^2)*x)*x)*x)*x - 96*c^2/a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\frac{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(5/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - a^2*c*x^2)^(5/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\sqrt{c}\,c^2\left(-105 asin(ax)+40\sqrt{-a^2x^2+1}\,a^5x^5+96\sqrt{-a^2x^2+1}\,a^4x^4-10\sqrt{-a^2x^2+1}\,a^3x^3\right.}{240a}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*c**2*( - 105*asin(a*x) + 40*sqrt( - a**2*x**2 + 1)*a**5*x**5 + 96
*sqrt( - a**2*x**2 + 1)*a**4*x**4 - 10*sqrt( - a**2*x**2 + 1)*a**3*x**3 -
192*sqrt( - a**2*x**2 + 1)*a**2*x**2 - 135*sqrt( - a**2*x**2 + 1)*a*x + 96
*sqrt( - a**2*x**2 + 1) - 96))/(240*a)
```

3.608 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{2(c-a^2cx^2)^{3/2}}{3a}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}-\frac{5c^{3/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}$$

output
```
-5/8*c*x*(-a^2*c*x^2+c)^(1/2)+2/3*(-a^2*c*x^2+c)^(3/2)/a+1/4*x*(-a^2*c*x^2
+c)^(3/2)-5/8*c^(3/2)*arctan(a*c^(1/2)*x/(-a^2*c*x^2+c)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}(16-25ax-7a^2x^2+10a^3x^3+6a^4x^4)+30\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-a}}{\sqrt{2}}\right.\right.}{24a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

output
```
(c*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(16 - 25*a*x - 7*a^2*x^2 + 10*a^3*x^
3 + 6*a^4*x^4) + 30*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(24*a*Sq
rt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6717, 6691, 469, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{3/2} e^{2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int (ax+1)^2\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{5}{4}\int (ax+1)\sqrt{c-a^2cx^2}dx-\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{5}{4}\left(\int \sqrt{c-a^2cx^2}dx-\frac{\left(c-a^2cx^2\right)^{3/2}}{3ac}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{5}{4}\left(\frac{1}{2}c\int \frac{1}{\sqrt{c-a^2cx^2}}dx-\frac{\left(c-a^2cx^2\right)^{3/2}}{3ac}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{5}{4}\left(\frac{1}{2}c\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}-\frac{\left(c-a^2cx^2\right)^{3/2}}{3ac}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{5}{4}\left(\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}-\frac{\left(c-a^2cx^2\right)^{3/2}}{3ac}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)-\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

output
```
-(c*(-1/4*((1 + a*x)*(c - a^2*c*x^2)^(3/2))/(a*c) + (5*((x*Sqrt[c - a^2*c*
x^2])/2 - (c - a^2*c*x^2)^(3/2)/(3*a*c) + (Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sq
rt[c - a^2*c*x^2]])/(2*a)))/4))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

method	result
risch	$\frac{(6a^3x^3+16a^2x^2+9ax-16)(a^2x^2-1)c^2}{24a\sqrt{-c(a^2x^2-1)}} - \frac{5\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)c^2}{8\sqrt{a^2c}}$
default	$\frac{x(-a^2cx^2+c)^{\frac{3}{2}}}{4} + \frac{3c\left(\frac{x\sqrt{-a^2cx^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)}{2\sqrt{a^2c}}\right)}{4} + \frac{\frac{2\left(-\left(x-\frac{1}{a}\right)^2a^2c-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}{3}-2ac\left(-\frac{\left(-2\left(x-\frac{1}{a}\right)a^2c-2ac\right)\ldots}{\ldots}\right.}{\ldots}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*a^3*x^3+16*a^2*x^2+9*a*x-16)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*
c^2-5/8/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.80

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\left[\frac{15\sqrt{-c}c\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\left(6a^3cx^3+16a^2cx^2+9acx-1\right.}{48a}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(15*sqrt(-c)*c*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
 - c) - 2*(6*a^3*c*x^3 + 16*a^2*c*x^2 + 9*a*c*x - 16*c)*sqrt(-a^2*c*x^2 +
c))/a, 1/24*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2
 - c)) - (6*a^3*c*x^3 + 16*a^2*c*x^2 + 9*a*c*x - 16*c)*sqrt(-a^2*c*x^2 + c
))/a]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(88) = 176$.

Time = 2.31 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.50

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\begin{cases}\frac{-2c\left(\begin{cases}\left(\frac{a^2x^2}{3}-\frac{1}{3}\right)\sqrt{-a^2cx^2+c} & \text{for } c\neq 0\\ \frac{a^2\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)-c\left(\left(\begin{cases}\frac{c\left(\begin{cases}\frac{\log\left(-2acx+2\sqrt{-c}\sqrt{-a^2cx^2+c}\right)}{\sqrt{-c}} & \text{for } c\neq\\ \frac{ax\log(ax)}{\sqrt{-a^2cx^2}} & \text{other}\end{cases}\right.}{8}\\ \frac{a^3\sqrt{c}x^3}{3}\end{cases}\right.\right.}{}\\ -c^{\frac{3}{2}}x\end{cases}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Piecewise(((-2*c*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c), Ne
(c, 0)), (a**2*sqrt(c)*x**2/2, True)) - c*Piecewise((c*Piecewise((log(-2*a
*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*
x)/sqrt(-a**2*c*x**2), True))/8 + (a**3*x**3/4 - a*x/8)*sqrt(-a**2*c*x**2
+ c), Ne(c, 0)), (a**3*sqrt(c)*x**3/3, True)) - c*Piecewise((a*x*sqrt(-a**
2*c*x**2 + c)/2 + c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2
 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/2, Ne
(c, 0)), (a*sqrt(c)*x, True)))/a, Ne(a, 0)), (-c**(3/2)*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.31

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{1}{4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}x-\sqrt{a^2cx^2-4\,acx+3\,c}cx+\frac{3}{8}\sqrt{-a^2cx^2+c}cx-\frac{c^3\arcsin\left(ax-2\right)}{a\left(-c\right)^{\frac{3}{2}}}+\frac{3\,c^{\frac{3}{2}}\arcsin\left(ax\right)}{8\,a}+\frac{2\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3\,a}+\frac{2\sqrt{a^2cx^2-4\,acx+3\,c}c}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(-a^2*c*x^2 + c)^(3/2)*x - sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c*x + 3/8*s
qrt(-a^2*c*x^2 + c)*c*x - c^3*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 3/8*c^(3/2)
*arcsin(a*x)/a + 2/3*(-a^2*c*x^2 + c)^(3/2)/a + 2*sqrt(a^2*c*x^2 - 4*a*c*x
 + 3*c)*c/a
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.85

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3\,a^2cx+8\,ac\right)x+9\,c\right)x-\frac{16\,c}{a}\right)+\frac{5\,c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8\sqrt{-c}|a|}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*a^2*c*x + 8*a*c)*x + 9*c)*x - 16*c/a) +
5/8*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^{3/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(3/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - a^2*c*x^2)^(3/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int e^{2\coth^{-1}(ax)}\big(c-a^2cx^2\big)^{3/2}\,dx=\frac{\sqrt{c}\,c\big(-15asin(ax)-6\sqrt{-a^2x^2+1}\,a^3x^3-16\sqrt{-a^2x^2+1}\,a^2x^2-9\sqrt{-a^2x^2+1}\,ax+16\sqrt{}}{24a}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*c*( - 15*asin(a*x) - 6*sqrt( - a**2*x**2 + 1)*a**3*x**3 - 16*sqrt
( - a**2*x**2 + 1)*a**2*x**2 - 9*sqrt( - a**2*x**2 + 1)*a*x + 16*sqrt( - a
**2*x**2 + 1) - 16))/(24*a)
```

3.609 $\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{2\sqrt{c-a^2cx^2}}{a}+\frac{1}{2}x\sqrt{c-a^2cx^2}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

output
```
2*(-a^2*c*x^2+c)^(1/2)/a+1/2*x*(-a^2*c*x^2+c)^(1/2)-3/2*c^(1/2)*arctan(a*c
^(1/2)*x/(-a^2*c*x^2+c)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left((4+ax)\sqrt{1-a^2x^2}+6\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]
/Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6691, 469, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int \frac{(ax+1)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{3}{2}\int \frac{ax+1}{\sqrt{c-a^2cx^2}}dx-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{ac}\right)-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{3}{2}\left(\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{ac}\right)-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{3}{2}\left(\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{ac}\right)-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2ac}\right)$$

input

```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
-(c*(-1/2*((1 + a*x)*Sqrt[c - a^2*c*x^2])/(a*c) + (3*(-(Sqrt[c - a^2*c*x^2
]/(a*c)) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]/(a*Sqrt[c])))/2))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

method	result	size
risch	$-\frac{(ax+4)(a^2x^2-1)c}{2a\sqrt{-c(a^2x^2-1)}}-\frac{3c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}$	69
default	$\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}+\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2\left(x-\frac{1}{a}\right)ac}-\frac{2ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2\left(x-\frac{1}{a}\right)ac}}\right)}{\sqrt{a^2c}}}{a}$	136

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x+4)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c-3/2*c/(a^2*c)^(1/2)*ar
ctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.74

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$$
$$=\left[\frac{2\sqrt{-a^2cx^2+c}(ax+4)+3\sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{4a},\frac{\sqrt{-a^2cx^2+c}(ax+4)+3}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x + 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sq
rt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x +
4) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a
]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{1}{2}\sqrt{-a^2cx^2+c}x - \frac{3\sqrt{c}\arcsin{(ax)}}{2a} + \frac{2\sqrt{-a^2cx^2+c}}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a + 2*sqrt(-a^2*c*x^2
 + c)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{1}{2}\sqrt{-a^2cx^2+c}\left(x+\frac{4}{a}\right) + \frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}|a|}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-a^2*c*x^2 + c)*(x + 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(
-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c}\left(-3asin(ax)+\sqrt{-a^2x^2+1}\,ax+4\sqrt{-a^2x^2+1}-4\right)}{2a}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*( - 3*asin(a*x) + sqrt( - a**2*x**2 + 1)*a*x + 4*sqrt( - a**2*x**
2 + 1) - 4))/(2*a)
```

3.610 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 59

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}+\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

output
```
(-2*a*x-2)/a/(-a^2*c*x^2+c)^(1/2)+arctan(a*c^(1/2)*x/(-a^2*c*x^2+c)^(1/2))
/a/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2\sqrt{1-a^2x^2}\left(\sqrt{1+ax}+\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-ax}\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

output
```
(-2*Sqrt[1 - a^2*x^2]*(Sqrt[1 + a*x] + Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/
Sqrt[2]]))/(a*Sqrt[1 - a*x]*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6717, 6691, 457, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int \frac{(ax+1)^2}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{457}$$

$$-c\left(\frac{2(ax+1)}{ac\sqrt{c-a^2cx^2}}-\frac{\int \frac{1}{\sqrt{c-a^2cx^2}}dx}{c}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{2(ax+1)}{ac\sqrt{c-a^2cx^2}}-\frac{\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}}{c}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{2(ax+1)}{ac\sqrt{c-a^2cx^2}}-\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{ac^{3/2}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

output
```
-(c*((2*(1 + a*x))/(a*c*Sqrt[c - a^2*c*x^2]) - ArcTan[(a*Sqrt[c]*x)/Sqrt[c
 - a^2*c*x^2]]/(a*c^(3/2))))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}+\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2\left(x-\frac{1}{a}\right)ac}}{a^2c\left(x-\frac{1}{a}\right)}$	79

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^2/c/(x-1/
a)*(-(x-1/a)^2*a^2*c-2*(x-1/a)*a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.59

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \left[-\frac{(ax-1)\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-4\,\sqrt{-a^2cx^2+c}}{2\,(a^2cx-ac)},\; -\frac{(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)-2\,\sqrt{-a^2cx^2+c}}{a^2cx-ac}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((a*x - 1)*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(
-c)*x - c) - 4*sqrt(-a^2*c*x^2 + c))/(a^2*c*x - a*c), -((a*x - 1)*sqrt(c)*
arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*c*x
^2 + c))/(a^2*c*x - a*c)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{ax+1}{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral((a*x + 1)/(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2\sqrt{-a^2cx^2+c}}{a^2cx-ac} + \frac{\arcsin(ax)}{a\sqrt{c}}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(-a^2*c*x^2 + c)/(a^2*c*x - a*c) + arcsin(a*x)/(a*sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{a\,x+1}{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}\,dx$$

input
```
int((a*x + 1)/((c - a^2*c*x^2)^(1/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - a^2*c*x^2)^(1/2)*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{c}\left(asin(ax)\tan\left(\frac{asin(ax)}{2}\right)-asin(ax)+4\tan\left(\frac{asin(ax)}{2}\right)\right)}{ac\left(\tan\left(\frac{asin(ax)}{2}\right)-1\right)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*(asin(a*x)*tan(asin(a*x)/2) - asin(a*x) + 4*tan(asin(a*x)/2)))/(a
*c*(tan(asin(a*x)/2) - 1))
```

3.611 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{2(1+ax)}{3a\,(c-a^2cx^2)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}$$

output
```
1/3*(-2*a*x-2)/a/(-a^2*c*x^2+c)^(3/2)-1/3*x/c/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.25

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{(2-ax)\sqrt{1+ax}\sqrt{1-a^2x^2}}{3ac(1-ax)^{3/2}\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-1/3*((2 - a*x)*Sqrt[1 + a*x]*Sqrt[1 - a^2*x^2])/(a*c*(1 - a*x)^(3/2)*Sqrt
[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6717, 6691, 457, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int \frac{(ax+1)^2}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{457}$$

$$-c\left(\frac{\int \frac{1}{(c-a^2cx^2)^{3/2}}dx}{3c}+\frac{2(ax+1)}{3ac\,(c-a^2cx^2)^{3/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$-c\left(\frac{x}{3c^2\sqrt{c-a^2cx^2}}+\frac{2(ax+1)}{3ac\,(c-a^2cx^2)^{3/2}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-(c*((2*(1 + a*x))/(3*a*c*(c - a^2*c*x^2)^(3/2)) + x/(3*c^2*Sqrt[c - a^2*c
*x^2])))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(ax+1)^2(ax-2)}{3a(-a^2cx^2+c)^{\frac{3}{2}}}$	31
orering	$\frac{(ax+1)^2(ax-2)}{3a(-a^2cx^2+c)^{\frac{3}{2}}}$	31
trager	$\frac{(ax-2)\sqrt{-a^2cx^2+c}}{3c^2(ax-1)^2a}$	34
default	$\frac{x}{c\sqrt{-a^2cx^2+c}}+\frac{\frac{2}{3ac\left(x-\frac{1}{a}\right)\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2\left(x-\frac{1}{a}\right)ac}}+\frac{2\left(-2\left(x-\frac{1}{a}\right)a^2c-2ac\right)}{3a\,c^2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2\left(x-\frac{1}{a}\right)ac}}}{a}$	127

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*x+1)^2*(a*x-2)/a/(-a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{-a^2cx^2+c}(ax-2)}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(-a^2*c*x^2 + c)*(a*x - 2)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{ax+1}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{x}{3\sqrt{-a^2cx^2+c}c} + \frac{2}{3\left(\sqrt{-a^2cx^2+c}a^2cx-\sqrt{-a^2cx^2+c}ac\right)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*x/(sqrt(-a^2*c*x^2 + c)*c) + 2/3/(sqrt(-a^2*c*x^2 + c)*a^2*c*x - sqrt
(-a^2*c*x^2 + c)*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(43) = 86$.

Time = 0.16 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.90

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{(ac-3\sqrt{-a^2c}\sqrt{c})\mathrm{sgn}(x)}{3\left(a^2c^{\frac{5}{2}}-\sqrt{-a^2c}ac^2\right)} - \frac{2\left(2\,a^2c+3\,a\sqrt{c}\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)+3\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^2\right)}{3\left(a\sqrt{c}+\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^3\mathrm{csgn}\,(x)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
1/3*(a*c - 3*sqrt(-a^2*c)*sqrt(c))*sgn(x)/(a^2*c^(5/2) - sqrt(-a^2*c)*a*c^
2) - 2/3*(2*a^2*c + 3*a*sqrt(c)*(sqrt(-a^2*c + c/x^2) - sqrt(c)/x) + 3*(sq
rt(-a^2*c + c/x^2) - sqrt(c)/x)^2)/((a*sqrt(c) + sqrt(-a^2*c + c/x^2) - sq
rt(c)/x)^3*c*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-2)}{3\,a\,c^2\,(a\,x-1)^2}$$

input
```
int((a*x + 1)/((c - a^2*c*x^2)^(3/2)*(a*x - 1)),x)
```

output
```
((c - a^2*c*x^2)^(1/2)*(a*x - 2))/(3*a*c^2*(a*x - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{2\sqrt{c}\left(\tan\left(\frac{asin(ax)}{2}\right)^3+1\right)}{3a\,c^2\left(\tan\left(\frac{asin(ax)}{2}\right)^3-3\tan\left(\frac{asin(ax)}{2}\right)^2+3\tan\left(\frac{asin(ax)}{2}\right)-1\right)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(2*sqrt(c)*(tan(asin(a*x)/2)**3 + 1))/(3*a*c**2*(tan(asin(a*x)/2)**3 - 3*t
an(asin(a*x)/2)**2 + 3*tan(asin(a*x)/2) - 1))
```

3.612 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{2(1+ax)}{5a(c-a^2cx^2)^{5/2}} - \frac{x}{5c(c-a^2cx^2)^{3/2}} - \frac{2x}{5c^2\sqrt{c-a^2cx^2}}$$

output
```
1/5*(-2*a*x-2)/a/(-a^2*c*x^2+c)^(5/2)-1/5*x/c/(-a^2*c*x^2+c)^(3/2)-2/5*x/c
^2/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{2+ax-4a^2x^2+2a^3x^3}{5ac^2(-1+ax)^2\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-1/5*(2 + a*x - 4*a^2*x^2 + 2*a^3*x^3)/(a*c^2*(-1 + a*x)^2*Sqrt[c - a^2*c*
x^2])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6717, 6691, 457, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int \frac{(ax+1)^2}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{457}$$

$$-c\left(\frac{3\int \frac{1}{(c-a^2cx^2)^{5/2}}dx}{5c}+\frac{2(ax+1)}{5ac\,(c-a^2cx^2)^{5/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{3\left(\frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}dx}{3c}+\frac{x}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}+\frac{2(ax+1)}{5ac\,(c-a^2cx^2)^{5/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$-c\left(\frac{3\left(\frac{2x}{3c^2\sqrt{c-a^2cx^2}}+\frac{x}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}+\frac{2(ax+1)}{5ac\,(c-a^2cx^2)^{5/2}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-(c*((2*(1 + a*x))/(5*a*c*(c - a^2*c*x^2)^(5/2)) + (3*(x/(3*c*(c - a^2*c*x
^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c - a^2*c*x^2])))/(5*c)))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result
gosper	$-\frac{(ax+1)^{2}(2a^{3}x^{3}-4a^{2}x^{2}+ax+2)}{5a(-a^{2}c\,x^{2}+c)^{\frac{5}{2}}}$
orering	$-\frac{(ax+1)^{2}(2a^{3}x^{3}-4a^{2}x^{2}+ax+2)}{5a(-a^{2}c\,x^{2}+c)^{\frac{5}{2}}}$
trager	$\frac{(2a^{3}x^{3}-4a^{2}x^{2}+ax+2)\sqrt{-a^{2}c\,x^{2}+c}}{5c^{3}(ax-1)^{3}a(ax+1)}$
default	$\frac{x}{3c(-a^{2}c\,x^{2}+c)^{\frac{3}{2}}}+\frac{2x}{3c^{2}\sqrt{-a^{2}c\,x^{2}+c}}+\frac{\frac{2}{5ac\left(x-\frac{1}{a}\right)\left(-\left(x-\frac{1}{a}\right)^{2}a^{2}c-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}-\frac{8a\left(-\frac{-2\left(x-\frac{1}{a}\right)a^{2}c-2ac}{6a^{2}c^{2}\left(-\left(x-\frac{1}{a}\right)^{2}a^{2}c-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}-\frac{}{3a^{2}}\right.}{5}}{a}$

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(a*x+1)^2*(2*a^3*x^3-4*a^2*x^2+a*x+2)/a/(-a^2*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\frac{(2\,a^3x^3-4\,a^2x^2+ax+2)\sqrt{-a^2cx^2+c}}{5\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/5*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 2)*sqrt(-a^2*c*x^2 + c)/(a^5*c^3*x^4 -
2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{ax+1}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{5}{2}}\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{2}{5\left((-a^2cx^2+c)^{\frac{3}{2}}a^2cx-(-a^2cx^2+c)^{\frac{3}{2}}ac\right)} - \frac{2\,x}{5\,\sqrt{-a^2cx^2+c}c^2} - \frac{x}{5\,(-a^2cx^2+c)^{\frac{3}{2}}c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
2/5/((-a^2*c*x^2 + c)^(3/2)*a^2*c*x - (-a^2*c*x^2 + c)^(3/2)*a*c) - 2/5*x/
(sqrt(-a^2*c*x^2 + c)*c^2) - 1/5*x/((-a^2*c*x^2 + c)^(3/2)*c)
```

Giac [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{ax+1}{(-a^2cx^2+c)^{\frac{5}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*(a*x - 1)), x)
```

Mupad [B] (verification not implemented)

Time = 13.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}\,\left(2\,a^3\,x^3-4\,a^2\,x^2+a\,x+2\right)}{5\,a\,c^3\,{\left(a\,x-1\right)}^3\,\left(a\,x+1\right)}$$

input
```
int((a*x + 1)/((c - a^2*c*x^2)^(5/2)*(a*x - 1)),x)
```

output
```
((c - a^2*c*x^2)^(1/2)*(a*x - 4*a^2*x^2 + 2*a^3*x^3 + 2))/(5*a*c^3*(a*x -
1)^3*(a*x + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.41

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,\left(\sqrt{-a^2x^2+1}\,a^2x^2-2\sqrt{-a^2x^2+1}\,ax+\sqrt{-a^2x^2+1}-4a^3x^3+8a^2x^2-2ax-\right.}{10\sqrt{-a^2x^2+1}\,a\,c^3\left(a^2x^2-2ax+1\right)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*(sqrt( - a**2*x**2 + 1)*a**2*x**2 - 2*sqrt( - a**2*x**2 + 1)*a*x
+ sqrt( - a**2*x**2 + 1) - 4*a**3*x**3 + 8*a**2*x**2 - 2*a*x - 4))/(10*sqr
t( - a**2*x**2 + 1)*a*c**3*(a**2*x**2 - 2*a*x + 1))
```

3.613 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{2(1+ax)}{7a(c-a^2cx^2)^{7/2}} - \frac{x}{7c(c-a^2cx^2)^{5/2}} - \frac{4x}{21c^2(c-a^2cx^2)^{3/2}} - \frac{8x}{21c^3\sqrt{c-a^2cx^2}}$$

output
```
1/7*(-2*a*x-2)/a/(-a^2*c*x^2+c)^(7/2)-1/7*x/c/(-a^2*c*x^2+c)^(5/2)-4/21*x/
c^2/(-a^2*c*x^2+c)^(3/2)-8/21*x/c^3/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\sqrt{1-a^2x^2}(6+9ax-24a^2x^2+4a^3x^3+16a^4x^4-8a^5x^5)}{21ac^3(1-ax)^{7/2}(1+ax)^{3/2}\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

output
```
-1/21*(Sqrt[1 - a^2*x^2]*(6 + 9*a*x - 24*a^2*x^2 + 4*a^3*x^3 + 16*a^4*x^4
- 8*a^5*x^5))/(a*c^3*(1 - a*x)^(7/2)*(1 + a*x)^(3/2)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6691, 457, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 6691}$$

$$-c\int \frac{(ax+1)^2}{(c-a^2cx^2)^{9/2}}dx$$

$$\Big\downarrow \text{ 457}$$

$$-c\left(\frac{5\int \frac{1}{(c-a^2cx^2)^{7/2}}dx}{7c}+\frac{2(ax+1)}{7ac\left(c-a^2cx^2\right)^{7/2}}\right)$$

$$\Big\downarrow \text{ 209}$$

$$-c\left(\frac{5\left(\frac{4\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}dx}{5c}+\frac{x}{5c(c-a^2cx^2)^{5/2}}\right)}{7c}+\frac{2(ax+1)}{7ac\left(c-a^2cx^2\right)^{7/2}}\right)$$

$$\Big\downarrow \text{ 209}$$

$$-c\left(\frac{5\left(\frac{4\left(\frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}dx}{3c}+\frac{x}{3c\left(c-a^2cx^2\right)^{3/2}}\right)}{5c}+\frac{x}{5c(c-a^2cx^2)^{5/2}}\right)}{7c}+\frac{2(ax+1)}{7ac\left(c-a^2cx^2\right)^{7/2}}\right)$$

$\big\downarrow$ 208

$$-c\left(\frac{5\left(\frac{4\left(\frac{2x}{3c^2\sqrt{c-a^2cx^2}}+\frac{x}{3c\left(c-a^2cx^2\right)^{3/2}}\right)}{5c}+\frac{x}{5c(c-a^2cx^2)^{5/2}}\right)}{7c}+\frac{2(ax+1)}{7ac\left(c-a^2cx^2\right)^{7/2}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

output
```
-(c*((2*(1 + a*x))/(7*a*c*(c - a^2*c*x^2)^(7/2)) + (5*(x/(5*c*(c - a^2*c*x
^2)^(5/2)) + (4*(x/(3*c*(c - a^2*c*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c - a^2
*c*x^2])))/(5*c)))/(7*c)))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.66

method	result
gosper	$\frac{(ax+1)^2(8a^5x^5-16a^4x^4-4a^3x^3+24a^2x^2-9ax-6)}{21a(-a^2cx^2+c)^{\frac{7}{2}}}$
orering	$\frac{(ax+1)^2(8a^5x^5-16a^4x^4-4a^3x^3+24a^2x^2-9ax-6)}{21a(-a^2cx^2+c)^{\frac{7}{2}}}$
trager	$\frac{(8a^5x^5-16a^4x^4-4a^3x^3+24a^2x^2-9ax-6)\sqrt{-a^2cx^2+c}}{21c^4(ax-1)^4(ax+1)^2a}$
default	$\frac{x}{5c(-a^2cx^2+c)^{\frac{5}{2}}}+\frac{\frac{4x}{15c\left(-a^2cx^2+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{-a^2cx^2+c}}}{c}+\frac{\frac{2}{7ac\left(x-\frac{1}{a}\right)\left(-\left(x-\frac{1}{a}\right)^2a^2c-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{5}{2}}}-\frac{12a\left(-\frac{-2\left(x-\frac{1}{a}\right)c}{10a^2c^2\left(-\left(x-\frac{1}{a}\right)^2a^2\right.}\right.}{}}{}$

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/21*(a*x+1)^2*(8*a^5*x^5-16*a^4*x^4-4*a^3*x^3+24*a^2*x^2-9*a*x-6)/a/(-a^2
*c*x^2+c)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{(8\,a^5x^5-16\,a^4x^4-4\,a^3x^3+24\,a^2x^2-9\,ax-6)\sqrt{-a^2cx^2+c}}{21\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
1/21*(8*a^5*x^5 - 16*a^4*x^4 - 4*a^3*x^3 + 24*a^2*x^2 - 9*a*x - 6)*sqrt(-a
^2*c*x^2 + c)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 -
 a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{ax+1}{(-c\,(ax-1)\,(ax+1))^{\frac{7}{2}}\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{2}{7\left((-a^2cx^2+c)^{\frac{5}{2}}a^2cx-(-a^2cx^2+c)^{\frac{5}{2}}ac\right)} - \frac{8\,x}{21\,\sqrt{-a^2cx^2+c}c^3} - \frac{4\,x}{21\,(-a^2cx^2+c)^{\frac{3}{2}}c^2} - \frac{x}{7\,(-a^2cx^2+c)^{\frac{5}{2}}c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
2/7/((-a^2*c*x^2 + c)^(5/2)*a^2*c*x - (-a^2*c*x^2 + c)^(5/2)*a*c) - 8/21*x
/(sqrt(-a^2*c*x^2 + c)*c^3) - 4/21*x/((-a^2*c*x^2 + c)^(3/2)*c^2) - 1/7*x/
((-a^2*c*x^2 + c)^(5/2)*c)
```

Giac [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{ax+1}{(-a^2cx^2+c)^{\frac{7}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*x + 1)/((-a^2*c*x^2 + c)^(7/2)*(a*x - 1)), x)
```

Mupad [B] (verification not implemented)

Time = 13.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.38

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}}{14\,a\,c^4\,(a\,x-1)^3} - \frac{\sqrt{c-a^2\,c\,x^2}}{28\,a\,c^4\,(a\,x-1)^4} - \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{11\,x}{42\,c^4}+\frac{5}{28\,a\,c^4}\right)}{(a\,x-1)^2\,(a\,x+1)^2} + \frac{8\,x\,\sqrt{c-a^2\,c\,x^2}}{21\,c^4\,(a\,x-1)\,(a\,x+1)}$$

input
```
int((a*x + 1)/((c - a^2*c*x^2)^(7/2)*(a*x - 1)),x)
```

output
```
(c - a^2*c*x^2)^(1/2)/(14*a*c^4*(a*x - 1)^3) - (c - a^2*c*x^2)^(1/2)/(28*a
*c^4*(a*x - 1)^4) - ((c - a^2*c*x^2)^(1/2)*((11*x)/(42*c^4) + 5/(28*a*c^4)
))/((a*x - 1)^2*(a*x + 1)^2) + (8*x*(c - a^2*c*x^2)^(1/2))/(21*c^4*(a*x -
1)*(a*x + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.55

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\sqrt{c}\,\left(9\sqrt{-a^2x^2+1}\,a^4x^4-18\sqrt{-a^2x^2+1}\,a^3x^3+18\sqrt{-a^2x^2+1}\,ax-9\sqrt{-a^2x^2+1}\right.}{42\sqrt{-a^2x^2+1}\,a\,c^4\left(a^4x^4-2a^3x^3+2a\right.}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x)
```

output
```
(sqrt(c)*(9*sqrt( - a**2*x**2 + 1)*a**4*x**4 - 18*sqrt( - a**2*x**2 + 1)*a
**3*x**3 + 18*sqrt( - a**2*x**2 + 1)*a*x - 9*sqrt( - a**2*x**2 + 1) - 16*a
**5*x**5 + 32*a**4*x**4 + 8*a**3*x**3 - 48*a**2*x**2 + 18*a*x + 12))/(42*s
qrt( - a**2*x**2 + 1)*a*c**4*(a**4*x**4 - 2*a**3*x**3 + 2*a*x - 1))
```

3.614 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = -\frac{2(1+ax)}{9a\,(c-a^2cx^2)^{9/2}} - \frac{x}{9c\,(c-a^2cx^2)^{7/2}} - \frac{2x}{15c^2\,(c-a^2cx^2)^{5/2}} - \frac{8x}{45c^3\,(c-a^2cx^2)^{3/2}} - \frac{16x}{45c^4\sqrt{c-a^2cx^2}}$$

output
```
1/9*(-2*a*x-2)/a/(-a^2*c*x^2+c)^(9/2)-1/9*x/c/(-a^2*c*x^2+c)^(7/2)-2/15*x/
c^2/(-a^2*c*x^2+c)^(5/2)-8/45*x/c^3/(-a^2*c*x^2+c)^(3/2)-16/45*x/c^4/(-a^2
*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{\sqrt{1-a^2x^2}(-10-25ax+60a^2x^2+10a^3x^3-80a^4x^4+24a^5x^5+32a^6x^6-16a^7x^7)}{45ac^4(1-ax)^{9/2}(1+ax)^{5/2}\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

output
```
(Sqrt[1 - a^2*x^2]*(-10 - 25*a*x + 60*a^2*x^2 + 10*a^3*x^3 - 80*a^4*x^4 +
24*a^5*x^5 + 32*a^6*x^6 - 16*a^7*x^7))/(45*a*c^4*(1 - a*x)^(9/2)*(1 + a*x)
^(5/2)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6717, 6691, 457, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{(c-a^2cx^2)^{9/2}}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int \frac{(ax+1)^2}{(c-a^2cx^2)^{11/2}}dx$$

$$\Big\downarrow \text{457}$$

$$-c\left(\frac{7\int \frac{1}{(c-a^2cx^2)^{9/2}}dx}{9c}+\frac{2(ax+1)}{9ac(c-a^2cx^2)^{9/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{7\left(\frac{6\int \frac{1}{(c-a^2cx^2)^{7/2}}dx}{7c}+\frac{x}{7c(c-a^2cx^2)^{7/2}}\right)}{9c}+\frac{2(ax+1)}{9ac(c-a^2cx^2)^{9/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{7\left(\frac{6\left(\frac{4\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}dx}{5c}+\frac{x}{5c\left(c-a^2cx^2\right)^{5/2}}\right)}{7c}+\frac{x}{7c(c-a^2cx^2)^{7/2}}\right)}{9c}+\frac{2(ax+1)}{9ac\left(c-a^2cx^2\right)^{9/2}}\right)$$

$\big\downarrow$ 209

$$-c\left(\frac{7\left(\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}dx}{3c}+\frac{x}{3c\left(c-a^2cx^2\right)^{3/2}}\right)}{5c}+\frac{x}{5c\left(c-a^2cx^2\right)^{5/2}}\right)}{7c}+\frac{x}{7c(c-a^2cx^2)^{7/2}}\right)}{9c}+\frac{2(ax+1)}{9ac\left(c-a^2cx^2\right)^{9/2}}\right)$$

$\big\downarrow$ 208

$$-c\left(\frac{7\left(\frac{6\left(\frac{4\left(\frac{2x}{3c^2\sqrt{c-a^2cx^2}}+\frac{x}{3c\left(c-a^2cx^2\right)^{3/2}}\right)}{5c}+\frac{x}{5c\left(c-a^2cx^2\right)^{5/2}}\right)}{7c}+\frac{x}{7c(c-a^2cx^2)^{7/2}}\right)}{9c}+\frac{2(ax+1)}{9ac\left(c-a^2cx^2\right)^{9/2}}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

output
```
-(c*((2*(1 + a*x))/(9*a*c*(c - a^2*c*x^2)^(9/2)) + (7*(x/(7*c*(c - a^2*c*x
^2)^(7/2)) + (6*(x/(5*c*(c - a^2*c*x^2)^(5/2)) + (4*(x/(3*c*(c - a^2*c*x^2
)^(3/2)) + (2*x)/(3*c^2*Sqrt[c - a^2*c*x^2])))/(5*c)))/(7*c)))/(9*c)))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67

method	result
gosper	$-\frac{(ax+1)^2(16a^7x^7-32x^6a^6-24a^5x^5+80a^4x^4-10a^3x^3-60a^2x^2+25ax+10)}{45a(-a^2cx^2+c)^{\frac{9}{2}}}$
orering	$-\frac{(ax+1)^2(16a^7x^7-32x^6a^6-24a^5x^5+80a^4x^4-10a^3x^3-60a^2x^2+25ax+10)}{45a(-a^2cx^2+c)^{\frac{9}{2}}}$
trager	$\frac{(16a^7x^7-32x^6a^6-24a^5x^5+80a^4x^4-10a^3x^3-60a^2x^2+25ax+10)\sqrt{-a^2cx^2+c}}{45c^5(ax-1)^5(ax+1)^3a}$
default	$\frac{x}{7c(-a^2cx^2+c)^{\frac{7}{2}}}+\frac{\frac{6x}{35c\left(-a^2cx^2+c\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15c\left(-a^2cx^2+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{-a^2cx^2+c}}\right)}{7c}}{c}+\frac{\frac{2}{9ac\left(x-\frac{1}{a}\right)\left(-\left(x-\frac{1}{a}\right)^2a^2c-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{7}{2}}}-\cdots}{\cdots}$ 16

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/45*(a*x+1)^2*(16*a^7*x^7-32*a^6*x^6-24*a^5*x^5+80*a^4*x^4-10*a^3*x^3-60
*a^2*x^2+25*a*x+10)/a/(-a^2*c*x^2+c)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.27

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{(16\,a^7x^7-32\,a^6x^6-24\,a^5x^5+80\,a^4x^4-10\,a^3x^3-60\,a^2x^2+25\,ax+10)\sqrt{-a^2cx^2}}{45\,(a^9c^5x^8-2\,a^8c^5x^7-2\,a^7c^5x^6+6\,a^6c^5x^5-6\,a^4c^5x^3+2\,a^3c^5x^2+2\,a^2c^5x-ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
1/45*(16*a^7*x^7 - 32*a^6*x^6 - 24*a^5*x^5 + 80*a^4*x^4 - 10*a^3*x^3 - 60*
a^2*x^2 + 25*a*x + 10)*sqrt(-a^2*c*x^2 + c)/(a^9*c^5*x^8 - 2*a^8*c^5*x^7 -
 2*a^7*c^5*x^6 + 6*a^6*c^5*x^5 - 6*a^4*c^5*x^3 + 2*a^3*c^5*x^2 + 2*a^2*c^5
*x - a*c^5)
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \int \frac{ax+1}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{9}{2}}\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(9/2),x)
```

output
```
Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(9/2)*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{2}{9\left((-a^2cx^2+c)^{\frac{7}{2}}a^2cx-(-a^2cx^2+c)^{\frac{7}{2}}ac\right)} - \frac{16\,x}{45\,\sqrt{-a^2cx^2+c}c^4} - \frac{8\,x}{45\,(-a^2cx^2+c)^{\frac{3}{2}}c^3} - \frac{2\,x}{15\,(-a^2cx^2+c)^{\frac{5}{2}}c^2} - \frac{x}{9\,(-a^2cx^2+c)^{\frac{7}{2}}c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
2/9/((-a^2*c*x^2 + c)^(7/2)*a^2*c*x - (-a^2*c*x^2 + c)^(7/2)*a*c) - 16/45*
x/(sqrt(-a^2*c*x^2 + c)*c^4) - 8/45*x/((-a^2*c*x^2 + c)^(3/2)*c^3) - 2/15*
x/((-a^2*c*x^2 + c)^(5/2)*c^2) - 1/9*x/((-a^2*c*x^2 + c)^(7/2)*c)
```

Giac [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \int \frac{ax+1}{(-a^2cx^2+c)^{\frac{9}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((a*x + 1)/((-a^2*c*x^2 + c)^(9/2)*(a*x - 1)), x)
```

Mupad [B] (verification not implemented)

Time = 13.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.48

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}}{72\,a\,c^5\,(a\,x-1)^5} - \frac{5\,\sqrt{c-a^2\,c\,x^2}}{144\,a\,c^5\,(a\,x-1)^4} + \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{31\,x}{120\,c^5}+\frac{5}{24\,a\,c^5}\right)}{(a\,x-1)^3\,(a\,x+1)^3} - \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{8\,x}{45\,c^5}-\frac{5}{144\,a\,c^5}\right)}{(a\,x-1)^2\,(a\,x+1)^2} + \frac{16\,x\,\sqrt{c-a^2\,c\,x^2}}{45\,c^5\,(a\,x-1)\,(a\,x+1)}$$

input
```
int((a*x + 1)/((c - a^2*c*x^2)^(9/2)*(a*x - 1)),x)
```

output
```
(c - a^2*c*x^2)^(1/2)/(72*a*c^5*(a*x - 1)^5) - (5*(c - a^2*c*x^2)^(1/2))/(
144*a*c^5*(a*x - 1)^4) + ((c - a^2*c*x^2)^(1/2)*((31*x)/(120*c^5) + 5/(24*
a*c^5)))/((a*x - 1)^3*(a*x + 1)^3) - ((c - a^2*c*x^2)^(1/2)*((8*x)/(45*c^5
) - 5/(144*a*c^5)))/((a*x - 1)^2*(a*x + 1)^2) + (16*x*(c - a^2*c*x^2)^(1/2
))/(45*c^5*(a*x - 1)*(a*x + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.06

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{\sqrt{c}\left(25\sqrt{-a^2x^2+1}\,a^6x^6-50\sqrt{-a^2x^2+1}\,a^5x^5-25\sqrt{-a^2x^2+1}\,a^4x^4+100\sqrt{-a^2x}\right.}{90}$$

input
```
int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x)
```

output
```
(sqrt(c)*(25*sqrt( - a**2*x**2 + 1)*a**6*x**6 - 50*sqrt( - a**2*x**2 + 1)*
a**5*x**5 - 25*sqrt( - a**2*x**2 + 1)*a**4*x**4 + 100*sqrt( - a**2*x**2 +
1)*a**3*x**3 - 25*sqrt( - a**2*x**2 + 1)*a**2*x**2 - 50*sqrt( - a**2*x**2
+ 1)*a*x + 25*sqrt( - a**2*x**2 + 1) - 32*a**7*x**7 + 64*a**6*x**6 + 48*a*
*5*x**5 - 160*a**4*x**4 + 20*a**3*x**3 + 120*a**2*x**2 - 50*a*x - 20))/(90
*sqrt( - a**2*x**2 + 1)*a*c**5*(a**6*x**6 - 2*a**5*x**5 - a**4*x**4 + 4*a*
*3*x**3 - a**2*x**2 - 2*a*x + 1))
```

3.615 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 185

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=-\frac{8(1+ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1+ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{2(1+ax)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1+ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

output
```
-8/7*(a*x+1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3/2*(a*x+
1)^8*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-2/3*(a*x+1)^9*(-a^2
*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(a*x+1)^10*(-a^2*c*x^2+c
)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{c^4(1+ax)^7\sqrt{c-a^2cx^2}(-44+98ax-77a^2x^2+21a^3x^3)}{210a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

output
```
(c^4*(1 + a*x)^7*Sqrt[c - a^2*c*x^2]*(-44 + 98*a*x - 77*a^2*x^2 + 21*a^3*x
^3))/(210*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2 c x^2\right)^{9/2} e^{3 \coth^{-1}(a x)} \, dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2 c x^2\right)^{9/2} \int e^{3 \coth^{-1}(a x)} \left(1-\frac{1}{a^2 x^2}\right)^{9/2} x^9 dx}{x^9 \left(1-\frac{1}{a^2 x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2 c x^2\right)^{9/2} \int -(1-a x)^3 (a x+1)^6 dx}{a^9 x^9 \left(1-\frac{1}{a^2 x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(c-a^2 c x^2\right)^{9/2} \int (1-a x)^3 (a x+1)^6 dx}{a^9 x^9 \left(1-\frac{1}{a^2 x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\left(c-a^2 c x^2\right)^{9/2} \int \left(-(a x+1)^9+6 (a x+1)^8-12 (a x+1)^7+8 (a x+1)^6\right) dx}{a^9 x^9 \left(1-\frac{1}{a^2 x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-\frac{(a x+1)^{10}}{10 a}+\frac{2 (a x+1)^9}{3 a}-\frac{3 (a x+1)^8}{2 a}+\frac{8 (a x+1)^7}{7 a}\right) \left(c-a^2 c x^2\right)^{9/2}}{a^9 x^9 \left(1-\frac{1}{a^2 x^2}\right)^{9/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

output
```
-(((c - a^2*c*x^2)^(9/2)*((8*(1 + a*x)^7)/(7*a) - (3*(1 + a*x)^8)/(2*a) +
(2*(1 + a*x)^9)/(3*a) - (1 + a*x)^10/(10*a)))/(a^9*(1 - 1/(a^2*x^2))^(9/2)
*x^9))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.54

method	result	size
gosper	$\frac{x(21a^9x^9+70a^8x^8-240x^6a^6-210a^5x^5+252a^4x^4+420a^3x^3-315ax-210)(-a^2cx^2+c)^{\frac{9}{2}}}{210(ax-1)^3(ax+1)^6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	100
orering	$\frac{x(21a^9x^9+70a^8x^8-240x^6a^6-210a^5x^5+252a^4x^4+420a^3x^3-315ax-210)(-a^2cx^2+c)^{\frac{9}{2}}}{210(ax-1)^3(ax+1)^6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	100
default	$\frac{(21a^9x^9+70a^8x^8-240x^6a^6-210a^5x^5+252a^4x^4+420a^3x^3-315ax-210)x\,c^4\sqrt{-c(a^2x^2-1)}\,(ax-1)}{210(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	102

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE
)
```

output
```
1/210*x*(21*a^9*x^9+70*a^8*x^8-240*a^6*x^6-210*a^5*x^5+252*a^4*x^4+420*a^3
*x^3-315*a*x-210)*(-a^2*c*x^2+c)^(9/2)/(a*x-1)^3/(a*x+1)^6/((a*x-1)/(a*x+1
))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.51

$$\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx = \frac{(21\,a^9c^4x^{10}+70\,a^8c^4x^9-240\,a^6c^4x^7-210\,a^5c^4x^6+252\,a^4c^4x^5+420\,a^3c^4x^4-315\,ac^4x^2}{210\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fri
cas")
```

output
```
1/210*(21*a^9*c^4*x^10 + 70*a^8*c^4*x^9 - 240*a^6*c^4*x^7 - 210*a^5*c^4*x^
6 + 252*a^4*c^4*x^5 + 420*a^3*c^4*x^4 - 315*a*c^4*x^2 - 210*c^4*x)*sqrt(-a
^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.10

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\left(21\,a^{11}\sqrt{-c}c^4x^{11}+49\,a^{10}\sqrt{-c}c^4x^{10}-70\,a^9\sqrt{-c}c^4x^9-240\,a^8\sqrt{-c}c^4x^8+30\,a^7\sqrt{-c}c^4x^7\right.}{21}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="max
ima")
```

output
```
1/210*(21*a^11*sqrt(-c)*c^4*x^11 + 49*a^10*sqrt(-c)*c^4*x^10 - 70*a^9*sqrt
(-c)*c^4*x^9 - 240*a^8*sqrt(-c)*c^4*x^8 + 30*a^7*sqrt(-c)*c^4*x^7 + 462*a^
6*sqrt(-c)*c^4*x^6 + 168*a^5*sqrt(-c)*c^4*x^5 - 420*a^4*sqrt(-c)*c^4*x^4 -
 315*a^3*sqrt(-c)*c^4*x^3 + 105*a^2*sqrt(-c)*c^4*x^2 + 210*sqrt(-c)*c^4)*(
a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1))
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,9,9,4,0]%%%}+%%%{3,[0,8,8,4,0]%%%}+%%%{-8,[0,6,6,4,0]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int \frac{\left(c-a^2\,c\,x^2\right)^{9/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.39

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\sqrt{c}\,c^4i(-21a^{10}x^{10}-70a^9x^9+240a^7x^7+210a^6x^6-252a^5x^5-420a^4x^4+315a^2x^2+210}{210a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x)
```

output
```
(sqrt(c)*c**4*i*( - 21*a**10*x**10 - 70*a**9*x**9 + 240*a**7*x**7 + 210*a*
*6*x**6 - 252*a**5*x**5 - 420*a**4*x**4 + 315*a**2*x**2 + 210*a*x - 212))/
(210*a)
```

3.616 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{2(1+ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{4(1+ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1+ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

output
```
2/3*(a*x+1)^6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-4/7*(a*x+1)
^7*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(a*x+1)^8*(-a^2*c*
x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.45

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{c^3(1+ax)^6\left(37-54ax+21a^2x^2\right)\sqrt{c-a^2cx^2}}{168a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

output
```
-1/168*(c^3*(1 + a*x)^6*(37 - 54*a*x + 21*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(a
^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.57, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{7/2}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7dx}{x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2}\int (1-ax)^2(ax+1)^5dx}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2}\int \left((ax+1)^7-4(ax+1)^6+4(ax+1)^5\right)dx}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{(ax+1)^8}{8a}-\frac{4(ax+1)^7}{7a}+\frac{2(ax+1)^6}{3a}\right)\left(c-a^2cx^2\right)^{7/2}}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

output
```
((c - a^2*c*x^2)^(7/2)*((2*(1 + a*x)^6)/(3*a) - (4*(1 + a*x)^7)/(7*a) + (1
 + a*x)^8/(8*a)))/(a^7*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{x\left(21a^{7}x^{7}+72x^{6}a^{6}+28a^{5}x^{5}-168a^{4}x^{4}-210a^{3}x^{3}+56a^{2}x^{2}+252ax+168\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{7}{2}}}{168\left(ax-1\right)^{2}\left(ax+1\right)^{5}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	100
orering	$\frac{x\left(21a^{7}x^{7}+72x^{6}a^{6}+28a^{5}x^{5}-168a^{4}x^{4}-210a^{3}x^{3}+56a^{2}x^{2}+252ax+168\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{7}{2}}}{168\left(ax-1\right)^{2}\left(ax+1\right)^{5}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	100
default	$-\frac{\left(21a^{7}x^{7}+72x^{6}a^{6}+28a^{5}x^{5}-168a^{4}x^{4}-210a^{3}x^{3}+56a^{2}x^{2}+252ax+168\right)x\,c^{3}\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax-1\right)}{168\left(ax+1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	102

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE
)
```

output
```
1/168*x*(21*a^7*x^7+72*a^6*x^6+28*a^5*x^5-168*a^4*x^4-210*a^3*x^3+56*a^2*x
^2+252*a*x+168)*(-a^2*c*x^2+c)^(7/2)/(a*x-1)^2/(a*x+1)^5/((a*x-1)/(a*x+1))
^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = -\frac{\left(21\,a^7c^3x^8+72\,a^6c^3x^7+28\,a^5c^3x^6-168\,a^4c^3x^5-210\,a^3c^3x^4+56\,a^2c^3x^3+252\,ac^3x^2+168\,c^3x\right)\sqrt{-a^2c}}{168\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fri
cas")
```

output
```
-1/168*(21*a^7*c^3*x^8 + 72*a^6*c^3*x^7 + 28*a^5*c^3*x^6 - 168*a^4*c^3*x^5
 - 210*a^3*c^3*x^4 + 56*a^2*c^3*x^3 + 252*a*c^3*x^2 + 168*c^3*x)*sqrt(-a^2
*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.24

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = -\frac{\left(21\,a^9\sqrt{-c}c^3x^9+51\,a^8\sqrt{-c}c^3x^8-44\,a^7\sqrt{-c}c^3x^7-196\,a^6\sqrt{-c}c^3x^6-42\,a^5\sqrt{-c}c^3x^5+266\,a^4\sqrt{-c}c^3x^4\right.}{168\,(a^3x^2+2\,a^2x+a)(ax-1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="max
ima")
```

output
```
-1/168*(21*a^9*sqrt(-c)*c^3*x^9 + 51*a^8*sqrt(-c)*c^3*x^8 - 44*a^7*sqrt(-c
)*c^3*x^7 - 196*a^6*sqrt(-c)*c^3*x^6 - 42*a^5*sqrt(-c)*c^3*x^5 + 266*a^4*s
qrt(-c)*c^3*x^4 + 196*a^3*sqrt(-c)*c^3*x^3 - 84*a^2*sqrt(-c)*c^3*x^2 - 168
*sqrt(-c)*c^3)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1))
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,7,7,3,0]%%%}+%%%{-3,[0,6,6,3,0]%%%}+%%%{-1,[0,5,5,3,
0]%%%}+%%
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int \frac{\left(c-a^2\,c\,x^2\right)^{7/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.53

$$\int e^{3\coth^{-1}(ax)}\big(c -a^2cx^2\big)^{7/2}\,dx=\frac{\sqrt{c}\,c^3 i(21a^8x^8+72a^7x^7+28a^6x^6-168a^5x^5-210a^4x^4+56a^3x^3+252a^2x^2+168ax-2}{168a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x)
```

output
```
(sqrt(c)*c**3*i*(21*a**8*x**8 + 72*a**7*x**7 + 28*a**6*x**6 - 168*a**5*x**
5 - 210*a**4*x**4 + 56*a**3*x**3 + 252*a**2*x**2 + 168*a*x - 219))/(168*a)
```

3.617 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\frac{2(1+ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1+ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

output
```
-2/5*(a*x+1)^5*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(a*x+1
)^6*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2(1+ax)^5(-7+5ax)\sqrt{c-a^2cx^2}}{30a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

output
```
(c^2*(1 + a*x)^5*(-7 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sqrt[1 - 1/(a^2
*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{5/2} e^{3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2} \int e^{3\coth^{-1}(ax)} \left(1-\frac{1}{a^2x^2}\right)^{5/2} x^5 dx}{x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2} \int -\left((1-ax)(ax+1)^4\right) dx}{a^5x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(c-a^2cx^2\right)^{5/2} \int (1-ax)(ax+1)^4 dx}{a^5x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\left(c-a^2cx^2\right)^{5/2} \int \left(2(ax+1)^4-(ax+1)^5\right) dx}{a^5x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\frac{2(ax+1)^5}{5a}-\frac{(ax+1)^6}{6a}\right) \left(c-a^2cx^2\right)^{5/2}}{a^5x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

output
```
-(((c - a^2*c*x^2)^(5/2)*((2*(1 + a*x)^5)/(5*a) - (1 + a*x)^6/(6*a)))/(a^5
*(1 - 1/(a^2*x^2))^(5/2)*x^5))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

method	result	size
gosper	$\frac{x\left(5a^5x^5+18a^4x^4+15a^3x^3-20a^2x^2-45ax-30\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{30(ax-1)(ax+1)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	84
orering	$\frac{x\left(5a^5x^5+18a^4x^4+15a^3x^3-20a^2x^2-45ax-30\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{30(ax-1)(ax+1)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	84
default	$\frac{\left(5a^5x^5+18a^4x^4+15a^3x^3-20a^2x^2-45ax-30\right)x\,c^2\sqrt{-c\left(a^2x^2-1\right)}\,(ax-1)}{30(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	86

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
1/30*x*(5*a^5*x^5+18*a^4*x^4+15*a^3*x^3-20*a^2*x^2-45*a*x-30)*(-a^2*c*x^2+
c)^(5/2)/(a*x-1)/(a*x+1)^4/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx=\frac{(5\,a^5c^2x^6+18\,a^4c^2x^5+15\,a^3c^2x^4-20\,a^2c^2x^3-45\,ac^2x^2-30\,c^2x)\sqrt{-a^2c}}{30\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

output
```
1/30*(5*a^5*c^2*x^6 + 18*a^4*c^2*x^5 + 15*a^3*c^2*x^4 - 20*a^2*c^2*x^3 - 4
5*a*c^2*x^2 - 30*c^2*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.51

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\left(5\,a^7\sqrt{-c}c^2x^7+13\,a^6\sqrt{-c}c^2x^6-3\,a^5\sqrt{-c}c^2x^5-35\,a^4\sqrt{-c}c^2x^4-25\,a^3\sqrt{-c}c^2x^3+15\,a^2\right.}{30\,(a^3x^2+2\,a^2x+a)(ax-1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

output
```
1/30*(5*a^7*sqrt(-c)*c^2*x^7 + 13*a^6*sqrt(-c)*c^2*x^6 - 3*a^5*sqrt(-c)*c^
2*x^5 - 35*a^4*sqrt(-c)*c^2*x^4 - 25*a^3*sqrt(-c)*c^2*x^3 + 15*a^2*sqrt(-c
)*c^2*x^2 + 30*sqrt(-c)*c^2)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1
))
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,5,5,2,0]%%%}+%%%{3,[0,4,4,2,0]%%%}+%%%{2,[0,3,3,2,0]%
%%}+%%%{-
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^{5/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\sqrt{c}\,c^2i(-5a^6x^6-18a^5x^5-15a^4x^4+20a^3x^3+45a^2x^2+30ax-57)}{30a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*c**2*i*( - 5*a**6*x**6 - 18*a**5*x**5 - 15*a**4*x**4 + 20*a**3*x**
*3 + 45*a**2*x**2 + 30*a*x - 57))/(30*a)
```

3.618 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{(1+ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

output
```
1/4*(a*x+1)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{c\sqrt{c-a^2cx^2}(4+6ax+4a^2x^2+a^3x^3)}{4a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

output
```
-1/4*(c*Sqrt[c - a^2*c*x^2]*(4 + 6*a*x + 4*a^2*x^2 + a^3*x^3))/(a*Sqrt[1 -
 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6746, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{3/2} e^{3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2} \int e^{3\coth^{-1}(ax)} \left(1-\frac{1}{a^2x^2}\right)^{3/2} x^3dx}{x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2} \int (ax+1)^3dx}{a^3x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{17}$$

$$\frac{(ax+1)^4 \left(c-a^2cx^2\right)^{3/2}}{4a^4x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

output
```
((1 + a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{(ax-1)(ax+1)^{2}\sqrt{-c(a^{2}x^{2}-1)}\,c}{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	48
gosper	$\frac{x(a^{3}x^{3}+4a^{2}x^{2}+6ax+4)\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{4(ax+1)^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	60
orering	$\frac{x(a^{3}x^{3}+4a^{2}x^{2}+6ax+4)\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{4(ax+1)^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	60

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/4/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*c/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{\left(a^3cx^4+4\,a^2cx^3+6\,acx^2+4\,cx\right)\sqrt{-a^2c}}{4\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

output
```
-1/4*(a^3*c*x^4 + 4*a^2*c*x^3 + 6*a*c*x^2 + 4*c*x)*sqrt(-a^2*c)/a
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\frac{\left(-c(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(40) = 80.

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.11

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{\left(a^5\sqrt{-c}cx^5+3\,a^4\sqrt{-c}cx^4+2\,a^3\sqrt{-c}cx^3-2\,a^2\sqrt{-c}cx^2-4\,\sqrt{-c}c\right)(ax+1)^2}{4\,(a^3x^2+2\,a^2x+a)(ax-1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

output
```
-1/4*(a^5*sqrt(-c)*c*x^5 + 3*a^4*sqrt(-c)*c*x^4 + 2*a^3*sqrt(-c)*c*x^3 - 2
*a^2*sqrt(-c)*c*x^2 - 4*sqrt(-c)*c)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(
a*x - 1))
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,3,3,1,0]%%%}+%%%{-3,[0,2,2,1,0]%%%}+%%%{-3,[0,1,1,1,
0]%%%}+%%
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{\sqrt{c}\,ci(a^4x^4+4a^3x^3+6a^2x^2+4ax-15)}{4a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*c*i*(a**4*x**4 + 4*a**3*x**3 + 6*a**2*x**2 + 4*a*x - 15))/(4*a)
```

3.619 $\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 113

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1
-1/a^2/x^2)^(1/2)+4*(-a^2*c*x^2+c)^(1/2)*ln(-a*x+1)/a^2/(1-1/a^2/x^2)^(1/2
)/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1-ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((3*x)/a + x^2/2 + (4*Log[1 - a*x])/a^2))/(Sqrt[1 - 1
/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-ax+\frac{4}{1-ax}-3\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}-3x\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a))/(a*Sqrt[1
- 1/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{(a^2x^2+6ax+8\ln(ax-1))\sqrt{-c(a^2x^2-1)}\,(ax-1)}{2a(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	67

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a/(a*x+1)^2
/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(a^2x^2+6\,ax+8\,\log{(ax-1)})\sqrt{-a^2c}}{2\,a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(-a^2*c)/a^2
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{1}{2}\sqrt{-c}\left(\frac{ax^2+6x}{\operatorname{sgn}(ax+1)}+\frac{a^3x^2\operatorname{sgn}(ax+1)+6a^2x\operatorname{sgn}(ax+1)}{a^2}+\frac{16\log(|ax-1|)}{a\operatorname{sgn}(ax+1)}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
1/2*sqrt(-c)*((a*x^2 + 6*x)/sgn(a*x + 1) + (a^3*x^2*sgn(a*x + 1) + 6*a^2*x
*sgn(a*x + 1))/a^2 + 16*log(abs(a*x - 1))/(a*sgn(a*x + 1)))
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.28

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c}\,i\left(-16\log\left(\sqrt{-ax+1}\right)-a^2x^2-6ax+7\right)}{2a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*i*( - 16*log(sqrt( - a*x + 1)) - a**2*x**2 - 6*a*x + 7))/(2*a)
```

3.620 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

output
```
2*(1-1/a^2/x^2)^(1/2)*x/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+(1-1/a^2/x^2)^(1/2)*
x*ln(-a*x+1)/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(\frac{2}{1-ax}+\log(1-ax)\right)}{\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(2/(1 - a*x) + Log[1 - a*x]))/Sqrt[c - a^2*c*x^2]
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\int \frac{ax+1}{(1-ax)^2}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{1}{ax-1}+\frac{2}{(ax-1)^2}\right)dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{2}{a(1-ax)}+\frac{\log(1-ax)}{a}\right)}{\sqrt{c-a^2cx^2}}$$

input
```
Int[E^(3*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x*(2/(a*(1 - a*x)) + Log[1 - a*x]/a))/Sqrt[c - a^
2*c*x^2]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\left(a \ln\left(a x -1\right) x -\ln\left(a x -1\right)-2\right)}{a c \left(a x +1\right)^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	64

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-(-c*(a^2*x^2-1))^(1/2)*(a*ln(a*x-1)*x-ln(a*x-1)-2)/a/c/(a*x+1)^2/((a*x-1)
/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=-\frac{\sqrt{-a^2c}((ax-1)\log(ax-1)-2)}{a^3cx-a^2c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
-sqrt(-a^2*c)*((a*x - 1)*log(a*x - 1) - 2)/(a^3*c*x - a^2*c)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c(ax-1)(ax+1)}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(((a*x - 1)/(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{1}{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(1/(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{1}{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.52

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2\sqrt{c}\,i\left(\log\left(\sqrt{-ax+1}\right)ax-\log\left(\sqrt{-ax+1}\right)-1\right)}{ac\,(ax-1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(2*sqrt(c)*i*(log(sqrt( - a*x + 1))*a*x - log(sqrt( - a*x + 1)) - 1))/(a*c
*(a*x - 1))
```

3.621 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)^2\left(c-a^2cx^2\right)^{3/2}}$$

output
```
-1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)^2/(-a^2*c*x^2+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-a^2cx^2}}{2c^2(-1+ax)^3(1+ax)}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-1/2*(Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])/(c^2*(-1 + a*x)^3*(1 + 
a*x))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6746, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{1}{(ax-1)^3}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)^2\,(c-a^2cx^2)^{3/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-1/2*(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/((1 - a*x)^2*(c - a^2*c*x^2)^(3/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{a x -1}{2 a \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}$	39
orering	$\frac{x \left(a x -2\right) \left(a x -1\right)}{2 \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}$	42
default	$-\frac{\sqrt{-c \left(a^{2} x^{2}-1\right)}}{2 \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(a x +1\right) \left(a^{2} x^{2}-1\right) c^{2} a}$	56

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/2*(a*x-1)/a/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{-a^2c}}{2\,(a^4c^2x^2-2\,a^3c^2x+a^2c^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

output
```
-1/2*sqrt(-a^2*c)/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.91

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(\frac{1}{2\,a^3\,c}+\frac{x}{2\,a^2\,c}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}+x^2\,\sqrt{c-a^2\,c\,x^2}-\frac{2\,x\,\sqrt{c-a^2\,c\,x^2}}{a}}$$

input
```
int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
((1/(2*a^3*c) + x/(2*a^2*c))*((a*x - 1)/(a*x + 1))^(1/2))/((c - a^2*c*x^2)
^(1/2)/a^2 + x^2*(c - a^2*c*x^2)^(1/2) - (2*x*(c - a^2*c*x^2)^(1/2))/a)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.55

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,i}{2a\,c^2\,(a^2x^2-2ax+1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*i)/(2*a*c**2*(a**2*x**2 - 2*a*x + 1))
```

3.622 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1-ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\text{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

output
```
1/6*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^3/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1
-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2
)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*
arctanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-10+9ax-3a^2x^2+3(-1+ax)^3\text{arctanh}(ax)\right)}{24c^2(-1+ax)^3\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(-10 + 9*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^3*ArcTanh
[a*x]))/(24*c^2*(-1 + a*x)^3*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6746, 6747, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{1}{(1-ax)^4(ax+1)}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{54}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(\frac{1}{8(ax-1)^2}-\frac{1}{4(ax-1)^3}+\frac{1}{2(ax-1)^4}-\frac{1}{8(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\text{arctanh}(ax)}{8a}+\frac{1}{8a(1-ax)}+\frac{1}{8a(1-ax)^2}+\frac{1}{6a(1-ax)^3}\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(6*a*(1 - a*x)^3) + 1/(8*a*(1 - a*x)^2
) + 1/(8*a*(1 - a*x)) + ArcTanh[a*x]/(8*a)))/(c - a^2*c*x^2)^(5/2)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.91

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(3\ln(ax+1)x^3a^3-3a^3\ln(ax-1)x^3-9\ln(ax+1)x^2a^2+9a^2\ln(ax-1)x^2-6a^2x^2+9\ln(ax+1)xa-9a\ln(ax-1)x+18}{48\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-1)(ax+1)(a^2x^2-1)c^3a}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/48/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln
(a*x+1)*x^3*a^3-3*a^3*ln(a*x-1)*x^3-9*ln(a*x+1)*x^2*a^2+9*a^2*ln(a*x-1)*x^
2-6*a^2*x^2+9*ln(a*x+1)*x*a-9*a*ln(a*x-1)*x+18*a*x-3*ln(a*x+1)+3*ln(a*x-1)
-20)/(a^2*x^2-1)/c^3/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.75

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{3\,(a^4x^3-3\,a^3x^2+3\,a^2x-a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\,(3\,a^2x^2-9\,ax+10)\sqrt{-a^2c}}{48\,(a^5c^3x^3-3\,a^4c^3x^2+3\,a^3c^3x-a^2c^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

output
```
-1/48*(3*(a^4*x^3 - 3*a^3*x^2 + 3*a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*s
qrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(3*a^2*x^2 - 9*a*x + 10)*sq
rt(-a^2*c))/(a^5*c^3*x^3 - 3*a^4*c^3*x^2 + 3*a^3*c^3*x - a^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(c-a^2\,c\,x^2)^{5/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.31

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,i\big(3\log\big(\sqrt{-ax+1}-\sqrt{2}\big)\,a^3x^3-9\log\big(\sqrt{-ax+1}-\sqrt{2}\big)\,a^2x^2+9\log\big(\sqrt{-ax+}}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*i*(3*log(sqrt( - a*x + 1) - sqrt(2))*a**3*x**3 - 9*log(sqrt( - a*
x + 1) - sqrt(2))*a**2*x**2 + 9*log(sqrt( - a*x + 1) - sqrt(2))*a*x - 3*lo
g(sqrt( - a*x + 1) - sqrt(2)) + 3*log(sqrt( - a*x + 1) + sqrt(2))*a**3*x**
3 - 9*log(sqrt( - a*x + 1) + sqrt(2))*a**2*x**2 + 9*log(sqrt( - a*x + 1) +
 sqrt(2))*a*x - 3*log(sqrt( - a*x + 1) + sqrt(2)) - 6*log(sqrt( - a*x + 1)
)*a**3*x**3 + 18*log(sqrt( - a*x + 1))*a**2*x**2 - 18*log(sqrt( - a*x + 1)
)*a*x + 6*log(sqrt( - a*x + 1)) - 6*a**2*x**2 + 18*a*x - 20))/(48*a*c**3*(
a**3*x**3 - 3*a**2*x**2 + 3*a*x - 1))
```

3.623 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 278

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^4\left(c-a^2cx^2\right)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1-ax)^3\left(c-a^2cx^2\right)^{7/2}} - \frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}} + \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)\left(c-a^2cx^2\right)^{7/2}} - \frac{5a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\text{arctanh}(ax)}{32\left(c-a^2cx^2\right)^{7/2}}$$

output
```
-1/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^4/(-a^2*c*x^2+c)^(7/2)-1/12*a^6
*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^3/(-a^2*c*x^2+c)^(7/2)-3/32*a^6*(1-1/a^2
/x^2)^(7/2)*x^7/(-a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(1-1/a^2/x^2)^(7/2
)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)+1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+
1)/(-a^2*c*x^2+c)^(7/2)-5/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^
2*c*x^2+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.36

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x(32-15ax-35a^2x^2+45a^3x^3-15a^4x^4+15(-1+ax)^4(1+ax)\text{arcta}}{96c^3(-1+ax)^4(1+ax)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(32 - 15*a*x - 35*a^2*x^2 + 45*a^3*x^3 - 15*a^4*x
^4 + 15*(-1 + a*x)^4*(1 + a*x)*ArcTanh[a*x]))/(96*c^3*(-1 + a*x)^4*(1 + a*
x)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.44, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6746, 6747, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int -\frac{1}{(1-ax)^5(ax+1)^2}dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int\frac{1}{(1-ax)^5(ax+1)^2}dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$\Big\downarrow$ 54

$$-\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int\left(\frac{1}{8(ax-1)^2}+\frac{1}{32(ax+1)^2}-\frac{3}{16(ax-1)^3}+\frac{1}{4(ax-1)^4}-\frac{1}{4(ax-1)^5}-\frac{5}{32(a^2x^2-1)}\right)dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$\Big\downarrow$ 2009

$$-\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(\frac{5\text{arctanh}(ax)}{32a}+\frac{1}{8a(1-ax)}-\frac{1}{32a(ax+1)}+\frac{3}{32a(1-ax)^2}+\frac{1}{12a(1-ax)^3}+\frac{1}{16a(1-ax)^4}\right)}{\left(c-a^2cx^2\right)^{7/2}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

output
```
-((a^7*(1 - 1/(a^2*x^2))^(7/2)*x^7*(1/(16*a*(1 - a*x)^4) + 1/(12*a*(1 - a*
x)^3) + 3/(32*a*(1 - a*x)^2) + 1/(8*a*(1 - a*x)) - 1/(32*a*(1 + a*x)) + (5
*ArcTanh[a*x])/(32*a)))/(c - a^2*c*x^2)^(7/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.87

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5-45\ln(ax+1)x^4a^4+45\ln(ax-1)x^4a^4-30a^4x^4+30\ln(ax+1)x^3a^3-30a^3\ln(}{192\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(a}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/192/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*
(15*ln(a*x+1)*x^5*a^5-15*ln(a*x-1)*x^5*a^5-45*ln(a*x+1)*x^4*a^4+45*ln(a*x-
1)*x^4*a^4-30*a^4*x^4+30*ln(a*x+1)*x^3*a^3-30*a^3*ln(a*x-1)*x^3+90*a^3*x^3
+30*ln(a*x+1)*x^2*a^2-30*a^2*ln(a*x-1)*x^2-70*a^2*x^2-45*ln(a*x+1)*x*a+45*
a*ln(a*x-1)*x-30*a*x+15*ln(a*x+1)-15*ln(a*x-1)+64)/(a^2*x^2-1)/c^4/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.68

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx =$$
$$-\frac{15\,(a^6x^5-3\,a^5x^4+2\,a^4x^3+2\,a^3x^2-3\,a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\,(15\,a^4x^4-45\,a^3x^3+}{192\,(a^7c^4x^5-3\,a^6c^4x^4+2\,a^5c^4x^3+2\,a^4c^4x^2-3\,a^3c^4x+a^2c^4)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fri
cas")
```

output
```
-1/192*(15*(a^6*x^5 - 3*a^5*x^4 + 2*a^4*x^3 + 2*a^3*x^2 - 3*a^2*x + a)*sqr
t(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(
15*a^4*x^4 - 45*a^3*x^3 + 35*a^2*x^2 + 15*a*x - 32)*sqrt(-a^2*c))/(a^7*c^4
*x^5 - 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 + 2*a^4*c^4*x^2 - 3*a^3*c^4*x + a^2*c
^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="max
ima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{1}{\left(c-a^2\,c\,x^2\right)^{7/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.42

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\sqrt{c}\,i\left(15\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^5x^5-45\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^4x^4+30\log\left(\sqrt{-ax\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x)
```

output
```
(sqrt(c)*i*(15*log(sqrt( - a*x + 1) - sqrt(2))*a**5*x**5 - 45*log(sqrt( -
a*x + 1) - sqrt(2))*a**4*x**4 + 30*log(sqrt( - a*x + 1) - sqrt(2))*a**3*x*
*3 + 30*log(sqrt( - a*x + 1) - sqrt(2))*a**2*x**2 - 45*log(sqrt( - a*x + 1
) - sqrt(2))*a*x + 15*log(sqrt( - a*x + 1) - sqrt(2)) + 15*log(sqrt( - a*x
 + 1) + sqrt(2))*a**5*x**5 - 45*log(sqrt( - a*x + 1) + sqrt(2))*a**4*x**4
+ 30*log(sqrt( - a*x + 1) + sqrt(2))*a**3*x**3 + 30*log(sqrt( - a*x + 1) +
 sqrt(2))*a**2*x**2 - 45*log(sqrt( - a*x + 1) + sqrt(2))*a*x + 15*log(sqrt
( - a*x + 1) + sqrt(2)) - 30*log(sqrt( - a*x + 1))*a**5*x**5 + 90*log(sqrt
( - a*x + 1))*a**4*x**4 - 60*log(sqrt( - a*x + 1))*a**3*x**3 - 60*log(sqrt
( - a*x + 1))*a**2*x**2 + 90*log(sqrt( - a*x + 1))*a*x - 30*log(sqrt( - a*
x + 1)) + 15*a**5*x**5 - 75*a**4*x**4 + 120*a**3*x**3 - 40*a**2*x**2 - 75*
a*x + 79))/(192*a*c**4*(a**5*x**5 - 3*a**4*x**4 + 2*a**3*x**3 + 2*a**2*x**
2 - 3*a*x + 1))
```

3.624 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 234

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{8(1-ax)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{32(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{8(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

output
```
8/3*(-a*x+1)^6*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-32/7*(-a*
x+1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3*(-a*x+1)^8*(-a^
2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-8/9*(-a*x+1)^9*(-a^2*c*x^2+c
)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(-a*x+1)^10*(-a^2*c*x^2+c)^(9/2)
/a^10/(1-1/a^2/x^2)^(9/2)/x^9
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{c^4(-1+ax)^6\sqrt{c-a^2cx^2}(193+528ax+588a^2x^2+308a^3x^3+63a^4x^4)}{630a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a^2*c*x^2)^(9/2)/E^ArcCoth[a*x],x]
```

output
```
(c^4*(-1 + a*x)^6*Sqrt[c - a^2*c*x^2]*(193 + 528*a*x + 588*a^2*x^2 + 308*a
^3*x^3 + 63*a^4*x^4))/(630*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.47, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{9/2}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{9/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9dx}{x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{9/2}\int -(1-ax)^5(ax+1)^4dx}{a^9x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(c-a^2cx^2\right)^{9/2}\int(1-ax)^5(ax+1)^4dx}{a^9x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\left(c-a^2cx^2\right)^{9/2}\int\left((1-ax)^9-8(1-ax)^8+24(1-ax)^7-32(1-ax)^6+16(1-ax)^5\right)dx}{a^9x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-\frac{(1-ax)^{10}}{10a}+\frac{8(1-ax)^9}{9a}-\frac{3(1-ax)^8}{a}+\frac{32(1-ax)^7}{7a}-\frac{8(1-ax)^6}{3a}\right)\left(c-a^2cx^2\right)^{9/2}}{a^9x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

input
```
Int[(c - a^2*c*x^2)^(9/2)/E^ArcCoth[a*x],x]
```

output
```
-(((c - a^2*c*x^2)^(9/2)*((-8*(1 - a*x)^6)/(3*a) + (32*(1 - a*x)^7)/(7*a)
 - (3*(1 - a*x)^8)/a + (8*(1 - a*x)^9)/(9*a) - (1 - a*x)^10/(10*a)))/(a^9*(
1 - 1/(a^2*x^2))^(9/2)*x^9))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\left(63a^{9}x^{9}-70a^{8}x^{8}-315a^{7}x^{7}+360x^{6}a^{6}+630a^{5}x^{5}-756a^{4}x^{4}-630a^{3}x^{3}+840a^{2}x^{2}+315ax-630\right)x\,c^{4}\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\sqrt{\frac{ax-1}{ax+1}}}{630ax-630}$	113
gosper	$\frac{x\left(63a^{9}x^{9}-70a^{8}x^{8}-315a^{7}x^{7}+360x^{6}a^{6}+630a^{5}x^{5}-756a^{4}x^{4}-630a^{3}x^{3}+840a^{2}x^{2}+315ax-630\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}}{630\left(ax+1\right)^{4}\left(ax-1\right)^{5}}$	116
orering	$\frac{x\left(63a^{9}x^{9}-70a^{8}x^{8}-315a^{7}x^{7}+360x^{6}a^{6}+630a^{5}x^{5}-756a^{4}x^{4}-630a^{3}x^{3}+840a^{2}x^{2}+315ax-630\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}}{630\left(ax+1\right)^{4}\left(ax-1\right)^{5}}$	116

input
```
int((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/630*(63*a^9*x^9-70*a^8*x^8-315*a^7*x^7+360*a^6*x^6+630*a^5*x^5-756*a^4*x
^4-630*a^3*x^3+840*a^2*x^2+315*a*x-630)*x*c^4*(-c*(a^2*x^2-1))^(1/2)*((a*x
-1)/(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.50

$$\int e^{-\coth^{-1}(ax)}\left(c-a^{2}cx^{2}\right)^{9/2}\,dx=\frac{\left(63\,a^{9}c^{4}x^{10}-70\,a^{8}c^{4}x^{9}-315\,a^{7}c^{4}x^{8}+360\,a^{6}c^{4}x^{7}+630\,a^{5}c^{4}x^{6}-756\,a^{4}c^{4}x^{5}-630\,a^{3}c^{4}x\right.}{630\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

output
```
1/630*(63*a^9*c^4*x^10 - 70*a^8*c^4*x^9 - 315*a^7*c^4*x^8 + 360*a^6*c^4*x^
7 + 630*a^5*c^4*x^6 - 756*a^4*c^4*x^5 - 630*a^3*c^4*x^4 + 840*a^2*c^4*x^3
+ 315*a*c^4*x^2 - 630*c^4*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(9/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((-a^2*c*x^2 + c)^(9/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.79

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{1}{630}\left(63\,a^9c^4x^{10}\text{sgn}(ax+1)-70\,a^8c^4x^9\text{sgn}(ax+1)-315\,a^7c^4x^8\text{sgn}(ax+1)+360\,a^6c^4\right.$$

input
```
integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

output
```
1/630*(63*a^9*c^4*x^10*sgn(a*x + 1) - 70*a^8*c^4*x^9*sgn(a*x + 1) - 315*a^
7*c^4*x^8*sgn(a*x + 1) + 360*a^6*c^4*x^7*sgn(a*x + 1) + 630*a^5*c^4*x^6*sg
n(a*x + 1) - 756*a^4*c^4*x^5*sgn(a*x + 1) - 630*a^3*c^4*x^4*sgn(a*x + 1) +
 840*a^2*c^4*x^3*sgn(a*x + 1) + 315*a*c^4*x^2*sgn(a*x + 1) - 630*c^4*x*sgn
(a*x + 1) - 319*c^4*sgn(a*x + 1)/a)*sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int \left(c-a^2\,c\,x^2\right)^{9/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.38

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\sqrt{c}\,c^4i(-63a^{10}x^{10}+70a^9x^9+315a^8x^8-360a^7x^7-630a^6x^6+756a^5x^5+630a^4x^4-840}{630a}$$

input
```
int((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*c**4*i*( - 63*a**10*x**10 + 70*a**9*x**9 + 315*a**8*x**8 - 360*a*
*7*x**7 - 630*a**6*x**6 + 756*a**5*x**5 + 630*a**4*x**4 - 840*a**3*x**3 -
315*a**2*x**2 + 630*a*x - 193))/(630*a)
```

3.625 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 187

$$\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx = -\frac{8(1-ax)^5(c-a^2cx^2)^{7/2}}{5a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7} + \frac{2(1-ax)^6(c-a^2cx^2)^{7/2}}{a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7} - \frac{6(1-ax)^7(c-a^2cx^2)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7} + \frac{(1-ax)^8(c-a^2cx^2)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

output
```
-8/5*(-a*x+1)^5*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+2*(-a*x+1
)^6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-6/7*(-a*x+1)^7*(-a^2*
c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(-a*x+1)^8*(-a^2*c*x^2+c)^(
7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx = -\frac{c^3(-1+ax)^5\sqrt{c-a^2cx^2}(93+185ax+135a^2x^2+35a^3x^3)}{280a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a^2*c*x^2)^(7/2)/E^ArcCoth[a*x],x]
```

output
```
-1/280*(c^3*(-1 + a*x)^5*Sqrt[c - a^2*c*x^2]*(93 + 185*a*x + 135*a^2*x^2 +
 35*a^3*x^3))/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{7/2}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7dx}{x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2}\int (1-ax)^4(ax+1)^3dx}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2}\int \left(-(1-ax)^7+6(1-ax)^6-12(1-ax)^5+8(1-ax)^4\right)dx}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{(1-ax)^8}{8a}-\frac{6(1-ax)^7}{7a}+\frac{2(1-ax)^6}{a}-\frac{8(1-ax)^5}{5a}\right)\left(c-a^2cx^2\right)^{7/2}}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

input
```
Int[(c - a^2*c*x^2)^(7/2)/E^ArcCoth[a*x],x]
```

output
```
((c - a^2*c*x^2)^(7/2)*((-8*(1 - a*x)^5)/(5*a) + (2*(1 - a*x)^6)/a - (6*(1
 - a*x)^7)/(7*a) + (1 - a*x)^8/(8*a)))/(a^7*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.52

method	result	size
default	$-\frac{\left(35 a^{7} x^{7}-40 x^{6} a^{6}-140 a^{5} x^{5}+168 a^{4} x^{4}+210 a^{3} x^{3}-280 a^{2} x^{2}-140 a x +280\right) x \,c^{3} \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \sqrt{\frac{a x -1}{a x +1}}}{280 \left(a x -1\right)}$	97
gosper	$\frac{x \left(35 a^{7} x^{7}-40 x^{6} a^{6}-140 a^{5} x^{5}+168 a^{4} x^{4}+210 a^{3} x^{3}-280 a^{2} x^{2}-140 a x +280\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{7}{2}} \sqrt{\frac{a x -1}{a x +1}}}{280 \left(a x +1\right)^{3} \left(a x -1\right)^{4}}$	100
orering	$\frac{x \left(35 a^{7} x^{7}-40 x^{6} a^{6}-140 a^{5} x^{5}+168 a^{4} x^{4}+210 a^{3} x^{3}-280 a^{2} x^{2}-140 a x +280\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{7}{2}} \sqrt{\frac{a x -1}{a x +1}}}{280 \left(a x +1\right)^{3} \left(a x -1\right)^{4}}$	100

input
```
int((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/280*(35*a^7*x^7-40*a^6*x^6-140*a^5*x^5+168*a^4*x^4+210*a^3*x^3-280*a^2*
x^2-140*a*x+280)*x*c^3*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x
-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.51

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = -\frac{(35\,a^7c^3x^8-40\,a^6c^3x^7-140\,a^5c^3x^6+168\,a^4c^3x^5+210\,a^3c^3x^4-280\,a^2c^3x^3-140\,ac^3x^2+280\,c^3x)\sqrt{-a}}{280\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

output
```
-1/280*(35*a^7*c^3*x^8 - 40*a^6*c^3*x^7 - 140*a^5*c^3*x^6 + 168*a^4*c^3*x^
5 + 210*a^3*c^3*x^4 - 280*a^2*c^3*x^3 - 140*a*c^3*x^2 + 280*c^3*x)*sqrt(-a
^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=$$
$$-\frac{1}{280}\left(35\,a^7c^3x^8\mathrm{sgn}(ax+1)-40\,a^6c^3x^7\mathrm{sgn}(ax+1)-140\,a^5c^3x^6\mathrm{sgn}(ax+1)+168\,a^4c^3x^5\mathrm{sgn}(ax+1)+\right.$$

input
```
integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

output
```
-1/280*(35*a^7*c^3*x^8*sgn(a*x + 1) - 40*a^6*c^3*x^7*sgn(a*x + 1) - 140*a^
5*c^3*x^6*sgn(a*x + 1) + 168*a^4*c^3*x^5*sgn(a*x + 1) + 210*a^3*c^3*x^4*sg
n(a*x + 1) - 280*a^2*c^3*x^3*sgn(a*x + 1) - 140*a*c^3*x^2*sgn(a*x + 1) + 2
80*c^3*x*sgn(a*x + 1) + 163*c^3*sgn(a*x + 1)/a)*sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int \left(c-a^2\,c\,x^2\right)^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.39

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{\sqrt{c}\,c^3i(35a^8x^8-40a^7x^7-140a^6x^6+168a^5x^5+210a^4x^4-280a^3x^3-140a^2x^2+280ax}{280a}$$

input
```
int((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*c**3*i*(35*a**8*x**8 - 40*a**7*x**7 - 140*a**6*x**6 + 168*a**5*x*
*5 + 210*a**4*x**4 - 280*a**3*x**3 - 140*a**2*x**2 + 280*a*x - 93))/(280*a
)
```

3.626 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{(1-ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}-\frac{4(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

output
```
(-a*x+1)^4*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5-4/5*(-a*x+1)^5
*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(-a*x+1)^6*(-a^2*c*x
^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.45

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2(-1+ax)^4\left(11+14ax+5a^2x^2\right)\sqrt{c-a^2cx^2}}{30a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a^2*c*x^2)^(5/2)/E^ArcCoth[a*x],x]
```

output
```
(c^2*(-1 + a*x)^4*(11 + 14*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(30*a^2*S
qrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.58, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{5/2} e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2} \int e^{-\coth^{-1}(ax)} \left(1-\frac{1}{a^2x^2}\right)^{5/2} x^5 dx}{x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2} \int -(1-ax)^3(ax+1)^2 dx}{a^5x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(c-a^2cx^2\right)^{5/2} \int (1-ax)^3(ax+1)^2 dx}{a^5x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\left(c-a^2cx^2\right)^{5/2} \int \left((1-ax)^5-4(1-ax)^4+4(1-ax)^3\right) dx}{a^5x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-\frac{(1-ax)^6}{6a}+\frac{4(1-ax)^5}{5a}-\frac{(1-ax)^4}{a}\right) \left(c-a^2cx^2\right)^{5/2}}{a^5x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

input
```
Int[(c - a^2*c*x^2)^(5/2)/E^ArcCoth[a*x],x]
```

output
```
-(((c - a^2*c*x^2)^(5/2)*(-((1 - a*x)^4/a) + (4*(1 - a*x)^5)/(5*a) - (1 -
a*x)^6/(6*a)))/(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{(5a^5x^5-6a^4x^4-15a^3x^3+20a^2x^2+15ax-30)x\,c^2\sqrt{-c(a^2x^2-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{30ax-30}$	81
gosper	$\frac{x(5a^5x^5-6a^4x^4-15a^3x^3+20a^2x^2+15ax-30)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}{30(ax+1)^2(ax-1)^3}$	84
orering	$\frac{x(5a^5x^5-6a^4x^4-15a^3x^3+20a^2x^2+15ax-30)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}{30(ax+1)^2(ax-1)^3}$	84

input
```
int((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*(5*a^5*x^5-6*a^4*x^4-15*a^3*x^3+20*a^2*x^2+15*a*x-30)*x*c^2*(-c*(a^2*
x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.53

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{(5\,a^5c^2x^6-6\,a^4c^2x^5-15\,a^3c^2x^4+20\,a^2c^2x^3+15\,ac^2x^2-30\,c^2x)\sqrt{-a^2c}}{30\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

output
```
1/30*(5*a^5*c^2*x^6 - 6*a^4*c^2*x^5 - 15*a^3*c^2*x^4 + 20*a^2*c^2*x^3 + 15
*a*c^2*x^2 - 30*c^2*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx=\frac{1}{30}\left(5\,a^5c^2x^6\text{sgn}(ax+1)-6\,a^4c^2x^5\text{sgn}(ax+1)-15\,a^3c^2x^4\text{sgn}(ax+1)+20\,a^2c^2x^3\text{sgn}(\right.$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

output
```
1/30*(5*a^5*c^2*x^6*sgn(a*x + 1) - 6*a^4*c^2*x^5*sgn(a*x + 1) - 15*a^3*c^2
*x^4*sgn(a*x + 1) + 20*a^2*c^2*x^3*sgn(a*x + 1) + 15*a*c^2*x^2*sgn(a*x + 1
) - 30*c^2*x*sgn(a*x + 1) - 21*c^2*sgn(a*x + 1)/a)*sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int \left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.41

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\sqrt{c}\,c^2i(-5a^6x^6+6a^5x^5+15a^4x^4-20a^3x^3-15a^2x^2+30ax-11)}{30a}$$

input
```
int((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*c**2*i*( - 5*a**6*x**6 + 6*a**5*x**5 + 15*a**4*x**4 - 20*a**3*x**
3 - 15*a**2*x**2 + 30*a*x - 11))/(30*a)
```

3.627 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{2(1-ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}+\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

output
```
-2/3*(-a*x+1)^3*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3+1/4*(-a*x
+1)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.56

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{c(-1+ax)^3(5+3ax)\sqrt{c-a^2cx^2}}{12a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a^2*c*x^2)^(3/2)/E^ArcCoth[a*x],x]
```

output
```
-1/12*(c*(-1 + a*x)^3*(5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^
2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{3/2}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3dx}{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2}\int (1-ax)^2(ax+1)dx}{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2}\int \left(2(1-ax)^2-(1-ax)^3\right)dx}{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{(1-ax)^4}{4a}-\frac{2(1-ax)^3}{3a}\right)\left(c-a^2cx^2\right)^{3/2}}{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Int[(c - a^2*c*x^2)^(3/2)/E^ArcCoth[a*x],x]
```

output
```
((c - a^2*c*x^2)^(3/2)*((-2*(1 - a*x)^3)/(3*a) + (1 - a*x)^4/(4*a)))/(a^3*
(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{\left(3 a^{3} x^{3}-4 a^{2} x^{2}-6 a x +12\right) x c \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \sqrt{\frac{a x -1}{a x +1}}}{12 \left(a x -1\right)}$	63
gosper	$\frac{x \left(3 a^{3} x^{3}-4 a^{2} x^{2}-6 a x +12\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \sqrt{\frac{a x -1}{a x +1}}}{12 \left(a x +1\right) \left(a x -1\right)^{2}}$	68
orering	$\frac{x \left(3 a^{3} x^{3}-4 a^{2} x^{2}-6 a x +12\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \sqrt{\frac{a x -1}{a x +1}}}{12 \left(a x +1\right) \left(a x -1\right)^{2}}$	68

input
```
int((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(3*a^3*x^3-4*a^2*x^2-6*a*x+12)*x*c*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(
a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.45

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{\left(3\,a^3cx^4-4\,a^2cx^3-6\,acx^2+12\,cx\right)\sqrt{-a^2c}}{12\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

output
```
-1/12*(3*a^3*c*x^4 - 4*a^2*c*x^3 - 6*a*c*x^2 + 12*c*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = -\frac{1}{12}\left(3\,a^3cx^4\mathrm{sgn}(ax+1)-4\,a^2cx^3\mathrm{sgn}(ax+1)-6\,acx^2\mathrm{sgn}(ax+1)+12\,cx\mathrm{sgn}(ax+1)+\frac{11\,c\mathrm{sgn}(ax+1)}{a}\right.$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

output
```
-1/12*(3*a^3*c*x^4*sgn(a*x + 1) - 4*a^2*c*x^3*sgn(a*x + 1) - 6*a*c*x^2*sgn
(a*x + 1) + 12*c*x*sgn(a*x + 1) + 11*c*sgn(a*x + 1)/a)*sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \int \left(c-a^2\,c\,x^2\right)^{3/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.41

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \frac{\sqrt{c}\,ci(3a^4x^4-4a^3x^3-6a^2x^2+12ax-5)}{12a}$$

input
```
int((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*c*i*(3*a**4*x**4 - 4*a**3*x**3 - 6*a**2*x**2 + 12*a*x - 5))/(12*a
)
```

3.628 $\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 47

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(1-ax)^2\sqrt{c-a^2cx^2}}{2a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

output
```
1/2*(-a*x+1)^2*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(-2+ax)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

output
```
((-2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6746, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int (ax-1)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{17}$$

$$\frac{(1-ax)^2\sqrt{c-a^2cx^2}}{2a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

output
```
((1 - a*x)^2*Sqrt[c - a^2*c*x^2])/(2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x(ax-2)\sqrt{-a^{2}c\,x^{2}+c}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	44
orering	$\frac{x(ax-2)\sqrt{-a^{2}c\,x^{2}+c}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	44
default	$\frac{(ax-2)x\sqrt{-c(a^{2}x^{2}-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	45

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(a*x-2)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.47

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{-a^2c}(ax^2-2\,x)}{2\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

output
```
1/2*sqrt(-a^2*c)*(a*x^2 - 2*x)/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{1}{2}\left(ax^2\mathrm{sgn}(ax+1)-2\,x\mathrm{sgn}(ax+1)-\frac{3\,\mathrm{sgn}(ax+1)}{a}\right)\sqrt{-c}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

output
```
1/2*(a*x^2*sgn(a*x + 1) - 2*x*sgn(a*x + 1) - 3*sgn(a*x + 1)/a)*sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.47

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c}\,i(-a^2x^2+2ax-1)}{2a}$$

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*i*( - a**2*x**2 + 2*a*x - 1))/(2*a)
```

3.629 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 37

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}} \, dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\, x \log(1+ax)}{\sqrt{c-a^2cx^2}}$$

output
```
(1-1/a^2/x^2)^(1/2)*x*ln(a*x+1)/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}} \, dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\, x \log(1+ax)}{\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2]),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6746, 6747, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\int \frac{1}{ax+1}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{\sqrt{c-a^2cx^2}}$$

input
```
Int[1/(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2]),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

method	result	size
default	$-\frac{\ln(ax+1)\sqrt{-c(a^2x^2-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{c(ax-1)a}$	51

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-ln(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/c/(a*x-1)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{\sqrt{-a^2c}\log{(ax+1)}}{a^2c}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

output
```
-sqrt(-a^2*c)*log(a*x + 1)/(a^2*c)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(-a^2*c*x^2 + c), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\log(|ax+1|)\,\mathrm{sgn}(ax+1)}{a\sqrt{-c}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

output
```
log(abs(a*x + 1))*sgn(a*x + 1)/(a*sqrt(-c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{c}\,i\left(\log\left(\sqrt{-ax+1}-\sqrt{2}\right)+\log\left(\sqrt{-ax+1}+\sqrt{2}\right)\right)}{ac}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*i*(log(sqrt( - a*x + 1) - sqrt(2)) + log(sqrt( - a*x + 1) + sqrt(
2))))/(a*c)
```

3.630 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)\,(c-a^2cx^2)^{3/2}} - \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\text{arctanh}(ax)}{2\,(c-a^2cx^2)^{3/2}}$$

output
```
1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/2*a^2*(1-1/
a^2/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}x(-1+(1+ax)\text{arctanh}(ax))}{2(c+acx)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + (1 + a*x)*ArcTanh[a*x]))/(2*(c + a*c*x)*Sqr
t[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6746, 6747, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int -\frac{1}{(1-ax)(ax+1)^2}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{1}{(1-ax)(ax+1)^2}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{54}$$

$$-\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(\frac{1}{2(ax+1)^2}-\frac{1}{2(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\text{arctanh}(ax)}{2a}-\frac{1}{2a(ax+1)}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
-((a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-1/2*1/(a*(1 + a*x)) + ArcTanh[a*x]/(2
*a)))/(c - a^2*c*x^2)^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(\ln\left(ax+1\right)xa-a\ln\left(ax-1\right)x+\ln\left(ax+1\right)-\ln\left(ax-1\right)-2\right)}{4\left(a^{2}x^{2}-1\right)c^{2}a}$	84

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*x*a-a*ln(a*
x-1)*x+ln(a*x+1)-ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{(a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\sqrt{-a^2c}}{4\left(a^3c^2x+a^2c^2\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

output
```
-1/4*((a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)
/(a^2*x^2 - 1)) - 2*sqrt(-a^2*c))/(a^3*c^2*x + a^2*c^2)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(\frac{\log(|ax+1|)}{ac} - \frac{\log(|ax-1|)}{ac} - \frac{2}{(ax+1)ac}\right)\operatorname{sgn}(ax+1)}{4\sqrt{-c}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

output
```
1/4*(log(abs(a*x + 1))/(a*c) - log(abs(a*x - 1))/(a*c) - 2/((a*x + 1)*a*c)
)*sgn(a*x + 1)/sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{(c-a^2\,c\,x^2)^{3/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,i\left(\log\left(\sqrt{-ax+1}-\sqrt{2}\right)ax+\log\left(\sqrt{-ax+1}-\sqrt{2}\right)+\log\left(\sqrt{-ax+1}+\sqrt{2}\right)ax\right.}{4a\,c^2\,(ax}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*i*(log(sqrt( - a*x + 1) - sqrt(2))*a*x + log(sqrt( - a*x + 1) - s
qrt(2)) + log(sqrt( - a*x + 1) + sqrt(2))*a*x + log(sqrt( - a*x + 1) + sqr
t(2)) - 2*log(sqrt( - a*x + 1))*a*x - 2*log(sqrt( - a*x + 1)) + a*x - 1))/
(4*a*c**2*(a*x + 1))
```

3.631 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 183

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1+ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

output
```
1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a^4*(1-1
/a^2/x^2)^(5/2)*x^5/(a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/4*a^4*(1-1/a^2/x^2)^(
5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+3/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arct
anh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.44

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(2-3ax-3a^2x^2+3(-1+ax)(1+ax)^2\operatorname{arctanh}(ax)\right)}{8(-1+ax)(c+acx)^2\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)*(1 + a*x)^2
*ArcTanh[a*x]))/(8*(-1 + a*x)*(c + a*c*x)^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6746, 6747, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{1}{(1-ax)^2(ax+1)^3}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{54}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(\frac{1}{8(ax-1)^2}+\frac{1}{4(ax+1)^2}+\frac{1}{4(ax+1)^3}-\frac{3}{8(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{3\text{arctanh}(ax)}{8a}+\frac{1}{8a(1-ax)}-\frac{1}{4a(ax+1)}-\frac{1}{8a(ax+1)^2}\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(8*a*(1 - a*x)) - 1/(8*a*(1 + a*x)^2)
- 1/(4*a*(1 + a*x)) + (3*ArcTanh[a*x])/(8*a)))/(c - a^2*c*x^2)^(5/2)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(a^2x^2-1)}\,(3\ln(ax+1)x^3a^3-3a^3\ln(ax-1)x^3+3\ln(ax+1)x^2a^2-3a^2\ln(ax-1)x^2-6a^2x^2-3\ln(ax+1)xa+3a\ln(ax}{16(ax+1)(a^2x^2-1)c^3a(ax-1)}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*((a*x-1)/(a*x+1))^(1/2)/(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x+1)*
x^3*a^3-3*a^3*ln(a*x-1)*x^3+3*ln(a*x+1)*x^2*a^2-3*a^2*ln(a*x-1)*x^2-6*a^2*
x^2-3*ln(a*x+1)*x*a+3*a*ln(a*x-1)*x-6*a*x-3*ln(a*x+1)+3*ln(a*x-1)+4)/(a^2*
x^2-1)/c^3/a/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.75

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{3\,(a^4x^3+a^3x^2-a^2x-a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\,(3\,a^2x^2+3\,ax-2)\sqrt{-a^2c}}{16\,(a^5c^3x^3+a^4c^3x^2-a^3c^3x-a^2c^3)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="frica
s")
```

output
```
-1/16*(3*(a^4*x^3 + a^3*x^2 - a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(
-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(3*a^2*x^2 + 3*a*x - 2)*sqrt(-a
^2*c))/(a^5*c^3*x^3 + a^4*c^3*x^2 - a^3*c^3*x - a^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.44

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\frac{\left(\frac{3\log(|ax+1|)}{ac^2}-\frac{3\log(|ax-1|)}{ac^2}-\frac{2\,(3\,a^2x^2+3\,ax-2)}{(ax+1)^2(ax-1)ac^2}\right)\mathrm{sgn}(ax+1)}{16\sqrt{-c}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac"
)
```

output
```
1/16*(3*log(abs(a*x + 1))/(a*c^2) - 3*log(abs(a*x - 1))/(a*c^2) - 2*(3*a^2
*x^2 + 3*a*x - 2)/((a*x + 1)^2*(a*x - 1)*a*c^2))*sgn(a*x + 1)/sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{(c-a^2\,c\,x^2)^{5/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(5/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.36

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,i\big(6\log\big(\sqrt{-ax+1}-\sqrt{2}\big)\,a^3x^3+6\log\big(\sqrt{-ax+1}-\sqrt{2}\big)\,a^2x^2-6\log\big(\sqrt{-ax+$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*i*(6*log(sqrt( - a*x + 1) - sqrt(2))*a**3*x**3 + 6*log(sqrt( - a*
x + 1) - sqrt(2))*a**2*x**2 - 6*log(sqrt( - a*x + 1) - sqrt(2))*a*x - 6*lo
g(sqrt( - a*x + 1) - sqrt(2)) + 6*log(sqrt( - a*x + 1) + sqrt(2))*a**3*x**
3 + 6*log(sqrt( - a*x + 1) + sqrt(2))*a**2*x**2 - 6*log(sqrt( - a*x + 1) +
 sqrt(2))*a*x - 6*log(sqrt( - a*x + 1) + sqrt(2)) - 12*log(sqrt( - a*x + 1
))*a**3*x**3 - 12*log(sqrt( - a*x + 1))*a**2*x**2 + 12*log(sqrt( - a*x + 1
))*a*x + 12*log(sqrt( - a*x + 1)) + 3*a**3*x**3 - 9*a**2*x**2 - 15*a*x + 5
))/(32*a*c**3*(a**3*x**3 + a**2*x**2 - a*x - 1))
```

3.632 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 276

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}} + \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1+ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2\left(c-a^2cx^2\right)^{7/2}} + \frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{5a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\operatorname{arctanh}(ax)}{16\left(c-a^2cx^2\right)^{7/2}}$$

output
```
-1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*
(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)+1/24*a^6*(1-1/a^2/x^
2)^(7/2)*x^7/(a*x+1)^3/(-a^2*c*x^2+c)^(7/2)+3/32*a^6*(1-1/a^2/x^2)^(7/2)*x
^7/(a*x+1)^2/(-a^2*c*x^2+c)^(7/2)+3/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)
/(-a^2*c*x^2+c)^(7/2)-5/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*
c*x^2+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.36

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x(-8+25ax+25a^2x^2-15a^3x^3-15a^4x^4+15(-1+ax)^2(1+ax)^3\text{arct}}{48(-1+ax)^2(c+acx)^3\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2)),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(-8 + 25*a*x + 25*a^2*x^2 - 15*a^3*x^3 - 15*a^4*x
^4 + 15*(-1 + a*x)^2*(1 + a*x)^3*ArcTanh[a*x]))/(48*(-1 + a*x)^2*(c + a*c*
x)^3*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.44, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6746, 6747, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int -\frac{1}{(1-ax)^3(ax+1)^4}dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int\frac{1}{(1-ax)^3(ax+1)^4}dx}{(c-a^2cx^2)^{7/2}}$$

$\big\downarrow$ 54

$$-\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int\left(\frac{1}{8(ax-1)^2}+\frac{3}{16(ax+1)^2}-\frac{1}{16(ax-1)^3}+\frac{3}{16(ax+1)^3}+\frac{1}{8(ax+1)^4}-\frac{5}{16(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{7/2}}$$

$\big\downarrow$ 2009

$$-\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(\frac{5\text{arctanh}(ax)}{16a}+\frac{1}{8a(1-ax)}-\frac{3}{16a(ax+1)}+\frac{1}{32a(1-ax)^2}-\frac{3}{32a(ax+1)^2}-\frac{1}{24a(ax+1)^3}\right)}{(c-a^2cx^2)^{7/2}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2)),x]
```

output
```
-((a^7*(1 - 1/(a^2*x^2))^(7/2)*x^7*(1/(32*a*(1 - a*x)^2) + 1/(8*a*(1 - a*x
)) - 1/(24*a*(1 + a*x)^3) - 3/(32*a*(1 + a*x)^2) - 3/(16*a*(1 + a*x)) + (5
*ArcTanh[a*x])/(16*a)))/(c - a^2*c*x^2)^(7/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.87

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(a^2x^2-1)}\,(15\ln(ax-1)x^5a^5-15\ln(ax+1)x^5a^5+15\ln(ax-1)x^4a^4-15\ln(ax+1)x^4a^4+30a^4x^4-30a^3\ln(ax-1)x^3+30}{96(ax+1)}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/96*((a*x-1)/(a*x+1))^(1/2)/(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x-1
)*x^5*a^5-15*ln(a*x+1)*x^5*a^5+15*ln(a*x-1)*x^4*a^4-15*ln(a*x+1)*x^4*a^4+3
0*a^4*x^4-30*a^3*ln(a*x-1)*x^3+30*ln(a*x+1)*x^3*a^3+30*a^3*x^3-30*a^2*ln(a
*x-1)*x^2+30*ln(a*x+1)*x^2*a^2-50*a^2*x^2+15*a*ln(a*x-1)*x-15*ln(a*x+1)*x*
a-50*a*x+15*ln(a*x-1)-15*ln(a*x+1)+16)/(a^2*x^2-1)/c^4/a/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.67

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx =$$
$$-\frac{15\,(a^6x^5+a^5x^4-2\,a^4x^3-2\,a^3x^2+a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\,(15\,a^4x^4+15\,a^3x^3-25}{96\,(a^7c^4x^5+a^6c^4x^4-2\,a^5c^4x^3-2\,a^4c^4x^2+a^3c^4x+a^2c^4)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="frica
s")
```

output
```
-1/96*(15*(a^6*x^5 + a^5*x^4 - 2*a^4*x^3 - 2*a^3*x^2 + a^2*x + a)*sqrt(-c)
*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(15*a^
4*x^4 + 15*a^3*x^3 - 25*a^2*x^2 - 25*a*x + 8)*sqrt(-a^2*c))/(a^7*c^4*x^5 +
 a^6*c^4*x^4 - 2*a^5*c^4*x^3 - 2*a^4*c^4*x^2 + a^3*c^4*x + a^2*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.35

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx=\frac{\left(\frac{15\log(|ax+1|)}{ac^3}-\frac{15\log(|ax-1|)}{ac^3}-\frac{2\left(15a^4x^4+15a^3x^3-25a^2x^2-25ax+8\right)}{(ax+1)^3(ax-1)^2ac^3}\right)\operatorname{sgn}(ax+1)}{96\sqrt{-c}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac"
)
```

output
```
1/96*(15*log(abs(a*x + 1))/(a*c^3) - 15*log(abs(a*x - 1))/(a*c^3) - 2*(15*
a^4*x^4 + 15*a^3*x^3 - 25*a^2*x^2 - 25*a*x + 8)/((a*x + 1)^3*(a*x - 1)^2*a
*c^3))*sgn(a*x + 1)/sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-a^2\,c\,x^2\right)}^{7/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(7/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.42

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx=\frac{\sqrt{c}\,i\left(15\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^5x^5+15\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^4x^4-30\log\left(\sqrt{-ax\right.\right.}{}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x)
```

output
```
(sqrt(c)*i*(15*log(sqrt( - a*x + 1) - sqrt(2))*a**5*x**5 + 15*log(sqrt( -
a*x + 1) - sqrt(2))*a**4*x**4 - 30*log(sqrt( - a*x + 1) - sqrt(2))*a**3*x*
*3 - 30*log(sqrt( - a*x + 1) - sqrt(2))*a**2*x**2 + 15*log(sqrt( - a*x + 1
) - sqrt(2))*a*x + 15*log(sqrt( - a*x + 1) - sqrt(2)) + 15*log(sqrt( - a*x
 + 1) + sqrt(2))*a**5*x**5 + 15*log(sqrt( - a*x + 1) + sqrt(2))*a**4*x**4
- 30*log(sqrt( - a*x + 1) + sqrt(2))*a**3*x**3 - 30*log(sqrt( - a*x + 1) +
 sqrt(2))*a**2*x**2 + 15*log(sqrt( - a*x + 1) + sqrt(2))*a*x + 15*log(sqrt
( - a*x + 1) + sqrt(2)) - 30*log(sqrt( - a*x + 1))*a**5*x**5 - 30*log(sqrt
( - a*x + 1))*a**4*x**4 + 60*log(sqrt( - a*x + 1))*a**3*x**3 + 60*log(sqrt
( - a*x + 1))*a**2*x**2 - 30*log(sqrt( - a*x + 1))*a*x - 30*log(sqrt( - a*
x + 1)) + 5*a**5*x**5 - 25*a**4*x**4 - 40*a**3*x**3 + 40*a**2*x**2 + 55*a*
x - 11))/(96*a*c**4*(a**5*x**5 + a**4*x**4 - 2*a**3*x**3 - 2*a**2*x**2 + a
*x + 1))
```

3.633 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{2(c-a^2cx^2)^{5/2}}{5a}+\frac{1}{6}x\left(c-a^2cx^2\right)^{5/2}-\frac{7c^{5/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}$$

output
```
-7/16*c^2*x*(-a^2*c*x^2+c)^(1/2)-7/24*c*x*(-a^2*c*x^2+c)^(3/2)-2/5*(-a^2*c
*x^2+c)^(5/2)/a+1/6*x*(-a^2*c*x^2+c)^(5/2)-7/16*c^(5/2)*arctan(a*c^(1/2)*x
/(-a^2*c*x^2+c)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2\sqrt{c-a^2cx^2}\left(-\sqrt{1+ax}(96+39ax-327a^2x^2+202a^3x^3+86a^4x^4-136a^5x^5+40a^6x^6\cdots\right.}{240a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

input
```
Integrate[(c - a^2*c*x^2)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^2*Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(96 + 39*a*x - 327*a^2*x^2 + 202
*a^3*x^3 + 86*a^4*x^4 - 136*a^5*x^5 + 40*a^6*x^6)) + 210*Sqrt[1 - a*x]*Arc
Sin[Sqrt[1 - a*x]/Sqrt[2]]))/(240*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6692, 469, 455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{5/2} e^{-2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)} \left(c-a^2cx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int (1-ax)^2 \left(c-a^2cx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{7}{6}\int (1-ax) \left(c-a^2cx^2\right)^{3/2}dx+\frac{(1-ax) \left(c-a^2cx^2\right)^{5/2}}{6ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{7}{6}\left(\int \left(c-a^2cx^2\right)^{3/2}dx+\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}\right)+\frac{(1-ax) \left(c-a^2cx^2\right)^{5/2}}{6ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{7}{6}\left(\frac{3}{4}c\int \sqrt{c-a^2cx^2}dx+\frac{\left(c-a^2cx^2\right)^{5/2}}{5ac}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\right)+\frac{(1-ax) \left(c-a^2cx^2\right)^{5/2}}{6ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{7}{6}\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{(c-a^2cx^2)^{5/2}}{5ac}+\frac{1}{4}x(c-a^2cx^2)^{3/2}\right)+\frac{(1-ax)\,(c-a^2}{6ac}$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{7}{6}\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{(c-a^2cx^2)^{5/2}}{5ac}+\frac{1}{4}x(c-a^2cx^2)^{3/2}\right)+\frac{(1-}{}$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{7}{6}\left(\frac{3}{4}c\left(\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{(c-a^2cx^2)^{5/2}}{5ac}+\frac{1}{4}x(c-a^2cx^2)^{3/2}\right)+\frac{(1-ax)\,(c-}{6a}$$

input
```
Int[(c - a^2*c*x^2)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*((((1 - a*x)*(c - a^2*c*x^2)^(5/2))/(6*a*c) + (7*((x*(c - a^2*c*x^2)^(3
/2))/4 + (c - a^2*c*x^2)^(5/2)/(5*a*c) + (3*c*((x*Sqrt[c - a^2*c*x^2])/2 +
 (Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)))/4))/6))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\left(40 a^{5} x^{5}-96 a^{4} x^{4}-10 a^{3} x^{3}+192 a^{2} x^{2}-135 a x -96\right) \left(a^{2} x^{2}-1\right) c^{3}}{240 a \sqrt{-c \left(a^{2} x^{2}-1\right)}}-\frac{7 \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right) c^{3}}{16 \sqrt{a^{2} c}}$
default	$\frac{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}{6}+\frac{5 c \left(\frac{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{4}+\frac{3 c \left(\frac{x \sqrt{-a^{2} c \,x^{2}+c}}{2}+\frac{c \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right)}{2 \sqrt{a^{2} c}}\right)}{4}\right)}{6}-\frac{2 \left(\frac{\left(-\left(x+\frac{1}{a}\right)^{2} a^{2} c+2 \left(x+\frac{1}{a}\right) a c \right)^{\frac{5}{2}}}{5}+ac \right.}{}$

input
```
int((a*x-1)*(-a^2*c*x^2+c)^(5/2)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(40*a^5*x^5-96*a^4*x^4-10*a^3*x^3+192*a^2*x^2-135*a*x-96)*(a^2*x^2-
1)/a/(-c*(a^2*x^2-1))^(1/2)*c^3-7/16/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/
(-a^2*c*x^2+c)^(1/2))*c^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.96

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx = \left[\frac{105\sqrt{-c}c^2\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)+2\left(40\,a^5c^2x^5-96\,a^4c^2x^4-10\,a\right.}{480\,a}\right.$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/480*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) + 2*(40*a^5*c^2*x^5 - 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 192*a^2*
c^2*x^2 - 135*a*c^2*x - 96*c^2)*sqrt(-a^2*c*x^2 + c))/a, 1/240*(105*c^(5/2
)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (40*a^5*c^2*x
^5 - 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 192*a^2*c^2*x^2 - 135*a*c^2*x - 96*
c^2)*sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(110) = 220.

Time = 2.64 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.55

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx = \begin{cases} \frac{2c^2\left(\begin{cases}\left(\frac{a^2x^2}{3}-\frac{1}{3}\right)\sqrt{-a^2cx^2+c} & \text{for } c\neq 0\\ \frac{a^2\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)-2c^2\left(\begin{cases}\sqrt{-a^2cx^2+c}\left(\frac{a^4x^4}{5}-\frac{a^2x^2}{15}-\frac{2}{15}\right) & \text{f}\\ \frac{a^4\sqrt{c}x^4}{4} & \text{o}\end{cases}\right.}{} \\ -c^{\frac{5}{2}}x \end{cases}$$

input
```
integrate((-a**2*c*x**2+c)**(5/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise(((2*c**2*Piecewise((((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c),
Ne(c, 0)), (a**2*sqrt(c)*x**2/2, True)) - 2*c**2*Piecewise((sqrt(-a**2*c*x
**2 + c)*(a**4*x**4/5 - a**2*x**2/15 - 2/15), Ne(c, 0)), (a**4*sqrt(c)*x**
4/4, True)) + c**2*Piecewise((c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(
-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2),
True))/16 + sqrt(-a**2*c*x**2 + c)*(a**5*x**5/6 - a**3*x**3/24 - a*x/16),
Ne(c, 0)), (a**5*sqrt(c)*x**5/5, True)) - c**2*Piecewise((a*x*sqrt(-a**2*c
*x**2 + c)/2 + c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 +
c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/2, Ne(c,
 0)), (a*sqrt(c)*x, True)))/a, Ne(a, 0)), (-c**(5/2)*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.25

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx &= \frac{1}{6}\left(-a^2cx^2+c\right)^{\frac{5}{2}}x-\frac{7}{24}\left(-a^2cx^2+c\right)^{\frac{3}{2}}cx\\ &-\frac{3}{4}\sqrt{a^2cx^2+4\,acx+3\,c}c^2x+\frac{5}{16}\sqrt{-a^2cx^2+c}c^2x+\frac{3\,c^4\arcsin\left(ax+2\right)}{4\,a\left(-c\right)^{\frac{3}{2}}}\\ &+\frac{5\,c^{\frac{5}{2}}\arcsin\left(ax\right)}{16\,a}-\frac{2\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{5\,a}-\frac{3\sqrt{a^2cx^2+4\,acx+3\,c}c^2}{2\,a}\end{aligned}$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/6*(-a^2*c*x^2 + c)^(5/2)*x - 7/24*(-a^2*c*x^2 + c)^(3/2)*c*x - 3/4*sqrt(
a^2*c*x^2 + 4*a*c*x + 3*c)*c^2*x + 5/16*sqrt(-a^2*c*x^2 + c)*c^2*x + 3/4*c
^4*arcsin(a*x + 2)/(a*(-c)^(3/2)) + 5/16*c^(5/2)*arcsin(a*x)/a - 2/5*(-a^2
*c*x^2 + c)^(5/2)/a - 3/2*sqrt(a^2*c*x^2 + 4*a*c*x + 3*c)*c^2/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{7\,c^3\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{16\,\sqrt{-c}|a|}-\frac{1}{240}\,\sqrt{-a^2cx^2+c}\left(\left(135\,c^2-2\,\left(96\,ac^2-\left(5\,a^2c^2-4\,\left(5\,a^4c^2x-12\,a^3c^2\right)x\right)x\right)x\right)x+\frac{96\,c^2}{a}\right)$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
7/16*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a)
) - 1/240*sqrt(-a^2*c*x^2 + c)*((135*c^2 - 2*(96*a*c^2 - (5*a^2*c^2 - 4*(5
*a^4*c^2*x - 12*a^3*c^2)*x)*x)*x)*x + 96*c^2/a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^{5/2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(5/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - a^2*c*x^2)^(5/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\sqrt{c}\,c^2\left(-105asin(ax)+40\sqrt{-a^2x^2+1}\,a^5x^5-96\sqrt{-a^2x^2+1}\,a^4x^4-10\sqrt{-a^2x^2+1}\,a^3x^3\right.}{240a}$$

input
```
int((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*c**2*( - 105*asin(a*x) + 40*sqrt( - a**2*x**2 + 1)*a**5*x**5 - 96
*sqrt( - a**2*x**2 + 1)*a**4*x**4 - 10*sqrt( - a**2*x**2 + 1)*a**3*x**3 +
192*sqrt( - a**2*x**2 + 1)*a**2*x**2 - 135*sqrt( - a**2*x**2 + 1)*a*x - 96
*sqrt( - a**2*x**2 + 1) + 96))/(240*a)
```

3.634 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{2(c-a^2cx^2)^{3/2}}{3a}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}-\frac{5c^{3/2}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}$$

output
```
-5/8*c*x*(-a^2*c*x^2+c)^(1/2)-2/3*(-a^2*c*x^2+c)^(3/2)/a+1/4*x*(-a^2*c*x^2
+c)^(3/2)-5/8*c^(3/2)*arctan(a*c^(1/2)*x/(-a^2*c*x^2+c)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}(-16+7ax+25a^2x^2-22a^3x^3+6a^4x^4)+30\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1\cdots}}{\sqrt{\cdots}}\right.\right.}{24a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

input
```
Integrate[(c - a^2*c*x^2)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(-16 + 7*a*x + 25*a^2*x^2 - 22*a^3*x
^3 + 6*a^4*x^4) + 30*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(24*a*S
qrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6717, 6692, 469, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{3/2} e^{-2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)} \left(c-a^2cx^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int (1-ax)^2\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{5}{4}\int (1-ax)\sqrt{c-a^2cx^2}dx+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{5}{4}\left(\int \sqrt{c-a^2cx^2}dx+\frac{\left(c-a^2cx^2\right)^{3/2}}{3ac}\right)+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

$$\Big\downarrow \text{211}$$

$$-c\left(\frac{5}{4}\left(\frac{1}{2}c\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{\left(c-a^2cx^2\right)^{3/2}}{3ac}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{5}{4}\left(\frac{1}{2}c\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{\left(c-a^2cx^2\right)^{3/2}}{3ac}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{5}{4}\left(\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}+\frac{\left(c-a^2cx^2\right)^{3/2}}{3ac}+\frac{1}{2}x\sqrt{c-a^2cx^2}\right)+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4ac}\right)$$

input
```
Int[(c - a^2*c*x^2)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(((1 - a*x)*(c - a^2*c*x^2)^(3/2))/(4*a*c) + (5*((x*Sqrt[c - a^2*c*x^2
])/2 + (c - a^2*c*x^2)^(3/2)/(3*a*c) + (Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(2*a)))/4))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

method	result
risch	$\frac{(6a^3x^3-16a^2x^2+9ax+16)(a^2x^2-1)c^2}{24a\sqrt{-c(a^2x^2-1)}}-\frac{5\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)c^2}{8\sqrt{a^2c}}$
default	$\frac{x(-a^2cx^2+c)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{-a^2cx^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)}{2\sqrt{a^2c}}\right)}{4}-\frac{2\left(\frac{\left(-\left(x+\frac{1}{a}\right)^2a^2c+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}{3}+ac\left(-\frac{\left(-2\left(x+\frac{1}{a}\right)a^2c+2ac\right.}{}\right.\right.}{}$

input
```
int((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*a^3*x^3-16*a^2*x^2+9*a*x+16)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*
c^2-5/8/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.80

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \left[\frac{15\sqrt{-c}c\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\left(6a^3cx^3-16a^2cx^2+9acx+1\right.}{48a}\right.$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/48*(15*sqrt(-c)*c*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
 - c) - 2*(6*a^3*c*x^3 - 16*a^2*c*x^2 + 9*a*c*x + 16*c)*sqrt(-a^2*c*x^2 +
c))/a, 1/24*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2
 - c)) - (6*a^3*c*x^3 - 16*a^2*c*x^2 + 9*a*c*x + 16*c)*sqrt(-a^2*c*x^2 + c
))/a]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(88) = 176$.

Time = 2.29 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.48

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \begin{cases} \frac{2c\left(\begin{cases}\left(\frac{a^2x^2}{3}-\frac{1}{3}\right)\sqrt{-a^2cx^2+c} & \text{for } c\neq 0\\ \frac{a^2\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)-c\left(\begin{cases}\frac{c\left(\begin{cases}\frac{\log\left(-2acx+2\sqrt{-c}\sqrt{-a^2cx^2+c}\right)}{\sqrt{-c}} & \text{for } c\neq \\ \frac{ax\log(ax)}{\sqrt{-a^2cx^2}} & \text{otherw}\end{cases}\right.}{8} \\ \frac{a^3\sqrt{c}x^3}{3} \end{cases}\right.}{} \\ -c^{\frac{3}{2}}x \end{cases}$$

input
```
integrate((-a**2*c*x**2+c)**(3/2)*(a*x-1)/(a*x+1),x)
```

output
```
Piecewise(((2*c*Piecewise(((a**2*x**2/3 - 1/3)*sqrt(-a**2*c*x**2 + c), Ne(
c, 0)), (a**2*sqrt(c)*x**2/2, True)) - c*Piecewise((c*Piecewise((log(-2*a*
c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2 + c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x
)/sqrt(-a**2*c*x**2), True))/8 + (a**3*x**3/4 - a*x/8)*sqrt(-a**2*c*x**2 +
 c), Ne(c, 0)), (a**3*sqrt(c)*x**3/3, True)) - c*Piecewise((a*x*sqrt(-a**2
*c*x**2 + c)/2 + c*Piecewise((log(-2*a*c*x + 2*sqrt(-c)*sqrt(-a**2*c*x**2
+ c))/sqrt(-c), Ne(c, 0)), (a*x*log(a*x)/sqrt(-a**2*c*x**2), True))/2, Ne(
c, 0)), (a*sqrt(c)*x, True)))/a, Ne(a, 0)), (-c**(3/2)*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.30

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{1}{4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}x-\sqrt{a^2cx^2+4\,acx+3\,c}cx+\frac{3}{8}\sqrt{-a^2cx^2+c}cx+\frac{c^3\arcsin\left(ax+2\right)}{a\left(-c\right)^{\frac{3}{2}}}+\frac{3\,c^{\frac{3}{2}}\arcsin\left(ax\right)}{8\,a}-\frac{2\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3\,a}-\frac{2\sqrt{a^2cx^2+4\,acx+3\,c}c}{a}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/4*(-a^2*c*x^2 + c)^(3/2)*x - sqrt(a^2*c*x^2 + 4*a*c*x + 3*c)*c*x + 3/8*s
qrt(-a^2*c*x^2 + c)*c*x + c^3*arcsin(a*x + 2)/(a*(-c)^(3/2)) + 3/8*c^(3/2)
*arcsin(a*x)/a - 2/3*(-a^2*c*x^2 + c)^(3/2)/a - 2*sqrt(a^2*c*x^2 + 4*a*c*x
 + 3*c)*c/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.85

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3\,a^2cx-8\,ac\right)x+9\,c\right)x+\frac{16\,c}{a}\right)+\frac{5\,c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8\sqrt{-c}|a|}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*a^2*c*x - 8*a*c)*x + 9*c)*x + 16*c/a) +
5/8*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^{3/2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(3/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - a^2*c*x^2)^(3/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{\sqrt{c}\,c\left(-15asin(ax)-6\sqrt{-a^2x^2+1}\,a^3x^3+16\sqrt{-a^2x^2+1}\,a^2x^2-9\sqrt{-a^2x^2+1}\,ax-16\sqrt{-a^2x^2+1}+16\right)}{24a}$$

input
```
int((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*c*( - 15*asin(a*x) - 6*sqrt( - a**2*x**2 + 1)*a**3*x**3 + 16*sqrt
( - a**2*x**2 + 1)*a**2*x**2 - 9*sqrt( - a**2*x**2 + 1)*a*x - 16*sqrt( - a
**2*x**2 + 1) + 16))/(24*a)
```

3.635 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=-\frac{2\sqrt{c-a^2cx^2}}{a}+\frac{1}{2}x\sqrt{c-a^2cx^2}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

output
```
-2*(-a^2*c*x^2+c)^(1/2)/a+1/2*x*(-a^2*c*x^2+c)^(1/2)-3/2*c^(1/2)*arctan(a*
c^(1/2)*x/(-a^2*c*x^2+c)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.30

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\sqrt{1+ax}(4-5ax+a^2x^2)+6\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2)) + 6*Sqrt[1 -
a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6717, 6692, 469, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int \frac{(1-ax)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{3}{2}\int \frac{1-ax}{\sqrt{c-a^2cx^2}}dx+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{\sqrt{c-a^2cx^2}}{ac}\right)+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{3}{2}\left(\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{\sqrt{c-a^2cx^2}}{ac}\right)+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{3}{2}\left(\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}+\frac{\sqrt{c-a^2cx^2}}{ac}\right)+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2ac}\right)$$

input
```
Int[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a*c) + (3*(Sqrt[c - a^2*c*x^2]/(a*
c) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]/(a*Sqrt[c])))/2))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

method	result	size
risch	$-\frac{(ax-4)(a^2x^2-1)c}{2a\sqrt{-c(a^2x^2-1)}}-\frac{3c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}$	69
default	$\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}-\frac{2\left(\sqrt{-\left(x+\frac{1}{a}\right)^2a^2c+2\left(x+\frac{1}{a}\right)ac}+\frac{ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-\left(x+\frac{1}{a}\right)^2a^2c+2\left(x+\frac{1}{a}\right)ac}}\right)}{\sqrt{a^2c}}\right)}{a}$	127

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x-4)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c-3/2*c/(a^2*c)^(1/2)*ar
ctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.74

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$$
$$=\left[\frac{2\sqrt{-a^2cx^2+c}(ax-4)+3\sqrt{-c}\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{4\,a},\ \frac{\sqrt{-a^2cx^2+c}(ax-4)+3\right.$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x - 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sq
rt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x -
4) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a
]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{ax+1}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{1}{2}\sqrt{-a^2cx^2+c}x-\frac{3\sqrt{c}\arcsin(ax)}{2a}-\frac{2\sqrt{-a^2cx^2+c}}{a}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a - 2*sqrt(-a^2*c*x^2
 + c)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x-\frac{4}{a}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}|a|}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/2*sqrt(-a^2*c*x^2 + c)*(x - 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(
-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c}\,\left(-3asin(ax)+\sqrt{-a^2x^2+1}\,ax-4\sqrt{-a^2x^2+1}+4\right)}{2a}$$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*( - 3*asin(a*x) + sqrt( - a**2*x**2 + 1)*a*x - 4*sqrt( - a**2*x**
2 + 1) + 4))/(2*a)
```

3.636 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2\sqrt{c-a^2cx^2}}{ac(1+ax)}+\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

output
```
2*(-a^2*c*x^2+c)^(1/2)/a/c/(a*x+1)+arctan(a*c^(1/2)*x/(-a^2*c*x^2+c)^(1/2)
)/a/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.56

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2\sqrt{1-a^2x^2}\left((-1+ax)\sqrt{1+ax}+\sqrt{1-ax}(1+ax)\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-ax}(1+ax)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

output
```
(-2*Sqrt[1 - a^2*x^2]*((-1 + a*x)*Sqrt[1 + a*x] + Sqrt[1 - a*x]*(1 + a*x)*
ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*Sqrt[1 - a*x]*(1 + a*x)*Sqrt[c - a^2*c*
x^2])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6717, 6692, 457, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int \frac{(1-ax)^2}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{457}$$

$$-c\left(-\frac{\int \frac{1}{\sqrt{c-a^2cx^2}}dx}{c}-\frac{2(1-ax)}{ac\sqrt{c-a^2cx^2}}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(-\frac{\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}}{c}-\frac{2(1-ax)}{ac\sqrt{c-a^2cx^2}}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(-\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{ac^{3/2}}-\frac{2(1-ax)}{ac\sqrt{c-a^2cx^2}}\right)$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

output
```
-(c*((-2*(1 - a*x))/(a*c*Sqrt[c - a^2*c*x^2]) - ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]]/(a*c^(3/2))))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}+\frac{2\sqrt{-\left(x+\frac{1}{a}\right)^2a^2c+2\left(x+\frac{1}{a}\right)ac}}{a^2c\left(x+\frac{1}{a}\right)}$	73

input
```
int((a*x-1)/(-a^2*c*x^2+c)^(1/2)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^2/c/(x+1/
a)*(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.36

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \left[-\frac{(ax+1)\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-4\,\sqrt{-a^2cx^2+c}}{2\,(a^2cx+ac)},\ -\frac{(ax+1)\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)-2\,\sqrt{-a^2cx^2+c}}{a^2cx+ac}\right]$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((a*x + 1)*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(
-c)*x - c) - 4*sqrt(-a^2*c*x^2 + c))/(a^2*c*x + a*c), -((a*x + 1)*sqrt(c)*
arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*c*x
^2 + c))/(a^2*c*x + a*c)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{ax-1}{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral((a*x - 1)/(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2\sqrt{-a^2cx^2+c}}{a^2cx+ac} + \frac{\arcsin(ax)}{a\sqrt{c}}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(-a^2*c*x^2 + c)/(a^2*c*x + a*c) + arcsin(a*x)/(a*sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{a\,x-1}{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)^(1/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - a^2*c*x^2)^(1/2)*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{c}\left(asin(ax)\tan\left(\frac{asin(ax)}{2}\right)+asin(ax)-4\tan\left(\frac{asin(ax)}{2}\right)\right)}{ac\left(\tan\left(\frac{asin(ax)}{2}\right)+1\right)}$$

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*(asin(a*x)*tan(asin(a*x)/2) + asin(a*x) - 4*tan(asin(a*x)/2)))/(a
*c*(tan(asin(a*x)/2) + 1))
```

3.637 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{\sqrt{c-a^2cx^2}}{3ac^2(1+ax)^2}+\frac{\sqrt{c-a^2cx^2}}{3ac^2(1+ax)}$$

output
```
1/3*(-a^2*c*x^2+c)^(1/2)/a/c^2/(a*x+1)^2+1/3*(-a^2*c*x^2+c)^(1/2)/a/c^2/(a
*x+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{\sqrt{1-ax}(2+ax)\sqrt{1-a^2x^2}}{3ac(1+ax)^{3/2}\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 - a*x]*(2 + a*x)*Sqrt[1 - a^2*x^2])/(3*a*c*(1 + a*x)^(3/2)*Sqrt[c
- a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6717, 6692, 457, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 6692}$$

$$-c\int \frac{(1-ax)^2}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 457}$$

$$-c\left(\frac{\int \frac{1}{(c-a^2cx^2)^{3/2}}dx}{3c}-\frac{2(1-ax)}{3ac\,(c-a^2cx^2)^{3/2}}\right)$$

$$\Big\downarrow \text{ 208}$$

$$-c\left(\frac{x}{3c^2\sqrt{c-a^2cx^2}}-\frac{2(1-ax)}{3ac\,(c-a^2cx^2)^{3/2}}\right)$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
-(c*((-2*(1 - a*x))/(3*a*c*(c - a^2*c*x^2)^(3/2)) + x/(3*c^2*Sqrt[c - a^2*
c*x^2])))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.48

method	result	size
gosper	$\frac{(ax-1)^{2}(ax+2)}{3a\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}$	31
orering	$\frac{(ax-1)^{2}(ax+2)}{3a\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}$	31
trager	$\frac{(ax+2)\sqrt{-a^{2}c\,x^{2}+c}}{3c^{2}(ax+1)^{2}a}$	34
default	$\frac{x}{c\sqrt{-a^{2}c\,x^{2}+c}}-\frac{2\left(-\frac{1}{3ac\left(x+\frac{1}{a}\right)\sqrt{-\left(x+\frac{1}{a}\right)^{2}a^{2}c+2\left(x+\frac{1}{a}\right)ac}}-\frac{-2\left(x+\frac{1}{a}\right)a^{2}c+2ac}{3a\,c^{2}\sqrt{-\left(x+\frac{1}{a}\right)^{2}a^{2}c+2\left(x+\frac{1}{a}\right)ac}}\right)}{a}$	115

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*x-1)^2*(a*x+2)/a/(-a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{-a^2cx^2+c}(ax+2)}{3\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(-a^2*c*x^2 + c)*(a*x + 2)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{ax-1}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}\,(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{x}{3\sqrt{-a^2cx^2+c}c} + \frac{2}{3\left(\sqrt{-a^2cx^2+c}a^2cx+\sqrt{-a^2cx^2+c}ac\right)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*x/(sqrt(-a^2*c*x^2 + c)*c) + 2/3/(sqrt(-a^2*c*x^2 + c)*a^2*c*x + sqrt
(-a^2*c*x^2 + c)*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(57) = 114$.

Time = 0.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.28

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{(ac+3\sqrt{-a^2c}\sqrt{c})\mathrm{sgn}(x)}{3\left(a^2c^{\frac{5}{2}}+\sqrt{-a^2c}ac^2\right)} + \frac{2\left(2a^2c-3a\sqrt{c}\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)+3\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^2\right)}{3\left(a\sqrt{c}-\sqrt{-a^2c+\frac{c}{x^2}}+\frac{\sqrt{c}}{x}\right)^3 c\,\mathrm{sgn}(x)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-1/3*(a*c + 3*sqrt(-a^2*c)*sqrt(c))*sgn(x)/(a^2*c^(5/2) + sqrt(-a^2*c)*a*c
^2) + 2/3*(2*a^2*c - 3*a*sqrt(c)*(sqrt(-a^2*c + c/x^2) - sqrt(c)/x) + 3*(s
qrt(-a^2*c + c/x^2) - sqrt(c)/x)^2)/((a*sqrt(c) - sqrt(-a^2*c + c/x^2) + s
qrt(c)/x)^3*c*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.51

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+2)}{3\,a\,c^2\,(a\,x+1)^2}$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)^(3/2)*(a*x + 1)),x)
```

output
```
((c - a^2*c*x^2)^(1/2)*(a*x + 2))/(3*a*c^2*(a*x + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{2\sqrt{c}\left(-\tan\left(\frac{asin(ax)}{2}\right)^3+1\right)}{3a\,c^2\left(\tan\left(\frac{asin(ax)}{2}\right)^3+3\tan\left(\frac{asin(ax)}{2}\right)^2+3\tan\left(\frac{asin(ax)}{2}\right)+1\right)}$$

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(2*sqrt(c)*( - tan(asin(a*x)/2)**3 + 1))/(3*a*c**2*(tan(asin(a*x)/2)**3 +
3*tan(asin(a*x)/2)**2 + 3*tan(asin(a*x)/2) + 1))
```

3.638 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{2x}{5c^2\sqrt{c-a^2cx^2}} + \frac{1}{5ac^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{1}{5ac^2(1+ax)\sqrt{c-a^2cx^2}}$$

output
```
-2/5*x/c^2/(-a^2*c*x^2+c)^(1/2)+1/5/a/c^2/(a*x+1)^2/(-a^2*c*x^2+c)^(1/2)+1
/5/a/c^2/(a*x+1)/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\sqrt{1-a^2x^2}(-2+ax+4a^2x^2+2a^3x^3)}{5ac^2\sqrt{1-ax}(1+ax)^{5/2}\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
-1/5*(Sqrt[1 - a^2*x^2]*(-2 + a*x + 4*a^2*x^2 + 2*a^3*x^3))/(a*c^2*Sqrt[1
- a*x]*(1 + a*x)^(5/2)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6717, 6692, 457, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 6692}$$

$$-c\int \frac{(1-ax)^2}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 457}$$

$$-c\left(\frac{3\int \frac{1}{(c-a^2cx^2)^{5/2}}dx}{5c}-\frac{2(1-ax)}{5ac\,(c-a^2cx^2)^{5/2}}\right)$$

$$\Big\downarrow \text{ 209}$$

$$-c\left(\frac{3\left(\frac{2\int \frac{1}{(c-a^2cx^2)^{3/2}}dx}{3c}+\frac{x}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}-\frac{2(1-ax)}{5ac\,(c-a^2cx^2)^{5/2}}\right)$$

$$\Big\downarrow \text{ 208}$$

$$-c\left(\frac{3\left(\frac{2x}{3c^2\sqrt{c-a^2cx^2}}+\frac{x}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}-\frac{2(1-ax)}{5ac\,(c-a^2cx^2)^{5/2}}\right)$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
-(c*((-2*(1 - a*x))/(5*a*c*(c - a^2*c*x^2)^(5/2)) + (3*(x/(3*c*(c - a^2*c*
x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c - a^2*c*x^2])))/(5*c)))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

method	result
gosper	$-\frac{(ax-1)^2(2a^3x^3+4a^2x^2+ax-2)}{5a(-a^2c\,x^2+c)^{\frac{5}{2}}}$
orering	$-\frac{(ax-1)^2(2a^3x^3+4a^2x^2+ax-2)}{5a(-a^2c\,x^2+c)^{\frac{5}{2}}}$
trager	$\frac{(2a^3x^3+4a^2x^2+ax-2)\sqrt{-a^2c\,x^2+c}}{5c^3(ax+1)^3a(ax-1)}$
default	$\frac{x}{3c(-a^2c\,x^2+c)^{\frac{3}{2}}}+\frac{2x}{3c^2\sqrt{-a^2c\,x^2+c}}-\frac{2\left(-\frac{1}{5ac\left(x+\frac{1}{a}\right)\left(-\left(x+\frac{1}{a}\right)^2a^2c+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}+\frac{4a\left(-\frac{-2\left(x+\frac{1}{a}\right)a^2c+2ac}{6a^2c^2\left(-\left(x+\frac{1}{a}\right)^2a^2c+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}\right.}{5}\right.}{a}$

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(a*x-1)^2*(2*a^3*x^3+4*a^2*x^2+a*x-2)/a/(-a^2*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{(2\,a^3x^3+4\,a^2x^2+ax-2)\sqrt{-a^2cx^2+c}}{5\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/5*(2*a^3*x^3 + 4*a^2*x^2 + a*x - 2)*sqrt(-a^2*c*x^2 + c)/(a^5*c^3*x^4 +
2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{ax-1}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}\,(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{2}{5\left((-a^2cx^2+c)^{\frac{3}{2}}a^2cx+(-a^2cx^2+c)^{\frac{3}{2}}ac\right)} - \frac{2\,x}{5\,\sqrt{-a^2cx^2+c}c^2} - \frac{x}{5\,(-a^2cx^2+c)^{\frac{3}{2}}c}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
2/5/((-a^2*c*x^2 + c)^(3/2)*a^2*c*x + (-a^2*c*x^2 + c)^(3/2)*a*c) - 2/5*x/
(sqrt(-a^2*c*x^2 + c)*c^2) - 1/5*x/((-a^2*c*x^2 + c)^(3/2)*c)
```

Giac [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{ax-1}{(-a^2cx^2+c)^{\frac{5}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*x - 1)/((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 13.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}\,\left(2\,a^3\,x^3+4\,a^2\,x^2+a\,x-2\right)}{5\,a\,c^3\,\left(a\,x-1\right)\,{\left(a\,x+1\right)}^3}$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)^(5/2)*(a*x + 1)),x)
```

output
```
((c - a^2*c*x^2)^(1/2)*(a*x + 4*a^2*x^2 + 2*a^3*x^3 - 2))/(5*a*c^3*(a*x -
1)*(a*x + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.22

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(-\sqrt{-a^2x^2+1}\,a^2x^2-2\sqrt{-a^2x^2+1}\,ax-\sqrt{-a^2x^2+1}-4a^3x^3-8a^2x^2-2ax}{10\sqrt{-a^2x^2+1}\,a\,c^3\left(a^2x^2+2ax+1\right)}$$

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - sqrt( - a**2*x**2 + 1)*a**2*x**2 - 2*sqrt( - a**2*x**2 + 1)*a
*x - sqrt( - a**2*x**2 + 1) - 4*a**3*x**3 - 8*a**2*x**2 - 2*a*x + 4))/(10*
sqrt( - a**2*x**2 + 1)*a*c**3*(a**2*x**2 + 2*a*x + 1))
```

3.639 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=-\frac{4x}{21c^2(c-a^2cx^2)^{3/2}}+\frac{1}{7ac^2(1+ax)^2(c-a^2cx^2)^{3/2}}+\frac{1}{7ac^2(1+ax)(c-a^2cx^2)^{3/2}}-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}$$

output
```
-4/21*x/c^2/(-a^2*c*x^2+c)^(3/2)+1/7/a/c^2/(a*x+1)^2/(-a^2*c*x^2+c)^(3/2)+
1/7/a/c^2/(a*x+1)/(-a^2*c*x^2+c)^(3/2)-8/21*x/c^3/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=-\frac{\sqrt{1-a^2x^2}(-6+9ax+24a^2x^2+4a^3x^3-16a^4x^4-8a^5x^5)}{21ac^3(1-ax)^{3/2}(1+ax)^{7/2}\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

output
```
-1/21*(Sqrt[1 - a^2*x^2]*(-6 + 9*a*x + 24*a^2*x^2 + 4*a^3*x^3 - 16*a^4*x^4
 - 8*a^5*x^5))/(a*c^3*(1 - a*x)^(3/2)*(1 + a*x)^(7/2)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6692, 457, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int \frac{(1-ax)^2}{(c-a^2cx^2)^{9/2}}dx$$

$$\Big\downarrow \text{457}$$

$$-c\left(\frac{5\int \frac{1}{(c-a^2cx^2)^{7/2}}dx}{7c}-\frac{2(1-ax)}{7ac\,(c-a^2cx^2)^{7/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{5\left(\frac{4\int \frac{1}{(c-a^2cx^2)^{5/2}}dx}{5c}+\frac{x}{5c(c-a^2cx^2)^{5/2}}\right)}{7c}-\frac{2(1-ax)}{7ac\,(c-a^2cx^2)^{7/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{5\left(\frac{4\left(\frac{2\int\frac{1}{\left(c-a^2cx^2\right)^{3/2}}dx}{3c}+\frac{x}{3c\left(c-a^2cx^2\right)^{3/2}}\right)}{5c}+\frac{x}{5c(c-a^2cx^2)^{5/2}}\right)}{7c}-\frac{2(1-ax)}{7ac\left(c-a^2cx^2\right)^{7/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$-c\left(\frac{5\left(\frac{4\left(\frac{2x}{3c^2\sqrt{c-a^2cx^2}}+\frac{x}{3c\left(c-a^2cx^2\right)^{3/2}}\right)}{5c}+\frac{x}{5c(c-a^2cx^2)^{5/2}}\right)}{7c}-\frac{2(1-ax)}{7ac\left(c-a^2cx^2\right)^{7/2}}\right)$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

output
```
-(c*((-2*(1 - a*x))/(7*a*c*(c - a^2*c*x^2)^(7/2)) + (5*(x/(5*c*(c - a^2*c*
x^2)^(5/2)) + (4*(x/(3*c*(c - a^2*c*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c - a^
2*c*x^2])))/(5*c)))/(7*c)))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

method	result
gosper	$\frac{(ax-1)^2(8a^5x^5+16a^4x^4-4a^3x^3-24a^2x^2-9ax+6)}{21a(-a^2cx^2+c)^{\frac{7}{2}}}$
orering	$\frac{(ax-1)^2(8a^5x^5+16a^4x^4-4a^3x^3-24a^2x^2-9ax+6)}{21a(-a^2cx^2+c)^{\frac{7}{2}}}$
trager	$\frac{(8a^5x^5+16a^4x^4-4a^3x^3-24a^2x^2-9ax+6)\sqrt{-a^2cx^2+c}}{21c^4(ax+1)^4(ax-1)^2a}$
default	$\frac{x}{5c(-a^2cx^2+c)^{\frac{5}{2}}}+\frac{\frac{4x}{15c\left(-a^2cx^2+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{-a^2cx^2+c}}}{c}-\frac{2\left(-\frac{1}{7ac\left(x+\frac{1}{a}\right)\left(-\left(x+\frac{1}{a}\right)^2a^2c+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{5}{2}}}+\frac{6a\left(-\frac{-2(x+\cdots}{10a^2c^2\left(-\left(x+\frac{1}{a}\cdots\right.\right.}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$

input
```
int((a*x-1)/(-a^2*c*x^2+c)^(7/2)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/21*(a*x-1)^2*(8*a^5*x^5+16*a^4*x^4-4*a^3*x^3-24*a^2*x^2-9*a*x+6)/a/(-a^2
*c*x^2+c)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.12

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{(8\,a^5x^5+16\,a^4x^4-4\,a^3x^3-24\,a^2x^2-9\,ax+6)\sqrt{-a^2cx^2+c}}{21\,(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4)}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
1/21*(8*a^5*x^5 + 16*a^4*x^4 - 4*a^3*x^3 - 24*a^2*x^2 - 9*a*x + 6)*sqrt(-a
^2*c*x^2 + c)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 -
 a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{ax-1}{(-c\,(ax-1)\,(ax+1))^{\frac{7}{2}}\,(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{2}{7\left((-a^2cx^2+c)^{\frac{5}{2}}a^2cx+(-a^2cx^2+c)^{\frac{5}{2}}ac\right)} - \frac{8\,x}{21\,\sqrt{-a^2cx^2+c}c^3} - \frac{4\,x}{21\,(-a^2cx^2+c)^{\frac{3}{2}}c^2} - \frac{x}{7\,(-a^2cx^2+c)^{\frac{5}{2}}c}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
2/7/((-a^2*c*x^2 + c)^(5/2)*a^2*c*x + (-a^2*c*x^2 + c)^(5/2)*a*c) - 8/21*x
/(sqrt(-a^2*c*x^2 + c)*c^3) - 4/21*x/((-a^2*c*x^2 + c)^(3/2)*c^2) - 1/7*x/
((-a^2*c*x^2 + c)^(5/2)*c)
```

Giac [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{ax-1}{(-a^2cx^2+c)^{\frac{7}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*x - 1)/((-a^2*c*x^2 + c)^(7/2)*(a*x + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 13.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.21

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\sqrt{c-a^2\,c\,x^2}}{14\,a\,c^4\,(a\,x+1)^3} + \frac{\sqrt{c-a^2\,c\,x^2}}{28\,a\,c^4\,(a\,x+1)^4} - \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{11\,x}{42\,c^4}-\frac{5}{28\,a\,c^4}\right)}{(a\,x-1)^2\,(a\,x+1)^2} + \frac{8\,x\,\sqrt{c-a^2\,c\,x^2}}{21\,c^4\,(a\,x-1)\,(a\,x+1)}$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)^(7/2)*(a*x + 1)),x)
```

output
```
(c - a^2*c*x^2)^(1/2)/(14*a*c^4*(a*x + 1)^3) + (c - a^2*c*x^2)^(1/2)/(28*a
*c^4*(a*x + 1)^4) - ((c - a^2*c*x^2)^(1/2)*((11*x)/(42*c^4) - 5/(28*a*c^4)
))/((a*x - 1)^2*(a*x + 1)^2) + (8*x*(c - a^2*c*x^2)^(1/2))/(21*c^4*(a*x -
1)*(a*x + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.35

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\sqrt{c}\left(-9\sqrt{-a^2x^2+1}\,a^4x^4-18\sqrt{-a^2x^2+1}\,a^3x^3+18\sqrt{-a^2x^2+1}\,ax+9\sqrt{-a^2x^2+}\right.}{42\sqrt{-a^2x^2+1}\,a\,c^4\left(a^4x^4+2a^3x^3-2\right.}$$

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x)
```

output
```
(sqrt(c)*( - 9*sqrt( - a**2*x**2 + 1)*a**4*x**4 - 18*sqrt( - a**2*x**2 + 1
)*a**3*x**3 + 18*sqrt( - a**2*x**2 + 1)*a*x + 9*sqrt( - a**2*x**2 + 1) - 1
6*a**5*x**5 - 32*a**4*x**4 + 8*a**3*x**3 + 48*a**2*x**2 + 18*a*x - 12))/(4
2*sqrt( - a**2*x**2 + 1)*a*c**4*(a**4*x**4 + 2*a**3*x**3 - 2*a*x - 1))
```

3.640 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = -\frac{2x}{15c^2(c-a^2cx^2)^{5/2}}+\frac{1}{9ac^2(1+ax)^2(c-a^2cx^2)^{5/2}} + \frac{1}{9ac^2(1+ax)(c-a^2cx^2)^{5/2}}-\frac{8x}{45c^3(c-a^2cx^2)^{3/2}}-\frac{16x}{45c^4\sqrt{c-a^2cx^2}}$$

output
```
-2/15*x/c^2/(-a^2*c*x^2+c)^(5/2)+1/9/a/c^2/(a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+
1/9/a/c^2/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-8/45*x/c^3/(-a^2*c*x^2+c)^(3/2)-16/
45*x/c^4/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{\sqrt{1-a^2x^2}(10-25ax-60a^2x^2+10a^3x^3+80a^4x^4+24a^5x^5-32a^6x^6-16a^7x^7)}{45ac^4(1-ax)^{5/2}(1+ax)^{9/2}\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2)),x]
```

output
```
(Sqrt[1 - a^2*x^2]*(10 - 25*a*x - 60*a^2*x^2 + 10*a^3*x^3 + 80*a^4*x^4 + 2
4*a^5*x^5 - 32*a^6*x^6 - 16*a^7*x^7))/(45*a*c^4*(1 - a*x)^(5/2)*(1 + a*x)^
(9/2)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6717, 6692, 457, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{(c-a^2cx^2)^{9/2}}dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int \frac{(1-ax)^2}{(c-a^2cx^2)^{11/2}}dx$$

$$\Big\downarrow \text{457}$$

$$-c\left(\frac{7\int \frac{1}{(c-a^2cx^2)^{9/2}}dx}{9c}-\frac{2(1-ax)}{9ac\,(c-a^2cx^2)^{9/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{7\left(\frac{6\int \frac{1}{\left(c-a^2cx^2\right)^{7/2}}dx}{7c}+\frac{x}{7c(c-a^2cx^2)^{7/2}}\right)}{9c}-\frac{2(1-ax)}{9ac\,(c-a^2cx^2)^{9/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-c\left(\frac{7\left(\frac{6\left(\frac{4\int\frac{1}{\left(c-a^2cx^2\right)^{5/2}}dx}{5c}+\frac{x}{5c\left(c-a^2cx^2\right)^{5/2}}\right)}{7c}+\frac{x}{7c(c-a^2cx^2)^{7/2}}\right)}{9c}-\frac{2(1-ax)}{9ac\left(c-a^2cx^2\right)^{9/2}}\right)$$

$$\Big\downarrow \; 209$$

$$-c\left(\frac{7\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{1}{\left(c-a^2cx^2\right)^{3/2}}dx}{3c}+\frac{x}{3c\left(c-a^2cx^2\right)^{3/2}}\right)}{5c}+\frac{x}{5c\left(c-a^2cx^2\right)^{5/2}}\right)}{7c}+\frac{x}{7c(c-a^2cx^2)^{7/2}}\right)}{9c}-\frac{2(1-ax)}{9ac\left(c-a^2cx^2\right)^{9/2}}\right)$$

$$\Big\downarrow \; 208$$

$$-c\left(\frac{7\left(\frac{6\left(\frac{4\left(\frac{2x}{3c^2\sqrt{c-a^2cx^2}}+\frac{x}{3c\left(c-a^2cx^2\right)^{3/2}}\right)}{5c}+\frac{x}{5c\left(c-a^2cx^2\right)^{5/2}}\right)}{7c}+\frac{x}{7c(c-a^2cx^2)^{7/2}}\right)}{9c}-\frac{2(1-ax)}{9ac\left(c-a^2cx^2\right)^{9/2}}\right)$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2)),x]
```

output
```
-(c*((-2*(1 - a*x))/(9*a*c*(c - a^2*c*x^2)^(9/2)) + (7*(x/(7*c*(c - a^2*c*
x^2)^(7/2)) + (6*(x/(5*c*(c - a^2*c*x^2)^(5/2)) + (4*(x/(3*c*(c - a^2*c*x^
2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c - a^2*c*x^2])))/(5*c)))/(7*c)))/(9*c)))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.60

method	result
gosper	$-\frac{(ax-1)^{2} (16a^{7}x^{7}+32x^{6}a^{6}-24a^{5}x^{5}-80a^{4}x^{4}-10a^{3}x^{3}+60a^{2}x^{2}+25ax-10)}{45a(-a^{2}c\,x^{2}+c)^{\frac{9}{2}}}$
orering	$-\frac{(ax-1)^{2} (16a^{7}x^{7}+32x^{6}a^{6}-24a^{5}x^{5}-80a^{4}x^{4}-10a^{3}x^{3}+60a^{2}x^{2}+25ax-10)}{45a(-a^{2}c\,x^{2}+c)^{\frac{9}{2}}}$
trager	$\frac{(16a^{7}x^{7}+32x^{6}a^{6}-24a^{5}x^{5}-80a^{4}x^{4}-10a^{3}x^{3}+60a^{2}x^{2}+25ax-10)\sqrt{-a^{2}c\,x^{2}+c}}{45c^{5}(ax+1)^{5}(ax-1)^{3}a}$
default	$\frac{x}{7c(-a^{2}c\,x^{2}+c)^{\frac{7}{2}}}+\frac{\frac{6x}{35c\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15c\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^{2}\sqrt{-a^{2}c\,x^{2}+c}}\right)}{7c}}{c}-\frac{2\left(-\frac{1}{9ac\left(x+\frac{1}{a}\right)\left(-\left(x+\frac{1}{a}\right)^{2}a^{2}c+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{7}{2}}} \cdots\right.}{\cdots}$

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/45*(a*x-1)^2*(16*a^7*x^7+32*a^6*x^6-24*a^5*x^5-80*a^4*x^4-10*a^3*x^3+60
*a^2*x^2+25*a*x-10)/a/(-a^2*c*x^2+c)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{(16\,a^7x^7+32\,a^6x^6-24\,a^5x^5-80\,a^4x^4-10\,a^3x^3+60\,a^2x^2+25\,ax-10)\sqrt{-a^2cx^2}}{45\,(a^9c^5x^8+2\,a^8c^5x^7-2\,a^7c^5x^6-6\,a^6c^5x^5+6\,a^4c^5x^3+2\,a^3c^5x^2-2\,a^2c^5x-ac^5}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
1/45*(16*a^7*x^7 + 32*a^6*x^6 - 24*a^5*x^5 - 80*a^4*x^4 - 10*a^3*x^3 + 60*
a^2*x^2 + 25*a*x - 10)*sqrt(-a^2*c*x^2 + c)/(a^9*c^5*x^8 + 2*a^8*c^5*x^7 -
 2*a^7*c^5*x^6 - 6*a^6*c^5*x^5 + 6*a^4*c^5*x^3 + 2*a^3*c^5*x^2 - 2*a^2*c^5
*x - a*c^5)
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \int \frac{ax-1}{(-c\,(ax-1)\,(ax+1))^{\frac{9}{2}}\,(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(9/2),x)
```

output
```
Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(9/2)*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{2}{9\left((-a^2cx^2+c)^{\frac{7}{2}}a^2cx+(-a^2cx^2+c)^{\frac{7}{2}}ac\right)} - \frac{16\,x}{45\,\sqrt{-a^2cx^2+c}c^4} - \frac{8\,x}{45\,(-a^2cx^2+c)^{\frac{3}{2}}c^3} - \frac{2\,x}{15\,(-a^2cx^2+c)^{\frac{5}{2}}c^2} - \frac{x}{9\,(-a^2cx^2+c)^{\frac{7}{2}}c}$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
2/9/((-a^2*c*x^2 + c)^(7/2)*a^2*c*x + (-a^2*c*x^2 + c)^(7/2)*a*c) - 16/45*
x/(sqrt(-a^2*c*x^2 + c)*c^4) - 8/45*x/((-a^2*c*x^2 + c)^(3/2)*c^3) - 2/15*
x/((-a^2*c*x^2 + c)^(5/2)*c^2) - 1/9*x/((-a^2*c*x^2 + c)^(7/2)*c)
```

Giac [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \int \frac{ax-1}{(-a^2cx^2+c)^{\frac{9}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((a*x - 1)/((-a^2*c*x^2 + c)^(9/2)*(a*x + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.32

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{5\sqrt{c-a^2\,c\,x^2}}{144\,a\,c^5\,(a\,x+1)^4} + \frac{\sqrt{c-a^2\,c\,x^2}}{72\,a\,c^5\,(a\,x+1)^5} + \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{31\,x}{120\,c^5}-\frac{5}{24\,a\,c^5}\right)}{(a\,x-1)^3\,(a\,x+1)^3} - \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{8\,x}{45\,c^5}+\frac{5}{144\,a\,c^5}\right)}{(a\,x-1)^2\,(a\,x+1)^2} + \frac{16\,x\,\sqrt{c-a^2\,c\,x^2}}{45\,c^5\,(a\,x-1)\,(a\,x+1)}$$

input
```
int((a*x - 1)/((c - a^2*c*x^2)^(9/2)*(a*x + 1)),x)
```

output
```
(5*(c - a^2*c*x^2)^(1/2))/(144*a*c^5*(a*x + 1)^4) + (c - a^2*c*x^2)^(1/2)/
(72*a*c^5*(a*x + 1)^5) + ((c - a^2*c*x^2)^(1/2)*((31*x)/(120*c^5) - 5/(24*
a*c^5)))/((a*x - 1)^3*(a*x + 1)^3) - ((c - a^2*c*x^2)^(1/2)*((8*x)/(45*c^5
) + 5/(144*a*c^5)))/((a*x - 1)^2*(a*x + 1)^2) + (16*x*(c - a^2*c*x^2)^(1/2
))/(45*c^5*(a*x - 1)*(a*x + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.84

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{\sqrt{c}\,\left(-25\sqrt{-a^2x^2+1}\,a^6x^6-50\sqrt{-a^2x^2+1}\,a^5x^5+25\sqrt{-a^2x^2+1}\,a^4x^4+100\sqrt{-a}\right.}{9}$$

input
```
int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x)
```

output
```
(sqrt(c)*( - 25*sqrt( - a**2*x**2 + 1)*a**6*x**6 - 50*sqrt( - a**2*x**2 +
1)*a**5*x**5 + 25*sqrt( - a**2*x**2 + 1)*a**4*x**4 + 100*sqrt( - a**2*x**2
 + 1)*a**3*x**3 + 25*sqrt( - a**2*x**2 + 1)*a**2*x**2 - 50*sqrt( - a**2*x*
*2 + 1)*a*x - 25*sqrt( - a**2*x**2 + 1) - 32*a**7*x**7 - 64*a**6*x**6 + 48
*a**5*x**5 + 160*a**4*x**4 + 20*a**3*x**3 - 120*a**2*x**2 - 50*a*x + 20))/
(90*sqrt( - a**2*x**2 + 1)*a*c**5*(a**6*x**6 + 2*a**5*x**5 - a**4*x**4 - 4
*a**3*x**3 - a**2*x**2 + 2*a*x + 1))
```

3.641 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 189

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=-\frac{8(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{2(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

output
```
-8/7*(-a*x+1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3/2*(-a*
x+1)^8*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-2/3*(-a*x+1)^9*(-
a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(-a*x+1)^10*(-a^2*c*x
^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{c^4(-1+ax)^7\sqrt{c-a^2cx^2}(44+98ax+77a^2x^2+21a^3x^3)}{210a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a^2*c*x^2)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^4*(-1 + a*x)^7*Sqrt[c - a^2*c*x^2]*(44 + 98*a*x + 77*a^2*x^2 + 21*a^3*x
^3))/(210*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{9/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{9/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9dx}{x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{9/2}\int (1-ax)^6(ax+1)^3dx}{a^9x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\left(c-a^2cx^2\right)^{9/2}\int \left(-(1-ax)^9+6(1-ax)^8-12(1-ax)^7+8(1-ax)^6\right)dx}{a^9x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{(1-ax)^{10}}{10a}-\frac{2(1-ax)^9}{3a}+\frac{3(1-ax)^8}{2a}-\frac{8(1-ax)^7}{7a}\right)\left(c-a^2cx^2\right)^{9/2}}{a^9x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

input
```
Int[(c - a^2*c*x^2)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
((c - a^2*c*x^2)^(9/2)*((-8*(1 - a*x)^7)/(7*a) + (3*(1 - a*x)^8)/(2*a) - (
2*(1 - a*x)^9)/(3*a) + (1 - a*x)^10/(10*a)))/(a^9*(1 - 1/(a^2*x^2))^(9/2)*
x^9)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.53

method	result	size
gosper	$\frac{x\left(21a^{9}x^{9}-70a^{8}x^{8}+240x^{6}a^{6}-210a^{5}x^{5}-252a^{4}x^{4}+420a^{3}x^{3}-315ax+210\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{210\left(ax+1\right)^{3}\left(ax-1\right)^{6}}$	100
orering	$\frac{x\left(21a^{9}x^{9}-70a^{8}x^{8}+240x^{6}a^{6}-210a^{5}x^{5}-252a^{4}x^{4}+420a^{3}x^{3}-315ax+210\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{210\left(ax+1\right)^{3}\left(ax-1\right)^{6}}$	100
default	$\frac{\left(21a^{9}x^{9}-70a^{8}x^{8}+240x^{6}a^{6}-210a^{5}x^{5}-252a^{4}x^{4}+420a^{3}x^{3}-315ax+210\right)x\,c^{4}\sqrt{-c\left(a^{2}x^{2}-1\right)}\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{210\left(ax-1\right)^{2}}$	102

input
```
int((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/210*x*(21*a^9*x^9-70*a^8*x^8+240*a^6*x^6-210*a^5*x^5-252*a^4*x^4+420*a^3
*x^3-315*a*x+210)*(-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)^3/(
a*x-1)^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.50

$$\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx = \frac{(21\,a^9c^4x^{10} - 70\,a^8c^4x^9 + 240\,a^6c^4x^7 - 210\,a^5c^4x^6 - 252\,a^4c^4x^5 + 420\,a^3c^4x^4 - 315\,ac^4x^2}{210\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

output
```
1/210*(21*a^9*c^4*x^10 - 70*a^8*c^4*x^9 + 240*a^6*c^4*x^7 - 210*a^5*c^4*x^
6 - 252*a^4*c^4*x^5 + 420*a^3*c^4*x^4 - 315*a*c^4*x^2 + 210*c^4*x)*sqrt(-a
^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.08

$$\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx = \frac{(21\,a^{11}\sqrt{-c}c^4x^{11} - 49\,a^{10}\sqrt{-c}c^4x^{10} - 70\,a^9\sqrt{-c}c^4x^9 + 240\,a^8\sqrt{-c}c^4x^8 + 30\,a^7\sqrt{-c}c^4x^7}{21}$$

input
```
integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

output
```
1/210*(21*a^11*sqrt(-c)*c^4*x^11 - 49*a^10*sqrt(-c)*c^4*x^10 - 70*a^9*sqrt
(-c)*c^4*x^9 + 240*a^8*sqrt(-c)*c^4*x^8 + 30*a^7*sqrt(-c)*c^4*x^7 - 462*a^
6*sqrt(-c)*c^4*x^6 + 168*a^5*sqrt(-c)*c^4*x^5 + 420*a^4*sqrt(-c)*c^4*x^4 -
 315*a^3*sqrt(-c)*c^4*x^3 - 105*a^2*sqrt(-c)*c^4*x^2 - 210*sqrt(-c)*c^4)*(
a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.80

$$\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx = \frac{1}{210}\left(21\,a^9c^4x^{10}\mathrm{sgn}(ax+1) - 70\,a^8c^4x^9\mathrm{sgn}(ax+1) + 240\,a^6c^4x^7\mathrm{sgn}(ax+1) - 210\,a^5c^4\right.$$

input
```
integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

output
```
1/210*(21*a^9*c^4*x^10*sgn(a*x + 1) - 70*a^8*c^4*x^9*sgn(a*x + 1) + 240*a^
6*c^4*x^7*sgn(a*x + 1) - 210*a^5*c^4*x^6*sgn(a*x + 1) - 252*a^4*c^4*x^5*sg
n(a*x + 1) + 420*a^3*c^4*x^4*sgn(a*x + 1) - 315*a*c^4*x^2*sgn(a*x + 1) + 2
10*c^4*x*sgn(a*x + 1) + 212*c^4*sgn(a*x + 1)/a)*sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\int \left(c-a^2\,c\,x^2\right)^{9/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.39

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\sqrt{c}\,c^4i(-21a^{10}x^{10}+70a^9x^9-240a^7x^7+210a^6x^6+252a^5x^5-420a^4x^4+315a^2x^2-210}{210a}$$

input
```
int((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*c**4*i*( - 21*a**10*x**10 + 70*a**9*x**9 - 240*a**7*x**7 + 210*a*
*6*x**6 + 252*a**5*x**5 - 420*a**4*x**4 + 315*a**2*x**2 - 210*a*x + 44))/(
210*a)
```

3.642 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 142

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{4(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

output
```
2/3*(-a*x+1)^6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-4/7*(-a*x+
1)^7*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(-a*x+1)^8*(-a^2
*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.44

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=-\frac{c^3(-1+ax)^6\left(37+54ax+21a^2x^2\right)\sqrt{c-a^2cx^2}}{168a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a^2*c*x^2)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-1/168*(c^3*(-1 + a*x)^6*(37 + 54*a*x + 21*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(
a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{7/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7dx}{x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\left(c-a^2cx^2\right)^{7/2}\int -(1-ax)^5(ax+1)^2dx}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(c-a^2cx^2\right)^{7/2}\int (1-ax)^5(ax+1)^2dx}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\left(c-a^2cx^2\right)^{7/2}\int \left((1-ax)^7-4(1-ax)^6+4(1-ax)^5\right)dx}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-\frac{(1-ax)^8}{8a}+\frac{4(1-ax)^7}{7a}-\frac{2(1-ax)^6}{3a}\right)\left(c-a^2cx^2\right)^{7/2}}{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

input
```
Int[(c - a^2*c*x^2)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-(((c - a^2*c*x^2)^(7/2)*((-2*(1 - a*x)^6)/(3*a) + (4*(1 - a*x)^7)/(7*a) -
 (1 - a*x)^8/(8*a)))/(a^7*(1 - 1/(a^2*x^2))^(7/2)*x^7))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{x\left(21a^{7}x^{7}-72x^{6}a^{6}+28a^{5}x^{5}+168a^{4}x^{4}-210a^{3}x^{3}-56a^{2}x^{2}+252ax-168\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{168\left(ax+1\right)^{2}\left(ax-1\right)^{5}}$	100
orering	$\frac{x\left(21a^{7}x^{7}-72x^{6}a^{6}+28a^{5}x^{5}+168a^{4}x^{4}-210a^{3}x^{3}-56a^{2}x^{2}+252ax-168\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{168\left(ax+1\right)^{2}\left(ax-1\right)^{5}}$	100
default	$-\frac{\left(21a^{7}x^{7}-72x^{6}a^{6}+28a^{5}x^{5}+168a^{4}x^{4}-210a^{3}x^{3}-56a^{2}x^{2}+252ax-168\right)x\,c^{3}\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{168\left(ax-1\right)^{2}}$	102

input
```
int((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/168*x*(21*a^7*x^7-72*a^6*x^6+28*a^5*x^5+168*a^4*x^4-210*a^3*x^3-56*a^2*x
^2+252*a*x-168)*(-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)^2/(a*
x-1)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = -\frac{\left(21\,a^7c^3x^8-72\,a^6c^3x^7+28\,a^5c^3x^6+168\,a^4c^3x^5-210\,a^3c^3x^4-56\,a^2c^3x^3+252\,ac^3x^2-168\,c^3x\right)\sqrt{-a^2c}}{168\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

output
```
-1/168*(21*a^7*c^3*x^8 - 72*a^6*c^3*x^7 + 28*a^5*c^3*x^6 + 168*a^4*c^3*x^5
 - 210*a^3*c^3*x^4 - 56*a^2*c^3*x^3 + 252*a*c^3*x^2 - 168*c^3*x)*sqrt(-a^2
*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.21

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=$$
$$-\frac{\left(21\,a^9\sqrt{-c}c^3x^9-51\,a^8\sqrt{-c}c^3x^8-44\,a^7\sqrt{-c}c^3x^7+196\,a^6\sqrt{-c}c^3x^6-42\,a^5\sqrt{-c}c^3x^5-266\,a^4\sqrt{-c}c^3x^4\right.}{168\,(a^3x^2-2\,a^2x+a)(ax+1)}$$

input
```
integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

output
```
-1/168*(21*a^9*sqrt(-c)*c^3*x^9 - 51*a^8*sqrt(-c)*c^3*x^8 - 44*a^7*sqrt(-c
)*c^3*x^7 + 196*a^6*sqrt(-c)*c^3*x^6 - 42*a^5*sqrt(-c)*c^3*x^5 - 266*a^4*s
qrt(-c)*c^3*x^4 + 196*a^3*sqrt(-c)*c^3*x^3 + 84*a^2*sqrt(-c)*c^3*x^2 + 168
*sqrt(-c)*c^3)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=$$
$$-\frac{1}{168}\left(21\,a^7c^3x^8\text{sgn}(ax+1)-72\,a^6c^3x^7\text{sgn}(ax+1)+28\,a^5c^3x^6\text{sgn}(ax+1)+168\,a^4c^3x^5\text{sgn}(ax+1)-\right.$$

input
```
integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

output
```
-1/168*(21*a^7*c^3*x^8*sgn(a*x + 1) - 72*a^6*c^3*x^7*sgn(a*x + 1) + 28*a^5
*c^3*x^6*sgn(a*x + 1) + 168*a^4*c^3*x^5*sgn(a*x + 1) - 210*a^3*c^3*x^4*sgn
(a*x + 1) - 56*a^2*c^3*x^3*sgn(a*x + 1) + 252*a*c^3*x^2*sgn(a*x + 1) - 168
*c^3*x*sgn(a*x + 1) - 219*c^3*sgn(a*x + 1)/a)*sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\int \left(c-a^2\,c\,x^2\right)^{7/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.51

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{\sqrt{c}\,c^3i(21a^8x^8-72a^7x^7+28a^6x^6+168a^5x^5-210a^4x^4-56a^3x^3+252a^2x^2-168ax+3}{168a}$$

input
```
int((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*c**3*i*(21*a**8*x**8 - 72*a**7*x**7 + 28*a**6*x**6 + 168*a**5*x**
5 - 210*a**4*x**4 - 56*a**3*x**3 + 252*a**2*x**2 - 168*a*x + 37))/(168*a)
```

3.643 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\frac{2(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

output
```
-2/5*(-a*x+1)^5*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(-a*x
+1)^6*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{c^2(-1+ax)^5(7+5ax)\sqrt{c-a^2cx^2}}{30a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(c - a^2*c*x^2)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^2*(-1 + a*x)^5*(7 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sqrt[1 - 1/(a^2
*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{5/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5dx}{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2}\int (1-ax)^4(ax+1)dx}{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\left(c-a^2cx^2\right)^{5/2}\int \left(2(1-ax)^4-(1-ax)^5\right)dx}{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(\frac{(1-ax)^6}{6a}-\frac{2(1-ax)^5}{5a}\right)\left(c-a^2cx^2\right)^{5/2}}{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

input
```
Int[(c - a^2*c*x^2)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
((c - a^2*c*x^2)^(5/2)*((-2*(1 - a*x)^5)/(5*a) + (1 - a*x)^6/(6*a)))/(a^5*
(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{x\left(5a^{5}x^{5}-18a^{4}x^{4}+15a^{3}x^{3}+20a^{2}x^{2}-45ax+30\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{30(ax+1)(ax-1)^{4}}$	84
orering	$\frac{x\left(5a^{5}x^{5}-18a^{4}x^{4}+15a^{3}x^{3}+20a^{2}x^{2}-45ax+30\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{30(ax+1)(ax-1)^{4}}$	84
default	$\frac{\left(5a^{5}x^{5}-18a^{4}x^{4}+15a^{3}x^{3}+20a^{2}x^{2}-45ax+30\right)x\,c^{2}\sqrt{-c\left(a^{2}x^{2}-1\right)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{30(ax-1)^{2}}$	86

input
```
int((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*x*(5*a^5*x^5-18*a^4*x^4+15*a^3*x^3+20*a^2*x^2-45*a*x+30)*(-a^2*c*x^2+
c)^(5/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/(a*x-1)^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx = \frac{(5\,a^5c^2x^6-18\,a^4c^2x^5+15\,a^3c^2x^4+20\,a^2c^2x^3-45\,ac^2x^2+30\,c^2x)\sqrt{-a^2c}}{30\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

output
```
1/30*(5*a^5*c^2*x^6 - 18*a^4*c^2*x^5 + 15*a^3*c^2*x^4 + 20*a^2*c^2*x^3 - 4
5*a*c^2*x^2 + 30*c^2*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.47

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\left(5\,a^7\sqrt{-c}c^2x^7-13\,a^6\sqrt{-c}c^2x^6-3\,a^5\sqrt{-c}c^2x^5+35\,a^4\sqrt{-c}c^2x^4-25\,a^3\sqrt{-c}c^2x^3-15\,a^2\right.}{30\,(a^3x^2-2\,a^2x+a)(ax+1)}$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

output
```
1/30*(5*a^7*sqrt(-c)*c^2*x^7 - 13*a^6*sqrt(-c)*c^2*x^6 - 3*a^5*sqrt(-c)*c^
2*x^5 + 35*a^4*sqrt(-c)*c^2*x^4 - 25*a^3*sqrt(-c)*c^2*x^3 - 15*a^2*sqrt(-c
)*c^2*x^2 - 30*sqrt(-c)*c^2)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1
))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.23

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{1}{30}\left(5\,a^5c^2x^6\mathrm{sgn}(ax+1)-18\,a^4c^2x^5\mathrm{sgn}(ax+1)+15\,a^3c^2x^4\mathrm{sgn}(ax+1)+20\,a^2c^2x^3\mathrm{sgn}\right.$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

output
```
1/30*(5*a^5*c^2*x^6*sgn(a*x + 1) - 18*a^4*c^2*x^5*sgn(a*x + 1) + 15*a^3*c^
2*x^4*sgn(a*x + 1) + 20*a^2*c^2*x^3*sgn(a*x + 1) - 45*a*c^2*x^2*sgn(a*x +
1) + 30*c^2*x*sgn(a*x + 1) + 57*c^2*sgn(a*x + 1)/a)*sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\int \left(c-a^2\,c\,x^2\right)^{5/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

input
```
int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.60

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\sqrt{c}\,c^2i(-5a^6x^6+18a^5x^5-15a^4x^4-20a^3x^3+45a^2x^2-30ax+7)}{30a}$$

input
```
int((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*c**2*i*( - 5*a**6*x**6 + 18*a**5*x**5 - 15*a**4*x**4 - 20*a**3*x**
*3 + 45*a**2*x**2 - 30*a*x + 7))/(30*a)
```

3.644 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 47

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

output
```
1/4*(-a*x+1)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{c\sqrt{c-a^2cx^2}\left(-4+6ax-4a^2x^2+a^3x^3\right)}{4a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(c - a^2*c*x^2)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-1/4*(c*Sqrt[c - a^2*c*x^2]*(-4 + 6*a*x - 4*a^2*x^2 + a^3*x^3))/(a*Sqrt[1
 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6746, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-a^2cx^2)^{3/2} e^{-3\coth^{-1}(ax)} dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{(c-a^2cx^2)^{3/2} \int e^{-3\coth^{-1}(ax)} \left(1-\frac{1}{a^2x^2}\right)^{3/2} x^3 dx}{x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{(c-a^2cx^2)^{3/2} \int (ax-1)^3 dx}{a^3x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{17}$$

$$\frac{(1-ax)^4 (c-a^2cx^2)^{3/2}}{4a^4x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Int[(c - a^2*c*x^2)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
((1 - a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} (ax-1)^{2} (ax+1) \sqrt{-c\left(a^{2} x^{2}-1\right)}\, c}{4a}$	48
gosper	$\frac{x\left(a^{3} x^{3}-4 a^{2} x^{2}+6 a x-4\right) \left(-a^{2} c\, x^{2}+c\right)^{\frac{3}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4 (ax-1)^{3}}$	60
orering	$\frac{x\left(a^{3} x^{3}-4 a^{2} x^{2}+6 a x-4\right) \left(-a^{2} c\, x^{2}+c\right)^{\frac{3}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4 (ax-1)^{3}}$	60

input
```
int((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*((a*x-1)/(a*x+1))^(3/2)*(a*x-1)^2*(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*c/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{\left(a^3cx^4-4\,a^2cx^3+6\,acx^2-4\,cx\right)\sqrt{-a^2c}}{4\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

output
```
-1/4*(a^3*c*x^4 - 4*a^2*c*x^3 + 6*a*c*x^2 - 4*c*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(40) = 80$.

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.06

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\frac{\left(a^5\sqrt{-c}cx^5-3\,a^4\sqrt{-c}cx^4+2\,a^3\sqrt{-c}cx^3+2\,a^2\sqrt{-c}cx^2+4\,\sqrt{-c}c\right)(ax-1)^2}{4\,(a^3x^2-2\,a^2x+a)(ax+1)}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

output
```
-1/4*(a^5*sqrt(-c)*c*x^5 - 3*a^4*sqrt(-c)*c*x^4 + 2*a^3*sqrt(-c)*c*x^3 + 2
*a^2*sqrt(-c)*c*x^2 + 4*sqrt(-c)*c)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(
a*x + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.53

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = -\frac{1}{4}\left(a^3cx^4\text{sgn}(ax+1)-4\,a^2cx^3\text{sgn}(ax+1)+6\,acx^2\text{sgn}(ax+1)-4\,cx\text{sgn}(ax+1)-\frac{15\,c\text{sgn}(ax+1)}{a}\right)$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

output
```
-1/4*(a^3*c*x^4*sgn(a*x + 1) - 4*a^2*c*x^3*sgn(a*x + 1) + 6*a*c*x^2*sgn(a*
x + 1) - 4*c*x*sgn(a*x + 1) - 15*c*sgn(a*x + 1)/a)*sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \int \left(c-a^2\,c\,x^2\right)^{3/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \frac{\sqrt{c}\,ci\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)}{4a}$$

input
```
int((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*c*i*(a**4*x**4 - 4*a**3*x**3 + 6*a**2*x**2 - 4*a*x + 1))/(4*a)
```

3.645 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
-3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(
1-1/a^2/x^2)^(1/2)+4*(-a^2*c*x^2+c)^(1/2)*ln(a*x+1)/a^2/(1-1/a^2/x^2)^(1/2
)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1+ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((-3*x)/a + x^2/2 + (4*Log[1 + a*x])/a^2))/(Sqrt[1 -
1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{ax+1}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(ax+\frac{4}{ax+1}-3\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{ax^2}{2}+\frac{4\log(ax+1)}{a}-3x\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-3*x + (a*x^2)/2 + (4*Log[1 + a*x])/a))/(a*Sqrt[1 -
1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\left(a^{2} x^{2}-6 a x +8 \ln \left(a x +1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{2 a \left(a x -1\right)^{2}}$	67

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a
*x+1))^(3/2)/a/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{(a^2x^2-6\,ax+8\,\log(ax+1))\sqrt{-a^2c}}{2\,a^2}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

output
```
1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(-a^2*c)/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{1}{2}\sqrt{-c}\left(\frac{8\log\left(|ax+1|\right)\operatorname{sgn}(ax+1)}{a}+\frac{a^3x^2\operatorname{sgn}(ax+1)-6a^2x\operatorname{sgn}(ax+1)}{a^2}\right)$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

output
```
1/2*sqrt(-c)*(8*log(abs(a*x + 1))*sgn(a*x + 1)/a + (a^3*x^2*sgn(a*x + 1) -
 6*a^2*x*sgn(a*x + 1))/a^2)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.27

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c}\,i(-8\log(ax+1)-a^2x^2+6ax-5)}{2a}$$

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*i*( - 8*log(a*x + 1) - a**2*x**2 + 6*a*x - 5))/(2*a)
```

3.646 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{2\sqrt{1-\frac{1}{a^2x^2}}x}{(1+ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}x\log(1+ax)}{\sqrt{c-a^2cx^2}}$$

output
```
2*(1-1/a^2/x^2)^(1/2)*x/(a*x+1)/(-a^2*c*x^2+c)^(1/2)+(1-1/a^2/x^2)^(1/2)*x
*ln(a*x+1)/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x\left(\frac{2}{1+ax}+\log(1+ax)\right)}{\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(2/(1 + a*x) + Log[1 + a*x]))/Sqrt[c - a^2*c*x^2]
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{1-ax}{(ax+1)^2}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\int \frac{1-ax}{(ax+1)^2}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{2}{(ax+1)^2}+\frac{1}{-ax-1}\right)dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(-\frac{2}{a(ax+1)}-\frac{\log(ax+1)}{a}\right)}{\sqrt{c-a^2cx^2}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

output
```
-((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2/(a*(1 + a*x)) - Log[1 + a*x]/a))/Sqrt[c -
 a^2*c*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(\ln \left(a x +1\right) x a +\ln \left(a x +1\right)+2\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{a \left(a x -1\right)^{2} c}$	62

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*x*a+ln(a*x+1)+2)*((a*x-1)/(a*x+1))^(3/2
)/a/(a*x-1)^2/c
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{\sqrt{-a^2c}((ax+1)\log(ax+1)+2)}{a^3cx+a^2c}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

output
```
-sqrt(-a^2*c)*((a*x + 1)*log(a*x + 1) + 2)/(a^3*c*x + a^2*c)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2)/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(-a^2*c*x^2 + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{c}\,i\left(\log\left(\sqrt{-ax+1}-\sqrt{2}\right)ax+\log\left(\sqrt{-ax+1}-\sqrt{2}\right)+\log\left(\sqrt{-ax+1}+\sqrt{2}\right)ax+\log\left(\sqrt{-ax+1}+\right.\right.}{ac\,(ax+1)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*i*(log(sqrt( - a*x + 1) - sqrt(2))*a*x + log(sqrt( - a*x + 1) - s
qrt(2)) + log(sqrt( - a*x + 1) + sqrt(2))*a*x + log(sqrt( - a*x + 1) + sqr
t(2)) - a*x + 1))/(a*c*(a*x + 1))
```

3.647 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)^2\,(c-a^2cx^2)^{3/2}}$$

output
```
-1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(a*x+1)^2/(-a^2*c*x^2+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-a^2cx^2}}{2c^2(-1+ax)(1+ax)^3}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
-1/2*(Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])/(c^2*(-1 + a*x)*(1 + a*
x)^3)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6746, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{1}{(ax+1)^3}dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{17}$$

$$-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(ax+1)^2\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
-1/2*(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/((1 + a*x)^2*(c - a^2*c*x^2)^(3/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
gosper	$-\frac{(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2a\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}$	39
orering	$\frac{x(ax+2)(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}$	42
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\left(a^{2}x^{2}-1\right)}}{2(ax-1)\left(a^{2}x^{2}-1\right)a\,c^{2}}$	56

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x+1)/a*((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{-a^2c}}{2\,(a^4c^2x^2+2\,a^3c^2x+a^2c^2)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

output
```
-1/2*sqrt(-a^2*c)/(a^4*c^2*x^2 + 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2)/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\operatorname{sgn}(ax+1)}{2\,(ax+1)^2a\sqrt{-c}c}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

output
```
1/2*sgn(a*x + 1)/((a*x + 1)^2*a*sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 13.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a^2\,c\,\left(x\,\sqrt{c-a^2\,c\,x^2}+\frac{\sqrt{c-a^2\,c\,x^2}}{a}\right)}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(3/2),x)
```

output
```
((a*x - 1)/(a*x + 1))^(1/2)/(2*a^2*c*(x*(c - a^2*c*x^2)^(1/2) + (c - a^2*c
*x^2)^(1/2)/a))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,i}{2a\,c^2\,(a^2x^2+2ax+1)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*i)/(2*a*c**2*(a**2*x**2 + 2*a*x + 1))
```

3.648 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 182

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}} \, dx = \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1+ax)^3(c-a^2cx^2)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2(c-a^2cx^2)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)(c-a^2cx^2)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\text{arctanh}(ax)}{8(c-a^2cx^2)^{5/2}}$$

output
```
1/6*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(a*x+1)^3/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-
1/a^2/x^2)^(5/2)*x^5/(a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^
(5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arc
tanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.39

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}} \, dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}x(-10-9ax-3a^2x^2+3(1+ax)^3\text{arctanh}(ax))}{24c^2(1+ax)^3\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
-1/24*(Sqrt[1 - 1/(a^2*x^2)]*x*(-10 - 9*a*x - 3*a^2*x^2 + 3*(1 + a*x)^3*Ar
cTanh[a*x]))/(c^2*(1 + a*x)^3*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.50, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6746, 6747, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{1}{(1-ax)(ax+1)^4}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{1}{(1-ax)(ax+1)^4}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{54}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(\frac{1}{8(ax+1)^2}+\frac{1}{4(ax+1)^3}+\frac{1}{2(ax+1)^4}-\frac{1}{8(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\text{arctanh}(ax)}{8a}-\frac{1}{8a(ax+1)}-\frac{1}{8a(ax+1)^2}-\frac{1}{6a(ax+1)^3}\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(-1/6*1/(a*(1 + a*x)^3) - 1/(8*a*(1 + a
*x)^2) - 1/(8*a*(1 + a*x)) + ArcTanh[a*x]/(8*a)))/(c - a^2*c*x^2)^(5/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.93

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} \sqrt{-c(a^2x^2-1)}\,(3\ln(ax+1)x^3a^3-3a^3\ln(ax-1)x^3+9\ln(ax+1)x^2a^2-9a^2\ln(ax-1)x^2-6a^2x^2+9\ln(ax+1)xa-9a\ln(ax-}{48(ax-1)(ax+1)(a^2x^2-1)c^3a}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(
a*x+1)*x^3*a^3-3*a^3*ln(a*x-1)*x^3+9*ln(a*x+1)*x^2*a^2-9*a^2*ln(a*x-1)*x^2
-6*a^2*x^2+9*ln(a*x+1)*x*a-9*a*ln(a*x-1)*x-18*a*x+3*ln(a*x+1)-3*ln(a*x-1)-
20)/(a^2*x^2-1)/c^3/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.75

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{3\,(a^4x^3+3\,a^3x^2+3\,a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\,(3\,a^2x^2+9\,ax+10)\sqrt{-a^2c}}{48\,(a^5c^3x^3+3\,a^4c^3x^2+3\,a^3c^3x+a^2c^3)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="frica
s")
```

output
```
-1/48*(3*(a^4*x^3 + 3*a^3*x^2 + 3*a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*s
qrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 + 9*a*x + 10)*sq
rt(-a^2*c))/(a^5*c^3*x^3 + 3*a^4*c^3*x^2 + 3*a^3*c^3*x + a^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.54

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\frac{3\log(|ax+1|)\text{sgn}(ax+1)}{ac^2} - \frac{3\log(|ax-1|)\text{sgn}(ax+1)}{ac^2} - \frac{2\left(3a^2x^2\text{sgn}(ax+1)+9ax\text{sgn}(ax+1)+10\,\text{sgn}(ax+1)\right)}{(ax+1)^3ac^2}}{48\sqrt{-c}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac"
)
```

output
```
-1/48*(3*log(abs(a*x + 1))*sgn(a*x + 1)/(a*c^2) - 3*log(abs(a*x - 1))*sgn(
a*x + 1)/(a*c^2) - 2*(3*a^2*x^2*sgn(a*x + 1) + 9*a*x*sgn(a*x + 1) + 10*sgn
(a*x + 1))/((a*x + 1)^3*a*c^2))/sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-a^2\,c\,x^2\right)^{5/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(5/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.37

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\frac{\sqrt{c}\,i\left(-3\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^3x^3-9\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^2x^2-9\log\left(\sqrt{-ax\right.$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*i*( - 3*log(sqrt( - a*x + 1) - sqrt(2))*a**3*x**3 - 9*log(sqrt( -
 a*x + 1) - sqrt(2))*a**2*x**2 - 9*log(sqrt( - a*x + 1) - sqrt(2))*a*x - 3
*log(sqrt( - a*x + 1) - sqrt(2)) - 3*log(sqrt( - a*x + 1) + sqrt(2))*a**3*
x**3 - 9*log(sqrt( - a*x + 1) + sqrt(2))*a**2*x**2 - 9*log(sqrt( - a*x + 1
) + sqrt(2))*a*x - 3*log(sqrt( - a*x + 1) + sqrt(2)) + 6*log(sqrt( - a*x +
 1))*a**3*x**3 + 18*log(sqrt( - a*x + 1))*a**2*x**2 + 18*log(sqrt( - a*x +
 1))*a*x + 6*log(sqrt( - a*x + 1)) - a**3*x**3 + 3*a**2*x**2 + 15*a*x + 19
))/(48*a*c**3*(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1))
```

3.649 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 275

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx=\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)^4\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1+ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1+ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{5a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\text{arctanh}(ax)}{32\left(c-a^2cx^2\right)^{7/2}}$$

output
```
1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)-1/16*a^6*(1
-1/a^2/x^2)^(7/2)*x^7/(a*x+1)^4/(-a^2*c*x^2+c)^(7/2)-1/12*a^6*(1-1/a^2/x^2
)^(7/2)*x^7/(a*x+1)^3/(-a^2*c*x^2+c)^(7/2)-3/32*a^6*(1-1/a^2/x^2)^(7/2)*x^
7/(a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)/(
-a^2*c*x^2+c)^(7/2)+5/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*c*
x^2+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.36

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(32+15ax-35a^2x^2-45a^3x^3-15a^4x^4+15(-1+ax)(1+ax)^4\text{arctanh}(ax))}{96c^3(-1+ax)(1+ax)^4\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

output
```
-1/96*(Sqrt[1 - 1/(a^2*x^2)]*x*(32 + 15*a*x - 35*a^2*x^2 - 45*a^3*x^3 - 15
*a^4*x^4 + 15*(-1 + a*x)*(1 + a*x)^4*ArcTanh[a*x]))/(c^3*(-1 + a*x)*(1 + a
*x)^4*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.43, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6746, 6747, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}dx}{(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int \frac{1}{(1-ax)^2(ax+1)^5}dx}{(c-a^2cx^2)^{7/2}}$$

$$\downarrow \text{54}$$

$$\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\int\left(\frac{1}{32(ax-1)^2}+\frac{1}{8(ax+1)^2}+\frac{3}{16(ax+1)^3}+\frac{1}{4(ax+1)^4}+\frac{1}{4(ax+1)^5}-\frac{5}{32(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{7/2}}$$

$$\downarrow \text{2009}$$

$$\frac{a^7x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(\frac{5\text{arctanh}(ax)}{32a}+\frac{1}{32a(1-ax)}-\frac{1}{8a(ax+1)}-\frac{3}{32a(ax+1)^2}-\frac{1}{12a(ax+1)^3}-\frac{1}{16a(ax+1)^4}\right)}{(c-a^2cx^2)^{7/2}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

output
```
(a^7*(1 - 1/(a^2*x^2))^(7/2)*x^7*(1/(32*a*(1 - a*x)) - 1/(16*a*(1 + a*x)^4
) - 1/(12*a*(1 + a*x)^3) - 3/(32*a*(1 + a*x)^2) - 1/(8*a*(1 + a*x)) + (5*A
rcTanh[a*x])/(32*a)))/(c - a^2*c*x^2)^(7/2)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.88

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c(a^2x^2-1)}\,(15\ln(ax+1)x^5a^5-15\ln(ax-1)x^5a^5+45\ln(ax+1)x^4a^4-45\ln(ax-1)x^4a^4-30a^4x^4+30\ln(ax+1)x^3a^3-}{192(ax-}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*(
15*ln(a*x+1)*x^5*a^5-15*ln(a*x-1)*x^5*a^5+45*ln(a*x+1)*x^4*a^4-45*ln(a*x-1
)*x^4*a^4-30*a^4*x^4+30*ln(a*x+1)*x^3*a^3-30*a^3*ln(a*x-1)*x^3-90*a^3*x^3-
30*ln(a*x+1)*x^2*a^2+30*a^2*ln(a*x-1)*x^2-70*a^2*x^2-45*ln(a*x+1)*x*a+45*a
*ln(a*x-1)*x+30*a*x-15*ln(a*x+1)+15*ln(a*x-1)+64)/(a^2*x^2-1)/c^4/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.70

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx =$$

$$-\frac{15\,(a^6x^5+3\,a^5x^4+2\,a^4x^3-2\,a^3x^2-3\,a^2x-a)\sqrt{-c}\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\,(15\,a^4x^4+45\,a^3x^3+}{192\,(a^7c^4x^5+3\,a^6c^4x^4+2\,a^5c^4x^3-2\,a^4c^4x^2-3\,a^3c^4x-a^2c^4)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="frica
s")
```

output
```
-1/192*(15*(a^6*x^5 + 3*a^5*x^4 + 2*a^4*x^3 - 2*a^3*x^2 - 3*a^2*x - a)*sqr
t(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(
15*a^4*x^4 + 45*a^3*x^3 + 35*a^2*x^2 - 15*a*x - 32)*sqrt(-a^2*c))/(a^7*c^4
*x^5 + 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 - 2*a^4*c^4*x^2 - 3*a^3*c^4*x - a^2*c
^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxim
a")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.48

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\frac{15\log(|ax+1|)\mathrm{sgn}(ax+1)}{ac^3} - \frac{15\log(|ax-1|)\mathrm{sgn}(ax+1)}{ac^3} - \frac{2\left(15a^4x^4\mathrm{sgn}(ax+1)+45a^3x^3\mathrm{sgn}(ax+1)+35a^2x^2\mathrm{sgn}(ax+1)-15ax\mathrm{sgn}(ax+1)-3\right.}{(ax+1)^4(ax-1)ac^3}}{192\sqrt{-c}}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac"
)
```

output
```
-1/192*(15*log(abs(a*x + 1))*sgn(a*x + 1)/(a*c^3) - 15*log(abs(a*x - 1))*s
gn(a*x + 1)/(a*c^3) - 2*(15*a^4*x^4*sgn(a*x + 1) + 45*a^3*x^3*sgn(a*x + 1)
 + 35*a^2*x^2*sgn(a*x + 1) - 15*a*x*sgn(a*x + 1) - 32*sgn(a*x + 1))/((a*x
+ 1)^4*(a*x - 1)*a*c^3))/sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{(c-a^2\,c\,x^2)^{7/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(7/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.43

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\sqrt{c}\,i\left(-60\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^5x^5-180\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^4x^4-120\log\left(\sqrt{\right.\right.}{}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x)
```

output
```
(sqrt(c)*i*( - 60*log(sqrt( - a*x + 1) - sqrt(2))*a**5*x**5 - 180*log(sqrt
( - a*x + 1) - sqrt(2))*a**4*x**4 - 120*log(sqrt( - a*x + 1) - sqrt(2))*a*
*3*x**3 + 120*log(sqrt( - a*x + 1) - sqrt(2))*a**2*x**2 + 180*log(sqrt( -
a*x + 1) - sqrt(2))*a*x + 60*log(sqrt( - a*x + 1) - sqrt(2)) - 60*log(sqrt
( - a*x + 1) + sqrt(2))*a**5*x**5 - 180*log(sqrt( - a*x + 1) + sqrt(2))*a*
*4*x**4 - 120*log(sqrt( - a*x + 1) + sqrt(2))*a**3*x**3 + 120*log(sqrt( -
a*x + 1) + sqrt(2))*a**2*x**2 + 180*log(sqrt( - a*x + 1) + sqrt(2))*a*x +
60*log(sqrt( - a*x + 1) + sqrt(2)) + 120*log(sqrt( - a*x + 1))*a**5*x**5 +
 360*log(sqrt( - a*x + 1))*a**4*x**4 + 240*log(sqrt( - a*x + 1))*a**3*x**3
 - 240*log(sqrt( - a*x + 1))*a**2*x**2 - 360*log(sqrt( - a*x + 1))*a*x - 1
20*log(sqrt( - a*x + 1)) - 15*a**5*x**5 + 75*a**4*x**4 + 330*a**3*x**3 + 3
10*a**2*x**2 - 75*a*x - 241))/(768*a*c**4*(a**5*x**5 + 3*a**4*x**4 + 2*a**
3*x**3 - 2*a**2*x**2 - 3*a*x - 1))
```

3.650 $\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 76

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/3*x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^3*(-a^2*c*x^2+c)^
(1/2)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{x^2(4+3ax)\sqrt{c-a^2cx^2}}{12a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*x^2*Sqrt[c - a^2*c*x^2],x]
```

output
```
(x^2*(4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{c-a^2 c x^2} e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\sqrt{c-a^2 c x^2} \int e^{\coth^{-1}(ax)} \sqrt{1-\frac{1}{a^2 x^2}} x^3 dx}{x \sqrt{1-\frac{1}{a^2 x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-a^2 c x^2} \int x^2 (ax+1) dx}{a x \sqrt{1-\frac{1}{a^2 x^2}}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{c-a^2 c x^2} \int \left(a x^3+x^2\right) dx}{a x \sqrt{1-\frac{1}{a^2 x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(\frac{a x^4}{4}+\frac{x^3}{3}\right) \sqrt{c-a^2 c x^2}}{a x \sqrt{1-\frac{1}{a^2 x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*x^2*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(x^3/3 + (a*x^4)/4))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{x^{3}(3ax+4)\sqrt{-a^{2}c\,x^{2}+c}}{12(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	47
orering	$\frac{x^{3}(3ax+4)\sqrt{-a^{2}c\,x^{2}+c}}{12(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	47
default	$\frac{(3ax+4)x^{3}\sqrt{-c(a^{2}x^{2}-1)}}{12(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	48

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/12*x^3*(3*a*x+4)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.33

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{(3\,ax^4+4\,x^3)\sqrt{-a^2c}}{12\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

output
```
1/12*(3*a*x^4 + 4*x^3)*sqrt(-a^2*c)/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \int \frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \int \frac{x^2\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.34

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c}\,i(-3a^4x^4-4a^3x^3+7)}{12a^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*i*( - 3*a**4*x**4 - 4*a**3*x**3 + 7))/(12*a**3)
```

3.651 $\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/2*x*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1
/2)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.58

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{x(3+2ax)\sqrt{c-a^2cx^2}}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*x*Sqrt[c - a^2*c*x^2],x]
```

output
```
(x*(3 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-a^2cx^2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int x(ax+1)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(ax^2+x\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{ax^3}{3}+\frac{x^2}{2}\right)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*x*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(x^2/2 + (a*x^3)/3))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x^{2}\left(2 a x+3\right) \sqrt{-a^{2} c \,x^{2}+c}}{6 \left(a x+1\right) \sqrt{\frac{a x-1}{a x+1}}}$	47
orering	$\frac{x^{2}\left(2 a x+3\right) \sqrt{-a^{2} c \,x^{2}+c}}{6 \left(a x+1\right) \sqrt{\frac{a x-1}{a x+1}}}$	47
default	$\frac{\left(2 a x+3\right) x^{2} \sqrt{-c \left(a^{2} x^{2}-1\right)}}{6 \left(a x+1\right) \sqrt{\frac{a x-1}{a x+1}}}$	48

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
1/6*x^2*(2*a*x+3)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.34

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{(2\,ax^3+3\,x^2)\sqrt{-a^2c}}{6\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

output
```
1/6*(2*a*x^3 + 3*x^2)*sqrt(-a^2*c)/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int \frac{x\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,2,1,0]%%%}+%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,0,1]%%%}
Error: Ba
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int \frac{x\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.35

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c}\,i(-2a^3x^3-3a^2x^2+5)}{6a^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*i*( - 2*a**3*x**3 - 3*a**2*x**2 + 5))/(6*a**2)
```

3.652 $\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(1+ax)^2\sqrt{c-a^2cx^2}}{2a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
1/2*(a*x+1)^2*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(2+ax)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

output
```
((2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6746, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \ 6746$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 6747$$

$$\frac{\sqrt{c-a^2cx^2}\int (ax+1)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 17$$

$$\frac{(ax+1)^2\sqrt{c-a^2cx^2}}{2a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

output
```
((1 + a*x)^2*Sqrt[c - a^2*c*x^2])/(2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{x(ax+2)\sqrt{-a^2c\,x^2+c}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	44
orering	$\frac{x(ax+2)\sqrt{-a^2c\,x^2+c}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	44
default	$\frac{(ax+2)x\sqrt{-c(a^2x^2-1)}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	45

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/2*x*(a*x+2)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{-a^2c}(ax^2+2\,x)}{2\,a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
1/2*sqrt(-a^2*c)*(a*x^2 + 2*x)/a
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{1}{2}\sqrt{-c}\left(\frac{ax^2+2\,x}{\operatorname{sgn}(ax+1)}+\frac{\operatorname{sgn}(ax+1)}{a}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
1/2*sqrt(-c)*((a*x^2 + 2*x)/sgn(a*x + 1) + sgn(a*x + 1)/a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c}\,i(-a^2x^2-2ax+3)}{2a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*i*( - a**2*x**2 - 2*a*x + 3))/(2*a)
```

3.653 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 69

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+(-a^2*c*x^2+c)^(1/2)*ln(x)/a/(1-1
/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\left(x+\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(x + Log[x]/a))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{ax+1}{x}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(a+\frac{1}{x}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c-a^2cx^2}(ax+\log(x))}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(a*x + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{(ax+\ln(x))\sqrt{-c(a^2x^2-1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	44

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBO
SE)
```

output
```
(a*x+ln(x))*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.26

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{-a^2c}(ax+\log{(x)})}{a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="f
ricas")
```

output
```
sqrt(-a^2*c)*(a*x + log(x))/a
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = 2\sqrt{-c}\left(\frac{ax}{\operatorname{sgn}(ax+1)} + \frac{\log(|x|)}{\operatorname{sgn}(ax+1)}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="g
iac")
```

output
```
2*sqrt(-c)*(a*x/sgn(a*x + 1) + log(abs(x))/sgn(a*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.49

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \sqrt{c}\,i\left(-\log\left(\sqrt{-ax+1}-1\right)-\log\left(\sqrt{-ax+1}+1\right)-ax+1\right)$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x)
```

output
```
sqrt(c)*i*( - log(sqrt( - a*x + 1) - 1) - log(sqrt( - a*x + 1) + 1) - a*x
+ 1)
```

3.654 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
-(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^2+(-a^2*c*x^2+c)^(1/2)*ln(x)
/(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{ax}+\log(x)\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x^2,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-(1/(a*x)) + Log[x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{ax+1}{x^2}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(\frac{a}{x}+\frac{1}{x^2}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(a\log(x)-\frac{1}{x}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x^2,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-x^(-1) + a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{(a\ln(x)x-1)\sqrt{-c(a^2x^2-1)}}{(ax+1)x\sqrt{\frac{ax-1}{ax+1}}}$	48

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x,method=_RETURNVER
BOSE)
```

output
```
(a*ln(x)*x-1)*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/x/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.30

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{-a^2c}(ax\log(x)-1)}{ax}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"fricas")
```

output
```
sqrt(-a^2*c)*(a*x*log(x) - 1)/(a*x)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-c(ax-1)(ax+1)}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.34

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\left(a\log\left(|x|\right)-\frac{1}{x}\right)\sqrt{-c}}{\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"giac")
```

output
```
(a*log(abs(x)) - 1/x)*sqrt(-c)/sgn(a*x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{x^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.56

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c}\,i\left(-\log\left(\sqrt{-ax+1}-1\right)ax-\log\left(\sqrt{-ax+1}+1\right)ax-ax+1\right)}{x}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

output
```
(sqrt(c)*i*( - log(sqrt( - a*x + 1) - 1)*a*x - log(sqrt( - a*x + 1) + 1)*a
*x - a*x + 1))/x
```

3.655 $\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 154

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{6\sqrt{c-a^2cx^2}}{5a^4}+\frac{3x\sqrt{c-a^2cx^2}}{4a^3}+\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

output
```
6/5*(-a^2*c*x^2+c)^(1/2)/a^4+3/4*x*(-a^2*c*x^2+c)^(1/2)/a^3+3/5*x^2*(-a^2*
c*x^2+c)^(1/2)/a^2+1/2*x^3*(-a^2*c*x^2+c)^(1/2)/a+1/5*x^4*(-a^2*c*x^2+c)^(
1/2)-3/4*c^(1/2)*arctan(a*c^(1/2)*x/(-a^2*c*x^2+c)^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.62

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}(24+15ax+12a^2x^2+10a^3x^3+4a^4x^4)+15\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{20a^4}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(24 + 15*a*x + 12*a^2*x^2 + 10*a^3*x^3 + 4*a^4*x^4) +
 15*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(2
0*a^4)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.25, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {6717, 6701, 541, 25, 27, 533, 27, 533, 27, 533, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}x^3\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{ 6701}$$

$$-c\int \frac{x^3(ax+1)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{ 541}$$

$$-c\left(-\frac{\int -\frac{a^2cx^3(10ax+9)}{\sqrt{c-a^2cx^2}}dx}{5a^2c}-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-c\left(\frac{\int \frac{a^2cx^3(10ax+9)}{\sqrt{c-a^2cx^2}}dx}{5a^2c}-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-c\left(\frac{1}{5}\int \frac{x^3(10ax+9)}{\sqrt{c-a^2cx^2}}dx-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{533}$$

$$-c\left(\frac{1}{5}\left(\frac{\int \frac{6acx^2(6ax+5)}{\sqrt{c-a^2cx^2}}dx}{4a^2c}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{5}\left(\frac{3\int \frac{x^2(6ax+5)}{\sqrt{c-a^2cx^2}}dx}{2a}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{533}$$

$$-c\left(\frac{1}{5}\left(\frac{3\left(\frac{\int \frac{3acx(5ax+4)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}-\frac{2x^2\sqrt{c-a^2cx^2}}{ac}\right)}{2a}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{5}\left(\frac{3\left(\frac{\int \frac{x(5ax+4)}{\sqrt{c-a^2cx^2}}dx}{a}-\frac{2x^2\sqrt{c-a^2cx^2}}{ac}\right)}{2a}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{533}$$

$$-c\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{\int \frac{ac(8ax+5)}{\sqrt{c-a^2cx^2}}dx}{2a^2c}-\frac{5x\sqrt{c-a^2cx^2}}{2ac}}{a}-\frac{2x^2\sqrt{c-a^2cx^2}}{ac}\right)}{2a}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{\int \frac{8ax+5}{\sqrt{c-a^2cx^2}}dx}{2a}-\frac{5x\sqrt{c-a^2cx^2}}{2ac}}{a}-\frac{2x^2\sqrt{c-a^2cx^2}}{ac}\right)}{2a}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{5\int \frac{1}{\sqrt{c-a^2cx^2}}dx-\frac{8\sqrt{c-a^2cx^2}}{ac}}{2a}-\frac{5x\sqrt{c-a^2cx^2}}{2ac}}{a}-\frac{2x^2\sqrt{c-a^2cx^2}}{ac}\right)}{2a}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \textbf{224}$$

$$-c\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{5\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}-\frac{8\sqrt{c-a^2cx^2}}{ac}}{2a}-\frac{5x\sqrt{c-a^2cx^2}}{2ac}}{a}-\frac{2x^2\sqrt{c-a^2cx^2}}{ac}\right)}{2a}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \textbf{216}$$

$$-c\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{\frac{5\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}-\frac{8\sqrt{c-a^2cx^2}}{ac}}{2a}-\frac{5x\sqrt{c-a^2cx^2}}{2ac}}{a}-\frac{2x^2\sqrt{c-a^2cx^2}}{ac}\right)}{2a}-\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

output
```
-(c*(-1/5*(x^4*Sqrt[c - a^2*c*x^2])/c + ((-5*x^3*Sqrt[c - a^2*c*x^2])/(2*a
*c) + (3*((-2*x^2*Sqrt[c - a^2*c*x^2])/(a*c) + ((-5*x*Sqrt[c - a^2*c*x^2])
/(2*a*c) + ((-8*Sqrt[c - a^2*c*x^2])/(a*c) + (5*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(a*Sqrt[c]))/(2*a))/a))/(2*a))/5))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{\left(4 a^{4} x^{4}+10 a^{3} x^{3}+12 a^{2} x^{2}+15 a x +24\right) \left(a^{2} x^{2}-1\right) c}{20 a^{4} \sqrt{-c \left(a^{2} x^{2}-1\right)}}-\frac{3 \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right) c}{4 a^{3} \sqrt{a^{2} c}}$
default	$-\frac{x^{2} \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{5 a^{2} c}-\frac{4 \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{5 c \,a^{4}}+\frac{x \sqrt{-a^{2} c \,x^{2}+c}+\frac{c \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right)}{\sqrt{a^{2} c}}}{a^{3}}+\frac{-\frac{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{2 a^{2} c}+\frac{\frac{x \sqrt{-a^{2} c \,x^{2}+c}}{2}+\frac{c \arctan}{}}{a}}{2a^2}$

input
```
int(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(4*a^4*x^4+10*a^3*x^3+12*a^2*x^2+15*a*x+24)*(a^2*x^2-1)/a^4/(-c*(a^2
*x^2-1))^(1/2)*c-3/4/a^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+
c)^(1/2))*c
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.19

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$$
$$=\left[\frac{2\,(4\,a^4x^4+10\,a^3x^3+12\,a^2x^2+15\,ax+24)\sqrt{-a^2cx^2+c}+15\,\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-\right.}{40\,a^4}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/40*(2*(4*a^4*x^4 + 10*a^3*x^3 + 12*a^2*x^2 + 15*a*x + 24)*sqrt(-a^2*c*x
^2 + c) + 15*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*
x - c))/a^4, 1/20*((4*a^4*x^4 + 10*a^3*x^3 + 12*a^2*x^2 + 15*a*x + 24)*sqr
t(-a^2*c*x^2 + c) + 15*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^
2*c*x^2 - c)))/a^4]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\frac{x^3\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**3*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.76

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=-\frac{(-a^2cx^2+c)^{\frac{3}{2}}x^2}{5\,a^2c}+\frac{5\,\sqrt{-a^2cx^2+c}x}{4\,a^3}-\frac{(-a^2cx^2+c)^{\frac{3}{2}}x}{2\,a^3c}-\frac{3\,\sqrt{c}\arcsin{(ax)}}{4\,a^4}+\frac{2\,\sqrt{-a^2cx^2+c}}{a^4}-\frac{4\,(-a^2cx^2+c)^{\frac{3}{2}}}{5\,a^4c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

output
```
-1/5*(-a^2*c*x^2 + c)^(3/2)*x^2/(a^2*c) + 5/4*sqrt(-a^2*c*x^2 + c)*x/a^3 -
 1/2*(-a^2*c*x^2 + c)^(3/2)*x/(a^3*c) - 3/4*sqrt(c)*arcsin(a*x)/a^4 + 2*sq
rt(-a^2*c*x^2 + c)/a^4 - 4/5*(-a^2*c*x^2 + c)^(3/2)/(a^4*c)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \int \frac{x^3\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.65

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$$
$$= \frac{\sqrt{c}\left(-15asin(ax)+4\sqrt{-a^2x^2+1}\,a^4x^4+10\sqrt{-a^2x^2+1}\,a^3x^3+12\sqrt{-a^2x^2+1}\,a^2x^2+15\sqrt{-a^2x^2+1}\,a\right.}{20a^4}$$

input
```
int(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*( - 15*asin(a*x) + 4*sqrt( - a**2*x**2 + 1)*a**4*x**4 + 10*sqrt(
- a**2*x**2 + 1)*a**3*x**3 + 12*sqrt( - a**2*x**2 + 1)*a**2*x**2 + 15*sqrt
( - a**2*x**2 + 1)*a*x + 24*sqrt( - a**2*x**2 + 1) - 24))/(20*a**4)
```

3.656 $\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 129

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{4\sqrt{c-a^2cx^2}}{3a^3}+\frac{7x\sqrt{c-a^2cx^2}}{8a^2}+\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{7\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

output
```
4/3*(-a^2*c*x^2+c)^(1/2)/a^3+7/8*x*(-a^2*c*x^2+c)^(1/2)/a^2+2/3*x^2*(-a^2*
c*x^2+c)^(1/2)/a+1/4*x^3*(-a^2*c*x^2+c)^(1/2)-7/8*c^(1/2)*arctan(a*c^(1/2)
*x/(-a^2*c*x^2+c)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.68

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}(32+21ax+16a^2x^2+6a^3x^3)+21\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{24a^3}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(32 + 21*a*x + 16*a^2*x^2 + 6*a^3*x^3) + 21*Sqrt[c]*A
rcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(24*a^3)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6717, 6701, 541, 25, 27, 533, 27, 533, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}x^2\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6701}$$

$$-c\int \frac{x^2(ax+1)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{541}$$

$$-c\left(-\frac{\int -\frac{a^2cx^2(8ax+7)}{\sqrt{c-a^2cx^2}}dx}{4a^2c}-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{a^2cx^2(8ax+7)}{\sqrt{c-a^2cx^2}}dx}{4a^2c}-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{4}\int \frac{x^2(8ax+7)}{\sqrt{c-a^2cx^2}}dx-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{533}$$

$$-c\left(\frac{1}{4}\left(\frac{\int \frac{acx(21ax+16)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}-\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \textbf{27}$$

$$-c\left(\frac{1}{4}\left(\frac{\int \frac{x(21ax+16)}{\sqrt{c-a^2cx^2}}dx}{3a}-\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \textbf{533}$$

$$-c\left(\frac{1}{4}\left(\frac{\frac{\int \frac{ac(32ax+21)}{\sqrt{c-a^2cx^2}}dx}{2a^2c}-\frac{21x\sqrt{c-a^2cx^2}}{2ac}}{3a}-\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \textbf{27}$$

$$-c\left(\frac{1}{4}\left(\frac{\frac{\int \frac{32ax+21}{\sqrt{c-a^2cx^2}}dx}{2a}-\frac{21x\sqrt{c-a^2cx^2}}{2ac}}{3a}-\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \textbf{455}$$

$$-c\left(\frac{1}{4}\left(\frac{\frac{21\int \frac{1}{\sqrt{c-a^2cx^2}}dx-\frac{32\sqrt{c-a^2cx^2}}{ac}}{2a}-\frac{21x\sqrt{c-a^2cx^2}}{2ac}}{3a}-\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \textbf{224}$$

$$-c\left(\frac{1}{4}\left(\frac{\frac{21\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}-\frac{32\sqrt{c-a^2cx^2}}{ac}}{2a}-\frac{21x\sqrt{c-a^2cx^2}}{2ac}}{3a}-\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \textbf{216}$$

$$-c\left(\frac{1}{4}\left(\frac{\frac{\frac{21\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}-\frac{32\sqrt{c-a^2cx^2}}{ac}}{2a}-\frac{21x\sqrt{c-a^2cx^2}}{2ac}}{3a}-\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

output
```
-(c*(-1/4*(x^3*Sqrt[c - a^2*c*x^2])/c + ((-8*x^2*Sqrt[c - a^2*c*x^2])/(3*a
*c) + ((-21*x*Sqrt[c - a^2*c*x^2])/(2*a*c) + ((-32*Sqrt[c - a^2*c*x^2])/(a
*c) + (21*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(a*Sqrt[c]))/(2*a))/(
3*a))/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\left(6 a^{3} x^{3}+16 a^{2} x^{2}+21 a x +32\right) \left(a^{2} x^{2}-1\right) c}{24 a^{3} \sqrt{-c \left(a^{2} x^{2}-1\right)}}-\frac{7 \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right) c}{8 a^{2} \sqrt{a^{2} c}}$
default	$-\frac{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{4 a^{2} c}+\frac{\frac{9 x \sqrt{-a^{2} c \,x^{2}+c}}{8}+\frac{9 c \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right)}{8 \sqrt{a^{2} c}}}{a^{2}}-\frac{2 \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{3 a^{3} c}+\frac{2 \sqrt{-\left(x -\frac{1}{a}\right)^{2} a^{2} c -2 \left(x -\frac{1}{a}\right) a c}-\frac{2 a c \arctan}{}}{a^{3}}$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x+1)*x^2/(a*x-1),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*a^3*x^3+16*a^2*x^2+21*a*x+32)*(a^2*x^2-1)/a^3/(-c*(a^2*x^2-1))^(1
/2)*c-7/8/a^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.30

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$$
$$= \left[\frac{2\,(6\,a^3x^3+16\,a^2x^2+21\,ax+32)\sqrt{-a^2cx^2+c}+21\,\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{48\,a^3},\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/48*(2*(6*a^3*x^3 + 16*a^2*x^2 + 21*a*x + 32)*sqrt(-a^2*c*x^2 + c) + 21*
sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^3,
1/24*((6*a^3*x^3 + 16*a^2*x^2 + 21*a*x + 32)*sqrt(-a^2*c*x^2 + c) + 21*sqr
t(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^3]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.72

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{9\sqrt{-a^2cx^2+c}x}{8a^2} - \frac{(-a^2cx^2+c)^{\frac{3}{2}}x}{4a^2c} - \frac{7\sqrt{c}\arcsin(ax)}{8a^3} + \frac{2\sqrt{-a^2cx^2+c}}{a^3} - \frac{2(-a^2cx^2+c)^{\frac{3}{2}}}{3a^3c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

output
```
9/8*sqrt(-a^2*c*x^2 + c)*x/a^2 - 1/4*(-a^2*c*x^2 + c)^(3/2)*x/(a^2*c) - 7/
8*sqrt(c)*arcsin(a*x)/a^3 + 2*sqrt(-a^2*c*x^2 + c)/a^3 - 2/3*(-a^2*c*x^2 +
 c)^(3/2)/(a^3*c)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.65

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3x+\frac{8}{a}\right)x+\frac{21}{a^2}\right)x+\frac{32}{a^3}\right) + \frac{7c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8a^2\sqrt{-c}|a|}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*x + 8/a)*x + 21/a^2)*x + 32/a^3) + 7/8*c*
log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^2*sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int \frac{x^2\,\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.63

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{\sqrt{c}\,\left(-21asin(ax)+6\sqrt{-a^2x^2+1}\,a^3x^3+16\sqrt{-a^2x^2+1}\,a^2x^2+21\sqrt{-a^2x^2+1}\,ax+32\sqrt{-a^2x^2+1}-3\right.}{24a^3}$$

input
```
int(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*( - 21*asin(a*x) + 6*sqrt( - a**2*x**2 + 1)*a**3*x**3 + 16*sqrt(
- a**2*x**2 + 1)*a**2*x**2 + 21*sqrt( - a**2*x**2 + 1)*a*x + 32*sqrt( - a*
*2*x**2 + 1) - 32))/(24*a**3)
```

3.657 $\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 99

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{5\sqrt{c-a^2cx^2}}{3a^2}+\frac{x\sqrt{c-a^2cx^2}}{a}+\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

output
```
5/3*(-a^2*c*x^2+c)^(1/2)/a^2+x*(-a^2*c*x^2+c)^(1/2)/a+1/3*x^2*(-a^2*c*x^2+
c)^(1/2)-c^(1/2)*arctan(a*c^(1/2)*x/(-a^2*c*x^2+c)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.80

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{(5+3ax+a^2x^2)\sqrt{c-a^2cx^2}+3\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{3a^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

output
```
((5 + 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2] + 3*Sqrt[c]*ArcTan[(a*x*Sqrt[c
- a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6717, 6701, 541, 25, 27, 533, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}x\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6701}$$

$$-c\int \frac{x(ax+1)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{541}$$

$$-c\left(-\frac{\int -\frac{a^2cx(6ax+5)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{a^2cx(6ax+5)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{3}\int \frac{x(6ax+5)}{\sqrt{c-a^2cx^2}}dx-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{533}$$

$$-c\left(\frac{1}{3}\left(\frac{\int \frac{2ac(5ax+3)}{\sqrt{c-a^2cx^2}}dx}{2a^2c}-\frac{3x\sqrt{c-a^2cx^2}}{ac}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{3}\left(\frac{\int \frac{5ax+3}{\sqrt{c-a^2cx^2}}dx}{a}-\frac{3x\sqrt{c-a^2cx^2}}{ac}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{1}{3}\left(\frac{3\int \frac{1}{\sqrt{c-a^2cx^2}}dx-\frac{5\sqrt{c-a^2cx^2}}{ac}}{a}-\frac{3x\sqrt{c-a^2cx^2}}{ac}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{1}{3}\left(\frac{3\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}-\frac{5\sqrt{c-a^2cx^2}}{ac}}{a}-\frac{3x\sqrt{c-a^2cx^2}}{ac}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{1}{3}\left(\frac{\frac{3\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}-\frac{5\sqrt{c-a^2cx^2}}{ac}}{a}-\frac{3x\sqrt{c-a^2cx^2}}{ac}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

output
```
-(c*(-1/3*(x^2*Sqrt[c - a^2*c*x^2])/c + ((-3*x*Sqrt[c - a^2*c*x^2])/(a*c)
+ ((-5*Sqrt[c - a^2*c*x^2])/(a*c) + (3*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c
*x^2]])/(a*Sqrt[c]))/a)/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{(a^2x^2+3ax+5)(a^2x^2-1)c}{3a^2\sqrt{-c(a^2x^2-1)}}-\frac{\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c}{a\sqrt{a^2c}}$
default	$-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{3a^2c}+\frac{x\sqrt{-a^2c\,x^2+c}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}}{a}+\frac{2\sqrt{-(x-\frac{1}{a})^2a^2c-2(x-\frac{1}{a})ac}-\frac{2ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-(x-\frac{1}{a})^2a^2c-2(x-}\right.}{\sqrt{a^2c}}}{a^2}$

input
```
int(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a^2*x^2+3*a*x+5)*(a^2*x^2-1)/a^2/(-c*(a^2*x^2-1))^(1/2)*c-1/a/(a^2*c
)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.52

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$$

$$=\left[\frac{2\sqrt{-a^2cx^2+c}(a^2x^2+3\,ax+5)+3\sqrt{-c}\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{6\,a^2},\frac{\sqrt{-a^2cx^2+c}(a^2\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 3*a*x + 5) + 3*sqrt(-c)*log(2*a^2*
c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^2, 1/3*(sqrt(-a^2*c*x^
2 + c)*(a^2*x^2 + 3*a*x + 5) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqr
t(c)*x/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\frac{x\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{-a^2cx^2+c}x}{a}-\frac{\sqrt{c}\arcsin{(ax)}}{a^2}+\frac{2\sqrt{-a^2cx^2+c}}{a^2}-\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3\,a^2c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(-a^2*c*x^2 + c)*x/a - sqrt(c)*arcsin(a*x)/a^2 + 2*sqrt(-a^2*c*x^2 + c
)/a^2 - 1/3*(-a^2*c*x^2 + c)^(3/2)/(a^2*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.73

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{1}{3}\sqrt{-a^2cx^2+c}\left(\left(x+\frac{3}{a}\right)x+\frac{5}{a^2}\right) + \frac{c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{a\sqrt{-c}|a|}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(-a^2*c*x^2 + c)*((x + 3/a)*x + 5/a^2) + c*log(abs(-sqrt(-a^2*c)*x
 + sqrt(-a^2*c*x^2 + c)))/(a*sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int \frac{x\,\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62

$$\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c}\left(-3asin(ax)+\sqrt{-a^2x^2+1}\,a^2x^2+3\sqrt{-a^2x^2+1}\,ax+5\sqrt{-a^2x^2+1}-5\right)}{3a^2}$$

input
```
int(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*( - 3*asin(a*x) + sqrt( - a**2*x**2 + 1)*a**2*x**2 + 3*sqrt( - a*
*2*x**2 + 1)*a*x + 5*sqrt( - a**2*x**2 + 1) - 5))/(3*a**2)
```

3.658 $\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{2\sqrt{c-a^2cx^2}}{a}+\frac{1}{2}x\sqrt{c-a^2cx^2}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

output
```
2*(-a^2*c*x^2+c)^(1/2)/a+1/2*x*(-a^2*c*x^2+c)^(1/2)-3/2*c^(1/2)*arctan(a*c
^(1/2)*x/(-a^2*c*x^2+c)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left((4+ax)\sqrt{1-a^2x^2}+6\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]
/Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6717, 6691, 469, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6691}$$

$$-c\int \frac{(ax+1)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{3}{2}\int \frac{ax+1}{\sqrt{c-a^2cx^2}}dx-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{ac}\right)-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{3}{2}\left(\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{ac}\right)-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{3}{2}\left(\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{ac}\right)-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2ac}\right)$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
-(c*(-1/2*((1 + a*x)*Sqrt[c - a^2*c*x^2])/(a*c) + (3*(-(Sqrt[c - a^2*c*x^2
]/(a*c)) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]/(a*Sqrt[c])))/2))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

method	result	size
risch	$-\frac{(ax+4)(a^2x^2-1)c}{2a\sqrt{-c(a^2x^2-1)}}-\frac{3c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}$	69
default	$\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}+\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2\left(x-\frac{1}{a}\right)ac}-\frac{2ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2\left(x-\frac{1}{a}\right)ac}}\right)}{\sqrt{a^2c}}}{a}$	136

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x+4)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c-3/2*c/(a^2*c)^(1/2)*ar
ctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.74

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$$
$$=\left[\frac{2\sqrt{-a^2cx^2+c}(ax+4)+3\sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{4a},\frac{\sqrt{-a^2cx^2+c}(ax+4)+3}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x + 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sq
rt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x +
4) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a
]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{1}{2}\sqrt{-a^2cx^2+c}x-\frac{3\sqrt{c}\arcsin{(ax)}}{2\,a}+\frac{2\sqrt{-a^2cx^2+c}}{a}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a + 2*sqrt(-a^2*c*x^2
 + c)/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x+\frac{4}{a}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}|a|}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-a^2*c*x^2 + c)*(x + 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(
-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c}\left(-3asin(ax)+\sqrt{-a^2x^2+1}\,ax+4\sqrt{-a^2x^2+1}-4\right)}{2a}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*( - 3*asin(a*x) + sqrt( - a**2*x**2 + 1)*a*x + 4*sqrt( - a**2*x**
2 + 1) - 4))/(2*a)
```

3.659 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 75

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \sqrt{c-a^2cx^2}-2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
(-a^2*c*x^2+c)^(1/2)-2*c^(1/2)*arctan(a*c^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+c^
(1/2)*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \sqrt{c-a^2cx^2}+2\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right)-\sqrt{c}\log(x)+\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

output
```
Sqrt[c - a^2*c*x^2] + 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*
(-1 + a^2*x^2))] - Sqrt[c]*Log[x] + Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c
*x^2]]
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6717, 6701, 541, 25, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x}dx$$

$$\Big\downarrow \text{ 6701}$$

$$-c\int \frac{(ax+1)^2}{x\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{ 541}$$

$$-c\left(-\frac{\int -\frac{a^2c(2ax+1)}{x\sqrt{c-a^2cx^2}}dx}{a^2c}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-c\left(\frac{\int \frac{a^2c(2ax+1)}{x\sqrt{c-a^2cx^2}}dx}{a^2c}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-c\left(\int \frac{2ax+1}{x\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{ 538}$$

$$-c\left(2a\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \textbf{224}$$

$$-c\left(\int \frac{1}{x\sqrt{c-a^2cx^2}}dx+2a\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \textbf{216}$$

$$-c\left(\int \frac{1}{x\sqrt{c-a^2cx^2}}dx+\frac{2\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \textbf{243}$$

$$-c\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2+\frac{2\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \textbf{73}$$

$$-c\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{a^2c}+\frac{2\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \textbf{221}$$

$$-c\left(\frac{2\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

output
```
-(c*(-(Sqrt[c - a^2*c*x^2]/c) + (2*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2
]])/Sqrt[c] - ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]/Sqrt[c]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(61) = 122$.

Time = 0.18 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.72

method	result
default	$-\sqrt{-a^2 c\,x^2+c}+\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)+2\sqrt{-\left(x-\frac{1}{a}\right)^2 a^2c-2\left(x-\frac{1}{a}\right)ac}-\frac{2ac\arctan\left(\frac{\cdots}{\sqrt{-\cdots}}\right)}{\cdots}$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-(-a^2*c*x^2+c)^(1/2)+c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)+2
*(-(x-1/a)^2*a^2*c-2*(x-1/a)*a*c)^(1/2)-2*a*c/(a^2*c)^(1/2)*arctan((a^2*c)
^(1/2)*x/(-(x-1/a)^2*a^2*c-2*(x-1/a)*a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.41

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \left[2\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right) + \frac{1}{2}\sqrt{c}\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right) + \sqrt{-a^2cx^2+c}, -\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{c}\right) + \sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right) + \sqrt{-a^2cx^2+c}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

output
```
[2*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2*
sqrt(c)*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sqr
t(-a^2*c*x^2 + c), -sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/c) + sqr
t(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) + sqrt(-a
^2*c*x^2 + c)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-c(ax-1)(ax+1)}(ax+1)}{x(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x*(a*x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = -a^2\left(\frac{\sqrt{c}\arcsin(ax)}{a^2}-\frac{\sqrt{-a^2cx^2+c}}{a^2}\right) - a\left(\frac{\sqrt{c}\arcsin(ax)}{a}-\frac{\sqrt{c}\log\left(\frac{2\sqrt{-a^2cx^2+c}\sqrt{c}}{|x|}+\frac{2c}{|x|}\right)}{a}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

output
```
-a^2*(sqrt(c)*arcsin(a*x)/a^2 - sqrt(-a^2*c*x^2 + c)/a^2) - a*(sqrt(c)*arc
sin(a*x)/a - sqrt(c)*log(2*sqrt(-a^2*c*x^2 + c)*sqrt(c)/abs(x) + 2*c/abs(x
))/a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.27

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = -\frac{2\,c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{2\,a\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|} + \sqrt{-a^2cx^2+c}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

output
```
-2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) -
2*a*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) + sqr
t(-a^2*c*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{x\,(a\,x-1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.43

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\sqrt{c}\left(-2asin(ax)+\sqrt{-a^2x^2+1}-\log\left(\tan\left(\frac{asin(ax)}{2}\right)\right)-1\right)$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x)
```

output
```
sqrt(c)*( - 2*asin(a*x) + sqrt( - a**2*x**2 + 1) - log(tan(asin(a*x)/2)) -
 1)
```

3.660 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+2a\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
(-a^2*c*x^2+c)^(1/2)/x-a*c^(1/2)*arctan(a*c^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+
2*a*c^(1/2)*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{\sqrt{c-a^2cx^2}}{x}+a\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right)-2a\sqrt{c}\log(x)+2a\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

output
```
Sqrt[c - a^2*c*x^2]/x + a*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c
]*(-1 + a^2*x^2))] - 2*a*Sqrt[c]*Log[x] + 2*a*Sqrt[c]*Log[c + Sqrt[c]*Sqrt
[c - a^2*c*x^2]]
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6717, 6701, 540, 25, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^2}dx$$

$$\Big\downarrow \text{6701}$$

$$-c\int \frac{(ax+1)^2}{x^2\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-c\left(-\frac{\int -\frac{ac(ax+2)}{x\sqrt{c-a^2cx^2}}dx}{c}-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{ac(ax+2)}{x\sqrt{c-a^2cx^2}}dx}{c}-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(a\int \frac{ax+2}{x\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{538}$$

$$-c\left(a\left(a\int \frac{1}{\sqrt{c-a^2cx^2}}dx+2\int \frac{1}{x\sqrt{c-a^2cx^2}}dx\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{ 224}$$

$$-c\left(a\left(2\int \frac{1}{x\sqrt{c-a^2cx^2}}dx+a\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{ 216}$$

$$-c\left(a\left(2\int \frac{1}{x\sqrt{c-a^2cx^2}}dx+\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{ 243}$$

$$-c\left(a\left(\int \frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2+\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{ 73}$$

$$-c\left(a\left(\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{2\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{a^2c}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{ 221}$$

$$-c\left(a\left(\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

output
```
-(c*(-(Sqrt[c - a^2*c*x^2]/(c*x)) + a*(ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c
*x^2]]/Sqrt[c] - (2*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c])))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{\left(a^{2} x^{2}-1\right) c}{x \sqrt{-c \left(a^{2} x^{2}-1\right)}}-\left(\frac{a^{2} \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right)}{\sqrt{a^{2} c}}-\frac{2 a \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{-a^{2} c \,x^{2}+c}}{x}\right)}{\sqrt{c}}\right) c$
default	$\frac{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{c x}+2 a^{2} \left(\frac{x \sqrt{-a^{2} c \,x^{2}+c}}{2}+\frac{c \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right)}{2 \sqrt{a^{2} c}}\right)-2 a \left(\sqrt{-a^{2} c \,x^{2}+c}-\sqrt{c}\, \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{-a^{2}}}{x}\right.\right.$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(a^2*x^2-1)/x/(-c*(a^2*x^2-1))^(1/2)*c-(a^2/(a^2*c)^(1/2)*arctan((a^2*c)^
(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2*a/c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)
^(1/2))/x))*c
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.43

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$$
$$= \left[\frac{a\sqrt{c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+a\sqrt{c}x\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+\sqrt{-a^2cx^2+c}}{x},\right.$$
$$\left. -\frac{4a\sqrt{-c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{c}\right)-a\sqrt{-c}x\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)-2\sqrt{-a^2cx^2+c}}{2x}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas"
)
```

output
```
[(a*sqrt(c)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + a
*sqrt(c)*x*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) +
sqrt(-a^2*c*x^2 + c))/x, -1/2*(4*a*sqrt(-c)*x*arctan(sqrt(-a^2*c*x^2 + c)*
sqrt(-c)/c) - a*sqrt(-c)*x*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt
(-c)*x - c) - 2*sqrt(-a^2*c*x^2 + c))/x]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-c(ax-1)(ax+1)}(ax+1)}{x^2(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**2*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{(ax-1)x^2}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.63

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = -\frac{4\,ac\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{a^2\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|} - \frac{2\,a^2\sqrt{-c}c}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a|}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-4*a*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
- a^2*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) - 2
*a^2*sqrt(-c)*c/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{x^2\,(a\,x-1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^2*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.46

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c}\left(-asin(ax)\,ax+\sqrt{-a^2x^2+1}-2\log\left(\tan\left(\frac{asin(ax)}{2}\right)\right)ax\right)}{x}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

output
```
(sqrt(c)*( - asin(a*x)*a*x + sqrt( - a**2*x**2 + 1) - 2*log(tan(asin(a*x)/
2))*a*x))/x
```

3.661 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x} + \frac{3}{2}a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
1/2*(-a^2*c*x^2+c)^(1/2)/x^2+2*a*(-a^2*c*x^2+c)^(1/2)/x+3/2*a^2*c^(1/2)*ar
ctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{1}{2}\left(\frac{(1+4ax)\sqrt{c-a^2cx^2}}{x^2}-3a^2\sqrt{c}\log(x) + 3a^2\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

output
```
(((1 + 4*a*x)*Sqrt[c - a^2*c*x^2])/x^2 - 3*a^2*Sqrt[c]*Log[x] + 3*a^2*Sqrt
[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/2
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6701, 540, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^3}dx$$

$$\Big\downarrow \text{6701}$$

$$-c\int \frac{(ax+1)^2}{x^3\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-c\left(-\frac{\int -\frac{ac(3ax+4)}{x^2\sqrt{c-a^2cx^2}}dx}{2c}-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{ac(3ax+4)}{x^2\sqrt{c-a^2cx^2}}dx}{2c}-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{2}a\int \frac{3ax+4}{x^2\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{534}$$

$$-c\left(\frac{1}{2}a\left(3a\int\frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{4\sqrt{c-a^2cx^2}}{cx}\right)-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{ 243}$$

$$-c\left(\frac{1}{2}a\left(\frac{3}{2}a\int\frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2-\frac{4\sqrt{c-a^2cx^2}}{cx}\right)-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{ 73}$$

$$-c\left(\frac{1}{2}a\left(-\frac{3\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{ac}-\frac{4\sqrt{c-a^2cx^2}}{cx}\right)-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{ 221}$$

$$-c\left(\frac{1}{2}a\left(-\frac{3a\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{4\sqrt{c-a^2cx^2}}{cx}\right)-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

output
```
-(c*(-1/2*Sqrt[c - a^2*c*x^2]/(c*x^2) + (a*((-4*Sqrt[c - a^2*c*x^2])/(c*x)
 - (3*a*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c]))/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{(4a^3x^3+a^2x^2-4ax-1)c}{2x^2\sqrt{-c(a^2x^2-1)}}+\frac{3a^2\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{2}$
default	$\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{2c\,x^2}-\frac{3a^2\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)\right)}{2}-2a\left(-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{cx}-2a^2\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\right.\right.$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(4*a^3*x^3+a^2*x^2-4*a*x-1)/x^2/(-c*(a^2*x^2-1))^(1/2)*c+3/2*a^2*c^(1
/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.77

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \left[\frac{3\,a^2\sqrt{c}x^2\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\,\sqrt{-a^2cx^2+c}(4\,ax+1)}{4\,x^2},\ -\frac{3\,a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{c}\right)-\sqrt{-a^2cx^2+c}(4\,ax+1)}{2\,x^2}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fricas"
)
```

output
```
[1/4*(3*a^2*sqrt(c)*x^2*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
 2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(4*a*x + 1))/x^2, -1/2*(3*a^2*sqrt(-c)
*x^2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/c) - sqrt(-a^2*c*x^2 + c)*(4*a*x
 + 1))/x^2]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{x^3\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**3*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{(ax-1)x^3}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(64) = 128$.

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.56

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = -\frac{3\,a^2c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^3a^2c-4\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2a\sqrt{-c}c|a|+\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right.}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-3*a^2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c
) + ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^2*c - 4*(sqrt(-a^2*c)*x -
 sqrt(-a^2*c*x^2 + c))^2*a*sqrt(-c)*c*abs(a) + (sqrt(-a^2*c)*x - sqrt(-a^2
*c*x^2 + c))*a^2*c^2 + 4*a*sqrt(-c)*c^2*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a
^2*c*x^2 + c))^2 - c)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{x^3\,(a\,x-1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^3*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$$
$$= \frac{\sqrt{c}\left(4\sqrt{-a^2x^2+1}\,ax+\sqrt{-a^2x^2+1}-3\log\left(\tan\left(\frac{asin(ax)}{2}\right)\right)a^2x^2\right)}{2x^2}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x)
```

output
```
(sqrt(c)*(4*sqrt( - a**2*x**2 + 1)*a*x + sqrt( - a**2*x**2 + 1) - 3*log(ta
n(asin(a*x)/2))*a**2*x**2))/(2*x**2)
```

3.662 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 99

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x} + a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
1/3*(-a^2*c*x^2+c)^(1/2)/x^3+a*(-a^2*c*x^2+c)^(1/2)/x^2+5/3*a^2*(-a^2*c*x^
2+c)^(1/2)/x+a^3*c^(1/2)*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{(1+3ax+5a^2x^2)\sqrt{c-a^2cx^2}}{3x^3}-a^3\sqrt{c}\log(x) + a^3\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

output
```
((1 + 3*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*x^3) - a^3*Sqrt[c]*Log[x]
 + a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6717, 6701, 540, 25, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^4}dx$$

$$\Big\downarrow \text{6701}$$

$$-c\int \frac{(ax+1)^2}{x^4\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-c\left(-\frac{\int -\frac{ac(5ax+6)}{x^3\sqrt{c-a^2cx^2}}dx}{3c}-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{ac(5ax+6)}{x^3\sqrt{c-a^2cx^2}}dx}{3c}-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{3}a\int \frac{5ax+6}{x^3\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{539}$$

$$-c\left(\frac{1}{3}a\left(-\frac{\int -\frac{2ac(3ax+5)}{x^2\sqrt{c-a^2cx^2}}dx}{2c}-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{3}a\left(a\int \frac{3ax+5}{x^2\sqrt{c-a^2cx^2}}dx-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{534}$$

$$-c\left(\frac{1}{3}a\left(a\left(3a\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{5\sqrt{c-a^2cx^2}}{cx}\right)-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{243}$$

$$-c\left(\frac{1}{3}a\left(a\left(\frac{3}{2}a\int \frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2-\frac{5\sqrt{c-a^2cx^2}}{cx}\right)-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{73}$$

$$-c\left(\frac{1}{3}a\left(a\left(-\frac{3\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{ac}-\frac{5\sqrt{c-a^2cx^2}}{cx}\right)-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{221}$$

$$-c\left(\frac{1}{3}a\left(a\left(-\frac{3a\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{5\sqrt{c-a^2cx^2}}{cx}\right)-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

output
```
-(c*(-1/3*Sqrt[c - a^2*c*x^2]/(c*x^3) + (a*((-3*Sqrt[c - a^2*c*x^2])/(c*x^
2) + a*((-5*Sqrt[c - a^2*c*x^2])/(c*x) - (3*a*ArcTanh[Sqrt[c - a^2*c*x^2]/
Sqrt[c]])/Sqrt[c])))/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{(5a^4x^4+3a^3x^3-4a^2x^2-3ax-1)c}{3x^3\sqrt{-c(a^2x^2-1)}}+a^3\sqrt{c}\ln\left(\frac{2c+2\sqrt{c}\sqrt{-a^2cx^2+c}}{x}\right)$
default	$\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3cx^3}-2a\left(-\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{2cx^2}-\frac{a^2\left(\sqrt{-a^2cx^2+c}-\sqrt{c}\ln\left(\frac{2c+2\sqrt{c}\sqrt{-a^2cx^2+c}}{x}\right)\right)}{2}\right)-2a^2\left(-\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{cx}\right.$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(5*a^4*x^4+3*a^3*x^3-4*a^2*x^2-3*a*x-1)/x^3/(-c*(a^2*x^2-1))^(1/2)*c+
a^3*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.56

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \left[\frac{3\,a^3\sqrt{c}x^3\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\,\sqrt{-a^2cx^2+c}(5\,a^2x^2+3\,ax+1)}{6\,x^3},\ -\frac{3\,a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{c}\right)-\sqrt{-a^2cx^2+c}(5\,a^2x^2+3\,ax+1)}{3\,x^3}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fricas"
)
```

output
```
[1/6*(3*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
 2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 + 3*a*x + 1))/x^3, -1/3*(3*
a^3*sqrt(-c)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/c) - sqrt(-a^2*c*x^2
 + c)*(5*a^2*x^2 + 3*a*x + 1))/x^3]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{x^4\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**4*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{(ax-1)x^4}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(83) = 166$.

Time = 0.14 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.53

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = -\frac{2\,a^3c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{2\left(3\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5a^3c-3\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^4a^2\sqrt{-c}c|a|+12\left(\sqrt{-a^2c}x-\sqrt{-c\cdots}\right.\right.}{3\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2\cdots\right.}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-2*a^3*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c
) + 2/3*(3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c - 3*(sqrt(-a^2*
c)*x - sqrt(-a^2*c*x^2 + c))^4*a^2*sqrt(-c)*c*abs(a) + 12*(sqrt(-a^2*c)*x
- sqrt(-a^2*c*x^2 + c))^2*a^2*sqrt(-c)*c^2*abs(a) - 3*(sqrt(-a^2*c)*x - sq
rt(-a^2*c*x^2 + c))*a^3*c^3 - 5*a^2*sqrt(-c)*c^3*abs(a))/((sqrt(-a^2*c)*x
- sqrt(-a^2*c*x^2 + c))^2 - c)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{x^4\,(a\,x-1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^4*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c}\left(5\sqrt{-a^2x^2+1}\,a^2x^2+3\sqrt{-a^2x^2+1}\,ax+\sqrt{-a^2x^2+1}-3\log\left(\tan\left(\frac{asin(ax)}{2}\right)\right)a^3x^3\right)}{3x^3}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x)
```

output
```
(sqrt(c)*(5*sqrt( - a**2*x**2 + 1)*a**2*x**2 + 3*sqrt( - a**2*x**2 + 1)*a*
x + sqrt( - a**2*x**2 + 1) - 3*log(tan(asin(a*x)/2))*a**3*x**3))/(3*x**3)
```

3.663 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 130

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{7}{8}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
1/4*(-a^2*c*x^2+c)^(1/2)/x^4+2/3*a*(-a^2*c*x^2+c)^(1/2)/x^3+7/8*a^2*(-a^2*
c*x^2+c)^(1/2)/x^2+4/3*a^3*(-a^2*c*x^2+c)^(1/2)/x+7/8*a^4*c^(1/2)*arctanh(
(-a^2*c*x^2+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}(6+16ax+21a^2x^2+32a^3x^3)}{24x^4}-\frac{7}{8}a^4\sqrt{c}\log(x)+\frac{7}{8}a^4\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(6 + 16*a*x + 21*a^2*x^2 + 32*a^3*x^3))/(24*x^4) - (7
*a^4*Sqrt[c]*Log[x])/8 + (7*a^4*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2
]])/8
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.15, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6717, 6701, 540, 25, 27, 539, 25, 27, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^5}dx$$

$$\Big\downarrow \text{ 6701}$$

$$-c\int \frac{(ax+1)^2}{x^5\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{ 540}$$

$$-c\left(-\frac{\int -\frac{ac(7ax+8)}{x^4\sqrt{c-a^2cx^2}}dx}{4c}-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-c\left(\frac{\int \frac{ac(7ax+8)}{x^4\sqrt{c-a^2cx^2}}dx}{4c}-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-c\left(\frac{1}{4}a\int \frac{7ax+8}{x^4\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \text{ 539}$$

$$-c\left(\frac{1}{4}a\left(-\frac{\int -\frac{ac(16ax+21)}{x^3\sqrt{c-a^2cx^2}}dx}{3c}-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \; 25$$

$$-c\left(\frac{1}{4}a\left(\frac{\int \frac{ac(16ax+21)}{x^3\sqrt{c-a^2cx^2}}dx}{3c}-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \; 27$$

$$-c\left(\frac{1}{4}a\left(\frac{1}{3}a\int \frac{16ax+21}{x^3\sqrt{c-a^2cx^2}}dx-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \; 539$$

$$-c\left(\frac{1}{4}a\left(\frac{1}{3}a\left(-\frac{\int -\frac{ac(21ax+32)}{x^2\sqrt{c-a^2cx^2}}dx}{2c}-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \; 25$$

$$-c\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{\int \frac{ac(21ax+32)}{x^2\sqrt{c-a^2cx^2}}dx}{2c}-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \; 27$$

$$-c\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\int \frac{21ax+32}{x^2\sqrt{c-a^2cx^2}}dx-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \; 534$$

$$-c\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\left(21a\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{32\sqrt{c-a^2cx^2}}{cx}\right)-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \; 243$$

$$-c\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\left(\frac{21}{2}a\int \frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2-\frac{32\sqrt{c-a^2cx^2}}{cx}\right)-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \; 73$$

$$-c\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\left(-\frac{21\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{ac}-\frac{32\sqrt{c-a^2cx^2}}{cx}\right)-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$\Big\downarrow$ 221

$$-c\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\left(-\frac{21a\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{32\sqrt{c-a^2cx^2}}{cx}\right)-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

output
```
-(c*(-1/4*Sqrt[c - a^2*c*x^2]/(c*x^4) + (a*((-8*Sqrt[c - a^2*c*x^2])/(3*c*
x^3) + (a*((-21*Sqrt[c - a^2*c*x^2])/(2*c*x^2) + (a*((-32*Sqrt[c - a^2*c*x
^2])/(c*x) - (21*a*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c]))/2))/3))
/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{(32a^5x^5+21a^4x^4-16a^3x^3-15a^2x^2-16ax-6)c}{24x^4\sqrt{-c(a^2x^2-1)}}+\frac{7a^4\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{8}$
default	$\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{4c\,x^4}-\frac{9a^2\left(-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{2c\,x^2}-\frac{a^2\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)\right)}{2}\right)}{4}+\frac{2a(-a^2c\,x^2+c)^{\frac{3}{2}}}{3c\,x^3}-2a^3\Bigg(-$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(32*a^5*x^5+21*a^4*x^4-16*a^3*x^3-15*a^2*x^2-16*a*x-6)/x^4/(-c*(a^2*
x^2-1))^(1/2)*c+7/8*a^4*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.31

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$$
$$=\left[\frac{21\,a^4\sqrt{c}x^4\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\,(32\,a^3x^3+21\,a^2x^2+16\,ax+6)\sqrt{-a^2cx^2+c}}{48\,x^4},\right.$$
$$\left.-\frac{21\,a^4\sqrt{-c}x^4\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{c}\right)-(32\,a^3x^3+21\,a^2x^2+16\,ax+6)\sqrt{-a^2cx^2+c}}{24\,x^4}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="fricas"
)
```

output
```
[1/48*(21*a^4*sqrt(c)*x^4*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
 - 2*c)/x^2) + 2*(32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqrt(-a^2*c*x^2 +
c))/x^4, -1/24*(21*a^4*sqrt(-c)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/c
) - (32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqrt(-a^2*c*x^2 + c))/x^4]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{x^5\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**5*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax+1)}{(ax-1)x^5}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(106) = 212$.

Time = 0.15 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.49

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = -\frac{7\,a^4c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}} + \frac{21\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^7a^4c-45\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5a^4c^2+96\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+}\right.}{}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-7/4*a^4*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(
-c) + 1/12*(21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^4*c - 45*(sqrt(
-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^2 + 96*(sqrt(-a^2*c)*x - sqrt(-a
^2*c*x^2 + c))^4*a^3*sqrt(-c)*c^2*abs(a) - 45*(sqrt(-a^2*c)*x - sqrt(-a^2*
c*x^2 + c))^3*a^4*c^3 - 128*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*
sqrt(-c)*c^3*abs(a) + 21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^4*c^4 +
 32*a^3*sqrt(-c)*c^4*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 -
c)^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{x^5\,(a\,x-1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^5*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.69

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$$
$$= \frac{\sqrt{c}\left(32\sqrt{-a^2x^2+1}\,a^3x^3+21\sqrt{-a^2x^2+1}\,a^2x^2+16\sqrt{-a^2x^2+1}\,ax+6\sqrt{-a^2x^2+1}-21\log\left(\tan\left(\frac{asin}{2}\right.\right.\right.}{24x^4}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x)
```

output
```
(sqrt(c)*(32*sqrt( - a**2*x**2 + 1)*a**3*x**3 + 21*sqrt( - a**2*x**2 + 1)*
a**2*x**2 + 16*sqrt( - a**2*x**2 + 1)*a*x + 6*sqrt( - a**2*x**2 + 1) - 21*
log(tan(asin(a*x)/2))*a**4*x**4))/(24*x**4)
```

3.664 $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 228

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^5\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

output
```
4*(-a^2*c*x^2+c)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)+2*x*(-a^2*c*x^2+c)^(1/2)/a^
3/(1-1/a^2/x^2)^(1/2)+4/3*x^2*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)
+3/4*x^3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/5*x^4*(-a^2*c*x^2+c)
^(1/2)/(1-1/a^2/x^2)^(1/2)+4*(-a^2*c*x^2+c)^(1/2)*ln(-a*x+1)/a^5/(1-1/a^2/
x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.38

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{4x}{a^4}+\frac{2x^2}{a^3}+\frac{4x^3}{3a^2}+\frac{3x^4}{4a}+\frac{x^5}{5}+\frac{4\log(1-ax)}{a^5}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((4*x)/a^4 + (2*x^2)/a^3 + (4*x^3)/(3*a^2) + (3*x^4)/
(4*a) + x^5/5 + (4*Log[1 - a*x])/a^5))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.39, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^4dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{x^3(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{x^3(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$\Big\downarrow$ 99

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-ax^4-3x^3-\frac{4x^2}{a}-\frac{4x}{a^2}-\frac{4}{a^3(ax-1)}-\frac{4}{a^3}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$\Big\downarrow$ 2009

$$-\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(1-ax)}{a^4}-\frac{4x}{a^3}-\frac{2x^2}{a^2}-\frac{ax^5}{5}-\frac{4x^3}{3a}-\frac{3x^4}{4}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*((-4*x)/a^3 - (2*x^2)/a^2 - (4*x^3)/(3*a) - (3*x^4)
/4 - (a*x^5)/5 - (4*Log[1 - a*x])/a^4))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.40

method	result	size
default	$\frac{(12a^5x^5+45a^4x^4+80a^3x^3+120a^2x^2+240ax+240\ln(ax-1))\sqrt{-c(a^2x^2-1)}\,(ax-1)}{60a^4(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	92

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/60*(12*a^5*x^5+45*a^4*x^4+80*a^3*x^3+120*a^2*x^2+240*a*x+240*ln(a*x-1))*
(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a^4/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.25

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{(12\,a^5x^5+45\,a^4x^4+80\,a^3x^3+120\,a^2x^2+240\,ax+240\,\log{(ax-1)})\sqrt{-a^2c}}{60\,a^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

output
```
1/60*(12*a^5*x^5 + 45*a^4*x^4 + 80*a^3*x^3 + 120*a^2*x^2 + 240*a*x + 240*l
og(a*x - 1))*sqrt(-a^2*c)/a^5
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \int \frac{x^3\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x
)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \int \frac{x^3\sqrt{c-a^2\,c\,x^2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((x^3*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^3*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.25

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c}\,i\left(-480\log\left(\sqrt{-ax+1}\right)-12a^5x^5-45a^4x^4-80a^3x^3-120a^2x^2-240ax+497\right)}{60a^4}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*i*( - 480*log(sqrt( - a*x + 1)) - 12*a**5*x**5 - 45*a**4*x**4 - 8
0*a**3*x**3 - 120*a**2*x**2 - 240*a*x + 497))/(60*a**4)
```

3.665 $\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 186

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
4*(-a^2*c*x^2+c)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x*(-a^2*c*x^2+c)^(1/2)/a^
2/(1-1/a^2/x^2)^(1/2)+x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x
^3*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*(-a^2*c*x^2+c)^(1/2)*ln(-a*x
+1)/a^4/(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.40

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{4x}{a^3}+\frac{2x^2}{a^2}+\frac{x^3}{a}+\frac{x^4}{4}+\frac{4\log(1-ax)}{a^4}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((4*x)/a^3 + (2*x^2)/a^2 + x^3/a + x^4/4 + (4*Log[1 -
 a*x])/a^4))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.41, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{x^2(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{x^2(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-ax^3-3x^2-\frac{4x}{a}-\frac{4}{a^2(ax-1)}-\frac{4}{a^2}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(1-ax)}{a^3}-\frac{4x}{a^2}-\frac{ax^4}{4}-\frac{2x^2}{a}-x^3\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*((-4*x)/a^2 - (2*x^2)/a - x^3 - (a*x^4)/4 - (4*Log[
1 - a*x])/a^3))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

method	result	size
default	$\frac{\left(a^{4} x^{4}+4 a^{3} x^{3}+8 a^{2} x^{2}+16 a x +16 \ln \left(a x -1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x -1\right)}{4 a^{3} \left(a x +1\right)^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	83

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/4*(a^4*x^4+4*a^3*x^3+8*a^2*x^2+16*a*x+16*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/
2)*(a*x-1)/a^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.26

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{(a^4x^4+4\,a^3x^3+8\,a^2x^2+16\,ax+16\,\log{(ax-1)})\sqrt{-a^2c}}{4\,a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

output
```
1/4*(a^4*x^4 + 4*a^3*x^3 + 8*a^2*x^2 + 16*a*x + 16*log(a*x - 1))*sqrt(-a^2
*c)/a^4
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x
)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.66

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{1}{4}\sqrt{-c}\left(\frac{a^3x^4+4\,a^2x^3+8\,ax^2+16\,x}{a^2\mathrm{sgn}\,(ax+1)}+\frac{a^5x^4\mathrm{sgn}(ax+1)+4\,a^4x^3\mathrm{sgn}(ax+1)+8\,a^3x^2\mathrm{sgn}(ax+1)+16}{a^4}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

output
```
1/4*sqrt(-c)*((a^3*x^4 + 4*a^2*x^3 + 8*a*x^2 + 16*x)/(a^2*sgn(a*x + 1)) +
(a^5*x^4*sgn(a*x + 1) + 4*a^4*x^3*sgn(a*x + 1) + 8*a^3*x^2*sgn(a*x + 1) +
16*a^2*x*sgn(a*x + 1))/a^4 + 32*log(abs(a*x - 1))/(a^3*sgn(a*x + 1)))
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int \frac{x^2\sqrt{c-a^2\,c\,x^2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.26

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c}\,i\left(-32\log\left(\sqrt{-ax+1}\right)-a^4x^4-4a^3x^3-8a^2x^2-16ax+29\right)}{4a^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*i*( - 32*log(sqrt( - a*x + 1)) - a**4*x**4 - 4*a**3*x**3 - 8*a**2
*x**2 - 16*a*x + 29))/(4*a**3)
```

3.666 $\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 152

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
4*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)+3/2*x*(-a^2*c*x^2+c)^(1/2)/
a/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*(
-a^2*c*x^2+c)^(1/2)*ln(-a*x+1)/a^3/(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}(ax(24+9ax+2a^2x^2)+24\log(1-ax))}{6a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(a*x*(24 + 9*a*x + 2*a^2*x^2) + 24*Log[1 - a*x]))/(6*
a^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.47, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6746, 6747, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{x(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{x(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{86}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-ax^2-3x-\frac{4}{a}-\frac{4}{a(ax-1)}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(1-ax)}{a^2}-\frac{ax^3}{3}-\frac{4x}{a}-\frac{3x^2}{2}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*((-4*x)/a - (3*x^2)/2 - (a*x^3)/3 - (4*Log[1 - a*x]
)/a^2))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{(2a^3x^3+9a^2x^2+24ax+24\ln(ax-1))\sqrt{-c(a^2x^2-1)}\,(ax-1)}{6a^2(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	76

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
1/6*(2*a^3*x^3+9*a^2*x^2+24*a*x+24*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-
1)/a^2/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.28

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{(2\,a^3x^3+9\,a^2x^2+24\,ax+24\,\log{(ax-1)})\sqrt{-a^2c}}{6\,a^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

output
```
1/6*(2*a^3*x^3 + 9*a^2*x^2 + 24*a*x + 24*log(a*x - 1))*sqrt(-a^2*c)/a^3
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int \frac{x\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{-a^2cx^2+cx}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.61

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{1}{6}\sqrt{-c}\left(\frac{2a^2x^3+9ax^2+24x}{a\text{sgn}(ax+1)}+\frac{48\log(|ax-1|)}{a^2\text{sgn}(ax+1)}+\frac{2a^4x^3+9a^3x^2+24a^2x}{a^3\text{sgn}(ax+1)}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

output
```
1/6*sqrt(-c)*((2*a^2*x^3 + 9*a*x^2 + 24*x)/(a*sgn(a*x + 1)) + 48*log(abs(a
*x - 1))/(a^2*sgn(a*x + 1)) + (2*a^4*x^3 + 9*a^3*x^2 + 24*a^2*x)/(a^3*sgn(
a*x + 1)))
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int \frac{x\sqrt{c-a^2\,c\,x^2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.26

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c}\,i\left(-48\log\left(\sqrt{-ax+1}\right)-2a^3x^3-9a^2x^2-24ax+35\right)}{6a^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*i*( - 48*log(sqrt( - a*x + 1)) - 2*a**3*x**3 - 9*a**2*x**2 - 24*a
*x + 35))/(6*a**2)
```

3.667 $\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 113

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1
-1/a^2/x^2)^(1/2)+4*(-a^2*c*x^2+c)^(1/2)*ln(-a*x+1)/a^2/(1-1/a^2/x^2)^(1/2
)/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1-ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((3*x)/a + x^2/2 + (4*Log[1 - a*x])/a^2))/(Sqrt[1 - 1
/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-ax+\frac{4}{1-ax}-3\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}-3x\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a))/(a*Sqrt[1
- 1/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{(a^2x^2+6ax+8\ln(ax-1))\sqrt{-c(a^2x^2-1)}\,(ax-1)}{2a(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	67

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a/(a*x+1)^2
/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{(a^2x^2+6\,ax+8\,\log{(ax-1)})\sqrt{-a^2c}}{2\,a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(-a^2*c)/a^2
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx =\frac{1}{2}\sqrt{-c}\left(\frac{ax^2+6x}{\mathrm{sgn}\left(ax+1\right)}+\frac{a^3x^2\mathrm{sgn}(ax+1)+6a^2x\mathrm{sgn}(ax+1)}{a^2}+\frac{16\log\left(|ax-1|\right)}{a\mathrm{sgn}\left(ax+1\right)}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
1/2*sqrt(-c)*((a*x^2 + 6*x)/sgn(a*x + 1) + (a^3*x^2*sgn(a*x + 1) + 6*a^2*x
*sgn(a*x + 1))/a^2 + 16*log(abs(a*x - 1))/(a*sgn(a*x + 1)))
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.28

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c}\,i\left(-16\log\left(\sqrt{-ax+1}\right)-a^2x^2-6ax+7\right)}{2a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*i*( - 16*log(sqrt( - a*x + 1)) - a**2*x**2 - 6*a*x + 7))/(2*a)
```

3.668 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 114

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)-(-a^2*c*x^2+c)^(1/2)*ln(x)/a/(1-1
/a^2/x^2)^(1/2)/x+4*(-a^2*c*x^2+c)^(1/2)*ln(-a*x+1)/a/(1-1/a^2/x^2)^(1/2)/
x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}\left(x-\frac{\log(x)}{a}+\frac{4\log(1-ax)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(x - Log[x]/a + (4*Log[1 - a*x])/a))/(Sqrt[1 - 1/(a^2
*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.46, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6746, 6747, 25, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{(ax+1)^2}{x(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{(ax+1)^2}{x(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4a}{ax-1}-a+\frac{1}{x}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}(-ax-4\log(1-ax)+\log(x))}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-(a*x) + Log[x] - 4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(
a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{(ax+4\ln(ax-1)-\ln(x))\sqrt{-c(a^2x^2-1)}\,(ax-1)}{(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	59

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBO
SE)
```

output
```
(a*x+4*ln(a*x-1)-ln(x))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/(a*x+1)^2/((a*x-1)/
(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.25

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{-a^2c}(ax+4\log(ax-1)-\log(x))}{a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="f
ricas")
```

output
```
sqrt(-a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/(x*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.43

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = 2\sqrt{-c}\left(\frac{ax}{\operatorname{sgn}(ax+1)} + \frac{4\log(|ax-1|)}{\operatorname{sgn}(ax+1)} - \frac{\log(|x|)}{\operatorname{sgn}(ax+1)}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="g
iac")
```

output
```
2*sqrt(-c)*(a*x/sgn(a*x + 1) + 4*log(abs(a*x - 1))/sgn(a*x + 1) - log(abs(
x))/sgn(a*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{x\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.35

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \sqrt{c}\,i\big(\log\big(\sqrt{-ax+1}-1\big)+\log\big(\sqrt{-ax+1}+1\big) - 8\log\big(\sqrt{-ax+1}\big)-ax+1\big)$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x)
```

output
```
sqrt(c)*i*(log(sqrt( - a*x + 1) - 1) + log(sqrt( - a*x + 1) + 1) - 8*log(s
qrt( - a*x + 1)) - a*x + 1)
```

3.669 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 114

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}x^2} - \frac{3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x} + \frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^2-3*(-a^2*c*x^2+c)^(1/2)*ln(x
)/(1-1/a^2/x^2)^(1/2)/x+4*(-a^2*c*x^2+c)^(1/2)*ln(-a*x+1)/(1-1/a^2/x^2)^(1
/2)/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}\left(\frac{1}{ax}-3\log(x)+4\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(1/(a*x) - 3*Log[x] + 4*Log[1 - a*x]))/(Sqrt[1 - 1/(a
^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \textbf{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{(ax+1)^2}{x^2(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{(ax+1)^2}{x^2(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{99}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4a^2}{ax-1}+\frac{3a}{x}+\frac{1}{x^2}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(3a\log(x)-4a\log(1-ax)-\frac{1}{x}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-x^(-1) + 3*a*Log[x] - 4*a*Log[1 - a*x]))/(a*Sqrt[
1 - 1/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{(4a\ln(ax-1)x-3a\ln(x)x+1)\sqrt{-c(a^2x^2-1)}\,(ax-1)}{(ax+1)^2x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	64

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x,method=_RETURNVER
BOSE)
```

output
```
(4*a*ln(a*x-1)*x-3*a*ln(x)*x+1)*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/(a*x+1)^2/x
/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{-a^2c}(4\,ax\log(ax-1) - 3\,ax\log(x) + 1)}{ax}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"fricas")
```

output
```
sqrt(-a^2*c)*(4*a*x*log(a*x - 1) - 3*a*x*log(x) + 1)/(a*x)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-c(ax-1)(ax+1)}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/(x**2*((a*x - 1)/(a*x + 1))**(3/2)),
 x)
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.30

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\left(4\,a\log\left(|ax-1|\right)-3\,a\log\left(|x|\right)+\frac{1}{x}\right)\sqrt{-c}}{\operatorname{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"giac")
```

output
```
(4*a*log(abs(a*x - 1)) - 3*a*log(abs(x)) + 1/x)*sqrt(-c)/sgn(a*x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{x^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.46

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c}\,i\left(3\log\left(\sqrt{-ax+1}-1\right)ax+3\log\left(\sqrt{-ax+1}+1\right)ax-8\log\left(\sqrt{-ax+1}\right)ax+ax-1\right)}{x}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

output
```
(sqrt(c)*i*(3*log(sqrt( - a*x + 1) - 1)*a*x + 3*log(sqrt( - a*x + 1) + 1)*
a*x - 8*log(sqrt( - a*x + 1))*a*x + a*x - 1))/x
```

3.670 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 153

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{3\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2} - \frac{4a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4a\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
1/2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^3+3*(-a^2*c*x^2+c)^(1/2)/
(1-1/a^2/x^2)^(1/2)/x^2-4*a*(-a^2*c*x^2+c)^(1/2)*ln(x)/(1-1/a^2/x^2)^(1/2)
/x+4*a*(-a^2*c*x^2+c)^(1/2)*ln(-a*x+1)/(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.42

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c-a^2cx^2}\left(\frac{1}{2ax^2}+\frac{3}{x}-4a\log(x)+4a\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(1/(2*a*x^2) + 3/x - 4*a*Log[x] + 4*a*Log[1 - a*x]))/
(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.46, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{(ax+1)^2}{x^3(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{(ax+1)^2}{x^3(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4a^3}{ax-1}+\frac{4a^2}{x}+\frac{3a}{x^2}+\frac{1}{x^3}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(4a^2\log(x)-4a^2\log(1-ax)-\frac{3a}{x}-\frac{1}{2x^2}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-1/2*1/x^2 - (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1
- a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{\left(8 a^{2} \ln \left(a x -1\right) x^{2}-8 a^{2} \ln \left(x \right) x^{2}+6 a x +1\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x -1\right)}{2 x^{2} \left(a x +1\right)^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	77

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x,method=_RETURNVER
BOSE)
```

output
```
1/2*(8*a^2*ln(a*x-1)*x^2-8*a^2*ln(x)*x^2+6*a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(
a*x-1)/x^2/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.59

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{8\,a^3\sqrt{-c}x^2\log\left(\frac{2\,a^3cx^2-2\,a^2cx+\sqrt{-a^2c}(2\,ax-1)\sqrt{-c}+ac}{ax^2-x}\right)+\sqrt{-a^2c}(6\,ax+1)}{2\,ax^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm=
"fricas")
```

output
```
1/2*(8*a^3*sqrt(-c)*x^2*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 - 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + sqrt(-a^2*c)*(6*a*x + 1))/(a*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.30

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\left(8\,a^2\log\left(|ax-1|\right)-8\,a^2\log\left(|x|\right)+\frac{6\,ax+1}{x^2}\right)\sqrt{-c}}{2\,\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm=
"giac")
```

output
```
1/2*(8*a^2*log(abs(a*x - 1)) - 8*a^2*log(abs(x)) + (6*a*x + 1)/x^2)*sqrt(-
c)/sgn(a*x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{x^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.48

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c}\,i\left(8\log\left(\sqrt{-ax+1}-1\right)a^2x^2+8\log\left(\sqrt{-ax+1}+1\right)a^2x^2-16\log\left(\sqrt{-ax+1}\right)a^2x^2+3a^2x^2-6ax\right.}{2x^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x)
```

output
```
(sqrt(c)*i*(8*log(sqrt( - a*x + 1) - 1)*a**2*x**2 + 8*log(sqrt( - a*x + 1)
 + 1)*a**2*x**2 - 16*log(sqrt( - a*x + 1))*a**2*x**2 + 3*a**2*x**2 - 6*a*x
 - 1))/(2*x**2)
```

3.671 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 194

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{3\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{4a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2} - \frac{4a^2\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4a^2\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

output

```
1/3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^4+3/2*(-a^2*c*x^2+c)^(1/2
)/(1-1/a^2/x^2)^(1/2)/x^3+4*a*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)/x^2
-4*a^2*(-a^2*c*x^2+c)^(1/2)*ln(x)/(1-1/a^2/x^2)^(1/2)/x+4*a^2*(-a^2*c*x^2+
c)^(1/2)*ln(-a*x+1)/(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.39

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}\left(\frac{1}{3ax^3}+\frac{3}{2x^2}+\frac{4a}{x}-4a^2\log(x)+4a^2\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(1/(3*a*x^3) + 3/(2*x^2) + (4*a)/x - 4*a^2*Log[x] + 4
*a^2*Log[1 - a*x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.41, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{(ax+1)^2}{x^4(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{(ax+1)^2}{x^4(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4a^4}{ax-1}+\frac{4a^3}{x}+\frac{4a^2}{x^2}+\frac{3a}{x^3}+\frac{1}{x^4}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(4a^3\log(x)-4a^3\log(1-ax)-\frac{4a^2}{x}-\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-1/3*1/x^3 - (3*a)/(2*x^2) - (4*a^2)/x + 4*a^3*Log
[x] - 4*a^3*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.44

method	result	size
default	$\frac{\left(24 a^{3} \ln \left(a x-1\right) x^{3}-24 \ln \left(x \right) x^{3} a^{3}+24 a^{2} x^{2}+9 a x+2\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x-1\right)}{6 x^{3} \left(a x+1\right)^{2} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}$	85

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x,method=_RETURNVER
BOSE)
```

output
```
1/6*(24*a^3*ln(a*x-1)*x^3-24*ln(x)*x^3*a^3+24*a^2*x^2+9*a*x+2)*(-c*(a^2*x^
2-1))^(1/2)*(a*x-1)/x^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.51

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{24\,a^4\sqrt{-c}x^3\log\left(\frac{2\,a^3cx^2-2\,a^2cx+\sqrt{-a^2c}(2\,ax-1)\sqrt{-c}+ac}{ax^2-x}\right)+(24\,a^2x^2+9\,ax+2)\sqrt{-a^2c}}{6\,ax^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm=
"fricas")
```

output
```
1/6*(24*a^4*sqrt(-c)*x^3*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*
x - 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + (24*a^2*x^2 + 9*a*x + 2)*sqrt(-a^2*c
))/(a*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.28

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\left(24\,a^3\log\left(|ax-1|\right)-24\,a^3\log\left(|x|\right)+\frac{24\,a^2x^2+9\,ax+2}{x^3}\right)\sqrt{-c}}{6\,\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm=
"giac")
```

output
```
1/6*(24*a^3*log(abs(a*x - 1)) - 24*a^3*log(abs(x)) + (24*a^2*x^2 + 9*a*x +
 2)/x^3)*sqrt(-c)/sgn(a*x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{x^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.42

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c}\,i\left(24\log\left(\sqrt{-ax+1}-1\right)a^3x^3+24\log\left(\sqrt{-ax+1}+1\right)a^3x^3-48\log\left(\sqrt{-ax+1}\right)a^3x^3+8a^3x^3-24}{6x^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x)
```

output
```
(sqrt(c)*i*(24*log(sqrt( - a*x + 1) - 1)*a**3*x**3 + 24*log(sqrt( - a*x +
1) + 1)*a**3*x**3 - 48*log(sqrt( - a*x + 1))*a**3*x**3 + 8*a**3*x**3 - 24*
a**2*x**2 - 9*a*x - 2))/(6*x**3)
```

3.672 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 228

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{2a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{4a^3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4a^3\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

output

```
1/4*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^5+(-a^2*c*x^2+c)^(1/2)/(1
-1/a^2/x^2)^(1/2)/x^4+2*a*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)/x^3+4*a
^2*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)/x^2-4*a^3*(-a^2*c*x^2+c)^(1/2)
*ln(x)/(1-1/a^2/x^2)^(1/2)/x+4*a^3*(-a^2*c*x^2+c)^(1/2)*ln(-a*x+1)/(1-1/a^
2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.35

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}\left(\frac{1}{4ax^4}+\frac{1}{x^3}+\frac{2a}{x^2}+\frac{4a^2}{x}-4a^3\log(x)+4a^3\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(1/(4*a*x^4) + x^(-3) + (2*a)/x^2 + (4*a^2)/x - 4*a^3
*Log[x] + 4*a^3*Log[1 - a*x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.38, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{(ax+1)^2}{x^5(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{(ax+1)^2}{x^5(1-ax)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4a^5}{ax-1}+\frac{4a^4}{x}+\frac{4a^3}{x^2}+\frac{4a^2}{x^3}+\frac{3a}{x^4}+\frac{1}{x^5}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(4a^4\log(x)-4a^4\log(1-ax)-\frac{4a^3}{x}-\frac{2a^2}{x^2}-\frac{a}{x^3}-\frac{1}{4x^4}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-1/4*1/x^4 - a/x^3 - (2*a^2)/x^2 - (4*a^3)/x + 4*a
^4*Log[x] - 4*a^4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{(16\ln(ax-1)x^4a^4-16\ln(x)x^4a^4+16a^3x^3+8a^2x^2+4ax+1)\sqrt{-c(a^2x^2-1)}\,(ax-1)}{4x^4(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	93

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x,method=_RETURNVER
BOSE)
```

output
```
1/4*(16*ln(a*x-1)*x^4*a^4-16*ln(x)*x^4*a^4+16*a^3*x^3+8*a^2*x^2+4*a*x+1)*(
-c*(a^2*x^2-1))^(1/2)*(a*x-1)/x^4/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.46

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{16\,a^5\sqrt{-c}x^4\log\left(\frac{2\,a^3cx^2-2\,a^2cx+\sqrt{-a^2c}(2\,ax-1)\sqrt{-c}+ac}{ax^2-x}\right)+(16\,a^3x^3+8\,a^2x^2+4\,ax+1)\sqrt{-a^2c}}{4\,ax^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm=
"fricas")
```

output
```
1/4*(16*a^5*sqrt(-c)*x^4*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*
x - 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + (16*a^3*x^3 + 8*a^2*x^2 + 4*a*x + 1)
*sqrt(-a^2*c))/(a*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.27

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\left(16\,a^4\log\left(|ax-1|\right)-16\,a^4\log\left(|x|\right)+\frac{16\,a^3x^3+8\,a^2x^2+4\,ax+1}{x^4}\right)\sqrt{-c}}{4\,\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm=
"giac")
```

output
```
1/4*(16*a^4*log(abs(a*x - 1)) - 16*a^4*log(abs(x)) + (16*a^3*x^3 + 8*a^2*x
^2 + 4*a*x + 1)/x^4)*sqrt(-c)/sgn(a*x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}}{x^5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.39

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$$
$$=\frac{\sqrt{c}\,i\left(16\log\left(\sqrt{-ax+1}-1\right)a^4x^4+16\log\left(\sqrt{-ax+1}+1\right)a^4x^4-32\log\left(\sqrt{-ax+1}\right)a^4x^4+4a^4x^4-16\right.}{4x^4}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x)
```

output
```
(sqrt(c)*i*(16*log(sqrt( - a*x + 1) - 1)*a**4*x**4 + 16*log(sqrt( - a*x +
1) + 1)*a**4*x**4 - 32*log(sqrt( - a*x + 1))*a**4*x**4 + 4*a**4*x**4 - 16*
a**3*x**3 - 8*a**2*x**2 - 4*a*x - 1))/(4*x**4)
```

3.673 $\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 211

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{a\,(c-a^2cx^2)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5}{2\,(c-a^2cx^2)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2a^2(1-ax)\,(c-a^2cx^2)^{3/2}} + \frac{7\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4a^2\,(c-a^2cx^2)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4a^2\,(c-a^2cx^2)^{3/2}}$$

output
```
(1-1/a^2/x^2)^(3/2)*x^4/a/(-a^2*c*x^2+c)^(3/2)+1/2*(1-1/a^2/x^2)^(3/2)*x^5
/(-a^2*c*x^2+c)^(3/2)+1/2*(1-1/a^2/x^2)^(3/2)*x^3/a^2/(-a*x+1)/(-a^2*c*x^2
+c)^(3/2)+7/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/a^2/(-a^2*c*x^2+c)^(3/2)+
1/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/a^2/(-a^2*c*x^2+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.39

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(2\left(\frac{2x}{a}+x^2+\frac{1}{a^2-a^3x}\right)+\frac{7\log(1-ax)}{a^2}+\frac{\log(1+ax)}{a^2}\right)}{4(c-a^2cx^2)^{3/2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(3/2)*x^3*(2*((2*x)/a + x^2 + (a^2 - a^3*x)^(-1)) + (7*
Log[1 - a*x])/a^2 + Log[1 + a*x]/a^2))/(4*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{x^4}{(1-ax)^2(ax+1)}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(\frac{x}{a^3}+\frac{7}{4a^4(ax-1)}+\frac{1}{4a^4(ax+1)}+\frac{1}{2a^4(ax-1)^2}+\frac{1}{a^4}\right)dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{2a^5(1-ax)}+\frac{7\log(1-ax)}{4a^5}+\frac{\log(ax+1)}{4a^5}+\frac{x}{a^4}+\frac{x^2}{2a^3}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(x/a^4 + x^2/(2*a^3) + 1/(2*a^5*(1 - a*x)
) + (7*Log[1 - a*x])/(4*a^5) + Log[1 + a*x]/(4*a^5)))/(c - a^2*c*x^2)^(3/2
)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(2a^3x^3+2a^2x^2+\ln(ax+1)xa+7a\ln(ax-1)x-4ax-\ln(ax+1)-7\ln(ax-1)-2)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-1)c^2a^5}$	106

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(2*a^3*x^3+2*a^2*x^2+ln
(a*x+1)*x*a+7*a*ln(a*x-1)*x-4*a*x-ln(a*x+1)-7*ln(a*x-1)-2)/(a^2*x^2-1)/c^2
/a^5
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \frac{(2\,a^3x^3+2\,a^2x^2-4\,ax+(ax-1)\log{(ax+1)}+7\,(ax-1)\log{(ax-1)}-2)\sqrt{-a^2}}{4\,(a^7c^2x-a^6c^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

output
```
1/4*(2*a^3*x^3 + 2*a^2*x^2 - 4*a*x + (a*x - 1)*log(a*x + 1) + 7*(a*x - 1)*
log(a*x - 1) - 2)*sqrt(-a^2*c)/(a^7*c^2*x - a^6*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^4}{\sqrt{\frac{ax-1}{ax+1}}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**4/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**4/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)),
 x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^4}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate(x^4/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^4}{(c-a^2\,c\,x^2)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x^4/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x^4/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.56

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,i\left(-\log\left(\sqrt{-ax+1}-\sqrt{2}\right)ax+\log\left(\sqrt{-ax+1}-\sqrt{2}\right)-\log\left(\sqrt{-ax+1}+\sqrt{2}\right)a\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*i*( - log(sqrt( - a*x + 1) - sqrt(2))*a*x + log(sqrt( - a*x + 1)
- sqrt(2)) - log(sqrt( - a*x + 1) + sqrt(2))*a*x + log(sqrt( - a*x + 1) +
sqrt(2)) - 14*log(sqrt( - a*x + 1))*a*x + 14*log(sqrt( - a*x + 1)) - 2*a**
3*x**3 - 2*a**2*x**2 + 10*a*x - 4))/(4*a**5*c**2*(a*x - 1))
```

3.674 $\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{(c-a^2cx^2)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2a(1-ax)\left(c-a^2cx^2\right)^{3/2}} + \frac{5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4a\left(c-a^2cx^2\right)^{3/2}} - \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4a\left(c-a^2cx^2\right)^{3/2}}$$

output
```
(1-1/a^2/x^2)^(3/2)*x^4/(-a^2*c*x^2+c)^(3/2)+1/2*(1-1/a^2/x^2)^(3/2)*x^3/a
/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+5/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/a/(-
a^2*c*x^2+c)^(3/2)-1/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/a/(-a^2*c*x^2+c)^
(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.43

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(x+\frac{1}{2a-2a^2x}+\frac{5\log(1-ax)}{4a}-\frac{\log(1+ax)}{4a}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(3/2)*x^3*(x + (2*a - 2*a^2*x)^(-1) + (5*Log[1 - a*x])/
(4*a) - Log[1 + a*x]/(4*a)))/(c - a^2*c*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{x^3}{(1-ax)^2(ax+1)}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(-\frac{1}{4a^3(ax+1)}+\frac{1}{a^3}+\frac{5}{4a^3(ax-1)}+\frac{1}{2a^3(ax-1)^2}\right)dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{2a^4(1-ax)}+\frac{5\log(1-ax)}{4a^4}-\frac{\log(ax+1)}{4a^4}+\frac{x}{a^3}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(x/a^3 + 1/(2*a^4*(1 - a*x)) + (5*Log[1 -
 a*x])/(4*a^4) - Log[1 + a*x]/(4*a^4)))/(c - a^2*c*x^2)^(3/2)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.57

method	result	size
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(-4 a^{2} x^{2}+\ln \left(a x +1\right) x a -5 a \ln \left(a x -1\right) x +4 a x -\ln \left(a x +1\right)+5 \ln \left(a x -1\right)+2\right)}{4 \sqrt{\frac{a x -1}{a x +1}}\, \left(a^{2} x^{2}-1\right) c^{2} a^{4}}$	98

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(-4*a^2*x^2+ln(a*x+1)*
x*a-5*a*ln(a*x-1)*x+4*a*x-ln(a*x+1)+5*ln(a*x-1)+2)/(a^2*x^2-1)/c^2/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.40

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \frac{(4\,a^2x^2-4\,ax-(ax-1)\log{(ax+1)}+5\,(ax-1)\log{(ax-1)}-2)\sqrt{-a^2c}}{4\,(a^6c^2x-a^5c^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

output
```
1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
 - 2)*sqrt(-a^2*c)/(a^6*c^2*x - a^5*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^3}{\sqrt{\frac{ax-1}{ax+1}}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**3/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)),
 x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^3}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate(x^3/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^3}{(c-a^2\,c\,x^2)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x^3/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x^3/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.65

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,i\left(\log\left(\sqrt{-ax+1}-\sqrt{2}\right)ax-\log\left(\sqrt{-ax+1}-\sqrt{2}\right)+\log\left(\sqrt{-ax+1}+\sqrt{2}\right)ax\right.}{4a^4}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*i*(log(sqrt( - a*x + 1) - sqrt(2))*a*x - log(sqrt( - a*x + 1) - s
qrt(2)) + log(sqrt( - a*x + 1) + sqrt(2))*a*x - log(sqrt( - a*x + 1) + sqr
t(2)) - 10*log(sqrt( - a*x + 1))*a*x + 10*log(sqrt( - a*x + 1)) - 4*a**2*x
**2 + 8*a*x - 2))/(4*a**4*c**2*(a*x - 1))
```

3.675 $\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} + \frac{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4\left(c-a^2cx^2\right)^{3/2}}$$

output
```
1/2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+3/4*(1-1/a^2/x^2
)^(3/2)*x^3*ln(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+1/4*(1-1/a^2/x^2)^(3/2)*x^3*ln
(a*x+1)/(-a^2*c*x^2+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.48

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(\frac{2}{1-ax}+3\log(1-ax)+\log(1+ax)\right)}{4\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(3/2)*x^3*(2/(1 - a*x) + 3*Log[1 - a*x] + Log[1 + a*x])
)/(4*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 6746$$

$$\frac{x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2} \int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2} x} dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \; 6747$$

$$\frac{a^3x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2} \int \frac{x^2}{(1-ax)^2(ax+1)} dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \; 99$$

$$\frac{a^3x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2} \int \left(\frac{1}{4a^2(ax+1)}+\frac{3}{4a^2(ax-1)}+\frac{1}{2a^2(ax-1)^2}\right) dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{a^3x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2} \left(\frac{1}{2a^3(1-ax)}+\frac{3\log(1-ax)}{4a^3}+\frac{\log(ax+1)}{4a^3}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(1/(2*a^3*(1 - a*x)) + (3*Log[1 - a*x])/(
4*a^3) + Log[1 + a*x]/(4*a^3)))/(c - a^2*c*x^2)^(3/2)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\left(\ln\left(a x +1\right) x a +3 a \ln\left(a x -1\right) x -\ln\left(a x +1\right)-3 \ln\left(a x -1\right)-2\right)}{4 \sqrt{\frac{a x -1}{a x +1}}\,\left(a^{2} x^{2}-1\right) c^{2} a^{3}}$	86

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*x*a+3*a*ln(a
*x-1)*x-ln(a*x+1)-3*ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.43

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{-a^2c}((ax-1)\log(ax+1)+3(ax-1)\log(ax-1)-2)}{4(a^5c^2x-a^4c^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

output
```
1/4*sqrt(-a^2*c)*((a*x - 1)*log(a*x + 1) + 3*(a*x - 1)*log(a*x - 1) - 2)/(
a^5*c^2*x - a^4*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{\frac{ax-1}{ax+1}}\,(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**2/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)),
 x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^2}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate(x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^2}{(c-a^2\,c\,x^2)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x^2/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x^2/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,i\left(-\log\left(\sqrt{-ax+1}-\sqrt{2}\right)ax+\log\left(\sqrt{-ax+1}-\sqrt{2}\right)-\log\left(\sqrt{-ax+1}+\sqrt{2}\right)a\right.}{4a^3c^2\,(ax-}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*i*( - log(sqrt( - a*x + 1) - sqrt(2))*a*x + log(sqrt( - a*x + 1)
- sqrt(2)) - log(sqrt( - a*x + 1) + sqrt(2))*a*x + log(sqrt( - a*x + 1) +
sqrt(2)) - 6*log(sqrt( - a*x + 1))*a*x + 6*log(sqrt( - a*x + 1)) + 2))/(4*
a**3*c**2*(a*x - 1))
```

3.676 $\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} - \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\text{arctanh}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

output
```
1/2*a*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/2*a*(1-1/a^2
/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(\frac{1}{1-ax}-\text{arctanh}(ax)\right)}{2\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(a*(1 - 1/(a^2*x^2))^(3/2)*x^3*((1 - a*x)^(-1) - ArcTanh[a*x]))/(2*(c - a^
2*c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6746, 6747, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{x}{(1-ax)^2(ax+1)}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(\frac{1}{2(a^2x^2-1)a}+\frac{1}{2(ax-1)^2a}\right)dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{2a^2(1-ax)}-\frac{\text{arctanh}(ax)}{2a^2}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(1/(2*a^2*(1 - a*x)) - ArcTanh[a*x]/(2*a^
2)))/(c - a^2*c*x^2)^(3/2)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\left(\ln \left(a x +1\right) x a -a \ln \left(a x -1\right) x -\ln \left(a x +1\right)+\ln \left(a x -1\right)+2\right)}{4 \sqrt{\frac{a x -1}{a x +1}}\,\left(a^{2} x^{2}-1\right) c^{2} a^{2}}$	84

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*x*a-a*ln(a*
x-1)*x-ln(a*x+1)+ln(a*x-1)+2)/(a^2*x^2-1)/c^2/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{(a^2x-a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\sqrt{-a^2c}}{4\,(a^4c^2x-a^3c^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

output
```
-1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)
/(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^4*c^2*x - a^3*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x}{\sqrt{\frac{ax-1}{ax+1}}\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

output
```
integrate(x/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x}{(c-a^2\,c\,x^2)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,i\left(\log\left(\sqrt{-ax+1}-\sqrt{2}\right)ax-\log\left(\sqrt{-ax+1}-\sqrt{2}\right)+\log\left(\sqrt{-ax+1}+\sqrt{2}\right)ax\right.}{4a^2c^2\,(ax-}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*i*(log(sqrt( - a*x + 1) - sqrt(2))*a*x - log(sqrt( - a*x + 1) - s
qrt(2)) + log(sqrt( - a*x + 1) + sqrt(2))*a*x - log(sqrt( - a*x + 1) + sqr
t(2)) - 2*log(sqrt( - a*x + 1))*a*x + 2*log(sqrt( - a*x + 1)) + 2))/(4*a**
2*c**2*(a*x - 1))
```

3.677 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\text{arctanh}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

output
```
1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+1/2*a^2*(1-1
/a^2/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3(-1+(-1+ax)\text{arctanh}(ax))}{(-2+2ax)\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-1 + (-1 + a*x)*ArcTanh[a*x]))/((-2 + 2*
a*x)*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6746, 6747, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{1}{(1-ax)^2(ax+1)}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{54}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(\frac{1}{2(ax-1)^2}-\frac{1}{2(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\text{arctanh}(ax)}{2a}+\frac{1}{2a(1-ax)}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a)))
/(c - a^2*c*x^2)^(3/2)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\left(\ln \left(a x +1\right) x a -a \ln \left(a x -1\right) x -\ln \left(a x +1\right)+\ln \left(a x -1\right)-2\right)}{4 \sqrt{\frac{a x -1}{a x +1}}\,\left(a^{2} x^{2}-1\right) c^{2} a}$	84

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*x*a-a*ln(a*x
-1)*x-ln(a*x+1)+ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{(a^2x-a)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)+2\sqrt{-a^2c}}{4\left(a^3c^2x-a^2c^2\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

output
```
-1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)
/(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^3*c^2*x - a^2*c^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(1/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{\left(c-a^2\,c\,x^2\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,i\left(-\log\left(\sqrt{-ax+1}-\sqrt{2}\right)ax+\log\left(\sqrt{-ax+1}-\sqrt{2}\right)-\log\left(\sqrt{-ax+1}+\sqrt{2}\right)a\right.}{4a\,c^2\,(ax-}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

output

```
(sqrt(c)*i*( - log(sqrt( - a*x + 1) - sqrt(2))*a*x + log(sqrt( - a*x + 1)
- sqrt(2)) - log(sqrt( - a*x + 1) + sqrt(2))*a*x + log(sqrt( - a*x + 1) +
sqrt(2)) + 2*log(sqrt( - a*x + 1))*a*x - 2*log(sqrt( - a*x + 1)) + 2))/(4*
a*c**2*(a*x - 1))
```

3.678 $\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx = \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(c-a^2cx^2)^{3/2}}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{(c-a^2cx^2)^{3/2}} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4(c-a^2cx^2)^{3/2}}-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4(c-a^2cx^2)^{3/2}}$$

output
```
1/2*a^3*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+a^3*(1-1/a^2
/x^2)^(3/2)*x^3*ln(x)/(-a^2*c*x^2+c)^(3/2)-3/4*a^3*(1-1/a^2/x^2)^(3/2)*x^3
*ln(-a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/4*a^3*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)
/(-a^2*c*x^2+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.38

$$\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx = \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(\frac{1}{2-2ax}+\log(x)-\frac{3}{4}\log(1-ax)-\frac{1}{4}\log(1+ax)\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*((2 - 2*a*x)^(-1) + Log[x] - (3*Log[1 - a
*x])/4 - Log[1 + a*x]/4))/(c - a^2*c*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.41, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{1}{x(1-ax)^2(ax+1)}dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{93}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(-\frac{3a}{4(ax-1)}-\frac{a}{4(ax+1)}+\frac{a}{2(ax-1)^2}+\frac{1}{x}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{2(1-ax)}-\frac{3}{4}\log(1-ax)-\frac{1}{4}\log(ax+1)+\log(x)\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Int[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(1/(2*(1 - a*x)) + Log[x] - (3*Log[1 - a*
x])/4 - Log[1 + a*x]/4))/(c - a^2*c*x^2)^(3/2)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\left(\ln \left(a x +1\right) x a +3 a \ln \left(a x -1\right) x -4 a \ln \left(x \right) x -\ln \left(a x +1\right)-3 \ln \left(a x -1\right)+4 \ln \left(x \right)+2\right)}{4 \sqrt{\frac{a x -1}{a x +1}}\,\left(a^{2} x^{2}-1\right) c^{2}}$	93

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*x*a+3*a*ln(
a*x-1)*x-4*a*ln(x)*x-ln(a*x+1)-3*ln(a*x-1)+4*ln(x)+2)/(a^2*x^2-1)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.36

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = -\frac{\sqrt{-a^2c}((ax-1)\log{(ax+1)}+3\,(ax-1)\log{(ax-1)}-4\,(ax-1)\log{(x)}+2)}{4\,(a^2c^2x-ac^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

output
```
-1/4*sqrt(-a^2*c)*((a*x - 1)*log(a*x + 1) + 3*(a*x - 1)*log(a*x - 1) - 4*(
a*x - 1)*log(x) + 2)/(a^2*c^2*x - a*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{x\,(c-a^2\,c\,x^2)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/(x*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/(x*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.84

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\,i\left(\log\left(\sqrt{-ax+1}-\sqrt{2}\right)ax-\log\left(\sqrt{-ax+1}-\sqrt{2}\right)-4\log\left(\sqrt{-ax+1}-1\right)a\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*i*(log(sqrt( - a*x + 1) - sqrt(2))*a*x - log(sqrt( - a*x + 1) - s
qrt(2)) - 4*log(sqrt( - a*x + 1) - 1)*a*x + 4*log(sqrt( - a*x + 1) - 1) +
log(sqrt( - a*x + 1) + sqrt(2))*a*x - log(sqrt( - a*x + 1) + sqrt(2)) - 4*
log(sqrt( - a*x + 1) + 1)*a*x + 4*log(sqrt( - a*x + 1) + 1) + 6*log(sqrt(
- a*x + 1))*a*x - 6*log(sqrt( - a*x + 1)) + 2))/(4*c**2*(a*x - 1))
```

3.679 $\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 214

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{3/2}}\,dx = &-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{(c-a^2cx^2)^{3/2}} \\ &+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(c-a^2cx^2)^{3/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{(c-a^2cx^2)^{3/2}} \\ &-\frac{5a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4(c-a^2cx^2)^{3/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4(c-a^2cx^2)^{3/2}}\end{aligned}$$

output

```
-a^3*(1-1/a^2/x^2)^(3/2)*x^2/(-a^2*c*x^2+c)^(3/2)+1/2*a^4*(1-1/a^2/x^2)^(3
/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+a^4*(1-1/a^2/x^2)^(3/2)*x^3*ln(x)/(-
a^2*c*x^2+c)^(3/2)-5/4*a^4*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/(-a^2*c*x^2+
c)^(3/2)+1/4*a^4*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/(-a^2*c*x^2+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.37

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx=\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(-\frac{4}{x}+\frac{2a}{1-ax}+4a\log(x)-5a\log(1-ax)+a\log(1+ax)\right)}{4\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-4/x + (2*a)/(1 - a*x) + 4*a*Log[x] - 5*
a*Log[1 - a*x] + a*Log[1 + a*x]))/(4*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5}dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{1}{x^2(1-ax)^2(ax+1)}dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(-\frac{5a^2}{4(ax-1)}+\frac{a^2}{4(ax+1)}+\frac{a^2}{2(ax-1)^2}+\frac{a}{x}+\frac{1}{x^2}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{a}{2(1-ax)}+a\log(x)-\frac{5}{4}a\log(1-ax)+\frac{1}{4}a\log(ax+1)-\frac{1}{x}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Int[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-x^(-1) + a/(2*(1 - a*x)) + a*Log[x] - (
5*a*Log[1 - a*x])/4 + (a*Log[1 + a*x])/4))/(c - a^2*c*x^2)^(3/2)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(\ln(ax+1)x^2a^2-5a^2\ln(ax-1)x^2+4a^2\ln(x)x^2-\ln(ax+1)xa+5a\ln(ax-1)x-4a\ln(x)x-6ax+4)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-1)c^2x}$	118

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*x^2*a^2-5*a^
2*ln(a*x-1)*x^2+4*a^2*ln(x)*x^2-ln(a*x+1)*x*a+5*a*ln(a*x-1)*x-4*a*ln(x)*x-
6*a*x+4)/(a^2*x^2-1)/c^2/x
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.43

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx = -\frac{\sqrt{-a^2c}(6\,ax-(a^2x^2-ax)\log(ax+1)+5\,(a^2x^2-ax)\log(ax-1)-4\,(a^2x^2-ax)\log(x)-4)}{4\,(a^2c^2x^2-ac^2x)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

output
```
-1/4*sqrt(-a^2*c)*(6*a*x - (a^2*x^2 - a*x)*log(a*x + 1) + 5*(a^2*x^2 - a*x
)*log(a*x - 1) - 4*(a^2*x^2 - a*x)*log(x) - 4)/(a^2*c^2*x^2 - a*c^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c-a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c - a^2cx^2)^{3/2}} \, dx = \int \frac{1}{x^2 (c - a^2 c x^2)^{3/2} \sqrt{\frac{a x-1}{a x+1}}} \, dx$$

input
```
int(1/(x^2*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/(x^2*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.91

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c - a^2cx^2)^{3/2}} \, dx = \frac{\sqrt{c}\, i\left(-\log\left(\sqrt{-ax+1}-\sqrt{2}\right) a^2x^2 + \log\left(\sqrt{-ax+1}-\sqrt{2}\right) ax - 4\log\left(\sqrt{-ax+\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*i*( - log(sqrt( - a*x + 1) - sqrt(2))*a**2*x**2 + log(sqrt( - a*x
 + 1) - sqrt(2))*a*x - 4*log(sqrt( - a*x + 1) - 1)*a**2*x**2 + 4*log(sqrt(
 - a*x + 1) - 1)*a*x - log(sqrt( - a*x + 1) + sqrt(2))*a**2*x**2 + log(sqr
t( - a*x + 1) + sqrt(2))*a*x - 4*log(sqrt( - a*x + 1) + 1)*a**2*x**2 + 4*l
og(sqrt( - a*x + 1) + 1)*a*x + 10*log(sqrt( - a*x + 1))*a**2*x**2 - 10*log
(sqrt( - a*x + 1))*a*x - 6*a**2*x**2 + 12*a*x - 4))/(4*c**2*x*(a*x - 1))
```

3.680 $\int \frac{e^{\coth^{-1}(ax)}}{x^3(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 252

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3(c-a^2cx^2)^{3/2}}\,dx = -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{2(c-a^2cx^2)^{3/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{(c-a^2cx^2)^{3/2}} + \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(c-a^2cx^2)^{3/2}} + \frac{2a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{(c-a^2cx^2)^{3/2}} - \frac{7a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4(c-a^2cx^2)^{3/2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4(c-a^2cx^2)^{3/2}}$$

output
```
-1/2*a^3*(1-1/a^2/x^2)^(3/2)*x/(-a^2*c*x^2+c)^(3/2)-a^4*(1-1/a^2/x^2)^(3/2
)*x^2/(-a^2*c*x^2+c)^(3/2)+1/2*a^5*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*
c*x^2+c)^(3/2)+2*a^5*(1-1/a^2/x^2)^(3/2)*x^3*ln(x)/(-a^2*c*x^2+c)^(3/2)-7/
4*a^5*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/4*a^5*(1-1
/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/(-a^2*c*x^2+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.37

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3 (c-a^2cx^2)^{3/2}}\,dx = \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} x^3 \left(-\frac{2}{x^2}-\frac{4a}{x}+\frac{2a^2}{1-ax}+8a^2\log(x)-7a^2\log(1-ax)-a^2\log(1+\right.}{4(c-a^2cx^2)^{3/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-2/x^2 - (4*a)/x + (2*a^2)/(1 - a*x) + 8
*a^2*Log[x] - 7*a^2*Log[1 - a*x] - a^2*Log[1 + a*x]))/(4*(c - a^2*c*x^2)^(
3/2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3 (c-a^2cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 6746$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^6}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \; 6747$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{1}{x^3(1-ax)^2(ax+1)}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \; 99$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int\left(-\frac{7a^3}{4(ax-1)}-\frac{a^3}{4(ax+1)}+\frac{a^3}{2(ax-1)^2}+\frac{2a^2}{x}+\frac{a}{x^2}+\frac{1}{x^3}\right)dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{a^2}{2(1-ax)}+2a^2\log(x)-\frac{7}{4}a^2\log(1-ax)-\frac{1}{4}a^2\log(ax+1)-\frac{a}{x}-\frac{1}{2x^2}\right)}{(c-a^2cx^2)^{3/2}}$$

input
```
Int[E^ArcCoth[a*x]/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-1/2*1/x^2 - a/x + a^2/(2*(1 - a*x)) + 2
*a^2*Log[x] - (7*a^2*Log[1 - a*x])/4 - (a^2*Log[1 + a*x])/4))/(c - a^2*c*x
^2)^(3/2)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.55

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(\ln(ax+1)x^3a^3+7a^3\ln(ax-1)x^3-8\ln(x)x^3a^3-\ln(ax+1)x^2a^2-7a^2\ln(ax-1)x^2+8a^2\ln(x)x^2+6a^2x^2-2ax-2)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-1)c^2x^2}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*x^3*a^3+7*a
^3*ln(a*x-1)*x^3-8*ln(x)*x^3*a^3-ln(a*x+1)*x^2*a^2-7*a^2*ln(a*x-1)*x^2+8*a
^2*ln(x)*x^2+6*a^2*x^2-2*a*x-2)/(a^2*x^2-1)/c^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3\,(c-a^2cx^2)^{3/2}}\,dx = -\frac{\left(6\,a^2x^2-2\,ax+(a^3x^3-a^2x^2)\log(ax+1)+7\,(a^3x^3-a^2x^2)\log(ax-1)-8\,(a^3x^3-a^2x^2)\log(x)-2\right)\dots}{4\,(a^2c^2x^3-ac^2x^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

output
```
-1/4*(6*a^2*x^2 - 2*a*x + (a^3*x^3 - a^2*x^2)*log(a*x + 1) + 7*(a^3*x^3 -
a^2*x^2)*log(a*x - 1) - 8*(a^3*x^3 - a^2*x^2)*log(x) - 2)*sqrt(-a^2*c)/(a^
2*c^2*x^3 - a*c^2*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3 (c - a^2cx^2)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**3/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3 (c - a^2cx^2)^{3/2}} \, dx = \int \frac{1}{(-a^2cx^2 + c)^{\frac{3}{2}} x^3 \sqrt{\frac{ax-1}{ax+1}}} \, dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3 (c - a^2cx^2)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3(c-a^2cx^2)^{3/2}}\,dx = \int \frac{1}{x^3\,(c-a^2\,c\,x^2)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/(x^3*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/(x^3*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.88

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,i\left(\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^3x^3-\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^2x^2-8\log\left(\sqrt{-ax+\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*i*(log(sqrt( - a*x + 1) - sqrt(2))*a**3*x**3 - log(sqrt( - a*x +
1) - sqrt(2))*a**2*x**2 - 8*log(sqrt( - a*x + 1) - 1)*a**3*x**3 + 8*log(sq
rt( - a*x + 1) - 1)*a**2*x**2 + log(sqrt( - a*x + 1) + sqrt(2))*a**3*x**3
- log(sqrt( - a*x + 1) + sqrt(2))*a**2*x**2 - 8*log(sqrt( - a*x + 1) + 1)*
a**3*x**3 + 8*log(sqrt( - a*x + 1) + 1)*a**2*x**2 + 14*log(sqrt( - a*x + 1
))*a**3*x**3 - 14*log(sqrt( - a*x + 1))*a**2*x**2 - 3*a**3*x**3 + 9*a**2*x
**2 - 2*a*x - 2))/(4*c**2*x**2*(a*x - 1))
```

3.681 $\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 262

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}{(c-a^2cx^2)^{5/2}}-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8a(1-ax)^2(c-a^2cx^2)^{5/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{a(1-ax)(c-a^2cx^2)^{5/2}}-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8a(1+ax)(c-a^2cx^2)^{5/2}}+\frac{23\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-ax)}{16a(c-a^2cx^2)^{5/2}}-\frac{7\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1+ax)}{16a(c-a^2cx^2)^{5/2}}$$

output
```
(1-1/a^2/x^2)^(5/2)*x^6/(-a^2*c*x^2+c)^(5/2)-1/8*(1-1/a^2/x^2)^(5/2)*x^5/a
/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+(1-1/a^2/x^2)^(5/2)*x^5/a/(-a*x+1)/(-a^2*
c*x^2+c)^(5/2)-1/8*(1-1/a^2/x^2)^(5/2)*x^5/a/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+
23/16*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/a/(-a^2*c*x^2+c)^(5/2)-7/16*(1-1/
a^2/x^2)^(5/2)*x^5*ln(a*x+1)/a/(-a^2*c*x^2+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.38

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(x-\frac{1}{8a(-1+ax)^2}+\frac{1}{a-a^2x}-\frac{1}{8a+8a^2x}+\frac{23\log(1-ax)}{16a}-\frac{7\log(1+ax)}{16a}\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x^5)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(5/2)*x^5*(x - 1/(8*a*(-1 + a*x)^2) + (a - a^2*x)^(-1)
- (8*a + 8*a^2*x)^(-1) + (23*Log[1 - a*x])/(16*a) - (7*Log[1 + a*x])/(16*a
)))/(c - a^2*c*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.43, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{x^5}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int\frac{x^5}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int\left(\frac{7}{16a^5(ax+1)}-\frac{1}{8a^5(ax+1)^2}-\frac{1}{a^5}-\frac{23}{16a^5(ax-1)}-\frac{1}{a^5(ax-1)^2}-\frac{1}{4a^5(ax-1)^3}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(-\frac{1}{a^6(1-ax)}+\frac{1}{8a^6(ax+1)}+\frac{1}{8a^6(1-ax)^2}-\frac{23\log(1-ax)}{16a^6}+\frac{7\log(ax+1)}{16a^6}-\frac{x}{a^5}\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x^5)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(-(x/a^5) + 1/(8*a^6*(1 - a*x)^2) - 1/(
a^6*(1 - a*x)) + 1/(8*a^6*(1 + a*x)) - (23*Log[1 - a*x])/(16*a^6) + (7*Log
[1 + a*x])/(16*a^6)))/(c - a^2*c*x^2)^(5/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.71

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(-16a^4x^4+7\ln(ax+1)x^3a^3-23a^3\ln(ax-1)x^3+16a^3x^3-7\ln(ax+1)x^2a^2+23a^2\ln(ax-1)x^2+34a^2x^2-7\ln(ax+1)x}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a^6(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(-16*a^4*x^4+7
*ln(a*x+1)*x^3*a^3-23*a^3*ln(a*x-1)*x^3+16*a^3*x^3-7*ln(a*x+1)*x^2*a^2+23*
a^2*ln(a*x-1)*x^2+34*a^2*x^2-7*ln(a*x+1)*x*a+23*a*ln(a*x-1)*x-18*a*x+7*ln(
a*x+1)-23*ln(a*x-1)-12)/(a^2*x^2-1)/c^3/a^6/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.53

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx =$$
$$-\frac{(16\,a^4x^4 - 16\,a^3x^3 - 34\,a^2x^2 + 18\,ax - 7\,(a^3x^3 - a^2x^2 - ax + 1)\log(ax+1) + 23\,(a^3x^3 - a^2x^2 - ax +}{16\,(a^{10}c^3x^3 - a^9c^3x^2 - a^8c^3x + a^7c^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

output
```
-1/16*(16*a^4*x^4 - 16*a^3*x^3 - 34*a^2*x^2 + 18*a*x - 7*(a^3*x^3 - a^2*x^
2 - a*x + 1)*log(a*x + 1) + 23*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1)
+ 12)*sqrt(-a^2*c)/(a^10*c^3*x^3 - a^9*c^3*x^2 - a^8*c^3*x + a^7*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x***5/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^5}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate(x^5/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^5}{\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x^5/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x^5/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.98

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,i\left(-7\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^3x^3+7\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^2x^2+7\log\left(\sqrt{-ax\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*i*( - 7*log(sqrt( - a*x + 1) - sqrt(2))*a**3*x**3 + 7*log(sqrt( -
 a*x + 1) - sqrt(2))*a**2*x**2 + 7*log(sqrt( - a*x + 1) - sqrt(2))*a*x - 7
*log(sqrt( - a*x + 1) - sqrt(2)) - 7*log(sqrt( - a*x + 1) + sqrt(2))*a**3*
x**3 + 7*log(sqrt( - a*x + 1) + sqrt(2))*a**2*x**2 + 7*log(sqrt( - a*x + 1
) + sqrt(2))*a*x - 7*log(sqrt( - a*x + 1) + sqrt(2)) + 46*log(sqrt( - a*x
+ 1))*a**3*x**3 - 46*log(sqrt( - a*x + 1))*a**2*x**2 - 46*log(sqrt( - a*x
+ 1))*a*x + 46*log(sqrt( - a*x + 1)) + 16*a**4*x**4 - 23*a**3*x**3 - 27*a*
*2*x**2 + 25*a*x + 5))/(16*a**6*c**3*(a**3*x**3 - a**2*x**2 - a*x + 1))
```

3.682 $\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} + \frac{3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{11\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-ax)}{16\left(c-a^2cx^2\right)^{5/2}} + \frac{5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1+ax)}{16\left(c-a^2cx^2\right)^{5/2}}$$

output

```
-1/8*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+3/4*(1-1/a^2/
x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*(1-1/a^2/x^2)^(5/2)*x^5/(
a*x+1)/(-a^2*c*x^2+c)^(5/2)+11/16*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/(-a^2
*c*x^2+c)^(5/2)+5/16*(1-1/a^2/x^2)^(5/2)*x^5*ln(a*x+1)/(-a^2*c*x^2+c)^(5/2
)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.39

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(-\frac{2(-6+3ax+5a^2x^2)}{(-1+ax)^2(1+ax)}+11\log(1-ax)+5\log(1+ax)\right)}{16\,(c-a^2cx^2)^{5/2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(5/2)*x^5*((-2*(-6 + 3*a*x + 5*a^2*x^2))/((-1 + a*x)^2*
(1 + a*x)) + 11*Log[1 - a*x] + 5*Log[1 + a*x]))/(16*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.50, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{x^4}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{x^4}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int\left(-\frac{5}{16a^4(ax+1)}+\frac{1}{8a^4(ax+1)^2}-\frac{11}{16a^4(ax-1)}-\frac{3}{4a^4(ax-1)^2}-\frac{1}{4a^4(ax-1)^3}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(-\frac{3}{4a^5(1-ax)}-\frac{1}{8a^5(ax+1)}+\frac{1}{8a^5(1-ax)^2}-\frac{11\log(1-ax)}{16a^5}-\frac{5\log(ax+1)}{16a^5}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(8*a^5*(1 - a*x)^2) - 3/(4*a^5*(1 -
a*x)) - 1/(8*a^5*(1 + a*x)) - (11*Log[1 - a*x])/(16*a^5) - (5*Log[1 + a*x]
)/(16*a^5)))/(c - a^2*c*x^2)^(5/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(5\ln\left(ax+1\right)x^{3}a^{3}+11a^{3}\ln\left(ax-1\right)x^{3}-5\ln\left(ax+1\right)x^{2}a^{2}-11a^{2}\ln\left(ax-1\right)x^{2}-10a^{2}x^{2}-5\ln\left(ax+1\right)xa-11a\ln\left(ax-1\right)\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,\left(ax-1\right)\left(a^{2}x^{2}-1\right)c^{3}a^{5}\left(ax+1\right)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(5*ln(a*x+1)*
x^3*a^3+11*a^3*ln(a*x-1)*x^3-5*ln(a*x+1)*x^2*a^2-11*a^2*ln(a*x-1)*x^2-10*a
^2*x^2-5*ln(a*x+1)*x*a-11*a*ln(a*x-1)*x-6*a*x+5*ln(a*x+1)+11*ln(a*x-1)+12)
/(a^2*x^2-1)/c^3/a^5/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.56

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(10\,a^2x^2+6\,ax-5\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)-11\left(a^3x^3-a^2x^2-ax+\right.\right.}{16\left(a^9c^3x^3-a^8c^3x^2-a^7c^3x+a^6c^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

output
```
1/16*(10*a^2*x^2 + 6*a*x - 5*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) -
11*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 12)*sqrt(-a^2*c)/(a^9*c^3*
x^3 - a^8*c^3*x^2 - a^7*c^3*x + a^6*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**4/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate(x^4/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{x^4}{(c-a^2\,c\,x^2)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x^4/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x^4/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.15

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx=\frac{\sqrt{c}\,i\left(5\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^3x^3-5\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^2x^2-5\log\left(\sqrt{-ax+\right.\right.}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*i*(5*log(sqrt( - a*x + 1) - sqrt(2))*a**3*x**3 - 5*log(sqrt( - a*
x + 1) - sqrt(2))*a**2*x**2 - 5*log(sqrt( - a*x + 1) - sqrt(2))*a*x + 5*lo
g(sqrt( - a*x + 1) - sqrt(2)) + 5*log(sqrt( - a*x + 1) + sqrt(2))*a**3*x**
3 - 5*log(sqrt( - a*x + 1) + sqrt(2))*a**2*x**2 - 5*log(sqrt( - a*x + 1) +
 sqrt(2))*a*x + 5*log(sqrt( - a*x + 1) + sqrt(2)) + 22*log(sqrt( - a*x + 1
))*a**3*x**3 - 22*log(sqrt( - a*x + 1))*a**2*x**2 - 22*log(sqrt( - a*x + 1
))*a*x + 22*log(sqrt( - a*x + 1)) + 5*a**3*x**3 - 15*a**2*x**2 - 11*a*x +
17))/(16*a**5*c**3*(a**3*x**3 - a**2*x**2 - a*x + 1))
```

3.683 $\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 176

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\text{arctanh}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

output
```
-1/8*a*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/2*a*(1-1/
a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a*(1-1/a^2/x^2)^(5/2)
*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-3/8*a*(1-1/a^2/x^2)^(5/2)*x^5*arctanh(a*
x)/(-a^2*c*x^2+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.41

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(\frac{2+ax-5a^2x^2}{(-1+ax)^2(1+ax)}-3\text{arctanh}(ax)\right)}{8\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(a*(1 - 1/(a^2*x^2))^(5/2)*x^5*((2 + a*x - 5*a^2*x^2)/((-1 + a*x)^2*(1 + a
*x)) - 3*ArcTanh[a*x]))/(8*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.53, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{x^3}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{x^3}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(-\frac{1}{2a^3(ax-1)^2}-\frac{1}{8a^3(ax+1)^2}-\frac{1}{4a^3(ax-1)^3}-\frac{3}{8a^3(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{3\text{arctanh}(ax)}{8a^4}-\frac{1}{2a^4(1-ax)}+\frac{1}{8a^4(ax+1)}+\frac{1}{8a^4(1-ax)^2}\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(8*a^4*(1 - a*x)^2) - 1/(2*a^4*(1 -
a*x)) + 1/(8*a^4*(1 + a*x)) + (3*ArcTanh[a*x])/(8*a^4)))/(c - a^2*c*x^2)^(
5/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.96

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(3\ln(ax+1)x^3a^3-3a^3\ln(ax-1)x^3-3\ln(ax+1)x^2a^2+3a^2\ln(ax-1)x^2+10a^2x^2-3\ln(ax+1)xa+3a\ln(ax-1)x-2ax}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a^4(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x+1)*x
^3*a^3-3*a^3*ln(a*x-1)*x^3-3*ln(a*x+1)*x^2*a^2+3*a^2*ln(a*x-1)*x^2+10*a^2*
x^2-3*ln(a*x+1)*x*a+3*a*ln(a*x-1)*x-2*a*x+3*ln(a*x+1)-3*ln(a*x-1)-4)/(a^2*
x^2-1)/c^3/a^4/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{3\left(a^4x^3-a^3x^2-a^2x+a\right)\sqrt{-c}\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\left(5\,a^2x^2-ax-2\right)\sqrt{-a^2c}}{16\left(a^8c^3x^3-a^7c^3x^2-a^6c^3x+a^5c^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

output
```
-1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(
-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(5*a^2*x^2 - a*x - 2)*sqrt(-a^2
*c))/(a^8*c^3*x^3 - a^7*c^3*x^2 - a^6*c^3*x + a^5*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^3}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate(x^3/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^3}{(c-a^2\,c\,x^2)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x^3/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x^3/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.42

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,i\left(-3\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^3x^3+3\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^2x^2+3\log\left(\sqrt{-ax+\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*i*( - 3*log(sqrt( - a*x + 1) - sqrt(2))*a**3*x**3 + 3*log(sqrt( -
 a*x + 1) - sqrt(2))*a**2*x**2 + 3*log(sqrt( - a*x + 1) - sqrt(2))*a*x - 3
*log(sqrt( - a*x + 1) - sqrt(2)) - 3*log(sqrt( - a*x + 1) + sqrt(2))*a**3*
x**3 + 3*log(sqrt( - a*x + 1) + sqrt(2))*a**2*x**2 + 3*log(sqrt( - a*x + 1
) + sqrt(2))*a*x - 3*log(sqrt( - a*x + 1) + sqrt(2)) + 6*log(sqrt( - a*x +
 1))*a**3*x**3 - 6*log(sqrt( - a*x + 1))*a**2*x**2 - 6*log(sqrt( - a*x + 1
))*a*x + 6*log(sqrt( - a*x + 1)) + 5*a**3*x**3 - 15*a**2*x**2 - 3*a*x + 9)
)/(16*a**4*c**3*(a**3*x**3 - a**2*x**2 - a*x + 1))
```

3.684 $\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 184

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(c-a^2cx^2)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)(c-a^2cx^2)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)(c-a^2cx^2)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8(c-a^2cx^2)^{5/2}}$$

output
```
-1/8*a^2*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/4*a^2*(
1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^2*(1-1/a^2/x^2)
^(5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^2*(1-1/a^2/x^2)^(5/2)*x^5*ar
ctanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.46

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(2-3ax-a^2x^2+(-1+ax)^2(1+ax)\operatorname{arctanh}(ax))}{8a^2c^2(-1+ax)^2(1+ax)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - a^2*x^2 + (-1 + a*x)^2*(1 + a*x)*Arc
Tanh[a*x]))/(8*a^2*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{\coth^{-1}(ax)}}{\left(c-a^2 c x^2\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5 \left(1-\frac{1}{a^2 x^2}\right)^{5/2} \int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2 x^2}\right)^{5/2} x^3}dx}{\left(c-a^2 c x^2\right)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5 x^5 \left(1-\frac{1}{a^2 x^2}\right)^{5/2} \int -\frac{x^2}{(1-ax)^3 (ax+1)^2}dx}{\left(c-a^2 c x^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^5 x^5 \left(1-\frac{1}{a^2 x^2}\right)^{5/2} \int \frac{x^2}{(1-ax)^3 (ax+1)^2}dx}{\left(c-a^2 c x^2\right)^{5/2}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^5 x^5 \left(1-\frac{1}{a^2 x^2}\right)^{5/2} \int \left(-\frac{1}{4 a^2 (ax-1)^2}+\frac{1}{8 a^2 (ax+1)^2}-\frac{1}{4 a^2 (ax-1)^3}+\frac{1}{8 a^2 \left(a^2 x^2-1\right)}\right)dx}{\left(c-a^2 c x^2\right)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5 x^5 \left(1-\frac{1}{a^2 x^2}\right)^{5/2} \left(-\frac{\text{arctanh}(ax)}{8 a^3}-\frac{1}{4 a^3 (1-ax)}-\frac{1}{8 a^3 (ax+1)}+\frac{1}{8 a^3 (1-ax)^2}\right)}{\left(c-a^2 c x^2\right)^{5/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(8*a^3*(1 - a*x)^2) - 1/(4*a^3*(1 -
a*x)) - 1/(8*a^3*(1 + a*x)) - ArcTanh[a*x]/(8*a^3)))/(c - a^2*c*x^2)^(5/2)
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.89

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(\ln(ax+1)x^3a^3-a^3\ln(ax-1)x^3-\ln(ax+1)x^2a^2+a^2\ln(ax-1)x^2-2a^2x^2-\ln(ax+1)xa+a\ln(ax-1)x-6ax+\ln(a}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a^3(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*x^
3*a^3-a^3*ln(a*x-1)*x^3-ln(a*x+1)*x^2*a^2+a^2*ln(a*x-1)*x^2-2*a^2*x^2-ln(a
*x+1)*x*a+a*ln(a*x-1)*x-6*a*x+ln(a*x+1)-ln(a*x-1)+4)/(a^2*x^2-1)/c^3/a^3/(
a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.73

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{(a^4x^3-a^3x^2-a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\,(a^2x^2+3\,ax-2)\sqrt{-a^2c}}{16\,(a^7c^3x^3-a^6c^3x^2-a^5c^3x+a^4c^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

output
```
-1/16*((a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a
^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(a^2*x^2 + 3*a*x - 2)*sqrt(-a^2*c
))/(a^7*c^3*x^3 - a^6*c^3*x^2 - a^5*c^3*x + a^4*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^2}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate(x^2/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^2}{(c-a^2\,c\,x^2)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x^2/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x^2/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.32

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,i\big(\log\big(\sqrt{-ax+1}-\sqrt{2}\big)\,a^3x^3-\log\big(\sqrt{-ax+1}-\sqrt{2}\big)\,a^2x^2-\log\big(\sqrt{-ax+1}-$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*i*(log(sqrt( - a*x + 1) - sqrt(2))*a**3*x**3 - log(sqrt( - a*x +
1) - sqrt(2))*a**2*x**2 - log(sqrt( - a*x + 1) - sqrt(2))*a*x + log(sqrt(
- a*x + 1) - sqrt(2)) + log(sqrt( - a*x + 1) + sqrt(2))*a**3*x**3 - log(sq
rt( - a*x + 1) + sqrt(2))*a**2*x**2 - log(sqrt( - a*x + 1) + sqrt(2))*a*x
+ log(sqrt( - a*x + 1) + sqrt(2)) - 2*log(sqrt( - a*x + 1))*a**3*x**3 + 2*
log(sqrt( - a*x + 1))*a**2*x**2 + 2*log(sqrt( - a*x + 1))*a*x - 2*log(sqrt
( - a*x + 1)) + a**3*x**3 - 3*a**2*x**2 - 7*a*x + 5))/(16*a**3*c**3*(a**3*
x**3 - a**2*x**2 - a*x + 1))
```

3.685 $\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\,(c-a^2cx^2)^{5/2}} - \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\,(c-a^2cx^2)^{5/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8\,(c-a^2cx^2)^{5/2}}$$

output
```
-1/8*a^3*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/8*a^3*(
1-1/a^2/x^2)^(5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^3*(1-1/a^2/x^2)^
(5/2)*x^5*arctanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(-\frac{1}{(-1+ax)^2}-\frac{1}{1+ax}+\operatorname{arctanh}(ax)\right)}{8\,(c-a^2cx^2)^{5/2}}$$

input
```
Integrate[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5*(-(-1 + a*x)^(-2) - (1 + a*x)^(-1) + ArcT
anh[a*x]))/(8*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6746, 6747, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^4}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{x}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{x}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{86}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(\frac{1}{8(a^2x^2-1)a}-\frac{1}{8(ax+1)^2a}-\frac{1}{4(ax-1)^3a}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(-\frac{\text{arctanh}(ax)}{8a^2}+\frac{1}{8a^2(ax+1)}+\frac{1}{8a^2(1-ax)^2}\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Int[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(8*a^2*(1 - a*x)^2) + 1/(8*a^2*(1 +
a*x)) - ArcTanh[a*x]/(8*a^2)))/(c - a^2*c*x^2)^(5/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.20

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(\ln(ax+1)x^3a^3-a^3\ln(ax-1)x^3-\ln(ax+1)x^2a^2+a^2\ln(ax-1)x^2-2a^2x^2-\ln(ax+1)xa+a\ln(ax-1)x+2ax+\ln(a}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a^2(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*x^
3*a^3-a^3*ln(a*x-1)*x^3-ln(a*x+1)*x^2*a^2+a^2*ln(a*x-1)*x^2-2*a^2*x^2-ln(a
*x+1)*x*a+a*ln(a*x-1)*x+2*a*x+ln(a*x+1)-ln(a*x-1)-4)/(a^2*x^2-1)/c^3/a^2/(
a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{(a^4x^3-a^3x^2-a^2x+a)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\left(a^2x^2-ax+2\right)\sqrt{-a^2c}}{16\left(a^6c^3x^3-a^5c^3x^2-a^4c^3x+a^3c^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="f
ricas")
```

output
```
-1/16*((a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a
^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(a^2*x^2 - a*x + 2)*sqrt(-a^2*c))
/(a^6*c^3*x^3 - a^5*c^3*x^2 - a^4*c^3*x + a^3*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="m
axima")
```

output
```
integrate(x/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="g
iac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x}{\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(x/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(x/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.77

$$\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,i\left(\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^3x^3-\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^2x^2-\log\left(\sqrt{-ax+1}-\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*i*(log(sqrt( - a*x + 1) - sqrt(2))*a**3*x**3 - log(sqrt( - a*x +
1) - sqrt(2))*a**2*x**2 - log(sqrt( - a*x + 1) - sqrt(2))*a*x + log(sqrt(
- a*x + 1) - sqrt(2)) + log(sqrt( - a*x + 1) + sqrt(2))*a**3*x**3 - log(sq
rt( - a*x + 1) + sqrt(2))*a**2*x**2 - log(sqrt( - a*x + 1) + sqrt(2))*a*x
+ log(sqrt( - a*x + 1) + sqrt(2)) - 2*log(sqrt( - a*x + 1))*a**3*x**3 + 2*
log(sqrt( - a*x + 1))*a**2*x**2 + 2*log(sqrt( - a*x + 1))*a*x - 2*log(sqrt
( - a*x + 1)) + a**3*x**3 - 3*a**2*x**2 + a*x - 3))/(16*a**2*c**3*(a**3*x*
*3 - a**2*x**2 - a*x + 1))
```

3.686 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 184

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\,(c-a^2cx^2)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\,(c-a^2cx^2)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\,(c-a^2cx^2)^{5/2}}-\frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\operatorname{arctanh}(ax)}{8\,(c-a^2cx^2)^{5/2}}$$

output
```
-1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/4*a^4*(
1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)
^(5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-3/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*ar
ctanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(2+3ax-3a^2x^2+3(-1+ax)^2(1+ax)\operatorname{arctanh}(ax))}{8c^2(-1+ax)^2(1+ax)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-1/8*(Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^2*(1 +
a*x)*ArcTanh[a*x]))/(c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6746, 6747, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{1}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{1}{(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{54}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(\frac{1}{4(ax-1)^2}+\frac{1}{8(ax+1)^2}-\frac{1}{4(ax-1)^3}-\frac{3}{8(a^2x^2-1)}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{3\text{arctanh}(ax)}{8a}+\frac{1}{4a(1-ax)}-\frac{1}{8a(ax+1)}+\frac{1}{8a(1-ax)^2}\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(8*a*(1 - a*x)^2) + 1/(4*a*(1 - a*x)
) - 1/(8*a*(1 + a*x)) + (3*ArcTanh[a*x])/(8*a)))/(c - a^2*c*x^2)^(5/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(3 \ln \left(a x +1\right) x^{3} a^{3}-3 a^{3} \ln \left(a x -1\right) x^{3}-3 \ln \left(a x +1\right) x^{2} a^{2}+3 a^{2} \ln \left(a x -1\right) x^{2}-6 a^{2} x^{2}-3 \ln \left(a x +1\right) x a +3 a \ln \left(a x -1\right) x +6 a x}{16 \sqrt{\frac{a x -1}{a x +1}}\, \left(a x -1\right) \left(a^{2} x^{2}-1\right) c^{3} a \left(a x +1\right)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x+1)*x
^3*a^3-3*a^3*ln(a*x-1)*x^3-3*ln(a*x+1)*x^2*a^2+3*a^2*ln(a*x-1)*x^2-6*a^2*x
^2-3*ln(a*x+1)*x*a+3*a*ln(a*x-1)*x+6*a*x+3*ln(a*x+1)-3*ln(a*x-1)+4)/(a^2*x
^2-1)/c^3/a/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\, dx = -\frac{3\left(a^{4} x^{3} - a^{3} x^{2} - a^{2} x + a\right)\sqrt{-c}\log\left(\frac{a^{2} c x^{2} + 2\sqrt{-a^{2} c}\sqrt{-c}x + c}{a^{2} x^{2} - 1}\right) + 2\left(3\,a^{2} x^{2} - 3\,a x - 2\right)\sqrt{-a^{2} c}}{16\left(a^{5} c^{3} x^{3} - a^{4} c^{3} x^{2} - a^{3} c^{3} x + a^{2} c^{3}\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

output
```
-1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(
-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 - 3*a*x - 2)*sqrt(-a
^2*c))/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.36

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,i\left(-3\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^3x^3+3\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^2x^2+3\log\left(\sqrt{-ax\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*i*( - 3*log(sqrt( - a*x + 1) - sqrt(2))*a**3*x**3 + 3*log(sqrt( -
 a*x + 1) - sqrt(2))*a**2*x**2 + 3*log(sqrt( - a*x + 1) - sqrt(2))*a*x - 3
*log(sqrt( - a*x + 1) - sqrt(2)) - 3*log(sqrt( - a*x + 1) + sqrt(2))*a**3*
x**3 + 3*log(sqrt( - a*x + 1) + sqrt(2))*a**2*x**2 + 3*log(sqrt( - a*x + 1
) + sqrt(2))*a*x - 3*log(sqrt( - a*x + 1) + sqrt(2)) + 6*log(sqrt( - a*x +
 1))*a**3*x**3 - 6*log(sqrt( - a*x + 1))*a**2*x**2 - 6*log(sqrt( - a*x + 1
))*a*x + 6*log(sqrt( - a*x + 1)) - 3*a**3*x**3 + 9*a**2*x**2 - 3*a*x - 7))
/(16*a*c**3*(a**3*x**3 - a**2*x**2 - a*x + 1))
```

3.687 $\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 271

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(x)}{\left(c-a^2cx^2\right)^{5/2}} + \frac{11a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-ax)}{16\left(c-a^2cx^2\right)^{5/2}} + \frac{5a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1+ax)}{16\left(c-a^2cx^2\right)^{5/2}}$$

output
```
-1/8*a^5*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/2*a^5*(
1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a^5*(1-1/a^2/x^2)
^(5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-a^5*(1-1/a^2/x^2)^(5/2)*x^5*ln(x)/
(-a^2*c*x^2+c)^(5/2)+11/16*a^5*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/(-a^2*c*
x^2+c)^(5/2)+5/16*a^5*(1-1/a^2/x^2)^(5/2)*x^5*ln(a*x+1)/(-a^2*c*x^2+c)^(5/
2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.32

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx=\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(-\frac{2}{(-1+ax)^2}+\frac{8}{-1+ax}-\frac{2}{1+ax}-16\log(x)+11\log(1-ax)+5\log(1+ax)\right)}{16\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(-2/(-1 + a*x)^2 + 8/(-1 + a*x) - 2/(1 +
a*x) - 16*Log[x] + 11*Log[1 - a*x] + 5*Log[1 + a*x]))/(16*(c - a^2*c*x^2)^
(5/2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.35, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int -\frac{1}{x(1-ax)^3(ax+1)^2}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{1}{x(1-ax)^3(ax+1)^2}dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(-\frac{11a}{16(ax-1)}-\frac{5a}{16(ax+1)}+\frac{a}{2(ax-1)^2}-\frac{a}{8(ax+1)^2}-\frac{a}{4(ax-1)^3}+\frac{1}{x}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{2(1-ax)}+\frac{1}{8(ax+1)}+\frac{1}{8(1-ax)^2}-\frac{11}{16}\log(1-ax)-\frac{5}{16}\log(ax+1)+\log(x)\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Int[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(1/(8*(1 - a*x)^2) + 1/(2*(1 - a*x)) +
1/(8*(1 + a*x)) + Log[x] - (11*Log[1 - a*x])/16 - (5*Log[1 + a*x])/16))/(c
 - a^2*c*x^2)^(5/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.72

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\,(5\ln(ax+1)x^3a^3+11a^3\ln(ax-1)x^3-16\ln(x)x^3a^3-5\ln(ax+1)x^2a^2-11a^2\ln(ax-1)x^2+16a^2\ln(x)x^2+6a^2x^2-5}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3(ax+1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(5*ln(a*x+1)*
x^3*a^3+11*a^3*ln(a*x-1)*x^3-16*ln(x)*x^3*a^3-5*ln(a*x+1)*x^2*a^2-11*a^2*l
n(a*x-1)*x^2+16*a^2*ln(x)*x^2+6*a^2*x^2-5*ln(a*x+1)*x*a-11*a*ln(a*x-1)*x+1
6*a*ln(x)*x+2*a*x+5*ln(a*x+1)+11*ln(a*x-1)-16*ln(x)-12)/(a^2*x^2-1)/c^3/(a
*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.54

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx =$$

$$-\frac{\left(6\,a^2x^2+2\,ax+5\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)+11\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax-1\right)-16\,(a}{16\left(a^4c^3x^3-a^3c^3x^2-a^2c^3x+ac^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="f
ricas")
```

output
```
-1/16*(6*a^2*x^2 + 2*a*x + 5*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) +
11*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 16*(a^3*x^3 - a^2*x^2 - a*
x + 1)*log(x) - 12)*sqrt(-a^2*c)/(a^4*c^3*x^3 - a^3*c^3*x^2 - a^2*c^3*x +
a*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="m
axima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="g
iac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \int \frac{1}{x\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/(x*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/(x*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.37

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{c}\,i\left(5\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^3x^3-5\log\left(\sqrt{-ax+1}-\sqrt{2}\right)a^2x^2-5\log\left(\sqrt{-ax}\right.\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x)
```

output

```
(sqrt(c)*i*(5*log(sqrt( - a*x + 1) - sqrt(2))*a**3*x**3 - 5*log(sqrt( - a*
x + 1) - sqrt(2))*a**2*x**2 - 5*log(sqrt( - a*x + 1) - sqrt(2))*a*x + 5*lo
g(sqrt( - a*x + 1) - sqrt(2)) - 16*log(sqrt( - a*x + 1) - 1)*a**3*x**3 + 1
6*log(sqrt( - a*x + 1) - 1)*a**2*x**2 + 16*log(sqrt( - a*x + 1) - 1)*a*x -
 16*log(sqrt( - a*x + 1) - 1) + 5*log(sqrt( - a*x + 1) + sqrt(2))*a**3*x**
3 - 5*log(sqrt( - a*x + 1) + sqrt(2))*a**2*x**2 - 5*log(sqrt( - a*x + 1) +
 sqrt(2))*a*x + 5*log(sqrt( - a*x + 1) + sqrt(2)) - 16*log(sqrt( - a*x + 1
) + 1)*a**3*x**3 + 16*log(sqrt( - a*x + 1) + 1)*a**2*x**2 + 16*log(sqrt( -
 a*x + 1) + 1)*a*x - 16*log(sqrt( - a*x + 1) + 1) + 22*log(sqrt( - a*x + 1
))*a**3*x**3 - 22*log(sqrt( - a*x + 1))*a**2*x**2 - 22*log(sqrt( - a*x + 1
))*a*x + 22*log(sqrt( - a*x + 1)) - 3*a**3*x**3 + 9*a**2*x**2 + 5*a*x - 15
))/(16*c**3*(a**3*x**3 - a**2*x**2 - a*x + 1))
```

3.688 $\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 307

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{5/2}}\,dx = \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^4}{(c-a^2cx^2)^{5/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(c-a^2cx^2)^{5/2}} - \frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)(c-a^2cx^2)^{5/2}} + \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)(c-a^2cx^2)^{5/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(x)}{(c-a^2cx^2)^{5/2}} + \frac{23a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-ax)}{16(c-a^2cx^2)^{5/2}} - \frac{7a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1+ax)}{16(c-a^2cx^2)^{5/2}}$$

output

```
a^5*(1-1/a^2/x^2)^(5/2)*x^4/(-a^2*c*x^2+c)^(5/2)-1/8*a^6*(1-1/a^2/x^2)^(5/
2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-3/4*a^6*(1-1/a^2/x^2)^(5/2)*x^5/(-a
*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^6*(1-1/a^2/x^2)^(5/2)*x^5/(a*x+1)/(-a^2*c
*x^2+c)^(5/2)-a^6*(1-1/a^2/x^2)^(5/2)*x^5*ln(x)/(-a^2*c*x^2+c)^(5/2)+23/16
*a^6*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-7/16*a^6*(1-1
/a^2/x^2)^(5/2)*x^5*ln(a*x+1)/(-a^2*c*x^2+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.32

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c-a^2cx^2)^{5/2}}\,dx = \frac{a^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2} x^5 \left(\frac{16}{x}-\frac{2a}{(-1+ax)^2}+\frac{12a}{-1+ax}+\frac{2a}{1+ax}-16a\log(x)+23a\log(1-ax)\right.}{16 (c-a^2cx^2)^{5/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(16/x - (2*a)/(-1 + a*x)^2 + (12*a)/(-1 +
 a*x) + (2*a)/(1 + a*x) - 16*a*Log[x] + 23*a*Log[1 - a*x] - 7*a*Log[1 + a*
x]))/(16*(c - a^2*c*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.35, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c-a^2cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 6746$$

$$\frac{x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2} \int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2} x^7}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \; 6747$$

$$\frac{a^5 x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2} \int -\frac{1}{x^2(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{1}{x^2(1-ax)^3(ax+1)^2}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(-\frac{23a^2}{16(ax-1)}+\frac{7a^2}{16(ax+1)}+\frac{3a^2}{4(ax-1)^2}+\frac{a^2}{8(ax+1)^2}-\frac{a^2}{4(ax-1)^3}+\frac{a}{x}+\frac{1}{x^2}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{3a}{4(1-ax)}-\frac{a}{8(ax+1)}+\frac{a}{8(1-ax)^2}+a\log(x)-\frac{23}{16}a\log(1-ax)+\frac{7}{16}a\log(ax+1)-\frac{1}{x}\right)}{(c-a^2cx^2)^{5/2}}$$

input
```
Int[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
-((a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(-x^(-1) + a/(8*(1 - a*x)^2) + (3*a)/(4
*(1 - a*x)) - a/(8*(1 + a*x)) + a*Log[x] - (23*a*Log[1 - a*x])/16 + (7*a*L
og[1 + a*x])/16))/(c - a^2*c*x^2)^(5/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.73

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,(7\ln(ax+1)x^4a^4-23\ln(ax-1)x^4a^4+16\ln(x)x^4a^4-7\ln(ax+1)x^3a^3+23a^3\ln(ax-1)x^3-16\ln(x)x^3a^3-30a^3x^3-7\,\mathrm{l}}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(7*ln(a*x+1)*x
^4*a^4-23*ln(a*x-1)*x^4*a^4+16*ln(x)*x^4*a^4-7*ln(a*x+1)*x^3*a^3+23*a^3*ln
(a*x-1)*x^3-16*ln(x)*x^3*a^3-30*a^3*x^3-7*ln(a*x+1)*x^2*a^2+23*a^2*ln(a*x-
1)*x^2-16*a^2*ln(x)*x^2+22*a^2*x^2+7*ln(a*x+1)*x*a-23*a*ln(a*x-1)*x+16*a*l
n(x)*x+28*a*x-16)/(a^2*x^2-1)/c^3/x/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.57

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{\left(30\,a^3x^3-22\,a^2x^2-28\,ax-7\left(a^4x^4-a^3x^3-a^2x^2+ax\right)\log\left(ax+1\right)+23\left(a^4x^4-a^3x^3-a^2x^2+ax\right)\mathrm{l}\right.}{16\left(a^4c^3x^4-a^3c^3x^3-a^2c^3x^2+ac^3x\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

output
```
-1/16*(30*a^3*x^3 - 22*a^2*x^2 - 28*a*x - 7*(a^4*x^4 - a^3*x^3 - a^2*x^2 +
 a*x)*log(a*x + 1) + 23*(a^4*x^4 - a^3*x^3 - a^2*x^2 + a*x)*log(a*x - 1) -
 16*(a^4*x^4 - a^3*x^3 - a^2*x^2 + a*x)*log(x) + 16)*sqrt(-a^2*c)/(a^4*c^3
*x^4 - a^3*c^3*x^3 - a^2*c^3*x^2 + a*c^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c-a^2cx^2)^{5/2}}\,dx = \int \frac{1}{x^2\,(c-a^2\,c\,x^2)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/(x^2*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/(x^2*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.34

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2 (c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,i\big(16-10a^4x^4-7\log\big(\sqrt{-ax+1}-\sqrt{2}\big)\,a^4x^4-7\log\big(\sqrt{-ax+1}+\sqrt{2}\big)\,a^4x^4}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*i*( - 7*log(sqrt( - a*x + 1) - sqrt(2))*a**4*x**4 + 7*log(sqrt( -
 a*x + 1) - sqrt(2))*a**3*x**3 + 7*log(sqrt( - a*x + 1) - sqrt(2))*a**2*x*
*2 - 7*log(sqrt( - a*x + 1) - sqrt(2))*a*x - 16*log(sqrt( - a*x + 1) - 1)*
a**4*x**4 + 16*log(sqrt( - a*x + 1) - 1)*a**3*x**3 + 16*log(sqrt( - a*x +
1) - 1)*a**2*x**2 - 16*log(sqrt( - a*x + 1) - 1)*a*x - 7*log(sqrt( - a*x +
 1) + sqrt(2))*a**4*x**4 + 7*log(sqrt( - a*x + 1) + sqrt(2))*a**3*x**3 + 7
*log(sqrt( - a*x + 1) + sqrt(2))*a**2*x**2 - 7*log(sqrt( - a*x + 1) + sqrt
(2))*a*x - 16*log(sqrt( - a*x + 1) + 1)*a**4*x**4 + 16*log(sqrt( - a*x + 1
) + 1)*a**3*x**3 + 16*log(sqrt( - a*x + 1) + 1)*a**2*x**2 - 16*log(sqrt( -
 a*x + 1) + 1)*a*x + 46*log(sqrt( - a*x + 1))*a**4*x**4 - 46*log(sqrt( - a
*x + 1))*a**3*x**3 - 46*log(sqrt( - a*x + 1))*a**2*x**2 + 46*log(sqrt( - a
*x + 1))*a*x - 10*a**4*x**4 + 40*a**3*x**3 - 12*a**2*x**2 - 38*a*x + 16))/
(16*c**3*x*(a**3*x**3 - a**2*x**2 - a*x + 1))
```

3.689 $\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 76

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=-\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/3*x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^3*(-a^2*c*x^2+c)
^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{x^2(-4+3ax)\sqrt{c-a^2cx^2}}{12a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

output
```
(x^2*(-4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-a^2cx^2}e^{-\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -x^2(1-ax)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int x^2(1-ax)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(x^2-ax^3\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\frac{x^3}{3}-\frac{ax^4}{4}\right)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(x^2*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(x^3/3 - (a*x^4)/4))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{x^{3}\left(3 a x -4\right) \sqrt{-a^{2} c \,x^{2}+c}\, \sqrt{\frac{a x -1}{a x +1}}}{12 a x -12}$	47
orering	$\frac{x^{3}\left(3 a x -4\right) \sqrt{-a^{2} c \,x^{2}+c}\, \sqrt{\frac{a x -1}{a x +1}}}{12 a x -12}$	47
default	$\frac{\left(3 a x -4\right) x^{3} \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \sqrt{\frac{a x -1}{a x +1}}}{12 a x -12}$	48

input
```
int(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
1/12*x^3*(3*a*x-4)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.33

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{(3\,ax^4-4\,x^3)\sqrt{-a^2c}}{12\,a}$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="f
ricas")
```

output
```
1/12*(3*a*x^4 - 4*x^3)*sqrt(-a^2*c)/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int x^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

input
```
integrate(x**2*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x**2*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+cx^2}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{1}{12}\left(3\,ax^4\text{sgn}(ax+1)-4\,x^3\text{sgn}(ax+1)-\frac{7\,\text{sgn}(ax+1)}{a^3}\right)\sqrt{-c}$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="g
iac")
```

output
```
1/12*(3*a*x^4*sgn(a*x + 1) - 4*x^3*sgn(a*x + 1) - 7*sgn(a*x + 1)/a^3)*sqrt
(-c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int x^2\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.34

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c}\,i(-3a^4x^4+4a^3x^3-1)}{12a^3}$$

input
```
int(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*i*( - 3*a**4*x**4 + 4*a**3*x**3 - 1))/(12*a**3)
```

3.690 $\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=-\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/2*x*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(
1/2)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.58

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{x(-3+2ax)\sqrt{c-a^2cx^2}}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(x*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

output
```
(x*(-3 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-a^2cx^2}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -x(1-ax)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int x(1-ax)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(x-ax^2\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\frac{x^2}{2}-\frac{ax^3}{3}\right)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(x*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(x^2/2 - (a*x^3)/3))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x^{2}\left(2 a x -3\right) \sqrt{-a^{2} c \,x^{2}+c}\, \sqrt{\frac{a x -1}{a x +1}}}{6 a x -6}$	47
orering	$\frac{x^{2}\left(2 a x -3\right) \sqrt{-a^{2} c \,x^{2}+c}\, \sqrt{\frac{a x -1}{a x +1}}}{6 a x -6}$	47
default	$\frac{\left(2 a x -3\right) x^{2} \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \sqrt{\frac{a x -1}{a x +1}}}{6 a x -6}$	48

input
```
int(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/6*x^2*(2*a*x-3)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.34

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{(2\,ax^3-3\,x^2)\sqrt{-a^2c}}{6\,a}$$

input
```
integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fri
cas")
```

output
```
1/6*(2*a*x^3 - 3*x^2)*sqrt(-a^2*c)/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int x\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

input
```
integrate(x*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(x*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.57

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{1}{6}\left(2\,ax^3\text{sgn}(ax+1)-3\,x^2\text{sgn}(ax+1)+\frac{5\,\text{sgn}(ax+1)}{a^2}\right)\sqrt{-c}$$

input
```
integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gia
c")
```

output
```
1/6*(2*a*x^3*sgn(a*x + 1) - 3*x^2*sgn(a*x + 1) + 5*sgn(a*x + 1)/a^2)*sqrt(
-c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int x\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.35

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c}\,i(-2a^3x^3+3a^2x^2-1)}{6a^2}$$

input
```
int(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*i*( - 2*a**3*x**3 + 3*a**2*x**2 - 1))/(6*a**2)
```

3.691 $\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 47

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(1-ax)^2\sqrt{c-a^2cx^2}}{2a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

output
```
1/2*(-a*x+1)^2*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{(-2+ax)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

output
```
((-2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6746, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int (ax-1)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{17}$$

$$\frac{(1-ax)^2\sqrt{c-a^2cx^2}}{2a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

output
```
((1 - a*x)^2*Sqrt[c - a^2*c*x^2])/(2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x(ax-2)\sqrt{-a^2c\,x^2+c}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	44
orering	$\frac{x(ax-2)\sqrt{-a^2c\,x^2+c}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	44
default	$\frac{(ax-2)x\sqrt{-c(a^2x^2-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	45

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(a*x-2)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.47

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{-a^2c}(ax^2-2\,x)}{2\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
```

output
```
1/2*sqrt(-a^2*c)*(a*x^2 - 2*x)/a
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{1}{2}\left(ax^2\mathrm{sgn}(ax+1)-2\,x\mathrm{sgn}(ax+1)-\frac{3\,\mathrm{sgn}(ax+1)}{a}\right)\sqrt{-c}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
```

output
```
1/2*(a*x^2*sgn(a*x + 1) - 2*x*sgn(a*x + 1) - 3*sgn(a*x + 1)/a)*sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int \sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.47

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c}\,i(-a^2x^2+2ax-1)}{2a}$$

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*i*( - a**2*x**2 + 2*a*x - 1))/(2*a)
```

3.692 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 70

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)-(-a^2*c*x^2+c)^(1/2)*ln(x)/a/(1-1
/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}\left(x-\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(x - Log[x]/a))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{1-ax}{x}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{1-ax}{x}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x}-a\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}(\log(x)-ax)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x),x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-(a*x) + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\left(-a x +\ln \left(x \right)\right) \sqrt{\frac{a x -1}{a x +1}}}{a x -1}$	46

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE
)
```

output
```
-(-c*(a^2*x^2-1))^(1/2)*(-a*x+ln(x))*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.29

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{-a^2c}(ax-\log(x))}{a}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fri
cas")
```

output
```
sqrt(-a^2*c)*(a*x - log(x))/a
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)(ax+1)}}{x}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/x, x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.39

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = (ax\mathrm{sgn}(ax+1) - \log(|x|)\,\mathrm{sgn}(ax+1))\sqrt{-c}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="gia
c")
```

output
```
(a*x*sgn(a*x + 1) - log(abs(x))*sgn(a*x + 1))*sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.20

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \sqrt{c}\,i(\log(ax) - ax + 1)$$

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x)
```

output
```
sqrt(c)*i*(log(a*x) - a*x + 1)
```

3.693 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 72

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^2+(-a^2*c*x^2+c)^(1/2)*ln(x)/
(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{1}{ax}+\log(x)\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x^2),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(1/(a*x) + Log[x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6746, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{1-ax}{x^2}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{1-ax}{x^2}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}-\frac{a}{x}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(-a\log(x)-\frac{1}{x}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x^2),x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(-x^(-1) - a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{(a\ln(x)x+1)\sqrt{-c(a^2x^2-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{x(ax-1)}$	48

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBO
SE)
```

output
```
(a*ln(x)*x+1)*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.31

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{-a^2c}(ax\log(x)+1)}{ax}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="f
ricas")
```

output
```
sqrt(-a^2*c)*(a*x*log(x) + 1)/(a*x)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)(ax+1)}}{x^2}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/x**2, x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.39

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \left(a\log\left(|x|\right)\operatorname{sgn}(ax+1) + \frac{\operatorname{sgn}(ax+1)}{x}\right)\sqrt{-c}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="g
iac")
```

output
```
(a*log(abs(x))*sgn(a*x + 1) + sgn(a*x + 1)/x)*sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x^2}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.28

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c}\,i(-\log(ax)\,ax+ax-1)}{x}$$

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

output
```
(sqrt(c)*i*( - log(a*x)*a*x + a*x - 1))/x
```

3.694 $\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 140

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{3x\sqrt{c-a^2cx^2}}{4a^3}+\frac{x(c-a^2cx^2)^{3/2}}{2a^3c}+\frac{(c-a^2cx^2)^{5/2}}{5a^4c^2}+\frac{(c-a^2cx^2)^{5/2}}{a^4c^2(1+ax)^2}+\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

output
```
3/4*x*(-a^2*c*x^2+c)^(1/2)/a^3+1/2*x*(-a^2*c*x^2+c)^(3/2)/a^3/c+1/5*(-a^2*
c*x^2+c)^(5/2)/a^4/c^2+(-a^2*c*x^2+c)^(5/2)/a^4/c^2/(a*x+1)^2+3/4*c^(1/2)*
arctan(a*c^(1/2)*x/(-a^2*c*x^2+c)^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.69

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}(24-15ax+12a^2x^2-10a^3x^3+4a^4x^4)-15\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{20a^4}$$

input
```
Integrate[(x^3*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(24 - 15*a*x + 12*a^2*x^2 - 10*a^3*x^3 + 4*a^4*x^4) -
 15*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(2
0*a^4)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.38, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6717, 6702, 541, 25, 27, 533, 27, 533, 27, 533, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x^3\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6702}$$

$$-c\int \frac{x^3(1-ax)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{541}$$

$$-c\left(-\frac{\int -\frac{a^2cx^3(9-10ax)}{\sqrt{c-a^2cx^2}}dx}{5a^2c}-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{a^2cx^3(9-10ax)}{\sqrt{c-a^2cx^2}}dx}{5a^2c}-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{5}\int \frac{x^3(9-10ax)}{\sqrt{c-a^2cx^2}}dx-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{533}$$

$$-c\left(\frac{1}{5}\left(\frac{\int -\frac{6acx^2(5-6ax)}{\sqrt{c-a^2cx^2}}dx}{4a^2c}+\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\int \frac{x^2(5-6ax)}{\sqrt{c-a^2cx^2}}dx}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{533}$$

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\left(\frac{\int -\frac{3acx(4-5ax)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}+\frac{2x^2\sqrt{c-a^2cx^2}}{ac}\right)}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\left(\frac{2x^2\sqrt{c-a^2cx^2}}{ac}-\frac{\int \frac{x(4-5ax)}{\sqrt{c-a^2cx^2}}dx}{a}\right)}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{533}$$

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\left(\frac{2x^2\sqrt{c-a^2cx^2}}{ac}-\frac{\frac{\int -\frac{ac(5-8ax)}{\sqrt{c-a^2cx^2}}dx}{2a^2c}+\frac{5x\sqrt{c-a^2cx^2}}{2ac}}{a}\right)}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\left(\frac{2x^2\sqrt{c-a^2cx^2}}{ac}-\frac{\frac{5x\sqrt{c-a^2cx^2}}{2ac}-\frac{\int \frac{ac(5-8ax)}{\sqrt{c-a^2cx^2}}dx}{2a^2c}}{a}\right)}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\left(\frac{2x^2\sqrt{c-a^2cx^2}}{ac}-\frac{\frac{5x\sqrt{c-a^2cx^2}}{2ac}-\frac{\int\frac{5-8ax}{\sqrt{c-a^2cx^2}}dx}{2a}}{a}\right)}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$\Big\downarrow$ 455

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\left(\frac{2x^2\sqrt{c-a^2cx^2}}{ac}-\frac{\frac{5x\sqrt{c-a^2cx^2}}{2ac}-\frac{5\int\frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{8\sqrt{c-a^2cx^2}}{ac}}{2a}}{a}\right)}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$\Big\downarrow$ 224

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\left(\frac{2x^2\sqrt{c-a^2cx^2}}{ac}-\frac{\frac{5x\sqrt{c-a^2cx^2}}{2ac}-\frac{5\int\frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{8\sqrt{c-a^2cx^2}}{ac}}{2a}}{a}\right)}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

$\Big\downarrow$ 216

$$-c\left(\frac{1}{5}\left(\frac{5x^3\sqrt{c-a^2cx^2}}{2ac}-\frac{3\left(\frac{2x^2\sqrt{c-a^2cx^2}}{ac}-\frac{\frac{5x\sqrt{c-a^2cx^2}}{2ac}-\frac{\frac{5\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}+\frac{8\sqrt{c-a^2cx^2}}{ac}}{2a}}{a}\right)}{2a}\right)-\frac{x^4\sqrt{c-a^2cx^2}}{5c}\right)$$

input
```
Int[(x^3*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(-1/5*(x^4*Sqrt[c - a^2*c*x^2])/c + ((5*x^3*Sqrt[c - a^2*c*x^2])/(2*a*
c) - (3*((2*x^2*Sqrt[c - a^2*c*x^2])/(a*c) - ((5*x*Sqrt[c - a^2*c*x^2])/(2
*a*c) - ((8*Sqrt[c - a^2*c*x^2])/(a*c) + (5*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
a^2*c*x^2]])/(a*Sqrt[c]))/(2*a))/a))/(2*a))/5))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\left(4 a^{4} x^{4}-10 a^{3} x^{3}+12 a^{2} x^{2}-15 a x +24\right) \left(a^{2} x^{2}-1\right) c}{20 a^{4} \sqrt{-c \left(a^{2} x^{2}-1\right)}}+\frac{3 \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right) c}{4 a^{3} \sqrt{a^{2} c}}$
default	$-\frac{x^{2} \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{5 a^{2} c}-\frac{4 \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{5 c \,a^{4}}-\frac{2 \left(\frac{x \sqrt{-a^{2} c \,x^{2}+c}}{2}+\frac{c \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right)}{2 \sqrt{a^{2} c}}\right)}{a^{3}}+\frac{2 \sqrt{-\left(x +\frac{1}{a}\right)^{2} a^{2} c +2 \left(x +\frac{1}{a}\right) a c +\frac{2 a c a}{}}}{a}$

input
```
int(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(4*a^4*x^4-10*a^3*x^3+12*a^2*x^2-15*a*x+24)*(a^2*x^2-1)/a^4/(-c*(a^2
*x^2-1))^(1/2)*c+3/4/a^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+
c)^(1/2))*c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.31

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$$
$$= \left[\frac{2\,(4\,a^4x^4-10\,a^3x^3+12\,a^2x^2-15\,ax+24)\sqrt{-a^2cx^2+c}+15\,\sqrt{-c}\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}a\sqrt{-}\right.}{40\,a^4}\right.$$

input
```
integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/40*(2*(4*a^4*x^4 - 10*a^3*x^3 + 12*a^2*x^2 - 15*a*x + 24)*sqrt(-a^2*c*x
^2 + c) + 15*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*
x - c))/a^4, 1/20*((4*a^4*x^4 - 10*a^3*x^3 + 12*a^2*x^2 - 15*a*x + 24)*sqr
t(-a^2*c*x^2 + c) - 15*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^
2*c*x^2 - c)))/a^4]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\frac{x^3\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{ax+1}\,dx$$

input
```
integrate(x**3*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = -\frac{(-a^2cx^2+c)^{\frac{3}{2}}x^2}{5\,a^2c} - \frac{5\,\sqrt{-a^2cx^2+c}x}{4\,a^3} + \frac{(-a^2cx^2+c)^{\frac{3}{2}}x}{2\,a^3c} + \frac{3\,\sqrt{c}\arcsin{(ax)}}{4\,a^4} + \frac{2\,\sqrt{-a^2cx^2+c}}{a^4} - \frac{4\,(-a^2cx^2+c)^{\frac{3}{2}}}{5\,a^4c}$$

input
```
integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-1/5*(-a^2*c*x^2 + c)^(3/2)*x^2/(a^2*c) - 5/4*sqrt(-a^2*c*x^2 + c)*x/a^3 +
 1/2*(-a^2*c*x^2 + c)^(3/2)*x/(a^3*c) + 3/4*sqrt(c)*arcsin(a*x)/a^4 + 2*sq
rt(-a^2*c*x^2 + c)/a^4 - 4/5*(-a^2*c*x^2 + c)^(3/2)/(a^4*c)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\frac{x^3\,\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{\sqrt{c}\,(15asin(ax)+4\sqrt{-a^2x^2+1}\,a^4x^4-10\sqrt{-a^2x^2+1}\,a^3x^3+12\sqrt{-a^2x^2+1}\,a^2x^2-15\sqrt{-a^2x^2+1}\,ax}{20a^4}$$

input
```
int(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*(15*asin(a*x) + 4*sqrt( - a**2*x**2 + 1)*a**4*x**4 - 10*sqrt( - a
**2*x**2 + 1)*a**3*x**3 + 12*sqrt( - a**2*x**2 + 1)*a**2*x**2 - 15*sqrt( -
 a**2*x**2 + 1)*a*x + 24*sqrt( - a**2*x**2 + 1) - 24))/(20*a**4)
```

3.695 $\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 129

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=-\frac{4\sqrt{c-a^2cx^2}}{3a^3}+\frac{7x\sqrt{c-a^2cx^2}}{8a^2}-\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{7\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

output
```
-4/3*(-a^2*c*x^2+c)^(1/2)/a^3+7/8*x*(-a^2*c*x^2+c)^(1/2)/a^2-2/3*x^2*(-a^2
*c*x^2+c)^(1/2)/a+1/4*x^3*(-a^2*c*x^2+c)^(1/2)-7/8*c^(1/2)*arctan(a*c^(1/2
)*x/(-a^2*c*x^2+c)^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.68

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}(-32+21ax-16a^2x^2+6a^3x^3)+21\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{24a^3}$$

input
```
Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-32 + 21*a*x - 16*a^2*x^2 + 6*a^3*x^3) + 21*Sqrt[c]*
ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(24*a^3)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.25, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {6717, 6702, 541, 25, 27, 533, 25, 27, 533, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x^2\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6702}$$

$$-c\int \frac{x^2(1-ax)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{541}$$

$$-c\left(-\frac{\int -\frac{a^2cx^2(7-8ax)}{\sqrt{c-a^2cx^2}}dx}{4a^2c}-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{a^2cx^2(7-8ax)}{\sqrt{c-a^2cx^2}}dx}{4a^2c}-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{4}\int \frac{x^2(7-8ax)}{\sqrt{c-a^2cx^2}}dx-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{533}$$

$$-c\left(\frac{1}{4}\left(\frac{\int -\frac{acx(16-21ax)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}+\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{1}{4}\left(\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}-\frac{\int \frac{acx(16-21ax)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{4}\left(\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}-\frac{\int \frac{x(16-21ax)}{\sqrt{c-a^2cx^2}}dx}{3a}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{533}$$

$$-c\left(\frac{1}{4}\left(\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}-\frac{\frac{\int -\frac{ac(21-32ax)}{\sqrt{c-a^2cx^2}}dx}{2a^2c}+\frac{21x\sqrt{c-a^2cx^2}}{2ac}}{3a}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{1}{4}\left(\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}-\frac{\frac{21x\sqrt{c-a^2cx^2}}{2ac}-\frac{\int \frac{ac(21-32ax)}{\sqrt{c-a^2cx^2}}dx}{2a^2c}}{3a}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{4}\left(\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}-\frac{\frac{21x\sqrt{c-a^2cx^2}}{2ac}-\frac{\int \frac{21-32ax}{\sqrt{c-a^2cx^2}}dx}{2a}}{3a}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{1}{4}\left(\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}-\frac{\frac{21x\sqrt{c-a^2cx^2}}{2ac}-\frac{21\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{32\sqrt{c-a^2cx^2}}{ac}}{2a}}{3a}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{1}{4}\left(\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}-\frac{\frac{21x\sqrt{c-a^2cx^2}}{2ac}-\frac{21\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{32\sqrt{c-a^2cx^2}}{ac}}{2a}}{3a}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{1}{4}\left(\frac{8x^2\sqrt{c-a^2cx^2}}{3ac}-\frac{\frac{21x\sqrt{c-a^2cx^2}}{2ac}-\frac{\frac{21\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}+\frac{32\sqrt{c-a^2cx^2}}{ac}}{2a}}{3a}\right)-\frac{x^3\sqrt{c-a^2cx^2}}{4c}\right)$$

input
```
Int[(x^2*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(-1/4*(x^3*Sqrt[c - a^2*c*x^2])/c + ((8*x^2*Sqrt[c - a^2*c*x^2])/(3*a*
c) - ((21*x*Sqrt[c - a^2*c*x^2])/(2*a*c) - ((32*Sqrt[c - a^2*c*x^2])/(a*c)
 + (21*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(a*Sqrt[c]))/(2*a))/(3*a
))/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{(6a^3x^3-16a^2x^2+21ax-32)(a^2x^2-1)c}{24a^3\sqrt{-c(a^2x^2-1)}}-\frac{7\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)c}{8a^2\sqrt{a^2c}}$
default	$-\frac{x(-a^2cx^2+c)^{\frac{3}{2}}}{4a^2c}+\frac{\frac{9x\sqrt{-a^2cx^2+c}}{8}+\frac{9c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)}{8\sqrt{a^2c}}}{a^2}-\frac{2\left(\sqrt{-\left(x+\frac{1}{a}\right)^2a^2c+2\left(x+\frac{1}{a}\right)ac}+\frac{ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-\left(x+\frac{1}{a}\right)^2a^2c+2}}\right)}{\sqrt{a^2c}}\right.}{a^3}$

input
```
int(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*a^3*x^3-16*a^2*x^2+21*a*x-32)*(a^2*x^2-1)/a^3/(-c*(a^2*x^2-1))^(1
/2)*c-7/8/a^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.30

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$$
$$= \left[\frac{2\left(6\,a^3x^3-16\,a^2x^2+21\,ax-32\right)\sqrt{-a^2cx^2+c}+21\,\sqrt{-c}\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{48\,a^3},\ \right.$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/48*(2*(6*a^3*x^3 - 16*a^2*x^2 + 21*a*x - 32)*sqrt(-a^2*c*x^2 + c) + 21*
sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^3,
1/24*((6*a^3*x^3 - 16*a^2*x^2 + 21*a*x - 32)*sqrt(-a^2*c*x^2 + c) + 21*sqr
t(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^3]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{ax+1}\,dx$$

input
```
integrate(x**2*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.72

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{9\sqrt{-a^2cx^2+c}\,x}{8\,a^2} - \frac{(-a^2cx^2+c)^{\frac{3}{2}}x}{4\,a^2c} - \frac{7\,\sqrt{c}\arcsin{(ax)}}{8\,a^3} - \frac{2\,\sqrt{-a^2cx^2+c}}{a^3} + \frac{2\,(-a^2cx^2+c)^{\frac{3}{2}}}{3\,a^3c}$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
9/8*sqrt(-a^2*c*x^2 + c)*x/a^2 - 1/4*(-a^2*c*x^2 + c)^(3/2)*x/(a^2*c) - 7/
8*sqrt(c)*arcsin(a*x)/a^3 - 2*sqrt(-a^2*c*x^2 + c)/a^3 + 2/3*(-a^2*c*x^2 +
 c)^(3/2)/(a^3*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.65

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{1}{24}\,\sqrt{-a^2cx^2+c}\left(\left(2\left(3\,x-\frac{8}{a}\right)x+\frac{21}{a^2}\right)x-\frac{32}{a^3}\right) + \frac{7\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8\,a^2\sqrt{-c}|a|}$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*x - 8/a)*x + 21/a^2)*x - 32/a^3) + 7/8*c*
log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^2*sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int \frac{x^2\,\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.63

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{\sqrt{c}\,\left(-21asin(ax)+6\sqrt{-a^2x^2+1}\,a^3x^3-16\sqrt{-a^2x^2+1}\,a^2x^2+21\sqrt{-a^2x^2+1}\,ax-32\sqrt{-a^2x^2+1}+3$$
$$24a^3$$

input
```
int(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*( - 21*asin(a*x) + 6*sqrt( - a**2*x**2 + 1)*a**3*x**3 - 16*sqrt(
- a**2*x**2 + 1)*a**2*x**2 + 21*sqrt( - a**2*x**2 + 1)*a*x - 32*sqrt( - a*
*2*x**2 + 1) + 32))/(24*a**3)
```

3.696 $\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 99

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{5\sqrt{c-a^2cx^2}}{3a^2}-\frac{x\sqrt{c-a^2cx^2}}{a}+\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

output
```
5/3*(-a^2*c*x^2+c)^(1/2)/a^2-x*(-a^2*c*x^2+c)^(1/2)/a+1/3*x^2*(-a^2*c*x^2+
c)^(1/2)+c^(1/2)*arctan(a*c^(1/2)*x/(-a^2*c*x^2+c)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.80

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{(5-3ax+a^2x^2)\sqrt{c-a^2cx^2}-3\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(-1+a^2x^2)}\right)}{3a^2}$$

input
```
Integrate[(x*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

output
```
((5 - 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2] - 3*Sqrt[c]*ArcTan[(a*x*Sqrt[c
- a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6717, 6702, 541, 25, 27, 533, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6702}$$

$$-c\int \frac{x(1-ax)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{541}$$

$$-c\left(-\frac{\int -\frac{a^2cx(5-6ax)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{a^2cx(5-6ax)}{\sqrt{c-a^2cx^2}}dx}{3a^2c}-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{3}\int \frac{x(5-6ax)}{\sqrt{c-a^2cx^2}}dx-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{533}$$

$$-c\left(\frac{1}{3}\left(\frac{\int -\frac{2ac(3-5ax)}{\sqrt{c-a^2cx^2}}dx}{2a^2c}+\frac{3x\sqrt{c-a^2cx^2}}{ac}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{1}{3}\left(\frac{3x\sqrt{c-a^2cx^2}}{ac}-\frac{\int \frac{3-5ax}{\sqrt{c-a^2cx^2}}dx}{a}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{1}{3}\left(\frac{3x\sqrt{c-a^2cx^2}}{ac}-\frac{3\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{5\sqrt{c-a^2cx^2}}{ac}}{a}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{1}{3}\left(\frac{3x\sqrt{c-a^2cx^2}}{ac}-\frac{3\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{5\sqrt{c-a^2cx^2}}{ac}}{a}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{1}{3}\left(\frac{3x\sqrt{c-a^2cx^2}}{ac}-\frac{\frac{3\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}+\frac{5\sqrt{c-a^2cx^2}}{ac}}{a}\right)-\frac{x^2\sqrt{c-a^2cx^2}}{3c}\right)$$

input
```
Int[(x*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(-1/3*(x^2*Sqrt[c - a^2*c*x^2])/c + ((3*x*Sqrt[c - a^2*c*x^2])/(a*c) -
 ((5*Sqrt[c - a^2*c*x^2])/(a*c) + (3*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x
^2]])/(a*Sqrt[c]))/a)/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{\left(a^{2} x^{2}-3 a x+5\right) \left(a^{2} x^{2}-1\right) c}{3 a^{2} \sqrt{-c \left(a^{2} x^{2}-1\right)}}+\frac{\arctan\left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right) c}{a \sqrt{a^{2} c}}$
default	$-\frac{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{3 a^{2} c}-\frac{2 \left(\frac{x \sqrt{-a^{2} c \,x^{2}+c}}{2}+\frac{c \arctan\left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right)}{2 \sqrt{a^{2} c}}\right)}{a}+\frac{2 \sqrt{-\left(x +\frac{1}{a}\right)^{2} a^{2} c +2 \left(x +\frac{1}{a}\right) a c}+\frac{2 a c \arctan\left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-\left(x +\frac{1}{a}\right)^{2} a^{2} c +2}}\right)}{\sqrt{a^{2} c}}}{a^{2}}$

input
```
int(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a^2*x^2-3*a*x+5)*(a^2*x^2-1)/a^2/(-c*(a^2*x^2-1))^(1/2)*c+1/a/(a^2*c
)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))*c
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.52

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$$
$$= \left[\frac{2\sqrt{-a^2cx^2+c}(a^2x^2-3\,ax+5)+3\sqrt{-c}\log\left(2\,a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{6\,a^2}, \frac{\sqrt{-a^2cx^2+c}(a^2\right.$$

input
```
integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/6*(2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 3*a*x + 5) + 3*sqrt(-c)*log(2*a^2*
c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^2, 1/3*(sqrt(-a^2*c*x^
2 + c)*(a^2*x^2 - 3*a*x + 5) - 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqr
t(c)*x/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int \frac{x\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{ax+1}\,dx$$

input
```
integrate(x*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = -\frac{\sqrt{-a^2cx^2+c}x}{a} + \frac{\sqrt{c}\arcsin(ax)}{a^2} + \frac{2\sqrt{-a^2cx^2+c}}{a^2} - \frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3\,a^2c}$$

input
```
integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
-sqrt(-a^2*c*x^2 + c)*x/a + sqrt(c)*arcsin(a*x)/a^2 + 2*sqrt(-a^2*c*x^2 +
c)/a^2 - 1/3*(-a^2*c*x^2 + c)^(3/2)/(a^2*c)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{1}{3}\sqrt{-a^2cx^2+c}\left(\left(x-\frac{3}{a}\right)x+\frac{5}{a^2}\right)-\frac{c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{a\sqrt{-c}|a|}$$

input
```
integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/3*sqrt(-a^2*c*x^2 + c)*((x - 3/a)*x + 5/a^2) - c*log(abs(-sqrt(-a^2*c)*x
 + sqrt(-a^2*c*x^2 + c)))/(a*sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\int\frac{x\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c}\left(3asin(ax)+\sqrt{-a^2x^2+1}\,a^2x^2-3\sqrt{-a^2x^2+1}\,ax+5\sqrt{-a^2x^2+1}-5\right)}{3a^2}$$

input
```
int(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*(3*asin(a*x) + sqrt( - a**2*x**2 + 1)*a**2*x**2 - 3*sqrt( - a**2*
x**2 + 1)*a*x + 5*sqrt( - a**2*x**2 + 1) - 5))/(3*a**2)
```

3.697 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=-\frac{2\sqrt{c-a^2cx^2}}{a}+\frac{1}{2}x\sqrt{c-a^2cx^2}-\frac{3\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

output
```
-2*(-a^2*c*x^2+c)^(1/2)/a+1/2*x*(-a^2*c*x^2+c)^(1/2)-3/2*c^(1/2)*arctan(a*
c^(1/2)*x/(-a^2*c*x^2+c)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.30

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\sqrt{1+ax}(4-5ax+a^2x^2)+6\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2)) + 6*Sqrt[1 -
a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6717, 6692, 469, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int \frac{(1-ax)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{469}$$

$$-c\left(\frac{3}{2}\int \frac{1-ax}{\sqrt{c-a^2cx^2}}dx+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{455}$$

$$-c\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\frac{\sqrt{c-a^2cx^2}}{ac}\right)+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{224}$$

$$-c\left(\frac{3}{2}\left(\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}+\frac{\sqrt{c-a^2cx^2}}{ac}\right)+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2ac}\right)$$

$$\Big\downarrow \text{216}$$

$$-c\left(\frac{3}{2}\left(\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}+\frac{\sqrt{c-a^2cx^2}}{ac}\right)+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2ac}\right)$$

input
```
Int[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a*c) + (3*(Sqrt[c - a^2*c*x^2]/(a*
c) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]/(a*Sqrt[c])))/2))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

method	result	size
risch	$-\frac{(ax-4)(a^2x^2-1)c}{2a\sqrt{-c(a^2x^2-1)}}-\frac{3c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}$	69
default	$\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}-\frac{2\left(\sqrt{-\left(x+\frac{1}{a}\right)^2a^2c+2\left(x+\frac{1}{a}\right)ac}+\frac{ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-\left(x+\frac{1}{a}\right)^2a^2c+2\left(x+\frac{1}{a}\right)ac}}\right)}{\sqrt{a^2c}}\right)}{a}$	127

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x-4)*(a^2*x^2-1)/a/(-c*(a^2*x^2-1))^(1/2)*c-3/2*c/(a^2*c)^(1/2)*ar
ctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.74

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$$
$$=\left[\frac{2\sqrt{-a^2cx^2+c}(ax-4)+3\sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{4a},\frac{\sqrt{-a^2cx^2+c}(ax-4)+3\right.$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x - 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sq
rt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x -
4) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a
]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{ax+1}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{1}{2}\sqrt{-a^2cx^2+c}x-\frac{3\sqrt{c}\arcsin{(ax)}}{2\,a}-\frac{2\sqrt{-a^2cx^2+c}}{a}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a - 2*sqrt(-a^2*c*x^2
 + c)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x-\frac{4}{a}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}|a|}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/2*sqrt(-a^2*c*x^2 + c)*(x - 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(
-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c}\,\left(-3asin(ax)+\sqrt{-a^2x^2+1}\,ax-4\sqrt{-a^2x^2+1}+4\right)}{2a}$$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*( - 3*asin(a*x) + sqrt( - a**2*x**2 + 1)*a*x - 4*sqrt( - a**2*x**
2 + 1) + 4))/(2*a)
```

3.698 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 75

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\sqrt{c-a^2cx^2}+2\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
(-a^2*c*x^2+c)^(1/2)+2*c^(1/2)*arctan(a*c^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+c^
(1/2)*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\sqrt{c-a^2cx^2}-2\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right)-\sqrt{c}\log(x)+\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
Sqrt[c - a^2*c*x^2] - 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*
(-1 + a^2*x^2))] - Sqrt[c]*Log[x] + Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c
*x^2]]
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6717, 6702, 541, 25, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x}dx$$

$$\Big\downarrow \text{6702}$$

$$-c\int \frac{(1-ax)^2}{x\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{541}$$

$$-c\left(-\frac{\int -\frac{a^2c(1-2ax)}{x\sqrt{c-a^2cx^2}}dx}{a^2c}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{a^2c(1-2ax)}{x\sqrt{c-a^2cx^2}}dx}{a^2c}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\int \frac{1-2ax}{x\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{538}$$

$$-c\left(-2a\int \frac{1}{\sqrt{c-a^2cx^2}}dx+\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{ 224}$$

$$-c\left(\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-2a\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{ 216}$$

$$-c\left(\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{2\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{ 243}$$

$$-c\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2-\frac{2\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{ 73}$$

$$-c\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{a^2c}-\frac{2\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

$$\Big\downarrow \text{ 221}$$

$$-c\left(-\frac{2\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{c-a^2cx^2}}{c}\right)$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
-(c*(-(Sqrt[c - a^2*c*x^2]/c) - (2*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2
]])/Sqrt[c] - ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]/Sqrt[c]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.61

method	result
default	$-\sqrt{-a^2 c\,x^2+c}+\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)+2\sqrt{-\left(x+\frac{1}{a}\right)^2 a^2 c+2\left(x+\frac{1}{a}\right)ac}+\frac{2ac\arctan\left(\frac{\cdots}{\sqrt{\cdots}}\right)}{\cdots}$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

output
```
-(-a^2*c*x^2+c)^(1/2)+c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)+2
*(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2)+2*a*c/(a^2*c)^(1/2)*arctan((a^2*c)
^(1/2)*x/(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.41

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \left[-2\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right) + \frac{1}{2}\sqrt{c}\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right) + \sqrt{-a^2cx^2+c}, -\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{c}\right) + \sqrt{-c}\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right) + \sqrt{-a^2cx^2+c}\right]$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

output
```
[-2*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2
*sqrt(c)*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sq
rt(-a^2*c*x^2 + c), -sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/c) + sq
rt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) + sqrt(-
a^2*c*x^2 + c)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{x\,(ax+1)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.15

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = a^2\left(\frac{\sqrt{c}\arcsin(ax)}{a^2}+\frac{\sqrt{-a^2cx^2+c}}{a^2}\right) + a\left(\frac{\sqrt{c}\arcsin(ax)}{a}+\frac{\sqrt{c}\log\left(\frac{2\sqrt{-a^2cx^2+c}\sqrt{c}}{|x|}+\frac{2c}{|x|}\right)}{a}\right)$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

output
```
a^2*(sqrt(c)*arcsin(a*x)/a^2 + sqrt(-a^2*c*x^2 + c)/a^2) + a*(sqrt(c)*arcs
in(a*x)/a + sqrt(c)*log(2*sqrt(-a^2*c*x^2 + c)*sqrt(c)/abs(x) + 2*c/abs(x)
)/a)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.27

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = -\frac{2\,c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{2\,a\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|} + \sqrt{-a^2cx^2+c}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

output
```
-2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) +
2*a*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) + sqr
t(-a^2*c*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{x\,(a\,x+1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.43

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \sqrt{c}\left(2asin(ax)+\sqrt{-a^2x^2+1} - \log\left(\tan\left(\frac{asin(ax)}{2}\right)\right)-1\right)$$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x)
```

output
```
sqrt(c)*(2*asin(a*x) + sqrt( - a**2*x**2 + 1) - log(tan(asin(a*x)/2)) - 1)
```

3.699 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}}{x} - a\sqrt{c}\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right) - 2a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
(-a^2*c*x^2+c)^(1/2)/x-a*c^(1/2)*arctan(a*c^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-
2*a*c^(1/2)*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}}{x} + a\sqrt{c}\arctan\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right) + 2a\sqrt{c}\log(x) - 2a\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
Sqrt[c - a^2*c*x^2]/x + a*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c
]*(-1 + a^2*x^2))] + 2*a*Sqrt[c]*Log[x] - 2*a*Sqrt[c]*Log[c + Sqrt[c]*Sqrt
[c - a^2*c*x^2]]
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6717, 6702, 540, 27, 538, 224, 216, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^2}dx$$

$$\Big\downarrow \text{ 6702}$$

$$-c\int \frac{(1-ax)^2}{x^2\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{ 540}$$

$$-c\left(-\frac{\int \frac{ac(2-ax)}{x\sqrt{c-a^2cx^2}}dx}{c}-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-c\left(-a\int \frac{2-ax}{x\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{ 538}$$

$$-c\left(-a\left(2\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-a\int \frac{1}{\sqrt{c-a^2cx^2}}dx\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \text{ 224}$$

$$-c\left(-a\left(2\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-a\int \frac{1}{\frac{a^2cx^2}{c-a^2cx^2}+1}d\frac{x}{\sqrt{c-a^2cx^2}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \ 216$$

$$-c\left(-a\left(2\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \ 243$$

$$-c\left(-a\left(\int \frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2-\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \ 73$$

$$-c\left(-a\left(-\frac{2\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{a^2c}-\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

$$\Big\downarrow \ 221$$

$$-c\left(-a\left(-\frac{\arctan\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{\sqrt{c}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\sqrt{c-a^2cx^2}}{cx}\right)$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
-(c*(-(Sqrt[c - a^2*c*x^2]/(c*x)) - a*(-(ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2
*c*x^2]]/Sqrt[c]) - (2*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c])))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{(a^2x^2-1)c}{x\sqrt{-c(a^2x^2-1)}}-\left(\frac{a^2\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}+\frac{2a\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{\sqrt{c}}\right)c$
default	$\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{cx}+2a^2\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)+2a\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a}}{x}\right.\right.$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(a^2*x^2-1)/x/(-c*(a^2*x^2-1))^(1/2)*c-(a^2/(a^2*c)^(1/2)*arctan((a^2*c)^
(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2*a/c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)
^(1/2))/x))*c
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.41

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$$
$$= \left[\frac{a\sqrt{c}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}a\sqrt{c}x}{a^2cx^2-c}\right)+a\sqrt{c}x\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+\sqrt{-a^2cx^2+c}}{x}, \frac{4a\sqrt{-c}x\arctan\right.$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

output
```
[(a*sqrt(c)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + a
*sqrt(c)*x*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) +
sqrt(-a^2*c*x^2 + c))/x, 1/2*(4*a*sqrt(-c)*x*arctan(sqrt(-a^2*c*x^2 + c)*s
qrt(-c)/c) + a*sqrt(-c)*x*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(
-c)*x - c) + 2*sqrt(-a^2*c*x^2 + c))/x]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{x^2\,(ax+1)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**2*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax-1)}{(ax+1)x^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.63

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{4\,ac\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{a^2\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|} - \frac{2\,a^2\sqrt{-c}c}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a|}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

output
```
4*a*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) -
 a^2*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) - 2*
a^2*sqrt(-c)*c/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*abs(a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{x^2\,(a\,x+1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^2*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.46

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c}\left(-asin(ax)\,ax+\sqrt{-a^2x^2+1}+2\log\left(\tan\left(\frac{asin(ax)}{2}\right)\right)ax\right)}{x}$$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x)
```

output
```
(sqrt(c)*( - asin(a*x)*a*x + sqrt( - a**2*x**2 + 1) + 2*log(tan(asin(a*x)/
2))*a*x))/x
```

3.700 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} + \frac{3}{2}a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
1/2*(-a^2*c*x^2+c)^(1/2)/x^2-2*a*(-a^2*c*x^2+c)^(1/2)/x+3/2*a^2*c^(1/2)*ar
ctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{1}{2}\left(\frac{(1-4ax)\sqrt{c-a^2cx^2}}{x^2} - 3a^2\sqrt{c}\log(x) + 3a^2\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)\right)$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
(((1 - 4*a*x)*Sqrt[c - a^2*c*x^2])/x^2 - 3*a^2*Sqrt[c]*Log[x] + 3*a^2*Sqrt
[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/2
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6717, 6702, 540, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^3}dx$$

$$\Big\downarrow \text{ 6702}$$

$$-c\int \frac{(1-ax)^2}{x^3\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{ 540}$$

$$-c\left(-\frac{\int \frac{ac(4-3ax)}{x^2\sqrt{c-a^2cx^2}}dx}{2c}-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-c\left(-\frac{1}{2}a\int \frac{4-3ax}{x^2\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{ 534}$$

$$-c\left(-\frac{1}{2}a\left(-3a\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{4\sqrt{c-a^2cx^2}}{cx}\right)-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \text{ 243}$$

$$-c\left(-\frac{1}{2}a\left(-\frac{3}{2}a\int \frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2-\frac{4\sqrt{c-a^2cx^2}}{cx}\right)-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \textbf{73}$$

$$-c\left(-\frac{1}{2}a\left(\frac{3\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{ac}-\frac{4\sqrt{c-a^2cx^2}}{cx}\right)-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

$$\Big\downarrow \textbf{221}$$

$$-c\left(-\frac{1}{2}a\left(\frac{3a\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{4\sqrt{c-a^2cx^2}}{cx}\right)-\frac{\sqrt{c-a^2cx^2}}{2cx^2}\right)$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
-(c*(-1/2*Sqrt[c - a^2*c*x^2]/(c*x^2) - (a*((-4*Sqrt[c - a^2*c*x^2])/(c*x)
 + (3*a*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c]))/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

method	result
risch	$\frac{(4a^3x^3-a^2x^2-4ax+1)c}{2x^2\sqrt{-c(a^2x^2-1)}}+\frac{3a^2\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{2}$
default	$\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{2c\,x^2}-\frac{3a^2\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)\right)}{2}+2a\left(-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{cx}-2a^2\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\right.\right.$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(4*a^3*x^3-a^2*x^2-4*a*x+1)/x^2/(-c*(a^2*x^2-1))^(1/2)*c+3/2*a^2*c^(1/
2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.76

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \left[\frac{3\,a^2\sqrt{c}x^2\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-2\sqrt{-a^2cx^2+c}(4\,ax-1)}{4\,x^2},\ -\frac{3\,a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{c}\right)+\sqrt{-a^2cx^2+c}(4\,ax-1)}{2\,x^2}\right]$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

output
```
[1/4*(3*a^2*sqrt(c)*x^2*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
 2*c)/x^2) - 2*sqrt(-a^2*c*x^2 + c)*(4*a*x - 1))/x^2, -1/2*(3*a^2*sqrt(-c)
*x^2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/c) + sqrt(-a^2*c*x^2 + c)*(4*a*x
 - 1))/x^2]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{x^3\,(ax+1)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**3*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax-1)}{(ax+1)x^3}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(64) = 128$.

Time = 0.15 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.56

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = -\frac{3\,a^2c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^3a^2c+4\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2a\sqrt{-c}c|a|+\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right.}{\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2-c\right)^2}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

output
```
-3*a^2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c
) + ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^2*c + 4*(sqrt(-a^2*c)*x -
 sqrt(-a^2*c*x^2 + c))^2*a*sqrt(-c)*c*abs(a) + (sqrt(-a^2*c)*x - sqrt(-a^2
*c*x^2 + c))*a^2*c^2 - 4*a*sqrt(-c)*c^2*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a
^2*c*x^2 + c))^2 - c)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{x^3\,(a\,x+1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^3*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c}\left(-4\sqrt{-a^2x^2+1}\,ax+\sqrt{-a^2x^2+1}-3\log\left(\tan\left(\frac{asin(ax)}{2}\right)\right)a^2x^2\right)}{2x^2}$$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x)
```

output
```
(sqrt(c)*( - 4*sqrt( - a**2*x**2 + 1)*a*x + sqrt( - a**2*x**2 + 1) - 3*log
(tan(asin(a*x)/2))*a**2*x**2))/(2*x**2)
```

3.701 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 101

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x} - a^3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
1/3*(-a^2*c*x^2+c)^(1/2)/x^3-a*(-a^2*c*x^2+c)^(1/2)/x^2+5/3*a^2*(-a^2*c*x^
2+c)^(1/2)/x-a^3*c^(1/2)*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{(1-3ax+5a^2x^2)\sqrt{c-a^2cx^2}}{3x^3}+a^3\sqrt{c}\log(x) - a^3\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
((1 - 3*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*x^3) + a^3*Sqrt[c]*Log[x]
 - a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6717, 6702, 540, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^4}dx$$

$$\Big\downarrow \text{6702}$$

$$-c\int \frac{(1-ax)^2}{x^4\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-c\left(-\frac{\int \frac{ac(6-5ax)}{x^3\sqrt{c-a^2cx^2}}dx}{3c}-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(-\frac{1}{3}a\int \frac{6-5ax}{x^3\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{539}$$

$$-c\left(-\frac{1}{3}a\left(-\frac{\int \frac{2ac(5-3ax)}{x^2\sqrt{c-a^2cx^2}}dx}{2c}-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(-\frac{1}{3}a\left(-a\int \frac{5-3ax}{x^2\sqrt{c-a^2cx^2}}dx-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \; 534$$

$$-c\left(-\frac{1}{3}a\left(-a\left(-3a\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{5\sqrt{c-a^2cx^2}}{cx}\right)-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \; 243$$

$$-c\left(-\frac{1}{3}a\left(-a\left(-\frac{3}{2}a\int \frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2-\frac{5\sqrt{c-a^2cx^2}}{cx}\right)-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \; 73$$

$$-c\left(-\frac{1}{3}a\left(-a\left(\frac{3\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{ac}-\frac{5\sqrt{c-a^2cx^2}}{cx}\right)-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

$$\Big\downarrow \; 221$$

$$-c\left(-\frac{1}{3}a\left(-a\left(\frac{3a\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{5\sqrt{c-a^2cx^2}}{cx}\right)-\frac{3\sqrt{c-a^2cx^2}}{cx^2}\right)-\frac{\sqrt{c-a^2cx^2}}{3cx^3}\right)$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
-(c*(-1/3*Sqrt[c - a^2*c*x^2]/(c*x^3) - (a*((-3*Sqrt[c - a^2*c*x^2])/(c*x^
2) - a*((-5*Sqrt[c - a^2*c*x^2])/(c*x) + (3*a*ArcTanh[Sqrt[c - a^2*c*x^2]/
Sqrt[c]])/Sqrt[c])))/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\left(5 a^{4} x^{4}-3 a^{3} x^{3}-4 a^{2} x^{2}+3 a x -1\right) c}{3 x^{3} \sqrt{-c \left(a^{2} x^{2}-1\right)}}-a^{3} \sqrt{c}\, \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{-a^{2} c \,x^{2}+c}}{x}\right)$
default	$\frac{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{3 c \,x^{3}}+2 a \left(-\frac{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{2 c \,x^{2}}-\frac{a^{2} \left(\sqrt{-a^{2} c \,x^{2}+c}-\sqrt{c}\, \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{-a^{2} c \,x^{2}+c}}{x}\right)\right)}{2}\right)-2 a^{2} \left(-\frac{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{c x}\right.$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(5*a^4*x^4-3*a^3*x^3-4*a^2*x^2+3*a*x-1)/x^3/(-c*(a^2*x^2-1))^(1/2)*c-
a^3*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.51

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \left[\frac{3\,a^3\sqrt{c}x^3\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\,\sqrt{-a^2cx^2+c}(5\,a^2x^2-3\,ax+1)}{6\,x^3}, \frac{3\,a^3\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-a^2}\right.}{}\right.$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

output
```
[1/6*(3*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
 2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 - 3*a*x + 1))/x^3, 1/3*(3*a
^3*sqrt(-c)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/c) + sqrt(-a^2*c*x^2
+ c)*(5*a^2*x^2 - 3*a*x + 1))/x^3]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{x^4\,(ax+1)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**4*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax-1)}{(ax+1)x^4}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(85) = 170$.

Time = 0.14 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.48

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{2\,a^3c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}$$
$$-\frac{2\left(3\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5a^3c+3\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^4a^2\sqrt{-c}c|a|-12\left(\sqrt{-a^2c}x-\sqrt{-c\cdots}\right.\right.}{3\left(\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^2\cdots\right.}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

output
```
2*a^3*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
 - 2/3*(3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c + 3*(sqrt(-a^2*c
)*x - sqrt(-a^2*c*x^2 + c))^4*a^2*sqrt(-c)*c*abs(a) - 12*(sqrt(-a^2*c)*x -
 sqrt(-a^2*c*x^2 + c))^2*a^2*sqrt(-c)*c^2*abs(a) - 3*(sqrt(-a^2*c)*x - sqr
t(-a^2*c*x^2 + c))*a^3*c^3 + 5*a^2*sqrt(-c)*c^3*abs(a))/((sqrt(-a^2*c)*x -
 sqrt(-a^2*c*x^2 + c))^2 - c)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{x^4\,(a\,x+1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^4*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.68

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$$
$$= \frac{\sqrt{c}\left(5\sqrt{-a^2x^2+1}\,a^2x^2-3\sqrt{-a^2x^2+1}\,ax+\sqrt{-a^2x^2+1}+3\log\left(\tan\left(\frac{asin(ax)}{2}\right)\right)a^3x^3\right)}{3x^3}$$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x)
```

output
```
(sqrt(c)*(5*sqrt( - a**2*x**2 + 1)*a**2*x**2 - 3*sqrt( - a**2*x**2 + 1)*a*
x + sqrt( - a**2*x**2 + 1) + 3*log(tan(asin(a*x)/2))*a**3*x**3))/(3*x**3)
```

3.702 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 130

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{7}{8}a^4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

output
```
1/4*(-a^2*c*x^2+c)^(1/2)/x^4-2/3*a*(-a^2*c*x^2+c)^(1/2)/x^3+7/8*a^2*(-a^2*
c*x^2+c)^(1/2)/x^2-4/3*a^3*(-a^2*c*x^2+c)^(1/2)/x+7/8*a^4*c^(1/2)*arctanh(
(-a^2*c*x^2+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}(6-16ax+21a^2x^2-32a^3x^3)}{24x^4}-\frac{7}{8}a^4\sqrt{c}\log(x)+\frac{7}{8}a^4\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^5),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(6 - 16*a*x + 21*a^2*x^2 - 32*a^3*x^3))/(24*x^4) - (7
*a^4*Sqrt[c]*Log[x])/8 + (7*a^4*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2
]])/8
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6717, 6702, 540, 27, 539, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-a^2cx^2}}{x^5}dx$$

$$\Big\downarrow \text{6702}$$

$$-c\int \frac{(1-ax)^2}{x^5\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{540}$$

$$-c\left(-\frac{\int \frac{ac(8-7ax)}{x^4\sqrt{c-a^2cx^2}}dx}{4c}-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(-\frac{1}{4}a\int \frac{8-7ax}{x^4\sqrt{c-a^2cx^2}}dx-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \text{539}$$

$$-c\left(-\frac{1}{4}a\left(-\frac{\int \frac{ac(21-16ax)}{x^3\sqrt{c-a^2cx^2}}dx}{3c}-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(-\frac{1}{4}a\left(-\frac{1}{3}a\int \frac{21-16ax}{x^3\sqrt{c-a^2cx^2}}dx-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$\Big\downarrow$ 539

$$-c\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{\int \frac{ac(32-21ax)}{x^2\sqrt{c-a^2cx^2}}dx}{2c}-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$\Big\downarrow$ 27

$$-c\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\int \frac{32-21ax}{x^2\sqrt{c-a^2cx^2}}dx-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\frac{\sqrt{c-a^2cx^2}}{4cx^4}\right)$$

$\Big\downarrow$ 534

$$-c\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\left(-21a\int \frac{1}{x\sqrt{c-a^2cx^2}}dx-\frac{32\sqrt{c-a^2cx^2}}{cx}\right)-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\right.$$

$\Big\downarrow$ 243

$$-c\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\left(-\frac{21}{2}a\int \frac{1}{x^2\sqrt{c-a^2cx^2}}dx^2-\frac{32\sqrt{c-a^2cx^2}}{cx}\right)-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\right.$$

$\Big\downarrow$ 73

$$-c\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\left(\frac{21\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2c}}d\sqrt{c-a^2cx^2}}{ac}-\frac{32\sqrt{c-a^2cx^2}}{cx}\right)-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\right.$$

$\Big\downarrow$ 221

$$-c\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\left(\frac{21a\text{arctanh}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{32\sqrt{c-a^2cx^2}}{cx}\right)-\frac{21\sqrt{c-a^2cx^2}}{2cx^2}\right)-\frac{8\sqrt{c-a^2cx^2}}{3cx^3}\right)-\right.$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^5),x]
```

output
```
-(c*(-1/4*Sqrt[c - a^2*c*x^2]/(c*x^4) - (a*((-8*Sqrt[c - a^2*c*x^2])/(3*c*
x^3) - (a*((-21*Sqrt[c - a^2*c*x^2])/(2*c*x^2) - (a*((-32*Sqrt[c - a^2*c*x
^2])/(c*x) + (21*a*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c]))/2))/3))
/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

method	result
risch	$\frac{(32a^5x^5-21a^4x^4-16a^3x^3+15a^2x^2-16ax+6)c}{24x^4\sqrt{-c(a^2x^2-1)}}+\frac{7a^4\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{8}$
default	$\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{4c\,x^4}-\frac{9a^2\left(-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{2c\,x^2}-\frac{a^2\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)\right)}{2}\right)}{4}-\frac{2a(-a^2c\,x^2+c)^{\frac{3}{2}}}{3c\,x^3}+2a^3\Bigg(-$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/24*(32*a^5*x^5-21*a^4*x^4-16*a^3*x^3+15*a^2*x^2-16*a*x+6)/x^4/(-c*(a^2*x
^2-1))^(1/2)*c+7/8*a^4*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.30

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$$
$$= \left[\frac{21\,a^4\sqrt{c}x^4\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-2\,(32\,a^3x^3-21\,a^2x^2+16\,ax-6)\sqrt{-a^2cx^2+c}}{48\,x^4},\right.$$
$$\left.-\frac{21\,a^4\sqrt{-c}x^4\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{c}\right)+(32\,a^3x^3-21\,a^2x^2+16\,ax-6)\sqrt{-a^2cx^2+c}}{24\,x^4}\right]$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

output
```
[1/48*(21*a^4*sqrt(c)*x^4*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
 - 2*c)/x^2) - 2*(32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqrt(-a^2*c*x^2 +
c))/x^4, -1/24*(21*a^4*sqrt(-c)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/c
) + (32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqrt(-a^2*c*x^2 + c))/x^4]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx=\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}(ax-1)}{x^5\,(ax+1)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**5*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-a^2cx^2+c}(ax-1)}{(ax+1)x^5}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(106) = 212.

Time = 0.13 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.49

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = -\frac{7\,a^4c\arctan\left(-\frac{\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}} + \frac{21\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^7a^4c-45\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+c}\right)^5a^4c^2-96\left(\sqrt{-a^2c}x-\sqrt{-a^2cx^2+\right.}}{}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

output
```
-7/4*a^4*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(
-c) + 1/12*(21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^4*c - 45*(sqrt(
-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^2 - 96*(sqrt(-a^2*c)*x - sqrt(-a
^2*c*x^2 + c))^4*a^3*sqrt(-c)*c^2*abs(a) - 45*(sqrt(-a^2*c)*x - sqrt(-a^2*
c*x^2 + c))^3*a^4*c^3 + 128*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*
sqrt(-c)*c^3*abs(a) + 21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^4*c^4 -
 32*a^3*sqrt(-c)*c^4*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 -
c)^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{x^5\,(a\,x+1)}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^5*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.69

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c}\left(-32\sqrt{-a^2x^2+1}\,a^3x^3+21\sqrt{-a^2x^2+1}\,a^2x^2-16\sqrt{-a^2x^2+1}\,ax+6\sqrt{-a^2x^2+1}-21\log\left(\tan\left(\frac{as}{}\right.\right.\right.}{24x^4}$$

input
```
int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x)
```

output
```
(sqrt(c)*( - 32*sqrt( - a**2*x**2 + 1)*a**3*x**3 + 21*sqrt( - a**2*x**2 +
1)*a**2*x**2 - 16*sqrt( - a**2*x**2 + 1)*a*x + 6*sqrt( - a**2*x**2 + 1) -
21*log(tan(asin(a*x)/2))*a**4*x**4))/(24*x**4)
```

3.703 $\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 227

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^5\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
4*(-a^2*c*x^2+c)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)-2*x*(-a^2*c*x^2+c)^(1/2)/a^
3/(1-1/a^2/x^2)^(1/2)+4/3*x^2*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)
-3/4*x^3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/5*x^4*(-a^2*c*x^2+c)
^(1/2)/(1-1/a^2/x^2)^(1/2)-4*(-a^2*c*x^2+c)^(1/2)*ln(a*x+1)/a^5/(1-1/a^2/x
^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.38

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(\frac{4x}{a^4}-\frac{2x^2}{a^3}+\frac{4x^3}{3a^2}-\frac{3x^4}{4a}+\frac{x^5}{5}-\frac{4\log(1+ax)}{a^5}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(x^3*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((4*x)/a^4 - (2*x^2)/a^3 + (4*x^3)/(3*a^2) - (3*x^4)/
(4*a) + x^5/5 - (4*Log[1 + a*x])/a^5))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^4dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{x^3(1-ax)^2}{ax+1}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-a^2cx^2}\int\left(ax^4-3x^3+\frac{4x^2}{a}-\frac{4x}{a^2}-\frac{4}{a^3(ax+1)}+\frac{4}{a^3}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(ax+1)}{a^4}+\frac{4x}{a^3}-\frac{2x^2}{a^2}+\frac{ax^5}{5}+\frac{4x^3}{3a}-\frac{3x^4}{4}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(x^3*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((4*x)/a^3 - (2*x^2)/a^2 + (4*x^3)/(3*a) - (3*x^4)/4
+ (a*x^5)/5 - (4*Log[1 + a*x])/a^4))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.41

method	result	size
default	$-\frac{\left(-12 a^{5} x^{5}+45 a^{4} x^{4}-80 a^{3} x^{3}+120 a^{2} x^{2}-240 a x+240 \ln \left(a x+1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x+1\right) \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{60 a^{4} \left(a x-1\right)^{2}}$	92

input
```
int(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/60*(-12*a^5*x^5+45*a^4*x^4-80*a^3*x^3+120*a^2*x^2-240*a*x+240*ln(a*x+1)
)*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^4/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.26

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{\left(12\,a^5x^5 - 45\,a^4x^4 + 80\,a^3x^3 - 120\,a^2x^2 + 240\,ax - 240\,\log\left(ax+1\right)\right)\sqrt{-a^2c}}{60\,a^5}$$

input
```
integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="f
ricas")
```

output
```
1/60*(12*a^5*x^5 - 45*a^4*x^4 + 80*a^3*x^3 - 120*a^2*x^2 + 240*a*x - 240*l
og(a*x + 1))*sqrt(-a^2*c)/a^5
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.44

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=-\frac{1}{60}\sqrt{-c}\left(\frac{240\log\left(|ax+1|\right)\operatorname{sgn}(ax+1)}{a^4}-\frac{12\,a^6x^5\operatorname{sgn}(ax+1)-45\,a^5x^4\operatorname{sgn}(ax+1)+80\,a^4x^3\operatorname{sgn}(ax}{a^5}\right.$$

input
```
integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="g
iac")
```

output
```
-1/60*sqrt(-c)*(240*log(abs(a*x + 1))*sgn(a*x + 1)/a^4 - (12*a^6*x^5*sgn(a
*x + 1) - 45*a^5*x^4*sgn(a*x + 1) + 80*a^4*x^3*sgn(a*x + 1) - 120*a^3*x^2*
sgn(a*x + 1) + 240*a^2*x*sgn(a*x + 1))/a^5)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\int x^3\,\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int(x^3*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x^3*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.24

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c}\,i(240\log(ax+1)-12a^5x^5+45a^4x^4-80a^3x^3+120a^2x^2-240ax+167)}{60a^4}$$

input
```
int(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*i*(240*log(a*x + 1) - 12*a**5*x**5 + 45*a**4*x**4 - 80*a**3*x**3
+ 120*a**2*x**2 - 240*a*x + 167))/(60*a**4)
```

3.704 $\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 186

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = -\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
-4*(-a^2*c*x^2+c)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x*(-a^2*c*x^2+c)^(1/2)/a
^2/(1-1/a^2/x^2)^(1/2)-x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*
x^3*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*(-a^2*c*x^2+c)^(1/2)*ln(a*x
+1)/a^4/(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.40

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\left(-\frac{4x}{a^3}+\frac{2x^2}{a^2}-\frac{x^3}{a}+\frac{x^4}{4}+\frac{4\log(1+ax)}{a^4}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((-4*x)/a^3 + (2*x^2)/a^2 - x^3/a + x^4/4 + (4*Log[1
+ a*x])/a^4))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{c-a^2cx^2} e^{-3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2} \int e^{-3\coth^{-1}(ax)} \sqrt{1-\frac{1}{a^2x^2}} x^3 dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2} \int \frac{x^2(1-ax)^2}{ax+1} dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-a^2cx^2} \int \left(ax^3-3x^2+\frac{4x}{a}+\frac{4}{a^2(ax+1)}-\frac{4}{a^2}\right) dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2} \left(\frac{4\log(ax+1)}{a^3}-\frac{4x}{a^2}+\frac{ax^4}{4}+\frac{2x^2}{a}-x^3\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(x^2*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((-4*x)/a^2 + (2*x^2)/a - x^3 + (a*x^4)/4 + (4*Log[1
+ a*x])/a^3))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

method	result	size
default	$\frac{\left(a^{4} x^{4}-4 a^{3} x^{3}+8 a^{2} x^{2}-16 a x +16 \ln \left(a x +1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{4 a^{3} \left(a x -1\right)^{2}}$	83

input
```
int(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
1/4*(a^4*x^4-4*a^3*x^3+8*a^2*x^2-16*a*x+16*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/
2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^3/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.26

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{(a^4x^4-4\,a^3x^3+8\,a^2x^2-16\,ax+16\,\log{(ax+1)})\sqrt{-a^2c}}{4\,a^4}$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="f
ricas")
```

output
```
1/4*(a^4*x^4 - 4*a^3*x^3 + 8*a^2*x^2 - 16*a*x + 16*log(a*x + 1))*sqrt(-a^2
*c)/a^4
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}\,x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.45

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{1}{4}\sqrt{-c}\left(\frac{16\log\left(|ax+1|\right)\operatorname{sgn}(ax+1)}{a^3}+\frac{a^5x^4\operatorname{sgn}(ax+1)-4a^4x^3\operatorname{sgn}(ax+1)+8a^3x^2\operatorname{sgn}(ax+1)-1}{a^4}\right.$$

input
```
integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="g
iac")
```

output
```
1/4*sqrt(-c)*(16*log(abs(a*x + 1))*sgn(a*x + 1)/a^3 + (a^5*x^4*sgn(a*x + 1
) - 4*a^4*x^3*sgn(a*x + 1) + 8*a^3*x^2*sgn(a*x + 1) - 16*a^2*x*sgn(a*x + 1
))/a^4)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx=\int x^2\,\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.25

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{\sqrt{c}\,i(-16\log(ax+1)-a^4x^4+4a^3x^3-8a^2x^2+16ax-11)}{4a^3}$$

input
```
int(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*i*( - 16*log(a*x + 1) - a**4*x**4 + 4*a**3*x**3 - 8*a**2*x**2 + 1
6*a*x - 11))/(4*a**3)
```

3.705 $\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 151

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
4*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)-3/2*x*(-a^2*c*x^2+c)^(1/2)/
a/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)-4*(
-a^2*c*x^2+c)^(1/2)*ln(a*x+1)/a^3/(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.43

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}(ax(24-9ax+2a^2x^2)-24\log(1+ax))}{6a^3\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(x*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(a*x*(24 - 9*a*x + 2*a^2*x^2) - 24*Log[1 + a*x]))/(6*
a^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{x(1-ax)^2}{ax+1}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{86}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(ax^2-3x+\frac{4}{a}-\frac{4}{a(ax+1)}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(ax+1)}{a^2}+\frac{ax^3}{3}+\frac{4x}{a}-\frac{3x^2}{2}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(x*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((4*x)/a - (3*x^2)/2 + (a*x^3)/3 - (4*Log[1 + a*x])/a
^2))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{\left(-2 a^{3} x^{3}+9 a^{2} x^{2}-24 a x +24 \ln \left(a x +1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{6 a^{2} \left(a x -1\right)^{2}}$	76

input
```
int(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/6*(-2*a^3*x^3+9*a^2*x^2-24*a*x+24*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*
x+1)*((a*x-1)/(a*x+1))^(3/2)/a^2/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.28

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{(2\,a^3x^3-9\,a^2x^2+24\,ax-24\,\log{(ax+1)})\sqrt{-a^2c}}{6\,a^3}$$

input
```
integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fri
cas")
```

output
```
1/6*(2*a^3*x^3 - 9*a^2*x^2 + 24*a*x - 24*log(a*x + 1))*sqrt(-a^2*c)/a^3
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.48

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx =$$
$$-\frac{1}{6}\sqrt{-c}\left(\frac{24\log\left(|ax+1|\right)\text{sgn}(ax+1)}{a^2}-\frac{2\,a^4x^3\text{sgn}(ax+1)-9\,a^3x^2\text{sgn}(ax+1)+24\,a^2x\text{sgn}(ax+1)}{a^3}\right.$$

input
```
integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gia
c")
```

output
```
-1/6*sqrt(-c)*(24*log(abs(a*x + 1))*sgn(a*x + 1)/a^2 - (2*a^4*x^3*sgn(a*x
+ 1) - 9*a^3*x^2*sgn(a*x + 1) + 24*a^2*x*sgn(a*x + 1))/a^3)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \int x\,\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.25

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c}\,i(24\log(ax+1)-2a^3x^3+9a^2x^2-24ax+17)}{6a^2}$$

input
```
int(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*i*(24*log(a*x + 1) - 2*a**3*x**3 + 9*a**2*x**2 - 24*a*x + 17))/(6
*a**2)
```

3.706 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
-3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(
1-1/a^2/x^2)^(1/2)+4*(-a^2*c*x^2+c)^(1/2)*ln(a*x+1)/a^2/(1-1/a^2/x^2)^(1/2
)/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1+ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((-3*x)/a + x^2/2 + (4*Log[1 + a*x])/a^2))/(Sqrt[1 -
1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6746, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{ax+1}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(ax+\frac{4}{ax+1}-3\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{ax^2}{2}+\frac{4\log(ax+1)}{a}-3x\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-3*x + (a*x^2)/2 + (4*Log[1 + a*x])/a))/(a*Sqrt[1 -
1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\left(a^{2} x^{2}-6 a x+8 \ln \left(a x+1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x+1\right) \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{2 a \left(a x-1\right)^{2}}$	67

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a
*x+1))^(3/2)/a/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{(a^2x^2 - 6\,ax + 8\,\log(ax+1))\sqrt{-a^2c}}{2\,a^2}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

output
```
1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(-a^2*c)/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{1}{2}\sqrt{-c}\left(\frac{8\,\log\left(|ax+1|\right)\text{sgn}(ax+1)}{a}+\frac{a^3x^2\text{sgn}(ax+1)-6\,a^2x\text{sgn}(ax+1)}{a^2}\right)$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

output
```
1/2*sqrt(-c)*(8*log(abs(a*x + 1))*sgn(a*x + 1)/a + (a^3*x^2*sgn(a*x + 1) -
 6*a^2*x*sgn(a*x + 1))/a^2)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.27

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c}\,i(-8\log(ax+1)-a^2x^2+6ax-5)}{2a}$$

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*i*( - 8*log(a*x + 1) - a**2*x**2 + 6*a*x - 5))/(2*a)
```

3.707 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 112

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+(-a^2*c*x^2+c)^(1/2)*ln(x)/a/(1-1
/a^2/x^2)^(1/2)/x-4*(-a^2*c*x^2+c)^(1/2)*ln(a*x+1)/a/(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.45

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx=\frac{\sqrt{c-a^2cx^2}(ax+\log(x)-4\log(1+ax))}{a\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x
^2)]*x)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6746, 6747, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x(ax+1)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4a}{ax+1}+a+\frac{1}{x}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c-a^2cx^2}(ax-4\log(ax+1)+\log(x))}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x
^2)]*x)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\left(-a x +4 \ln \left(a x +1\right)-\ln \left(x \right)\right) \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{\left(a x -1\right)^{2}}$	61

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE
)
```

output
```
-(-c*(a^2*x^2-1))^(1/2)*(-a*x+4*ln(a*x+1)-ln(x))*(a*x+1)*((a*x-1)/(a*x+1))
^(3/2)/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.23

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{-a^2c}(ax-4\log{(ax+1)}+\log{(x)})}{a}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fri
cas")
```

output
```
sqrt(-a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\,(ax-1)\,(ax+1)}}{x}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

output
```
Integral(((a*x - 1)/(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/x, x)
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.37

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$$
$$= (ax\text{sgn}(ax+1) - 4\log(|ax+1|)\,\text{sgn}(ax+1) + \log(|x|)\,\text{sgn}(ax+1))\sqrt{-c}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="gia
c")
```

output
```
(a*x*sgn(a*x + 1) - 4*log(abs(a*x + 1))*sgn(a*x + 1) + log(abs(x))*sgn(a*x
 + 1))*sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.21

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \sqrt{c}\,i(4\log(ax+1)-\log(ax)-ax+1)$$

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x)
```

output
```
sqrt(c)*i*(4*log(a*x + 1) - log(a*x) - a*x + 1)
```

3.708 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 114

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
-(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^2-3*(-a^2*c*x^2+c)^(1/2)*ln(
x)/(1-1/a^2/x^2)^(1/2)/x+4*(-a^2*c*x^2+c)^(1/2)*ln(a*x+1)/(1-1/a^2/x^2)^(1
/2)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{ax}-3\log(x)+4\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-(1/(a*x)) - 3*Log[x] + 4*Log[1 + a*x]))/(Sqrt[1 - 1
/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x^2(ax+1)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(\frac{4a^2}{ax+1}-\frac{3a}{x}+\frac{1}{x^2}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(-3a\log(x)+4a\log(ax+1)-\frac{1}{x}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-x^(-1) - 3*a*Log[x] + 4*a*Log[1 + a*x]))/(a*Sqrt[1
- 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{(4\ln(ax+1)xa-3a\ln(x)x-1)\sqrt{-c(a^2x^2-1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(ax-1)^2x}$	64

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBO
SE)
```

output
```
(4*ln(a*x+1)*x*a-3*a*ln(x)*x-1)*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a
*x+1))^(3/2)/(a*x-1)^2/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{-a^2c}(4\,ax\log{(ax+1)} - 3\,ax\log{(x)} - 1)}{ax}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="f
ricas")
```

output
```
sqrt(-a^2*c)*(4*a*x*log(a*x + 1) - 3*a*x*log(x) - 1)/(a*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.40

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \left(4\,a\log\left(|ax+1|\right)\operatorname{sgn}(ax+1) - 3\,a\log\left(|x|\right)\operatorname{sgn}(ax+1) - \frac{\operatorname{sgn}(ax+1)}{x}\right)\sqrt{-c}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="g
iac")
```

output
```
(4*a*log(abs(a*x + 1))*sgn(a*x + 1) - 3*a*log(abs(x))*sgn(a*x + 1) - sgn(a
*x + 1)/x)*sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^2}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.27

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c}\,i(-4\log(ax+1)\,ax+3\log(ax)\,ax-ax+1)}{x}$$

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

output
```
(sqrt(c)*i*( - 4*log(a*x + 1)*a*x + 3*log(a*x)*a*x - a*x + 1))/x
```

3.709 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 152

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = -\frac{\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

output
```
-1/2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^3+3*(-a^2*c*x^2+c)^(1/2)
/(1-1/a^2/x^2)^(1/2)/x^2+4*a*(-a^2*c*x^2+c)^(1/2)*ln(x)/(1-1/a^2/x^2)^(1/2
)/x-4*a*(-a^2*c*x^2+c)^(1/2)*ln(a*x+1)/(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{2ax^2}+\frac{3}{x}+4a\log(x)-4a\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^3),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-1/2*1/(a*x^2) + 3/x + 4*a*Log[x] - 4*a*Log[1 + a*x]
))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x^3(ax+1)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4a^3}{ax+1}+\frac{4a^2}{x}-\frac{3a}{x^2}+\frac{1}{x^3}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(4a^2\log(x)-4a^2\log(ax+1)+\frac{3a}{x}-\frac{1}{2x^2}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^3),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-1/2*1/x^2 + (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1 +
a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{\left(8 a^{2} \ln \left(x \right) x^{2}-8 \ln \left(a x +1\right) x^{2} a^{2}+6 a x -1\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{2 x^{2} \left(a x -1\right)^{2}}$	77

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBO
SE)
```

output
```
1/2*(8*a^2*ln(x)*x^2-8*ln(a*x+1)*x^2*a^2+6*a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(
a*x+1)*((a*x-1)/(a*x+1))^(3/2)/x^2/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.58

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{8\,a^3\sqrt{-c}x^2\log\left(\frac{2\,a^3cx^2+2\,a^2cx+\sqrt{-a^2c}(2\,ax+1)\sqrt{-c}+ac}{ax^2+x}\right)+\sqrt{-a^2c}(6\,ax-1)}{2\,ax^2}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="f
ricas")
```

output
```
1/2*(8*a^3*sqrt(-c)*x^2*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 + 1)*sqrt(-c) + a*c)/(a*x^2 + x)) + sqrt(-a^2*c)*(6*a*x - 1))/(a*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.42

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = -\frac{1}{2}\left(8\,a^2\log\left(|ax+1|\right)\mathrm{sgn}(ax+1) - 8\,a^2\log\left(|x|\right)\mathrm{sgn}(ax+1) - \frac{6\,ax\mathrm{sgn}(ax+1) - \mathrm{sgn}(ax+1)}{x^2}\right)\sqrt{-c}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="g
iac")
```

output
```
-1/2*(8*a^2*log(abs(a*x + 1))*sgn(a*x + 1) - 8*a^2*log(abs(x))*sgn(a*x + 1
) - (6*a*x*sgn(a*x + 1) - sgn(a*x + 1))/x^2)*sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^3}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3, x)
```

Reduce [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \int \frac{\sqrt{-a^2c\,x^2+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}dx$$

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

output
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

3.710 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 193

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = -\frac{\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{3\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}x^3}-\frac{4a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{4a^2\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4a^2\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

output
```
-1/3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^4+3/2*(-a^2*c*x^2+c)^(1/
2)/(1-1/a^2/x^2)^(1/2)/x^3-4*a*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)/x^
2-4*a^2*(-a^2*c*x^2+c)^(1/2)*ln(x)/(1-1/a^2/x^2)^(1/2)/x+4*a^2*(-a^2*c*x^2
+c)^(1/2)*ln(a*x+1)/(1-1/a^2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.39

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{3ax^3}+\frac{3}{2x^2}-\frac{4a}{x}-4a^2\log(x)+4a^2\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-1/3*1/(a*x^3) + 3/(2*x^2) - (4*a)/x - 4*a^2*Log[x]
+ 4*a^2*Log[1 + a*x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x^4(ax+1)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(\frac{4a^4}{ax+1}-\frac{4a^3}{x}+\frac{4a^2}{x^2}-\frac{3a}{x^3}+\frac{1}{x^4}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(-4a^3\log(x)+4a^3\log(ax+1)-\frac{4a^2}{x}+\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-1/3*1/x^3 + (3*a)/(2*x^2) - (4*a^2)/x - 4*a^3*Log[x
] + 4*a^3*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.44

method	result	size
default	$\frac{\left(24 \ln \left(a x +1\right) x^{3} a^{3}-24 \ln \left(x \right) x^{3} a^{3}-24 a^{2} x^{2}+9 a x -2\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{6 \left(a x -1\right)^{2} x^{3}}$	85

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBO
SE)
```

output
```
1/6*(24*ln(a*x+1)*x^3*a^3-24*ln(x)*x^3*a^3-24*a^2*x^2+9*a*x-2)*(-c*(a^2*x^
2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.51

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{24\,a^4\sqrt{-c}x^3\log\left(\frac{2\,a^3cx^2+2\,a^2cx-\sqrt{-a^2c}(2\,ax+1)\sqrt{-c}+ac}{ax^2+x}\right)-(24\,a^2x^2-9\,ax+2)\sqrt{-a^2c}}{6\,ax^3}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="f
ricas")
```

output
```
1/6*(24*a^4*sqrt(-c)*x^3*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(-a^2*c)*(2*a*
x + 1)*sqrt(-c) + a*c)/(a*x^2 + x)) - (24*a^2*x^2 - 9*a*x + 2)*sqrt(-a^2*c
))/(a*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.40

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$$
$$= \frac{1}{6}\left(24\,a^3\log\left(|ax+1|\right)\operatorname{sgn}(ax+1) - 24\,a^3\log\left(|x|\right)\operatorname{sgn}(ax+1) - \frac{24\,a^2x^2\operatorname{sgn}(ax+1) - 9\,ax\operatorname{sgn}(ax+1}{x^3}\right.$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="g
iac")
```

output
```
1/6*(24*a^3*log(abs(a*x + 1))*sgn(a*x + 1) - 24*a^3*log(abs(x))*sgn(a*x +
1) - (24*a^2*x^2*sgn(a*x + 1) - 9*a*x*sgn(a*x + 1) + 2*sgn(a*x + 1))/x^3)*
sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^4}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.29

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c}\,i(-24\log(ax+1)\,a^3x^3+24\log(ax)\,a^3x^3-8a^3x^3+24a^2x^2-9ax+2)}{6x^3}$$

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

output
```
(sqrt(c)*i*( - 24*log(a*x + 1)*a**3*x**3 + 24*log(a*x)*a**3*x**3 - 8*a**3*
x**3 + 24*a**2*x**2 - 9*a*x + 2))/(6*x**3)
```

3.711 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 227

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx =& -\frac{\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^4}\\&-\frac{2a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}\\&+\frac{4a^3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4a^3\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}x}\end{aligned}$$

output
```
-1/4*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^5+(-a^2*c*x^2+c)^(1/2)/(
1-1/a^2/x^2)^(1/2)/x^4-2*a*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)/x^3+4*
a^2*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)/x^2+4*a^3*(-a^2*c*x^2+c)^(1/2
)*ln(x)/(1-1/a^2/x^2)^(1/2)/x-4*a^3*(-a^2*c*x^2+c)^(1/2)*ln(a*x+1)/(1-1/a^
2/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.35

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{4ax^4}+\frac{1}{x^3}-\frac{2a}{x^2}+\frac{4a^2}{x}+4a^3\log(x)-4a^3\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-1/4*1/(a*x^4) + x^(-3) - (2*a)/x^2 + (4*a^2)/x + 4*
a^3*Log[x] - 4*a^3*Log[1 + a*x]))/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.37, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}}{x^5}\,dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x^5(ax+1)}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\sqrt{c-a^2cx^2}\int\left(-\frac{4a^5}{ax+1}+\frac{4a^4}{x}-\frac{4a^3}{x^2}+\frac{4a^2}{x^3}-\frac{3a}{x^4}+\frac{1}{x^5}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(4a^4\log(x)-4a^4\log(ax+1)+\frac{4a^3}{x}-\frac{2a^2}{x^2}+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-1/4*1/x^4 + a/x^3 - (2*a^2)/x^2 + (4*a^3)/x + 4*a^4
*Log[x] - 4*a^4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.41

method	result	size
default	$-\frac{\left(16\ln\left(ax+1\right)x^{4}a^{4}-16\ln\left(x\right)x^{4}a^{4}-16a^{3}x^{3}+8a^{2}x^{2}-4ax+1\right)\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4x^{4}\left(ax-1\right)^{2}}$	93

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBO
SE)
```

output
```
-1/4*(16*ln(a*x+1)*x^4*a^4-16*ln(x)*x^4*a^4-16*a^3*x^3+8*a^2*x^2-4*a*x+1)*
(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/x^4/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.46

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$$
$$=\frac{16\,a^5\sqrt{-c}x^4\log\left(\frac{2\,a^3cx^2+2\,a^2cx+\sqrt{-a^2c}(2\,ax+1)\sqrt{-c}+ac}{ax^2+x}\right)+(16\,a^3x^3-8\,a^2x^2+4\,ax-1)\sqrt{-a^2c}}{4\,ax^4}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="f
ricas")
```

output
```
1/4*(16*a^5*sqrt(-c)*x^4*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(-a^2*c)*(2*a*
x + 1)*sqrt(-c) + a*c)/(a*x^2 + x)) + (16*a^3*x^3 - 8*a^2*x^2 + 4*a*x - 1)
*sqrt(-a^2*c))/(a*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx =$$
$$-\frac{1}{4}\left(16\,a^4\log\left(|ax+1|\right)\operatorname{sgn}(ax+1) - 16\,a^4\log\left(|x|\right)\operatorname{sgn}(ax+1) - \frac{16\,a^3x^3\operatorname{sgn}(ax+1) - 8\,a^2x^2\operatorname{sgn}(ax\right.$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="g
iac")
```

output
```
-1/4*(16*a^4*log(abs(a*x + 1))*sgn(a*x + 1) - 16*a^4*log(abs(x))*sgn(a*x +
 1) - (16*a^3*x^3*sgn(a*x + 1) - 8*a^2*x^2*sgn(a*x + 1) + 4*a*x*sgn(a*x +
1) - sgn(a*x + 1))/x^4)*sqrt(-c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^5}\,dx$$

input
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
```

output
```
int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.28

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c}\,i(16\log(ax+1)\,a^4x^4-16\log(ax)\,a^4x^4+4a^4x^4-16a^3x^3+8a^2x^2-4ax+1)}{4x^4}$$

input
```
int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x)
```

output
```
(sqrt(c)*i*(16*log(a*x + 1)*a**4*x**4 - 16*log(a*x)*a**4*x**4 + 4*a**4*x**
4 - 16*a**3*x**3 + 8*a**2*x**2 - 4*a*x + 1))/(4*x**4)
```

3.712 $\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 136

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}} - \frac{4x^m\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}(1,1+m,2+m,ax)}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
3*x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2
+c)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)-4*x^m*(-a^2*c*x^2+c)^(1/2)*hypergeom([
1, 1+m],[2+m],a*x)/a/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{x^m\sqrt{c-a^2cx^2}(6+ax+m(3+ax)-4(2+m)\,\text{Hypergeometric2F1}(1,1+m,2+m,ax))}{a(1+m)(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

output
```
(x^m*Sqrt[c - a^2*c*x^2]*(6 + a*x + m*(3 + a*x) - 4*(2 + m)*Hypergeometric
2F1[1, 1 + m, 2 + m, a*x]))/(a*(1 + m)*(2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.63, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6746, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\sqrt{c-a^2cx^2}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^{m+1}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -\frac{x^m(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{x^m(ax+1)^2}{1-ax}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(\frac{4x^m}{1-ax}-3x^m-ax^{m+1}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(\frac{4x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,ax)}{m+1}-\frac{ax^{m+2}}{m+2}-\frac{3x^{m+1}}{m+1}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*((-3*x^(1 + m))/(1 + m) - (a*x^(2 + m))/(2 + m) + (
4*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(1 + m)))/(a*Sqrt[1 -
 1/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [F]

$$\int \frac{x^m\sqrt{-a^2c\,x^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{-a^2cx^2+c}x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 2*a*x + 1)*x^m*sqrt((a*x - 1)/(a*
x + 1))/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \int \frac{\sqrt{-a^2cx^2+c}x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int \frac{x^m\sqrt{c-a^2\,c\,x^2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((x^m*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^m*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [F]

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{\sqrt{c}\left(x^m\sqrt{ax-1}\,\sqrt{-ax+1}\,a^2m^2x^2+x^m\sqrt{ax-1}\,\sqrt{-ax+1}\,a^2m\,x^2+3x^m\sqrt{ax-1}\,\sqrt{-ax+1}\,a\,m^2x+\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*(x**m*sqrt(a*x - 1)*sqrt( - a*x + 1)*a**2*m**2*x**2 + x**m*sqrt(a
*x - 1)*sqrt( - a*x + 1)*a**2*m*x**2 + 3*x**m*sqrt(a*x - 1)*sqrt( - a*x +
1)*a*m**2*x + 6*x**m*sqrt(a*x - 1)*sqrt( - a*x + 1)*a*m*x + 4*x**m*sqrt(a*
x - 1)*sqrt( - a*x + 1)*m**2 + 12*x**m*sqrt(a*x - 1)*sqrt( - a*x + 1)*m +
8*x**m*sqrt(a*x - 1)*sqrt( - a*x + 1) + 4*int((x**m*sqrt(a*x - 1)*sqrt( -
a*x + 1))/(a**2*x**3 - 2*a*x**2 + x),x)*a*m**3*x + 12*int((x**m*sqrt(a*x -
 1)*sqrt( - a*x + 1))/(a**2*x**3 - 2*a*x**2 + x),x)*a*m**2*x + 8*int((x**m
*sqrt(a*x - 1)*sqrt( - a*x + 1))/(a**2*x**3 - 2*a*x**2 + x),x)*a*m*x - 4*i
nt((x**m*sqrt(a*x - 1)*sqrt( - a*x + 1))/(a**2*x**3 - 2*a*x**2 + x),x)*m**
3 - 12*int((x**m*sqrt(a*x - 1)*sqrt( - a*x + 1))/(a**2*x**3 - 2*a*x**2 + x
),x)*m**2 - 8*int((x**m*sqrt(a*x - 1)*sqrt( - a*x + 1))/(a**2*x**3 - 2*a*x
**2 + x),x)*m))/(a*m*(a*m**2*x + 3*a*m*x + 2*a*x - m**2 - 3*m - 2))
```

3.713 $\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 172

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}\\ &\quad-\frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}}\\ &\quad-\frac{2acx^{2+m}\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2x^2\right)}{(2+m)\sqrt{c-a^2cx^2}}\end{aligned}$$

output
```
x^(1+m)*(-a^2*c*x^2+c)^(1/2)/(2+m)-c*(3+2*m)*x^(1+m)*(-a^2*x^2+1)^(1/2)*hy
pergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)/(2+m)/(-a^2*c*x^2+c)^(
1/2)-2*a*c*x^(2+m)*(-a^2*x^2+1)^(1/2)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],a
^2*x^2)/(2+m)/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.75

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$$
$$=\frac{x^{1+m}\left(\frac{2\sqrt{1-ax}\sqrt{-c(1+ax)}\,\text{AppellF1}\left(1+m,\frac{1}{2},-\frac{1}{2},2+m,ax,-ax\right)}{\sqrt{-1+ax}\sqrt{1+ax}}+\frac{\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2x^2\right)}{\sqrt{1-a^2x^2}}\right)}{1+m}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

output
```
(x^(1 + m)*((2*Sqrt[1 - a*x]*Sqrt[-(c*(1 + a*x))]*AppellF1[1 + m, 1/2, -1/
2, 2 + m, a*x, -(a*x)])/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[c - a^2*c*x
^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/Sqrt[1 - a^2*x
^2]))/(1 + m)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6717, 6701, 559, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\sqrt{c-a^2cx^2}e^{2\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{6717}$$
$$-\int e^{2\text{arctanh}(ax)}x^m\sqrt{c-a^2cx^2}dx$$
$$\Big\downarrow \text{6701}$$
$$-c\int \frac{x^m(ax+1)^2}{\sqrt{c-a^2cx^2}}dx$$
$$\Big\downarrow \text{559}$$

$$-c\left(-\frac{\int -\frac{a^2cx^m(2m+2a(m+2)x+3)}{\sqrt{c-a^2cx^2}}dx}{a^2c(m+2)}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\int \frac{a^2cx^m(2m+2a(m+2)x+3)}{\sqrt{c-a^2cx^2}}dx}{a^2c(m+2)}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{\int \frac{x^m(2m+2a(m+2)x+3)}{\sqrt{c-a^2cx^2}}dx}{m+2}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$$\Big\downarrow \text{557}$$

$$-c\left(\frac{2a(m+2)\int \frac{x^{m+1}}{\sqrt{c-a^2cx^2}}dx+(2m+3)\int \frac{x^m}{\sqrt{c-a^2cx^2}}dx}{m+2}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$$\Big\downarrow \text{279}$$

$$-c\left(\frac{\frac{2a(m+2)\sqrt{1-a^2x^2}\int \frac{x^{m+1}}{\sqrt{1-a^2x^2}}dx}{\sqrt{c-a^2cx^2}}+\frac{(2m+3)\sqrt{1-a^2x^2}\int \frac{x^m}{\sqrt{1-a^2x^2}}dx}{\sqrt{c-a^2cx^2}}}{m+2}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$$\Big\downarrow \text{278}$$

$$-c\left(\frac{\frac{(2m+3)\sqrt{1-a^2x^2}x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{(m+1)\sqrt{c-a^2cx^2}}+\frac{2a\sqrt{1-a^2x^2}x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{\sqrt{c-a^2cx^2}}}{m+2}-\frac{x^m}{}\right.$$

input
```
Int[E^(2*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

output
```
-(c*(-((x^(1 + m)*Sqrt[c - a^2*c*x^2])/(c*(2 + m))) + (((3 + 2*m)*x^(1 + m
)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])
/((1 + m)*Sqrt[c - a^2*c*x^2]) + (2*a*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeo
metric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/Sqrt[c - a^2*c*x^2])/(2 + m
)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 559
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^n*(e*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*e^(n - 1)*(
m + n + 2*p + 1))), x] + Simp[1/(b*(m + n + 2*p + 1))   Int[(e*x)^m*(a + b*
x^2)^p*ExpandToSum[b*(m + n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1
)*x^n - a*d^n*(m + n - 1)*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && IGtQ[n, 1] &&  !IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 6701
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^(n/2)   Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
 /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ
[c, 0]) && IGtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [F]

$$\int \frac{(ax+1)\, x^{m} \sqrt{-a^{2} c\, x^{2}+c}}{ax-1}dx$$

input
```
int(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int e^{2\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\, dx = \int \frac{\sqrt{-a^2cx^2+c}(ax+1)x^m}{ax-1}\, dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/(a*x - 1), x)
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\, dx = \int \frac{x^m \sqrt{-c\,(ax-1)\,(ax+1)}(ax+1)}{ax-1}\, dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int\frac{\sqrt{-a^2cx^2+c}(ax+1)x^m}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/(a*x - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int\frac{x^m\,\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [F]

$$\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\sqrt{c}\left(\left(\int\frac{x^m\sqrt{-a^2x^2+1}\,x}{ax-1}dx\right)a+\int\frac{x^m\sqrt{-a^2x^2+1}}{ax-1}dx\right)$$

input
```
int(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
sqrt(c)*(int((x**m*sqrt( - a**2*x**2 + 1)*x)/(a*x - 1),x)*a + int((x**m*sq
rt( - a**2*x**2 + 1))/(a*x - 1),x))
```

3.714 $\int e^{\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 82

$$\int e^{\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx = \frac{x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2+c
)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.68

$$\int e^{\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx = \frac{x^m(2+m+ax+amx)\sqrt{c-a^2cx^2}}{a(1+m)(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*x^m*Sqrt[c - a^2*c*x^2],x]
```

output
```
(x^m*(2 + m + a*x + a*m*x)*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*(2 + m)*Sqrt[1
- 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6746, 6747, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{c-a^2cx^2}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^{m+1}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int x^m(ax+1)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{c-a^2cx^2}\int \left(x^m+ax^{m+1}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{ax^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*x^m*Sqrt[c - a^2*c*x^2],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(x^(1 + m)/(1 + m) + (a*x^(2 + m))/(2 + m)))/(a*Sqrt[
1 - 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

method	result	size
orering	$\frac{x(amx+ax+m+2)x^{m}\sqrt{-a^{2}c\,x^{2}+c}}{(2+m)(1+m)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	61
gosper	$\frac{x^{1+m}\sqrt{-a^{2}c\,x^{2}+c}\,(amx+ax+m+2)}{(1+m)(2+m)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	62
risch	$-\frac{\sqrt{-\frac{c\left(a^{2}x^{2}-1\right)}{(ax-1)(ax+1)}}\,(ax-1)c(amx+ax+m+2)x\,x^{m}}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\sqrt{-c}\,(2+m)(1+m)}$	95

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
x*(a*m*x+a*x+m+2)*x^m*(-a^2*c*x^2+c)^(1/2)/(2+m)/(1+m)/(a*x+1)/((a*x-1)/(a
*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = -\frac{\sqrt{-a^2cx^2+c}((am+a)x^2+(m+2)x)x^m\sqrt{\frac{ax-1}{ax+1}}}{m^2-(am^2+3\,am+2\,a)x+3\,m+2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

output
```
-sqrt(-a^2*c*x^2 + c)*((a*m + a)*x^2 + (m + 2)*x)*x^m*sqrt((a*x - 1)/(a*x
+ 1))/(m^2 - (a*m^2 + 3*a*m + 2*a)*x + 3*m + 2)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \int\frac{x^m\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{\left(a\sqrt{-c}(m+1)x^2+\sqrt{-c}(m+2)x\right)(ax+1)x^m}{(m^2+3\,m+2)ax+m^2+3\,m+2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

output
```
(a*sqrt(-c)*(m + 1)*x^2 + sqrt(-c)*(m + 2)*x)*(a*x + 1)*x^m/((m^2 + 3*m +
2)*a*x + m^2 + 3*m + 2)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,1,1,0]%%%}+%%%{1,[0,1,0,0,0]%%%} / %%%{1,[0,0,0,0,1
]%%%} Err
```

Mupad [B] (verification not implemented)

Time = 13.55 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.13

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(\frac{x^m\,x^2\,\sqrt{c-a^2\,c\,x^2}\,(m+1)}{m^2+3\,m+2}+\frac{x\,x^m\,\sqrt{c-a^2\,c\,x^2}\,(m+2)}{a\,(m^2+3\,m+2)}\right)}{x-\frac{1}{a}}$$

input
```
int((x^m*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(((a*x - 1)/(a*x + 1))^(1/2)*((x^m*x^2*(c - a^2*c*x^2)^(1/2)*(m + 1))/(3*m
 + m^2 + 2) + (x*x^m*(c - a^2*c*x^2)^(1/2)*(m + 2))/(a*(3*m + m^2 + 2))))/
(x - 1/a)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.39

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\frac{x^m\sqrt{c}\,ix(-amx-ax-m-2)}{m^2+3m+2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
(x**m*sqrt(c)*i*x*( - a*m*x - a*x - m - 2))/(m**2 + 3*m + 2)
```

3.715 $\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 83

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=-\frac{x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2+
c)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\left(-\frac{x^{1+m}}{a(1+m)}+\frac{x^{2+m}}{2+m}\right)}{\sqrt{1-\frac{1}{a^2x^2}}x}$$

input
```
Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - a^2*c*x^2]*(-(x^(1 + m)/(a*(1 + m))) + x^(2 + m)/(2 + m)))/(Sqrt
[1 - 1/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6746, 6747, 25, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{c-a^2cx^2}e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^{m+1}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int -x^m(1-ax)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-a^2cx^2}\int x^m(1-ax)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{53}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(x^m-ax^{m+1}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(\frac{x^{m+1}}{m+1}-\frac{ax^{m+2}}{m+2}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(x^m*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(x^(1 + m)/(1 + m) - (a*x^(2 + m))/(2 + m)))/(a*Sqr
t[1 - 1/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

method	result	size
orering	$\frac{x(amx+ax-m-2)x^{m}\sqrt{-a^{2}c\,x^{2}+c}\,\sqrt{\frac{ax-1}{ax+1}}}{(2+m)(1+m)(ax-1)}$	63
gosper	$\frac{x^{1+m}\sqrt{-a^{2}c\,x^{2}+c}\,(amx+ax-m-2)\sqrt{\frac{ax-1}{ax+1}}}{(1+m)(2+m)(ax-1)}$	64
risch	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-\frac{c\left(a^{2}x^{2}-1\right)}{(ax-1)(ax+1)}}\,(ax+1)c(amx+ax-m-2)x\,x^{m}}{\sqrt{-c(a^{2}x^{2}-1)}\,\sqrt{-c}\,(2+m)(1+m)}$	97

input
```
int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
x*(a*m*x+a*x-m-2)*x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(2+m)/(
1+m)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int e^{-\coth^{-1}(ax)}x^{m}\sqrt{c-a^{2}cx^{2}}\,dx=-\frac{\sqrt{-a^{2}cx^{2}+c}((am+a)x^{2}-(m+2)x)x^{m}\sqrt{\frac{ax-1}{ax+1}}}{m^{2}-(am^{2}+3\,am+2\,a)x+3\,m+2}$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="f
ricas")
```

output
```
-sqrt(-a^2*c*x^2 + c)*((a*m + a)*x^2 - (m + 2)*x)*x^m*sqrt((a*x - 1)/(a*x
+ 1))/(m^2 - (a*m^2 + 3*a*m + 2*a)*x + 3*m + 2)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

$$\int e^{-\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx = \frac{\left(a\sqrt{-c}(m+1)x^2 - \sqrt{-c}(m+2)x\right)(ax-1)x^m}{(m^2+3\,m+2)ax - m^2 - 3\,m - 2}$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="m
axima")
```

output
```
(a*sqrt(-c)*(m + 1)*x^2 - sqrt(-c)*(m + 2)*x)*(a*x - 1)*x^m/((m^2 + 3*m +
2)*a*x - m^2 - 3*m - 2)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx = \int \sqrt{-a^2cx^2+c}\,x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 13.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(\frac{x^m\,x^2\,\sqrt{c-a^2\,c\,x^2}\,(m+1)}{m^2+3\,m+2}-\frac{x\,x^m\,\sqrt{c-a^2\,c\,x^2}\,(m+2)}{a\,(m^2+3\,m+2)}\right)}{x-\frac{1}{a}}$$

input
```
int(x^m*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(((a*x - 1)/(a*x + 1))^(1/2)*((x^m*x^2*(c - a^2*c*x^2)^(1/2)*(m + 1))/(3*m
 + m^2 + 2) - (x*x^m*(c - a^2*c*x^2)^(1/2)*(m + 2))/(a*(3*m + m^2 + 2))))/
(x - 1/a)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.36

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{x^m\sqrt{c}\,ix(-amx-ax+m+2)}{m^2+3m+2}$$

input
```
int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(x**m*sqrt(c)*i*x*( - a*m*x - a*x + m + 2))/(m**2 + 3*m + 2)
```

3.716 $\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 172

$$\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} - \frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}} + \frac{2acx^{2+m}\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2x^2\right)}{(2+m)\sqrt{c-a^2cx^2}}$$

output

```
x^(1+m)*(-a^2*c*x^2+c)^(1/2)/(2+m)-c*(3+2*m)*x^(1+m)*(-a^2*x^2+1)^(1/2)*hy
pergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)/(2+m)/(-a^2*c*x^2+c)^(
1/2)+2*a*c*x^(2+m)*(-a^2*x^2+1)^(1/2)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],a
^2*x^2)/(2+m)/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.64

$$\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{x^{1+m}\left(-\frac{2\sqrt{c-acx}\operatorname{AppellF1}\left(1+m,\frac{1}{2},-\frac{1}{2},2+m,-ax,ax\right)}{\sqrt{1-ax}}+\frac{\sqrt{c-a^2cx^2}\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},a^2x^2\right)}{\sqrt{1-a^2x^2}}\right)}{1+m}$$

input
```
Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

output
```
(x^(1 + m)*((-2*Sqrt[c - a*c*x]*AppellF1[1 + m, 1/2, -1/2, 2 + m, -(a*x),
a*x])/Sqrt[1 - a*x] + (Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[-1/2, (1 + m)
/2, (3 + m)/2, a^2*x^2])/Sqrt[1 - a^2*x^2]))/(1 + m)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6717, 6702, 559, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\sqrt{c-a^2cx^2}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{-2\text{arctanh}(ax)}x^m\sqrt{c-a^2cx^2}dx$$

$$\Big\downarrow \text{ 6702}$$

$$-c\int \frac{x^m(1-ax)^2}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{ 559}$$

$$-c\left(-\frac{\int -\frac{a^2cx^m(2m-2a(m+2)x+3)}{\sqrt{c-a^2cx^2}}dx}{a^2c(m+2)}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$\Big\downarrow$ 25

$$-c\left(\frac{\int \frac{a^2cx^m(2m-2a(m+2)x+3)}{\sqrt{c-a^2cx^2}}dx}{a^2c(m+2)}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$\Big\downarrow$ 27

$$-c\left(\frac{\int \frac{x^m(2m-2a(m+2)x+3)}{\sqrt{c-a^2cx^2}}dx}{m+2}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$\Big\downarrow$ 557

$$-c\left(\frac{(2m+3)\int \frac{x^m}{\sqrt{c-a^2cx^2}}dx-2a(m+2)\int \frac{x^{m+1}}{\sqrt{c-a^2cx^2}}dx}{m+2}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$\Big\downarrow$ 279

$$-c\left(\frac{\frac{(2m+3)\sqrt{1-a^2x^2}\int \frac{x^m}{\sqrt{1-a^2x^2}}dx}{\sqrt{c-a^2cx^2}}-\frac{2a(m+2)\sqrt{1-a^2x^2}\int \frac{x^{m+1}}{\sqrt{1-a^2x^2}}dx}{\sqrt{c-a^2cx^2}}}{m+2}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{c(m+2)}\right)$$

$\Big\downarrow$ 278

$$-c\left(\frac{\frac{(2m+3)\sqrt{1-a^2x^2}x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{(m+1)\sqrt{c-a^2cx^2}}-\frac{2a\sqrt{1-a^2x^2}x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{\sqrt{c-a^2cx^2}}}{m+2}-\frac{x^m}{}\right.$$

input
```
Int[(x^m*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c*(-((x^(1 + m)*Sqrt[c - a^2*c*x^2])/(c*(2 + m))) + (((3 + 2*m)*x^(1 + m
)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])
/((1 + m)*Sqrt[c - a^2*c*x^2]) - (2*a*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeo
metric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/Sqrt[c - a^2*c*x^2])/(2 + m
)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 559
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[d^n*(e*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*e^(n - 1)*(
m + n + 2*p + 1))), x] + Simp[1/(b*(m + n + 2*p + 1))   Int[(e*x)^m*(a + b*
x^2)^p*ExpandToSum[b*(m + n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1
)*x^n - a*d^n*(m + n - 1)*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && IGtQ[n, 1] &&  !IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 6702
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^(n/2)   Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x]
, x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
 GtQ[c, 0]) && ILtQ[n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [F]

$$\int \frac{x^m \sqrt{-a^2 c\, x^2 + c}\, (a x - 1)}{a x + 1} dx$$

input
```
int(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
int(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

Fricas [F]

$$\int e^{-2\coth^{-1}(ax)} x^m \sqrt{c - a^2 c x^2}\, dx = \int \frac{\sqrt{-a^2 c x^2 + c}(a x - 1) x^m}{a x + 1}\, dx$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m/(a*x + 1), x)
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)} x^m \sqrt{c - a^2 c x^2}\, dx = \int \frac{x^m \sqrt{-c\,(a x - 1)\,(a x + 1)}(a x - 1)}{a x + 1}\, dx$$

input
```
integrate(x**m*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int \frac{\sqrt{-a^2cx^2+c}(ax-1)x^m}{ax+1}\,dx$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int \frac{x^m\,\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [F]

$$\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\sqrt{c}\left(\left(\left(\int\frac{x^m\sqrt{-a^2x^2+1}\,x}{ax+1}dx\right)a-\left(\int\frac{x^m\sqrt{-a^2x^2+1}}{ax+1}dx\right)\right)\right.$$

input
```
int(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
sqrt(c)*(int((x**m*sqrt( - a**2*x**2 + 1)*x)/(a*x + 1),x)*a - int((x**m*sq
rt( - a**2*x**2 + 1))/(a*x + 1),x))
```

3.717 $\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 137

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = -\frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4x^m\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}(1,1+m,2+m,-ax)}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-3*x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^
2+c)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)+4*x^m*(-a^2*c*x^2+c)^(1/2)*hypergeom(
[1, 1+m],[2+m],-a*x)/a/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.55

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{x^m\sqrt{c-a^2cx^2}(-6+ax+m(-3+ax)+4(2+m)\,\text{Hypergeometric2F1}(1,1+m,2+m,-ax))}{a(1+m)(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

output
```
(x^m*Sqrt[c - a^2*c*x^2]*(-6 + a*x + m*(-3 + a*x) + 4*(2 + m)*Hypergeometr
ic2F1[1, 1 + m, 2 + m, -(a*x)]))/(a*(1 + m)*(2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6746, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\sqrt{c-a^2cx^2}e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^{m+1}dx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{x^m(1-ax)^2}{ax+1}dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-a^2cx^2}\int\left(\frac{4x^m}{ax+1}-3x^m+ax^{m+1}\right)dx}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{4x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-ax)}{m+1}+\frac{ax^{m+2}}{m+2}-\frac{3x^{m+1}}{m+1}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(x^m*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - a^2*c*x^2]*((-3*x^(1 + m))/(1 + m) + (a*x^(2 + m))/(2 + m) + (4*
x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/(1 + m)))/(a*Sqrt[1
- 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Maple [F]

$$\int x^m \sqrt{-a^2 c\,x^2+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

input
```
int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

Fricas [F]

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="f
ricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x
 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\text{Timed out}$$

input
```
integrate(x**m*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int \sqrt{-a^2cx^2+c}x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="m
axima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int \sqrt{-a^2cx^2+c}x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="g
iac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\int x^m\,\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int(x^m*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x^m*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [F]

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\sqrt{c}\left(\left(\int\frac{x^m\sqrt{ax-1}\,\sqrt{-a^2x^2+1}\,x}{\sqrt{ax+1}\,ax+\sqrt{ax+1}}dx\right)a-\left(\int\frac{x^m\sqrt{ax-1}\,\sqrt{-a^2x^2+1}}{\sqrt{ax+1}\,ax+\sqrt{ax+1}}dx\right)\right)$$

input
```
int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
sqrt(c)*(int((x**m*sqrt(a*x - 1)*sqrt( - a**2*x**2 + 1)*x)/(sqrt(a*x + 1)*
a*x + sqrt(a*x + 1)),x)*a - int((x**m*sqrt(a*x - 1)*sqrt( - a**2*x**2 + 1)
)/(sqrt(a*x + 1)*a*x + sqrt(a*x + 1)),x))
```

3.718 $\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx=\frac{32\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-5+n)}(c-a^2cx^2)^{3/2}\,\text{Hypergeometric2F1}\left(5,\frac{5-n}{2},\frac{7-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^4(5-n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

output
```
32*(1-1/a/x)^(5/2-1/2*n)*(1+1/a/x)^(-5/2+1/2*n)*(-a^2*c*x^2+c)^(3/2)*hyper
geom([5, 5/2-1/2*n],[7/2-1/2*n],(a-1/x)/(a+1/x))/a^4/(5-n)/(1-1/a^2/x^2)^(
3/2)/x^3
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 280 vs. $2(116)=232$.

Time = 3.06 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.41

$$\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx=\frac{c^2\left(96a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(ae^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x(n+ax)+2e^{(1+n)\coth^{-1}(ax)}(-1+n)\text{H}\right.\right.}{}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

output
```
(c^2*(96*a^3*c*(1 - 1/(a^2*x^2))^(3/2)*x^3*(a*E^(n*ArcCoth[a*x])*Sqrt[1 -
1/(a^2*x^2)]*x*(n + a*x) + 2*E^((1 + n)*ArcCoth[a*x])*(-1 + n)*Hypergeomet
ric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])]) - c*(-1 + a^2*x^2)*(2
*E^(n*ArcCoth[a*x])*(-1 + a^2*x^2)^2*(-(a*(-21 + n^2)*x) + 2*n*(1 - n^2 +
(3 + n^2)*Cosh[2*ArcCoth[a*x]]) + a*(3 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh
[3*ArcCoth[a*x]]) + 16*a*E^((1 + n)*ArcCoth[a*x])*(-3 + 3*n - n^2 + n^3)*S
qrt[1 - 1/(a^2*x^2)]*x*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*Arc
Coth[a*x])])))/(192*a*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6746, 6749, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^{3/2}e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3dx}{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{ 6749}$$

$$-\frac{\left(c-a^2cx^2\right)^{3/2}\int \left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n+3}{2}}x^5d\frac{1}{x}}{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{ 141}$$

$$\frac{32\left(c-a^2cx^2\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-5}{2}}\text{Hypergeometric2F1}\left(5,\frac{5-n}{2},\frac{7-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^4(5-n)x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

output
```
(32*(1 - 1/(a*x))^((5 - n)/2)*(1 + 1/(a*x))^((-5 + n)/2)*(c - a^2*c*x^2)^(
3/2)*Hypergeometric2F1[5, (5 - n)/2, (7 - n)/2, (a - x^(-1))/(a + x^(-1))]
)/(a^4*(5 - n)*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6749
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/
x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d,
0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
 n/2] && IntegerQ[m]
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}dx$$

input
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*
n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(-c(ax-1)\,(ax+1)\right)^{\frac{3}{2}}e^{n\operatorname{acoth}(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\int\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a^2*c*x^2 + c)^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \int e^{n\,\mathrm{acoth}(a\,x)}\,\left(c-a^2\,c\,x^2\right)^{3/2}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = \sqrt{c}\,c\left(-\left(\int e^{acoth(ax)n}\sqrt{-a^2x^2+1}\,x^2dx\right)a^2 + \int e^{acoth(ax)n}\sqrt{-a^2x^2+1}dx\right)$$

input
```
int(exp(n*acoth(a*x))*(-a^2*c*x^2+c)^(3/2),x)
```

output
```
sqrt(c)*c*( - int(e**(acoth(a*x)*n)*sqrt( - a**2*x**2 + 1)*x**2,x)*a**2 +
int(e**(acoth(a*x)*n)*sqrt( - a**2*x**2 + 1),x))
```

3.719 $\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{8\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(3,\frac{3-n}{2},\frac{5-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(3-n)\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

output
```
8*(1-1/a/x)^(3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*(-a^2*c*x^2+c)^(1/2)*hyperg
eom([3, 3/2-1/2*n],[5/2-1/2*n],(a-1/x)/(a+1/x))/a^2/(3-n)/(1-1/a^2/x^2)^(1
/2)/x
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.87

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = -\frac{ce^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(n+ax)+2e^{\coth^{-1}(ax)}(-1+n)\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3-}{}\right.\right.}{2\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
-1/2*(c*E^(n*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*x*(a*Sqrt[1 - 1/(a^2*x^2)
]*x*(n + a*x) + 2*E^ArcCoth[a*x]*(-1 + n)*Hypergeometric2F1[1, (1 + n)/2,
(3 + n)/2, E^(2*ArcCoth[a*x])]))/Sqrt[c - a^2*c*x^2]
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6746, 6749, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-a^2cx^2}e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{\sqrt{c-a^2cx^2}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6749}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n+1}{2}}x^3d\frac{1}{x}}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{141}$$

$$\frac{8\sqrt{c-a^2cx^2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\text{Hypergeometric2F1}\left(3,\frac{3-n}{2},\frac{5-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(3-n)x\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(n*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

output
```
(8*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*Sqrt[c - a^2*c*x^2
]*Hypergeometric2F1[3, (3 - n)/2, (5 - n)/2, (a - x^(-1))/(a + x^(-1))])/(
a^2*(3 - n)*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6749
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/
x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d,
0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
 n/2] && IntegerQ[m]
```

Maple [F]

$$\int \mathrm{e}^{n\ \mathrm{arccoth}(ax)}\sqrt{-a^2c\,x^2+c}dx$$

input
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-c\,(ax-1)\,(ax+1)}e^{n\operatorname{acoth}(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\int\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \int e^{n\,\mathrm{acoth}(a\,x)}\,\sqrt{c-a^2\,c\,x^2}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \sqrt{c}\left(\int e^{acoth(ax)n}\sqrt{-a^2x^2+1}dx\right)$$

input
```
int(exp(n*acoth(a*x))*(-a^2*c*x^2+c)^(1/2),x)
```

output
```
sqrt(c)*int(e**(acoth(a*x)*n)*sqrt( - a**2*x**2 + 1),x)
```

3.720 $\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x\,\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

output
```
2*(1-1/a^2/x^2)^(1/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x*hyper
geom([1, 1/2-1/2*n],[3/2-1/2*n],(a-1/x)/(a+1/x))/(1-n)/(-a^2*c*x^2+c)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.73

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2e^{(1+n)\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},e^{2\coth^{-1}(ax)}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,(a^2cx+a^2cnx)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

output
```
(-2*E^((1 + n)*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[1, (1 +
 n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(Sqrt[1 - 1/(a^2*x^2)]*(a^2*c*x + a
^2*c*n*x))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6746, 6749, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}dx}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 6749}$$

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}xd\frac{1}{x}}{\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 141}$$

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

output
```
(2*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)
/2)*x*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))
])/((1 - n)*Sqrt[c - a^2*c*x^2])
```

Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6749
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/
x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d,
0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
 n/2] && IntegerQ[m]
```

Maple [F]

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{\sqrt{-a^2c\,x^2 + c}}dx$$

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 -
c), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{e^{n\operatorname{acoth}(ax)}}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\int \frac{e^{acoth(ax)n}}{\sqrt{-a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(exp(n*acoth(a*x))/(-a^2*c*x^2+c)^(1/2),x)
```

output
```
int(e**(acoth(a*x)*n)/sqrt( - a**2*x**2 + 1),x)/sqrt(c)
```

3.721 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=-\frac{e^{n \coth^{-1}(ax)}(n-ax)}{ac(1-n^2)\sqrt{c-a^2cx^2}}$$

output
```
-exp(n*arccoth(a*x))*(-a*x+n)/a/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\frac{e^{n \coth^{-1}(ax)}(n-ax)}{ac(-1+n^2)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 6738}$$
$$-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac(1-n^2)\sqrt{c-a^2cx^2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))
```

Defintions of rubi rules used

rule 6738
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
gosper	$\frac{(ax-1)(ax+1)(ax-n)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{(n^2-1)a(-a^2c\,x^2+c)^{\frac{3}{2}}}$	49
orering	$\frac{(ax-1)(ax+1)(ax-n)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{(n^2-1)a(-a^2c\,x^2+c)^{\frac{3}{2}}}$	49

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)*(a*x+1)*(a*x-n)*exp(n*arccoth(a*x))/(n^2-1)/a/(-a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{-a^2cx^2+c}(ax-n)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^2n^2-ac^2-(a^3c^2n^2-a^3c^2)x^2}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(-a^2*c*x^2 + c)*(a*x - n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n^2 -
 a*c^2 - (a^3*c^2*n^2 - a^3*c^2)*x^2)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.70

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx=-\frac{\left(\frac{x}{c\,(n^2-1)}-\frac{n}{a\,c\,(n^2-1)}\right)\,\left(\frac{a\,x+1}{a\,x}\right)^{n/2}}{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(3/2),x)
```

output
```
-((x/(c*(n^2 - 1)) - n/(a*c*(n^2 - 1)))*((a*x + 1)/(a*x))^(n/2))/((c - a^2
*c*x^2)^(1/2)*((a*x - 1)/(a*x))^(n/2))
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\int \frac{e^{acoth(ax)n}}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(exp(n*acoth(a*x))/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
( - int(e**(acoth(a*x)*n)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2
*x**2 + 1)),x))/(sqrt(c)*c)
```

3.722 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac(9-n^2)(c-a^2cx^2)^{3/2}} - \frac{6e^{n\coth^{-1}(ax)}(n-ax)}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}$$

output
```
-exp(n*arccoth(a*x))*(-3*a*x+n)/a/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)-6*exp(n*
arccoth(a*x))*(-a*x+n)/a/c^2/(-n^2+1)/(-n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.80 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}\left(-26n+2n^3+27ax-3an^2x-2n(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)+3a\right.}{4ac^2(9-10n^2+n^4)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(-26*n + 2*n^3 + 27*a*x - 3*a*n^2*x - 2*n*(-1 + n^2)*C
osh[2*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCot
h[a*x]]))/(4*a*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6739, 6738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{6\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx}{c\,(9-n^2)} - \frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac\,(9-n^2)\,(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6738}$$

$$-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2\,(1-n^2)\,(9-n^2)\,\sqrt{c-a^2cx^2}} - \frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac\,(9-n^2)\,(c-a^2cx^2)^{3/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)))
- (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2
*c*x^2])
```

Defintions of rubi rules used

rule 6738
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{(ax-1)(ax+1)\left(6a^3x^3-6n\,x^2a^2+3n^2xa-n^3-9ax+7n\right){\mathrm e}^{n\,\operatorname{arccoth}(ax)}}{a\left(n^4-10n^2+9\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}$	84
orering	$\frac{(ax-1)(ax+1)\left(6a^3x^3-6n\,x^2a^2+3n^2xa-n^3-9ax+7n\right){\mathrm e}^{n\,\operatorname{arccoth}(ax)}}{a\left(n^4-10n^2+9\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}$	84

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)*(a*x+1)*(6*a^3*x^3-6*a^2*n*x^2+3*a*n^2*x-n^3-9*a*x+7*n)*exp(n*arcc
oth(a*x))/a/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.62

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{(6\,a^3x^3-6\,a^2nx^2-n^3+3\,(an^2-3\,a)x+7\,n)\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^3n^4-10\,ac^3n^2+(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3)x^4+9\,ac^3-2\,(a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3)x^2}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-(6*a^3*x^3 - 6*a^2*n*x^2 - n^3 + 3*(a*n^2 - 3*a)*x + 7*n)*sqrt(-a^2*c*x^2
 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^4 - 10*a*c^3*n^2 + (a^5*c^3*n
^4 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^4 + 9*a*c^3 - 2*(a^3*c^3*n^4 - 10*a^3*c
^3*n^2 + 9*a^3*c^3)*x^2)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{e^{n\operatorname{acoth}(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.39 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.70

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{6\,x^3}{c^2\,(n^4-10\,n^2+9)}+\frac{7\,n-n^3}{a^3\,c^2\,(n^4-10\,n^2+9)}+\frac{3\,x\,(n^2-3)}{a^2\,c^2\,(n^4-10\,n^2+9)}-\frac{6\,n\,x^2}{a\,c^2\,(n^4-10\,n^2+9)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(5/2),x)
```

output
```
-(((a*x + 1)/(a*x))^(n/2)*((6*x^3)/(c^2*(n^4 - 10*n^2 + 9)) + (7*n - n^3)/
(a^3*c^2*(n^4 - 10*n^2 + 9)) + (3*x*(n^2 - 3))/(a^2*c^2*(n^4 - 10*n^2 + 9)
) - (6*n*x^2)/(a*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2 - x
^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{e^{acoth(ax)n}}{\sqrt{-a^2x^2+1}\,a^4x^4-2\sqrt{-a^2x^2+1}\,a^2x^2+\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(exp(n*acoth(a*x))/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
int(e**(acoth(a*x)*n)/(sqrt( - a**2*x**2 + 1)*a**4*x**4 - 2*sqrt( - a**2*x
**2 + 1)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

3.723 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 166

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx &= -\frac{e^{n\coth^{-1}(ax)}(n-5ax)}{ac(25-n^2)(c-a^2cx^2)^{5/2}}\\ &\quad -\frac{20e^{n\coth^{-1}(ax)}(n-3ax)}{ac^2(9-n^2)(25-n^2)(c-a^2cx^2)^{3/2}}\\ &\quad -\frac{120e^{n\coth^{-1}(ax)}(n-ax)}{ac^3(1-n^2)(9-n^2)(25-n^2)\sqrt{c-a^2cx^2}}\end{aligned}$$

output
```
-exp(n*arccoth(a*x))*(-5*a*x+n)/a/c/(-n^2+25)/(-a^2*c*x^2+c)^(5/2)-20*exp(
n*arccoth(a*x))*(-3*a*x+n)/a/c^2/(-n^2+9)/(-n^2+25)/(-a^2*c*x^2+c)^(3/2)-1
20*exp(n*arccoth(a*x))*(-a*x+n)/a/c^3/(-n^2+1)/(-n^2+9)/(-n^2+25)/(-a^2*c*
x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.80

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx =$$

$$-\frac{a^2e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(-\frac{10(225-34n^2+n^4)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2250n}{a\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{340n^3}{a\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{10n^5}{a\sqrt{1-\frac{1}{a^2x^2}}x}+15(25-26n^2+\cdots\right.}{\cdots}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

output
```
-1/16*(a^2*E^(n*ArcCoth[a*x])*(1 - 1/(a^2*x^2))^(3/2)*x^3*((-10*(225 - 34*
n^2 + n^4))/Sqrt[1 - 1/(a^2*x^2)] + (2250*n)/(a*Sqrt[1 - 1/(a^2*x^2)]*x) -
 (340*n^3)/(a*Sqrt[1 - 1/(a^2*x^2)]*x) + (10*n^5)/(a*Sqrt[1 - 1/(a^2*x^2)]
*x) + 15*(25 - 26*n^2 + n^4)*Cosh[3*ArcCoth[a*x]] - 45*Cosh[5*ArcCoth[a*x]
] + 50*n^2*Cosh[5*ArcCoth[a*x]] - 5*n^4*Cosh[5*ArcCoth[a*x]] - 125*n*Sinh[
3*ArcCoth[a*x]] + 130*n^3*Sinh[3*ArcCoth[a*x]] - 5*n^5*Sinh[3*ArcCoth[a*x]
] + 9*n*Sinh[5*ArcCoth[a*x]] - 10*n^3*Sinh[5*ArcCoth[a*x]] + n^5*Sinh[5*Ar
cCoth[a*x]]))/(c^2*(-5 + n)*(-3 + n)*(-1 + n)*(1 + n)*(3 + n)*(5 + n)*(c -
 a^2*c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6739, 6739, 6738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{20\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx}{c\,(25-n^2)}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac\,(25-n^2)\,(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \; 6739$$

$$\frac{20\left(\frac{6\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx}{c(9-n^2)}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}\right)}{c\,(25-n^2)}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac\,(25-n^2)\,(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \; 6738$$

$$\frac{20\left(-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}\right)}{c\,(25-n^2)}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac\,(25-n^2)\,(c-a^2cx^2)^{5/2}}$$

input

```
Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

output

```
-((E^(n*ArcCoth[a*x])*(n - 5*a*x))/(a*c*(25 - n^2)*(c - a^2*c*x^2)^(5/2)))
 + (20*(-((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^
(3/2))) - (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt
[c - a^2*c*x^2])))/(c*(25 - n^2))
```

Defintions of rubi rules used

rule 6738

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

rule 6739

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

method	result
gosper	$\frac{(ax-1)(ax+1)\left(120a^5x^5-120n\,a^4x^4+60a^3n^2x^3-20a^2n^3x^2-300a^3x^3+5a\,n^4x+260n\,x^2a^2-n^5-110n^2xa+30n^3+225ax-149n\right)\mathrm{e}^{n}}{a\left(n^6-35n^4+259n^2-225\right)\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}$
orering	$\frac{(ax-1)(ax+1)\left(120a^5x^5-120n\,a^4x^4+60a^3n^2x^3-20a^2n^3x^2-300a^3x^3+5a\,n^4x+260n\,x^2a^2-n^5-110n^2xa+30n^3+225ax-149n\right)\mathrm{e}^{n}}{a\left(n^6-35n^4+259n^2-225\right)\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}$

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)*(a*x+1)*(120*a^5*x^5-120*a^4*n*x^4+60*a^3*n^2*x^3-20*a^2*n^3*x^2-3
00*a^3*x^3+5*a*n^4*x+260*a^2*n*x^2-n^5-110*a*n^2*x+30*n^3+225*a*x-149*n)*e
xp(n*arccoth(a*x))/a/(n^6-35*n^4+259*n^2-225)/(-a^2*c*x^2+c)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.75

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx =$$

$$-\frac{(120\,a^5x^5-120\,a^4nx^4-n^5+60\,(a^3n^2-5\,a^3)x^3+30\,n^3-20\,(a^2n^3-13\,a^2n}{ac^4n^6-35\,ac^4n^4+259\,ac^4n^2-(a^7c^4n^6-35\,a^7c^4n^4+259\,a^7c^4n^2-225\,a^7c^4)x^6-225\,ac^4+3\,(a^5c^4n^6-}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
-(120*a^5*x^5 - 120*a^4*n*x^4 - n^5 + 60*(a^3*n^2 - 5*a^3)*x^3 + 30*n^3 -
20*(a^2*n^3 - 13*a^2*n)*x^2 + 5*(a*n^4 - 22*a*n^2 + 45*a)*x - 149*n)*sqrt(
-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^4*n^6 - 35*a*c^4*n^4 +
259*a*c^4*n^2 - (a^7*c^4*n^6 - 35*a^7*c^4*n^4 + 259*a^7*c^4*n^2 - 225*a^7*
c^4)*x^6 - 225*a*c^4 + 3*(a^5*c^4*n^6 - 35*a^5*c^4*n^4 + 259*a^5*c^4*n^2 -
 225*a^5*c^4)*x^4 - 3*(a^3*c^4*n^6 - 35*a^3*c^4*n^4 + 259*a^3*c^4*n^2 - 22
5*a^3*c^4)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c - a^2cx^2)^{7/2}} \, dx = \text{Timed out}$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c - a^2cx^2)^{7/2}} \, dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2 + c)^{\frac{7}{2}}} \, dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c - a^2cx^2)^{7/2}} \, dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2 + c)^{\frac{7}{2}}} \, dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.74

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx =$$

$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{120\,x^5}{c^3\,(n^6-35\,n^4+259\,n^2-225)}-\frac{120\,n\,x^4}{a\,c^3\,(n^6-35\,n^4+259\,n^2-225)}+\frac{x^3\,(60\,n^2-300)}{a^2\,c^3\,(n^6-35\,n^4+259\,n^2-225)}-\frac{n\,(n^4-30\,n^2+149)}{a^5\,c^3\,(n^6-35\,n^4+259\,n^2}\right.}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^4}+x^4\,\sqrt{c-a^2\,c\,x^2}-\frac{2\,x^2\,\sqrt{c-a^2\,c\,x^2}}{a^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right.}$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(7/2),x)
```

output
```
-(((a*x + 1)/(a*x))^(n/2)*((120*x^5)/(c^3*(259*n^2 - 35*n^4 + n^6 - 225))
- (120*n*x^4)/(a*c^3*(259*n^2 - 35*n^4 + n^6 - 225)) + (x^3*(60*n^2 - 300)
)/(a^2*c^3*(259*n^2 - 35*n^4 + n^6 - 225)) - (n*(n^4 - 30*n^2 + 149))/(a^5
*c^3*(259*n^2 - 35*n^4 + n^6 - 225)) + (5*x*(n^4 - 22*n^2 + 45))/(a^4*c^3*
(259*n^2 - 35*n^4 + n^6 - 225)) - (20*n*x^2*(n^2 - 13))/(a^3*c^3*(259*n^2
- 35*n^4 + n^6 - 225))))/(((c - a^2*c*x^2)^(1/2)/a^4 + x^4*(c - a^2*c*x^2)
^(1/2) - (2*x^2*(c - a^2*c*x^2)^(1/2))/a^2)*((a*x - 1)/(a*x))^(n/2))
```

Reduce [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\int \frac{e^{acoth(ax)n}}{\sqrt{-a^2x^2+1}\,a^6x^6-3\sqrt{-a^2x^2+1}\,a^4x^4+3\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c^3}$$

input
```
int(exp(n*acoth(a*x))/(-a^2*c*x^2+c)^(7/2),x)
```

output
```
( - int(e**(acoth(a*x)*n)/(sqrt( - a**2*x**2 + 1)*a**6*x**6 - 3*sqrt( - a*
*2*x**2 + 1)*a**4*x**4 + 3*sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2
*x**2 + 1)),x))/(sqrt(c)*c**3)
```

3.724 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 239

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx &= -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac(49-n^2)(c-a^2cx^2)^{7/2}} \\ &\quad -\frac{42e^{n\coth^{-1}(ax)}(n-5ax)}{ac^2(25-n^2)(49-n^2)(c-a^2cx^2)^{5/2}} \\ &\quad -\frac{840e^{n\coth^{-1}(ax)}(n-3ax)}{ac^3(9-n^2)(25-n^2)(49-n^2)(c-a^2cx^2)^{3/2}} \\ &\quad -\frac{5040e^{n\coth^{-1}(ax)}(n-ax)}{ac^4(1-n^2)(9-n^2)(25-n^2)(49-n^2)\sqrt{c-a^2cx^2}}\end{aligned}$$

output

```
-exp(n*arccoth(a*x))*(-7*a*x+n)/a/c/(-n^2+49)/(-a^2*c*x^2+c)^(7/2)-42*exp(
n*arccoth(a*x))*(-5*a*x+n)/a/c^2/(-n^2+25)/(-n^2+49)/(-a^2*c*x^2+c)^(5/2)-
840*exp(n*arccoth(a*x))*(-3*a*x+n)/a/c^3/(-n^2+9)/(-n^2+25)/(-n^2+49)/(-a^
2*c*x^2+c)^(3/2)-5040*exp(n*arccoth(a*x))*(-a*x+n)/a/c^4/(-n^2+1)/(-n^2+9)
/(-n^2+25)/(-n^2+49)/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.95 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.09

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{ae^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\left(-\frac{35n}{-1+n^2}+\frac{35ax}{-1+n^2}-\frac{63a\sqrt{1-\frac{1}{a^2x^2}}\,x\cosh(3\coth^{-1}(ax))}{-9+n^2}+\frac{35a\sqrt{1-\cdots}}{\cdots}\right.}{\cdots}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

output
```
(a*E^(n*ArcCoth[a*x])*(1 - 1/(a^2*x^2))*x^2*((-35*n)/(-1 + n^2) + (35*a*x)
/(-1 + n^2) - (63*a*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]])/(-9 + n^
2) + (35*a*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[5*ArcCoth[a*x]])/(-25 + n^2) - (7*
a*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[7*ArcCoth[a*x]])/(-49 + n^2) + (21*a*n*Sqrt
[1 - 1/(a^2*x^2)]*x*Sinh[3*ArcCoth[a*x]])/(-9 + n^2) - (7*a*n*Sqrt[1 - 1/(
a^2*x^2)]*x*Sinh[5*ArcCoth[a*x]])/(-25 + n^2) + (a*n*Sqrt[1 - 1/(a^2*x^2)]
*x*Sinh[7*ArcCoth[a*x]])/(-49 + n^2)))/(64*c^3*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6739, 6739, 6739, 6738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{42\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}dx}{c(49-n^2)}-\frac{(n-7ax)e^{n\coth^{-1}(ax)}}{ac(49-n^2)(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{6739}$$

$$\frac{42\left(\frac{20\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx}{c(25-n^2)}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac(25-n^2)(c-a^2cx^2)^{5/2}}\right)}{c(49-n^2)}-\frac{(n-7ax)e^{n\coth^{-1}(ax)}}{ac(49-n^2)(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{6739}$$

$$\frac{42\left(\frac{20\left(\frac{6\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx}{c(9-n^2)}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}\right)}{c(25-n^2)}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac(25-n^2)(c-a^2cx^2)^{5/2}}\right)}{c(49-n^2)}-\frac{(n-7ax)e^{n\coth^{-1}(ax)}}{ac(49-n^2)(c-a^2cx^2)^{7/2}}$$

$$\Big\downarrow \text{6738}$$

$$\frac{42\left(\frac{20\left(-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}\right)}{c(25-n^2)}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac(25-n^2)(c-a^2cx^2)^{5/2}}\right)}{c(49-n^2)}-\frac{(n-7ax)e^{n\coth^{-1}(ax)}}{ac(49-n^2)(c-a^2cx^2)^{7/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(n - 7*a*x))/(a*c*(49 - n^2)*(c - a^2*c*x^2)^(7/2)))
 + (42*(-((E^(n*ArcCoth[a*x])*(n - 5*a*x))/(a*c*(25 - n^2)*(c - a^2*c*x^2)
^(5/2))) + (20*(-((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2
*c*x^2)^(3/2))) - (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n
^2)*Sqrt[c - a^2*c*x^2])))/(c*(25 - n^2))))/(c*(49 - n^2))
```

Defintions of rubi rules used

rule 6738
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.91

method	result
gosper	$\frac{(ax-1)(ax+1)(5040a^7x^7-5040n\,a^6x^6+2520a^5n^2x^5-840a^4n^3x^4-17640a^5x^5+210a^3n^4x^3+15960n\,a^4x^4-42a^2n^5x^2-7140a^3n^2x}{a(n^8-84n^6+1974n^4-12916}$
orering	$\frac{(ax-1)(ax+1)(5040a^7x^7-5040n\,a^6x^6+2520a^5n^2x^5-840a^4n^3x^4-17640a^5x^5+210a^3n^4x^3+15960n\,a^4x^4-42a^2n^5x^2-7140a^3n^2x}{a(n^8-84n^6+1974n^4-12916}$

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)*(a*x+1)*(5040*a^7*x^7-5040*a^6*n*x^6+2520*a^5*n^2*x^5-840*a^4*n^3*
x^4-17640*a^5*x^5+210*a^3*n^4*x^3+15960*a^4*n*x^4-42*a^2*n^5*x^2-7140*a^3*
n^2*x^3+7*a*n^6*x+2100*a^2*n^3*x^2-n^7+22050*a^3*x^3-455*a*n^4*x-17178*a^2
*n*x^2+77*n^5+6433*a*n^2*x-1519*n^3-11025*a*x+6483*n)*exp(n*arccoth(a*x))/
a/(n^8-84*n^6+1974*n^4-12916*n^2+11025)/(-a^2*c*x^2+c)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.90

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx =$$

$$-\frac{(5040\,a^7x^7-5040\,a^6nx^6-n^7+2520\,(a^5n^2-7\,a^5)x^5}{ac^5n^8-84\,ac^5n^6+1974\,ac^5n^4+(a^9c^5n^8-84\,a^9c^5n^6+1974\,a^9c^5n^4-12916\,a^9c^5n^2+11025\,a^9c^5)x^8-1}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
-(5040*a^7*x^7 - 5040*a^6*n*x^6 - n^7 + 2520*(a^5*n^2 - 7*a^5)*x^5 + 77*n^
5 - 840*(a^4*n^3 - 19*a^4*n)*x^4 + 210*(a^3*n^4 - 34*a^3*n^2 + 105*a^3)*x^
3 - 1519*n^3 - 42*(a^2*n^5 - 50*a^2*n^3 + 409*a^2*n)*x^2 + 7*(a*n^6 - 65*a
*n^4 + 919*a*n^2 - 1575*a)*x + 6483*n)*sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*
x - 1))^(1/2*n)/(a*c^5*n^8 - 84*a*c^5*n^6 + 1974*a*c^5*n^4 + (a^9*c^5*n^8
- 84*a^9*c^5*n^6 + 1974*a^9*c^5*n^4 - 12916*a^9*c^5*n^2 + 11025*a^9*c^5)*x
^8 - 12916*a*c^5*n^2 - 4*(a^7*c^5*n^8 - 84*a^7*c^5*n^6 + 1974*a^7*c^5*n^4
- 12916*a^7*c^5*n^2 + 11025*a^7*c^5)*x^6 + 11025*a*c^5 + 6*(a^5*c^5*n^8 -
84*a^5*c^5*n^6 + 1974*a^5*c^5*n^4 - 12916*a^5*c^5*n^2 + 11025*a^5*c^5)*x^4
 - 4*(a^3*c^5*n^8 - 84*a^3*c^5*n^6 + 1974*a^3*c^5*n^4 - 12916*a^3*c^5*n^2
+ 11025*a^3*c^5)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(9/2), x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.62 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.85

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx =$$

$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{5040\,x^7}{c^4\,(n^8-84\,n^6+1974\,n^4-12916\,n^2+11025)}+\frac{-n^7+77\,n^5-1519\,n^3+6483\,n}{a^7\,c^4\,(n^8-84\,n^6+1974\,n^4-12916\,n^2+11025)}-\frac{5040\,n\,x^6}{a\,c^4\,(n^8-84\,n^6+1974\,n^4-1291}\right.}{}$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(9/2),x)
```

output
```
-((((a*x + 1)/(a*x))^(n/2)*((5040*x^7)/(c^4*(1974*n^4 - 12916*n^2 - 84*n^6
+ n^8 + 11025)) + (6483*n - 1519*n^3 + 77*n^5 - n^7)/(a^7*c^4*(1974*n^4 -
12916*n^2 - 84*n^6 + n^8 + 11025)) - (5040*n*x^6)/(a*c^4*(1974*n^4 - 12916
*n^2 - 84*n^6 + n^8 + 11025)) + (x^5*(2520*n^2 - 17640))/(a^2*c^4*(1974*n^
4 - 12916*n^2 - 84*n^6 + n^8 + 11025)) + (x^3*(210*n^4 - 7140*n^2 + 22050)
)/(a^4*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 11025)) + (7*x*(919*n^2
- 65*n^4 + n^6 - 1575))/(a^6*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 11
025)) - (840*n*x^4*(n^2 - 19))/(a^3*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n
^8 + 11025)) - (42*n*x^2*(n^4 - 50*n^2 + 409))/(a^5*c^4*(1974*n^4 - 12916*
n^2 - 84*n^6 + n^8 + 11025))))/(((a*x - 1)/(a*x))^(n/2)*((c - a^2*c*x^2)^(
1/2)/a^6 - x^6*(c - a^2*c*x^2)^(1/2) + (3*x^4*(c - a^2*c*x^2)^(1/2))/a^2 -
 (3*x^2*(c - a^2*c*x^2)^(1/2))/a^4))
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = \frac{\int \frac{e^{acoth(ax)n}}{\sqrt{-a^2x^2+1}\,a^8x^8-4\sqrt{-a^2x^2+1}\,a^6x^6+6\sqrt{-a^2x^2+1}\,a^4x^4-4\sqrt{-a^2x^2+1}\,a^2x^2+\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c^4}$$

input
```
int(exp(n*acoth(a*x))/(-a^2*c*x^2+c)^(9/2),x)
```

output
```
int(e**(acoth(a*x)*n)/(sqrt( - a**2*x**2 + 1)*a**8*x**8 - 4*sqrt( - a**2*x
**2 + 1)*a**6*x**6 + 6*sqrt( - a**2*x**2 + 1)*a**4*x**4 - 4*sqrt( - a**2*x
**2 + 1)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)/(sqrt(c)*c**4)
```

3.725 $\int \frac{e^{n \coth^{-1}(ax)} x^3}{(c-a^2cx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 356

$$\int \frac{e^{n \coth^{-1}(ax)} x^3}{(c-a^2cx^2)^{3/2}} \, dx = -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{(2+2n+n^2)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a\left(1-n^2\right)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^4}{\left(c-a^2cx^2\right)^{3/2}} - \frac{2n\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3 \text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{a(1-n)\left(c-a^2cx^2\right)^{3/2}}$$

output
```
-(2+n)*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x
^3/a/(1+n)/(-a^2*c*x^2+c)^(3/2)+(n^2+2*n+2)*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^
(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^3/a/(-n^2+1)/(-a^2*c*x^2+c)^(3/2)+(1-
1/a^2/x^2)^(3/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^4/(-a^2*c
*x^2+c)^(3/2)-2*n*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/
2+1/2*n)*x^3*hypergeom([1, -1/2+1/2*n],[1/2+1/2*n],(a+1/x)/(a-1/x))/a/(1-n
)/(-a^2*c*x^2+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.37

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\frac{ce^{n\coth^{-1}(ax)}(-1+anx)}{-1+n^2}-\frac{c(-1+a^2x^2)\left(e^{n\coth^{-1}(ax)}(1+n)+\frac{2e^{(1+n)\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},e^{2\coth^{-1}(ax)}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}x}\right)}{1+n}}{a^4c^2\sqrt{c-a^2cx^2}}$$

input
```
Integrate[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
((c*E^(n*ArcCoth[a*x])*(-1 + a*n*x))/(-1 + n^2) - (c*(-1 + a^2*x^2)*(E^(n*
ArcCoth[a*x])*(1 + n) + (2*E^((1 + n)*ArcCoth[a*x])*n*Hypergeometric2F1[1,
 (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(a*Sqrt[1 - 1/(a^2*x^2)]*x)))/
(1 + n))/(a^4*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.77, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6746, 6748, 144, 25, 27, 172, 25, 27, 172, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6748}$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}x^2d\frac{1}{x}}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{144}$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(x\left(-\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}-\int -\frac{\left(an+\frac{2}{x}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}x}{a^2}d\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\int \frac{\left(an+\frac{2}{x}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\int \left(an+\frac{2}{x}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{172}$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\frac{a(n+2)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{n+1}-\frac{a\int -\frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}\left(an(n+1)+\frac{n+2}{x}\right)x}{a}d\frac{1}{x}}{n+1}}{a^2}-x\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-}\right.}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\frac{a\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}\left(an(n+1)+\frac{n+2}{x}\right)x}{a}d\frac{1}{x}}{n+1}+\frac{a(n+2)\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{n+1}}{a^2}-x\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1}\right.}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\frac{\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}\left(an(n+1)+\frac{n+2}{x}\right)xd\frac{1}{x}}{n+1}+\frac{a(n+2)\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{n+1}}{a^2}-x\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\right.\right.}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{172}$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\frac{\frac{a\int n\left(1-n^2\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}xd\frac{1}{x}}{1-n}-\frac{a\left(n^2+2n+2\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{1-n}}{n+1}}{a^2}+\frac{a(n+2)\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-}}{n+1}\right.}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\frac{\frac{an\left(1-n^2\right)\int\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}xd\frac{1}{x}}{1-n}-\frac{a\left(n^2+2n+2\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{1-n}}{n+1}}{a^2}+\frac{a(n+2)\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1}}{n+1}\right.}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{141}$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{\frac{\frac{2an\left(1-n^2\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-1}{2},\frac{n+1}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{(1-n)^2}-\frac{a\left(n^2+2n+2\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{1-n}}{n+1}}{a^2}+\frac{a}{}\right.}{\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Int[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-(((1 - 1/(a^2*x^2))^(3/2)*x^3*(-((1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))
^((-1 + n)/2)*x) + ((a*(2 + n)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((
-1 + n)/2))/(1 + n) + (-((a*(2 + 2*n + n^2)*(1 - 1/(a*x))^((1 - n)/2)*(1 +
 1/(a*x))^((-1 + n)/2))/(1 - n)) + (2*a*n*(1 - n^2)*(1 - 1/(a*x))^((1 - n)
/2)*(1 + 1/(a*x))^((-1 + n)/2)*Hypergeometric2F1[1, (-1 + n)/2, (1 + n)/2,
 (a + x^(-1))/(a - x^(-1))])/(1 - n)^2)/(1 + n))/a^2))/(c - a^2*c*x^2)^(3/
2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 144
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[b*(a + b*x)^(m + 1)*(
c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x]
+ Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2))
- b*d*f*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && NeQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [F]

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)} x^3}{\left(-a^2 c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x
^4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^3e^{n\operatorname{acoth}(ax)}}{(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*x**3/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**3*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{x^3\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\left(c-a^2\,c\,x^2\right)^{3/2}}\,dx$$

input
```
int((x^3*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2),x)
```

output
```
int((x^3*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{\int \frac{e^{acoth(ax)n}x^3}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(exp(n*acoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
( - int((e**(acoth(a*x)*n)*x**3)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt(
 - a**2*x**2 + 1)),x))/(sqrt(c)*c)
```

3.726 $\int \frac{e^{n \coth^{-1}(ax)} x^2}{(c-a^2cx^2)^{3/2}}\, dx$

Optimal result

Integrand size = 27, antiderivative size = 164

$$\int \frac{e^{n \coth^{-1}(ax)} x^2}{(c-a^2cx^2)^{3/2}}\, dx = -\frac{e^{n \coth^{-1}(ax)}(n-ax)}{a^3c(1-n^2)\sqrt{c-a^2cx^2}} - \frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)} x\, \text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2c(1-n)\sqrt{c-a^2cx^2}}$$

output
```
-exp(n*arccoth(a*x))*(-a*x+n)/a^3/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)-2*(1-1/a
^2/x^2)^(1/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x*hypergeom([1,
 1/2-1/2*n],[3/2-1/2*n],(a-1/x)/(a+1/x))/a^2/c/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.77

$$\int \frac{e^{n \coth^{-1}(ax)} x^2}{(c-a^2cx^2)^{3/2}}\, dx = -\frac{e^{n \coth^{-1}(ax)}\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-n+ax) + 2e^{\coth^{-1}(ax)}(-1+n)\left(-1+a^2x^2\right) \text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+}{2}\right.\right.}{a^4c(-1+n)(1+n)\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-a^2cx^2}}$$

input
```
Integrate[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-n + a*x) + 2*E^ArcCoth[
a*x]*(-1 + n)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^
(2*ArcCoth[a*x])]))/(a^4*c*(-1 + n)*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c
 - a^2*c*x^2]))
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6743, 6746, 6749, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{6743}$$

$$-\frac{\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}dx}{a^2c}-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{a^3c(1-n^2)\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{6746}$$

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}x}dx}{a^2c\sqrt{c-a^2cx^2}}-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{a^3c(1-n^2)\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{6749}$$

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}xd\frac{1}{x}}{a^2c\sqrt{c-a^2cx^2}}-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{a^3c(1-n^2)\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{141}$$

$$-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2c(1-n)\sqrt{c-a^2cx^2}}-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{a^3c\,(1-n^2)\sqrt{c-a^2cx^2}}$$

input

```
Int[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

output

```
-((E^(n*ArcCoth[a*x])*(n - a*x))/(a^3*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])) -
(2*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)
/2)*x*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))
])/(a^2*c*(1 - n)*Sqrt[c - a^2*c*x^2])
```

Defintions of rubi rules used

rule 141

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6743

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a^
3*c*(n^2 - 4*(p + 1)^2))), x] - Simp[(n^2 + 2*(p + 1))/(a^2*c*(n^2 - 4*(p +
 1)^2))   Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c
, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[n^
2 + 2*(p + 1), 0] && NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !Integer
Q[n])
```

rule 6746

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6749
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/
x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d,
0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
 n/2] && IntegerQ[m]
```

Maple [F]

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)} x^{2}}{\left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}d x$$

input
```
int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x^{2} \left(\frac{a x + 1}{a x - 1}\right)^{\frac{1}{2} \, n}}{{\left(-a^{2} c x^{2} + c\right)}^{\frac{3}{2}}}\,{d x}$$

input
```
integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x
^4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^2e^{n\,\mathrm{acoth}\,(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**2*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

output
```
integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{(c-a^2\,c\,x^2)^{3/2}}\,dx$$

input
```
int((x^2*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2),x)
```

output
```
int((x^2*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\int \frac{e^{acoth(ax)n}x^2}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(exp(n*acoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
( - int((e**(acoth(a*x)*n)*x**2)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt(
 - a**2*x**2 + 1)),x))/(sqrt(c)*c)
```

3.727 $\int \frac{e^{n \coth^{-1}(ax)} x}{(c-a^2cx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 46

$$\int \frac{e^{n \coth^{-1}(ax)} x}{(c-a^2cx^2)^{3/2}} \, dx = \frac{e^{n \coth^{-1}(ax)} (1-anx)}{a^2c(1-n^2)\sqrt{c-a^2cx^2}}$$

output
```
exp(n*arccoth(a*x))*(-a*n*x+1)/a^2/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{e^{n \coth^{-1}(ax)} x}{(c-a^2cx^2)^{3/2}} \, dx = \frac{e^{n \coth^{-1}(ax)} (-1+anx)}{a^2c(-1+n^2)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(-1 + a*n*x))/(a^2*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {6740}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 6740}$$

$$\frac{(1-anx)e^{n\coth^{-1}(ax)}}{a^2c(1-n^2)\sqrt{c-a^2cx^2}}$$

input
```
Int[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(1 - a*n*x))/(a^2*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])
```

Defintions of rubi rules used

rule 6740
```
Int[(E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[(-(1 - a*n*x))*(E^(n*ArcCoth[a*x])/(a^2*c*(n^2 - 1)*Sqrt[c + d*
x^2])), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
gosper	$-\frac{(ax-1)(ax+1)(nax-1){\mathrm e}^{n\,\operatorname{arccoth}(ax)}}{a^2(n^2-1)(-a^2c\,x^2+c)^{\frac{3}{2}}}$	49
orering	$-\frac{(ax-1)(ax+1)(nax-1){\mathrm e}^{n\,\operatorname{arccoth}(ax)}}{a^2(n^2-1)(-a^2c\,x^2+c)^{\frac{3}{2}}}$	49

input
```
int(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)*(a*x+1)*(a*n*x-1)*exp(n*arccoth(a*x))/a^2/(n^2-1)/(-a^2*c*x^2+c)^
(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int \frac{e^{n \coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{-a^2cx^2+c}(anx-1)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2c^2n^2-a^2c^2-(a^4c^2n^2-a^4c^2)x^2}$$

input
```
integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas"
)
```

output
```
sqrt(-a^2*c*x^2 + c)*(a*n*x - 1)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*n^
2 - a^2*c^2 - (a^4*c^2*n^2 - a^4*c^2)*x^2)
```

Sympy [F]

$$\int \frac{e^{n \coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{xe^{n\,\mathrm{acoth}\,(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*x/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx=-\frac{\left(\frac{1}{a^2\,c\,(n^2-1)}-\frac{n\,x}{a\,c\,(n^2-1)}\right)\,\left(\frac{a\,x+1}{a\,x}\right)^{n/2}}{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input
```
int((x*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2),x)
```

output
```
-((1/(a^2*c*(n^2 - 1)) - (n*x)/(a*c*(n^2 - 1)))*((a*x + 1)/(a*x))^(n/2))/(
(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x))^(n/2))
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\int \frac{e^{acoth(ax)n}x}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(exp(n*acoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
( - int((e**(acoth(a*x)*n)*x)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( -
a**2*x**2 + 1)),x))/(sqrt(c)*c)
```

3.728 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} \, dx = -\frac{e^{n \coth^{-1}(ax)}(n-ax)}{ac(1-n^2)\sqrt{c-a^2cx^2}}$$

output
```
-exp(n*arccoth(a*x))*(-a*x+n)/a/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} \, dx = \frac{e^{n \coth^{-1}(ax)}(n-ax)}{ac(-1+n^2)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 6738$$

$$-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac\,(1-n^2)\,\sqrt{c-a^2cx^2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))
```

Defintions of rubi rules used

rule 6738
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
gosper	$\frac{(ax-1)(ax+1)(ax-n)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{(n^2-1)a(-a^2cx^2+c)^{\frac{3}{2}}}$	49
orering	$\frac{(ax-1)(ax+1)(ax-n)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{(n^2-1)a(-a^2cx^2+c)^{\frac{3}{2}}}$	49

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)*(a*x+1)*(a*x-n)*exp(n*arccoth(a*x))/(n^2-1)/a/(-a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{-a^2cx^2+c}(ax-n)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^2n^2-ac^2-(a^3c^2n^2-a^3c^2)x^2}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(-a^2*c*x^2 + c)*(a*x - n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n^2 -
 a*c^2 - (a^3*c^2*n^2 - a^3*c^2)*x^2)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{e^{n\operatorname{acoth}(ax)}}{(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.70

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\left(\frac{x}{c\,(n^2-1)}-\frac{n}{a\,c\,(n^2-1)}\right)\,\left(\frac{a\,x+1}{a\,x}\right)^{n/2}}{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(3/2),x)
```

output
```
-((x/(c*(n^2 - 1)) - n/(a*c*(n^2 - 1)))*((a*x + 1)/(a*x))^(n/2))/((c - a^2
*c*x^2)^(1/2)*((a*x - 1)/(a*x))^(n/2))
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\int \frac{e^{acoth(ax)n}}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(exp(n*acoth(a*x))/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
( - int(e**(acoth(a*x)*n)/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2
*x**2 + 1)),x))/(sqrt(c)*c)
```

3.729 $\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 277

$$\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx = -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1+n)(c-a^2cx^2)^{3/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1-n^2)(c-a^2cx^2)^{3/2}} - \frac{2^{\frac{1+n}{2}}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}x^3\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{(1-n)(c-a^2cx^2)^{3/2}}$$

output
```
-a^3*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^3
/(1+n)/(-a^2*c*x^2+c)^(3/2)+a^3*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1/2-1/2*n)*
(1+1/a/x)^(-1/2+1/2*n)*x^3/(-n^2+1)/(-a^2*c*x^2+c)^(3/2)-2^(1/2+1/2*n)*a^3
*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1/2-1/2*n)*x^3*hypergeom([1/2-1/2*n, 1/2-1
/2*n],[3/2-1/2*n],1/2*(a-1/x)/a)/(1-n)/(-a^2*c*x^2+c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.46

$$\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}\left(a\sqrt{1-\frac{1}{a^2x^2}}x(-1+anx)-2e^{\coth^{-1}(ax)}(-1+n)(-1+a^2x^2)\,\text{Hyperge}\right.}{ac(-1+n)(1+n)\sqrt{1-\frac{1}{a^2x^2}}x\sqrt{c-a^2cx}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
(E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*n*x) - 2*E^ArcCoth[
a*x]*(-1 + n)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -E
^(2*ArcCoth[a*x])]))/(a*c*(-1 + n)*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c
- a^2*c*x^2])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6746, 6749, 100, 27, 88, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 6746}$$

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}dx}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 6749}$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}}{x^2}d\frac{1}{x}}{(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 100}$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{a^3\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{n+1}-\frac{a^3\int\frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}\left(an+\frac{n+1}{x}\right)}{a^2}d\frac{1}{x}}{n+1}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{a^3\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{n+1}-\frac{a\int\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}\left(an+\frac{n+1}{x}\right)d\frac{1}{x}}{n+1}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 88$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{a^3\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{n+1}-\frac{a\left(a(n+1)\int\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}d\frac{1}{x}+\frac{a^2\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{1-n}\right)}{n+1}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 79$$

$$-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{a^3\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{n+1}-\frac{a\left(\frac{a^2\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}}{1-n}-\frac{a^2 2^{\frac{n+1}{2}}(n+1)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-}{2}\right.}{1-n}\right.}{n+1}}{\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

output
```
-(((1 - 1/(a^2*x^2))^(3/2)*x^3*((a^3*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*
x))^((-1 + n)/2))/(1 + n) - (a*((a^2*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x
))^((-1 + n)/2))/(1 - n) - (2^((1 + n)/2)*a^2*(1 + n)*(1 - 1/(a*x))^((1 -
n)/2)*Hypergeometric2F1[(1 - n)/2, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(2*a
)])/(1 - n)))/(1 + n)))/(c - a^2*c*x^2)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6749
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/
x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d,
0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
 n/2] && IntegerQ[m]
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{x\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}dx$$

input
```
int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^5 -
 2*a^2*c^2*x^3 + c^2*x), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\,(c-a^2cx^2)^{3/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/x/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(x*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\,(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\,(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx=\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{x\,\left(c-a^2\,c\,x^2\right)^{3/2}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(3/2)),x)
```

output
```
int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{\int \frac{e^{acoth(ax)n}}{\sqrt{-a^2x^2+1}\,a^2x^3-\sqrt{-a^2x^2+1}\,x}dx}{\sqrt{c}\,c}$$

input
```
int(exp(n*acoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
( - int(e**(acoth(a*x)*n)/(sqrt( - a**2*x**2 + 1)*a**2*x**3 - sqrt( - a**2
*x**2 + 1)*x),x))/(sqrt(c)*c)
```

3.730 $\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 460

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx &= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}} \\
&- \frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(1+n)(3+n)\,(c-a^2cx^2)^{5/2}} \\
&+ \frac{(15+6n+n^2)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(1-n^2)\,(c-a^2cx^2)^{5/2}} \\
&- \frac{(18+7n-2n^2-n^3)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(9-10n^2+n^4)\,(c-a^2cx^2)^{5/2}} \\
&- \frac{2\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^5\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{(1-n)\,(c-a^2cx^2)^{5/2}}\end{aligned}$$

output
```
-(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(3+
n)/(-a^2*c*x^2+c)^(5/2)-(6+n)*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(
1+1/a/x)^(-3/2+1/2*n)*x^5/(1+n)/(3+n)/(-a^2*c*x^2+c)^(5/2)+(n^2+6*n+15)*(1
-1/a^2/x^2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(3+n)/(
-n^2+1)/(-a^2*c*x^2+c)^(5/2)-(-n^3-2*n^2+7*n+18)*(1-1/a^2/x^2)^(5/2)*(1-1/
a/x)^(3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^
(5/2)-2*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x
^5*hypergeom([1, -1/2+1/2*n],[1/2+1/2*n],(a+1/x)/(a-1/x))/(1-n)/(-a^2*c*x^
2+c)^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.51 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.44

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \frac{(-1+a^2x^2)\left(\frac{8e^{n\coth^{-1}(ax)}(n-ax)}{-1+n^2}+\frac{e^{n\coth^{-1}(ax)}\left(26n-2n^3-27ax+3an^2x+2n(-1+n^2)\cosh(2\coth^{-1}\right.}{9-10n^2+n^4}\right.}{}$$

input
```
Integrate[(E^(n*ArcCoth[a*x])*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
((-1 + a^2*x^2)*((8*E^(n*ArcCoth[a*x])*(n - a*x))/(-1 + n^2) + (E^(n*ArcCo
th[a*x])*(26*n - 2*n^3 - 27*a*x + 3*a*n^2*x + 2*n*(-1 + n^2)*Cosh[2*ArcCot
h[a*x]] - 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]]))/(9
 - 10*n^2 + n^4) - (8*a*E^((1 + n)*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*x*H
ypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(1 + n)))/(
4*a^5*c*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.79, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {6746, 6749, 144, 25, 27, 172, 25, 27, 172, 27, 172, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 6746$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \; 6749$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}xd\frac{1}{x}}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \; 144$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}}{n+3}-\frac{a\int -\frac{\left(a(n+3)+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}x}{a^2}d\frac{1}{x}}{n+3}\right)}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{a\int \frac{\left(a(n+3)+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}x}{a^2}d\frac{1}{x}}{n+3}+\frac{\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{n+3}\right)}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\int \left(a(n+3)+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}xd\frac{1}{x}}{a(n+3)}+\frac{\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{n+3}\right)}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \; 172$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\frac{a(n+6)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}}{n+1}-\frac{a\int -\frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}\left(a(n+1)(n+3)+\frac{2(n+6)}{x}\right)x}{a}d\frac{1}{x}}{n+1}}{a(n+3)}+\frac{\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}}{}\right.}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\frac{a\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}\left(a(n+1)(n+3)+\frac{2(n+6)}{x}\right)x}{a}d\frac{1}{x}}{n+1}+\frac{a(n+6)\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{n+1}}{a(n+3)}+\frac{\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}(}{n}\right.}{(c-a^2cx^2)^{5/2}}$$

$\big\downarrow$ 27

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\frac{\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}\left(a(n+1)(n+3)+\frac{2(n+6)}{x}\right)xd\frac{1}{x}}{n+1}+\frac{a(n+6)\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{n+1}}{a(n+3)}+\frac{\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}(1-}{n+3}\right.}{(c-a^2cx^2)^{5/2}}$$

$\big\downarrow$ 172

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\frac{\frac{a\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}\left(a(1-n)(n+1)(n+3)-\frac{n^2+6n+15}{x}\right)x}{a}d\frac{1}{x}}{1-n}-\frac{a\left(n^2+6n+15\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}}{1-n}}{n+1}+\frac{a(n+6)\left(\frac{1}{a}\right.}{}}{a(n+3)}\right.}{(c-a^2cx^2)^{5/2}}$$

$\big\downarrow$ 27

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\frac{\frac{\int \left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}\left(a(1-n)(n+1)(n+3)-\frac{n^2+6n+15}{x}\right)xd\frac{1}{x}}{1-n}-\frac{a\left(n^2+6n+15\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}}{1-n}}{n+1}+\frac{a(n+6)\left(\frac{1}{ax}\right.}{}}{a(n+3)}\right.}{(c-a^2cx^2)^{5/2}}$$

$\big\downarrow$ 172

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\frac{\frac{\frac{a\int \left(n^4-10n^2+9\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}xd\frac{1}{x}}{3-n}+\frac{a\left(-n^3-2n^2+7n+18\right)\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}}{3-n}}{1-n}-\frac{a\left(n^2+6n+15\right)\left(1-\frac{1}{ax}\right)}{1-n}}{n+1}}{a(n+3)}\right.}{(c-a^2cx^2)^{5/2}}$$

$\big\downarrow$ 27

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\frac{\frac{\frac{a\left(n^4-10n^2+9\right)\int\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}xd\frac{1}{x}}{3-n}+\frac{a\left(-n^3-2n^2+7n+18\right)\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}}{3-n}}{1-n}-\frac{a\left(n^2+6n+15\right)\left(1-\frac{1}{ax}\right)}{1-n}}{n+1}}{a(n+3)}\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{141}$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{\frac{\frac{\frac{2a\left(n^4-10n^2+9\right)\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{n-1}{2},\frac{n+1}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{(1-n)(3-n)}+\frac{a\left(-n^3-2n^2+7n+18\right)\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\right.}{3-n}}{1-n}}{n+1}}{a(n+3)}\right.}{\left(c-a^2cx^2\right)^{5/}}$$

input

```
Int[(E^(n*ArcCoth[a*x])*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

output

```
-(((1 - 1/(a^2*x^2))^(5/2)*x^5*(((1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^
((-3 + n)/2))/(3 + n) + ((a*(6 + n)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x
))^((-3 + n)/2))/(1 + n) + (-((a*(15 + 6*n + n^2)*(1 - 1/(a*x))^((1 - n)/2
)*(1 + 1/(a*x))^((-3 + n)/2))/(1 - n)) + ((a*(18 + 7*n - 2*n^2 - n^3)*(1 -
 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2))/(3 - n) + (2*a*(9 - 10*n
^2 + n^4)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*Hypergeomet
ric2F1[1, (-1 + n)/2, (1 + n)/2, (a + x^(-1))/(a - x^(-1))])/((1 - n)*(3 -
 n)))/(1 - n))/(1 + n))/(a*(3 + n))))/(c - a^2*c*x^2)^(5/2))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 144
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[b*(a + b*x)^(m + 1)*(
c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x]
+ Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2))
- b*d*f*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && NeQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
])))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6749
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/
x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d,
0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
 n/2] && IntegerQ[m]
```

Maple [F]

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)} x^4}{(-a^2 c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
int(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)} x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="frica
s")
```

output
```
integral(-sqrt(-a^2*c*x^2 + c)*x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^6*c^3*
x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)} x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*x**4/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**4*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate(x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac"
)
```

output
```
integrate(x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{(c-a^2\,c\,x^2)^{5/2}}\,dx$$

input
```
int((x^4*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

output
```
int((x^4*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{e^{acoth(ax)n}x^4}{\sqrt{-a^2x^2+1}\,a^4x^4-2\sqrt{-a^2x^2+1}\,a^2x^2+\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(exp(n*acoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
int((e**(acoth(a*x)*n)*x**4)/(sqrt( - a**2*x**2 + 1)*a**4*x**4 - 2*sqrt( -
 a**2*x**2 + 1)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

3.731 $\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 330

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}} - \frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+4n+n^2)\,(c-a^2cx^2)^{5/2}} + \frac{6a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(1-n^2)\,(c-a^2cx^2)^{5/2}} - \frac{6a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(9-10n^2+n^4)\,(c-a^2cx^2)^{5/2}}$$

output
```
-a*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(
3+n)/(-a^2*c*x^2+c)^(5/2)-3*a*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(
1+1/a/x)^(-3/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+6*a*(1-1/a^2/x^
2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(3+n)/(-n^2+1)/(
-a^2*c*x^2+c)^(5/2)-6*a*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(3/2-1/2*n)*(1+1/a/x
)^(-3/2+1/2*n)*x^5/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.33

$$\int \frac{e^{n \coth^{-1}(ax)} x^3}{(c-a^2cx^2)^{5/2}}\, dx =$$
$$-\frac{e^{n\coth^{-1}(ax)}\left(3(10-2n^2-9anx+an^3x)-6(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)+an(-1+n^2)\sqrt{1-\frac{1}{a^2x^2}}x\, c}{4a^4c^2\,(9-10n^2+n^4)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-1/4*(E^(n*ArcCoth[a*x])*(3*(10 - 2*n^2 - 9*a*n*x + a*n^3*x) - 6*(-1 + n^2
)*Cosh[2*ArcCoth[a*x]] + a*n*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*Arc
Coth[a*x]]))/(a^4*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.72, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6746, 6749, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\, dx$$
$$\Big\downarrow \; 6746$$
$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^2}dx}{(c-a^2cx^2)^{5/2}}$$
$$\Big\downarrow \; 6749$$
$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}d\frac{1}{x}}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow 55$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{3\int\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}d\frac{1}{x}}{n+3}+\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{n+3}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow 55$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{3\left(\frac{2\int\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}d\frac{1}{x}}{n+1}+\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{n+1}\right)}{n+3}+\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{n+3}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow 55$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{3\left(\frac{2\left(-\frac{\int\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}d\frac{1}{x}}{1-n}-\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{1-n}\right)}{n+1}+\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{n+1}\right)}{n+3}+\frac{a\left(\frac{1}{ax}+1\right)^{n}}{}\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow 48$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{n+3}+\frac{3\left(\frac{a\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{n+1}+\frac{2\left(\frac{a\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}}{(1-n)(3-n)}-\frac{a\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\right.}{1-n}\right.}{n+1}\right.}{n+3}\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Int[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((((3*((2*(-((a*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2))/(1
- n)) + (a*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2))/((1 - n)*
(3 - n))))/(1 + n) + (a*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)
/2))/(1 + n)))/(3 + n) + (a*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-3
+ n)/2))/(3 + n))*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(c - a^2*c*x^2)^(5/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6749
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/
x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d,
0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
 n/2] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.28

method	result	size
gosper	$-\frac{(ax-1)(ax+1)\left(a^{3}n^{3}x^{3}-7a^{3}x^{3}n-3a^{2}n^{2}x^{2}+9a^{2}x^{2}+6nax-6\right)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{a^{4}\left(n^{4}-10n^{2}+9\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}$	93
orering	$-\frac{(ax-1)(ax+1)\left(a^{3}n^{3}x^{3}-7a^{3}x^{3}n-3a^{2}n^{2}x^{2}+9a^{2}x^{2}+6nax-6\right)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{a^{4}\left(n^{4}-10n^{2}+9\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}$	93

input
```
int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)*(a*x+1)*(a^3*n^3*x^3-7*a^3*n*x^3-3*a^2*n^2*x^2+9*a^2*x^2+6*a*n*x-
6)*exp(n*arccoth(a*x))/a^4/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.53

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{-a^2cx^2+c}\left((a^3n^3-7\,a^3n)x^3+6\,anx-3\,(a^2n^2-3\,a^2)x^2-6\right)\left(\frac{ax+1}{ax-1}\right)}{a^4c^3n^4-10\,a^4c^3n^2+9\,a^4c^3+(a^8c^3n^4-10\,a^8c^3n^2+9\,a^8c^3)x^4-2\,(a^6c^3n^4-10\,a^6c}$$

input
```
integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="frica
s")
```

output
```
sqrt(-a^2*c*x^2 + c)*((a^3*n^3 - 7*a^3*n)*x^3 + 6*a*n*x - 3*(a^2*n^2 - 3*a
^2)*x^2 - 6)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^3*n^4 - 10*a^4*c^3*n^2 +
 9*a^4*c^3 + (a^8*c^3*n^4 - 10*a^8*c^3*n^2 + 9*a^8*c^3)*x^4 - 2*(a^6*c^3*n
^4 - 10*a^6*c^3*n^2 + 9*a^6*c^3)*x^2)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^3e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*x**3/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**3*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.30 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.53

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{6}{a^6\,c^2\,(n^4-10\,n^2+9)}-\frac{6\,n\,x}{a^5\,c^2\,(n^4-10\,n^2+9)}+\frac{x^2\,(3\,n^2-9)}{a^4\,c^2\,(n^4-10\,n^2+9)}-\frac{n\,x^3\,(n^2-7)}{a^3\,c^2\,(n^4-10\,n^2+9)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input
```
int((x^3*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

output
```
-(((a*x + 1)/(a*x))^(n/2)*(6/(a^6*c^2*(n^4 - 10*n^2 + 9)) - (6*n*x)/(a^5*c
^2*(n^4 - 10*n^2 + 9)) + (x^2*(3*n^2 - 9))/(a^4*c^2*(n^4 - 10*n^2 + 9)) -
(n*x^3*(n^2 - 7))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a
^2 - x^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{e^{acoth(ax)n}x^3}{\sqrt{-a^2x^2+1}\,a^4x^4-2\sqrt{-a^2x^2+1}\,a^2x^2+\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(exp(n*acoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
int((e**(acoth(a*x)*n)*x**3)/(sqrt( - a**2*x**2 + 1)*a**4*x**4 - 2*sqrt( -
 a**2*x**2 + 1)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

3.732 $\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 102

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx=-\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{a^3c(9-n^2)(c-a^2cx^2)^{3/2}}+\frac{e^{n\coth^{-1}(ax)}(3-n^2)(n-ax)}{a^3c^2(9-10n^2+n^4)\sqrt{c-a^2cx^2}}$$

output
```
-exp(n*arccoth(a*x))*(-3*a*x+n)/a^3/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)+exp(n*
arccoth(a*x))*(-n^2+3)*(-a*x+n)/a^3/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2
)
```

Mathematica [A] (warning: unable to verify)

Time = 1.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx=\frac{e^{n\coth^{-1}(ax)}\left(10n-2n^3-9ax+an^2x-2n(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)+3a(-1\right.}{4a^3c^2(9-10n^2+n^4)\sqrt{c-a^2cx^2}}$$

input
```
Integrate[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(10*n - 2*n^3 - 9*a*x + a*n^2*x - 2*n*(-1 + n^2)*Cosh[
2*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*
x]]))/(4*a^3*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6743, 6738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{6743}$$

$$-\frac{(3-n^2)\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx}{a^2c(9-n^2)}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{a^3c(9-n^2)(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6738}$$

$$\frac{(3-n^2)(n-ax)e^{n\coth^{-1}(ax)}}{a^3c^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{a^3c(9-n^2)(c-a^2cx^2)^{3/2}}$$

input
```
Int[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a^3*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2))
) + (E^(n*ArcCoth[a*x])*(3 - n^2)*(n - a*x))/(a^3*c^2*(1 - n^2)*(9 - n^2)*
Sqrt[c - a^2*c*x^2])
```

Defintions of rubi rules used

rule 6738
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

rule 6743
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a^
3*c*(n^2 - 4*(p + 1)^2))), x] - Simp[(n^2 + 2*(p + 1))/(a^2*c*(n^2 - 4*(p +
 1)^2))   Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c
, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[n^
2 + 2*(p + 1), 0] && NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !Integer
Q[n])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{(ax-1)(ax+1)\left(a^{3} n^{2} x^{3}-a^{2} n^{3} x^{2}-3 a^{3} x^{3}+3 n \,x^{2} a^{2}+2 n^{2} x a-2 n\right) {\mathrm e}^{n \,\operatorname{arccoth}(a x)}}{\left(n^{4}-10 n^{2}+9\right) a^{3} \left(-a^{2} c \,x^{2}+c\right)^{\frac{5}{2}}}$	96
orering	$\frac{(ax-1)(ax+1)\left(a^{3} n^{2} x^{3}-a^{2} n^{3} x^{2}-3 a^{3} x^{3}+3 n \,x^{2} a^{2}+2 n^{2} x a-2 n\right) {\mathrm e}^{n \,\operatorname{arccoth}(a x)}}{\left(n^{4}-10 n^{2}+9\right) a^{3} \left(-a^{2} c \,x^{2}+c\right)^{\frac{5}{2}}}$	96

input
```
int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)*(a*x+1)*(a^3*n^2*x^3-a^2*n^3*x^2-3*a^3*x^3+3*a^2*n*x^2+2*a*n^2*x-2
*n)*exp(n*arccoth(a*x))/(n^4-10*n^2+9)/a^3/(-a^2*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.76

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\sqrt{-a^2cx^2+c}(2\,an^2x+(a^3n^2-3\,a^3)x^3-(a^2n^3-3\,a^2n)x^2-2\,n)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3+(a^7c^3n^4-10\,a^7c^3n^2+9\,a^7c^3)x^4-2\,(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3)x^2}$$

input
```
integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="frica
s")
```

output
```
-sqrt(-a^2*c*x^2 + c)*(2*a*n^2*x + (a^3*n^2 - 3*a^3)*x^3 - (a^2*n^3 - 3*a^
2*n)*x^2 - 2*n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^3*c^3*n^4 - 10*a^3*c^3*n^
2 + 9*a^3*c^3 + (a^7*c^3*n^4 - 10*a^7*c^3*n^2 + 9*a^7*c^3)*x^4 - 2*(a^5*c^
3*n^4 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^2)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^2e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*x**2/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**2*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac"
)
```

output
```
integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.82 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.72

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{2\,n}{a^5\,c^2\,(n^4-10\,n^2+9)}-\frac{x^3\,(n^2-3)}{a^2\,c^2\,(n^4-10\,n^2+9)}-\frac{2\,n^2\,x}{a^4\,c^2\,(n^4-10\,n^2+9)}+\frac{n\,x^2\,(n^2-3)}{a^3\,c^2\,(n^4-10\,n^2+9)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input
```
int((x^2*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

output
```
(((a*x + 1)/(a*x))^(n/2)*((2*n)/(a^5*c^2*(n^4 - 10*n^2 + 9)) - (x^3*(n^2 -
 3))/(a^2*c^2*(n^4 - 10*n^2 + 9)) - (2*n^2*x)/(a^4*c^2*(n^4 - 10*n^2 + 9))
 + (n*x^2*(n^2 - 3))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2
)/a^2 - x^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{e^{acoth(ax)n}x^2}{\sqrt{-a^2x^2+1}\,a^4x^4-2\sqrt{-a^2x^2+1}\,a^2x^2+\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(exp(n*acoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
int((e**(acoth(a*x)*n)*x**2)/(sqrt( - a**2*x**2 + 1)*a**4*x**4 - 2*sqrt( -
 a**2*x**2 + 1)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

3.733 $\int \frac{e^{n \coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{e^{n \coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx=\frac{e^{n \coth^{-1}(ax)}(3-anx)}{a^2c\,(9-n^2)\,(c-a^2cx^2)^{3/2}}+\frac{2e^{n \coth^{-1}(ax)}n(n-ax)}{a^2c^2\,(9-10n^2+n^4)\,\sqrt{c-a^2cx^2}}$$

output
```
exp(n*arccoth(a*x))*(-a*n*x+3)/a^2/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)+2*exp(n
*arccoth(a*x))*n*(-a*x+n)/a^2/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.80 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

$$\int \frac{e^{n \coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx=\frac{e^{n \coth^{-1}(ax)}\left(6+2n^2-9anx+an^3x+6(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)-an(-1+\right.}{4a^2c^2\,(9-10n^2+n^4)\,\sqrt{c-a^2cx^2}}$$

input
```
Integrate[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(6 + 2*n^2 - 9*a*n*x + a*n^3*x + 6*(-1 + n^2)*Cosh[2*A
rcCoth[a*x]] - a*n*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]]
))/(4*a^2*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6741, 6738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{6741}$$

$$\frac{(3-anx)e^{n\coth^{-1}(ax)}}{a^2c\,(9-n^2)\,(c-a^2cx^2)^{3/2}}-\frac{2n\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx}{ac\,(9-n^2)}$$

$$\Big\downarrow \text{6738}$$

$$\frac{2n(n-ax)e^{n\coth^{-1}(ax)}}{a^2c^2\,(1-n^2)\,(9-n^2)\,\sqrt{c-a^2cx^2}}+\frac{(3-anx)e^{n\coth^{-1}(ax)}}{a^2c\,(9-n^2)\,(c-a^2cx^2)^{3/2}}$$

input
```
Int[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(3 - a*n*x))/(a^2*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)) +
 (2*E^(n*ArcCoth[a*x])*n*(n - a*x))/(a^2*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c -
a^2*c*x^2])
```

Defintions of rubi rules used

rule 6738
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

rule 6741
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(2*(p + 1) + a*n*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a^2*
c*(n^2 - 4*(p + 1)^2))), x] - Simp[n*((2*p + 3)/(a*c*(n^2 - 4*(p + 1)^2)))
  Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
 x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[p, -3/2] &
& NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{(ax-1)(ax+1)\left(2a^{3}x^{3}n-2a^{2}n^{2}x^{2}+a\,n^{3}x-3nax-n^{2}+3\right){\mathrm e}^{n\,\operatorname{arccoth}(ax)}}{a^{2}\left(n^{4}-10n^{2}+9\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}$	86
orering	$-\frac{(ax-1)(ax+1)\left(2a^{3}x^{3}n-2a^{2}n^{2}x^{2}+a\,n^{3}x-3nax-n^{2}+3\right){\mathrm e}^{n\,\operatorname{arccoth}(ax)}}{a^{2}\left(n^{4}-10n^{2}+9\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}$	86

input
```
int(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)*(a*x+1)*(2*a^3*n*x^3-2*a^2*n^2*x^2+a*n^3*x-3*a*n*x-n^2+3)*exp(n*a
rccoth(a*x))/a^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.76

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \frac{(2\,a^3nx^3-2\,a^2n^2x^2-n^2+(an^3-3\,an)x+3)\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}}}{a^2c^3n^4-10\,a^2c^3n^2+9\,a^2c^3+(a^6c^3n^4-10\,a^6c^3n^2+9\,a^6c^3)x^4-2\,(a^4c^3n^4-10\,a^4c}$$

input
```
integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas"
)
```

output
```
(2*a^3*n*x^3 - 2*a^2*n^2*x^2 - n^2 + (a*n^3 - 3*a*n)*x + 3)*sqrt(-a^2*c*x^
2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^3*n^4 - 10*a^2*c^3*n^2 + 9*a^2
*c^3 + (a^6*c^3*n^4 - 10*a^6*c^3*n^2 + 9*a^6*c^3)*x^4 - 2*(a^4*c^3*n^4 - 1
0*a^4*c^3*n^2 + 9*a^4*c^3)*x^2)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{xe^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*x/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima"
)
```

output
```
integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx=\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.45 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.81

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{n^2-3}{a^4\,c^2\,(n^4-10\,n^2+9)}+\frac{2\,n^2\,x^2}{a^2\,c^2\,(n^4-10\,n^2+9)}-\frac{2\,n\,x^3}{a\,c^2\,(n^4-10\,n^2+9)}-\frac{n\,x\,(n^2-3)}{a^3\,c^2\,(n^4-10\,n^2+9)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input
```
int((x*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

output
```
-(((a*x + 1)/(a*x))^(n/2)*((n^2 - 3)/(a^4*c^2*(n^4 - 10*n^2 + 9)) + (2*n^2
*x^2)/(a^2*c^2*(n^4 - 10*n^2 + 9)) - (2*n*x^3)/(a*c^2*(n^4 - 10*n^2 + 9))
- (n*x*(n^2 - 3))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a
^2 - x^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\int \frac{e^{acoth(ax)n}x}{\sqrt{-a^2x^2+1}\,a^4x^4-2\sqrt{-a^2x^2+1}\,a^2x^2+\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(exp(n*acoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
int((e**(acoth(a*x)*n)*x)/(sqrt( - a**2*x**2 + 1)*a**4*x**4 - 2*sqrt( - a*
*2*x**2 + 1)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

3.734 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}} \, dx = -\frac{e^{n \coth^{-1}(ax)}(n-3ax)}{ac\,(9-n^2)\,(c-a^2cx^2)^{3/2}} - \frac{6e^{n \coth^{-1}(ax)}(n-ax)}{ac^2\,(1-n^2)\,(9-n^2)\,\sqrt{c-a^2cx^2}}$$

output
```
-exp(n*arccoth(a*x))*(-3*a*x+n)/a/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)-6*exp(n*
arccoth(a*x))*(-a*x+n)/a/c^2/(-n^2+1)/(-n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}} \, dx = \frac{e^{n \coth^{-1}(ax)}\left(-26n+2n^3+27ax-3an^2x-2n(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)+3a\right.}{4ac^2\,(9-10n^2+n^4)\,\sqrt{c-a^2cx^2}}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(E^(n*ArcCoth[a*x])*(-26*n + 2*n^3 + 27*a*x - 3*a*n^2*x - 2*n*(-1 + n^2)*C
osh[2*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCot
h[a*x]]))/(4*a*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6739, 6738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{6739}$$

$$\frac{6\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}dx}{c(9-n^2)}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{6738}$$

$$-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)))
- (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2
*c*x^2])
```

Defintions of rubi rules used

rule 6738
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

rule 6739
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2)))
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{(ax-1)(ax+1)(6a^3x^3-6n\,x^2a^2+3n^2xa-n^3-9ax+7n)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{a(n^4-10n^2+9)(-a^2c\,x^2+c)^{\frac{5}{2}}}$	84
orering	$\frac{(ax-1)(ax+1)(6a^3x^3-6n\,x^2a^2+3n^2xa-n^3-9ax+7n)\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{a(n^4-10n^2+9)(-a^2c\,x^2+c)^{\frac{5}{2}}}$	84

input
```
int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)*(a*x+1)*(6*a^3*x^3-6*a^2*n*x^2+3*a*n^2*x-n^3-9*a*x+7*n)*exp(n*arcc
oth(a*x))/a/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.62

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx =$$
$$-\frac{(6\,a^3x^3-6\,a^2nx^2-n^3+3\,(an^2-3\,a)x+7\,n)\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^3n^4-10\,ac^3n^2+(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3)x^4+9\,ac^3-2\,(a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3)x^2}$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-(6*a^3*x^3 - 6*a^2*n*x^2 - n^3 + 3*(a*n^2 - 3*a)*x + 7*n)*sqrt(-a^2*c*x^2
 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^4 - 10*a*c^3*n^2 + (a^5*c^3*n
^4 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^4 + 9*a*c^3 - 2*(a^3*c^3*n^4 - 10*a^3*c
^3*n^2 + 9*a^3*c^3)*x^2)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.70

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{6\,x^3}{c^2\,(n^4-10\,n^2+9)}+\frac{7\,n-n^3}{a^3\,c^2\,(n^4-10\,n^2+9)}+\frac{3\,x\,(n^2-3)}{a^2\,c^2\,(n^4-10\,n^2+9)}-\frac{6\,n\,x^2}{a\,c^2\,(n^4-10\,n^2+9)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

input
```
int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(5/2),x)
```

output
```
-(((a*x + 1)/(a*x))^(n/2)*((6*x^3)/(c^2*(n^4 - 10*n^2 + 9)) + (7*n - n^3)/
(a^3*c^2*(n^4 - 10*n^2 + 9)) + (3*x*(n^2 - 3))/(a^2*c^2*(n^4 - 10*n^2 + 9)
) - (6*n*x^2)/(a*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2 - x
^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{e^{acoth(ax)n}}{\sqrt{-a^2x^2+1}\,a^4x^4-2\sqrt{-a^2x^2+1}\,a^2x^2+\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(exp(n*acoth(a*x))/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
int(e**(acoth(a*x)*n)/(sqrt( - a**2*x**2 + 1)*a**4*x**4 - 2*sqrt( - a**2*x
**2 + 1)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

3.735 $\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 944

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

output

```
-a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5
/(3+n)/(-a^2*c*x^2+c)^(5/2)-3*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*
n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+6*a^5*(1-1/
a^2/x^2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(3+n)/(-n^
2+1)/(-a^2*c*x^2+c)^(5/2)-6*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(3/2-1/2*n)*
(1+1/a/x)^(-3/2+1/2*n)*x^5/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)+4*a^5*(1-1/
a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^5/(3+n)/(-a
^2*c*x^2+c)^(5/2)+8*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/
x)^(-1/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)-8*a^5*(1-1/a^2/x^2)^(
5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^5/(3+n)/(-n^2+1)/(-a^2
*c*x^2+c)^(5/2)-6*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)
^(1/2+1/2*n)*x^5/(3+n)/(-a^2*c*x^2+c)^(5/2)-6*a^5*(1-1/a^2/x^2)^(5/2)*(1-1
/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(1/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5
/2)+4*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(3/2+1/2*n)
*x^5/(3+n)/(-a^2*c*x^2+c)^(5/2)-2^(5/2+1/2*n)*a^5*(1-1/a^2/x^2)^(5/2)*(1-1
/a/x)^(-3/2-1/2*n)*x^5*hypergeom([-3/2-1/2*n, -3/2-1/2*n],[-1/2-1/2*n],1/2
*(a-1/x)/a)/(3+n)/(-a^2*c*x^2+c)^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.21 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.23

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\,(c-a^2cx^2)^{5/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}\left(a\sqrt{1-\frac{1}{a^2x^2}}x(42-2n^2-45anx+5an^3x)+6a(-1+n^2)\sqrt{1-\frac{1}{a^2x^2}}\right.}{}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
(E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(42 - 2*n^2 - 45*a*n*x + 5*
a*n^3*x) + 6*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[2*ArcCoth[a*x]] - n
*(-1 + n^2)*(-1 + a^2*x^2)*Cosh[3*ArcCoth[a*x]]) - 8*E^((1 + n)*ArcCoth[a*
x])*(9 - 9*n - n^2 + n^3)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (
3 + n)/2, -E^(2*ArcCoth[a*x])])/(4*a*c^2*(-1 + n)*(1 + n)*(-9 + n^2)*Sqrt[
1 - 1/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 624, normalized size of antiderivative = 0.66, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6746, 6749, 137, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\,(c-a^2cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{6746}$$

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}dx}{(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \text{6749}$$

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \frac{\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}}}{x^4}d\frac{1}{x}}{\left(c-a^2cx^2\right)^{5/2}}$$

$\Big\downarrow$ 137

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\int \left(-4a^4\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}+1}+6a^4\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-5)}\left(1+\frac{1}{ax}\right)^{\frac{n-5}{2}+2}-4a^4\left(1-\frac{1}{ax}\right)^{\frac{1}{2}}\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

$\Big\downarrow$ 2009

$$-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{a^5 2^{\frac{n+5}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-3),\frac{1}{2}(-n-3),\frac{1}{2}(-n-1),\frac{a-\frac{1}{x}}{2a}\right)}{n+3}+\frac{3a^5\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-}}{n^2+4n+3}\right.}{}$$

input
```
Int[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(5/2)),x]
```

output
```
-(((1 - 1/(a^2*x^2))^(5/2)*x^5*((a^5*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*
x))^((-3 + n)/2))/(3 + n) + (3*a^5*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x)
)^((-3 + n)/2))/(3 + 4*n + n^2) - (6*a^5*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/
(a*x))^((-3 + n)/2))/((3 + n)*(1 - n^2)) + (6*a^5*(1 - 1/(a*x))^((3 - n)/2
)*(1 + 1/(a*x))^((-3 + n)/2))/(9 - 10*n^2 + n^4) - (4*a^5*(1 - 1/(a*x))^((
-3 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2))/(3 + n) - (8*a^5*(1 - 1/(a*x))^((-1
 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2))/(3 + 4*n + n^2) + (8*a^5*(1 - 1/(a*x)
)^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2))/((3 + n)*(1 - n^2)) + (6*a^5*(1
- 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((1 + n)/2))/(3 + n) + (6*a^5*(1 - 1
/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((1 + n)/2))/(3 + 4*n + n^2) - (4*a^5*(
1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((3 + n)/2))/(3 + n) + (2^((5 + n)
/2)*a^5*(1 - 1/(a*x))^((-3 - n)/2)*Hypergeometric2F1[(-3 - n)/2, (-3 - n)/
2, (-1 - n)/2, (a - x^(-1))/(2*a)])/(3 + n)))/(c - a^2*c*x^2)^(5/2))
```

Defintions of rubi rules used

rule 137
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (IGtQ[m, 0] || (ILtQ[m, 0] &&
ILtQ[n, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6749
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/
x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d,
0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
 n/2] && IntegerQ[m]
```

Maple [F]

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{x\left(-a^{2} c\, x^{2}+c\right)^{\frac{5}{2}}}dx$$

input
```
int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^6*c^3*x^7
- 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 - c^3*x), x)
```

Sympy [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx=\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/x/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/(x*(-c*(a*x - 1)*(a*x + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima"
)
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-a^2cx^2+c)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{x\,(c-a^2\,c\,x^2)^{5/2}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(5/2)),x)
```

output
```
int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{e^{acoth(ax)n}}{\sqrt{-a^2x^2+1}\,a^4x^5-2\sqrt{-a^2x^2+1}\,a^2x^3+\sqrt{-a^2x^2+1}\,x}dx}{\sqrt{c}\,c^2}$$

input
```
int(exp(n*acoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
int(e**(acoth(a*x)*n)/(sqrt( - a**2*x**2 + 1)*a**4*x**5 - 2*sqrt( - a**2*x
**2 + 1)*a**2*x**3 + sqrt( - a**2*x**2 + 1)*x),x)/(sqrt(c)*c**2)
```

3.736 $\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$$
$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}+p}\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}+p}x(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(-1-2p,\frac{1}{2}(\right.}{1+2p}$$

output
```
((a-1/x)/(a+1/x))^(1/2*n-p)*(1-1/a/x)^(-1/2*n+p)*(1+1/a/x)^(1+1/2*n+p)*x*(
-a^2*c*x^2+c)^p*hypergeom([-1-2*p, 1/2*n-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((
1-1/a^2/x^2)^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx =$$
$$-\frac{e^{(-2+n)\coth^{-1}(ax)}\left(-1+e^{2\coth^{-1}(ax)}\right)(-1+a^2x^2)(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(1,-\frac{n}{2}-p,2-\frac{n}{2}+\right.}{a(n-2(1+p))}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
-((E^((-2 + n)*ArcCoth[a*x])*(-1 + E^(2*ArcCoth[a*x]))*(-1 + a^2*x^2)*(c -
 a^2*c*x^2)^p*Hypergeometric2F1[1, -1/2*n - p, 2 - n/2 + p, E^(-2*ArcCoth[
a*x])])/(a*(n - 2*(1 + p))))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6746, 6750, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^p e^{n\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6746}$$

$$x^{-2p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}dx$$

$$\Big\downarrow \text{ 6750}$$

$$\left(\frac{1}{x}\right)^{2p}\left(-\left(1-\frac{1}{a^2x^2}\right)^{-p}\right)\left(c-a^2cx^2\right)^p\int \left(1-\frac{1}{ax}\right)^{p-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n}{2}+p}\left(\frac{1}{x}\right)^{-2(p+1)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 142}$$

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{p-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+p+1}\text{Hypergeometric2F1}\left(-2p-1,\frac{1}{2}(n-2p)\right.}{2p+1}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
(((a - x^(-1))/(a + x^(-1)))^((n - 2*p)/2)*(1 - 1/(a*x))^(-1/2*n + p)*(1 +
 1/(a*x))^(1 + n/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, (n
 - 2*p)/2, -2*p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6750
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Simp[(-c^p)*x^m*(1/x)^m   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/
a)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
 EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !In
tegersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)} \left(-a^2 c\,x^2+c\right)^p dx$$

input
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x)
```

output
```
int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx=\int(-a^2cx^2+c)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

output
```
integral((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx=\int(-c(ax-1)(ax+1))^p e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**p,x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**p*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx=\int(-a^2cx^2+c)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

output
```
integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,\left(c-a^2\,c\,x^2\right)^p\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^p,x)
```

output
```
int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^p, x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int e^{acoth(ax)n}\left(-a^2c\,x^2+c\right)^p\,dx$$

input
```
int(exp(n*acoth(a*x))*(-a^2*c*x^2+c)^p,x)
```

output
```
int(e**(acoth(a*x)*n)*( - a**2*c*x**2 + c)**p,x)
```

3.737 $\int e^{2p\coth^{-1}(ax)}(c-a^2cx^2)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p \, dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+2p}x\left(c-a^2cx^2\right)^p}{1+2p}$$

output
```
(1+1/a/x)^(1+2*p)*x*(-a^2*c*x^2+c)^p/(1+2*p)/((1-1/a^2/x^2)^p)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p \, dx=\frac{e^{2p\coth^{-1}(ax)}(1+ax)\left(c-a^2cx^2\right)^p}{a+2ap}$$

input
```
Integrate[E^(2*p*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
(E^(2*p*ArcCoth[a*x])*(1 + a*x)*(c - a^2*c*x^2)^p)/(a + 2*a*p)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6746, 6750, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2cx^2\right)^p e^{2p\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6746}$$

$$x^{-2p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}dx$$

$$\Big\downarrow \text{ 6750}$$

$$\left(\frac{1}{x}\right)^{2p}\left(-\left(1-\frac{1}{a^2x^2}\right)^{-p}\right)\left(c-a^2cx^2\right)^p\int \left(1+\frac{1}{ax}\right)^{2p}\left(\frac{1}{x}\right)^{-2(p+1)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{ax}+1\right)^{2p+1}\left(c-a^2cx^2\right)^p}{2p+1}$$

input
```
Int[E^(2*p*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
((1 + 1/(a*x))^(1 + 2*p)*x*(c - a^2*c*x^2)^p)/((1 + 2*p)*(1 - 1/(a^2*x^2))
^p)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6750
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Simp[(-c^p)*x^m*(1/x)^m   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/
a)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
 EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !In
tegersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

method	result
gosper	$\frac{(ax+1)\mathrm{e}^{2p\ \mathrm{arccoth}(ax)}\left(-a^{2}c\,x^{2}+c\right)^{p}}{a(2p+1)}$
orering	$\frac{(ax+1)\mathrm{e}^{2p\ \mathrm{arccoth}(ax)}\left(-a^{2}c\,x^{2}+c\right)^{p}}{a(2p+1)}$
parallelrisch	$-\frac{-\mathrm{e}^{2p\ \mathrm{arccoth}(ax)}x\left(-a^{2}c\,x^{2}+c\right)^{p}a-\mathrm{e}^{2p\ \mathrm{arccoth}(ax)}\left(-a^{2}c\,x^{2}+c\right)^{p}}{a(2p+1)}$
risch	$(ax+1)(ax+1)^{2p}(ax-1)^{-p}c^{p}(ax-1)^{p}\mathrm{e}^{-\frac{ip\pi\left(\mathrm{csgn}(i(ax-1)(ax+1))^{3}-\mathrm{csgn}(i(ax-1)(ax+1))^{2}\,\mathrm{csgn}(i(ax+1))-\mathrm{csgn}(i(ax-1)(ax+1))^{2}\,\mathrm{c}\right.}{}}$

input
```
int(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x,method=_RETURNVERBOSE)
```

output
```
(a*x+1)/a/(2*p+1)*exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{(ax+1)(-a^2cx^2+c)^p\left(\frac{ax+1}{ax-1}\right)^p}{2\,ap+a}$$

input
```
integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

output
```
(a*x + 1)*(-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^p/(2*a*p + a)
```

Sympy [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$
$$=\begin{cases}-\frac{ix}{\sqrt{c}} & \text{for}\ a=0\wedge p=-\frac{1}{2}\\ c^pxe^{i\pi p} & \text{for}\ a=0\\ \int\frac{e^{-\operatorname{acoth}(ax)}}{\sqrt{-c(ax-1)(ax+1)}}\,dx & \text{for}\ p=-\frac{1}{2}\\ \frac{ax(-a^2cx^2+c)^pe^{2p\operatorname{acoth}(ax)}}{2ap+a}+\frac{(-a^2cx^2+c)^pe^{2p\operatorname{acoth}(ax)}}{2ap+a} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(2*p*acoth(a*x))*(-a**2*c*x**2+c)**p,x)
```

output
```
Piecewise((-I*x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x*exp(I*pi*p), Eq(
a, 0)), (Integral(exp(-acoth(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x), Eq(p,
 -1/2)), (a*x*(-a**2*c*x**2 + c)**p*exp(2*p*acoth(a*x))/(2*a*p + a) + (-a*
*2*c*x**2 + c)**p*exp(2*p*acoth(a*x))/(2*a*p + a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{(a(-c)^p x+(-c)^p)(ax+1)^{2\,p}}{a(2\,p+1)}$$

input
```
integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

output
```
(a*(-c)^p*x + (-c)^p)*(a*x + 1)^(2*p)/(a*(2*p + 1))
```

Giac [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

input
```
integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [B] (verification not implemented)

Time = 13.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{\left(c-a^2\,c\,x^2\right)^p\,\left(a\,x+1\right)\,\left(\frac{a\,x+1}{a\,x}\right)^p}{a\,\left(2\,p+1\right)\,\left(\frac{a\,x-1}{a\,x}\right)^p}$$

input
```
int(exp(2*p*acoth(a*x))*(c - a^2*c*x^2)^p,x)
```

output
```
((c - a^2*c*x^2)^p*(a*x + 1)*((a*x + 1)/(a*x))^p)/(a*(2*p + 1)*((a*x - 1)/
(a*x))^p)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

$$\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{e^{2acoth(ax)p}(-a^2c\,x^2+c)^p\,(ax-1)}{a\,(2p+1)}$$

input
```
int(exp(2*p*acoth(a*x))*(-a^2*c*x^2+c)^p,x)
```

output
```
(e**(2*acoth(a*x)*p)*( - a**2*c*x**2 + c)**p*(a*x - 1))/(a*(2*p + 1))
```

3.738 $\int e^{-2p\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 52

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{1+2p}x(c-a^2cx^2)^p}{1+2p}$$

output
```
(1-1/a/x)^(1+2*p)*x*(-a^2*c*x^2+c)^p/(1+2*p)/((1-1/a^2/x^2)^p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{e^{-2p\coth^{-1}(ax)}(-1+ax)\left(c-a^2cx^2\right)^p}{a+2ap}$$

input
```
Integrate[(c - a^2*c*x^2)^p/E^(2*p*ArcCoth[a*x]),x]
```

output
```
((-1 + a*x)*(c - a^2*c*x^2)^p)/(E^(2*p*ArcCoth[a*x])*(a + 2*a*p))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6746, 6750, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-a^2cx^2)^p e^{-2p\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \ 6746$$

$$x^{-2p}\left(1-\frac{1}{a^2x^2}\right)^{-p}(c-a^2cx^2)^p\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}dx$$

$$\Big\downarrow \ 6750$$

$$\left(\frac{1}{x}\right)^{2p}\left(-\left(1-\frac{1}{a^2x^2}\right)^{-p}\right)(c-a^2cx^2)^p\int \left(1-\frac{1}{ax}\right)^{2p}\left(\frac{1}{x}\right)^{-2(p+1)}d\frac{1}{x}$$

$$\Big\downarrow \ 48$$

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{2p+1}(c-a^2cx^2)^p}{2p+1}$$

input
```
Int[(c - a^2*c*x^2)^p/E^(2*p*ArcCoth[a*x]),x]
```

output
```
((1 - 1/(a*x))^(1 + 2*p)*x*(c - a^2*c*x^2)^p)/((1 + 2*p)*(1 - 1/(a^2*x^2))
^p)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6750
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Simp[(-c^p)*x^m*(1/x)^m   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/
a)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
 EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !In
tegersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

method	result
gosper	$\frac{(ax-1)\left(-a^{2}c\,x^{2}+c\right)^{p}\mathrm{e}^{-2p\ \mathrm{arccoth}(ax)}}{a(2p+1)}$
orering	$\frac{(ax-1)\left(-a^{2}c\,x^{2}+c\right)^{p}\mathrm{e}^{-2p\ \mathrm{arccoth}(ax)}}{a(2p+1)}$
parallelrisch	$\frac{\left(x\left(-a^{2}c\,x^{2}+c\right)^{p}a-\left(-a^{2}c\,x^{2}+c\right)^{p}\right)\mathrm{e}^{-2p\ \mathrm{arccoth}(ax)}}{a(2p+1)}$
risch	$\frac{(ax-1)\left(\left(ax-1\right)^{p}\right)^{2}c^{p}\mathrm{e}^{-\frac{ip\pi\left(\mathrm{csgn}(i(ax-1)(ax+1))^{3}-\mathrm{csgn}(i(ax-1)(ax+1))^{2}\,\mathrm{csgn}(i(ax+1))-\mathrm{csgn}(i(ax-1)(ax+1))^{2}\,\mathrm{csgn}(i(ax-1))+\mathrm{csgn}\cdots\right.}{}}}{}$

input
```
int((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)/a/(2*p+1)*(-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{(ax-1)(-a^2cx^2+c)^p}{(2\,ap+a)\left(\frac{ax+1}{ax-1}\right)^p}$$

input
```
integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")
```

output
```
(a*x - 1)*(-a^2*c*x^2 + c)^p/((2*a*p + a)*((a*x + 1)/(a*x - 1))^p)
```

Sympy [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$
$$=\begin{cases}\frac{ix}{\sqrt{c}} & \text{for } a=0\wedge p=-\frac{1}{2}\\ c^p x e^{-i\pi p} & \text{for } a=0\\ \int\frac{e^{\operatorname{acoth}(ax)}}{\sqrt{-c(ax-1)(ax+1)}}\,dx & \text{for } p=-\frac{1}{2}\\ \frac{ax(-a^2cx^2+c)^p}{2ape^{2p\operatorname{acoth}(ax)}+ae^{2p\operatorname{acoth}(ax)}}-\frac{(-a^2cx^2+c)^p}{2ape^{2p\operatorname{acoth}(ax)}+ae^{2p\operatorname{acoth}(ax)}} & \text{otherwise}\end{cases}$$

input
```
integrate((-a**2*c*x**2+c)**p/exp(2*p*acoth(a*x)),x)
```

output
```
Piecewise((I*x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x*exp(-I*pi*p), Eq(
a, 0)), (Integral(exp(acoth(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x), Eq(p,
-1/2)), (a*x*(-a**2*c*x**2 + c)**p/(2*a*p*exp(2*p*acoth(a*x)) + a*exp(2*p*
acoth(a*x))) - (-a**2*c*x**2 + c)**p/(2*a*p*exp(2*p*acoth(a*x)) + a*exp(2*
p*acoth(a*x))), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{(a(-c)^p x-(-c)^p)(ax-1)^{2p}}{a(2p+1)}$$

input
```
integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
```

output
```
(a*(-c)^p*x - (-c)^p)*(a*x - 1)^(2*p)/(a*(2*p + 1))
```

Giac [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(-a^2cx^2+c)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^p/((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{(c-a^2\,c\,x^2)^p\,(a\,x-1)\,\left(\frac{a\,x-1}{a\,x}\right)^p}{a\,(2\,p+1)\,\left(\frac{a\,x+1}{a\,x}\right)^p}$$

input
```
int(exp(-2*p*acoth(a*x))*(c - a^2*c*x^2)^p,x)
```

output
```
((c - a^2*c*x^2)^p*(a*x - 1)*((a*x - 1)/(a*x))^p)/(a*(2*p + 1)*((a*x + 1)/
(a*x))^p)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\frac{\left(-a^2c\,x^2+c\right)^p(ax+1)}{e^{2acoth(ax)p}a\,(2p+1)}$$

input
```
int((-a^2*c*x^2+c)^p/exp(2*p*acoth(a*x)),x)
```

output
```
(( - a**2*c*x**2 + c)**p*(a*x + 1))/(e**(2*acoth(a*x)*p)*a*(2*p + 1))
```

3.739 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{2^{2+p}c(1+ax)^{1-p}\left(c-a^2cx^2\right)^{-1+p}\text{Hypergeometric2F1}\left(-2-p,-1+p,p,\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

output
```
2^(2+p)*c*(a*x+1)^(1-p)*(-a^2*c*x^2+c)^(-1+p)*hypergeom([-1+p, -2-p],[p],-
1/2*a*x+1/2)/a/(1-p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = -\frac{2^{2+p}(1-ax)^{-1+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-2-p,-1+p,p,\frac{1}{2}(1-ax)\right)}{a(-1+p)}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
-((2^(2 + p)*(1 - a*x)^(-1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-2 - p
, -1 + p, p, (1 - a*x)/2])/(a*(-1 + p)*(1 - a^2*x^2)^p))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6717, 6691, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int e^{4\text{arctanh}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{ 6691}$$

$$c^2\int (ax+1)^4\left(c-a^2cx^2\right)^{p-2}dx$$

$$\Big\downarrow \text{ 473}$$

$$c^2(ax+1)^{1-p}(c-acx)^{1-p}\left(c-a^2cx^2\right)^{p-1}\int (ax+1)^{p+2}(c-acx)^{p-2}dx$$

$$\Big\downarrow \text{ 79}$$

$$\frac{c2^{p+2}(ax+1)^{1-p}\left(c-a^2cx^2\right)^{p-1}\text{Hypergeometric2F1}\left(-p-2,p-1,p,\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
(2^(2 + p)*c*(1 + a*x)^(1 - p)*(c - a^2*c*x^2)^(-1 + p)*Hypergeometric2F1[
-2 - p, -1 + p, p, (1 - a*x)/2])/(a*(1 - p))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [F]

$$\int \frac{(ax+1)^2 (-a^2c\,x^2+c)^p}{(ax-1)^2}dx$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x)
```

output
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x)
```

Fricas [F]

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{(ax+1)^2(-a^2cx^2+c)^p}{(ax-1)^2}\,dx$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

output
```
integral((a^2*x^2 + 2*a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 2*a*x + 1), x
)
```

Sympy [F]

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{(-c(ax-1)(ax+1))^p(ax+1)^2}{(ax-1)^2}\,dx$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**p,x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**2/(a*x - 1)**2, x)
```

Maxima [F]

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{(ax+1)^2(-a^2cx^2+c)^p}{(ax-1)^2}\,dx$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)^2*(-a^2*c*x^2 + c)^p/(a*x - 1)^2, x)
```

Giac [F]

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax+1)^2(-a^2cx^2+c)^p}{(ax-1)^2}\,dx$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
integrate((a*x + 1)^2*(-a^2*c*x^2 + c)^p/(a*x - 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(c-a^2\,c\,x^2)^p\,(a\,x+1)^2}{(a\,x-1)^2}\,dx$$

input
```
int(((c - a^2*c*x^2)^p*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
int(((c - a^2*c*x^2)^p*(a*x + 1)^2)/(a*x - 1)^2, x)
```

Reduce [F]

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$
$$=\frac{(-a^2c\,x^2+c)^p\,a^2p\,x^2+3(-a^2c\,x^2+c)^p\,apx+2(-a^2c\,x^2+c)^p\,ax-2(-a^2c\,x^2+c)^p\,p^2-4(-a^2c\,x^2+c}{}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x)
```

output
```
(( - a**2*c*x**2 + c)**p*a**2*p*x**2 + 3*( - a**2*c*x**2 + c)**p*a*p*x + 2
*( - a**2*c*x**2 + c)**p*a*x - 2*( - a**2*c*x**2 + c)**p*p**2 - 4*( - a**2
*c*x**2 + c)**p*p - 2*( - a**2*c*x**2 + c)**p + 8*int((( - a**2*c*x**2 + c
)**p*x)/(2*a**3*p*x**3 + a**3*x**3 - 2*a**2*p*x**2 - a**2*x**2 - 2*a*p*x -
 a*x + 2*p + 1),x)*a**3*p**4*x + 28*int((( - a**2*c*x**2 + c)**p*x)/(2*a**
3*p*x**3 + a**3*x**3 - 2*a**2*p*x**2 - a**2*x**2 - 2*a*p*x - a*x + 2*p + 1
),x)*a**3*p**3*x + 28*int((( - a**2*c*x**2 + c)**p*x)/(2*a**3*p*x**3 + a**
3*x**3 - 2*a**2*p*x**2 - a**2*x**2 - 2*a*p*x - a*x + 2*p + 1),x)*a**3*p**2
*x + 8*int((( - a**2*c*x**2 + c)**p*x)/(2*a**3*p*x**3 + a**3*x**3 - 2*a**2
*p*x**2 - a**2*x**2 - 2*a*p*x - a*x + 2*p + 1),x)*a**3*p*x - 8*int((( - a*
*2*c*x**2 + c)**p*x)/(2*a**3*p*x**3 + a**3*x**3 - 2*a**2*p*x**2 - a**2*x**
2 - 2*a*p*x - a*x + 2*p + 1),x)*a**2*p**4 - 28*int((( - a**2*c*x**2 + c)**
p*x)/(2*a**3*p*x**3 + a**3*x**3 - 2*a**2*p*x**2 - a**2*x**2 - 2*a*p*x - a*
x + 2*p + 1),x)*a**2*p**3 - 28*int((( - a**2*c*x**2 + c)**p*x)/(2*a**3*p*x
**3 + a**3*x**3 - 2*a**2*p*x**2 - a**2*x**2 - 2*a*p*x - a*x + 2*p + 1),x)*
a**2*p**2 - 8*int((( - a**2*c*x**2 + c)**p*x)/(2*a**3*p*x**3 + a**3*x**3 -
 2*a**2*p*x**2 - a**2*x**2 - 2*a*p*x - a*x + 2*p + 1),x)*a**2*p)/(a*p*(2*a
*p*x + a*x - 2*p - 1))
```

3.740 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{2^{1+p}(1+ax)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-1-p,p,1+p,\frac{1}{2}(1-ax)\right)}{ap}$$

output
```
2^(p+1)*(-a^2*c*x^2+c)^p*hypergeom([p, -1-p],[p+1],-1/2*a*x+1/2)/a/p/((a*x
+1)^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{2^{1+p}(1-ax)^p\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-1-p,p,1+p,\frac{1}{2}(1-ax)\right)}{ap}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
(2^(1 + p)*(1 - a*x)^p*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 +
p, (1 - a*x)/2])/(a*p*(1 - a^2*x^2)^p)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6717, 6691, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^p dx$$

$$\Big\downarrow \text{ 6691}$$

$$-c\int (ax+1)^2\left(c-a^2cx^2\right)^{p-1}dx$$

$$\Big\downarrow \text{ 473}$$

$$-c(ax+1)^{-p}(c-acx)^{-p}\left(c-a^2cx^2\right)^p\int (ax+1)^{p+1}(c-acx)^{p-1}dx$$

$$\Big\downarrow \text{ 79}$$

$$\frac{2^{p+1}(ax+1)^{-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-p-1,p,p+1,\frac{1}{2}(1-ax)\right)}{ap}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
(2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 - a*x)
/2])/(a*p*(1 + a*x)^p)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) && !Gt
Q[a, 0] && !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 6691
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^(n/2)   Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c
, d, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/
2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [F]

$$\int \frac{(ax+1)\left(-a^{2}c\,x^{2}+c\right)^{p}}{ax-1}dx$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x)
```

output
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x)
```

Fricas [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax+1)(-a^2cx^2+c)^p}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

output
```
integral((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.40 (sec) , antiderivative size = 648, normalized size of antiderivative = 12.00

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\text{Too large to display}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**p,x)
```

output
```
a*Piecewise((0**p*x/a - 0**p*log(1/(a**2*x**2))/(2*a**2) + 0**p*log(-1 + 1
/(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 - a**(2*p - 1)*c**p*p*x*
*(2*p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (
1/2 - p,), 1/(a**2*x**2))/(2*gamma(1/2 - p)*gamma(p + 1)) + c**p*x**2*gamm
a(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi)
)/(2*gamma(-p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a - 0**p*log(
1/(a**2*x**2))/(2*a**2) + 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atan
h(1/(a*x))/a**2 - a**(2*p - 1)*c**p*p*x**(2*p + 1)*exp(I*pi*p)*gamma(p)*ga
mma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), 1/(a**2*x**2))/(2*gamma
(1/2 - p)*gamma(p + 1)) + c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 -
 p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)), True
)) + Piecewise((0**p*log(a**2*x**2 - 1)/(2*a) - 0**p*acoth(a*x)/a + a*c**p
*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_pol
ar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*p - 2)*c**p*p*x**(2*p - 1)*
exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/
(a**2*x**2))/(2*gamma(3/2 - p)*gamma(p + 1)), Abs(a**2*x**2) > 1), (0**p*l
og(-a**2*x**2 + 1)/(2*a) - 0**p*atanh(a*x)/a + a*c**p*x**2*gamma(p)*gamma(
1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(
-p)*gamma(p + 1)) - a**(2*p - 2)*c**p*p*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*
gamma(1/2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2*ga...
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{(ax+1)(-a^2cx^2+c)^p}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

Giac [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax+1)(-a^2cx^2+c)^p}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(c-a^2\,c\,x^2)^p\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - a^2*c*x^2)^p*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - a^2*c*x^2)^p*(a*x + 1))/(a*x - 1), x)
```

Reduce [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$
$$=\frac{(-a^2c\,x^2+c)^p\,apx+2(-a^2c\,x^2+c)^p\,p+(-a^2c\,x^2+c)^p+4\left(\int\frac{(-a^2c\,x^2+c)^p}{2a^2p\,x^2+a^2x^2-2p-1}dx\right)a\,p^3+6\left(\int\frac{(-a^2c}{2a^2p\,x^2+a}\right.}{ap\,(2p+1)}$$

input
```
int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x)
```

output
```
(( - a**2*c*x**2 + c)**p*a*p*x + 2*( - a**2*c*x**2 + c)**p*p + ( - a**2*c*
x**2 + c)**p + 4*int(( - a**2*c*x**2 + c)**p/(2*a**2*p*x**2 + a**2*x**2 -
2*p - 1),x)*a*p**3 + 6*int(( - a**2*c*x**2 + c)**p/(2*a**2*p*x**2 + a**2*x
**2 - 2*p - 1),x)*a*p**2 + 2*int(( - a**2*c*x**2 + c)**p/(2*a**2*p*x**2 +
a**2*x**2 - 2*p - 1),x)*a*p)/(a*p*(2*p + 1))
```

3.741 $\int (c - a^2cx^2)^p \, dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int (c - a^2cx^2)^p \, dx = x(1 - a^2x^2)^{-p} (c - a^2cx^2)^p \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, a^2x^2\right)$$

output
```
x*(-a^2*c*x^2+c)^p*hypergeom([1/2, -p],[3/2],a^2*x^2)/((-a^2*x^2+1)^p)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (c - a^2cx^2)^p \, dx = x(1 - a^2x^2)^{-p} (c - a^2cx^2)^p \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, a^2x^2\right)$$

input
```
Integrate[(c - a^2*c*x^2)^p,x]
```

output
```
(x*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, a^2*x^2])/(1 - a^2*x^
2)^p
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-a^2 c x^2\right)^p \, dx$$

$$\Big\downarrow \; 238$$

$$\left(1-a^2x^2\right)^{-p} \left(c-a^2 c x^2\right)^p \int \left(1-a^2x^2\right)^p dx$$

$$\Big\downarrow \; 237$$

$$x\left(1-a^2x^2\right)^{-p} \left(c-a^2 c x^2\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},a^2x^2\right)$$

input
```
Int[(c - a^2*c*x^2)^p,x]
```

output
```
(x*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, a^2*x^2])/(1 - a^2*x^
2)^p
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int (-a^2c\,x^2+c)^p\,dx$$

input
```
int((-a^2*c*x^2+c)^p,x)
```

output
```
int((-a^2*c*x^2+c)^p,x)
```

Fricas [F]

$$\int (c-a^2cx^2)^p\ dx = \int (-a^2cx^2+c)^p\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

output
```
integral((-a^2*c*x^2 + c)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int (c-a^2cx^2)^p\ dx = c^p x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\right|a^2x^2e^{2i\pi}\right)$$

input
```
integrate((-a**2*c*x**2+c)**p,x)
```

output
```
c**p*x*hyper((1/2, -p), (3/2,), a**2*x**2*exp_polar(2*I*pi))
```

Maxima [F]

$$\int \left(c-a^2cx^2\right)^p\,dx=\int \left(-a^2cx^2+c\right)^p\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

output
```
integrate((-a^2*c*x^2 + c)^p, x)
```

Giac [F]

$$\int \left(c-a^2cx^2\right)^p\,dx=\int \left(-a^2cx^2+c\right)^p\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^p, x)
```

Mupad [B] (verification not implemented)

Time = 13.73 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \left(c-a^2cx^2\right)^p\,dx=\frac{x\,{\left(c-a^2\,c\,x^2\right)}^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ a^2\,x^2\right)}{{\left(1-a^2\,x^2\right)}^p}$$

input
```
int((c - a^2*c*x^2)^p,x)
```

output
```
(x*(c - a^2*c*x^2)^p*hypergeom([1/2, -p], 3/2, a^2*x^2))/(1 - a^2*x^2)^p
```

Reduce [F]

$$\int \left(c-a^2cx^2\right)^p\,dx$$
$$=\frac{\left(-a^2c\,x^2+c\right)^p x-4\left(\int \frac{\left(-a^2c\,x^2+c\right)^p}{2a^2p\,x^2+a^2x^2-2p-1}dx\right)p^2-2\left(\int \frac{\left(-a^2c\,x^2+c\right)^p}{2a^2p\,x^2+a^2x^2-2p-1}dx\right)p}{2p+1}$$

input
```
int((-a^2*c*x^2+c)^p,x)
```

output
```
(( - a**2*c*x**2 + c)**p*x - 4*int(( - a**2*c*x**2 + c)**p/(2*a**2*p*x**2
+ a**2*x**2 - 2*p - 1),x)*p**2 - 2*int(( - a**2*c*x**2 + c)**p/(2*a**2*p*x
**2 + a**2*x**2 - 2*p - 1),x)*p)/(2*p + 1)
```

3.742 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = \frac{2^{-1+p}(1+ax)^{-2-p}(c-a^2cx^2)^{2+p}\,\text{Hypergeometric2F1}\left(1-p,2+p,3+p,\frac{1}{2}(1-ax)\right)}{ac^2(2+p)}$$

output
```
2^(-1+p)*(a*x+1)^(-2-p)*(-a^2*c*x^2+c)^(2+p)*hypergeom([2+p, 1-p],[3+p],-1
/2*a*x+1/2)/a/c^2/(2+p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12

$$\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = \frac{2^{-1+p}(1-ax)^{2+p}(1-a^2x^2)^{-p}(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(1-p,2+p,3+p,\frac{1}{2}(1-ax)\right)}{a(2+p)}$$

input
```
Integrate[(c - a^2*c*x^2)^p/E^(2*ArcCoth[a*x]),x]
```

output
```
(2^(-1 + p)*(1 - a*x)^(2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1 - p, 2
 + p, 3 + p, (1 - a*x)/2])/(a*(2 + p)*(1 - a^2*x^2)^p)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6717, 6692, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{6692}$$

$$-c\int (1-ax)^2\left(c-a^2cx^2\right)^{p-1}dx$$

$$\Big\downarrow \text{473}$$

$$-c(1-ax)^{-p}(acx+c)^{-p}\left(c-a^2cx^2\right)^p\int (1-ax)^{p+1}(axc+c)^{p-1}dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{p+1}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-p-1,p,p+1,\frac{1}{2}(ax+1)\right)}{ap}$$

input
```
Int[(c - a^2*c*x^2)^p/E^(2*ArcCoth[a*x]),x]
```

output
```
-((2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 + a*
x)/2])/(a*p*(1 - a*x)^p))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [F]

$$\int \frac{(-a^2c\,x^2+c)^p\,(ax-1)}{ax+1}dx$$

input
```
int((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x)
```

output
```
int((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x)
```

Fricas [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax-1)(-a^2cx^2+c)^p}{ax+1}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
integral((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.36 (sec) , antiderivative size = 648, normalized size of antiderivative = 9.97

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\text{Too large to display}$$

input
```
integrate((-a**2*c*x**2+c)**p*(a*x-1)/(a*x+1),x)
```

output
```
a*Piecewise((0**p*x/a + 0**p*log(1/(a**2*x**2))/(2*a**2) - 0**p*log(-1 + 1
/(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 - a**(2*p - 1)*c**p*p*x*
*(2*p + 1)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (
1/2 - p,), 1/(a**2*x**2))/(2*gamma(1/2 - p)*gamma(p + 1)) - c**p*x**2*gamm
a(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi)
)/(2*gamma(-p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a + 0**p*log(
1/(a**2*x**2))/(2*a**2) - 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atan
h(1/(a*x))/a**2 - a**(2*p - 1)*c**p*p*x**(2*p + 1)*exp(I*pi*p)*gamma(p)*ga
mma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), 1/(a**2*x**2))/(2*gamma
(1/2 - p)*gamma(p + 1)) - c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 -
 p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)), True
)) - Piecewise((0**p*log(a**2*x**2 - 1)/(2*a) + 0**p*acoth(a*x)/a + a*c**p
*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_pol
ar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) + a**(2*p - 2)*c**p*p*x**(2*p - 1)*
exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/
(a**2*x**2))/(2*gamma(3/2 - p)*gamma(p + 1)), Abs(a**2*x**2) > 1), (0**p*l
og(-a**2*x**2 + 1)/(2*a) + 0**p*atanh(a*x)/a + a*c**p*x**2*gamma(p)*gamma(
1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(
-p)*gamma(p + 1)) + a**(2*p - 2)*c**p*p*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*
gamma(1/2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2*ga...
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{(ax-1)(-a^2cx^2+c)^p}{ax+1}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

Giac [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax-1)(-a^2cx^2+c)^p}{ax+1}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
integrate((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{\left(c-a^2\,c\,x^2\right)^p\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

input
```
int(((c - a^2*c*x^2)^p*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - a^2*c*x^2)^p*(a*x - 1))/(a*x + 1), x)
```

Reduce [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$
$$=\frac{(-a^2c\,x^2+c)^p\,apx-2(-a^2c\,x^2+c)^p\,p-(-a^2c\,x^2+c)^p+4\left(\int\frac{(-a^2c\,x^2+c)^p}{2a^2p\,x^2+a^2x^2-2p-1}dx\right)a\,p^3+6\left(\int\frac{(-a^2c}{2a^2p\,x^2+a}\right.}{ap\,(2p+1)}$$

input
```
int((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x)
```

output
```
(( - a**2*c*x**2 + c)**p*a*p*x - 2*( - a**2*c*x**2 + c)**p*p - ( - a**2*c*
x**2 + c)**p + 4*int(( - a**2*c*x**2 + c)**p/(2*a**2*p*x**2 + a**2*x**2 -
2*p - 1),x)*a*p**3 + 6*int(( - a**2*c*x**2 + c)**p/(2*a**2*p*x**2 + a**2*x
**2 - 2*p - 1),x)*a*p**2 + 2*int(( - a**2*c*x**2 + c)**p/(2*a**2*p*x**2 +
a**2*x**2 - 2*p - 1),x)*a*p)/(a*p*(2*p + 1))
```

3.743 $\int e^{-4\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int e^{-4\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = -\frac{2^{-2+p}(1+ax)^{-3-p}(c-a^2cx^2)^{3+p}\,\text{Hypergeometric2F1}\left(2-p,3+p,4+p,\frac{1}{2}(1-ax)\right)}{ac^3(3+p)}$$

output
```
-2^(-2+p)*(a*x+1)^(-3-p)*(-a^2*c*x^2+c)^(3+p)*hypergeom([2-p, 3+p],[4+p],-
1/2*a*x+1/2)/a/c^3/(3+p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

$$\int e^{-4\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = -\frac{2^{-2+p}(1-ax)^{3+p}(1-a^2x^2)^{-p}(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(2-p,3+p,4+p,\frac{1}{2}(1-ax)\right)}{a(3+p)}$$

input
```
Integrate[(c - a^2*c*x^2)^p/E^(4*ArcCoth[a*x]),x]
```

output
```
-((2^(-2 + p)*(1 - a*x)^(3 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[2 - p,
 3 + p, 4 + p, (1 - a*x)/2])/(a*(3 + p)*(1 - a^2*x^2)^p))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6717, 6692, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int e^{-4\text{arctanh}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{ 6692}$$

$$c^2\int (1-ax)^4\left(c-a^2cx^2\right)^{p-2}dx$$

$$\Big\downarrow \text{ 473}$$

$$c^2(1-ax)^{1-p}(acx+c)^{1-p}\left(c-a^2cx^2\right)^{p-1}\int (1-ax)^{p+2}(axc+c)^{p-2}dx$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{c2^{p+2}(1-ax)^{1-p}\left(c-a^2cx^2\right)^{p-1}\text{Hypergeometric2F1}\left(-p-2,p-1,p,\frac{1}{2}(ax+1)\right)}{a(1-p)}$$

input
```
Int[(c - a^2*c*x^2)^p/E^(4*ArcCoth[a*x]),x]
```

output
```
-((2^(2 + p)*c*(1 - a*x)^(1 - p)*(c - a^2*c*x^2)^(-1 + p)*Hypergeometric2F
1[-2 - p, -1 + p, p, (1 + a*x)/2])/(a*(1 - p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 6692
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[1/c^(n/2)   Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
 c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[
n/2, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [F]

$$\int \frac{(-a^2c\,x^2+c)^p\,(ax-1)^2}{(ax+1)^2}dx$$

input
```
int((-a^2*c*x^2+c)^p*(a*x-1)^2/(a*x+1)^2,x)
```

output
```
int((-a^2*c*x^2+c)^p*(a*x-1)^2/(a*x+1)^2,x)
```

Fricas [F]

$$\int e^{-4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax-1)^2(-a^2cx^2+c)^p}{(ax+1)^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*(a*x-1)^2/(a*x+1)^2,x, algorithm="fricas")
```

output
```
integral((a^2*x^2 - 2*a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^2 + 2*a*x + 1), x
)
```

Sympy [F]

$$\int e^{-4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(-c(ax-1)\,(ax+1))^p\,(ax-1)^2}{(ax+1)^2}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**p*(a*x-1)**2/(a*x+1)**2,x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x - 1)**2/(a*x + 1)**2, x)
```

Maxima [F]

$$\int e^{-4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax-1)^2(-a^2cx^2+c)^p}{(ax+1)^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*(a*x-1)^2/(a*x+1)^2,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)^2*(-a^2*c*x^2 + c)^p/(a*x + 1)^2, x)
```

Giac [F]

$$\int e^{-4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(ax-1)^2(-a^2cx^2+c)^p}{(ax+1)^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*(a*x-1)^2/(a*x+1)^2,x, algorithm="giac")
```

output
```
integrate((a*x - 1)^2*(-a^2*c*x^2 + c)^p/(a*x + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(c-a^2\,c\,x^2)^p\,(a\,x-1)^2}{(a\,x+1)^2}\,dx$$

input
```
int(((c - a^2*c*x^2)^p*(a*x - 1)^2)/(a*x + 1)^2,x)
```

output
```
int(((c - a^2*c*x^2)^p*(a*x - 1)^2)/(a*x + 1)^2, x)
```

Reduce [F]

$$\int e^{-4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$
$$=\frac{(-a^2c\,x^2+c)^p\,a^2p\,x^2-3(-a^2c\,x^2+c)^p\,apx-2(-a^2c\,x^2+c)^p\,ax-2(-a^2c\,x^2+c)^p\,p^2-4(-a^2c\,x^2+c}{}$$

input
```
int((-a^2*c*x^2+c)^p*(a*x-1)^2/(a*x+1)^2,x)
```

output
```
(( - a**2*c*x**2 + c)**p*a**2*p*x**2 - 3*( - a**2*c*x**2 + c)**p*a*p*x - 2
*( - a**2*c*x**2 + c)**p*a*x - 2*( - a**2*c*x**2 + c)**p*p**2 - 4*( - a**2
*c*x**2 + c)**p*p - 2*( - a**2*c*x**2 + c)**p + 8*int((( - a**2*c*x**2 + c
)**p*x)/(2*a**3*p*x**3 + a**3*x**3 + 2*a**2*p*x**2 + a**2*x**2 - 2*a*p*x -
 a*x - 2*p - 1),x)*a**3*p**4*x + 28*int((( - a**2*c*x**2 + c)**p*x)/(2*a**
3*p*x**3 + a**3*x**3 + 2*a**2*p*x**2 + a**2*x**2 - 2*a*p*x - a*x - 2*p - 1
),x)*a**3*p**3*x + 28*int((( - a**2*c*x**2 + c)**p*x)/(2*a**3*p*x**3 + a**
3*x**3 + 2*a**2*p*x**2 + a**2*x**2 - 2*a*p*x - a*x - 2*p - 1),x)*a**3*p**2
*x + 8*int((( - a**2*c*x**2 + c)**p*x)/(2*a**3*p*x**3 + a**3*x**3 + 2*a**2
*p*x**2 + a**2*x**2 - 2*a*p*x - a*x - 2*p - 1),x)*a**3*p*x + 8*int((( - a*
*2*c*x**2 + c)**p*x)/(2*a**3*p*x**3 + a**3*x**3 + 2*a**2*p*x**2 + a**2*x**
2 - 2*a*p*x - a*x - 2*p - 1),x)*a**2*p**4 + 28*int((( - a**2*c*x**2 + c)**
p*x)/(2*a**3*p*x**3 + a**3*x**3 + 2*a**2*p*x**2 + a**2*x**2 - 2*a*p*x - a*
x - 2*p - 1),x)*a**2*p**3 + 28*int((( - a**2*c*x**2 + c)**p*x)/(2*a**3*p*x
**3 + a**3*x**3 + 2*a**2*p*x**2 + a**2*x**2 - 2*a*p*x - a*x - 2*p - 1),x)*
a**2*p**2 + 8*int((( - a**2*c*x**2 + c)**p*x)/(2*a**3*p*x**3 + a**3*x**3 +
 2*a**2*p*x**2 + a**2*x**2 - 2*a*p*x - a*x - 2*p - 1),x)*a**2*p)/(a*p*(2*a
*p*x + a*x + 2*p + 1))
```

3.744 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(\frac{-1+ax}{1+ax}\right)^{-\frac{1}{2}-p}\left(c-a^2cx^2\right)^p\,\text{Hypergeometric2F1}\left(-1-2p,\frac{3}{2}-p,-2p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+2p}$$

output
```
(1-1/a^2/x^2)^(1/2)*x*((a*x-1)/(a*x+1))^(-1/2-p)*(-a^2*c*x^2+c)^p*hypergeo
m([-1-2*p, 3/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = -\frac{4^{1+p}e^{5\coth^{-1}(ax)}\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}\right)^{2p}\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)^{-2p}\left(c-a^2cx^2\right)^p\,\text{Hypergeometri}}{5a+2ap}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
-((4^(1 + p)*E^(5*ArcCoth[a*x])*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[
a*x]/(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[
5/2 + p, 2 + 2*p, 7/2 + p, E^(2*ArcCoth[a*x])])/((5*a + 2*a*p)*(a*Sqrt[1 -
 1/(a^2*x^2)]*x)^(2*p)))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6746, 6750, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{ 6746}$$

$$x^{-2p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}dx$$

$$\Big\downarrow \text{ 6750}$$

$$\left(\frac{1}{x}\right)^{2p}\left(-\left(1-\frac{1}{a^2x^2}\right)^{-p}\right)\left(c-a^2cx^2\right)^p\int\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\left(1+\frac{1}{ax}\right)^{p+\frac{3}{2}}\left(\frac{1}{x}\right)^{-2(p+1)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 142}$$

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{5}{2}}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-2p-1,\frac{3}{2}-p,-2p,\frac{2}{(a+\ldots}\right.}{2p+1}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

output
```
(((a - x^(-1))/(a + x^(-1)))^(3/2 - p)*(1 - 1/(a*x))^(-3/2 + p)*(1 + 1/(a*
x))^(5/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, 3/2 - p, -2*
p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6750
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Simp[(-c^p)*x^m*(1/x)^m   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/
a)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
 EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !In
tegersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(-a^2c\,x^2+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x)
```

Fricas [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{\left(-a^2cx^2+c\right)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="fricas"
)
```

output
```
integral((a^2*x^2 + 2*a*x + 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)
)/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{\left(-c(ax-1)\left(ax+1\right)\right)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**p,x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**p/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{\left(-a^2cx^2+c\right)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="maxima"
)
```

output
```
integrate((-a^2*c*x^2 + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{\left(-a^2cx^2+c\right)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{\left(c-a^2\,c\,x^2\right)^p}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\int \frac{\sqrt{ax+1}\,\left(-a^2c\,x^2+c\right)^p x}{\sqrt{ax-1}\,ax-\sqrt{ax-1}}dx\right)a+\int \frac{\sqrt{ax+1}\,\left(-a^2c\,x^2+c\right)^p}{\sqrt{ax-1}\,ax-\sqrt{ax-1}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x)
```

output
```
int((sqrt(a*x + 1)*( - a**2*c*x**2 + c)**p*x)/(sqrt(a*x - 1)*a*x - sqrt(a*
x - 1)),x)*a + int((sqrt(a*x + 1)*( - a**2*c*x**2 + c)**p)/(sqrt(a*x - 1)*
a*x - sqrt(a*x - 1)),x)
```

3.745 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 57

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2}-p,-2p,\frac{2}{1+ax}\right)}{1+2p}$$

output
```
(1-1/a^2/x^2)^(1/2)*x*(-a^2*c*x^2+c)^p*hypergeom([1, -1/2-p],[-2*p],2/(a*x
+1))/(1+2*p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 122 vs. 2(57) = 114.

Time = 0.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.14

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = -\frac{4^{1+p}e^{3\coth^{-1}(ax)}\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}\right)^{2p}\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)^{-2p}(c-a^2cx^2)^p\,\text{Hypergeometri}}{3a+2ap}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^p,x]
```

output
```
-((4^(1 + p)*E^(3*ArcCoth[a*x])*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[
a*x]/(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[
3/2 + p, 2 + 2*p, 5/2 + p, E^(2*ArcCoth[a*x])])/((3*a + 2*a*p)*(a*Sqrt[1 -
 1/(a^2*x^2)]*x)^(2*p)))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 118 vs. $2(57) = 114$.

Time = 0.67 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = $\{6746, 6750, 142\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{ 6746}$$

$$x^{-2p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}dx$$

$$\Big\downarrow \text{ 6750}$$

$$\left(\frac{1}{x}\right)^{2p}\left(-\left(1-\frac{1}{a^2x^2}\right)^{-p}\right)\left(c-a^2cx^2\right)^p\int\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\left(1+\frac{1}{ax}\right)^{p+\frac{1}{2}}\left(\frac{1}{x}\right)^{-2(p+1)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 142}$$

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{3}{2}}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-2p-1,\frac{1}{2}-p,-2p,\frac{2}{(a+\ldots}\right.}{2p+1}$$

input
```
Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^p,x]
```

output
```
(((a - x^(-1))/(a + x^(-1)))^(1/2 - p)*(1 - 1/(a*x))^(-1/2 + p)*(1 + 1/(a*
x))^(3/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, 1/2 - p, -2*
p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] && !IntegerQ[n]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

rule 6750
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Simp[(-c^p)*x^m*(1/x)^m   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/
a)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
 EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !In
tegersQ[2*p, p + n/2] && !IntegerQ[m]
```

Maple [F]

$$\int \frac{(-a^2c\,x^2+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x)
```

Fricas [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(-a^2cx^2+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="fricas"
)
```

output
```
integral((a*x + 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1),
 x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(-c(ax-1)(ax+1))^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**p,x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{(-a^2cx^2+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="maxima"
)
```

output
```
integrate((-a^2*c*x^2 + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{\left(-a^2cx^2+c\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{\left(c-a^2\,c\,x^2\right)^p}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \frac{\sqrt{ax+1}\left(-a^2c\,x^2+c\right)^p}{\sqrt{ax-1}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x)
```

output
```
int((sqrt(a*x + 1)*( - a**2*c*x**2 + c)**p)/sqrt(a*x - 1),x)
```

3.746 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(\frac{-1+ax}{1+ax}\right)^{-\frac{1}{2}-p}(c-a^2cx^2)^p\,\text{Hypergeometric2F1}\left(-1-2p,-\frac{1}{2}-p,-2p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+2p}$$

output
```
(1-1/a^2/x^2)^(1/2)*x*((a*x-1)/(a*x+1))^(-1/2-p)*(-a^2*c*x^2+c)^p*hypergeo
m([-1-2*p, -1/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

$$\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = -\frac{4^{1+p}e^{\coth^{-1}(ax)}\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}\right)^{2p}\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)^{-2p}(c-a^2cx^2)^p\,\text{Hypergeometric}}{a+2ap}$$

input
```
Integrate[(c - a^2*c*x^2)^p/E^ArcCoth[a*x],x]
```

output
```
-((4^(1 + p)*E^ArcCoth[a*x]*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a*x]
/(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2
+ p, 2 + 2*p, 3/2 + p, E^(2*ArcCoth[a*x])])/((a + 2*a*p)*(a*Sqrt[1 - 1/(a^
2*x^2)]*x)^(2*p)))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6746, 6750, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{6746}$$

$$x^{-2p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}dx$$

$$\Big\downarrow \text{6750}$$

$$\left(\frac{1}{x}\right)^{2p}\left(-\left(1-\frac{1}{a^2x^2}\right)^{-p}\right)\left(c-a^2cx^2\right)^p\int\left(1-\frac{1}{ax}\right)^{p+\frac{1}{2}}\left(1+\frac{1}{ax}\right)^{p-\frac{1}{2}}\left(\frac{1}{x}\right)^{-2(p+1)}d\frac{1}{x}$$

$$\Big\downarrow \text{142}$$

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{1}{2}}\left(1-\frac{1}{ax}\right)^{p+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{1}{2}}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-2p-1,-p-\frac{1}{2},-2p,\ldots\right)}{2p+1}$$

input
```
Int[(c - a^2*c*x^2)^p/E^ArcCoth[a*x],x]
```

output
```
(((a - x^(-1))/(a + x^(-1)))^(-1/2 - p)*(1 - 1/(a*x))^(1/2 + p)*(1 + 1/(a*
x))^(1/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -1/2 - p, -2
*p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

rule 6750
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Simp[(-c^p)*x^m*(1/x)^m   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/
a)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
 EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !In
tegersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Maple [F]

$$\int \left(-a^2c\,x^2+c\right)^p \sqrt{\frac{ax-1}{ax+1}}dx$$

input
```
int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

Fricas [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
integral((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\sqrt{\frac{ax-1}{ax+1}}(-c(ax-1)\,(ax+1))^p\,dx$$

input
```
integrate((-a**2*c*x**2+c)**p*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p, x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(c-a^2\,c\,x^2\right)^p\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\frac{\sqrt{ax-1}\,\left(-a^2c\,x^2+c\right)^p}{\sqrt{ax+1}}dx$$

input
```
int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
int((sqrt(a*x - 1)*( - a**2*c*x**2 + c)**p)/sqrt(a*x + 1),x)
```

3.747 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(\frac{-1+ax}{1+ax}\right)^{-\frac{1}{2}-p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-1-2p,-\frac{3}{2}-p,-2p,\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+2p}$$

output
```
(1-1/a^2/x^2)^(1/2)*x*((a*x-1)/(a*x+1))^(-1/2-p)*(-a^2*c*x^2+c)^p*hypergeo
m([-1-2*p, -3/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \frac{4^{1+p}e^{-\coth^{-1}(ax)}\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}\right)^{2p}\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)^{-2p}\left(c-a^2cx^2\right)^p \text{Hypergeometric2}}{a-2ap}$$

input
```
Integrate[(c - a^2*c*x^2)^p/E^(3*ArcCoth[a*x]),x]
```

output
```
(4^(1 + p)*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a*x]/(-1 + E^(2*ArcCo
th[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2 + p, 2 + 2*p, 1/
2 + p, E^(2*ArcCoth[a*x])])/(E^ArcCoth[a*x]*(a - 2*a*p)*(a*Sqrt[1 - 1/(a^2
*x^2)]*x)^(2*p))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6746, 6750, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$

$$\Big\downarrow \text{6746}$$

$$x^{-2p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}dx$$

$$\Big\downarrow \text{6750}$$

$$\left(\frac{1}{x}\right)^{2p}\left(-\left(1-\frac{1}{a^2x^2}\right)^{-p}\right)\left(c-a^2cx^2\right)^p\int\left(1-\frac{1}{ax}\right)^{p+\frac{3}{2}}\left(1+\frac{1}{ax}\right)^{p-\frac{3}{2}}\left(\frac{1}{x}\right)^{-2(p+1)}d\frac{1}{x}$$

$$\Big\downarrow \text{142}$$

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{3}{2}}\left(1-\frac{1}{ax}\right)^{p+\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{p-\frac{1}{2}}\left(c-a^2cx^2\right)^p\text{Hypergeometric2F1}\left(-2p-1,-p-\frac{3}{2},-2p,\ldots\right)}{2p+1}$$

input
```
Int[(c - a^2*c*x^2)^p/E^(3*ArcCoth[a*x]),x]
```

output
```
(((a - x^(-1))/(a + x^(-1)))^(-3/2 - p)*(1 - 1/(a*x))^(3/2 + p)*(1 + 1/(a*
x))^(-1/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -3/2 - p, -
2*p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] && !IntegerQ[n]
```

rule 6746
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p)   Int[u*x^(2*p)*(1 -
 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

rule 6750
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Simp[(-c^p)*x^m*(1/x)^m   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/
a)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
 EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !In
tegersQ[2*p, p + n/2] && !IntegerQ[m]
```

Maple [F]

$$\int (-a^2c\,x^2+c)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

input
```
int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

Fricas [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \left(-a^2cx^2+c\right)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
integral((a*x - 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**p*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int \left(-a^2cx^2+c\right)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(-a^2cx^2+c\right)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int\left(c-a^2\,c\,x^2\right)^p\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\int\frac{\sqrt{ax-1}\,\left(-a^2c\,x^2+c\right)^p x}{\sqrt{ax+1}\,ax+\sqrt{ax+1}}dx\right)a-\left(\int\frac{\sqrt{ax-1}\,\left(-a^2c\,x^2+c\right)^p}{\sqrt{ax+1}\,ax+\sqrt{ax+1}}dx\right)$$

input
```
int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
int((sqrt(a*x - 1)*( - a**2*c*x**2 + c)**p*x)/(sqrt(a*x + 1)*a*x + sqrt(a*
x + 1)),x)*a - int((sqrt(a*x - 1)*( - a**2*c*x**2 + c)**p)/(sqrt(a*x + 1)*
a*x + sqrt(a*x + 1)),x)
```

3.748 $\int e^{\coth^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right)^4 dx$

Optimal result

Integrand size = 20, antiderivative size = 170

$$\int e^{\coth^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right)^4 dx = -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(6a-\frac{35}{x}\right)}{30a^2} - \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(8a-\frac{35}{x}\right)}{24a^2} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{35}{x}\right)}{16a^2} + \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(7a-\frac{1}{x}\right)x}{7a} + \frac{35c^4\csc^{-1}(ax)}{16a} + \frac{c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-1/30*c^4*(1-1/a^2/x^2)^(5/2)*(6*a-35/x)/a^2-1/24*c^4*(1-1/a^2/x^2)^(3/2)*
(8*a-35/x)/a^2-1/16*c^4*(1-1/a^2/x^2)^(1/2)*(16*a-35/x)/a^2+1/7*c^4*(1-1/a
^2/x^2)^(7/2)*(7*a-1/x)*x/a+35/16*c^4*arccsc(a*x)/a+c^4*arctanh((1-1/a^2/x
^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.71

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$= \frac{c^4\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(240+280ax-1056a^2x^2-1330a^3x^3+1952a^4x^4+3045a^5x^5-2816a^6x^6+1680a^7x^7)}{x^6}+3675a^6\arcsin\left(\frac{1}{ax}\right)+1680a^6\,\mathrm{l}\right.}{1680a^7}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4,x]
```

output
```
(c^4*((Sqrt[1 - 1/(a^2*x^2)]*(240 + 280*a*x - 1056*a^2*x^2 - 1330*a^3*x^3
+ 1952*a^4*x^4 + 3045*a^5*x^5 - 2816*a^6*x^6 + 1680*a^7*x^7))/x^6 + 3675*a
^6*ArcSin[1/(a*x)] + 1680*a^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1680*a
^7)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.98, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.150$, Rules used = {6748, 108, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^4 e^{\coth^{-1}(ax)} dx$$
$$\Big\downarrow \text{6748}$$
$$-c^4\int \left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x^2d\frac{1}{x}$$
$$\Big\downarrow \text{108}$$
$$-c^4\left(\int \frac{\left(a-\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)$$

$$\Big\downarrow \; 27$$

$$-c^4\left(\frac{\int \left(a-\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)$$

$$\Big\downarrow \; 171$$

$$-c^4\left(\frac{\frac{1}{7}a\int \frac{\left(7a-\frac{47}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x}{a}d\frac{1}{x}-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)$$

$$\Big\downarrow \; 27$$

$$-c^4\left(\frac{\frac{1}{7}\int \left(7a-\frac{47}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}xd\frac{1}{x}-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)$$

$$\Big\downarrow \; 171$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{6}a\int \frac{3\left(14a-\frac{61}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x}{a}d\frac{1}{x}-\frac{47}{6}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}}{a^2}-x\Big(1\ldots\right.$$

$$\Big\downarrow \; 27$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\int \left(14a-\frac{61}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}xd\frac{1}{x}-\frac{47}{6}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}}{a^2}-\ldots\right.$$

$$\Big\downarrow \; 171$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}a\int \frac{\left(70a-\frac{131}{x}\right)\left(1+\frac{1}{ax}\right)^{7/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{61}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{47}{6}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{8}{7}a\Big(1-\ldots}{a^2}\right.$$

$$\Big\downarrow \; 27$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\int \frac{\left(70a-\frac{131}{x}\right)\left(1+\frac{1}{ax}\right)^{7/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{61}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{47}{6}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right.}{a^2}\right.$$

$$\Big\downarrow \color{blue}{171}$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{131}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{1}{4}a\int -\frac{7\left(40a-\frac{91}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{61}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{47}{6}a\left(1\right.\right.}{a^2}\right.$$

$$\Big\downarrow \color{blue}{27}$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\int \frac{\left(40a-\frac{91}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{131}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{61}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{47}{6}a\left(1-\frac{1}{a}\right.\right.}{a^2}\right.$$

$$\Big\downarrow \color{blue}{171}$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{91}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{3}a\int -\frac{5\left(24a-\frac{67}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{131}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{61}{5}\right.\right.\right.}{a^2}\right.$$

$$\Big\downarrow \color{blue}{27}$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\int \frac{\left(24a-\frac{67}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{91}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{131}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{61}{5}a\sqrt{1}\right.\right.\right.}{a^2}\right.$$

$$\Big\downarrow \color{blue}{171}$$

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{67}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}a\int -\frac{3\left(16a-\frac{51}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{91}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{131}{4}\cdots}{}\right.$$

$\downarrow$ 27

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\int \frac{\left(16a-\frac{51}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{67}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{91}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{131}{4}a\sqrt{\cdots}}{}\right.$$

$\downarrow$ 171

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(51a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int -\frac{\left(16a-\frac{35}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{67}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{91}{3}a\sqrt{\cdots}}{}\right.$$

$\downarrow$ 25

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(a\int \frac{\left(16a-\frac{35}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+51a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{67}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{91}{3}a\sqrt{1\cdots}}{}\right.$$

$\downarrow$ 27

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{\left(16a-\frac{35}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+51a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{67}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{91}{3}a\sqrt{1-\cdots}}{}\right.$$

$\downarrow$ 175

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(16a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-35\int\frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+51a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{67}{2}a\sqrt{1-\frac{1}{a}}\right.\right.\right.\right.\right.\right.}{}\right.$$

$\downarrow$ 39

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-35\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+16a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+51a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{67}{2}a\sqrt{1-\frac{1}{ax}}\right.\right.\right.\right.\right.\right.}{}\right.$$

$\downarrow$ 103

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-35\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+51a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{67}{2}\right.\right.\right.\right.\right.\right.}{}\right.$$

$\downarrow$ 221

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-35\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+51a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{67}{2}a\right.\right.\right.\right.\right.\right.}{}\right.$$

$\downarrow$ 223

$$-c^4\left(\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-35a\arcsin\left(\frac{1}{ax}\right)-16a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+51a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{67}{2}a\sqrt{1-}\right.\right.\right.\right.\right.\right.}{}\right.$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4,x]
```

output
```
-(c^4*(-((1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(9/2)*x) + ((-8*a*(1 - 1/(a*x))
^(5/2)*(1 + 1/(a*x))^(9/2))/7 + ((-47*a*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^
(9/2))/6 + ((-61*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/5 + ((131*a*Sqrt
[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/4 + (7*((91*a*Sqrt[1 - 1/(a*x)]*(1 + 1/
(a*x))^(5/2))/3 + (5*((67*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2))/2 + (3*
(51*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)] - 35*a*ArcSin[1/(a*x)] - 16*a*Ar
cTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]))/2))/3))/4)/5)/2)/7)/a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{(ax-1)(2816x^6a^6-3045a^5x^5-1952a^4x^4+1330a^3x^3+1056a^2x^2-280ax-240)c^4}{1680x^7a^8\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{35a^7\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{16}+\frac{a^8\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{c\cdots}\right.}{\sqrt{a^2}}\cdots\right.}{a^8\sqrt{\cdots}}$
default	$\frac{(ax-1)c^4\left(-1680\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^8x^8+1680(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6+3675a^7x^7\sqrt{a^2}\,\sqrt{a^2x^2-1}+1680\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^8x^7+\cdots\right.}{\cdots}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/1680*(a*x-1)*(2816*a^6*x^6-3045*a^5*x^5-1952*a^4*x^4+1330*a^3*x^3+1056*
a^2*x^2-280*a*x-240)/x^7*c^4/a^8/((a*x-1)/(a*x+1))^(1/2)+(35/16*a^7*arctan
(1/(a^2*x^2-1)^(1/2))+a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1
/2)+a^7*((a*x-1)*(a*x+1))^(1/2))*c^4/a^8/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*
(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.18

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4dx =$$
$$-\frac{7350\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(168\cdots}{\cdots}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
-1/1680*(7350*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) - 1680*a^7*c^4
*x^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 1680*a^7*c^4*x^7*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) - (1680*a^8*c^4*x^8 - 1136*a^7*c^4*x^7 + 229*a^6*c^4*x
^6 + 4997*a^5*c^4*x^5 + 622*a^4*c^4*x^4 - 2386*a^3*c^4*x^3 - 776*a^2*c^4*x
^2 + 520*a*c^4*x + 240*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\left(\int \frac{a^8}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \frac{1}{x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^2}{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int \frac{6a^4}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{}{x^2\sqrt{\frac{}{ax}}}\right.}{a^8}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**4,x)
```

output
```
c**4*(Integral(a**8/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x*
*8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-4*a**2/(x**6*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(6*a**4/(x**4*sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))), x) + Integral(-4*a**6/(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))), x))/a**8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(150) = 300$.

Time = 0.12 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.24

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\frac{1}{840}\left(\frac{3675\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{5355\,c^4\left(\frac{ax-1}{ax+1}\right.}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
-1/840*(3675*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 840*c^4*log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/a^2 + 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
/a^2 - (5355*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 31465*c^4*((a*x - 1)/(a*x
+ 1))^(13/2) + 72051*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 71801*c^4*((a*x -
1)/(a*x + 1))^(9/2) + 4569*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 17619*c^4*((a
*x - 1)/(a*x + 1))^(5/2) + 10185*c^4*((a*x - 1)/(a*x + 1))^(3/2) + 1995*c^
4*sqrt((a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a
^2/(a*x + 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x
+ 1)^5 - 14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 -
(a*x - 1)^8*a^2/(a*x + 1)^8 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(150) = 300$.

Time = 0.15 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.71

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$= -\frac{35\,c^4 \arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{8\,a\text{sgn}\left(ax+1\right)} - \frac{c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\text{sgn}\left(ax+1\right)} + \frac{\sqrt{a^2x^2-1}c^4}{a\text{sgn}\left(ax+1\right)}$$
$$- \frac{3045\left(x|a|-\sqrt{a^2x^2-1}\right)^{13}c^4|a| + 6720\left(x|a|-\sqrt{a^2x^2-1}\right)^{12}ac^4 + 6860\left(x|a|-\sqrt{a^2x^2-1}\right)^{11}c^4|a| + \cdots}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
-35/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c^4*log
(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2
- 1)*c^4/(a*sgn(a*x + 1)) - 1/840*(3045*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*
c^4*abs(a) + 6720*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4 + 6860*(x*abs(a)
 - sqrt(a^2*x^2 - 1))^11*c^4*abs(a) + 20160*(x*abs(a) - sqrt(a^2*x^2 - 1))
^10*a*c^4 + 9065*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^4*abs(a) + 49280*(x*ab
s(a) - sqrt(a^2*x^2 - 1))^8*a*c^4 + 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6
*a*c^4 - 9065*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a) + 38976*(x*abs(a
) - sqrt(a^2*x^2 - 1))^4*a*c^4 - 6860*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^4
*abs(a) + 12992*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^4 - 3045*(x*abs(a) -
sqrt(a^2*x^2 - 1))*c^4*abs(a) + 2816*a*c^4)/(((x*abs(a) - sqrt(a^2*x^2 - 1
))^2 + 1)^7*a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.95

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{\frac{19\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{97\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{8}+\frac{839\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{40}+\frac{1523\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{280}+\frac{71801\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{840}+\frac{3431\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{40}+899\cdots}{a+\frac{6\,a\,(a\,x-1)}{a\,x+1}+\frac{14\,a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}+\frac{14\,a\,{(a\,x-1)}^3}{{(a\,x+1)}^3}-\frac{14\,a\,{(a\,x-1)}^5}{{(a\,x+1)}^5}-\frac{14\,a\,{(a\,x-1)}^6}{{(a\,x+1)}^6}-\frac{6\,a\,{(a\,x-1)}^7}{{(a\,x+1)}^7}-\cdots}$$
$$-\frac{35\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}+\frac{2\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((19*c^4*((a*x - 1)/(a*x + 1))^(1/2))/8 + (97*c^4*((a*x - 1)/(a*x + 1))^(3
/2))/8 + (839*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 + (1523*c^4*((a*x - 1)/(
a*x + 1))^(7/2))/280 + (71801*c^4*((a*x - 1)/(a*x + 1))^(9/2))/840 + (3431
*c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 + (899*c^4*((a*x - 1)/(a*x + 1))^(13
/2))/24 + (51*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/(a
*x + 1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^3
- (14*a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a*(
a*x - 1)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (35*c^4*atan(((a*
x - 1)/(a*x + 1))^(1/2)))/(8*a) + (2*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)
))/a
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.43

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$= \frac{c^4\left(-7350atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^7x^7+7350atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^7x^7+1680\sqrt{ax-}\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x)
```

output
```
(c**4*( - 7350*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**7*x**7 + 7350*at
an(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**7*x**7 + 1680*sqrt(a*x + 1)*sqrt(
a*x - 1)*a**7*x**7 - 2816*sqrt(a*x + 1)*sqrt(a*x - 1)*a**6*x**6 + 3045*sqr
t(a*x + 1)*sqrt(a*x - 1)*a**5*x**5 + 1952*sqrt(a*x + 1)*sqrt(a*x - 1)*a**4
*x**4 - 1330*sqrt(a*x + 1)*sqrt(a*x - 1)*a**3*x**3 - 1056*sqrt(a*x + 1)*sq
rt(a*x - 1)*a**2*x**2 + 280*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 240*sqrt(a*x
 + 1)*sqrt(a*x - 1) + 3360*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*
*7*x**7 + 896*a**7*x**7))/(1680*a**8*x**7)
```

3.749 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx=-\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(4a-\frac{15}{x}\right)}{12a^2}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(8a-\frac{15}{x}\right)}{8a^2}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(5a-\frac{1}{x}\right)x}{5a}+\frac{15c^3\csc^{-1}(ax)}{8a}+\frac{c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-1/12*c^3*(1-1/a^2/x^2)^(3/2)*(4*a-15/x)/a^2-1/8*c^3*(1-1/a^2/x^2)^(1/2)*(
8*a-15/x)/a^2+1/5*c^3*(1-1/a^2/x^2)^(5/2)*(5*a-1/x)*x/a+15/8*c^3*arccsc(a*
x)/a+c^3*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{c^3\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(-24-30ax+88a^2x^2+135a^3x^3-184a^4x^4+120a^5x^5)}{x^4}+225a^4\arcsin\left(\frac{1}{ax}\right)+120a^4\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right.\right.}{120a^5}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3,x]
```

output
```
(c^3*((Sqrt[1 - 1/(a^2*x^2)]*(-24 - 30*a*x + 88*a^2*x^2 + 135*a^3*x^3 - 18
4*a^4*x^4 + 120*a^5*x^5))/x^4 + 225*a^4*ArcSin[1/(a*x)] + 120*a^4*Log[(1 +
 Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a^5)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.91, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.950$, Rules used = {6748, 108, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int\left(c-\frac{c}{a^2x^2}\right)^3e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6748}$$

$$-c^3\int\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$-c^3\left(\int\frac{\left(a-\frac{6}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c^3\left(\frac{\int \left(a-\frac{6}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow \text{ 171}$$

$$-c^3\left(\frac{\frac{1}{5}a\int \frac{\left(5a-\frac{23}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x}{a}d\frac{1}{x}-\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-c^3\left(\frac{\frac{1}{5}\int \left(5a-\frac{23}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}xd\frac{1}{x}-\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow \text{ 171}$$

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}a\int \frac{\left(20a-\frac{43}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{23}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\right.$$

$$\Big\downarrow \text{ 27}$$

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}\int \frac{\left(20a-\frac{43}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{23}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\right.$$

$$\Big\downarrow \text{ 171}$$

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{43}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{3}a\int -\frac{5\left(12a-\frac{31}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{23}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{6}{5}a\left(1-\frac{1}{ax}\right.}{a^2}\right.$$

$$\Big\downarrow \text{ 27}$$

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\int\frac{\left(12a-\frac{31}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{43}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)-\frac{23}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{3/2}}{a^2}\right.$$

$\downarrow$ 171

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{31}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}a\int-\frac{3\left(8a-\frac{23}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{43}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)-\frac{23}{4}a\sqrt{1-}}{a^2}\right.$$

$\downarrow$ 27

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\int\frac{\left(8a-\frac{23}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{31}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{43}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)-\frac{23}{4}a\sqrt{1-\frac{1}{ax}}\left(\right.}{a^2}\right.$$

$\downarrow$ 171

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(23a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int-\frac{\left(8a-\frac{15}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{31}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{43}{3}a\sqrt{1-\frac{1}{ax}}}{a^2}\right.$$

$\downarrow$ 25

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(a\int\frac{\left(8a-\frac{15}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+23a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{31}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{43}{3}a\sqrt{1-\frac{1}{ax}}\left(\right.}{a^2}\right.$$

$\downarrow$ 27

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{\left(8a-\frac{15}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+23a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{31}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{43}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}\right.}{a^2}\right.$$

$$\downarrow 175$$

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(8a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-15\int \frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+23a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{31}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+\right.}{a^2}\right.$$

$$\downarrow 39$$

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-15\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+8a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+23a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{31}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3}\right.}{a^2}\right.$$

$$\downarrow 103$$

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-15\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+23a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{31}{2}a\sqrt{1-\right.}{a^2}\right.$$

$$\downarrow 221$$

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-15\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+23a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{31}{2}a\sqrt{1-\frac{1}{ax}}\right.}{a^2}\right.$$

$$\downarrow 223$$

$$-c^3\left(\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-15a\arcsin\left(\frac{1}{ax}\right)-8a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+23a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{31}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}\right.}{a^2}\right.$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3,x]
```

output
```
-(c^3*(-((1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)*x) + ((-6*a*(1 - 1/(a*x))
^(3/2)*(1 + 1/(a*x))^(7/2))/5 + ((-23*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7
/2))/4 + ((43*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2))/3 + (5*((31*a*Sqrt[
1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2))/2 + (3*(23*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 +
1/(a*x)] - 15*a*ArcSin[1/(a*x)] - 8*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1
/(a*x)]]))/2))/3)/4)/5)/a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{(ax-1)(184a^4x^4-135a^3x^3-88a^2x^2+30ax+24)c^3}{120x^5a^6\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{15a^5\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{8}+\frac{a^6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a^5\sqrt{(ax-1)(ax+1)}\right.}{a^6\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)c^3\left(-120\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^6x^6+120(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4+225\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^5x^5+225\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}\,a^5x^5+120\right.}{120\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/120*(a*x-1)*(184*a^4*x^4-135*a^3*x^3-88*a^2*x^2+30*a*x+24)/x^5*c^3/a^6/
((a*x-1)/(a*x+1))^(1/2)+(15/8*a^5*arctan(1/(a^2*x^2-1)^(1/2))+a^6*ln(a^2*x
/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+a^5*((a*x-1)*(a*x+1))^(1/2))*c
^3/a^6/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.31

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx =$$
$$-\frac{450\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(120\,a^6c^3}{120\,a^6x^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
-1/120*(450*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) - 120*a^5*c^3*x^
5*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/
(a*x + 1)) - 1) - (120*a^6*c^3*x^6 - 64*a^5*c^3*x^5 - 49*a^4*c^3*x^4 + 223
*a^3*c^3*x^3 + 58*a^2*c^3*x^2 - 54*a*c^3*x - 24*c^3)*sqrt((a*x - 1)/(a*x +
 1)))/(a^6*x^5)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{c^3\left(\int \frac{a^6}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{3a^2}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^4}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^6}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**3,x)
```

output
```
c**3*(Integral(a**6/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x
**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(3*a**2/(x**4*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-3*a**4/(x**2*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))), x))/a**6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(121) = 242.

Time = 0.11 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.20

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{1}{60}\left(\frac{225\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{345\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
-1/60*(225*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 60*c^3*log(sqrt((a*
x - 1)/(a*x + 1)) + 1)/a^2 + 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2
 - (345*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 1345*c^3*((a*x - 1)/(a*x + 1))^
(9/2) + 1654*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 86*c^3*((a*x - 1)/(a*x + 1)
)^(5/2) + 305*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 105*c^3*sqrt((a*x - 1)/(a*
x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a
*x - 1)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^
2/(a*x + 1)^6 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(121) = 242$.

Time = 0.16 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.59

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{15\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{4\,a\mathrm{sgn}\left(ax+1\right)}-\frac{c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}c^3}{a\mathrm{sgn}\left(ax+1\right)} - \frac{135\left(x|a|-\sqrt{a^2x^2-1}\right)^9c^3|a|+360\left(x|a|-\sqrt{a^2x^2-1}\right)^8ac^3+150\left(x|a|-\sqrt{a^2x^2-1}\right)^7c^3|a|+720\left(x\right.}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
-15/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c^3*log
(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2
- 1)*c^3/(a*sgn(a*x + 1)) - 1/60*(135*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3
*abs(a) + 360*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3 + 150*(x*abs(a) - sqr
t(a^2*x^2 - 1))^7*c^3*abs(a) + 720*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^3
+ 1120*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^3 - 150*(x*abs(a) - sqrt(a^2*x
^2 - 1))^3*c^3*abs(a) + 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3 - 135*(
x*abs(a) - sqrt(a^2*x^2 - 1))*c^3*abs(a) + 184*a*c^3)/(((x*abs(a) - sqrt(a
^2*x^2 - 1))^2 + 1)^5*a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.32 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.88

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{\frac{7\,c^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{61\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{12}+\frac{43\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{30}+\frac{827\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{30}+\frac{269\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{12}+\frac{23\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{4}}{a+\frac{4\,a\,(a\,x-1)}{a\,x+1}+\frac{5\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{a\,(a\,x-1)^6}{(a\,x+1)^6}} - \frac{15\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}+\frac{2\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((7*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (61*c^3*((a*x - 1)/(a*x + 1))^(3/
2))/12 + (43*c^3*((a*x - 1)/(a*x + 1))^(5/2))/30 + (827*c^3*((a*x - 1)/(a*
x + 1))^(7/2))/30 + (269*c^3*((a*x - 1)/(a*x + 1))^(9/2))/12 + (23*c^3*((a
*x - 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x -
 1)^2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*
x + 1)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (15*c^3*atan(((a*x - 1)/(a*x + 1
))^(1/2)))/(4*a) + (2*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.48

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$$
$$= \frac{c^3\left(-450atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^5x^5+450atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^5x^5+120\sqrt{ax+1}\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x)
```

output
```
(c**3*( - 450*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**5*x**5 + 450*atan
(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**5*x**5 + 120*sqrt(a*x + 1)*sqrt(a*x
 - 1)*a**5*x**5 - 184*sqrt(a*x + 1)*sqrt(a*x - 1)*a**4*x**4 + 135*sqrt(a*x
 + 1)*sqrt(a*x - 1)*a**3*x**3 + 88*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 -
 30*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x - 24*sqrt(a*x + 1)*sqrt(a*x - 1) + 240
*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**5*x**5 + 40*a**5*x**5))/(
120*a**6*x**5)
```

3.750 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 104

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=-\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right)}{2a^2}+\frac{c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a-\frac{1}{x}\right)x}{3a}+\frac{3c^2\csc^{-1}(ax)}{2a}+\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-1/2*c^2*(1-1/a^2/x^2)^(1/2)*(2*a-3/x)/a^2+1/3*c^2*(1-1/a^2/x^2)^(3/2)*(3*
a-1/x)*x/a+3/2*c^2*arccsc(a*x)/a+c^2*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}(2+3ax-8a^2x^2+6a^3x^3)+9a^2x^2\arcsin\left(\frac{1}{ax}\right)+6a^2x^2\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a^3x^2}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2,x]
```

output
```
(c^2*(Sqrt[1 - 1/(a^2*x^2)]*(2 + 3*a*x - 8*a^2*x^2 + 6*a^3*x^3) + 9*a^2*x^
2*ArcSin[1/(a*x)] + 6*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3*
x^2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.81, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6748, 108, 27, 171, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^2 e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6748}$$

$$-c^2 \int \left(1-\frac{1}{ax}\right)^{3/2} \left(1+\frac{1}{ax}\right)^{5/2} x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$-c^2 \left(\int \frac{\left(a-\frac{4}{x}\right) \sqrt{1-\frac{1}{ax}} \left(1+\frac{1}{ax}\right)^{3/2} x}{a^2} d\frac{1}{x} - x \left(1-\frac{1}{ax}\right)^{3/2} \left(\frac{1}{ax}+1\right)^{5/2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c^2 \left(\frac{\int \left(a-\frac{4}{x}\right) \sqrt{1-\frac{1}{ax}} \left(1+\frac{1}{ax}\right)^{3/2} x d\frac{1}{x}}{a^2} - x \left(1-\frac{1}{ax}\right)^{3/2} \left(\frac{1}{ax}+1\right)^{5/2}\right)$$

$$\Big\downarrow \text{171}$$

$$-c^2 \left(\frac{\frac{1}{3} a \int \frac{\left(3a-\frac{7}{x}\right) \left(1+\frac{1}{ax}\right)^{3/2} x}{a \sqrt{1-\frac{1}{ax}}} d\frac{1}{x} - \frac{4}{3} a \sqrt{1-\frac{1}{ax}} \left(\frac{1}{ax}+1\right)^{5/2}}{a^2} - x \left(1-\frac{1}{ax}\right)^{3/2} \left(\frac{1}{ax}+1\right)^{5/2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c^2\left(\frac{\frac{1}{3}\int \frac{\left(3a-\frac{7}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}\right)$$

$$\Big\downarrow \text{171}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}a\int -\frac{3\left(2a-\frac{5}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\int \frac{\left(2a-\frac{5}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right.\right.$$

$$\Big\downarrow \text{171}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int -\frac{\left(2a-\frac{3}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5}}{a^2}\right.$$

$$\Big\downarrow \text{25}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(a\int \frac{\left(2a-\frac{3}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}\right.$$

$$\Big\downarrow \text{27}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{\left(2a-\frac{3}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-\right.$$

$\Big\downarrow$ 175

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(2a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-3\int \frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-}{a^2}\right.$$

$\Big\downarrow$ 39

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(-3\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+2a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)-\frac{4}{3}a}{a^2}\right.$$

$\Big\downarrow$ 103

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(-3\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-2\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)}{a^2}\right.$$

$\Big\downarrow$ 221

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(-3\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-2a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3}}{a^2}\right.$$

$\Big\downarrow$ 223

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(-3a\arcsin\left(\frac{1}{ax}\right)-2a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{7}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{a^2}\right.$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2,x]
```

output
```
-(c^2*(-((1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x) + ((-4*a*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(5/2))/3 + ((7*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2))
/2 + (3*(5*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)] - 3*a*ArcSin[1/(a*x)] - 2
*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]))/2)/3)/a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.50

method	result
risch	$-\frac{(ax-1)(8a^2x^2-3ax-2)c^2}{6x^3a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+\frac{a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a^3\sqrt{(ax-1)(ax+1)}\right)c^2\sqrt{(ax-1)(ax+1)}}{a^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)c^2\left(-6\sqrt{a^2}\sqrt{a^2x^2-1}\,a^4x^4+6\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^2x^2+9\sqrt{a^2}\sqrt{a^2x^2-1}\,a^3x^3+6\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a^4x^3+9a^3\sqrt{a^2}\,x^3\,\text{ar}\right.}{6\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(ax+1)}\,a^4x^3\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(a*x-1)*(8*a^2*x^2-3*a*x-2)/x^3*c^2/a^4/((a*x-1)/(a*x+1))^(1/2)+(3/2*
a^3*arctan(1/(a^2*x^2-1)^(1/2))+a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2)
)/(a^2)^(1/2)+a^3*((a*x-1)*(a*x+1))^(1/2))*c^2/a^4/((a*x-1)/(a*x+1))^(1/2)
*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.51

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\frac{18\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(6\,a^4c^2x^4-2\,a}{6\,a^4x^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(18*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 6*a^3*c^2*x^3*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) + 6*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x +
1)) - 1) - (6*a^4*c^2*x^4 - 2*a^3*c^2*x^3 - 5*a^2*c^2*x^2 + 5*a*c^2*x + 2*
c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{c^2\left(\int \frac{a^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \frac{1}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{2a^2}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**2,x)
```

output
```
c**2*(Integral(a**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x*
*4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-2*a**2/(x**2*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))), x))/a**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(92) = 184.

Time = 0.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.14

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\frac{1}{3}a\left(\frac{9\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{15\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+29\,c^2\left(\right.}{\frac{2\,(ax-1)a^2}{ax+1}-2\,(}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
-1/3*a*(9*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 3*c^2*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 + 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
(15*c^2*((a*x - 1)/(a*x + 1))^(7/2) + 29*c^2*((a*x - 1)/(a*x + 1))^(5/2) +
 c^2*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*
x - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*
x + 1)^4 + a^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(92) = 184$.

Time = 0.15 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.39

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$$
$$= -\frac{3\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\text{sgn}\left(ax+1\right)} - \frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\text{sgn}\left(ax+1\right)} + \frac{\sqrt{a^2x^2-1}c^2}{a\text{sgn}\left(ax+1\right)}$$
$$- \frac{3\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^2|a| + 12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^2 + 12\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2 - 3\left(x|a|-\sqrt{a\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|\text{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
-3*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c^2*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1
)*c^2/(a*sgn(a*x + 1)) - 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a
) + 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2 + 12*(x*abs(a) - sqrt(a^2*x^
2 - 1))^2*a*c^2 - 3*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^2*abs(a) + 8*a*c^2)/(
((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.62 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.76

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = \frac{c^2\sqrt{\frac{a\,x-1}{a\,x+1}} + \frac{c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3} + \frac{29\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{3} + 5\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{a + \frac{2\,a\,(a\,x-1)}{a\,x+1} - \frac{2\,a\,(a\,x-1)^3}{(a\,x+1)^3} - \frac{a\,(a\,x-1)^4}{(a\,x+1)^4}}$$
$$- \frac{3\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a} + \frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(c^2*((a*x - 1)/(a*x + 1))^(1/2) + (c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 + (
29*c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 + 5*c^2*((a*x - 1)/(a*x + 1))^(7/2))
/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x
- 1)^4)/(a*x + 1)^4) - (3*c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (2*c^
2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.49

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$$

$$= \frac{c^2\left(-18atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^3x^3+18atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^3x^3+6\sqrt{ax+1}\,\sqrt{ax}\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x)
```

output
```
(c**2*( - 18*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**3*x**3 + 18*atan(s
qrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**3*x**3 + 6*sqrt(a*x + 1)*sqrt(a*x - 1
)*a**3*x**3 - 8*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 + 3*sqrt(a*x + 1)*sq
rt(a*x - 1)*a*x + 2*sqrt(a*x + 1)*sqrt(a*x - 1) + 12*log((sqrt(a*x - 1) +
sqrt(a*x + 1))/sqrt(2))*a**3*x**3))/(6*a**4*x**3)
```

3.751 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 57

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{c\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a}+\frac{c\csc^{-1}(ax)}{a}+\frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
c*(1-1/a^2/x^2)^(1/2)*(a-1/x)*x/a+c*arccsc(a*x)/a+c*arctanh((1-1/a^2/x^2)^
(1/2))/a
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(-1+ax)+\arcsin\left(\frac{1}{ax}\right)+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{a}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2)),x]
```

output
```
(c*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) + ArcSin[1/(a*x)] + Log[(1 + Sqrt[1 -
 1/(a^2*x^2)])*x]))/a
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6748, 108, 27, 171, 25, 27, 35, 140, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right) e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6748}$$

$$-c\int \sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$-c\left(\int \frac{\left(a-\frac{2}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a^2\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c\left(\frac{\int \frac{\left(a-\frac{2}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{171}$$

$$-c\left(\frac{2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int -\frac{\left(a-\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{a\int \frac{\left(a-\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow 27$$

$$-c\left(\frac{\int \frac{\left(a-\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow 35$$

$$-c\left(\frac{a\int \frac{\sqrt{1-\frac{1}{ax}}x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow 140$$

$$-c\left(\frac{a\left(\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{\int \frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow 39$$

$$-c\left(\frac{a\left(\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow 103$$

$$-c\left(\frac{a\left(-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\frac{\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$-c\left(\frac{a\left(-\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{223}$$

$$-c\left(\frac{a\left(-\arcsin\left(\frac{1}{ax}\right)-\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2)),x]
```

output
```
-(c*(-(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)*x) + (2*a*Sqrt[1 - 1/(a*x)]*S
qrt[1 + 1/(a*x)] + a*(-ArcSin[1/(a*x)] - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1
+ 1/(a*x)]]))/a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(53) = 106$.

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.23

method	result
risch	$-\frac{(ax-1)c}{x\,a^{2}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{\sqrt{a^{2}}}+\sqrt{(ax-1)(ax+1)}+\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)\right)c\sqrt{(ax-1)(ax+1)}}{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)c\left(-\sqrt{a^{2}}\,\sqrt{a^{2}x^{2}-1}\,a^{2}x^{2}+\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}+\sqrt{a^{2}}\,\sqrt{a^{2}x^{2}-1}\,ax+\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{2}x+a\sqrt{a^{2}}\,x\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^{2}x\sqrt{a^{2}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
-(a*x-1)/x*c/a^2/((a*x-1)/(a*x+1))^(1/2)+1/a*(a*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)+((a*x-1)*(a*x+1))^(1/2)+arctan(1/(a^2*x^2-1)^(1/
2)))*c/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.82

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = -\frac{2\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(a^2cx^2-c\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
-(2*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - a*c*x*log(sqrt((a*x - 1)/(a*
x + 1)) + 1) + a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c*x^2 - c)*
sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{c\left(\int\frac{a^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2),x)
```

output
```
c*(Integral(a**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x**2
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(53) = 106$.

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.05

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = -\left(\frac{4\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
-(4*c*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*
c*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1)/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(53) = 106$.

Time = 0.17 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.28

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = -\frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\text{sgn}\left(ax+1\right)} - \frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\text{sgn}\left(ax+1\right)} + \frac{\sqrt{a^2x^2-1}c}{a\text{sgn}\left(ax+1\right)} - \frac{2\,c}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|\text{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
-2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c*log(abs(-x
*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*c/
(a*sgn(a*x + 1)) - 2*c/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a)*sgn(
a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.47

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{2\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}$$

input
```
int((c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(2*c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c*atan(((a*x - 1)/(a*x + 1
))^(1/2)))/a + (4*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x
 + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.81

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$$
$$=\frac{c\left(-2atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)ax+2atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)ax+\sqrt{ax+1}\,\sqrt{ax-1}\,ax-\right.}{a^2x}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x)
```

output
```
(c*( - 2*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a*x + 2*atan(sqrt(a*x - 1
) + sqrt(a*x + 1) + 1)*a*x + sqrt(a*x + 1)*sqrt(a*x - 1)*a*x - sqrt(a*x +
1)*sqrt(a*x - 1) + 2*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x - a*
x))/(a**2*x)
```

3.752 $\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=-\frac{a+\frac{1}{x}}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
-(a+1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)+(1-1/a^2/x^2)^(1/2)*x/c+arctanh((1-1/a^
2/x^2)^(1/2))/a/c
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-2+ax)}{-1+ax}+\frac{\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{c}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2)),x]
```

output
```
((Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x))/(-1 + a*x) + Log[(1 + Sqrt[1 - 1/(a^
2*x^2)])*x]/a)/c
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.46, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6748, 114, 25, 27, 35, 105, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int -\frac{\left(a+\frac{1}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{x\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}}{c}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\left(a+\frac{1}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{x\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\left(a+\frac{1}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}}{c}$$

$$\Big\downarrow \text{35}$$

$$-\frac{\frac{\int \frac{\sqrt{1+\frac{1}{ax}}x}{\left(1-\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}}{c}$$

$$\Big\downarrow \text{105}$$

$$-\frac{\frac{\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}}{c}$$

$$\Big\downarrow \; 103$$

$$-\frac{\frac{\frac{2\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}-\frac{\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}}{c}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\frac{2\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}-\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}}{c}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2)),x]
```

output
```
-((-((Sqrt[1 + 1/(a*x)]*x)/Sqrt[1 - 1/(a*x)]) + ((2*Sqrt[1 + 1/(a*x)])/Sqr
t[1 - 1/(a*x)] - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a)/c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(63) = 126$.

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.09

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{a^4\left(x-\frac{1}{a}\right)}\right)a^2\sqrt{(ax-1)(ax+1)}}{c\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{2\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2+3\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^2x^2-4\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x-((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}}{2a\sqrt{a^2}\,(ax-1)c\sqrt{(ax-1)(ax+1)}\,\sqrt{\frac{ax-1}{ax+1}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c/((a*x-1)/(a*x+1))^(1/2)+(1/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2
-1)^(1/2))/(a^2)^(1/2)-1/a^4/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2))/c*
a^2/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.35

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2x^2-ax-2)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x -
1)/(a*x + 1)) - 1) + (a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c
*x - a*c)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{a^2\int\frac{x^2}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2),x)
```

output
```
a**2*Integral(x**2/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))), x)/c
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.68

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=-a\left(\frac{\frac{3(ax-1)}{ax+1}-1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
-a*((3*(a*x - 1)/(a*x + 1) - 1)/(a^2*c*((a*x - 1)/(a*x + 1))^(3/2) - a^2*c
*sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) +
 log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c))
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{2\,a\,x+4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-4}{2\,a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

input
```
int(1/((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(2*a*x + 4*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2)
- 4)/(2*a*c*((a*x - 1)/(a*x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{4\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)-3\sqrt{ax-1}+2\sqrt{ax+1}\,ax-4\sqrt{ax+1}}{2\sqrt{ax-1}\,ac}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x)
```

output
```
(4*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) - 3*sqrt(a*x
 - 1) + 2*sqrt(a*x + 1)*a*x - 4*sqrt(a*x + 1))/(2*sqrt(a*x - 1)*a*c)
```

3.753 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} dx = -\frac{a+\frac{1}{x}}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{3a+\frac{5}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

output
```
-1/3*(a+1/x)/a^2/c^2/(1-1/a^2/x^2)^(3/2)-1/3*(3*a+5/x)/a^2/c^2/(1-1/a^2/x^
2)^(1/2)+(1-1/a^2/x^2)^(1/2)*x/c^2+arctanh((1-1/a^2/x^2)^(1/2))/a/c^2
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.80

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(8-5ax-7a^2x^2+3a^3x^3)}{3(-1+ax)^2(1+ax)} + \log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^2}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^2,x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 - 5*a*x - 7*a^2*x^2 + 3*a^3*x^3))/(3*(-1 +
a*x)^2*(1 + a*x)) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.63, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6748, 114, 25, 27, 169, 25, 27, 169, 25, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int -\frac{\left(a+\frac{3}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\left(a+\frac{3}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\left(a+\frac{3}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \text{169}$$

$$-\frac{\frac{\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{1}{3}a\int -\frac{\left(3a+\frac{8}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{1}{3}a\int \frac{\left(3a+\frac{8}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{3}\int \frac{\left(3a+\frac{8}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{11a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-a\int -\frac{\left(3a+\frac{11}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}\right)+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{1}{3}\left(a\int \frac{\left(3a+\frac{11}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{11a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{3}\left(\int \frac{\left(3a+\frac{11}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{11a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{1}{3}\left(a\int \frac{3x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{11a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{3}\left(3a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{11a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow 103$$

$$-\frac{\frac{\frac{1}{3}\left(-3\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)-\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{11a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

$$\Big\downarrow 221$$

$$-\frac{\frac{\frac{1}{3}\left(-3a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{11a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}}{c^2}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^2,x]
```

output
```
-((-(x/((1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])) + ((4*a)/(3*(1 - 1/(a*x))^
(3/2)*Sqrt[1 + 1/(a*x)]) + ((11*a)/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) -
 (8*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)] - 3*a*ArcTanh[Sqrt[1 - 1/(a*x)]
*Sqrt[1 + 1/(a*x)]])/3)/a^2)/c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(92) = 184$.

Time = 0.16 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.10

method	result
risch	$\frac{ax-1}{a\,c^{2}\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{4}\sqrt{a^{2}}}-\frac{19\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2a\left(x-\frac{1}{a}\right)}}{12a^{6}\left(x-\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2a\left(x-\frac{1}{a}\right)}}{6a^{7}\left(x-\frac{1}{a}\right)^{2}}+\frac{\sqrt{a^{2}\left(x+\frac{1}{a}\right)^{2}-2a\left(x+\frac{1}{a}\right)}}{4a^{6}\left(x+\frac{1}{a}\right)}\right)a^{4}\sqrt{(ax-1}}{c^{2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-45\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,a^{5}x^{5}-24\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{6}x^{5}+21\sqrt{a^{2}}\,((ax-1)(ax+1))^{\frac{3}{2}}a^{3}x^{3}+45\sqrt{(ax-1)(ax+1)}\,\sqrt{}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)+(1/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-19/12/a^6/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/
2)-1/6/a^7/(x-1/a)^2*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)+1/4/a^6/(x+1/a)*(a^
2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^4/c^2/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(
a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.29

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{3\left(a^{2}x^{2}-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^{2}x^{2}-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^{3}x^{3}-7\,a^{2}x^{2}-5\,ax\right.}{3\left(a^{3}c^{2}x^{2}-2\,a^{2}c^{2}x+ac^{2}\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x
^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 7*a^2*x^
2 - 5*a*x + 8)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c
^2)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{x^4}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**2,x)
```

output
```
a**4*Integral(x**4/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a**2*x
**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1)))
, x)/c**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.54

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$
$$= \frac{1}{12}a\left(\frac{\frac{17\,(ax-1)}{ax+1}-\frac{42\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}+\frac{3\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
1/12*a*((17*(a*x - 1)/(a*x + 1) - 42*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^2
*((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)) + 12*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 12*log(sqrt((a*x - 1)/(a*x +
 1)) - 1)/(a^2*c^2) + 3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2))
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
integrate(1/((c - c/(a^2*x^2))^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.23

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^2}-\frac{\frac{17\,(a\,x-1)}{3\,(a\,x+1)}-\frac{14\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{4\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-4\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}+\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}$$

input
```
int(1/((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
((a*x - 1)/(a*x + 1))^(1/2)/(4*a*c^2) - ((17*(a*x - 1))/(3*(a*x + 1)) - (1
4*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(4*a*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 4
*a*c^2*((a*x - 1)/(a*x + 1))^(5/2)) + (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)
))/(a*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.50

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$
$$=\frac{24\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^2x^2-24\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)-11\sqrt{ax-1}\,a^2x^2+11\sqrt{ax-1}+}{12\sqrt{ax-1}\,a\,c^2\left(a^2x^2-1\right)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x)
```

output
```
(24*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**2*x**2 -
 24*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) - 11*sqrt(a
*x - 1)*a**2*x**2 + 11*sqrt(a*x - 1) + 12*sqrt(a*x + 1)*a**3*x**3 - 28*sqr
t(a*x + 1)*a**2*x**2 - 20*sqrt(a*x + 1)*a*x + 32*sqrt(a*x + 1))/(12*sqrt(a
*x - 1)*a*c**2*(a**2*x**2 - 1))
```

3.754 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{a+\frac{1}{x}}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{5a+\frac{9}{x}}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} -\frac{5a+\frac{11}{x}}{5a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

output

```
-1/5*(a+1/x)/a^2/c^3/(1-1/a^2/x^2)^(5/2)-1/15*(5*a+9/x)/a^2/c^3/(1-1/a^2/x
^2)^(3/2)-1/5*(5*a+11/x)/a^2/c^3/(1-1/a^2/x^2)^(1/2)+(1-1/a^2/x^2)^(1/2)*x
/c^3+arctanh((1-1/a^2/x^2)^(1/2))/a/c^3
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.72

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-48+33ax+87a^2x^2-52a^3x^3-38a^4x^4+15a^5x^5\right)}{15(-1+ax)^3(1+ax)^2}+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^3}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^3,x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-48 + 33*a*x + 87*a^2*x^2 - 52*a^3*x^3 - 38*a
^4*x^4 + 15*a^5*x^5))/(15*(-1 + a*x)^3*(1 + a*x)^2) + Log[(1 + Sqrt[1 - 1/
(a^2*x^2)])*x])/(a*c^3)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.69, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6748, 114, 25, 27, 169, 25, 27, 169, 27, 169, 25, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int -\frac{\left(a+\frac{5}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\left(a+\frac{5}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\left(a+\frac{5}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow \text{169}$$

$$-\frac{\frac{\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1}{5}a\int -\frac{\left(5a+\frac{24}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{1}{5}a\int \frac{\left(5a+\frac{24}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{5}\int \frac{\left(5a+\frac{24}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{1}{5}\left(\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1}{3}a\int -\frac{3\left(5a+\frac{29}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{5}\left(\int \frac{\left(5a+\frac{29}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{1}{5}\left(a\left(-\int -\frac{\left(5a+\frac{68}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}\right)+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{1}{5}\left(a\int \frac{\left(5a+\frac{68}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{5}\left(\int \frac{\left(5a+\frac{68}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{1}{5}\left(\frac{1}{3}a\int \frac{3\left(5a+\frac{21}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{21a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{5}\left(\int \frac{\left(5a+\frac{21}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{21a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{1}{5}\left(a\int \frac{5x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{16a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{21a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{5}\left(5a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{16a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{21a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^3}$$

$$\Big\downarrow \; 103$$

$$-\frac{\frac{\frac{1}{5}\left(-5\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)-\frac{16a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{21a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\ldots}{c^3}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\frac{1}{5}\left(-5a\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-\frac{16a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{21a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{34a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{29a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-}{c^3}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^3,x]
```

output
```
-((-(x/((1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2))) + ((6*a)/(5*(1 - 1/(a*x)
)^(5/2)*(1 + 1/(a*x))^(3/2)) + ((29*a)/(3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x)
)^(3/2)) + (34*a)/(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) - (21*a*Sqrt[1 -
 1/(a*x)])/(1 + 1/(a*x))^(3/2) - (16*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)
] - 5*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/5)/a^2)/c^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(121) = 242$.

Time = 0.17 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.13

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^6\sqrt{a^2}}-\frac{493\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{240a^8\left(x-\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{20a^{10}\left(x-\frac{1}{a}\right)^3}-\frac{23\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{60a^9\left(x-\frac{1}{a}\right)^2}-\frac{\sqrt{a^2\left(x+\right.}}{24a}\right.}{c^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-525\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^7x^7-240\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^8x^7+285((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5+525\sqrt{(ax-1)(ax+}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^3/((a*x-1)/(a*x+1))^(1/2)+(1/a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-493/240/a^8/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(
1/2)-1/20/a^10/(x-1/a)^3*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-23/60/a^9/(x-1/
a)^2*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-1/24/a^9/(x+1/a)^2*(a^2*(x+1/a)^2-2
*a*(x+1/a))^(1/2)+25/48/a^8/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^6
/c^3/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.30

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{15\,(a^4x^4-2\,a^3x^3+2\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,(a^4x^4-2\,a^3x^3+2\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(15}{15\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
1/15*(15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
 1) - 15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) -
 1) + (15*a^5*x^5 - 38*a^4*x^4 - 52*a^3*x^3 + 87*a^2*x^2 + 33*a*x - 48)*sq
rt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^
3)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{a^6\int \frac{x^6}{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**3,x)
```

output
```
a**6*Integral(x**6/(a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**4*x
**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.42

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{1}{240}\,a\left(\frac{\frac{37\,(ax-1)}{ax+1}+\frac{410\,(ax-1)^2}{(ax+1)^2}-\frac{930\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{5\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+24\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^3}+\frac{240\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{24}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
1/240*a*((37*(a*x - 1)/(a*x + 1) + 410*(a*x - 1)^2/(a*x + 1)^2 - 930*(a*x
- 1)^3/(a*x + 1)^3 + 3)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^3*((a
*x - 1)/(a*x + 1))^(5/2)) + 5*(((a*x - 1)/(a*x + 1))^(3/2) + 24*sqrt((a*x
- 1)/(a*x + 1)))/(a^2*c^3) + 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c
^3) - 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
integrate(1/((c - c/(a^2*x^2))^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.25

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a\,c^3} - \frac{\frac{82\,(a\,x-1)^2}{3\,(a\,x+1)^2} - \frac{62\,(a\,x-1)^3}{(a\,x+1)^3} + \frac{37\,(a\,x-1)}{15\,(a\,x+1)} + \frac{1}{5}}{16\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2} - 16\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}} + \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{48\,a\,c^3} - \frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,2\mathrm{i}}{a\,c^3}$$

input
```
int(1/((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
((a*x - 1)/(a*x + 1))^(1/2)/(2*a*c^3) - ((82*(a*x - 1)^2)/(3*(a*x + 1)^2)
- (62*(a*x - 1)^3)/(a*x + 1)^3 + (37*(a*x - 1))/(15*(a*x + 1)) + 1/5)/(16*
a*c^3*((a*x - 1)/(a*x + 1))^(5/2) - 16*a*c^3*((a*x - 1)/(a*x + 1))^(7/2))
+ ((a*x - 1)/(a*x + 1))^(3/2)/(48*a*c^3) - (atan(((a*x - 1)/(a*x + 1))^(1/
2)*1i)*2i)/(a*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.74

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{120\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^4x^4 - 240\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^2x^2 + 120\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{}}{\sqrt{2}}\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x)
```

output
```
(120*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**4*x**4
- 240*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**2*x**2
 + 120*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) - 133*sq
rt(a*x - 1)*a**4*x**4 + 266*sqrt(a*x - 1)*a**2*x**2 - 133*sqrt(a*x - 1) +
60*sqrt(a*x + 1)*a**5*x**5 - 152*sqrt(a*x + 1)*a**4*x**4 - 208*sqrt(a*x +
1)*a**3*x**3 + 348*sqrt(a*x + 1)*a**2*x**2 + 132*sqrt(a*x + 1)*a*x - 192*s
qrt(a*x + 1))/(60*sqrt(a*x - 1)*a*c**3*(a**4*x**4 - 2*a**2*x**2 + 1))
```

3.755 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 170

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=-\frac{a+\frac{1}{x}}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{7a+\frac{13}{x}}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{35a+\frac{87}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{35a+\frac{93}{x}}{35a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

output
```
-1/7*(a+1/x)/a^2/c^4/(1-1/a^2/x^2)^(7/2)-1/35*(7*a+13/x)/a^2/c^4/(1-1/a^2/
x^2)^(5/2)-1/105*(35*a+87/x)/a^2/c^4/(1-1/a^2/x^2)^(3/2)-1/35*(35*a+93/x)/
a^2/c^4/(1-1/a^2/x^2)^(1/2)+(1-1/a^2/x^2)^(1/2)*x/c^4+arctanh((1-1/a^2/x^2
)^(1/2))/a/c^4
```

Mathematica [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.68

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}x(384-279ax-1065a^2x^2+715a^3x^3+965a^4x^4-559a^5x^5-281a^6x^6+105a^7x^7)}{105(-1+ax)^4(1+ax)^3}+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^4}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^4,x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(384 - 279*a*x - 1065*a^2*x^2 + 715*a^3*x^3 +
965*a^4*x^4 - 559*a^5*x^5 - 281*a^6*x^6 + 105*a^7*x^7))/(105*(-1 + a*x)^4*
(1 + a*x)^3) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.81, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {6748, 114, 25, 27, 169, 25, 27, 169, 27, 169, 25, 27, 169, 27, 169, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int -\frac{\left(a+\frac{7}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{\left(a+\frac{7}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\int \frac{\left(a+\frac{7}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{1}{7}a\int -\frac{\left(7a+\frac{48}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\frac{1}{7}a\int \frac{\left(7a+\frac{48}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{1}{7}\int \frac{\left(7a+\frac{48}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{1}{5}a\int -\frac{5\left(7a+\frac{55}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{1}{7}\left(\int \frac{\left(7a+\frac{55}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{\frac{1}{7}\left(-\frac{1}{3}a\int -\frac{\left(21a+\frac{248}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)}}{c^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}a\int \frac{\left(21a+\frac{248}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}\int \frac{\left(21a+\frac{248}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^4}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}\left(\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}-a\int -\frac{3\left(7a+\frac{269}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}\left(3\int \frac{\left(7a+\frac{269}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{}{(1-}}{c^4}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}\left(3\left(\frac{1}{5}a\int \frac{\left(35a+\frac{524}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{262a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{}{7\left(1-\frac{1}{ax}\right.}}{a^2}}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}\left(3\left(\frac{1}{5}\int\frac{\left(35a+\frac{524}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{262a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{}{7\left(1-\frac{1}{ax}\right)^{7/}}}{a^2}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}\left(3\left(\frac{1}{5}\left(\frac{1}{3}a\int\frac{3\left(35a+\frac{163}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{163a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)-\frac{262a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}\right.}}{a^2}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}\left(3\left(\frac{1}{5}\left(\int\frac{\left(35a+\frac{163}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{163a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)-\frac{262a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11a}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)}}{a^2}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}\left(3\left(\frac{1}{5}\left(a\int\frac{35x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{128a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{163a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)-\frac{262a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{}{\left(1-\frac{1}{ax}\right.}}{a^2}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}\left(3\left(\frac{1}{5}\left(35a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{128a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{163a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)-\frac{262a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{}{\left(1-\right.}}{a^2}}{c^4}$$

$$\Big\downarrow \ 103$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}\left(3\left(\frac{1}{5}\left(-35\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)-\frac{128a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{163a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)-\frac{262a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)}}{a^2}}{c^4}$$

$$\Big\downarrow \ 221$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{1}{3}\left(3\left(\frac{1}{5}\left(-35a\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-\frac{128a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{163a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)-\frac{262a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{269a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{62a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)}}{a^2}}{c^4}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^4,x]
```

output
```
-((-(x/((1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2))) + ((8*a)/(7*(1 - 1/(a*x)
)^(7/2)*(1 + 1/(a*x))^(5/2)) + ((11*a)/((1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^
(5/2)) + (62*a)/(3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)) + ((269*a)/(Sq
rt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)) + 3*((-262*a*Sqrt[1 - 1/(a*x)])/(5*(1
 + 1/(a*x))^(5/2)) + ((-163*a*Sqrt[1 - 1/(a*x)])/(1 + 1/(a*x))^(3/2) - (12
8*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)] - 35*a*ArcTanh[Sqrt[1 - 1/(a*x)]*
Sqrt[1 + 1/(a*x)]])/5))/3)/7)/a^2)/c^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(150) = 300$.

Time = 0.20 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.15

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{\ln\left(\frac{a^2 x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^8\sqrt{a^2}}-\frac{1657\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{672a^{10}\left(x-\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{56a^{13}\left(x-\frac{1}{a}\right)^4}-\frac{17\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{112a^{12}\left(x-\frac{1}{a}\right)^3}-\frac{211\sqrt{(}}{3}\right.}{c^4\sqrt{}}$
default	Expression too large to display

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^4/((a*x-1)/(a*x+1))^(1/2)+(1/a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-1657/672/a^10/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))
^(1/2)-1/56/a^13/(x-1/a)^4*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-17/112/a^12/(
x-1/a)^3*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-211/336/a^11/(x-1/a)^2*((x-1/a)
^2*a^2+2*a*(x-1/a))^(1/2)+1/80/a^12/(x+1/a)^3*(a^2*(x+1/a)^2-2*a*(x+1/a))^
(1/2)-7/60/a^11/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+379/480/a^10/(
x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^8/c^4/((a*x-1)/(a*x+1))^(1/2)*
((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.61

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{105\left(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^6x^6-2\,a^5x^5-a^4x^4+\cdots\right.}{105\left(a^7c^4x^6-2\,a^6\cdots\right.}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
1/105*(105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x +
1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4
 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (
105*a^7*x^7 - 281*a^6*x^6 - 559*a^5*x^5 + 965*a^4*x^4 + 715*a^3*x^3 - 1065
*a^2*x^2 - 279*a*x + 384)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 - 2*a^6*
c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{x^8}{a^8x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**4,x)
```

output
```
a**8*Integral(x**8/(a**8*x**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**6*x
**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**4*x**4*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1)) - 4*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))), x)/c**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.35

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{1}{6720}a\left(\frac{5\left(\frac{39\,(ax-1)}{ax+1}+\frac{287\,(ax-1)^2}{(ax+1)^2}+\frac{2611\,(ax-1)^3}{(ax+1)^3}-\frac{5628\,(ax-1)^4}{(ax+1)^4}+3\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{7\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+50\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+705\sqrt{}\right.}{a^2c^4}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
1/6720*a*(5*(39*(a*x - 1)/(a*x + 1) + 287*(a*x - 1)^2/(a*x + 1)^2 + 2611*(
a*x - 1)^3/(a*x + 1)^3 - 5628*(a*x - 1)^4/(a*x + 1)^4 + 3)/(a^2*c^4*((a*x
- 1)/(a*x + 1))^(9/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2)) + 7*(3*((a*x
- 1)/(a*x + 1))^(5/2) + 50*((a*x - 1)/(a*x + 1))^(3/2) + 705*sqrt((a*x - 1
)/(a*x + 1)))/(a^2*c^4) + 6720*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4
) - 6720*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
integrate(1/((c - c/(a^2*x^2))^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B] (verification not implemented)

Time = 13.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.24

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{47\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4}-\frac{\frac{41\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}+\frac{373\,{\left(a\,x-1\right)}^3}{3\,{\left(a\,x+1\right)}^3}-\frac{268\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}+\frac{13\,\left(a\,x-1\right)}{7\,\left(a\,x+1\right)}+\frac{1}{7}}{64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}-64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}+\frac{5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{96\,a\,c^4}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{320\,a\,c^4}-\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}}{a\,c^4}$$

input
```
int(1/((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
(47*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - ((41*(a*x - 1)^2)/(3*(a*x +
1)^2) + (373*(a*x - 1)^3)/(3*(a*x + 1)^3) - (268*(a*x - 1)^4)/(a*x + 1)^4
+ (13*(a*x - 1))/(7*(a*x + 1)) + 1/7)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(7/2
) - 64*a*c^4*((a*x - 1)/(a*x + 1))^(9/2)) + (5*((a*x - 1)/(a*x + 1))^(3/2)
)/(96*a*c^4) + ((a*x - 1)/(a*x + 1))^(5/2)/(320*a*c^4) - (atan(((a*x - 1)/
(a*x + 1))^(1/2)*1i)*2i)/(a*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.89

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{840\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^6x^6-2520\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^4x^4+2520\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}}{\quad}\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x)
```

output
```
(840*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**6*x**6
- 2520*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**4*x**
4 + 2520*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**2*x
**2 - 840*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) - 150
1*sqrt(a*x - 1)*a**6*x**6 + 4503*sqrt(a*x - 1)*a**4*x**4 - 4503*sqrt(a*x -
 1)*a**2*x**2 + 1501*sqrt(a*x - 1) + 420*sqrt(a*x + 1)*a**7*x**7 - 1124*sq
rt(a*x + 1)*a**6*x**6 - 2236*sqrt(a*x + 1)*a**5*x**5 + 3860*sqrt(a*x + 1)*
a**4*x**4 + 2860*sqrt(a*x + 1)*a**3*x**3 - 4260*sqrt(a*x + 1)*a**2*x**2 -
1116*sqrt(a*x + 1)*a*x + 1536*sqrt(a*x + 1))/(420*sqrt(a*x - 1)*a*c**4*(a*
*6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1))
```

3.756 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=-\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}+\frac{3c^5}{a^2x}+c^5x+\frac{2c^5\log(x)}{a}$$

output
```
-1/9*c^5/a^10/x^9-1/4*c^5/a^9/x^8+3/7*c^5/a^8/x^7+4/3*c^5/a^7/x^6-2/5*c^5/
a^6/x^5-3*c^5/a^5/x^4-2/3*c^5/a^4/x^3+4*c^5/a^3/x^2+3*c^5/a^2/x+c^5*x+2*c^
5*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=-\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}+\frac{3c^5}{a^2x}+c^5x+\frac{2c^5\log(x)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

output
```
-1/9*c^5/(a^10*x^9) - c^5/(4*a^9*x^8) + (3*c^5)/(7*a^8*x^7) + (4*c^5)/(3*a
^7*x^6) - (2*c^5)/(5*a^6*x^5) - (3*c^5)/(a^5*x^4) - (2*c^5)/(3*a^4*x^3) +
(4*c^5)/(a^3*x^2) + (3*c^5)/(a^2*x) + c^5*x + (2*c^5*Log[x])/a
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^5 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^5 e^{2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^5}{a^{10}}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^5\int e^{2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^5dx}{a^{10}}$$

$$\Big\downarrow \text{6707}$$

$$\frac{c^5\int \frac{e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^5}{x^{10}}dx}{a^{10}}$$

$$\Big\downarrow \text{6700}$$

$$\frac{c^5\int \frac{(1-ax)^4(ax+1)^6}{x^{10}}dx}{a^{10}}$$

$$\Big\downarrow \text{99}$$

$$\frac{c^5\int \left(a^{10}+\frac{2a^9}{x}-\frac{3a^8}{x^2}-\frac{8a^7}{x^3}+\frac{2a^6}{x^4}+\frac{12a^5}{x^5}+\frac{2a^4}{x^6}-\frac{8a^3}{x^7}-\frac{3a^2}{x^8}+\frac{2a}{x^9}+\frac{1}{x^{10}}\right)dx}{a^{10}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^5\left(a^{10}x+2a^9\log(x)+\frac{3a^8}{x}+\frac{4a^7}{x^2}-\frac{2a^6}{3x^3}-\frac{3a^5}{x^4}-\frac{2a^4}{5x^5}+\frac{4a^3}{3x^6}+\frac{3a^2}{7x^7}-\frac{a}{4x^8}-\frac{1}{9x^9}\right)}{a^{10}}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

output
```
(c^5*(-1/9*1/x^9 - a/(4*x^8) + (3*a^2)/(7*x^7) + (4*a^3)/(3*x^6) - (2*a^4)
/(5*x^5) - (3*a^5)/x^4 - (2*a^6)/(3*x^3) + (4*a^7)/x^2 + (3*a^8)/x + a^10*
x + 2*a^9*Log[x]))/a^10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

method	result
default	$\frac{c^{5}\left(x\,a^{10}+\frac{4a^{3}}{3x^{6}}+\frac{4a^{7}}{x^{2}}+\frac{3a^{8}}{x}+\frac{3a^{2}}{7x^{7}}-\frac{1}{9x^{9}}-\frac{3a^{5}}{x^{4}}+2a^{9}\ln\left(x\right)-\frac{2a^{6}}{3x^{3}}-\frac{2a^{4}}{5x^{5}}-\frac{a}{4x^{8}}\right)}{a^{10}}$
risch	$c^{5}x+\frac{3a^{8}c^{5}x^{8}+4a^{7}c^{5}x^{7}-\frac{2}{3}a^{6}c^{5}x^{6}-3a^{5}c^{5}x^{5}-\frac{2}{5}a^{4}c^{5}x^{4}+\frac{4}{3}a^{3}c^{5}x^{3}+\frac{3}{7}a^{2}c^{5}x^{2}-\frac{1}{4}a\,c^{5}x-\frac{1}{9}c^{5}}{a^{10}x^{9}}+\frac{2c^{5}\ln\left(x\right)}{a}$
norman	$\frac{a^{9}c^{5}x^{10}-\frac{c^{5}}{9a}-\frac{c^{5}x}{4}+\frac{4a^{2}c^{5}x^{3}}{3}-3a^{4}c^{5}x^{5}-\frac{2a^{5}c^{5}x^{6}}{3}+4a^{6}c^{5}x^{7}+3a^{7}c^{5}x^{8}+\frac{3c^{5}a\,x^{2}}{7}-\frac{2c^{5}a^{3}x^{4}}{5}}{a^{9}x^{9}}+\frac{2c^{5}\ln\left(x\right)}{a}$
parallelrisch	$\frac{1260a^{10}c^{5}x^{10}+2520c^{5}\ln\left(x\right)a^{9}x^{9}+3780a^{8}c^{5}x^{8}+5040a^{7}c^{5}x^{7}-840a^{6}c^{5}x^{6}-3780a^{5}c^{5}x^{5}-504a^{4}c^{5}x^{4}+1680a^{3}c^{5}x^{3}+540a^{2}c^{5}x}{1260a^{10}x^{9}}$
meijerg	$-\frac{c^{5}\left(-ax-\ln\left(-ax+1\right)\right)}{a}+\frac{5c^{5}\left(\ln\left(x\right)+\ln\left(-a\right)-\ln\left(-ax+1\right)\right)}{a}-\frac{10c^{5}\left(-\frac{1}{2a^{2}x^{2}}-\frac{1}{ax}+\ln\left(x\right)+\ln\left(-a\right)-\ln\left(-ax+1\right)\right)}{a}+\frac{10c^{5}}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x,method=_RETURNVERBOSE)
```

output
```
c^5/a^10*(x*a^10+4/3*a^3/x^6+4*a^7/x^2+3*a^8/x+3/7*a^2/x^7-1/9/x^9-3*a^5/x
^4+2*a^9*ln(x)-2/3*a^6/x^3-2/5*a^4/x^5-1/4*a/x^8)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^{2}x^{2}}\right)^{5}dx$$
$$=\frac{1260\,a^{10}c^{5}x^{10}+2520\,a^{9}c^{5}x^{9}\log\left(x\right)+3780\,a^{8}c^{5}x^{8}+5040\,a^{7}c^{5}x^{7}-840\,a^{6}c^{5}x^{6}-3780\,a^{5}c^{5}x^{5}-504\,a^{4}c^{5}x}{1260\,a^{10}x^{9}}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="fricas")
```

output
```
1/1260*(1260*a^10*c^5*x^10 + 2520*a^9*c^5*x^9*log(x) + 3780*a^8*c^5*x^8 +
5040*a^7*c^5*x^7 - 840*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 +
1680*a^3*c^5*x^3 + 540*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx = \frac{a^{10}c^5x+2a^9c^5\log{(x)}+\frac{3780a^8c^5x^8+5040a^7c^5x^7-840a^6c^5x^6-3780a^5c^5x^5-504a^4c^5x^4+1680a^3c^5x^3+540a^2c^5x^2-315ac^5x-140c^5}{1260x^9}}{a^{10}}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**5,x)
```

output
```
(a**10*c**5*x + 2*a**9*c**5*log(x) + (3780*a**8*c**5*x**8 + 5040*a**7*c**5
*x**7 - 840*a**6*c**5*x**6 - 3780*a**5*c**5*x**5 - 504*a**4*c**5*x**4 + 16
80*a**3*c**5*x**3 + 540*a**2*c**5*x**2 - 315*a*c**5*x - 140*c**5)/(1260*x*
*9))/a**10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx = c^5x+\frac{2\,c^5\log{(x)}}{a} + \frac{3780\,a^8c^5x^8+5040\,a^7c^5x^7-840\,a^6c^5x^6-3780\,a^5c^5x^5-504\,a^4c^5x^4+1680\,a^3c^5x^3+540\,a^2c^5x^2-315}{1260\,a^{10}x^9}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="maxima")
```

output
```
c^5*x + 2*c^5*log(x)/a + 1/1260*(3780*a^8*c^5*x^8 + 5040*a^7*c^5*x^7 - 840
*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1680*a^3*c^5*x^3 + 540
*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx = c^5x+\frac{2c^5\log(|x|)}{a} + \frac{3780a^8c^5x^8+5040a^7c^5x^7-840a^6c^5x^6-3780a^5c^5x^5-504a^4c^5x^4+1680a^3c^5x^3+540a^2c^5x^2-315}{1260a^{10}x^9}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="giac")
```

output
```
c^5*x + 2*c^5*log(abs(x))/a + 1/1260*(3780*a^8*c^5*x^8 + 5040*a^7*c^5*x^7
- 840*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1680*a^3*c^5*x^3
+ 540*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx = \frac{c^5\left(\frac{3a^2x^2}{7}-\frac{ax}{4}+\frac{4a^3x^3}{3}-\frac{2a^4x^4}{5}-3a^5x^5-\frac{2a^6x^6}{3}+4a^7x^7+3a^8x^8+a^{10}x^{10}+2a^9x^9\ln(x)-\frac{1}{9}\right)}{a^{10}x^9}$$

input
```
int(((c - c/(a^2*x^2))^5*(a*x + 1))/(a*x - 1),x)
```

output
```
(c^5*((3*a^2*x^2)/7 - (a*x)/4 + (4*a^3*x^3)/3 - (2*a^4*x^4)/5 - 3*a^5*x^5
- (2*a^6*x^6)/3 + 4*a^7*x^7 + 3*a^8*x^8 + a^10*x^10 + 2*a^9*x^9*log(x) - 1
/9))/(a^10*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.72

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$$
$$=\frac{c^5(2520\log(x)\,a^9x^9+1260a^{10}x^{10}+3780a^8x^8+5040a^7x^7-840a^6x^6-3780a^5x^5-504a^4x^4+1680a^3x^3}{1260a^{10}x^9}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x)
```

output
```
(c**5*(2520*log(x)*a**9*x**9 + 1260*a**10*x**10 + 3780*a**8*x**8 + 5040*a*
*7*x**7 - 840*a**6*x**6 - 3780*a**5*x**5 - 504*a**4*x**4 + 1680*a**3*x**3
+ 540*a**2*x**2 - 315*a*x - 140))/(1260*a**10*x**9)
```

3.757 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x+\frac{2c^4\log(x)}{a}$$

output
```
1/7*c^4/a^8/x^7+1/3*c^4/a^7/x^6-2/5*c^4/a^6/x^5-3/2*c^4/a^5/x^4+3*c^4/a^3/
x^2+2*c^4/a^2/x+c^4*x+2*c^4*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x+\frac{2c^4\log(x)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

output
```
c^4/(7*a^8*x^7) + c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) - (3*c^4)/(2*a^5*x
^4) + (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x + (2*c^4*Log[x])/a
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^4 e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^4 e^{2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^4}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\int e^{2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^4dx}{a^8}$$

$$\Big\downarrow \text{6707}$$

$$-\frac{c^4\int \frac{e^{2\text{arctanh}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$

$$\Big\downarrow \text{6700}$$

$$-\frac{c^4\int \frac{(1-ax)^3(ax+1)^5}{x^8}dx}{a^8}$$

$$\Big\downarrow \text{99}$$

$$-\frac{c^4\int \left(-a^8-\frac{2a^7}{x}+\frac{2a^6}{x^2}+\frac{6a^5}{x^3}-\frac{6a^3}{x^5}-\frac{2a^2}{x^6}+\frac{2a}{x^7}+\frac{1}{x^8}\right)dx}{a^8}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^4\left(a^8(-x)-2a^7\log(x)-\frac{2a^6}{x}-\frac{3a^5}{x^2}+\frac{3a^3}{2x^4}+\frac{2a^2}{5x^5}-\frac{a}{3x^6}-\frac{1}{7x^7}\right)}{a^8}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

output
```
-((c^4*(-1/7*1/x^7 - a/(3*x^6) + (2*a^2)/(5*x^5) + (3*a^3)/(2*x^4) - (3*a^
5)/x^2 - (2*a^6)/x - a^8*x - 2*a^7*Log[x]))/a^8)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.71

method	result
default	$\frac{c^4\left(x\,a^8+\frac{a}{3x^6}+\frac{3a^5}{x^2}+\frac{2a^6}{x}+\frac{1}{7x^7}-\frac{3a^3}{2x^4}+2a^7\ln(x)-\frac{2a^2}{5x^5}\right)}{a^8}$
risch	$c^4x+\frac{2a^6c^4x^6+3a^5c^4x^5-\frac{3}{2}a^3c^4x^3-\frac{2}{5}a^2c^4x^2+\frac{1}{3}a\,c^4x+\frac{1}{7}c^4}{a^8x^7}+\frac{2c^4\ln(x)}{a}$
norman	$\frac{a^7c^4x^8+\frac{c^4}{7a}+\frac{c^4x}{3}-\frac{2a\,c^4x^2}{5}-\frac{3a^2c^4x^3}{2}+3a^4c^4x^5+2a^5c^4x^6}{a^7x^7}+\frac{2c^4\ln(x)}{a}$
parallelrisch	$\frac{210a^8c^4x^8+420c^4\ln(x)a^7x^7+420a^6c^4x^6+630a^5c^4x^5-315a^3c^4x^3-84a^2c^4x^2+70a\,c^4x+30c^4}{210a^8x^7}$
meijerg	$-\frac{c^4(-ax-\ln(-ax+1))}{a}+\frac{4c^4(\ln(x)+\ln(-a)-\ln(-ax+1))}{a}-\frac{6c^4\left(-\frac{1}{2a^2x^2}-\frac{1}{ax}+\ln(x)+\ln(-a)-\ln(-ax+1)\right)}{a}+\frac{4c^4\left(-\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
c^4/a^8*(x*a^8+1/3*a/x^6+3*a^5/x^2+2*a^6/x+1/7/x^7-3/2*a^3/x^4+2*a^7*ln(x)
-2/5*a^2/x^5)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{210\,a^8c^4x^8+420\,a^7c^4x^7\log(x)+420\,a^6c^4x^6+630\,a^5c^4x^5-315\,a^3c^4x^3-84\,a^2c^4x^2+70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
1/210*(210*a^8*c^4*x^8 + 420*a^7*c^4*x^7*log(x) + 420*a^6*c^4*x^6 + 630*a^
5*c^4*x^5 - 315*a^3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x
^7)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = \frac{a^8c^4x+2a^7c^4\log(x)+\frac{420a^6c^4x^6+630a^5c^4x^5-315a^3c^4x^3-84a^2c^4x^2+70ac^4x+30c^4}{210x^7}}{a^8}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**4,x)
```

output
```
(a**8*c**4*x + 2*a**7*c**4*log(x) + (420*a**6*c**4*x**6 + 630*a**5*c**4*x*
*5 - 315*a**3*c**4*x**3 - 84*a**2*c**4*x**2 + 70*a*c**4*x + 30*c**4)/(210*
x**7))/a**8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = c^4x+\frac{2\,c^4\log(x)}{a} + \frac{420\,a^6c^4x^6+630\,a^5c^4x^5-315\,a^3c^4x^3-84\,a^2c^4x^2+70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
c^4*x + 2*c^4*log(x)/a + 1/210*(420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 315*a^
3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$= c^4x + \frac{2\,c^4\log\left(|x|\right)}{a} + \frac{420\,a^6c^4x^6 + 630\,a^5c^4x^5 - 315\,a^3c^4x^3 - 84\,a^2c^4x^2 + 70\,ac^4x + 30\,c^4}{210\,a^8x^7}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
c^4*x + 2*c^4*log(abs(x))/a + 1/210*(420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 3
15*a^3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.72

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$= \frac{c^4\left(\frac{a\,x}{3} - \frac{2\,a^2\,x^2}{5} - \frac{3\,a^3\,x^3}{2} + 3\,a^5\,x^5 + 2\,a^6\,x^6 + a^8\,x^8 + 2\,a^7\,x^7\,\ln\left(x\right) + \frac{1}{7}\right)}{a^8\,x^7}$$

input
```
int(((c - c/(a^2*x^2))^4*(a*x + 1))/(a*x - 1),x)
```

output
```
(c^4*((a*x)/3 - (2*a^2*x^2)/5 - (3*a^3*x^3)/2 + 3*a^5*x^5 + 2*a^6*x^6 + a^
8*x^8 + 2*a^7*x^7*log(x) + 1/7))/(a^8*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$=\frac{c^4(420\log(x)\,a^7x^7+210a^8x^8+420a^6x^6+630a^5x^5-315a^3x^3-84a^2x^2+70ax+30)}{210a^8x^7}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x)
```

output
```
(c**4*(420*log(x)*a**7*x**7 + 210*a**8*x**8 + 420*a**6*x**6 + 630*a**5*x**
5 - 315*a**3*x**3 - 84*a**2*x**2 + 70*a*x + 30))/(210*a**8*x**7)
```

3.758 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=-\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{2c^3\log(x)}{a}$$

output
```
-1/5*c^3/a^6/x^5-1/2*c^3/a^5/x^4+1/3*c^3/a^4/x^3+2*c^3/a^3/x^2+c^3/a^2/x+c
^3*x+2*c^3*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=-\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{2c^3\log(x)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

output
```
-1/5*c^3/(a^6*x^5) - c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) + (2*c^3)/(a^3*x^2)
 + c^3/(a^2*x) + c^3*x + (2*c^3*Log[x])/a
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^3 e^{2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^3 e^{2\text{arctanh}(ax)} \left(a^2-\frac{1}{x^2}\right)^3}{a^6}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3 \int e^{2\text{arctanh}(ax)} \left(a^2-\frac{1}{x^2}\right)^3 dx}{a^6}$$

$$\Big\downarrow \text{6707}$$

$$\frac{c^3 \int \frac{e^{2\text{arctanh}(ax)} \left(1-a^2x^2\right)^3}{x^6}dx}{a^6}$$

$$\Big\downarrow \text{6700}$$

$$\frac{c^3 \int \frac{(1-ax)^2(ax+1)^4}{x^6}dx}{a^6}$$

$$\Big\downarrow \text{99}$$

$$\frac{c^3 \int \left(a^6+\frac{2a^5}{x}-\frac{a^4}{x^2}-\frac{4a^3}{x^3}-\frac{a^2}{x^4}+\frac{2a}{x^5}+\frac{1}{x^6}\right)dx}{a^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3 \left(a^6x+2a^5\log(x)+\frac{a^4}{x}+\frac{2a^3}{x^2}+\frac{a^2}{3x^3}-\frac{a}{2x^4}-\frac{1}{5x^5}\right)}{a^6}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

output
```
(c^3*(-1/5*1/x^5 - a/(2*x^4) + a^2/(3*x^3) + (2*a^3)/x^2 + a^4/x + a^6*x +
 2*a^5*Log[x]))/a^6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.72

method	result
default	$\frac{c^3\left(x\,a^6+\frac{2a^3}{x^2}+\frac{a^4}{x}-\frac{a}{2x^4}+2a^5\ln(x)+\frac{a^2}{3x^3}-\frac{1}{5x^5}\right)}{a^6}$
risch	$c^3x+\frac{a^4c^3x^4+2a^3c^3x^3+\frac{1}{3}a^2c^3x^2-\frac{1}{2}a\,c^3x-\frac{1}{5}c^3}{a^6x^5}+\frac{2c^3\ln(x)}{a}$
norman	$\frac{a^3c^3x^4+a^5c^3x^6-\frac{c^3}{5a}-\frac{c^3x}{2}+\frac{a\,c^3x^2}{3}+2a^2c^3x^3}{a^5x^5}+\frac{2c^3\ln(x)}{a}$
parallelrisch	$\frac{30a^6c^3x^6+60c^3\ln(x)a^5x^5+30a^4c^3x^4+60a^3c^3x^3+10a^2c^3x^2-15a\,c^3x-6c^3}{30a^6x^5}$
meijerg	$-\frac{c^3(-ax-\ln(-ax+1))}{a}+\frac{3c^3(\ln(x)+\ln(-a)-\ln(-ax+1))}{a}-\frac{3c^3\left(-\frac{1}{2a^2x^2}-\frac{1}{ax}+\ln(x)+\ln(-a)-\ln(-ax+1)\right)}{a}+\frac{c^3\left(-\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
c^3/a^6*(x*a^6+2*a^3/x^2+a^4/x-1/2*a/x^4+2*a^5*ln(x)+1/3*a^2/x^3-1/5/x^5)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{30\,a^6c^3x^6+60\,a^5c^3x^5\log(x)+30\,a^4c^3x^4+60\,a^3c^3x^3+10\,a^2c^3x^2-15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
1/30*(30*a^6*c^3*x^6 + 60*a^5*c^3*x^5*log(x) + 30*a^4*c^3*x^4 + 60*a^3*c^3
*x^3 + 10*a^2*c^3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=\frac{a^6c^3x+2a^5c^3\log(x)+\frac{30a^4c^3x^4+60a^3c^3x^3+10a^2c^3x^2-15ac^3x-6c^3}{30x^5}}{a^6}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**3,x)
```

output
```
(a**6*c**3*x + 2*a**5*c**3*log(x) + (30*a**4*c**3*x**4 + 60*a**3*c**3*x**3
 + 10*a**2*c**3*x**2 - 15*a*c**3*x - 6*c**3)/(30*x**5))/a**6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=c^3x+\frac{2\,c^3\log(x)}{a}+\frac{30\,a^4c^3x^4+60\,a^3c^3x^3+10\,a^2c^3x^2-15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
c^3*x + 2*c^3*log(x)/a + 1/30*(30*a^4*c^3*x^4 + 60*a^3*c^3*x^3 + 10*a^2*c^
3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=c^3x+\frac{2\,c^3\log(|x|)}{a}+\frac{30\,a^4c^3x^4+60\,a^3c^3x^3+10\,a^2c^3x^2-15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
c^3*x + 2*c^3*log(abs(x))/a + 1/30*(30*a^4*c^3*x^4 + 60*a^3*c^3*x^3 + 10*a
^2*c^3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{c^3\left(\frac{a^2\,x^2}{3}-\frac{a\,x}{2}+2\,a^3\,x^3+a^4\,x^4+a^6\,x^6+2\,a^5\,x^5\,\ln\left(x\right)-\frac{1}{5}\right)}{a^6\,x^5}$$

input
```
int(((c - c/(a^2*x^2))^3*(a*x + 1))/(a*x - 1),x)
```

output
```
(c^3*((a^2*x^2)/3 - (a*x)/2 + 2*a^3*x^3 + a^4*x^4 + a^6*x^6 + 2*a^5*x^5*lo
g(x) - 1/5))/(a^6*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{c^3(60\log(x)\,a^5x^5+30a^6x^6+30a^4x^4+60a^3x^3+10a^2x^2-15ax-6)}{30a^6x^5}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x)
```

output
```
(c**3*(60*log(x)*a**5*x**5 + 30*a**6*x**6 + 30*a**4*x**4 + 60*a**3*x**3 +
10*a**2*x**2 - 15*a*x - 6))/(30*a**6*x**5)
```

3.759 $\int e^{2 \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 39

$$\int e^{2 \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^2 \, dx = \frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+c^2x+\frac{2c^2\log(x)}{a}$$

output
```
1/3*c^2/a^4/x^3+c^2/a^3/x^2+c^2*x+2*c^2*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int e^{2 \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^2 \, dx = \frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+c^2x+\frac{2c^2\log(x)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

output
```
c^2/(3*a^4*x^3) + c^2/(a^3*x^2) + c^2*x + (2*c^2*Log[x])/a
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^2 e^{2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{c^2 e^{2\text{arctanh}(ax)} \left(a^2-\frac{1}{x^2}\right)^2}{a^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^2 \int e^{2\text{arctanh}(ax)} \left(a^2-\frac{1}{x^2}\right)^2dx}{a^4}$$

$$\Big\downarrow \text{ 6707}$$

$$-\frac{c^2 \int \frac{e^{2\text{arctanh}(ax)}(1-a^2x^2)^2}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{ 6700}$$

$$-\frac{c^2 \int \frac{(1-ax)(ax+1)^3}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{ 84}$$

$$-\frac{c^2 \int \left(-a^4-\frac{2a^3}{x}+\frac{2a}{x^3}+\frac{1}{x^4}\right)dx}{a^4}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{c^2 \left(a^4(-x)-2a^3\log(x)-\frac{a}{x^2}-\frac{1}{3x^3}\right)}{a^4}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

output
```
-((c^2*(-1/3*1/x^3 - a/x^2 - a^4*x - 2*a^3*Log[x]))/a^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result
default	$\frac{c^2\left(x\,a^4+\frac{a}{x^2}+2a^3\ln\left(x\right)+\frac{1}{3x^3}\right)}{a^4}$
risch	$c^2x+\frac{a\,c^2x+\frac{1}{3}c^2}{a^4x^3}+\frac{2c^2\ln\left(x\right)}{a}$
norman	$\frac{c^2x+a^3c^2x^4+\frac{c^2}{3a}}{a^3x^3}+\frac{2c^2\ln\left(x\right)}{a}$
parallelrisch	$\frac{3a^4c^2x^4+6c^2\ln\left(x\right)a^3x^3+3a\,c^2x+c^2}{3a^4x^3}$
meijerg	$-\frac{c^2\left(-ax-\ln\left(-ax+1\right)\right)}{a}+\frac{2c^2\left(\ln\left(x\right)+\ln\left(-a\right)-\ln\left(-ax+1\right)\right)}{a}-\frac{c^2\left(-\frac{1}{2a^2x^2}-\frac{1}{ax}+\ln\left(x\right)+\ln\left(-a\right)-\ln\left(-ax+1\right)\right)}{a}+\frac{c^2\ln(-}{a}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2/a^4*(x*a^4+a/x^2+2*a^3*ln(x)+1/3/x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{3\,a^4c^2x^4+6\,a^3c^2x^3\log\left(x\right)+3\,ac^2x+c^2}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*c^2*x^4 + 6*a^3*c^2*x^3*log(x) + 3*a*c^2*x + c^2)/(a^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{a^4c^2x+2a^3c^2\log{(x)}+\frac{3ac^2x+c^2}{3x^3}}{a^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**2,x)
```

output
```
(a**4*c**2*x + 2*a**3*c**2*log(x) + (3*a*c**2*x + c**2)/(3*x**3))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=c^2x+\frac{2\,c^2\log{(x)}}{a}+\frac{3\,ac^2x+c^2}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
c^2*x + 2*c^2*log(x)/a + 1/3*(3*a*c^2*x + c^2)/(a^4*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=c^2x+\frac{2\,c^2\log{(|x|)}}{a}+\frac{3\,ac^2x+c^2}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
c^2*x + 2*c^2*log(abs(x))/a + 1/3*(3*a*c^2*x + c^2)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2\left(a\,x+a^4\,x^4+2\,a^3\,x^3\,\ln\left(x\right)+\frac{1}{3}\right)}{a^4\,x^3}$$

input
```
int(((c - c/(a^2*x^2))^2*(a*x + 1))/(a*x - 1),x)
```

output
```
(c^2*(a*x + a^4*x^4 + 2*a^3*x^3*log(x) + 1/3))/(a^4*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2(6\log(x)\,a^3x^3+3a^4x^4+3ax+1)}{3a^4x^3}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x)
```

output
```
(c**2*(6*log(x)*a**3*x**3 + 3*a**4*x**4 + 3*a*x + 1))/(3*a**4*x**3)
```

3.760 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 21

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c}{a^2x}+cx+\frac{2c\log(x)}{a}$$

output
```
-c/a^2/x+c*x+2*c*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c}{a^2x}+cx+\frac{2c\log(x)}{a}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

output
```
-(c/(a^2*x)) + c*x + (2*c*Log[x])/a
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6717, 27, 6707, 6700, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{ce^{2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c\int e^{2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{ 6707}$$

$$\frac{c\int \frac{e^{2\text{arctanh}(ax)}(1-a^2x^2)}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{ 6700}$$

$$\frac{c\int \frac{(ax+1)^2}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{c\int \left(a^2+\frac{2a}{x}+\frac{1}{x^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c\left(a^2x+2a\log(x)-\frac{1}{x}\right)}{a^2}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

output
```
(c*(-x^(-1) + a^2*x + 2*a*Log[x]))/a^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{c\left(a^{2} x-\frac{1}{x}+2\ln\left(x\right) a\right)}{a^{2}}$	22
risch	$-\frac{c}{a^{2} x}+x c+\frac{2 c \ln\left(x\right)}{a}$	22
parallelrisch	$\frac{a^{2} c\,x^{2}+2 c \ln\left(x\right) a x-c}{a^{2} x}$	27
norman	$\frac{a c\,x^{2}-\frac{c}{a}}{a x}+\frac{2 c \ln\left(x\right)}{a}$	30
meijerg	$-\frac{c\left(-a x-\ln\left(-a x+1\right)\right)}{a}+\frac{c\left(\ln\left(x\right)+\ln\left(-a\right)-\ln\left(-a x+1\right)\right)}{a}+\frac{c \ln\left(-a x+1\right)}{a}-\frac{c\left(\frac{1}{a x}-\ln\left(x\right)-\ln\left(-a\right)+\ln\left(-a x+1\right)\right)}{a}$	86

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
c/a^2*(a^2*x-1/x+2*ln(x)*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{a^2cx^2+2\,acx\log\left(x\right)-c}{a^2x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
(a^2*c*x^2 + 2*a*c*x*log(x) - c)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{a^2cx+2ac\log{(x)}-\frac{c}{x}}{a^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2),x)
```

output
```
(a**2*c*x + 2*a*c*log(x) - c/x)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=cx+\frac{2\,c\log{(x)}}{a}-\frac{c}{a^2x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
c*x + 2*c*log(x)/a - c/(a^2*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=cx+\frac{2\,c\log{(|x|)}}{a}-\frac{c}{a^2x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
c*x + 2*c*log(abs(x))/a - c/(a^2*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\,(a^2\,x^2+2\,a\,x\,\ln(x)-1)}{a^2\,x}$$

input
```
int(((c - c/(a^2*x^2))*(a*x + 1))/(a*x - 1),x)
```

output
```
(c*(a^2*x^2 + 2*a*x*log(x) - 1))/(a^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c(2\log(x)\,ax+a^2x^2-1)}{a^2x}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x)
```

output
```
(c*(2*log(x)*a*x + a**2*x**2 - 1))/(a**2*x)
```

3.761 $\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 36

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}} dx = \frac{x}{c}+\frac{1}{ac(1-ax)}+\frac{2\log(1-ax)}{ac}$$

output
```
x/c+1/a/c/(-a*x+1)+2*ln(-a*x+1)/a/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}} dx = \frac{x+\frac{1}{a-a^2x}+\frac{2\log(1-ax)}{a}}{c}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

output
```
(x + (a - a^2*x)^(-1) + (2*Log[1 - a*x])/a)/c
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{a^2e^{2\text{arctanh}(ax)}}{c\left(a^2-\frac{1}{x^2}\right)}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^2\int \frac{e^{2\text{arctanh}(ax)}}{a^2-\frac{1}{x^2}}dx}{c}$$

$$\Big\downarrow \text{ 6707}$$

$$\frac{a^2\int \frac{e^{2\text{arctanh}(ax)}x^2}{1-a^2x^2}dx}{c}$$

$$\Big\downarrow \text{ 6700}$$

$$\frac{a^2\int \frac{x^2}{(1-ax)^2}dx}{c}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{a^2\int \left(\frac{1}{a^2}+\frac{2}{a^2(ax-1)}+\frac{1}{a^2(ax-1)^2}\right)dx}{c}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2\left(\frac{1}{a^3(1-ax)}+\frac{2\log(1-ax)}{a^3}+\frac{x}{a^2}\right)}{c}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

output
```
(a^2*(x/a^2 + 1/(a^3*(1 - a*x)) + (2*Log[1 - a*x])/a^3))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{x}{c}-\frac{1}{ac(ax-1)}+\frac{2\ln(ax-1)}{ac}$	36
default	$\frac{a^2\left(\frac{x}{a^2}-\frac{1}{(ax-1)a^3}+\frac{2\ln(ax-1)}{a^3}\right)}{c}$	37
norman	$\frac{\frac{a\,x^2}{c}-\frac{2x}{c}}{ax-1}+\frac{2\ln(ax-1)}{ac}$	39
parallelrisch	$\frac{a^2x^2+2a\ln(ax-1)x-2ax-2\ln(ax-1)}{c(ax-1)a}$	45

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
x/c-1/a/c/(a*x-1)+2/a/c*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{a^2x^2-ax+2\,(ax-1)\log\left(ax-1\right)-1}{a^2cx-ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
(a^2*x^2 - a*x + 2*(a*x - 1)*log(a*x - 1) - 1)/(a^2*c*x - a*c)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = a^2\left(-\frac{1}{a^4cx-a^3c}+\frac{x}{a^2c}+\frac{2\log{(ax-1)}}{a^3c}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2),x)
```

output
```
a**2*(-1/(a**4*c*x - a**3*c) + x/(a**2*c) + 2*log(a*x - 1)/(a**3*c))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{x}{c}-\frac{1}{a^2cx-ac}+\frac{2\log{(ax-1)}}{ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
x/c - 1/(a^2*c*x - a*c) + 2*log(a*x - 1)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{x}{c}+\frac{2\log{(|ax-1|)}}{ac}-\frac{1}{(ax-1)ac}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
x/c + 2*log(abs(a*x - 1))/(a*c) - 1/((a*x - 1)*a*c)
```

Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}+\frac{1}{a\,\left(c-a\,c\,x\right)}+\frac{2\,\ln\left(a\,x-1\right)}{a\,c}$$

input
```
int((a*x + 1)/((c - c/(a^2*x^2))*(a*x - 1)),x)
```

output
```
x/c + 1/(a*(c - a*c*x)) + (2*log(a*x - 1))/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{2\log(ax-1)\,ax-2\log(ax-1)+a^2x^2-2ax}{ac\,(ax-1)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x)
```

output
```
(2*log(a*x - 1)*a*x - 2*log(a*x - 1) + a**2*x**2 - 2*a*x)/(a*c*(a*x - 1))
```

3.762 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{x}{c^2}-\frac{1}{4ac^2(1-ax)^2}+\frac{7}{4ac^2(1-ax)}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(1+ax)}{8ac^2}$$

output
```
x/c^2-1/4/a/c^2/(-a*x+1)^2+7/4/a/c^2/(-a*x+1)+17/8*ln(-a*x+1)/a/c^2-1/8*ln
(a*x+1)/a/c^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{x}{c^2}-\frac{1}{4ac^2(1-ax)^2}+\frac{7}{4ac^2(1-ax)}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(1+ax)}{8ac^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

output
```
x/c^2 - 1/(4*a*c^2*(1 - a*x)^2) + 7/(4*a*c^2*(1 - a*x)) + (17*Log[1 - a*x]
)/(8*a*c^2) - Log[1 + a*x]/(8*a*c^2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{a^4e^{2\text{arctanh}(ax)}}{c^2\left(a^2-\frac{1}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^4\int \frac{e^{2\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 6707}$$

$$-\frac{a^4\int \frac{e^{2\text{arctanh}(ax)}x^4}{(1-a^2x^2)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 6700}$$

$$-\frac{a^4\int \frac{x^4}{(1-ax)^3(ax+1)}dx}{c^2}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{a^4\int \left(\frac{1}{8a^4(ax+1)}-\frac{1}{a^4}-\frac{17}{8a^4(ax-1)}-\frac{7}{4a^4(ax-1)^2}-\frac{1}{2a^4(ax-1)^3}\right)dx}{c^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^4\left(-\frac{7}{4a^5(1-ax)}+\frac{1}{4a^5(1-ax)^2}-\frac{17\log(1-ax)}{8a^5}+\frac{\log(ax+1)}{8a^5}-\frac{x}{a^4}\right)}{c^2}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

output
```
-((a^4*(-(x/a^4) + 1/(4*a^5*(1 - a*x)^2) - 7/(4*a^5*(1 - a*x)) - (17*Log[1
 - a*x])/(8*a^5) + Log[1 + a*x]/(8*a^5)))/c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{a^{4}\left(\frac{x}{a^{4}}-\frac{\ln(ax+1)}{8a^{5}}+\frac{17\ln(ax-1)}{8a^{5}}-\frac{1}{4a^{5}(ax-1)^{2}}-\frac{7}{4a^{5}(ax-1)}\right)}{c^{2}}$	60
risch	$\frac{x}{c^{2}}+\frac{-\frac{7c^{2}x}{4}+\frac{3c^{2}}{2a}}{c^{4}(ax-1)^{2}}+\frac{17\ln(-ax+1)}{8a\,c^{2}}-\frac{\ln(ax+1)}{8a\,c^{2}}$	62
norman	$\frac{\frac{a^{3}x^{4}}{c}+\frac{9x}{4c}-\frac{5a\,x^{2}}{4c}-\frac{5a^{2}x^{3}}{2c}}{c(ax+1)(ax-1)^{2}}+\frac{17\ln(ax-1)}{8a\,c^{2}}-\frac{\ln(ax+1)}{8a\,c^{2}}$	85
parallelrisch	$\frac{8a^{3}x^{3}+17a^{2}\ln(ax-1)x^{2}-\ln(ax+1)x^{2}a^{2}-28a^{2}x^{2}-34a\ln(ax-1)x+2\ln(ax+1)xa+18ax+17\ln(ax-1)-\ln(ax+1)}{8c^{2}(ax-1)^{2}a}$	101

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a^4/c^2*(x/a^4-1/8*ln(a*x+1)/a^5+17/8/a^5*ln(a*x-1)-1/4/a^5/(a*x-1)^2-7/4/
a^5/(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.24

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{8\,a^{3}x^{3}-16\,a^{2}x^{2}-6\,ax-(a^{2}x^{2}-2\,ax+1)\log(ax+1)+17\,(a^{2}x^{2}-2\,ax+1)\log(ax-1)+12}{8\,(a^{3}c^{2}x^{2}-2\,a^{2}c^{2}x+ac^{2})}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
 17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^3*c^2*x^2 - 2*a^2*c^2*x +
a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = a^4\left(\frac{-7ax+6}{4a^7c^2x^2-8a^6c^2x+4a^5c^2}+\frac{x}{a^4c^2}+\frac{\frac{17\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**2,x)
```

output
```
a**4*((-7*a*x + 6)/(4*a**7*c**2*x**2 - 8*a**6*c**2*x + 4*a**5*c**2) + x/(a
**4*c**2) + (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{7\,ax-6}{4\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}+\frac{x}{c^2}-\frac{\log(ax+1)}{8\,ac^2}+\frac{17\,\log(ax-1)}{8\,ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(7*a*x - 6)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) + x/c^2 - 1/8*log(a*x
 + 1)/(a*c^2) + 17/8*log(a*x - 1)/(a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{x}{c^2}-\frac{\log(|ax+1|)}{8\,ac^2}+\frac{17\,\log(|ax-1|)}{8\,ac^2}-\frac{7\,ax-6}{4\,(ax-1)^2ac^2}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
x/c^2 - 1/8*log(abs(a*x + 1))/(a*c^2) + 17/8*log(abs(a*x - 1))/(a*c^2) - 1
/4*(7*a*x - 6)/((a*x - 1)^2*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{x}{c^2} - \frac{\frac{7\,x}{4}-\frac{3}{2\,a}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2} + \frac{17\,\ln\left(a\,x-1\right)}{8\,a\,c^2} - \frac{\ln\left(a\,x+1\right)}{8\,a\,c^2}$$

input
```
int((a*x + 1)/((c - c/(a^2*x^2))^2*(a*x - 1)),x)
```

output
```
x/c^2 - ((7*x)/4 - 3/(2*a))/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) + (17*log(a*x
- 1))/(8*a*c^2) - log(a*x + 1)/(8*a*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.40

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{17\log(ax-1)\,a^2x^2-34\log(ax-1)\,ax+17\log(ax-1)-\log(ax+1)\,a^2x^2+2\log(ax+1)\,ax-\log(a}{8a\,c^2\,(a^2x^2-2ax+1)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x)
```

output
```
(17*log(a*x - 1)*a**2*x**2 - 34*log(a*x - 1)*a*x + 17*log(a*x - 1) - log(a
*x + 1)*a**2*x**2 + 2*log(a*x + 1)*a*x - log(a*x + 1) + 8*a**3*x**3 - 19*a
**2*x**2 + 9)/(8*a*c**2*(a**2*x**2 - 2*a*x + 1))
```

3.763 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{x}{c^3}+\frac{1}{12ac^3(1-ax)^3}-\frac{5}{8ac^3(1-ax)^2}+\frac{39}{16ac^3(1-ax)}-\frac{1}{16ac^3(1+ax)}+\frac{9\log(1-ax)}{4ac^3}-\frac{\log(1+ax)}{4ac^3}$$

output
```
x/c^3+1/12/a/c^3/(-a*x+1)^3-5/8/a/c^3/(-a*x+1)^2+39/16/a/c^3/(-a*x+1)-1/16
/a/c^3/(a*x+1)+9/4*ln(-a*x+1)/a/c^3-1/4*ln(a*x+1)/a/c^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{\frac{2(-11+7ax+24a^2x^2-15a^3x^3-12a^4x^4+6a^5x^5)}{(-1+ax)^3(1+ax)}+27\log(1-ax)-3\log(1+ax)}{12ac^3}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

output
```
((2*(-11 + 7*a*x + 24*a^2*x^2 - 15*a^3*x^3 - 12*a^4*x^4 + 6*a^5*x^5))/((-1
 + a*x)^3*(1 + a*x)) + 27*Log[1 - a*x] - 3*Log[1 + a*x])/(12*a*c^3)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^6e^{2\text{arctanh}(ax)}}{c^3\left(a^2-\frac{1}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^6\int \frac{e^{2\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^3}dx}{c^3}$$

$$\Big\downarrow \text{6707}$$

$$\frac{a^6\int \frac{e^{2\text{arctanh}(ax)}x^6}{(1-a^2x^2)^3}dx}{c^3}$$

$$\Big\downarrow \text{6700}$$

$$\frac{a^6\int \frac{x^6}{(1-ax)^4(ax+1)^2}dx}{c^3}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^6\int \left(-\frac{1}{4a^6(ax+1)}+\frac{1}{16a^6(ax+1)^2}+\frac{1}{a^6}+\frac{9}{4a^6(ax-1)}+\frac{39}{16a^6(ax-1)^2}+\frac{5}{4a^6(ax-1)^3}+\frac{1}{4a^6(ax-1)^4}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^6\left(\frac{39}{16a^7(1-ax)} - \frac{1}{16a^7(ax+1)} - \frac{5}{8a^7(1-ax)^2} + \frac{1}{12a^7(1-ax)^3} + \frac{9\log(1-ax)}{4a^7} - \frac{\log(ax+1)}{4a^7} + \frac{x}{a^6}\right)}{c^3}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

output
```
(a^6*(x/a^6 + 1/(12*a^7*(1 - a*x)^3) - 5/(8*a^7*(1 - a*x)^2) + 39/(16*a^7*
(1 - a*x)) - 1/(16*a^7*(1 + a*x)) + (9*Log[1 - a*x])/(4*a^7) - Log[1 + a*x
]/(4*a^7)))/c^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

method	result
default	$\frac{a^{6}\left(\frac{x}{a^{6}}-\frac{\ln(ax+1)}{4a^{7}}-\frac{1}{16a^{7}(ax+1)}-\frac{1}{12a^{7}(ax-1)^{3}}-\frac{5}{8a^{7}(ax-1)^{2}}-\frac{39}{16a^{7}(ax-1)}+\frac{9\ln(ax-1)}{4a^{7}}\right)}{c^{3}}$
risch	$\frac{x}{c^{3}}+\frac{-\frac{5a^{2}c^{3}x^{3}}{2}+2a\,c^{3}x^{2}+\frac{13c^{3}x}{6}-\frac{11c^{3}}{6a}}{c^{6}(ax-1)^{2}(a^{2}x^{2}-1)}+\frac{9\ln(-ax+1)}{4a\,c^{3}}-\frac{\ln(ax+1)}{4a\,c^{3}}$
norman	$\frac{\frac{a^{5}x^{6}}{c}-\frac{5x}{2c}+\frac{3a\,x^{2}}{2c}+\frac{31a^{2}x^{3}}{6c}-\frac{8a^{3}x^{4}}{3c}-\frac{17a^{4}x^{5}}{6c}}{(ax-1)^{3}c^{2}(ax+1)^{2}}+\frac{9\ln(ax-1)}{4a\,c^{3}}-\frac{\ln(ax+1)}{4a\,c^{3}}$
parallelrisch	$\frac{12a^{5}x^{5}+27\ln(ax-1)x^{4}a^{4}-3\ln(ax+1)x^{4}a^{4}-46a^{4}x^{4}-54a^{3}\ln(ax-1)x^{3}+6\ln(ax+1)x^{3}a^{3}+14a^{3}x^{3}+48a^{2}x^{2}+54a\ln(ax-1)x}{12c^{3}(ax-1)^{2}(a^{2}x^{2}-1)a}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
a^6/c^3*(x/a^6-1/4*ln(a*x+1)/a^7-1/16/a^7/(a*x+1)-1/12/a^7/(a*x-1)^3-5/8/a
^7/(a*x-1)^2-39/16/a^7/(a*x-1)+9/4/a^7*ln(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$=\frac{12\,a^5x^5-24\,a^4x^4-30\,a^3x^3+48\,a^2x^2+14\,ax-3\,(a^4x^4-2\,a^3x^3+2\,ax-1)\log(ax+1)+27\,(a^4x^4-}{12\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
1/12*(12*a^5*x^5 - 24*a^4*x^4 - 30*a^3*x^3 + 48*a^2*x^2 + 14*a*x - 3*(a^4*
x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x + 1) + 27*(a^4*x^4 - 2*a^3*x^3 + 2*a*
x - 1)*log(a*x - 1) - 22)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c
^3)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = a^6\left(\frac{-15a^3x^3+12a^2x^2+13ax-11}{6a^{11}c^3x^4-12a^{10}c^3x^3+12a^8c^3x-6a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{9\log\left(x-\frac{1}{a}\right)}{4}-\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^7c^3}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**3,x)
```

output
```
a**6*((-15*a**3*x**3 + 12*a**2*x**2 + 13*a*x - 11)/(6*a**11*c**3*x**4 - 12
*a**10*c**3*x**3 + 12*a**8*c**3*x - 6*a**7*c**3) + x/(a**6*c**3) + (9*log(
x - 1/a)/4 - log(x + 1/a)/4)/(a**7*c**3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{15\,a^3x^3-12\,a^2x^2-13\,ax+11}{6\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}+\frac{x}{c^3}-\frac{\log\left(ax+1\right)}{4\,ac^3}+\frac{9\,\log\left(ax-1\right)}{4\,ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
-1/6*(15*a^3*x^3 - 12*a^2*x^2 - 13*a*x + 11)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3
+ 2*a^2*c^3*x - a*c^3) + x/c^3 - 1/4*log(a*x + 1)/(a*c^3) + 9/4*log(a*x -
1)/(a*c^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{x}{c^3} - \frac{\log\left(|ax+1|\right)}{4\,ac^3} + \frac{9\,\log\left(|ax-1|\right)}{4\,ac^3} - \frac{15\,a^3x^3 - 12\,a^2x^2 - 13\,ax + 11}{6\,(ax+1)(ax-1)^3ac^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
x/c^3 - 1/4*log(abs(a*x + 1))/(a*c^3) + 9/4*log(abs(a*x - 1))/(a*c^3) - 1/
6*(15*a^3*x^3 - 12*a^2*x^2 - 13*a*x + 11)/((a*x + 1)*(a*x - 1)^3*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{x}{c^3} - \frac{\frac{13\,x}{6} + 2\,a\,x^2 - \frac{11}{6\,a} - \frac{5\,a^2\,x^3}{2}}{-a^4\,c^3\,x^4 + 2\,a^3\,c^3\,x^3 - 2\,a\,c^3\,x + c^3} + \frac{9\,\ln\left(a\,x-1\right)}{4\,a\,c^3} - \frac{\ln\left(a\,x+1\right)}{4\,a\,c^3}$$

input
```
int((a*x + 1)/((c - c/(a^2*x^2))^3*(a*x - 1)),x)
```

output
```
x/c^3 - ((13*x)/6 + 2*a*x^2 - 11/(6*a) - (5*a^2*x^3)/2)/(c^3 + 2*a^3*c^3*x
^3 - a^4*c^3*x^4 - 2*a*c^3*x) + (9*log(a*x - 1))/(4*a*c^3) - log(a*x + 1)/
(4*a*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.39

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$= \frac{27\log(ax-1)\,a^4x^4-54\log(ax-1)\,a^3x^3+54\log(ax-1)\,ax-27\log(ax-1)-3\log(ax+1)\,a^4x^4+6}{12a\,c^3\left(a^4x^4-2a^3x^3\right.}$$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x)
```

output
```
(27*log(a*x - 1)*a**4*x**4 - 54*log(a*x - 1)*a**3*x**3 + 54*log(a*x - 1)*a
*x - 27*log(a*x - 1) - 3*log(a*x + 1)*a**4*x**4 + 6*log(a*x + 1)*a**3*x**3
 - 6*log(a*x + 1)*a*x + 3*log(a*x + 1) + 12*a**5*x**5 - 39*a**4*x**4 + 48*
a**2*x**2 - 16*a*x - 7)/(12*a*c**3*(a**4*x**4 - 2*a**3*x**3 + 2*a*x - 1))
```

3.764 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{x}{c^4}-\frac{1}{32ac^4(1-ax)^4}+\frac{13}{48ac^4(1-ax)^3}-\frac{35}{32ac^4(1-ax)^2}+\frac{99}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+ax)^2}-\frac{11}{64ac^4(1+ax)}+\frac{303\log(1-ax)}{128ac^4}-\frac{47\log(1+ax)}{128ac^4}$$

output
```
x/c^4-1/32/a/c^4/(-a*x+1)^4+13/48/a/c^4/(-a*x+1)^3-35/32/a/c^4/(-a*x+1)^2+
99/32/a/c^4/(-a*x+1)+1/64/a/c^4/(a*x+1)^2-11/64/a/c^4/(a*x+1)+303/128*ln(-
a*x+1)/a/c^4-47/128*ln(a*x+1)/a/c^4
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.68

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{2(400-275ax-1258a^2x^2+866a^3x^3+1254a^4x^4-819a^5x^5-384a^6x^6+192a^7x^7)}{(-1+ax)^4(1+ax)^2}+909\log(1-ax)-141\log(1+ax)}{384ac^4}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

output
```
((2*(400 - 275*a*x - 1258*a^2*x^2 + 866*a^3*x^3 + 1254*a^4*x^4 - 819*a^5*x
^5 - 384*a^6*x^6 + 192*a^7*x^7))/((-1 + a*x)^4*(1 + a*x)^2) + 909*Log[1 -
a*x] - 141*Log[1 + a*x])/(384*a*c^4)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^8 e^{2\text{arctanh}(ax)}}{c^4\left(a^2-\frac{1}{x^2}\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^8\int \frac{e^{2\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^4}dx}{c^4}$$

$$\Big\downarrow \text{6707}$$

$$-\frac{a^8 \int \frac{e^{2\text{arctanh}(ax)}x^8}{(1-a^2x^2)^4}dx}{c^4}$$

$$\Big\downarrow \text{ 6700}$$

$$-\frac{a^8 \int \frac{x^8}{(1-ax)^5(ax+1)^3}dx}{c^4}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{a^8 \int \left(\frac{47}{128a^8(ax+1)} - \frac{11}{64a^8(ax+1)^2} + \frac{1}{32a^8(ax+1)^3} - \frac{1}{a^8} - \frac{303}{128a^8(ax-1)} - \frac{99}{32a^8(ax-1)^2} - \frac{35}{16a^8(ax-1)^3} - \frac{13}{16a^8(ax-1)^4} - 8a^8 \right.}{c^4}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^8 \left(-\frac{99}{32a^9(1-ax)} + \frac{11}{64a^9(ax+1)} + \frac{35}{32a^9(1-ax)^2} - \frac{1}{64a^9(ax+1)^2} - \frac{13}{48a^9(1-ax)^3} + \frac{1}{32a^9(1-ax)^4} - \frac{303\log(1-ax)}{128a^9} + \frac{47\log(ax+1}{128a^9} \right.}{c^4}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

output
```
-((a^8*(-(x/a^8) + 1/(32*a^9*(1 - a*x)^4) - 13/(48*a^9*(1 - a*x)^3) + 35/(
32*a^9*(1 - a*x)^2) - 99/(32*a^9*(1 - a*x)) - 1/(64*a^9*(1 + a*x)^2) + 11/
(64*a^9*(1 + a*x)) - (303*Log[1 - a*x])/(128*a^9) + (47*Log[1 + a*x])/(128
*a^9)))/c^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result
default	$\frac{a^{8}\left(\frac{x}{a^{8}}-\frac{47\ln\left(ax+1\right)}{128a^{9}}+\frac{1}{64a^{9}\left(ax+1\right)^{2}}-\frac{11}{64a^{9}\left(ax+1\right)}-\frac{13}{48a^{9}\left(ax-1\right)^{3}}-\frac{35}{32a^{9}\left(ax-1\right)^{2}}-\frac{99}{32a^{9}\left(ax-1\right)}+\frac{303\ln\left(ax-1\right)}{128a^{9}}-\frac{1}{32a^{9}\left(ax-1\right)^{4}}\right)}{c^{4}}$
risch	$\frac{x}{c^{4}}+\frac{-\frac{209a^{4}c^{4}x^{5}}{64}+\frac{81a^{3}c^{4}x^{4}}{32}+\frac{529a^{2}c^{4}x^{3}}{96}-\frac{437a\,c^{4}x^{2}}{96}-\frac{467c^{4}x}{192}+\frac{25c^{4}}{12a}}{c^{8}\left(ax-1\right)^{2}\left(a^{2}x^{2}-1\right)^{2}}+\frac{303\ln\left(-ax+1\right)}{128a\,c^{4}}-\frac{47\ln\left(ax+1\right)}{128a\,c^{4}}$
norman	$\frac{\frac{a^{7}x^{8}}{c}+\frac{175x}{64c}-\frac{111a\,x^{2}}{64c}-\frac{199a^{2}x^{3}}{24c}+\frac{115a^{3}x^{4}}{24c}+\frac{545a^{4}x^{5}}{64c}-\frac{803a^{5}x^{6}}{192c}-\frac{37a^{6}x^{7}}{12c}}{c^{3}\left(ax-1\right)^{4}\left(ax+1\right)^{3}}+\frac{303\ln\left(ax-1\right)}{128a\,c^{4}}-\frac{47\ln\left(ax+1\right)}{128a\,c^{4}}$
parallelrisch	$-1468a^{3}x^{3}+1050ax+3308a^{4}x^{4}-38a^{5}x^{5}-1568x^{6}a^{6}+282\ln\left(ax+1\right)xa+384a^{7}x^{7}-141\ln\left(ax+1\right)-1716a^{2}x^{2}+909\ln\left(ax-1\right)+$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
a^8/c^4*(x/a^8-47/128/a^9*ln(a*x+1)+1/64/a^9/(a*x+1)^2-11/64/a^9/(a*x+1)-1
3/48/a^9/(a*x-1)^3-35/32/a^9/(a*x-1)^2-99/32/a^9/(a*x-1)+303/128/a^9*ln(a*
x-1)-1/32/a^9/(a*x-1)^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.61

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{384\,a^7x^7-768\,a^6x^6-1638\,a^5x^5+2508\,a^4x^4+1732\,a^3x^3-2516\,a^2x^2-550\,ax-141\,(a^6x^6-2\,a^5x^5-\cdots}{384\,(a^7c^4x^6-2\,a^6c^4x^5-\cdots}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
1/384*(384*a^7*x^7 - 768*a^6*x^6 - 1638*a^5*x^5 + 2508*a^4*x^4 + 1732*a^3*
x^3 - 2516*a^2*x^2 - 550*a*x - 141*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*
x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + 909*(a^6*x^6 - 2*a^5*x^5 - a^4*x
^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 800)/(a^7*c^4*x^6 - 2
*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a
*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = a^8\left(\frac{-627a^5x^5+486a^4x^4+1058a^3x^3-874a^2x^2-467ax+400}{192a^{15}c^4x^6-384a^{14}c^4x^5-192a^{13}c^4x^4+768a^{12}c^4x^3-192a^{11}c^4x^2-384a^{10}c^4x+192a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{303\log\left(x-\frac{1}{a}\right)}{128}-\frac{47\log\left(x+\frac{1}{a}\right)}{128}}{a^9c^4}\right)$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**4,x)
```

output
```
a**8*((-627*a**5*x**5 + 486*a**4*x**4 + 1058*a**3*x**3 - 874*a**2*x**2 - 4
67*a*x + 400)/(192*a**15*c**4*x**6 - 384*a**14*c**4*x**5 - 192*a**13*c**4*
x**4 + 768*a**12*c**4*x**3 - 192*a**11*c**4*x**2 - 384*a**10*c**4*x + 192*
a**9*c**4) + x/(a**8*c**4) + (303*log(x - 1/a)/128 - 47*log(x + 1/a)/128)/
(a**9*c**4))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{627\,a^5x^5-486\,a^4x^4-1058\,a^3x^3+874\,a^2x^2+467\,ax-400}{192\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)} + \frac{x}{c^4}-\frac{47\,\log\left(ax+1\right)}{128\,ac^4}+\frac{303\,\log\left(ax-1\right)}{128\,ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
-1/192*(627*a^5*x^5 - 486*a^4*x^4 - 1058*a^3*x^3 + 874*a^2*x^2 + 467*a*x -
 400)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4
*x^2 - 2*a^2*c^4*x + a*c^4) + x/c^4 - 47/128*log(a*x + 1)/(a*c^4) + 303/12
8*log(a*x - 1)/(a*c^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.66

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{x}{c^4}-\frac{47\,\log\left(|ax+1|\right)}{128\,ac^4}+\frac{303\,\log\left(|ax-1|\right)}{128\,ac^4} - \frac{627\,a^5x^5-486\,a^4x^4-1058\,a^3x^3+874\,a^2x^2+467\,ax-400}{192\,(ax+1)^2(ax-1)^4ac^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
x/c^4 - 47/128*log(abs(a*x + 1))/(a*c^4) + 303/128*log(abs(a*x - 1))/(a*c^
4) - 1/192*(627*a^5*x^5 - 486*a^4*x^4 - 1058*a^3*x^3 + 874*a^2*x^2 + 467*a
*x - 400)/((a*x + 1)^2*(a*x - 1)^4*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.98

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{x}{c^4} + \frac{\frac{467\,x}{192}+\frac{437\,a\,x^2}{96}-\frac{25}{12\,a}-\frac{529\,a^2\,x^3}{96}-\frac{81\,a^3\,x^4}{32}+\frac{209\,a^4\,x^5}{64}}{-a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5+a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3+a^2\,c^4\,x^2+2\,a\,c^4\,x-c^4} + \frac{303\,\ln\left(a\,x-1\right)}{128\,a\,c^4} - \frac{47\,\ln\left(a\,x+1\right)}{128\,a\,c^4}$$

input
```
int((a*x + 1)/((c - c/(a^2*x^2))^4*(a*x - 1)),x)
```

output
```
x/c^4 + ((467*x)/192 + (437*a*x^2)/96 - 25/(12*a) - (529*a^2*x^3)/96 - (81
*a^3*x^4)/32 + (209*a^4*x^5)/64)/(a^2*c^4*x^2 - c^4 - 4*a^3*c^4*x^3 + a^4*
c^4*x^4 + 2*a^5*c^4*x^5 - a^6*c^4*x^6 + 2*a*c^4*x) + (303*log(a*x - 1))/(1
28*a*c^4) - (47*log(a*x + 1))/(128*a*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.91

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{909\log(ax-1)\,a^6x^6 - 1818\log(ax-1)\,a^5x^5 - 909\log(ax-1)\,a^4x^4 + 3636\log(ax-1)\,a^3x^3 - 909\log(}{}$$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x)
```

output
```
(909*log(a*x - 1)*a**6*x**6 - 1818*log(a*x - 1)*a**5*x**5 - 909*log(a*x -
1)*a**4*x**4 + 3636*log(a*x - 1)*a**3*x**3 - 909*log(a*x - 1)*a**2*x**2 -
1818*log(a*x - 1)*a*x + 909*log(a*x - 1) - 141*log(a*x + 1)*a**6*x**6 + 28
2*log(a*x + 1)*a**5*x**5 + 141*log(a*x + 1)*a**4*x**4 - 564*log(a*x + 1)*a
**3*x**3 + 141*log(a*x + 1)*a**2*x**2 + 282*log(a*x + 1)*a*x - 141*log(a*x
 + 1) + 384*a**7*x**7 - 1587*a**6*x**6 + 3327*a**4*x**4 - 1544*a**3*x**3 -
 1697*a**2*x**2 + 1088*a*x - 19)/(384*a*c**4*(a**6*x**6 - 2*a**5*x**5 - a*
*4*x**4 + 4*a**3*x**3 - a**2*x**2 - 2*a*x + 1))
```

3.765 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 180

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx &= \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{7a}-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(6a-\frac{5}{x}\right)}{10a^2}\\ &\quad-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(8a-\frac{5}{x}\right)}{8a^2}-\frac{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{5}{x}\right)}{16a^2}\\ &\quad+c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}x+\frac{15c^4\csc^{-1}(ax)}{16a}+\frac{3c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}\end{aligned}$$

output
```
1/7*c^4*(1-1/a^2/x^2)^(7/2)/a-1/10*c^4*(1-1/a^2/x^2)^(5/2)*(6*a-5/x)/a^2-1
/8*c^4*(1-1/a^2/x^2)^(3/2)*(8*a-5/x)/a^2-3/16*c^4*(1-1/a^2/x^2)^(1/2)*(16*
a-5/x)/a^2+c^4*(1-1/a^2/x^2)^(7/2)*x+15/16*c^4*arccsc(a*x)/a+3*c^4*arctanh
((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.70

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$=\frac{c^4\left(\sqrt{1-\frac{1}{a^2x^2}}(-80-280ax-96a^2x^2+770a^3x^3+992a^4x^4-525a^5x^5-2496a^6x^6+560a^7x^7)+525a^6\right.}{560a^7x^6}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

output
```
(c^4*(Sqrt[1 - 1/(a^2*x^2)]*(-80 - 280*a*x - 96*a^2*x^2 + 770*a^3*x^3 + 99
2*a^4*x^4 - 525*a^5*x^5 - 2496*a^6*x^6 + 560*a^7*x^7) + 525*a^6*x^6*ArcSin
[1/(a*x)] + 1680*a^6*x^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(560*a^7*x^6
)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.87, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.091, Rules used = {6748, 108, 27, 171, 27, 171, 27, 171, 25, 27, 171, 27, 171, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^4 e^{3\coth^{-1}(ax)} dx$$
$$\Big\downarrow \text{ 6748}$$
$$-c^4\int \left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x^2d\frac{1}{x}$$
$$\Big\downarrow \text{ 108}$$
$$-c^4\left(\int \frac{\left(3a-\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}\right)$$

$$\Big\downarrow \; 27$$

$$-c^4\left(\frac{\int \left(3a-\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}\right)$$

$$\Big\downarrow \; 171$$

$$-c^4\left(\frac{\frac{1}{7}a\int \frac{3\left(7a-\frac{15}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x}{a}d\frac{1}{x}-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}\right)$$

$$\Big\downarrow \; 27$$

$$-c^4\left(\frac{\frac{3}{7}\int \left(7a-\frac{15}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}xd\frac{1}{x}-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}\right)$$

$$\Big\downarrow \; 171$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{6}a\int \frac{3\left(14a-\frac{19}{x}\right)\left(1+\frac{1}{ax}\right)^{9/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{5}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/}\right.$$

$$\Big\downarrow \; 27$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\int \frac{\left(14a-\frac{19}{x}\right)\left(1+\frac{1}{ax}\right)^{9/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{5}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{5/2}\right.$$

$$\Big\downarrow \; 171$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{19}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{5}a\int -\frac{\left(70a-\frac{101}{x}\right)\left(1+\frac{1}{ax}\right)^{7/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{5}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right.}{a^2}\right.$$

$$\Big\downarrow \; 25$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}a\int\frac{\left(70a-\frac{101}{x}\right)\left(1+\frac{1}{ax}\right)^{7/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{19}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{5}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{3}}{a^2}\right.$$

$$\downarrow 27$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\int\frac{\left(70a-\frac{101}{x}\right)\left(1+\frac{1}{ax}\right)^{7/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{19}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{5}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}\right)-\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{3/}}{a^2}\right.$$

$$\downarrow 171$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{101}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{1}{4}a\int-\frac{7\left(40a-\frac{61}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{19}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{5}{2}a\sqrt{1}}{a^2}\right.$$

$$\downarrow 27$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\int\frac{\left(40a-\frac{61}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{101}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)+\frac{19}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)-\frac{5}{2}a\sqrt{1-\frac{1}{a}}}{a^2}\right.$$

$$\downarrow 171$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{61}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{3}a\int-\frac{5\left(24a-\frac{37}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{101}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)+\frac{19}{5}}{a^2}\right.$$

$$\downarrow 27$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\int\frac{\left(24a-\frac{37}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{61}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{101}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)+\frac{19}{5}a\sqrt{1}\right.\right.\right.}{a^2}\right.$$

$$\Big\downarrow \; 171$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{37}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}a\int-\frac{3\left(16a-\frac{21}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{61}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{101}{4}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \; 27$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\int\frac{\left(16a-\frac{21}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{37}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{61}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{101}{4}a\sqrt{}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \; 171$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(21a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int-\frac{\left(16a-\frac{5}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{37}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{61}{3}a\sqrt{}\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow \; 25$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(a\int\frac{\left(16a-\frac{5}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+21a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{37}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{61}{3}a\sqrt{1}\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow \; 27$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{\left(16a-\frac{5}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+21a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{37}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{61}{3}a\sqrt{1-\right.\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow 175$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(16a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-5\int \frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+21a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{37}{2}a\sqrt{1-\frac{1}{ax}\right.\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow 39$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-5\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+16a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+21a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{37}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}\right.\right.\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow 103$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-5\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+21a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{37}{2}a\right.\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow 221$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-5\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+21a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{37}{2}a\right.\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow 223$$

$$-c^4\left(\frac{\frac{3}{7}\left(\frac{1}{2}\left(\frac{1}{5}\left(\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-5a\arcsin\left(\frac{1}{ax}\right)-16a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+21a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{37}{2}a\sqrt{1-\right.\right.\right.\right.\right.}{}\right.$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

output
```
-(c^4*(-((1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(11/2)*x) + ((-8*a*(1 - 1/(a*x)
)^(3/2)*(1 + 1/(a*x))^(11/2))/7 + (3*((-5*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x)
)^(11/2))/2 + ((19*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/5 + ((101*a*Sq
rt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/4 + (7*((61*a*Sqrt[1 - 1/(a*x)]*(1 +
1/(a*x))^(5/2))/3 + (5*((37*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2))/2 + (
3*(21*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)] - 5*a*ArcSin[1/(a*x)] - 16*a*A
rcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]))/2))/3))/4)/5)/2))/7)/a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00

method	result
risch	$\frac{(ax-1)(560a^7x^7-2496x^6a^6-525a^5x^5+992a^4x^4+770a^3x^3-96a^2x^2-280ax-80)c^4}{560x^7a^8\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{15a^7\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{16}+\frac{3a^8\ln\left(\frac{a^2x}{\sqrt{a^2}}+\right.}{\sqrt{a^2}}\right.}{a^8(ax+1)\sqrt{}}$
default	$\frac{(ax-1)^2c^4\left(-1680\sqrt{a^2x^2-1}\sqrt{a^2}\,a^8x^8+1680(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6+525a^7x^7\sqrt{a^2}\sqrt{a^2x^2-1}+525a^7x^7\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+1\right.}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/560*(a*x-1)*(560*a^7*x^7-2496*a^6*x^6-525*a^5*x^5+992*a^4*x^4+770*a^3*x^
3-96*a^2*x^2-280*a*x-80)/x^7*c^4/a^8/((a*x-1)/(a*x+1))^(1/2)+(15/16*a^7*ar
ctan(1/(a^2*x^2-1)^(1/2))+3*a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a
^2)^(1/2))*c^4/a^8/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.12

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx =$$
$$-\frac{1050\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(560}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
-1/560*(1050*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) - 1680*a^7*c^4*
x^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 1680*a^7*c^4*x^7*log(sqrt((a*x -
1)/(a*x + 1)) - 1) - (560*a^8*c^4*x^8 - 1936*a^7*c^4*x^7 - 3021*a^6*c^4*x^
6 + 467*a^5*c^4*x^5 + 1762*a^4*c^4*x^4 + 674*a^3*c^4*x^3 - 376*a^2*c^4*x^2
 - 360*a*c^4*x - 80*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{c^4\left(\int\left(-\frac{4a^2}{\frac{ax^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{6a^4}{\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{4a^6}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x}{}}\right.\right.}{a^8}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**4,x)
```

output
```
c**4*(Integral(-4*a**2/(a*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
 - x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**4
/(a*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**4*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-4*a**6/(a*x**3*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(
a*x + 1)), x) + Integral(a**8/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
+ 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*
*9*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**8*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1)), x))/a**8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(158) = 316$.

Time = 0.12 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.11

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=$$
$$-\frac{1}{280}\left(\frac{525\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2205\,c^4\left(\frac{ax-1}{ax+1}\right)}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
-1/280*(525*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 840*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 + 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
a^2 - (2205*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 13615*c^4*((a*x - 1)/(a*x +
 1))^(13/2) + 33621*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 39071*c^4*((a*x - 1
)/(a*x + 1))^(9/2) + 12799*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 20811*c^4*((a
*x - 1)/(a*x + 1))^(5/2) - 7665*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 1155*c^4
*sqrt((a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^
2/(a*x + 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x +
 1)^5 - 14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (
a*x - 1)^8*a^2/(a*x + 1)^8 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(158) = 316$.

Time = 0.16 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.56

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$= -\frac{15\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{8\,a\text{sgn}\,(ax+1)} - \frac{3\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\text{sgn}\,(ax+1)} + \frac{\sqrt{a^2x^2-1}c^4}{a\text{sgn}\,(ax+1)}$$
$$+ \frac{525\left(x|a|-\sqrt{a^2x^2-1}\right)^{13}c^4|a| - 4480\left(x|a|-\sqrt{a^2x^2-1}\right)^{12}ac^4 - 980\left(x|a|-\sqrt{a^2x^2-1}\right)^{11}c^4|a| - 20}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
-15/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 3*c^4*l
og(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^
2 - 1)*c^4/(a*sgn(a*x + 1)) + 1/280*(525*(x*abs(a) - sqrt(a^2*x^2 - 1))^13
*c^4*abs(a) - 4480*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4 - 980*(x*abs(a)
 - sqrt(a^2*x^2 - 1))^11*c^4*abs(a) - 20160*(x*abs(a) - sqrt(a^2*x^2 - 1))
^10*a*c^4 + 945*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^4*abs(a) - 38080*(x*abs
(a) - sqrt(a^2*x^2 - 1))^8*a*c^4 - 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*
a*c^4 - 945*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a) - 32256*(x*abs(a)
- sqrt(a^2*x^2 - 1))^4*a*c^4 + 980*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^4*ab
s(a) - 12992*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^4 - 525*(x*abs(a) - sqrt
(a^2*x^2 - 1))*c^4*abs(a) - 2496*a*c^4)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2
 + 1)^7*a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.84

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = \frac{\frac{12799\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{280}-\frac{219\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{8}-\frac{2973\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{40}-\frac{33\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{39071\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{280}+\frac{4803\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{40}+\cdots}{a+\frac{6\,a\,(a\,x-1)}{a\,x+1}+\frac{14\,a\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{14\,a\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{14\,a\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{14\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{6\,a\,(a\,x-1)^7}{(a\,x+1)^7}-\cdots} - \frac{15\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}+\frac{6\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((12799*c^4*((a*x - 1)/(a*x + 1))^(7/2))/280 - (219*c^4*((a*x - 1)/(a*x +
1))^(3/2))/8 - (2973*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 - (33*c^4*((a*x -
 1)/(a*x + 1))^(1/2))/8 + (39071*c^4*((a*x - 1)/(a*x + 1))^(9/2))/280 + (4
803*c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 + (389*c^4*((a*x - 1)/(a*x + 1))^
(13/2))/8 + (63*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/
(a*x + 1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^
3 - (14*a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a
*(a*x - 1)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (15*c^4*atan(((
a*x - 1)/(a*x + 1))^(1/2)))/(8*a) + (6*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/
2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.35

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{c^4\left(-1050atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^7x^7+1050atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^7x^7+560\sqrt{ax+\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x)
```

output
```
(c**4*( - 1050*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**7*x**7 + 1050*at
an(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**7*x**7 + 560*sqrt(a*x + 1)*sqrt(a
*x - 1)*a**7*x**7 - 2496*sqrt(a*x + 1)*sqrt(a*x - 1)*a**6*x**6 - 525*sqrt(
a*x + 1)*sqrt(a*x - 1)*a**5*x**5 + 992*sqrt(a*x + 1)*sqrt(a*x - 1)*a**4*x*
*4 + 770*sqrt(a*x + 1)*sqrt(a*x - 1)*a**3*x**3 - 96*sqrt(a*x + 1)*sqrt(a*x
 - 1)*a**2*x**2 - 280*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x - 80*sqrt(a*x + 1)*s
qrt(a*x - 1) + 3360*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**7*x**7
 + 1216*a**7*x**7))/(560*a**8*x**7)
```

3.766 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$

Optimal result

Integrand size = 22, antiderivative size = 147

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5a}-\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(4a-\frac{1}{x}\right)}{4a^2} - \frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}\left(8a-\frac{1}{x}\right)}{8a^2}+c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x + \frac{3c^3\csc^{-1}(ax)}{8a}+\frac{3c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
1/5*c^3*(1-1/a^2/x^2)^(5/2)/a-1/4*c^3*(1-1/a^2/x^2)^(3/2)*(4*a-1/x)/a^2-3/
8*c^3*(1-1/a^2/x^2)^(1/2)*(8*a-1/x)/a^2+c^3*(1-1/a^2/x^2)^(5/2)*x+3/8*c^3*
arccsc(a*x)/a+3*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.75

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}(8+30ax+24a^2x^2-55a^3x^3-152a^4x^4+40a^5x^5)+15a^4x^4\arcsin\left(\frac{1}{ax}\right)+120a^4x^4\log\left(\left(\right.\right.\right.}{40a^5x^4}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

output
```
(c^3*(Sqrt[1 - 1/(a^2*x^2)]*(8 + 30*a*x + 24*a^2*x^2 - 55*a^3*x^3 - 152*a^
4*x^4 + 40*a^5*x^5) + 15*a^4*x^4*ArcSin[1/(a*x)] + 120*a^4*x^4*Log[(1 + Sq
rt[1 - 1/(a^2*x^2)])*x]))/(40*a^5*x^4)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.79, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {6748, 108, 27, 171, 27, 171, 25, 27, 171, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int\left(c-\frac{c}{a^2x^2}\right)^3e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6748$$

$$-c^3\int\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \; 108$$

$$-c^3\left(\int\frac{3\left(a-\frac{2}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)$$

$$\Big\downarrow \; 27$$

$$-c^3\left(\frac{3\int \left(a-\frac{2}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)$$

$\Big\downarrow$ 171

$$-c^3\left(\frac{3\left(\frac{1}{5}a\int \frac{\left(5a-\frac{7}{x}\right)\left(1+\frac{1}{ax}\right)^{7/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{2}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)$$

$\Big\downarrow$ 27

$$-c^3\left(\frac{3\left(\frac{1}{5}\int \frac{\left(5a-\frac{7}{x}\right)\left(1+\frac{1}{ax}\right)^{7/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{2}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}\right)$$

$\Big\downarrow$ 171

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{7}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{1}{4}a\int -\frac{\left(20a-\frac{29}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)-\frac{2}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)}{a^2}-x\left(1-\frac{1}{ax}\right)^{3}\right.$$

$\Big\downarrow$ 25

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}a\int \frac{\left(20a-\frac{29}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{7}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{2}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\right.$$

$\Big\downarrow$ 27

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\int \frac{\left(20a-\frac{29}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{7}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{2}{5}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}\right)}{a^2}-x\left(1-\frac{1}{ax}\right)^{3/2}\right.$$

$\Big\downarrow$ 171

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{29}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{3}a\int -\frac{5\left(12a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{7}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{2}{5}a\sqrt{1-}\right.}{a^2}\right.$$

$\downarrow$ 27

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\int \frac{\left(12a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{29}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{7}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)-\frac{2}{5}a\sqrt{1-\frac{1}{ax}}\right.}{a^2}\right.$$

$\downarrow$ 171

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}a\int -\frac{3\left(8a-\frac{9}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{29}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{7}{4}a\sqrt{1-}\right.\right.}{a^2}\right.$$

$\downarrow$ 27

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\int \frac{\left(8a-\frac{9}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{29}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{7}{4}a\sqrt{1-\frac{1}{ax}}\right.\right.}{a^2}\right.$$

$\downarrow$ 171

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int -\frac{\left(8a-\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{29}{3}a\sqrt{1-}\right.\right.\right.}{a^2}\right.$$

$\downarrow$ 25

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(a\int\frac{\left(8a-\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{29}{3}a\sqrt{1-\frac{1}{ax}}\right.\right.\right.\right.}{a^2}\right.$$

$$\Big\downarrow \; 27$$

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(\int\frac{\left(8a-\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{29}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{a}\right.\right.\right.\right.}{a^2}\right.$$

$$\Big\downarrow \; 175$$

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(8a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\int\frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right.\right.\right.\right.}{a^2}\right.$$

$$\Big\downarrow \; 39$$

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+8a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3}\right.\right.\right.\right.}{a^2}\right.$$

$$\Big\downarrow \; 103$$

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{17}{2}a\sqrt{1-\frac{1}{a}}\right.\right.\right.\right.}{a^2}\right.$$

$$\Big\downarrow \; 221$$

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(-\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\right.\right.\right.\right.\right.}{a^2}\right.$$

$\downarrow$ 223

$$-c^3\left(\frac{3\left(\frac{1}{5}\left(\frac{1}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(a\left(-\arcsin\left(\frac{1}{ax}\right)\right)-8a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+9a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{17}{2}a\sqrt{1-\frac{1}{ax}}\right.\right.\right.\right.\right.}{a^2}\right.$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

output
```
-(c^3*(-((1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)*x) + (3*((-2*a*Sqrt[1 - 1
/(a*x)]*(1 + 1/(a*x))^(9/2))/5 + ((7*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/
2))/4 + ((29*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2))/3 + (5*((17*a*Sqrt[1
 - 1/(a*x)]*(1 + 1/(a*x))^(3/2))/2 + (3*(9*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/
(a*x)] - a*ArcSin[1/(a*x)] - 8*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x
)]]))/2))/3)/4)/5))/a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{(ax-1)(152a^4x^4+55a^3x^3-24a^2x^2-30ax-8)c^3}{40x^5a^6\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3a^5\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{8}+\frac{3a^6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a^5\sqrt{(ax-1)(ax+1)}\right)c}{a^6\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$
default	$\frac{(ax-1)^2c^3\left(-120\sqrt{a^2x^2-1}\sqrt{a^2}a^6x^6+120(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}a^4x^4+15\sqrt{a^2}\sqrt{a^2x^2-1}a^5x^5+15\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}a^5x^5+120\ln\right.}{40\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/40*(a*x-1)*(152*a^4*x^4+55*a^3*x^3-24*a^2*x^2-30*a*x-8)/x^5*c^3/a^6/((a
*x-1)/(a*x+1))^(1/2)+(3/8*a^5*arctan(1/(a^2*x^2-1)^(1/2))+3*a^6*ln(a^2*x/(
a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+a^5*((a*x-1)*(a*x+1))^(1/2))*c^3
/a^6/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.22

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3dx=$$
$$-\frac{30a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-120a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+120a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(40a^6c^3x^6}{40a^6x^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
-1/40*(30*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) - 120*a^5*c^3*x^5*
log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) - (40*a^6*c^3*x^6 - 112*a^5*c^3*x^5 - 207*a^4*c^3*x^4 - 31*a
^3*c^3*x^3 + 54*a^2*c^3*x^2 + 38*a*c^3*x + 8*c^3)*sqrt((a*x - 1)/(a*x + 1)
))/(a^6*x^5)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$$
$$= \frac{c^3\left(\int \frac{3a^2}{\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{3a^4}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^6}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}}}{ax\cdots}}\cdots\right.}{a^6}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**3,x)
```

output
```
c**3*(Integral(3*a**2/(a*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**4
/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**6/(a*x*sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)),
x) + Integral(-1/(a*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**
6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(129) = 258.

Time = 0.11 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.05

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx =$$
$$-\frac{1}{20}\left(\frac{15\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{135\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+5\cdots}{4\cdots}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
-1/20*(15*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 60*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 + 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2
- (135*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 575*c^3*((a*x - 1)/(a*x + 1))^(9
/2) + 842*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 298*c^3*((a*x - 1)/(a*x + 1))^
(5/2) - 465*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^3*sqrt((a*x - 1)/(a*x
+ 1)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x
 - 1)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/
(a*x + 1)^6 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(129) = 258$.

Time = 0.15 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.41

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$$
$$= -\frac{3\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{4\,a\mathrm{sgn}\left(ax+1\right)} - \frac{3\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)} + \frac{\sqrt{a^2x^2-1}c^3}{a\mathrm{sgn}\left(ax+1\right)}$$
$$+ \frac{55\left(x|a|-\sqrt{a^2x^2-1}\right)^9c^3|a| - 200\left(x|a|-\sqrt{a^2x^2-1}\right)^8ac^3 - 10\left(x|a|-\sqrt{a^2x^2-1}\right)^7c^3|a| - 720\left(x|a|\right.}{}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
-3/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 3*c^3*lo
g(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2
 - 1)*c^3/(a*sgn(a*x + 1)) + 1/20*(55*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3
*abs(a) - 200*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3 - 10*(x*abs(a) - sqrt
(a^2*x^2 - 1))^7*c^3*abs(a) - 720*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^3 -
 800*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^3 + 10*(x*abs(a) - sqrt(a^2*x^2
- 1))^3*c^3*abs(a) - 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3 - 55*(x*ab
s(a) - sqrt(a^2*x^2 - 1))*c^3*abs(a) - 152*a*c^3)/(((x*abs(a) - sqrt(a^2*x
^2 - 1))^2 + 1)^5*a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.76

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{\frac{149\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{10}-\frac{93\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{4}-\frac{21\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{421\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{10}+\frac{115\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}+\frac{27\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{4}}{a+\frac{4\,a\,(a\,x-1)}{a\,x+1}+\frac{5\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{a\,(a\,x-1)^6}{(a\,x+1)^6}}$$
$$-\frac{3\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}+\frac{6\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((149*c^3*((a*x - 1)/(a*x + 1))^(5/2))/10 - (93*c^3*((a*x - 1)/(a*x + 1))^
(3/2))/4 - (21*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (421*c^3*((a*x - 1)/(a
*x + 1))^(7/2))/10 + (115*c^3*((a*x - 1)/(a*x + 1))^(9/2))/4 + (27*c^3*((a
*x - 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x -
 1)^2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*
x + 1)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (3*c^3*atan(((a*x - 1)/(a*x + 1)
)^(1/2)))/(4*a) + (6*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.38

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{c^3\left(-30atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^5x^5+30atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^5x^5+40\sqrt{ax+1}\,\sqrt{ax\right.}}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x)
```

output
```
(c**3*( - 30*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**5*x**5 + 30*atan(s
qrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**5*x**5 + 40*sqrt(a*x + 1)*sqrt(a*x -
1)*a**5*x**5 - 152*sqrt(a*x + 1)*sqrt(a*x - 1)*a**4*x**4 - 55*sqrt(a*x + 1
)*sqrt(a*x - 1)*a**3*x**3 + 24*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 + 30*
sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 8*sqrt(a*x + 1)*sqrt(a*x - 1) + 240*log(
(sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**5*x**5 + 72*a**5*x**5))/(40*a*
*6*x**5)
```

3.767 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}-\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(6a+\frac{1}{x}\right)}{2a^2}+c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x-\frac{c^2\csc^{-1}(ax)}{2a}+\frac{3c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
1/3*c^2*(1-1/a^2/x^2)^(3/2)/a-1/2*c^2*(1-1/a^2/x^2)^(1/2)*(6*a+1/x)/a^2+c^
2*(1-1/a^2/x^2)^(3/2)*x-1/2*c^2*arccsc(a*x)/a+3*c^2*arctanh((1-1/a^2/x^2)^
(1/2))/a
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.84

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$$
$$=\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}(-2-9ax-16a^2x^2+6a^3x^3)-3a^2x^2\arcsin\left(\frac{1}{ax}\right)+18a^2x^2\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a^3x^2}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

output
```
(c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-2 - 9*a*x - 16*a^2*x^2 + 6*a^3*x^3) - 3*a^2*
x^2*ArcSin[1/(a*x)] + 18*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a
^3*x^2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.67, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {6748, 108, 27, 171, 25, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^2 e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \; 6748$$

$$-c^2\int \sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \; 108$$

$$-c^2\left(\int \frac{\left(3a-\frac{4}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{a^2\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow \; 27$$

$$-c^2\left(\frac{\int \frac{\left(3a-\frac{4}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$\Big\downarrow$ 171

$$-c^2\left(\frac{\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{3}a\int -\frac{\left(9a-\frac{11}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$\Big\downarrow$ 25

$$-c^2\left(\frac{\frac{1}{3}a\int \frac{\left(9a-\frac{11}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$\Big\downarrow$ 27

$$-c^2\left(\frac{\frac{1}{3}\int \frac{\left(9a-\frac{11}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$\Big\downarrow$ 171

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}a\int -\frac{3\left(6a-\frac{5}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right.\right.$$

$\Big\downarrow$ 27

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\int \frac{\left(6a-\frac{5}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right.$$

$\Big\downarrow$ 171

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int -\frac{\left(6a+\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)}{a^2}\right.$$

$$\downarrow 25$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(a\int \frac{\left(6a+\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/}}{a^2}\right.$$

$$\downarrow 27$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{\left(6a+\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{4}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}\right.$$

$$\downarrow 175$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(6a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\int \frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\cdots}{a^2}\right.$$

$$\downarrow 39$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+6a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{4}{3}a\sqrt{\cdots}}{a^2}\right.$$

$$\downarrow 103$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-6\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/}}{a^2}\right.$$

$$\Big\downarrow \textbf{221}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-6a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{a^2}\right.$$

$$\Big\downarrow \textbf{223}$$

$$-c^2\left(\frac{\frac{1}{3}\left(\frac{3}{2}\left(a\arcsin\left(\frac{1}{ax}\right)-6a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)}{a^2}\right.$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

output
```
-(c^2*(-(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x) + ((4*a*Sqrt[1 - 1/(a*x)
]*(1 + 1/(a*x))^(5/2))/3 + ((11*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2))/2
 + (3*(5*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)] + a*ArcSin[1/(a*x)] - 6*a*A
rcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]))/2)/3)/a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.40

method	result
risch	$-\frac{(ax-1)(16a^2x^2+9ax+2)c^2}{6x^3a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(-\frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+\frac{3a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a^3\sqrt{(ax-1)(ax+1)}\right)c^2\sqrt{(ax-1)(ax+1)}}{a^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)^2c^2\left(-18\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^4x^4+18\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^2x^2-3\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^3x^3-3a^3\sqrt{a^2}\,x^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+18\ln\left(\frac{a^2x}{}\right.\right.}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a^4x^3\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(a*x-1)*(16*a^2*x^2+9*a*x+2)/x^3*c^2/a^4/((a*x-1)/(a*x+1))^(1/2)+(-1/
2*a^3*arctan(1/(a^2*x^2-1)^(1/2))+3*a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(
1/2))/(a^2)^(1/2)+a^3*((a*x-1)*(a*x+1))^(1/2))*c^2/a^4/((a*x-1)/(a*x+1))^(
1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.39

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = \frac{6\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(6\,a^4c^2x^4-10\,a}{6\,a^4x^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
1/6*(6*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 18*a^3*c^2*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) - 18*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x +
1)) - 1) + (6*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 25*a^2*c^2*x^2 - 11*a*c^2*x -
 2*c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2dx$$
$$=\frac{c^2\left(\int\left(-\frac{2a^2}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^4\sqrt{\frac{ax}{ax+1}}}{ax+}}\right.}{a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**2,x)
```

output
```
c**2*(Integral(-2*a**2/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
 - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**4/(
a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(
a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1) - x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a
**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(98) = 196$.

Time = 0.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.99

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2dx$$
$$=\frac{1}{3}a\left(\frac{3c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{15c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+37c^2\left(\frac{ax-}{ax+}\right.}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(}{(}}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
1/3*a*(3*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 9*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^2 - 9*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (
15*c^2*((a*x - 1)/(a*x + 1))^(7/2) + 37*c^2*((a*x - 1)/(a*x + 1))^(5/2) +
17*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 21*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*
(a*x - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/
(a*x + 1)^4 + a^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(98) = 196$.

Time = 0.15 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.21

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$$
$$= \frac{c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\text{sgn}\left(ax+1\right)} - \frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\text{sgn}\left(ax+1\right)} + \frac{\sqrt{a^2x^2-1}c^2}{a\text{sgn}\left(ax+1\right)}$$
$$+ \frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^2|a| - 12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^2 - 36\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2 - 9\left(x|a|-\sqrt{a\ldots\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|\text{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 3*c^2*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)
*c^2/(a*sgn(a*x + 1)) + 1/3*(9*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a)
 - 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2 - 36*(x*abs(a) - sqrt(a^2*x^2
 - 1))^2*a*c^2 - 9*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^2*abs(a) - 16*a*c^2)/(
((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a)*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.23 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.63

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$$
$$=\frac{\frac{17\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}-7\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{37\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{3}+5\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{a+\frac{2\,a\,(a\,x-1)}{a\,x+1}-\frac{2\,a\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{a\,(a\,x-1)^4}{(a\,x+1)^4}}$$
$$+\frac{c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{6\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((17*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 - 7*c^2*((a*x - 1)/(a*x + 1))^(1/2
) + (37*c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 + 5*c^2*((a*x - 1)/(a*x + 1))^(
7/2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*
(a*x - 1)^4)/(a*x + 1)^4) + (c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (6
*c^2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.46

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$$
$$=\frac{c^2\left(6atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^3x^3-6atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^3x^3+6\sqrt{ax+1}\,\sqrt{ax-1}\,\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x)
```

output
```
(c**2*(6*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**3*x**3 - 6*atan(sqrt(a
*x - 1) + sqrt(a*x + 1) + 1)*a**3*x**3 + 6*sqrt(a*x + 1)*sqrt(a*x - 1)*a**
3*x**3 - 16*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 - 9*sqrt(a*x + 1)*sqrt(a
*x - 1)*a*x - 2*sqrt(a*x + 1)*sqrt(a*x - 1) + 36*log((sqrt(a*x - 1) + sqrt
(a*x + 1))/sqrt(2))*a**3*x**3 + 8*a**3*x**3))/(6*a**4*x**3)
```

3.768 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}}{a}+c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{3c\csc^{-1}(ax)}{a}+\frac{3c\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
c*(1-1/a^2/x^2)^(1/2)/a+c*(1-1/a^2/x^2)^(1/2)*x-3*c*arccsc(a*x)/a+3*c*arct
anh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(1+ax)-3\arcsin\left(\frac{1}{ax}\right)+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{a}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

output
```
(c*(Sqrt[1 - 1/(a^2*x^2)]*(1 + a*x) - 3*ArcSin[1/(a*x)] + 3*Log[(1 + Sqrt[
1 - 1/(a^2*x^2)])*x]))/a
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6748, 109, 27, 140, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6748}$$

$$-c\int \frac{\left(1+\frac{1}{ax}\right)^{5/2}x^2}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 109}$$

$$-c\left(x\left(-\sqrt{1-\frac{1}{ax}}\right)\left(\frac{1}{ax}+1\right)^{3/2}-\int -\frac{3\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-c\left(\frac{3\int \frac{\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}}{a}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 140}$$

$$-c\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}+\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 39}$$

$$-c\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}+\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{103}$$

$$-c\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\frac{\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\right)}{a}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$-c\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)}{a}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{223}$$

$$-c\left(\frac{3\left(\arcsin\left(\frac{1}{ax}\right)-\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)}{a}-x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

output
```
-(c*(-(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)*x) + (3*(ArcSin[1/(a*x)] - Ar
cTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]))/a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(62) = 124$.

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.90

method	result
risch	$\frac{(ax-1)c}{x\,a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\sqrt{(ax-1)(ax+1)}-3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\frac{3a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}\right)c\sqrt{(ax-1)(ax+1)}}{a(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)^2c\left(\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^2x^2+4\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,ax-(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-3\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x-\right.}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-1)/x*c/a^2/((a*x-1)/(a*x+1))^(1/2)+1/a*(((a*x-1)*(a*x+1))^(1/2)-3*arc
tan(1/(a^2*x^2-1)^(1/2))+3*a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)
^(1/2))*c/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.56

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{6\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2cx^2+2\,acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
(6*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + 3*a*c*x*log(sqrt((a*x - 1)/(a
*x + 1)) + 1) - 3*a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c*x^2 +
2*a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{c\left(\int\frac{a^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)}{a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2),x)
```

output
```
c*(Integral(a**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x**3*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
)/(a*x + 1)), x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.74

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = -a\left(\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
-a*(4*c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 6*
c*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1)
) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(62) = 124.

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.91

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{6\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)} - \frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)} + \frac{\sqrt{a^2x^2-1}c}{a\mathrm{sgn}\left(ax+1\right)} + \frac{2\,c}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
6*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 3*c*log(abs(-
x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*c
/(a*sgn(a*x + 1)) + 2*c/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a)*sgn
(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.24

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{6\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{6\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)^2}{(a\,x+1)^2}}$$

input
```
int((c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(6*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (6*c*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a + (4*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1)^2)/(a*x
 + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.49

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$$
$$=\frac{c\Big(6atan\big(\sqrt{ax-1}+\sqrt{ax+1}-1\big)\,ax-6atan\big(\sqrt{ax-1}+\sqrt{ax+1}+1\big)\,ax+\sqrt{ax+1}\,\sqrt{ax-1}\,ax+\sqrt{}}{a^2x}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x)
```

output
```
(c*(6*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a*x - 6*atan(sqrt(a*x - 1) +
 sqrt(a*x + 1) + 1)*a*x + sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + sqrt(a*x + 1)*
sqrt(a*x - 1) + 6*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x + a*x))
/(a**2*x)
```

3.769 $\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{9a+\frac{11}{x}}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}+\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
1/3*(-4*a-4/x)/a^2/c/(1-1/a^2/x^2)^(3/2)-1/3*(9*a+11/x)/a^2/c/(1-1/a^2/x^2
)^(1/2)+(1-1/a^2/x^2)^(1/2)*x/c+3*arctanh((1-1/a^2/x^2)^(1/2))/a/c
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.66

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(14-19ax+3a^2x^2)}{(-1+ax)^2}+\frac{9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{3c}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

output
```
((Sqrt[1 - 1/(a^2*x^2)]*x*(14 - 19*a*x + 3*a^2*x^2))/(-1 + a*x)^2 + (9*Log
[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.33, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6748, 110, 27, 169, 25, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{\sqrt{1+\frac{1}{ax}}x^2}{\left(1-\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{110}$$

$$-\frac{\int \frac{\left(3a+\frac{2}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\left(3a+\frac{2}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{169}$$

$$-\frac{\frac{\frac{5a\sqrt{\frac{1}{ax}+1}}{3\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{1}{3}a\int -\frac{\left(9a+\frac{5}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\frac{1}{3}a\int \frac{\left(9a+\frac{5}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{5a\sqrt{\frac{1}{ax}+1}}{3\left(1-\frac{1}{ax}\right)^{3/2}}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{3}\int \frac{\left(9a+\frac{5}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{5a\sqrt{\frac{1}{ax}+1}}{3\left(1-\frac{1}{ax}\right)^{3/2}}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{14a\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}-a\int -\frac{9x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{5a\sqrt{\frac{1}{ax}+1}}{3\left(1-\frac{1}{ax}\right)^{3/2}}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{3}\left(9a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{14a\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}\right)+\frac{5a\sqrt{\frac{1}{ax}+1}}{3\left(1-\frac{1}{ax}\right)^{3/2}}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

$$\Big\downarrow \; 103$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{14a\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}-9\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)\right)+\frac{5a\sqrt{\frac{1}{ax}+1}}{3\left(1-\frac{1}{ax}\right)^{3/2}}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{14a\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}-9a\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{5a\sqrt{\frac{1}{ax}+1}}{3\left(1-\frac{1}{ax}\right)^{3/2}}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{c}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

output
```
-((-((Sqrt[1 + 1/(a*x)]*x)/(1 - 1/(a*x))^(3/2)) + ((5*a*Sqrt[1 + 1/(a*x)])
/(3*(1 - 1/(a*x))^(3/2)) + ((14*a*Sqrt[1 + 1/(a*x)])/Sqrt[1 - 1/(a*x)] - 9
*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/3)/a^2)/c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.76

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{2}\sqrt{a^{2}}}-\frac{13\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2a\left(x-\frac{1}{a}\right)}}{3a^{4}\left(x-\frac{1}{a}\right)}-\frac{2\sqrt{\left(x-\frac{1}{a}\right)^{2}a^{2}+2a\left(x-\frac{1}{a}\right)}}{3a^{5}\left(x-\frac{1}{a}\right)^{2}}\right)a^{2}\sqrt{(ax-1)(ax+1)}}{c\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-9\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,a^{3}x^{3}-9\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{4}x^{3}+6\sqrt{a^{2}}\,((ax-1)(ax+1))^{\frac{3}{2}}ax+27\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,a^{2}x}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c/((a*x-1)/(a*x+1))^(1/2)+(3/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2
-1)^(1/2))/(a^2)^(1/2)-13/3/a^4/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-
2/3/a^5/(x-1/a)^2*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2))/c*a^2/((a*x-1)/(a*x+1
))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.22

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{9\left(a^{2}x^{2}-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\left(a^{2}x^{2}-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^{3}x^{3}-16\,a^{2}x^{2}-5\,ax\right.}{3\left(a^{3}cx^{2}-2\,a^{2}cx+ac\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
1/3*(9*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a^2*x
^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 16*a^2*x
^2 - 5*a*x + 14)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{a^2\int\frac{x^2}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2),x)
```

output
```
a**2*Integral(x**2/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
 1)), x)/c
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.27

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{1}{3}\,a\left(\frac{\frac{11\,(ax-1)}{ax+1}-\frac{18\,(ax-1)^2}{(ax+1)^2}+1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
1/3*a*((11*(a*x - 1)/(a*x + 1) - 18*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c*((
a*x - 1)/(a*x + 1))^(5/2) - a^2*c*((a*x - 1)/(a*x + 1))^(3/2)) + 9*log(sqr
t((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
/(a^2*c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c} - \frac{\frac{11\,(a\,x-1)}{3\,(a\,x+1)} - \frac{6\,(a\,x-1)^2}{(a\,x+1)^2} + \frac{1}{3}}{a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} - a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

input
```
int(1/((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
(6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c) - ((11*(a*x - 1))/(3*(a*x + 1
)) - (6*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(a*c*((a*x - 1)/(a*x + 1))^(3/2) -
 a*c*((a*x - 1)/(a*x + 1))^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.24

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$$
$$= \frac{18\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax - 18\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right) + 4\sqrt{ax-1}\,ax - 4\sqrt{ax-1} + 3\sqrt{ax}}{3\sqrt{ax-1}\,ac\,(ax-1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x)
```

output
```
(18*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x - 18*sq
rt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 4*sqrt(a*x - 1)
*a*x - 4*sqrt(a*x - 1) + 3*sqrt(a*x + 1)*a**2*x**2 - 19*sqrt(a*x + 1)*a*x
+ 14*sqrt(a*x + 1))/(3*sqrt(a*x - 1)*a*c*(a*x - 1))
```

3.770 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 138

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

output
```
1/5*(-4*a-4/x)/a^2/c^2/(1-1/a^2/x^2)^(5/2)-1/5*(5*a+7/x)/a^2/c^2/(1-1/a^2/
x^2)^(3/2)-1/5*(15*a+19/x)/a^2/c^2/(1-1/a^2/x^2)^(1/2)+(1-1/a^2/x^2)^(1/2)
*x/c^2+3*arctanh((1-1/a^2/x^2)^(1/2))/a/c^2
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-24+57ax-39a^2x^2+5a^3x^3\right)}{5(-1+ax)^3} + 3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^2}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-24 + 57*a*x - 39*a^2*x^2 + 5*a^3*x^3))/(5*(-
1 + a*x)^3) + 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {6748, 114, 27, 35, 110, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int -\frac{3\left(a+\frac{1}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{3\int \frac{\left(a+\frac{1}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{35}$$

$$-\frac{\frac{3\int \frac{\sqrt{1+\frac{1}{ax}}x}{\left(1-\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{110}$$

$$-\frac{\frac{3\left(\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{2}{5}\int -\frac{\left(5a+\frac{4}{x}\right)x}{2a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{3\left(\frac{\int \frac{\left(5a+\frac{4}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{5a}+\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(1-\frac{1}{ax}\right)^{5/2}}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{ 169}$$

$$-\frac{\frac{3\left(\frac{\frac{3a\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{1}{3}a\int -\frac{3\left(5a+\frac{3}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{5a}+\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(1-\frac{1}{ax}\right)^{5/2}}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{3\left(\frac{\int \frac{\left(5a+\frac{3}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{3a\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{5a}+\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(1-\frac{1}{ax}\right)^{5/2}}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{ 169}$$

$$-\frac{\frac{3\left(\frac{a\left(-\int -\frac{5x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{8a\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}+\frac{3a\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{5a}+\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(1-\frac{1}{ax}\right)^{5/2}}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{3\left(\frac{5a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{8a\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}+\frac{3a\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{5a}+\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(1-\frac{1}{ax}\right)^{5/2}}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{ 103}$$

$$-\frac{\frac{3\left(\frac{-5\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+\frac{8a\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}+\frac{3a\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{5a}+\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(1-\frac{1}{ax}\right)^{5/2}}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

$\Big\downarrow$ 221

$$-\frac{\frac{3\left(\frac{-5a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{8a\sqrt{\frac{1}{ax}+1}}{\sqrt{1-\frac{1}{ax}}}+\frac{3a\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{3/2}}}{5a}+\frac{2\sqrt{\frac{1}{ax}+1}}{5\left(1-\frac{1}{ax}\right)^{5/2}}\right)}{a}-\frac{x\sqrt{\frac{1}{ax}+1}}{\left(1-\frac{1}{ax}\right)^{5/2}}}{c^2}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

output
```
-((-((Sqrt[1 + 1/(a*x)]*x)/(1 - 1/(a*x))^(5/2)) + (3*((2*Sqrt[1 + 1/(a*x)]
)/(5*(1 - 1/(a*x))^(5/2)) + ((3*a*Sqrt[1 + 1/(a*x)])/(1 - 1/(a*x))^(3/2) +
 (8*a*Sqrt[1 + 1/(a*x)])/Sqrt[1 - 1/(a*x)] - 5*a*ArcTanh[Sqrt[1 - 1/(a*x)]
*Sqrt[1 + 1/(a*x)]])/(5*a)))/a)/c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.63

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2 a^2+2a\left(x-\frac{1}{a}\right)}}{5a^8\left(x-\frac{1}{a}\right)^3}-\frac{6\sqrt{\left(x-\frac{1}{a}\right)^2 a^2+2a\left(x-\frac{1}{a}\right)}}{5a^7\left(x-\frac{1}{a}\right)^2}-\frac{24\sqrt{\left(x-\frac{1}{a}\right)^2 a^2+2a\left(x-\frac{1}{a}\right)}}{5a^6\left(x-\frac{1}{a}\right)}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}\right)a^4\sqrt{(a}}{c^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-125\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^4x^4-120\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^5x^4+85\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2+500\sqrt{a^2}\,\sqrt{(ax-1)(}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)+(-1/5/a^8/(x-1/a)^3*((x-1/a)^2*a^2
+2*a*(x-1/a))^(1/2)-6/5/a^7/(x-1/a)^2*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-24
/5/a^6/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)+3/a^4*ln(a^2*x/(a^2)^(1/2
)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2))*a^4/c^2/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)
*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.23

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$
$$=\frac{15\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(5\,a}{5\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
1/5*(15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) -
1) + (5*a^4*x^4 - 34*a^3*x^3 + 18*a^2*x^2 + 33*a*x - 24)*sqrt((a*x - 1)/(a
*x + 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{x^4}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**2,x)
```

output
```
a**4*Integral(x**4/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a**3*x**3*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 2*a**2*x**2*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1) + a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{1}{20}a\left(\frac{\frac{9(ax-1)}{ax+1}+\frac{75(ax-1)^2}{(ax+1)^2}-\frac{125(ax-1)^3}{(ax+1)^3}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{60\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{60\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
1/20*a*((9*(a*x - 1)/(a*x + 1) + 75*(a*x - 1)^2/(a*x + 1)^2 - 125*(a*x - 1
)^3/(a*x + 1)^3 + 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^2*((a*x
- 1)/(a*x + 1))^(5/2)) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) -
 60*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c^2|a|\text{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}}{ac^2\text{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
-3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c^2*abs(a)*sgn(a*x + 1)) + sqr
t(a^2*x^2 - 1)/(a*c^2*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{6\,\text{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}-\frac{\frac{15\,{(a\,x-1)}^2}{{(a\,x+1)}^2}-\frac{25\,{(a\,x-1)}^3}{{(a\,x+1)}^3}+\frac{9\,(a\,x-1)}{5\,(a\,x+1)}+\frac{1}{5}}{4\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-4\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

input
```
int(1/((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
(6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^2) - ((15*(a*x - 1)^2)/(a*x +
1)^2 - (25*(a*x - 1)^3)/(a*x + 1)^3 + (9*(a*x - 1))/(5*(a*x + 1)) + 1/5)/(
4*a*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 4*a*c^2*((a*x - 1)/(a*x + 1))^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.44

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{60\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^2x^2-120\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax+60\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right.}{10\sqrt{ax}}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x)
```

output
```
(60*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**2*x**2 -
 120*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x + 60*s
qrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 33*sqrt(a*x -
1)*a**2*x**2 - 66*sqrt(a*x - 1)*a*x + 33*sqrt(a*x - 1) + 10*sqrt(a*x + 1)*
a**3*x**3 - 78*sqrt(a*x + 1)*a**2*x**2 + 114*sqrt(a*x + 1)*a*x - 48*sqrt(a
*x + 1))/(10*sqrt(a*x - 1)*a*c**2*(a**2*x**2 - 2*a*x + 1))
```

3.771 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 171

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{4\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{21a+\frac{31}{x}}{35a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{3\left(35a+\frac{47}{x}\right)}{35a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{35a+\frac{53}{x}}{35a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

output
```
1/7*(-4*a-4/x)/a^2/c^3/(1-1/a^2/x^2)^(7/2)-1/35*(21*a+31/x)/a^2/c^3/(1-1/a
^2/x^2)^(5/2)-3/35*(35*a+47/x)/a^2/c^3/(1-1/a^2/x^2)^(1/2)-1/35*(35*a+53/x
)/a^2/c^3/(1-1/a^2/x^2)^(3/2)+(1-1/a^2/x^2)^(1/2)*x/c^3+3*arctanh((1-1/a^2
/x^2)^(1/2))/a/c^3
```

Mathematica [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.59

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(176-423ax+125a^2x^2+368a^3x^3-286a^4x^4+35a^5x^5\right)}{35(-1+ax)^4(1+ax)}+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^3}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(176 - 423*a*x + 125*a^2*x^2 + 368*a^3*x^3 - 2
86*a^4*x^4 + 35*a^5*x^5))/(35*(-1 + a*x)^4*(1 + a*x)) + 3*Log[(1 + Sqrt[1
- 1/(a^2*x^2)])*x])/(a*c^3)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.43, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.864$, Rules used = {6748, 114, 25, 27, 169, 25, 27, 169, 27, 169, 25, 27, 169, 25, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int -\frac{\left(3a+\frac{5}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow 25$$

$$-\frac{\int \frac{\left(3a+\frac{5}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\int \frac{\left(3a+\frac{5}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow 169$$

$$-\frac{\frac{\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}-\frac{1}{7}a\int -\frac{\left(21a+\frac{32}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{\frac{1}{7}a\int \frac{\left(21a+\frac{32}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\frac{1}{7}\int \frac{\left(21a+\frac{32}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow 169$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}-\frac{1}{5}a\int -\frac{3\left(35a+\frac{53}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\int \frac{\left(35a+\frac{53}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow 169$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{1}{3}a\int -\frac{\left(105a+\frac{176}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}a\int \frac{\left(105a+\frac{176}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\int \frac{\left(105a+\frac{176}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{281a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-a\int -\frac{\left(105a+\frac{281}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}\right)+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(a\int \frac{\left(105a+\frac{281}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{281a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\int \frac{\left(105a+\frac{281}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{281a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{c^3}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(a\int \frac{105x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{176a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{281a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}-}{c^3}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(105a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{176a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{281a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}}{a^2}}{c^3}$$

$\downarrow$ 103

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(-105\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)-\frac{176a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{281a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/}}}{a^2}}{c^3}$$

$\downarrow$ 221

$$-\frac{\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(-105a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-\frac{176a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{281a}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}\right)+\frac{88a}{3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{53a}{5\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}\right)+\frac{8a}{7\left(1-\frac{1}{ax}\right)^{7/}}}{a^2}}{c^3}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

output
```
-((-(x/((1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])) + ((8*a)/(7*(1 - 1/(a*x))^
(7/2)*Sqrt[1 + 1/(a*x)]) + ((53*a)/(5*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)
]) + (3*((88*a)/(3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) + ((281*a)/(Sqrt
[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (176*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a
*x)] - 105*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/3))/5)/7)/a^2)/
c^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.75

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^6\sqrt{a^2}}-\frac{2931\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{560a^8\left(x-\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{14a^{11}\left(x-\frac{1}{a}\right)^4}-\frac{71\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{140a^{10}\left(x-\frac{1}{a}\right)^3}-\frac{477\sqrt{(}}{}\right.}{c^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-3675\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^7x^7-3360\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^8x^7+2555((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5+11025\sqrt{(ax-1}}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^3/((a*x-1)/(a*x+1))^(1/2)+(3/a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-2931/560/a^8/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^
(1/2)-1/14/a^11/(x-1/a)^4*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-71/140/a^10/(x
-1/a)^3*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-477/280/a^9/(x-1/a)^2*((x-1/a)^2
*a^2+2*a*(x-1/a))^(1/2)+1/16/a^8/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)
)*a^6/c^3/((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.19

$$\int\frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{105\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log}{35\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-\right.}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
1/35*(105*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1)
/(a*x + 1)) + 1) - 105*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(s
qrt((a*x - 1)/(a*x + 1)) - 1) + (35*a^5*x^5 - 286*a^4*x^4 + 368*a^3*x^3 +
125*a^2*x^2 - 423*a*x + 176)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 4*a
^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$= \frac{a^6\int \frac{x^6}{\frac{a^7x^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax}{}}}{c^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**3,x)
```

output
```
a**6*Integral(x**6/(a**7*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3*a**5*x**5*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a**4*x**4*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1) + 3*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a
*x + 1) - 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x +
1))/(a*x + 1)), x)/c**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.12

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$= \frac{1}{560}a\left(\frac{\frac{51(ax-1)}{ax+1}+\frac{294(ax-1)^2}{(ax+1)^2}+\frac{2170(ax-1)^3}{(ax+1)^3}-\frac{3640(ax-1)^4}{(ax+1)^4}+5}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{1680\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{1680\log\left(\sqrt{\frac{ax}{ax}}\right.}{a^2c^3}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
1/560*a*((51*(a*x - 1)/(a*x + 1) + 294*(a*x - 1)^2/(a*x + 1)^2 + 2170*(a*x
 - 1)^3/(a*x + 1)^3 - 3640*(a*x - 1)^4/(a*x + 1)^4 + 5)/(a^2*c^3*((a*x - 1
)/(a*x + 1))^(9/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2)) + 1680*log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 1680*log(sqrt((a*x - 1)/(a*x + 1)) -
 1)/(a^2*c^3) + 35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3))
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
integrate(1/((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{16\,a\,c^3}-\frac{\frac{42\,{\left(a\,x-1\right)}^2}{5\,{\left(a\,x+1\right)}^2}+\frac{62\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{104\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}+\frac{51\,\left(a\,x-1\right)}{35\,\left(a\,x+1\right)}+\frac{1}{7}}{16\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}-16\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}+\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3}$$

input
```
int(1/((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
((a*x - 1)/(a*x + 1))^(1/2)/(16*a*c^3) - ((42*(a*x - 1)^2)/(5*(a*x + 1)^2)
 + (62*(a*x - 1)^3)/(a*x + 1)^3 - (104*(a*x - 1)^4)/(a*x + 1)^4 + (51*(a*x
 - 1))/(35*(a*x + 1)) + 1/7)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 16*a*
c^3*((a*x - 1)/(a*x + 1))^(9/2)) + (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/
(a*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.65

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$= \frac{840\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^4x^4 - 1680\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^3x^3 + 1680\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}}{}\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x)
```

output
```
(840*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**4*x**4
- 1680*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**3*x**
3 + 1680*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x -
840*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 549*sqrt(
a*x - 1)*a**4*x**4 - 1098*sqrt(a*x - 1)*a**3*x**3 + 1098*sqrt(a*x - 1)*a*x
 - 549*sqrt(a*x - 1) + 140*sqrt(a*x + 1)*a**5*x**5 - 1144*sqrt(a*x + 1)*a*
*4*x**4 + 1472*sqrt(a*x + 1)*a**3*x**3 + 500*sqrt(a*x + 1)*a**2*x**2 - 169
2*sqrt(a*x + 1)*a*x + 704*sqrt(a*x + 1))/(140*sqrt(a*x - 1)*a*c**3*(a**4*x
**4 - 2*a**3*x**3 + 2*a*x - 1))
```

3.772 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 204

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{4\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{27a+\frac{41}{x}}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{63a+\frac{103}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{315a+\frac{517}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{945a+\frac{1349}{x}}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4}+\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

output
```
1/9*(-4*a-4/x)/a^2/c^4/(1-1/a^2/x^2)^(9/2)-1/63*(27*a+41/x)/a^2/c^4/(1-1/a
^2/x^2)^(7/2)-1/105*(63*a+103/x)/a^2/c^4/(1-1/a^2/x^2)^(5/2)-1/315*(315*a+
517/x)/a^2/c^4/(1-1/a^2/x^2)^(3/2)-1/315*(945*a+1349/x)/a^2/c^4/(1-1/a^2/x
^2)^(1/2)+(1-1/a^2/x^2)^(1/2)*x/c^4+3*arctanh((1-1/a^2/x^2)^(1/2))/a/c^4
```

Mathematica [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.57

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}x(-1664+4047ax+339a^2x^2-7399a^3x^3+4029a^4x^4+2967a^5x^5-2669a^6x^6+315a^7x^7)}{315(-1+ax)^5(1+ax)^2}+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^4}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1664 + 4047*a*x + 339*a^2*x^2 - 7399*a^3*x^3
 + 4029*a^4*x^4 + 2967*a^5*x^5 - 2669*a^6*x^6 + 315*a^7*x^7))/(315*(-1 + a
*x)^5*(1 + a*x)^2) + 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.53, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.045$, Rules used = {6748, 114, 25, 27, 169, 27, 169, 25, 27, 169, 25, 27, 169, 27, 169, 25, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int -\frac{\left(3a+\frac{7}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\left(3a+\frac{7}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{\left(3a+\frac{7}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^4}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1}{9}a\int -\frac{3\left(9a+\frac{20}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{3}\int \frac{\left(9a+\frac{20}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^4}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1}{7}a\int -\frac{\left(63a+\frac{145}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}\right)+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}a\int \frac{\left(63a+\frac{145}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\int \frac{\left(63a+\frac{145}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^4}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1}{5}a\int -\frac{\left(315a+\frac{832}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}\right.}}{c^4}$$

$\downarrow$ 25

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}a\int \frac{\left(315a+\frac{832}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+\right.}}{c^4}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\int \frac{\left(315a+\frac{832}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)}}{c^4}$$

$\downarrow$ 169

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1}{3}a\int -\frac{3\left(315a+\frac{1147}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{1}{9\left(1-\frac{1}{ax}\right)^{9/}}}{a^2}}{c^4}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\int \frac{\left(315a+\frac{1147}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{10a}{9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+\right.}}{a^2}}{c^4}$$

$\downarrow$ 169

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(a\left(-\int -\frac{\left(315a+\frac{2924}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}\right)+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+\right.}}{a^2}}{c^4}$$

$\downarrow$ 25

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(a\int \frac{\left(315a+\frac{2924}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)}{a^2}}{c^4}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\int \frac{\left(315a+\frac{2924}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{29a}{7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+}{a^2}}{c^4}$$

$\downarrow$ 169

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}a\int \frac{\left(945a+\frac{2609}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{2609a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{}{7\left(1-\frac{1}{ax}\right.}\right.\right.}{a^2}}{c^4}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{\left(945a+\frac{2609}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{2609a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{}{7\left(1-\frac{1}{ax}\right)^{7/}}\right.}{a^2}}{c^4}$$

$\downarrow$ 169

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(a\int \frac{945x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{1664a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2609a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/}}\right.\right.}{a^2}}{c^4}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(945a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{1664a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2609a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{208a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+\right.}\right.\right.}{a^2}}{c^4}$$

$\downarrow$ 103

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-945\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)-\frac{1664a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2609a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{}{5\left(1-\frac{1}{ax}\right.}\right.\right.}{a^2}}{c^4}$$

$\Big\downarrow$ 221

$$-\frac{\frac{\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-945a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1664a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)-\frac{2609a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1462a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1147a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{}{5\left(1-\frac{1}{ax}\right.}\right.\right.}{a^2}}{c^4}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

output
```
-((-(x/((1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(3/2))) + ((10*a)/(9*(1 - 1/(a*x
))^(9/2)*(1 + 1/(a*x))^(3/2)) + ((29*a)/(7*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x
))^(3/2)) + ((208*a)/(5*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)) + ((1147*
a)/(3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)) + (1462*a)/(Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)) - (2609*a*Sqrt[1 - 1/(a*x)])/(3*(1 + 1/(a*x))^(3/2
)) + ((-1664*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)] - 945*a*ArcTanh[Sqrt[1
 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/3)/5)/7)/3)/a^2)/c^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(184) = 368$.

Time = 0.19 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.83

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^8\sqrt{a^2}}-\frac{113591\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{20160a^{10}\left(x-\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{36a^{14}\left(x-\frac{1}{a}\right)^5}-\frac{59\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{252a^{13}\left(x-\frac{1}{a}\right)^4}-\frac{1507}{}\right.}{c^4}$
default	$-\frac{-138915\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^9x^9-120960\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^{10}x^9+98595((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^7x^7+416745}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x-1)/c^4/((a*x-1)/(a*x+1))^(1/2)+(3/a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2-1)^(1/2))/(a^2)^(1/2)-113591/20160/a^10/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1
/a))^(1/2)-1/36/a^14/(x-1/a)^5*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-59/252/a^
13/(x-1/a)^4*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-1507/1680/a^12/(x-1/a)^3*((
x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-691/315/a^11/(x-1/a)^2*((x-1/a)^2*a^2+2*a*
(x-1/a))^(1/2)-1/96/a^11/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+31/19
2/a^10/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^8/c^4/((a*x-1)/(a*x+1)
)^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.22

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$=\frac{945\left(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2+4\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-945\left(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2\right.}{315\left(a^7c^4x^6-4\,a\right.}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output

```
1/315*(945*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) - 945*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a
^2*x^2 + 4*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (315*a^7*x^7 - 26
69*a^6*x^6 + 2967*a^5*x^5 + 4029*a^4*x^4 - 7399*a^3*x^3 + 339*a^2*x^2 + 40
47*a*x - 1664)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5
*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

Sympy [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$= \frac{a^8\int \frac{x^8}{\frac{a^9x^9\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^8x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4a^7x^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{4a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{6a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{6a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-4a}{c^4}$$

input

```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**4,x)
```

output

```
a**8*Integral(x**8/(a**9*x**9*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- a**8*x**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a**7*x**7*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 4*a**6*x**6*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1) + 6*a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a
*x + 1) - 6*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a**3
*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 4*a**2*x**2*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1) + a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/
(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.11

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$= \frac{1}{20160}a\left(\frac{\frac{415(ax-1)}{ax+1}+\frac{2511(ax-1)^2}{(ax+1)^2}+\frac{11739(ax-1)^3}{(ax+1)^3}+\frac{80745(ax-1)^4}{(ax+1)^4}-\frac{135765(ax-1)^5}{(ax+1)^5}+35}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}+\frac{105\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+30\right.}{a^2c^4}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
1/20160*a*((415*(a*x - 1)/(a*x + 1) + 2511*(a*x - 1)^2/(a*x + 1)^2 + 11739
*(a*x - 1)^3/(a*x + 1)^3 + 80745*(a*x - 1)^4/(a*x + 1)^4 - 135765*(a*x - 1
)^5/(a*x + 1)^5 + 35)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(11/2) - a^2*c^4*((a*
x - 1)/(a*x + 1))^(9/2)) + 105*(((a*x - 1)/(a*x + 1))^(3/2) + 30*sqrt((a*x
 - 1)/(a*x + 1)))/(a^2*c^4) + 60480*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^
2*c^4) - 60480*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Giac [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
integrate(1/((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{5\sqrt{\frac{a\,x-1}{a\,x+1}}}{32\,a\,c^4}-\frac{\frac{279\,{\left(a\,x-1\right)}^2}{35\,{\left(a\,x+1\right)}^2}+\frac{559\,{\left(a\,x-1\right)}^3}{15\,{\left(a\,x+1\right)}^3}+\frac{769\,{\left(a\,x-1\right)}^4}{3\,{\left(a\,x+1\right)}^4}-\frac{431\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{83\,\left(a\,x-1\right)}{63\,\left(a\,x+1\right)}+\frac{1}{9}}{64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}-64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{192\,a\,c^4}-\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,6{}\mathrm{i}}{a\,c^4}$$

input
```
int(1/((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
(5*((a*x - 1)/(a*x + 1))^(1/2))/(32*a*c^4) - ((279*(a*x - 1)^2)/(35*(a*x +
 1)^2) + (559*(a*x - 1)^3)/(15*(a*x + 1)^3) + (769*(a*x - 1)^4)/(3*(a*x +
1)^4) - (431*(a*x - 1)^5)/(a*x + 1)^5 + (83*(a*x - 1))/(63*(a*x + 1)) + 1/
9)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(9/2) - 64*a*c^4*((a*x - 1)/(a*x + 1))^
(11/2)) + ((a*x - 1)/(a*x + 1))^(3/2)/(192*a*c^4) - (atan(((a*x - 1)/(a*x
+ 1))^(1/2)*1i)*6i)/(a*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.33

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{7560\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^6x^6 - 15120\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^5x^5 - 7560\sqrt{ax-1}\log\left(\frac{\sqrt{ax}}{}\right.}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x)
```

output
```
(7560*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**6*x**6
 - 15120*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**5*x
**5 - 7560*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**4
*x**4 + 30240*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a
**3*x**3 - 7560*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))
*a**2*x**2 - 15120*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(
2))*a*x + 7560*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))
+ 4691*sqrt(a*x - 1)*a**6*x**6 - 9382*sqrt(a*x - 1)*a**5*x**5 - 4691*sqrt(
a*x - 1)*a**4*x**4 + 18764*sqrt(a*x - 1)*a**3*x**3 - 4691*sqrt(a*x - 1)*a*
*2*x**2 - 9382*sqrt(a*x - 1)*a*x + 4691*sqrt(a*x - 1) + 1260*sqrt(a*x + 1)
*a**7*x**7 - 10676*sqrt(a*x + 1)*a**6*x**6 + 11868*sqrt(a*x + 1)*a**5*x**5
 + 16116*sqrt(a*x + 1)*a**4*x**4 - 29596*sqrt(a*x + 1)*a**3*x**3 + 1356*sq
rt(a*x + 1)*a**2*x**2 + 16188*sqrt(a*x + 1)*a*x - 6656*sqrt(a*x + 1))/(126
0*sqrt(a*x - 1)*a*c**4*(a**6*x**6 - 2*a**5*x**5 - a**4*x**4 + 4*a**3*x**3
- a**2*x**2 - 2*a*x + 1))
```

3.773 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$

Optimal result

Integrand size = 22, antiderivative size = 116

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+c^5x+\frac{4c^5\log(x)}{a}$$

output
```
1/9*c^5/a^10/x^9+1/2*c^5/a^9/x^8+3/7*c^5/a^8/x^7-4/3*c^5/a^7/x^6-14/5*c^5/
a^6/x^5+14/3*c^5/a^4/x^3+4*c^5/a^3/x^2-3*c^5/a^2/x+c^5*x+4*c^5*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+c^5x+\frac{4c^5\log(x)}{a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

output
```
c^5/(9*a^10*x^9) + c^5/(2*a^9*x^8) + (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*
x^6) - (14*c^5)/(5*a^6*x^5) + (14*c^5)/(3*a^4*x^3) + (4*c^5)/(a^3*x^2) - (
3*c^5)/(a^2*x) + c^5*x + (4*c^5*Log[x])/a
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^5 e^{4\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{c^5 \left(a^2-\frac{1}{x^2}\right)^5 e^{4\text{arctanh}(ax)}}{a^{10}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c^5 \int e^{4\text{arctanh}(ax)} \left(a^2-\frac{1}{x^2}\right)^5 dx}{a^{10}}$$

$$\Big\downarrow \text{ 6707}$$

$$-\frac{c^5 \int \frac{e^{4\text{arctanh}(ax)} \left(1-a^2x^2\right)^5}{x^{10}}dx}{a^{10}}$$

$$\Big\downarrow \text{ 6700}$$

$$-\frac{c^5 \int \frac{(1-ax)^3(ax+1)^7}{x^{10}}dx}{a^{10}}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{c^5 \int \left(-a^{10}-\frac{4a^9}{x}-\frac{3a^8}{x^2}+\frac{8a^7}{x^3}+\frac{14a^6}{x^4}-\frac{14a^4}{x^6}-\frac{8a^3}{x^7}+\frac{3a^2}{x^8}+\frac{4a}{x^9}+\frac{1}{x^{10}}\right)dx}{a^{10}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{c^5 \left(a^{10}(-x)-4a^9\log(x)+\frac{3a^8}{x}-\frac{4a^7}{x^2}-\frac{14a^6}{3x^3}+\frac{14a^4}{5x^5}+\frac{4a^3}{3x^6}-\frac{3a^2}{7x^7}-\frac{a}{2x^8}-\frac{1}{9x^9}\right)}{a^{10}}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

output
```
-((c^5*(-1/9*1/x^9 - a/(2*x^8) - (3*a^2)/(7*x^7) + (4*a^3)/(3*x^6) + (14*a
^4)/(5*x^5) - (14*a^6)/(3*x^3) - (4*a^7)/x^2 + (3*a^8)/x - a^10*x - 4*a^9*
Log[x]))/a^10)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

method	result
default	$\frac{c^5\left(x a^{10}-\frac{4a^3}{3x^6}+\frac{4a^7}{x^2}-\frac{3a^8}{x}+\frac{3a^2}{7x^7}+\frac{1}{9x^9}+4a^9\ln(x)+\frac{14a^6}{3x^3}-\frac{14a^4}{5x^5}+\frac{a}{2x^8}\right)}{a^{10}}$
risch	$c^5x+\frac{-3a^8c^5x^8+4a^7c^5x^7+\frac{14}{3}a^6c^5x^6-\frac{14}{5}a^4c^5x^4-\frac{4}{3}a^3c^5x^3+\frac{3}{7}a^2c^5x^2+\frac{1}{2}a\,c^5x+\frac{1}{9}c^5}{a^{10}x^9}+\frac{4c^5\ln(x)}{a}$
parallelrisch	$\frac{630a^{10}c^5x^{10}+2520c^5\ln(x)a^9x^9-1890a^8c^5x^8+2520a^7c^5x^7+2940a^6c^5x^6-1764a^4c^5x^4-840a^3c^5x^3+270a^2c^5x^2+315a\,c^5x+}{630a^{10}x^9}$
norman	$\frac{-4a^9c^5x^{10}+c^5a^{10}x^{11}-\frac{c^5}{9a}-\frac{7c^5x}{18}+\frac{37a^2c^5x^3}{21}-\frac{14a^4c^5x^5}{5}-\frac{14a^5c^5x^6}{3}+\frac{2a^6c^5x^7}{3}+7a^7c^5x^8+\frac{c^5a\,x^2}{14}+\frac{22c^5a^3x^4}{15}}{(ax-1)a^9x^9}+\frac{4c^5\ln(x)}{a}$
meijerg	$-\frac{c^5\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{4c^5x}{-ax+1}-\frac{5c^5\left(\frac{1}{ax}-1-2\ln(x)-2\ln(-a)-\frac{3ax}{-3ax+3}+2\ln(-ax+1)\right)}{a}+\frac{5c^5\left(\frac{1}{5x^5a^5}\right.}{}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x,method=_RETURNVERBOSE)
```

output
```
c^5/a^10*(x*a^10-4/3*a^3/x^6+4*a^7/x^2-3*a^8/x+3/7*a^2/x^7+1/9/x^9+4*a^9*l
n(x)+14/3*a^6/x^3-14/5*a^4/x^5+1/2*a/x^8)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx = \frac{630\,a^{10}c^5x^{10}+2520\,a^9c^5x^9\log(x)-1890\,a^8c^5x^8+2520\,a^7c^5x^7+2940\,a^6c^5x^6-1764\,a^4c^5x^4-840\,a^3c^5x}{630\,a^{10}x^9}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="fricas")
```

output
```
1/630*(630*a^10*c^5*x^10 + 2520*a^9*c^5*x^9*log(x) - 1890*a^8*c^5*x^8 + 25
20*a^7*c^5*x^7 + 2940*a^6*c^5*x^6 - 1764*a^4*c^5*x^4 - 840*a^3*c^5*x^3 + 2
70*a^2*c^5*x^2 + 315*a*c^5*x + 70*c^5)/(a^10*x^9)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx = \frac{a^{10}c^5x+4a^9c^5\log(x)+\frac{-1890a^8c^5x^8+2520a^7c^5x^7+2940a^6c^5x^6-1764a^4c^5x^4-840a^3c^5x^3+270a^2c^5x^2+315ac^5x+70c^5}{630x^9}}{a^{10}}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**5,x)
```

output
```
(a**10*c**5*x + 4*a**9*c**5*log(x) + (-1890*a**8*c**5*x**8 + 2520*a**7*c**
5*x**7 + 2940*a**6*c**5*x**6 - 1764*a**4*c**5*x**4 - 840*a**3*c**5*x**3 +
270*a**2*c**5*x**2 + 315*a*c**5*x + 70*c**5)/(630*x**9))/a**10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx = c^5x+\frac{4\,c^5\log(x)}{a} - \frac{1890\,a^8c^5x^8-2520\,a^7c^5x^7-2940\,a^6c^5x^6+1764\,a^4c^5x^4+840\,a^3c^5x^3-270\,a^2c^5x^2-315\,ac^5x-70\,c^5}{630\,a^{10}x^9}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="maxima")
```

output
```
c^5*x + 4*c^5*log(x)/a - 1/630*(1890*a^8*c^5*x^8 - 2520*a^7*c^5*x^7 - 2940
*a^6*c^5*x^6 + 1764*a^4*c^5*x^4 + 840*a^3*c^5*x^3 - 270*a^2*c^5*x^2 - 315*
a*c^5*x - 70*c^5)/(a^10*x^9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.59

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx = -\frac{4c^5\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4c^5\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a} + \frac{\left(630c^5+\frac{4049c^5}{ax-1}+\frac{6201c^5}{(ax-1)^2}-\frac{18036c^5}{(ax-1)^3}-\frac{89124c^5}{(ax-1)^4}-\frac{160146c^5}{(ax-1)^5}-\frac{153090c^5}{(ax-1)^6}-\frac{80220c^5}{(ax-1)^7}-\frac{21420c^5}{(ax-1)^8}-\frac{2520c^5}{(ax-1)^9}\right)(ax-}{630a\left(\frac{1}{ax-1}+1\right)^9}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="giac")
```

output
```
-4*c^5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^5*log(abs(-1/(a*x -
1) - 1))/a + 1/630*(630*c^5 + 4049*c^5/(a*x - 1) + 6201*c^5/(a*x - 1)^2 -
18036*c^5/(a*x - 1)^3 - 89124*c^5/(a*x - 1)^4 - 160146*c^5/(a*x - 1)^5 - 1
53090*c^5/(a*x - 1)^6 - 80220*c^5/(a*x - 1)^7 - 21420*c^5/(a*x - 1)^8 - 25
20*c^5/(a*x - 1)^9)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^9)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx = \frac{c^5\left(\frac{ax}{2}+\frac{3a^2x^2}{7}-\frac{4a^3x^3}{3}-\frac{14a^4x^4}{5}+\frac{14a^6x^6}{3}+4a^7x^7-3a^8x^8+a^{10}x^{10}+4a^9x^9\ln(x)+\frac{1}{9}\right)}{a^{10}x^9}$$

input
```
int(((c - c/(a^2*x^2))^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(c^5*((a*x)/2 + (3*a^2*x^2)/7 - (4*a^3*x^3)/3 - (14*a^4*x^4)/5 + (14*a^6*x
^6)/3 + 4*a^7*x^7 - 3*a^8*x^8 + a^10*x^10 + 4*a^9*x^9*log(x) + 1/9))/(a^10
*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx = \frac{c^5(2520\log(x)\,a^9x^9+630a^{10}x^{10}-1890a^8x^8+2520a^7x^7+2940a^6x^6-1764a^4x^4-840a^3x^3+270a^2x^2}{630a^{10}x^9}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x)
```

output
```
(c**5*(2520*log(x)*a**9*x**9 + 630*a**10*x**10 - 1890*a**8*x**8 + 2520*a**
7*x**7 + 2940*a**6*x**6 - 1764*a**4*x**4 - 840*a**3*x**3 + 270*a**2*x**2 +
 315*a*x + 70))/(630*a**10*x**9)
```

3.774 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+c^4x+\frac{4c^4\log(x)}{a}$$

output
```
-1/7*c^4/a^8/x^7-2/3*c^4/a^7/x^6-4/5*c^4/a^6/x^5+c^4/a^5/x^4+10/3*c^4/a^4/
x^3+2*c^4/a^3/x^2-4*c^4/a^2/x+c^4*x+4*c^4*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+c^4x+\frac{4c^4\log(x)}{a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

output
```
-1/7*c^4/(a^8*x^7) - (2*c^4)/(3*a^7*x^6) - (4*c^4)/(5*a^6*x^5) + c^4/(a^5*
x^4) + (10*c^4)/(3*a^4*x^3) + (2*c^4)/(a^3*x^2) - (4*c^4)/(a^2*x) + c^4*x
+ (4*c^4*Log[x])/a
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^4 e^{4\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{c^4\left(a^2-\frac{1}{x^2}\right)^4 e^{4\text{arctanh}(ax)}}{a^8}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{c^4\int e^{4\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^4dx}{a^8}$$

$$\Big\downarrow \text{6707}$$

$$\frac{c^4\int \frac{e^{4\text{arctanh}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$

$$\Big\downarrow \text{6700}$$

$$\frac{c^4\int \frac{(1-ax)^2(ax+1)^6}{x^8}dx}{a^8}$$

$$\Big\downarrow \text{99}$$

$$\frac{c^4\int \left(a^8+\frac{4a^7}{x}+\frac{4a^6}{x^2}-\frac{4a^5}{x^3}-\frac{10a^4}{x^4}-\frac{4a^3}{x^5}+\frac{4a^2}{x^6}+\frac{4a}{x^7}+\frac{1}{x^8}\right)dx}{a^8}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^4\left(a^8x+4a^7\log(x)-\frac{4a^6}{x}+\frac{2a^5}{x^2}+\frac{10a^4}{3x^3}+\frac{a^3}{x^4}-\frac{4a^2}{5x^5}-\frac{2a}{3x^6}-\frac{1}{7x^7}\right)}{a^8}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

output
```
(c^4*(-1/7*1/x^7 - (2*a)/(3*x^6) - (4*a^2)/(5*x^5) + a^3/x^4 + (10*a^4)/(3
*x^3) + (2*a^5)/x^2 - (4*a^6)/x + a^8*x + 4*a^7*Log[x]))/a^8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.71

method	result
default	$\frac{c^{4}\left(x\,a^{8}-\frac{2a}{3x^{6}}+\frac{2a^{5}}{x^{2}}-\frac{4a^{6}}{x}-\frac{1}{7x^{7}}+\frac{a^{3}}{x^{4}}+4a^{7}\ln\left(x\right)+\frac{10a^{4}}{3x^{3}}-\frac{4a^{2}}{5x^{5}}\right)}{a^{8}}$
risch	$c^{4}x+\frac{-4a^{6}c^{4}x^{6}+2a^{5}c^{4}x^{5}+\frac{10}{3}a^{4}c^{4}x^{4}+a^{3}c^{4}x^{3}-\frac{4}{5}a^{2}c^{4}x^{2}-\frac{2}{3}a\,c^{4}x-\frac{1}{7}c^{4}}{a^{8}x^{7}}+\frac{4c^{4}\ln\left(x\right)}{a}$
parallelrisch	$\frac{105a^{8}c^{4}x^{8}+420c^{4}\ln\left(x\right)a^{7}x^{7}-420a^{6}c^{4}x^{6}+210a^{5}c^{4}x^{5}+350a^{4}c^{4}x^{4}+105a^{3}c^{4}x^{3}-84a^{2}c^{4}x^{2}-70a\,c^{4}x-15c^{4}}{105a^{8}x^{7}}$
norman	$\frac{-5a^{7}c^{4}x^{8}+a^{8}c^{4}x^{9}+\frac{c^{4}}{7a}+\frac{11c^{4}x}{21}+\frac{2a\,c^{4}x^{2}}{15}-\frac{9a^{2}c^{4}x^{3}}{5}-\frac{7a^{3}c^{4}x^{4}}{3}+\frac{4a^{4}c^{4}x^{5}}{3}+6a^{5}c^{4}x^{6}}{\left(ax-1\right)a^{7}x^{7}}+\frac{4c^{4}\ln\left(x\right)}{a}$
meijerg	$-\frac{c^{4}\left(-\frac{ax\left(-3ax+6\right)}{3\left(-ax+1\right)}-2\ln\left(-ax+1\right)\right)}{a}-\frac{3c^{4}x}{-ax+1}-\frac{2c^{4}\left(\frac{1}{ax}-1-2\ln\left(x\right)-2\ln\left(-a\right)-\frac{3ax}{-3ax+3}+2\ln\left(-ax+1\right)\right)}{a}-\frac{2c^{4}\left(\frac{1}{3x^{3}a^{3}}\right.}{}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
c^4/a^8*(x*a^8-2/3*a/x^6+2*a^5/x^2-4*a^6/x-1/7/x^7+a^3/x^4+4*a^7*ln(x)+10/
3*a^4/x^3-4/5*a^2/x^5)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = \frac{105\,a^{8}c^{4}x^{8}+420\,a^{7}c^{4}x^{7}\log\left(x\right)-420\,a^{6}c^{4}x^{6}+210\,a^{5}c^{4}x^{5}+350\,a^{4}c^{4}x^{4}+105\,a^{3}c^{4}x^{3}-84\,a^{2}c^{4}x^{2}-70\,ac}{105\,a^{8}x^{7}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
1/105*(105*a^8*c^4*x^8 + 420*a^7*c^4*x^7*log(x) - 420*a^6*c^4*x^6 + 210*a^
5*c^4*x^5 + 350*a^4*c^4*x^4 + 105*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*
x - 15*c^4)/(a^8*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = \frac{a^8c^4x+4a^7c^4\log(x)+\frac{-420a^6c^4x^6+210a^5c^4x^5+350a^4c^4x^4+105a^3c^4x^3-84a^2c^4x^2-70ac^4x-15c^4}{105x^7}}{a^8}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**4,x)
```

output
```
(a**8*c**4*x + 4*a**7*c**4*log(x) + (-420*a**6*c**4*x**6 + 210*a**5*c**4*x
**5 + 350*a**4*c**4*x**4 + 105*a**3*c**4*x**3 - 84*a**2*c**4*x**2 - 70*a*c
**4*x - 15*c**4)/(105*x**7))/a**8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = c^4x+\frac{4\,c^4\log(x)}{a} - \frac{420\,a^6c^4x^6-210\,a^5c^4x^5-350\,a^4c^4x^4-105\,a^3c^4x^3+84\,a^2c^4x^2+70\,ac^4x+15\,c^4}{105\,a^8x^7}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
c^4*x + 4*c^4*log(x)/a - 1/105*(420*a^6*c^4*x^6 - 210*a^5*c^4*x^5 - 350*a^
4*c^4*x^4 - 105*a^3*c^4*x^3 + 84*a^2*c^4*x^2 + 70*a*c^4*x + 15*c^4)/(a^8*x
^7)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.60

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$= -\frac{4\,c^4\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^4\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}$$
$$+\frac{\left(105\,c^4+\frac{659\,c^4}{ax-1}+\frac{1253\,c^4}{(ax-1)^2}-\frac{231\,c^4}{(ax-1)^3}-\frac{3885\,c^4}{(ax-1)^4}-\frac{5250\,c^4}{(ax-1)^5}-\frac{2730\,c^4}{(ax-1)^6}-\frac{420\,c^4}{(ax-1)^7}\right)(ax-1)}{105\,a\left(\frac{1}{ax-1}+1\right)^7}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
-4*c^4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^4*log(abs(-1/(a*x -
1) - 1))/a + 1/105*(105*c^4 + 659*c^4/(a*x - 1) + 1253*c^4/(a*x - 1)^2 - 2
31*c^4/(a*x - 1)^3 - 3885*c^4/(a*x - 1)^4 - 5250*c^4/(a*x - 1)^5 - 2730*c^
4/(a*x - 1)^6 - 420*c^4/(a*x - 1)^7)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^7)
```

Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.72

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$=\frac{c^4\left(a^3\,x^3-\frac{4\,a^2\,x^2}{5}-\frac{2\,a\,x}{3}+\frac{10\,a^4\,x^4}{3}+2\,a^5\,x^5-4\,a^6\,x^6+a^8\,x^8+4\,a^7\,x^7\,\ln(x)-\frac{1}{7}\right)}{a^8\,x^7}$$

input
```
int(((c - c/(a^2*x^2))^4*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(c^4*(a^3*x^3 - (4*a^2*x^2)/5 - (2*a*x)/3 + (10*a^4*x^4)/3 + 2*a^5*x^5 - 4
*a^6*x^6 + a^8*x^8 + 4*a^7*x^7*log(x) - 1/7))/(a^8*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.75

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$=\frac{c^4(420\log(x)\,a^7x^7+105a^8x^8-420a^6x^6+210a^5x^5+350a^4x^4+105a^3x^3-84a^2x^2-70ax-15)}{105a^8x^7}$$

input

```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x)
```

output

```
(c**4*(420*log(x)*a**7*x**7 + 105*a**8*x**8 - 420*a**6*x**6 + 210*a**5*x**
5 + 350*a**4*x**4 + 105*a**3*x**3 - 84*a**2*x**2 - 70*a*x - 15))/(105*a**8
*x**7)
```

3.775 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=\frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+c^3x+\frac{4c^3\log(x)}{a}$$

output
```
1/5*c^3/a^6/x^5+c^3/a^5/x^4+5/3*c^3/a^4/x^3-5*c^3/a^2/x+c^3*x+4*c^3*ln(x)/
a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=\frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+c^3x+\frac{4c^3\log(x)}{a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

output
```
c^3/(5*a^6*x^5) + c^3/(a^5*x^4) + (5*c^3)/(3*a^4*x^3) - (5*c^3)/(a^2*x) +
c^3*x + (4*c^3*Log[x])/a
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^3 e^{4\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{c^3\left(a^2-\frac{1}{x^2}\right)^3 e^{4\text{arctanh}(ax)}}{a^6}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{c^3\int e^{4\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^3dx}{a^6}$$

$$\Big\downarrow \text{6707}$$

$$-\frac{c^3\int \frac{e^{4\text{arctanh}(ax)}\left(1-a^2x^2\right)^3}{x^6}dx}{a^6}$$

$$\Big\downarrow \text{6700}$$

$$-\frac{c^3\int \frac{(1-ax)(ax+1)^5}{x^6}dx}{a^6}$$

$$\Big\downarrow \text{84}$$

$$-\frac{c^3\int \left(-a^6-\frac{4a^5}{x}-\frac{5a^4}{x^2}+\frac{5a^2}{x^4}+\frac{4a}{x^5}+\frac{1}{x^6}\right)dx}{a^6}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^3\left(a^6(-x)-4a^5\log(x)+\frac{5a^4}{x}-\frac{5a^2}{3x^3}-\frac{a}{x^4}-\frac{1}{5x^5}\right)}{a^6}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

output
```
-((c^3*(-1/5*1/x^5 - a/x^4 - (5*a^2)/(3*x^3) + (5*a^4)/x - a^6*x - 4*a^5*L
og[x]))/a^6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

method	result
default	$\frac{c^{3}\left(x\,a^{6}-\frac{5a^{4}}{x}+\frac{a}{x^{4}}+4a^{5}\ln\left(x\right)+\frac{5a^{2}}{3x^{3}}+\frac{1}{5x^{5}}\right)}{a^{6}}$
risch	$c^{3}x+\frac{-5a^{4}c^{3}x^{4}+\frac{5}{3}a^{2}c^{3}x^{2}+a\,c^{3}x+\frac{1}{5}c^{3}}{a^{6}x^{5}}+\frac{4c^{3}\ln\left(x\right)}{a}$
parallelrisch	$\frac{15a^{6}c^{3}x^{6}+60c^{3}\ln\left(x\right)a^{5}x^{5}-75a^{4}c^{3}x^{4}+25a^{2}c^{3}x^{2}+15a\,c^{3}x+3c^{3}}{15a^{6}x^{5}}$
norman	$\frac{-6a^{5}c^{3}x^{6}+a^{6}c^{3}x^{7}-\frac{c^{3}}{5a}-\frac{4c^{3}x}{5}-\frac{2a\,c^{3}x^{2}}{3}+\frac{5a^{2}c^{3}x^{3}}{3}+5a^{3}c^{3}x^{4}}{\left(ax-1\right)a^{5}x^{5}}+\frac{4c^{3}\ln\left(x\right)}{a}$
meijerg	$-\frac{c^{3}\left(-\frac{ax\left(-3ax+6\right)}{3\left(-ax+1\right)}-2\ln\left(-ax+1\right)\right)}{a}-\frac{2c^{3}x}{-ax+1}-\frac{2c^{3}\left(\frac{1}{3x^{3}a^{3}}+\frac{1}{a^{2}x^{2}}+\frac{3}{ax}-1-4\ln\left(x\right)-4\ln\left(-a\right)-\frac{5ax}{-5ax+5}+4\ln\left(-ax+1\right)\right)}{a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
c^3/a^6*(x*a^6-5*a^4/x+a/x^4+4*a^5*ln(x)+5/3*a^2/x^3+1/5/x^5)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{15\,a^{6}c^{3}x^{6}+60\,a^{5}c^{3}x^{5}\log\left(x\right)-75\,a^{4}c^{3}x^{4}+25\,a^{2}c^{3}x^{2}+15\,ac^{3}x+3\,c^{3}}{15\,a^{6}x^{5}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
1/15*(15*a^6*c^3*x^6 + 60*a^5*c^3*x^5*log(x) - 75*a^4*c^3*x^4 + 25*a^2*c^3
*x^2 + 15*a*c^3*x + 3*c^3)/(a^6*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=\frac{a^6c^3x+4a^5c^3\log(x)+\frac{-75a^4c^3x^4+25a^2c^3x^2+15ac^3x+3c^3}{15x^5}}{a^6}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**3,x)
```

output
```
(a**6*c**3*x + 4*a**5*c**3*log(x) + (-75*a**4*c**3*x**4 + 25*a**2*c**3*x**
2 + 15*a*c**3*x + 3*c**3)/(15*x**5))/a**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=c^3x+\frac{4\,c^3\log(x)}{a}-\frac{75\,a^4c^3x^4-25\,a^2c^3x^2-15\,ac^3x-3\,c^3}{15\,a^6x^5}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
c^3*x + 4*c^3*log(x)/a - 1/15*(75*a^4*c^3*x^4 - 25*a^2*c^3*x^2 - 15*a*c^3*
x - 3*c^3)/(a^6*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(59) = 118$.

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.16

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx&=-\frac{4\,c^3\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^3\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}\\&\quad+\frac{\left(15\,c^3+\frac{107\,c^3}{ax-1}+\frac{235\,c^3}{(ax-1)^2}+\frac{170\,c^3}{(ax-1)^3}-\frac{30\,c^3}{(ax-1)^4}-\frac{60\,c^3}{(ax-1)^5}\right)(ax-1)}{15\,a\left(\frac{1}{ax-1}+1\right)^5}\end{aligned}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
-4*c^3*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^3*log(abs(-1/(a*x -
1) - 1))/a + 1/15*(15*c^3 + 107*c^3/(a*x - 1) + 235*c^3/(a*x - 1)^2 + 170*
c^3/(a*x - 1)^3 - 30*c^3/(a*x - 1)^4 - 60*c^3/(a*x - 1)^5)*(a*x - 1)/(a*(1
/(a*x - 1) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 13.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=\frac{c^3\left(a\,x+\frac{5\,a^2\,x^2}{3}-5\,a^4\,x^4+a^6\,x^6+4\,a^5\,x^5\,\ln\left(x\right)+\frac{1}{5}\right)}{a^6\,x^5}$$

input
```
int(((c - c/(a^2*x^2))^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
(c^3*(a*x + (5*a^2*x^2)/3 - 5*a^4*x^4 + a^6*x^6 + 4*a^5*x^5*log(x) + 1/5))
/(a^6*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=\frac{c^3(60\log(x)\,a^5x^5+15a^6x^6-75a^4x^4+25a^2x^2+15ax+3)}{15a^6x^5}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x)
```

output
```
(c**3*(60*log(x)*a**5*x**5 + 15*a**6*x**6 - 75*a**4*x**4 + 25*a**2*x**2 +
15*a*x + 3))/(15*a**6*x**5)
```

3.776 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=-\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}$$

output
```
-1/3*c^2/a^4/x^3-2*c^2/a^3/x^2-6*c^2/a^2/x+c^2*x+4*c^2*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=-\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

output
```
-1/3*c^2/(a^4*x^3) - (2*c^2)/(a^3*x^2) - (6*c^2)/(a^2*x) + c^2*x + (4*c^2*
Log[x])/a
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^2 e^{4\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{c^2\left(a^2-\frac{1}{x^2}\right)^2 e^{4\text{arctanh}(ax)}}{a^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c^2\int e^{4\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^2dx}{a^4}$$

$$\Big\downarrow \text{ 6707}$$

$$\frac{c^2\int \frac{e^{4\text{arctanh}(ax)}(1-a^2x^2)^2}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{ 6700}$$

$$\frac{c^2\int \frac{(ax+1)^4}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{c^2\int \left(a^4+\frac{4a^3}{x}+\frac{6a^2}{x^2}+\frac{4a}{x^3}+\frac{1}{x^4}\right)dx}{a^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^2\left(a^4x+4a^3\log(x)-\frac{6a^2}{x}-\frac{2a}{x^2}-\frac{1}{3x^3}\right)}{a^4}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

output
```
(c^2*(-1/3*1/x^3 - (2*a)/x^2 - (6*a^2)/x + a^4*x + 4*a^3*Log[x]))/a^4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

method	result
default	$\frac{c^{2}\left(x\,a^{4}-\frac{2a}{x^{2}}-\frac{6a^{2}}{x}+4a^{3}\ln\left(x\right)-\frac{1}{3x^{3}}\right)}{a^{4}}$
risch	$c^{2}x+\frac{-6c^{2}a^{2}x^{2}-2a\,c^{2}x-\frac{1}{3}c^{2}}{a^{4}x^{3}}+\frac{4c^{2}\ln\left(x\right)}{a}$
parallelrisch	$\frac{3a^{4}c^{2}x^{4}+12c^{2}\ln\left(x\right)a^{3}x^{3}-18c^{2}a^{2}x^{2}-6a\,c^{2}x-c^{2}}{3a^{4}x^{3}}$
norman	$\frac{-7a^{3}c^{2}x^{4}+c^{2}x^{5}a^{4}+\frac{c^{2}}{3a}+\frac{5c^{2}x}{3}+4a\,c^{2}x^{2}}{(ax-1)a^{3}x^{3}}+\frac{4c^{2}\ln\left(x\right)}{a}$
meijerg	$-\frac{c^{2}\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln\left(-ax+1\right)\right)}{a}-\frac{c^{2}x}{-ax+1}+\frac{c^{2}\left(\frac{1}{ax}-1-2\ln\left(x\right)-2\ln\left(-a\right)-\frac{3ax}{-3ax+3}+2\ln\left(-ax+1\right)\right)}{a}+\frac{2c^{2}\left(\frac{ax}{-ax+1}\right.}{}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2/a^4*(x*a^4-2*a/x^2-6*a^2/x+4*a^3*ln(x)-1/3/x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^{2}x^{2}}\right)^{2}\,dx=\frac{3\,a^{4}c^{2}x^{4}+12\,a^{3}c^{2}x^{3}\log\left(x\right)-18\,a^{2}c^{2}x^{2}-6\,ac^{2}x-c^{2}}{3\,a^{4}x^{3}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*c^2*x^4 + 12*a^3*c^2*x^3*log(x) - 18*a^2*c^2*x^2 - 6*a*c^2*x -
c^2)/(a^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{a^4c^2x+4a^3c^2\log\left(x\right)+\frac{-18a^2c^2x^2-6ac^2x-c^2}{3x^3}}{a^4}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**2,x)
```

output
```
(a**4*c**2*x + 4*a**3*c**2*log(x) + (-18*a**2*c**2*x**2 - 6*a*c**2*x - c**
2)/(3*x**3))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=c^2x+\frac{4\,c^2\log\left(x\right)}{a}-\frac{18\,a^2c^2x^2+6\,ac^2x+c^2}{3\,a^4x^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
c^2*x + 4*c^2*log(x)/a - 1/3*(18*a^2*c^2*x^2 + 6*a*c^2*x + c^2)/(a^4*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(49) = 98.

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.20

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=-\frac{4\,c^2\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^2\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(3\,c^2+\frac{34\,c^2}{ax-1}+\frac{66\,c^2}{(ax-1)^2}+\frac{36\,c^2}{(ax-1)^3}\right)(ax-1)}{3\,a\left(\frac{1}{ax-1}+1\right)^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
-4*c^2*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^2*log(abs(-1/(a*x -
1) - 1))/a + 1/3*(3*c^2 + 34*c^2/(a*x - 1) + 66*c^2/(a*x - 1)^2 + 36*c^2/(
a*x - 1)^3)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=-\frac{c^2\,\left(6\,a\,x+18\,a^2\,x^2-3\,a^4\,x^4-12\,a^3\,x^3\,\ln\left(x\right)+1\right)}{3\,a^4\,x^3}$$

input
```
int(((c - c/(a^2*x^2))^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
-(c^2*(6*a*x + 18*a^2*x^2 - 3*a^4*x^4 - 12*a^3*x^3*log(x) + 1))/(3*a^4*x^3
)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2(12\log(x)\,a^3x^3+3a^4x^4-18a^2x^2-6ax-1)}{3a^4x^3}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x)
```

output
```
(c**2*(12*log(x)*a**3*x**3 + 3*a**4*x**4 - 18*a**2*x**2 - 6*a*x - 1))/(3*a
**4*x**3)
```

3.777 $\int e^{4 \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 33

$$\int e^{4 \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right) \, dx=\frac{c}{a^2x}+cx-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}$$

output
```
c/a^2/x+c*x-4*c*ln(x)/a+8*c*ln(-a*x+1)/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{4 \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right) \, dx=\frac{c}{a^2x}+cx-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}$$

input
```
Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

output
```
c/(a^2*x) + c*x - (4*c*Log[x])/a + (8*c*Log[1 - a*x])/a
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right) e^{4\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{c\left(a^2-\frac{1}{x^2}\right) e^{4\text{arctanh}(ax)}}{a^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\int e^{4\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{ 6707}$$

$$-\frac{c\int \frac{e^{4\text{arctanh}(ax)}(1-a^2x^2)}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{ 6700}$$

$$-\frac{c\int \frac{(ax+1)^3}{x^2(1-ax)}dx}{a^2}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{c\int \left(-\frac{8a^2}{ax-1}-a^2+\frac{4a}{x}+\frac{1}{x^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{c\left(a^2(-x)+4a\log(x)-8a\log(1-ax)-\frac{1}{x}\right)}{a^2}$$

input
```
Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

output
```
-((c*(-x^(-1) - a^2*x + 4*a*Log[x] - 8*a*Log[1 - a*x]))/a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result
default	$\frac{c\left(a^{2}x+8a\ln(ax-1)+\frac{1}{x}-4\ln(x)a\right)}{a^{2}}$
risch	$\frac{c}{a^{2}x}+xc-\frac{4c\ln(x)}{a}+\frac{8c\ln(-ax+1)}{a}$
parallelrisch	$-\frac{-a^{2}c\,x^{2}+4c\ln(x)ax-8c\ln(ax-1)ax-c}{a^{2}x}$
norman	$\frac{a^{2}c\,x^{3}-\frac{c}{a}}{ax(ax-1)}-\frac{4c\ln(x)}{a}+\frac{8c\ln(ax-1)}{a}$
meijerg	$-\frac{c\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}+\frac{2c\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}-\frac{2c\left(1+\ln(x)+\ln(-a)+\frac{2ax}{-2ax+2}-\ln(-ax+1)\right)}{a}+c\left(\frac{1}{ax}\right.$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
c/a^2*(a^2*x+8*a*ln(a*x-1)+1/x-4*ln(x)*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^{2}x^{2}}\right)\,dx=\frac{a^{2}cx^{2}+8\,acx\log\left(ax-1\right)-4\,acx\log\left(x\right)+c}{a^{2}x}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
(a^2*c*x^2 + 8*a*c*x*log(a*x - 1) - 4*a*c*x*log(x) + c)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = cx+\frac{4c\left(-\log{(x)}+2\log{\left(x-\frac{1}{a}\right)}\right)}{a}+\frac{c}{a^2x}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2),x)
```

output
```
c*x + 4*c*(-log(x) + 2*log(x - 1/a))/a + c/(a**2*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = cx+\frac{8\,c\log\left(ax-1\right)}{a}-\frac{4\,c\log\left(x\right)}{a}+\frac{c}{a^2x}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
c*x + 8*c*log(a*x - 1)/a - 4*c*log(x)/a + c/(a^2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = -\frac{4\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{4\,c\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{(ax-1)c}{a\left(\frac{1}{ax-1}+1\right)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
-4*c*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - 4*c*log(abs(-1/(a*x - 1) -
 1))/a + (a*x - 1)*c/(a*(1/(a*x - 1) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = c\,x+\frac{c}{a^2\,x}-\frac{4\,c\,\ln\left(x\right)}{a}+\frac{8\,c\,\ln\left(a\,x-1\right)}{a}$$

input
```
int(((c - c/(a^2*x^2))*(a*x + 1)^2)/(a*x - 1)^2,x)
```

output
```
c*x + c/(a^2*x) - (4*c*log(x))/a + (8*c*log(a*x - 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{c(8\log(ax-1)\,ax-4\log(x)\,ax+a^2x^2+1)}{a^2x}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x)
```

output
```
(c*(8*log(a*x - 1)*a*x - 4*log(x)*a*x + a**2*x**2 + 1))/(a**2*x)
```

3.778 $\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{x}{c}-\frac{1}{ac(1-ax)^2}+\frac{5}{ac(1-ax)}+\frac{4\log(1-ax)}{ac}$$

output
```
x/c-1/a/c/(-a*x+1)^2+5/a/c/(-a*x+1)+4*ln(-a*x+1)/a/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{x}{c}-\frac{1}{ac(1-ax)^2}+\frac{5}{ac(1-ax)}+\frac{4\log(1-ax)}{ac}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

output
```
x/c - 1/(a*c*(1 - a*x)^2) + 5/(a*c*(1 - a*x)) + (4*Log[1 - a*x])/(a*c)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{a^2e^{4\text{arctanh}(ax)}}{c\left(a^2-\frac{1}{x^2}\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2\int \frac{e^{4\text{arctanh}(ax)}}{a^2-\frac{1}{x^2}}dx}{c}$$

$$\Big\downarrow \text{6707}$$

$$-\frac{a^2\int \frac{e^{4\text{arctanh}(ax)}x^2}{1-a^2x^2}dx}{c}$$

$$\Big\downarrow \text{6700}$$

$$-\frac{a^2\int \frac{x^2(ax+1)}{(1-ax)^3}dx}{c}$$

$$\Big\downarrow \text{86}$$

$$-\frac{a^2\int \left(-\frac{1}{a^2}-\frac{4}{a^2(ax-1)}-\frac{5}{a^2(ax-1)^2}-\frac{2}{a^2(ax-1)^3}\right)dx}{c}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\left(-\frac{5}{a^3(1-ax)}+\frac{1}{a^3(1-ax)^2}-\frac{4\log(1-ax)}{a^3}-\frac{x}{a^2}\right)}{c}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

output
```
-((a^2*(-(x/a^2) + 1/(a^3*(1 - a*x)^2) - 5/(a^3*(1 - a*x)) - (4*Log[1 - a*
x])/a^3))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{x}{c}+\frac{-5xc+\frac{4c}{a}}{c^{2}\left(ax-1\right)^{2}}+\frac{4\ln\left(ax-1\right)}{ac}$	43
default	$\frac{a^{2}\left(\frac{x}{a^{2}}-\frac{5}{\left(ax-1\right)a^{3}}+\frac{4\ln\left(ax-1\right)}{a^{3}}-\frac{1}{a^{3}\left(ax-1\right)^{2}}\right)}{c}$	49
norman	$\frac{\frac{a^{2}x^{3}}{c}-\frac{6a\,x^{2}}{c}+\frac{4x}{c}}{\left(ax-1\right)^{2}}+\frac{4\ln\left(ax-1\right)}{ac}$	50
parallelrisch	$\frac{a^{3}x^{3}+4a^{2}\ln\left(ax-1\right)x^{2}-6a^{2}x^{2}-8a\ln\left(ax-1\right)x+4ax+4\ln\left(ax-1\right)}{\left(ax-1\right)^{2}ca}$	67

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
x/c+(-5*x*c+4*c/a)/c^2/(a*x-1)^2+4/a/c*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{a^3x^3-2\,a^2x^2-4\,ax+4\,(a^2x^2-2\,ax+1)\log\left(ax-1\right)+4}{a^3cx^2-2\,a^2cx+ac}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
(a^3*x^3 - 2*a^2*x^2 - 4*a*x + 4*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 4)/(
a^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{-5ax+4}{a^3cx^2-2a^2cx+ac}+\frac{x}{c}+\frac{4\log(ax-1)}{ac}$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2),x)
```

output
```
(-5*a*x + 4)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + x/c + 4*log(a*x - 1)/(a*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{5\,ax-4}{a^3cx^2-2\,a^2cx+ac}+\frac{x}{c}+\frac{4\,\log(ax-1)}{ac}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
-(5*a*x - 4)/(a^3*c*x^2 - 2*a^2*c*x + a*c) + x/c + 4*log(a*x - 1)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{ax-1}{ac}-\frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac}-\frac{\frac{5\,a^3c}{ax-1}+\frac{a^3c}{(ax-1)^2}}{a^4c^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
(a*x - 1)/(a*c) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) - (5*a^3*
c/(a*x - 1) + a^3*c/(a*x - 1)^2)/(a^4*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{x}{c} - \frac{5\,x-\frac{4}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c} + \frac{4\,\ln\left(a\,x-1\right)}{a\,c}$$

input
```
int((a*x + 1)^2/((c - c/(a^2*x^2))*(a*x - 1)^2),x)
```

output
```
x/c - (5*x - 4/a)/(c + a^2*c*x^2 - 2*a*c*x) + (4*log(a*x - 1))/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{4\log(ax-1)\,a^2x^2-8\log(ax-1)\,ax+4\log(ax-1)+a^3x^3-4a^2x^2+2}{ac\left(a^2x^2-2ax+1\right)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x)
```

output
```
(4*log(a*x - 1)*a**2*x**2 - 8*log(a*x - 1)*a*x + 4*log(a*x - 1) + a**3*x**
3 - 4*a**2*x**2 + 2)/(a*c*(a**2*x**2 - 2*a*x + 1))
```

3.779 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{x}{c^2}+\frac{1}{3ac^2(1-ax)^3}-\frac{2}{ac^2(1-ax)^2}+\frac{6}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}$$

output
```
x/c^2+1/3/a/c^2/(-a*x+1)^3-2/a/c^2/(-a*x+1)^2+6/a/c^2/(-a*x+1)+4*ln(-a*x+1
)/a/c^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{-13+27ax-9a^2x^2-9a^3x^3+3a^4x^4+12(-1+ax)^3\log(1-ax)}{3ac^2(-1+ax)^3}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

output
```
(-13 + 27*a*x - 9*a^2*x^2 - 9*a^3*x^3 + 3*a^4*x^4 + 12*(-1 + a*x)^3*Log[1
- a*x])/(3*a*c^2*(-1 + a*x)^3)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{a^4e^{4\text{arctanh}(ax)}}{c^2\left(a^2-\frac{1}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{a^4\int \frac{e^{4\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^2}dx}{c^2}$$

$$\Big\downarrow \text{6707}$$

$$\frac{a^4\int \frac{e^{4\text{arctanh}(ax)}x^4}{(1-a^2x^2)^2}dx}{c^2}$$

$$\Big\downarrow \text{6700}$$

$$\frac{a^4\int \frac{x^4}{(1-ax)^4}dx}{c^2}$$

$$\Big\downarrow \text{49}$$

$$\frac{a^4\int \left(\frac{1}{a^4}+\frac{4}{a^4(ax-1)}+\frac{6}{a^4(ax-1)^2}+\frac{4}{a^4(ax-1)^3}+\frac{1}{a^4(ax-1)^4}\right)dx}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4\left(\frac{6}{a^5(1-ax)}-\frac{2}{a^5(1-ax)^2}+\frac{1}{3a^5(1-ax)^3}+\frac{4\log(1-ax)}{a^5}+\frac{x}{a^4}\right)}{c^2}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

output
```
(a^4*(x/a^4 + 1/(3*a^5*(1 - a*x)^3) - 2/(a^5*(1 - a*x)^2) + 6/(a^5*(1 - a*
x)) + (4*Log[1 - a*x])/a^5))/c^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x}{c^{2}}+\frac{-6a\,c^{2}x^{2}+10c^{2}x-\frac{13c^{2}}{3a}}{c^{4}\left(ax-1\right)^{3}}+\frac{4\ln\left(ax-1\right)}{a\,c^{2}}$	56
default	$\frac{a^{4}\left(\frac{x}{a^{4}}-\frac{6}{a^{5}\left(ax-1\right)}+\frac{4\ln\left(ax-1\right)}{a^{5}}-\frac{2}{a^{5}\left(ax-1\right)^{2}}-\frac{1}{3a^{5}\left(ax-1\right)^{3}}\right)}{c^{2}}$	61
norman	$\frac{\frac{a^{4}x^{5}}{c}+\frac{6a\,x^{2}}{c}-\frac{4x}{c}+\frac{8a^{2}x^{3}}{3c}-\frac{19a^{3}x^{4}}{3c}}{c\left(ax+1\right)\left(ax-1\right)^{3}}+\frac{4\ln\left(ax-1\right)}{a\,c^{2}}$	82
parallelrisch	$\frac{3a^{4}x^{4}+12a^{3}\ln\left(ax-1\right)x^{3}-22a^{3}x^{3}-36a^{2}\ln\left(ax-1\right)x^{2}+30a^{2}x^{2}+36a\ln\left(ax-1\right)x-12ax-12\ln\left(ax-1\right)}{3\left(ax-1\right)^{3}c^{2}a}$	91

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x/c^2+(-6*a*c^2*x^2+10*c^2*x-13/3*c^2/a)/c^4/(a*x-1)^3+4/a/c^2*ln(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{3\,a^{4}x^{4}-9\,a^{3}x^{3}-9\,a^{2}x^{2}+27\,ax+12\,\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(ax-1\right)-13}{3\,\left(a^{4}c^{2}x^{3}-3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x-ac^{2}\right)}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*x^4 - 9*a^3*x^3 - 9*a^2*x^2 + 27*a*x + 12*(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)*log(a*x - 1) - 13)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
 - a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = a^4\left(\frac{-18a^2x^2+30ax-13}{3a^8c^2x^3-9a^7c^2x^2+9a^6c^2x-3a^5c^2}+\frac{x}{a^4c^2}+\frac{4\log(ax-1)}{a^5c^2}\right)$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**2,x)
```

output
```
a**4*((-18*a**2*x**2 + 30*a*x - 13)/(3*a**8*c**2*x**3 - 9*a**7*c**2*x**2 +
 9*a**6*c**2*x - 3*a**5*c**2) + x/(a**4*c**2) + 4*log(a*x - 1)/(a**5*c**2)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{18\,a^2x^2-30\,ax+13}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}+\frac{x}{c^2}+\frac{4\,\log(ax-1)}{ac^2}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
-1/3*(18*a^2*x^2 - 30*a*x + 13)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
 - a*c^2) + x/c^2 + 4*log(a*x - 1)/(a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.31

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{ax-1}{ac^2}-\frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^2}-\frac{\frac{18\,a^5c^4}{ax-1}+\frac{6\,a^5c^4}{(ax-1)^2}+\frac{a^5c^4}{(ax-1)^3}}{3\,a^6c^6}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
(a*x - 1)/(a*c^2) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^2) - 1/3
*(18*a^5*c^4/(a*x - 1) + 6*a^5*c^4/(a*x - 1)^2 + a^5*c^4/(a*x - 1)^3)/(a^6
*c^6)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{6\,a\,x^2-10\,x+\frac{13}{3\,a}}{-a^3\,c^2\,x^3+3\,a^2\,c^2\,x^2-3\,a\,c^2\,x+c^2}+\frac{x}{c^2}+\frac{4\,\ln\left(a\,x-1\right)}{a\,c^2}$$

input
```
int((a*x + 1)^2/((c - c/(a^2*x^2))^2*(a*x - 1)^2),x)
```

output
```
(6*a*x^2 - 10*x + 13/(3*a))/(c^2 + 3*a^2*c^2*x^2 - a^3*c^2*x^3 - 3*a*c^2*x
) + x/c^2 + (4*log(a*x - 1))/(a*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$
$$= \frac{12\log(ax-1)\,a^3x^3-36\log(ax-1)\,a^2x^2+36\log(ax-1)\,ax-12\log(ax-1)+3a^4x^4-12a^3x^3+18a}{3a\,c^2\left(a^3x^3-3a^2x^2+3ax-1\right)}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x)
```

output
```
(12*log(a*x - 1)*a**3*x**3 - 36*log(a*x - 1)*a**2*x**2 + 36*log(a*x - 1)*a
*x - 12*log(a*x - 1) + 3*a**4*x**4 - 12*a**3*x**3 + 18*a*x - 10)/(3*a*c**2
*(a**3*x**3 - 3*a**2*x**2 + 3*a*x - 1))
```

3.780 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{x}{c^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{11}{12ac^3(1-ax)^3}-\frac{49}{16ac^3(1-ax)^2}+\frac{111}{16ac^3(1-ax)}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(1+ax)}{32ac^3}$$

output
```
x/c^3-1/8/a/c^3/(-a*x+1)^4+11/12/a/c^3/(-a*x+1)^3-49/16/a/c^3/(-a*x+1)^2+1
11/16/a/c^3/(-a*x+1)+129/32*ln(-a*x+1)/a/c^3-1/32*ln(a*x+1)/a/c^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.80

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$=\frac{2(224-701ax+660a^2x^2-45a^3x^3-192a^4x^4+48a^5x^5)+387(-1+ax)^4\log(1-ax)-3(-1+ax)^4\,\mathrm{l}}{96ac^3(-1+ax)^4}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

output
```
(2*(224 - 701*a*x + 660*a^2*x^2 - 45*a^3*x^3 - 192*a^4*x^4 + 48*a^5*x^5) +
 387*(-1 + a*x)^4*Log[1 - a*x] - 3*(-1 + a*x)^4*Log[1 + a*x])/(96*a*c^3*(-
1 + a*x)^4)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$\int \frac{a^6e^{4\text{arctanh}(ax)}}{c^3\left(a^2-\frac{1}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{a^6\int \frac{e^{4\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^3}dx}{c^3}$$

$$\Big\downarrow \text{6707}$$

$$-\frac{a^6\int \frac{e^{4\text{arctanh}(ax)}x^6}{(1-a^2x^2)^3}dx}{c^3}$$

$$\Big\downarrow \text{6700}$$

$$-\frac{a^6\int \frac{x^6}{(1-ax)^5(ax+1)}dx}{c^3}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^6\int \left(\frac{1}{32a^6(ax+1)}-\frac{1}{a^6}-\frac{129}{32a^6(ax-1)}-\frac{111}{16a^6(ax-1)^2}-\frac{49}{8a^6(ax-1)^3}-\frac{11}{4a^6(ax-1)^4}-\frac{1}{2a^6(ax-1)^5}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6\left(-\frac{111}{16a^7(1-ax)}+\frac{49}{16a^7(1-ax)^2}-\frac{11}{12a^7(1-ax)^3}+\frac{1}{8a^7(1-ax)^4}-\frac{129\log(1-ax)}{32a^7}+\frac{\log(ax+1)}{32a^7}-\frac{x}{a^6}\right)}{c^3}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

output
```
-((a^6*(-(x/a^6) + 1/(8*a^7*(1 - a*x)^4) - 11/(12*a^7*(1 - a*x)^3) + 49/(1
6*a^7*(1 - a*x)^2) - 111/(16*a^7*(1 - a*x)) - (129*Log[1 - a*x])/(32*a^7)
+ Log[1 + a*x]/(32*a^7)))/c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.74

method	result
risch	$\frac{x}{c^{3}}+\frac{-\frac{111a^{2}c^{3}x^{3}}{16}+\frac{71a\,c^{3}x^{2}}{4}-\frac{749c^{3}x}{48}+\frac{14c^{3}}{3a}}{c^{6}\left(ax-1\right)^{4}}+\frac{129\ln\left(-ax+1\right)}{32a\,c^{3}}-\frac{\ln\left(ax+1\right)}{32a\,c^{3}}$
default	$\frac{a^{6}\left(\frac{x}{a^{6}}-\frac{\ln\left(ax+1\right)}{32a^{7}}-\frac{1}{8a^{7}\left(ax-1\right)^{4}}-\frac{11}{12a^{7}\left(ax-1\right)^{3}}-\frac{49}{16a^{7}\left(ax-1\right)^{2}}-\frac{111}{16a^{7}\left(ax-1\right)}+\frac{129\ln\left(ax-1\right)}{32a^{7}}\right)}{c^{3}}$
norman	$\frac{\frac{a^{6}x^{7}}{c}+\frac{65x}{16c}-\frac{49a\,x^{2}}{8c}-\frac{161a^{2}x^{3}}{24c}+\frac{301a^{3}x^{4}}{24c}+\frac{67a^{4}x^{5}}{48c}-\frac{20a^{5}x^{6}}{3c}}{c^{2}\left(ax+1\right)^{2}\left(ax-1\right)^{4}}+\frac{129\ln\left(ax-1\right)}{32a\,c^{3}}-\frac{\ln\left(ax+1\right)}{32a\,c^{3}}$
parallelrisch	$\frac{1702a^{3}x^{3}+390ax-832a^{4}x^{4}+96a^{5}x^{5}+12\ln\left(ax+1\right)xa-3\ln\left(ax+1\right)-1368a^{2}x^{2}+387\ln\left(ax-1\right)-18\ln\left(ax+1\right)x^{2}a^{2}+387\ln(ax-}{96\left(ax-1\right)^{4}c^{3}a}$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
x/c^3+(-111/16*a^2*c^3*x^3+71/4*a*c^3*x^2-749/48*c^3*x+14/3*c^3/a)/c^6/(a*
x-1)^4+129/32*ln(-a*x+1)/a/c^3-1/32*ln(a*x+1)/a/c^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.47

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{96\,a^5x^5-384\,a^4x^4-90\,a^3x^3+1320\,a^2x^2-1402\,ax-3\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(ax+1\right.}{96\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+\right.}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
1/96*(96*a^5*x^5 - 384*a^4*x^4 - 90*a^3*x^3 + 1320*a^2*x^2 - 1402*a*x - 3*
(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x + 1) + 387*(a^4*x^4
- 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 448)/(a^5*c^3*x^4 - 4*
a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.03

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = a^6\left(\frac{-333a^3x^3+852a^2x^2-749ax+224}{48a^{11}c^3x^4-192a^{10}c^3x^3+288a^9c^3x^2-192a^8c^3x+48a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{129\log\left(x-\frac{1}{a}\right)}{32}-\frac{\log\left(x+\frac{1}{a}\right)}{32}}{a^7c^3}\right)$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**3,x)
```

output
```
a**6*((-333*a**3*x**3 + 852*a**2*x**2 - 749*a*x + 224)/(48*a**11*c**3*x**4
 - 192*a**10*c**3*x**3 + 288*a**9*c**3*x**2 - 192*a**8*c**3*x + 48*a**7*c*
*3) + x/(a**6*c**3) + (129*log(x - 1/a)/32 - log(x + 1/a)/32)/(a**7*c**3))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{333\,a^3x^3-852\,a^2x^2+749\,ax-224}{48\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}+\frac{x}{c^3}-\frac{\log(ax+1)}{32\,ac^3}+\frac{129\,\log(ax-1)}{32\,ac^3}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
-1/48*(333*a^3*x^3 - 852*a^2*x^2 + 749*a*x - 224)/(a^5*c^3*x^4 - 4*a^4*c^3
*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + x/c^3 - 1/32*log(a*x + 1)/(a
*c^3) + 129/32*log(a*x - 1)/(a*c^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{ax-1}{ac^3} - \frac{4\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^3} - \frac{\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{32\,ac^3} - \frac{\frac{333\,a^{11}c^9}{ax-1}+\frac{147\,a^{11}c^9}{(ax-1)^2}+\frac{44\,a^{11}c^9}{(ax-1)^3}+\frac{6\,a^{11}c^9}{(ax-1)^4}}{48\,a^{12}c^{12}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
(a*x - 1)/(a*c^3) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^3) - 1/3
2*log(abs(-2/(a*x - 1) - 1))/(a*c^3) - 1/48*(333*a^11*c^9/(a*x - 1) + 147*
a^11*c^9/(a*x - 1)^2 + 44*a^11*c^9/(a*x - 1)^3 + 6*a^11*c^9/(a*x - 1)^4)/(
a^12*c^12)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{x}{c^3} - \frac{\frac{749\,x}{48}-\frac{71\,a\,x^2}{4}-\frac{14}{3\,a}+\frac{111\,a^2\,x^3}{16}}{a^4\,c^3\,x^4-4\,a^3\,c^3\,x^3+6\,a^2\,c^3\,x^2-4\,a\,c^3\,x+c^3} + \frac{129\,\ln(a\,x-1)}{32\,a\,c^3} - \frac{\ln(a\,x+1)}{32\,a\,c^3}$$

input
```
int((a*x + 1)^2/((c - c/(a^2*x^2))^3*(a*x - 1)^2),x)
```

output
```
x/c^3 - ((749*x)/48 - (71*a*x^2)/4 - 14/(3*a) + (111*a^2*x^3)/16)/(c^3 + 6
*a^2*c^3*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x) + (129*log(a*x - 1
))/(32*a*c^3) - log(a*x + 1)/(32*a*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.70

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$= \frac{774\log(ax-1)\,a^4x^4 - 3096\log(ax-1)\,a^3x^3 + 4644\log(ax-1)\,a^2x^2 - 3096\log(ax-1)\,ax + 774\log(a}{}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x)
```

output
```
(774*log(a*x - 1)*a**4*x**4 - 3096*log(a*x - 1)*a**3*x**3 + 4644*log(a*x -
 1)*a**2*x**2 - 3096*log(a*x - 1)*a*x + 774*log(a*x - 1) - 6*log(a*x + 1)*
a**4*x**4 + 24*log(a*x + 1)*a**3*x**3 - 36*log(a*x + 1)*a**2*x**2 + 24*log
(a*x + 1)*a*x - 6*log(a*x + 1) + 192*a**5*x**5 - 813*a**4*x**4 + 2370*a**2
*x**2 - 2624*a*x + 851)/(192*a*c**3*(a**4*x**4 - 4*a**3*x**3 + 6*a**2*x**2
 - 4*a*x + 1))
```

3.781 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 146

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{x}{c^4}+\frac{1}{20ac^4(1-ax)^5}-\frac{7}{16ac^4(1-ax)^4}+\frac{83}{48ac^4(1-ax)^3}-\frac{67}{16ac^4(1-ax)^2}+\frac{501}{64ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)}+\frac{261\log(1-ax)}{64ac^4}-\frac{5\log(1+ax)}{64ac^4}$$

output
```
x/c^4+1/20/a/c^4/(-a*x+1)^5-7/16/a/c^4/(-a*x+1)^4+83/48/a/c^4/(-a*x+1)^3-6
7/16/a/c^4/(-a*x+1)^2+501/64/a/c^4/(-a*x+1)-1/64/a/c^4/(a*x+1)+261/64*ln(-
a*x+1)/a/c^4-5/64*ln(a*x+1)/a/c^4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.67

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{2(-2384+7541ax-4900a^2x^2-6800a^3x^3+9300a^4x^4-1365a^5x^5-1920a^6x^6+480a^7x^7)}{(-1+ax)^5(1+ax)}+3915\log(1-ax)-75\log(1+ax)}{960ac^4}$$

input
```
Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

output
```
((2*(-2384 + 7541*a*x - 4900*a^2*x^2 - 6800*a^3*x^3 + 9300*a^4*x^4 - 1365*
a^5*x^5 - 1920*a^6*x^6 + 480*a^7*x^7))/((-1 + a*x)^5*(1 + a*x)) + 3915*Log
[1 - a*x] - 75*Log[1 + a*x])/(960*a*c^4)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}dx$$

$$\Big\downarrow \text{ 6717}$$

$$\int \frac{a^8e^{4\text{arctanh}(ax)}}{c^4\left(a^2-\frac{1}{x^2}\right)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a^8\int \frac{e^{4\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^4}dx}{c^4}$$

$$\Big\downarrow \text{ 6707}$$

$$\frac{a^8 \int \frac{e^{4\text{arctanh}(ax)}x^8}{(1-a^2x^2)^4}dx}{c^4}$$

$$\Big\downarrow \text{6700}$$

$$\frac{a^8 \int \frac{x^8}{(1-ax)^6(ax+1)^2}dx}{c^4}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^8 \int \left(-\frac{5}{64a^8(ax+1)} + \frac{1}{64a^8(ax+1)^2} + \frac{1}{a^8} + \frac{261}{64a^8(ax-1)} + \frac{501}{64a^8(ax-1)^2} + \frac{67}{8a^8(ax-1)^3} + \frac{83}{16a^8(ax-1)^4} + \frac{7}{4a^8(ax-1)^5} + \frac{1}{4a^8(ax-\cdots}\right.}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^8 \left(\frac{501}{64a^9(1-ax)} - \frac{1}{64a^9(ax+1)} - \frac{67}{16a^9(1-ax)^2} + \frac{83}{48a^9(1-ax)^3} - \frac{7}{16a^9(1-ax)^4} + \frac{1}{20a^9(1-ax)^5} + \frac{261\log(1-ax)}{64a^9} - \frac{5\log(ax+1)}{64a^9} + \cdots\right.}{c^4}$$

input
```
Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

output
```
(a^8*(x/a^8 + 1/(20*a^9*(1 - a*x)^5) - 7/(16*a^9*(1 - a*x)^4) + 83/(48*a^9
*(1 - a*x)^3) - 67/(16*a^9*(1 - a*x)^2) + 501/(64*a^9*(1 - a*x)) - 1/(64*a
^9*(1 + a*x)) + (261*Log[1 - a*x])/(64*a^9) - (5*Log[1 + a*x])/(64*a^9)))/
c^4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result
default	$\frac{a^{8}\left(\frac{x}{a^{8}}-\frac{1}{64 a^{9}\left(a x +1\right)}-\frac{5 \ln \left(a x +1\right)}{64 a^{9}}-\frac{1}{20 a^{9}\left(a x -1\right)^{5}}-\frac{7}{16 a^{9}\left(a x -1\right)^{4}}-\frac{83}{48 a^{9}\left(a x -1\right)^{3}}-\frac{67}{16 a^{9}\left(a x -1\right)^{2}}-\frac{501}{64 a^{9}\left(a x -1\right)}+\frac{261 \ln \left(a x -1\right)}{64 a^{9}}\right)}{c^{4}}$
risch	$\frac{x}{c^{4}}+\frac{-\frac{251 a^{4} c^{4} x^{5}}{32}+\frac{155 a^{3} c^{4} x^{4}}{8}-\frac{55 a^{2} c^{4} x^{3}}{6}-\frac{341 a \,c^{4} x^{2}}{24}+\frac{8021 c^{4} x}{480}-\frac{149 c^{4}}{30 a}}{c^{8} \left(a x -1\right)^{4} \left(a^{2} x^{2}-1\right)}+\frac{261 \ln \left(-a x +1\right)}{64 a \,c^{4}}-\frac{5 \ln \left(a x +1\right)}{64 a \,c^{4}}$
norman	$\frac{\frac{a^{8} x^{9}}{c}-\frac{115 a^{3} x^{4}}{6 c}-\frac{133 x}{32 c}+\frac{101 a \,x^{2}}{16 c}+\frac{1049 a^{2} x^{3}}{96 c}-\frac{3869 a^{4} x^{5}}{480 c}+\frac{4709 a^{5} x^{6}}{240 c}+\frac{43 a^{6} x^{7}}{480 c}-\frac{209 a^{7} x^{8}}{30 c}}{c^{3} \left(a x +1\right)^{3} \left(a x -1\right)^{5}}+\frac{261 \ln \left(a x -1\right)}{64 a \,c^{4}}-\frac{5 \ln \left(a x +1\right)}{64 a \,c^{4}}$
parallelrisch	$-13600 a^{3} x^{3}-3990 a x -5240 a^{4} x^{4}+16342 a^{5} x^{5}-8608 x^{6} a^{6}-300 \ln \left(a x +1\right) x a +960 a^{7} x^{7}+75 \ln \left(a x +1\right)+14040 a^{2} x^{2}-3915 \ln (a x$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
a^8/c^4*(x/a^8-1/64/a^9/(a*x+1)-5/64/a^9*ln(a*x+1)-1/20/a^9/(a*x-1)^5-7/16
/a^9/(a*x-1)^4-83/48/a^9/(a*x-1)^3-67/16/a^9/(a*x-1)^2-501/64/a^9/(a*x-1)+
261/64/a^9*ln(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.42

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$= \frac{960\,a^7x^7 - 3840\,a^6x^6 - 2730\,a^5x^5 + 18600\,a^4x^4 - 13600\,a^3x^3 - 9800\,a^2x^2 + 15082\,ax - 75\,(a^6x^6 - 4\,a^5}{960\,(a^7c^4x^6 - 4\,a^6c^4x^5 +}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
1/960*(960*a^7*x^7 - 3840*a^6*x^6 - 2730*a^5*x^5 + 18600*a^4*x^4 - 13600*a
^3*x^3 - 9800*a^2*x^2 + 15082*a*x - 75*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 -
5*a^2*x^2 + 4*a*x - 1)*log(a*x + 1) + 3915*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^
4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x - 1) - 4768)/(a^7*c^4*x^6 - 4*a^6*c^4*x
^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$= a^8\left(\frac{-3765a^5x^5 + 9300a^4x^4 - 4400a^3x^3 - 6820a^2x^2 + 8021ax - 2384}{480a^{15}c^4x^6 - 1920a^{14}c^4x^5 + 2400a^{13}c^4x^4 - 2400a^{11}c^4x^2 + 1920a^{10}c^4x - 480a^9c^4} + \frac{x}{a^8c^4} + \frac{\frac{261\log\left(x-\frac{1}{a}\right)}{64} - \frac{5\log\left(x+\frac{1}{a}\right)}{64}}{a^9c^4}\right)$$

input
```
integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**4,x)
```

output
```
a**8*((-3765*a**5*x**5 + 9300*a**4*x**4 - 4400*a**3*x**3 - 6820*a**2*x**2
+ 8021*a*x - 2384)/(480*a**15*c**4*x**6 - 1920*a**14*c**4*x**5 + 2400*a**1
3*c**4*x**4 - 2400*a**11*c**4*x**2 + 1920*a**10*c**4*x - 480*a**9*c**4) +
x/(a**8*c**4) + (261*log(x - 1/a)/64 - 5*log(x + 1/a)/64)/(a**9*c**4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{3765\,a^5x^5 - 9300\,a^4x^4 + 4400\,a^3x^3 + 6820\,a^2x^2 - 8021\,ax + 2384}{480\,(a^7c^4x^6 - 4\,a^6c^4x^5 + 5\,a^5c^4x^4 - 5\,a^3c^4x^2 + 4\,a^2c^4x - ac^4)} + \frac{x}{c^4} - \frac{5\,\log\left(ax+1\right)}{64\,ac^4} + \frac{261\,\log\left(ax-1\right)}{64\,ac^4}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
-1/480*(3765*a^5*x^5 - 9300*a^4*x^4 + 4400*a^3*x^3 + 6820*a^2*x^2 - 8021*a
*x + 2384)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 +
4*a^2*c^4*x - a*c^4) + x/c^4 - 5/64*log(a*x + 1)/(a*c^4) + 261/64*log(a*x
- 1)/(a*c^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{(ax-1)\left(\frac{257}{ax-1}+128\right)}{128\,ac^4\left(\frac{2}{ax-1}+1\right)} - \frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^4} - \frac{5\,\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{64\,ac^4} - \frac{\frac{7515\,a^{19}c^{16}}{ax-1} + \frac{4020\,a^{19}c^{16}}{(ax-1)^2} + \frac{1660\,a^{19}c^{16}}{(ax-1)^3} + \frac{420\,a^{19}c^{16}}{(ax-1)^4} + \frac{48\,a^{19}c^{16}}{(ax-1)^5}}{960\,a^{20}c^{20}}$$

input
```
integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
1/128*(a*x - 1)*(257/(a*x - 1) + 128)/(a*c^4*(2/(a*x - 1) + 1)) - 4*log(ab
s(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^4) - 5/64*log(abs(-2/(a*x - 1) - 1))
/(a*c^4) - 1/960*(7515*a^19*c^16/(a*x - 1) + 4020*a^19*c^16/(a*x - 1)^2 +
1660*a^19*c^16/(a*x - 1)^3 + 420*a^19*c^16/(a*x - 1)^4 + 48*a^19*c^16/(a*x
 - 1)^5)/(a^20*c^20)
```

Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{341\,a\,x^2}{24}-\frac{8021\,x}{480}+\frac{149}{30\,a}+\frac{55\,a^2\,x^3}{6}-\frac{155\,a^3\,x^4}{8}+\frac{251\,a^4\,x^5}{32}}{-a^6\,c^4\,x^6+4\,a^5\,c^4\,x^5-5\,a^4\,c^4\,x^4+5\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4} + \frac{x}{c^4}+\frac{261\,\ln\left(a\,x-1\right)}{64\,a\,c^4}-\frac{5\,\ln\left(a\,x+1\right)}{64\,a\,c^4}$$

input
```
int((a*x + 1)^2/((c - c/(a^2*x^2))^4*(a*x - 1)^2),x)
```

output
```
((341*a*x^2)/24 - (8021*x)/480 + 149/(30*a) + (55*a^2*x^3)/6 - (155*a^3*x^
4)/8 + (251*a^4*x^5)/32)/(c^4 + 5*a^2*c^4*x^2 - 5*a^4*c^4*x^4 + 4*a^5*c^4*
x^5 - a^6*c^4*x^6 - 4*a*c^4*x) + x/c^4 + (261*log(a*x - 1))/(64*a*c^4) - (
5*log(a*x + 1))/(64*a*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.65

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$= \frac{7830\log(ax-1)\,a^6x^6 - 31320\log(ax-1)\,a^5x^5 + 39150\log(ax-1)\,a^4x^4 - 39150\log(ax-1)\,a^2x^2 + 31}{}$$

input
```
int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x)
```

output
```
(7830*log(a*x - 1)*a**6*x**6 - 31320*log(a*x - 1)*a**5*x**5 + 39150*log(a*
x - 1)*a**4*x**4 - 39150*log(a*x - 1)*a**2*x**2 + 31320*log(a*x - 1)*a*x -
 7830*log(a*x - 1) - 150*log(a*x + 1)*a**6*x**6 + 600*log(a*x + 1)*a**5*x*
*5 - 750*log(a*x + 1)*a**4*x**4 + 750*log(a*x + 1)*a**2*x**2 - 600*log(a*x
 + 1)*a*x + 150*log(a*x + 1) + 1920*a**7*x**7 - 9045*a**6*x**6 + 30375*a**
4*x**4 - 27200*a**3*x**3 - 12775*a**2*x**2 + 24704*a*x - 8171)/(1920*a*c**
4*(a**6*x**6 - 4*a**5*x**5 + 5*a**4*x**4 - 5*a**2*x**2 + 4*a*x - 1))
```

3.782 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$

Optimal result

Integrand size = 22, antiderivative size = 169

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx=\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(6a+\frac{35}{x}\right)}{30a^2}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(8a+\frac{35}{x}\right)}{24a^2}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{35}{x}\right)}{16a^2}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(7a+\frac{1}{x}\right)x}{7a}+\frac{35c^4\csc^{-1}(ax)}{16a}-\frac{c^4\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
1/30*c^4*(1-1/a^2/x^2)^(5/2)*(6*a+35/x)/a^2+1/24*c^4*(1-1/a^2/x^2)^(3/2)*(
8*a+35/x)/a^2+1/16*c^4*(1-1/a^2/x^2)^(1/2)*(16*a+35/x)/a^2+1/7*c^4*(1-1/a^
2/x^2)^(7/2)*(7*a+1/x)*x/a+35/16*c^4*arccsc(a*x)/a-c^4*arctanh((1-1/a^2/x^
2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.71

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}(-240+280ax+1056a^2x^2-1330a^3x^3-1952a^4x^4+3045a^5x^5+2816a^6x^6+1680a^7x^7)}{x^6}+3675a^6\arcsin\left(\frac{1}{ax}\right)-1680a^6\right.}{1680a^7}$$

input
```
Integrate[(c - c/(a^2*x^2))^4/E^ArcCoth[a*x],x]
```

output
```
(c^4*((Sqrt[1 - 1/(a^2*x^2)]*(-240 + 280*a*x + 1056*a^2*x^2 - 1330*a^3*x^3
 - 1952*a^4*x^4 + 3045*a^5*x^5 + 2816*a^6*x^6 + 1680*a^7*x^7))/x^6 + 3675*
a^6*ArcSin[1/(a*x)] - 1680*a^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1680*
a^7)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.94, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.136$, Rules used = {6748, 108, 25, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 25, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^4 e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6748}$$

$$-c^4\int \left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 108}$$

$$-c^4\left(\int -\frac{\left(a+\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{7/2}\right)$$

$$\Big\downarrow 25$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\int\frac{\left(a+\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x}{a^2}d\frac{1}{x}\right)$$

$$\Big\downarrow 27$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\int\left(a+\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}xd\frac{1}{x}}{a^2}\right)$$

$$\Big\downarrow 171$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{7}a\int\frac{7\left(a+\frac{7}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x}{a}d\frac{1}{x}+\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}}{a^2}\right)$$

$$\Big\downarrow 27$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\int\left(a+\frac{7}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}xd\frac{1}{x}+\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}}{a^2}\right)$$

$$\Big\downarrow 171$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}a\int\frac{\left(6a+\frac{29}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x}{a}d\frac{1}{x}+\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{7}{6}a}{a^2}\right.$$

$$\Big\downarrow 27$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\int\left(6a+\frac{29}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}xd\frac{1}{x}+\frac{8}{7}a\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7}}{a^2}\right.$$

$$\Big\downarrow 171$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{1}{5}a\int\frac{3\left(10a+\frac{19}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x}{a}d\frac{1}{x}+\frac{29}{5}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}\right)}{a^2}\right.$$

$$\Big\downarrow 27$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\int\left(10a+\frac{19}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}xd\frac{1}{x}+\frac{29}{5}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right.\right.}{a^2}\right.$$

$$\Big\downarrow \text{171}$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}a\int\frac{\left(40a-\frac{21}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{19}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)+\frac{29}{5}a\left(\right.\right.}{}\right.$$

$$\Big\downarrow \text{27}$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\int\frac{\left(40a-\frac{21}{x}\right)\left(1+\frac{1}{ax}\right)^{5/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{19}{4}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}\right)+\frac{29}{5}a\left(1\right.\right.}{}\right.$$

$$\Big\downarrow \text{171}$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(7a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{3}a\int-\frac{15\left(8a+\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\right.\right.}{}\right.$$

$$\Big\downarrow \text{27}$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5\int\frac{\left(8a+\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+7a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{19}{4}a\sqrt{\right.\right.}{}\right.$$

$$\Big\downarrow \text{171}$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5\left(-\frac{1}{2}a\int-\frac{\left(16a+\frac{19}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)\right.\right.\right.}{}\right.$$

$$\downarrow 25$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5\left(\frac{1}{2}a\int\frac{\left(16a+\frac{19}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+7\right.\right.\right.\right.}{}\right.\right.$$

$$\downarrow 27$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5\left(\frac{1}{2}\int\frac{\left(16a+\frac{19}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-\frac{1}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+7a\right.\right.\right.\right.}{}\right.\right.$$

$$\downarrow 171$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5\left(\frac{1}{2}\left(a\left(-\int-\frac{\left(16a+\frac{35}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)-19a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right.\right.\right.\right.\right.}{}\right.\right.$$

$$\downarrow 25$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5\left(\frac{1}{2}\left(a\int\frac{\left(16a+\frac{35}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-19a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}a\right.\right.\right.\right.}{}\right.\right.$$

$$\downarrow 27$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5\left(\frac{1}{2}\left(\int\frac{\left(16a+\frac{35}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-19a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-\frac{1}{2}a\sqrt{\right.\right.\right.\right.}{}\right.\right.$$

$$\downarrow 175$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5\left(\frac{1}{2}\left(16a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+35\int\frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-19a\right.\right.\right.\right.\right.\right.}{}\right.\right.$$

$\downarrow$ 39

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5\left(\frac{1}{2}\left(35\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+16a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-19a\sqrt{1-\right.\right.\right.\right.\right.\right.}{}\right.\right.$$

$\downarrow$ 103

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5\left(\frac{1}{2}\left(35\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)\right.\right.\right.\right.\right.\right.}{}\right.\right.$$

$\downarrow$ 221

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5\left(\frac{1}{2}\left(35\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-\right.\right.\right.\right.\right.\right.}{}\right.\right.$$

$\downarrow$ 223

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{7/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{\frac{1}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(5\left(\frac{1}{2}\left(35a\arcsin\left(\frac{1}{ax}\right)-16a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-19\right.\right.\right.\right.\right.\right.}{}\right.\right.$$

input
```
Int[(c - c/(a^2*x^2))^4/E^ArcCoth[a*x],x]
```

output
```
-(c^4*(-((1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(7/2)*x) - ((7*a*(1 - 1/(a*x))^
(5/2)*(1 + 1/(a*x))^(7/2))/6 + (8*a*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(7/2
))/7 + ((29*a*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2))/5 + (3*((19*a*Sqrt[
1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/4 + (7*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2) + 5*(-1/2*(a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) + (-19*a*Sqrt[1
 - 1/(a*x)]*Sqrt[1 + 1/(a*x)] + 35*a*ArcSin[1/(a*x)] - 16*a*ArcTanh[Sqrt[1
 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/2))/4))/5)/6)/a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.12

method	result
risch	$\frac{(ax+1)\left(2816x^6a^6+3045a^5x^5-1952a^4x^4-1330a^3x^3+1056a^2x^2+280ax-240\right)c^4\sqrt{\frac{ax-1}{ax+1}}}{1680x^7a^8}+\frac{\left(\frac{35a^7\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{16}-\frac{a^8\ln\left(\frac{a^2}{\sqrt{a}}\right.}{}\right.}{}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^4\left(-1680\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^8x^8+1680\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6-3675a^7x^7\sqrt{a^2}\,\sqrt{a^2x^2-1}-3675a^7x^7\sqrt{a^2}\,\arctan\left(\frac{}{\sqrt{}}\right.\right.}{}$

input
```
int((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1680*(a*x+1)*(2816*a^6*x^6+3045*a^5*x^5-1952*a^4*x^4-1330*a^3*x^3+1056*a
^2*x^2+280*a*x-240)/x^7*c^4/a^8*((a*x-1)/(a*x+1))^(1/2)+(35/16*a^7*arctan(
1/(a^2*x^2-1)^(1/2))-a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/
2)+a^7*((a*x-1)*(a*x+1))^(1/2))*c^4/a^8*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(
a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.19

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\frac{7350\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(168\cdots}{\cdots}$$

input
```
integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-1/1680*(7350*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) + 1680*a^7*c^4
*x^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 1680*a^7*c^4*x^7*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) - (1680*a^8*c^4*x^8 + 4496*a^7*c^4*x^7 + 5861*a^6*c^4*
x^6 + 1093*a^5*c^4*x^5 - 3282*a^4*c^4*x^4 - 274*a^3*c^4*x^3 + 1336*a^2*c^4
*x^2 + 40*a*c^4*x - 240*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\left(\int a^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^8}\,dx+\int\left(-\frac{4a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^6}\right)dx+\int\frac{6a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\cdots\right.\right.}{a^8}$$

input
```
integrate((c-c/a**2/x**2)**4*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c**4*(Integral(a**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/x**8, x) + Integral(-4*a**2*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/x**6, x) + Integral(6*a**4*sqrt(a*x/(a*x + 1) - 1/(a*x +
1))/x**4, x) + Integral(-4*a**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x**2, x)
)/a**8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(149) = 298$.

Time = 0.11 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.25

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx =$$

$$-\frac{1}{840}\left(\frac{3675\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{1995\,c^4\left(\frac{ax-1}{ax+1}\right.}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/840*(3675*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 840*c^4*log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/a^2 - 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
/a^2 - (1995*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 10185*c^4*((a*x - 1)/(a*x
+ 1))^(13/2) + 17619*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 4569*c^4*((a*x - 1
)/(a*x + 1))^(9/2) + 71801*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 72051*c^4*((a
*x - 1)/(a*x + 1))^(5/2) + 31465*c^4*((a*x - 1)/(a*x + 1))^(3/2) + 5355*c^
4*sqrt((a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a
^2/(a*x + 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x
+ 1)^5 - 14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 -
(a*x - 1)^8*a^2/(a*x + 1)^8 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(149) = 298$.

Time = 0.16 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.10

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = \text{Too large to display}$$

input
```
integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
-35/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^4*log(a
bs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)
*c^4*sgn(a*x + 1)/a - 1/840*(3045*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*c^4*ab
s(a)*sgn(a*x + 1) - 6720*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4*sgn(a*x +
 1) + 6860*(x*abs(a) - sqrt(a^2*x^2 - 1))^11*c^4*abs(a)*sgn(a*x + 1) - 201
60*(x*abs(a) - sqrt(a^2*x^2 - 1))^10*a*c^4*sgn(a*x + 1) + 9065*(x*abs(a) -
 sqrt(a^2*x^2 - 1))^9*c^4*abs(a)*sgn(a*x + 1) - 49280*(x*abs(a) - sqrt(a^2
*x^2 - 1))^8*a*c^4*sgn(a*x + 1) - 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a
*c^4*sgn(a*x + 1) - 9065*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*sgn(a
*x + 1) - 38976*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) - 6860
*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^4*abs(a)*sgn(a*x + 1) - 12992*(x*abs(a
) - sqrt(a^2*x^2 - 1))^2*a*c^4*sgn(a*x + 1) - 3045*(x*abs(a) - sqrt(a^2*x^
2 - 1))*c^4*abs(a)*sgn(a*x + 1) - 2816*a*c^4*sgn(a*x + 1))/(((x*abs(a) - s
qrt(a^2*x^2 - 1))^2 + 1)^7*a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.96

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$= \frac{\frac{51\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{899\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{24}+\frac{3431\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{40}+\frac{71801\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{840}+\frac{1523\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{280}+\frac{839\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{40}+\frac{97}{}}{a+\frac{6\,a\,(a\,x-1)}{a\,x+1}+\frac{14\,a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}+\frac{14\,a\,{(a\,x-1)}^3}{{(a\,x+1)}^3}-\frac{14\,a\,{(a\,x-1)}^5}{{(a\,x+1)}^5}-\frac{14\,a\,{(a\,x-1)}^6}{{(a\,x+1)}^6}-\frac{6\,a\,{(a\,x-1)}^7}{{(a\,x+1)}^7}-\frac{a\,(}{(}}$$
$$-\frac{35\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}-\frac{2\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((51*c^4*((a*x - 1)/(a*x + 1))^(1/2))/8 + (899*c^4*((a*x - 1)/(a*x + 1))^(
3/2))/24 + (3431*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 + (71801*c^4*((a*x -
1)/(a*x + 1))^(7/2))/840 + (1523*c^4*((a*x - 1)/(a*x + 1))^(9/2))/280 + (8
39*c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 + (97*c^4*((a*x - 1)/(a*x + 1))^(1
3/2))/8 + (19*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/(a
*x + 1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^3
- (14*a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a*(
a*x - 1)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (35*c^4*atan(((a*
x - 1)/(a*x + 1))^(1/2)))/(8*a) - (2*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)
))/a
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.44

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$= \frac{c^4\left(-7350atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^7x^7+7350atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^7x^7+1680\sqrt{ax-}\right.}{}$$

input
```
int((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(c**4*( - 7350*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**7*x**7 + 7350*at
an(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**7*x**7 + 1680*sqrt(a*x + 1)*sqrt(
a*x - 1)*a**7*x**7 + 2816*sqrt(a*x + 1)*sqrt(a*x - 1)*a**6*x**6 + 3045*sqr
t(a*x + 1)*sqrt(a*x - 1)*a**5*x**5 - 1952*sqrt(a*x + 1)*sqrt(a*x - 1)*a**4
*x**4 - 1330*sqrt(a*x + 1)*sqrt(a*x - 1)*a**3*x**3 + 1056*sqrt(a*x + 1)*sq
rt(a*x - 1)*a**2*x**2 + 280*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x - 240*sqrt(a*x
 + 1)*sqrt(a*x - 1) - 3360*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*
*7*x**7 - 896*a**7*x**7))/(1680*a**8*x**7)
```

3.783 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 136

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(4a+\frac{15}{x}\right)}{12a^2}+\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(8a+\frac{15}{x}\right)}{8a^2}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(5a+\frac{1}{x}\right)x}{5a}+\frac{15c^3\csc^{-1}(ax)}{8a}-\frac{c^3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
1/12*c^3*(1-1/a^2/x^2)^(3/2)*(4*a+15/x)/a^2+1/8*c^3*(1-1/a^2/x^2)^(1/2)*(8
*a+15/x)/a^2+1/5*c^3*(1-1/a^2/x^2)^(5/2)*(5*a+1/x)*x/a+15/8*c^3*arccsc(a*x
)/a-c^3*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}(24-30ax-88a^2x^2+135a^3x^3+184a^4x^4+120a^5x^5)+225a^4x^4\arcsin\left(\frac{1}{ax}\right)-120a^4x^4\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{120a^5x^4}$$

input
```
Integrate[(c - c/(a^2*x^2))^3/E^ArcCoth[a*x],x]
```

output
```
(c^3*(Sqrt[1 - 1/(a^2*x^2)]*(24 - 30*a*x - 88*a^2*x^2 + 135*a^3*x^3 + 184*
a^4*x^4 + 120*a^5*x^5) + 225*a^4*x^4*ArcSin[1/(a*x)] - 120*a^4*x^4*Log[(1
+ Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a^5*x^4)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.90, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {6748, 108, 25, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^3 e^{-\coth^{-1}(ax)} dx$$

$$\Big\downarrow \text{6748}$$

$$-c^3\int \left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$-c^3\left(\int -\frac{\left(a+\frac{6}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}\right)$$

$$\Big\downarrow \text{25}$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\int\frac{\left(a+\frac{6}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{a^2}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\int\left(a+\frac{6}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}}{a^2}\right)$$

$$\Big\downarrow \text{171}$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{5}a\int\frac{5\left(a+\frac{5}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{a}d\frac{1}{x}+\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\int\left(a+\frac{5}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}+\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}\right)$$

$$\Big\downarrow \text{171}$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}a\int\frac{\left(4a+\frac{11}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{a}d\frac{1}{x}+\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{5}{4}a(1}{a^2}\right.$$

$$\Big\downarrow \text{27}$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\int\left(4a+\frac{11}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}+\frac{6}{5}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}{a^2}\right.$$

$$\Big\downarrow \text{171}$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\left(\frac{1}{3}a\int\frac{\left(12a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{11}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{6}{5}a(1-}{a^2}\right.$$

$$\Big\downarrow \text{27}$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\left(\frac{1}{3}\int\frac{\left(12a-\frac{1}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{11}{3}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{6}{5}a\left(1-\frac{1}{ax}\right.}{a^2}\right.$$

$$\Big\downarrow \; 171$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}a\int-\frac{3\left(8a+\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{11}{3}a\sqrt{1\right.}{}\right.$$

$$\Big\downarrow \; 27$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{\left(8a+\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{1}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}\right)+\frac{11}{3}a\sqrt{1-\right.}{}\right.$$

$$\Big\downarrow \; 171$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(a\left(-\int-\frac{\left(8a+\frac{15}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)-7a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{1}{2}a\right.\right.}{}\right.$$

$$\Big\downarrow \; 25$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(a\int\frac{\left(8a+\frac{15}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-7a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{1}{2}a\sqrt{1-\right.\right.}{}\right.$$

$$\Big\downarrow \; 27$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(\int\frac{\left(8a+\frac{15}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-7a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{1}{2}a\sqrt{1-\frac{1}{ax}}\left(\right.\right.\right.}{}\right.$$

$$\downarrow 175$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(8a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+15\int\frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-7a\sqrt{1-\frac{1}{ax}}\right.\right.\right.}{}\right.$$

$$\downarrow 39$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(15\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+8a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-7a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}}\right.\right.\right.}{}\right.$$

$$\downarrow 103$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(15\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)-7a\sqrt{}\right.\right.\right.}{}\right.$$

$$\downarrow 221$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(15\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-7a\sqrt{1}\right.\right.\right.}{}\right.$$

$$\downarrow 223$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(15a\arcsin\left(\frac{1}{ax}\right)-8a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-7a\sqrt{1-}\right.\right.\right.}{}\right.$$

input
```
Int[(c - c/(a^2*x^2))^3/E^ArcCoth[a*x],x]
```

output
```
-(c^3*(-((1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)*x) - ((5*a*(1 - 1/(a*x))^
(3/2)*(1 + 1/(a*x))^(5/2))/4 + (6*a*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2
))/5 + ((11*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2))/3 + ((a*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/2 + (3*(-7*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]
 + 15*a*ArcSin[1/(a*x)] - 8*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]
))/2)/3)/4)/a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.27

method	result
risch	$\frac{(ax+1)\left(184a^4x^4+135a^3x^3-88a^2x^2-30ax+24\right)c^3\sqrt{\frac{ax-1}{ax+1}}}{120x^5a^6}+\frac{\left(\frac{15a^5\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{8}-\frac{a^6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a^5\sqrt{(ax-1)(a}\right.}{a^6(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^3\left(-120\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^6x^6+120\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4-225\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^5x^5-225\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}\right.}{120\sqrt{(ax}}$

input
```
int((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/120*(a*x+1)*(184*a^4*x^4+135*a^3*x^3-88*a^2*x^2-30*a*x+24)/x^5*c^3/a^6*(
(a*x-1)/(a*x+1))^(1/2)+(15/8*a^5*arctan(1/(a^2*x^2-1)^(1/2))-a^6*ln(a^2*x/
(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+a^5*((a*x-1)*(a*x+1))^(1/2))*c^
3/a^6*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.32

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{450\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(120\,a^6c^3}{120\,a^6x^5}$$

input
```
integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-1/120*(450*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) + 120*a^5*c^3*x^
5*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/
(a*x + 1)) - 1) - (120*a^6*c^3*x^6 + 304*a^5*c^3*x^5 + 319*a^4*c^3*x^4 + 4
7*a^3*c^3*x^3 - 118*a^2*c^3*x^2 - 6*a*c^3*x + 24*c^3)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^6*x^5)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{c^3\left(\int a^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^6}\right)dx+\int\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\frac{3a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\right)dx\right)}{a^6}$$

input
```
integrate((c-c/a**2/x**2)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c**3*(Integral(a**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/x**6, x) + Integral(3*a**2*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/x**4, x) + Integral(-3*a**4*sqrt(a*x/(a*x + 1) - 1/(a*x +
 1))/x**2, x))/a**6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(120) = 240.

Time = 0.13 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.22

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = -\frac{1}{60}\left(\frac{225\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+\cdots}{\cdots}\right.$$

input
```
integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/60*(225*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 60*c^3*log(sqrt((a*
x - 1)/(a*x + 1)) + 1)/a^2 - 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2
 - (105*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 305*c^3*((a*x - 1)/(a*x + 1))^(
9/2) + 86*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 1654*c^3*((a*x - 1)/(a*x + 1))
^(5/2) + 1345*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 345*c^3*sqrt((a*x - 1)/(a*
x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a
*x - 1)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^
2/(a*x + 1)^6 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(120) = 240.

Time = 0.14 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.90

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{15\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{4\,a} + \frac{c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c^3\operatorname{sgn}(ax+1)}{a} - \frac{135\left(x|a|-\sqrt{a^2x^2-1}\right)^9 c^3|a|\operatorname{sgn}(ax+1) - 360\left(x|a|-\sqrt{a^2x^2-1}\right)^8 ac^3\operatorname{sgn}(ax+1) + 150\left(x|a|-\sqrt{a^2x^2-1}\right.}{}$$

input
```
integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
-15/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^3*log(a
bs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)
*c^3*sgn(a*x + 1)/a - 1/60*(135*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3*abs(a
)*sgn(a*x + 1) - 360*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3*sgn(a*x + 1) +
 150*(x*abs(a) - sqrt(a^2*x^2 - 1))^7*c^3*abs(a)*sgn(a*x + 1) - 720*(x*abs
(a) - sqrt(a^2*x^2 - 1))^6*a*c^3*sgn(a*x + 1) - 1120*(x*abs(a) - sqrt(a^2*
x^2 - 1))^4*a*c^3*sgn(a*x + 1) - 150*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*
abs(a)*sgn(a*x + 1) - 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x +
 1) - 135*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) - 184*a*c
^3*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^5*a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.90

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{\frac{23\,c^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{269\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{12}+\frac{827\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{30}+\frac{43\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{30}+\frac{61\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{12}+\frac{7\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{4}}{a+\frac{4\,a\,(a\,x-1)}{a\,x+1}+\frac{5\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{a\,(a\,x-1)^6}{(a\,x+1)^6}} - \frac{15\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a} - \frac{2\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
((23*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (269*c^3*((a*x - 1)/(a*x + 1))^(
3/2))/12 + (827*c^3*((a*x - 1)/(a*x + 1))^(5/2))/30 + (43*c^3*((a*x - 1)/(
a*x + 1))^(7/2))/30 + (61*c^3*((a*x - 1)/(a*x + 1))^(9/2))/12 + (7*c^3*((a
*x - 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x -
 1)^2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*
x + 1)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (15*c^3*atan(((a*x - 1)/(a*x + 1
))^(1/2)))/(4*a) - (2*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.49

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{c^3\left(-450atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^5x^5+450atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^5x^5+120\sqrt{ax+1}\right.}{}$$

input
```
int((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(c**3*( - 450*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**5*x**5 + 450*atan
(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**5*x**5 + 120*sqrt(a*x + 1)*sqrt(a*x
 - 1)*a**5*x**5 + 184*sqrt(a*x + 1)*sqrt(a*x - 1)*a**4*x**4 + 135*sqrt(a*x
 + 1)*sqrt(a*x - 1)*a**3*x**3 - 88*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 -
 30*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 24*sqrt(a*x + 1)*sqrt(a*x - 1) - 240
*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**5*x**5 - 40*a**5*x**5))/(
120*a**6*x**5)
```

3.784 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a}+\frac{3c^2\csc^{-1}(ax)}{2a}-\frac{c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
1/2*c^2*(1-1/a^2/x^2)^(1/2)*(2*a+3/x)/a^2+1/3*c^2*(1-1/a^2/x^2)^(3/2)*(3*a
+1/x)*x/a+3/2*c^2*arccsc(a*x)/a-c^2*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.91

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}\left(-2+3ax+8a^2x^2+6a^3x^3\right)+9a^2x^2\arcsin\left(\frac{1}{ax}\right)-6a^2x^2\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a^3x^2}$$

input
```
Integrate[(c - c/(a^2*x^2))^2/E^ArcCoth[a*x],x]
```

output
```
(c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-2 + 3*a*x + 8*a^2*x^2 + 6*a^3*x^3) + 9*a^2*x
^2*ArcSin[1/(a*x)] - 6*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3
*x^2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.79, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {6748, 108, 25, 27, 171, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^2 e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6748}$$

$$-c^2\int \left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 108}$$

$$-c^2\left(\int -\frac{\left(a+\frac{4}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\int \frac{\left(a+\frac{4}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{a^2}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\int \left(a+\frac{4}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}}{a^2}\right)$$

$$\Big\downarrow \text{ 171}$$

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{3}a\int \frac{3\left(a+\frac{3}{x}\right)\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{a}d\frac{1}{x}+\frac{4}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{a^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\int\left(a+\frac{3}{x}\right)\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}+\frac{4}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{a^2}\right)$$

$$\Big\downarrow \text{171}$$

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}a\int\frac{\left(2a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{4}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{3}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}\right.}{a^2}\right.$$

$$\Big\downarrow \text{27}$$

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\int\frac{\left(2a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{4}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{3}{2}a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}\right.}{a^2}\right.$$

$$\Big\downarrow \text{171}$$

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\left(a\left(-\int-\frac{\left(2a+\frac{3}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)-a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{4}{3}a\left(1-\frac{1}{ax}\right.}{a^2}\right.$$

$$\Big\downarrow \text{25}$$

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\left(a\int\frac{\left(2a+\frac{3}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{4}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}\right.}{a^2}\right.$$

$$\Big\downarrow \text{27}$$

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\left(\int \frac{\left(2a+\frac{3}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{4}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+\right.}{a^2}\right.$$

$\downarrow$ 175

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\left(2a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+3\int \frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right.}{a^2}\right.$$

$\downarrow$ 39

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\left(3\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+2a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+}{a^2}\right.$$

$\downarrow$ 103

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\left(3\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-2\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+a\left(-\sqrt{1-\frac{1}{ax}}\right.\right.}{a}\right.$$

$\downarrow$ 221

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\left(3\int \frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-2a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{a}}\right.}{a^2}\right.$$

$\downarrow$ 223

$$-c^2\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{5/2}-\frac{\frac{1}{2}\left(3a\arcsin\left(\frac{1}{ax}\right)-2a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}}\right.}{a^2}\right.$$

input
```
Int[(c - c/(a^2*x^2))^2/E^ArcCoth[a*x],x]
```

output
```
-(c^2*(-((1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x) - ((3*a*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2))/2 + (4*a*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2))
/3 + (-(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + 3*a*ArcSin[1/(a*x)] - 2*a
*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/2)/a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.52

method	result
risch	$\frac{(ax+1)\left(8a^2x^2+3ax-2\right)c^2\sqrt{\frac{ax-1}{ax+1}}}{6x^3a^4}+\frac{\left(\frac{3a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}-\frac{a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a^3\sqrt{(ax-1)(ax+1)}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-}}{a^4(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^2\left(-6\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^4x^4+6\sqrt{a^2}\,\left(a^2x^2-1\right)^{\frac{3}{2}}a^2x^2-9\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^3x^3-9a^3\sqrt{a^2}\,x^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+6\ln\right.}{6\sqrt{(ax-1)(ax+1)}\,a^4x^3\sqrt{a^2}}$

input
```
int((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x+1)*(8*a^2*x^2+3*a*x-2)/x^3*c^2/a^4*((a*x-1)/(a*x+1))^(1/2)+(3/2*a
^3*arctan(1/(a^2*x^2-1)^(1/2))-a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))
/(a^2)^(1/2)+a^3*((a*x-1)*(a*x+1))^(1/2))*c^2/a^4*((a*x-1)/(a*x+1))^(1/2)*
((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.51

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = -\frac{18\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(6\,a^4c^2x^4+14}{6\,a^4x^3}$$

input
```
integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(18*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 6*a^3*c^2*x^3*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x +
1)) - 1) - (6*a^4*c^2*x^4 + 14*a^3*c^2*x^3 + 11*a^2*c^2*x^2 + a*c^2*x - 2*
c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = \frac{c^2\left(\int a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\frac{2a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\right)dx\right)}{a^4}$$

input
```
integrate((c-c/a**2/x**2)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c**2*(Integral(a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/x**4, x) + Integral(-2*a**2*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/x**2, x))/a**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(91) = 182.

Time = 0.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.17

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\frac{1}{3}a\left(\frac{9\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{3\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+c^2\left(\frac{ax-1}{ax+1}\right.}{\frac{2\,(ax-1)a^2}{ax+1}-2(}\right.$$

input
```
integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-1/3*a*(9*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^2*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
(3*c^2*((a*x - 1)/(a*x + 1))^(7/2) + c^2*((a*x - 1)/(a*x + 1))^(5/2) + 29*
c^2*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*
x - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*
x + 1)^4 + a^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(91) = 182.

Time = 0.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.56

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\frac{3\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}(ax+1)}{a} + \frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c^2\mathrm{sgn}(ax+1)}{a} - \frac{3\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^2|a|\mathrm{sgn}(ax+1)-12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^2\mathrm{sgn}(ax+1)-12\left(x|a|-\sqrt{a^2x^2}\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right.}$$

input
```
integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
-3*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^2*log(abs(
-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^
2*sgn(a*x + 1)/a - 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a)*sgn(
a*x + 1) - 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2*sgn(a*x + 1) - 12*(x*
abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^2*sgn(a*x + 1) - 3*(x*abs(a) - sqrt(a^2*
x^2 - 1))*c^2*abs(a)*sgn(a*x + 1) - 8*a*c^2*sgn(a*x + 1))/(((x*abs(a) - sq
rt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.78

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{5\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{29\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}+c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{3\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(5*c^2*((a*x - 1)/(a*x + 1))^(1/2) + (29*c^2*((a*x - 1)/(a*x + 1))^(3/2))/
3 + (c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 + c^2*((a*x - 1)/(a*x + 1))^(7/2))
/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x
- 1)^4)/(a*x + 1)^4) - (3*c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c^
2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.50

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = \frac{c^2\left(-18atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^3x^3+18atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^3x^3+6\sqrt{ax+1}\,\sqrt{ax}\right.}{}$$

input
```
int((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(c**2*( - 18*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**3*x**3 + 18*atan(s
qrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**3*x**3 + 6*sqrt(a*x + 1)*sqrt(a*x - 1
)*a**3*x**3 + 8*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 + 3*sqrt(a*x + 1)*sq
rt(a*x - 1)*a*x - 2*sqrt(a*x + 1)*sqrt(a*x - 1) - 12*log((sqrt(a*x - 1) +
sqrt(a*x + 1))/sqrt(2))*a**3*x**3))/(6*a**4*x**3)
```

3.785 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 56

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a}+\frac{c\csc^{-1}(ax)}{a}-\frac{c\,\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
c*(1-1/a^2/x^2)^(1/2)*(a+1/x)*x/a+c*arccsc(a*x)/a-c*arctanh((1-1/a^2/x^2)^
(1/2))/a
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(1+ax)+\arcsin\left(\frac{1}{ax}\right)-\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{a}$$

input
```
Integrate[(c - c/(a^2*x^2))/E^ArcCoth[a*x],x]
```

output
```
(c*(Sqrt[1 - 1/(a^2*x^2)]*(1 + a*x) + ArcSin[1/(a*x)] - Log[(1 + Sqrt[1 -
1/(a^2*x^2)])*x]))/a
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6748, 108, 25, 27, 171, 140, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right) e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6748}$$

$$-c\int \left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 108}$$

$$-c\left(\int -\frac{\left(a+\frac{2}{x}\right)\sqrt{1-\frac{1}{ax}}x}{a^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\int \frac{\left(a+\frac{2}{x}\right)\sqrt{1-\frac{1}{ax}}x}{a^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{\int \frac{\left(a+\frac{2}{x}\right)\sqrt{1-\frac{1}{ax}}x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2}\right)$$

$$\Big\downarrow \text{ 171}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{a\int\frac{\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}\right)$$

$$\Big\downarrow \text{ 140}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{a\left(\frac{\int\frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}+\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}\right)$$

$$\Big\downarrow \text{ 39}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{a\left(\frac{\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}+\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}\right)$$

$$\Big\downarrow \text{ 103}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{a\left(\frac{\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\frac{\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}\right)$$

$$\Big\downarrow \text{ 221}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{a\left(\frac{\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}\right)$$

$$\Big\downarrow \text{ 223}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{a\left(\arcsin\left(\frac{1}{ax}\right)-\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+2a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a^2}\right)$$

input
```
Int[(c - c/(a^2*x^2))/E^ArcCoth[a*x],x]
```

output
```
-(c*(-((1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x) - (2*a*Sqrt[1 - 1/(a*x)]*S
qrt[1 + 1/(a*x)] + a*(ArcSin[1/(a*x)] - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 +
 1/(a*x)]]))/a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(52) = 104$.

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.27

method	result
risch	$\frac{(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{x\,a^2}+\frac{\left(\sqrt{(ax-1)(ax+1)}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-\frac{a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{a(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c\left(-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax-a\sqrt{a^2}\,x\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}}{\sqrt{a^2}}\right.\right.}{\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

input
```
int((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x+1)/x*c/a^2*((a*x-1)/(a*x+1))^(1/2)+1/a*(((a*x-1)*(a*x+1))^(1/2)+arcta
n(1/(a^2*x^2-1)^(1/2))-a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/
2))*c*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(52) = 104$.

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.91

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx = -\frac{2\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2cx^2+2\,acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

input
```
integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

output
```
-(2*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + a*c*x*log(sqrt((a*x - 1)/(a*
x + 1)) + 1) - a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c*x^2 + 2*a
*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\left(\int a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\right)dx\right)}{a^2}$$

input
```
integrate((c-c/a**2/x**2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
c*(Integral(a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/x**2, x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(52) = 104$.

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.09

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=$$
$$-a\left(\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

input
```
integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

output
```
-a*(4*c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*
c*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1)/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(52) = 104$.

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.16

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = -\frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a} + \frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c\operatorname{sgn}(ax+1)}{a} + \frac{2\,c\operatorname{sgn}(ax+1)}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|}$$

input
```
integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
-2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c*log(abs(-x*a
bs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn(
a*x + 1)/a + 2*c*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(
a))
```

Mupad [B] (verification not implemented)

Time = 13.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.50

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = \frac{4\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}} - \frac{2\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a} - \frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(4*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (2*c
*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c*atan(((a*x - 1)/(a*x + 1))^(
1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.80

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$$
$$=\frac{c\left(-2atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)ax+2atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)ax+\sqrt{ax+1}\,\sqrt{ax-1}\,ax+\right.}{a^2x}$$

input
```
int((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(c*( - 2*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a*x + 2*atan(sqrt(a*x - 1
) + sqrt(a*x + 1) + 1)*a*x + sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + sqrt(a*x +
1)*sqrt(a*x - 1) - 2*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x + a*
x))/(a**2*x)
```

3.786 $\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}} dx = \frac{a-\frac{1}{x}}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
(a-1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)+(1-1/a^2/x^2)^(1/2)*x/c-arctanh((1-1/a^2
/x^2)^(1/2))/a/c
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}} dx = \frac{\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(2+ax)}{1+ax} - \frac{\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{c}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))),x]
```

output
```
((Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x))/(1 + a*x) - Log[(1 + Sqrt[1 - 1/(a^2*
x^2)])*x]/a)/c
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.44, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6748, 114, 27, 35, 105, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{ 6748}$$

$$-\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 114}$$

$$-\frac{-\int \frac{\left(a-\frac{1}{x}\right)x}{a^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}-\frac{x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{-\frac{\int \frac{\left(a-\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}}{c}$$

$$\Big\downarrow \text{ 35}$$

$$-\frac{-\frac{\int \frac{\sqrt{1-\frac{1}{ax}}x}{\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}}{c}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{-\frac{\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{2\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}}{c}$$

$$\Big\downarrow \text{ 103}$$

$$-\frac{-\frac{\frac{2\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}}{c}$$

$$\Big\downarrow \text{221}$$

$$-\frac{-\frac{\frac{2\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}}{c}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))),x]
```

output
```
-((-((Sqrt[1 - 1/(a*x)]*x)/Sqrt[1 + 1/(a*x)]) - ((2*Sqrt[1 - 1/(a*x)])/Sqr
t[1 + 1/(a*x)] - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a)/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(65) = 130$.

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.94

method	result
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}+\frac{\left(-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}+\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^4\left(x+\frac{1}{a}\right)}\right)a^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c(ax-1)}$
default	$-\frac{\left(-3\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^2x^2+2\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-6\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,ax+4\,l}{2a\sqrt{a^2}\,(ax+1)c\sqrt{(ax-1)(ax}}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/c+(-1/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)+1/a^4/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))/c
*a^2*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{(ax+2)\sqrt{\frac{ax-1}{ax+1}}-\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
((a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)
+ log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{a^2\int \frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx}{c}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2),x)
```

output
```
a**2*Integral(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x)/c
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.70

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
-a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) + log(
sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x + 1)) - 1
)/(a^2*c) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{c-\frac{c}{a^2x^2}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 13.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.21

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}}+\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2)),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1)) + ((a*x
- 1)/(a*x + 1))^(1/2)/(a*c) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.34

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$$
$$=\frac{2\sqrt{ax+1}\,\sqrt{ax-1}\,ax+4\sqrt{ax+1}\,\sqrt{ax-1}-4\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax-4\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)+3ax+3}{2ac\,(ax+1)}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x)
```

output
```
(2*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 4*sqrt(a*x + 1)*sqrt(a*x - 1) - 4*log
((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x - 4*log((sqrt(a*x - 1) + sqr
t(a*x + 1))/sqrt(2)) + 3*a*x + 3)/(2*a*c*(a*x + 1))
```

3.787 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{3a-\frac{5}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{a-\frac{1}{x}}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

output
```
1/3*(3*a-5/x)/a^2/c^2/(1-1/a^2/x^2)^(1/2)+1/3*(a-1/x)/a^2/c^2/(1-1/a^2/x^2
)^(3/2)+(1-1/a^2/x^2)^(1/2)*x/c^2-arctanh((1-1/a^2/x^2)^(1/2))/a/c^2
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-8-5ax+7a^2x^2+3a^3x^3)}{3(-1+ax)(1+ax)^2}-\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^2}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2),x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-8 - 5*a*x + 7*a^2*x^2 + 3*a^3*x^3))/(3*(-1 +
 a*x)*(1 + a*x)^2) - Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.60, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {6748, 114, 27, 169, 25, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int \frac{\left(a-\frac{3}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{\left(a-\frac{3}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$$\Big\downarrow \text{169}$$

$$-\frac{-\frac{a\left(-\int -\frac{\left(a-\frac{4}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}\right)-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{a\int \frac{\left(a-\frac{4}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int \frac{\left(a-\frac{4}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$\Big\downarrow$ 169

$$-\frac{-\frac{\frac{1}{3}a\int \frac{\left(3a-\frac{5}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\frac{1}{3}\int \frac{\left(3a-\frac{5}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$\Big\downarrow$ 169

$$-\frac{-\frac{\frac{1}{3}\left(a\int \frac{3x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\frac{1}{3}\left(3a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$\Big\downarrow$ 103

$$-\frac{-\frac{\frac{1}{3}\left(\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-3\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)\right)+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

$\Big\downarrow$ 221

$$-\frac{-\frac{\frac{1}{3}\left(\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-3a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}}{c^2}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2),x]
```

output
```
-((-(x/(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2))) - ((-2*a)/(Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)) + (5*a*Sqrt[1 - 1/(a*x)])/(3*(1 + 1/(a*x))^(3/2))
+ ((8*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)] - 3*a*ArcTanh[Sqrt[1 - 1/(a*x
)]*Sqrt[1 + 1/(a*x)]])/3)/a^2)/c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(95) = 190$.

Time = 0.16 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.99

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2}+\frac{\left(-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{4a^6\left(x-\frac{1}{a}\right)}+\frac{19\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{12a^6\left(x+\frac{1}{a}\right)}-\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{6a^7\left(x+\frac{1}{a}\right)^2}\right)a^4\sqrt{\cdots}}{c^2(ax-1)}$
default	$-\frac{\left(-45\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^5x^5+24\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^6x^5+21\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^3x^3-45\sqrt{(ax-1)(ax+1)}\cdots\right.}{\cdots}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^2*((a*x-1)/(a*x+1))^(1/2)+(-1/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)-1/4/a^6/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2
)+19/12/a^6/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-1/6/a^7/(x+1/a)^2*(a
^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^4/c^2*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*
(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.11

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{3\,(a^2x^2-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,(a^2x^2-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(3\,a^3x^3+7\,a^2x^2-5\,ax-8)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^2x^2-ac^2)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(3*(a^2*x^2 - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x^2 - 1)
*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (3*a^3*x^3 + 7*a^2*x^2 - 5*a*x - 8)*
sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - a*c^2)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**2,x)
```

output
```
a**4*Integral(x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**2*x
**2 + 1), x)/c**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.52

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx =$$

$$-\frac{1}{12}\,a\left(\frac{3\left(\frac{9\,(ax-1)}{ax+1}-1\right)}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+18\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-\right.}{a^2c^2}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
-1/12*a*(3*(9*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2
) - a^2*c^2*sqrt((a*x - 1)/(a*x + 1))) - (((a*x - 1)/(a*x + 1))^(3/2) + 18
*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^2) + 12*log(sqrt((a*x - 1)/(a*x + 1)) +
 1)/(a^2*c^2) - 12*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.28

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{\frac{9\,(a\,x-1)}{a\,x+1}-1}{4\,a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}-4\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}+\frac{3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a\,c^2}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12\,a\,c^2}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}}{a\,c^2}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^2,x)
```

output
```
((9*(a*x - 1))/(a*x + 1) - 1)/(4*a*c^2*((a*x - 1)/(a*x + 1))^(1/2) - 4*a*c
^2*((a*x - 1)/(a*x + 1))^(3/2)) + (3*((a*x - 1)/(a*x + 1))^(1/2))/(2*a*c^2
) + ((a*x - 1)/(a*x + 1))^(3/2)/(12*a*c^2) + (atan(((a*x - 1)/(a*x + 1))^(
1/2)*1i)*2i)/(a*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.86

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\frac{-24\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^2x^2-48\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)ax-24\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+}}{\sqrt{2}}\right.}{12\sqrt{ax}}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x)
```

output
```
( - 24*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**2*x**
2 - 48*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x - 24
*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 11*sqrt(a*x
- 1)*a**2*x**2 + 22*sqrt(a*x - 1)*a*x + 11*sqrt(a*x - 1) + 12*sqrt(a*x + 1
)*a**3*x**3 + 28*sqrt(a*x + 1)*a**2*x**2 - 20*sqrt(a*x + 1)*a*x - 32*sqrt(
a*x + 1))/(12*sqrt(a*x - 1)*a*c**2*(a**2*x**2 + 2*a*x + 1))
```

3.788 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 140

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{5a-\frac{11}{x}}{5a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5a-\frac{9}{x}}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{a-\frac{1}{x}}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

output
```
1/5*(5*a-11/x)/a^2/c^3/(1-1/a^2/x^2)^(1/2)+1/15*(5*a-9/x)/a^2/c^3/(1-1/a^2
/x^2)^(3/2)+1/5*(a-1/x)/a^2/c^3/(1-1/a^2/x^2)^(5/2)+(1-1/a^2/x^2)^(1/2)*x/
c^3-arctanh((1-1/a^2/x^2)^(1/2))/a/c^3
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(48+33ax-87a^2x^2-52a^3x^3+38a^4x^4+15a^5x^5\right)}{15(-1+ax)^2(1+ax)^3}-\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^3}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3),x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(48 + 33*a*x - 87*a^2*x^2 - 52*a^3*x^3 + 38*a^
4*x^4 + 15*a^5*x^5))/(15*(-1 + a*x)^2*(1 + a*x)^3) - Log[(1 + Sqrt[1 - 1/(
a^2*x^2)])*x])/(a*c^3)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.72, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {6748, 114, 27, 169, 25, 27, 169, 27, 169, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int \frac{\left(a-\frac{5}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{\left(a-\frac{5}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \text{169}$$

$$-\frac{-\frac{-\frac{1}{3}a\int -\frac{\left(3a-\frac{16}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{-\frac{\frac{1}{3}a\int \frac{\left(3a-\frac{16}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\frac{1}{3}\int \frac{\left(3a-\frac{16}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \ 169$$

$$-\frac{-\frac{\frac{1}{3}\left(a\left(-\int -\frac{3\left(a-\frac{13}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\frac{1}{3}\left(3\int \frac{\left(a-\frac{13}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \ 169$$

$$-\frac{-\frac{\frac{1}{3}\left(3\left(\frac{1}{5}a\int \frac{\left(5a-\frac{28}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{14a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\frac{1}{3}\left(3\left(\frac{1}{5}\int \frac{\left(5a-\frac{28}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{14a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{c^3}$$

$$\Big\downarrow \ 169$$

$$-\frac{-\frac{\frac{1}{3}\left(3\left(\frac{1}{5}\left(\frac{1}{3}a\int \frac{3\left(5a-\frac{11}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{11a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{14a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}\right.}}{c^3}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\frac{1}{3}\left(3\left(\frac{1}{5}\left(\int \frac{\left(5a-\frac{11}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{11a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{14a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)}}{c^3}$$

$\Big\downarrow$ 169

$$-\frac{-\frac{\frac{1}{3}\left(3\left(\frac{1}{5}\left(a\int \frac{5x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{16a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{11a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{14a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{}{\left(1-\frac{1}{ax}\right)}}{c^3}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\frac{1}{3}\left(3\left(\frac{1}{5}\left(5a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{16a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{11a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{14a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}-\frac{}{\left(1-\frac{1}{ax}\right.}}{c^3}$$

$\Big\downarrow$ 103

$$-\frac{-\frac{\frac{1}{3}\left(3\left(\frac{1}{5}\left(-5\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+\frac{16a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{11a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{14a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}}{c^3}$$

$\Big\downarrow$ 221

$$-\frac{-\frac{\frac{1}{3}\left(3\left(\frac{1}{5}\left(-5a\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{16a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{11a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{14a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{13a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}\right)-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}}{a^2}}{c^3}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3),x]
```

output
```
-((-(x/((1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2))) - ((-4*a)/(3*(1 - 1/(a*x
))^(3/2)*(1 + 1/(a*x))^(5/2)) + ((-13*a)/(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^
(5/2)) + 3*((14*a*Sqrt[1 - 1/(a*x)])/(5*(1 + 1/(a*x))^(5/2)) + ((11*a*Sqrt
[1 - 1/(a*x)])/(1 + 1/(a*x))^(3/2) + (16*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(
a*x)] - 5*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/5))/3)/a^2)/c^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(124) = 248$.

Time = 0.16 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.05

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3}+\frac{\left(-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^6\sqrt{a^2}}-\frac{25\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{48a^8\left(x-\frac{1}{a}\right)}+\frac{493\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{240a^8\left(x+\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{24a^9\left(x-\frac{1}{a}\right)^2}+\sqrt{a \cdots}\right.}{c^3(ax-1)}$
default	$-\frac{\left(-525\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^7x^7+240\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^8x^7+285((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5-525\sqrt{(ax-1)(ax \cdots}\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^3*((a*x-1)/(a*x+1))^(1/2)+(-1/a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)-25/48/a^8/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1
/2)+493/240/a^8/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-1/24/a^9/(x-1/a)
^2*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)+1/20/a^10/(x+1/a)^3*(a^2*(x+1/a)^2-2*
a*(x+1/a))^(1/2)-23/60/a^9/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^
6/c^3*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.15

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{15\,(a^4x^4-2\,a^2x^2+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,(a^4x^4-2\,a^2x^2+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(15\,a^5x^5+38\,a}{15\,(a^5c^3x^4-2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
-1/15*(15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 1
5*(a^4*x^4 - 2*a^2*x^2 + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (15*a^5*x
^5 + 38*a^4*x^4 - 52*a^3*x^3 - 87*a^2*x^2 + 33*a*x + 48)*sqrt((a*x - 1)/(a
*x + 1)))/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{a^6\int \frac{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**3,x)
```

output
```
a**6*Integral(x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 3*a**4*x
**4 + 3*a**2*x**2 - 1), x)/c**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.41

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$=\frac{1}{240}\,a\left(\frac{5\left(\frac{23\,(ax-1)}{ax+1}-\frac{120\,(ax-1)^2}{(ax+1)^2}+1\right)}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+40\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+450\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}-\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
1/240*a*(5*(23*(a*x - 1)/(a*x + 1) - 120*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2
*c^3*((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)) +
(3*((a*x - 1)/(a*x + 1))^(5/2) + 40*((a*x - 1)/(a*x + 1))^(3/2) + 450*sqrt
((a*x - 1)/(a*x + 1)))/(a^2*c^3) - 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/
(a^2*c^3) + 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.27

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{15\sqrt{\frac{a\,x-1}{a\,x+1}}}{8\,a\,c^3} - \frac{\frac{23\,(a\,x-1)}{3\,(a\,x+1)} - \frac{40\,(a\,x-1)^2}{(a\,x+1)^2} + \frac{1}{3}}{16\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} - 16\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}} + \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6\,a\,c^3} + \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{80\,a\,c^3} + \frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,2\mathrm{i}}{a\,c^3}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^3,x)
```

output
```
(15*((a*x - 1)/(a*x + 1))^(1/2))/(8*a*c^3) - ((23*(a*x - 1))/(3*(a*x + 1))
 - (40*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(3/
2) - 16*a*c^3*((a*x - 1)/(a*x + 1))^(5/2)) + ((a*x - 1)/(a*x + 1))^(3/2)/(
6*a*c^3) + ((a*x - 1)/(a*x + 1))^(5/2)/(80*a*c^3) + (atan(((a*x - 1)/(a*x
+ 1))^(1/2)*1i)*2i)/(a*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.01

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{-120\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^4x^4 - 240\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^3x^3 + 240\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}\cdots}{\sqrt{\cdots}}\right.}{}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x)
```

output
```
( - 120*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**4*x*
*4 - 240*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**3*x
**3 + 240*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x +
 120*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 133*sqrt
(a*x - 1)*a**4*x**4 + 266*sqrt(a*x - 1)*a**3*x**3 - 266*sqrt(a*x - 1)*a*x
- 133*sqrt(a*x - 1) + 60*sqrt(a*x + 1)*a**5*x**5 + 152*sqrt(a*x + 1)*a**4*
x**4 - 208*sqrt(a*x + 1)*a**3*x**3 - 348*sqrt(a*x + 1)*a**2*x**2 + 132*sqr
t(a*x + 1)*a*x + 192*sqrt(a*x + 1))/(60*sqrt(a*x - 1)*a*c**3*(a**4*x**4 +
2*a**3*x**3 - 2*a*x - 1))
```

3.789 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 173

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{35a-\frac{93}{x}}{35a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{35a-\frac{87}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{7a-\frac{13}{x}}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{a-\frac{1}{x}}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

output

```
1/35*(35*a-93/x)/a^2/c^4/(1-1/a^2/x^2)^(1/2)+1/105*(35*a-87/x)/a^2/c^4/(1-
1/a^2/x^2)^(3/2)+1/35*(7*a-13/x)/a^2/c^4/(1-1/a^2/x^2)^(5/2)+1/7*(a-1/x)/a
^2/c^4/(1-1/a^2/x^2)^(7/2)+(1-1/a^2/x^2)^(1/2)*x/c^4-arctanh((1-1/a^2/x^2)
^(1/2))/a/c^4
```

Mathematica [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.68

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-384-279ax+1065a^2x^2+715a^3x^3-965a^4x^4-559a^5x^5+281a^6x^6+105a^7x^7)}{105(-1+ax)^3(1+ax)^4}-\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^4}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4),x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-384 - 279*a*x + 1065*a^2*x^2 + 715*a^3*x^3 -
 965*a^4*x^4 - 559*a^5*x^5 + 281*a^6*x^6 + 105*a^7*x^7))/(105*(-1 + a*x)^3
*(1 + a*x)^4) - Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.80, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.955$, Rules used = {6748, 114, 27, 169, 25, 27, 169, 27, 169, 25, 27, 169, 27, 169, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int \frac{\left(a-\frac{7}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{\left(a-\frac{7}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \; 169$$

$$-\frac{-\frac{-\frac{1}{5}a\int -\frac{\left(5a-\frac{36}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\frac{1}{5}a\int \frac{\left(5a-\frac{36}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{1}{5}\int \frac{\left(5a-\frac{36}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \; 169$$

$$-\frac{-\frac{\frac{1}{5}\left(-\frac{1}{3}a\int -\frac{5\left(3a-\frac{31}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\int \frac{\left(3a-\frac{31}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \; 169$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(a\left(-\int -\frac{\left(3a-\frac{112}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}\right)-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(a\int \frac{\left(3a-\frac{112}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\int \frac{\left(3a-\frac{112}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^4}$$

$$\Big\downarrow \; 169$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{1}{7}a\int \frac{3\left(7a-\frac{115}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{}{\left(1-\frac{1}{ax}\right)}}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\int \frac{\left(7a-\frac{115}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{}{\left(1-\frac{1}{ax}\right)^{5/}}}{c^4}$$

$$\Big\downarrow \; 169$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\left(\frac{1}{5}a\int \frac{\left(35a-\frac{244}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{122a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}\right.}}{a^2}}{c^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\left(\frac{1}{5}\int \frac{\left(35a-\frac{244}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{122a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{6a}{5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+\right.}}{a^2}}{c^4}$$

$$\Big\downarrow \; 169$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}a\int \frac{3\left(35a-\frac{93}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{93a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{122a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}}{c^4}$$

$\big\downarrow$ 27

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\left(\frac{1}{5}\left(\int \frac{\left(35a-\frac{93}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{93a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{122a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-}{a^2}}{c^4}$$

$\big\downarrow$ 169

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\left(\frac{1}{5}\left(a\int \frac{35x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{128a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{93a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{122a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}\right.}}{a^2}}{c^4}$$

$\big\downarrow$ 27

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\left(\frac{1}{5}\left(35a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{128a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{93a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{122a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{31a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\right.}}{a^2}}{c^4}$$

$\big\downarrow$ 103

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\left(\frac{1}{5}\left(-35\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+\frac{128a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{93a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{122a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{}{3(}}{a^2}}{c^4}$$

$\big\downarrow$ 221

$$-\frac{-\frac{\frac{1}{5}\left(\frac{5}{3}\left(\frac{3}{7}\left(\frac{1}{5}\left(-35a\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{128a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{93a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{122a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{115a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}\right)-\frac{}{3(1}}{a^2}}{c^4}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4),x]
```

output
```
-((-(x/((1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2))) - ((-6*a)/(5*(1 - 1/(a*x
))^(5/2)*(1 + 1/(a*x))^(7/2)) + ((-31*a)/(3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*
x))^(7/2)) + (5*((-28*a)/(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)) + (115*a*
Sqrt[1 - 1/(a*x)])/(7*(1 + 1/(a*x))^(7/2)) + (3*((122*a*Sqrt[1 - 1/(a*x)])
/(5*(1 + 1/(a*x))^(5/2)) + ((93*a*Sqrt[1 - 1/(a*x)])/(1 + 1/(a*x))^(3/2) +
 (128*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)] - 35*a*ArcTanh[Sqrt[1 - 1/(a*
x)]*Sqrt[1 + 1/(a*x)]])/5))/7))/3)/5)/a^2)/c^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(153) = 306$.

Time = 0.18 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.09

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^4}+\frac{\left(-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^8\sqrt{a^2}}-\frac{379\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{480a^{10}\left(x-\frac{1}{a}\right)}+\frac{1657\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{672a^{10}\left(x+\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{80a^{12}\left(x-\frac{1}{a}\right)^3}-\cdots\right.}{}$
default	Expression too large to display

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^4*((a*x-1)/(a*x+1))^(1/2)+(-1/a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)-379/480/a^10/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))
^(1/2)+1657/672/a^10/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-1/80/a^12/(
x-1/a)^3*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)-7/60/a^11/(x-1/a)^2*((x-1/a)^2*
a^2+2*a*(x-1/a))^(1/2)-1/56/a^13/(x+1/a)^4*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/
2)+17/112/a^12/(x+1/a)^3*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-211/336/a^11/(x
+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^8/c^4*((a*x-1)/(a*x+1))^(1/2)
*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{105\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-\right.}{105\left(a^7c^4x^6-3\,a^5c^4x^4\right.}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
-1/105*(105*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(sqrt((a*x - 1)/(a*x
+ 1)) + 1) - 105*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(sqrt((a*x - 1)/
(a*x + 1)) - 1) - (105*a^7*x^7 + 281*a^6*x^6 - 559*a^5*x^5 - 965*a^4*x^4 +
 715*a^3*x^3 + 1065*a^2*x^2 - 279*a*x - 384)*sqrt((a*x - 1)/(a*x + 1)))/(a
^7*c^4*x^6 - 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2 - a*c^4)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^8x^8-4a^6x^6+6a^4x^4-4a^2x^2+1}\,dx}{c^4}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**4,x)
```

output
```
a**8*Integral(x**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**8*x**8 - 4*a**6*x
**6 + 6*a**4*x**4 - 4*a**2*x**2 + 1), x)/c**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.34

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{1}{6720}a\left(\frac{7\left(\frac{47\,(ax-1)}{ax+1}+\frac{655\,(ax-1)^2}{(ax+1)^2}-\frac{2625\,(ax-1)^3}{(ax+1)^3}+3\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{5\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+42\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+329\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+2940\right.}{a^2c^4}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
1/6720*a*(7*(47*(a*x - 1)/(a*x + 1) + 655*(a*x - 1)^2/(a*x + 1)^2 - 2625*(
a*x - 1)^3/(a*x + 1)^3 + 3)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^4
*((a*x - 1)/(a*x + 1))^(5/2)) + 5*(3*((a*x - 1)/(a*x + 1))^(7/2) + 42*((a*
x - 1)/(a*x + 1))^(5/2) + 329*((a*x - 1)/(a*x + 1))^(3/2) + 2940*sqrt((a*x
 - 1)/(a*x + 1)))/(a^2*c^4) - 6720*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2
*c^4) + 6720*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.25

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{35\sqrt{\frac{a\,x-1}{a\,x+1}}}{16\,a\,c^4}-\frac{\frac{131\,(a\,x-1)^2}{3\,(a\,x+1)^2}-\frac{175\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{47\,(a\,x-1)}{15\,(a\,x+1)}+\frac{1}{5}}{64\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}-64\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}+\frac{47\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{192\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{32\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{448\,a\,c^4}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,2\mathrm{i}}{a\,c^4}$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^4,x)
```

output
```
(35*((a*x - 1)/(a*x + 1))^(1/2))/(16*a*c^4) - ((131*(a*x - 1)^2)/(3*(a*x +
 1)^2) - (175*(a*x - 1)^3)/(a*x + 1)^3 + (47*(a*x - 1))/(15*(a*x + 1)) + 1
/5)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 64*a*c^4*((a*x - 1)/(a*x + 1))
^(7/2)) + (47*((a*x - 1)/(a*x + 1))^(3/2))/(192*a*c^4) + ((a*x - 1)/(a*x +
 1))^(5/2)/(32*a*c^4) + ((a*x - 1)/(a*x + 1))^(7/2)/(448*a*c^4) + (atan(((
a*x - 1)/(a*x + 1))^(1/2)*1i)*2i)/(a*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.75

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$=\frac{-840\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^6x^6-1680\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^5x^5+840\sqrt{ax-1}\,\log\left(\frac{\sqrt{ax-}}{}\right.}{}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x)
```

output
```
( - 840*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**6*x*
*6 - 1680*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**5*
x**5 + 840*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**4
*x**4 + 3360*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*
*3*x**3 + 840*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a
**2*x**2 - 1680*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))
*a*x - 840*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) + 15
01*sqrt(a*x - 1)*a**6*x**6 + 3002*sqrt(a*x - 1)*a**5*x**5 - 1501*sqrt(a*x
- 1)*a**4*x**4 - 6004*sqrt(a*x - 1)*a**3*x**3 - 1501*sqrt(a*x - 1)*a**2*x*
*2 + 3002*sqrt(a*x - 1)*a*x + 1501*sqrt(a*x - 1) + 420*sqrt(a*x + 1)*a**7*
x**7 + 1124*sqrt(a*x + 1)*a**6*x**6 - 2236*sqrt(a*x + 1)*a**5*x**5 - 3860*
sqrt(a*x + 1)*a**4*x**4 + 2860*sqrt(a*x + 1)*a**3*x**3 + 4260*sqrt(a*x + 1
)*a**2*x**2 - 1116*sqrt(a*x + 1)*a*x - 1536*sqrt(a*x + 1))/(420*sqrt(a*x -
 1)*a*c**4*(a**6*x**6 + 2*a**5*x**5 - a**4*x**4 - 4*a**3*x**3 - a**2*x**2
+ 2*a*x + 1))
```

3.790 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x-\frac{2c^4\log(x)}{a}$$

output
```
1/7*c^4/a^8/x^7-1/3*c^4/a^7/x^6-2/5*c^4/a^6/x^5+3/2*c^4/a^5/x^4-3*c^4/a^3/
x^2+2*c^4/a^2/x+c^4*x-2*c^4*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x-\frac{2c^4\log(x)}{a}$$

input
```
Integrate[(c - c/(a^2*x^2))^4/E^(2*ArcCoth[a*x]),x]
```

output
```
c^4/(7*a^8*x^7) - c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) + (3*c^4)/(2*a^5*x
^4) - (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x - (2*c^4*Log[x])/a
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^4 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{c^4e^{-2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^4}{a^8}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^4\int e^{-2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^4dx}{a^8}$$

$$\Big\downarrow \text{ 6707}$$

$$-\frac{c^4\int \frac{e^{-2\text{arctanh}(ax)}(1-a^2x^2)^4}{x^8}dx}{a^8}$$

$$\Big\downarrow \text{ 6700}$$

$$-\frac{c^4\int \frac{(1-ax)^5(ax+1)^3}{x^8}dx}{a^8}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{c^4\int \left(-a^8+\frac{2a^7}{x}+\frac{2a^6}{x^2}-\frac{6a^5}{x^3}+\frac{6a^3}{x^5}-\frac{2a^2}{x^6}-\frac{2a}{x^7}+\frac{1}{x^8}\right)dx}{a^8}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{c^4\left(a^8(-x)+2a^7\log(x)-\frac{2a^6}{x}+\frac{3a^5}{x^2}-\frac{3a^3}{2x^4}+\frac{2a^2}{5x^5}+\frac{a}{3x^6}-\frac{1}{7x^7}\right)}{a^8}$$

input
```
Int[(c - c/(a^2*x^2))^4/E^(2*ArcCoth[a*x]),x]
```

output
```
-((c^4*(-1/7*1/x^7 + a/(3*x^6) + (2*a^2)/(5*x^5) - (3*a^3)/(2*x^4) + (3*a^
5)/x^2 - (2*a^6)/x - a^8*x + 2*a^7*Log[x]))/a^8)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.71

method	result
default	$\frac{c^{4}\left(x\,a^{8}-\frac{a}{3x^{6}}-\frac{3a^{5}}{x^{2}}+\frac{2a^{6}}{x}+\frac{1}{7x^{7}}+\frac{3a^{3}}{2x^{4}}-2a^{7}\ln(x)-\frac{2a^{2}}{5x^{5}}\right)}{a^{8}}$
risch	$c^{4}x+\frac{2a^{6}c^{4}x^{6}-3a^{5}c^{4}x^{5}+\frac{3}{2}a^{3}c^{4}x^{3}-\frac{2}{5}a^{2}c^{4}x^{2}-\frac{1}{3}a\,c^{4}x+\frac{1}{7}c^{4}}{a^{8}x^{7}}-\frac{2c^{4}\ln(x)}{a}$
norman	$\frac{a^{7}c^{4}x^{8}+\frac{c^{4}}{7a}-\frac{c^{4}x}{3}-\frac{2a\,c^{4}x^{2}}{5}+\frac{3a^{2}c^{4}x^{3}}{2}-3a^{4}c^{4}x^{5}+2a^{5}c^{4}x^{6}}{a^{7}x^{7}}-\frac{2c^{4}\ln(x)}{a}$
parallelrisch	$-\frac{-210a^{8}c^{4}x^{8}+420c^{4}\ln(x)a^{7}x^{7}-420a^{6}c^{4}x^{6}+630a^{5}c^{4}x^{5}-315a^{3}c^{4}x^{3}+84a^{2}c^{4}x^{2}+70a\,c^{4}x-30c^{4}}{210a^{8}x^{7}}$
meijerg	$\frac{c^{4}(ax-\ln(ax+1))}{a}-\frac{c^{4}\ln(ax+1)}{a}-\frac{4c^{4}(\ln(x)+\ln(a)-\ln(ax+1))}{a}+\frac{4c^{4}\left(-\frac{1}{ax}-\ln(x)-\ln(a)+\ln(ax+1)\right)}{a}+\frac{6c^{4}\left(-\frac{1}{2a^{2}x}\right.}{}$

input
```
int((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^4/a^8*(x*a^8-1/3*a/x^6-3*a^5/x^2+2*a^6/x+1/7/x^7+3/2*a^3/x^4-2*a^7*ln(x)
-2/5*a^2/x^5)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{210\,a^8c^4x^8-420\,a^7c^4x^7\log(x)+420\,a^6c^4x^6-630\,a^5c^4x^5+315\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

input
```
integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/210*(210*a^8*c^4*x^8 - 420*a^7*c^4*x^7*log(x) + 420*a^6*c^4*x^6 - 630*a^
5*c^4*x^5 + 315*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x
^7)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = \frac{a^8c^4x-2a^7c^4\log(x)+\frac{420a^6c^4x^6-630a^5c^4x^5+315a^3c^4x^3-84a^2c^4x^2-70ac^4x+30c^4}{210x^7}}{a^8}$$

input
```
integrate((c-c/a**2/x**2)**4*(a*x-1)/(a*x+1),x)
```

output
```
(a**8*c**4*x - 2*a**7*c**4*log(x) + (420*a**6*c**4*x**6 - 630*a**5*c**4*x*
*5 + 315*a**3*c**4*x**3 - 84*a**2*c**4*x**2 - 70*a*c**4*x + 30*c**4)/(210*
x**7))/a**8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = c^4x-\frac{2\,c^4\log(x)}{a} + \frac{420\,a^6c^4x^6-630\,a^5c^4x^5+315\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

input
```
integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
c^4*x - 2*c^4*log(x)/a + 1/210*(420*a^6*c^4*x^6 - 630*a^5*c^4*x^5 + 315*a^
3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=c^4x-\frac{2\,c^4\log(|x|)}{a}+\frac{420\,a^6c^4x^6-630\,a^5c^4x^5+315\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

input
```
integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
c^4*x - 2*c^4*log(abs(x))/a + 1/210*(420*a^6*c^4*x^6 - 630*a^5*c^4*x^5 + 3
15*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

Mupad [B] (verification not implemented)

Time = 13.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=-\frac{c^4\left(\frac{a\,x}{3}+\frac{2\,a^2\,x^2}{5}-\frac{3\,a^3\,x^3}{2}+3\,a^5\,x^5-2\,a^6\,x^6-a^8\,x^8+2\,a^7\,x^7\,\ln(x)-\frac{1}{7}\right)}{a^8\,x^7}$$

input
```
int(((c - c/(a^2*x^2))^4*(a*x - 1))/(a*x + 1),x)
```

output
```
-(c^4*((a*x)/3 + (2*a^2*x^2)/5 - (3*a^3*x^3)/2 + 3*a^5*x^5 - 2*a^6*x^6 - a
^8*x^8 + 2*a^7*x^7*log(x) - 1/7))/(a^8*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{c^4(-420\log(x)\,a^7x^7+210a^8x^8+420a^6x^6-630a^5x^5+315a^3x^3-84a^2x^2-70ax+30)}{210a^8x^7}$$

input
```
int((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x)
```

output
```
(c**4*( - 420*log(x)*a**7*x**7 + 210*a**8*x**8 + 420*a**6*x**6 - 630*a**5*
x**5 + 315*a**3*x**3 - 84*a**2*x**2 - 70*a*x + 30))/(210*a**8*x**7)
```

3.791 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=-\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{2c^3\log(x)}{a}$$

output
```
-1/5*c^3/a^6/x^5+1/2*c^3/a^5/x^4+1/3*c^3/a^4/x^3-2*c^3/a^3/x^2+c^3/a^2/x+c
^3*x-2*c^3*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=-\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{2c^3\log(x)}{a}$$

input
```
Integrate[(c - c/(a^2*x^2))^3/E^(2*ArcCoth[a*x]),x]
```

output
```
-1/5*c^3/(a^6*x^5) + c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) - (2*c^3)/(a^3*x^2)
 + c^3/(a^2*x) + c^3*x - (2*c^3*Log[x])/a
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^3 e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{c^3e^{-2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^3}{a^6}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\int e^{-2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^3dx}{a^6}$$

$$\Big\downarrow \text{6707}$$

$$\frac{c^3\int \frac{e^{-2\text{arctanh}(ax)}(1-a^2x^2)^3}{x^6}dx}{a^6}$$

$$\Big\downarrow \text{6700}$$

$$\frac{c^3\int \frac{(1-ax)^4(ax+1)^2}{x^6}dx}{a^6}$$

$$\Big\downarrow \text{99}$$

$$\frac{c^3\int \left(a^6-\frac{2a^5}{x}-\frac{a^4}{x^2}+\frac{4a^3}{x^3}-\frac{a^2}{x^4}-\frac{2a}{x^5}+\frac{1}{x^6}\right)dx}{a^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3\left(a^6x-2a^5\log(x)+\frac{a^4}{x}-\frac{2a^3}{x^2}+\frac{a^2}{3x^3}+\frac{a}{2x^4}-\frac{1}{5x^5}\right)}{a^6}$$

input
```
Int[(c - c/(a^2*x^2))^3/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^3*(-1/5*1/x^5 + a/(2*x^4) + a^2/(3*x^3) - (2*a^3)/x^2 + a^4/x + a^6*x -
 2*a^5*Log[x]))/a^6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.72

method	result
default	$\frac{c^3\left(x\,a^6-\frac{2a^3}{x^2}+\frac{a^4}{x}+\frac{a}{2x^4}-2a^5\ln\left(x\right)+\frac{a^2}{3x^3}-\frac{1}{5x^5}\right)}{a^6}$
risch	$c^3x+\frac{a^4c^3x^4-2a^3c^3x^3+\frac{1}{3}a^2c^3x^2+\frac{1}{2}a\,c^3x-\frac{1}{5}c^3}{a^6x^5}-\frac{2c^3\ln\left(x\right)}{a}$
norman	$\frac{a^3c^3x^4+a^5c^3x^6-\frac{c^3}{5a}+\frac{c^3x}{2}+\frac{a\,c^3x^2}{3}-2a^2c^3x^3}{a^5x^5}-\frac{2c^3\ln\left(x\right)}{a}$
parallelrisch	$-\frac{-30a^6c^3x^6+60c^3\ln\left(x\right)a^5x^5-30a^4c^3x^4+60a^3c^3x^3-10a^2c^3x^2-15a\,c^3x+6c^3}{30a^6x^5}$
meijerg	$\frac{c^3\left(ax-\ln\left(ax+1\right)\right)}{a}-\frac{c^3\ln\left(ax+1\right)}{a}-\frac{3c^3\left(\ln\left(x\right)+\ln\left(a\right)-\ln\left(ax+1\right)\right)}{a}+\frac{3c^3\left(-\frac{1}{ax}-\ln\left(x\right)-\ln\left(a\right)+\ln\left(ax+1\right)\right)}{a}+\frac{3c^3\left(-\frac{1}{2a^2x}\right.}{}$

input
```
int((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^3/a^6*(x*a^6-2*a^3/x^2+a^4/x+1/2*a/x^4-2*a^5*ln(x)+1/3*a^2/x^3-1/5/x^5)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{30\,a^6c^3x^6-60\,a^5c^3x^5\log\left(x\right)+30\,a^4c^3x^4-60\,a^3c^3x^3+10\,a^2c^3x^2+15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

input
```
integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/30*(30*a^6*c^3*x^6 - 60*a^5*c^3*x^5*log(x) + 30*a^4*c^3*x^4 - 60*a^3*c^3
*x^3 + 10*a^2*c^3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{a^6c^3x-2a^5c^3\log(x)+\frac{30a^4c^3x^4-60a^3c^3x^3+10a^2c^3x^2+15ac^3x-6c^3}{30x^5}}{a^6}$$

input
```
integrate((c-c/a**2/x**2)**3*(a*x-1)/(a*x+1),x)
```

output
```
(a**6*c**3*x - 2*a**5*c**3*log(x) + (30*a**4*c**3*x**4 - 60*a**3*c**3*x**3
 + 10*a**2*c**3*x**2 + 15*a*c**3*x - 6*c**3)/(30*x**5))/a**6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = c^3x-\frac{2\,c^3\log(x)}{a}+\frac{30\,a^4c^3x^4-60\,a^3c^3x^3+10\,a^2c^3x^2+15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

input
```
integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
c^3*x - 2*c^3*log(x)/a + 1/30*(30*a^4*c^3*x^4 - 60*a^3*c^3*x^3 + 10*a^2*c^
3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = c^3x - \frac{2\,c^3\log\left(|x|\right)}{a} + \frac{30\,a^4c^3x^4 - 60\,a^3c^3x^3 + 10\,a^2c^3x^2 + 15\,ac^3x - 6\,c^3}{30\,a^6x^5}$$

input
```
integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
c^3*x - 2*c^3*log(abs(x))/a + 1/30*(30*a^4*c^3*x^4 - 60*a^3*c^3*x^3 + 10*a
^2*c^3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{c^3\left(\frac{a\,x}{2}+\frac{a^2\,x^2}{3}-2\,a^3\,x^3+a^4\,x^4+a^6\,x^6-2\,a^5\,x^5\,\ln\left(x\right)-\frac{1}{5}\right)}{a^6\,x^5}$$

input
```
int(((c - c/(a^2*x^2))^3*(a*x - 1))/(a*x + 1),x)
```

output
```
(c^3*((a*x)/2 + (a^2*x^2)/3 - 2*a^3*x^3 + a^4*x^4 + a^6*x^6 - 2*a^5*x^5*lo
g(x) - 1/5))/(a^6*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$$
$$=\frac{c^3(-60\log(x)\,a^5x^5+30a^6x^6+30a^4x^4-60a^3x^3+10a^2x^2+15ax-6)}{30a^6x^5}$$

input

```
int((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x)
```

output

```
(c**3*( - 60*log(x)*a**5*x**5 + 30*a**6*x**6 + 30*a**4*x**4 - 60*a**3*x**3
 + 10*a**2*x**2 + 15*a*x - 6))/(30*a**6*x**5)
```

3.792 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 40

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}+c^2x-\frac{2c^2\log(x)}{a}$$

output
```
1/3*c^2/a^4/x^3-c^2/a^3/x^2+c^2*x-2*c^2*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}+c^2x-\frac{2c^2\log(x)}{a}$$

input
```
Integrate[(c - c/(a^2*x^2))^2/E^(2*ArcCoth[a*x]),x]
```

output
```
c^2/(3*a^4*x^3) - c^2/(a^3*x^2) + c^2*x - (2*c^2*Log[x])/a
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^2 e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{c^2 e^{-2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^2}{a^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c^2\int e^{-2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)^2dx}{a^4}$$

$$\Big\downarrow \text{ 6707}$$

$$-\frac{c^2\int \frac{e^{-2\text{arctanh}(ax)}(1-a^2x^2)^2}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{ 6700}$$

$$-\frac{c^2\int \frac{(1-ax)^3(ax+1)}{x^4}dx}{a^4}$$

$$\Big\downarrow \text{ 84}$$

$$-\frac{c^2\int \left(-a^4+\frac{2a^3}{x}-\frac{2a}{x^3}+\frac{1}{x^4}\right)dx}{a^4}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{c^2\left(a^4(-x)+2a^3\log(x)+\frac{a}{x^2}-\frac{1}{3x^3}\right)}{a^4}$$

input

```
Int[(c - c/(a^2*x^2))^2/E^(2*ArcCoth[a*x]),x]
```

output
```
-((c^2*(-1/3*1/x^3 + a/x^2 - a^4*x + 2*a^3*Log[x]))/a^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result
default	$\frac{c^2\left(x\,a^4-\frac{a}{x^2}-2a^3\ln(x)+\frac{1}{3x^3}\right)}{a^4}$
risch	$c^2x+\frac{-a\,c^2x+\frac{1}{3}c^2}{a^4x^3}-\frac{2c^2\ln(x)}{a}$
norman	$\frac{a^3c^2x^4+\frac{c^2}{3a}-c^2x}{a^3x^3}-\frac{2c^2\ln(x)}{a}$
parallelrisch	$-\frac{-3a^4c^2x^4+6c^2\ln(x)a^3x^3+3a\,c^2x-c^2}{3a^4x^3}$
meijerg	$\frac{c^2(ax-\ln(ax+1))}{a}-\frac{c^2\ln(ax+1)}{a}-\frac{2c^2(\ln(x)+\ln(a)-\ln(ax+1))}{a}+\frac{2c^2\left(-\frac{1}{ax}-\ln(x)-\ln(a)+\ln(ax+1)\right)}{a}+\frac{c^2\left(-\frac{1}{2a^2x^2}\right.}{}$

input
```
int((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c^2/a^4*(x*a^4-a/x^2-2*a^3*ln(x)+1/3/x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{3\,a^4c^2x^4-6\,a^3c^2x^3\log\left(x\right)-3\,ac^2x+c^2}{3\,a^4x^3}$$

input
```
integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
1/3*(3*a^4*c^2*x^4 - 6*a^3*c^2*x^3*log(x) - 3*a*c^2*x + c^2)/(a^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{a^4c^2x-2a^3c^2\log(x)+\frac{-3ac^2x+c^2}{3x^3}}{a^4}$$

input
```
integrate((c-c/a**2/x**2)**2*(a*x-1)/(a*x+1),x)
```

output
```
(a**4*c**2*x - 2*a**3*c**2*log(x) + (-3*a*c**2*x + c**2)/(3*x**3))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=c^2x-\frac{2\,c^2\log(x)}{a}-\frac{3\,ac^2x-c^2}{3\,a^4x^3}$$

input
```
integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
c^2*x - 2*c^2*log(x)/a - 1/3*(3*a*c^2*x - c^2)/(a^4*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=c^2x-\frac{2\,c^2\log(|x|)}{a}-\frac{3\,ac^2x-c^2}{3\,a^4x^3}$$

input
```
integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
c^2*x - 2*c^2*log(abs(x))/a - 1/3*(3*a*c^2*x - c^2)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 13.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=-\frac{c^2\,\left(3\,a\,x-3\,a^4\,x^4+6\,a^3\,x^3\,\ln\left(x\right)-1\right)}{3\,a^4\,x^3}$$

input
```
int(((c - c/(a^2*x^2))^2*(a*x - 1))/(a*x + 1),x)
```

output
```
-(c^2*(3*a*x - 3*a^4*x^4 + 6*a^3*x^3*log(x) - 1))/(3*a^4*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2(-6\log(x)\,a^3x^3+3a^4x^4-3ax+1)}{3a^4x^3}$$

input
```
int((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x)
```

output
```
(c**2*( - 6*log(x)*a**3*x**3 + 3*a**4*x**4 - 3*a*x + 1))/(3*a**4*x**3)
```

3.793 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 21

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c}{a^2x}+cx-\frac{2c\log(x)}{a}$$

output
```
-c/a^2/x+c*x-2*c*ln(x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c}{a^2x}+cx-\frac{2c\log(x)}{a}$$

input
```
Integrate[(c - c/(a^2*x^2))/E^(2*ArcCoth[a*x]),x]
```

output
```
-(c/(a^2*x)) + c*x - (2*c*Log[x])/a
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6717, 27, 6707, 6700, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right) e^{-2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{ce^{-2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\int e^{-2\text{arctanh}(ax)}\left(a^2-\frac{1}{x^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{6707}$$

$$\frac{c\int \frac{e^{-2\text{arctanh}(ax)}(1-a^2x^2)}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{6700}$$

$$\frac{c\int \frac{(1-ax)^2}{x^2}dx}{a^2}$$

$$\Big\downarrow \text{49}$$

$$\frac{c\int \left(a^2-\frac{2a}{x}+\frac{1}{x^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\left(a^2x-2a\log(x)-\frac{1}{x}\right)}{a^2}$$

input
```
Int[(c - c/(a^2*x^2))/E^(2*ArcCoth[a*x]),x]
```

output
```
(c*(-x^(-1) + a^2*x - 2*a*Log[x]))/a^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{c\left(a^{2}x-\frac{1}{x}-2\ln\left(x\right)a\right)}{a^{2}}$	22
risch	$-\frac{c}{a^{2}x}+xc-\frac{2c\ln\left(x\right)}{a}$	22
parallelrisch	$-\frac{-a^{2}c\,x^{2}+2c\ln\left(x\right)ax+c}{a^{2}x}$	27
norman	$\frac{ac\,x^{2}-\frac{c}{a}}{ax}-\frac{2c\ln\left(x\right)}{a}$	30
meijerg	$\frac{c\left(ax-\ln\left(ax+1\right)\right)}{a}-\frac{c\ln\left(ax+1\right)}{a}-\frac{c\left(\ln\left(x\right)+\ln\left(a\right)-\ln\left(ax+1\right)\right)}{a}+\frac{c\left(-\frac{1}{ax}-\ln\left(x\right)-\ln\left(a\right)+\ln\left(ax+1\right)\right)}{a}$	78

input
```
int((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
c/a^2*(a^2*x-1/x-2*ln(x)*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{a^2cx^2-2\,acx\log\left(x\right)-c}{a^2x}$$

input
```
integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
(a^2*c*x^2 - 2*a*c*x*log(x) - c)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{a^2cx-2ac\log(x)-\frac{c}{x}}{a^2}$$

input
```
integrate((c-c/a**2/x**2)*(a*x-1)/(a*x+1),x)
```

output
```
(a**2*c*x - 2*a*c*log(x) - c/x)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=cx-\frac{2\,c\log(x)}{a}-\frac{c}{a^2x}$$

input
```
integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
c*x - 2*c*log(x)/a - c/(a^2*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=cx-\frac{2\,c\log(|x|)}{a}-\frac{c}{a^2x}$$

input
```
integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
c*x - 2*c*log(abs(x))/a - c/(a^2*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c\,(2\,a\,x\,\ln{(x)}-a^2\,x^2+1)}{a^2\,x}$$

input
```
int(((c - c/(a^2*x^2))*(a*x - 1))/(a*x + 1),x)
```

output
```
-(c*(2*a*x*log(x) - a^2*x^2 + 1))/(a^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c(-2\log(x)\,ax+a^2x^2-1)}{a^2x}$$

input
```
int((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x)
```

output
```
(c*( - 2*log(x)*a*x + a**2*x**2 - 1))/(a**2*x)
```

3.794 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 35

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}-\frac{1}{ac(1+ax)}-\frac{2\log(1+ax)}{ac}$$

output
```
x/c-1/a/c/(a*x+1)-2*ln(a*x+1)/a/c
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x-\frac{1}{a+a^2x}-\frac{2\log(1+ax)}{a}}{c}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

output
```
(x - (a + a^2*x)^(-1) - (2*Log[1 + a*x])/a)/c
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{a^2e^{-2\text{arctanh}(ax)}}{c\left(a^2-\frac{1}{x^2}\right)}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^2\int \frac{e^{-2\text{arctanh}(ax)}}{a^2-\frac{1}{x^2}}dx}{c}$$

$$\Big\downarrow \text{ 6707}$$

$$\frac{a^2\int \frac{e^{-2\text{arctanh}(ax)}x^2}{1-a^2x^2}dx}{c}$$

$$\Big\downarrow \text{ 6700}$$

$$\frac{a^2\int \frac{x^2}{(ax+1)^2}dx}{c}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{a^2\int \left(\frac{1}{a^2}-\frac{2}{a^2(ax+1)}+\frac{1}{a^2(ax+1)^2}\right)dx}{c}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2\left(-\frac{1}{a^3(ax+1)}-\frac{2\log(ax+1)}{a^3}+\frac{x}{a^2}\right)}{c}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

output
```
(a^2*(x/a^2 - 1/(a^3*(1 + a*x)) - (2*Log[1 + a*x])/a^3))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
risch	$\frac{x}{c}-\frac{1}{ac(ax+1)}-\frac{2\ln(ax+1)}{ac}$	36
default	$\frac{a^2\left(\frac{x}{a^2}-\frac{2\ln(ax+1)}{a^3}-\frac{1}{a^3(ax+1)}\right)}{c}$	37
norman	$\frac{\frac{a\,x^2}{c}+\frac{2x}{c}}{ax+1}-\frac{2\ln(ax+1)}{ac}$	39
parallelrisch	$\frac{a^2x^2-2\ln(ax+1)xa+2ax-2\ln(ax+1)}{c(ax+1)a}$	45

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
x/c-1/a/c/(a*x+1)-2*ln(a*x+1)/a/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{a^2x^2+ax-2\,(ax+1)\log\left(ax+1\right)-1}{a^2cx+ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
(a^2*x^2 + a*x - 2*(a*x + 1)*log(a*x + 1) - 1)/(a^2*c*x + a*c)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = a^2\left(-\frac{1}{a^4cx+a^3c}+\frac{x}{a^2c}-\frac{2\log(ax+1)}{a^3c}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2),x)
```

output
```
a**2*(-1/(a**4*c*x + a**3*c) + x/(a**2*c) - 2*log(a*x + 1)/(a**3*c))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{x}{c}-\frac{1}{a^2cx+ac}-\frac{2\log(ax+1)}{ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
x/c - 1/(a^2*c*x + a*c) - 2*log(a*x + 1)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{x}{c}-\frac{2\log(|ax+1|)}{ac}-\frac{1}{(ax+1)ac}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
x/c - 2*log(abs(a*x + 1))/(a*c) - 1/((a*x + 1)*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{x}{c}-\frac{1}{a\,\left(c+a\,c\,x\right)}-\frac{2\,\ln\left(a\,x+1\right)}{a\,c}$$

input
```
int((a*x - 1)/((c - c/(a^2*x^2))*(a*x + 1)),x)
```

output
```
x/c - 1/(a*(c + a*c*x)) - (2*log(a*x + 1))/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{-2\log(ax+1)\,ax-2\log(ax+1)+a^2x^2+2ax}{ac\,(ax+1)}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x)
```

output
```
( - 2*log(a*x + 1)*a*x - 2*log(a*x + 1) + a**2*x**2 + 2*a*x)/(a*c*(a*x + 1
))
```

3.795 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} \, dx = \frac{x}{c^2}+\frac{1}{4ac^2(1+ax)^2}-\frac{7}{4ac^2(1+ax)}+\frac{\log(1-ax)}{8ac^2}-\frac{17\log(1+ax)}{8ac^2}$$

output
```
x/c^2+1/4/a/c^2/(a*x+1)^2-7/4/a/c^2/(a*x+1)+1/8*ln(-a*x+1)/a/c^2-17/8*ln(a
*x+1)/a/c^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} \, dx = \frac{2(-6-3ax+8a^2x^2+4a^3x^3)+(1+ax)^2\log(1-ax)-17(1+ax)^2\log(1+ax)}{8a(c+acx)^2}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

output
```
(2*(-6 - 3*a*x + 8*a^2*x^2 + 4*a^3*x^3) + (1 + a*x)^2*Log[1 - a*x] - 17*(1
 + a*x)^2*Log[1 + a*x])/(8*a*(c + a*c*x)^2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^4e^{-2\text{arctanh}(ax)}}{c^2\left(a^2-\frac{1}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\int \frac{e^{-2\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^2}dx}{c^2}$$

$$\Big\downarrow \text{6707}$$

$$-\frac{a^4\int \frac{e^{-2\text{arctanh}(ax)}x^4}{(1-a^2x^2)^2}dx}{c^2}$$

$$\Big\downarrow \text{6700}$$

$$-\frac{a^4\int \frac{x^4}{(1-ax)(ax+1)^3}dx}{c^2}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^4\int \left(\frac{17}{8a^4(ax+1)}-\frac{7}{4a^4(ax+1)^2}+\frac{1}{2a^4(ax+1)^3}-\frac{1}{a^4}-\frac{1}{8a^4(ax-1)}\right)dx}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4\left(\frac{7}{4a^5(ax+1)}-\frac{1}{4a^5(ax+1)^2}-\frac{\log(1-ax)}{8a^5}+\frac{17\log(ax+1)}{8a^5}-\frac{x}{a^4}\right)}{c^2}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

output
```
-((a^4*(-(x/a^4) - 1/(4*a^5*(1 + a*x)^2) + 7/(4*a^5*(1 + a*x)) - Log[1 - a
*x]/(8*a^5) + (17*Log[1 + a*x])/(8*a^5)))/c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{a^{4}\left(\frac{x}{a^{4}}-\frac{17 \ln \left(a x +1\right)}{8 a^{5}}+\frac{1}{4 a^{5} \left(a x +1\right)^{2}}-\frac{7}{4 a^{5} \left(a x +1\right)}+\frac{\ln \left(a x -1\right)}{8 a^{5}}\right)}{c^{2}}$	60
risch	$\frac{x}{c^{2}}+\frac{-\frac{7 c^{2} x}{4}-\frac{3 c^{2}}{2 a}}{c^{4} \left(a x +1\right)^{2}}-\frac{17 \ln \left(a x +1\right)}{8 a \,c^{2}}+\frac{\ln \left(-a x +1\right)}{8 a \,c^{2}}$	62
norman	$\frac{\frac{a^{3} x^{4}}{c}-\frac{9 x}{4 c}-\frac{5 a \,x^{2}}{4 c}+\frac{5 a^{2} x^{3}}{2 c}}{c \left(a x +1\right)^{2} \left(a x -1\right)}+\frac{\ln \left(a x -1\right)}{8 a \,c^{2}}-\frac{17 \ln \left(a x +1\right)}{8 a \,c^{2}}$	85
parallelrisch	$\frac{8 a^{3} x^{3}+a^{2} \ln \left(a x -1\right) x^{2}-17 \ln \left(a x +1\right) x^{2} a^{2}+28 a^{2} x^{2}+2 a \ln \left(a x -1\right) x -34 \ln \left(a x +1\right) x a +18 a x +\ln \left(a x -1\right)-17 \ln \left(a x +1\right)}{8 c^{2} \left(a x +1\right)^{2} a}$	98

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a^4/c^2*(x/a^4-17/8*ln(a*x+1)/a^5+1/4/a^5/(a*x+1)^2-7/4/a^5/(a*x+1)+1/8/a^
5*ln(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{8\,a^{3}x^{3} + 16\,a^{2}x^{2} - 6\,ax - 17\,\left(a^{2}x^{2} + 2\,ax + 1\right)\log\left(ax + 1\right) + \left(a^{2}x^{2} + 2\,ax + 1\right)\log\left(ax - 1\right) - 12}{8\,\left(a^{3}c^{2}x^{2} + 2\,a^{2}c^{2}x + ac^{2}\right)}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
1/8*(8*a^3*x^3 + 16*a^2*x^2 - 6*a*x - 17*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1
) + (a^2*x^2 + 2*a*x + 1)*log(a*x - 1) - 12)/(a^3*c^2*x^2 + 2*a^2*c^2*x +
a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = a^4\left(\frac{-7ax-6}{4a^7c^2x^2+8a^6c^2x+4a^5c^2}+\frac{x}{a^4c^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{17\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**2,x)
```

output
```
a**4*((-7*a*x - 6)/(4*a**7*c**2*x**2 + 8*a**6*c**2*x + 4*a**5*c**2) + x/(a
**4*c**2) + (log(x - 1/a)/8 - 17*log(x + 1/a)/8)/(a**5*c**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{7\,ax+6}{4\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}+\frac{x}{c^2}-\frac{17\,\log\left(ax+1\right)}{8\,ac^2}+\frac{\log\left(ax-1\right)}{8\,ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(7*a*x + 6)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2) + x/c^2 - 17/8*log(a*
x + 1)/(a*c^2) + 1/8*log(a*x - 1)/(a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{x}{c^2}-\frac{17\,\log\left(|ax+1|\right)}{8\,ac^2}+\frac{\log\left(|ax-1|\right)}{8\,ac^2}-\frac{7\,ax+6}{4\,(ax+1)^2ac^2}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
x/c^2 - 17/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) - 1
/4*(7*a*x + 6)/((a*x + 1)^2*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 13.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{x}{c^2} - \frac{\frac{7\,x}{4}+\frac{3}{2\,a}}{a^2\,c^2\,x^2+2\,a\,c^2\,x+c^2} + \frac{\ln\left(a\,x-1\right)}{8\,a\,c^2} - \frac{17\,\ln\left(a\,x+1\right)}{8\,a\,c^2}$$

input
```
int((a*x - 1)/((c - c/(a^2*x^2))^2*(a*x + 1)),x)
```

output
```
x/c^2 - ((7*x)/4 + 3/(2*a))/(c^2 + a^2*c^2*x^2 + 2*a*c^2*x) + log(a*x - 1)
/(8*a*c^2) - (17*log(a*x + 1))/(8*a*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

$$= \frac{\log(ax-1)\,a^2x^2+2\log(ax-1)\,ax+\log(ax-1)-17\log(ax+1)\,a^2x^2-34\log(ax+1)\,ax-17\log(a}{8a\,c^2\left(a^2x^2+2ax+1\right)}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x)
```

output
```
(log(a*x - 1)*a**2*x**2 + 2*log(a*x - 1)*a*x + log(a*x - 1) - 17*log(a*x +
 1)*a**2*x**2 - 34*log(a*x + 1)*a*x - 17*log(a*x + 1) + 8*a**3*x**3 + 19*a
**2*x**2 - 9)/(8*a*c**2*(a**2*x**2 + 2*a*x + 1))
```

3.796 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 108

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{x}{c^3}+\frac{1}{16ac^3(1-ax)}-\frac{1}{12ac^3(1+ax)^3}+\frac{5}{8ac^3(1+ax)^2}-\frac{39}{16ac^3(1+ax)}+\frac{\log(1-ax)}{4ac^3}-\frac{9\log(1+ax)}{4ac^3}$$

output
```
x/c^3+1/16/a/c^3/(-a*x+1)-1/12/a/c^3/(a*x+1)^3+5/8/a/c^3/(a*x+1)^2-39/16/a
/c^3/(a*x+1)+1/4*ln(-a*x+1)/a/c^3-9/4*ln(a*x+1)/a/c^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.96

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{2(11+7ax-24a^2x^2-15a^3x^3+12a^4x^4+6a^5x^5)+3(-1+ax)(1+ax)^3\log(1-ax)-27(-1+ax)(1}{12a(-1+ax)(c+acx)^3}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

output
```
(2*(11 + 7*a*x - 24*a^2*x^2 - 15*a^3*x^3 + 12*a^4*x^4 + 6*a^5*x^5) + 3*(-1
 + a*x)*(1 + a*x)^3*Log[1 - a*x] - 27*(-1 + a*x)*(1 + a*x)^3*Log[1 + a*x])
/(12*a*(-1 + a*x)*(c + a*c*x)^3)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{a^6e^{-2\text{arctanh}(ax)}}{c^3\left(a^2-\frac{1}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^6\int \frac{e^{-2\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^3}dx}{c^3}$$

$$\Big\downarrow \text{6707}$$

$$\frac{a^6\int \frac{e^{-2\text{arctanh}(ax)}x^6}{(1-a^2x^2)^3}dx}{c^3}$$

$$\Big\downarrow \text{6700}$$

$$\frac{a^6\int \frac{x^6}{(1-ax)^2(ax+1)^4}dx}{c^3}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^6\int \left(-\frac{9}{4a^6(ax+1)}+\frac{39}{16a^6(ax+1)^2}-\frac{5}{4a^6(ax+1)^3}+\frac{1}{4a^6(ax+1)^4}+\frac{1}{a^6}+\frac{1}{4a^6(ax-1)}+\frac{1}{16a^6(ax-1)^2}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^6\left(\frac{1}{16a^7(1-ax)}-\frac{39}{16a^7(ax+1)}+\frac{5}{8a^7(ax+1)^2}-\frac{1}{12a^7(ax+1)^3}+\frac{\log(1-ax)}{4a^7}-\frac{9\log(ax+1)}{4a^7}+\frac{x}{a^6}\right)}{c^3}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

output
```
(a^6*(x/a^6 + 1/(16*a^7*(1 - a*x)) - 1/(12*a^7*(1 + a*x)^3) + 5/(8*a^7*(1
+ a*x)^2) - 39/(16*a^7*(1 + a*x)) + Log[1 - a*x]/(4*a^7) - (9*Log[1 + a*x]
)/(4*a^7)))/c^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.78

method	result
default	$\frac{a^{6}\left(\frac{x}{a^{6}}-\frac{9\ln(ax+1)}{4a^{7}}-\frac{1}{12a^{7}(ax+1)^{3}}+\frac{5}{8a^{7}(ax+1)^{2}}-\frac{39}{16a^{7}(ax+1)}-\frac{1}{16a^{7}(ax-1)}+\frac{\ln(ax-1)}{4a^{7}}\right)}{c^{3}}$
risch	$\frac{x}{c^{3}}+\frac{-\frac{5a^{2}c^{3}x^{3}}{2}-2a\,c^{3}x^{2}+\frac{13c^{3}x}{6}+\frac{11c^{3}}{6a}}{c^{6}(ax+1)^{2}(a^{2}x^{2}-1)}-\frac{9\ln(ax+1)}{4a\,c^{3}}+\frac{\ln(-ax+1)}{4a\,c^{3}}$
norman	$\frac{\frac{a^{5}x^{6}}{c}+\frac{5x}{2c}+\frac{3a\,x^{2}}{2c}-\frac{31a^{2}x^{3}}{6c}-\frac{8a^{3}x^{4}}{3c}+\frac{17a^{4}x^{5}}{6c}}{c^{2}(ax-1)^{2}(ax+1)^{3}}+\frac{\ln(ax-1)}{4a\,c^{3}}-\frac{9\ln(ax+1)}{4a\,c^{3}}$
parallelrisch	$\frac{12a^{5}x^{5}+3\ln(ax-1)x^{4}a^{4}-27\ln(ax+1)x^{4}a^{4}+46a^{4}x^{4}+6a^{3}\ln(ax-1)x^{3}-54\ln(ax+1)x^{3}a^{3}+14a^{3}x^{3}-48a^{2}x^{2}-6a\ln(ax-1)x-}{12c^{3}(ax+1)^{2}(a^{2}x^{2}-1)a}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
a^6/c^3*(x/a^6-9/4*ln(a*x+1)/a^7-1/12/a^7/(a*x+1)^3+5/8/a^7/(a*x+1)^2-39/1
6/a^7/(a*x+1)-1/16/a^7/(a*x-1)+1/4/a^7*ln(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.27

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^{2}x^{2}}\right)^{3}}\,dx$$
$$=\frac{12\,a^{5}x^{5}+24\,a^{4}x^{4}-30\,a^{3}x^{3}-48\,a^{2}x^{2}+14\,ax-27\left(a^{4}x^{4}+2\,a^{3}x^{3}-2\,ax-1\right)\log\left(ax+1\right)+3\left(a^{4}x^{4}+\right.}{12\left(a^{5}c^{3}x^{4}+2\,a^{4}c^{3}x^{3}-2\,a^{2}c^{3}x-ac^{3}\right)}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
1/12*(12*a^5*x^5 + 24*a^4*x^4 - 30*a^3*x^3 - 48*a^2*x^2 + 14*a*x - 27*(a^4
*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x + 1) + 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*
x - 1)*log(a*x - 1) + 22)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c
^3)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = a^6\left(\frac{-15a^3x^3-12a^2x^2+13ax+11}{6a^{11}c^3x^4+12a^{10}c^3x^3-12a^8c^3x-6a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{4}-\frac{9\log\left(x+\frac{1}{a}\right)}{4}}{a^7c^3}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**3,x)
```

output
```
a**6*((-15*a**3*x**3 - 12*a**2*x**2 + 13*a*x + 11)/(6*a**11*c**3*x**4 + 12
*a**10*c**3*x**3 - 12*a**8*c**3*x - 6*a**7*c**3) + x/(a**6*c**3) + (log(x
- 1/a)/4 - 9*log(x + 1/a)/4)/(a**7*c**3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{15\,a^3x^3+12\,a^2x^2-13\,ax-11}{6\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}+\frac{x}{c^3}-\frac{9\,\log\left(ax+1\right)}{4\,ac^3}+\frac{\log\left(ax-1\right)}{4\,ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
-1/6*(15*a^3*x^3 + 12*a^2*x^2 - 13*a*x - 11)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3
- 2*a^2*c^3*x - a*c^3) + x/c^3 - 9/4*log(a*x + 1)/(a*c^3) + 1/4*log(a*x -
1)/(a*c^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.74

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{x}{c^3}-\frac{9\log\left(|ax+1|\right)}{4\,ac^3}+\frac{\log\left(|ax-1|\right)}{4\,ac^3}-\frac{15\,a^3x^3+12\,a^2x^2-13\,ax-11}{6\,(ax+1)^3(ax-1)ac^3}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
x/c^3 - 9/4*log(abs(a*x + 1))/(a*c^3) + 1/4*log(abs(a*x - 1))/(a*c^3) - 1/
6*(15*a^3*x^3 + 12*a^2*x^2 - 13*a*x - 11)/((a*x + 1)^3*(a*x - 1)*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 13.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.87

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{x}{c^3}-\frac{\frac{13\,x}{6}-2\,a\,x^2+\frac{11}{6\,a}-\frac{5\,a^2\,x^3}{2}}{-a^4\,c^3\,x^4-2\,a^3\,c^3\,x^3+2\,a\,c^3\,x+c^3}+\frac{\ln\left(a\,x-1\right)}{4\,a\,c^3}-\frac{9\,\ln\left(a\,x+1\right)}{4\,a\,c^3}$$

input
```
int((a*x - 1)/((c - c/(a^2*x^2))^3*(a*x + 1)),x)
```

output
```
x/c^3 - ((13*x)/6 - 2*a*x^2 + 11/(6*a) - (5*a^2*x^3)/2)/(c^3 - 2*a^3*c^3*x
^3 - a^4*c^3*x^4 + 2*a*c^3*x) + log(a*x - 1)/(4*a*c^3) - (9*log(a*x + 1))/
(4*a*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.42

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$= \frac{3\log(ax-1)\,a^4x^4+6\log(ax-1)\,a^3x^3-6\log(ax-1)\,ax-3\log(ax-1)-27\log(ax+1)\,a^4x^4-54\log}{12a\,c^3\left(a^4x^4+2a^3x^3\right.}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x)
```

output
```
(3*log(a*x - 1)*a**4*x**4 + 6*log(a*x - 1)*a**3*x**3 - 6*log(a*x - 1)*a*x
- 3*log(a*x - 1) - 27*log(a*x + 1)*a**4*x**4 - 54*log(a*x + 1)*a**3*x**3 +
 54*log(a*x + 1)*a*x + 27*log(a*x + 1) + 12*a**5*x**5 + 39*a**4*x**4 - 48*
a**2*x**2 - 16*a*x + 7)/(12*a*c**3*(a**4*x**4 + 2*a**3*x**3 - 2*a*x - 1))
```

3.797 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 143

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{x}{c^4}-\frac{1}{64ac^4(1-ax)^2}+\frac{11}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}-\frac{13}{48ac^4(1+ax)^3}+\frac{35}{32ac^4(1+ax)^2}-\frac{99}{32ac^4(1+ax)}+\frac{47\log(1-ax)}{128ac^4}-\frac{303\log(1+ax)}{128ac^4}$$

output
```
x/c^4-1/64/a/c^4/(-a*x+1)^2+11/64/a/c^4/(-a*x+1)+1/32/a/c^4/(a*x+1)^4-13/4
8/a/c^4/(a*x+1)^3+35/32/a/c^4/(a*x+1)^2-99/32/a/c^4/(a*x+1)+47/128*ln(-a*x
+1)/a/c^4-303/128*ln(a*x+1)/a/c^4
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.87

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$=\frac{2(-400-275ax+1258a^2x^2+866a^3x^3-1254a^4x^4-819a^5x^5+384a^6x^6+192a^7x^7)+141(-1+ax)^2}{384a(-1+ax)^2(c+acx)^4}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

output
```
(2*(-400 - 275*a*x + 1258*a^2*x^2 + 866*a^3*x^3 - 1254*a^4*x^4 - 819*a^5*x
^5 + 384*a^6*x^6 + 192*a^7*x^7) + 141*(-1 + a*x)^2*(1 + a*x)^4*Log[1 - a*x
] - 909*(-1 + a*x)^2*(1 + a*x)^4*Log[1 + a*x])/(384*a*(-1 + a*x)^2*(c + a*
c*x)^4)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6717, 27, 6707, 6700, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$\Big\downarrow \text{6717}$$
$$-\int \frac{a^8e^{-2\text{arctanh}(ax)}}{c^4\left(a^2-\frac{1}{x^2}\right)^4}dx$$
$$\Big\downarrow \text{27}$$
$$-\frac{a^8\int \frac{e^{-2\text{arctanh}(ax)}}{\left(a^2-\frac{1}{x^2}\right)^4}dx}{c^4}$$

$$\Big\downarrow \text{6707}$$

$$-\frac{a^8\int \frac{e^{-2\text{arctanh}(ax)}x^8}{(1-a^2x^2)^4}dx}{c^4}$$

$$\Big\downarrow \text{6700}$$

$$-\frac{a^8\int \frac{x^8}{(1-ax)^3(ax+1)^5}dx}{c^4}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^8\int \left(\frac{303}{128a^8(ax+1)}-\frac{99}{32a^8(ax+1)^2}+\frac{35}{16a^8(ax+1)^3}-\frac{13}{16a^8(ax+1)^4}+\frac{1}{8a^8(ax+1)^5}-\frac{1}{a^8}-\frac{47}{128a^8(ax-1)}-\frac{11}{64a^8(ax-1)^2}-\frac{}{32a^8}\right.}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^8\left(-\frac{11}{64a^9(1-ax)}+\frac{99}{32a^9(ax+1)}+\frac{1}{64a^9(1-ax)^2}-\frac{35}{32a^9(ax+1)^2}+\frac{13}{48a^9(ax+1)^3}-\frac{1}{32a^9(ax+1)^4}-\frac{47\log(1-ax)}{128a^9}+\frac{303\log(ax+1}{128a^9}\right.}{c^4}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

output
```
-((a^8*(-(x/a^8) + 1/(64*a^9*(1 - a*x)^2) - 11/(64*a^9*(1 - a*x)) - 1/(32*
a^9*(1 + a*x)^4) + 13/(48*a^9*(1 + a*x)^3) - 35/(32*a^9*(1 + a*x)^2) + 99/
(32*a^9*(1 + a*x)) - (47*Log[1 - a*x])/(128*a^9) + (303*Log[1 + a*x])/(128
*a^9)))/c^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6700
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[c^p   Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
 x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
 GtQ[c, 0])
```

rule 6707
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[d^p   Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x
] /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.76

method	result
default	$\frac{a^{8} \left(\frac{x}{a^{8}}-\frac{303 \ln \left(a x +1\right)}{128 a^{9}}+\frac{1}{32 a^{9} \left(a x +1\right)^{4}}-\frac{13}{48 a^{9} \left(a x +1\right)^{3}}+\frac{35}{32 a^{9} \left(a x +1\right)^{2}}-\frac{99}{32 a^{9} \left(a x +1\right)}-\frac{1}{64 a^{9} \left(a x -1\right)^{2}}-\frac{11}{64 a^{9} \left(a x -1\right)}+\frac{47 \ln \left(a x -1\right)}{128 a^{9}}\right)}{c^{4}}$
risch	$\frac{x}{c^{4}}+\frac{-\frac{209 a^{4} c^{4} x^{5}}{64}-\frac{81 a^{3} c^{4} x^{4}}{32}+\frac{529 a^{2} c^{4} x^{3}}{96}+\frac{437 a \,c^{4} x^{2}}{96}-\frac{467 c^{4} x}{192}-\frac{25 c^{4}}{12 a}}{c^{8} \left(a x +1\right)^{2} \left(a^{2} x^{2}-1\right)^{2}}+\frac{47 \ln \left(-a x +1\right)}{128 a \,c^{4}}-\frac{303 \ln \left(a x +1\right)}{128 a \,c^{4}}$
norman	$\frac{\frac{a^{7} x^{8}}{c}-\frac{175 x}{64 c}-\frac{111 a \,x^{2}}{64 c}+\frac{199 a^{2} x^{3}}{24 c}+\frac{115 a^{3} x^{4}}{24 c}-\frac{545 a^{4} x^{5}}{64 c}-\frac{803 a^{5} x^{6}}{192 c}+\frac{37 a^{6} x^{7}}{12 c}}{c^{3} \left(a x -1\right)^{3} \left(a x +1\right)^{4}}+\frac{47 \ln \left(a x -1\right)}{128 a \,c^{4}}-\frac{303 \ln \left(a x +1\right)}{128 a \,c^{4}}$
parallelrisch	$-1468 a^{3} x^{3}+1050 a x -3308 a^{4} x^{4}-38 a^{5} x^{5}+1568 x^{6} a^{6}-1818 \ln \left(a x +1\right) x a +384 a^{7} x^{7}-909 \ln \left(a x +1\right)+1716 a^{2} x^{2}+141 \ln \left(a x -1\right)$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
a^8/c^4*(x/a^8-303/128/a^9*ln(a*x+1)+1/32/a^9/(a*x+1)^4-13/48/a^9/(a*x+1)^
3+35/32/a^9/(a*x+1)^2-99/32/a^9/(a*x+1)-1/64/a^9/(a*x-1)^2-11/64/a^9/(a*x-
1)+47/128/a^9*ln(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.63

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{384\,a^7x^7+768\,a^6x^6-1638\,a^5x^5-2508\,a^4x^4+1732\,a^3x^3+2516\,a^2x^2-550\,ax-909\,(a^6x^6+2\,a^5x^5-}{384\,(a^7c^4x^6+2\,a^6c^4x^5-}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
1/384*(384*a^7*x^7 + 768*a^6*x^6 - 1638*a^5*x^5 - 2508*a^4*x^4 + 1732*a^3*
x^3 + 2516*a^2*x^2 - 550*a*x - 909*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*
x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x + 1) + 141*(a^6*x^6 + 2*a^5*x^5 - a^4*x
^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x - 1) - 800)/(a^7*c^4*x^6 + 2
*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a
*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.09

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = a^8\left(\frac{-627a^5x^5-486a^4x^4+1058a^3x^3+874a^2x^2-467ax-400}{192a^{15}c^4x^6+384a^{14}c^4x^5-192a^{13}c^4x^4-768a^{12}c^4x^3-192a^{11}c^4x^2+384a^{10}c^4x+192a^9c^4} + \frac{x}{a^8c^4} + \frac{\frac{47\log\left(x-\frac{1}{a}\right)}{128}-\frac{303\log\left(x+\frac{1}{a}\right)}{128}}{a^9c^4}\right)$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**4,x)
```

output
```
a**8*((-627*a**5*x**5 - 486*a**4*x**4 + 1058*a**3*x**3 + 874*a**2*x**2 - 4
67*a*x - 400)/(192*a**15*c**4*x**6 + 384*a**14*c**4*x**5 - 192*a**13*c**4*
x**4 - 768*a**12*c**4*x**3 - 192*a**11*c**4*x**2 + 384*a**10*c**4*x + 192*
a**9*c**4) + x/(a**8*c**4) + (47*log(x - 1/a)/128 - 303*log(x + 1/a)/128)/
(a**9*c**4))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx &= -\frac{627\,a^5x^5+486\,a^4x^4-1058\,a^3x^3-874\,a^2x^2+467\,ax+400}{192\,(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4)}\\&\quad+\frac{x}{c^4}-\frac{303\,\log(ax+1)}{128\,ac^4}+\frac{47\,\log(ax-1)}{128\,ac^4}\end{aligned}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
-1/192*(627*a^5*x^5 + 486*a^4*x^4 - 1058*a^3*x^3 - 874*a^2*x^2 + 467*a*x +
 400)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4
*x^2 + 2*a^2*c^4*x + a*c^4) + x/c^4 - 303/128*log(a*x + 1)/(a*c^4) + 47/12
8*log(a*x - 1)/(a*c^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.67

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx &= \frac{x}{c^4}-\frac{303\,\log(|ax+1|)}{128\,ac^4}+\frac{47\,\log(|ax-1|)}{128\,ac^4}\\&\quad-\frac{627\,a^5x^5+486\,a^4x^4-1058\,a^3x^3-874\,a^2x^2+467\,ax+400}{192\,(ax+1)^4(ax-1)^2ac^4}\end{aligned}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
x/c^4 - 303/128*log(abs(a*x + 1))/(a*c^4) + 47/128*log(abs(a*x - 1))/(a*c^
4) - 1/192*(627*a^5*x^5 + 486*a^4*x^4 - 1058*a^3*x^3 - 874*a^2*x^2 + 467*a
*x + 400)/((a*x + 1)^4*(a*x - 1)^2*a*c^4)
```

Mupad [B] (verification not implemented)

Time = 13.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{x}{c^4}-\frac{\frac{467\,x}{192}-\frac{437\,a\,x^2}{96}+\frac{25}{12\,a}-\frac{529\,a^2\,x^3}{96}+\frac{81\,a^3\,x^4}{32}+\frac{209\,a^4\,x^5}{64}}{a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5-a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3-a^2\,c^4\,x^2+2\,a\,c^4\,x+c^4} + \frac{47\,\ln\left(a\,x-1\right)}{128\,a\,c^4}-\frac{303\,\ln\left(a\,x+1\right)}{128\,a\,c^4}$$

input
```
int((a*x - 1)/((c - c/(a^2*x^2))^4*(a*x + 1)),x)
```

output
```
x/c^4 - ((467*x)/192 - (437*a*x^2)/96 + 25/(12*a) - (529*a^2*x^3)/96 + (81
*a^3*x^4)/32 + (209*a^4*x^5)/64)/(c^4 - a^2*c^4*x^2 - 4*a^3*c^4*x^3 - a^4*
c^4*x^4 + 2*a^5*c^4*x^5 + a^6*c^4*x^6 + 2*a*c^4*x) + (47*log(a*x - 1))/(12
8*a*c^4) - (303*log(a*x + 1))/(128*a*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.94

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{141\log(ax-1)\,a^6x^6+282\log(ax-1)\,a^5x^5-141\log(ax-1)\,a^4x^4-564\log(ax-1)\,a^3x^3-141\log(ax}{}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x)
```

output
```
(141*log(a*x - 1)*a**6*x**6 + 282*log(a*x - 1)*a**5*x**5 - 141*log(a*x - 1
)*a**4*x**4 - 564*log(a*x - 1)*a**3*x**3 - 141*log(a*x - 1)*a**2*x**2 + 28
2*log(a*x - 1)*a*x + 141*log(a*x - 1) - 909*log(a*x + 1)*a**6*x**6 - 1818*
log(a*x + 1)*a**5*x**5 + 909*log(a*x + 1)*a**4*x**4 + 3636*log(a*x + 1)*a*
*3*x**3 + 909*log(a*x + 1)*a**2*x**2 - 1818*log(a*x + 1)*a*x - 909*log(a*x
 + 1) + 384*a**7*x**7 + 1587*a**6*x**6 - 3327*a**4*x**4 - 1544*a**3*x**3 +
 1697*a**2*x**2 + 1088*a*x + 19)/(384*a*c**4*(a**6*x**6 + 2*a**5*x**5 - a*
*4*x**4 - 4*a**3*x**3 - a**2*x**2 + 2*a*x + 1))
```

3.798 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$

Optimal result

Integrand size = 22, antiderivative size = 180

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx &= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{7a}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(6a+\frac{5}{x}\right)}{10a^2}\\&+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(8a+\frac{5}{x}\right)}{8a^2}+\frac{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{5}{x}\right)}{16a^2}\\&+c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}x+\frac{15c^4\csc^{-1}(ax)}{16a}-\frac{3c^4\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}\end{aligned}$$

output
```
-1/7*c^4*(1-1/a^2/x^2)^(7/2)/a+1/10*c^4*(1-1/a^2/x^2)^(5/2)*(6*a+5/x)/a^2+
1/8*c^4*(1-1/a^2/x^2)^(3/2)*(8*a+5/x)/a^2+3/16*c^4*(1-1/a^2/x^2)^(1/2)*(16
*a+5/x)/a^2+c^4*(1-1/a^2/x^2)^(7/2)*x+15/16*c^4*arccsc(a*x)/a-3*c^4*arctan
h((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.70

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx$$
$$= \frac{c^4\left(\sqrt{1-\frac{1}{a^2x^2}}(80-280ax+96a^2x^2+770a^3x^3-992a^4x^4-525a^5x^5+2496a^6x^6+560a^7x^7)+525a^6x^6\right.}{560a^7x^6}$$

input
```
Integrate[(c - c/(a^2*x^2))^4/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^4*(Sqrt[1 - 1/(a^2*x^2)]*(80 - 280*a*x + 96*a^2*x^2 + 770*a^3*x^3 - 992
*a^4*x^4 - 525*a^5*x^5 + 2496*a^6*x^6 + 560*a^7*x^7) + 525*a^6*x^6*ArcSin[
1/(a*x)] - 1680*a^6*x^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(560*a^7*x^6)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.81, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.136$, Rules used = {6748, 108, 25, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 25, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^4 e^{-3\coth^{-1}(ax)} dx$$

$$\Big\downarrow \text{6748}$$

$$-c^4\int \left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2} x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$-c^4\left(\int -\frac{\left(3a+\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2} x}{a^2} d\frac{1}{x} - x\left(1-\frac{1}{ax}\right)^{11/2}\left(\frac{1}{ax}+1\right)^{5/2}\right)$$

$$\Big\downarrow \text{25}$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\int \frac{\left(3a+\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{a^2}d\frac{1}{x}\right)$$

$\Big\downarrow$ 27

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\int \left(3a+\frac{8}{x}\right)\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}}{a^2}\right)$$

$\Big\downarrow$ 171

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{1}{7}a\int \frac{3\left(7a+\frac{17}{x}\right)\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{a}d\frac{1}{x}+\frac{8}{7}a\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{9/2}}{a^2}\right)$$

$\Big\downarrow$ 27

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\int \left(7a+\frac{17}{x}\right)\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}+\frac{8}{7}a\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)}{a^2}\right.$$

$\Big\downarrow$ 171

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{1}{6}a\int \frac{7\left(6a+\frac{11}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{a}d\frac{1}{x}+\frac{17}{6}a\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\right.}{a^2}\right.$$

$\Big\downarrow$ 27

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\int \left(6a+\frac{11}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}+\frac{17}{6}a\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\right.\right.}{a^2}\right.$$

$\Big\downarrow$ 171

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\frac{1}{5}a\int \frac{5\left(6a+\frac{5}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{a}d\frac{1}{x}+\frac{11}{5}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)\right.\right.}{a^2}\right.$$

$\Big\downarrow$ 27

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\int\left(6a+\frac{5}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}+\frac{11}{5}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}\right.\right.\right.}{a}\right.$$

$\Big\downarrow$ 171

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\frac{1}{4}a\int\frac{3\left(8a-\frac{3}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{a}d\frac{1}{x}+\frac{11}{5}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}\right.\right.}{}\right.$$

$\Big\downarrow$ 27

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\frac{3}{4}\int\left(8a-\frac{3}{x}\right)\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}xd\frac{1}{x}+\frac{11}{5}a\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+\right.\right.\right.}{}\right.$$

$\Big\downarrow$ 171

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\frac{3}{4}\left(\frac{1}{3}a\int\frac{3\left(8a-\frac{9}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{11}{5}a\right.\right.}{}\right.$$

$\Big\downarrow$ 27

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\frac{3}{4}\left(\int\frac{\left(8a-\frac{9}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}\right)+\frac{11}{5}a\left(1-\right.\right.\right.}{}\right.$$

$\Big\downarrow$ 171

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\frac{3}{4}\left(-\frac{1}{2}a\int-\frac{\left(16a-\frac{11}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{9}{2}a\right.\right.\right.}{}\right.$$

$\Big\downarrow$ 25

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\frac{3}{4}\left(\frac{1}{2}a\int\frac{\left(16a-\frac{11}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{9}{2}a\sqrt{1\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow 27$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\left(16a-\frac{11}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}-a\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{9}{2}a\sqrt{1-\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow 171$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(11a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int-\frac{\left(16a+\frac{5}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)-a\sqrt{\right.\right.\right.}{}\right.$$

$$\Big\downarrow 25$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(a\int\frac{\left(16a+\frac{5}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+11a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-a\sqrt{1\right.\right.\right.}{}\right.$$

$$\Big\downarrow 27$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(\int\frac{\left(16a+\frac{5}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+11a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)-a\sqrt{1-\right.\right.\right.}{}\right.$$

$$\Big\downarrow 175$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(16a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5\int\frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+11a\sqrt{1\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{39}$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(5\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+16a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+11a\sqrt{1-\frac{1}{ax}}\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{103}$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(5\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+1\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{221}$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(5\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-16a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+11\right.\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{223}$$

$$-c^4\left(x\left(-\left(\frac{1}{ax}+1\right)^{5/2}\right)\left(1-\frac{1}{ax}\right)^{11/2}-\frac{\frac{3}{7}\left(\frac{7}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(5a\arcsin\left(\frac{1}{ax}\right)-16a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+11a\sqrt{\right.\right.\right.\right.}{}\right.$$

input
```
Int[(c - c/(a^2*x^2))^4/E^(3*ArcCoth[a*x]),x]
```

output
```
-(c^4*(-((1 - 1/(a*x))^(11/2)*(1 + 1/(a*x))^(5/2)*x) - ((8*a*(1 - 1/(a*x))
^(9/2)*(1 + 1/(a*x))^(5/2))/7 + (3*((17*a*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x)
)^(5/2))/6 + (7*((5*a*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2))/4 + (11*a*(
1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2))/5 + (3*((9*a*Sqrt[1 - 1/(a*x)]*(1
+ 1/(a*x))^(3/2))/2 - a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2) + (11*a*Sqrt
[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)] + 5*a*ArcSin[1/(a*x)] - 16*a*ArcTanh[Sqrt[
1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/2))/4))/6))/7)/a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00

method	result
risch	$\frac{(ax+1)\left(560a^7x^7+2496x^6a^6-525a^5x^5-992a^4x^4+770a^3x^3+96a^2x^2-280ax+80\right)c^4\sqrt{\frac{ax-1}{ax+1}}}{560x^7a^8}+\frac{\left(\frac{15a^7\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{16}-\frac{3a^8\ln\left(\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^4\left(-1680\sqrt{a^2x^2-1}\sqrt{a^2}\,a^8x^8+1680\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6-525a^7x^7\sqrt{a^2}\sqrt{a^2x^2-1}-525a^7x^7\sqrt{a^2}\arctan\left(\cdots\right.\right.}{\cdots}$

input
```
int((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/560*(a*x+1)*(560*a^7*x^7+2496*a^6*x^6-525*a^5*x^5-992*a^4*x^4+770*a^3*x^
3+96*a^2*x^2-280*a*x+80)/x^7*c^4/a^8*((a*x-1)/(a*x+1))^(1/2)+(15/16*a^7*ar
ctan(1/(a^2*x^2-1)^(1/2))-3*a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a
^2)^(1/2))*c^4/a^8*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.12

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\frac{1050\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(560}{}$$

input
```
integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-1/560*(1050*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) + 1680*a^7*c^4*
x^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 1680*a^7*c^4*x^7*log(sqrt((a*x -
1)/(a*x + 1)) - 1) - (560*a^8*c^4*x^8 + 3056*a^7*c^4*x^7 + 1971*a^6*c^4*x^
6 - 1517*a^5*c^4*x^5 - 222*a^4*c^4*x^4 + 866*a^3*c^4*x^3 - 184*a^2*c^4*x^2
 - 200*a*c^4*x + 80*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**4*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(158) = 316$.

Time = 0.12 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.11

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = -\frac{1}{280}\left(\frac{525\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{1155\,c^4\left(\frac{ax-1}{ax+1}\right)\right.$$

input
```
integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-1/280*(525*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 840*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 - 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
a^2 + (1155*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 7665*c^4*((a*x - 1)/(a*x +
1))^(13/2) + 20811*c^4*((a*x - 1)/(a*x + 1))^(11/2) - 12799*c^4*((a*x - 1)
/(a*x + 1))^(9/2) - 39071*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 33621*c^4*((a*
x - 1)/(a*x + 1))^(5/2) - 13615*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 2205*c^4
*sqrt((a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^
2/(a*x + 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x +
 1)^5 - 14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (
a*x - 1)^8*a^2/(a*x + 1)^8 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(158) = 316$.

Time = 0.16 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.92

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = \text{Too large to display}$$

input
```
integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
-15/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^4*log
(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 -
1)*c^4*sgn(a*x + 1)/a + 1/280*(525*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*c^4*a
bs(a)*sgn(a*x + 1) + 4480*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4*sgn(a*x
+ 1) - 980*(x*abs(a) - sqrt(a^2*x^2 - 1))^11*c^4*abs(a)*sgn(a*x + 1) + 201
60*(x*abs(a) - sqrt(a^2*x^2 - 1))^10*a*c^4*sgn(a*x + 1) + 945*(x*abs(a) -
sqrt(a^2*x^2 - 1))^9*c^4*abs(a)*sgn(a*x + 1) + 38080*(x*abs(a) - sqrt(a^2*
x^2 - 1))^8*a*c^4*sgn(a*x + 1) + 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*
c^4*sgn(a*x + 1) - 945*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*sgn(a*x
 + 1) + 32256*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) + 980*(x
*abs(a) - sqrt(a^2*x^2 - 1))^3*c^4*abs(a)*sgn(a*x + 1) + 12992*(x*abs(a) -
 sqrt(a^2*x^2 - 1))^2*a*c^4*sgn(a*x + 1) - 525*(x*abs(a) - sqrt(a^2*x^2 -
1))*c^4*abs(a)*sgn(a*x + 1) + 2496*a*c^4*sgn(a*x + 1))/(((x*abs(a) - sqrt(
a^2*x^2 - 1))^2 + 1)^7*a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 13.21 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.84

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=\frac{\frac{63\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{389\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{8}+\frac{4803\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{40}+\frac{39071\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{280}+\frac{12799\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{280}-\frac{2973\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{40}-}{a+\frac{6\,a\,(a\,x-1)}{a\,x+1}+\frac{14\,a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}+\frac{14\,a\,{(a\,x-1)}^3}{{(a\,x+1)}^3}-\frac{14\,a\,{(a\,x-1)}^5}{{(a\,x+1)}^5}-\frac{14\,a\,{(a\,x-1)}^6}{{(a\,x+1)}^6}-\frac{6\,a\,{(a\,x-1)}^7}{{(a\,x+1)}^7}-}$$
$$-\frac{15\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}-\frac{6\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((63*c^4*((a*x - 1)/(a*x + 1))^(1/2))/8 + (389*c^4*((a*x - 1)/(a*x + 1))^(
3/2))/8 + (4803*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 + (39071*c^4*((a*x - 1
)/(a*x + 1))^(7/2))/280 + (12799*c^4*((a*x - 1)/(a*x + 1))^(9/2))/280 - (2
973*c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 - (219*c^4*((a*x - 1)/(a*x + 1))^
(13/2))/8 - (33*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/
(a*x + 1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^
3 - (14*a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a
*(a*x - 1)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (15*c^4*atan(((
a*x - 1)/(a*x + 1))^(1/2)))/(8*a) - (6*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/
2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.35

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\left(-1050atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^7x^7+1050atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^7x^7+560\sqrt{ax+\cdots}\right.}{\cdots}$$

input
```
int((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(c**4*( - 1050*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**7*x**7 + 1050*at
an(sqrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**7*x**7 + 560*sqrt(a*x + 1)*sqrt(a
*x - 1)*a**7*x**7 + 2496*sqrt(a*x + 1)*sqrt(a*x - 1)*a**6*x**6 - 525*sqrt(
a*x + 1)*sqrt(a*x - 1)*a**5*x**5 - 992*sqrt(a*x + 1)*sqrt(a*x - 1)*a**4*x*
*4 + 770*sqrt(a*x + 1)*sqrt(a*x - 1)*a**3*x**3 + 96*sqrt(a*x + 1)*sqrt(a*x
 - 1)*a**2*x**2 - 280*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 80*sqrt(a*x + 1)*s
qrt(a*x - 1) - 3360*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**7*x**7
 - 1216*a**7*x**7))/(560*a**8*x**7)
```

3.799 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 143

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx &= -\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5a}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(4a+\frac{1}{x}\right)}{4a^2}\\ &\quad+\frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}\left(8a+\frac{1}{x}\right)}{8a^2}+c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x\\ &\quad+\frac{3c^3\csc^{-1}(ax)}{8a}-\frac{3c^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}\end{aligned}$$

output

```
-1/5*c^3*(1-1/a^2/x^2)^(5/2)/a+1/4*c^3*(1-1/a^2/x^2)^(3/2)*(4*a+1/x)/a^2+3
/8*c^3*(1-1/a^2/x^2)^(1/2)*(8*a+1/x)/a^2+c^3*(1-1/a^2/x^2)^(5/2)*x+3/8*c^3
*arccsc(a*x)/a-3*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.77

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$
$$=\frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}(-8+30ax-24a^2x^2-55a^3x^3+152a^4x^4+40a^5x^5)+15a^4x^4\arcsin\left(\frac{1}{ax}\right)-120a^4x^4\log\left(\right.\right.}{40a^5x^4}$$

input
```
Integrate[(c - c/(a^2*x^2))^3/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^3*(Sqrt[1 - 1/(a^2*x^2)]*(-8 + 30*a*x - 24*a^2*x^2 - 55*a^3*x^3 + 152*a
^4*x^4 + 40*a^5*x^5) + 15*a^4*x^4*ArcSin[1/(a*x)] - 120*a^4*x^4*Log[(1 + S
qrt[1 - 1/(a^2*x^2)])*x]))/(40*a^5*x^4)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.80, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.864, Rules used = {6748, 108, 27, 171, 27, 171, 27, 171, 27, 171, 27, 171, 25, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^3 e^{-3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6748}$$

$$-c^3\int \left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$-c^3\left(\int -\frac{3\left(a+\frac{2}{x}\right)\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\int\left(a+\frac{2}{x}\right)\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}}{a^2}\right)$$

$\downarrow$ 171

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}a\int\frac{\left(5a+\frac{9}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}x}{a}d\frac{1}{x}+\frac{2}{5}a\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{7/2}\right)}{a^2}\right)$$

$\downarrow$ 27

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\int\left(5a+\frac{9}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}+\frac{2}{5}a\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{7/}\right.}{a^2}\right.$$

$\downarrow$ 171

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{1}{4}a\int\frac{5\left(4a+\frac{5}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}x}{a}d\frac{1}{x}+\frac{9}{4}a\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\right)\right.}{a^2}\right.$$

$\downarrow$ 27

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\int\left(4a+\frac{5}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}+\frac{9}{4}a\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right.\right.\right.}{a^2}\right.$$

$\downarrow$ 171

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\left(\frac{1}{3}a\int\frac{3\left(4a+\frac{1}{x}\right)\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x}{a}d\frac{1}{x}+\frac{5}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}\right)\right.\right.}{a^2}\right.$$

$\downarrow$ 27

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\left(\int\left(4a+\frac{1}{x}\right)\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}xd\frac{1}{x}+\frac{5}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3}\right.\right.\right.}{a^2}\right.$$

$\downarrow$ 171

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\left(\frac{1}{2}a\int\frac{\left(8a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}x}{a\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{5}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{1}{2}a\sqrt{\right.\right.\right.}{}\right.$$

$\downarrow$ 27

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\left(\frac{1}{2}\int\frac{\left(8a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}x}{\sqrt{1-\frac{1}{ax}}}d\frac{1}{x}+\frac{5}{3}a\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{1}{2}a\sqrt{1\right.\right.\right.}{}\right.$$

$\downarrow$ 171

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\left(\frac{1}{2}\left(7a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-a\int-\frac{\left(8a+\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)+\frac{5}{3}a(1\right.\right.\right.}{}\right.$$

$\downarrow$ 25

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\left(\frac{1}{2}\left(a\int\frac{\left(8a+\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+7a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{5}{3}a(1-\right.\right.\right.}{}\right.$$

$\downarrow$ 27

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\left(\frac{1}{2}\left(\int\frac{\left(8a+\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+7a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{5}{3}a\left(1-\frac{1}{ax}\right.\right.\right.\right.}{}\right.$$

$\Big\downarrow$ 175

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\left(\frac{1}{2}\left(8a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\int\frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+7a\sqrt{1-\frac{1}{ax}}\right.\right.\right.\right.\right.}{}\right.$$

$\Big\downarrow$ 39

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\left(\frac{1}{2}\left(\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+8a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+7a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}}\right.\right.\right.\right.\right.}{}\right.$$

$\Big\downarrow$ 103

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\left(\frac{1}{2}\left(\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+7a\sqrt{}\right.\right.\right.\right.\right.}{}\right.$$

$\Big\downarrow$ 221

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\left(\frac{1}{2}\left(\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-8a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+7a\sqrt{1}\right.\right.\right.\right.\right.}{}\right.$$

$\Big\downarrow$ 223

$$-c^3\left(x\left(-\left(\frac{1}{ax}+1\right)^{3/2}\right)\left(1-\frac{1}{ax}\right)^{9/2}-\frac{3\left(\frac{1}{5}\left(\frac{5}{4}\left(\frac{1}{2}\left(a\arcsin\left(\frac{1}{ax}\right)-8a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+7a\sqrt{1-}\right.\right.\right.\right.}{}\right.$$

input
```
Int[(c - c/(a^2*x^2))^3/E^(3*ArcCoth[a*x]),x]
```

output
```
-(c^3*(-((1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(3/2)*x) - (3*((2*a*(1 - 1/(a*x
))^(7/2)*(1 + 1/(a*x))^(3/2))/5 + ((9*a*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^
(3/2))/4 + (5*((a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2))/2 + (5*a*(1 - 1/(
a*x))^(3/2)*(1 + 1/(a*x))^(3/2))/3 + (7*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*
x)] + a*ArcSin[1/(a*x)] - 8*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]
)/2))/4)/5))/a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.21

method	result
risch	$\frac{(ax+1)\left(152a^4x^4-55a^3x^3-24a^2x^2+30ax-8\right)c^3\sqrt{\frac{ax-1}{ax+1}}}{40x^5a^6}+\frac{\left(\frac{3a^5\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{8}-\frac{3a^6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a^5\sqrt{(ax-1)(ax+\cdots}\cdots\right.}{a^6(ax-1)\cdots}$
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^3\left(-120\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^6x^6+120\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,a^4x^4-15\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^5x^5-15\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2\cdots}\right.}{40(ax-1)\sqrt{(a\cdots}}$

input
```
int((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/40*(a*x+1)*(152*a^4*x^4-55*a^3*x^3-24*a^2*x^2+30*a*x-8)/x^5*c^3/a^6*((a*
x-1)/(a*x+1))^(1/2)+(3/8*a^5*arctan(1/(a^2*x^2-1)^(1/2))-3*a^6*ln(a^2*x/(a
^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2)^(1/2)+a^5*((a*x-1)*(a*x+1))^(1/2))*c^3/
a^6*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*((a*x-1)*(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.25

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = -\frac{30\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(40\,a^6c^3x^6}{40\,a^6x^5}$$

input
```
integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
-1/40*(30*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) + 120*a^5*c^3*x^5*
log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) - (40*a^6*c^3*x^6 + 192*a^5*c^3*x^5 + 97*a^4*c^3*x^4 - 79*a^
3*c^3*x^3 + 6*a^2*c^3*x^2 + 22*a*c^3*x - 8*c^3)*sqrt((a*x - 1)/(a*x + 1)))
/(a^6*x^5)
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = \frac{c^3\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^7+x^6}\,dx+\int\left(-\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^6+x^5}\right)dx+\int\left(-\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^5+x^4}\right)dx+\int\frac{3a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\frac{3a^4}{}\right.}{a^6}$$

input
```
integrate((c-c/a**2/x**2)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**7 + x**6), x) + Int
egral(-a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**6 + x**5), x) + Integral(
-3*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + Integral(3
*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integral(3*a
**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(-3*a*
*5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**6*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**7*x*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(125) = 250$.

Time = 0.11 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.10

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{1}{20}\left(\frac{15\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+4\right.$$

input
```
integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-1/20*(15*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 60*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 - 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2
+ (105*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 465*c^3*((a*x - 1)/(a*x + 1))^(9
/2) - 298*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 842*c^3*((a*x - 1)/(a*x + 1))^
(5/2) - 575*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 135*c^3*sqrt((a*x - 1)/(a*x
+ 1)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x
 - 1)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/
(a*x + 1)^6 + a^2))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(125) = 250$.

Time = 0.15 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.76

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{3\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{4\,a} + \frac{3\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c^3\operatorname{sgn}(ax+1)}{a} + \frac{55\left(x|a|-\sqrt{a^2x^2-1}\right)^9c^3|a|\operatorname{sgn}(ax+1)+200\left(x|a|-\sqrt{a^2x^2-1}\right)^8ac^3\operatorname{sgn}(ax+1)-10\left(x|a|-\sqrt{a^2x}\right.}{}$$

input
```
integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
-3/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^3*log(
abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1
)*c^3*sgn(a*x + 1)/a + 1/20*(55*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3*abs(a
)*sgn(a*x + 1) + 200*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3*sgn(a*x + 1) -
 10*(x*abs(a) - sqrt(a^2*x^2 - 1))^7*c^3*abs(a)*sgn(a*x + 1) + 720*(x*abs(
a) - sqrt(a^2*x^2 - 1))^6*a*c^3*sgn(a*x + 1) + 800*(x*abs(a) - sqrt(a^2*x^
2 - 1))^4*a*c^3*sgn(a*x + 1) + 10*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs
(a)*sgn(a*x + 1) + 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x + 1)
 - 55*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) + 152*a*c^3*s
gn(a*x + 1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^5*a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.80

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \frac{\frac{27\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{115\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{4}+\frac{421\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{10}+\frac{149\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{10}-\frac{93\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{4}-\frac{21\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{4}}{a+\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{5\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}} - \frac{3\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a} - \frac{6\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
((27*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (115*c^3*((a*x - 1)/(a*x + 1))^(
3/2))/4 + (421*c^3*((a*x - 1)/(a*x + 1))^(5/2))/10 + (149*c^3*((a*x - 1)/(
a*x + 1))^(7/2))/10 - (93*c^3*((a*x - 1)/(a*x + 1))^(9/2))/4 - (21*c^3*((a
*x - 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x -
 1)^2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*
x + 1)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (3*c^3*atan(((a*x - 1)/(a*x + 1)
)^(1/2)))/(4*a) - (6*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.42

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$$

$$=\frac{c^3\left(-30atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^5x^5+30atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^5x^5+40\sqrt{ax+1}\,\sqrt{ax}\right.}{}$$

input
```
int((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(c**3*( - 30*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**5*x**5 + 30*atan(s
qrt(a*x - 1) + sqrt(a*x + 1) + 1)*a**5*x**5 + 40*sqrt(a*x + 1)*sqrt(a*x -
1)*a**5*x**5 + 152*sqrt(a*x + 1)*sqrt(a*x - 1)*a**4*x**4 - 55*sqrt(a*x + 1
)*sqrt(a*x - 1)*a**3*x**3 - 24*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 + 30*
sqrt(a*x + 1)*sqrt(a*x - 1)*a*x - 8*sqrt(a*x + 1)*sqrt(a*x - 1) - 240*log(
(sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**5*x**5 - 72*a**5*x**5))/(40*a*
*6*x**5)
```

3.800 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 114

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=-\frac{c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}+\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2}+c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x-\frac{c^2\csc^{-1}(ax)}{2a}-\frac{3c^2\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-1/3*c^2*(1-1/a^2/x^2)^(3/2)/a+1/2*c^2*(1-1/a^2/x^2)^(1/2)*(6*a-1/x)/a^2+c
^2*(1-1/a^2/x^2)^(3/2)*x-1/2*c^2*arccsc(a*x)/a-3*c^2*arctanh((1-1/a^2/x^2)
^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}(2-9ax+16a^2x^2+6a^3x^3)-3a^2x^2\arcsin\left(\frac{1}{ax}\right)-18a^2x^2\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a^3x^2}$$

input
```
Integrate[(c - c/(a^2*x^2))^2/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^2*(Sqrt[1 - 1/(a^2*x^2)]*(2 - 9*a*x + 16*a^2*x^2 + 6*a^3*x^3) - 3*a^2*x
^2*ArcSin[1/(a*x)] - 18*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^
3*x^2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.66, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6748, 108, 25, 27, 171, 27, 171, 27, 171, 27, 175, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^2 e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6748}$$

$$-c^2\int \left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$-c^2\left(\int -\frac{\left(3a+\frac{4}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}x}{a^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}\right)$$

$$\Big\downarrow \text{25}$$

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\int\frac{\left(3a+\frac{4}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}x}{a^2\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\int\frac{\left(3a+\frac{4}{x}\right)\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a^2}\right)$$

$$\Big\downarrow \text{171}$$

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}a\int\frac{\left(9a+\frac{11}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}x}{a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4}{3}a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{a^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\int\frac{\left(9a+\frac{11}{x}\right)\left(1-\frac{1}{ax}\right)^{3/2}x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{4}{3}a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{a^2}\right)$$

$$\Big\downarrow \text{171}$$

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\left(\frac{1}{2}a\int\frac{3\left(6a+\frac{5}{x}\right)\sqrt{1-\frac{1}{ax}}x}{a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{11}{2}a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}\right)+\frac{4}{3}a\sqrt{\frac{1}{ax}+1}(1-}{a^2}\right.$$

$$\Big\downarrow \text{27}$$

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\left(\frac{3}{2}\int\frac{\left(6a+\frac{5}{x}\right)\sqrt{1-\frac{1}{ax}}x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{11}{2}a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}\right)+\frac{4}{3}a\sqrt{\frac{1}{ax}+1}(1-}{a^2}\right.$$

$$\Big\downarrow \text{171}$$

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\left(\frac{3}{2}\left(a\int\frac{\left(6a-\frac{1}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{\frac{1}{ax}+1}(1-\right.}{a^2}\right.$$

$\downarrow$ 27

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\left(\frac{3}{2}\left(\int\frac{\left(6a-\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{11}{2}a\sqrt{\frac{1}{ax}+1}(1-\frac{1}{ax}\right.}{a^2}\right.$$

$\downarrow$ 175

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\left(\frac{3}{2}\left(6a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\int\frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right.}{a^2}\right.$$

$\downarrow$ 39

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\left(\frac{3}{2}\left(-\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}+6a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+5a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\right.}{a^2}\right.$$

$\downarrow$ 103

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\left(\frac{3}{2}\left(-\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-6\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{\right.\right.}{a^2}\right.$$

$\downarrow$ 221

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\left(\frac{3}{2}\left(-\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}-6a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{}\right.\right.}{a^2}\right.$$

$\Big\downarrow$ 223

$$-c^2\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{7/2}-\frac{\frac{1}{3}\left(\frac{3}{2}\left(a\left(-\arcsin\left(\frac{1}{ax}\right)\right)-6a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+5a\sqrt{1-\frac{1}{ax}}\sqrt{}\right.\right.}{a^2}\right.$$

input
```
Int[(c - c/(a^2*x^2))^2/E^(3*ArcCoth[a*x]),x]
```

output
```
-(c^2*(-((1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x) - ((4*a*(1 - 1/(a*x))^(5
/2)*Sqrt[1 + 1/(a*x)])/3 + ((11*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/2
 + (3*(5*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)] - a*ArcSin[1/(a*x)] - 6*a*A
rcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]))/2)/3)/a^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.38

method	result
risch	$\frac{(ax+1)(16a^2x^2-9ax+2)c^2\sqrt{\frac{ax-1}{ax+1}}}{6x^3a^4}+\frac{\left(-\frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}-\frac{3a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a^3\sqrt{(ax-1)(ax+1)}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{(a}}{a^4(ax-1)}$
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^2\left(-18\sqrt{a^2}\sqrt{a^2x^2-1}\,a^4x^4+18\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^2x^2+3\sqrt{a^2}\sqrt{a^2x^2-1}\,a^3x^3+3a^3\sqrt{a^2}\,x^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right.\right.}{6(ax-1)\sqrt{(ax-1)(ax+1)}\,a^4x^3\sqrt{a^2}}$

input
```
int((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x+1)*(16*a^2*x^2-9*a*x+2)/x^3*c^2/a^4*((a*x-1)/(a*x+1))^(1/2)+(-1/2
*a^3*arctan(1/(a^2*x^2-1)^(1/2))-3*a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1
/2))/(a^2)^(1/2)+a^3*((a*x-1)*(a*x+1))^(1/2))*c^2/a^4*((a*x-1)/(a*x+1))^(1
/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.37

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2dx$$
$$=\frac{6\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(6\,a^4c^2x^4+22\,a}{6\,a^4x^3}$$

input
```
integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
1/6*(6*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 18*a^3*c^2*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) + 18*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x +
1)) - 1) + (6*a^4*c^2*x^4 + 22*a^3*c^2*x^3 + 7*a^2*c^2*x^2 - 7*a*c^2*x + 2
*c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$$
$$= \frac{c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^5+x^4}\right)dx+\int\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}dx+\int\frac{2a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}dx+\int\left(-\frac{2a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\right)dx+\int\left(-\right.\right.}{a^4}$$

input
```
integrate((c-c/a**2/x**2)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + In
tegral(a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integral(
2*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(-2
*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**4*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**5*x*sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(100) = 200$.

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.96

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$$
$$= \frac{1}{3}a\left(\frac{3c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{21c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-17c^2\left(\frac{ax-}{ax+}\right.}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(}{(}}\right.$$

input
```
integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
1/3*a*(3*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 9*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^2 + 9*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (
21*c^2*((a*x - 1)/(a*x + 1))^(7/2) - 17*c^2*((a*x - 1)/(a*x + 1))^(5/2) -
37*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*
(a*x - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/
(a*x + 1)^4 + a^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(100) = 200$.

Time = 0.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.32

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = \frac{c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a} + \frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}c^2\operatorname{sgn}(ax+1)}{a} + \frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^2|a|\operatorname{sgn}(ax+1)+12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^2\operatorname{sgn}(ax+1)+36\left(x|a|-\sqrt{a^2x^2-1}\right.}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+\right.}$$

input
```
integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^2*log(abs(-
x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^2
*sgn(a*x + 1)/a + 1/3*(9*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a)*sgn(a
*x + 1) + 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2*sgn(a*x + 1) + 36*(x*a
bs(a) - sqrt(a^2*x^2 - 1))^2*a*c^2*sgn(a*x + 1) - 9*(x*abs(a) - sqrt(a^2*x
^2 - 1))*c^2*abs(a)*sgn(a*x + 1) + 16*a*c^2*sgn(a*x + 1))/(((x*abs(a) - sq
rt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.61

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = \frac{5\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{37\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{17\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-7\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}} + \frac{c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a} - \frac{6\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

input
```
int((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(5*c^2*((a*x - 1)/(a*x + 1))^(1/2) + (37*c^2*((a*x - 1)/(a*x + 1))^(3/2))/
3 + (17*c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 - 7*c^2*((a*x - 1)/(a*x + 1))^(
7/2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*
(a*x - 1)^4)/(a*x + 1)^4) + (c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6
*c^2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.43

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$$
$$= \frac{c^2\left(6atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)a^3x^3 - 6atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)a^3x^3 + 6\sqrt{ax+1}\sqrt{ax-1}\right.}{}$$

input
```
int((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(c**2*(6*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a**3*x**3 - 6*atan(sqrt(a
*x - 1) + sqrt(a*x + 1) + 1)*a**3*x**3 + 6*sqrt(a*x + 1)*sqrt(a*x - 1)*a**
3*x**3 + 16*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 - 9*sqrt(a*x + 1)*sqrt(a
*x - 1)*a*x + 2*sqrt(a*x + 1)*sqrt(a*x - 1) - 36*log((sqrt(a*x - 1) + sqrt
(a*x + 1))/sqrt(2))*a**3*x**3 - 8*a**3*x**3))/(6*a**4*x**3)
```

3.801 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\frac{c\sqrt{1-\frac{1}{a^2x^2}}}{a}+c\sqrt{1-\frac{1}{a^2x^2}}x-\frac{3c\csc^{-1}(ax)}{a}-\frac{3\text{carctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-c*(1-1/a^2/x^2)^(1/2)/a+c*(1-1/a^2/x^2)^(1/2)*x-3*c*arccsc(a*x)/a-3*c*arc
tanh((1-1/a^2/x^2)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(-1+ax)-3\arcsin\left(\frac{1}{ax}\right)-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{a}$$

input
```
Integrate[(c - c/(a^2*x^2))/E^(3*ArcCoth[a*x]),x]
```

output
```
(c*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) - 3*ArcSin[1/(a*x)] - 3*Log[(1 + Sqrt
[1 - 1/(a^2*x^2)])*x]))/a
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6748, 109, 27, 140, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6748}$$

$$-c\int \frac{\left(1-\frac{1}{ax}\right)^{5/2}x^2}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 109}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\int \frac{3\sqrt{1-\frac{1}{ax}}x}{a\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3\int \frac{\sqrt{1-\frac{1}{ax}}x}{\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}\right)$$

$$\Big\downarrow \text{ 140}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3\left(\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{\int \frac{1}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}}{a}\right)}{a}\right)$$

$$\Big\downarrow \text{ 39}$$

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3\left(\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}-\frac{\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}\right)}{a}\right)$$

$\downarrow$ 103

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3\left(-\frac{\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\frac{\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}\right)}{a}\right)$$

$\downarrow$ 221

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3\left(-\frac{\int\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}d\frac{1}{x}}{a}-\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)}{a}\right)$$

$\downarrow$ 223

$$-c\left(x\left(-\sqrt{\frac{1}{ax}+1}\right)\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3\left(-\arcsin\left(\frac{1}{ax}\right)-\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)}{a}\right)$$

input
```
Int[(c - c/(a^2*x^2))/E^(3*ArcCoth[a*x]),x]
```

output
```
-(c*(-((1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x) - (3*(-ArcSin[1/(a*x)] - A
rcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]))/a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(63) = 126$.

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.88

method	result
risch	$-\frac{(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{x\,a^2}+\frac{\left(\sqrt{(ax-1)(ax+1)}-3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-\frac{3a\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{a(ax-1)}$
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c\left(-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^2x^2+4\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,ax+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-3\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}}{\sqrt{a^2}}\right.\right.}{(ax-1)\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$

input
```
int((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*x+1)/x*c/a^2*((a*x-1)/(a*x+1))^(1/2)+1/a*(((a*x-1)*(a*x+1))^(1/2)-3*ar
ctan(1/(a^2*x^2-1)^(1/2))-3*a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2-1)^(1/2))/(a^2
)^(1/2))*c*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.49

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$$
$$=\frac{6\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2cx^2-c)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

input
```
integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

output
```
(6*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - 3*a*c*x*log(sqrt((a*x - 1)/(a
*x + 1)) + 1) + 3*a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c*x^2 -
c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$$
$$=\frac{c\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\,dx+\int\left(-\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\right)dx+\int\left(-\frac{a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^3x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a^2}$$

input
```
integrate((c-c/a**2/x**2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integr
al(-a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**2*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x*sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.71

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=$$
$$-\left(\frac{4\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

input
```
integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

output
```
-(4*c*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 6*
c*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)
) + 1)/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.77

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{6\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}c\operatorname{sgn}(ax+1)}{a}-\frac{2\,c\operatorname{sgn}(ax+1)}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|}$$

input
```
integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
6*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c*log(abs(-x*
abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn
(a*x + 1)/a - 2*c*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs
(a))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{6\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}$$

input
```
int((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
(6*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6*c*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a + (4*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x
 + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.49

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$$
$$=\frac{c\left(6atan\left(\sqrt{ax-1}+\sqrt{ax+1}-1\right)ax-6atan\left(\sqrt{ax-1}+\sqrt{ax+1}+1\right)ax+\sqrt{ax+1}\,\sqrt{ax-1}\,ax-\right.}{a^2x}$$

input
```
int((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(c*(6*atan(sqrt(a*x - 1) + sqrt(a*x + 1) - 1)*a*x - 6*atan(sqrt(a*x - 1) +
 sqrt(a*x + 1) + 1)*a*x + sqrt(a*x + 1)*sqrt(a*x - 1)*a*x - sqrt(a*x + 1)*
sqrt(a*x - 1) - 6*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a*x - a*x))
/(a**2*x)
```

3.802 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{9a-\frac{11}{x}}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\left(a-\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}-\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

output
```
1/3*(9*a-11/x)/a^2/c/(1-1/a^2/x^2)^(1/2)+4/3*(a-1/x)/a^2/c/(1-1/a^2/x^2)^(
3/2)+(1-1/a^2/x^2)^(1/2)*x/c-3*arctanh((1-1/a^2/x^2)^(1/2))/a/c
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.64

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(14+19ax+3a^2x^2)}{(1+ax)^2}-\frac{9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{3c}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

output
```
((Sqrt[1 - 1/(a^2*x^2)]*x*(14 + 19*a*x + 3*a^2*x^2))/(1 + a*x)^2 - (9*Log[
(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.32, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {6748, 110, 25, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{\sqrt{1-\frac{1}{ax}}x^2}{\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{110}$$

$$-\frac{\int -\frac{\left(3a-\frac{2}{x}\right)x}{a^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\int \frac{\left(3a-\frac{2}{x}\right)x}{a^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{\left(3a-\frac{2}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{169}$$

$$-\frac{-\frac{\frac{1}{3}a\int \frac{\left(9a-\frac{5}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\frac{1}{3}\int \frac{\left(9a-\frac{5}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{169}$$

$$-\frac{-\frac{\frac{1}{3}\left(a\int \frac{9x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{14a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\frac{1}{3}\left(9a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{14a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{103}$$

$$-\frac{-\frac{\frac{1}{3}\left(\frac{14a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-9\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)\right)+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

$$\Big\downarrow \text{221}$$

$$-\frac{-\frac{\frac{1}{3}\left(\frac{14a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-9a\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{5a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{c}$$

input

```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

output

```
-((-((Sqrt[1 - 1/(a*x)]*x)/(1 + 1/(a*x))^(3/2)) - ((5*a*Sqrt[1 - 1/(a*x)])
/(3*(1 + 1/(a*x))^(3/2)) + ((14*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)] - 9
*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/3)/a^2)/c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.62

method	result
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}+\frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}-\frac{2\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{3a^5\left(x+\frac{1}{a}\right)^2}+\frac{13\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{3a^4\left(x+\frac{1}{a}\right)}\right)a^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}}{c(ax-1)}$
default	$-\frac{\left(9\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^4x^3-9\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,a^3x^3+27\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2+6\sqrt{a^2}\,((ax-1)(a\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/c+(-3/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2-1)^(1/2))/(a^2)^(1/2)-2/3/a^5/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2
)+13/3/a^4/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))/c*a^2*((a*x-1)/(a*x+
1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{9\,(ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,(ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(3\,a^2x^2+19\,ax+14)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^2cx+ac)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
-1/3*(9*(a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a*x + 1)*log(sqr
t((a*x - 1)/(a*x + 1)) - 1) - (3*a^2*x^2 + 19*a*x + 14)*sqrt((a*x - 1)/(a*
x + 1)))/(a^2*c*x + a*c)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{a^2\left(\int\left(-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\right)dx+\int\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\,dx\right)}{c}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2),x)
```

output
```
a**2*(Integral(-x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 + a**2*x
**2 - a*x - 1), x) + Integral(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*
*3*x**3 + a**2*x**2 - a*x - 1), x))/c
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.31

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{1}{3}\,a\left(\frac{6\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+12\sqrt{\frac{ax-1}{ax+1}}}{a^2c}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
-1/3*a*(6*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) -
(((a*x - 1)/(a*x + 1))^(3/2) + 12*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c) + 9*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 9*log(sqrt((a*x - 1)/(a*x + 1)
) - 1)/(a^2*c))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.55

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{3\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\,\mathrm{sgn}(ax+1)}{c|a|}+\frac{\sqrt{a^2x^2-1}\mathrm{sgn}(ax+1)}{ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c*abs(a)) + sqrt(a
^2*x^2 - 1)*sgn(a*x + 1)/(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}}+\frac{4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3\,a\,c}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,6\mathrm{i}}{a\,c}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2)),x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1)) + (4*((a
*x - 1)/(a*x + 1))^(1/2))/(a*c) + ((a*x - 1)/(a*x + 1))^(3/2)/(3*a*c) + (a
tan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*6i)/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.48

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$$
$$= \frac{3\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2+19\sqrt{ax+1}\,\sqrt{ax-1}\,ax+14\sqrt{ax+1}\,\sqrt{ax-1}-18\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^2x^2 -}{3ac\,(a^2x^2+2ax+1)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x)
```

output
```
(3*sqrt(a*x + 1)*sqrt(a*x - 1)*a**2*x**2 + 19*sqrt(a*x + 1)*sqrt(a*x - 1)*
a*x + 14*sqrt(a*x + 1)*sqrt(a*x - 1) - 18*log((sqrt(a*x - 1) + sqrt(a*x +
1))/sqrt(2))*a**2*x**2 - 36*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a
*x - 18*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) - 4*a**2*x**2 - 8*a*x
 - 4)/(3*a*c*(a**2*x**2 + 2*a*x + 1))
```

3.803 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 140

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} \, dx = \frac{15a-\frac{19}{x}}{5a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5a-\frac{7}{x}}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{4\left(a-\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}-\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

output
```
1/5*(15*a-19/x)/a^2/c^2/(1-1/a^2/x^2)^(1/2)+1/5*(5*a-7/x)/a^2/c^2/(1-1/a^2
/x^2)^(3/2)+4/5*(a-1/x)/a^2/c^2/(1-1/a^2/x^2)^(5/2)+(1-1/a^2/x^2)^(1/2)*x/
c^2-3*arctanh((1-1/a^2/x^2)^(1/2))/a/c^2
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.56

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} \, dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(24+57ax+39a^2x^2+5a^3x^3)}{5(1+ax)^3}-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^2}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(24 + 57*a*x + 39*a^2*x^2 + 5*a^3*x^3))/(5*(1
+ a*x)^3) - 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {6748, 114, 27, 35, 110, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int \frac{3\left(a-\frac{1}{x}\right)x}{a^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{3\int \frac{\left(a-\frac{1}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{a^2}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{35}$$

$$-\frac{-\frac{3\int \frac{\sqrt{1-\frac{1}{ax}}x}{\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{110}$$

$$-\frac{-\frac{3\left(\frac{2\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{2}{5}\int -\frac{\left(5a-\frac{4}{x}\right)x}{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{3\left(\frac{\int \frac{\left(5a-\frac{4}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}}{5a}+\frac{2\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \ 169$$

$$-\frac{-\frac{3\left(\frac{\frac{1}{3}a\int \frac{3\left(5a-\frac{3}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{3a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{5a}+\frac{2\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{3\left(\frac{\int \frac{\left(5a-\frac{3}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{3a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{5a}+\frac{2\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \ 169$$

$$-\frac{-\frac{3\left(\frac{a\int \frac{5x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{3a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{5a}+\frac{2\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{3\left(\frac{5a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{3a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{5a}+\frac{2\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \ 103$$

$$-\frac{-\frac{3\left(\frac{-5\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)+\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{3a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{5a}+\frac{2\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{-\frac{3\left(\frac{-5a\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)+\frac{8a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{3a\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{3/2}}}{5a}+\frac{2\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)}{a}-\frac{x\sqrt{1-\frac{1}{ax}}}{\left(\frac{1}{ax}+1\right)^{5/2}}}{c^2}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

output
```
-((-((Sqrt[1 - 1/(a*x)]*x)/(1 + 1/(a*x))^(5/2)) - (3*((2*Sqrt[1 - 1/(a*x)]
)/(5*(1 + 1/(a*x))^(5/2)) + ((3*a*Sqrt[1 - 1/(a*x)])/(1 + 1/(a*x))^(3/2) +
 (8*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)] - 5*a*ArcTanh[Sqrt[1 - 1/(a*x)]
*Sqrt[1 + 1/(a*x)]])/(5*a)))/a)/c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.48

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2}+\frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}+\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{5a^8\left(x+\frac{1}{a}\right)^3}-\frac{6\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{5a^7\left(x+\frac{1}{a}\right)^2}+\frac{24\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{5a^6\left(x+\frac{1}{a}\right)}\right)a^4\sqrt{}}{c^2(ax-1)}$
default	$-\frac{\left(-125\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^4x^4+120\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^5x^4+85\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}a^2x^2-500\sqrt{a^2}\,\sqrt{(ax-1)}\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^2*((a*x-1)/(a*x+1))^(1/2)+(-3/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)+1/5/a^8/(x+1/a)^3*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1
/2)-6/5/a^7/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+24/5/a^6/(x+1/a)*(
a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^4/c^2*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)
*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{15\,(a^2x^2+2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,(a^2x^2+2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(5\,a^3x^3+39\,a^2x^2+}{5\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
-1/5*(15*(a^2*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*(a^
2*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (5*a^3*x^3 + 39*a^
2*x^2 + 57*a*x + 24)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 + 2*a^2*c^2*x
 + a*c^2)
```

Sympy [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\left(\int\left(-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\right)dx+\int\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\,dx\right)}{c^2}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**2,x)
```

output
```
a**4*(Integral(-x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 + a**4*x
**4 - 2*a**3*x**3 - 2*a**2*x**2 + a*x + 1), x) + Integral(a*x**5*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 + a**4*x**4 - 2*a**3*x**3 - 2*a**2*x**
2 + a*x + 1), x))/c**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.15

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{1}{20}a\left(\frac{40\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c^2}{ax+1}-a^2c^2}-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+10\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+85\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}+\frac{60\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{60\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
-1/20*a*(40*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c^2/(a*x + 1) - a^2*c
^2) - (((a*x - 1)/(a*x + 1))^(5/2) + 10*((a*x - 1)/(a*x + 1))^(3/2) + 85*s
qrt((a*x - 1)/(a*x + 1)))/(a^2*c^2) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1
)/(a^2*c^2) - 60*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.42

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{3\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{c^2|a|} + \frac{\sqrt{a^2x^2-1}\mathrm{sgn}(ax+1)}{ac^2}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c^2*abs(a)) + sqrt
(a^2*x^2 - 1)*sgn(a*x + 1)/(a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2-\frac{a\,c^2\,(a\,x-1)}{a\,x+1}} + \frac{17\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^2} + \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{2\,a\,c^2} + \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{20\,a\,c^2} + \frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,6\mathrm{i}}{a\,c^2}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^2,x)
```

output
```
(2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c^2 - (a*c^2*(a*x - 1))/(a*x + 1)) + (1
7*((a*x - 1)/(a*x + 1))^(1/2))/(4*a*c^2) + ((a*x - 1)/(a*x + 1))^(3/2)/(2*
a*c^2) + ((a*x - 1)/(a*x + 1))^(5/2)/(20*a*c^2) + (atan(((a*x - 1)/(a*x +
1))^(1/2)*1i)*6i)/(a*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.58

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$
$$= \frac{10\sqrt{ax+1}\,\sqrt{ax-1}\,a^3x^3+78\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x^2+114\sqrt{ax+1}\,\sqrt{ax-1}\,ax+48\sqrt{ax+1}\,\sqrt{ax-1}}{}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x)
```

output
```
(10*sqrt(a*x + 1)*sqrt(a*x - 1)*a**3*x**3 + 78*sqrt(a*x + 1)*sqrt(a*x - 1)
*a**2*x**2 + 114*sqrt(a*x + 1)*sqrt(a*x - 1)*a*x + 48*sqrt(a*x + 1)*sqrt(a
*x - 1) - 60*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**3*x**3 - 180*
log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**2*x**2 - 180*log((sqrt(a*x
 - 1) + sqrt(a*x + 1))/sqrt(2))*a*x - 60*log((sqrt(a*x - 1) + sqrt(a*x + 1
))/sqrt(2)) - 33*a**3*x**3 - 99*a**2*x**2 - 99*a*x - 33)/(10*a*c**2*(a**3*
x**3 + 3*a**2*x**2 + 3*a*x + 1))
```

3.804 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 173

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx=\frac{35a-\frac{53}{x}}{35a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{3\left(35a-\frac{47}{x}\right)}{35a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{21a-\frac{31}{x}}{35a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{4\left(a-\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3}-\frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

output
```
1/35*(35*a-53/x)/a^2/c^3/(1-1/a^2/x^2)^(3/2)+3/35*(35*a-47/x)/a^2/c^3/(1-1
/a^2/x^2)^(1/2)+1/35*(21*a-31/x)/a^2/c^3/(1-1/a^2/x^2)^(5/2)+4/7*(a-1/x)/a
^2/c^3/(1-1/a^2/x^2)^(7/2)+(1-1/a^2/x^2)^(1/2)*x/c^3-3*arctanh((1-1/a^2/x^
2)^(1/2))/a/c^3
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.58

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-176-423ax-125a^2x^2+368a^3x^3+286a^4x^4+35a^5x^5)}{35(-1+ax)(1+ax)^4}-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^3}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-176 - 423*a*x - 125*a^2*x^2 + 368*a^3*x^3 +
286*a^4*x^4 + 35*a^5*x^5))/(35*(-1 + a*x)*(1 + a*x)^4) - 3*Log[(1 + Sqrt[1
 - 1/(a^2*x^2)])*x])/(a*c^3)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.42, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {6748, 114, 27, 169, 25, 27, 169, 27, 169, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}}{c^3}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int \frac{\left(3a-\frac{5}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{\left(3a-\frac{5}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow \; 169$$

$$-\frac{-\frac{a\left(-\int -\frac{\left(3a-\frac{8}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}\right)-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{a\int \frac{\left(3a-\frac{8}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{\left(3a-\frac{8}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow \; 169$$

$$-\frac{-\frac{\frac{1}{7}a\int \frac{3\left(7a-\frac{11}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{3}{7}\int \frac{\left(7a-\frac{11}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow \; 169$$

$$-\frac{-\frac{\frac{3}{7}\left(\frac{1}{5}a\int \frac{\left(35a-\frac{36}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{18a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{3}{7}\left(\frac{1}{5}\int \frac{\left(35a-\frac{36}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{18a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$\big\downarrow$ 169

$$-\frac{-\frac{\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}a\int \frac{\left(105a-\frac{71}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{71a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{18a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$\big\downarrow$ 27

$$-\frac{-\frac{\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{\left(105a-\frac{71}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{71a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{18a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{c^3}$$

$\big\downarrow$ 169

$$-\frac{-\frac{\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(a\int \frac{105x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{176a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{71a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{18a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)}}{c^3}$$

$\big\downarrow$ 27

$$-\frac{-\frac{\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(105a\int \frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{176a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{71a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{18a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{x}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+\right.}}{c^3}$$

$\big\downarrow$ 103

$$-\frac{-\frac{\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{176a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-105\int \frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)\right)+\frac{71a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{18a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{}{\sqrt{1-}}}{c^3}$$

$\big\downarrow$ 221

$$-\frac{-\frac{\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{176a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-105a\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{71a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{18a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{11a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}}{a^2}-\frac{\cdots}{\sqrt{1-\cdots}}}{c^3}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

output
```
-((-(x/(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))) - ((-2*a)/(Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(7/2)) + (11*a*Sqrt[1 - 1/(a*x)])/(7*(1 + 1/(a*x))^(7/2))
 + (3*((18*a*Sqrt[1 - 1/(a*x)])/(5*(1 + 1/(a*x))^(5/2)) + ((71*a*Sqrt[1 -
1/(a*x)])/(3*(1 + 1/(a*x))^(3/2)) + ((176*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/
(a*x)] - 105*a*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/3)/5))/7)/a^2
)/c^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.62

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3}+\frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^6\sqrt{a^2}}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{16a^8\left(x-\frac{1}{a}\right)}-\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{14a^{11}\left(x+\frac{1}{a}\right)^4}+\frac{71\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{140a^{10}\left(x+\frac{1}{a}\right)^3}-\frac{477\sqrt{}}{}\right.}{c^3(ax-1)}$
default	$-\frac{\left(-3675\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^7x^7+3360\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^8x^7+2555((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5-11025\sqrt{(ax-\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^3*((a*x-1)/(a*x+1))^(1/2)+(-3/a^6*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)-1/16/a^8/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/
2)-1/14/a^11/(x+1/a)^4*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+71/140/a^10/(x+1/
a)^3*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-477/280/a^9/(x+1/a)^2*(a^2*(x+1/a)^
2-2*a*(x+1/a))^(1/2)+2931/560/a^8/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2
))*a^6/c^3*((a*x-1)/(a*x+1))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.03

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{105\,(a^4x^4+2\,a^3x^3-2\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,(a^4x^4+2\,a^3x^3-2\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-}{35\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

output
```
-1/35*(105*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1))
 + 1) - 105*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) - (35*a^5*x^5 + 286*a^4*x^4 + 368*a^3*x^3 - 125*a^2*x^2 - 423*a*x -
 176)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*
x - a*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.15

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx =$$

$$-\frac{1}{560}a\left(\frac{35\left(\frac{33(ax-1)}{ax+1}-1\right)}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+56\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+350\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+2520\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{1680\,\mathrm{lo}}{}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

output
```
-1/560*a*(35*(33*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(
3/2) - a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) - (5*((a*x - 1)/(a*x + 1))^(7/2)
 + 56*((a*x - 1)/(a*x + 1))^(5/2) + 350*((a*x - 1)/(a*x + 1))^(3/2) + 2520
*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3) + 1680*log(sqrt((a*x - 1)/(a*x + 1))
 + 1)/(a^2*c^3) - 1680*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^3, x)
```

Mupad [B] (verification not implemented)

Time = 13.13 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.06

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{\frac{33\,(a\,x-1)}{a\,x+1}-1}{16\,a\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}-16\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}+\frac{9\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a\,c^3}+\frac{5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{8\,a\,c^3}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{10\,a\,c^3}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{112\,a\,c^3}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,6{}\mathrm{i}}{a\,c^3}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^3,x)
```

output
```
((33*(a*x - 1))/(a*x + 1) - 1)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(1/2) - 16*
a*c^3*((a*x - 1)/(a*x + 1))^(3/2)) + (9*((a*x - 1)/(a*x + 1))^(1/2))/(2*a*
c^3) + (5*((a*x - 1)/(a*x + 1))^(3/2))/(8*a*c^3) + ((a*x - 1)/(a*x + 1))^(
5/2)/(10*a*c^3) + ((a*x - 1)/(a*x + 1))^(7/2)/(112*a*c^3) + (atan(((a*x -
1)/(a*x + 1))^(1/2)*1i)*6i)/(a*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.95

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \frac{-840\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^4x^4-3360\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^3x^3-5040\sqrt{ax-1}\log\left(\frac{\sqrt{ax-}}{}\right.}{}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x)
```

output
```
( - 840*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**4*x*
*4 - 3360*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**3*
x**3 - 5040*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**
2*x**2 - 3360*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a
*x - 840*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)) - 549*
sqrt(a*x - 1)*a**4*x**4 - 2196*sqrt(a*x - 1)*a**3*x**3 - 3294*sqrt(a*x - 1
)*a**2*x**2 - 2196*sqrt(a*x - 1)*a*x - 549*sqrt(a*x - 1) + 140*sqrt(a*x +
1)*a**5*x**5 + 1144*sqrt(a*x + 1)*a**4*x**4 + 1472*sqrt(a*x + 1)*a**3*x**3
 - 500*sqrt(a*x + 1)*a**2*x**2 - 1692*sqrt(a*x + 1)*a*x - 704*sqrt(a*x + 1
))/(140*sqrt(a*x - 1)*a*c**3*(a**4*x**4 + 4*a**3*x**3 + 6*a**2*x**2 + 4*a*
x + 1))
```

3.805 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 206

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{945a-\frac{1349}{x}}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{315a-\frac{517}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{63a-\frac{103}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{27a-\frac{41}{x}}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{4\left(a-\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4}-\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

output
```
1/315*(945*a-1349/x)/a^2/c^4/(1-1/a^2/x^2)^(1/2)+1/315*(315*a-517/x)/a^2/c
^4/(1-1/a^2/x^2)^(3/2)+1/105*(63*a-103/x)/a^2/c^4/(1-1/a^2/x^2)^(5/2)+1/63
*(27*a-41/x)/a^2/c^4/(1-1/a^2/x^2)^(7/2)+4/9*(a-1/x)/a^2/c^4/(1-1/a^2/x^2)
^(9/2)+(1-1/a^2/x^2)^(1/2)*x/c^4-3*arctanh((1-1/a^2/x^2)^(1/2))/a/c^4
```

Mathematica [A] (warning: unable to verify)

Time = 0.74 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.57

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}x(1664+4047ax-339a^2x^2-7399a^3x^3-4029a^4x^4+2967a^5x^5+2669a^6x^6+315a^7x^7)}{315(-1+ax)^2(1+ax)^5}-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^4}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

output
```
((a*Sqrt[1 - 1/(a^2*x^2)]*x*(1664 + 4047*a*x - 339*a^2*x^2 - 7399*a^3*x^3
- 4029*a^4*x^4 + 2967*a^5*x^5 + 2669*a^6*x^6 + 315*a^7*x^7))/(315*(-1 + a*
x)^2*(1 + a*x)^5) - 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.52, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {6748, 114, 27, 169, 27, 169, 25, 27, 169, 27, 169, 27, 169, 27, 169, 27, 169, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \frac{x^2}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}}d\frac{1}{x}}{c^4}$$

$$\Big\downarrow \text{114}$$

$$-\frac{-\int \frac{\left(3a-\frac{7}{x}\right)x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}}d\frac{1}{x}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{-\frac{\int \frac{\left(3a-\frac{7}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}}d\frac{1}{x}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \textbf{169}$$

$$-\frac{-\frac{-\frac{1}{3}a\int -\frac{3\left(3a-\frac{8}{x}\right)x}{a\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}d\frac{1}{x}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{-\frac{\int \frac{\left(3a-\frac{8}{x}\right)x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}d\frac{1}{x}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \textbf{169}$$

$$-\frac{-\frac{a\left(-\int -\frac{\left(3a-\frac{25}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}d\frac{1}{x}\right)-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{-\frac{a\int \frac{\left(3a-\frac{25}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}d\frac{1}{x}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{-\frac{\int \frac{\left(3a-\frac{25}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}d\frac{1}{x}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \textbf{169}$$

$$-\frac{-\frac{\frac{1}{9}a\int \frac{\left(27a-\frac{112}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{-\frac{\frac{1}{9}\int \frac{\left(27a-\frac{112}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}d\frac{1}{x}+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{-\frac{\frac{1}{9}\left(\frac{1}{7}a\int \frac{3\left(63a-\frac{139}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\right)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\frac{1}{9}\left(\frac{3}{7}\int \frac{\left(63a-\frac{139}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}d\frac{1}{x}+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\right)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-\frac{x}{\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}a\int \frac{\left(315a-\frac{404}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{202a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\right)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\int \frac{\left(315a-\frac{404}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}d\frac{1}{x}+\frac{202a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\right)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}}{a^2}-}{c^4}$$

$$\Big\downarrow \ 169$$

$$-\frac{-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}a\int \frac{\left(945a-\frac{719}{x}\right)x}{a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{719a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{202a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\right)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-}{a^2}}{c^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{\left(945a-\frac{719}{x}\right)x}{\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}d\frac{1}{x}+\frac{719a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{202a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\right)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4a}{3\left(1-\frac{1}{ax}\right)^{3/2}\left(\cdots\right.}}{a^2}}{c^4}$$

$\downarrow$ 169

$$-\frac{-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(a\int\frac{945x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{1664a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{719a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{202a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\right)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}\cdots}{a^2}}{c^4}$$

$\downarrow$ 27

$$-\frac{-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(945a\int\frac{x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}d\frac{1}{x}+\frac{1664a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right)+\frac{719a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{202a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\right)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5a}{\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)\cdots}}{a^2}}{c^4}$$

$\downarrow$ 103

$$-\frac{-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{1664a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-945\int\frac{1}{\frac{1}{a}-\frac{1}{ax^2}}d\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)\right)+\frac{719a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{202a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\right)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\cdots}{a^2}}{c^4}$$

$\downarrow$ 221

$$-\frac{-\frac{\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{1664a\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-945a\,\text{arctanh}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)\right)+\frac{719a\sqrt{1-\frac{1}{ax}}}{3\left(\frac{1}{ax}+1\right)^{3/2}}\right)+\frac{202a\sqrt{1-\frac{1}{ax}}}{5\left(\frac{1}{ax}+1\right)^{5/2}}\right)+\frac{139a\sqrt{1-\frac{1}{ax}}}{7\left(\frac{1}{ax}+1\right)^{7/2}}\right)+\frac{28a\sqrt{1-\frac{1}{ax}}}{9\left(\frac{1}{ax}+1\right)^{9/2}}-\cdots}{a^2}}{c^4}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

output
```
-((-(x/((1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2))) - ((-4*a)/(3*(1 - 1/(a*x
))^(3/2)*(1 + 1/(a*x))^(9/2)) - (5*a)/(Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/
2)) + (28*a*Sqrt[1 - 1/(a*x)])/(9*(1 + 1/(a*x))^(9/2)) + ((139*a*Sqrt[1 -
1/(a*x)])/(7*(1 + 1/(a*x))^(7/2)) + (3*((202*a*Sqrt[1 - 1/(a*x)])/(5*(1 +
1/(a*x))^(5/2)) + ((719*a*Sqrt[1 - 1/(a*x)])/(3*(1 + 1/(a*x))^(3/2)) + ((1
664*a*Sqrt[1 - 1/(a*x)])/Sqrt[1 + 1/(a*x)] - 945*a*ArcTanh[Sqrt[1 - 1/(a*x
)]*Sqrt[1 + 1/(a*x)]])/3)/5))/7)/9)/a^2)/c^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.72

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^4}+\frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^8\sqrt{a^2}}-\frac{31\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{192a^{10}\left(x-\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2+2a\left(x-\frac{1}{a}\right)}}{96a^{11}\left(x-\frac{1}{a}\right)^2}+\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{36a^{14}\left(x+\frac{1}{a}\right)^5}-\frac{59\sqrt{\cdots}}{\cdots}\right.}{\cdots}$
default	$-\frac{\left(-138915\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,a^9x^9+120960\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^{10}x^9+98595((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}\,a^7x^7-41674\cdots\right.}{\cdots}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1)/c^4*((a*x-1)/(a*x+1))^(1/2)+(-3/a^8*ln(a^2*x/(a^2)^(1/2)+(a^2*
x^2-1)^(1/2))/(a^2)^(1/2)-31/192/a^10/(x-1/a)*((x-1/a)^2*a^2+2*a*(x-1/a))^
(1/2)-1/96/a^11/(x-1/a)^2*((x-1/a)^2*a^2+2*a*(x-1/a))^(1/2)+1/36/a^14/(x+1
/a)^5*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)-59/252/a^13/(x+1/a)^4*(a^2*(x+1/a)
^2-2*a*(x+1/a))^(1/2)+1507/1680/a^12/(x+1/a)^3*(a^2*(x+1/a)^2-2*a*(x+1/a))
^(1/2)-691/315/a^11/(x+1/a)^2*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2)+113591/201
60/a^10/(x+1/a)*(a^2*(x+1/a)^2-2*a*(x+1/a))^(1/2))*a^8/c^4*((a*x-1)/(a*x+1
))^(1/2)*((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.33

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{945\left(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-945\left(a^6x^6+2\,a^5x^5-a^4x^4\right.}{315\left(a^7c^4x^6\right.}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

output
```
-1/315*(945*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x +
 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 945*(a^6*x^6 + 2*a^5*x^5 - a^4*x^
4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) -
(315*a^7*x^7 + 2669*a^6*x^6 + 2967*a^5*x^5 - 4029*a^4*x^4 - 7399*a^3*x^3 -
 339*a^2*x^2 + 4047*a*x + 1664)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 +
2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x +
a*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.12

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{1}{20160}a\left(\frac{105\left(\frac{29\,(ax-1)}{ax+1}-\frac{414\,(ax-1)^2}{(ax+1)^2}+1\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{35\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+450\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+2961\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+14700\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{a^2c^4}\right.$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

output
```
1/20160*a*(105*(29*(a*x - 1)/(a*x + 1) - 414*(a*x - 1)^2/(a*x + 1)^2 + 1)/
(a^2*c^4*((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2)
) + (35*((a*x - 1)/(a*x + 1))^(9/2) + 450*((a*x - 1)/(a*x + 1))^(7/2) + 29
61*((a*x - 1)/(a*x + 1))^(5/2) + 14700*((a*x - 1)/(a*x + 1))^(3/2) + 95445
*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^4) - 60480*log(sqrt((a*x - 1)/(a*x + 1)
) + 1)/(a^2*c^4) + 60480*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Giac [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^4, x)
```

Mupad [B] (verification not implemented)

Time = 13.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.09

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx=\frac{303\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4}-\frac{\frac{29\,(a\,x-1)}{3\,(a\,x+1)}-\frac{138\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{64\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-64\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}+\frac{35\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{48\,a\,c^4}+\frac{47\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{320\,a\,c^4}+\frac{5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{224\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{576\,a\,c^4}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,6{}\mathrm{i}}{a\,c^4}$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^4,x)
```

output
```
(303*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - ((29*(a*x - 1))/(3*(a*x + 1
)) - (138*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(64*a*c^4*((a*x - 1)/(a*x + 1))^
(3/2) - 64*a*c^4*((a*x - 1)/(a*x + 1))^(5/2)) + (35*((a*x - 1)/(a*x + 1))^
(3/2))/(48*a*c^4) + (47*((a*x - 1)/(a*x + 1))^(5/2))/(320*a*c^4) + (5*((a*
x - 1)/(a*x + 1))^(7/2))/(224*a*c^4) + ((a*x - 1)/(a*x + 1))^(9/2)/(576*a*
c^4) + (atan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*6i)/(a*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.04

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$= \frac{-7560\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^6x^6-30240\sqrt{ax-1}\log\left(\frac{\sqrt{ax-1}+\sqrt{ax+1}}{\sqrt{2}}\right)a^5x^5-37800\sqrt{ax-1}\log\left(\right.}{}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x)
```

output
```
( - 7560*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**6*x
**6 - 30240*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*a**
5*x**5 - 37800*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2))*
a**4*x**4 + 37800*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2
))*a**2*x**2 + 30240*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqr
t(2))*a*x + 7560*sqrt(a*x - 1)*log((sqrt(a*x - 1) + sqrt(a*x + 1))/sqrt(2)
) - 4691*sqrt(a*x - 1)*a**6*x**6 - 18764*sqrt(a*x - 1)*a**5*x**5 - 23455*s
qrt(a*x - 1)*a**4*x**4 + 23455*sqrt(a*x - 1)*a**2*x**2 + 18764*sqrt(a*x -
1)*a*x + 4691*sqrt(a*x - 1) + 1260*sqrt(a*x + 1)*a**7*x**7 + 10676*sqrt(a*
x + 1)*a**6*x**6 + 11868*sqrt(a*x + 1)*a**5*x**5 - 16116*sqrt(a*x + 1)*a**
4*x**4 - 29596*sqrt(a*x + 1)*a**3*x**3 - 1356*sqrt(a*x + 1)*a**2*x**2 + 16
188*sqrt(a*x + 1)*a*x + 6656*sqrt(a*x + 1))/(1260*sqrt(a*x - 1)*a*c**4*(a*
*6*x**6 + 4*a**5*x**5 + 5*a**4*x**4 - 5*a**2*x**2 - 4*a*x - 1))
```

3.806 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 321

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/(1-1/a^2/x^2)^(1/2)/x^6+1/5*c^3*(c-c/a^2/x
^2)^(1/2)/a^6/(1-1/a^2/x^2)^(1/2)/x^5-3/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/(1-1
/a^2/x^2)^(1/2)/x^4-c^3*(c-c/a^2/x^2)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)/x^3+3/
2*c^3*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)/x^2+3*c^3*(c-c/a^2/x^2)^
(1/2)/a^2/(1-1/a^2/x^2)^(1/2)/x+c^3*(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1
/2)+c^3*(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.29

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6a^7x^6}+\frac{1}{5a^6x^5}-\frac{3}{4a^5x^4}-\frac{1}{a^4x^3}+\frac{3}{2a^3x^2}+\frac{3}{a^2x}+x+\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2),x]
```

output
```
((c - c/(a^2*x^2))^(7/2)*(1/(6*a^7*x^6) + 1/(5*a^6*x^5) - 3/(4*a^5*x^4) -
1/(a^4*x^3) + 3/(2*a^3*x^2) + 3/(a^2*x) + x + Log[x]/a))/(1 - 1/(a^2*x^2))
^(7/2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.31, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{7/2}e^{\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(1-ax)^3(ax+1)^4}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^3(ax+1)^4}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int \left(-a^7-\frac{a^6}{x}+\frac{3a^5}{x^2}+\frac{3a^4}{x^3}-\frac{3a^3}{x^4}-\frac{3a^2}{x^5}+\frac{a}{x^6}+\frac{1}{x^7}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(a^7(-x)-a^6\log(x)-\frac{3a^5}{x}-\frac{3a^4}{2x^2}+\frac{a^3}{x^3}+\frac{3a^2}{4x^4}-\frac{a}{5x^5}-\frac{1}{6x^6}\right)}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2),x]
```

output
```
-((c^3*Sqrt[c - c/(a^2*x^2)]*(-1/6*1/x^6 - a/(5*x^5) + (3*a^2)/(4*x^4) + a
^3/x^3 - (3*a^4)/(2*x^2) - (3*a^5)/x - a^7*x - a^6*Log[x]))/(a^7*Sqrt[1 -
1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\left(60 a^{7} x^{7}+60 a^{6} \ln \left(x \right) x^{6}+180 a^{5} x^{5}+90 a^{4} x^{4}-60 a^{3} x^{3}-45 a^{2} x^{2}+12 a x +10\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{7}{2}} x}{60 \left(a x +1\right) \left(a^{2} x^{2}-1\right)^{3} \sqrt{\frac{a x -1}{a x +1}}}$	112

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*(60*a^7*x^7+60*a^6*ln(x)*x^6+180*a^5*x^5+90*a^4*x^4-60*a^3*x^3-45*a^2
*x^2+12*a*x+10)*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a*x+1)/(a^2*x^2-1)^3/((a*
x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{7/2} dx = \frac{(60\,a^7c^3x^7 + 60\,a^6c^3x^6\log(x) + 180\,a^5c^3x^5 + 90\,a^4c^3x^4 - 60\,a^3c^3x^3 - 45\,a^2c^3x^2 + 12\,ac^3x}{60\,a^8x^6}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="fric
as")
```

output
```
1/60*(60*a^7*c^3*x^7 + 60*a^6*c^3*x^6*log(x) + 180*a^5*c^3*x^5 + 90*a^4*c^
3*x^4 - 60*a^3*c^3*x^3 - 45*a^2*c^3*x^2 + 12*a*c^3*x + 10*c^3)*sqrt(a^2*c)
/(a^8*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxi
ma")
```

output
```
integrate((c - c/(a^2*x^2))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.41

$$\int e^{\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{7/2} dx = \frac{1}{30}\left(\frac{60\,c^3x\mathrm{sgn}(x)}{a\mathrm{sgn}\,(ax+1)} + \frac{60\,c^3\log\left(|x|\right)\mathrm{sgn}(x)}{a^2\mathrm{sgn}\,(ax+1)} + \frac{180\,a^5c^3x^5\mathrm{sgn}(x) + 90\,a^4c^3x^4\mathrm{sgn}(x) - 60\,a^3\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac
")
```

output
```
1/30*(60*c^3*x*sgn(x)/(a*sgn(a*x + 1)) + 60*c^3*log(abs(x))*sgn(x)/(a^2*sg
n(a*x + 1)) + (180*a^5*c^3*x^5*sgn(x) + 90*a^4*c^3*x^4*sgn(x) - 60*a^3*c^3
*x^3*sgn(x) - 45*a^2*c^3*x^2*sgn(x) + 12*a*c^3*x*sgn(x) + 10*c^3*sgn(x))/(
a^8*x^6*sgn(a*x + 1)))*sqrt(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{7/2} dx = \int \frac{\left(c - \frac{c}{a^2\,x^2}\right)^{7/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.21

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\sqrt{c}\,c^3(60\log(x)\,a^6x^6+60a^7x^7+180a^5x^5+90a^4x^4-60a^3x^3-45a^2x^2+12ax+10)}{60a^7x^6}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x)
```

output
```
(sqrt(c)*c**3*(60*log(x)*a**6*x**6 + 60*a**7*x**7 + 180*a**5*x**5 + 90*a**
4*x**4 - 60*a**3*x**3 - 45*a**2*x**2 + 12*a*x + 10))/(60*a**7*x**6)
```

3.807 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 236

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output

```
-1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/(1-1/a^2/x^2)^(1/2)/x^4-1/3*c^2*(c-c/a^2/
x^2)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)/x^3+c^2*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^
2/x^2)^(1/2)/x^2+2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)/x+c^2*(
c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)^(1/2)*ln(x)/a/(
1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.31

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2} dx = \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4a^5x^4}-\frac{1}{3a^4x^3}+\frac{1}{a^3x^2}+\frac{2}{a^2x}+x+\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2),x]
```

output
```
((c - c/(a^2*x^2))^(5/2)*(-1/4*1/(a^5*x^4) - 1/(3*a^4*x^3) + 1/(a^3*x^2) +
 2/(a^2*x) + x + Log[x]/a))/(1 - 1/(a^2*x^2))^(5/2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.33, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{5/2} e^{\coth^{-1}(ax)} dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2(ax+1)^3}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int \left(a^5+\frac{a^4}{x}-\frac{2a^3}{x^2}-\frac{2a^2}{x^3}+\frac{a}{x^4}+\frac{1}{x^5}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(a^5x+a^4\log(x)+\frac{2a^3}{x}+\frac{a^2}{x^2}-\frac{a}{3x^3}-\frac{1}{4x^4}\right)}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2),x]
```

output
```
(c^2*Sqrt[c - c/(a^2*x^2)]*(-1/4*1/x^4 - a/(3*x^3) + a^2/x^2 + (2*a^3)/x +
 a^5*x + a^4*Log[x]))/(a^5*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{(12a^5x^5+12\ln(x)x^4a^4+24a^3x^3+12a^2x^2-4ax-3)\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}x}{12(ax+1)(a^2x^2-1)^2\sqrt{\frac{ax-1}{ax+1}}}$	96

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(12*a^5*x^5+12*ln(x)*x^4*a^4+24*a^3*x^3+12*a^2*x^2-4*a*x-3)*(c*(a^2*x
^2-1)/a^2/x^2)^(5/2)*x/(a*x+1)/(a^2*x^2-1)^2/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{(12a^5c^2x^5+12a^4c^2x^4\log(x)+24a^3c^2x^3+12a^2c^2x^2-4ac^2x-3c^2)\sqrt{a^2c}}{12a^6x^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="fric
as")
```

output
```
1/12*(12*a^5*c^2*x^5 + 12*a^4*c^2*x^4*log(x) + 24*a^3*c^2*x^3 + 12*a^2*c^2
*x^2 - 4*a*c^2*x - 3*c^2)*sqrt(a^2*c)/(a^6*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate((c - c/(a^2*x^2))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.45

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{1}{6}\left(\frac{12\,c^2x\mathrm{sgn}(x)}{a\mathrm{sgn}\,(ax+1)}+\frac{12\,c^2\log\left(|x|\right)\mathrm{sgn}(x)}{a^2\mathrm{sgn}\,(ax+1)}+\frac{24\,a^3c^2x^3\mathrm{sgn}(x)+12\,a^2c^2x^2\mathrm{sgn}(x)-4\,ac^2x\mathrm{s}}{a^6x^4\mathrm{sgn}\,(ax+1)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac
")
```

output
```
1/6*(12*c^2*x*sgn(x)/(a*sgn(a*x + 1)) + 12*c^2*log(abs(x))*sgn(x)/(a^2*sgn
(a*x + 1)) + (24*a^3*c^2*x^3*sgn(x) + 12*a^2*c^2*x^2*sgn(x) - 4*a*c^2*x*sg
n(x) - 3*c^2*sgn(x))/(a^6*x^4*sgn(a*x + 1)))*sqrt(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.22

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{\sqrt{c}\,c^2(12\log(x)\,a^4x^4+12a^5x^5+24a^3x^3+12a^2x^2-4ax-3)}{12a^5x^4}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x)
```

output
```
(sqrt(c)*c**2*(12*log(x)*a**4*x**4 + 12*a**5*x**5 + 24*a**3*x**3 + 12*a**2
*x**2 - 4*a*x - 3))/(12*a**5*x**4)
```

3.808 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 146

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)/x^2+c*(c-c/a^2/x^2)^(1/2
)/a^2/(1-1/a^2/x^2)^(1/2)/x+c*(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1/2)+c*
(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.42

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{3}{2a}+\frac{1}{2a^3x^2}+\frac{1}{a^2x}+x+\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2),x]
```

output
```
((c - c/(a^2*x^2))^(3/2)*(3/(2*a) + 1/(2*a^3*x^2) + 1/(a^2*x) + x + Log[x]
/a))/(1 - 1/(a^2*x^2))^(3/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.42, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6751, 6747, 25, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{3/2} e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}} \int e^{\coth^{-1}(ax)} \left(1-\frac{1}{a^2x^2}\right)^{3/2} dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}} \int -\frac{(1-ax)(ax+1)^2}{x^3} dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}} \int \frac{(1-ax)(ax+1)^2}{x^3} dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{84}$$

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}} \int \left(-a^3-\frac{a^2}{x}+\frac{a}{x^2}+\frac{1}{x^3}\right) dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}} \left(a^3(-x)-a^2\log(x)-\frac{a}{x}-\frac{1}{2x^2}\right)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2),x]
```

output
```
-((c*Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 - a/x - a^3*x - a^2*Log[x]))/(a^3*S
qrt[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\left(2 a^{3} x^{3}+2 a^{2} \ln \left(x \right) x^{2}+2 a x +1\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{3}{2}} x}{2 \left(a x +1\right) \left(a^{2} x^{2}-1\right) \sqrt{\frac{a x -1}{a x +1}}}$	80

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*a^3*x^3+2*a^2*ln(x)*x^2+2*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a
*x+1)/(a^2*x^2-1)/((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\left(2\,a^3cx^3+2\,a^2cx^2\log\left(x\right)+2\,acx+c\right)\sqrt{a^2c}}{2\,a^4x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="fric
as")
```

output
```
1/2*(2*a^3*c*x^3 + 2*a^2*c*x^2*log(x) + 2*a*c*x + c)*sqrt(a^2*c)/(a^4*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int \frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((c - c/(a^2*x^2))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.48

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\sqrt{c}\left(\frac{2\,cx\mathrm{sgn}(x)}{a\mathrm{sgn}\,(ax+1)}+\frac{2\,c\log\left(|x|\right)\mathrm{sgn}(x)}{a^2\mathrm{sgn}\,(ax+1)}+\frac{2\,acx\mathrm{sgn}(x)+c\mathrm{sgn}(x)}{a^4x^2\mathrm{sgn}\,(ax+1)}\right)|a|$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac
")
```

output
```
sqrt(c)*(2*c*x*sgn(x)/(a*sgn(a*x + 1)) + 2*c*log(abs(x))*sgn(x)/(a^2*sgn(a
*x + 1)) + (2*a*c*x*sgn(x) + c*sgn(x))/(a^4*x^2*sgn(a*x + 1)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.24

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\sqrt{c}\,c(2\log(x)\,a^2x^2+2a^3x^3+2ax+1)}{2a^3x^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x)
```

output
```
(sqrt(c)*c*(2*log(x)*a**2*x**2 + 2*a**3*x**3 + 2*a*x + 1))/(2*a**3*x**2)
```

3.809 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1/2)+(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1
/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x+\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x + Log[x]/a))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6751, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{ax+1}{x}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{(ax+\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	50

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.25

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax+\log(x))}{a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

output
```
sqrt(a^2*c)*(a*x + log(x))/a^2
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)),
x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = 2\sqrt{c}\left(\frac{x\mathrm{sgn}(x)}{a\mathrm{sgn}\,(ax+1)}+\frac{\log\left(|x|\right)\mathrm{sgn}(x)}{a^2\mathrm{sgn}\,(ax+1)}\right)|a|$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

output
```
2*sqrt(c)*(x*sgn(x)/(a*sgn(a*x + 1)) + log(abs(x))*sgn(x)/(a^2*sgn(a*x + 1
)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.18

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c}\,(\log(x)+ax)}{a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x)
```

output
```
(sqrt(c)*(log(x) + a*x))/a
```

3.810 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/(c-c/a^2/x^2)^(1/2)+(1-1/a^2/x^2)^(1/2)*ln(-a*x+1)/a
/(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(x+\frac{\log(1-ax)}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*(x + Log[1 - a*x]/a))/Sqrt[c - c/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6751, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x}{1-ax}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x}{1-ax}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \left(-\frac{1}{(ax-1)a}-\frac{1}{a}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(-\frac{\log(1-ax)}{a^2}-\frac{x}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
-((a*Sqrt[1 - 1/(a^2*x^2)]*(-(x/a) - Log[1 - a*x]/a^2))/Sqrt[c - c/(a^2*x^
2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{(ax-1)(ax+\ln(ax-1))}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,x\,a^{2}}$	57

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a^2*(a*x
+ln(a*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.33

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{a^2c}(ax+\log(ax-1))}{a^2c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

output
```
sqrt(a^2*c)*(a*x + log(a*x - 1))/(a^2*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.26

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{c}\,(\log(ax-1)+ax)}{ac}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x)
```

output
```
(sqrt(c)*(log(a*x - 1) + a*x))/(a*c)
```

3.811 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 173

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/c/(c-c/a^2/x^2)^(1/2)+1/2*(1-1/a^2/x^2)^(1/2)/a/c/(c
-c/a^2/x^2)^(1/2)/(-a*x+1)+5/4*(1-1/a^2/x^2)^(1/2)*ln(-a*x+1)/a/c/(c-c/a^2
/x^2)^(1/2)-1/4*(1-1/a^2/x^2)^(1/2)*ln(a*x+1)/a/c/(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.41

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(x+\frac{1}{2a-2a^2x}+\frac{5\log(1-ax)}{4a}-\frac{\log(1+ax)}{4a}\right)}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(3/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(3/2)*(x + (2*a - 2*a^2*x)^(-1) + (5*Log[1 - a*x])/(4*a
) - Log[1 + a*x]/(4*a)))/(c - c/(a^2*x^2))^(3/2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 6751$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \; 6747$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^3}{(1-ax)^2(ax+1)}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \; 99$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int\left(-\frac{1}{4a^3(ax+1)}+\frac{1}{a^3}+\frac{5}{4a^3(ax-1)}+\frac{1}{2a^3(ax-1)^2}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{2a^4(1-ax)}+\frac{5\log(1-ax)}{4a^4}-\frac{\log(ax+1)}{4a^4}+\frac{x}{a^3}\right)}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(3/2),x]
```

output
```
(a^3*Sqrt[1 - 1/(a^2*x^2)]*(x/a^3 + 1/(2*a^4*(1 - a*x)) + (5*Log[1 - a*x])
/(4*a^4) - Log[1 + a*x]/(4*a^4)))/(c*Sqrt[c - c/(a^2*x^2)])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.59

method	result	size
default	$-\frac{(ax-1)\left(-4a^{2}x^{2}+\ln(ax+1)xa-5a\ln(ax-1)x+4ax-\ln(ax+1)+5\ln(ax-1)+2\right)(ax+1)}{4\sqrt{\frac{ax-1}{ax+1}}\,a^{4}x^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}}$	102

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(-4*a^2*x^2+ln(a*x+1)*x*a-5*a*ln(a*x-
1)*x+4*a*x-ln(a*x+1)+5*ln(a*x-1)+2)*(a*x+1)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2
)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.39

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(4\,a^{2}x^{2}-4\,ax-(ax-1)\log(ax+1)+5\,(ax-1)\log(ax-1)-2\right)\sqrt{a^{2}c}}{4\,(a^{3}c^{2}x-a^{2}c^{2})}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fric
as")
```

output
```
1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
 - 2)*sqrt(a^2*c)/(a^3*c^2*x - a^2*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.37

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\sqrt{c}\,(5\log(ax-1)\,ax-5\log(ax-1)-\log(ax+1)\,ax+\log(ax+1)+4a^2x^2-6ax)}{4a\,c^2\,(ax-1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x)
```

output
```
(sqrt(c)*(5*log(a*x - 1)*a*x - 5*log(a*x - 1) - log(a*x + 1)*a*x + log(a*x
 + 1) + 4*a**2*x**2 - 6*a*x))/(4*a*c**2*(a*x - 1))
```

3.812 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 263

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}\\ &+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}\\ &+\frac{23\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{7\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}\end{aligned}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/c^2/(c-c/a^2/x^2)^(1/2)-1/8*(1-1/a^2/x^2)^(1/2)/a/c^
2/(c-c/a^2/x^2)^(1/2)/(-a*x+1)^2+(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(
1/2)/(-a*x+1)-1/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)/(a*x+1)+23
/16*(1-1/a^2/x^2)^(1/2)*ln(-a*x+1)/a/c^2/(c-c/a^2/x^2)^(1/2)-7/16*(1-1/a^2
/x^2)^(1/2)*ln(a*x+1)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.37

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(x-\frac{1}{8a(-1+ax)^2}+\frac{1}{a-a^2x}-\frac{1}{8a+8a^2x}+\frac{23\log(1-ax)}{16a}-\frac{7\log(1+ax)}{16a}\right)}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(5/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(5/2)*(x - 1/(8*a*(-1 + a*x)^2) + (a - a^2*x)^(-1) - (8
*a + 8*a^2*x)^(-1) + (23*Log[1 - a*x])/(16*a) - (7*Log[1 + a*x])/(16*a)))/
(c - c/(a^2*x^2))^(5/2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.43, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x^5}{(1-ax)^3(ax+1)^2}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^5}{(1-ax)^3(ax+1)^2}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{7}{16a^5(ax+1)}-\frac{1}{8a^5(ax+1)^2}-\frac{1}{a^5}-\frac{23}{16a^5(ax-1)}-\frac{1}{a^5(ax-1)^2}-\frac{1}{4a^5(ax-1)^3}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\left(-\frac{1}{a^6(1-ax)}+\frac{1}{8a^6(ax+1)}+\frac{1}{8a^6(1-ax)^2}-\frac{23\log(1-ax)}{16a^6}+\frac{7\log(ax+1)}{16a^6}-\frac{x}{a^5}\right)}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(5/2),x]
```

output
```
-((a^5*Sqrt[1 - 1/(a^2*x^2)]*(-(x/a^5) + 1/(8*a^6*(1 - a*x)^2) - 1/(a^6*(1
 - a*x)) + 1/(8*a^6*(1 + a*x)) - (23*Log[1 - a*x])/(16*a^6) + (7*Log[1 + a
*x])/(16*a^6)))/(c^2*Sqrt[c - c/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.67

method	result
default	$-\frac{(ax-1)(ax+1)\left(-16a^4x^4+7\ln(ax+1)x^3a^3-23a^3\ln(ax-1)x^3+16a^3x^3-7\ln(ax+1)x^2a^2+23a^2\ln(ax-1)x^2+34a^2x^2-7\ln(ax+1\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,a^6x^5\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(a*x+1)*(-16*a^4*x^4+7*ln(a*x+1)*x^3
*a^3-23*a^3*ln(a*x-1)*x^3+16*a^3*x^3-7*ln(a*x+1)*x^2*a^2+23*a^2*ln(a*x-1)*
x^2+34*a^2*x^2-7*ln(a*x+1)*x*a+23*a*ln(a*x-1)*x-18*a*x+7*ln(a*x+1)-23*ln(a
*x-1)-12)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.52

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(16\,a^4x^4-16\,a^3x^3-34\,a^2x^2+18\,ax-7\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)+23\left(\right.\right.}{16\left(a^5c^3x^3-a^4c^3x^2-a^3c^3x+a^2c^3\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fric
as")
```

output
```
1/16*(16*a^4*x^4 - 16*a^3*x^3 - 34*a^2*x^2 + 18*a*x - 7*(a^3*x^3 - a^2*x^2
 - a*x + 1)*log(a*x + 1) + 23*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) +
 12)*sqrt(a^2*c)/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.56

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{\sqrt{c}\,(23\log(ax-1)\,a^3x^3-23\log(ax-1)\,a^2x^2-23\log(ax-1)\,ax+23\log(ax-1)}{16}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x)
```

output
```
(sqrt(c)*(23*log(a*x - 1)*a**3*x**3 - 23*log(a*x - 1)*a**2*x**2 - 23*log(a
*x - 1)*a*x + 23*log(a*x - 1) - 7*log(a*x + 1)*a**3*x**3 + 7*log(a*x + 1)*
a**2*x**2 + 7*log(a*x + 1)*a*x - 7*log(a*x + 1) + 16*a**4*x**4 - 50*a**3*x
**3 + 52*a*x - 22))/(16*a*c**3*(a**3*x**3 - a**2*x**2 - a*x + 1))
```

3.813 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 359

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}} dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^3}\\ &-\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}\\ &-\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{51\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{19\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}\end{aligned}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/c^3/(c-c/a^2/x^2)^(1/2)+1/24*(1-1/a^2/x^2)^(1/2)/a/c
^3/(c-c/a^2/x^2)^(1/2)/(-a*x+1)^3-11/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2
/x^2)^(1/2)/(-a*x+1)^2+3/2*(1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)/(
-a*x+1)+1/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)/(a*x+1)^2-5/16*
(1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)/(a*x+1)+51/32*(1-1/a^2/x^2)^
(1/2)*ln(-a*x+1)/a/c^3/(c-c/a^2/x^2)^(1/2)-19/32*(1-1/a^2/x^2)^(1/2)*ln(a*
x+1)/a/c^3/(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.33

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(96x-\frac{4}{a(-1+ax)^3}-\frac{33}{a(-1+ax)^2}+\frac{3}{a(1+ax)^2}+\frac{144}{a-a^2x}-\frac{30}{a+a^2x}+\frac{153\log(1-ax)}{a}\right.}{96\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

input
```
Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(7/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(7/2)*(96*x - 4/(a*(-1 + a*x)^3) - 33/(a*(-1 + a*x)^2)
+ 3/(a*(1 + a*x)^2) + 144/(a - a^2*x) - 30/(a + a^2*x) + (153*Log[1 - a*x]
)/a - (57*Log[1 + a*x])/a))/(96*(c - c/(a^2*x^2))^(7/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^7}{(1-ax)^4(ax+1)^3}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int\left(-\frac{19}{32a^7(ax+1)}+\frac{5}{16a^7(ax+1)^2}-\frac{1}{16a^7(ax+1)^3}+\frac{1}{a^7}+\frac{51}{32a^7(ax-1)}+\frac{3}{2a^7(ax-1)^2}+\frac{11}{16a^7(ax-1)^3}+\frac{1}{8a^7(ax-1)}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{3}{2a^8(1-ax)}-\frac{5}{16a^8(ax+1)}-\frac{11}{32a^8(1-ax)^2}+\frac{1}{32a^8(ax+1)^2}+\frac{1}{24a^8(1-ax)^3}+\frac{51\log(1-ax)}{32a^8}-\frac{19\log(ax+1)}{32a^8}+\frac{x}{a^7}\right)}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(7/2),x]
```

output
```
(a^7*Sqrt[1 - 1/(a^2*x^2)]*(x/a^7 + 1/(24*a^8*(1 - a*x)^3) - 11/(32*a^8*(1
 - a*x)^2) + 3/(2*a^8*(1 - a*x)) + 1/(32*a^8*(1 + a*x)^2) - 5/(16*a^8*(1 +
 a*x)) + (51*Log[1 - a*x])/(32*a^8) - (19*Log[1 + a*x])/(32*a^8)))/(c^3*Sq
rt[c - c/(a^2*x^2)])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.69

method	result
default	$-\frac{(ax-1)(ax+1)(-96x^6a^6+57\ln(ax+1)x^5a^5-153\ln(ax-1)x^5a^5+96a^5x^5-57\ln(ax+1)x^4a^4+153\ln(ax-1)x^4a^4+366a^4x^4-114}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(a*x+1)*(-96*x^6*a^6+57*ln(a*x+1)*x^
5*a^5-153*ln(a*x-1)*x^5*a^5+96*a^5*x^5-57*ln(a*x+1)*x^4*a^4+153*ln(a*x-1)*
x^4*a^4+366*a^4*x^4-114*ln(a*x+1)*x^3*a^3+306*a^3*ln(a*x-1)*x^3-222*a^3*x^
3+114*ln(a*x+1)*x^2*a^2-306*a^2*ln(a*x-1)*x^2-338*a^2*x^2+57*ln(a*x+1)*x*a
-153*a*ln(a*x-1)*x+122*a*x-57*ln(a*x+1)+153*ln(a*x-1)+88)/a^8/x^7/(c*(a^2*
x^2-1)/a^2/x^2)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.57

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\frac{(96\,a^6x^6-96\,a^5x^5-366\,a^4x^4+222\,a^3x^3+338\,a^2x^2-122\,ax-57\,(a^5x^5-a^4x^4-2}{96\,(a^7c^4x^5-a^6c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fric
as")
```

output
```
1/96*(96*a^6*x^6 - 96*a^5*x^5 - 366*a^4*x^4 + 222*a^3*x^3 + 338*a^2*x^2 -
122*a*x - 57*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x
 + 1) + 153*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x
- 1) - 88)*sqrt(a^2*c)/(a^7*c^4*x^5 - a^6*c^4*x^4 - 2*a^5*c^4*x^3 + 2*a^4*
c^4*x^2 + a^3*c^4*x - a^2*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.65

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{c}\,(153\log(ax-1)\,a^5x^5 - 153\log(ax-1)\,a^4x^4 - 306\log(ax-1)\,a^3x^3 + 306\log(a}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x)
```

output
```
(sqrt(c)*(153*log(a*x - 1)*a**5*x**5 - 153*log(a*x - 1)*a**4*x**4 - 306*lo
g(a*x - 1)*a**3*x**3 + 306*log(a*x - 1)*a**2*x**2 + 153*log(a*x - 1)*a*x -
 153*log(a*x - 1) - 57*log(a*x + 1)*a**5*x**5 + 57*log(a*x + 1)*a**4*x**4
+ 114*log(a*x + 1)*a**3*x**3 - 114*log(a*x + 1)*a**2*x**2 - 57*log(a*x + 1
)*a*x + 57*log(a*x + 1) + 96*a**6*x**6 - 462*a**5*x**5 + 954*a**3*x**3 - 3
94*a**2*x**2 - 488*a*x + 278))/(96*a*c**4*(a**5*x**5 - a**4*x**4 - 2*a**3*
x**3 + 2*a**2*x**2 + a*x - 1))
```

3.814 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 190

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=-\frac{c\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(12a-\frac{25}{x}\right)}{30a^2}-\frac{c^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(16a-\frac{25}{x}\right)}{24a^2}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(32a-\frac{25}{x}\right)}{16a^2}+\left(c-\frac{c}{a^2x^2}\right)^{7/2}x+\frac{25c^{7/2}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\right)}{16a}+\frac{2c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{a}$$

output
```
-1/30*c*(c-c/a^2/x^2)^(5/2)*(12*a-25/x)/a^2-1/24*c^2*(c-c/a^2/x^2)^(3/2)*(
16*a-25/x)/a^2-1/16*c^3*(c-c/a^2/x^2)^(1/2)*(32*a-25/x)/a^2+(c-c/a^2/x^2)^
(7/2)*x+25/16*c^(7/2)*arctan(c^(1/2)/a/(c-c/a^2/x^2)^(1/2)/x)/a+2*c^(7/2)*
arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.79

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(-40-96ax+70a^2x^2+352a^3x^3+105a^4x^4-736a^5x^5+240a^6x^6)\right.}{240a^6x^5\sqrt{-1+a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

output
```
(c^3*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-40 - 96*a*x + 70*a^2*x^2
+ 352*a^3*x^3 + 105*a^4*x^4 - 736*a^5*x^5 + 240*a^6*x^6) + 375*a^6*x^6*Arc
Tan[1/Sqrt[-1 + a^2*x^2]] + 480*a^6*x^6*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(2
40*a^6*x^5*Sqrt[-1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6717, 6709, 540, 25, 27, 537, 25, 537, 27, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{7/2}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\int \frac{(ax+1)^2(1-a^2x^2)^{5/2}}{x^7}dx}{(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}\int -\frac{a(5ax+12)\left(1-a^2x^2\right)^{5/2}}{x^6}dx-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6}\int \frac{a(5ax+12)\left(1-a^2x^2\right)^{5/2}}{x^6}dx-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6}a\int \frac{(5ax+12)\left(1-a^2x^2\right)^{5/2}}{x^6}dx-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \text{537}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6}a\left(\frac{1}{4}a^2\int -\frac{(25ax+48)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{(25ax+48)\left(1-a^2x^2\right)^{5/2}}{20x^5}\right)-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6}a\left(-\frac{1}{4}a^2\int \frac{(25ax+48)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{(25ax+48)\left(1-a^2x^2\right)^{5/2}}{20x^5}\right)-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \text{537}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6}a\left(-\frac{1}{4}a^2\left(\frac{1}{2}a^2\int -\frac{3(25ax+32)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(25ax+32)\left(1-a^2x^2\right)^{3/2}}{2x^3}\right)-\frac{(25ax+48)\left(1-a^2x^2\right)^{5/2}}{20x^5}\right)-(1\ldots}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6}a\left(-\frac{1}{4}a^2\left(-\frac{3}{2}a^2\int \frac{(25ax+32)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(25ax+32)\left(1-a^2x^2\right)^{3/2}}{2x^3}\right)-\frac{(25ax+48)\left(1-a^2x^2\right)^{5/2}}{20x^5}\right)-(1-\ldots}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \text{536}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6}a\left(-\frac{1}{4}a^2\left(-\frac{3}{2}a^2\left(\int \frac{25a-32a^2x}{x\sqrt{1-a^2x^2}}dx-\frac{(32-25ax)\sqrt{1-a^2x^2}}{x}\right)-\frac{(25ax+32)\left(1-a^2x^2\right)^{3/2}}{2x^3}\right)-\frac{(25ax+48)(1-\ldots}{20x^5}\ldots}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \text{538}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6}a\left(-\frac{1}{4}a^2\left(-\frac{3}{2}a^2\left(-32a^2\int\frac{1}{\sqrt{1-a^2x^2}}dx+25a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{(32-25ax)\sqrt{1-a^2x^2}}{x}\right)-\frac{(25ax+32)(\cdots}{2a\cdots}\right.\right.\right.}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6}a\left(-\frac{1}{4}a^2\left(-\frac{3}{2}a^2\left(25a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}(32-25ax)}{x}-32a\arcsin(ax)\right)-\frac{(25ax+32)(1-a^2x\cdots}{2x^3}\right.\right.\right.}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6}a\left(-\frac{1}{4}a^2\left(-\frac{3}{2}a^2\left(\frac{25}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}(32-25ax)}{x}-32a\arcsin(ax)\right)-\frac{(25ax+32)(1-a\cdots}{2x^3}\right.\right.\right.}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6}a\left(-\frac{1}{4}a^2\left(-\frac{3}{2}a^2\left(-\frac{25\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}(32-25ax)}{x}-32a\arcsin(ax)\right)-\frac{(25ax+32)(\cdots}{2\cdots}\right.\right.\right.}{\left(1-a^2x^2\right)^{7/2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6}a\left(-\frac{1}{4}a^2\left(-\frac{3}{2}a^2\left(-25a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}(32-25ax)}{x}-32a\arcsin(ax)\right)-\frac{(25ax+\cdots}{\cdots}\right.\right.\right.}{\left(1-a^2x^2\right)^{7/2}}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

output
```
-(((c - c/(a^2*x^2))^(7/2)*x^7*(-1/6*(1 - a^2*x^2)^(7/2)/x^6 + (a*(-1/20*(
(48 + 25*a*x)*(1 - a^2*x^2)^(5/2))/x^5 - (a^2*(-1/2*((32 + 25*a*x)*(1 - a^
2*x^2)^(3/2))/x^3 - (3*a^2*(-(((32 - 25*a*x)*Sqrt[1 - a^2*x^2])/x) - 32*a*
ArcSin[a*x] - 25*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2))/4))/6))/(1 - a^2*x^2)^
(7/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{\left(736 a^{7} x^{7}-105 x^{6} a^{6}-1088 a^{5} x^{5}+35 a^{4} x^{4}+448 a^{3} x^{3}+110 a^{2} x^{2}-96 a x-40\right) c^{3} \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}{240 x^{5} a^{6} \left(a^{2} x^{2}-1\right)}+\frac{\left(\frac{25 a^{6} \ln\left(\frac{-2 c+2 \sqrt{-c}\, \sqrt{a^{2} c\, x^{2}-c}}{x}\right)}{16 \sqrt{-c}}\right.}{}$
default	$-\frac{\left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{7}{2}} x \left(-2016 \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{7}{2}} \sqrt{-\frac{c}{a^{2}}}\, a^{9} c\, x^{7}+2016 \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{9}{2}} \sqrt{-\frac{c}{a^{2}}}\, a^{9} x^{5}+375 \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{7}{2}} \sqrt{-\frac{c}{a^{2}}}\, a^{8} c\, x\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(736*a^7*x^7-105*a^6*x^6-1088*a^5*x^5+35*a^4*x^4+448*a^3*x^3+110*a^
2*x^2-96*a*x-40)/x^5*c^3/a^6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)+(25
/16*a^6/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)+2*a^7*ln(
a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+a^6/c*(c*(a^2*x^2
-1))^(1/2))*c^3/a^6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)*x*(c*(a^2*x^
2-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.24

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \left[-\frac{960\,a^5\sqrt{-c}c^3x^5\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-375\,a^5\sqrt{-c}c^3x^5\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{x^2}\right.}{}\right.$$

$$-\frac{375\,a^5c^{\frac{7}{2}}x^5\arctan\left(\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\sqrt{c}}\right)-240\,a^5c^{\frac{7}{2}}x^5\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-(240\,a^6c^3x^6-7}{240\,a^6x^5}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

output
```
[-1/480*(960*a^5*sqrt(-c)*c^3*x^5*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2
- c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 375*a^5*sqrt(-c)*c^3*x^5*log(-(a^2*c*x^
2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(240*a^
6*c^3*x^6 - 736*a^5*c^3*x^5 + 105*a^4*c^3*x^4 + 352*a^3*c^3*x^3 + 70*a^2*c
^3*x^2 - 96*a*c^3*x - 40*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5),
-1/240*(375*a^5*c^(7/2)*x^5*arctan(a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/sqr
t(c)) - 240*a^5*c^(7/2)*x^5*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*
c*x^2 - c)/(a^2*x^2)) - c) - (240*a^6*c^3*x^6 - 736*a^5*c^3*x^5 + 105*a^4*
c^3*x^4 + 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 - 96*a*c^3*x - 40*c^3)*sqrt((a^
2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.38 (sec) , antiderivative size = 1059, normalized size of antiderivative = 5.57

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(7/2),x)
```

output
```
c**3*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*s
qrt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1)
, (I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*s
qrt(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**3*Piecewise((-a*sqrt
(c)*x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x*
*2 - 1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqr
t(c)*asin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - c**3*Pie
cewise((I*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x*
*2))) - I*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2)
 > 1), (-a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x)
, True))/a**2 - 4*c**3*Piecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))*
*(3/2)/(3*c), True))/a**3 - c**3*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/
8 - I*a**2*sqrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sq
rt(-1 + 1/(a**2*x**2))) - I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2)))
, 1/Abs(a**2*x**2) > 1), (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*
x*sqrt(1 - 1/(a**2*x**2))) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) +
sqrt(c)/(4*a**2*x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**4 + 2*c**3*Piecew
ise((2*a**3*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x) + a*sqrt(c)*sqrt(a**2*x**2
- 1)/(15*x**3) - sqrt(c)*sqrt(a**2*x**2 - 1)/(5*a*x**5), Abs(a**2*x**2) >
1), (2*I*a**3*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x) + I*a*sqrt(c)*sqrt(-a...
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\int \frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c - c/(a^2*x^2))^(7/2)/(a*x - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(162) = 324$.

Time = 0.28 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=$$
$$-\frac{1}{120}\left(\frac{375\,c^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}+\frac{240\,c^{\frac{7}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}-\frac{120\sqrt{}\cdots}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

output
```
-1/120*(375*c^(7/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x)/a^2 + 240*c^(7/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*s
gn(x)/(a*abs(a)) - 120*sqrt(a^2*c*x^2 - c)*c^3*sgn(x)/a^2 + (105*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^11*c^4*abs(a)*sgn(x) + 1440*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^10*a*c^(9/2)*sgn(x) + 595*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))^9*c^5*abs(a)*sgn(x) + 4320*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c)
)^8*a*c^(11/2)*sgn(x) - 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^6*ab
s(a)*sgn(x) + 7360*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(13/2)*sgn(
x) + 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^7*abs(a)*sgn(x) + 6720*
(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(15/2)*sgn(x) - 595*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - c))^3*c^8*abs(a)*sgn(x) + 2976*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*a*c^(17/2)*sgn(x) - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x
^2 - c))*c^9*abs(a)*sgn(x) + 736*a*c^(19/2)*sgn(x))/(((sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^2 + c)^6*a^2*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(7/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(7/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.97

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\frac{\sqrt{c}\,c^3\left(-750atan\left(\sqrt{a^2x^2-1}+ax\right)a^6x^6+240\sqrt{a^2x^2-1}\,a^6x^6-736\sqrt{a^2x^2-1}\,a^5x^5+10\right.}{}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x)
```

output
```
(sqrt(c)*c**3*( - 750*atan(sqrt(a**2*x**2 - 1) + a*x)*a**6*x**6 + 240*sqrt
(a**2*x**2 - 1)*a**6*x**6 - 736*sqrt(a**2*x**2 - 1)*a**5*x**5 + 105*sqrt(a
**2*x**2 - 1)*a**4*x**4 + 352*sqrt(a**2*x**2 - 1)*a**3*x**3 + 70*sqrt(a**2
*x**2 - 1)*a**2*x**2 - 96*sqrt(a**2*x**2 - 1)*a*x - 40*sqrt(a**2*x**2 - 1)
 + 480*log(sqrt(a**2*x**2 - 1) + a*x)*a**6*x**6 + 256*a**6*x**6))/(240*a**
7*x**6)
```

3.815 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=-\frac{c\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(8a-\frac{9}{x}\right)}{12a^2}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(16a-\frac{9}{x}\right)}{8a^2}+\left(c-\frac{c}{a^2x^2}\right)^{5/2}x+\frac{9c^{5/2}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\right)}{8a}+\frac{2c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{a}$$

output
```
-1/12*c*(c-c/a^2/x^2)^(3/2)*(8*a-9/x)/a^2-1/8*c^2*(c-c/a^2/x^2)^(1/2)*(16*
a-9/x)/a^2+(c-c/a^2/x^2)^(5/2)*x+9/8*c^(5/2)*arctan(c^(1/2)/a/(c-c/a^2/x^2
)^(1/2)/x)/a+2*c^(5/2)*arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(6+16ax-3a^2x^2-64a^3x^3+24a^4x^4)+27a^4x^4\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+48a^4x^4\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{24a^4x^3\sqrt{-1+a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

output
```
(c^2*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 + 16*a*x - 3*a^2*x^2 - 6
4*a^3*x^3 + 24*a^4*x^4) + 27*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 48*a^4
*x^4*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^4*x^3*Sqrt[-1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.542$, Rules used = {6717, 6709, 540, 25, 27, 537, 25, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int\left(c-\frac{c}{a^2x^2}\right)^{5/2}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\int\frac{(ax+1)^2\left(1-a^2x^2\right)^{3/2}}{x^5}dx}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}\int -\frac{a(3ax+8)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}\int \frac{a(3ax+8)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}a\int \frac{(3ax+8)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 537$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}a\left(\frac{1}{2}a^2\int -\frac{(9ax+16)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(9ax+16)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}a\left(-\frac{1}{2}a^2\int \frac{(9ax+16)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(9ax+16)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 536$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}a\left(-\frac{1}{2}a^2\left(\int \frac{9a-16a^2x}{x\sqrt{1-a^2x^2}}dx-\frac{(16-9ax)\sqrt{1-a^2x^2}}{x}\right)-\frac{(9ax+16)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 538$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}a\left(-\frac{1}{2}a^2\left(-16a^2\int \frac{1}{\sqrt{1-a^2x^2}}dx+9a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{(16-9ax)\sqrt{1-a^2x^2}}{x}\right)-\frac{(9ax+16)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right.\right.}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \ 223$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}a\left(-\frac{1}{2}a^2\left(9a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}(16-9ax)}{x}-16a\arcsin(ax)\right)-\frac{(9ax+16)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)-\right.}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow 243$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}a\left(-\frac{1}{2}a^2\left(\frac{9}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}(16-9ax)}{x}-16a\arcsin(ax)\right)-\frac{(9ax+16)(1-a^2x^2)^{3/2}}{6x^3}\right)-\right.}{(1-a^2x^2)^{5/2}}$$

$$\Big\downarrow 73$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}a\left(-\frac{1}{2}a^2\left(-\frac{9\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}(16-9ax)}{x}-16a\arcsin(ax)\right)-\frac{(9ax+16)(1-a^2x^2)^{3/2}}{6x^3}\right.\right.}{(1-a^2x^2)^{5/2}}$$

$$\Big\downarrow 221$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4}a\left(-\frac{1}{2}a^2\left(-9a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}(16-9ax)}{x}-16a\arcsin(ax)\right)-\frac{(9ax+16)(1-a^2x^2)}{6x^3}\right.\right.}{(1-a^2x^2)^{5/2}}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

output
```
-(((c - c/(a^2*x^2))^(5/2)*x^5*(-1/4*(1 - a^2*x^2)^(5/2)/x^4 + (a*(-1/6*((
16 + 9*a*x)*(1 - a^2*x^2)^(3/2))/x^3 - (a^2*(-(((16 - 9*a*x)*Sqrt[1 - a^2*
x^2])/x) - 16*a*ArcSin[a*x] - 9*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2))/4))/(1
- a^2*x^2)^(5/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.50

method	result
risch	$-\frac{(64a^5x^5+3a^4x^4-80a^3x^3-9a^2x^2+16ax+6)c^2\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{24x^3a^4(a^2x^2-1)}+\frac{\left(\frac{9a^4\ln\left(\frac{-2c+2\sqrt{-c}\sqrt{a^2c\,x^2-c}}{x}\right)}{8\sqrt{-c}}+\frac{2a^5\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}+\right.}{a^4(a^2x^2-1)}$
default	$\frac{\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{5}{2}}x\left(-80\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{5}{2}}a^7cx^5+80\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{7}{2}}a^7x^3+48\sqrt{-\frac{c}{a^2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{5}{2}}a^6cx^4+27\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(64*a^5*x^5+3*a^4*x^4-80*a^3*x^3-9*a^2*x^2+16*a*x+6)/x^3*c^2/a^4*(c*
(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)+(9/8*a^4/(-c)^(1/2)*ln((-2*c+2*(-c)
^(1/2)*(a^2*c*x^2-c)^(1/2))/x)+2*a^5*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c
)^(1/2))/(a^2*c)^(1/2)+a^4/c*(c*(a^2*x^2-1))^(1/2))*c^2/a^4*(c*(a^2*x^2-1)
/a^2/x^2)^(1/2)/(a^2*x^2-1)*x*(c*(a^2*x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.44

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \left[-\frac{96\,a^3\sqrt{-c}c^2x^3\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-27\,a^3\sqrt{-c}c^2x^3\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{x^2}\right.}{48\,a^4x^3}\right.$$

$$-\frac{27\,a^3c^{\frac{5}{2}}x^3\arctan\left(\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\sqrt{c}}\right)-24\,a^3c^{\frac{5}{2}}x^3\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-(24\,a^4c^2x^4-64\,a^3}{24\,a^4x^3}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/48*(96*a^3*sqrt(-c)*c^2*x^3*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 27*a^3*sqrt(-c)*c^2*x^3*log(-(a^2*c*x^2 -
 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(24*a^4*c^
2*x^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x
^2 - c)/(a^2*x^2)))/(a^4*x^3), -1/24*(27*a^3*c^(5/2)*x^3*arctan(a*x*sqrt((
a^2*c*x^2 - c)/(a^2*x^2))/sqrt(c)) - 24*a^3*c^(5/2)*x^3*log(2*a^2*c*x^2 +
2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) - (24*a^4*c^2*x^4 -
 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)))/(a^4*x^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.64 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.21

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = c^2\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}+\frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } |a^2x^2|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right) + \frac{2c^2\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}(ax)+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{for } |a^2x^2|>1\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}(ax)-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}\right)}{a} - \frac{2c^2\left(\begin{cases}0 & \text{for } c=0\\ \frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)}{a^3} - \frac{c^2\left(\begin{cases}\frac{ia^3\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{8}-\frac{ia^2\sqrt{c}}{8x\sqrt{-1+\frac{1}{a^2x^2}}}+\frac{3i\sqrt{c}}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{i\sqrt{c}}{4a^2x^5\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{a^3\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{8}+\frac{a^2\sqrt{c}}{8x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c}}{8x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c}}{4a^2x^5\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)}{a^4}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(5/2),x)
```

output
```
c**2*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*s
qrt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1)
, (I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*s
qrt(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**2*Piecewise((-a*sqrt
(c)*x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x*
*2 - 1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqr
t(c)*asin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - 2*c**2*P
iecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c), True))/a**
3 - c**2*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*sqrt(c)/(8*x*
sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2)))
- I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1),
 (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2)
)) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*x**5*sqr
t(1 - 1/(a**2*x**2))), True))/a**4
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c - c/(a^2*x^2))^(5/2)/(a*x - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(132) = 264$.

Time = 0.22 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.67

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=$$

$$-\frac{1}{12}\left(\frac{27\,c^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}+\frac{24\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{12\,\sqrt{a^2cx^2}}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

output
```
-1/12*(27*c^(5/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*s
gn(x)/a^2 + 24*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(
x)/(a*abs(a)) - 12*sqrt(a^2*c*x^2 - c)*c^2*sgn(x)/a^2 - (3*(sqrt(a^2*c)*x
- sqrt(a^2*c*x^2 - c))^7*c^3*abs(a)*sgn(x) - 96*(sqrt(a^2*c)*x - sqrt(a^2*
c*x^2 - c))^6*a*c^(7/2)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
5*c^4*abs(a)*sgn(x) - 192*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(9/2
)*sgn(x) + 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*c^5*abs(a)*sgn(x) -
160*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(11/2)*sgn(x) - 3*(sqrt(a^
2*c)*x - sqrt(a^2*c*x^2 - c))*c^6*abs(a)*sgn(x) - 64*a*c^(13/2)*sgn(x))/((
(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^4*a^2*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(5/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(5/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{\sqrt{c}\,c^2\left(-54atan\left(\sqrt{a^2x^2-1}+ax\right)a^4x^4+24\sqrt{a^2x^2-1}\,a^4x^4-64\sqrt{a^2x^2-1}\,a^3x^3-3\sqrt{a^2}\cdots\right.}{24a^5\cdots}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x)
```

output
```
(sqrt(c)*c**2*( - 54*atan(sqrt(a**2*x**2 - 1) + a*x)*a**4*x**4 + 24*sqrt(a
**2*x**2 - 1)*a**4*x**4 - 64*sqrt(a**2*x**2 - 1)*a**3*x**3 - 3*sqrt(a**2*x
**2 - 1)*a**2*x**2 + 16*sqrt(a**2*x**2 - 1)*a*x + 6*sqrt(a**2*x**2 - 1) +
48*log(sqrt(a**2*x**2 - 1) + a*x)*a**4*x**4 + 16*a**4*x**4))/(24*a**5*x**4
)
```

3.816 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 122

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(4a-\frac{1}{x}\right)}{2a^2}+\left(c-\frac{c}{a^2x^2}\right)^{3/2}x+\frac{c^{3/2}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\right)}{2a}+\frac{2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{a}$$

output
```
-1/2*c*(c-c/a^2/x^2)^(1/2)*(4*a-1/x)/a^2+(c-c/a^2/x^2)^(3/2)*x+1/2*c^(3/2)
*arctan(c^(1/2)/a/(c-c/a^2/x^2)^(1/2)/x)/a+2*c^(3/2)*arctanh((c-c/a^2/x^2)
^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(-1-4ax+2a^2x^2)+a^2x^2\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+4a^2x^2\log\left(ax\right.\right.}{2a^2x\sqrt{-1+a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

output
```
(c*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-1 - 4*a*x + 2*a^2*x^2) + a^
2*x^2*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 4*a^2*x^2*Log[a*x + Sqrt[-1 + a^2*x^2
]]))/(2*a^2*x*Sqrt[-1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6717, 6709, 540, 25, 27, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{3/2} e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\int \frac{(ax+1)^2\sqrt{1-a^2x^2}}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}\int -\frac{a(ax+4)\sqrt{1-a^2x^2}}{x^2}dx-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2}\int \frac{a(ax+4)\sqrt{1-a^2x^2}}{x^2}dx-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2}a\int\frac{(ax+4)\sqrt{1-a^2x^2}}{x^2}dx-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{536}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2}a\left(\int\frac{a-4a^2x}{x\sqrt{1-a^2x^2}}dx-\frac{(4-ax)\sqrt{1-a^2x^2}}{x}\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{538}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2}a\left(-4a^2\int\frac{1}{\sqrt{1-a^2x^2}}dx+a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{(4-ax)\sqrt{1-a^2x^2}}{x}\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2}a\left(a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}(4-ax)}{x}-4a\arcsin(ax)\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2}a\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}(4-ax)}{x}-4a\arcsin(ax)\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2}a\left(-\frac{\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}(4-ax)}{x}-4a\arcsin(ax)\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2}a\left(-a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}(4-ax)}{x}-4a\arcsin(ax)\right)-\frac{\left(1-a^2x^2\right)^{3/2}}{2x^2}\right)}{\left(1-a^2x^2\right)^{3/2}}$$

input
```
Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

output
```
-(((c - c/(a^2*x^2))^(3/2)*x^3*(-1/2*(1 - a^2*x^2)^(3/2)/x^2 + (a*(-(((4 -
 a*x)*Sqrt[1 - a^2*x^2])/x) - 4*a*ArcSin[a*x] - a*ArcTanh[Sqrt[1 - a^2*x^2
]]))/2))/(1 - a^2*x^2)^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.66

method	result
risch	$\frac{\left(2 a^{4} x^{4}-4 a^{3} x^{3}-3 a^{2} x^{2}+4 a x +1\right) c \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}{2 x \,a^{2} \left(a^{2} x^{2}-1\right)}+\frac{\left(\frac{a^{2} \ln \left(\frac{-2 c +2 \sqrt{-c}\, \sqrt{a^{2} c \,x^{2}-c}}{x}\right)}{2 \sqrt{-c}}+\frac{2 a^{3} \ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}-c}\right)}{\sqrt{a^{2} c}}\right) c \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x}{a^{2} \left(a^{2} x^{2}-1\right)}$
default	$-\frac{\left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{3}{2}} x \left(-12 \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} \sqrt{-\frac{c}{a^{2}}}\, a^{5} c \,x^{3}+12 \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{5}{2}} \sqrt{-\frac{c}{a^{2}}}\, a^{5} x +\left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} \sqrt{-\frac{c}{a^{2}}}\, a^{4} c \,x^{2}-4 \sqrt{-\frac{c}{a^{2}}}\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*a^4*x^4-4*a^3*x^3-3*a^2*x^2+4*a*x+1)/x*c/a^2*(c*(a^2*x^2-1)/a^2/x^2
)^(1/2)/(a^2*x^2-1)+(1/2*a^2/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c
)^(1/2))/x)+2*a^3*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1
/2))*c/a^2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-
1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.49

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\left[-\frac{8\,a\sqrt{-c}cx\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-a\sqrt{-c}cx\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2}{4\,a^2x}\right.$$

$$-\frac{ac^{\frac{3}{2}}x\arctan\left(\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\sqrt{c}}\right)-2\,ac^{\frac{3}{2}}x\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-(2\,a^2cx^2-4\,acx-c)\sqrt{\frac{a^2}{c}}}{2\,a^2x}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(8*a*sqrt(-c)*c*x*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*
x^2))/(a^2*c*x^2 - c)) - a*sqrt(-c)*c*x*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(2*a^2*c*x^2 - 4*a*c*x - c)
*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x), -1/2*(a*c^(3/2)*x*arctan(a*x*sq
rt((a^2*c*x^2 - c)/(a^2*x^2))/sqrt(c)) - 2*a*c^(3/2)*x*log(2*a^2*c*x^2 + 2
*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) - (2*a^2*c*x^2 - 4*a
*c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.21 (sec) , antiderivative size = 376, normalized size of antiderivative = 3.08

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = c\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}+\frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } |a^2x^2|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right) + \frac{2c\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}(ax)+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{for } |a^2x^2|>1\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}(ax)-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}\right)}{a} + \frac{c\left(\begin{cases}\frac{ia\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}+\frac{i\sqrt{c}}{2x\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{i\sqrt{c}}{2a^2x^3\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{a\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)}{a^2}$$

input

```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(3/2),x)
```

output

```
c*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sqrt
(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1), (
I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c*Piecewise((-a*sqrt(c)*x/
sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 - 1
)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + c*Piecewise((I
*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - I
*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-
a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/
a**2
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\int\frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(c - c/(a^2*x^2))^(3/2)/(a*x - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(102) = 204$.

Time = 0.19 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.18

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=$$
$$-\left(\frac{c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}+\frac{2\,c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{\sqrt{a^2cx^2-c}c\operatorname{sgn}(}{a^2}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

output
```
-(c^(3/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^
2 + 2*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs
(a)) - sqrt(a^2*c*x^2 - c)*c*sgn(x)/a^2 - ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - c))^3*c^2*abs(a)*sgn(x) - 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c
^(5/2)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^3*abs(a)*sgn(x) -
4*a*c^(7/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^2*a^2*a
bs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(3/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(3/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int e^{2\coth^{-1}(ax)}\Big(c-\frac{c}{a^2x^2}\Big)^{3/2}\,dx=\frac{\sqrt{c}\,c\left(-2atan\left(\sqrt{a^2x^2-1}+ax\right)a^2x^2+2\sqrt{a^2x^2-1}\,a^2x^2-4\sqrt{a^2x^2-1}\,ax-\sqrt{a^2x^2-1}-\cdots\right.}{2a^3x^2}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x)
```

output
```
(sqrt(c)*c*( - 2*atan(sqrt(a**2*x**2 - 1) + a*x)*a**2*x**2 + 2*sqrt(a**2*x
**2 - 1)*a**2*x**2 - 4*sqrt(a**2*x**2 - 1)*a*x - sqrt(a**2*x**2 - 1) + 4*l
og(sqrt(a**2*x**2 - 1) + a*x)*a**2*x**2))/(2*a**3*x**2)
```

3.817 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\sqrt{c-\frac{c}{a^2x^2}}x-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}x}\right)}{a}+\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{a}$$

output
```
(c-c/a^2/x^2)^(1/2)*x-c^(1/2)*arctan(c^(1/2)/a/(c-c/a^2/x^2)^(1/2)/x)/a+2*
c^(1/2)*arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}-\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]
] + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6717, 6709, 541, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{541}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int -\frac{a^2(2ax+1)}{x\sqrt{1-a^2x^2}}dx}{a^2}-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{\int \frac{a^2(2ax+1)}{x\sqrt{1-a^2x^2}}dx}{a^2}-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{2ax+1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 538}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(2a\int \frac{1}{\sqrt{1-a^2x^2}}dx+\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}+2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\sqrt{1-a^2x^2}+2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a^2}-\sqrt{1-a^2x^2}+2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\sqrt{1-a^2x^2}+2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-Sqrt[1 - a^2*x^2] + 2*ArcSin[a*x] - ArcTanh[S
qrt[1 - a^2*x^2]]))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(74) = 148$.

Time = 0.21 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.24

method	result
default	$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(2\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}+2\sqrt{c}\,\ln\left(\frac{\sqrt{c}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+xc}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^2}}-\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2-c\ln\left(\frac{2\sqrt{-\frac{c}{a^2}} \cdots\right.\right.}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)*a^2*(-c/a
^2)^(1/2)+2*c^(1/2)*ln((c^(1/2)*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)+x*c)/c^(1/2)
)*a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-
c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^
(1/2)/a^2/(-c/a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.86

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-4\,\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a}, \frac{ax\sqrt{\frac{a^2cx^2-}{a^2x^2}}}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 4*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a
^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a,
(a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + sqrt(c)*arctan(a*x*sqrt((a^2*c*x^2
- c)/(a^2*x^2))/sqrt(c)) + sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/(a*x - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.58

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c}\left(2atan\left(\sqrt{a^2x^2-1}+ax\right)+\sqrt{a^2x^2-1}+2\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{a}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x)
```

output
```
(sqrt(c)*(2*atan(sqrt(a**2*x**2 - 1) + a*x) + sqrt(a**2*x**2 - 1) + 2*log(
sqrt(a**2*x**2 - 1) + a*x)))/a
```

3.818 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 78

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=-\frac{2\left(a+\frac{1}{x}\right)}{a^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{c}+\frac{2\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

output
```
(-2*a-2/x)/a^2/(c-c/a^2/x^2)^(1/2)+(c-c/a^2/x^2)^(1/2)*x/c+2*arctanh((c-c/
a^2/x^2)^(1/2)/c^(1/2))/a/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{-3-2ax+a^2x^2+2\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1+a^2x^2}\right)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(-3 - 2*a*x + a^2*x^2 + 2*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]]
)/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6717, 6709, 527, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{\sqrt{1-a^2x^2}\int \frac{x(ax+1)^2}{(1-a^2x^2)^{3/2}}dx}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{527}$$

$$-\frac{\sqrt{1-a^2x^2}\left(\frac{2(ax+1)}{a^2\sqrt{1-a^2x^2}}-\frac{\int \frac{ax+2}{\sqrt{1-a^2x^2}}dx}{a}\right)}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{455}$$

$$-\frac{\sqrt{1-a^2x^2}\left(\frac{2(ax+1)}{a^2\sqrt{1-a^2x^2}}-\frac{2\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{a}}{a}\right)}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\sqrt{1-a^2x^2}\left(\frac{2(ax+1)}{a^2\sqrt{1-a^2x^2}}-\frac{\frac{2\arcsin(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}}{a}\right)}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
-((Sqrt[1 - a^2*x^2]*((2*(1 + a*x))/(a^2*Sqrt[1 - a^2*x^2]) - (-(Sqrt[1 -
a^2*x^2]/a) + (2*ArcSin[a*x])/a)/a))/(Sqrt[c - c/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 527
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_S
ymbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a +
b*x^2])), x] + Simp[1/(b*d^(m - 2))   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[(
2^(n - 1)*c^(m + n - 1) - d^m*x^m*(c + d*x)^(n - 1))/(c - d*x), x], x], x]
/; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && EqQ[b*c^2 + a*d^2,
 0]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(71) = 142$.

Time = 0.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.05

method	result
risch	$\frac{a^2x^2-1}{a^2x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}+\frac{\left(\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a\sqrt{a^2c}}-\frac{2\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+2\left(x-\frac{1}{a}\right)ac}}{a^3c\left(x-\frac{1}{a}\right)}\right)\sqrt{c(a^2x^2-1)}}{x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}$
default	$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\left(\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\sqrt{c}\,a^2x+2\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)acx-\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a\sqrt{c}-2a\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\sqrt{c}-2\ln\left(x\sqrt{c}+\cdots\right.\right.}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\,c^{\frac{3}{2}}a(ax-1)}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a^2*x^2-1)/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(2/a*ln(a^2*c*x/(a^2*c)^
(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)-2/a^3/c/(x-1/a)*((x-1/a)^2*a^2*c+
2*(x-1/a)*a*c)^(1/2))*(c*(a^2*x^2-1))^(1/2)/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.77

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \left[\frac{(ax-1)\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(a^2x^2-3\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx-ac},\; -\frac{2\,(ax-1)\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(a^2x^2-3\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx-ac}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[((a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)) - c) + (a^2*x^2 - 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a
^2*c*x - a*c), -(2*(a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (a^2*x^2 - 3*a*x)*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)))/(a^2*c*x - a*c)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{ax+1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral((a*x + 1)/(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{ax+1}{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)/((a*x - 1)*sqrt(c - c/(a^2*x^2))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{a\,x+1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}\,dx$$

input
```
int((a*x + 1)/((c - c/(a^2*x^2))^(1/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - c/(a^2*x^2))^(1/2)*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{c}\,\left(2\sqrt{a^2x^2-1}\,ax-6\sqrt{a^2x^2-1}+4\log\left(\sqrt{a^2x^2-1}+ax\right)ax-4\log\left(\sqrt{a^2x^2-1}+ax\right)-5ax+5\right)}{2ac\,(ax-1)}$$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x)
```

output
```
(sqrt(c)*(2*sqrt(a**2*x**2 - 1)*a*x - 6*sqrt(a**2*x**2 - 1) + 4*log(sqrt(a
**2*x**2 - 1) + a*x)*a*x - 4*log(sqrt(a**2*x**2 - 1) + a*x) - 5*a*x + 5))/
(2*a*c*(a*x - 1))
```

3.819 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = -\frac{2\left(a+\frac{1}{x}\right)}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{10\sqrt{c-\frac{c}{a^2x^2}}\,x}{3c^2}-\frac{\left(7a+\frac{6}{x}\right)x}{3ac\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{ac^{3/2}}$$

output
```
1/3*(-2*a-2/x)/a^2/(c-c/a^2/x^2)^(3/2)+10/3*(c-c/a^2/x^2)^(1/2)*x/c^2-1/3*
(7*a+6/x)*x/a/c/(c-c/a^2/x^2)^(1/2)+2*arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))
/a/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{10-4ax-11a^2x^2+3a^3x^3+6(-1+ax)\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1+a^2x^2}\right)}{3a^2c\sqrt{c-\frac{c}{a^2x^2}}x(-1+ax)}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

output
```
(10 - 4*a*x - 11*a^2*x^2 + 3*a^3*x^3 + 6*(-1 + a*x)*Sqrt[-1 + a^2*x^2]*Log
[a*x + Sqrt[-1 + a^2*x^2]])/(3*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6717, 6709, 529, 2166, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\int \frac{x^3(ax+1)^2}{\left(1-a^2x^2\right)^{5/2}}dx}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{529}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{(ax+1)^2}{3a^4(1-a^2x^2)^{3/2}}-\frac{1}{3}\int\frac{(ax+1)\left(\frac{3x^2}{a}+\frac{3x}{a^2}+\frac{2}{a^3}\right)}{(1-a^2x^2)^{3/2}}dx\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{2166}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{1}{3}\left(\int\frac{3(ax+2)}{a^3\sqrt{1-a^2x^2}}dx-\frac{8(ax+1)}{a^4\sqrt{1-a^2x^2}}\right)+\frac{(ax+1)^2}{3a^4(1-a^2x^2)^{3/2}}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{1}{3}\left(\frac{3\int\frac{ax+2}{\sqrt{1-a^2x^2}}dx}{a^3}-\frac{8(ax+1)}{a^4\sqrt{1-a^2x^2}}\right)+\frac{(ax+1)^2}{3a^4(1-a^2x^2)^{3/2}}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{455}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{1}{3}\left(\frac{3\left(2\int\frac{1}{\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^3}-\frac{8(ax+1)}{a^4\sqrt{1-a^2x^2}}\right)+\frac{(ax+1)^2}{3a^4(1-a^2x^2)^{3/2}}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{(ax+1)^2}{3a^4(1-a^2x^2)^{3/2}}+\frac{1}{3}\left(\frac{3\left(\frac{2\arcsin(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^3}-\frac{8(ax+1)}{a^4\sqrt{1-a^2x^2}}\right)\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

output
```
-((((1 - a^2*x^2)^(3/2)*((1 + a*x)^2/(3*a^4*(1 - a^2*x^2)^(3/2)) + ((-8*(1
+ a*x))/(a^4*Sqrt[1 - a^2*x^2]) + (3*(-(Sqrt[1 - a^2*x^2]/a) + (2*ArcSin[a
*x])/a))/a^3)/3))/((c - c/(a^2*x^2))^(3/2)*x^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(104) = 208.

Time = 0.18 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.81

method	result
risch	$\frac{a^2x^2-1}{a^2cx\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}+\frac{\left(-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+2\left(x-\frac{1}{a}\right)ac}}{3a^6c\left(x-\frac{1}{a}\right)^2}-\frac{8\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+2\left(x-\frac{1}{a}\right)ac}}{3a^5c\left(x-\frac{1}{a}\right)}+\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^3\sqrt{a^2c}}\right)a^2\sqrt{c(a^2x^2-1)}}{cx\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}$
default	$\frac{\left(3c^{\frac{3}{2}}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a^3x^3-15x^2a^2c^{\frac{3}{2}}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+4c^{\frac{3}{2}}\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2x^2+6\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,\sqrt{\cdots}\right.}{3\sqrt{\frac{c(ax-\cdots}{}}\cdots}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a^2*x^2-1)/c/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(-1/3/a^6/c/(x-1/a)^2*
((x-1/a)^2*a^2*c+2*(x-1/a)*a*c)^(1/2)-8/3/a^5/c/(x-1/a)*((x-1/a)^2*a^2*c+2
*(x-1/a)*a*c)^(1/2)+2/a^3*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a
^2*c)^(1/2))*a^2/c/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.37

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \left[\frac{3\left(a^2x^2-2\,ax+1\right)\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(3\,a^3x^3-14\,a^2x^2\right.}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}\right.$$

$$\left.-\frac{6\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\left(3\,a^3x^3-14\,a^2x^2+10\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*
sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (3*a^3*x^3 - 14*a^2*x^2 + 10*a*x)*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/3*
(6*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2
- c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (3*a^3*x^3 - 14*a^2*x^2 + 10*a*x)*sqrt(
(a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(3/2),x)
```

output
```
Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2)*(a*x - 1)), x
)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{a\,x+1}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,(a\,x-1)}\,dx$$

input
```
int((a*x + 1)/((c - c/(a^2*x^2))^(3/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - c/(a^2*x^2))^(3/2)*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.20

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\sqrt{c}\left(3\sqrt{a^2x^2-1}\,a^2x^2-14\sqrt{a^2x^2-1}\,ax+10\sqrt{a^2x^2-1}+6\log\left(\sqrt{a^2x^2-1}+ax\right)a^2}{3a\,c^2\left(a^2x^2-\right.}$$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x)
```

output
```
(sqrt(c)*(3*sqrt(a**2*x**2 - 1)*a**2*x**2 - 14*sqrt(a**2*x**2 - 1)*a*x + 1
0*sqrt(a**2*x**2 - 1) + 6*log(sqrt(a**2*x**2 - 1) + a*x)*a**2*x**2 - 12*lo
g(sqrt(a**2*x**2 - 1) + a*x)*a*x + 6*log(sqrt(a**2*x**2 - 1) + a*x) + 3*a*
*2*x**2 - 6*a*x + 3))/(3*a*c**2*(a**2*x**2 - 2*a*x + 1))
```

3.820 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 152

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = -\frac{2\left(a+\frac{1}{x}\right)}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}} - \frac{2\left(15a+\frac{14}{x}\right)}{15a^2c^2\sqrt{c-\frac{c}{a^2x^2}}} + \frac{28\sqrt{c-\frac{c}{a^2x^2}}\,x}{15c^3} - \frac{\left(13a+\frac{10}{x}\right)x}{15ac\left(c-\frac{c}{a^2x^2}\right)^{3/2}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{ac^{5/2}}$$

output
```
1/5*(-2*a-2/x)/a^2/(c-c/a^2/x^2)^(5/2)-2/15*(15*a+14/x)/a^2/c^2/(c-c/a^2/x
^2)^(1/2)+28/15*(c-c/a^2/x^2)^(1/2)*x/c^3-1/15*(13*a+10/x)*x/a/c/(c-c/a^2/
x^2)^(3/2)+2*arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))/a/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.69

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{-56+82ax+32a^2x^2-76a^3x^3+15a^4x^4+30(-1+ax)^2\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-\right.}}{15a^2c^2\sqrt{c-\frac{c}{a^2x^2}}\,x(-1+ax)^2}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

output
```
(-56 + 82*a*x + 32*a^2*x^2 - 76*a^3*x^3 + 15*a^4*x^4 + 30*(-1 + a*x)^2*Sqr
t[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(15*a^2*c^2*Sqrt[c - c/(a^2
*x^2)]*x*(-1 + a*x)^2)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6717, 6709, 529, 2166, 2345, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\int \frac{x^5(ax+1)^2}{\left(1-a^2x^2\right)^{7/2}}dx}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{ 529}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{(ax+1)^2}{5a^6(1-a^2x^2)^{5/2}}-\frac{1}{5}\int\frac{(ax+1)\left(\frac{5x^4}{a}+\frac{5x^3}{a^2}+\frac{5x^2}{a^3}+\frac{5x}{a^4}+\frac{2}{a^5}\right)}{(1-a^2x^2)^{5/2}}dx\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \textbf{2166}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{\frac{15x^3}{a^2}+\frac{30x^2}{a^3}+\frac{45x}{a^4}+\frac{16}{a^5}}{(1-a^2x^2)^{3/2}}dx-\frac{22(ax+1)}{3a^6(1-a^2x^2)^{3/2}}\right)+\frac{(ax+1)^2}{5a^6(1-a^2x^2)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \textbf{2345}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(23ax+30)}{a^6\sqrt{1-a^2x^2}}-\int\frac{15(ax+2)}{a^5\sqrt{1-a^2x^2}}dx\right)-\frac{22(ax+1)}{3a^6(1-a^2x^2)^{3/2}}\right)+\frac{(ax+1)^2}{5a^6(1-a^2x^2)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(23ax+30)}{a^6\sqrt{1-a^2x^2}}-\frac{15\int\frac{ax+2}{\sqrt{1-a^2x^2}}dx}{a^5}\right)-\frac{22(ax+1)}{3a^6(1-a^2x^2)^{3/2}}\right)+\frac{(ax+1)^2}{5a^6(1-a^2x^2)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \textbf{455}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(23ax+30)}{a^6\sqrt{1-a^2x^2}}-\frac{15\left(2\int\frac{1}{\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^5}\right)-\frac{22(ax+1)}{3a^6(1-a^2x^2)^{3/2}}\right)+\frac{(ax+1)^2}{5a^6(1-a^2x^2)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \textbf{223}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{(ax+1)^2}{5a^6(1-a^2x^2)^{5/2}}+\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(23ax+30)}{a^6\sqrt{1-a^2x^2}}-\frac{15\left(\frac{2\arcsin(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^5}\right)-\frac{22(ax+1)}{3a^6(1-a^2x^2)^{3/2}}\right)\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

output
```
-(((1 - a^2*x^2)^(5/2)*((1 + a*x)^2/(5*a^6*(1 - a^2*x^2)^(5/2)) + ((-22*(1
 + a*x))/(3*a^6*(1 - a^2*x^2)^(3/2)) + ((2*(30 + 23*a*x))/(a^6*Sqrt[1 - a^
2*x^2]) - (15*(-(Sqrt[1 - a^2*x^2]/a) + (2*ArcSin[a*x])/a))/a^5)/3)/5))/((
c - c/(a^2*x^2))^(5/2)*x^5))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(134) = 268$.

Time = 0.19 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.96

method	result
risch	$\frac{a^2x^2-1}{a^2c^2x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}+\frac{\left(\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^5\sqrt{a^2c}}-\frac{383\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+2\left(x-\frac{1}{a}\right)ac}}{120a^7c\left(x-\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+2\left(x-\frac{1}{a}\right)ac}}{10a^9c\left(x-\frac{1}{a}\right)^3}-\frac{41\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+2\left(x\right.}}{60a^8c\left(x-\frac{1}{a}\right)^2}\right.}{c^2x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}$
default	$\frac{\left(15\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}c^{\frac{5}{2}}a^5x^5-16\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}c^{\frac{5}{2}}a^4x^4-45x^4c^{\frac{5}{2}}a^4\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}+16\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}c^{\frac{5}{2}}a^3x^3-60\left(\frac{c(ax\right.}{}\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a^2*x^2-1)/c^2/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(2/a^5*ln(a^2*c*x/(a
^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)-383/120/a^7/c/(x-1/a)*((x-1
/a)^2*a^2*c+2*(x-1/a)*a*c)^(1/2)-1/10/a^9/c/(x-1/a)^3*((x-1/a)^2*a^2*c+2*(
x-1/a)*a*c)^(1/2)-41/60/a^8/c/(x-1/a)^2*((x-1/a)^2*a^2*c+2*(x-1/a)*a*c)^(1
/2)+1/8/a^7/c/(x+1/a)*((x+1/a)^2*a^2*c-2*(x+1/a)*a*c)^(1/2))*a^4/c^2/x/(c*
(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.32

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \left[\frac{15\,(a^4x^4-2\,a^3x^3+2\,ax-1)\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(15\,a^5x^5 \cdots}{15\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-a\cdots}\right.$$

$$-\frac{30\,(a^4x^4-2\,a^3x^3+2\,ax-1)\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(15\,a^5x^5-76\,a^4x^4+32\,a^3x^3+82\,a^2x^2 \cdots}{15\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/15*(15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^
2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (15*a^5*x^5 - 76*a^4*
x^4 + 32*a^3*x^3 + 82*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(
a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3), -1/15*(30*(a^4*x^4 - 2
*a^3*x^3 + 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2))/(a^2*c*x^2 - c)) - (15*a^5*x^5 - 76*a^4*x^4 + 32*a^3*x^3 + 82
*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^3*x^4 - 2*a^4*c
^3*x^3 + 2*a^2*c^3*x - a*c^3)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(5/2),x)
```

output
```
Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(5/2)*(a*x - 1)), x
)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{a\,x+1}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,(a\,x-1)}\,dx$$

input
```
int((a*x + 1)/((c - c/(a^2*x^2))^(5/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - c/(a^2*x^2))^(5/2)*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.43

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{\sqrt{c}\,\left(30\sqrt{a^2x^2-1}\,a^4x^4-152\sqrt{a^2x^2-1}\,a^3x^3+64\sqrt{a^2x^2-1}\,a^2x^2+164\sqrt{a^2x^2-1}\,a\right.}{}$$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x)
```

output
```
(sqrt(c)*(30*sqrt(a**2*x**2 - 1)*a**4*x**4 - 152*sqrt(a**2*x**2 - 1)*a**3*
x**3 + 64*sqrt(a**2*x**2 - 1)*a**2*x**2 + 164*sqrt(a**2*x**2 - 1)*a*x - 11
2*sqrt(a**2*x**2 - 1) + 60*log(sqrt(a**2*x**2 - 1) + a*x)*a**4*x**4 - 120*
log(sqrt(a**2*x**2 - 1) + a*x)*a**3*x**3 + 120*log(sqrt(a**2*x**2 - 1) + a
*x)*a*x - 60*log(sqrt(a**2*x**2 - 1) + a*x) + 47*a**4*x**4 - 94*a**3*x**3
+ 94*a*x - 47))/(30*a*c**3*(a**4*x**4 - 2*a**3*x**3 + 2*a*x - 1))
```

3.821 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 187

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = -\frac{2\left(a+\frac{1}{x}\right)}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{2\left(35a+\frac{27}{x}\right)}{105a^2c^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{144\sqrt{c-\frac{c}{a^2x^2}}\,x}{35c^4}-\frac{2\left(9a+\frac{7}{x}\right)x}{7ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\left(19a+\frac{14}{x}\right)x}{35ac\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{ac^{7/2}}$$

output

```
1/7*(-2*a-2/x)/a^2/(c-c/a^2/x^2)^(7/2)-2/105*(35*a+27/x)/a^2/c^2/(c-c/a^2/
x^2)^(3/2)+144/35*(c-c/a^2/x^2)^(1/2)*x/c^4-2/7*(9*a+7/x)*x/a/c^3/(c-c/a^2
/x^2)^(1/2)-1/35*(19*a+14/x)*x/a/c/(c-c/a^2/x^2)^(5/2)+2*arctanh((c-c/a^2/
x^2)^(1/2)/c^(1/2))/a/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.71

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{432-654ax-636a^2x^2+1226a^3x^3+74a^4x^4-562a^5x^5+105a^6x^6+210(-1+ax)}{105a^2c^3\sqrt{c-\frac{c}{a^2x^2}}\,x(-1+ax)^3(1+ax}$$

input
```
Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

output
```
(432 - 654*a*x - 636*a^2*x^2 + 1226*a^3*x^3 + 74*a^4*x^4 - 562*a^5*x^5 + 1
05*a^6*x^6 + 210*(-1 + a*x)^3*(1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[
-1 + a^2*x^2]])/(105*a^2*c^3*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)^3*(1 + a*x
))
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6717, 6709, 529, 2166, 2345, 2345, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\int \frac{x^7(ax+1)^2}{(1-a^2x^2)^{9/2}}dx}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{ 529}$$

$$-\frac{(1-a^2x^2)^{7/2}\left(\frac{(ax+1)^2}{7a^8(1-a^2x^2)^{7/2}}-\frac{1}{7}\int \frac{(ax+1)\left(\frac{7x^6}{a}+\frac{7x^5}{a^2}+\frac{7x^4}{a^3}+\frac{7x^3}{a^4}+\frac{7x^2}{a^5}+\frac{7x}{a^6}+\frac{2}{a^7}\right)}{(1-a^2x^2)^{7/2}}dx\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\downarrow 2166$$

$$-\frac{(1-a^2x^2)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\int \frac{\frac{35x^5}{a^2}+\frac{70x^4}{a^3}+\frac{105x^3}{a^4}+\frac{140x^2}{a^5}+\frac{175x}{a^6}+\frac{34}{a^7}}{(1-a^2x^2)^{5/2}}dx-\frac{44(ax+1)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{(ax+1)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\downarrow 2345$$

$$-\frac{(1-a^2x^2)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{244ax+315}{3a^8(1-a^2x^2)^{3/2}}-\frac{1}{3}\int \frac{\frac{105x^3}{a^4}+\frac{210x^2}{a^5}+\frac{420x}{a^6}+\frac{142}{a^7}}{(1-a^2x^2)^{3/2}}dx\right)-\frac{44(ax+1)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{(ax+1)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\downarrow 2345$$

$$-\frac{(1-a^2x^2)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{105(ax+2)}{a^7\sqrt{1-a^2x^2}}dx-\frac{352ax+525}{a^8\sqrt{1-a^2x^2}}\right)+\frac{244ax+315}{3a^8(1-a^2x^2)^{3/2}}\right)-\frac{44(ax+1)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{(ax+1)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\downarrow 27$$

$$-\frac{(1-a^2x^2)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{105\int \frac{ax+2}{\sqrt{1-a^2x^2}}dx}{a^7}-\frac{352ax+525}{a^8\sqrt{1-a^2x^2}}\right)+\frac{244ax+315}{3a^8(1-a^2x^2)^{3/2}}\right)-\frac{44(ax+1)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{(ax+1)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\downarrow 455$$

$$-\frac{(1-a^2x^2)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{105\left(2\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^7}-\frac{352ax+525}{a^8\sqrt{1-a^2x^2}}\right)+\frac{244ax+315}{3a^8(1-a^2x^2)^{3/2}}\right)-\frac{44(ax+1)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{\cdots}{7a\cdots}\right.}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\downarrow 223$$

$$-\frac{(1-a^2x^2)^{7/2}\left(\frac{(ax+1)^2}{7a^8(1-a^2x^2)^{7/2}}+\frac{1}{7}\left(\frac{1}{5}\left(\frac{244ax+315}{3a^8(1-a^2x^2)^{3/2}}+\frac{1}{3}\left(\frac{105\left(\frac{2\arcsin(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^7}-\frac{352ax+525}{a^8\sqrt{1-a^2x^2}}\right)\right)-\frac{44(ax\cdots}{5a^8(1-a\cdots}\right.\right.}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

input
```
Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

output
```
-(((1 - a^2*x^2)^(7/2)*((1 + a*x)^2/(7*a^8*(1 - a^2*x^2)^(7/2)) + ((-44*(1
 + a*x))/(5*a^8*(1 - a^2*x^2)^(5/2)) + ((315 + 244*a*x)/(3*a^8*(1 - a^2*x^
2)^(3/2)) + (-((525 + 352*a*x)/(a^8*Sqrt[1 - a^2*x^2])) + (105*(-(Sqrt[1 -
 a^2*x^2]/a) + (2*ArcSin[a*x])/a))/a^7)/3)/5)/7))/((c - c/(a^2*x^2))^(7/2)
*x^7))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(165) = 330$.

Time = 0.21 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.04

method	result
risch	$\frac{a^2x^2-1}{a^2c^3x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}+\frac{\left(\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^7\sqrt{a^2c}}-\frac{3061\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+2\left(x-\frac{1}{a}\right)ac}}{840a^9c\left(x-\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+2\left(x-\frac{1}{a}\right)ac}}{28a^{12}c\left(x-\frac{1}{a}\right)^4}-\frac{39\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+2(}}{140a^{11}c\left(x-\frac{1}{a}\right)}\right.}{c^3x\sqrt{}}$
default	$\frac{\left(105\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}a^7x^7+96\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}a^6x^6-553x^6c^{\frac{7}{2}}a^6\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{5}{2}}-96\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}a^5x^5-392\left(\right.\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a^2*x^2-1)/c^3/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(2/a^7*ln(a^2*c*x/(a
^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)-3061/840/a^9/c/(x-1/a)*((x-
1/a)^2*a^2*c+2*(x-1/a)*a*c)^(1/2)-1/28/a^12/c/(x-1/a)^4*((x-1/a)^2*a^2*c+2
*(x-1/a)*a*c)^(1/2)-39/140/a^11/c/(x-1/a)^3*((x-1/a)^2*a^2*c+2*(x-1/a)*a*c
)^(1/2)-1753/1680/a^10/c/(x-1/a)^2*((x-1/a)^2*a^2*c+2*(x-1/a)*a*c)^(1/2)-1
/48/a^10/c/(x+1/a)^2*((x+1/a)^2*a^2*c-2*(x+1/a)*a*c)^(1/2)+7/24/a^9/c/(x+1
/a)*((x+1/a)^2*a^2*c-2*(x+1/a)*a*c)^(1/2))*a^6/c^3/x/(c*(a^2*x^2-1)/a^2/x^
2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.65

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \left[\frac{105\,(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1)\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\right.}{105\,(a^7c^4x^6-2\,a^6c^4x^5-}\right.$$

$$-\frac{210\,(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1)\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(105\,a^7x^7-562}{105\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

output
```
[1/105*(105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x +
 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)) - c) + (105*a^7*x^7 - 562*a^6*x^6 + 74*a^5*x^5 + 1226*a^4*x^4 - 636*
a^3*x^3 - 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^7*c^4
*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c
^4*x + a*c^4), -1/105*(210*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^
2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(
a^2*x^2))/(a^2*c*x^2 - c)) - (105*a^7*x^7 - 562*a^6*x^6 + 74*a^5*x^5 + 122
6*a^4*x^4 - 636*a^3*x^3 - 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2
*x^2)))/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c
^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(7/2),x)
```

output
```
Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(7/2)*(a*x - 1)), x
)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \int \frac{a\,x+1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,(a\,x-1)}\,dx$$

input
```
int((a*x + 1)/((c - c/(a^2*x^2))^(7/2)*(a*x - 1)),x)
```

output
```
int((a*x + 1)/((c - c/(a^2*x^2))^(7/2)*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.98

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{c}\left(420\sqrt{a^2x^2-1}\,a^6x^6-2248\sqrt{a^2x^2-1}\,a^5x^5+296\sqrt{a^2x^2-1}\,a^4x^4+4904\sqrt{a^2x^2-\right.}{}$$

input
```
int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x)
```

output
```
(sqrt(c)*(420*sqrt(a**2*x**2 - 1)*a**6*x**6 - 2248*sqrt(a**2*x**2 - 1)*a**
5*x**5 + 296*sqrt(a**2*x**2 - 1)*a**4*x**4 + 4904*sqrt(a**2*x**2 - 1)*a**3
*x**3 - 2544*sqrt(a**2*x**2 - 1)*a**2*x**2 - 2616*sqrt(a**2*x**2 - 1)*a*x
+ 1728*sqrt(a**2*x**2 - 1) + 840*log(sqrt(a**2*x**2 - 1) + a*x)*a**6*x**6
- 1680*log(sqrt(a**2*x**2 - 1) + a*x)*a**5*x**5 - 840*log(sqrt(a**2*x**2 -
 1) + a*x)*a**4*x**4 + 3360*log(sqrt(a**2*x**2 - 1) + a*x)*a**3*x**3 - 840
*log(sqrt(a**2*x**2 - 1) + a*x)*a**2*x**2 - 1680*log(sqrt(a**2*x**2 - 1) +
 a*x)*a*x + 840*log(sqrt(a**2*x**2 - 1) + a*x) + 463*a**6*x**6 - 926*a**5*
x**5 - 463*a**4*x**4 + 1852*a**3*x**3 - 463*a**2*x**2 - 926*a*x + 463))/(4
20*a*c**4*(a**6*x**6 - 2*a**5*x**5 - a**4*x**4 + 4*a**3*x**3 - a**2*x**2 -
 2*a*x + 1))
```

3.822 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 322

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx &= \frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9\sqrt{1-\frac{1}{a^2x^2}}\,x^8}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8\sqrt{1-\frac{1}{a^2x^2}}\,x^7}\\ &-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}-\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}\\ &+\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

output
```
1/8*c^4*(c-c/a^2/x^2)^(1/2)/a^9/(1-1/a^2/x^2)^(1/2)/x^8+3/7*c^4*(c-c/a^2/x
^2)^(1/2)/a^8/(1-1/a^2/x^2)^(1/2)/x^7-8/5*c^4*(c-c/a^2/x^2)^(1/2)/a^6/(1-1
/a^2/x^2)^(1/2)/x^5-3/2*c^4*(c-c/a^2/x^2)^(1/2)/a^5/(1-1/a^2/x^2)^(1/2)/x^
4+2*c^4*(c-c/a^2/x^2)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)/x^3+4*c^4*(c-c/a^2/x^2
)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)/x^2+c^4*(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2
)^(1/2)+3*c^4*(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(\frac{1}{8a^9x^8}+\frac{3}{7a^8x^7}-\frac{8}{5a^6x^5}-\frac{3}{2a^5x^4}+\frac{2}{a^4x^3}+\frac{4}{a^3x^2}+x+\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

output
```
((c - c/(a^2*x^2))^(9/2)*(1/(8*a^9*x^8) + 3/(7*a^8*x^7) - 8/(5*a^6*x^5) -
3/(2*a^5*x^4) + 2/(a^4*x^3) + 4/(a^3*x^2) + x + (3*Log[x])/a))/(1 - 1/(a^2
*x^2))^(9/2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.32, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{9/2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(1-ax)^3(ax+1)^6}{x^9}dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\int\frac{(1-ax)^3(ax+1)^6}{x^9}dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\int\left(-a^9-\frac{3a^8}{x}+\frac{8a^6}{x^3}+\frac{6a^5}{x^4}-\frac{6a^4}{x^5}-\frac{8a^3}{x^6}+\frac{3a}{x^8}+\frac{1}{x^9}\right)dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\left(a^9(-x)-3a^8\log(x)-\frac{4a^6}{x^2}-\frac{2a^5}{x^3}+\frac{3a^4}{2x^4}+\frac{8a^3}{5x^5}-\frac{3a}{7x^7}-\frac{1}{8x^8}\right)}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

output
```
-((c^4*Sqrt[c - c/(a^2*x^2)]*(-1/8*1/x^8 - (3*a)/(7*x^7) + (8*a^3)/(5*x^5)
 + (3*a^4)/(2*x^4) - (2*a^5)/x^3 - (4*a^6)/x^2 - a^9*x - 3*a^8*Log[x]))/(a
^9*Sqrt[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\left(280 a^{9} x^{9}+840 a^{8} \ln \left(x \right) x^{8}+1120 x^{6} a^{6}+560 a^{5} x^{5}-420 a^{4} x^{4}-448 a^{3} x^{3}+120 a x +35\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{9}{2}} x}{280 \left(a x +1\right)^{3} \left(a^{2} x^{2}-1\right)^{3} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	112

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/280*(280*a^9*x^9+840*a^8*ln(x)*x^8+1120*x^6*a^6+560*a^5*x^5-420*a^4*x^4-
448*a^3*x^3+120*a*x+35)*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/(a*x+1)^3/(a^2*x^2
-1)^3/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\frac{(280\,a^9c^4x^9+840\,a^8c^4x^8\log(x)+1120\,a^6c^4x^6+560\,a^5c^4x^5-420\,a^4c^4x^4-448\,a^3c^4x^3+}{280\,a^{10}x^8}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="fric
as")
```

output
```
1/280*(280*a^9*c^4*x^9 + 840*a^8*c^4*x^8*log(x) + 1120*a^6*c^4*x^6 + 560*a
^5*c^4*x^5 - 420*a^4*c^4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x + 35*c^4)*sqr
t(a^2*c)/(a^10*x^8)
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="maxi
ma")
```

output
```
integrate((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.41

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{1}{140}\left(\frac{280\,c^4x\mathrm{sgn}(x)}{a\mathrm{sgn}\,(ax+1)}+\frac{840\,c^4\log\left(|x|\right)\mathrm{sgn}(x)}{a^2\mathrm{sgn}\,(ax+1)}+\frac{1120\,a^6c^4x^6\mathrm{sgn}(x)+560\,a^5c^4x^5\mathrm{sgn}(x)-\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="giac
")
```

output
```
1/140*(280*c^4*x*sgn(x)/(a*sgn(a*x + 1)) + 840*c^4*log(abs(x))*sgn(x)/(a^2
*sgn(a*x + 1)) + (1120*a^6*c^4*x^6*sgn(x) + 560*a^5*c^4*x^5*sgn(x) - 420*a
^4*c^4*x^4*sgn(x) - 448*a^3*c^4*x^3*sgn(x) + 120*a*c^4*x*sgn(x) + 35*c^4*s
gn(x))/(a^10*x^8*sgn(a*x + 1)))*sqrt(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{9/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.21

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\sqrt{c}\,c^4(840\log(x)\,a^8x^8+280a^9x^9+1120a^6x^6+560a^5x^5-420a^4x^4-448a^3x^3+120ax+}{280a^9x^8}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x)
```

output
```
(sqrt(c)*c**4*(840*log(x)*a**8*x**8 + 280*a**9*x**9 + 1120*a**6*x**6 + 560
*a**5*x**5 - 420*a**4*x**4 - 448*a**3*x**3 + 120*a*x + 35))/(280*a**9*x**8
)
```

3.823 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 324

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}x^6}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}x^5}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/(1-1/a^2/x^2)^(1/2)/x^6-3/5*c^3*(c-c/a^2/
x^2)^(1/2)/a^6/(1-1/a^2/x^2)^(1/2)/x^5-1/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/(1-
1/a^2/x^2)^(1/2)/x^4+5/3*c^3*(c-c/a^2/x^2)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)/x
^3+5/2*c^3*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)/x^2-c^3*(c-c/a^2/x^
2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)/x+c^3*(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)
^(1/2)+3*c^3*(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6a^7x^6}-\frac{3}{5a^6x^5}-\frac{1}{4a^5x^4}+\frac{5}{3a^4x^3}+\frac{5}{2a^3x^2}-\frac{1}{a^2x}+x+\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

output
```
((c - c/(a^2*x^2))^(7/2)*(-1/6*1/(a^7*x^6) - 3/(5*a^6*x^5) - 1/(4*a^5*x^4)
 + 5/(3*a^4*x^3) + 5/(2*a^3*x^2) - 1/(a^2*x) + x + (3*Log[x])/a))/(1 - 1/(
a^2*x^2))^(7/2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{7/2} e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2(ax+1)^5}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int\left(a^7+\frac{3a^6}{x}+\frac{a^5}{x^2}-\frac{5a^4}{x^3}-\frac{5a^3}{x^4}+\frac{a^2}{x^5}+\frac{3a}{x^6}+\frac{1}{x^7}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(a^7x+3a^6\log(x)-\frac{a^5}{x}+\frac{5a^4}{2x^2}+\frac{5a^3}{3x^3}-\frac{a^2}{4x^4}-\frac{3a}{5x^5}-\frac{1}{6x^6}\right)}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

output
```
(c^3*Sqrt[c - c/(a^2*x^2)]*(-1/6*1/x^6 - (3*a)/(5*x^5) - a^2/(4*x^4) + (5*
a^3)/(3*x^3) + (5*a^4)/(2*x^2) - a^5/x + a^7*x + 3*a^6*Log[x]))/(a^7*Sqrt[
1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\left(60a^{7}x^{7}+180a^{6}\ln\left(x\right)x^{6}-60a^{5}x^{5}+150a^{4}x^{4}+100a^{3}x^{3}-15a^{2}x^{2}-36ax-10\right)\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{7}{2}}x}{60\left(ax+1\right)^{3}\left(a^{2}x^{2}-1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	112

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*(60*a^7*x^7+180*a^6*ln(x)*x^6-60*a^5*x^5+150*a^4*x^4+100*a^3*x^3-15*a
^2*x^2-36*a*x-10)*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a*x+1)^3/(a^2*x^2-1)^2/
((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(60\,a^{7}c^{3}x^{7}+180\,a^{6}c^{3}x^{6}\log\left(x\right)-60\,a^{5}c^{3}x^{5}+150\,a^{4}c^{3}x^{4}+100\,a^{3}c^{3}x^{3}-15\,a^{2}c^{3}x^{2}-36\,a\cdots\right.}{60\,a^{8}x^{6}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="fric
as")
```

output
```
1/60*(60*a^7*c^3*x^7 + 180*a^6*c^3*x^6*log(x) - 60*a^5*c^3*x^5 + 150*a^4*c
^3*x^4 + 100*a^3*c^3*x^3 - 15*a^2*c^3*x^2 - 36*a*c^3*x - 10*c^3)*sqrt(a^2*
c)/(a^8*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxi
ma")
```

output
```
integrate((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.41

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\frac{1}{30}\left(\frac{60\,c^3x\mathrm{sgn}(x)}{a\mathrm{sgn}\,(ax+1)}+\frac{180\,c^3\log\left(|x|\right)\mathrm{sgn}(x)}{a^2\mathrm{sgn}\,(ax+1)}-\frac{60\,a^5c^3x^5\mathrm{sgn}(x)-150\,a^4c^3x^4\mathrm{sgn}(x)-100$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac
")
```

output
```
1/30*(60*c^3*x*sgn(x)/(a*sgn(a*x + 1)) + 180*c^3*log(abs(x))*sgn(x)/(a^2*s
gn(a*x + 1)) - (60*a^5*c^3*x^5*sgn(x) - 150*a^4*c^3*x^4*sgn(x) - 100*a^3*c
^3*x^3*sgn(x) + 15*a^2*c^3*x^2*sgn(x) + 36*a*c^3*x*sgn(x) + 10*c^3*sgn(x))
/(a^8*x^6*sgn(a*x + 1)))*sqrt(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.21

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\frac{\sqrt{c}\,c^3(180\log(x)\,a^6x^6+60a^7x^7-60a^5x^5+150a^4x^4+100a^3x^3-15a^2x^2-36ax-10)}{60a^7x^6}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x)
```

output
```
(sqrt(c)*c**3*(180*log(x)*a**6*x**6 + 60*a**7*x**7 - 60*a**5*x**5 + 150*a*
*4*x**4 + 100*a**3*x**3 - 15*a**2*x**2 - 36*a*x - 10))/(60*a**7*x**6)
```

3.824 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 234

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/(1-1/a^2/x^2)^(1/2)/x^4+c^2*(c-c/a^2/x^2)^
(1/2)/a^4/(1-1/a^2/x^2)^(1/2)/x^3+c^2*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2
)^(1/2)/x^2-2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)/x+c^2*(c-c/a
^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1/2)+3*c^2*(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1
/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.33

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{5}{4a}+\frac{1}{4a^5x^4}+\frac{1}{a^4x^3}+\frac{1}{a^3x^2}-\frac{2}{a^2x}+x+\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

output
```
((c - c/(a^2*x^2))^(5/2)*(5/(4*a) + 1/(4*a^5*x^4) + 1/(a^4*x^3) + 1/(a^3*x
^2) - 2/(a^2*x) + x + (3*Log[x])/a))/(1 - 1/(a^2*x^2))^(5/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.34, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{5/2}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(1-ax)(ax+1)^4}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)(ax+1)^4}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 84}$$

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int \left(-a^5-\frac{3a^4}{x}-\frac{2a^3}{x^2}+\frac{2a^2}{x^3}+\frac{3a}{x^4}+\frac{1}{x^5}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(a^5(-x)-3a^4\log(x)+\frac{2a^3}{x}-\frac{a^2}{x^2}-\frac{a}{x^3}-\frac{1}{4x^4}\right)}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

output
```
-((c^2*Sqrt[c - c/(a^2*x^2)]*(-1/4*1/x^4 - a/x^3 - a^2/x^2 + (2*a^3)/x - a
^5*x - 3*a^4*Log[x]))/(a^5*Sqrt[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{(4a^5x^5+12\ln(x)x^4a^4-8a^3x^3+4a^2x^2+4ax+1)\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{5}{2}}x}{4(ax+1)^3(a^2x^2-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	96

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*a^5*x^5+12*ln(x)*x^4*a^4-8*a^3*x^3+4*a^2*x^2+4*a*x+1)*(c*(a^2*x^2-1
)/a^2/x^2)^(5/2)*x/(a*x+1)^3/(a^2*x^2-1)/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.31

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \frac{(4a^5c^2x^5+12a^4c^2x^4\log(x)-8a^3c^2x^3+4a^2c^2x^2+4ac^2x+c^2)\sqrt{a^2c}}{4a^6x^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="fric
as")
```

output
```
1/4*(4*a^5*c^2*x^5 + 12*a^4*c^2*x^4*log(x) - 8*a^3*c^2*x^3 + 4*a^2*c^2*x^2
 + 4*a*c^2*x + c^2)*sqrt(a^2*c)/(a^6*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.46

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{1}{2}\left(\frac{4\,c^2x\mathrm{sgn}(x)}{a\mathrm{sgn}\,(ax+1)}+\frac{12\,c^2\log\left(|x|\right)\mathrm{sgn}(x)}{a^2\mathrm{sgn}\,(ax+1)}-\frac{8\,a^3c^2x^3\mathrm{sgn}(x)-4\,a^2c^2x^2\mathrm{sgn}(x)-4\,ac^2x\mathrm{sgn}}{a^6x^4\mathrm{sgn}\,(ax+1)}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac
")
```

output
```
1/2*(4*c^2*x*sgn(x)/(a*sgn(a*x + 1)) + 12*c^2*log(abs(x))*sgn(x)/(a^2*sgn(
a*x + 1)) - (8*a^3*c^2*x^3*sgn(x) - 4*a^2*c^2*x^2*sgn(x) - 4*a*c^2*x*sgn(x
) - c^2*sgn(x))/(a^6*x^4*sgn(a*x + 1)))*sqrt(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.23

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{\sqrt{c}\,c^2(12\log(x)\,a^4x^4+4a^5x^5-8a^3x^3+4a^2x^2+4ax+1)}{4a^5x^4}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x)
```

output
```
(sqrt(c)*c**2*(12*log(x)*a**4*x***4 + 4*a**5*x***5 - 8*a**3*x**3 + 4*a**2*x*
*2 + 4*a*x + 1))/(4*a**5*x**4)
```

3.825 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 148

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)/x^2-3*c*(c-c/a^2/x^2)^(
1/2)/a^2/(1-1/a^2/x^2)^(1/2)/x+c*(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1/2)
+3*c*(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.39

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2a^3x^2}-\frac{3}{a^2x}+x+\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

output
```
((c - c/(a^2*x^2))^(3/2)*(-1/2*1/(a^3*x^2) - 3/(a^2*x) + x + (3*Log[x])/a)
)/(1 - 1/(a^2*x^2))^(3/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.41, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{3/2} e^{3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^3}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int \left(a^3+\frac{3a^2}{x}+\frac{3a}{x^2}+\frac{1}{x^3}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(a^3x+3a^2\log(x)-\frac{3a}{x}-\frac{1}{2x^2}\right)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

output
```
(c*Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 - (3*a)/x + a^3*x + 3*a^2*Log[x]))/(a
^3*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{\left(2 a^{3} x^{3}+6 a^{2} \ln \left(x \right) x^{2}-6 a x -1\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{3}{2}} x}{2 \left(a x +1\right)^{3} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	69

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*a^3*x^3+6*a^2*ln(x)*x^2-6*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a
*x+1)^3/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.30

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{(2\,a^3cx^3+6\,a^2cx^2\log{(x)}-6\,acx-c)\sqrt{a^2c}}{2\,a^4x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="fric
as")
```

output
```
1/2*(2*a^3*c*x^3 + 6*a^2*c*x^2*log(x) - 6*a*c*x - c)*sqrt(a^2*c)/(a^4*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.48

$$\int e^{3\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{3/2} dx = \sqrt{c}\left(\frac{2\,cx\mathrm{sgn}(x)}{a\mathrm{sgn}\,(ax+1)} + \frac{6\,c\log\left(|x|\right)\mathrm{sgn}(x)}{a^2\mathrm{sgn}\,(ax+1)} - \frac{6\,acx\mathrm{sgn}(x) + c\mathrm{sgn}(x)}{a^4x^2\mathrm{sgn}\,(ax+1)}\right)|a|$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac
")
```

output
```
sqrt(c)*(2*c*x*sgn(x)/(a*sgn(a*x + 1)) + 6*c*log(abs(x))*sgn(x)/(a^2*sgn(a
*x + 1)) - (6*a*c*x*sgn(x) + c*sgn(x))/(a^4*x^2*sgn(a*x + 1)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{3/2} dx = \int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.24

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\sqrt{c}\,c(6\log(x)\,a^2x^2+2a^3x^3-6ax-1)}{2a^3x^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x)
```

output
```
(sqrt(c)*c*(6*log(x)*a**2*x**2 + 2*a**3*x**3 - 6*a*x - 1))/(2*a**3*x**2)
```

3.826 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1/2)-(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1
/a^2/x^2)^(1/2)+4*(c-c/a^2/x^2)^(1/2)*ln(-a*x+1)/a/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x-\frac{\log(x)}{a}+\frac{4\log(1-ax)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x - Log[x]/a + (4*Log[1 - a*x])/a))/Sqrt[1 - 1/(a^
2*x^2)]
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.46, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6751, 6747, 25, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ax+1)^2}{x(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a}{ax-1}-a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}(-ax-4\log(1-ax)+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-(a*x) + Log[x] - 4*Log[1 - a*x]))/(a*Sqrt[1 - 1
/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{(ax+4\ln(ax-1)-\ln(x))x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,(ax-1)}{(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	65

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x+4*ln(a*x-1)-ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/
((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.25

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax+4\log{(ax-1)}-\log{(x)})}{a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

output
```
sqrt(a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.58

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = 2\sqrt{c}\left(\frac{x\mathrm{sgn}(x)}{a\mathrm{sgn}\,(ax+1)}+\frac{4\,\log\left(|ax-1|\right)\mathrm{sgn}(x)}{a^2\mathrm{sgn}\,(ax+1)}-\frac{\log\left(|x|\right)\mathrm{sgn}(x)}{a^2\mathrm{sgn}\,(ax+1)}\right)|a|$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

output
```
2*sqrt(c)*(x*sgn(x)/(a*sgn(a*x + 1)) + 4*log(abs(a*x - 1))*sgn(x)/(a^2*sgn
(a*x + 1)) - log(abs(x))*sgn(x)/(a^2*sgn(a*x + 1)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.20

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c}\,(4\log(ax-1)-\log(x)+ax)}{a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x)
```

output
```
(sqrt(c)*(4*log(a*x - 1) - log(x) + a*x))/a
```

3.827 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/(c-c/a^2/x^2)^(1/2)+2*(1-1/a^2/x^2)^(1/2)/a/(c-c/a^2
/x^2)^(1/2)/(-a*x+1)+3*(1-1/a^2/x^2)^(1/2)*ln(-a*x+1)/a/(c-c/a^2/x^2)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(x+\frac{2}{a(1-ax)}+\frac{3\log(1-ax)}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*(x + 2/(a*(1 - a*x)) + (3*Log[1 - a*x])/a))/Sqrt[c
- c/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x(ax+1)}{(1-ax)^2}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{86}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{3}{(ax-1)a}+\frac{2}{(ax-1)^2a}+\frac{1}{a}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{2}{a^2(1-ax)}+\frac{3\log(1-ax)}{a^2}+\frac{x}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*(x/a + 2/(a^2*(1 - a*x)) + (3*Log[1 - a*x])/a^2))
/Sqrt[c - c/(a^2*x^2)]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{(ax-1)\left(a^{2}x^{2}+3a\ln\left(ax-1\right)x-ax-3\ln\left(ax-1\right)-2\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,x\,a^{2}}$	85

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/
a^2*(a^2*x^2+3*a*ln(a*x-1)*x-a*x-3*ln(a*x-1)-2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.43

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{(a^2x^2-ax+3\,(ax-1)\log{(ax-1)}-2)\sqrt{a^2c}}{a^3cx-a^2c}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

output
```
(a^2*x^2 - a*x + 3*(a*x - 1)*log(a*x - 1) - 2)*sqrt(a^2*c)/(a^3*c*x - a^2*
c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.40

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{c}\,(3\log(ax-1)\,ax-3\log(ax-1)+a^2x^2-3ax)}{ac\,(ax-1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x)
```

output
```
(sqrt(c)*(3*log(a*x - 1)*a*x - 3*log(a*x - 1) + a**2*x**2 - 3*a*x))/(a*c*(
a*x - 1))
```

3.828 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 171

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/c/(c-c/a^2/x^2)^(1/2)-1/2*(1-1/a^2/x^2)^(1/2)/a/c/(c
-c/a^2/x^2)^(1/2)/(-a*x+1)^2+3*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
/(-a*x+1)+3*(1-1/a^2/x^2)^(1/2)*ln(-a*x+1)/a/c/(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.37

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(x+\frac{5-6ax}{2a(-1+ax)^2}+\frac{3\log(1-ax)}{a}\right)}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(3/2)*(x + (5 - 6*a*x)/(2*a*(-1 + a*x)^2) + (3*Log[1 -
a*x])/a))/(c - c/(a^2*x^2))^(3/2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.49, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x^3}{(1-ax)^3}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^3}{(1-ax)^3}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \left(-\frac{1}{a^3}-\frac{3}{a^3(ax-1)}-\frac{3}{a^3(ax-1)^2}-\frac{1}{a^3(ax-1)^3}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(-\frac{3}{a^4(1-ax)}+\frac{1}{2a^4(1-ax)^2}-\frac{3\log(1-ax)}{a^4}-\frac{x}{a^3}\right)}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

output
```
-((a^3*Sqrt[1 - 1/(a^2*x^2)]*(-(x/a^3) + 1/(2*a^4*(1 - a*x)^2) - 3/(a^4*(1
 - a*x)) - (3*Log[1 - a*x])/a^4))/(c*Sqrt[c - c/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{(ax-1)\left(2a^{3}x^{3}+6a^{2}\ln(ax-1)x^{2}-4a^{2}x^{2}-12a\ln(ax-1)x-4ax+6\ln(ax-1)+5\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a^{4}x^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}}$	102

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(2*a^3*x^3+6*a^2*ln(a*x-1)*x^2-4*a^2*x
^2-12*a*ln(a*x-1)*x-4*a*x+6*ln(a*x-1)+5)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(
3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.47

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(2\,a^{3}x^{3}-4\,a^{2}x^{2}-4\,ax+6\,\left(a^{2}x^{2}-2\,ax+1\right)\log\left(ax-1\right)+5\right)\sqrt{a^{2}c}}{2\,\left(a^{4}c^{2}x^{2}-2\,a^{3}c^{2}x+a^{2}c^{2}\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fric
as")
```

output
```
1/2*(2*a^3*x^3 - 4*a^2*x^2 - 4*a*x + 6*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1)
+ 5)*sqrt(a^2*c)/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.44

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\sqrt{c}\,(6\log(ax-1)\,a^2x^2-12\log(ax-1)\,ax+6\log(ax-1)+2a^3x^3-6a^2x^2+3)}{2a\,c^2\,(a^2x^2-2ax+1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x)
```

output
```
(sqrt(c)*(6*log(a*x - 1)*a**2*x**2 - 12*log(a*x - 1)*a*x + 6*log(a*x - 1)
+ 2*a**3*x**3 - 6*a**2*x**2 + 3))/(2*a*c**2*(a**2*x**2 - 2*a*x + 1))
```

3.829 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 267

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^3}\\ &\quad-\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}+\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}\\ &\quad+\frac{49\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}\end{aligned}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/c^2/(c-c/a^2/x^2)^(1/2)+1/6*(1-1/a^2/x^2)^(1/2)/a/c^
2/(c-c/a^2/x^2)^(1/2)/(-a*x+1)^3-9/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^
2)^(1/2)/(-a*x+1)^2+31/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)/(-a
*x+1)+49/16*(1-1/a^2/x^2)^(1/2)*ln(-a*x+1)/a/c^2/(c-c/a^2/x^2)^(1/2)-1/16*
(1-1/a^2/x^2)^(1/2)*ln(a*x+1)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.36

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(48x-\frac{8}{a(-1+ax)^3}-\frac{54}{a(-1+ax)^2}+\frac{186}{a-a^2x}+\frac{147\log(1-ax)}{a}-\frac{3\log(1+ax)}{a}\right)}{48\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(5/2)*(48*x - 8/(a*(-1 + a*x)^3) - 54/(a*(-1 + a*x)^2)
+ 186/(a - a^2*x) + (147*Log[1 - a*x])/a - (3*Log[1 + a*x])/a))/(48*(c - c
/(a^2*x^2))^(5/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.43, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^5}{(1-ax)^4(ax+1)}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int\left(-\frac{1}{16a^5(ax+1)}+\frac{1}{a^5}+\frac{49}{16a^5(ax-1)}+\frac{31}{8a^5(ax-1)^2}+\frac{9}{4a^5(ax-1)^3}+\frac{1}{2a^5(ax-1)^4}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{31}{8a^6(1-ax)}-\frac{9}{8a^6(1-ax)^2}+\frac{1}{6a^6(1-ax)^3}+\frac{49\log(1-ax)}{16a^6}-\frac{\log(ax+1)}{16a^6}+\frac{x}{a^5}\right)}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

output
```
(a^5*Sqrt[1 - 1/(a^2*x^2)]*(x/a^5 + 1/(6*a^6*(1 - a*x)^3) - 9/(8*a^6*(1 -
a*x)^2) + 31/(8*a^6*(1 - a*x)) + (49*Log[1 - a*x])/(16*a^6) - Log[1 + a*x]
/(16*a^6)))/(c^2*Sqrt[c - c/(a^2*x^2)])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.66

method	result
default	$-\frac{(ax-1)(ax+1)(-48a^4x^4+3\ln(ax+1)x^3a^3-147a^3\ln(ax-1)x^3+144a^3x^3-9\ln(ax+1)x^2a^2+441a^2\ln(ax-1)x^2+42a^2x^2+9\ln(a}{48\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a^6x^5\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{5}{2}}}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-48*a^4*x^4+3*ln(a*x+1)*x^3
*a^3-147*a^3*ln(a*x-1)*x^3+144*a^3*x^3-9*ln(a*x+1)*x^2*a^2+441*a^2*ln(a*x-
1)*x^2+42*a^2*x^2+9*ln(a*x+1)*x*a-441*a*ln(a*x-1)*x-270*a*x-3*ln(a*x+1)+14
7*ln(a*x-1)+140)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.52

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{(48\,a^4x^4-144\,a^3x^3-42\,a^2x^2+270\,ax-3\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log(ax+1)+}{48\,(a^5c^3x^3-3\,a^4c^3x^2+3\,a^3c^3x-}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fric
as")
```

output
```
1/48*(48*a^4*x^4 - 144*a^3*x^3 - 42*a^2*x^2 + 270*a*x - 3*(a^3*x^3 - 3*a^2
*x^2 + 3*a*x - 1)*log(a*x + 1) + 147*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log
(a*x - 1) - 140)*sqrt(a^2*c)/(a^5*c^3*x^3 - 3*a^4*c^3*x^2 + 3*a^3*c^3*x -
a^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.55

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{\sqrt{c}\,(147\log(ax-1)\,a^3x^3-441\log(ax-1)\,a^2x^2+441\log(ax-1)\,ax-147\log(ax$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x)
```

output
```
(sqrt(c)*(147*log(a*x - 1)*a**3*x**3 - 441*log(a*x - 1)*a**2*x**2 + 441*lo
g(a*x - 1)*a*x - 147*log(a*x - 1) - 3*log(a*x + 1)*a**3*x**3 + 9*log(a*x +
 1)*a**2*x**2 - 9*log(a*x + 1)*a*x + 3*log(a*x + 1) + 48*a**4*x**4 - 158*a
**3*x**3 + 228*a*x - 126))/(48*a*c**3*(a**3*x**3 - 3*a**2*x**2 + 3*a*x - 1
))
```

3.830 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 360

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^4}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^3}-\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}+\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{201\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{9\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/c^3/(c-c/a^2/x^2)^(1/2)-1/16*(1-1/a^2/x^2)^(1/2)/a/c
^3/(c-c/a^2/x^2)^(1/2)/(-a*x+1)^4+1/2*(1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x
^2)^(1/2)/(-a*x+1)^3-59/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)/(
-a*x+1)^2+75/16*(1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)/(-a*x+1)-1/3
2*(1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)/(a*x+1)+201/64*(1-1/a^2/x^
2)^(1/2)*ln(-a*x+1)/a/c^3/(c-c/a^2/x^2)^(1/2)-9/64*(1-1/a^2/x^2)^(1/2)*ln(
a*x+1)/a/c^3/(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.32

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(\frac{2(104-207ax-59a^2x^2+309a^3x^3-87a^4x^4-96a^5x^5+32a^6x^6)}{(-1+ax)^4(1+ax)}+201\log(1-ax)-9\log(1+ax)\right)}{64a\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

output
```
((1 - 1/(a^2*x^2))^(7/2)*((2*(104 - 207*a*x - 59*a^2*x^2 + 309*a^3*x^3 - 8
7*a^4*x^4 - 96*a^5*x^5 + 32*a^6*x^6))/((-1 + a*x)^4*(1 + a*x)) + 201*Log[1
 - a*x] - 9*Log[1 + a*x]))/(64*a*(c - c/(a^2*x^2))^(7/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.40, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x^7}{(1-ax)^5(ax+1)^2}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^7}{(1-ax)^5(ax+1)^2}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \textbf{99}$$

$$-\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{9}{64a^7(ax+1)}-\frac{1}{32a^7(ax+1)^2}-\frac{1}{a^7}-\frac{201}{64a^7(ax-1)}-\frac{75}{16a^7(ax-1)^2}-\frac{59}{16a^7(ax-1)^3}-\frac{3}{2a^7(ax-1)^4}-\frac{1}{4a^7(ax-1)}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\left(-\frac{75}{16a^8(1-ax)}+\frac{1}{32a^8(ax+1)}+\frac{59}{32a^8(1-ax)^2}-\frac{1}{2a^8(1-ax)^3}+\frac{1}{16a^8(1-ax)^4}-\frac{201\log(1-ax)}{64a^8}+\frac{9\log(ax+1)}{64a^8}-\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

output
```
-((a^7*Sqrt[1 - 1/(a^2*x^2)]*(-(x/a^7) + 1/(16*a^8*(1 - a*x)^4) - 1/(2*a^8
*(1 - a*x)^3) + 59/(32*a^8*(1 - a*x)^2) - 75/(16*a^8*(1 - a*x)) + 1/(32*a^
8*(1 + a*x)) - (201*Log[1 - a*x])/(64*a^8) + (9*Log[1 + a*x])/(64*a^8)))/(
c^3*Sqrt[c - c/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.69

method	result
default	$-\frac{(ax-1)(ax+1)\left(-64x^6a^6+9\ln(ax+1)x^5a^5-201\ln(ax-1)x^5a^5+192a^5x^5-27\ln(ax+1)x^4a^4+603\ln(ax-1)x^4a^4+174a^4x^4+18\ln\right.}{}$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-64*x^6*a^6+9*ln(a*x+1)*x^5
*a^5-201*ln(a*x-1)*x^5*a^5+192*a^5*x^5-27*ln(a*x+1)*x^4*a^4+603*ln(a*x-1)*
x^4*a^4+174*a^4*x^4+18*ln(a*x+1)*x^3*a^3-402*a^3*ln(a*x-1)*x^3-618*a^3*x^3
+18*ln(a*x+1)*x^2*a^2-402*a^2*ln(a*x-1)*x^2+118*a^2*x^2-27*ln(a*x+1)*x*a+6
03*a*ln(a*x-1)*x+414*a*x+9*ln(a*x+1)-201*ln(a*x-1)-208)/a^8/x^7/(c*(a^2*x^
2-1)/a^2/x^2)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.58

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{(64\,a^6x^6 - 192\,a^5x^5 - 174\,a^4x^4 + 618\,a^3x^3 - 118\,a^2x^2 - 414\,ax - 9\,(a^5x^5 - 3\,a^4x^4 +}{64\,(a^7c^4x^5 - 3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fric
as")
```

output
```
1/64*(64*a^6*x^6 - 192*a^5*x^5 - 174*a^4*x^4 + 618*a^3*x^3 - 118*a^2*x^2 -
 414*a*x - 9*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 - 3*a*x + 1)*log
(a*x + 1) + 201*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 - 3*a*x + 1)*
log(a*x - 1) + 208)*sqrt(a^2*c)/(a^7*c^4*x^5 - 3*a^6*c^4*x^4 + 2*a^5*c^4*x
^3 + 2*a^4*c^4*x^2 - 3*a^3*c^4*x + a^2*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.65

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{c}\,(201\log(ax-1)\,a^5x^5-603\log(ax-1)\,a^4x^4+402\log(ax-1)\,a^3x^3+402\log(ax}{}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x)
```

output
```
(sqrt(c)*(201*log(a*x - 1)*a**5*x**5 - 603*log(a*x - 1)*a**4*x**4 + 402*lo
g(a*x - 1)*a**3*x**3 + 402*log(a*x - 1)*a**2*x**2 - 603*log(a*x - 1)*a*x +
 201*log(a*x - 1) - 9*log(a*x + 1)*a**5*x**5 + 27*log(a*x + 1)*a**4*x**4 -
 18*log(a*x + 1)*a**3*x**3 - 18*log(a*x + 1)*a**2*x**2 + 27*log(a*x + 1)*a
*x - 9*log(a*x + 1) + 64*a**6*x**6 - 250*a**5*x**5 + 502*a**3*x**3 - 234*a
**2*x**2 - 240*a*x + 150))/(64*a*c**4*(a**5*x**5 - 3*a**4*x**4 + 2*a**3*x*
*3 + 2*a**2*x**2 - 3*a*x + 1))
```

3.831 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 322

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}x^6}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output

```
-1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/(1-1/a^2/x^2)^(1/2)/x^6+1/5*c^3*(c-c/a^2/
x^2)^(1/2)/a^6/(1-1/a^2/x^2)^(1/2)/x^5+3/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/(1-
1/a^2/x^2)^(1/2)/x^4-c^3*(c-c/a^2/x^2)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)/x^3-3
/2*c^3*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)/x^2+3*c^3*(c-c/a^2/x^2)
^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)/x+c^3*(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(
1/2)-c^3*(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6a^7x^6}+\frac{1}{5a^6x^5}+\frac{3}{4a^5x^4}-\frac{1}{a^4x^3}-\frac{3}{2a^3x^2}+\frac{3}{a^2x}+x-\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(7/2)/E^ArcCoth[a*x],x]
```

output
```
((c - c/(a^2*x^2))^(7/2)*(-1/6*1/(a^7*x^6) + 1/(5*a^6*x^5) + 3/(4*a^5*x^4)
 - 1/(a^4*x^3) - 3/(2*a^3*x^2) + 3/(a^2*x) + x - Log[x]/a))/(1 - 1/(a^2*x^
2))^(7/2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{7/2}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^4(ax+1)^3}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int\left(a^7-\frac{a^6}{x}-\frac{3a^5}{x^2}+\frac{3a^4}{x^3}+\frac{3a^3}{x^4}-\frac{3a^2}{x^5}-\frac{a}{x^6}+\frac{1}{x^7}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(a^7x-a^6\log(x)+\frac{3a^5}{x}-\frac{3a^4}{2x^2}-\frac{a^3}{x^3}+\frac{3a^2}{4x^4}+\frac{a}{5x^5}-\frac{1}{6x^6}\right)}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(c - c/(a^2*x^2))^(7/2)/E^ArcCoth[a*x],x]
```

output
```
(c^3*Sqrt[c - c/(a^2*x^2)]*(-1/6*1/x^6 + a/(5*x^5) + (3*a^2)/(4*x^4) - a^3
/x^3 - (3*a^4)/(2*x^2) + (3*a^5)/x + a^7*x - a^6*Log[x]))/(a^7*Sqrt[1 - 1/
(a^2*x^2)])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$-\frac{\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,x\left(-60a^{7}x^{7}+60a^{6}\ln\left(x\right)x^{6}-180a^{5}x^{5}+90a^{4}x^{4}+60a^{3}x^{3}-45a^{2}x^{2}-12ax+10\right)}{60\left(ax-1\right)\left(a^{2}x^{2}-1\right)^{3}}$	112

input
```
int((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2)*x*(-60*a^7*x^7
+60*a^6*ln(x)*x^6-180*a^5*x^5+90*a^4*x^4+60*a^3*x^3-45*a^2*x^2-12*a*x+10)/
(a*x-1)/(a^2*x^2-1)^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \frac{(60\,a^7c^3x^7-60\,a^6c^3x^6\log{(x)}+180\,a^5c^3x^5-90\,a^4c^3x^4-60\,a^3c^3x^3+45\,a^2c^3x^2+12\,ac^3x}{60\,a^8x^6}$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

output
```
1/60*(60*a^7*c^3*x^7 - 60*a^6*c^3*x^6*log(x) + 180*a^5*c^3*x^5 - 90*a^4*c^
3*x^4 - 60*a^3*c^3*x^3 + 45*a^2*c^3*x^2 + 12*a*c^3*x - 10*c^3)*sqrt(a^2*c)
/(a^8*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \int \left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

output
```
integrate((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.48

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = \frac{1}{60}\left(\frac{60\,c^3x\operatorname{sgn}(ax+1)\operatorname{sgn}(x)}{a}-\frac{60\,c^3\log\left(|x|\right)\operatorname{sgn}(ax+1)\operatorname{sgn}(x)}{a^2}+\frac{180\,a^5c^3x^5\operatorname{sgn}(ax+\right.$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
1/60*(60*c^3*x*sgn(a*x + 1)*sgn(x)/a - 60*c^3*log(abs(x))*sgn(a*x + 1)*sgn
(x)/a^2 + (180*a^5*c^3*x^5*sgn(a*x + 1)*sgn(x) - 90*a^4*c^3*x^4*sgn(a*x +
1)*sgn(x) - 60*a^3*c^3*x^3*sgn(a*x + 1)*sgn(x) + 45*a^2*c^3*x^2*sgn(a*x +
1)*sgn(x) + 12*a*c^3*x*sgn(a*x + 1)*sgn(x) - 10*c^3*sgn(a*x + 1)*sgn(x))/(
a^8*x^6))*sqrt(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int \left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.25

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\frac{\sqrt{c}\,c^3(-60\log(ax)\,a^6x^6+60a^7x^7-240a^6x^6+180a^5x^5-90a^4x^4-60a^3x^3+45a^2x^2+12ax-10)}{60a^7x^6}$$

input
```
int((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*c**3*( - 60*log(a*x)*a**6*x**6 + 60*a**7*x**7 - 240*a**6*x**6 + 1
80*a**5*x**5 - 90*a**4*x**4 - 60*a**3*x**3 + 45*a**2*x**2 + 12*a*x - 10))/
(60*a**7*x**6)
```

3.832 $\int e^{-\coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^{5/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 238

$$\int e^{-\coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^{5/2} dx = \frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4} - \frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}x^3} - \frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2} + \frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x} + \frac{c^2\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/(1-1/a^2/x^2)^(1/2)/x^4-1/3*c^2*(c-c/a^2/x
^2)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)/x^3-c^2*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2
/x^2)^(1/2)/x^2+2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)/x+c^2*(c
-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1/2)-c^2*(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1
-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.32

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4a^5x^4}-\frac{1}{3a^4x^3}-\frac{1}{a^3x^2}+\frac{2}{a^2x}+x-\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(5/2)/E^ArcCoth[a*x],x]
```

output
```
((c - c/(a^2*x^2))^(5/2)*(1/(4*a^5*x^4) - 1/(3*a^4*x^3) - 1/(a^3*x^2) + 2/
(a^2*x) + x - Log[x]/a))/(1 - 1/(a^2*x^2))^(5/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.34, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{5/2}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(1-ax)^3(ax+1)^2}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^3(ax+1)^2}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int\left(-a^5+\frac{a^4}{x}+\frac{2a^3}{x^2}-\frac{2a^2}{x^3}-\frac{a}{x^4}+\frac{1}{x^5}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(a^5(-x)+a^4\log(x)-\frac{2a^3}{x}+\frac{a^2}{x^2}+\frac{a}{3x^3}-\frac{1}{4x^4}\right)}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(c - c/(a^2*x^2))^(5/2)/E^ArcCoth[a*x],x]
```

output
```
-((c^2*Sqrt[c - c/(a^2*x^2)]*(-1/4*1/x^4 + a/(3*x^3) + a^2/x^2 - (2*a^3)/x
 - a^5*x + a^4*Log[x]))/(a^5*Sqrt[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.40

method	result	size
default	$-\frac{\left(\frac{c\left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{5}{2}} \sqrt{\frac{a x -1}{a x +1}}\, x \left(-12 a^{5} x^{5}+12 \ln \left(x \right) x^{4} a^{4}-24 a^{3} x^{3}+12 a^{2} x^{2}+4 a x -3\right)}{12 \left(a x -1\right) \left(a^{2} x^{2}-1\right)^{2}}$	96

input
```
int((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2)*x*(-12*a^5*x^5
+12*ln(x)*x^4*a^4-24*a^3*x^3+12*a^2*x^2+4*a*x-3)/(a*x-1)/(a^2*x^2-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{\left(12\,a^{5}c^{2}x^{5}-12\,a^{4}c^{2}x^{4}\log\left(x\right)+24\,a^{3}c^{2}x^{3}-12\,a^{2}c^{2}x^{2}-4\,ac^{2}x+3\,c^{2}\right)\sqrt{a^{2}c}}{12\,a^{6}x^{4}}$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

output
```
1/12*(12*a^5*c^2*x^5 - 12*a^4*c^2*x^4*log(x) + 24*a^3*c^2*x^3 - 12*a^2*c^2
*x^2 - 4*a*c^2*x + 3*c^2)*sqrt(a^2*c)/(a^6*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \int \left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

output
```
integrate((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.50

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \frac{1}{12}\left(\frac{12\,c^2x\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a} - \frac{12\,c^2\log\left(|x|\right)\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a^2} + \frac{24\,a^3c^2x^3\mathrm{sgn}(ax+1)}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
1/12*(12*c^2*x*sgn(a*x + 1)*sgn(x)/a - 12*c^2*log(abs(x))*sgn(a*x + 1)*sgn
(x)/a^2 + (24*a^3*c^2*x^3*sgn(a*x + 1)*sgn(x) - 12*a^2*c^2*x^2*sgn(a*x + 1
)*sgn(x) - 4*a*c^2*x*sgn(a*x + 1)*sgn(x) + 3*c^2*sgn(a*x + 1)*sgn(x))/(a^6
*x^4))*sqrt(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int \left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.26

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{\sqrt{c}\,c^2(-12\log(ax)\,a^4x^4+12a^5x^5-36a^4x^4+24a^3x^3-12a^2x^2-4ax+3)}{12a^5x^4}$$

input
```
int((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*c**2*( - 12*log(a*x)*a**4*x**4 + 12*a**5*x**5 - 36*a**4*x**4 + 24
*a**3*x**3 - 12*a**2*x**2 - 4*a*x + 3))/(12*a**5*x**4)
```

3.833 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 147

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)/x^2+c*(c-c/a^2/x^2)^(1/
2)/a^2/(1-1/a^2/x^2)^(1/2)/x+c*(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1/2)-c
*(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.43

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{3}{2a}-\frac{1}{2a^3x^2}+\frac{1}{a^2x}+x-\frac{\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(3/2)/E^ArcCoth[a*x],x]
```

output
```
((c - c/(a^2*x^2))^(3/2)*(-3/(2*a) - 1/(2*a^3*x^2) + 1/(a^2*x) + x - Log[x
]/a))/(1 - 1/(a^2*x^2))^(3/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6751, 6747, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{3/2} e^{-\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2(ax+1)}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{84}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int \left(a^3-\frac{a^2}{x}-\frac{a}{x^2}+\frac{1}{x^3}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(a^3x-a^2\log(x)+\frac{a}{x}-\frac{1}{2x^2}\right)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(c - c/(a^2*x^2))^(3/2)/E^ArcCoth[a*x],x]
```

output
```
(c*Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 + a/x + a^3*x - a^2*Log[x]))/(a^3*Sqr
t[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{\left(\frac{c\left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{3}{2}} \sqrt{\frac{a x -1}{a x +1}}\, x \left(-2 a^{3} x^{3}+2 a^{2} \ln \left(x \right) x^{2}-2 a x +1\right)}{2 \left(a x -1\right) \left(a^{2} x^{2}-1\right)}$	80

input
```
int((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2)*x*(-2*a^3*x^3+2
*a^2*ln(x)*x^2-2*a*x+1)/(a*x-1)/(a^2*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.30

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{(2\,a^3cx^3-2\,a^2cx^2\log{(x)}+2\,acx-c)\sqrt{a^2c}}{2\,a^4x^2}$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

output
```
1/2*(2*a^3*c*x^3 - 2*a^2*c*x^2*log(x) + 2*a*c*x - c)*sqrt(a^2*c)/(a^4*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

output
```
integrate((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.49

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{1}{2}\left(\frac{2\,cx\text{sgn}(ax+1)\,\text{sgn}(x)}{a}-\frac{2\,c\log(|x|)\,\text{sgn}(ax+1)\,\text{sgn}(x)}{a^2}+\frac{2\,acx\text{sgn}(ax+1)\,\text{sgn}(x)}{a^4x}\right.$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
1/2*(2*c*x*sgn(a*x + 1)*sgn(x)/a - 2*c*log(abs(x))*sgn(a*x + 1)*sgn(x)/a^2
 + (2*a*c*x*sgn(a*x + 1)*sgn(x) - c*sgn(a*x + 1)*sgn(x))/(a^4*x^2))*sqrt(c
)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\int\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.31

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\sqrt{c}\,c(-2\log(ax)\,a^2x^2+2a^3x^3-4a^2x^2+2ax-1)}{2a^3x^2}$$

input
```
int((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*c*( - 2*log(a*x)*a**2*x**2 + 2*a**3*x**3 - 4*a**2*x**2 + 2*a*x -
1))/(2*a**3*x**2)
```

3.834 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1/2)-(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1
/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x-\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x - Log[x]/a))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{1-ax}{x}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1-ax}{x}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x}-a\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}(\log(x)-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-(a*x) + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{(-ax+\ln(x))x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$	52

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x
-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.28

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax-\log(x))}{a^2}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

output
```
sqrt(a^2*c)*(a*x - log(x))/a^2
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))),
x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.53

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \left(\frac{x\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a}-\frac{\log\left(|x|\right)\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a^2}\right)\sqrt{c}|a|$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
(x*sgn(a*x + 1)*sgn(x)/a - log(abs(x))*sgn(a*x + 1)*sgn(x)/a^2)*sqrt(c)*ab
s(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \int \sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.25

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c}\,(-\log(ax)+ax-1)}{a}$$

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*( - log(a*x) + a*x - 1))/a
```

3.835 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/(c-c/a^2/x^2)^(1/2)-(1-1/a^2/x^2)^(1/2)*ln(a*x+1)/a/
(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(x-\frac{\log(1+ax)}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*(x - Log[1 + a*x]/a))/Sqrt[c - c/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x}{ax+1}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{1}{a}-\frac{1}{a(ax+1)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{x}{a}-\frac{\log(ax+1)}{a^2}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*(x/a - Log[1 + a*x]/a^2))/Sqrt[c - c/(a^2*x^2)]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)(-ax+\ln(ax+1))}{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\,a^2}$	59

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*x+ln(a*x+1))/(c*(a^2*x^2-1)/a^2/x^2)^
(1/2)/x/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.36

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{a^2c}(ax-\log{(ax+1)})}{a^2c}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas
")
```

output
```
sqrt(a^2*c)*(a*x - log(a*x + 1))/(a^2*c)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))),
x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a^2*x^2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(1/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.29

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{c}\,(-\log(ax+1)+ax)}{ac}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x)
```

output
```
(sqrt(c)*( - log(a*x + 1) + a*x))/(a*c)
```

3.836 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1+ax)} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}} - \frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/c/(c-c/a^2/x^2)^(1/2)-1/2*(1-1/a^2/x^2)^(1/2)/a/c/(c
-c/a^2/x^2)^(1/2)/(a*x+1)+1/4*(1-1/a^2/x^2)^(1/2)*ln(-a*x+1)/a/c/(c-c/a^2/
x^2)^(1/2)-5/4*(1-1/a^2/x^2)^(1/2)*ln(a*x+1)/a/c/(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.41

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(4x-\frac{2}{a+a^2x}+\frac{\log(1-ax)}{a}-\frac{5\log(1+ax)}{a}\right)}{4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
```

output
```
((1 - 1/(a^2*x^2))^(3/2)*(4*x - 2/(a + a^2*x) + Log[1 - a*x]/a - (5*Log[1
+ a*x])/a))/(4*(c - c/(a^2*x^2))^(3/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.49, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x^3}{(1-ax)(ax+1)^2}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^3}{(1-ax)(ax+1)^2}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{5}{4a^3(ax+1)}-\frac{1}{2a^3(ax+1)^2}-\frac{1}{a^3}-\frac{1}{4a^3(ax-1)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{2a^4(ax+1)}-\frac{\log(1-ax)}{4a^4}+\frac{5\log(ax+1)}{4a^4}-\frac{x}{a^3}\right)}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
```

output
```
-((a^3*Sqrt[1 - 1/(a^2*x^2)]*(-(x/a^3) + 1/(2*a^4*(1 + a*x)) - Log[1 - a*x
]/(4*a^4) + (5*Log[1 + a*x])/(4*a^4)))/(c*Sqrt[c - c/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-4a^{2}x^{2}+5\ln\left(ax+1\right)xa-a\ln\left(ax-1\right)x-4ax+5\ln\left(ax+1\right)-\ln\left(ax-1\right)+2\right)\left(ax-1\right)}{4a^{4}x^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}}$	103

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-4*a^2*x^2+5*ln(a*x+1)*x*a-a*ln(a*x-
1)*x-4*a*x+5*ln(a*x+1)-ln(a*x-1)+2)*(a*x-1)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2
)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.38

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(4\,a^{2}x^{2}+4\,ax-5\,(ax+1)\log\left(ax+1\right)+(ax+1)\log\left(ax-1\right)-2\right)\sqrt{a^{2}c}}{4\,\left(a^{3}c^{2}x+a^{2}c^{2}\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas
")
```

output
```
1/4*(4*a^2*x^2 + 4*a*x - 5*(a*x + 1)*log(a*x + 1) + (a*x + 1)*log(a*x - 1)
 - 2)*sqrt(a^2*c)/(a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(3/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.37

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\sqrt{c}\,(\log(ax-1)\,ax+\log(ax-1)-5\log(ax+1)\,ax-5\log(ax+1)+4a^2x^2+6ax)}{4a\,c^2\,(ax+1)}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x)
```

output
```
(sqrt(c)*(log(a*x - 1)*a*x + log(a*x - 1) - 5*log(a*x + 1)*a*x - 5*log(a*x
 + 1) + 4*a**2*x**2 + 6*a*x))/(4*a*c**2*(a*x + 1))
```

3.837 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 263

$$\begin{aligned}\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}\\ &+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}\\ &+\frac{7\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{23\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}\end{aligned}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/c^2/(c-c/a^2/x^2)^(1/2)+1/8*(1-1/a^2/x^2)^(1/2)/a/c^
2/(c-c/a^2/x^2)^(1/2)/(-a*x+1)+1/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)
^(1/2)/(a*x+1)^2-(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)/(a*x+1)+7/1
6*(1-1/a^2/x^2)^(1/2)*ln(-a*x+1)/a/c^2/(c-c/a^2/x^2)^(1/2)-23/16*(1-1/a^2/
x^2)^(1/2)*ln(a*x+1)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.36

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(2\left(8x+\frac{1}{a(1+ax)^2}+\frac{1}{a-a^2x}-\frac{8}{a+a^2x}\right)+\frac{7\log(1-ax)}{a}-\frac{23\log(1+ax)}{a}\right)}{16\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2)),x]
```

output
```
((1 - 1/(a^2*x^2))^(5/2)*(2*(8*x + 1/(a*(1 + a*x)^2) + (a - a^2*x)^(-1) -
8/(a + a^2*x)) + (7*Log[1 - a*x])/a - (23*Log[1 + a*x])/a))/(16*(c - c/(a^
2*x^2))^(5/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.42, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^5}{(1-ax)^2(ax+1)^3}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int \left(-\frac{23}{16a^5(ax+1)}+\frac{1}{a^5(ax+1)^2}-\frac{1}{4a^5(ax+1)^3}+\frac{1}{a^5}+\frac{7}{16a^5(ax-1)}+\frac{1}{8a^5(ax-1)^2}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{8a^6(1-ax)}-\frac{1}{a^6(ax+1)}+\frac{1}{8a^6(ax+1)^2}+\frac{7\log(1-ax)}{16a^6}-\frac{23\log(ax+1)}{16a^6}+\frac{x}{a^5}\right)}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2)),x]
```

output
```
(a^5*Sqrt[1 - 1/(a^2*x^2)]*(x/a^5 + 1/(8*a^6*(1 - a*x)) + 1/(8*a^6*(1 + a*
x)^2) - 1/(a^6*(1 + a*x)) + (7*Log[1 - a*x])/(16*a^6) - (23*Log[1 + a*x])/
(16*a^6)))/(c^2*Sqrt[c - c/(a^2*x^2)])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.67

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)(ax-1)\left(-16a^4x^4+23\ln(ax+1)x^3a^3-7a^3\ln(ax-1)x^3-16a^3x^3+23\ln(ax+1)x^2a^2-7a^2\ln(ax-1)x^2+34a^2x^2-2\right.}{16a^6x^5\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(a*x-1)*(-16*a^4*x^4+23*ln(a*x+1)*x^
3*a^3-7*a^3*ln(a*x-1)*x^3-16*a^3*x^3+23*ln(a*x+1)*x^2*a^2-7*a^2*ln(a*x-1)*
x^2+34*a^2*x^2-23*ln(a*x+1)*x*a+7*a*ln(a*x-1)*x+18*a*x-23*ln(a*x+1)+7*ln(a
*x-1)-12)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.51

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(16\,a^4x^4+16\,a^3x^3-34\,a^2x^2-18\,ax-23\,(a^3x^3+a^2x^2-ax-1)\log(ax+1)+7\,(a\right.}{16\,(a^5c^3x^3+a^4c^3x^2-a^3c^3x-a^2c^3)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas
")
```

output
```
1/16*(16*a^4*x^4 + 16*a^3*x^3 - 34*a^2*x^2 - 18*a*x - 23*(a^3*x^3 + a^2*x^
2 - a*x - 1)*log(a*x + 1) + 7*(a^3*x^3 + a^2*x^2 - a*x - 1)*log(a*x - 1) +
 12)*sqrt(a^2*c)/(a^5*c^3*x^3 + a^4*c^3*x^2 - a^3*c^3*x - a^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(5/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.56

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{\sqrt{c}\,(7\log(ax-1)\,a^3x^3+7\log(ax-1)\,a^2x^2-7\log(ax-1)\,ax-7\log(ax-1)-23}{16}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x)
```

output
```
(sqrt(c)*(7*log(a*x - 1)*a**3*x**3 + 7*log(a*x - 1)*a**2*x**2 - 7*log(a*x
- 1)*a*x - 7*log(a*x - 1) - 23*log(a*x + 1)*a**3*x**3 - 23*log(a*x + 1)*a*
*2*x**2 + 23*log(a*x + 1)*a*x + 23*log(a*x + 1) + 16*a**4*x**4 + 50*a**3*x
**3 - 52*a*x - 22))/(16*a*c**3*(a**3*x**3 + a**2*x**2 - a*x - 1))
```

3.838 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 358

$$\begin{aligned}\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}\\
&+\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^3}\\
&+\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}\\
&+\frac{19\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{51\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}\end{aligned}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/c^3/(c-c/a^2/x^2)^(1/2)-1/32*(1-1/a^2/x^2)^(1/2)/a/c
^3/(c-c/a^2/x^2)^(1/2)/(-a*x+1)^2+5/16*(1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/
x^2)^(1/2)/(-a*x+1)-1/24*(1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)/(a*
x+1)^3+11/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)/(a*x+1)^2-3/2*(
1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)/(a*x+1)+19/32*(1-1/a^2/x^2)^(
1/2)*ln(-a*x+1)/a/c^3/(c-c/a^2/x^2)^(1/2)-51/32*(1-1/a^2/x^2)^(1/2)*ln(a*x
+1)/a/c^3/(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.33

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(96x-\frac{3}{a(-1+ax)^2}-\frac{4}{a(1+ax)^3}+\frac{33}{a(1+ax)^2}+\frac{30}{a-a^2x}-\frac{144}{a+a^2x}+\frac{57\log(1-ax)}{a}-\right.}{96\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

input
```
Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2)),x]
```

output
```
((1 - 1/(a^2*x^2))^(7/2)*(96*x - 3/(a*(-1 + a*x)^2) - 4/(a*(1 + a*x)^3) +
33/(a*(1 + a*x)^2) + 30/(a - a^2*x) - 144/(a + a^2*x) + (57*Log[1 - a*x])/
a - (153*Log[1 + a*x])/a))/(96*(c - c/(a^2*x^2))^(7/2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.40, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x^7}{(1-ax)^3(ax+1)^4}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^7}{(1-ax)^3(ax+1)^4}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{51}{32a^7(ax+1)}-\frac{3}{2a^7(ax+1)^2}+\frac{11}{16a^7(ax+1)^3}-\frac{1}{8a^7(ax+1)^4}-\frac{1}{a^7}-\frac{19}{32a^7(ax-1)}-\frac{5}{16a^7(ax-1)^2}-\frac{1}{16a^7(ax-1)}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\left(-\frac{5}{16a^8(1-ax)}+\frac{3}{2a^8(ax+1)}+\frac{1}{32a^8(1-ax)^2}-\frac{11}{32a^8(ax+1)^2}+\frac{1}{24a^8(ax+1)^3}-\frac{19\log(1-ax)}{32a^8}+\frac{51\log(ax+1)}{32a^8}-\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2)),x]
```

output
```
-((a^7*Sqrt[1 - 1/(a^2*x^2)]*(-(x/a^7) + 1/(32*a^8*(1 - a*x)^2) - 5/(16*a^
8*(1 - a*x)) + 1/(24*a^8*(1 + a*x)^3) - 11/(32*a^8*(1 + a*x)^2) + 3/(2*a^8
*(1 + a*x)) - (19*Log[1 - a*x])/(32*a^8) + (51*Log[1 + a*x])/(32*a^8)))/(c
^3*Sqrt[c - c/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.69

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)(ax-1)\left(-96x^6a^6+153\ln(ax+1)x^5a^5-57\ln(ax-1)x^5a^5-96a^5x^5+153\ln(ax+1)x^4a^4-57\ln(ax-1)x^4a^4+366a^4\right.}{}$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(a*x-1)*(-96*x^6*a^6+153*ln(a*x+1)*x
^5*a^5-57*ln(a*x-1)*x^5*a^5-96*a^5*x^5+153*ln(a*x+1)*x^4*a^4-57*ln(a*x-1)*
x^4*a^4+366*a^4*x^4-306*ln(a*x+1)*x^3*a^3+114*a^3*ln(a*x-1)*x^3+222*a^3*x^
3-306*ln(a*x+1)*x^2*a^2+114*a^2*ln(a*x-1)*x^2-338*a^2*x^2+153*ln(a*x+1)*x*
a-57*a*ln(a*x-1)*x-122*a*x+153*ln(a*x+1)-57*ln(a*x-1)+88)/a^8/x^7/(c*(a^2*
x^2-1)/a^2/x^2)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.56

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{(96\,a^6x^6+96\,a^5x^5-366\,a^4x^4-222\,a^3x^3+338\,a^2x^2+122\,ax-153\,(a^5x^5+a^4x^4-}{96\,(a^7c^4x^5+a^6c}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas
")
```

output
```
1/96*(96*a^6*x^6 + 96*a^5*x^5 - 366*a^4*x^4 - 222*a^3*x^3 + 338*a^2*x^2 +
122*a*x - 153*(a^5*x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*log(a*
x + 1) + 57*(a^5*x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*log(a*x
- 1) - 88)*sqrt(a^2*c)/(a^7*c^4*x^5 + a^6*c^4*x^4 - 2*a^5*c^4*x^3 - 2*a^4*
c^4*x^2 + a^3*c^4*x + a^2*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(7/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.65

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{c}\,(57\log(ax-1)\,a^5x^5+57\log(ax-1)\,a^4x^4-114\log(ax-1)\,a^3x^3-114\log(ax-}{}$$

input
```
int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x)
```

output
```
(sqrt(c)*(57*log(a*x - 1)*a**5*x**5 + 57*log(a*x - 1)*a**4*x**4 - 114*log(
a*x - 1)*a**3*x**3 - 114*log(a*x - 1)*a**2*x**2 + 57*log(a*x - 1)*a*x + 57
*log(a*x - 1) - 153*log(a*x + 1)*a**5*x**5 - 153*log(a*x + 1)*a**4*x**4 +
306*log(a*x + 1)*a**3*x**3 + 306*log(a*x + 1)*a**2*x**2 - 153*log(a*x + 1)
*a*x - 153*log(a*x + 1) + 96*a**6*x**6 + 462*a**5*x**5 - 954*a**3*x**3 - 3
94*a**2*x**2 + 488*a*x + 278))/(96*a*c**4*(a**5*x**5 + a**4*x**4 - 2*a**3*
x**3 - 2*a**2*x**2 + a*x + 1))
```

3.839 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 190

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{c\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(12a+\frac{25}{x}\right)}{30a^2}+\frac{c^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(16a+\frac{25}{x}\right)}{24a^2}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(32a+\frac{25}{x}\right)}{16a^2}+\left(c-\frac{c}{a^2x^2}\right)^{7/2}x+\frac{25c^{7/2}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\right)}{16a}-\frac{2c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{a}$$

output
```
1/30*c*(c-c/a^2/x^2)^(5/2)*(12*a+25/x)/a^2+1/24*c^2*(c-c/a^2/x^2)^(3/2)*(1
6*a+25/x)/a^2+1/16*c^3*(c-c/a^2/x^2)^(1/2)*(32*a+25/x)/a^2+(c-c/a^2/x^2)^(
7/2)*x+25/16*c^(7/2)*arctan(c^(1/2)/a/(c-c/a^2/x^2)^(1/2)/x)/a-2*c^(7/2)*a
rctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.79

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(-40+96ax+70a^2x^2-352a^3x^3+105a^4x^4+736a^5x^5+240a^6x^6)+375a^6x^6\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)-480a^6x^6\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{240a^6x^5\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^3*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-40 + 96*a*x + 70*a^2*x^2
- 352*a^3*x^3 + 105*a^4*x^4 + 736*a^5*x^5 + 240*a^6*x^6) + 375*a^6*x^6*Arc
Tan[1/Sqrt[-1 + a^2*x^2]] - 480*a^6*x^6*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(2
40*a^6*x^5*Sqrt[-1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6717, 6709, 570, 540, 27, 537, 25, 537, 27, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{7/2}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\int \frac{(1-a^2x^2)^{9/2}}{x^7(ax+1)^2}dx}{(1-a^2x^2)^{7/2}}$$

$$\Big\downarrow \text{ 570}$$

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\int\frac{(1-ax)^2\left(1-a^2x^2\right)^{5/2}}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}$$

$\Big\downarrow$ 540

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}\int\frac{a(12-5ax)\left(1-a^2x^2\right)^{5/2}}{x^6}dx-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$\Big\downarrow$ 27

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}a\int\frac{(12-5ax)\left(1-a^2x^2\right)^{5/2}}{x^6}dx-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$\Big\downarrow$ 537

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}a\left(\frac{1}{4}a^2\int-\frac{(48-25ax)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{(48-25ax)\left(1-a^2x^2\right)^{5/2}}{20x^5}\right)-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$\Big\downarrow$ 25

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}a\left(-\frac{1}{4}a^2\int\frac{(48-25ax)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{(48-25ax)\left(1-a^2x^2\right)^{5/2}}{20x^5}\right)-\frac{\left(1-a^2x^2\right)^{7/2}}{6x^6}\right)}{\left(1-a^2x^2\right)^{7/2}}$$

$\Big\downarrow$ 537

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}a\left(-\frac{1}{4}a^2\left(\frac{1}{2}a^2\int-\frac{3(32-25ax)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(32-25ax)\left(1-a^2x^2\right)^{3/2}}{2x^3}\right)-\frac{(48-25ax)\left(1-a^2x^2\right)^{5/2}}{20x^5}\right)-\right.}{\left(1-a^2x^2\right)^{7/2}}$$

$\Big\downarrow$ 27

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}a\left(-\frac{1}{4}a^2\left(-\frac{3}{2}a^2\int\frac{(32-25ax)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(32-25ax)\left(1-a^2x^2\right)^{3/2}}{2x^3}\right)-\frac{(48-25ax)\left(1-a^2x^2\right)^{5/2}}{20x^5}\right)-\right.}{\left(1-a^2x^2\right)^{7/2}}$$

$\Big\downarrow$ 536

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}a\left(-\frac{1}{4}a^2\left(-\frac{3}{2}a^2\left(\int\frac{-32xa^2-25a}{x\sqrt{1-a^2x^2}}dx-\frac{(25ax+32)\sqrt{1-a^2x^2}}{x}\right)-\frac{(32-25ax)\left(1-a^2x^2\right)^{3/2}}{2x^3}\right)-\frac{(48-25ax)}{2}\right.\right.}{\left(1-a^2x^2\right)^{7/2}}$$

$\Big\downarrow$ 538

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}a\left(-\frac{1}{4}a^2\left(-\frac{3}{2}a^2\left(-32a^2\int\frac{1}{\sqrt{1-a^2x^2}}dx-25a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{(25ax+32)\sqrt{1-a^2x^2}}{x}\right)-\frac{(32-25ax}{}\right.\right.\right.}{(1-a^2x^2)^{7/2}}$$

$\Big\downarrow$ 223

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}a\left(-\frac{1}{4}a^2\left(-\frac{3}{2}a^2\left(-25a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}(25ax+32)}{x}-32a\arcsin(ax)\right)-\frac{(32-25ax)(1-}{2x^3}\right.\right.\right.}{(1-a^2x^2)^{7/2}}$$

$\Big\downarrow$ 243

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}a\left(-\frac{1}{4}a^2\left(-\frac{3}{2}a^2\left(-\frac{25}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}(25ax+32)}{x}-32a\arcsin(ax)\right)-\frac{(32-25ax)(}{2}\right.\right.\right.}{(1-a^2x^2)^{7/2}}$$

$\Big\downarrow$ 73

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}a\left(-\frac{1}{4}a^2\left(-\frac{3}{2}a^2\left(\frac{25\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}(25ax+32)}{x}-32a\arcsin(ax)\right)-\frac{(32-25ax)(}{2}\right.\right.\right.}{(1-a^2x^2)^{7/2}}$$

$\Big\downarrow$ 221

$$-\frac{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6}a\left(-\frac{1}{4}a^2\left(-\frac{3}{2}a^2\left(25a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}(25ax+32)}{x}-32a\arcsin(ax)\right)-\frac{(32-25}{}\right.\right.\right.}{(1-a^2x^2)^{7/2}}$$

input
```
Int[(c - c/(a^2*x^2))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((c - c/(a^2*x^2))^(7/2)*x^7*(-1/6*(1 - a^2*x^2)^(7/2)/x^6 - (a*(-1/20*(
(48 - 25*a*x)*(1 - a^2*x^2)^(5/2))/x^5 - (a^2*(-1/2*((32 - 25*a*x)*(1 - a^
2*x^2)^(3/2))/x^3 - (3*a^2*(-(((32 + 25*a*x)*Sqrt[1 - a^2*x^2])/x) - 32*a*
ArcSin[a*x] + 25*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2))/4))/6))/(1 - a^2*x^2)^
(7/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.32

method	result
risch	$\frac{\left(736a^7x^7+105x^6a^6-1088a^5x^5-35a^4x^4+448a^3x^3-110a^2x^2-96ax+40\right)c^3\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{240x^5a^6\left(a^2x^2-1\right)}+\frac{\left(\frac{25a^6\ln\left(\frac{-2c+2\sqrt{-c}\sqrt{a^2c\,x^2-c}}{x}\right)}{16\sqrt{-c}}-\cdots\right.}{}$
default	$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{7}{2}}x\left(2016\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\,a^9cx^7-2016\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{9}{2}}\sqrt{-\frac{c}{a^2}}\,a^9x^5+375\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\,a^8cx^6\cdots\right.}{}$

input
```
int((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/240*(736*a^7*x^7+105*a^6*x^6-1088*a^5*x^5-35*a^4*x^4+448*a^3*x^3-110*a^2
*x^2-96*a*x+40)/x^5*c^3/a^6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)+(25/
16*a^6/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)-2*a^7*ln(a
^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+a^6/c*(c*(a^2*x^2-
1))^(1/2))*c^3/a^6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)*x*(c*(a^2*x^2
-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.24

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\left[\frac{960\,a^5\sqrt{-c}c^3x^5\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+375\,a^5\sqrt{-c}c^3x^5\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}\cdots}{x^2}\right.}{}\right.$$

$$-\frac{375\,a^5c^{\frac{7}{2}}x^5\arctan\left(\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\sqrt{c}}\right)-240\,a^5c^{\frac{7}{2}}x^5\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-(240\,a^6c^3x^6+7\cdots}{240\,a^6x^5}$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/480*(960*a^5*sqrt(-c)*c^3*x^5*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 375*a^5*sqrt(-c)*c^3*x^5*log(-(a^2*c*x^2
 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(240*a^6
*c^3*x^6 + 736*a^5*c^3*x^5 + 105*a^4*c^3*x^4 - 352*a^3*c^3*x^3 + 70*a^2*c^
3*x^2 + 96*a*c^3*x - 40*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5), -
1/240*(375*a^5*c^(7/2)*x^5*arctan(a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/sqrt
(c)) - 240*a^5*c^(7/2)*x^5*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c
*x^2 - c)/(a^2*x^2)) - c) - (240*a^6*c^3*x^6 + 736*a^5*c^3*x^5 + 105*a^4*c
^3*x^4 - 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 + 96*a*c^3*x - 40*c^3)*sqrt((a^2
*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.48 (sec) , antiderivative size = 1059, normalized size of antiderivative = 5.57

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2} dx = \text{Too large to display}$$

input
```
integrate((c-c/a**2/x**2)**(7/2)*(a*x-1)/(a*x+1),x)
```

output
```
c**3*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*s
qrt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1)
, (I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*s
qrt(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c**3*Piecewise((-a*sqrt
(c)*x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x*
*2 - 1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqr
t(c)*asin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - c**3*Pie
cewise((I*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x*
*2))) - I*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2)
 > 1), (-a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x)
, True))/a**2 + 4*c**3*Piecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))*
*(3/2)/(3*c), True))/a**3 - c**3*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/
8 - I*a**2*sqrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sq
rt(-1 + 1/(a**2*x**2))) - I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2)))
, 1/Abs(a**2*x**2) > 1), (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*
x*sqrt(1 - 1/(a**2*x**2))) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) +
sqrt(c)/(4*a**2*x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**4 - 2*c**3*Piecew
ise((2*a**3*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x) + a*sqrt(c)*sqrt(a**2*x**2
- 1)/(15*x**3) - sqrt(c)*sqrt(a**2*x**2 - 1)/(5*a*x**5), Abs(a**2*x**2) >
1), (2*I*a**3*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x) + I*a*sqrt(c)*sqrt(-a...
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\int\frac{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{ax+1}dx$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(c - c/(a^2*x^2))^(7/2)/(a*x + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(162) = 324$.

Time = 0.27 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.95

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx =$$

$$-\frac{1}{120}\left(\frac{375\,c^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{240\,c^{\frac{7}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-120\sqrt{\right.$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-1/120*(375*c^(7/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x)/a^2 - 240*c^(7/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*s
gn(x)/(a*abs(a)) - 120*sqrt(a^2*c*x^2 - c)*c^3*sgn(x)/a^2 + (105*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^11*c^4*abs(a)*sgn(x) - 1440*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^10*a*c^(9/2)*sgn(x) + 595*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))^9*c^5*abs(a)*sgn(x) - 4320*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c)
)^8*a*c^(11/2)*sgn(x) - 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^6*ab
s(a)*sgn(x) - 7360*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(13/2)*sgn(
x) + 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^7*abs(a)*sgn(x) - 6720*
(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(15/2)*sgn(x) - 595*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - c))^3*c^8*abs(a)*sgn(x) - 2976*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*a*c^(17/2)*sgn(x) - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x
^2 - c))*c^9*abs(a)*sgn(x) - 736*a*c^(19/2)*sgn(x))/(((sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^2 + c)^6*a^2*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(7/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(7/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.97

$$\int e^{-2\coth^{-1}(ax)}\left(c - \frac{c}{a^2x^2}\right)^{7/2} dx = \frac{\sqrt{c}\,c^3\left(-750atan\left(\sqrt{a^2x^2-1}+ax\right)a^6x^6+240\sqrt{a^2x^2-1}\,a^6x^6+736\sqrt{a^2x^2-1}\,a^5x^5+10\right.}{}$$

input
```
int((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*c**3*( - 750*atan(sqrt(a**2*x**2 - 1) + a*x)*a**6*x**6 + 240*sqrt
(a**2*x**2 - 1)*a**6*x**6 + 736*sqrt(a**2*x**2 - 1)*a**5*x**5 + 105*sqrt(a
**2*x**2 - 1)*a**4*x**4 - 352*sqrt(a**2*x**2 - 1)*a**3*x**3 + 70*sqrt(a**2
*x**2 - 1)*a**2*x**2 + 96*sqrt(a**2*x**2 - 1)*a*x - 40*sqrt(a**2*x**2 - 1)
 - 480*log(sqrt(a**2*x**2 - 1) + a*x)*a**6*x**6 - 256*a**6*x**6))/(240*a**
7*x**6)
```

3.840 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{c\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(8a+\frac{9}{x}\right)}{12a^2}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(16a+\frac{9}{x}\right)}{8a^2}+\left(c-\frac{c}{a^2x^2}\right)^{5/2}x+\frac{9c^{5/2}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\right)}{8a}-\frac{2c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{a}$$

output
```
1/12*c*(c-c/a^2/x^2)^(3/2)*(8*a+9/x)/a^2+1/8*c^2*(c-c/a^2/x^2)^(1/2)*(16*a
+9/x)/a^2+(c-c/a^2/x^2)^(5/2)*x+9/8*c^(5/2)*arctan(c^(1/2)/a/(c-c/a^2/x^2)
^(1/2)/x)/a-2*c^(5/2)*arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(6-16ax-3a^2x^2+64a^3x^3+24a^4x^4)+27a^4x^4\arctan\left(\frac{1}{\sqrt{-1+\cdots}}\right.\right.}{24a^4x^3\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c^2*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 - 16*a*x - 3*a^2*x^2 + 6
4*a^3*x^3 + 24*a^4*x^4) + 27*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 48*a^4
*x^4*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^4*x^3*Sqrt[-1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {6717, 6709, 570, 540, 27, 537, 25, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{5/2} e^{-2\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)} \left(c-\frac{c}{a^2x^2}\right)^{5/2} dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x^5 \left(c-\frac{c}{a^2x^2}\right)^{5/2} \int \frac{\left(1-a^2x^2\right)^{7/2}}{x^5(ax+1)^2}dx}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{x^5 \left(c-\frac{c}{a^2x^2}\right)^{5/2} \int \frac{(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{x^5}dx}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x^5 \left(c-\frac{c}{a^2x^2}\right)^{5/2} \left(-\frac{1}{4}\int \frac{a(8-3ax)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}a\int\frac{(8-3ax)\left(1-a^2x^2\right)^{3/2}}{x^4}dx-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{537}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}a\left(\frac{1}{2}a^2\int-\frac{(16-9ax)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(16-9ax)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}a\left(-\frac{1}{2}a^2\int\frac{(16-9ax)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(16-9ax)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{536}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}a\left(-\frac{1}{2}a^2\left(\int\frac{-16xa^2-9a}{x\sqrt{1-a^2x^2}}dx-\frac{(9ax+16)\sqrt{1-a^2x^2}}{x}\right)-\frac{(16-9ax)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)-\frac{\left(1-a^2x^2\right)^{5/2}}{4x^4}\right)}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{538}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}a\left(-\frac{1}{2}a^2\left(-16a^2\int\frac{1}{\sqrt{1-a^2x^2}}dx-9a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{(9ax+16)\sqrt{1-a^2x^2}}{x}\right)-\frac{(16-9ax)\left(1-a^2x^2\right)^{3/}}{6x^3}\right.\right.}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}a\left(-\frac{1}{2}a^2\left(-9a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}(9ax+16)}{x}-16a\arcsin(ax)\right)-\frac{(16-9ax)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right)\right.}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}a\left(-\frac{1}{2}a^2\left(-\frac{9}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}(9ax+16)}{x}-16a\arcsin(ax)\right)-\frac{(16-9ax)\left(1-a^2x^2\right)^{3/2}}{6x^3}\right.\right.}{\left(1-a^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}a\left(-\frac{1}{2}a^2\left(\frac{9\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}(9ax+16)}{x}-16a\arcsin(ax)\right)-\frac{(16-9ax)(1-a^2x^2)^{3/2}}{6x^3}\right)\right)}{(1-a^2x^2)^{5/2}}$$

$\Big\downarrow$ 221

$$-\frac{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4}a\left(-\frac{1}{2}a^2\left(9a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}(9ax+16)}{x}-16a\arcsin(ax)\right)-\frac{(16-9ax)(1-a^2x^2)}{6x^3}\right)\right)}{(1-a^2x^2)^{5/2}}$$

input
```
Int[(c - c/(a^2*x^2))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((c - c/(a^2*x^2))^(5/2)*x^5*(-1/4*(1 - a^2*x^2)^(5/2)/x^4 - (a*(-1/6*((
16 - 9*a*x)*(1 - a^2*x^2)^(3/2))/x^3 - (a^2*(-(((16 + 9*a*x)*Sqrt[1 - a^2*
x^2])/x) - 16*a*ArcSin[a*x] + 9*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2))/4))/(1
- a^2*x^2)^(5/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.50

method	result
risch	$\frac{(64a^5x^5-3a^4x^4-80a^3x^3+9a^2x^2+16ax-6)c^2\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{24x^3a^4(a^2x^2-1)}+\frac{\left(\frac{9a^4\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)}{8\sqrt{-c}}-\frac{2a^5\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}+\frac{a^4}{\cdots}\right.}{a^4(a^2x^2-1)}$
default	$-\frac{\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{5}{2}}x\left(-80\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{5}{2}}a^7cx^5+80\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{7}{2}}a^7x^3-48\sqrt{-\frac{c}{a^2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{5}{2}}a^6cx^4-\cdots\right.}{\cdots}$

input
```
int((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/24*(64*a^5*x^5-3*a^4*x^4-80*a^3*x^3+9*a^2*x^2+16*a*x-6)/x^3*c^2/a^4*(c*(
a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)+(9/8*a^4/(-c)^(1/2)*ln((-2*c+2*(-c)^
(1/2)*(a^2*c*x^2-c)^(1/2))/x)-2*a^5*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)
^(1/2))/(a^2*c)^(1/2)+a^4/c*(c*(a^2*x^2-1))^(1/2))*c^2/a^4*(c*(a^2*x^2-1)/
a^2/x^2)^(1/2)/(a^2*x^2-1)*x*(c*(a^2*x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.44

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \left[\frac{96\,a^3\sqrt{-c}c^2x^3\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+27\,a^3\sqrt{-c}c^2x^3\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\cdots}{x^2}\right.\cdots}{48\,a^4x^3}\right.$$

$$-\frac{27\,a^3c^{\frac{5}{2}}x^3\arctan\left(\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\sqrt{c}}\right)-24\,a^3c^{\frac{5}{2}}x^3\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-\left(24\,a^4c^2x^4+64\,a^3\cdots\right.}{24\,a^4x^3}$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/48*(96*a^3*sqrt(-c)*c^2*x^3*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2))/(a^2*c*x^2 - c)) + 27*a^3*sqrt(-c)*c^2*x^3*log(-(a^2*c*x^2 -
2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*c^2
*x^4 + 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)))/(a^4*x^3), -1/24*(27*a^3*c^(5/2)*x^3*arctan(a*x*sqrt((a
^2*c*x^2 - c)/(a^2*x^2))/sqrt(c)) - 24*a^3*c^(5/2)*x^3*log(2*a^2*c*x^2 - 2
*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) - (24*a^4*c^2*x^4 +
64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)))/(a^4*x^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.18 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.21

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = c^2\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}+\frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } |a^2x^2|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right) - \frac{2c^2\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}(ax)+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{for } |a^2x^2|>1\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}(ax)-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}\right)}{a} + \frac{2c^2\left(\begin{cases}0 & \text{for } c=0\\ \frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)}{a^3} - \frac{c^2\left(\begin{cases}\frac{ia^3\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{8}-\frac{ia^2\sqrt{c}}{8x\sqrt{-1+\frac{1}{a^2x^2}}}+\frac{3i\sqrt{c}}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{i\sqrt{c}}{4a^2x^5\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{a^3\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{8}+\frac{a^2\sqrt{c}}{8x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c}}{8x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c}}{4a^2x^5\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)}{a^4}$$

input
```
integrate((c-c/a**2/x**2)**(5/2)*(a*x-1)/(a*x+1),x)
```

output
```
c**2*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*s
qrt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1)
, (I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*s
qrt(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c**2*Piecewise((-a*sqrt
(c)*x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x*
*2 - 1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqr
t(c)*asin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + 2*c**2*P
iecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c), True))/a**
3 - c**2*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*sqrt(c)/(8*x*
sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2)))
- I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1),
 (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2)
)) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*x**5*sqr
t(1 - 1/(a**2*x**2))), True))/a**4
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\int\frac{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{ax+1}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(c - c/(a^2*x^2))^(5/2)/(a*x + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(132) = 264$.

Time = 0.24 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.67

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=$$

$$-\frac{1}{12}\left(\frac{27\,c^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{24\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{12\sqrt{a^2cx^2}}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-1/12*(27*c^(5/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*s
gn(x)/a^2 - 24*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(
x)/(a*abs(a)) - 12*sqrt(a^2*c*x^2 - c)*c^2*sgn(x)/a^2 - (3*(sqrt(a^2*c)*x
- sqrt(a^2*c*x^2 - c))^7*c^3*abs(a)*sgn(x) + 96*(sqrt(a^2*c)*x - sqrt(a^2*
c*x^2 - c))^6*a*c^(7/2)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
5*c^4*abs(a)*sgn(x) + 192*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(9/2
)*sgn(x) + 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*c^5*abs(a)*sgn(x) +
160*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(11/2)*sgn(x) - 3*(sqrt(a^
2*c)*x - sqrt(a^2*c*x^2 - c))*c^6*abs(a)*sgn(x) + 64*a*c^(13/2)*sgn(x))/((
(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^4*a^2*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(5/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(5/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{\sqrt{c}\,c^2\left(-54atan\left(\sqrt{a^2x^2-1}+ax\right)a^4x^4+24\sqrt{a^2x^2-1}\,a^4x^4+64\sqrt{a^2x^2-1}\,a^3x^3-3\sqrt{a^2}\cdots\right.}{24a^5\cdots}$$

input
```
int((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*c**2*( - 54*atan(sqrt(a**2*x**2 - 1) + a*x)*a**4*x**4 + 24*sqrt(a
**2*x**2 - 1)*a**4*x**4 + 64*sqrt(a**2*x**2 - 1)*a**3*x**3 - 3*sqrt(a**2*x
**2 - 1)*a**2*x**2 - 16*sqrt(a**2*x**2 - 1)*a*x + 6*sqrt(a**2*x**2 - 1) -
48*log(sqrt(a**2*x**2 - 1) + a*x)*a**4*x**4 - 16*a**4*x**4))/(24*a**5*x**4
)
```

3.841 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2}+\left(c-\frac{c}{a^2x^2}\right)^{3/2}x+\frac{c^{3/2}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\right)}{2a}-\frac{2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{a}$$

output
```
1/2*c*(c-c/a^2/x^2)^(1/2)*(4*a+1/x)/a^2+(c-c/a^2/x^2)^(3/2)*x+1/2*c^(3/2)*
arctan(c^(1/2)/a/(c-c/a^2/x^2)^(1/2)/x)/a-2*c^(3/2)*arctanh((c-c/a^2/x^2)^
(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.96

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(-1+4ax+2a^2x^2)+a^2x^2\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)-4a^2x^2\log\left(ax\right.\right.}{2a^2x\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(c*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-1 + 4*a*x + 2*a^2*x^2) + a^
2*x^2*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 4*a^2*x^2*Log[a*x + Sqrt[-1 + a^2*x^2
]]))/(2*a^2*x*Sqrt[-1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6717, 6709, 570, 540, 27, 536, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{3/2} e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\int \frac{(1-a^2x^2)^{5/2}}{x^3(ax+1)^2}dx}{(1-a^2x^2)^{3/2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\int \frac{(1-ax)^2\sqrt{1-a^2x^2}}{x^3}dx}{(1-a^2x^2)^{3/2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}\int \frac{a(4-ax)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(1-a^2x^2)^{3/2}}{2x^2}\right)}{(1-a^2x^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}a\int\frac{(4-ax)\sqrt{1-a^2x^2}}{x^2}dx-\frac{(1-a^2x^2)^{3/2}}{2x^2}\right)}{(1-a^2x^2)^{3/2}}$$

$\Big\downarrow$ 536

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}a\left(\int\frac{-4xa^2-a}{x\sqrt{1-a^2x^2}}dx-\frac{(ax+4)\sqrt{1-a^2x^2}}{x}\right)-\frac{(1-a^2x^2)^{3/2}}{2x^2}\right)}{(1-a^2x^2)^{3/2}}$$

$\Big\downarrow$ 538

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}a\left(-4a^2\int\frac{1}{\sqrt{1-a^2x^2}}dx-a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{(ax+4)\sqrt{1-a^2x^2}}{x}\right)-\frac{(1-a^2x^2)^{3/2}}{2x^2}\right)}{(1-a^2x^2)^{3/2}}$$

$\Big\downarrow$ 223

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}a\left(-a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}(ax+4)}{x}-4a\arcsin(ax)\right)-\frac{(1-a^2x^2)^{3/2}}{2x^2}\right)}{(1-a^2x^2)^{3/2}}$$

$\Big\downarrow$ 243

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}a\left(-\frac{1}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}(ax+4)}{x}-4a\arcsin(ax)\right)-\frac{(1-a^2x^2)^{3/2}}{2x^2}\right)}{(1-a^2x^2)^{3/2}}$$

$\Big\downarrow$ 73

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}a\left(\frac{\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\sqrt{1-a^2x^2}(ax+4)}{x}-4a\arcsin(ax)\right)-\frac{(1-a^2x^2)^{3/2}}{2x^2}\right)}{(1-a^2x^2)^{3/2}}$$

$\Big\downarrow$ 221

$$-\frac{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2}a\left(a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}(ax+4)}{x}-4a\arcsin(ax)\right)-\frac{(1-a^2x^2)^{3/2}}{2x^2}\right)}{(1-a^2x^2)^{3/2}}$$

input
```
Int[(c - c/(a^2*x^2))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-(((c - c/(a^2*x^2))^(3/2)*x^3*(-1/2*(1 - a^2*x^2)^(3/2)/x^2 - (a*(-(((4 +
 a*x)*Sqrt[1 - a^2*x^2])/x) - 4*a*ArcSin[a*x] + a*ArcTanh[Sqrt[1 - a^2*x^2
]]))/2))/(1 - a^2*x^2)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(100) = 200$.

Time = 0.24 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.68

method	result
risch	$\frac{\left(2a^4x^4+4a^3x^3-3a^2x^2-4ax+1\right)c\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{2x\,a^2\left(a^2x^2-1\right)}+\frac{\left(\frac{a^2\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)}{2\sqrt{-c}}-\frac{2a^3\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}\right)c\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x}{a^2\left(a^2x^2-1\right)}$
default	$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}x\left(12\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^5cx^3-12\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}\sqrt{-\frac{c}{a^2}}\,a^5x-4\sqrt{-\frac{c}{a^2}}\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}a^4cx^2+\sqrt{-}\right.}{}$

input
```
int((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*a^4*x^4+4*a^3*x^3-3*a^2*x^2-4*a*x+1)/x*c/a^2*(c*(a^2*x^2-1)/a^2/x^2
)^(1/2)/(a^2*x^2-1)+(1/2*a^2/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c
)^(1/2))/x)-2*a^3*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1
/2))*c/a^2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-
1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.52

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\left[\frac{8\,a\sqrt{-c}cx\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+a\sqrt{-c}cx\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(2}{4\,a^2x}\right.$$

$$-\frac{ac^{\frac{3}{2}}x\arctan\left(\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\sqrt{c}}\right)-2\,ac^{\frac{3}{2}}x\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-\left(2\,a^2cx^2+4\,acx-c\right)\sqrt{\frac{a^2}{}}}{2\,a^2x}$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/4*(8*a*sqrt(-c)*c*x*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x
^2))/(a^2*c*x^2 - c)) + a*sqrt(-c)*c*x*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(2*a^2*c*x^2 + 4*a*c*x - c)*
sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x), -1/2*(a*c^(3/2)*x*arctan(a*x*sqr
t((a^2*c*x^2 - c)/(a^2*x^2))/sqrt(c)) - 2*a*c^(3/2)*x*log(2*a^2*c*x^2 - 2*
a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) - (2*a^2*c*x^2 + 4*a*
c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.05 (sec) , antiderivative size = 376, normalized size of antiderivative = 3.13

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = c\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}+\frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } |a^2x^2|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right) - \frac{2c\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}(ax)+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{for } |a^2x^2|>1\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}(ax)-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}\right)}{a} + \frac{c\left(\begin{cases}\frac{ia\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}+\frac{i\sqrt{c}}{2x\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{i\sqrt{c}}{2a^2x^3\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{a\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)}{a^2}$$

input
```
integrate((c-c/a**2/x**2)**(3/2)*(a*x-1)/(a*x+1),x)
```

output
```
c*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sqrt
(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1), (
I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c*Piecewise((-a*sqrt(c)*x/
sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 - 1
)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + c*Piecewise((I
*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - I
*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-
a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/
a**2
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\int\frac{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{ax+1}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*(c - c/(a^2*x^2))^(3/2)/(a*x + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(100) = 200.

Time = 0.18 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.22

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=-\left(\frac{c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\text{sgn}(x)}{a^2}-\frac{2\,c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\text{sgn}(x)}{a|a|}-\frac{\sqrt{a^2cx^2-c}c\text{sgn}(}{a^2}\right.$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
-(c^(3/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^
2 - 2*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs
(a)) - sqrt(a^2*c*x^2 - c)*c*sgn(x)/a^2 - ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - c))^3*c^2*abs(a)*sgn(x) + 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c
^(5/2)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^3*abs(a)*sgn(x) +
4*a*c^(7/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^2*a^2*a
bs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(3/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(3/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.85

$$\int e^{-2\coth^{-1}(ax)}\Big(c-\frac{c}{a^2x^2}\Big)^{3/2}dx=\frac{\sqrt{c}\,c\left(-2atan\left(\sqrt{a^2x^2-1}+ax\right)a^2x^2+2\sqrt{a^2x^2-1}\,a^2x^2+4\sqrt{a^2x^2-1}\,ax-\sqrt{a^2x^2-1}-\cdots\right.}{2a^3x^2}$$

input
```
int((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*c*( - 2*atan(sqrt(a**2*x**2 - 1) + a*x)*a**2*x**2 + 2*sqrt(a**2*x
**2 - 1)*a**2*x**2 + 4*sqrt(a**2*x**2 - 1)*a*x - sqrt(a**2*x**2 - 1) - 4*l
og(sqrt(a**2*x**2 - 1) + a*x)*a**2*x**2))/(2*a**3*x**2)
```

3.842 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\sqrt{c-\frac{c}{a^2x^2}}x-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}x}\right)}{a}-\frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{a}$$

output
```
(c-c/a^2/x^2)^(1/2)*x-c^(1/2)*arctan(c^(1/2)/a/(c-c/a^2/x^2)^(1/2)/x)/a-2*
c^(1/2)*arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}-\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)-2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]
] - 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6717, 6709, 570, 541, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{\left(1-a^2x^2\right)^{3/2}}{x(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{541}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int -\frac{a^2(1-2ax)}{x\sqrt{1-a^2x^2}}dx}{a^2}-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{\int \frac{a^2(1-2ax)}{x\sqrt{1-a^2x^2}}dx}{a^2}-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{1-2ax}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 538}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-2a\int \frac{1}{\sqrt{1-a^2x^2}}dx+\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}-2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\sqrt{1-a^2x^2}-2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a^2}-\sqrt{1-a^2x^2}-2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\sqrt{1-a^2x^2}-2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-Sqrt[1 - a^2*x^2] - 2*ArcSin[a*x] - ArcTanh[S
qrt[1 - a^2*x^2]]))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(74) = 148$.

Time = 0.21 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.23

method	result
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2-2\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}+2\sqrt{c}\,\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+xc}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^2}}+c\ln\left(\frac{2\sqrt{-}\cdots}{}\right.\right.}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}}$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)
*a^2-2*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)*a^2*(-c/a^2)^(1/2)+2*c^(1/2)*ln((c^(1
/2)*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)+x*c)/c^(1/2))*a*(-c/a^2)^(1/2)+c*ln(2*((
-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)
^(1/2)/a^2/(-c/a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.86

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+4\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a}, \frac{ax\sqrt{\frac{a^2cx^2-}{a^2x^2}}}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 4*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a
^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a,
(a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + sqrt(c)*arctan(a*x*sqrt((a^2*c*x^2
- c)/(a^2*x^2))/sqrt(c)) + sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.58

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$
$$=\frac{\sqrt{c}\left(2atan\left(\sqrt{a^2x^2-1}+ax\right)+\sqrt{a^2x^2-1}-2\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{a}$$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*(2*atan(sqrt(a**2*x**2 - 1) + a*x) + sqrt(a**2*x**2 - 1) - 2*log(
sqrt(a**2*x**2 - 1) + a*x)))/a
```

3.843 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{2\sqrt{c-\frac{c}{a^2x^2}}}{c\left(a+\frac{1}{x}\right)}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{c}-\frac{2\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

output
```
2*(c-c/a^2/x^2)^(1/2)/c/(a+1/x)+(c-c/a^2/x^2)^(1/2)*x/c-2*arctanh((c-c/a^2
/x^2)^(1/2)/c^(1/2))/a/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{-3+2ax+a^2x^2-2\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1+a^2x^2}\right)}{a^2\sqrt{c-\frac{c}{a^2x^2}}x}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

output
```
(-3 + 2*a*x + a^2*x^2 - 2*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]]
)/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6717, 6709, 563, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{\sqrt{1-a^2x^2}\int \frac{x\sqrt{1-a^2x^2}}{(ax+1)^2}dx}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{563}$$

$$-\frac{\sqrt{1-a^2x^2}\left(\frac{\int \frac{2-ax}{\sqrt{1-a^2x^2}}dx}{a}+\frac{2\sqrt{1-a^2x^2}}{a^2(ax+1)}\right)}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{455}$$

$$-\frac{\sqrt{1-a^2x^2}\left(\frac{2\int \frac{1}{\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}}{a}}{a}+\frac{2\sqrt{1-a^2x^2}}{a^2(ax+1)}\right)}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\sqrt{1-a^2x^2}\left(\frac{\frac{\sqrt{1-a^2x^2}}{a}+\frac{2\arcsin(ax)}{a}}{a}+\frac{2\sqrt{1-a^2x^2}}{a^2(ax+1)}\right)}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

output
```
-((Sqrt[1 - a^2*x^2]*((2*Sqrt[1 - a^2*x^2])/(a^2*(1 + a*x)) + (Sqrt[1 - a^
2*x^2]/a + (2*ArcSin[a*x])/a)/a))/(Sqrt[c - c/(a^2*x^2)]*x))
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(70) = 140$.

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.92

method	result
risch	$\frac{a^2x^2-1}{a^2x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}+\frac{\left(-\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a\sqrt{a^2c}}+\frac{2\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-2\left(x+\frac{1}{a}\right)ac}}{a^3c\left(x+\frac{1}{a}\right)}\right)\sqrt{c(a^2x^2-1)}}{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2}}\left(-\sqrt{\frac{c(a^2x^2-1)}{a^2}}\sqrt{c}\,a^2x+2\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)acx-2a\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\sqrt{c}-\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a\sqrt{c}+2\ln\left(x\sqrt{c}\right.\right.}{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\,c^{\frac{3}{2}}a(ax+1)}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a^2*x^2-1)/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(-2/a*ln(a^2*c*x/(a^2*c)
^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+2/a^3/c/(x+1/a)*((x+1/a)^2*a^2*c
-2*(x+1/a)*a*c)^(1/2))/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.65

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

$$= \left[\frac{(ax+1)\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(a^2x^2+3\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx+ac}, \frac{2\,(ax+1)\sqrt{-c}\arctan\left(\right.}{}\right.$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[((a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)) - c) + (a^2*x^2 + 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a
^2*c*x + a*c), (2*(a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x
^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (a^2*x^2 + 3*a*x)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)))/(a^2*c*x + a*c)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{ax-1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral((a*x - 1)/(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{ax-1}{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*sqrt(c - c/(a^2*x^2))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{a\,x-1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a^2*x^2))^(1/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a^2*x^2))^(1/2)*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{c}\left(2\sqrt{a^2x^2-1}\,ax+6\sqrt{a^2x^2-1}-4\log\left(\sqrt{a^2x^2-1}+ax\right)ax-4\log\left(\sqrt{a^2x^2-1}+ax\right)+5ax+5\right)}{2ac\,(ax+1)}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x)
```

output
```
(sqrt(c)*(2*sqrt(a**2*x**2 - 1)*a*x + 6*sqrt(a**2*x**2 - 1) - 4*log(sqrt(a
**2*x**2 - 1) + a*x)*a*x - 4*log(sqrt(a**2*x**2 - 1) + a*x) + 5*a*x + 5))/
(2*a*c*(a*x + 1))
```

3.844 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{2}{3c\sqrt{c-\frac{c}{a^2x^2}}\left(a+\frac{1}{x}\right)} + \frac{10\sqrt{c-\frac{c}{a^2x^2}}\,x}{3c^2} - \frac{\left(7a-\frac{6}{x}\right)x}{3ac\sqrt{c-\frac{c}{a^2x^2}}} - \frac{2\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{ac^{3/2}}$$

output
```
2/3/c/(c-c/a^2/x^2)^(1/2)/(a+1/x)+10/3*(c-c/a^2/x^2)^(1/2)*x/c^2-1/3*(7*a-
6/x)*x/a/c/(c-c/a^2/x^2)^(1/2)-2*arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))/a/c^
(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{-10-4ax+11a^2x^2+3a^3x^3-6(1+ax)\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1+a^2x^2}\right)}{3a^2c\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

output
```
(-10 - 4*a*x + 11*a^2*x^2 + 3*a^3*x^3 - 6*(1 + a*x)*Sqrt[-1 + a^2*x^2]*Log
[a*x + Sqrt[-1 + a^2*x^2]])/(3*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6717, 6709, 570, 529, 25, 2166, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \ 6717$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \ 6709$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\int \frac{x^3}{(ax+1)^2\sqrt{1-a^2x^2}}dx}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \ 570$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\int \frac{x^3(1-ax)^2}{(1-a^2x^2)^{5/2}}dx}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \; 529$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{(1-ax)^2}{3a^4(1-a^2x^2)^{3/2}}-\frac{1}{3}\int -\frac{(1-ax)\left(\frac{3x^2}{a}-\frac{3x}{a^2}+\frac{2}{a^3}\right)}{(1-a^2x^2)^{3/2}}dx\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{1}{3}\int \frac{(1-ax)\left(\frac{3x^2}{a}-\frac{3x}{a^2}+\frac{2}{a^3}\right)}{(1-a^2x^2)^{3/2}}dx+\frac{(1-ax)^2}{3a^4(1-a^2x^2)^{3/2}}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \; 2166$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{1}{3}\left(-\int \frac{3(2-ax)}{a^3\sqrt{1-a^2x^2}}dx-\frac{8(1-ax)}{a^4\sqrt{1-a^2x^2}}\right)+\frac{(1-ax)^2}{3a^4(1-a^2x^2)^{3/2}}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{1}{3}\left(-\frac{3\int \frac{2-ax}{\sqrt{1-a^2x^2}}dx}{a^3}-\frac{8(1-ax)}{a^4\sqrt{1-a^2x^2}}\right)+\frac{(1-ax)^2}{3a^4(1-a^2x^2)^{3/2}}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \; 455$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{1}{3}\left(-\frac{3\left(2\int \frac{1}{\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^3}-\frac{8(1-ax)}{a^4\sqrt{1-a^2x^2}}\right)+\frac{(1-ax)^2}{3a^4(1-a^2x^2)^{3/2}}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

$$\Big\downarrow \; 223$$

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\frac{(1-ax)^2}{3a^4(1-a^2x^2)^{3/2}}+\frac{1}{3}\left(-\frac{8(1-ax)}{a^4\sqrt{1-a^2x^2}}-\frac{3\left(\frac{\sqrt{1-a^2x^2}}{a}+\frac{2\arcsin(ax)}{a}\right)}{a^3}\right)\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

output
```
-(((1 - a^2*x^2)^(3/2)*((1 - a*x)^2/(3*a^4*(1 - a^2*x^2)^(3/2)) + ((-8*(1
- a*x))/(a^4*Sqrt[1 - a^2*x^2]) - (3*(Sqrt[1 - a^2*x^2]/a + (2*ArcSin[a*x]
)/a))/a^3)/3))/((c - c/(a^2*x^2))^(3/2)*x^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.68

method	result
risch	$\frac{a^{2} x^{2}-1}{a^{2} c x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}+\frac{\left(-\frac{\sqrt{\left(x+\frac{1}{a}\right)^{2} a^{2} c-2 \left(x+\frac{1}{a}\right) a c}}{3 a^{6} c \left(x+\frac{1}{a}\right)^{2}}+\frac{8 \sqrt{\left(x+\frac{1}{a}\right)^{2} a^{2} c-2 \left(x+\frac{1}{a}\right) a c}}{3 a^{5} c \left(x+\frac{1}{a}\right)}-\frac{2 \ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}-c}\right)}{a^{3} \sqrt{a^{2} c}}\right) a^{2} \sqrt{c \left(a^{2} x^{2}-1\right)}}{c x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}$
default	$\frac{\left(3 \sqrt{\frac{c (a x-1)(a x+1)}{a^{2}}}\, c^{\frac{3}{2}} a^{3} x^{3}+15 x^{2} a^{2} c^{\frac{3}{2}} \sqrt{\frac{c (a x-1)(a x+1)}{a^{2}}}-4 c^{\frac{3}{2}} \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a^{2} x^{2}-6 \sqrt{\frac{c (a x-1)(a x+1)}{a^{2}}}\, \ln \left(x \sqrt{c}+\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right) \sqrt{\cdots}\right.}{3 \sqrt{\frac{c (a x-\cdots}{\cdots}}}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a^2*x^2-1)/c/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(-1/3/a^6/c/(x+1/a)^2*
((x+1/a)^2*a^2*c-2*(x+1/a)*a*c)^(1/2)+8/3/a^5/c/(x+1/a)*((x+1/a)^2*a^2*c-2
*(x+1/a)*a*c)^(1/2)-2/a^3*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a
^2*c)^(1/2))*a^2/c/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.32

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \left[\frac{3\left(a^2x^2+2\,ax+1\right)\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(3\,a^3x^3+14\,a^2x^2\right.}{3\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}\right.$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(a^2*x^2 + 2*a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*
sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (3*a^3*x^3 + 14*a^2*x^2 + 10*a*x)*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2), 1/3*(
6*(a^2*x^2 + 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (3*a^3*x^3 + 14*a^2*x^2 + 10*a*x)*sqrt((
a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(3/2),x)
```

output
```
Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2)*(a*x + 1)), x
)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{a\,x-1}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a^2*x^2))^(3/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a^2*x^2))^(3/2)*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.18

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\sqrt{c}\left(3\sqrt{a^2x^2-1}\,a^2x^2+14\sqrt{a^2x^2-1}\,ax+10\sqrt{a^2x^2-1}-6\log\left(\sqrt{a^2x^2-1}+ax\right)a^2\right.}{3a\,c^2\left(a^2x^2+\right.}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x)
```

output
```
(sqrt(c)*(3*sqrt(a**2*x**2 - 1)*a**2*x**2 + 14*sqrt(a**2*x**2 - 1)*a*x + 1
0*sqrt(a**2*x**2 - 1) - 6*log(sqrt(a**2*x**2 - 1) + a*x)*a**2*x**2 - 12*lo
g(sqrt(a**2*x**2 - 1) + a*x)*a*x - 6*log(sqrt(a**2*x**2 - 1) + a*x) - 3*a*
*2*x**2 - 6*a*x - 3))/(3*a*c**2*(a**2*x**2 + 2*a*x + 1))
```

3.845 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 154

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{2\left(15a-\frac{14}{x}\right)}{15a^2c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2}{5c\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(a+\frac{1}{x}\right)}+\frac{28\sqrt{c-\frac{c}{a^2x^2}}\,x}{15c^3}-\frac{\left(13a-\frac{10}{x}\right)x}{15ac\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{ac^{5/2}}$$

output
```
2/15*(15*a-14/x)/a^2/c^2/(c-c/a^2/x^2)^(1/2)+2/5/c/(c-c/a^2/x^2)^(3/2)/(a+
1/x)+28/15*(c-c/a^2/x^2)^(1/2)*x/c^3-1/15*(13*a-10/x)*x/a/c/(c-c/a^2/x^2)^
(3/2)-2*arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))/a/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.68

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{-56-82ax+32a^2x^2+76a^3x^3+15a^4x^4-30(1+ax)^2\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1}\right.}{15a^2c^2\sqrt{c-\frac{c}{a^2x^2}}x(1+ax)^2}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

output
```
(-56 - 82*a*x + 32*a^2*x^2 + 76*a^3*x^3 + 15*a^4*x^4 - 30*(1 + a*x)^2*Sqrt
[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(15*a^2*c^2*Sqrt[c - c/(a^2*
x^2)]*x*(1 + a*x)^2)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6717, 6709, 570, 529, 25, 2166, 2345, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\int \frac{x^5}{(ax+1)^2\left(1-a^2x^2\right)^{3/2}}dx}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{ 570}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\int \frac{x^5(1-ax)^2}{(1-a^2x^2)^{7/2}}dx}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{529}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{(1-ax)^2}{5a^6(1-a^2x^2)^{5/2}}-\frac{1}{5}\int -\frac{(1-ax)\left(\frac{5x^4}{a}-\frac{5x^3}{a^2}+\frac{5x^2}{a^3}-\frac{5x}{a^4}+\frac{2}{a^5}\right)}{(1-a^2x^2)^{5/2}}dx\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\int \frac{(1-ax)\left(\frac{5x^4}{a}-\frac{5x^3}{a^2}+\frac{5x^2}{a^3}-\frac{5x}{a^4}+\frac{2}{a^5}\right)}{(1-a^2x^2)^{5/2}}dx+\frac{(1-ax)^2}{5a^6(1-a^2x^2)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{2166}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\left(-\frac{1}{3}\int \frac{-\frac{15x^3}{a^2}+\frac{30x^2}{a^3}-\frac{45x}{a^4}+\frac{16}{a^5}}{(1-a^2x^2)^{3/2}}dx-\frac{22(1-ax)}{3a^6(1-a^2x^2)^{3/2}}\right)+\frac{(1-ax)^2}{5a^6(1-a^2x^2)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{15(2-ax)}{a^5\sqrt{1-a^2x^2}}dx+\frac{2(30-23ax)}{a^6\sqrt{1-a^2x^2}}\right)-\frac{22(1-ax)}{3a^6(1-a^2x^2)^{3/2}}\right)+\frac{(1-ax)^2}{5a^6(1-a^2x^2)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{15\int \frac{2-ax}{\sqrt{1-a^2x^2}}dx}{a^5}+\frac{2(30-23ax)}{a^6\sqrt{1-a^2x^2}}\right)-\frac{22(1-ax)}{3a^6(1-a^2x^2)^{3/2}}\right)+\frac{(1-ax)^2}{5a^6(1-a^2x^2)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{455}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{15\left(2\int \frac{1}{\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^5}+\frac{2(30-23ax)}{a^6\sqrt{1-a^2x^2}}\right)-\frac{22(1-ax)}{3a^6(1-a^2x^2)^{3/2}}\right)+\frac{(1-ax)^2}{5a^6(1-a^2x^2)^{5/2}}\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\left(1-a^2x^2\right)^{5/2}\left(\frac{(1-ax)^2}{5a^6(1-a^2x^2)^{5/2}}+\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(30-23ax)}{a^6\sqrt{1-a^2x^2}}+\frac{15\left(\frac{\sqrt{1-a^2x^2}}{a}+\frac{2\arcsin(ax)}{a}\right)}{a^5}\right)-\frac{22(1-ax)}{3a^6(1-a^2x^2)^{3/2}}\right)\right)}{x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

output
```
-(((1 - a^2*x^2)^(5/2)*((1 - a*x)^2/(5*a^6*(1 - a^2*x^2)^(5/2)) + ((-22*(1
 - a*x))/(3*a^6*(1 - a^2*x^2)^(3/2)) + ((2*(30 - 23*a*x))/(a^6*Sqrt[1 - a^
2*x^2]) + (15*(Sqrt[1 - a^2*x^2]/a + (2*ArcSin[a*x])/a))/a^5)/3)/5))/((c -
 c/(a^2*x^2))^(5/2)*x^5))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(132) = 264.

Time = 0.30 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.86

method	result
risch	$\frac{a^2x^2-1}{a^2c^2x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}+\frac{\left(-\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^5\sqrt{a^2c}}+\frac{383\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-2\left(x+\frac{1}{a}\right)ac}}{120a^7c\left(x+\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+2\left(x-\frac{1}{a}\right)ac}}{8a^7c\left(x-\frac{1}{a}\right)}+\frac{\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-2\left(x-\right.}}{10a^9c\left(x+\frac{1}{a}\right)^3}\right.}{c^2x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}$
default	$-\frac{\left(-15\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}c^{\frac{5}{2}}a^5x^5-16\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}c^{\frac{5}{2}}a^4x^4-45x^4c^{\frac{5}{2}}a^4\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{3}{2}}-16\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}c^{\frac{5}{2}}a^3x^3+60\right.}{}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a^2*x^2-1)/c^2/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(-2/a^5*ln(a^2*c*x/(
a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+383/120/a^7/c/(x+1/a)*((x+
1/a)^2*a^2*c-2*(x+1/a)*a*c)^(1/2)-1/8/a^7/c/(x-1/a)*((x-1/a)^2*a^2*c+2*(x-
1/a)*a*c)^(1/2)+1/10/a^9/c/(x+1/a)^3*((x+1/a)^2*a^2*c-2*(x+1/a)*a*c)^(1/2)
-41/60/a^8/c/(x+1/a)^2*((x+1/a)^2*a^2*c-2*(x+1/a)*a*c)^(1/2))*a^4/c^2/x/(c
*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.28

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\left[\frac{15\,(a^4x^4+2\,a^3x^3-2\,ax-1)\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(15\,a^5x^5}{15\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-a}\right.$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/15*(15*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^
2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (15*a^5*x^5 + 76*a^4*
x^4 + 32*a^3*x^3 - 82*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(
a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3), 1/15*(30*(a^4*x^4 + 2*
a^3*x^3 - 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)
/(a^2*x^2))/(a^2*c*x^2 - c)) + (15*a^5*x^5 + 76*a^4*x^4 + 32*a^3*x^3 - 82*
a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^3*x^4 + 2*a^4*c^
3*x^3 - 2*a^2*c^3*x - a*c^3)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(5/2),x)
```

output
```
Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(5/2)*(a*x + 1)), x
)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{a\,x-1}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a^2*x^2))^(5/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a^2*x^2))^(5/2)*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.41

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\sqrt{c}\left(30\sqrt{a^2x^2-1}\,a^4x^4+152\sqrt{a^2x^2-1}\,a^3x^3+64\sqrt{a^2x^2-1}\,a^2x^2-164\sqrt{a^2x^2-1}\,a\right.}{}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x)
```

output
```
(sqrt(c)*(30*sqrt(a**2*x**2 - 1)*a**4*x**4 + 152*sqrt(a**2*x**2 - 1)*a**3*
x**3 + 64*sqrt(a**2*x**2 - 1)*a**2*x**2 - 164*sqrt(a**2*x**2 - 1)*a*x - 11
2*sqrt(a**2*x**2 - 1) - 60*log(sqrt(a**2*x**2 - 1) + a*x)*a**4*x**4 - 120*
log(sqrt(a**2*x**2 - 1) + a*x)*a**3*x**3 + 120*log(sqrt(a**2*x**2 - 1) + a
*x)*a*x + 60*log(sqrt(a**2*x**2 - 1) + a*x) - 47*a**4*x**4 - 94*a**3*x**3
+ 94*a*x + 47))/(30*a*c**3*(a**4*x**4 + 2*a**3*x**3 - 2*a*x - 1))
```

3.846 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 189

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{2\left(35a-\frac{27}{x}\right)}{105a^2c^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{2}{7c\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(a+\frac{1}{x}\right)} + \frac{144\sqrt{c-\frac{c}{a^2x^2}}\,x}{35c^4}-\frac{\left(19a-\frac{14}{x}\right)x}{35ac\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2\left(9a-\frac{7}{x}\right)x}{7ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{ac^{7/2}}$$

output
```
2/105*(35*a-27/x)/a^2/c^2/(c-c/a^2/x^2)^(3/2)+2/7/c/(c-c/a^2/x^2)^(5/2)/(a
+1/x)+144/35*(c-c/a^2/x^2)^(1/2)*x/c^4-1/35*(19*a-14/x)*x/a/c/(c-c/a^2/x^2
)^(5/2)-2/7*(9*a-7/x)*x/a/c^3/(c-c/a^2/x^2)^(1/2)-2*arctanh((c-c/a^2/x^2)^
(1/2)/c^(1/2))/a/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.69

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\frac{432+654ax-636a^2x^2-1226a^3x^3+74a^4x^4+562a^5x^5+105a^6x^6-210(-1+ax)}{105a^2\sqrt{c-\frac{c}{a^2x^2}}\,x(-1+ax)(c+acx)^3}$$

input
```
Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

output
```
(432 + 654*a*x - 636*a^2*x^2 - 1226*a^3*x^3 + 74*a^4*x^4 + 562*a^5*x^5 + 1
05*a^6*x^6 - 210*(-1 + a*x)*(1 + a*x)^3*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[
-1 + a^2*x^2]])/(105*a^2*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)*(c + a*c*x)^3)
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6717, 6709, 570, 529, 25, 2166, 2345, 2345, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\int \frac{x^7}{(ax+1)^2\left(1-a^2x^2\right)^{5/2}}dx}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$$\Big\downarrow \text{ 570}$$

$$-\frac{\left(1-a^2x^2\right)^{7/2}\int\frac{x^7(1-ax)^2}{(1-a^2x^2)^{9/2}}dx}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$\Big\downarrow$ 529

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{(1-ax)^2}{7a^8(1-a^2x^2)^{7/2}}-\frac{1}{7}\int-\frac{(1-ax)\left(\frac{7x^6}{a}-\frac{7x^5}{a^2}+\frac{7x^4}{a^3}-\frac{7x^3}{a^4}+\frac{7x^2}{a^5}-\frac{7x}{a^6}+\frac{2}{a^7}\right)}{(1-a^2x^2)^{7/2}}dx\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$\Big\downarrow$ 25

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{1}{7}\int\frac{(1-ax)\left(\frac{7x^6}{a}-\frac{7x^5}{a^2}+\frac{7x^4}{a^3}-\frac{7x^3}{a^4}+\frac{7x^2}{a^5}-\frac{7x}{a^6}+\frac{2}{a^7}\right)}{(1-a^2x^2)^{7/2}}dx+\frac{(1-ax)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$\Big\downarrow$ 2166

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{1}{7}\left(-\frac{1}{5}\int\frac{-\frac{35x^5}{a^2}+\frac{70x^4}{a^3}-\frac{105x^3}{a^4}+\frac{140x^2}{a^5}-\frac{175x}{a^6}+\frac{34}{a^7}}{(1-a^2x^2)^{5/2}}dx-\frac{44(1-ax)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{(1-ax)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$\Big\downarrow$ 2345

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{-\frac{105x^3}{a^4}+\frac{210x^2}{a^5}-\frac{420x}{a^6}+\frac{142}{a^7}}{(1-a^2x^2)^{3/2}}dx+\frac{315-244ax}{3a^8(1-a^2x^2)^{3/2}}\right)-\frac{44(1-ax)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{(1-ax)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$\Big\downarrow$ 2345

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-\int\frac{105(2-ax)}{a^7\sqrt{1-a^2x^2}}dx-\frac{525-352ax}{a^8\sqrt{1-a^2x^2}}\right)+\frac{315-244ax}{3a^8(1-a^2x^2)^{3/2}}\right)-\frac{44(1-ax)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{(1-ax)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$\Big\downarrow$ 27

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-\frac{105\int\frac{2-ax}{\sqrt{1-a^2x^2}}dx}{a^7}-\frac{525-352ax}{a^8\sqrt{1-a^2x^2}}\right)+\frac{315-244ax}{3a^8(1-a^2x^2)^{3/2}}\right)-\frac{44(1-ax)}{5a^8(1-a^2x^2)^{5/2}}\right)+\frac{(1-ax)^2}{7a^8(1-a^2x^2)^{7/2}}\right)}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$\Big\downarrow$ 455

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-\frac{105\left(2\int\frac{1}{\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}}{a}\right)}{a^7}-\frac{525-352ax}{a^8\sqrt{1-a^2x^2}}\right)+\frac{315-244ax}{3a^8(1-a^2x^2)^{3/2}}\right)-\frac{44(1-ax)}{5a^8(1-a^2x^2)^{5/2}}\right)+\right.}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

$\big\downarrow$ 223

$$-\frac{\left(1-a^2x^2\right)^{7/2}\left(\frac{(1-ax)^2}{7a^8(1-a^2x^2)^{7/2}}+\frac{1}{7}\left(\frac{1}{5}\left(\frac{315-244ax}{3a^8(1-a^2x^2)^{3/2}}+\frac{1}{3}\left(-\frac{525-352ax}{a^8\sqrt{1-a^2x^2}}-\frac{105\left(\frac{\sqrt{1-a^2x^2}}{a}+\frac{2\arcsin(ax)}{a}\right)}{a^7}\right)\right)-\frac{44(1-}{5a^8(1-}\right.\right.}{x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

input
```
Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

output
```
-(((1 - a^2*x^2)^(7/2)*((1 - a*x)^2/(7*a^8*(1 - a^2*x^2)^(7/2)) + ((-44*(1
 - a*x))/(5*a^8*(1 - a^2*x^2)^(5/2)) + ((315 - 244*a*x)/(3*a^8*(1 - a^2*x^
2)^(3/2)) + (-((525 - 352*a*x)/(a^8*Sqrt[1 - a^2*x^2])) - (105*(Sqrt[1 - a
^2*x^2]/a + (2*ArcSin[a*x])/a))/a^7)/3)/5)/7))/((c - c/(a^2*x^2))^(7/2)*x^
7))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 529
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a*d + b*c*x, x], R = PolynomialRem
ainder[x^m, a*d + b*c*x, x]}, Simp[(-c)*R*(c + d*x)^n*((a + b*x^2)^(p + 1)/
(2*a*d*(p + 1))), x] + Simp[c/(2*a*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b*
x^2)^(p + 1)*ExpandToSum[2*a*d*(p + 1)*Qx + R*(n + 2*p + 2), x], x], x]] /;
 FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 1] && LtQ[p, -1] && EqQ[b*
c^2 + a*d^2, 0]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(163) = 326$.

Time = 0.21 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.96

method	result
risch	$\frac{a^2x^2-1}{a^2c^3x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}+\frac{\left(-\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^7\sqrt{a^2c}}+\frac{3061\sqrt{\left(x+\frac{1}{a}\right)^2a^2c-2\left(x+\frac{1}{a}\right)ac}}{840a^9c\left(x+\frac{1}{a}\right)}-\frac{\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+2\left(x-\frac{1}{a}\right)ac}}{48a^{10}c\left(x-\frac{1}{a}\right)^2}-\frac{7\sqrt{\left(x-\frac{1}{a}\right)^2a^2c+2\left(\right.}}{24a^9c\left(x-\frac{1}{a}\right.}}{c^3x\sqrt{}}$
default	$-\frac{\left(-105\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}a^7x^7+96\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}a^6x^6-553x^6c^{\frac{7}{2}}a^6\left(\frac{c(ax-1)(ax+1)}{a^2}\right)^{\frac{5}{2}}+96\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}a^5x^5+39\right.}{}$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a^2*x^2-1)/c^3/x/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(-2/a^7*ln(a^2*c*x/(
a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)+3061/840/a^9/c/(x+1/a)*((x
+1/a)^2*a^2*c-2*(x+1/a)*a*c)^(1/2)-1/48/a^10/c/(x-1/a)^2*((x-1/a)^2*a^2*c+
2*(x-1/a)*a*c)^(1/2)-7/24/a^9/c/(x-1/a)*((x-1/a)^2*a^2*c+2*(x-1/a)*a*c)^(1
/2)-1/28/a^12/c/(x+1/a)^4*((x+1/a)^2*a^2*c-2*(x+1/a)*a*c)^(1/2)+39/140/a^1
1/c/(x+1/a)^3*((x+1/a)^2*a^2*c-2*(x+1/a)*a*c)^(1/2)-1753/1680/a^10/c/(x+1/
a)^2*((x+1/a)^2*a^2*c-2*(x+1/a)*a*c)^(1/2))*a^6/c^3/x/(c*(a^2*x^2-1)/a^2/x
^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.62

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \left[\frac{105\left(a^6x^6+2a^5x^5-a^4x^4-4a^3x^3-a^2x^2+2ax+1\right)\sqrt{c}\log\left(2a^2cx^2-2a^2\sqrt{c}x^2\right.}{105\left(a^7c^4x^6+2a^6c^4x^5-\right.}\right.$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

output
```
[1/105*(105*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x +
 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)) - c) + (105*a^7*x^7 + 562*a^6*x^6 + 74*a^5*x^5 - 1226*a^4*x^4 - 636*
a^3*x^3 + 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^7*c^4
*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c
^4*x + a*c^4), 1/105*(210*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2
*x^2 + 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a
^2*x^2))/(a^2*c*x^2 - c)) + (105*a^7*x^7 + 562*a^6*x^6 + 74*a^5*x^5 - 1226
*a^4*x^4 - 636*a^3*x^3 + 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)))/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^
4*x^2 + 2*a^2*c^4*x + a*c^4)]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(7/2),x)
```

output
```
Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(7/2)*(a*x + 1)), x
)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \int \frac{a\,x-1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,(a\,x+1)}\,dx$$

input
```
int((a*x - 1)/((c - c/(a^2*x^2))^(7/2)*(a*x + 1)),x)
```

output
```
int((a*x - 1)/((c - c/(a^2*x^2))^(7/2)*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.96

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{c}\,\big(420\sqrt{a^2x^2-1}\,a^6x^6+2248\sqrt{a^2x^2-1}\,a^5x^5+296\sqrt{a^2x^2-1}\,a^4x^4-4904\sqrt{a^2x^2-}}{}$$

input
```
int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x)
```

output
```
(sqrt(c)*(420*sqrt(a**2*x**2 - 1)*a**6*x**6 + 2248*sqrt(a**2*x**2 - 1)*a**
5*x**5 + 296*sqrt(a**2*x**2 - 1)*a**4*x**4 - 4904*sqrt(a**2*x**2 - 1)*a**3
*x**3 - 2544*sqrt(a**2*x**2 - 1)*a**2*x**2 + 2616*sqrt(a**2*x**2 - 1)*a*x
+ 1728*sqrt(a**2*x**2 - 1) - 840*log(sqrt(a**2*x**2 - 1) + a*x)*a**6*x**6
- 1680*log(sqrt(a**2*x**2 - 1) + a*x)*a**5*x**5 + 840*log(sqrt(a**2*x**2 -
 1) + a*x)*a**4*x**4 + 3360*log(sqrt(a**2*x**2 - 1) + a*x)*a**3*x**3 + 840
*log(sqrt(a**2*x**2 - 1) + a*x)*a**2*x**2 - 1680*log(sqrt(a**2*x**2 - 1) +
 a*x)*a*x - 840*log(sqrt(a**2*x**2 - 1) + a*x) - 463*a**6*x**6 - 926*a**5*
x**5 + 463*a**4*x**4 + 1852*a**3*x**3 + 463*a**2*x**2 - 926*a*x - 463))/(4
20*a*c**4*(a**6*x**6 + 2*a**5*x**5 - a**4*x**4 - 4*a**3*x**3 - a**2*x**2 +
 2*a*x + 1))
```

3.847 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 322

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx &= -\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9\sqrt{1-\frac{1}{a^2x^2}}x^8}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8\sqrt{1-\frac{1}{a^2x^2}}x^7}\\ &-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}\\ &-\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

output
```
-1/8*c^4*(c-c/a^2/x^2)^(1/2)/a^9/(1-1/a^2/x^2)^(1/2)/x^8+3/7*c^4*(c-c/a^2/
x^2)^(1/2)/a^8/(1-1/a^2/x^2)^(1/2)/x^7-8/5*c^4*(c-c/a^2/x^2)^(1/2)/a^6/(1-
1/a^2/x^2)^(1/2)/x^5+3/2*c^4*(c-c/a^2/x^2)^(1/2)/a^5/(1-1/a^2/x^2)^(1/2)/x
^4+2*c^4*(c-c/a^2/x^2)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)/x^3-4*c^4*(c-c/a^2/x^
2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)/x^2+c^4*(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^
2)^(1/2)-3*c^4*(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(-\frac{1}{8a^9x^8}+\frac{3}{7a^8x^7}-\frac{8}{5a^6x^5}+\frac{3}{2a^5x^4}+\frac{2}{a^4x^3}-\frac{4}{a^3x^2}+x-\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
((c - c/(a^2*x^2))^(9/2)*(-1/8*1/(a^9*x^8) + 3/(7*a^8*x^7) - 8/(5*a^6*x^5)
 + 3/(2*a^5*x^4) + 2/(a^4*x^3) - 4/(a^3*x^2) + x - (3*Log[x])/a))/(1 - 1/(
a^2*x^2))^(9/2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{9/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^6(ax+1)^3}{x^9}dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\int\left(a^9-\frac{3a^8}{x}+\frac{8a^6}{x^3}-\frac{6a^5}{x^4}-\frac{6a^4}{x^5}+\frac{8a^3}{x^6}-\frac{3a}{x^8}+\frac{1}{x^9}\right)dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\left(a^9x-3a^8\log(x)-\frac{4a^6}{x^2}+\frac{2a^5}{x^3}+\frac{3a^4}{2x^4}-\frac{8a^3}{5x^5}+\frac{3a}{7x^7}-\frac{1}{8x^8}\right)}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(c - c/(a^2*x^2))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^4*Sqrt[c - c/(a^2*x^2)]*(-1/8*1/x^8 + (3*a)/(7*x^7) - (8*a^3)/(5*x^5) +
 (3*a^4)/(2*x^4) + (2*a^5)/x^3 - (4*a^6)/x^2 + a^9*x - 3*a^8*Log[x]))/(a^9
*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$-\frac{(-280a^9x^9+840a^8\ln(x)x^8+1120x^6a^6-560a^5x^5-420a^4x^4+448a^3x^3-120ax+35)x\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{280(ax-1)^3(a^2x^2-1)^3}$	112

input
```
int((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/280*(-280*a^9*x^9+840*a^8*ln(x)*x^8+1120*x^6*a^6-560*a^5*x^5-420*a^4*x^
4+448*a^3*x^3-120*a*x+35)*x*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1)
)^(3/2)/(a*x-1)^3/(a^2*x^2-1)^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx = \frac{(280\,a^9c^4x^9-840\,a^8c^4x^8\log(x)-1120\,a^6c^4x^6+560\,a^5c^4x^5+420\,a^4c^4x^4-448\,a^3c^4x^3+}{280\,a^{10}x^8}$$

input
```
integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

output
```
1/280*(280*a^9*c^4*x^9 - 840*a^8*c^4*x^8*log(x) - 1120*a^6*c^4*x^6 + 560*a
^5*c^4*x^5 + 420*a^4*c^4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x - 35*c^4)*sqr
t(a^2*c)/(a^10*x^8)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

input
```
integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

output
```
integrate((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.49

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{1}{280}\left(\frac{280\,c^4x\operatorname{sgn}(ax+1)\operatorname{sgn}(x)}{a}-\frac{840\,c^4\log(|x|)\operatorname{sgn}(ax+1)\operatorname{sgn}(x)}{a^2}-\frac{1120\,a^6c^4x^6\operatorname{sgn}(\cdots}{\cdots}\right.$$

input
```
integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
1/280*(280*c^4*x*sgn(a*x + 1)*sgn(x)/a - 840*c^4*log(abs(x))*sgn(a*x + 1)*
sgn(x)/a^2 - (1120*a^6*c^4*x^6*sgn(a*x + 1)*sgn(x) - 560*a^5*c^4*x^5*sgn(a
*x + 1)*sgn(x) - 420*a^4*c^4*x^4*sgn(a*x + 1)*sgn(x) + 448*a^3*c^4*x^3*sgn
(a*x + 1)*sgn(x) - 120*a*c^4*x*sgn(a*x + 1)*sgn(x) + 35*c^4*sgn(a*x + 1)*s
gn(x))/(a^10*x^8))*sqrt(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\int\left(c-\frac{c}{a^2\,x^2}\right)^{9/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.25

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\frac{\sqrt{c}\,c^4(-840\log(ax)\,a^8x^8+280a^9x^9-1260a^8x^8-1120a^6x^6+560a^5x^5+420a^4x^4-448a}{280a^9x^8}$$

input
```
int((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*c**4*( - 840*log(a*x)*a**8*x**8 + 280*a**9*x**9 - 1260*a**8*x**8
- 1120*a**6*x**6 + 560*a**5*x**5 + 420*a**4*x**4 - 448*a**3*x**3 + 120*a*x
 - 35))/(280*a**9*x**8)
```

3.848 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 324

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/(1-1/a^2/x^2)^(1/2)/x^6-3/5*c^3*(c-c/a^2/x
^2)^(1/2)/a^6/(1-1/a^2/x^2)^(1/2)/x^5+1/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/(1-1
/a^2/x^2)^(1/2)/x^4+5/3*c^3*(c-c/a^2/x^2)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)/x^
3-5/2*c^3*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)/x^2-c^3*(c-c/a^2/x^2
)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)/x+c^3*(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^
(1/2)-3*c^3*(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6a^7x^6}-\frac{3}{5a^6x^5}+\frac{1}{4a^5x^4}+\frac{5}{3a^4x^3}-\frac{5}{2a^3x^2}-\frac{1}{a^2x}+x-\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
((c - c/(a^2*x^2))^(7/2)*(1/(6*a^7*x^6) - 3/(5*a^6*x^5) + 1/(4*a^5*x^4) +
5/(3*a^4*x^3) - 5/(2*a^3*x^2) - 1/(a^2*x) + x - (3*Log[x])/a))/(1 - 1/(a^2
*x^2))^(7/2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.32, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{7/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(1-ax)^5(ax+1)^2}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^5(ax+1)^2}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\int \left(-a^7+\frac{3a^6}{x}-\frac{a^5}{x^2}-\frac{5a^4}{x^3}+\frac{5a^3}{x^4}+\frac{a^2}{x^5}-\frac{3a}{x^6}+\frac{1}{x^7}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(a^7(-x)+3a^6\log(x)+\frac{a^5}{x}+\frac{5a^4}{2x^2}-\frac{5a^3}{3x^3}-\frac{a^2}{4x^4}+\frac{3a}{5x^5}-\frac{1}{6x^6}\right)}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(c - c/(a^2*x^2))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c^3*Sqrt[c - c/(a^2*x^2)]*(-1/6*1/x^6 + (3*a)/(5*x^5) - a^2/(4*x^4) - (
5*a^3)/(3*x^3) + (5*a^4)/(2*x^2) + a^5/x - a^7*x + 3*a^6*Log[x]))/(a^7*Sqr
t[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

method	result	size
default	$-\frac{\left(-60 a^{7} x^{7}+180 a^{6} \ln \left(x \right) x^{6}+60 a^{5} x^{5}+150 a^{4} x^{4}-100 a^{3} x^{3}-15 a^{2} x^{2}+36 a x -10\right) x \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{7}{2}} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{60 \left(a x -1\right)^{3} \left(a^{2} x^{2}-1\right)^{2}}$	112

input
```
int((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*(-60*a^7*x^7+180*a^6*ln(x)*x^6+60*a^5*x^5+150*a^4*x^4-100*a^3*x^3-15
*a^2*x^2+36*a*x-10)*x*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2
)/(a*x-1)^3/(a^2*x^2-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\frac{(60\,a^7c^3x^7-180\,a^6c^3x^6\log(x)-60\,a^5c^3x^5-150\,a^4c^3x^4+100\,a^3c^3x^3+15\,a^2c^3x^2-36\,a}{60\,a^8x^6}$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

output
```
1/60*(60*a^7*c^3*x^7 - 180*a^6*c^3*x^6*log(x) - 60*a^5*c^3*x^5 - 150*a^4*c
^3*x^4 + 100*a^3*c^3*x^3 + 15*a^2*c^3*x^2 - 36*a*c^3*x + 10*c^3)*sqrt(a^2*
c)/(a^8*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

output
```
integrate((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.48

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{1}{60}\left(\frac{60\,c^3x\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a}-\frac{180\,c^3\log\left(|x|\right)\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a^2}-\frac{60\,a^5c^3x^5\mathrm{sgn}(ax+\right.$$

input
```
integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
1/60*(60*c^3*x*sgn(a*x + 1)*sgn(x)/a - 180*c^3*log(abs(x))*sgn(a*x + 1)*sg
n(x)/a^2 - (60*a^5*c^3*x^5*sgn(a*x + 1)*sgn(x) + 150*a^4*c^3*x^4*sgn(a*x +
 1)*sgn(x) - 100*a^3*c^3*x^3*sgn(a*x + 1)*sgn(x) - 15*a^2*c^3*x^2*sgn(a*x
+ 1)*sgn(x) + 36*a*c^3*x*sgn(a*x + 1)*sgn(x) - 10*c^3*sgn(a*x + 1)*sgn(x))
/(a^8*x^6))*sqrt(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\int\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}dx$$

input
```
int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.24

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\sqrt{c}\,c^3(-180\log(ax)\,a^6x^6+60a^7x^7-200a^6x^6-60a^5x^5-150a^4x^4+100a^3x^3+15a^2x^2-}{60a^7x^6}$$

input
```
int((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*c**3*( - 180*log(a*x)*a**6*x**6 + 60*a**7*x**7 - 200*a**6*x**6 -
60*a**5*x**5 - 150*a**4*x**4 + 100*a**3*x**3 + 15*a**2*x**2 - 36*a*x + 10)
)/(60*a**7*x**6)
```

3.849 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 235

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}x^3}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/(1-1/a^2/x^2)^(1/2)/x^4+c^2*(c-c/a^2/x^2)
^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)/x^3-c^2*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^
2)^(1/2)/x^2-2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)/x+c^2*(c-c/
a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1/2)-3*c^2*(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-
1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{5}{4a}-\frac{1}{4a^5x^4}+\frac{1}{a^4x^3}-\frac{1}{a^3x^2}-\frac{2}{a^2x}+x-\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
((c - c/(a^2*x^2))^(5/2)*(-5/(4*a) - 1/(4*a^5*x^4) + 1/(a^4*x^3) - 1/(a^3*
x^2) - 2/(a^2*x) + x - (3*Log[x])/a))/(1 - 1/(a^2*x^2))^(5/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.33, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{5/2}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^4(ax+1)}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 84}$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\int\left(a^5-\frac{3a^4}{x}+\frac{2a^3}{x^2}+\frac{2a^2}{x^3}-\frac{3a}{x^4}+\frac{1}{x^5}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 2009$$

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(a^5x-3a^4\log(x)-\frac{2a^3}{x}-\frac{a^2}{x^2}+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(c - c/(a^2*x^2))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(c^2*Sqrt[c - c/(a^2*x^2)]*(-1/4*1/x^4 + a/x^3 - a^2/x^2 - (2*a^3)/x + a^5
*x - 3*a^4*Log[x]))/(a^5*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.41

method	result	size
default	$-\frac{\left(-4a^{5}x^{5}+12\ln\left(x\right)x^{4}a^{4}+8a^{3}x^{3}+4a^{2}x^{2}-4ax+1\right)x\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4\left(ax-1\right)^{3}\left(a^{2}x^{2}-1\right)}$	96

input
```
int((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-4*a^5*x^5+12*ln(x)*x^4*a^4+8*a^3*x^3+4*a^2*x^2-4*a*x+1)*x*(c*(a^2*x
^2-1)/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3/(a^2*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{\left(4\,a^{5}c^{2}x^{5}-12\,a^{4}c^{2}x^{4}\log\left(x\right)-8\,a^{3}c^{2}x^{3}-4\,a^{2}c^{2}x^{2}+4\,ac^{2}x-c^{2}\right)\sqrt{a^{2}c}}{4\,a^{6}x^{4}}$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

output
```
1/4*(4*a^5*c^2*x^5 - 12*a^4*c^2*x^4*log(x) - 8*a^3*c^2*x^3 - 4*a^2*c^2*x^2
 + 4*a*c^2*x - c^2)*sqrt(a^2*c)/(a^6*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

output
```
integrate((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.50

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{1}{4}\left(\frac{4\,c^2x\text{sgn}(ax+1)\,\text{sgn}(x)}{a}-\frac{12\,c^2\log\left(|x|\right)\text{sgn}(ax+1)\,\text{sgn}(x)}{a^2}-\frac{8\,a^3c^2x^3\text{sgn}(ax+1)\,\text{sg}}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
1/4*(4*c^2*x*sgn(a*x + 1)*sgn(x)/a - 12*c^2*log(abs(x))*sgn(a*x + 1)*sgn(x
)/a^2 - (8*a^3*c^2*x^3*sgn(a*x + 1)*sgn(x) + 4*a^2*c^2*x^2*sgn(a*x + 1)*sg
n(x) - 4*a*c^2*x*sgn(a*x + 1)*sgn(x) + c^2*sgn(a*x + 1)*sgn(x))/(a^6*x^4))
*sqrt(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\int \left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.27

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx=\frac{\sqrt{c}\,c^2(-12\log(ax)\,a^4x^4+4a^5x^5-8a^4x^4-8a^3x^3-4a^2x^2+4ax-1)}{4a^5x^4}$$

input
```
int((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*c**2*( - 12*log(a*x)*a**4*x**4 + 4*a**5*x**5 - 8*a**4*x**4 - 8*a*
*3*x**3 - 4*a**2*x**2 + 4*a*x - 1))/(4*a**5*x**4)
```

3.850 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 148

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)/x^2-3*c*(c-c/a^2/x^2)^(1
/2)/a^2/(1-1/a^2/x^2)^(1/2)/x+c*(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1/2)-
3*c*(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.39

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2a^3x^2}-\frac{3}{a^2x}+x-\frac{3\log(x)}{a}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[(c - c/(a^2*x^2))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
((c - c/(a^2*x^2))^(3/2)*(1/(2*a^3*x^2) - 3/(a^2*x) + x - (3*Log[x])/a))/(
1 - 1/(a^2*x^2))^(3/2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.42, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6751, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{3/2} e^{-3\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \; 6751$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 6747$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(1-ax)^3}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 25$$

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^3}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 49$$

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int \left(-a^3+\frac{3a^2}{x}-\frac{3a}{x^2}+\frac{1}{x^3}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(a^3(-x)+3a^2\log(x)+\frac{3a}{x}-\frac{1}{2x^2}\right)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(c - c/(a^2*x^2))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

output
```
-((c*Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 + (3*a)/x - a^3*x + 3*a^2*Log[x]))/
(a^3*Sqrt[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{\left(-2a^{3}x^{3}+6a^{2}\ln\left(x\right)x^{2}+6ax-1\right)x\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2\left(ax-1\right)^{3}}$	69

input
```
int((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*a^3*x^3+6*a^2*ln(x)*x^2+6*a*x-1)*x*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*
((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.28

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\left(2\,a^{3}cx^{3}-6\,a^{2}cx^{2}\log\left(x\right)-6\,acx+c\right)\sqrt{a^{2}c}}{2\,a^{4}x^{2}}$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

output
```
1/2*(2*a^3*c*x^3 - 6*a^2*c*x^2*log(x) - 6*a*c*x + c)*sqrt(a^2*c)/(a^4*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

output
```
integrate((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.49

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{1}{2}\left(\frac{2\,cx\text{sgn}(ax+1)\,\text{sgn}(x)}{a}-\frac{6\,c\log\left(|x|\right)\text{sgn}(ax+1)\,\text{sgn}(x)}{a^2}-\frac{6\,acx\text{sgn}(ax+1)\,\text{sgn}(x)}{a^4x}\right.$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
1/2*(2*c*x*sgn(a*x + 1)*sgn(x)/a - 6*c*log(abs(x))*sgn(a*x + 1)*sgn(x)/a^2
 - (6*a*c*x*sgn(a*x + 1)*sgn(x) - c*sgn(a*x + 1)*sgn(x))/(a^4*x^2))*sqrt(c
)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\int\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}dx$$

input
```
int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.25

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\sqrt{c}\,c(-6\log(ax)\,a^2x^2+2a^3x^3-6ax+1)}{2a^3x^2}$$

input
```
int((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*c*( - 6*log(a*x)*a**2*x**2 + 2*a**3*x**3 - 6*a*x + 1))/(2*a**3*x*
*2)
```

3.851 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1/2)+(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1
/a^2/x^2)^(1/2)-4*(c-c/a^2/x^2)^(1/2)*ln(a*x+1)/a/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}(ax+\log(x)-4\log(1+ax))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2
*x^2)])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x(ax+1)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a}{ax+1}+a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax-4\log(ax+1)+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2
*x^2)])
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

method	result	size
default	$-\frac{\left(-a x+4 \ln \left(a x+1\right)-\ln \left(x\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x+1\right) \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{\left(a x-1\right)^{2}}$	67

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a*x+4*ln(a*x+1)-ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)
/(a*x+1))^(3/2)/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.23

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax-4\log(ax+1)+\log(x))}{a^2}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

output
```
sqrt(a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$
$$=\left(\frac{x\text{sgn}(ax+1)\,\text{sgn}(x)}{a}-\frac{4\,\log\left(|ax+1|\right)\text{sgn}(ax+1)\,\text{sgn}(x)}{a^2}+\frac{\log\left(|x|\right)\text{sgn}(ax+1)\,\text{sgn}(x)}{a^2}\right)\sqrt{c}|a|$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
(x*sgn(a*x + 1)*sgn(x)/a - 4*log(abs(a*x + 1))*sgn(a*x + 1)*sgn(x)/a^2 + 1
og(abs(x))*sgn(a*x + 1)*sgn(x)/a^2)*sqrt(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.21

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c}\,(-4\log(ax+1)+\log(ax)+ax-1)}{a}$$

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*( - 4*log(a*x + 1) + log(a*x) + a*x - 1))/a
```

3.852 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 113

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}} - \frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{a^2x^2}}(1+ax)} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/(c-c/a^2/x^2)^(1/2)-2*(1-1/a^2/x^2)^(1/2)/a/(c-c/a^2
/x^2)^(1/2)/(a*x+1)-3*(1-1/a^2/x^2)^(1/2)*ln(a*x+1)/a/(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.49

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(x-\frac{2}{a(1+ax)}-\frac{3\log(1+ax)}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

output
```
(Sqrt[1 - 1/(a^2*x^2)]*(x - 2/(a*(1 + a*x)) - (3*Log[1 + a*x])/a))/Sqrt[c
 - c/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6751, 6747, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x(1-ax)}{(ax+1)^2}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x(1-ax)}{(ax+1)^2}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{86}$$

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{3}{(ax+1)a}-\frac{2}{(ax+1)^2a}-\frac{1}{a}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{2}{a^2(ax+1)}+\frac{3\log(ax+1)}{a^2}-\frac{x}{a}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

output
```
-((a*Sqrt[1 - 1/(a^2*x^2)]*(-(x/a) + 2/(a^2*(1 + a*x)) + (3*Log[1 + a*x])/
a^2))/Sqrt[c - c/(a^2*x^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}} (a x+1) \left(-a^{2} x^{2}+3 \ln \left(a x+1\right) x a-a x+3 \ln \left(a x+1\right)+2\right)}{\left(a x-1\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x\, a^{2}}$	87

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((a*x-1)/(a*x+1))^(3/2)/(a*x-1)*(a*x+1)*(-a^2*x^2+3*ln(a*x+1)*x*a-a*x+3*l
n(a*x+1)+2)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.42

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\left(a^2x^2 + ax - 3\,(ax+1)\log{(ax+1)} - 2\right)\sqrt{a^2c}}{a^3cx + a^2c}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas
")
```

output
```
(a^2*x^2 + a*x - 3*(a*x + 1)*log(a*x + 1) - 2)*sqrt(a^2*c)/(a^3*c*x + a^2*
c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(c - c/(a^2*x^2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(1/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\sqrt{c}\,(-3\log(ax+1)\,ax-3\log(ax+1)+a^2x^2+3ax)}{ac\,(ax+1)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x)
```

output
```
(sqrt(c)*( - 3*log(a*x + 1)*a*x - 3*log(a*x + 1) + a**2*x**2 + 3*a*x))/(a*
c*(a*x + 1))
```

3.853 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/c/(c-c/a^2/x^2)^(1/2)+1/2*(1-1/a^2/x^2)^(1/2)/a/c/(c
-c/a^2/x^2)^(1/2)/(a*x+1)^2-3*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)/
(a*x+1)-3*(1-1/a^2/x^2)^(1/2)*ln(a*x+1)/a/c/(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.38

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(x-\frac{5+6ax}{2a(1+ax)^2}-\frac{3\log(1+ax)}{a}\right)}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

output
```
((1 - 1/(a^2*x^2))^(3/2)*(x - (5 + 6*a*x)/(2*a*(1 + a*x)^2) - (3*Log[1 + a
*x])/a))/(c - c/(a^2*x^2))^(3/2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^3}{(ax+1)^3}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int\left(\frac{1}{a^3}-\frac{3}{a^3(ax+1)}+\frac{3}{a^3(ax+1)^2}-\frac{1}{a^3(ax+1)^3}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \; 2009$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(-\frac{3}{a^4(ax+1)}+\frac{1}{2a^4(ax+1)^2}-\frac{3\log(ax+1)}{a^4}+\frac{x}{a^3}\right)}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

output
```
(a^3*Sqrt[1 - 1/(a^2*x^2)]*(x/a^3 + 1/(2*a^4*(1 + a*x)^2) - 3/(a^4*(1 + a*
x)) - (3*Log[1 + a*x])/a^4))/(c*Sqrt[c - c/(a^2*x^2)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.61

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\left(-2a^{3}x^{3}+6\ln(ax+1)x^{2}a^{2}-4a^{2}x^{2}+12\ln(ax+1)xa+4ax+6\ln(ax+1)+5\right)}{2a^{4}x^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}}$	102

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(-2*a^3*x^3+6*ln(a*x+1)*x^2*a^2-4*a^2
*x^2+12*ln(a*x+1)*x*a+4*a*x+6*ln(a*x+1)+5)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)
^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.48

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(2\,a^{3}x^{3}+4\,a^{2}x^{2}-4\,ax-6\,\left(a^{2}x^{2}+2\,ax+1\right)\log\left(ax+1\right)-5\right)\sqrt{a^{2}c}}{2\,\left(a^{4}c^{2}x^{2}+2\,a^{3}c^{2}x+a^{2}c^{2}\right)}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas
")
```

output
```
1/2*(2*a^3*x^3 + 4*a^2*x^2 - 4*a*x - 6*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1)
- 5)*sqrt(a^2*c)/(a^4*c^2*x^2 + 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima
")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\frac{\sqrt{c}\,(-6\log(ax+1)\,a^2x^2-12\log(ax+1)\,ax-6\log(ax+1)+2a^3x^3+6a^2x^2-3)}{2a\,c^2\,(a^2x^2+2ax+1)}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x)
```

output
```
(sqrt(c)*( - 6*log(a*x + 1)*a**2*x**2 - 12*log(a*x + 1)*a*x - 6*log(a*x +
1) + 2*a**3*x**3 + 6*a**2*x**2 - 3))/(2*a*c**2*(a**2*x**2 + 2*a*x + 1))
```

3.854 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 264

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^3}\\
&+\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}-\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}\\
&+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{49\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}\end{aligned}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/c^2/(c-c/a^2/x^2)^(1/2)-1/6*(1-1/a^2/x^2)^(1/2)/a/c^
2/(c-c/a^2/x^2)^(1/2)/(a*x+1)^3+9/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2
)^(1/2)/(a*x+1)^2-31/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)/(a*x+
1)+1/16*(1-1/a^2/x^2)^(1/2)*ln(-a*x+1)/a/c^2/(c-c/a^2/x^2)^(1/2)-49/16*(1-
1/a^2/x^2)^(1/2)*ln(a*x+1)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.36

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(48x-\frac{8}{a(1+ax)^3}+\frac{54}{a(1+ax)^2}-\frac{186}{a+a^2x}+\frac{3\log(1-ax)}{a}-\frac{147\log(1+ax)}{a}\right)}{48\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

output
```
((1 - 1/(a^2*x^2))^(5/2)*(48*x - 8/(a*(1 + a*x)^3) + 54/(a*(1 + a*x)^2) -
186/(a + a^2*x) + (3*Log[1 - a*x])/a - (147*Log[1 + a*x])/a))/(48*(c - c/(
a^2*x^2))^(5/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.43, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x^5}{(1-ax)(ax+1)^4}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^5}{(1-ax)(ax+1)^4}dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\int \left(\frac{49}{16a^5(ax+1)}-\frac{31}{8a^5(ax+1)^2}+\frac{9}{4a^5(ax+1)^3}-\frac{1}{2a^5(ax+1)^4}-\frac{1}{a^5}-\frac{1}{16a^5(ax-1)}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{31}{8a^6(ax+1)}-\frac{9}{8a^6(ax+1)^2}+\frac{1}{6a^6(ax+1)^3}-\frac{\log(1-ax)}{16a^6}+\frac{49\log(ax+1)}{16a^6}-\frac{x}{a^5}\right)}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

output
```
-((a^5*Sqrt[1 - 1/(a^2*x^2)]*(-(x/a^5) + 1/(6*a^6*(1 + a*x)^3) - 9/(8*a^6*
(1 + a*x)^2) + 31/(8*a^6*(1 + a*x)) - Log[1 - a*x]/(16*a^6) + (49*Log[1 +
a*x])/(16*a^6)))/(c^2*Sqrt[c - c/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.66

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-1)(ax+1)\left(-48a^4x^4+147\ln(ax+1)x^3a^3-3a^3\ln(ax-1)x^3-144a^3x^3+441\ln(ax+1)x^2a^2-9a^2\ln(ax-1)x^2+42a^2\right.}{48a^6x^5\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-48*a^4*x^4+147*ln(a*x+1)*x
^3*a^3-3*a^3*ln(a*x-1)*x^3-144*a^3*x^3+441*ln(a*x+1)*x^2*a^2-9*a^2*ln(a*x-
1)*x^2+42*a^2*x^2+441*ln(a*x+1)*x*a-9*a*ln(a*x-1)*x+270*a*x+147*ln(a*x+1)-
3*ln(a*x-1)+140)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.52

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(48\,a^4x^4+144\,a^3x^3-42\,a^2x^2-270\,ax-147\left(a^3x^3+3\,a^2x^2+3\,ax+1\right)\log\left(ax+1\right.\right.}{48\left(a^5c^3x^3+3\,a^4c^3x^2+3\,a^3c^3x+\right.}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas
")
```

output
```
1/48*(48*a^4*x^4 + 144*a^3*x^3 - 42*a^2*x^2 - 270*a*x - 147*(a^3*x^3 + 3*a
^2*x^2 + 3*a*x + 1)*log(a*x + 1) + 3*(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)*log
(a*x - 1) - 140)*sqrt(a^2*c)/(a^5*c^3*x^3 + 3*a^4*c^3*x^2 + 3*a^3*c^3*x +
a^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima
")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.56

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\sqrt{c}\,(3\log(ax-1)\,a^3x^3+9\log(ax-1)\,a^2x^2+9\log(ax-1)\,ax+3\log(ax-1)-14}{}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x)
```

output
```
(sqrt(c)*(3*log(a*x - 1)*a**3*x**3 + 9*log(a*x - 1)*a**2*x**2 + 9*log(a*x
- 1)*a*x + 3*log(a*x - 1) - 147*log(a*x + 1)*a**3*x**3 - 441*log(a*x + 1)*
a**2*x**2 - 441*log(a*x + 1)*a*x - 147*log(a*x + 1) + 48*a**4*x**4 + 158*a
**3*x**3 - 228*a*x - 126))/(48*a*c**3*(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1
))
```

3.855 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 357

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1-ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^4}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^3}+\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}-\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}(1+ax)}+\frac{9\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{201\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
(1-1/a^2/x^2)^(1/2)*x/c^3/(c-c/a^2/x^2)^(1/2)+1/32*(1-1/a^2/x^2)^(1/2)/a/c
^3/(c-c/a^2/x^2)^(1/2)/(-a*x+1)+1/16*(1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^
2)^(1/2)/(a*x+1)^4-1/2*(1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)/(a*x+
1)^3+59/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)/(a*x+1)^2-75/16*(
1-1/a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)/(a*x+1)+9/64*(1-1/a^2/x^2)^(1
/2)*ln(-a*x+1)/a/c^3/(c-c/a^2/x^2)^(1/2)-201/64*(1-1/a^2/x^2)^(1/2)*ln(a*x
+1)/a/c^3/(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.30

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(64x+\frac{208+478ax+74a^2x^2-490a^3x^3-302a^4x^4}{a(-1+ax)(1+ax)^4}+\frac{9\log(1-ax)}{a}-\frac{201\log(1+ax)}{a}\right)}{64\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

input
```
Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

output
```
((1 - 1/(a^2*x^2))^(7/2)*(64*x + (208 + 478*a*x + 74*a^2*x^2 - 490*a^3*x^3
 - 302*a^4*x^4)/(a*(-1 + a*x)*(1 + a*x)^4) + (9*Log[1 - a*x])/a - (201*Log
[1 + a*x])/a))/(64*(c - c/(a^2*x^2))^(7/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^7}{(1-ax)^2(ax+1)^5}dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\int\left(-\frac{201}{64a^7(ax+1)}+\frac{75}{16a^7(ax+1)^2}-\frac{59}{16a^7(ax+1)^3}+\frac{3}{2a^7(ax+1)^4}-\frac{1}{4a^7(ax+1)^5}+\frac{1}{a^7}+\frac{9}{64a^7(ax-1)}+\frac{1}{32a^7(ax-1)}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^7\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{32a^8(1-ax)}-\frac{75}{16a^8(ax+1)}+\frac{59}{32a^8(ax+1)^2}-\frac{1}{2a^8(ax+1)^3}+\frac{1}{16a^8(ax+1)^4}+\frac{9\log(1-ax)}{64a^8}-\frac{201\log(ax+1)}{64a^8}+\frac{x}{a^7}\right)}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

output
```
(a^7*Sqrt[1 - 1/(a^2*x^2)]*(x/a^7 + 1/(32*a^8*(1 - a*x)) + 1/(16*a^8*(1 +
a*x)^4) - 1/(2*a^8*(1 + a*x)^3) + 59/(32*a^8*(1 + a*x)^2) - 75/(16*a^8*(1
+ a*x)) + (9*Log[1 - a*x])/(64*a^8) - (201*Log[1 + a*x])/(64*a^8)))/(c^3*S
qrt[c - c/(a^2*x^2)])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.69

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-1)(ax+1)(-64x^6a^6+201\ln(ax+1)x^5a^5-9\ln(ax-1)x^5a^5-192a^5x^5+603\ln(ax+1)x^4a^4-27\ln(ax-1)x^4a^4+174}{}$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64*((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-64*x^6*a^6+201*ln(a*x+1)*x
^5*a^5-9*ln(a*x-1)*x^5*a^5-192*a^5*x^5+603*ln(a*x+1)*x^4*a^4-27*ln(a*x-1)*
x^4*a^4+174*a^4*x^4+402*ln(a*x+1)*x^3*a^3-18*a^3*ln(a*x-1)*x^3+618*a^3*x^3
-402*ln(a*x+1)*x^2*a^2+18*a^2*ln(a*x-1)*x^2+118*a^2*x^2-603*ln(a*x+1)*x*a+
27*a*ln(a*x-1)*x-414*a*x-201*ln(a*x+1)+9*ln(a*x-1)-208)/a^8/x^7/(c*(a^2*x^
2-1)/a^2/x^2)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.58

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{(64\,a^6x^6 + 192\,a^5x^5 - 174\,a^4x^4 - 618\,a^3x^3 - 118\,a^2x^2 + 414\,ax - 201\,(a^5x^5 + 3\,a^4x^4}{64\,(a^7c^4x^5 + 3}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas
")
```

output
```
1/64*(64*a^6*x^6 + 192*a^5*x^5 - 174*a^4*x^4 - 618*a^3*x^3 - 118*a^2*x^2 +
 414*a*x - 201*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 - 3*a*x - 1)*l
og(a*x + 1) + 9*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 - 3*a*x - 1)*
log(a*x - 1) + 208)*sqrt(a^2*c)/(a^7*c^4*x^5 + 3*a^6*c^4*x^4 + 2*a^5*c^4*x
^3 - 2*a^4*c^4*x^2 - 3*a^3*c^4*x - a^2*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima
")
```

output
```
integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}}\,dx$$

input
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2),x)
```

output
```
int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.66

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{c}\,(9\log(ax-1)\,a^5x^5+27\log(ax-1)\,a^4x^4+18\log(ax-1)\,a^3x^3-18\log(ax-1)}{}$$

input
```
int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x)
```

output
```
(sqrt(c)*(9*log(a*x - 1)*a**5*x**5 + 27*log(a*x - 1)*a**4*x**4 + 18*log(a*
x - 1)*a**3*x**3 - 18*log(a*x - 1)*a**2*x**2 - 27*log(a*x - 1)*a*x - 9*log
(a*x - 1) - 201*log(a*x + 1)*a**5*x**5 - 603*log(a*x + 1)*a**4*x**4 - 402*
log(a*x + 1)*a**3*x**3 + 402*log(a*x + 1)*a**2*x**2 + 603*log(a*x + 1)*a*x
 + 201*log(a*x + 1) + 64*a**6*x**6 + 250*a**5*x**5 - 502*a**3*x**3 - 234*a
**2*x**2 + 240*a*x + 150))/(64*a*c**4*(a**5*x**5 + 3*a**4*x**4 + 2*a**3*x*
*3 - 2*a**2*x**2 - 3*a*x - 1))
```

3.856 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 76

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/2*(c-c/a^2/x^2)^(1/2)*x^2/a/(1-1/a^2/x^2)^(1/2)+1/3*(c-c/a^2/x^2)^(1/2)*
x^3/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(3+2ax)}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x^2*(3 + 2*a*x))/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6751, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{a^2x^2}}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \ 6751$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 6747$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int x(ax+1)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 49$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(ax^2+x\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 2009$$

$$\frac{\left(\frac{ax^3}{3}+\frac{x^2}{2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x^2/2 + (a*x^3)/3))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.62

method	result	size
orering	$\frac{x^{3}(2ax+3)\sqrt{c-\frac{c}{a^{2}x^{2}}}}{6(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	47
gosper	$\frac{x^{3}(2ax+3)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{6(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	53
default	$\frac{x^{3}(2ax+3)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{6(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	53

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)*x^2,x,method=_RETURNVERB
OSE)
```

output
```
1/6*x^3*(2*a*x+3)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.32

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{(2\,ax^3+3\,x^2)\sqrt{a^2c}}{6\,a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)*x^2,x, algorithm="
fricas")
```

output
```
1/6*(2*a*x^3 + 3*x^2)*sqrt(a^2*c)/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2)*x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)*x^2,x, algorithm="
maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)*x^2,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,2,1,0]%%%}+%%%{1,[0,1,1,0,0]%%%} / %%%{1,[0,0,0,2,1
]%%%} Err
```

Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = \frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,(2\,a\,x+3)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,(a\,x-1)}$$

input
```
int((x^2*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(x^3*(c - c/(a^2*x^2))^(1/2)*(2*a*x + 3)*((a*x - 1)/(a*x + 1))^(1/2))/(6*(
a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.21

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c}\,x^2(2ax+3)}{6a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)*x^2,x)
```

output
```
(sqrt(c)*x**2*(2*a*x + 3))/(6*a)
```

3.857 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal result

Integrand size = 23, antiderivative size = 43

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{2a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/2*(c-c/a^2/x^2)^(1/2)*(a*x+1)^2/a^2/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{x}{a}+\frac{x^2}{2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x/a + x^2/2))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6751, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{a^2x^2}}e^{\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int (ax+1)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{2a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(2*a^2*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

method	result	size
orering	$\frac{x^{2}(ax+2)\sqrt{c-\frac{c}{a^{2}x^{2}}}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	46
gosper	$\frac{x^{2}(ax+2)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	52
default	$\frac{x^{2}(ax+2)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	52

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)*x,x,method=_RETURNVERBOS
E)
```

output
```
1/2*x^2*(a*x+2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.49

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{a^2c}(ax^2+2\,x)}{2\,a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)*x,x, algorithm="fr
icas")
```

output
```
1/2*sqrt(a^2*c)*(a*x^2 + 2*x)/a^2
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2)*x,x)
```

output
```
Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1))
, x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)*x,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)*x,x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,1,1,0]%%%}+%%%{1,[0,1,0,0,0]%%%} / %%%{1,[0,0,0,2,1
]%%%} Err
```

Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = \frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+2)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,(a\,x-1)}$$

input
```
int((x*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 2)*((a*x - 1)/(a*x + 1))^(1/2))/(2*(a*
x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.30

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = \frac{\sqrt{c}\,x(ax+2)}{2a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)*x,x)
```

output
```
(sqrt(c)*x*(a*x + 2))/(2*a)
```

3.858 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1/2)+(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1
/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x+\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x + Log[x]/a))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6751, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{\coth^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}} \int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}} \int \frac{ax+1}{x}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}} \int \left(a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{(ax+\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	50

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.25

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax+\log(x))}{a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

output
```
sqrt(a^2*c)*(a*x + log(x))/a^2
```

Sympy [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)),
x)
```

Maxima [F]

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = 2\sqrt{c}\left(\frac{x\text{sgn}(x)}{a\text{sgn}(ax+1)}+\frac{\log(|x|)\,\text{sgn}(x)}{a^2\text{sgn}(ax+1)}\right)|a|$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

output
```
2*sqrt(c)*(x*sgn(x)/(a*sgn(a*x + 1)) + log(abs(x))*sgn(x)/(a^2*sgn(a*x + 1
)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.18

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c}\,(\log(x)+ax)}{a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x)
```

output
```
(sqrt(c)*(log(x) + a*x))/a
```

3.859 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 70

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x+(c-c/a^2/x^2)^(1/2)*ln(x)/(1-
1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{ax}+\log(x)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-(1/(a*x)) + Log[x]))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6751, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{ax+1}{x^2}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{a}{x}+\frac{1}{x^2}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(a\log(x)-\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-x^(-1) + a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{(a\ln(x)x-1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	50

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x,method=_RETURNVERBOS
E)
```

output
```
(a*ln(x)*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.30

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{a^2c}(ax\log{(x)}-1)}{a^2x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fr
icas")
```

output
```
sqrt(a^2*c)*(a*x*log(x) - 1)/(a^2*x)
```

Sympy [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2)/x,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/(x*sqrt((a*x - 1)/(a*x + 1)
)), x)
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\left(a\log\left(|x|\right)\operatorname{sgn}(x)-\frac{\operatorname{sgn}(x)}{x}\right)\sqrt{c}|a|}{a^2\operatorname{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="gi
ac")
```

output
```
(a*log(abs(x))*sgn(x) - sgn(x)/x)*sqrt(c)*abs(a)/(a^2*sgn(a*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.23

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c}\,(\log(x)\,ax-1)}{ax}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x)
```

output
```
(sqrt(c)*(log(x)*a*x - 1))/(a*x)
```

3.860 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 46

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}(1+ax)^2}{2a\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

output
```
-1/2*(c-c/a^2/x^2)^(1/2)*(a*x+1)^2/a/(1-1/a^2/x^2)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2ax^2}-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/2*1/(a*x^2) - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6751, 6747, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{ax+1}{x^3}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

output
```
-1/2*(Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

method	result	size
orering	$-\frac{(2ax+1)\sqrt{c-\frac{c}{a^{2}x^{2}}}}{2x(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	47
gosper	$-\frac{(2ax+1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{2x(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	53
default	$-\frac{(2ax+1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{2x(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	53

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x,method=_RETURNVERB
OSE)
```

output
```
-1/2*(2*a*x+1)/x/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.46

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\frac{\sqrt{a^2c}(2\,ax+1)}{2\,a^2x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
fricas")
```

output
```
-1/2*sqrt(a^2*c)*(2*a*x + 1)/(a^2*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\frac{(2\,ax\mathrm{sgn}(x)+\mathrm{sgn}(x))\sqrt{c}|a|}{2\,a^2x^2\mathrm{sgn}\,(ax+1)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
giac")
```

output
```
-1/2*(2*a*x*sgn(x) + sgn(x))*sqrt(c)*abs(a)/(a^2*x^2*sgn(a*x + 1))
```

Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\left(x\,\sqrt{c-\frac{c}{a^2\,x^2}}+\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{2\,a}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{x}{a}-x^2}$$

input
```
int((c - c/(a^2*x^2))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
((x*(c - c/(a^2*x^2))^(1/2) + (c - c/(a^2*x^2))^(1/2)/(2*a))*((a*x - 1)/(a
*x + 1))^(1/2))/(x/a - x^2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.35

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c}\,(-2ax-1)}{2a\,x^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x)
```

output
```
(sqrt(c)*( - 2*a*x - 1))/(2*a*x**2)
```

3.861 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx$

Optimal result

Integrand size = 27, antiderivative size = 155

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx=\frac{6\sqrt{c-\frac{c}{a^2x^2}}x}{5a^4}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^2}{4a^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^3}{5a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^4}{2a}+\frac{1}{5}\sqrt{c-\frac{c}{a^2x^2}}x^5+\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{4a^5}$$

output
```
6/5*(c-c/a^2/x^2)^(1/2)*x/a^4+3/4*(c-c/a^2/x^2)^(1/2)*x^2/a^3+3/5*(c-c/a^2
/x^2)^(1/2)*x^3/a^2+1/2*(c-c/a^2/x^2)^(1/2)*x^4/a+1/5*(c-c/a^2/x^2)^(1/2)*
x^5+3/4*c^(1/2)*arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))/a^5
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.65

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}(24+15ax+12a^2x^2+10a^3x^3+4a^4x^4)+15\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{20a^4\sqrt{-1+a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^4,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(24 + 15*a*x + 12*a^2*x^2 + 1
0*a^3*x^3 + 4*a^4*x^4) + 15*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(20*a^4*Sqrt[-
1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {6717, 6709, 541, 25, 27, 533, 27, 533, 27, 533, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int\frac{x^3(ax+1)^2}{\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{541}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int -\frac{a^2x^3(10ax+9)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{\int \frac{a^2x^3(10ax+9)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\int \frac{x^3(10ax+9)}{\sqrt{1-a^2x^2}}dx-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \ 533$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\left(\frac{\int \frac{6ax^2(6ax+5)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{5x^3\sqrt{1-a^2x^2}}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\left(\frac{3\int \frac{x^2(6ax+5)}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{5x^3\sqrt{1-a^2x^2}}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \ 533$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\left(\frac{3\left(\frac{\int \frac{3ax(5ax+4)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{2x^2\sqrt{1-a^2x^2}}{a}\right)}{2a}-\frac{5x^3\sqrt{1-a^2x^2}}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\left(\frac{3\left(\frac{\int \frac{x(5ax+4)}{\sqrt{1-a^2x^2}}dx}{a}-\frac{2x^2\sqrt{1-a^2x^2}}{a}\right)}{2a}-\frac{5x^3\sqrt{1-a^2x^2}}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \ 533$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{\int\frac{a(8ax+5)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{5x\sqrt{1-a^2x^2}}{2a}}{a}-\frac{2x^2\sqrt{1-a^2x^2}}{a}\right)}{2a}-\frac{5x^3\sqrt{1-a^2x^2}}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$\downarrow$ 27

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{\int\frac{8ax+5}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{5x\sqrt{1-a^2x^2}}{2a}}{a}-\frac{2x^2\sqrt{1-a^2x^2}}{a}\right)}{2a}-\frac{5x^3\sqrt{1-a^2x^2}}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$\downarrow$ 455

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{5\int\frac{1}{\sqrt{1-a^2x^2}}dx-\frac{8\sqrt{1-a^2x^2}}{a}}{2a}-\frac{5x\sqrt{1-a^2x^2}}{2a}}{a}-\frac{2x^2\sqrt{1-a^2x^2}}{a}\right)}{2a}-\frac{5x^3\sqrt{1-a^2x^2}}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$\downarrow$ 223

$$-\frac{x\left(\frac{1}{5}\left(\frac{3\left(\frac{\frac{\frac{5\arcsin(ax)}{a}-\frac{8\sqrt{1-a^2x^2}}{a}}{2a}-\frac{5x\sqrt{1-a^2x^2}}{2a}}{a}-\frac{2x^2\sqrt{1-a^2x^2}}{a}\right)}{2a}-\frac{5x^3\sqrt{1-a^2x^2}}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^4,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/5*(x^4*Sqrt[1 - a^2*x^2]) + ((-5*x^3*Sqrt[1
 - a^2*x^2])/(2*a) + (3*((-2*x^2*Sqrt[1 - a^2*x^2])/a + ((-5*x*Sqrt[1 - a^
2*x^2])/(2*a) + ((-8*Sqrt[1 - a^2*x^2])/a + (5*ArcSin[a*x])/a)/(2*a))/a))/
(2*a))/5))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.92

method	result
risch	$\frac{(4a^4x^4+10a^3x^3+12a^2x^2+15ax+24)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x}{20a^4}+\frac{3\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{4a^3\sqrt{a^2c}\,(a^2x^2-1)}$
default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\left(4x^2\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^5+10x\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^4+16\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3+25\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2cx-25c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\right.\right.}{20\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^5c}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/20*(4*a^4*x^4+10*a^3*x^3+12*a^2*x^2+15*a*x+24)/a^4*(c*(a^2*x^2-1)/a^2/x^
2)^(1/2)*x+3/4/a^3*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(
1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.54

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx$$

$$=\left[\frac{2\left(4a^5x^5+10a^4x^4+12a^3x^3+15a^2x^2+24ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+15\sqrt{c}\log\left(2a^2cx^2+2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\right.}{40a^5}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x^4,x, algorithm="fricas")
```

output
```
[1/40*(2*(4*a^5*x^5 + 10*a^4*x^4 + 12*a^3*x^3 + 15*a^2*x^2 + 24*a*x)*sqrt(
(a^2*c*x^2 - c)/(a^2*x^2)) + 15*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^
2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^5, 1/20*((4*a^5*x^5 + 10*a^4*x^4
 + 12*a^3*x^3 + 15*a^2*x^2 + 24*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 15*
sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^
2 - c)))/a^5]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx=\int\frac{x^4\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)*x**4,x)
```

output
```
Integral(x**4*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x
)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}x^4}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x^4,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^4/(a*x - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx$$
$$=\frac{1}{40}\left(2\sqrt{a^2cx^2-c}\left(\left(2\left(x\left(\frac{2\,x\text{sgn}(x)}{a^2}+\frac{5\,\text{sgn}(x)}{a^3}\right)+\frac{6\,\text{sgn}(x)}{a^4}\right)x+\frac{15\,\text{sgn}(x)}{a^5}\right)x+\frac{24\,\text{sgn}(x)}{a^6}\right)-\frac{30\,\sqrt{}}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x^4,x, algorithm="giac")
```

output
```
1/40*(2*sqrt(a^2*c*x^2 - c)*((2*(x*(2*x*sgn(x)/a^2 + 5*sgn(x)/a^3) + 6*sgn
(x)/a^4)*x + 15*sgn(x)/a^5)*x + 24*sgn(x)/a^6) - 30*sqrt(c)*log(abs(-sqrt(
a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^5*abs(a)) + 3*(5*a*sqrt(c)*log(
abs(c)) - 16*sqrt(-c)*abs(a))*sgn(x)/(a^6*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx=\int\frac{x^4\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

input
```
int((x^4*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x^4*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.68

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx$$
$$=\frac{\sqrt{c}\left(4\sqrt{a^2x^2-1}\,a^4x^4+10\sqrt{a^2x^2-1}\,a^3x^3+12\sqrt{a^2x^2-1}\,a^2x^2+15\sqrt{a^2x^2-1}\,ax+24\sqrt{a^2x^2-1}+15\right.}{20a^5}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x^4,x)
```

output
```
(sqrt(c)*(4*sqrt(a**2*x**2 - 1)*a**4*x**4 + 10*sqrt(a**2*x**2 - 1)*a**3*x*
*3 + 12*sqrt(a**2*x**2 - 1)*a**2*x**2 + 15*sqrt(a**2*x**2 - 1)*a*x + 24*sq
rt(a**2*x**2 - 1) + 15*log(sqrt(a**2*x**2 - 1) + a*x)))/(20*a**5)
```

3.862 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 130

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{3a^3}+\frac{7\sqrt{c-\frac{c}{a^2x^2}}x^2}{8a^2}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x^3}{3a}+\frac{1}{4}\sqrt{c-\frac{c}{a^2x^2}}x^4+\frac{7\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{8a^4}$$

output
```
4/3*(c-c/a^2/x^2)^(1/2)*x/a^3+7/8*(c-c/a^2/x^2)^(1/2)*x^2/a^2+2/3*(c-c/a^2
/x^2)^(1/2)*x^3/a+1/4*(c-c/a^2/x^2)^(1/2)*x^4+7/8*c^(1/2)*arctanh((c-c/a^2
/x^2)^(1/2)/c^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.72

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}(32+21ax+16a^2x^2+6a^3x^3)+21\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{24a^3\sqrt{-1+a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(32 + 21*a*x + 16*a^2*x^2 + 6
*a^3*x^3) + 21*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^3*Sqrt[-1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6717, 6709, 541, 25, 27, 533, 27, 533, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int\frac{x^2(ax+1)^2}{\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{541}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int-\frac{a^2x^2(8ax+7)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{\int\frac{a^2x^2(8ax+7)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\int\frac{x^2(8ax+7)}{\sqrt{1-a^2x^2}}dx-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{533}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{\int \frac{ax(21ax+16)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{8x^2\sqrt{1-a^2x^2}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{\int \frac{x(21ax+16)}{\sqrt{1-a^2x^2}}dx}{3a}-\frac{8x^2\sqrt{1-a^2x^2}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{533}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{a(32ax+21)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{21x\sqrt{1-a^2x^2}}{2a}}{3a}-\frac{8x^2\sqrt{1-a^2x^2}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{32ax+21}{\sqrt{1-a^2x^2}}dx}{2a}-\frac{21x\sqrt{1-a^2x^2}}{2a}}{3a}-\frac{8x^2\sqrt{1-a^2x^2}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{455}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{\frac{21\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{32\sqrt{1-a^2x^2}}{a}}{2a}-\frac{21x\sqrt{1-a^2x^2}}{2a}}{3a}-\frac{8x^2\sqrt{1-a^2x^2}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x\left(\frac{1}{4}\left(\frac{\frac{\frac{21\arcsin(ax)}{a}-\frac{32\sqrt{1-a^2x^2}}{a}}{2a}-\frac{21x\sqrt{1-a^2x^2}}{2a}}{3a}-\frac{8x^2\sqrt{1-a^2x^2}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/4*(x^3*Sqrt[1 - a^2*x^2]) + ((-8*x^2*Sqrt[1
 - a^2*x^2])/(3*a) + ((-21*x*Sqrt[1 - a^2*x^2])/(2*a) + ((-32*Sqrt[1 - a^2
*x^2])/a + (21*ArcSin[a*x])/a)/(2*a))/(3*a))/4))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03

method	result
risch	$\frac{(6a^3x^3+16a^2x^2+21ax+32)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x}{24a^3}+\frac{7\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2cx^2-c}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{8a^2\sqrt{a^2c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\left(-6x\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^4-16\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3-27\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2cx+27c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)-48c^{\frac{3}{2}}\ \mathrm{l}\right.}{24\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^4c}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*a^3*x^3+16*a^2*x^2+21*a*x+32)/a^3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x+
7/8/a^2*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*(c*(a^
2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.71

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$=\left[\frac{2\left(6a^4x^4+16a^3x^3+21a^2x^2+32ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+21\sqrt{c}\log\left(2a^2cx^2+2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{48a^4},\ \frac{(6a^4\ \cdots}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x^3,x, algorithm="fricas")
```

output
```
[1/48*(2*(6*a^4*x^4 + 16*a^3*x^3 + 21*a^2*x^2 + 32*a*x)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)) + 21*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c
*x^2 - c)/(a^2*x^2)) - c))/a^4, 1/24*((6*a^4*x^4 + 16*a^3*x^3 + 21*a^2*x^2
 + 32*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 21*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^4]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx = \int \frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)*x**3,x)
```

output
```
Integral(x**3*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x
)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}x^3}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x^3,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^3/(a*x - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$=\frac{1}{48}\left(2\sqrt{a^2cx^2-c}\left(\left(2x\left(\frac{3x\text{sgn}(x)}{a^2}+\frac{8\,\text{sgn}(x)}{a^3}\right)+\frac{21\,\text{sgn}(x)}{a^4}\right)x+\frac{32\,\text{sgn}(x)}{a^5}\right)-\frac{42\sqrt{c}\log\left(\left|-\sqrt{a^2c}x\right.\right.}{c}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x^3,x, algorithm="giac")
```

output
```
1/48*(2*sqrt(a^2*c*x^2 - c)*((2*x*(3*x*sgn(x)/a^2 + 8*sgn(x)/a^3) + 21*sgn
(x)/a^4)*x + 32*sgn(x)/a^5) - 42*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2
*c*x^2 - c)))*sgn(x)/(a^4*abs(a)) + (21*a*sqrt(c)*log(abs(c)) - 64*sqrt(-c
)*abs(a))*sgn(x)/(a^5*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\frac{x^3\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.67

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$=\frac{\sqrt{c}\left(6\sqrt{a^2x^2-1}\,a^3x^3+16\sqrt{a^2x^2-1}\,a^2x^2+21\sqrt{a^2x^2-1}\,ax+32\sqrt{a^2x^2-1}+21\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{24a^4}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x^3,x)
```

output
```
(sqrt(c)*(6*sqrt(a**2*x**2 - 1)*a**3*x**3 + 16*sqrt(a**2*x**2 - 1)*a**2*x*
*2 + 21*sqrt(a**2*x**2 - 1)*a*x + 32*sqrt(a**2*x**2 - 1) + 21*log(sqrt(a**
2*x**2 - 1) + a*x)))/(24*a**4)
```

3.863 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 99

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{5\sqrt{c-\frac{c}{a^2x^2}}x}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{a}+\frac{1}{3}\sqrt{c-\frac{c}{a^2x^2}}x^3+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{a^3}$$

output
```
5/3*(c-c/a^2/x^2)^(1/2)*x/a^2+(c-c/a^2/x^2)^(1/2)*x^2/a+1/3*(c-c/a^2/x^2)^
(1/2)*x^3+c^(1/2)*arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}(5+3ax+a^2x^2)+3\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{3a^2\sqrt{-1+a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(5 + 3*a*x + a^2*x^2) + 3*Log
[a*x + Sqrt[-1 + a^2*x^2]]))/(3*a^2*Sqrt[-1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6709, 541, 25, 27, 533, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int\frac{x(ax+1)^2}{\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{541}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int-\frac{a^2x(6ax+5)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{1}{3}x^2\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{\int\frac{a^2x(6ax+5)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{1}{3}x^2\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}\int\frac{x(6ax+5)}{\sqrt{1-a^2x^2}}dx-\frac{1}{3}x^2\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{533}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}\left(\frac{\int \frac{2a(5ax+3)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{3x\sqrt{1-a^2x^2}}{a}\right)-\frac{1}{3}x^2\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}\left(\frac{\int \frac{5ax+3}{\sqrt{1-a^2x^2}}dx}{a}-\frac{3x\sqrt{1-a^2x^2}}{a}\right)-\frac{1}{3}x^2\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{455}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}\left(\frac{3\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{a}}{a}-\frac{3x\sqrt{1-a^2x^2}}{a}\right)-\frac{1}{3}x^2\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x\left(\frac{1}{3}\left(\frac{\frac{3\arcsin(ax)}{a}-\frac{5\sqrt{1-a^2x^2}}{a}}{a}-\frac{3x\sqrt{1-a^2x^2}}{a}\right)-\frac{1}{3}x^2\sqrt{1-a^2x^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/3*(x^2*Sqrt[1 - a^2*x^2]) + ((-3*x*Sqrt[1 -
 a^2*x^2])/a + ((-5*Sqrt[1 - a^2*x^2])/a + (3*ArcSin[a*x])/a)/a)/3))/Sqrt[
1 - a^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.25

method	result
risch	$\frac{(a^2x^2+3ax+5)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x}{3a^2}+\frac{\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{a\sqrt{a^2c}\,(a^2x^2-1)}$
default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\left(\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3+3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2cx-3c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)+6c^{\frac{3}{2}}\ln\left(\frac{\sqrt{c}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+xc}{\sqrt{c}}\right)+6\sqrt{}\right.}{3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^3c}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(a^2*x^2+3*a*x+5)/a^2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x+1/a*ln(a^2*c*x/(
a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(1
/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.06

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$$
$$=\left[\frac{2\,(a^3x^3+3\,a^2x^2+5\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{6\,a^3},\ \frac{(a^3x^3+3\,a^2x^2+5\,a}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x^2,x, algorithm="fricas")
```

output
```
[1/6*(2*(a^3*x^3 + 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*
sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
) - c))/a^3, 1/3*((a^3*x^3 + 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)) - 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
/(a^2*c*x^2 - c)))/a^3]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)*x**2,x)
```

output
```
Integral(x**2*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x
)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}x^2}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x^2,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^2/(a*x - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$$
$$=\frac{1}{6}\left(2\sqrt{a^2cx^2-c}\left(x\left(\frac{x\mathrm{sgn}(x)}{a^2}+\frac{3\,\mathrm{sgn}(x)}{a^3}\right)+\frac{5\,\mathrm{sgn}(x)}{a^4}\right)-\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^3|a|}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x^2,x, algorithm="giac")
```

output
```
1/6*(2*sqrt(a^2*c*x^2 - c)*(x*(x*sgn(x)/a^2 + 3*sgn(x)/a^3) + 5*sgn(x)/a^4
) - 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^3*a
bs(a)) + (3*a*sqrt(c)*log(abs(c)) - 10*sqrt(-c)*abs(a))*sgn(x)/(a^4*abs(a)
))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.69

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c}\left(\sqrt{a^2x^2-1}\,a^2x^2+3\sqrt{a^2x^2-1}\,ax+5\sqrt{a^2x^2-1}+3\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{3a^3}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x^2,x)
```

output
```
(sqrt(c)*(sqrt(a**2*x**2 - 1)*a**2*x**2 + 3*sqrt(a**2*x**2 - 1)*a*x + 5*sq
rt(a**2*x**2 - 1) + 3*log(sqrt(a**2*x**2 - 1) + a*x)))/(3*a**3)
```

3.864 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{2\sqrt{c-\frac{c}{a^2x^2}}x}{a}+\frac{1}{2}\sqrt{c-\frac{c}{a^2x^2}}x^2+\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{2a^2}$$

output
```
2*(c-c/a^2/x^2)^(1/2)*x/a+1/2*(c-c/a^2/x^2)^(1/2)*x^2+3/2*c^(1/2)*arctanh(
(c-c/a^2/x^2)^(1/2)/c^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left((4+ax)\sqrt{1-a^2x^2}+6\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 -
a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6717, 6709, 469, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}xdx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{469}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{3}{2}\int \frac{ax+1}{\sqrt{1-a^2x^2}}dx-\frac{(ax+1)\sqrt{1-a^2x^2}}{2a}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{455}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{a}\right)-\frac{(ax+1)\sqrt{1-a^2x^2}}{2a}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x\left(\frac{3}{2}\left(\frac{\arcsin(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}\right)-\frac{(ax+1)\sqrt{1-a^2x^2}}{2a}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/2*((1 + a*x)*Sqrt[1 - a^2*x^2])/a + (3*(-(S
qrt[1 - a^2*x^2]/a) + ArcSin[a*x]/a))/2))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.46

method	result	s
risch	$\frac{(ax+4)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x}{2a}+\frac{3\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{2\sqrt{a^2c}\,(a^2x^2-1)}$	
default	$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2-\sqrt{c}\,\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)+4\sqrt{c}\,\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+xc}{\sqrt{c}}\right)+4\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a\right)}{2\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2}$	

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x+4)/a*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x+3/2*ln(a^2*c*x/(a^2*c)^(1/2)
+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*
x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.41

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = \left[\frac{2\,(a^2x^2+4\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{4\,a^2}, \frac{(a^2x^2+4\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\sqrt{}}{2}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x,x, algorithm="fricas")
```

output
```
[1/4*(2*(a^2*x^2 + 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*sqrt(c)*log(
2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^2,
 1/2*((a^2*x^2 + 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(-c)*arcta
n(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = \int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)*x,x)
```

output
```
Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}x}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x/(a*x - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.36

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$
$$=\frac{1}{4}\left(2\sqrt{a^2cx^2-c}\left(\frac{x\mathrm{sgn}(x)}{a^2}+\frac{4\,\mathrm{sgn}(x)}{a^3}\right)-\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^2|a|}+\frac{(3\,a\sqrt{c}\log\left(|c\right.}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x,x, algorithm="giac")
```

output
```
1/4*(2*sqrt(a^2*c*x^2 - c)*(x*sgn(x)/a^2 + 4*sgn(x)/a^3) - 6*sqrt(c)*log(a
bs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c))*sgn(x)/(a^2*abs(a)) + (3*a*sqrt(
c)*log(abs(c)) - 8*sqrt(-c)*abs(a))*sgn(x)/(a^3*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int((x*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int((x*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = \frac{\sqrt{c}\left(\sqrt{a^2x^2-1}\,ax+4\sqrt{a^2x^2-1}+3\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{2a^2}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)*x,x)
```

output
```
(sqrt(c)*(sqrt(a**2*x**2 - 1)*a*x + 4*sqrt(a**2*x**2 - 1) + 3*log(sqrt(a**
2*x**2 - 1) + a*x)))/(2*a**2)
```

3.865 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\sqrt{c-\frac{c}{a^2x^2}}x-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}x}\right)}{a}+\frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{a}$$

output
```
(c-c/a^2/x^2)^(1/2)*x-c^(1/2)*arctan(c^(1/2)/a/(c-c/a^2/x^2)^(1/2)/x)/a+2*
c^(1/2)*arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}-\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

input
```
Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]
] + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6717, 6709, 541, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 541}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int -\frac{a^2(2ax+1)}{x\sqrt{1-a^2x^2}}dx}{a^2}-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{\int \frac{a^2(2ax+1)}{x\sqrt{1-a^2x^2}}dx}{a^2}-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{2ax+1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{538}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(2a\int \frac{1}{\sqrt{1-a^2x^2}}dx+\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}+2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\sqrt{1-a^2x^2}+2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a^2}-\sqrt{1-a^2x^2}+2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\sqrt{1-a^2x^2}+2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

input
```
Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-Sqrt[1 - a^2*x^2] + 2*ArcSin[a*x] - ArcTanh[S
qrt[1 - a^2*x^2]]))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(74) = 148$.

Time = 0.22 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.23

method	result
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2-2\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}-2\sqrt{c}\,\ln\left(\frac{\sqrt{c}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+xc}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^2}}+c\ln\left(\frac{2\sqrt{-\cdots}}{\cdots}\right)\right)}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)
*a^2-2*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)*a^2*(-c/a^2)^(1/2)-2*c^(1/2)*ln((c^(1
/2)*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)+x*c)/c^(1/2))*a*(-c/a^2)^(1/2)+c*ln(2*((
-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)
^(1/2)/a^2/(-c/a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.86

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-4\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a}, \frac{ax\sqrt{\frac{a^2cx^2-}{a^2x^2}}}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 4*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a
^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a,
(a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + sqrt(c)*arctan(a*x*sqrt((a^2*c*x^2
- c)/(a^2*x^2))/sqrt(c)) + sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Maxima [F]

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/(a*x - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{a\,x-1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.58

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c}\,\left(2atan\left(\sqrt{a^2x^2-1}+ax\right)+\sqrt{a^2x^2-1}+2\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{a}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x)
```

output
```
(sqrt(c)*(2*atan(sqrt(a**2*x**2 - 1) + a*x) + sqrt(a**2*x**2 - 1) + 2*log(
sqrt(a**2*x**2 - 1) + a*x)))/a
```

3.866 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 79

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \sqrt{c-\frac{c}{a^2x^2}} - 2\sqrt{c}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}x}\right) + \sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)$$

output
```
(c-c/a^2/x^2)^(1/2)-2*c^(1/2)*arctan(c^(1/2)/a/(c-c/a^2/x^2)^(1/2)/x)+c^(1
/2)*arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2} - 2ax\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right) + ax\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2] - 2*a*x*ArcTan[1/Sqrt[-1 + a^2*
x^2]] + a*x*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6717, 6709, 540, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^2\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\int -\frac{a(ax+2)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{a(ax+2)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(a\int \frac{ax+2}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \ 538$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(a\left(a\int\frac{1}{\sqrt{1-a^2x^2}}dx+2\int\frac{1}{x\sqrt{1-a^2x^2}}dx\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \ 223$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(a\left(2\int\frac{1}{x\sqrt{1-a^2x^2}}dx+\arcsin(ax)\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \ 243$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(a\left(\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2+\arcsin(ax)\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \ 73$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(a\left(\arcsin(ax)-\frac{2\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a^2}\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \ 221$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(a\left(\arcsin(ax)-2\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-(Sqrt[1 - a^2*x^2]/x) + a*(ArcSin[a*x] - 2*Ar
cTanh[Sqrt[1 - a^2*x^2]])))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(65) = 130$.

Time = 0.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.84

method	result
risch	$\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}+\frac{\left(\frac{a^2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}-\frac{2a\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)}{\sqrt{-c}}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{a^2x^2-1}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^3cx^2+a^3\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}+c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)\sqrt{-\frac{c}{a^2}}\,ax-2c^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\right.}{a\sqrt{\frac{c(a}{}}}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(a^2*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^
(1/2))/(a^2*c)^(1/2)-2*a/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1
/2))/x))*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.01

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \left[-\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + \sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right) + \sqrt{\frac{a^2cx^2-c}{a^2x^2}}, 2\sqrt{c}\arctan\left(\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\sqrt{c}}\right) + \frac{1}{2}\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right) + \sqrt{\frac{a^2cx^2-c}{a^2x^2}}\right]$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
[-sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*
x^2 - c)) + sqrt(-c)*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)) - 2*c)/x^2) + sqrt((a^2*c*x^2 - c)/(a^2*x^2)), 2*sqrt(c)*arcta
n(a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/sqrt(c)) + 1/2*sqrt(c)*log(2*a^2*c*x
^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + sqrt((a^2*c*
x^2 - c)/(a^2*x^2))]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{x\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x*(a*x - 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.61

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \left(\frac{4\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a}-\frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{|a|}+\frac{2c^{\frac{3}{2}}}{\left(\left(\sqrt{a^2c}x-\sqrt{c}\right.\right.}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
(4*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
 - sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/abs(a) +
2*c^(3/2)*sgn(x)/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)*abs(a)))*a
bs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{x\,(a\,x-1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c}\left(4atan\left(\sqrt{a^2x^2-1}+ax\right)ax+\sqrt{a^2x^2-1}+\log\left(\sqrt{a^2x^2-1}+ax\right)ax+ax\right)}{ax}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x)
```

output
```
(sqrt(c)*(4*atan(sqrt(a**2*x**2 - 1) + a*x)*a*x + sqrt(a**2*x**2 - 1) + lo
g(sqrt(a**2*x**2 - 1) + a*x)*a*x + a*x))/(a*x)
```

3.867 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 79

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=2a\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2x}-\frac{3}{2}a\sqrt{c}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\right)$$

output
```
2*a*(c-c/a^2/x^2)^(1/2)+1/2*(c-c/a^2/x^2)^(1/2)/x-3/2*a*c^(1/2)*arctan(c^(
1/2)/a/(c-c/a^2/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx =\frac{\sqrt{c-\frac{c}{a^2x^2}}\left((1+4ax)\sqrt{-1+a^2x^2}-3a^2x^2\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{2x\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*((1 + 4*a*x)*Sqrt[-1 + a^2*x^2] - 3*a^2*x^2*ArcTan[
1/Sqrt[-1 + a^2*x^2]]))/(2*x*Sqrt[-1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6709, 540, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}dx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^3\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2}\int -\frac{a(3ax+4)}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}\int \frac{a(3ax+4)}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}a\int \frac{3ax+4}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 534}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}a\left(3a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{4\sqrt{1-a^2x^2}}{x}\right)-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}a\left(\frac{3}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{4\sqrt{1-a^2x^2}}{x}\right)-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}a\left(-\frac{3\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{4\sqrt{1-a^2x^2}}{x}\right)-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{x\left(\frac{1}{2}a\left(-3a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{4\sqrt{1-a^2x^2}}{x}\right)-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/2*Sqrt[1 - a^2*x^2]/x^2 + (a*((-4*Sqrt[1 -
a^2*x^2])/x - 3*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(65) = 130$.

Time = 0.25 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.80

method	result
risch	$\frac{(4a^3x^3+a^2x^2-4ax-1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{2x(a^2x^2-1)} - \frac{3a^2\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2cx^2-c}}{x}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{2\sqrt{-c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\left(-4\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^3cx^3+4\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,a^3x+3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^2cx^2+4\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(x\right.\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(4*a^3*x^3+a^2*x^2-4*a*x-1)/x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1
)-3/2*a^2/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)*(c*(a^2
*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.05

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$$
$$= \left[\frac{3\,a\sqrt{-c}x\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(4\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,x}, \frac{3\,a\sqrt{c}x\arctan\left(\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\sqrt{c}}\right)+(4\,a\right.}{2\,x}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/4*(3*a*sqrt(-c)*x*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)) - 2*c)/x^2) + 2*(4*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x
, 1/2*(3*a*sqrt(c)*x*arctan(a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/sqrt(c)) +
 (4*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{x^2\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**2*(a*x - 1)),
 x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^2}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(65) = 130$.

Time = 0.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.46

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$$
$$= \left(3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x) - \frac{\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^3 ac\,\mathrm{sgn}(x) - 4\left(\sqrt{a^2c}x - \sqrt{a\right.}{\left(\left(\sqrt{c\right.\right.}}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
(3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) -
 ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*a*c*sgn(x) - 4*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^2*c^(3/2)*abs(a)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))*a*c^2*sgn(x) - 4*c^(5/2)*abs(a)*sgn(x))/(((sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^2 + c)^2*a))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{x^2\,(a\,x-1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^2*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$$
$$=\frac{\sqrt{c}\left(6atan\left(\sqrt{a^2x^2-1}+ax\right)a^2x^2+4\sqrt{a^2x^2-1}\,ax+\sqrt{a^2x^2-1}\right)}{2a\,x^2}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x)
```

output
```
(sqrt(c)*(6*atan(sqrt(a**2*x**2 - 1) + a*x)*a**2*x**2 + 4*sqrt(a**2*x**2 -
 1)*a*x + sqrt(a**2*x**2 - 1)))/(2*a*x**2)
```

3.868 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 103

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{5}{3}a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{x} - a^2\sqrt{c}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}x}\right)$$

output
```
5/3*a^2*(c-c/a^2/x^2)^(1/2)+1/3*(c-c/a^2/x^2)^(1/2)/x^2+a*(c-c/a^2/x^2)^(1
/2)/x-a^2*c^(1/2)*arctan(c^(1/2)/a/(c-c/a^2/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(1+3ax+5a^2x^2)-3a^3x^3\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{3x^2\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1 + 3*a*x + 5*a^2*x^2) - 3*a^3
*x^3*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(3*x^2*Sqrt[-1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6717, 6709, 540, 25, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^4\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3}\int -\frac{a(5ax+6)}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}\int \frac{a(5ax+6)}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}a\int \frac{5ax+6}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}a\left(-\frac{1}{2}\int-\frac{2a(3ax+5)}{x^2\sqrt{1-a^2x^2}}dx-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}a\left(a\int\frac{3ax+5}{x^2\sqrt{1-a^2x^2}}dx-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 534}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}a\left(a\left(3a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{x}\right)-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}a\left(a\left(\frac{3}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{5\sqrt{1-a^2x^2}}{x}\right)-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3}a\left(a\left(-\frac{3\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{5\sqrt{1-a^2x^2}}{x}\right)-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{x\left(\frac{1}{3}a\left(a\left(-3a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{5\sqrt{1-a^2x^2}}{x}\right)-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/3*Sqrt[1 - a^2*x^2]/x^3 + (a*((-3*Sqrt[1 -
a^2*x^2])/x^2 + a*((-5*Sqrt[1 - a^2*x^2])/x - 3*a*ArcTanh[Sqrt[1 - a^2*x^2
]])))/3))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.47

method	result
risch	$\frac{(5a^4x^4+3a^3x^3-4a^2x^2-3ax-1)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{3x^2(a^2x^2-1)}-\frac{a^3\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{\sqrt{-c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,a\left(-6\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^3c\,x^4+6\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,a^3x^2+3\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^2c\,x^3+6\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(\right.\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(5*a^4*x^4+3*a^3*x^3-4*a^2*x^2-3*a*x-1)/x^2*(c*(a^2*x^2-1)/a^2/x^2)^(1
/2)/(a^2*x^2-1)-a^3/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/
x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.81

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$$
$$= \left[\frac{3\,a^2\sqrt{-c}x^2\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(5\,a^2x^2+3\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{6\,x^2},\ \frac{3\,a^2\sqrt{c}x^2\arctan\left(\frac{ax\sqrt{a^2}}{\sqrt{}}\right.}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/6*(3*a^2*sqrt(-c)*x^2*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
- c)/(a^2*x^2)) - 2*c)/x^2) + 2*(5*a^2*x^2 + 3*a*x + 1)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)))/x^2, 1/3*(3*a^2*sqrt(c)*x^2*arctan(a*x*sqrt((a^2*c*x^2 - c)
/(a^2*x^2))/sqrt(c)) + (5*a^2*x^2 + 3*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x
^2)))/x^2]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{x^3\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**3*(a*x - 1)),
 x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^3}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(87) = 174.

Time = 0.17 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.24

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{2}{3}\left(3\,a\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x) - \frac{3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^5 ac\operatorname{sgn}(x) - 3\left(\sqrt{a^2c}x -\right.}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
2/3*(3*a*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sg
n(x) - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a*c*sgn(x) - 3*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^4*c^(3/2)*abs(a)*sgn(x) - 12*(sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - c))^2*c^(5/2)*abs(a)*sgn(x) - 3*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))*a*c^3*sgn(x) - 5*c^(7/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2 + c)^3)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{x^3\,(a\,x-1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^3*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c}\,\left(6atan\left(\sqrt{a^2x^2-1}+ax\right)a^3x^3+5\sqrt{a^2x^2-1}\,a^2x^2+3\sqrt{a^2x^2-1}\,ax+\sqrt{a^2x^2-1}-3a^3x^3\right)}{3a\,x^3}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x)
```

output
```
(sqrt(c)*(6*atan(sqrt(a**2*x**2 - 1) + a*x)*a**3*x**3 + 5*sqrt(a**2*x**2 -
 1)*a**2*x**2 + 3*sqrt(a**2*x**2 - 1)*a*x + sqrt(a**2*x**2 - 1) - 3*a**3*x
**3))/(3*a*x**3)
```

3.869 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 133

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{7}{8}a^3\sqrt{c}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\right)$$

output

```
4/3*a^3*(c-c/a^2/x^2)^(1/2)+1/4*(c-c/a^2/x^2)^(1/2)/x^3+2/3*a*(c-c/a^2/x^2
)^(1/2)/x^2+7/8*a^2*(c-c/a^2/x^2)^(1/2)/x-7/8*a^3*c^(1/2)*arctan(c^(1/2)/a
/(c-c/a^2/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.71

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(6+16ax+21a^2x^2+32a^3x^3)-21a^4x^4\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{24x^3\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 + 16*a*x + 21*a^2*x^2 + 32*a
^3*x^3) - 21*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(24*x^3*Sqrt[-1 + a^2*
x^2])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6717, 6709, 540, 25, 27, 539, 25, 27, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^5\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}\int -\frac{a(7ax+8)}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\int \frac{a(7ax+8)}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\int\frac{7ax+8}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$\big\downarrow$ 539

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(-\frac{1}{3}\int-\frac{a(16ax+21)}{x^3\sqrt{1-a^2x^2}}dx-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$\big\downarrow$ 25

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(\frac{1}{3}\int\frac{a(16ax+21)}{x^3\sqrt{1-a^2x^2}}dx-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$\big\downarrow$ 27

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(\frac{1}{3}a\int\frac{16ax+21}{x^3\sqrt{1-a^2x^2}}dx-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$\big\downarrow$ 539

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(\frac{1}{3}a\left(-\frac{1}{2}\int-\frac{a(21ax+32)}{x^2\sqrt{1-a^2x^2}}dx-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$\big\downarrow$ 25

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}\int\frac{a(21ax+32)}{x^2\sqrt{1-a^2x^2}}dx-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$\big\downarrow$ 27

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\int\frac{21ax+32}{x^2\sqrt{1-a^2x^2}}dx-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$\big\downarrow$ 534

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\left(21a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{32\sqrt{1-a^2x^2}}{x}\right)-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$\big\downarrow$ 243

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\left(\frac{21}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{32\sqrt{1-a^2x^2}}{x}\right)-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$\big\downarrow$ 73

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\left(-\frac{21\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{32\sqrt{1-a^2x^2}}{x}\right)-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{x\left(\frac{1}{4}a\left(\frac{1}{3}a\left(\frac{1}{2}a\left(-21a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{32\sqrt{1-a^2x^2}}{x}\right)-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/4*Sqrt[1 - a^2*x^2]/x^4 + (a*((-8*Sqrt[1 -
a^2*x^2])/(3*x^3) + (a*((-21*Sqrt[1 - a^2*x^2])/(2*x^2) + (a*((-32*Sqrt[1
- a^2*x^2])/x - 21*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2))/3))/4))/Sqrt[1 - a^2
*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20

method	result
risch	$\frac{\left(32 a^{5} x^{5}+21 a^{4} x^{4}-16 a^{3} x^{3}-15 a^{2} x^{2}-16 a x -6\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}}{24 x^{3} \left(a^{2} x^{2}-1\right)}-\frac{7 a^{4} \ln \left(\frac{-2 c +2 \sqrt{-c}\, \sqrt{a^{2} c \,x^{2}-c}}{x}\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \sqrt{c \left(a^{2} x^{2}-1\right)}\, x}{8 \sqrt{-c}\, \left(a^{2} x^{2}-1\right)}$
default	$-\frac{\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, a^{2} \left(-48 \sqrt{-\frac{c}{a^{2}}}\, \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a^{3} c \,x^{5}+48 \sqrt{-\frac{c}{a^{2}}}\, \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{3} x^{3}+48 \sqrt{-\frac{c}{a^{2}}}\, c^{\frac{3}{2}} \ln \left(x \sqrt{c}+\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right) a \,x^{4}\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/24*(32*a^5*x^5+21*a^4*x^4-16*a^3*x^3-15*a^2*x^2-16*a*x-6)/x^3*(c*(a^2*x^
2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)-7/8*a^4/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(
a^2*c*x^2-c)^(1/2))/x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)
/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.52

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$
$$= \left[\frac{21\,a^3\sqrt{-c}x^3\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(32\,a^3x^3+21\,a^2x^2+16\,ax+6)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{48\,x^3}, \frac{21\,a^3\sqrt{c}x^3}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(21*a^3*sqrt(-c)*x^3*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3, 1/24*(21*a^3*sqrt(c)*x^3*arctan(a*x*s
qrt((a^2*c*x^2 - c)/(a^2*x^2))/sqrt(c)) + (32*a^3*x^3 + 21*a^2*x^2 + 16*a*
x + 6)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{x^4\left(ax-1\right)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**4*(a*x - 1)),
 x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx=\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^4}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(109) = 218.

Time = 0.18 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.38

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$
$$= \frac{1}{12}\left(21\,a^2\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x) - \frac{21\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^7 a^2c\operatorname{sgn}(x)+45\left(\sqrt{}\right.}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/12*(21*a^2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x) - (21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^2*c*sgn(x) + 45*(
sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a^2*c^2*sgn(x) - 96*(sqrt(a^2*c)*x
- sqrt(a^2*c*x^2 - c))^4*a*c^(5/2)*abs(a)*sgn(x) - 45*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*a^2*c^3*sgn(x) - 128*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 c))^2*a*c^(7/2)*abs(a)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*
a^2*c^4*sgn(x) - 32*a*c^(9/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*
x^2 - c))^2 + c)^4)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{x^4\,(a\,x-1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^4*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.14

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$
$$= \frac{\sqrt{c}\left(18atan\left(\sqrt{a^2x^2-1}+ax\right)a^4x^4+12atan\left(\frac{\sqrt{a^2x^2-1}\,ax+a^2x^2-1}{\sqrt{a^2x^2-1}+ax}\right)a^4x^4+32\sqrt{a^2x^2-1}\,a^3x^3+21\sqrt{a^2x^2-\cdots}\right.}{24a\,x^4}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x)
```

output
```
(sqrt(c)*(18*atan(sqrt(a**2*x**2 - 1) + a*x)*a**4*x**4 + 12*atan((sqrt(a**
2*x**2 - 1)*a*x + a**2*x**2 - 1)/(sqrt(a**2*x**2 - 1) + a*x))*a**4*x**4 +
32*sqrt(a**2*x**2 - 1)*a**3*x**3 + 21*sqrt(a**2*x**2 - 1)*a**2*x**2 + 16*s
qrt(a**2*x**2 - 1)*a*x + 6*sqrt(a**2*x**2 - 1) - 32*a**4*x**4))/(24*a*x**4
)
```

3.870 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 158

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}-\frac{3}{4}a^4\sqrt{c}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\right)$$

output
```
6/5*a^4*(c-c/a^2/x^2)^(1/2)+1/5*(c-c/a^2/x^2)^(1/2)/x^4+1/2*a*(c-c/a^2/x^2
)^(1/2)/x^3+3/5*a^2*(c-c/a^2/x^2)^(1/2)/x^2+3/4*a^3*(c-c/a^2/x^2)^(1/2)/x-
3/4*a^4*c^(1/2)*arctan(c^(1/2)/a/(c-c/a^2/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.65

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(4+10ax+12a^2x^2+15a^3x^3+24a^4x^4)-15a^5x^5\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{20x^4\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(4 + 10*a*x + 12*a^2*x^2 + 15*a
^3*x^3 + 24*a^4*x^4) - 15*a^5*x^5*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(20*x^4*S
qrt[-1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {6717, 6709, 540, 25, 27, 539, 27, 539, 27, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{2\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^6\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}\int -\frac{a(9ax+10)}{x^5\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\int \frac{a(9ax+10)}{x^5\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\int \frac{9ax+10}{x^5\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(-\frac{1}{4}\int -\frac{6a(5ax+6)}{x^4\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\int \frac{5ax+6}{x^4\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\left(-\frac{1}{3}\int -\frac{3a(4ax+5)}{x^3\sqrt{1-a^2x^2}}dx-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\left(a\int \frac{4ax+5}{x^3\sqrt{1-a^2x^2}}dx-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\left(a\left(-\frac{1}{2}\int -\frac{a(5ax+8)}{x^2\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\left(a\left(\frac{1}{2}\int \frac{a(5ax+8)}{x^2\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\left(a\left(\frac{1}{2}a\int\frac{5ax+8}{x^2\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$\downarrow$ 534

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\left(a\left(\frac{1}{2}a\left(5a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{8\sqrt{1-a^2x^2}}{x}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$\downarrow$ 243

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\left(a\left(\frac{1}{2}a\left(\frac{5}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{8\sqrt{1-a^2x^2}}{x}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$\downarrow$ 73

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}a\left(\frac{3}{2}a\left(a\left(\frac{1}{2}a\left(-\frac{5\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{8\sqrt{1-a^2x^2}}{x}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\right.}{\sqrt{1-a^2x^2}}$$

$\downarrow$ 221

$$-\frac{x\left(\frac{1}{5}a\left(\frac{3}{2}a\left(a\left(\frac{1}{2}a\left(-5a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{x}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

input
```
Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/5*Sqrt[1 - a^2*x^2]/x^5 + (a*((-5*Sqrt[1 -
a^2*x^2])/(2*x^4) + (3*a*((-2*Sqrt[1 - a^2*x^2])/x^3 + a*((-5*Sqrt[1 - a^2
*x^2])/(2*x^2) + (a*((-8*Sqrt[1 - a^2*x^2])/x - 5*a*ArcTanh[Sqrt[1 - a^2*x
^2]]))/2)))/2))/5))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06

method	result
risch	$\frac{\left(24x^6a^6+15a^5x^5-12a^4x^4-5a^3x^3-8a^2x^2-10ax-4\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{20x^4\left(a^2x^2-1\right)}-\frac{3a^5\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c\left(a^2x^2-1\right)}}{4\sqrt{-c}\left(a^2x^2-1\right)}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,a^2\left(-40\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^4c\,x^6+40\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^4x^4+40\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)a^2x\right.}{}$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/20*(24*a^6*x^6+15*a^5*x^5-12*a^4*x^4-5*a^3*x^3-8*a^2*x^2-10*a*x-4)/x^4*(
c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)-3/4*a^5/(-c)^(1/2)*ln((-2*c+2*(-c
)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-
1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.38

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \left[\frac{15\,a^4\sqrt{-c}x^4\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(24\,a^4x^4+15\,a^3x^3+12\,a^2x^2+10\,ax+4)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{40\,x^4},\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[1/40*(15*a^4*sqrt(-c)*x^4*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*x^4 + 15*a^3*x^3 + 12*a^2*x^2 +
10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4, 1/20*(15*a^4*sqrt(c)*x^4
*arctan(a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/sqrt(c)) + (24*a^4*x^4 + 15*a^
3*x^3 + 12*a^2*x^2 + 10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4]
```

Sympy [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax+1)}{x^5\,(ax-1)}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**5*(a*x - 1)),
 x)
```

Maxima [F]

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^5}\,dx$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(130) = 260$.

Time = 0.25 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.29

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$
$$=\frac{1}{10}\left(15\,a^3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\text{sgn}(x)-\frac{15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^9a^3c\text{sgn}(x)+70\left(\sqrt{}\right.}{}\right.$$

input
```
integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/10*(15*a^3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x) - (15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*a^3*c*sgn(x) + 70*(
sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^3*c^2*sgn(x) - 40*(sqrt(a^2*c)*x
- sqrt(a^2*c*x^2 - c))^6*a^2*c^(5/2)*abs(a)*sgn(x) - 200*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^4*a^2*c^(7/2)*abs(a)*sgn(x) - 70*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*a^3*c^4*sgn(x) - 120*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 c))^2*a^2*c^(9/2)*abs(a)*sgn(x) - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c)
)*a^3*c^5*sgn(x) - 24*a^2*c^(11/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - c))^2 + c)^5)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{x^5\,(a\,x-1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^5*(a*x - 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.77

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$
$$=\frac{\sqrt{c}\,\left(30atan\left(\sqrt{a^2x^2-1}+ax\right)a^5x^5+24\sqrt{a^2x^2-1}\,a^4x^4+15\sqrt{a^2x^2-1}\,a^3x^3+12\sqrt{a^2x^2-1}\,a^2x^2+10\sqrt{}\right.}{20a\,x^5}$$

input
```
int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x)
```

output
```
(sqrt(c)*(30*atan(sqrt(a**2*x**2 - 1) + a*x)*a**5*x**5 + 24*sqrt(a**2*x**2
 - 1)*a**4*x**4 + 15*sqrt(a**2*x**2 - 1)*a**3*x**3 + 12*sqrt(a**2*x**2 - 1
)*a**2*x**2 + 10*sqrt(a**2*x**2 - 1)*a*x + 4*sqrt(a**2*x**2 - 1) - 24*a**5
*x**5))/(20*a*x**5)
```

3.871 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 186

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^4}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
4*(c-c/a^2/x^2)^(1/2)*x/a^3/(1-1/a^2/x^2)^(1/2)+2*(c-c/a^2/x^2)^(1/2)*x^2/
a^2/(1-1/a^2/x^2)^(1/2)+(c-c/a^2/x^2)^(1/2)*x^3/a/(1-1/a^2/x^2)^(1/2)+1/4*
(c-c/a^2/x^2)^(1/2)*x^4/(1-1/a^2/x^2)^(1/2)+4*(c-c/a^2/x^2)^(1/2)*ln(-a*x+
1)/a^4/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{4x}{a^3}+\frac{2x^2}{a^2}+\frac{x^3}{a}+\frac{x^4}{4}+\frac{4\log(1-ax)}{a^4}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*((4*x)/a^3 + (2*x^2)/a^2 + x^3/a + x^4/4 + (4*Log[1
 - a*x])/a^4))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.40, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \ 6751$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 6747$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{x^2(ax+1)^2}{1-ax}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 25$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x^2(ax+1)^2}{1-ax}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 99$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-ax^3-3x^2-\frac{4x}{a}-\frac{4}{a^2(ax-1)}-\frac{4}{a^2}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4\log(1-ax)}{a^3}-\frac{4x}{a^2}-\frac{ax^4}{4}-\frac{2x^2}{a}-x^3\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*((-4*x)/a^2 - (2*x^2)/a - x^3 - (a*x^4)/4 - (4*Lo
g[1 - a*x])/a^3))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\left(a^{4} x^{4}+4 a^{3} x^{3}+8 a^{2} x^{2}+16 a x+16 \ln \left(a x-1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x-1\right)}{4 a^{3} \left(a x+1\right)^{2} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}$	89

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)*x^3,x,method=_RETURNVERB
OSE)
```

output
```
1/4*(a^4*x^4+4*a^3*x^3+8*a^2*x^2+16*a*x+16*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2
/x^2)^(1/2)*(a*x-1)/a^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.26

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{(a^4x^4+4\,a^3x^3+8\,a^2x^2+16\,ax+16\,\log{(ax-1)})\sqrt{a^2c}}{4\,a^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)*x^3,x, algorithm="
fricas")
```

output
```
1/4*(a^4*x^4 + 4*a^3*x^3 + 8*a^2*x^2 + 16*a*x + 16*log(a*x - 1))*sqrt(a^2*
c)/a^5
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)*x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)*x^3,x, algorithm="
maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.76

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$=\frac{1}{4}\sqrt{c}\left(\frac{a^3x^4\text{sgn}(x)+4a^2x^3\text{sgn}(x)+8ax^2\text{sgn}(x)+16x\text{sgn}(x)}{a^4\text{sgn}(ax+1)}+\frac{32\log(|ax-1|)\text{sgn}(x)}{a^5\text{sgn}(ax+1)}+\frac{a^{11}x^4\text{sgn}(a}{}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)*x^3,x, algorithm="
giac")
```

output
```
1/4*sqrt(c)*((a^3*x^4*sgn(x) + 4*a^2*x^3*sgn(x) + 8*a*x^2*sgn(x) + 16*x*sg
n(x))/(a^4*sgn(a*x + 1)) + 32*log(abs(a*x - 1))*sgn(x)/(a^5*sgn(a*x + 1))
+ (a^11*x^4*sgn(a*x + 1)*sgn(x) + 4*a^10*x^3*sgn(a*x + 1)*sgn(x) + 8*a^9*x
^2*sgn(a*x + 1)*sgn(x) + 16*a^8*x*sgn(a*x + 1)*sgn(x))/a^12)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int \frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((x^3*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^3*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.23

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{\sqrt{c}\,(16\log(ax-1)+a^4x^4+4a^3x^3+8a^2x^2+16ax)}{4a^4}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)*x^3,x)
```

output
```
(sqrt(c)*(16*log(a*x - 1) + a**4*x**4 + 4*a**3*x**3 + 8*a**2*x**2 + 16*a*x
))/(4*a**4)
```

3.872 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 152

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
4*(c-c/a^2/x^2)^(1/2)*x/a^2/(1-1/a^2/x^2)^(1/2)+3/2*(c-c/a^2/x^2)^(1/2)*x^
2/a/(1-1/a^2/x^2)^(1/2)+1/3*(c-c/a^2/x^2)^(1/2)*x^3/(1-1/a^2/x^2)^(1/2)+4*
(c-c/a^2/x^2)^(1/2)*ln(-a*x+1)/a^3/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.41

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax(24+9ax+2a^2x^2)+24\log(1-ax))}{6a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(a*x*(24 + 9*a*x + 2*a^2*x^2) + 24*Log[1 - a*x]))/(
6*a^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.45, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6751, 6747, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{x(ax+1)^2}{1-ax}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x(ax+1)^2}{1-ax}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{86}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-ax^2-3x-\frac{4}{a}-\frac{4}{a(ax-1)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4\log(1-ax)}{a^2}-\frac{ax^3}{3}-\frac{4x}{a}-\frac{3x^2}{2}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*((-4*x)/a - (3*x^2)/2 - (a*x^3)/3 - (4*Log[1 - a*
x])/a^2))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{\left(2a^{3}x^{3}+9a^{2}x^{2}+24ax+24\ln\left(ax-1\right)\right)x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax-1\right)}{6a^{2}\left(ax+1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	82

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)*x^2,x,method=_RETURNVERB
OSE)
```

output
```
1/6*(2*a^3*x^3+9*a^2*x^2+24*a*x+24*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1
/2)*(a*x-1)/a^2/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.27

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\left(2\,a^3x^3+9\,a^2x^2+24\,ax+24\,\log\left(ax-1\right)\right)\sqrt{a^2c}}{6\,a^4}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)*x^2,x, algorithm="
fricas")
```

output
```
1/6*(2*a^3*x^3 + 9*a^2*x^2 + 24*a*x + 24*log(a*x - 1))*sqrt(a^2*c)/a^4
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)*x***2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)*x^2,x, algorithm="
maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.70

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$$
$$=\frac{1}{6}\sqrt{c}\left(\frac{2\,a^2x^3\mathrm{sgn}(x)+9\,ax^2\mathrm{sgn}(x)+24\,x\mathrm{sgn}(x)}{a^3\mathrm{sgn}\,(ax+1)}+\frac{48\,\log\left(|ax-1|\right)\mathrm{sgn}(x)}{a^4\mathrm{sgn}\,(ax+1)}+\frac{2\,a^8x^3\mathrm{sgn}(x)+9\,a^7x^2\mathrm{sgn}(\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)*x^2,x, algorithm="
giac")
```

output
```
1/6*sqrt(c)*((2*a^2*x^3*sgn(x) + 9*a*x^2*sgn(x) + 24*x*sgn(x))/(a^3*sgn(a*
x + 1)) + 48*log(abs(a*x - 1))*sgn(x)/(a^4*sgn(a*x + 1)) + (2*a^8*x^3*sgn(
x) + 9*a^7*x^2*sgn(x) + 24*a^6*x*sgn(x))/(a^9*sgn(a*x + 1)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int \frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((x^2*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x^2*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.24

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c}\,(24\log(ax-1)+2a^3x^3+9a^2x^2+24ax)}{6a^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)*x^2,x)
```

output
```
(sqrt(c)*(24*log(a*x - 1) + 2*a**3*x**3 + 9*a**2*x**2 + 24*a*x))/(6*a**3)
```

3.873 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
3*(c-c/a^2/x^2)^(1/2)*x/a/(1-1/a^2/x^2)^(1/2)+1/2*(c-c/a^2/x^2)^(1/2)*x^2/
(1-1/a^2/x^2)^(1/2)+4*(c-c/a^2/x^2)^(1/2)*ln(-a*x+1)/a^2/(1-1/a^2/x^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1-ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*((3*x)/a + x^2/2 + (4*Log[1 - a*x])/a^2))/Sqrt[1 -
1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6751, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ax+1)^2}{1-ax}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{1-ax}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-ax+\frac{4}{1-ax}-3\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}-3x\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a))/(a*Sqrt[
1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\left(a^{2} x^{2}+6 a x+8 \ln \left(a x-1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x-1\right)}{2 a \left(a x+1\right)^{2} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}$	73

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)*x,x,method=_RETURNVERBOS
E)
```

output
```
1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/a/
(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.28

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = \frac{(a^2x^2+6\,ax+8\,\log{(ax-1)})\sqrt{a^2c}}{2\,a^3}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)*x,x, algorithm="fr
icas")
```

output
```
1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(a^2*c)/a^3
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)*x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)*x,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$
$$=\frac{1}{2}\sqrt{c}\left(\frac{ax^2\mathrm{sgn}(x)+6\,x\mathrm{sgn}(x)}{a^2\mathrm{sgn}\,(ax+1)}+\frac{16\,\log\left(|ax-1|\right)\mathrm{sgn}(x)}{a^3\mathrm{sgn}\,(ax+1)}+\frac{a^5x^2\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)+6\,a^4x\mathrm{sgn}(ax+1)}{a^6}\right.$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)*x,x, algorithm="gi
ac")
```

output
```
1/2*sqrt(c)*((a*x^2*sgn(x) + 6*x*sgn(x))/(a^2*sgn(a*x + 1)) + 16*log(abs(a
*x - 1))*sgn(x)/(a^3*sgn(a*x + 1)) + (a^5*x^2*sgn(a*x + 1)*sgn(x) + 6*a^4*
x*sgn(a*x + 1)*sgn(x))/a^6)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int \frac{x\sqrt{c-\frac{c}{a^2\,x^2}}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((x*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((x*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.24

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c}\,(8\log(ax-1)+a^2x^2+6ax)}{2a^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)*x,x)
```

output
```
(sqrt(c)*(8*log(a*x - 1) + a**2*x**2 + 6*a*x))/(2*a**2)
```

3.874 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1/2)-(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1
/a^2/x^2)^(1/2)+4*(c-c/a^2/x^2)^(1/2)*ln(-a*x+1)/a/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x-\frac{\log(x)}{a}+\frac{4\log(1-ax)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x - Log[x]/a + (4*Log[1 - a*x])/a))/Sqrt[1 - 1/(a^
2*x^2)]
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.46, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6751, 6747, 25, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ax+1)^2}{x(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a}{ax-1}-a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}(-ax-4\log(1-ax)+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-(a*x) + Log[x] - 4*Log[1 - a*x]))/(a*Sqrt[1 - 1
/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{(ax+4\ln(ax-1)-\ln(x))x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,(ax-1)}{(ax+1)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	65

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x+4*ln(a*x-1)-ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/
((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.25

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax+4\log{(ax-1)}-\log{(x)})}{a^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

output
```
sqrt(a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.58

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=2\sqrt{c}\left(\frac{x\mathrm{sgn}(x)}{a\mathrm{sgn}\left(ax+1\right)}+\frac{4\log\left(|ax-1|\right)\mathrm{sgn}(x)}{a^2\mathrm{sgn}\left(ax+1\right)}-\frac{\log\left(|x|\right)\mathrm{sgn}(x)}{a^2\mathrm{sgn}\left(ax+1\right)}\right)|a|$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

output
```
2*sqrt(c)*(x*sgn(x)/(a*sgn(a*x + 1)) + 4*log(abs(a*x - 1))*sgn(x)/(a^2*sgn
(a*x + 1)) - log(abs(x))*sgn(x)/(a^2*sgn(a*x + 1)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.20

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c}\,(4\log(ax-1)-\log(x)+ax)}{a}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x)
```

output
```
(sqrt(c)*(4*log(a*x - 1) - log(x) + a*x))/a
```

3.875 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 108

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x-3*(c-c/a^2/x^2)^(1/2)*ln(x)/(1
-1/a^2/x^2)^(1/2)+4*(c-c/a^2/x^2)^(1/2)*ln(-a*x+1)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{ax}-3\log(x)+4\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1/(a*x) - 3*Log[x] + 4*Log[1 - a*x]))/Sqrt[1 - 1/(
a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ax+1)^2}{x^2(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^2(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a^2}{ax-1}+\frac{3a}{x}+\frac{1}{x^2}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(3a\log(x)-4a\log(1-ax)-\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-x^(-1) + 3*a*Log[x] - 4*a*Log[1 - a*x]))/(a*Sqr
t[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{(4a\ln(ax-1)x-3a\ln(x)x+1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,(ax-1)}{(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	66

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x,method=_RETURNVERBOS
E)
```

output
```
(4*a*ln(a*x-1)*x-3*a*ln(x)*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x
+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.30

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{a^2c}(4\,ax\log{(ax-1)}-3\,ax\log{(x)}+1)}{a^2x}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fr
icas")
```

output
```
sqrt(a^2*c)*(4*a*x*log(a*x - 1) - 3*a*x*log(x) + 1)/(a^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.41

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\left(4\,a\log\left(|ax-1|\right)\mathrm{sgn}(x)-3\,a\log\left(|x|\right)\mathrm{sgn}(x)+\frac{\mathrm{sgn}(x)}{x}\right)\sqrt{c}|a|}{a^2\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="gi
ac")
```

output
```
(4*a*log(abs(a*x - 1))*sgn(x) - 3*a*log(abs(x))*sgn(x) + sgn(x)/x)*sqrt(c)
*abs(a)/(a^2*sgn(a*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.25

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{c}\,(4\log(ax-1)\,ax-3\log(x)\,ax+1)}{ax}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x)
```

output
```
(sqrt(c)*(4*log(a*x - 1)*a*x - 3*log(x)*a*x + 1))/(a*x)
```

3.876 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x} - \frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/2*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^2+3*(c-c/a^2/x^2)^(1/2)/(1
-1/a^2/x^2)^(1/2)/x-4*a*(c-c/a^2/x^2)^(1/2)*ln(x)/(1-1/a^2/x^2)^(1/2)+4*a*
(c-c/a^2/x^2)^(1/2)*ln(-a*x+1)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.41

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2ax^2}+\frac{3}{x}-4a\log(x)+4a\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1/(2*a*x^2) + 3/x - 4*a*Log[x] + 4*a*Log[1 - a*x])
)/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.46, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ax+1)^2}{x^3(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^3(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a^3}{ax-1}+\frac{4a^2}{x}+\frac{3a}{x^2}+\frac{1}{x^3}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(4a^2\log(x)-4a^2\log(1-ax)-\frac{3a}{x}-\frac{1}{2x^2}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 - (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[
1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{\left(8a^{2}\ln\left(ax-1\right)x^{2}-8a^{2}\ln\left(x\right)x^{2}+6ax+1\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax-1\right)}{2\left(ax+1\right)^{2}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	82

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x,method=_RETURNVERB
OSE)
```

output
```
1/2*(8*a^2*ln(a*x-1)*x^2-8*a^2*ln(x)*x^2+6*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^
(1/2)*(a*x-1)/(a*x+1)^2/x/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{8\,a^3\sqrt{c}x^2\log\left(\frac{2\,a^3cx^2-2\,a^2cx-\sqrt{a^2c}(2\,ax-1)\sqrt{c}+ac}{ax^2-x}\right)+\sqrt{a^2c}(6\,ax+1)}{2\,a^2x^2}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
fricas")
```

output
```
1/2*(8*a^3*sqrt(c)*x^2*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x -
 1)*sqrt(c) + a*c)/(a*x^2 - x)) + sqrt(a^2*c)*(6*a*x + 1))/(a^2*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.38

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\left(8\,a^2\log\left(|ax-1|\right)\operatorname{sgn}(x)-8\,a^2\log\left(|x|\right)\operatorname{sgn}(x)+\frac{6\,ax\operatorname{sgn}(x)+\operatorname{sgn}(x)}{x^2}\right)\sqrt{c}|a|}{2\,a^2\operatorname{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
giac")
```

output
```
1/2*(8*a^2*log(abs(a*x - 1))*sgn(x) - 8*a^2*log(abs(x))*sgn(x) + (6*a*x*sg
n(x) + sgn(x))/x^2)*sqrt(c)*abs(a)/(a^2*sgn(a*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.27

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\frac{\sqrt{c}\,(8\log(ax-1)\,a^2x^2-8\log(x)\,a^2x^2+6ax+1)}{2a\,x^2}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x)
```

output
```
(sqrt(c)*(8*log(a*x - 1)*a**2*x**2 - 8*log(x)*a**2*x**2 + 6*a*x + 1))/(2*a
*x**2)
```

3.877 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 188

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3a\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output

```
1/3*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^3+3/2*(c-c/a^2/x^2)^(1/2)/
(1-1/a^2/x^2)^(1/2)/x^2+4*a*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)/x-4*a^
2*(c-c/a^2/x^2)^(1/2)*ln(x)/(1-1/a^2/x^2)^(1/2)+4*a^2*(c-c/a^2/x^2)^(1/2)*
ln(-a*x+1)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.39

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3ax^3}+\frac{3}{2x^2}+\frac{4a}{x}-4a^2\log(x)+4a^2\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1/(3*a*x^3) + 3/(2*x^2) + (4*a)/x - 4*a^2*Log[x] +
 4*a^2*Log[1 - a*x]))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.41, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}}{x^3}\,dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ax+1)^2}{x^4(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^4(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a^4}{ax-1}+\frac{4a^3}{x}+\frac{4a^2}{x^2}+\frac{3a}{x^3}+\frac{1}{x^4}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(4a^3\log(x)-4a^3\log(1-ax)-\frac{4a^2}{x}-\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-1/3*1/x^3 - (3*a)/(2*x^2) - (4*a^2)/x + 4*a^3*L
og[x] - 4*a^3*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\left(24 a^{3} \ln \left(a x -1\right) x^{3}-24 \ln \left(x \right) x^{3} a^{3}+24 a^{2} x^{2}+9 a x +2\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x -1\right)}{6 \left(a x +1\right)^{2} x^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	90

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x,method=_RETURNVERB
OSE)
```

output
```
1/6*(24*a^3*ln(a*x-1)*x^3-24*ln(x)*x^3*a^3+24*a^2*x^2+9*a*x+2)*(c*(a^2*x^2
-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^2/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.49

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{24\,a^{4}\sqrt{c}x^{3}\log\left(\frac{2\,a^{3}cx^{2}-2\,a^{2}cx-\sqrt{a^{2}c}(2\,ax-1)\sqrt{c}+ac}{ax^{2}-x}\right)+(24\,a^{2}x^{2}+9\,ax+2)\sqrt{a^{2}c}}{6\,a^{2}x^{3}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="
fricas")
```

output
```
1/6*(24*a^4*sqrt(c)*x^3*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x
- 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (24*a^2*x^2 + 9*a*x + 2)*sqrt(a^2*c))/(
a^2*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="
maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.36

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\left(24\,a^3\log\left(|ax-1|\right)\mathrm{sgn}(x)-24\,a^3\log\left(|x|\right)\mathrm{sgn}(x)+\frac{24\,a^2x^2\mathrm{sgn}(x)+9\,ax\mathrm{sgn}(x)+2\,\mathrm{sgn}(x)}{x^3}\right)\sqrt{c}|a|}{6\,a^2\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="
giac")
```

output
```
1/6*(24*a^3*log(abs(a*x - 1))*sgn(x) - 24*a^3*log(abs(x))*sgn(x) + (24*a^2
*x^2*sgn(x) + 9*a*x*sgn(x) + 2*sgn(x))/x^3)*sqrt(c)*abs(a)/(a^2*sgn(a*x +
1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.26

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c}\,(24\log(ax-1)\,a^3x^3-24\log(x)\,a^3x^3+24a^2x^2+9ax+2)}{6a\,x^3}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x)
```

output
```
(sqrt(c)*(24*log(a*x - 1)*a**3*x**3 - 24*log(x)*a**3*x**3 + 24*a**2*x**2 +
 9*a*x + 2))/(6*a*x**3)
```

3.878 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 222

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4a\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/4*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^4+(c-c/a^2/x^2)^(1/2)/(1-1
/a^2/x^2)^(1/2)/x^3+2*a*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)/x^2+4*a^2*
(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)/x-4*a^3*(c-c/a^2/x^2)^(1/2)*ln(x)/
(1-1/a^2/x^2)^(1/2)+4*a^3*(c-c/a^2/x^2)^(1/2)*ln(-a*x+1)/(1-1/a^2/x^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.35

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4ax^4}+\frac{1}{x^3}+\frac{2a}{x^2}+\frac{4a^2}{x}-4a^3\log(x)+4a^3\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1/(4*a*x^4) + x^(-3) + (2*a)/x^2 + (4*a^2)/x - 4*a
^3*Log[x] + 4*a^3*Log[1 - a*x]))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.37, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ax+1)^2}{x^5(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^5(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a^5}{ax-1}+\frac{4a^4}{x}+\frac{4a^3}{x^2}+\frac{4a^2}{x^3}+\frac{3a}{x^4}+\frac{1}{x^5}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(4a^4\log(x)-4a^4\log(1-ax)-\frac{4a^3}{x}-\frac{2a^2}{x^2}-\frac{a}{x^3}-\frac{1}{4x^4}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-1/4*1/x^4 - a/x^3 - (2*a^2)/x^2 - (4*a^3)/x + 4
*a^4*Log[x] - 4*a^4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.44

method	result	size
default	$\frac{\left(16 \ln \left(a x -1\right) x^{4} a^{4}-16 \ln \left(x \right) x^{4} a^{4}+16 a^{3} x^{3}+8 a^{2} x^{2}+4 a x +1\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x -1\right)}{4 \left(a x +1\right)^{2} x^{3} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	98

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x,method=_RETURNVERB
OSE)
```

output
```
1/4*(16*ln(a*x-1)*x^4*a^4-16*ln(x)*x^4*a^4+16*a^3*x^3+8*a^2*x^2+4*a*x+1)*(
c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^3/((a*x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.45

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{16\,a^{5}\sqrt{c}x^{4}\log\left(\frac{2\,a^{3}cx^{2}-2\,a^{2}cx-\sqrt{a^{2}c}(2\,ax-1)\sqrt{c}+ac}{ax^{2}-x}\right)+\left(16\,a^{3}x^{3}+8\,a^{2}x^{2}+4\,ax+1\right)\sqrt{a^{2}c}}{4\,a^{2}x^{4}}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="
fricas")
```

output
```
1/4*(16*a^5*sqrt(c)*x^4*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x
- 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (16*a^3*x^3 + 8*a^2*x^2 + 4*a*x + 1)*sq
rt(a^2*c))/(a^2*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="
maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.34

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\left(16\,a^4\log\left(|ax-1|\right)\operatorname{sgn}(x)-16\,a^4\log\left(|x|\right)\operatorname{sgn}(x)+\frac{16\,a^3x^3\operatorname{sgn}(x)+8\,a^2x^2\operatorname{sgn}(x)+4\,ax\operatorname{sgn}(x)+\operatorname{sgn}(x)}{x^4}\right)\sqrt{c}|a|}{4\,a^2\operatorname{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="
giac")
```

output
```
1/4*(16*a^4*log(abs(a*x - 1))*sgn(x) - 16*a^4*log(abs(x))*sgn(x) + (16*a^3
*x^3*sgn(x) + 8*a^2*x^2*sgn(x) + 4*a*x*sgn(x) + sgn(x))/x^4)*sqrt(c)*abs(a
)/(a^2*sgn(a*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.25

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\sqrt{c}\,(16\log(ax-1)\,a^4x^4-16\log(x)\,a^4x^4+16a^3x^3+8a^2x^2+4ax+1)}{4a\,x^4}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x)
```

output
```
(sqrt(c)*(16*log(a*x - 1)*a**4*x**4 - 16*log(x)*a**4*x**4 + 16*a**3*x**3 +
 8*a**2*x**2 + 4*a*x + 1))/(4*a*x**4)
```

3.879 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 264

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5a\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output

```
1/5*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^5+3/4*(c-c/a^2/x^2)^(1/2)/
(1-1/a^2/x^2)^(1/2)/x^4+4/3*a*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)/x^3+
2*a^2*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)/x^2+4*a^3*(c-c/a^2/x^2)^(1/2
)/(1-1/a^2/x^2)^(1/2)/x-4*a^4*(c-c/a^2/x^2)^(1/2)*ln(x)/(1-1/a^2/x^2)^(1/2
)+4*a^4*(c-c/a^2/x^2)^(1/2)*ln(-a*x+1)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.34

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5ax^5}+\frac{3}{4x^4}+\frac{4a}{3x^3}+\frac{2a^2}{x^2}+\frac{4a^3}{x}-4a^4\log(x)+4a^4\log(1-ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1/(5*a*x^5) + 3/(4*x^4) + (4*a)/(3*x^3) + (2*a^2)/
x^2 + (4*a^3)/x - 4*a^4*Log[x] + 4*a^4*Log[1 - a*x]))/Sqrt[1 - 1/(a^2*x^2)
]
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.36, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6751, 6747, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{3\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \ 6751$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^5}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 6747$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ax+1)^2}{x^6(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ax+1)^2}{x^6(1-ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a^6}{ax-1}+\frac{4a^5}{x}+\frac{4a^4}{x^2}+\frac{4a^3}{x^3}+\frac{4a^2}{x^4}+\frac{3a}{x^5}+\frac{1}{x^6}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(4a^5\log(x)-4a^5\log(1-ax)-\frac{4a^4}{x}-\frac{2a^3}{x^2}-\frac{4a^2}{3x^3}-\frac{3a}{4x^4}-\frac{1}{5x^5}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-1/5*1/x^5 - (3*a)/(4*x^4) - (4*a^2)/(3*x^3) - (
2*a^3)/x^2 - (4*a^4)/x + 4*a^5*Log[x] - 4*a^5*Log[1 - a*x]))/(a*Sqrt[1 - 1
/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.40

method	result	size
default	$\frac{\left(240 \ln \left(a x -1\right) x^{5} a^{5}-240 a^{5} \ln \left(x \right) x^{5}+240 a^{4} x^{4}+120 a^{3} x^{3}+80 a^{2} x^{2}+45 a x +12\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x -1\right)}{60 \left(a x +1\right)^{2} x^{4} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	106

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x,method=_RETURNVERB
OSE)
```

output
```
1/60*(240*ln(a*x-1)*x^5*a^5-240*a^5*ln(x)*x^5+240*a^4*x^4+120*a^3*x^3+80*a
^2*x^2+45*a*x+12)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^4/((a*
x-1)/(a*x+1))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.41

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{240\,a^6\sqrt{c}x^5\log\left(\frac{2\,a^3cx^2-2\,a^2cx-\sqrt{a^2c}(2\,ax-1)\sqrt{c}+ac}{ax^2-x}\right)+(240\,a^4x^4+120\,a^3x^3+80\,a^2x^2+45\,ax+12)\sqrt{a^2c}}{60\,a^2x^5}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="
fricas")
```

output
```
1/60*(240*a^6*sqrt(c)*x^5*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*
x - 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (240*a^4*x^4 + 120*a^3*x^3 + 80*a^2*x
^2 + 45*a*x + 12)*sqrt(a^2*c))/(a^2*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="
maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.33

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$
$$= \frac{\Big(240\,a^5\log\left(|ax-1|\right)\mathrm{sgn}(x) - 240\,a^5\log\left(|x|\right)\mathrm{sgn}(x) + \frac{240\,a^4x^4\mathrm{sgn}(x)+120\,a^3x^3\mathrm{sgn}(x)+80\,a^2x^2\mathrm{sgn}(x)+45\,ax\mathrm{sgn}(x)+1}{x^5}}{60\,a^2\mathrm{sgn}\left(ax+1\right)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="
giac")
```

output
```
1/60*(240*a^5*log(abs(a*x - 1))*sgn(x) - 240*a^5*log(abs(x))*sgn(x) + (240
*a^4*x^4*sgn(x) + 120*a^3*x^3*sgn(x) + 80*a^2*x^2*sgn(x) + 45*a*x*sgn(x) +
 12*sgn(x))/x^5)*sqrt(c)*abs(a)/(a^2*sgn(a*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.24

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$
$$=\frac{\sqrt{c}\,(240\log(ax-1)\,a^5x^5-240\log(x)\,a^5x^5+240a^4x^4+120a^3x^3+80a^2x^2+45ax+12)}{60a\,x^5}$$

input
```
int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x)
```

output
```
(sqrt(c)*(240*log(a*x - 1)*a**5*x**5 - 240*log(x)*a**5*x**5 + 240*a**4*x**
4 + 120*a**3*x**3 + 80*a**2*x**2 + 45*a*x + 12))/(60*a*x**5)
```

3.880 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 76

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/2*(c-c/a^2/x^2)^(1/2)*x^2/a/(1-1/a^2/x^2)^(1/2)+1/3*(c-c/a^2/x^2)^(1/2)
*x^3/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2(-3+2ax)}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x^2*(-3 + 2*a*x))/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.66, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6751, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{a^2x^2}}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -x(1-ax)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int x(1-ax)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(x-ax^2\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\frac{x^2}{2}-\frac{ax^3}{3}\right)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^ArcCoth[a*x],x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(x^2/2 - (a*x^3)/3))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.62

method	result	size
orering	$\frac{x^{3}(2ax-3)\sqrt{c-\frac{c}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$	47
gosper	$\frac{x^{3}(2ax-3)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$	53
default	$\frac{x^{3}(2ax-3)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$	53

input
```
int((c-c/a^2/x^2)^(1/2)*x^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/6*x^3*(2*a*x-3)/(a*x-1)*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.32

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{(2\,ax^3-3\,x^2)\sqrt{a^2c}}{6\,a^2}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fr
icas")
```

output
```
1/6*(2*a*x^3 - 3*x^2)*sqrt(a^2*c)/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*x**2*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$$
$$=\frac{1}{6}\sqrt{c}\left(\frac{2\,ax^3\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)-3\,x^2\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a^2}-\frac{5\,\mathrm{sgn}(ax+1)\,\mathrm{sgn}(a)}{a^4}\right)|a|$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gi
ac")
```

output
```
1/6*sqrt(c)*((2*a*x^3*sgn(a*x + 1)*sgn(x) - 3*x^2*sgn(a*x + 1)*sgn(x))/a^2
 - 5*sgn(a*x + 1)*sgn(a)/a^4)*abs(a)
```

Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(2\,a\,x-3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,\left(a\,x-1\right)}$$

input
```
int(x^2*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(x^3*(c - c/(a^2*x^2))^(1/2)*(2*a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))/(6*(
a*x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.33

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c}\,(2a^3x^3-3a^2x^2+1)}{6a^3}$$

input
```
int((c-c/a^2/x^2)^(1/2)*x^2*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*(2*a**3*x**3 - 3*a**2*x**2 + 1))/(6*a**3)
```

3.881 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 44

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{2a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
1/2*(c-c/a^2/x^2)^(1/2)*(-a*x+1)^2/a^2/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{x}{a}+\frac{x^2}{2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-(x/a) + x^2/2))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6751, 6747, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{a^2x^2}}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int (ax-1)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{17}$$

$$\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{2a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(2*a^2*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

method	result	size
orering	$\frac{x^2(ax-2)\sqrt{c-\frac{c}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	46
gosper	$\frac{x^2(ax-2)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	52
default	$\frac{x^2(ax-2)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	52

input
```
int((c-c/a^2/x^2)^(1/2)*x*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*(a*x-2)/(a*x-1)*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{a^2c}(ax^2-2\,x)}{2\,a^2}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fric
as")
```

output
```
1/2*sqrt(a^2*c)*(a*x^2 - 2*x)/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*x*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$
$$=\frac{1}{2}\sqrt{c}\left(\frac{ax^2\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)-2\,x\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a^2}+\frac{3\,\mathrm{sgn}(ax+1)\,\mathrm{sgn}(a)}{a^3}\right)|a|$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac
")
```

output
```
1/2*sqrt(c)*((a*x^2*sgn(a*x + 1)*sgn(x) - 2*x*sgn(a*x + 1)*sgn(x))/a^2 + 3
*sgn(a*x + 1)*sgn(a)/a^3)*abs(a)
```

Mupad [B] (verification not implemented)

Time = 13.74 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-2\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,\left(a\,x-1\right)}$$

input
```
int(x*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
(x^2*(c - c/(a^2*x^2))^(1/2)*(a*x - 2)*((a*x - 1)/(a*x + 1))^(1/2))/(2*(a*
x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.45

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c}\,(a^2x^2-2ax+1)}{2a^2}$$

input
```
int((c-c/a^2/x^2)^(1/2)*x*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*(a**2*x**2 - 2*a*x + 1))/(2*a**2)
```

3.882 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1/2)-(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1
/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x-\frac{\log(x)}{a}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(x - Log[x]/a))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6751, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{-\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{1-ax}{x}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1-ax}{x}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x}-a\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}(\log(x)-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-(a*x) + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{(-ax+\ln(x))x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$	52

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x
-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.28

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax-\log{(x)})}{a^2}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

output
```
sqrt(a^2*c)*(a*x - log(x))/a^2
```

Sympy [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))),
x)
```

Maxima [F]

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.53

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \left(\frac{x\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a} - \frac{\log(|x|)\,\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a^2}\right)\sqrt{c}|a|$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

output
```
(x*sgn(a*x + 1)*sgn(x)/a - log(abs(x))*sgn(a*x + 1)*sgn(x)/a^2)*sqrt(c)*ab
s(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \int \sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.25

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c}\,(-\log(ax)+ax-1)}{a}$$

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(sqrt(c)*( - log(a*x) + a*x - 1))/a
```

3.883 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 69

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x+(c-c/a^2/x^2)^(1/2)*ln(x)/(1-1
/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.58

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{ax}+\log(x)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1/(a*x) + Log[x]))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6751, 6747, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{1-ax}{x^2}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1-ax}{x^2}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^2}-\frac{a}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-a\log(x)-\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*(-x^(-1) - a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{(a\ln(x)x+1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$	50

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(a*ln(x)*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.30

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{a^2c}(ax\log(x)+1)}{a^2x}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fric
as")
```

output
```
sqrt(a^2*c)*(a*x*log(x) + 1)/(a^2*x)
```

Sympy [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

output
```
Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/x
, x)
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \left(\frac{\log(|x|)\operatorname{sgn}(ax+1)\operatorname{sgn}(x)}{a} + \frac{\operatorname{sgn}(ax+1)\operatorname{sgn}(x)}{a^2x}\right)\sqrt{c}|a|$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac
")
```

output
```
(log(abs(x))*sgn(a*x + 1)*sgn(x)/a + sgn(a*x + 1)*sgn(x)/(a^2*x))*sqrt(c)*
abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.32

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c}\,(\log(ax)\,ax-ax+1)}{ax}$$

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x)
```

output
```
(sqrt(c)*(log(a*x)*a*x - a*x + 1))/(a*x)
```

3.884 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 47

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}{2a\sqrt{1-\frac{1}{a^2x^2}}x^2}$$

output
```
1/2*(c-c/a^2/x^2)^(1/2)*(-a*x+1)^2/a/(1-1/a^2/x^2)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2ax^2}-\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x^2),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1/(2*a*x^2) - x^(-1)))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6751, 6747, 25, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{1-ax}{x^3}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1-ax}{x^3}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{48}$$

$$\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x^2),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result	size
orering	$-\frac{(2ax-1)\sqrt{c-\frac{c}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{2x(ax-1)}$	47
gosper	$-\frac{(2ax-1)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{2x(ax-1)}$	53
default	$-\frac{(2ax-1)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{2x(ax-1)}$	53

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOS
E)
```

output
```
-1/2*(2*a*x-1)/x/(a*x-1)*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.45

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\frac{\sqrt{a^2c}(2\,ax-1)}{2\,a^2x^2}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fr
icas")
```

output
```
-1/2*sqrt(a^2*c)*(2*a*x - 1)/(a^2*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$$
$$=\frac{1}{2}\left(3\,\mathrm{sgn}(ax+1)\,\mathrm{sgn}(a)-\frac{2\,ax\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)-\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a^2x^2}\right)\sqrt{c}|a|$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="gi
ac")
```

output
```
1/2*(3*sgn(a*x + 1)*sgn(a) - (2*a*x*sgn(a*x + 1)*sgn(x) - sgn(a*x + 1)*sgn
(x))/(a^2*x^2))*sqrt(c)*abs(a)
```

Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\frac{\left(x\,\sqrt{c-\frac{c}{a^2\,x^2}}-\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{2\,a}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{x}{a}-x^2}$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
```

output
```
((x*(c - c/(a^2*x^2))^(1/2) - (c - c/(a^2*x^2))^(1/2)/(2*a))*((a*x - 1)/(a
*x + 1))^(1/2))/(x/a - x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.49

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c}\,(a^2x^2-2ax+1)}{2a\,x^2}$$

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

output
```
(sqrt(c)*(a**2*x**2 - 2*a*x + 1))/(2*a*x**2)
```

3.885 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx$

Optimal result

Integrand size = 27, antiderivative size = 155

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx=\frac{6\sqrt{c-\frac{c}{a^2x^2}}x}{5a^4}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^2}{4a^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^3}{5a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^4}{2a}+\frac{1}{5}\sqrt{c-\frac{c}{a^2x^2}}x^5-\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{4a^5}$$

output
```
6/5*(c-c/a^2/x^2)^(1/2)*x/a^4-3/4*(c-c/a^2/x^2)^(1/2)*x^2/a^3+3/5*(c-c/a^2
/x^2)^(1/2)*x^3/a^2-1/2*(c-c/a^2/x^2)^(1/2)*x^4/a+1/5*(c-c/a^2/x^2)^(1/2)*
x^5-3/4*c^(1/2)*arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))/a^5
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.65

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}(24-15ax+12a^2x^2-10a^3x^3+4a^4x^4)-15\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{20a^4\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x^4)/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(24 - 15*a*x + 12*a^2*x^2 - 1
0*a^3*x^3 + 4*a^4*x^4) - 15*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(20*a^4*Sqrt[-
1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6717, 6709, 570, 541, 25, 27, 533, 27, 533, 27, 533, 25, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{6717}$$
$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4dx$$
$$\Big\downarrow \text{6709}$$
$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x^3(1-a^2x^2)^{3/2}}{(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$
$$\Big\downarrow \text{570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x^3(1-ax)^2}{\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{541}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int -\frac{a^2x^3(9-10ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{\int \frac{a^2x^3(9-10ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\int \frac{x^3(9-10ax)}{\sqrt{1-a^2x^2}}dx-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{533}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\left(\frac{\int -\frac{6ax^2(5-6ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{5x^3\sqrt{1-a^2x^2}}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\left(\frac{5x^3\sqrt{1-a^2x^2}}{2a}-\frac{3\int \frac{x^2(5-6ax)}{\sqrt{1-a^2x^2}}dx}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{533}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\left(\frac{5x^3\sqrt{1-a^2x^2}}{2a}-\frac{3\left(\frac{\int -\frac{3ax(4-5ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{2x^2\sqrt{1-a^2x^2}}{a}\right)}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\left(\frac{5x^3\sqrt{1-a^2x^2}}{2a}-\frac{3\left(\frac{2x^2\sqrt{1-a^2x^2}}{a}-\frac{\int\frac{x(4-5ax)}{\sqrt{1-a^2x^2}}dx}{a}\right)}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{533}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\left(\frac{5x^3\sqrt{1-a^2x^2}}{2a}-\frac{3\left(\frac{2x^2\sqrt{1-a^2x^2}}{a}-\frac{\frac{\int-\frac{a(5-8ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{5x\sqrt{1-a^2x^2}}{2a}}{a}\right)}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\left(\frac{5x^3\sqrt{1-a^2x^2}}{2a}-\frac{3\left(\frac{2x^2\sqrt{1-a^2x^2}}{a}-\frac{\frac{5x\sqrt{1-a^2x^2}}{2a}-\frac{\int\frac{a(5-8ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}}{a}\right)}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\left(\frac{5x^3\sqrt{1-a^2x^2}}{2a}-\frac{3\left(\frac{2x^2\sqrt{1-a^2x^2}}{a}-\frac{\frac{5x\sqrt{1-a^2x^2}}{2a}-\frac{\int\frac{5-8ax}{\sqrt{1-a^2x^2}}dx}{2a}}{a}\right)}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{455}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{5}\left(\frac{5x^3\sqrt{1-a^2x^2}}{2a}-\frac{3\left(\frac{2x^2\sqrt{1-a^2x^2}}{a}-\frac{\frac{5x\sqrt{1-a^2x^2}}{2a}-\frac{5\int\frac{1}{\sqrt{1-a^2x^2}}dx+\frac{8\sqrt{1-a^2x^2}}{a}}{2a}}{a}\right)}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x\left(\frac{1}{5}\left(\frac{5x^3\sqrt{1-a^2x^2}}{2a}-\frac{3\left(\frac{2x^2\sqrt{1-a^2x^2}}{a}-\frac{\frac{5x\sqrt{1-a^2x^2}}{2a}-\frac{\frac{8\sqrt{1-a^2x^2}}{a}+\frac{5\arcsin(ax)}{a}}{2a}}{a}\right)}{2a}\right)-\frac{1}{5}x^4\sqrt{1-a^2x^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x^4)/E^(2*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/5*(x^4*Sqrt[1 - a^2*x^2]) + ((5*x^3*Sqrt[1
- a^2*x^2])/(2*a) - (3*((2*x^2*Sqrt[1 - a^2*x^2])/a - ((5*x*Sqrt[1 - a^2*x
^2])/(2*a) - ((8*Sqrt[1 - a^2*x^2])/a + (5*ArcSin[a*x])/a)/(2*a))/a))/(2*a
))/5))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.92

method	result
risch	$\frac{\left(4 a^{4} x^{4}-10 a^{3} x^{3}+12 a^{2} x^{2}-15 a x+24\right) \sqrt{\frac{c\left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x}{20 a^{4}}-\frac{3 \ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c\, x^{2}-c}\right) \sqrt{\frac{c\left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \sqrt{c\left(a^{2} x^{2}-1\right)}\, x}{4 a^{3} \sqrt{a^{2} c}\,\left(a^{2} x^{2}-1\right)}$
default	$\frac{\sqrt{\frac{c\left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x\left(4 x^{2}\left(\frac{c\left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{5}-10 x\left(\frac{c\left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{4}+16\left(\frac{c\left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{3}-25 \sqrt{\frac{c\left(a^{2} x^{2}-1\right)}{a^{2}}}\, a^{2} c x+25 c^{\frac{3}{2}} \ln \left(x \sqrt{c}+\sqrt{\right.}\right.}{20 \sqrt{\frac{c\left(a^{2} x^{2}-1\right)}{a^{2}}}\, a^{5} c}$

input
```
int((c-c/a^2/x^2)^(1/2)*x^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/20*(4*a^4*x^4-10*a^3*x^3+12*a^2*x^2-15*a*x+24)/a^4*(c*(a^2*x^2-1)/a^2/x^
2)^(1/2)*x-3/4/a^3*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(
1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.54

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx$$
$$=\left[\frac{2\,(4\,a^5x^5-10\,a^4x^4+12\,a^3x^3-15\,a^2x^2+24\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+15\,\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\right.}{40\,a^5}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/40*(2*(4*a^5*x^5 - 10*a^4*x^4 + 12*a^3*x^3 - 15*a^2*x^2 + 24*a*x)*sqrt(
(a^2*c*x^2 - c)/(a^2*x^2)) + 15*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^
2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^5, 1/20*((4*a^5*x^5 - 10*a^4*x^4
 + 12*a^3*x^3 - 15*a^2*x^2 + 24*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 15*
sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^
2 - c)))/a^5]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx=\int\frac{x^4\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*x**4*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x**4*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x
)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}x^4}{ax+1}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x^4/(a*x + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx$$
$$=\frac{1}{40}\left(2\sqrt{a^2cx^2-c}\left(\left(2\left(x\left(\frac{2\,x\mathrm{sgn}(x)}{a^2}-\frac{5\,\mathrm{sgn}(x)}{a^3}\right)+\frac{6\,\mathrm{sgn}(x)}{a^4}\right)x-\frac{15\,\mathrm{sgn}(x)}{a^5}\right)x+\frac{24\,\mathrm{sgn}(x)}{a^6}\right)+\frac{30\,\sqrt{}}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/40*(2*sqrt(a^2*c*x^2 - c)*((2*(x*(2*x*sgn(x)/a^2 - 5*sgn(x)/a^3) + 6*sgn
(x)/a^4)*x - 15*sgn(x)/a^5)*x + 24*sgn(x)/a^6) + 30*sqrt(c)*log(abs(-sqrt(
a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^5*abs(a)) - 3*(5*a*sqrt(c)*log(
abs(c)) + 16*sqrt(-c)*abs(a))*sgn(x)/(a^6*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx=\int\frac{x^4\,\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x^4*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x^4*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.68

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^4\,dx = \frac{\sqrt{c}\left(4\sqrt{a^2x^2-1}\,a^4x^4-10\sqrt{a^2x^2-1}\,a^3x^3+12\sqrt{a^2x^2-1}\,a^2x^2-15\sqrt{a^2x^2-1}\,ax+24\sqrt{a^2x^2-1}-15\right.}{20a^5}$$

input
```
int((c-c/a^2/x^2)^(1/2)*x^4*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*(4*sqrt(a**2*x**2 - 1)*a**4*x**4 - 10*sqrt(a**2*x**2 - 1)*a**3*x*
*3 + 12*sqrt(a**2*x**2 - 1)*a**2*x**2 - 15*sqrt(a**2*x**2 - 1)*a*x + 24*sq
rt(a**2*x**2 - 1) - 15*log(sqrt(a**2*x**2 - 1) + a*x)))/(20*a**5)
```

3.886 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 130

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=-\frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{3a^3}+\frac{7\sqrt{c-\frac{c}{a^2x^2}}x^2}{8a^2}-\frac{2\sqrt{c-\frac{c}{a^2x^2}}x^3}{3a}+\frac{1}{4}\sqrt{c-\frac{c}{a^2x^2}}x^4+\frac{7\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{8a^4}$$

output
```
-4/3*(c-c/a^2/x^2)^(1/2)*x/a^3+7/8*(c-c/a^2/x^2)^(1/2)*x^2/a^2-2/3*(c-c/a^
2/x^2)^(1/2)*x^3/a+1/4*(c-c/a^2/x^2)^(1/2)*x^4+7/8*c^(1/2)*arctanh((c-c/a^
2/x^2)^(1/2)/c^(1/2))/a^4
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.72

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}(-32+21ax-16a^2x^2+6a^3x^3)+21\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{24a^3\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(-32 + 21*a*x - 16*a^2*x^2 +
6*a^3*x^3) + 21*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^3*Sqrt[-1 + a^2*x^2]
)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {6717, 6709, 570, 541, 25, 27, 533, 25, 27, 533, 25, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x^2\left(1-a^2x^2\right)^{3/2}}{(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x^2(1-ax)^2}{\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{541}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int -\frac{a^2x^2(7-8ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{\int \frac{a^2x^2(7-8ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\int \frac{x^2(7-8ax)}{\sqrt{1-a^2x^2}}dx-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{533}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{\int -\frac{ax(16-21ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{8x^2\sqrt{1-a^2x^2}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{8x^2\sqrt{1-a^2x^2}}{3a}-\frac{\int \frac{ax(16-21ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{8x^2\sqrt{1-a^2x^2}}{3a}-\frac{\int \frac{x(16-21ax)}{\sqrt{1-a^2x^2}}dx}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{533}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{8x^2\sqrt{1-a^2x^2}}{3a}-\frac{\frac{\int -\frac{a(21-32ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{21x\sqrt{1-a^2x^2}}{2a}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{8x^2\sqrt{1-a^2x^2}}{3a}-\frac{\frac{21x\sqrt{1-a^2x^2}}{2a}-\frac{\int \frac{a(21-32ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{8x^2\sqrt{1-a^2x^2}}{3a}-\frac{\frac{21x\sqrt{1-a^2x^2}}{2a}-\frac{\int \frac{21-32ax}{\sqrt{1-a^2x^2}}dx}{2a}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{455}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4}\left(\frac{8x^2\sqrt{1-a^2x^2}}{3a}-\frac{\frac{21x\sqrt{1-a^2x^2}}{2a}-\frac{21\int\frac{1}{\sqrt{1-a^2x^2}}dx+\frac{32\sqrt{1-a^2x^2}}{a}}{2a}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x\left(\frac{1}{4}\left(\frac{8x^2\sqrt{1-a^2x^2}}{3a}-\frac{\frac{21x\sqrt{1-a^2x^2}}{2a}-\frac{\frac{32\sqrt{1-a^2x^2}}{a}+\frac{21\arcsin(ax)}{a}}{2a}}{3a}\right)-\frac{1}{4}x^3\sqrt{1-a^2x^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/4*(x^3*Sqrt[1 - a^2*x^2]) + ((8*x^2*Sqrt[1
- a^2*x^2])/(3*a) - ((21*x*Sqrt[1 - a^2*x^2])/(2*a) - ((32*Sqrt[1 - a^2*x^
2])/a + (21*ArcSin[a*x])/a)/(2*a))/(3*a))/4))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 541

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570

```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03

method	result
risch	$\frac{(6a^3x^3-16a^2x^2+21ax-32)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x}{24a^3}+\frac{7\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{8a^2\sqrt{a^2c}\,(a^2x^2-1)}$
default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\left(6x\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^4-16\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3+27\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2cx-27c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)+48c^{\frac{3}{2}}\ln\left(\right.\right.}{24\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^4c}$

input
```
int((c-c/a^2/x^2)^(1/2)*x^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*a^3*x^3-16*a^2*x^2+21*a*x-32)/a^3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x+
7/8/a^2*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*(c*(a^
2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.71

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$=\left[\frac{2\left(6a^4x^4-16a^3x^3+21a^2x^2-32ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+21\sqrt{c}\log\left(2a^2cx^2+2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{48a^4},\ \frac{(6a^4}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/48*(2*(6*a^4*x^4 - 16*a^3*x^3 + 21*a^2*x^2 - 32*a*x)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)) + 21*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c
*x^2 - c)/(a^2*x^2)) - c))/a^4, 1/24*((6*a^4*x^4 - 16*a^3*x^3 + 21*a^2*x^2
 - 32*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 21*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^4]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*x**3*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x**3*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x
)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}x^3}{ax+1}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x^3/(a*x + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$=\frac{1}{48}\left(2\sqrt{a^2cx^2-c}\left(\left(2x\left(\frac{3x\,\mathrm{sgn}(x)}{a^2}-\frac{8\,\mathrm{sgn}(x)}{a^3}\right)+\frac{21\,\mathrm{sgn}(x)}{a^4}\right)x-\frac{32\,\mathrm{sgn}(x)}{a^5}\right)-\frac{42\sqrt{c}\log\left(\left|-\sqrt{a^2c}x\right.\right.}{a}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/48*(2*sqrt(a^2*c*x^2 - c)*((2*x*(3*x*sgn(x)/a^2 - 8*sgn(x)/a^3) + 21*sgn
(x)/a^4)*x - 32*sgn(x)/a^5) - 42*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2
*c*x^2 - c)))*sgn(x)/(a^4*abs(a)) + (21*a*sqrt(c)*log(abs(c)) + 64*sqrt(-c
)*abs(a))*sgn(x)/(a^5*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int\frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.67

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$=\frac{\sqrt{c}\,\big(6\sqrt{a^2x^2-1}\,a^3x^3-16\sqrt{a^2x^2-1}\,a^2x^2+21\sqrt{a^2x^2-1}\,ax-32\sqrt{a^2x^2-1}+21\log\big(\sqrt{a^2x^2-1}+ax\big)}{24a^4}$$

input
```
int((c-c/a^2/x^2)^(1/2)*x^3*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*(6*sqrt(a**2*x**2 - 1)*a**3*x**3 - 16*sqrt(a**2*x**2 - 1)*a**2*x*
*2 + 21*sqrt(a**2*x**2 - 1)*a*x - 32*sqrt(a**2*x**2 - 1) + 21*log(sqrt(a**
2*x**2 - 1) + a*x)))/(24*a**4)
```

3.887 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 101

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{5\sqrt{c-\frac{c}{a^2x^2}}x}{3a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{a}+\frac{1}{3}\sqrt{c-\frac{c}{a^2x^2}}x^3-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{a^3}$$

output
```
5/3*(c-c/a^2/x^2)^(1/2)*x/a^2-(c-c/a^2/x^2)^(1/2)*x^2/a+1/3*(c-c/a^2/x^2)^
(1/2)*x^3-c^(1/2)*arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))/a^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}(5-3ax+a^2x^2)-3\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{3a^2\sqrt{-1+a^2x^2}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(5 - 3*a*x + a^2*x^2) - 3*Log
[a*x + Sqrt[-1 + a^2*x^2]]))/(3*a^2*Sqrt[-1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6717, 6709, 571, 466, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x(1-a^2x^2)^{3/2}}{(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{571}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{2\int \frac{(1-a^2x^2)^{3/2}}{ax+1}dx}{a}-\frac{(1-a^2x^2)^{5/2}}{a^2(ax+1)^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{466}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{2\left(\int \sqrt{1-a^2x^2}dx+\frac{(1-a^2x^2)^{3/2}}{3a}\right)}{a}-\frac{(1-a^2x^2)^{5/2}}{a^2(ax+1)^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{211}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{2\left(\frac{1}{2}\int\frac{1}{\sqrt{1-a^2x^2}}dx+\frac{\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}x\sqrt{1-a^2x^2}\right)}{a}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(ax+1)^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x\left(-\frac{2\left(\frac{\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}x\sqrt{1-a^2x^2}+\frac{\arcsin(ax)}{2a}\right)}{a}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(ax+1)^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-((1 - a^2*x^2)^(5/2)/(a^2*(1 + a*x)^2)) - (2*
((x*Sqrt[1 - a^2*x^2])/2 + (1 - a^2*x^2)^(3/2)/(3*a) + ArcSin[a*x]/(2*a)))
/a))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 571
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*(n + p + 1))), x] + Simp[n/(2*d*
(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ((LtQ[n, -1] &&  !IGtQ[n + p +
1, 0]) || (LtQ[n, 0] && LtQ[p, -1]) || EqQ[n + 2*p + 2, 0]) && NeQ[n + p +
1, 0]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

method	result
risch	$\frac{\left(a^{2} x^{2}-3 a x +5\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x}{3 a^{2}}-\frac{\ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}-c}\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \sqrt{c \left(a^{2} x^{2}-1\right)}\, x}{a \sqrt{a^{2} c}\, \left(a^{2} x^{2}-1\right)}$
default	$\frac{\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x \left(\left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{3}-3 \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a^{2} c x +3 c^{\frac{3}{2}} \ln \left(x \sqrt{c}+\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right)-6 c^{\frac{3}{2}} \ln \left(\frac{\sqrt{c}\, \sqrt{\frac{c \left(a x -1\right) \left(a x +1\right)}{a^{2}}}+x c}{\sqrt{c}}\right)+6 \cdots}{3 \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a^{3} c}$

input
```
int((c-c/a^2/x^2)^(1/2)*x^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a^2*x^2-3*a*x+5)/a^2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x-1/a*ln(a^2*c*x/(
a^2*c)^(1/2)+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(1
/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.02

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$$
$$=\left[\frac{2\,(a^3x^3-3\,a^2x^2+5\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{6\,a^3},\frac{(a^3x^3-3\,a^2x^2+5\,a\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/6*(2*(a^3*x^3 - 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*
sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
) - c))/a^3, 1/3*((a^3*x^3 - 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)) + 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
/(a^2*c*x^2 - c)))/a^3]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*x**2*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x**2*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x
)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}x^2}{ax+1}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x^2/(a*x + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.16

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$$
$$=\frac{1}{6}\left(2\sqrt{a^2cx^2-c}\left(x\left(\frac{x\mathrm{sgn}(x)}{a^2}-\frac{3\,\mathrm{sgn}(x)}{a^3}\right)+\frac{5\,\mathrm{sgn}(x)}{a^4}\right)+\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^3|a|}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/6*(2*sqrt(a^2*c*x^2 - c)*(x*(x*sgn(x)/a^2 - 3*sgn(x)/a^3) + 5*sgn(x)/a^4
) + 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^3*a
bs(a)) - (3*a*sqrt(c)*log(abs(c)) + 10*sqrt(-c)*abs(a))*sgn(x)/(a^4*abs(a)
))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int\frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$$
$$=\frac{\sqrt{c}\left(\sqrt{a^2x^2-1}\,a^2x^2-3\sqrt{a^2x^2-1}\,ax+5\sqrt{a^2x^2-1}-3\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{3a^3}$$

input
```
int((c-c/a^2/x^2)^(1/2)*x^2*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*(sqrt(a**2*x**2 - 1)*a**2*x**2 - 3*sqrt(a**2*x**2 - 1)*a*x + 5*sq
rt(a**2*x**2 - 1) - 3*log(sqrt(a**2*x**2 - 1) + a*x)))/(3*a**3)
```

3.888 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=-\frac{2\sqrt{c-\frac{c}{a^2x^2}}x}{a}+\frac{1}{2}\sqrt{c-\frac{c}{a^2x^2}}x^2+\frac{3\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{2a^2}$$

output
```
-2*(c-c/a^2/x^2)^(1/2)*x/a+1/2*(c-c/a^2/x^2)^(1/2)*x^2+3/2*c^(1/2)*arctanh
((c-c/a^2/x^2)^(1/2)/c^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.28

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{1+ax}(4-5ax+a^2x^2)-6\sqrt{1-ax}\arcsin\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^(2*ArcCoth[a*x]),x]
```

output
```
-1/2*(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2) - 6*Sqr
t[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x
^2])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6717, 6709, 466, 466, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}xdx$$

$$\Big\downarrow \text{ 6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-a^2x^2)^{3/2}}{(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 466}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{3}{2}\int \frac{\sqrt{1-a^2x^2}}{ax+1}dx+\frac{(1-a^2x^2)^{3/2}}{2a(ax+1)}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 466}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-a^2x^2}}dx+\frac{\sqrt{1-a^2x^2}}{a}\right)+\frac{(1-a^2x^2)^{3/2}}{2a(ax+1)}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{x\left(\frac{3}{2}\left(\frac{\sqrt{1-a^2x^2}}{a}+\frac{\arcsin(ax)}{a}\right)+\frac{(1-a^2x^2)^{3/2}}{2a(ax+1)}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^(2*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*((1 - a^2*x^2)^(3/2)/(2*a*(1 + a*x)) + (3*(Sqrt
[1 - a^2*x^2]/a + ArcSin[a*x]/a))/2))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.46

method	result
risch	$\frac{(ax-4)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x}{2a}+\frac{3\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{2\sqrt{a^2c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(-x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2+\sqrt{c}\,\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)-4\sqrt{c}\,\ln\left(\frac{\sqrt{c}\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+xc}{\sqrt{c}}\right)+4\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a\right)}{2\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2}$

input
```
int((c-c/a^2/x^2)^(1/2)*x*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x-4)/a*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x+3/2*ln(a^2*c*x/(a^2*c)^(1/2)
+(a^2*c*x^2-c)^(1/2))/(a^2*c)^(1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*
x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.41

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$
$$=\left[\frac{2\,(a^2x^2-4\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{4\,a^2},\frac{(a^2x^2-4\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{}}{2}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/4*(2*(a^2*x^2 - 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*sqrt(c)*log(
2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^2,
 1/2*((a^2*x^2 - 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(-c)*arcta
n(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*x*(a*x-1)/(a*x+1),x)
```

output
```
Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}x}{ax+1}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x/(a*x + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.36

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$
$$=\frac{1}{4}\left(2\sqrt{a^2cx^2-c}\left(\frac{x\mathrm{sgn}(x)}{a^2}-\frac{4\,\mathrm{sgn}(x)}{a^3}\right)-\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^2|a|}+\frac{(3\,a\sqrt{c}\log\left(|c\right.}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(a^2*c*x^2 - c)*(x*sgn(x)/a^2 - 4*sgn(x)/a^3) - 6*sqrt(c)*log(a
bs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^2*abs(a)) + (3*a*sqrt(
c)*log(abs(c)) + 8*sqrt(-c)*abs(a))*sgn(x)/(a^3*abs(a)))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int \frac{x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int((x*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int((x*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$
$$=\frac{\sqrt{c}\,\left(\sqrt{a^2x^2-1}\,ax-4\sqrt{a^2x^2-1}+3\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{2a^2}$$

input
```
int((c-c/a^2/x^2)^(1/2)*x*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*(sqrt(a**2*x**2 - 1)*a*x - 4*sqrt(a**2*x**2 - 1) + 3*log(sqrt(a**
2*x**2 - 1) + a*x)))/(2*a**2)
```

3.889 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\sqrt{c-\frac{c}{a^2x^2}}x-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}x}\right)}{a}-\frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)}{a}$$

output
```
(c-c/a^2/x^2)^(1/2)*x-c^(1/2)*arctan(c^(1/2)/a/(c-c/a^2/x^2)^(1/2)/x)/a-2*
c^(1/2)*arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}x\left(\sqrt{-1+a^2x^2}-\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)-2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]
] - 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6717, 6709, 570, 541, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{\left(1-a^2x^2\right)^{3/2}}{x(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{541}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int -\frac{a^2(1-2ax)}{x\sqrt{1-a^2x^2}}dx}{a^2}-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{\int \frac{a^2(1-2ax)}{x\sqrt{1-a^2x^2}}dx}{a^2}-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{1-2ax}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{538}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-2a\int \frac{1}{\sqrt{1-a^2x^2}}dx+\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\sqrt{1-a^2x^2}-2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\sqrt{1-a^2x^2}-2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a^2}-\sqrt{1-a^2x^2}-2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\sqrt{1-a^2x^2}-2\arcsin(ax)\right)}{\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-Sqrt[1 - a^2*x^2] - 2*ArcSin[a*x] - ArcTanh[S
qrt[1 - a^2*x^2]]))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 541
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[d^n*x^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*(m + n + 2*p + 1))), x
] + Simp[1/(b*(m + n + 2*p + 1))   Int[x^m*(a + b*x^2)^p*ExpandToSum[b*(m +
 n + 2*p + 1)*(c + d*x)^n - b*d^n*(m + n + 2*p + 1)*x^n - a*d^n*(m + n - 1)
*x^(n - 2), x], x], x] /; FreeQ[{a, b, c, d, m, p}, x] && IGtQ[n, 1] && IGt
Q[m, -2] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(74) = 148$.

Time = 0.20 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.23

method	result
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2-2\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}+2\sqrt{c}\,\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{c(ax-1)(ax+1)}{a^2}}+xc}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^2}}+c\ln\left(2\sqrt{-\right.\right.}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}}$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)
*a^2-2*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)*a^2*(-c/a^2)^(1/2)+2*c^(1/2)*ln((c^(1
/2)*(c*(a*x-1)*(a*x+1)/a^2)^(1/2)+x*c)/c^(1/2))*a*(-c/a^2)^(1/2)+c*ln(2*((
-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)
^(1/2)/a^2/(-c/a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.86

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$
$$=\left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+4\,\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},\ \frac{ax\sqrt{\frac{a^2cx^2-}{a^2x^2}}}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 4*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a
^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a,
(a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + sqrt(c)*arctan(a*x*sqrt((a^2*c*x^2
- c)/(a^2*x^2))/sqrt(c)) + sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{ax+1}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Maxima [F]

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{a\,x+1}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.58

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c}\left(2atan\left(\sqrt{a^2x^2-1}+ax\right)+\sqrt{a^2x^2-1}-2\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{a}$$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x)
```

output
```
(sqrt(c)*(2*atan(sqrt(a**2*x**2 - 1) + a*x) + sqrt(a**2*x**2 - 1) - 2*log(
sqrt(a**2*x**2 - 1) + a*x)))/a
```

3.890 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 79

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\sqrt{c-\frac{c}{a^2x^2}}+2\sqrt{c}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}x}\right)+\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{c}}\right)$$

output
```
(c-c/a^2/x^2)^(1/2)+2*c^(1/2)*arctan(c^(1/2)/a/(c-c/a^2/x^2)^(1/2)/x)+c^(1
/2)*arctanh((c-c/a^2/x^2)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}+2ax\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+ax\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2] + 2*a*x*ArcTan[1/Sqrt[-1 + a^2*
x^2]] + a*x*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6717, 6709, 570, 540, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-a^2x^2)^{3/2}}{x^2(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^2\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\int \frac{a(2-ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-a\int \frac{2-ax}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 538$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-a\left(2\int \frac{1}{x\sqrt{1-a^2x^2}}dx-a\int \frac{1}{\sqrt{1-a^2x^2}}dx\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 223$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-a\left(2\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\arcsin(ax)\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 243$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-a\left(\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\arcsin(ax)\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 73$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-a\left(-\frac{2\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a^2}-\arcsin(ax)\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 221$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-a\left(-2\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\arcsin(ax)\right)-\frac{\sqrt{1-a^2x^2}}{x}\right)}{\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-(Sqrt[1 - a^2*x^2]/x) - a*(-ArcSin[a*x] - 2*A
rcTanh[Sqrt[1 - a^2*x^2]])))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(65) = 130$.

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.84

method	result
risch	$\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}+\frac{\left(\frac{a^2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}+\frac{2a\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)}{\sqrt{-c}}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{a^2x^2-1}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^3cx^2+a^3\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}-2\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,xca^2\sqrt{-\frac{c}{a^2}}+c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2\right.}{a^2}}\right.\right.}{a\sqrt{\frac{c\left(a\right.}{}}}$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

output
```
(c*(a^2*x^2-1)/a^2/x^2)^(1/2)+(a^2*ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2-c)^
(1/2))/(a^2*c)^(1/2)+2*a/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1
/2))/x))*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.01

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \left[-\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + \sqrt{-c}\log\left(-\frac{a^2cx^2-2a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{x^2}\right) + \sqrt{\frac{a^2cx^2-c}{a^2x^2}}, -2\sqrt{c}\arctan\left(\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\sqrt{c}}\right) + \frac{1}{2}\sqrt{c}\log\left(2a^2cx^2+2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right) + \sqrt{\frac{a^2cx^2-c}{a^2x^2}}\right]$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

output
```
[-sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*
x^2 - c)) + sqrt(-c)*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)) - 2*c)/x^2) + sqrt((a^2*c*x^2 - c)/(a^2*x^2)), -2*sqrt(c)*arct
an(a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/sqrt(c)) + 1/2*sqrt(c)*log(2*a^2*c*
x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + sqrt((a^2*c
*x^2 - c)/(a^2*x^2))]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{x\,(ax+1)}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.61

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx= -\left(\frac{4\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a}+\frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{|a|}-\frac{2}{\left(\left(\sqrt{a^2c}x-\right.\right.}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

output
```
-(4*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/
a + sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/abs(a) -
 2*c^(3/2)*sgn(x)/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)*abs(a)))*
abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{x\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$$
$$=\frac{\sqrt{c}\left(-4atan\left(\sqrt{a^2x^2-1}+ax\right)ax+\sqrt{a^2x^2-1}+\log\left(\sqrt{a^2x^2-1}+ax\right)ax+ax\right)}{ax}$$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x)
```

output
```
(sqrt(c)*( - 4*atan(sqrt(a**2*x**2 - 1) + a*x)*a*x + sqrt(a**2*x**2 - 1) +
 log(sqrt(a**2*x**2 - 1) + a*x)*a*x + a*x))/(a*x)
```

3.891 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 79

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=-2a\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2x}-\frac{3}{2}a\sqrt{c}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\right)$$

output
```
-2*a*(c-c/a^2/x^2)^(1/2)+1/2*(c-c/a^2/x^2)^(1/2)/x-3/2*a*c^(1/2)*arctan(c^
(1/2)/a/(c-c/a^2/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left((-1+4ax)\sqrt{-1+a^2x^2}+3a^2x^2\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{2x\sqrt{-1+a^2x^2}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
-1/2*(Sqrt[c - c/(a^2*x^2)]*((-1 + 4*a*x)*Sqrt[-1 + a^2*x^2] + 3*a^2*x^2*A
rcTan[1/Sqrt[-1 + a^2*x^2]]))/(x*Sqrt[-1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6717, 6709, 570, 540, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-a^2x^2)^{3/2}}{x^3(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^3\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2}\int \frac{a(4-3ax)}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2}a\int \frac{4-3ax}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{534}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2}a\left(-3a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{4\sqrt{1-a^2x^2}}{x}\right)-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2}a\left(-\frac{3}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{4\sqrt{1-a^2x^2}}{x}\right)-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2}a\left(\frac{3\int\frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{4\sqrt{1-a^2x^2}}{x}\right)-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{x\left(-\frac{1}{2}a\left(3a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{4\sqrt{1-a^2x^2}}{x}\right)-\frac{\sqrt{1-a^2x^2}}{2x^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/2*Sqrt[1 - a^2*x^2]/x^2 - (a*((-4*Sqrt[1 -
a^2*x^2])/x + 3*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(65) = 130$.

Time = 0.22 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.81

method	result
risch	$-\frac{(4a^3x^3-a^2x^2-4ax+1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{2x(a^2x^2-1)}-\frac{3a^2\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{2\sqrt{-c}\,(a^2x^2-1)}$
default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\left(-4\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^3cx^3+4\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,a^3x-3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^2cx^2+4\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(x\sqrt{c}\right.\right.}{}$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(4*a^3*x^3-a^2*x^2-4*a*x+1)/x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-
1)-3/2*a^2/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)*(c*(a^
2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.06

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \left[\frac{3\,a\sqrt{-c}x\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\,(4\,ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,x},\ \frac{3\,a\sqrt{c}x\arctan\left(\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\sqrt{c}}\right)-(4\,a\ldots}{2\,x}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

output
```
[1/4*(3*a*sqrt(-c)*x*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)) - 2*c)/x^2) - 2*(4*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x
, 1/2*(3*a*sqrt(c)*x*arctan(a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/sqrt(c)) -
 (4*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{x^2\,(ax+1)}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**2*(a*x + 1)),
 x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^2}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(65) = 130$.

Time = 0.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.46

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$$
$$= \left(3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x) - \frac{\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^3 ac\operatorname{sgn}(x)+4\left(\sqrt{a^2c}x-\sqrt{a\right.}{\left(\left(\sqrt{a\right.\right.}}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

output
```
(3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) -
 ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*a*c*sgn(x) + 4*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^2*c^(3/2)*abs(a)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))*a*c^2*sgn(x) + 4*c^(5/2)*abs(a)*sgn(x))/(((sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^2 + c)^2*a))*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{x^2\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c}\left(6atan\left(\sqrt{a^2x^2-1}+ax\right)a^2x^2-4\sqrt{a^2x^2-1}\,ax+\sqrt{a^2x^2-1}\right)}{2a\,x^2}$$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x)
```

output
```
(sqrt(c)*(6*atan(sqrt(a**2*x**2 - 1) + a*x)*a**2*x**2 - 4*sqrt(a**2*x**2 -
 1)*a*x + sqrt(a**2*x**2 - 1)))/(2*a*x**2)
```

3.892 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 103

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{5}{3}a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{x} + a^2\sqrt{c}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}x}\right)$$

output
```
5/3*a^2*(c-c/a^2/x^2)^(1/2)+1/3*(c-c/a^2/x^2)^(1/2)/x^2-a*(c-c/a^2/x^2)^(1
/2)/x+a^2*c^(1/2)*arctan(c^(1/2)/a/(c-c/a^2/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(1-3ax+5a^2x^2)+3a^3x^3\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{3x^2\sqrt{-1+a^2x^2}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1 - 3*a*x + 5*a^2*x^2) + 3*a^3
*x^3*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(3*x^2*Sqrt[-1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6717, 6709, 570, 540, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-a^2x^2)^{3/2}}{x^4(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^4\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3}\int \frac{a(6-5ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3}a\int \frac{6-5ax}{x^3\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3}a\left(-\frac{1}{2}\int \frac{2a(5-3ax)}{x^2\sqrt{1-a^2x^2}}dx-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3}a\left(-a\int \frac{5-3ax}{x^2\sqrt{1-a^2x^2}}dx-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 534$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3}a\left(-a\left(-3a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{x}\right)-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 243$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3}a\left(-a\left(-\frac{3}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{5\sqrt{1-a^2x^2}}{x}\right)-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 73$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3}a\left(-a\left(\frac{3\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{5\sqrt{1-a^2x^2}}{x}\right)-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 221$$

$$-\frac{x\left(-\frac{1}{3}a\left(-a\left(3a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{5\sqrt{1-a^2x^2}}{x}\right)-\frac{3\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{3x^3}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/3*Sqrt[1 - a^2*x^2]/x^3 - (a*((-3*Sqrt[1 -
a^2*x^2])/x^2 - a*((-5*Sqrt[1 - a^2*x^2])/x + 3*a*ArcTanh[Sqrt[1 - a^2*x^2
]])))/3))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.46

method	result
risch	$\frac{\left(5a^4x^4-3a^3x^3-4a^2x^2+3ax-1\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{3x^2\left(a^2x^2-1\right)}+\frac{a^3\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,x}{\sqrt{-c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,a\left(-6\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^3c\,x^4+6\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^3x^2+6\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)a\,x^3-6\sqrt{}\right.}{}$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(5*a^4*x^4-3*a^3*x^3-4*a^2*x^2+3*a*x-1)/x^2*(c*(a^2*x^2-1)/a^2/x^2)^(1
/2)/(a^2*x^2-1)+a^3/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*(a^2*c*x^2-c)^(1/2))/
x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.82

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \left[\frac{3\,a^2\sqrt{-c}x^2\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(5\,a^2x^2-3\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{6\,x^2}, -\frac{3\,a^2\sqrt{c}x^2\arctan\left(\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\sqrt{c}}\right)-(5\,a^2x^2-3\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\,x^2}\right]$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

output
```
[1/6*(3*a^2*sqrt(-c)*x^2*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
- c)/(a^2*x^2)) - 2*c)/x^2) + 2*(5*a^2*x^2 - 3*a*x + 1)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)))/x^2, -1/3*(3*a^2*sqrt(c)*x^2*arctan(a*x*sqrt((a^2*c*x^2 - c
)/(a^2*x^2))/sqrt(c)) - (5*a^2*x^2 - 3*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)))/x^2]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{x^3\,(ax+1)}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**3*(a*x + 1)),
 x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^3}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(87) = 174.

Time = 0.19 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.24

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=$$

$$-\frac{2}{3}\left(3\,a\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)-\frac{3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^5ac\operatorname{sgn}(x)+3\left(\sqrt{a^2c\ldots\right.}{\ldots}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

output
```
-2/3*(3*a*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*s
gn(x) - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a*c*sgn(x) + 3*(sqrt(a^
2*c)*x - sqrt(a^2*c*x^2 - c))^4*c^(3/2)*abs(a)*sgn(x) + 12*(sqrt(a^2*c)*x
- sqrt(a^2*c*x^2 - c))^2*c^(5/2)*abs(a)*sgn(x) - 3*(sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - c))*a*c^3*sgn(x) + 5*c^(7/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^2 + c)^3)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{x^3\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c}\left(-6atan\left(\sqrt{a^2x^2-1}+ax\right)a^3x^3+5\sqrt{a^2x^2-1}\,a^2x^2-3\sqrt{a^2x^2-1}\,ax+\sqrt{a^2x^2-1}-3a^3x^3\right)}{3a\,x^3}$$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x)
```

output
```
(sqrt(c)*( - 6*atan(sqrt(a**2*x**2 - 1) + a*x)*a**3*x**3 + 5*sqrt(a**2*x**
2 - 1)*a**2*x**2 - 3*sqrt(a**2*x**2 - 1)*a*x + sqrt(a**2*x**2 - 1) - 3*a**
3*x**3))/(3*a*x**3)
```

3.893 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 133

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{7}{8}a^3\sqrt{c}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\right)$$

output
```
-4/3*a^3*(c-c/a^2/x^2)^(1/2)+1/4*(c-c/a^2/x^2)^(1/2)/x^3-2/3*a*(c-c/a^2/x^
2)^(1/2)/x^2+7/8*a^2*(c-c/a^2/x^2)^(1/2)/x-7/8*a^3*c^(1/2)*arctan(c^(1/2)/
a/(c-c/a^2/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.71

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = -\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(-6+16ax-21a^2x^2+32a^3x^3)+21a^4x^4\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{24x^3\sqrt{-1+a^2x^2}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
-1/24*(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-6 + 16*a*x - 21*a^2*x^2
 + 32*a^3*x^3) + 21*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(x^3*Sqrt[-1 +
a^2*x^2])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {6717, 6709, 570, 540, 27, 539, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-a^2x^2)^{3/2}}{x^5(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^5\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}\int \frac{a(8-7ax)}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}a\int \frac{8-7ax}{x^4\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}a\left(-\frac{1}{3}\int \frac{a(21-16ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}a\left(-\frac{1}{3}a\int \frac{21-16ax}{x^3\sqrt{1-a^2x^2}}dx-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}\int \frac{a(32-21ax)}{x^2\sqrt{1-a^2x^2}}dx-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\int \frac{32-21ax}{x^2\sqrt{1-a^2x^2}}dx-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{534}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\left(-21a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{32\sqrt{1-a^2x^2}}{x}\right)-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\left(-\frac{21}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{32\sqrt{1-a^2x^2}}{x}\right)-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\left(\frac{21\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{32\sqrt{1-a^2x^2}}{x}\right)-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{x\left(-\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{1}{2}a\left(21a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{32\sqrt{1-a^2x^2}}{x}\right)-\frac{21\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{3x^3}\right)-\frac{\sqrt{1-a^2x^2}}{4x^4}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/4*Sqrt[1 - a^2*x^2]/x^4 - (a*((-8*Sqrt[1 -
a^2*x^2])/(3*x^3) - (a*((-21*Sqrt[1 - a^2*x^2])/(2*x^2) - (a*((-32*Sqrt[1
- a^2*x^2])/x + 21*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2))/3))/4))/Sqrt[1 - a^2
*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{(32a^5x^5-21a^4x^4-16a^3x^3+15a^2x^2-16ax+6)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{24x^3(a^2x^2-1)}-\frac{7a^4\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\sqrt{c(a^2x^2-1)}}{8\sqrt{-c}\,(a^2x^2-1)}$
default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,a^2\left(-48\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^3cx^5+48\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3x^3+48\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)a\,x^4-\right.}{}$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(32*a^5*x^5-21*a^4*x^4-16*a^3*x^3+15*a^2*x^2-16*a*x+6)/x^3*(c*(a^2*x
^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)-7/8*a^4/(-c)^(1/2)*ln((-2*c+2*(-c)^(1/2)*
(a^2*c*x^2-c)^(1/2))/x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(c*(a^2*x^2-1))^(1
/2)/(a^2*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.53

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \left[\frac{21\,a^3\sqrt{-c}x^3\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\,(32\,a^3x^3-21\,a^2x^2+16\,ax-6)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{48\,x^3},\ \frac{21\,a^3\sqrt{c}x^3\cdots}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(21*a^3*sqrt(-c)*x^3*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3, 1/24*(21*a^3*sqrt(c)*x^3*arctan(a*x*s
qrt((a^2*c*x^2 - c)/(a^2*x^2))/sqrt(c)) - (32*a^3*x^3 - 21*a^2*x^2 + 16*a*
x - 6)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{x^4\,(ax+1)}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**4*(a*x + 1)),
 x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^4}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(109) = 218$.

Time = 0.18 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.38

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$

$$= \frac{1}{12}\left(21\,a^2\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x) - \frac{21\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^7 a^2c\mathrm{sgn}(x) + 45\left(\sqrt{\right.}}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

output
```
1/12*(21*a^2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x) - (21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^2*c*sgn(x) + 45*(
sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a^2*c^2*sgn(x) + 96*(sqrt(a^2*c)*x
 - sqrt(a^2*c*x^2 - c))^4*a*c^(5/2)*abs(a)*sgn(x) - 45*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*a^2*c^3*sgn(x) + 128*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 c))^2*a*c^(7/2)*abs(a)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*
a^2*c^4*sgn(x) + 32*a*c^(9/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*
x^2 - c))^2 + c)^4)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{x^4\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.14

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$
$$= \frac{\sqrt{c}\left(18atan\left(\sqrt{a^2x^2-1}+ax\right)a^4x^4+12atan\left(\frac{\sqrt{a^2x^2-1}\,ax+a^2x^2-1}{\sqrt{a^2x^2-1}+ax}\right)a^4x^4-32\sqrt{a^2x^2-1}\,a^3x^3+21\sqrt{a^2x^2-}\right.}{24a\,x^4}$$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x)
```

output
```
(sqrt(c)*(18*atan(sqrt(a**2*x**2 - 1) + a*x)*a**4*x**4 + 12*atan((sqrt(a**
2*x**2 - 1)*a*x + a**2*x**2 - 1)/(sqrt(a**2*x**2 - 1) + a*x))*a**4*x**4 -
32*sqrt(a**2*x**2 - 1)*a**3*x**3 + 21*sqrt(a**2*x**2 - 1)*a**2*x**2 - 16*s
qrt(a**2*x**2 - 1)*a*x + 6*sqrt(a**2*x**2 - 1) + 32*a**4*x**4))/(24*a*x**4
)
```

3.894 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 148

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\frac{a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{5c^2}+\frac{a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{c^2\left(a+\frac{1}{x}\right)^2}+\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}+\frac{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2cx}+\frac{3}{4}a^4\sqrt{c}\arctan\left(\frac{\sqrt{c}}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\right)$$

output
```
1/5*a^4*(c-c/a^2/x^2)^(5/2)/c^2+a^6*(c-c/a^2/x^2)^(5/2)/c^2/(a+1/x)^2+3/4*
a^3*(c-c/a^2/x^2)^(1/2)/x+1/2*a^3*(c-c/a^2/x^2)^(3/2)/c/x+3/4*a^4*c^(1/2)*
arctan(c^(1/2)/a/(c-c/a^2/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.69

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}(4-10ax+12a^2x^2-15a^3x^3+24a^4x^4)+15a^5x^5\arctan\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{20x^4\sqrt{-1+a^2x^2}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(4 - 10*a*x + 12*a^2*x^2 - 15*a
^3*x^3 + 24*a^4*x^4) + 15*a^5*x^5*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(20*x^4*S
qrt[-1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6717, 6709, 570, 540, 27, 539, 27, 539, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-2\coth^{-1}(ax)}}{x^5}dx$$

$$\Big\downarrow \text{6717}$$

$$-\int \frac{e^{-2\text{arctanh}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}dx$$

$$\Big\downarrow \text{6709}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-a^2x^2)^{3/2}}{x^6(ax+1)^2}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{570}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^6\sqrt{1-a^2x^2}}dx}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{540}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}\int \frac{a(10-9ax)}{x^5\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\int\frac{10-9ax}{x^5\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{1}{4}\int\frac{6a(6-5ax)}{x^4\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{3}{2}a\int\frac{6-5ax}{x^4\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{3}{2}a\left(-\frac{1}{3}\int\frac{3a(5-4ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{3}{2}a\left(-a\int\frac{5-4ax}{x^3\sqrt{1-a^2x^2}}dx-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{539}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{3}{2}a\left(-a\left(-\frac{1}{2}\int\frac{a(8-5ax)}{x^2\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{3}{2}a\left(-a\left(-\frac{1}{2}a\int\frac{8-5ax}{x^2\sqrt{1-a^2x^2}}dx-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a^2x^2}}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{534}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{3}{2}a\left(-a\left(-\frac{1}{2}a\left(-5a\int\frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{8\sqrt{1-a^2x^2}}{x}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\right.}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{3}{2}a\left(-a\left(-\frac{1}{2}a\left(-\frac{5}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{8\sqrt{1-a^2x^2}}{x}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)\right.}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 73$$

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5}a\left(-\frac{3}{2}a\left(-a\left(-\frac{1}{2}a\left(\frac{5\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{8\sqrt{1-a^2x^2}}{x}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\cdots\right.\right.}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 221$$

$$-\frac{x\left(-\frac{1}{5}a\left(-\frac{3}{2}a\left(-a\left(-\frac{1}{2}a\left(5a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{8\sqrt{1-a^2x^2}}{x}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^2}\right)-\frac{2\sqrt{1-a^2x^2}}{x^3}\right)-\frac{5\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\sqrt{1-a\cdots}}{5x^5}\cdots\right.}{\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

output
```
-((Sqrt[c - c/(a^2*x^2)]*x*(-1/5*Sqrt[1 - a^2*x^2]/x^5 - (a*((-5*Sqrt[1 -
a^2*x^2])/(2*x^4) - (3*a*((-2*Sqrt[1 - a^2*x^2])/x^3 - a*((-5*Sqrt[1 - a^2
*x^2])/(2*x^2) - (a*((-8*Sqrt[1 - a^2*x^2])/x + 5*a*ArcTanh[Sqrt[1 - a^2*x
^2]]))/2)))/2))/5))/Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 570
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, -1] &&  !(IGtQ[m, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

rule 6709
```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[x^(2*p)*((c + d/x^2)^p/(1 - a^2*x^2)^p)   Int[u*((1 + a*x)^n/(x^
(2*p)*(1 - a^2*x^2)^(n/2 - p))), x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c
+ a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0]
```

rule 6717
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Simp[(-1)^(n/2)   Int[
u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.13

method	result
risch	$\frac{(24x^6a^6-15a^5x^5-12a^4x^4+5a^3x^3-8a^2x^2+10ax-4)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{20x^4(a^2x^2-1)}+\frac{3a^5\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c(a^2x^2-1)}\,}{4\sqrt{-c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,a^2\left(-40\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^4cx^6+40\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^4x^4+40\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)a^2x\right.}{}$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/20*(24*a^6*x^6-15*a^5*x^5-12*a^4*x^4+5*a^3*x^3-8*a^2*x^2+10*a*x-4)/x^4*(
c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a^2*x^2-1)+3/4*a^5/(-c)^(1/2)*ln((-2*c+2*(-c
)^(1/2)*(a^2*c*x^2-c)^(1/2))/x)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(c*(a^2*x^2-
1))^(1/2)/(a^2*x^2-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.48

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \left[\frac{15\,a^4\sqrt{-c}x^4\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(24\,a^4x^4-15\,a^3x^3+12\,a^2x^2-10\,ax+4)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{40\,x^4}, -\frac{15\,a^4\sqrt{c}x^4\arctan\left(\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\sqrt{c}}\right)-(24\,a^4x^4-15\,a^3x^3+12\,a^2x^2-10\,ax+4)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{20\,x^4}\right]$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

output
```
[1/40*(15*a^4*sqrt(-c)*x^4*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*x^4 - 15*a^3*x^3 + 12*a^2*x^2 -
10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4, -1/20*(15*a^4*sqrt(c)*x^
4*arctan(a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/sqrt(c)) - (24*a^4*x^4 - 15*a
^3*x^3 + 12*a^2*x^2 - 10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4]
```

Sympy [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}(ax-1)}{x^5\left(ax+1\right)}\,dx$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**5*(a*x + 1)),
 x)
```

Maxima [F]

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^5}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

output
```
integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(126) = 252$.

Time = 0.26 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.45

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = -\frac{1}{10}\left(15\,a^3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-c}\right)^9a^3c\mathrm{sgn}(x)+70\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

output
```
-1/10*(15*a^3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c
))*sgn(x) - (15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*a^3*c*sgn(x) + 70*
(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^3*c^2*sgn(x) + 40*(sqrt(a^2*c)*x
 - sqrt(a^2*c*x^2 - c))^6*a^2*c^(5/2)*abs(a)*sgn(x) + 200*(sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - c))^4*a^2*c^(7/2)*abs(a)*sgn(x) - 70*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^3*a^3*c^4*sgn(x) + 120*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
- c))^2*a^2*c^(9/2)*abs(a)*sgn(x) - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))*a^3*c^5*sgn(x) + 24*a^2*c^(11/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(
a^2*c*x^2 - c))^2 + c)^5)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{x^5\,(a\,x+1)}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$
$$= \frac{\sqrt{c}\left(-30atan\left(\sqrt{a^2x^2-1}+ax\right)a^5x^5+24\sqrt{a^2x^2-1}\,a^4x^4-15\sqrt{a^2x^2-1}\,a^3x^3+12\sqrt{a^2x^2-1}\,a^2x^2-10\right.}{20a\,x^5}$$

input
```
int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x)
```

output
```
(sqrt(c)*( - 30*atan(sqrt(a**2*x**2 - 1) + a*x)*a**5*x**5 + 24*sqrt(a**2*x
**2 - 1)*a**4*x**4 - 15*sqrt(a**2*x**2 - 1)*a**3*x**3 + 12*sqrt(a**2*x**2
- 1)*a**2*x**2 - 10*sqrt(a**2*x**2 - 1)*a*x + 4*sqrt(a**2*x**2 - 1) - 24*a
**5*x**5))/(20*a*x**5)
```

3.895 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 186

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx = -\frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^4}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-4*(c-c/a^2/x^2)^(1/2)*x/a^3/(1-1/a^2/x^2)^(1/2)+2*(c-c/a^2/x^2)^(1/2)*x^2
/a^2/(1-1/a^2/x^2)^(1/2)-(c-c/a^2/x^2)^(1/2)*x^3/a/(1-1/a^2/x^2)^(1/2)+1/4
*(c-c/a^2/x^2)^(1/2)*x^4/(1-1/a^2/x^2)^(1/2)+4*(c-c/a^2/x^2)^(1/2)*ln(a*x+
1)/a^4/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.38

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4x}{a^3}+\frac{2x^2}{a^2}-\frac{x^3}{a}+\frac{x^4}{4}+\frac{4\log(1+ax)}{a^4}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*((-4*x)/a^3 + (2*x^2)/a^2 - x^3/a + x^4/4 + (4*Log[
1 + a*x])/a^4))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^3dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x^2(1-ax)^2}{ax+1}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(ax^3-3x^2+\frac{4x}{a}+\frac{4}{a^2(ax+1)}-\frac{4}{a^2}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{4\log(ax+1)}{a^3}-\frac{4x}{a^2}+\frac{ax^4}{4}+\frac{2x^2}{a}-x^3\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*((-4*x)/a^2 + (2*x^2)/a - x^3 + (a*x^4)/4 + (4*Log[
1 + a*x])/a^3))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\left(a^{4} x^{4}-4 a^{3} x^{3}+8 a^{2} x^{2}-16 a x +16 \ln \left(a x +1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{4 a^{3} \left(a x -1\right)^{2}}$	89

input
```
int((c-c/a^2/x^2)^(1/2)*x^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/4*(a^4*x^4-4*a^3*x^3+8*a^2*x^2-16*a*x+16*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2
/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^3/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.26

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx = \frac{(a^4x^4-4\,a^3x^3+8\,a^2x^2-16\,ax+16\,\log(ax+1))\sqrt{a^2c}}{4\,a^5}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fr
icas")
```

output
```
1/4*(a^4*x^4 - 4*a^3*x^3 + 8*a^2*x^2 - 16*a*x + 16*log(a*x + 1))*sqrt(a^2*
c)/a^5
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx = \text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*x**3*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int \sqrt{c-\frac{c}{a^2x^2}}x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.51

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$=\frac{1}{4}\sqrt{c}\left(\frac{16\log(|ax+1|)\operatorname{sgn}(ax+1)\operatorname{sgn}(x)}{a^5}+\frac{a^{11}x^4\operatorname{sgn}(ax+1)\operatorname{sgn}(x)-4a^{10}x^3\operatorname{sgn}(ax+1)\operatorname{sgn}(x)+8}{a^{12}}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gi
ac")
```

output
```
1/4*sqrt(c)*(16*log(abs(a*x + 1))*sgn(a*x + 1)*sgn(x)/a^5 + (a^11*x^4*sgn(
a*x + 1)*sgn(x) - 4*a^10*x^3*sgn(a*x + 1)*sgn(x) + 8*a^9*x^2*sgn(a*x + 1)*
sgn(x) - 16*a^8*x*sgn(a*x + 1)*sgn(x))/a^12)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx=\int x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int(x^3*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x^3*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.24

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^3\,dx$$
$$=\frac{\sqrt{c}\,(16\log(ax+1)+a^4x^4-4a^3x^3+8a^2x^2-16ax+11)}{4a^4}$$

input
```
int((c-c/a^2/x^2)^(1/2)*x^3*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*(16*log(a*x + 1) + a**4*x**4 - 4*a**3*x**3 + 8*a**2*x**2 - 16*a*x
 + 11))/(4*a**4)
```

3.896 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 151

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{4\sqrt{c-\frac{c}{a^2x^2}}x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
4*(c-c/a^2/x^2)^(1/2)*x/a^2/(1-1/a^2/x^2)^(1/2)-3/2*(c-c/a^2/x^2)^(1/2)*x^
2/a/(1-1/a^2/x^2)^(1/2)+1/3*(c-c/a^2/x^2)^(1/2)*x^3/(1-1/a^2/x^2)^(1/2)-4*
(c-c/a^2/x^2)^(1/2)*ln(a*x+1)/a^3/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.41

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax(24-9ax+2a^2x^2)-24\log(1+ax))}{6a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(a*x*(24 - 9*a*x + 2*a^2*x^2) - 24*Log[1 + a*x]))/(
6*a^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6751, 6747, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}x^2dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x(1-ax)^2}{ax+1}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{86}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(ax^2-3x+\frac{4}{a}-\frac{4}{a(ax+1)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4\log(ax+1)}{a^2}+\frac{ax^3}{3}+\frac{4x}{a}-\frac{3x^2}{2}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*((4*x)/a - (3*x^2)/2 + (a*x^3)/3 - (4*Log[1 + a*x])
/a^2))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{\left(-2 a^{3} x^{3}+9 a^{2} x^{2}-24 a x +24 \ln \left(a x +1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{6 a^{2} \left(a x -1\right)^{2}}$	82

input
```
int((c-c/a^2/x^2)^(1/2)*x^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/6*(-2*a^3*x^3+9*a^2*x^2-24*a*x+24*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/x^2)^
(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^2/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.27

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = \frac{(2\,a^3x^3-9\,a^2x^2+24\,ax-24\,\log(ax+1))\sqrt{a^2c}}{6\,a^4}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fr
icas")
```

output
```
1/6*(2*a^3*x^3 - 9*a^2*x^2 + 24*a*x - 24*log(a*x + 1))*sqrt(a^2*c)/a^4
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = \text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*x**2*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int \sqrt{c-\frac{c}{a^2x^2}}x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.53

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx = -\frac{1}{6}\sqrt{c}\left(\frac{24\log\left(|ax+1|\right)\text{sgn}(ax+1)\,\text{sgn}(x)}{a^4}-\frac{2\,a^8x^3\text{sgn}(ax+1)\,\text{sgn}(x)-9\,a^7x^2\text{sgn}(ax+1)\,\text{sgn}(x)+}{a^9}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gi
ac")
```

output
```
-1/6*sqrt(c)*(24*log(abs(a*x + 1))*sgn(a*x + 1)*sgn(x)/a^4 - (2*a^8*x^3*sg
n(a*x + 1)*sgn(x) - 9*a^7*x^2*sgn(a*x + 1)*sgn(x) + 24*a^6*x*sgn(a*x + 1)*
sgn(x))/a^9)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\int x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

input
```
int(x^2*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x^2*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.25

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x^2\,dx=\frac{\sqrt{c}\,(-24\log(ax+1)+2a^3x^3-9a^2x^2+24ax-17)}{6a^3}$$

input
```
int((c-c/a^2/x^2)^(1/2)*x^2*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*( - 24*log(a*x + 1) + 2*a**3*x**3 - 9*a**2*x**2 + 24*a*x - 17))/(
6*a**3)
```

3.897 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = -\frac{3\sqrt{c-\frac{c}{a^2x^2}}x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-3*(c-c/a^2/x^2)^(1/2)*x/a/(1-1/a^2/x^2)^(1/2)+1/2*(c-c/a^2/x^2)^(1/2)*x^2
/(1-1/a^2/x^2)^(1/2)+4*(c-c/a^2/x^2)^(1/2)*ln(a*x+1)/a^2/(1-1/a^2/x^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.49

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{3x}{a}+\frac{x^2}{2}+\frac{4\log(1+ax)}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*((-3*x)/a + x^2/2 + (4*Log[1 + a*x])/a^2))/Sqrt[1 -
 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6751, 6747, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}xdx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{ax+1}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(ax+\frac{4}{ax+1}-3\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax^2}{2}+\frac{4\log(ax+1)}{a}-3x\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-3*x + (a*x^2)/2 + (4*Log[1 + a*x])/a))/(a*Sqrt[1
- 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\left(a^{2} x^{2}-6 a x+8 \ln \left(a x+1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x+1\right) \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{2 a \left(a x-1\right)^{2}}$	73

input
```
int((c-c/a^2/x^2)^(1/2)*x*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((
a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.29

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{(a^2x^2-6\,ax+8\,\log{(ax+1)})\sqrt{a^2c}}{2\,a^3}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fric
as")
```

output
```
1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(a^2*c)/a^3
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*x*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx$$
$$=\frac{1}{2}\sqrt{c}\left(\frac{8\log\left(|ax+1|\right)\text{sgn}(ax+1)\,\text{sgn}(x)}{a^3}+\frac{a^5x^2\text{sgn}(ax+1)\,\text{sgn}(x)-6\,a^4x\text{sgn}(ax+1)\,\text{sgn}(x)}{a^6}\right)|a|$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac
")
```

output
```
1/2*sqrt(c)*(8*log(abs(a*x + 1))*sgn(a*x + 1)*sgn(x)/a^3 + (a^5*x^2*sgn(a*
x + 1)*sgn(x) - 6*a^4*x*sgn(a*x + 1)*sgn(x))/a^6)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\int x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

input
```
int(x*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int(x*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.25

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}x\,dx=\frac{\sqrt{c}\,(8\log(ax+1)+a^2x^2-6ax+5)}{2a^2}$$

input
```
int((c-c/a^2/x^2)^(1/2)*x*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*(8*log(a*x + 1) + a**2*x**2 - 6*a*x + 5))/(2*a**2)
```

3.898 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(c-c/a^2/x^2)^(1/2)*x/(1-1/a^2/x^2)^(1/2)+(c-c/a^2/x^2)^(1/2)*ln(x)/a/(1-1
/a^2/x^2)^(1/2)-4*(c-c/a^2/x^2)^(1/2)*ln(a*x+1)/a/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}(ax+\log(x)-4\log(1+ax))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2
*x^2)])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6751, 6747, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x(ax+1)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{93}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a}{ax+1}+a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}(ax-4\log(ax+1)+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2
*x^2)])
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

method	result	size
default	$-\frac{\left(-ax+4\ln\left(ax+1\right)-\ln\left(x\right)\right)x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(ax-1\right)^{2}}$	67

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-a*x+4*ln(a*x+1)-ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)
/(a*x+1))^(3/2)/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.23

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{a^2c}(ax-4\log(ax+1)+\log(x))}{a^2}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

output
```
sqrt(a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$
$$=\left(\frac{x\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a}-\frac{4\,\log\left(|ax+1|\right)\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a^2}+\frac{\log\left(|x|\right)\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a^2}\right)\sqrt{c}|a|$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

output
```
(x*sgn(a*x + 1)*sgn(x)/a - 4*log(abs(a*x + 1))*sgn(a*x + 1)*sgn(x)/a^2 + 1
og(abs(x))*sgn(a*x + 1)*sgn(x)/a^2)*sqrt(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int \sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.21

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c}\,(-4\log(ax+1)+\log(ax)+ax-1)}{a}$$

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

output
```
(sqrt(c)*( - 4*log(a*x + 1) + log(a*x) + a*x - 1))/a
```

3.899 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 108

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}x}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x-3*(c-c/a^2/x^2)^(1/2)*ln(x)/(
1-1/a^2/x^2)^(1/2)+4*(c-c/a^2/x^2)^(1/2)*ln(a*x+1)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{ax}-3\log(x)+4\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-(1/(a*x)) - 3*Log[x] + 4*Log[1 + a*x]))/Sqrt[1 -
1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}}{x}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^2(ax+1)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{4a^2}{ax+1}-\frac{3a}{x}+\frac{1}{x^2}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-3a\log(x)+4a\log(ax+1)-\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-x^(-1) - 3*a*Log[x] + 4*a*Log[1 + a*x]))/(a*Sqrt[
1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\left(4 \ln \left(a x +1\right) x a -3 a \ln \left(x \right) x -1\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{\left(a x -1\right)^{2}}$	66

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(4*ln(a*x+1)*x*a-3*a*ln(x)*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*
x-1)/(a*x+1))^(3/2)/(a*x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.30

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\frac{\sqrt{a^2c}(4\,ax\log{(ax+1)}-3\,ax\log{(x)}-1)}{a^2x}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fric
as")
```

output
```
sqrt(a^2*c)*(4*a*x*log(a*x + 1) - 3*a*x*log(x) - 1)/(a^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.55

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \left(\frac{4\log(|ax+1|)\,\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a}-\frac{3\log(|x|)\,\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a}-\frac{\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)}{a^2x}\right)\sqrt{c}|a|$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac
")
```

output
```
(4*log(abs(a*x + 1))*sgn(a*x + 1)*sgn(x)/a - 3*log(abs(x))*sgn(a*x + 1)*sg
n(x)/a - sgn(a*x + 1)*sgn(x)/(a^2*x))*sqrt(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.30

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c}\,(4\log(ax+1)\,ax-3\log(ax)\,ax+ax-1)}{ax}$$

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x)
```

output
```
(sqrt(c)*(4*log(a*x + 1)*a*x - 3*log(a*x)*a*x + a*x - 1))/(a*x)
```

3.900 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 146

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x} + \frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/2*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^2+3*(c-c/a^2/x^2)^(1/2)/(
1-1/a^2/x^2)^(1/2)/x+4*a*(c-c/a^2/x^2)^(1/2)*ln(x)/(1-1/a^2/x^2)^(1/2)-4*a
*(c-c/a^2/x^2)^(1/2)*ln(a*x+1)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2ax^2}+\frac{3}{x}+4a\log(x)-4a\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/2*1/(a*x^2) + 3/x + 4*a*Log[x] - 4*a*Log[1 + a*
x]))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^3(ax+1)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4a^3}{ax+1}+\frac{4a^2}{x}-\frac{3a}{x^2}+\frac{1}{x^3}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(4a^2\log(x)-4a^2\log(ax+1)+\frac{3a}{x}-\frac{1}{2x^2}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 + (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1
+ a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{\left(8 \ln \left(a x +1\right) x^{2} a^{2}-8 a^{2} \ln \left(x \right) x^{2}-6 a x +1\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{2 \left(a x -1\right)^{2} x}$	82

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOS
E)
```

output
```
-1/2*(8*ln(a*x+1)*x^2*a^2-8*a^2*ln(x)*x^2-6*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)
^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.57

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{8\,a^3\sqrt{c}x^2\log\left(\frac{2\,a^3cx^2+2\,a^2cx-\sqrt{a^2c}(2\,ax+1)\sqrt{c}+ac}{ax^2+x}\right)+\sqrt{a^2c}(6\,ax-1)}{2\,a^2x^2}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fr
icas")
```

output
```
1/2*(8*a^3*sqrt(c)*x^2*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(a^2*c)*(2*a*x +
 1)*sqrt(c) + a*c)/(a*x^2 + x)) + sqrt(a^2*c)*(6*a*x - 1))/(a^2*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\frac{1}{2}\left(8\,\log\left(|ax+1|\right)\operatorname{sgn}(ax+1)\operatorname{sgn}(x) - 8\,\log\left(|x|\right)\operatorname{sgn}(ax+1)\operatorname{sgn}(x) - \frac{6\,ax\operatorname{sgn}(ax+1)\operatorname{sgn}(x) - \operatorname{sg}}{a^2x^2}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="gi
ac")
```

output
```
-1/2*(8*log(abs(a*x + 1))*sgn(a*x + 1)*sgn(x) - 8*log(abs(x))*sgn(a*x + 1)
*sgn(x) - (6*a*x*sgn(a*x + 1)*sgn(x) - sgn(a*x + 1)*sgn(x))/(a^2*x^2))*sqr
t(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^2}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.34

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx=\frac{\sqrt{c}\,(-8\log(ax+1)\,a^2x^2+8\log(ax)\,a^2x^2-3a^2x^2+6ax-1)}{2a\,x^2}$$

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

output
```
(sqrt(c)*( - 8*log(a*x + 1)*a**2*x**2 + 8*log(a*x)*a**2*x**2 - 3*a**2*x**2
 + 6*a*x - 1))/(2*a*x**2)
```

3.901 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 187

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-1/3*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^3+3/2*(c-c/a^2/x^2)^(1/2)
/(1-1/a^2/x^2)^(1/2)/x^2-4*a*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)/x-4*a
^2*(c-c/a^2/x^2)^(1/2)*ln(x)/(1-1/a^2/x^2)^(1/2)+4*a^2*(c-c/a^2/x^2)^(1/2)
*ln(a*x+1)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3ax^3}+\frac{3}{2x^2}-\frac{4a}{x}-4a^2\log(x)+4a^2\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/3*1/(a*x^3) + 3/(2*x^2) - (4*a)/x - 4*a^2*Log[x
] + 4*a^2*Log[1 + a*x]))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}}{x^3}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^4(ax+1)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(\frac{4a^4}{ax+1}-\frac{4a^3}{x}+\frac{4a^2}{x^2}-\frac{3a}{x^3}+\frac{1}{x^4}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-4a^3\log(x)+4a^3\log(ax+1)-\frac{4a^2}{x}+\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/3*1/x^3 + (3*a)/(2*x^2) - (4*a^2)/x - 4*a^3*Log
[x] + 4*a^3*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\left(24 \ln \left(a x+1\right) x^{3} a^{3}-24 \ln \left(x \right) x^{3} a^{3}-24 a^{2} x^{2}+9 a x-2\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x+1\right) \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{6 \left(a x-1\right)^{2} x^{2}}$	90

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOS
E)
```

output
```
1/6*(24*ln(a*x+1)*x^3*a^3-24*ln(x)*x^3*a^3-24*a^2*x^2+9*a*x-2)*(c*(a^2*x^2
-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.49

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{24\,a^4\sqrt{c}x^3\log\left(\frac{2\,a^3cx^2+2\,a^2cx+\sqrt{a^2c}(2\,ax+1)\sqrt{c}+ac}{ax^2+x}\right)-(24\,a^2x^2-9\,ax+2)\sqrt{a^2c}}{6\,a^2x^3}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fr
icas")
```

output
```
1/6*(24*a^4*sqrt(c)*x^3*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(a^2*c)*(2*a*x
+ 1)*sqrt(c) + a*c)/(a*x^2 + x)) - (24*a^2*x^2 - 9*a*x + 2)*sqrt(a^2*c))/(
a^2*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$$
$$= \frac{1}{6}\left(24\,a\log\left(|ax+1|\right)\operatorname{sgn}(ax+1)\operatorname{sgn}(x) - 24\,a\log\left(|x|\right)\operatorname{sgn}(ax+1)\operatorname{sgn}(x) - \frac{24\,a^2x^2\operatorname{sgn}(ax+1)\operatorname{sgn}(x}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="gi
ac")
```

output
```
1/6*(24*a*log(abs(a*x + 1))*sgn(a*x + 1)*sgn(x) - 24*a*log(abs(x))*sgn(a*x
 + 1)*sgn(x) - (24*a^2*x^2*sgn(a*x + 1)*sgn(x) - 9*a*x*sgn(a*x + 1)*sgn(x)
 + 2*sgn(a*x + 1)*sgn(x))/(a^2*x^3))*sqrt(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^3}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.31

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx =\frac{\sqrt{c}\,(24\log(ax+1)\,a^3x^3-24\log(ax)\,a^3x^3+8a^3x^3-24a^2x^2+9ax-2)}{6a\,x^3}$$

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

output
```
(sqrt(c)*(24*log(a*x + 1)*a**3*x**3 - 24*log(a*x)*a**3*x**3 + 8*a**3*x**3
- 24*a**2*x**2 + 9*a*x - 2))/(6*a*x**3)
```

3.902 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 221

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4a\sqrt{1-\frac{1}{a^2x^2}}x^4}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

output

```
-1/4*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^4+(c-c/a^2/x^2)^(1/2)/(1-
1/a^2/x^2)^(1/2)/x^3-2*a*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)/x^2+4*a^2
*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)/x+4*a^3*(c-c/a^2/x^2)^(1/2)*ln(x)
/(1-1/a^2/x^2)^(1/2)-4*a^3*(c-c/a^2/x^2)^(1/2)*ln(a*x+1)/(1-1/a^2/x^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.34

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4ax^4}+\frac{1}{x^3}-\frac{2a}{x^2}+\frac{4a^2}{x}+4a^3\log(x)-4a^3\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/4*1/(a*x^4) + x^(-3) - (2*a)/x^2 + (4*a^2)/x +
4*a^3*Log[x] - 4*a^3*Log[1 + a*x]))/Sqrt[1 - 1/(a^2*x^2)]
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.36, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}}{x^4}\,dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^5(ax+1)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(-\frac{4a^5}{ax+1}+\frac{4a^4}{x}-\frac{4a^3}{x^2}+\frac{4a^2}{x^3}-\frac{3a}{x^4}+\frac{1}{x^5}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(4a^4\log(x)-4a^4\log(ax+1)+\frac{4a^3}{x}-\frac{2a^2}{x^2}+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/4*1/x^4 + a/x^3 - (2*a^2)/x^2 + (4*a^3)/x + 4*a
^4*Log[x] - 4*a^4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.44

method	result	size
default	$-\frac{\left(16\ln \left(a x +1\right) x^{4} a^{4}-16\ln \left(x \right) x^{4} a^{4}-16 a^{3} x^{3}+8 a^{2} x^{2}-4 a x +1\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{4 \left(a x -1\right)^{2} x^{3}}$	98

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOS
E)
```

output
```
-1/4*(16*ln(a*x+1)*x^4*a^4-16*ln(x)*x^4*a^4-16*a^3*x^3+8*a^2*x^2-4*a*x+1)*
(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^
3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.45

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{16\,a^5\sqrt{c}x^4\log\left(\frac{2\,a^3cx^2+2\,a^2cx-\sqrt{a^2c}(2\,ax+1)\sqrt{c}+ac}{ax^2+x}\right)+\left(16\,a^3x^3-8\,a^2x^2+4\,ax-1\right)\sqrt{a^2c}}{4\,a^2x^4}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fr
icas")
```

output
```
1/4*(16*a^5*sqrt(c)*x^4*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(a^2*c)*(2*a*x
+ 1)*sqrt(c) + a*c)/(a*x^2 + x)) + (16*a^3*x^3 - 8*a^2*x^2 + 4*a*x - 1)*sq
rt(a^2*c))/(a^2*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.48

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx =$$
$$-\frac{1}{4}\left(16\,a^2\log\left(|ax+1|\right)\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)-16\,a^2\log\left(|x|\right)\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)-\frac{16\,a^3x^3\mathrm{sgn}(ax+1)\,\mathrm{s}}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="gi
ac")
```

output
```
-1/4*(16*a^2*log(abs(a*x + 1))*sgn(a*x + 1)*sgn(x) - 16*a^2*log(abs(x))*sg
n(a*x + 1)*sgn(x) - (16*a^3*x^3*sgn(a*x + 1)*sgn(x) - 8*a^2*x^2*sgn(a*x +
1)*sgn(x) + 4*a*x*sgn(a*x + 1)*sgn(x) - sgn(a*x + 1)*sgn(x))/(a^2*x^4))*sq
rt(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^4}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.30

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$
$$= \frac{\sqrt{c}\left(-16\log(ax+1)\,a^4x^4+16\log(ax)\,a^4x^4-4a^4x^4+16a^3x^3-8a^2x^2+4ax-1\right)}{4a\,x^4}$$

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

output
```
(sqrt(c)*( - 16*log(a*x + 1)*a**4*x**4 + 16*log(a*x)*a**4*x**4 - 4*a**4*x*
*4 + 16*a**3*x**3 - 8*a**2*x**2 + 4*a*x - 1))/(4*a*x**4)
```

3.903 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 263

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx =& -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5a\sqrt{1-\frac{1}{a^2x^2}}x^5}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}x^4}\\ &-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}x^3}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x^2}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}x}\\ &-\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

output
```
-1/5*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)/x^5+3/4*(c-c/a^2/x^2)^(1/2)
/(1-1/a^2/x^2)^(1/2)/x^4-4/3*a*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)/x^3
+2*a^2*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)/x^2-4*a^3*(c-c/a^2/x^2)^(1/
2)/(1-1/a^2/x^2)^(1/2)/x-4*a^4*(c-c/a^2/x^2)^(1/2)*ln(x)/(1-1/a^2/x^2)^(1/
2)+4*a^4*(c-c/a^2/x^2)^(1/2)*ln(a*x+1)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.34

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{5ax^5}+\frac{3}{4x^4}-\frac{4a}{3x^3}+\frac{2a^2}{x^2}-\frac{4a^3}{x}-4a^4\log(x)+4a^4\log(1+ax)\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/5*1/(a*x^5) + 3/(4*x^4) - (4*a)/(3*x^3) + (2*a^
2)/x^2 - (4*a^3)/x - 4*a^4*Log[x] + 4*a^4*Log[1 + a*x]))/Sqrt[1 - 1/(a^2*x
^2)]
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.35, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6751, 6747, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}e^{-3\coth^{-1}(ax)}}{x^5}\,dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^5}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1-ax)^2}{x^6(ax+1)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(\frac{4a^6}{ax+1}-\frac{4a^5}{x}+\frac{4a^4}{x^2}-\frac{4a^3}{x^3}+\frac{4a^2}{x^4}-\frac{3a}{x^5}+\frac{1}{x^6}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-4a^5\log(x)+4a^5\log(ax+1)-\frac{4a^4}{x}+\frac{2a^3}{x^2}-\frac{4a^2}{3x^3}+\frac{3a}{4x^4}-\frac{1}{5x^5}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(-1/5*1/x^5 + (3*a)/(4*x^4) - (4*a^2)/(3*x^3) + (2*
a^3)/x^2 - (4*a^4)/x - 4*a^5*Log[x] + 4*a^5*Log[1 + a*x]))/(a*Sqrt[1 - 1/(
a^2*x^2)])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.40

method	result	size
default	$\frac{\left(240 \ln \left(a x +1\right) x^{5} a^{5}-240 a^{5} \ln \left(x \right) x^{5}-240 a^{4} x^{4}+120 a^{3} x^{3}-80 a^{2} x^{2}+45 a x -12\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{60 \left(a x -1\right)^{2} x^{4}}$	106

input
```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOS
E)
```

output
```
1/60*(240*ln(a*x+1)*x^5*a^5-240*a^5*ln(x)*x^5-240*a^4*x^4+120*a^3*x^3-80*a
^2*x^2+45*a*x-12)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^
(3/2)/(a*x-1)^2/x^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{240\,a^{6}\sqrt{c}x^{5}\log\left(\frac{2\,a^{3}cx^{2}+2\,a^{2}cx+\sqrt{a^{2}c}(2\,ax+1)\sqrt{c}+ac}{ax^{2}+x}\right)-(240\,a^{4}x^{4}-120\,a^{3}x^{3}+80\,a^{2}x^{2}-45\,ax+12)\sqrt{a^{2}c}}{60\,a^{2}x^{5}}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fr
icas")
```

output
```
1/60*(240*a^6*sqrt(c)*x^5*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(a^2*c)*(2*a*
x + 1)*sqrt(c) + a*c)/(a*x^2 + x)) - (240*a^4*x^4 - 120*a^3*x^3 + 80*a^2*x
^2 - 45*a*x + 12)*sqrt(a^2*c))/(a^2*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.47

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$
$$=\frac{1}{60}\left(240\,a^3\log\left(|ax+1|\right)\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)-240\,a^3\log\left(|x|\right)\mathrm{sgn}(ax+1)\,\mathrm{sgn}(x)-\frac{240\,a^4x^4\mathrm{sgn}(ax+1}{}\right.$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="gi
ac")
```

output
```
1/60*(240*a^3*log(abs(a*x + 1))*sgn(a*x + 1)*sgn(x) - 240*a^3*log(abs(x))*
sgn(a*x + 1)*sgn(x) - (240*a^4*x^4*sgn(a*x + 1)*sgn(x) - 120*a^3*x^3*sgn(a
*x + 1)*sgn(x) + 80*a^2*x^2*sgn(a*x + 1)*sgn(x) - 45*a*x*sgn(a*x + 1)*sgn(
x) + 12*sgn(a*x + 1)*sgn(x))/(a^2*x^5))*sqrt(c)*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^5}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.28

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\sqrt{c}\,(240\log(ax+1)\,a^5x^5-240\log(ax)\,a^5x^5+48a^5x^5-240a^4x^4+120a^3x^3-80a^2x^2+45ax-12)}{60a\,x^5}$$

input

```
int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x)
```

output

```
(sqrt(c)*(240*log(a*x + 1)*a**5*x**5 - 240*log(a*x)*a**5*x**5 + 48*a**5*x*
*5 - 240*a**4*x**4 + 120*a**3*x**3 - 80*a**2*x**2 + 45*a*x - 12))/(60*a*x*
*5)
```

3.904 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx$

Optimal result

Integrand size = 27, antiderivative size = 189

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx=\frac{ce^2\sqrt{c-\frac{c}{a^2x^2}}(ex)^{-2+m}}{a^3(2-m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{ce\sqrt{c-\frac{c}{a^2x^2}}(ex)^{-1+m}}{a^2(1-m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}(ex)^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}(ex)^{1+m}}{e(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
c*e^2*(c-c/a^2/x^2)^(1/2)*(e*x)^(-2+m)/a^3/(2-m)/(1-1/a^2/x^2)^(1/2)+c*e*(
c-c/a^2/x^2)^(1/2)*(e*x)^(-1+m)/a^2/(1-m)/(1-1/a^2/x^2)^(1/2)+c*(c-c/a^2/x
^2)^(1/2)*(e*x)^m/a/m/(1-1/a^2/x^2)^(1/2)+c*(c-c/a^2/x^2)^(1/2)*(e*x)^(1+m
)/e/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.51

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\left(-(1+ax)^3+\frac{a(-1+2m)x(m-2ax-a^2mx^2+(m+amx)^2)}{m(-1+m^2)}\right)}{a^3(-2+m)\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}$$

input
```
Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2)*(e*x)^m,x]
```

output
```
((c - c/(a^2*x^2))^(3/2)*(e*x)^m*(-(1 + a*x)^3 + (a*(-1 + 2*m)*x*(m - 2*a*
x - a^2*m*x^2 + (m + a*m*x)^2))/(m*(-1 + m^2))))/(a^3*(-2 + m)*(1 - 1/(a^2
*x^2))^(3/2)*x^2)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.60, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6751, 6747, 8, 25, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{3/2} e^{\coth^{-1}(ax)}(ex)^m\,dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}(ex)^m dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ex)^m(1-ax)(ax+1)^2}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{8}$$

$$\frac{ce^3\sqrt{c-\frac{c}{a^2x^2}}\int -(ex)^{m-3}(1-ax)(ax+1)^2dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{ce^3\sqrt{c-\frac{c}{a^2x^2}}\int (ex)^{m-3}(1-ax)(ax+1)^2dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{84}$$

$$-\frac{ce^3\sqrt{c-\frac{c}{a^2x^2}}\int\left((ex)^{m-3}+\frac{a(ex)^{m-2}}{e}-\frac{a^2(ex)^{m-1}}{e^2}-\frac{a^3(ex)^m}{e^3}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ce^3\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{a^3(ex)^{m+1}}{e^4(m+1)}-\frac{a^2(ex)^m}{e^3m}-\frac{a(ex)^{m-1}}{e^2(1-m)}-\frac{(ex)^{m-2}}{e(2-m)}\right)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2)*(e*x)^m,x]
```

output
```
-((c*e^3*Sqrt[c - c/(a^2*x^2)]*(-((e*x)^(-2 + m)/(e*(2 - m))) - (a*(e*x)^(
-1 + m))/(e^2*(1 - m)) - (a^2*(e*x)^m)/(e^3*m) - (a^3*(e*x)^(1 + m))/(e^4*
(1 + m))))/(a^3*Sqrt[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.85

method	result
orering	$\frac{\left(a^{3} m^{3} x^{3}-3 a^{3} m^{2} x^{3}+2 a^{3} m\,x^{3}+a^{2} x^{2} m^{3}-2 a^{2} m^{2} x^{2}-a^{2} m\,x^{2}-a\,m^{3} x+2 a^{2} x^{2}+a x\,m^{2}+2 a m x-m^{3}+m\right) x \left(c-\frac{c}{a^{2} x^{2}}\right)^{\frac{3}{2}} \left(e x\right)^{m}}{\left(1+m\right) m \left(m-1\right) \left(-2+m\right) \left(a x+1\right)^{2} \left(a x-1\right) \sqrt{\frac{a x-1}{a x+1}}}$
gosper	$\frac{x \left(a^{3} m^{3} x^{3}-3 a^{3} m^{2} x^{3}+2 a^{3} m\,x^{3}+a^{2} x^{2} m^{3}-2 a^{2} m^{2} x^{2}-a^{2} m\,x^{2}-a\,m^{3} x+2 a^{2} x^{2}+a x\,m^{2}+2 a m x-m^{3}+m\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{3}{2}} \left(e x\right)^{m}}{\left(1+m\right) m \left(m-1\right) \left(-2+m\right) \left(a x+1\right)^{2} \left(a x-1\right) \sqrt{\frac{a x-1}{a x+1}}}$
risch	$\frac{\sqrt{c}\, \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{\left(a x-1\right) \left(a x+1\right)}}\, \left(a x-1\right) \left(a^{3} m^{3} x^{3}-3 a^{3} m^{2} x^{3}+2 a^{3} m\,x^{3}+a^{2} x^{2} m^{3}-2 a^{2} m^{2} x^{2}-a^{2} m\,x^{2}-a\,m^{3} x+2 a^{2} x^{2}+a x\,m^{2}+2 a\right.}{\sqrt{\frac{a x-1}{a x+1}}\, \left(a^{2} x^{2}-1\right) a^{2} x m \left(1+m\right) \left(m-1\right) \left(-2+m\right)}$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2)*(e*x)^m,x,method=_RETURN
VERBOSE)
```

output
```
(a^3*m^3*x^3-3*a^3*m^2*x^3+2*a^3*m*x^3+a^2*m^3*x^2-2*a^2*m^2*x^2-a^2*m*x^2
-a*m^3*x+2*a^2*x^2+a*m^2*x+2*a*m*x-m^3+m)/(1+m)/m/(m-1)/(-2+m)/(a*x+1)^2/(
a*x-1)*x/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2)*(e*x)^m
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.11

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx =$$

$$-\frac{\left(cm^3 - (a^3cm^3 - 3\,a^3cm^2 + 2\,a^3cm)x^3 - (a^2cm^3 - 2\,a^2cm^2 - a^2cm + 2\,a^2c)x^2 - cm + (acm^3 - acm^2 -\right.}{(a^3m^4 - 2\,a^3m^3 - a^3m^2 + 2\,a^3m)x^2 - (a^2m^4 - 2\,a^2m^3 - a^2m^2 + 2\,a^2m)}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2)*(e*x)^m,x, algorit
hm="fricas")
```

output
```
-(c*m^3 - (a^3*c*m^3 - 3*a^3*c*m^2 + 2*a^3*c*m)*x^3 - (a^2*c*m^3 - 2*a^2*c
*m^2 - a^2*c*m + 2*a^2*c)*x^2 - c*m + (a*c*m^3 - a*c*m^2 - 2*a*c*m)*x)*(e*
x)^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/((a^3*m^4 -
 2*a^3*m^3 - a^3*m^2 + 2*a^3*m)*x^2 - (a^2*m^4 - 2*a^2*m^3 - a^2*m^2 + 2*a
^2*m)*x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(3/2)*(e*x)**m,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.09

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx=\frac{\left((m^3-3\,m^2+2\,m)a^3c^{\frac{3}{2}}e^mx^3+(m^3-2\,m^2-m+2)a^2c^{\frac{3}{2}}e^mx^2-(m^3-m^2-2\,m\right.}{(m^4-2\,m^3-m^2+2\,m)a^6x^5+(m^4-2\,m^3-m^2+2\,m)a^5x^4-(m^4-2\,m^3-m}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2)*(e*x)^m,x, algorit
hm="maxima")
```

output
```
((m^3 - 3*m^2 + 2*m)*a^3*c^(3/2)*e^m*x^3 + (m^3 - 2*m^2 - m + 2)*a^2*c^(3/
2)*e^m*x^2 - (m^3 - m^2 - 2*m)*a*c^(3/2)*e^m*x - (m^3 - m)*c^(3/2)*e^m)*(a
*x + 1)^2*(a*x - 1)*x^m/((m^4 - 2*m^3 - m^2 + 2*m)*a^6*x^5 + (m^4 - 2*m^3
- m^2 + 2*m)*a^5*x^4 - (m^4 - 2*m^3 - m^2 + 2*m)*a^4*x^3 - (m^4 - 2*m^3 -
m^2 + 2*m)*a^3*x^2)
```

Giac [F]

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx=\int\frac{(ex)^m\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2)*(e*x)^m,x, algorit
hm="giac")
```

output
```
integrate((e*x)^m*(c - c/(a^2*x^2))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx=\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,{(e\,x)}^m}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(3/2)*(e*x)^m)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(((c - c/(a^2*x^2))^(3/2)*(e*x)^m)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.67

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx=\frac{x^m e^m\sqrt{c}\,c(a^3m^3x^3-3a^3m^2x^3+2a^3m\,x^3+a^2m^3x^2-2a^2m^2x^2-a^2m\,x^2-a\,m^3x}{a^3m\,x^2\,(m^3-2m^2-m+2)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2)*(e*x)^m,x)
```

output
```
(x**m*e**m*sqrt(c)*c*(a**3*m**3*x**3 - 3*a**3*m**2*x**3 + 2*a**3*m*x**3 +
a**2*m**3*x**2 - 2*a**2*m**2*x**2 - a**2*m*x**2 + 2*a**2*x**2 - a*m**3*x +
 a*m**2*x + 2*a*m*x - m**3 + m))/(a**3*m*x**2*(m**3 - 2*m**2 - m + 2))
```

3.905 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx$

Optimal result

Integrand size = 27, antiderivative size = 87

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}(ex)^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(ex)^{1+m}}{e(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
(c-c/a^2/x^2)^(1/2)*(e*x)^m/a/m/(1-1/a^2/x^2)^(1/2)+(c-c/a^2/x^2)^(1/2)*(e
*x)^(1+m)/e/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.61

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}(ex)^m(1+m+amx)}{am(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*(e*x)^m,x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(e*x)^m*(1 + m + a*m*x))/(a*m*(1 + m)*Sqrt[1 - 1/(a
^2*x^2)])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6751, 6747, 8, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{\coth^{-1}(ax)}(ex)^m\,dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}(ex)^mdx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ex)^m(ax+1)}{x}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 8}$$

$$\frac{e\sqrt{c-\frac{c}{a^2x^2}}\int (ex)^{m-1}(ax+1)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{e\sqrt{c-\frac{c}{a^2x^2}}\int \left((ex)^{m-1}+\frac{a(ex)^m}{e}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{a(ex)^{m+1}}{e^2(m+1)}+\frac{(ex)^m}{em}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*(e*x)^m,x]
```

output
```
(e*Sqrt[c - c/(a^2*x^2)]*((e*x)^m/(e*m) + (a*(e*x)^(1 + m))/(e^2*(1 + m)))
)/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

method	result	size
orering	$\frac{(amx+m+1)x\sqrt{c-\frac{c}{a^2x^2}}\,(ex)^m}{(1+m)m(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	58
gosper	$\frac{x(amx+m+1)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,(ex)^m}{(1+m)m(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	64
risch	$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\sqrt{\frac{c\left(a^2x^2-1\right)}{(ax-1)(ax+1)}}\,(ax-1)(amx+m+1)(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{c}\,(a^2x^2-1)(1+m)m}$	103

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)*(e*x)^m,x,method=_RETURN
VERBOSE)
```

output
```
(a*m*x+m+1)/(1+m)/m/(a*x+1)*x/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)*
(e*x)^m
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx = -\frac{(amx^2+(m+1)x)(ex)^m\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{m^2-(am^2+am)x+m}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)*(e*x)^m,x, algorit
hm="fricas")
```

output
```
-(a*m*x^2 + (m + 1)*x)*(e*x)^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2))/(m^2 - (a*m^2 + a*m)*x + m)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2)*(e*x)**m,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.57

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx = \frac{\left(a\sqrt{c}e^m mx+\sqrt{c}e^m(m+1)\right)(ax+1)x^m}{(m^2+m)a^2x+(m^2+m)a}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)*(e*x)^m,x, algorit
hm="maxima")
```

output
```
(a*sqrt(c)*e^m*m*x + sqrt(c)*e^m*(m + 1))*(a*x + 1)*x^m/((m^2 + m)*a^2*x +
 (m^2 + m)*a)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)*(e*x)^m,x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx=\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(e\,x\right)}^m}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int(((c - c/(a^2*x^2))^(1/2)*(e*x)^m)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int(((c - c/(a^2*x^2))^(1/2)*(e*x)^m)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.31

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx=\frac{x^m e^m \sqrt{c}\,(amx+m+1)}{am\,(m+1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)*(e*x)^m,x)
```

output
```
(x**m*e**m*sqrt(c)*(a*m*x + m + 1))/(a*m*(m + 1))
```

3.906 $\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 58

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = -\frac{a\sqrt{1-\frac{1}{a^2x^2}}(ex)^{2+m}\,\text{Hypergeometric2F1}(1,2+m,3+m,ax)}{e^2(2+m)\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
-a*(1-1/a^2/x^2)^(1/2)*(e*x)^(2+m)*hypergeom([1, 2+m],[3+m],a*x)/e^2/(2+m)
/(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = -\frac{a\sqrt{1-\frac{1}{a^2x^2}}x^2(ex)^m\,\text{Hypergeometric2F1}(1,2+m,3+m,ax)}{(2+m)\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Integrate[(E^ArcCoth[a*x]*(e*x)^m)/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
-((a*Sqrt[1 - 1/(a^2*x^2)]*x^2*(e*x)^m*Hypergeometric2F1[1, 2 + m, 3 + m,
a*x])/((2 + m)*Sqrt[c - c/(a^2*x^2)]))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6751, 6747, 8, 25, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{1-\frac{1}{a^2x^2}}}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x(ex)^m}{1-ax}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 8}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{(ex)^{m+1}}{1-ax}dx}{e\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \frac{(ex)^{m+1}}{1-ax}dx}{e\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 74}$$

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}(ex)^{m+2}\text{Hypergeometric2F1}(1,m+2,m+3,ax)}{e^2(m+2)\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[(E^ArcCoth[a*x]*(e*x)^m)/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
-((a*Sqrt[1 - 1/(a^2*x^2)]*(e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m
, a*x])/(e^2*(2 + m)*Sqrt[c - c/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [F]

$$\int \frac{(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{c-\frac{c}{a^2x^2}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(c-c/a^2/x^2)^(1/2),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(c-c/a^2/x^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \int \frac{(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(c-c/a^2/x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
integral((e*x)^m*a^2*x^2*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 - c)/(a
^2*x^2))/(a^2*c*x^2 - 2*a*c*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(e*x)**m/(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(c-c/a^2/x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((e*x)^m/(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(c-c/a^2/x^2)^(1/2),x, algorit
hm="giac")
```

output
```
integrate((e*x)^m/(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{(e\,x)^m}{\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((e*x)^m/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((e*x)^m/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$
$$= \frac{e^m\sqrt{c}\,\left(x^m amx + x^m m + x^m + \left(\int \frac{x^m}{a\,x^2-x}dx\right)m^2 + \left(\int \frac{x^m}{a\,x^2-x}dx\right)m\right)}{acm\,(m+1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(c-c/a^2/x^2)^(1/2),x)
```

output
```
(e**m*sqrt(c)*(x**m*a*m*x + x**m*m + x**m + int(x**m/(a*x**2 - x),x)*m**2
+ int(x**m/(a*x**2 - x),x)*m))/(a*c*m*(m + 1))
```

3.907 $\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 149

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}(ex)^{4+m}\,\text{Hypergeometric2F1}\left(2,\frac{4+m}{2},\frac{6+m}{2},a^2x^2\right)}{ce^4(4+m)\sqrt{c-\frac{c}{a^2x^2}}} + \frac{a^4\sqrt{1-\frac{1}{a^2x^2}}(ex)^{5+m}\,\text{Hypergeometric2F1}\left(2,\frac{5+m}{2},\frac{7+m}{2},a^2x^2\right)}{ce^5(5+m)\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
a^3*(1-1/a^2/x^2)^(1/2)*(e*x)^(4+m)*hypergeom([2, 2+1/2*m],[3+1/2*m],a^2*x
^2)/c/e^4/(4+m)/(c-c/a^2/x^2)^(1/2)+a^4*(1-1/a^2/x^2)^(1/2)*(e*x)^(5+m)*hy
pergeom([2, 5/2+1/2*m],[7/2+1/2*m],a^2*x^2)/c/e^5/(5+m)/(c-c/a^2/x^2)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}x^4(ex)^m\left((5+m)\,\text{Hypergeometric2F1}\left(2,2+\frac{m}{2},3+\frac{m}{2},a^2x^2\right)+a(4\right.}{c(4+m)(5+m)\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Integrate[(E^ArcCoth[a*x]*(e*x)^m)/(c - c/(a^2*x^2))^(3/2),x]
```

output
```
(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^4*(e*x)^m*((5 + m)*Hypergeometric2F1[2, 2 + m
/2, 3 + m/2, a^2*x^2] + a*(4 + m)*x*Hypergeometric2F1[2, (5 + m)/2, (7 + m
)/2, a^2*x^2]))/(c*(4 + m)*(5 + m)*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6751, 6747, 8, 92, 82, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x^3(ex)^m}{(1-ax)^2(ax+1)}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{8}$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \frac{(ex)^{m+3}}{(1-ax)^2(ax+1)}dx}{ce^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$\Big\downarrow$ 92

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(\int \frac{(ex)^{m+3}}{(1-ax)^2(ax+1)^2}dx+\frac{a\int \frac{(ex)^{m+4}}{(1-ax)^2(ax+1)^2}dx}{e}\right)}{ce^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$\Big\downarrow$ 82

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(\int \frac{(ex)^{m+3}}{(1-a^2x^2)^2}dx+\frac{a\int \frac{(ex)^{m+4}}{(1-a^2x^2)^2}dx}{e}\right)}{ce^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$\Big\downarrow$ 278

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{a(ex)^{m+5}\text{Hypergeometric2F1}\left(2,\frac{m+5}{2},\frac{m+7}{2},a^2x^2\right)}{e^2(m+5)}+\frac{(ex)^{m+4}\text{Hypergeometric2F1}\left(2,\frac{m+4}{2},\frac{m+6}{2},a^2x^2\right)}{e(m+4)}\right)}{ce^3\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[(E^ArcCoth[a*x]*(e*x)^m)/(c - c/(a^2*x^2))^(3/2),x]
```

output
```
(a^3*Sqrt[1 - 1/(a^2*x^2)]*(((e*x)^(4 + m)*Hypergeometric2F1[2, (4 + m)/2,
 (6 + m)/2, a^2*x^2])/(e*(4 + m)) + (a*(e*x)^(5 + m)*Hypergeometric2F1[2,
(5 + m)/2, (7 + m)/2, a^2*x^2])/(e^2*(5 + m))))/(c*e^3*Sqrt[c - c/(a^2*x^2
)])
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 92
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Simp[a   Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Simp[b/f   In
t[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, f, m,
 n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[p] &&
!IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [F]

$$\int \frac{(ex)^m}{\sqrt{\frac{ax-1}{ax+1}}\,\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(c-c/a^2/x^2)^(3/2),x)
```

output
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(c-c/a^2/x^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(c-c/a^2/x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
integral((e*x)^m*a^4*x^4*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 - c)/(a
^2*x^2))/(a^4*c^2*x^4 - 2*a^3*c^2*x^3 + 2*a*c^2*x - c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/((a*x-1)/(a*x+1))**(1/2)*(e*x)**m/(c-c/a**2/x**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(c-c/a^2/x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
integrate((e*x)^m/((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(c-c/a^2/x^2)^(3/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{(e\,x)^m}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

input
```
int((e*x)^m/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

output
```
int((e*x)^m/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{e^m\sqrt{c}\left(x^m amx + x^m m + x^m - \left(\int \frac{x^m}{a^3x^4-a^2x^3-a\,x^2+x}dx\right)m^2 - \left(\int \frac{x^m}{a^3x^4-a^2x^3-a\,x^2+x}dx\right.\right.}{a\,c^2m\,(m+1)}$$

input
```
int(1/((a*x-1)/(a*x+1))^(1/2)*(e*x)^m/(c-c/a^2/x^2)^(3/2),x)
```

output
```
(e**m*sqrt(c)*(x**m*a*m*x + x**m*m + x**m - int(x**m/(a**3*x**4 - a**2*x**
3 - a*x**2 + x),x)*m**2 - int(x**m/(a**3*x**4 - a**2*x**3 - a*x**2 + x),x)
*m + 2*int((x**m*x)/(a**3*x**3 - a**2*x**2 - a*x + 1),x)*a**2*m**2 + 2*int
((x**m*x)/(a**3*x**3 - a**2*x**2 - a*x + 1),x)*a**2*m))/(a*c**2*m*(m + 1))
```

3.908 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx$

Optimal result

Integrand size = 29, antiderivative size = 191

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx=-\frac{ce^2\sqrt{c-\frac{c}{a^2x^2}}(ex)^{-2+m}}{a^3(2-m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{ce\sqrt{c-\frac{c}{a^2x^2}}(ex)^{-1+m}}{a^2(1-m)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}(ex)^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}(ex)^{1+m}}{e(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-c*e^2*(c-c/a^2/x^2)^(1/2)*(e*x)^(-2+m)/a^3/(2-m)/(1-1/a^2/x^2)^(1/2)+c*e*
(c-c/a^2/x^2)^(1/2)*(e*x)^(-1+m)/a^2/(1-m)/(1-1/a^2/x^2)^(1/2)-c*(c-c/a^2/
x^2)^(1/2)*(e*x)^m/a/m/(1-1/a^2/x^2)^(1/2)+c*(c-c/a^2/x^2)^(1/2)*(e*x)^(1+
m)/e/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.65

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx=\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^{-m}(ex)^m\left(\frac{x^{-2+m}(-1+ax)^3}{2-m}-\frac{(-a(2-m)+a(1+m))\left(-\frac{x^{-1+m}}{1-m}-\frac{2ax^m}{m}+\frac{a^2x^{1+m}}{1+m}\right)}{2-m}\right)}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

input
```
Integrate[((c - c/(a^2*x^2))^(3/2)*(e*x)^m)/E^ArcCoth[a*x],x]
```

output
```
((c - c/(a^2*x^2))^(3/2)*(e*x)^m*((x^(-2 + m)*(-1 + a*x)^3)/(2 - m) - ((-(
a*(2 - m)) + a*(1 + m))*(-(x^(-1 + m)/(1 - m)) - (2*a*x^m)/m + (a^2*x^(1 +
 m))/(1 + m)))/(2 - m)))/(a^3*(1 - 1/(a^2*x^2))^(3/2)*x^m)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.58, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {6751, 6747, 8, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^{3/2} e^{-\coth^{-1}(ax)}(ex)^m \, dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}(ex)^m dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{6747}$$

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(ex)^m(1-ax)^2(ax+1)}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{8}$$

$$\frac{ce^3\sqrt{c-\frac{c}{a^2x^2}}\int (ex)^{m-3}(1-ax)^2(ax+1)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{84}$$

$$\frac{ce^3\sqrt{c-\frac{c}{a^2x^2}}\int \left((ex)^{m-3}-\frac{a(ex)^{m-2}}{e}-\frac{a^2(ex)^{m-1}}{e^2}+\frac{a^3(ex)^m}{e^3}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ce^3\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{a^3(ex)^{m+1}}{e^4(m+1)}-\frac{a^2(ex)^m}{e^3m}+\frac{a(ex)^{m-1}}{e^2(1-m)}-\frac{(ex)^{m-2}}{e(2-m)}\right)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[((c - c/(a^2*x^2))^(3/2)*(e*x)^m)/E^ArcCoth[a*x],x]
```

output
```
(c*e^3*Sqrt[c - c/(a^2*x^2)]*(-((e*x)^(-2 + m)/(e*(2 - m))) + (a*(e*x)^(-1
 + m))/(e^2*(1 - m)) - (a^2*(e*x)^m)/(e^3*m) + (a^3*(e*x)^(1 + m))/(e^4*(1
 + m))))/(a^3*Sqrt[1 - 1/(a^2*x^2)])
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.84

method	result
orering	$\frac{(a^3m^3x^3-3a^3m^2x^3+2a^3m\,x^3-a^2x^2m^3+2a^2m^2x^2+a^2m\,x^2-a\,m^3x-2a^2x^2+ax\,m^2+2amx+m^3-m)x\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}(ex)^m\sqrt{\frac{ax-1}{ax+1}}}{(1+m)m(m-1)(-2+m)(ax+1)(ax-1)^2}$
gosper	$\frac{x(a^3m^3x^3-3a^3m^2x^3+2a^3m\,x^3-a^2x^2m^3+2a^2m^2x^2+a^2m\,x^2-a\,m^3x-2a^2x^2+ax\,m^2+2amx+m^3-m)\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{3}{2}}(ex)^m\sqrt{}}{(1+m)m(m-1)(-2+m)(ax+1)(ax-1)^2}$
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{c}\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\sqrt{\frac{c(a^2x^2-1)}{(ax-1)(ax+1)}}(ax+1)(a^3m^3x^3-3a^3m^2x^3+2a^3m\,x^3-a^2x^2m^3+2a^2m^2x^2+a^2m\,x^2-a\,m^3x-2a^2x^2+a}{(a^2x^2-1)a^2xm(1+m)(m-1)(-2+m)}$

input
```
int((c-c/a^2/x^2)^(3/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
(a^3*m^3*x^3-3*a^3*m^2*x^3+2*a^3*m*x^3-a^2*m^3*x^2+2*a^2*m^2*x^2+a^2*m*x^2
-a*m^3*x-2*a^2*x^2+a*m^2*x+2*a*m*x+m^3-m)/(1+m)/m/(m-1)/(-2+m)/(a*x+1)/(a*
x-1)^2*x*(c-c/a^2/x^2)^(3/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.09

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx=\frac{(cm^3+(a^3cm^3-3\,a^3cm^2+2\,a^3cm)x^3-(a^2cm^3-2\,a^2cm^2-a^2cm+2\,a^2c)x^2-c}{(a^3m^4-2\,a^3m^3-a^3m^2+2\,a^3m)x^2-(a^2m^4-2\,a^2}$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm
="fricas")
```

output
```
(c*m^3 + (a^3*c*m^3 - 3*a^3*c*m^2 + 2*a^3*c*m)*x^3 - (a^2*c*m^3 - 2*a^2*c*
m^2 - a^2*c*m + 2*a^2*c)*x^2 - c*m - (a*c*m^3 - a*c*m^2 - 2*a*c*m)*x)*(e*x
)^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/((a^3*m^4 -
2*a^3*m^3 - a^3*m^2 + 2*a^3*m)*x^2 - (a^2*m^4 - 2*a^2*m^3 - a^2*m^2 + 2*a^
2*m)*x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx=\text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(3/2)*(e*x)**m*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.08

$$\int e^{-\coth^{-1}(ax)}\Big(c-\frac{c}{a^2x^2}\Big)^{3/2}(ex)^m\,dx=\frac{\left((m^3-3\,m^2+2\,m)a^3c^{\frac{3}{2}}e^mx^3-(m^3-2\,m^2-m+2)a^2c^{\frac{3}{2}}e^mx^2-(m^3-m^2-2\,m\right.}{(m^4-2\,m^3-m^2+2\,m)a^6x^5-(m^4-2\,m^3-m^2+2\,m)a^5x^4-(m^4-2\,m^3-m}$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm
="maxima")
```

output
```
((m^3 - 3*m^2 + 2*m)*a^3*c^(3/2)*e^m*x^3 - (m^3 - 2*m^2 - m + 2)*a^2*c^(3/
2)*e^m*x^2 - (m^3 - m^2 - 2*m)*a*c^(3/2)*e^m*x + (m^3 - m)*c^(3/2)*e^m)*(a
*x + 1)*(a*x - 1)^2*x^m/((m^4 - 2*m^3 - m^2 + 2*m)*a^6*x^5 - (m^4 - 2*m^3
- m^2 + 2*m)*a^5*x^4 - (m^4 - 2*m^3 - m^2 + 2*m)*a^4*x^3 + (m^4 - 2*m^3 -
m^2 + 2*m)*a^3*x^2)
```

Giac [F]

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx=\int (ex)^m\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

input
```
integrate((c-c/a^2/x^2)^(3/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm
="giac")
```

output
```
integrate((e*x)^m*(c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx=\int \left(c-\frac{c}{a^2\,x^2}\right)^{3/2}(e\,x)^m\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(3/2)*(e*x)^m*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(3/2)*(e*x)^m*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.66

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ex)^m\,dx=\frac{x^m e^m\sqrt{c}\,c(a^3m^3x^3-3a^3m^2x^3+2a^3m\,x^3-a^2m^3x^2+2a^2m^2x^2+a^2m\,x^2-a\,m^3x}{a^3m\,x^2\,(m^3-2m^2-m+2)}$$

input
```
int((c-c/a^2/x^2)^(3/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2),x)
```

output

```
(x**m*e**m*sqrt(c)*c*(a**3*m**3*x**3 - 3*a**3*m**2*x**3 + 2*a**3*m*x**3 -
a**2*m**3*x**2 + 2*a**2*m**2*x**2 + a**2*m*x**2 - 2*a**2*x**2 - a*m**3*x +
 a*m**2*x + 2*a*m*x + m**3 - m))/(a**3*m*x**2*(m**3 - 2*m**2 - m + 2))
```

3.909 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx$

Optimal result

Integrand size = 29, antiderivative size = 88

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx=-\frac{\sqrt{c-\frac{c}{a^2x^2}}(ex)^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}(ex)^{1+m}}{e(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

output
```
-(c-c/a^2/x^2)^(1/2)*(e*x)^m/a/m/(1-1/a^2/x^2)^(1/2)+(c-c/a^2/x^2)^(1/2)*(
e*x)^(1+m)/e/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}(ex)^m(-1+m(-1+ax))}{am(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Integrate[(Sqrt[c - c/(a^2*x^2)]*(e*x)^m)/E^ArcCoth[a*x],x]
```

output
```
(Sqrt[c - c/(a^2*x^2)]*(e*x)^m*(-1 + m*(-1 + a*x)))/(a*m*(1 + m)*Sqrt[1 -
1/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6751, 6747, 8, 25, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{-\coth^{-1}(ax)}(ex)^m\,dx$$

$$\Big\downarrow \ 6751$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}(ex)^mdx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 6747$$

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int -\frac{(ex)^m(1-ax)}{x}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 8$$

$$\frac{e\sqrt{c-\frac{c}{a^2x^2}}\int -(ex)^{m-1}(1-ax)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 25$$

$$-\frac{e\sqrt{c-\frac{c}{a^2x^2}}\int (ex)^{m-1}(1-ax)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 53$$

$$-\frac{e\sqrt{c-\frac{c}{a^2x^2}}\int \left((ex)^{m-1}-\frac{a(ex)^m}{e}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{e\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{(ex)^m}{em}-\frac{a(ex)^{m+1}}{e^2(m+1)}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

input
```
Int[(Sqrt[c - c/(a^2*x^2)]*(e*x)^m)/E^ArcCoth[a*x],x]
```

output
```
-((e*Sqrt[c - c/(a^2*x^2)]*((e*x)^m/(e*m) - (a*(e*x)^(1 + m))/(e^2*(1 + m)
)))/(a*Sqrt[1 - 1/(a^2*x^2)]))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.68

method	result	size
orering	$\frac{(amx-m-1)x\sqrt{c-\frac{c}{a^{2}x^{2}}}\,(ex)^{m}\sqrt{\frac{ax-1}{ax+1}}}{m(1+m)(ax-1)}$	60
gosper	$\frac{x(amx-m-1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,(ex)^{m}\sqrt{\frac{ax-1}{ax+1}}}{m(1+m)(ax-1)}$	66
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{(ax-1)(ax+1)}}\,(ax+1)(amx-m-1)(ex)^{m}}{\sqrt{c}\,(a^{2}x^{2}-1)m(1+m)}$	105

input
```
int((c-c/a^2/x^2)^(1/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
(a*m*x-m-1)/m/(1+m)/(a*x-1)*x*(c-c/a^2/x^2)^(1/2)*(e*x)^m*((a*x-1)/(a*x+1)
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^{2}x^{2}}}(ex)^{m}\,dx=-\frac{(amx^{2}-(m+1)x)(ex)^{m}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{m^{2}-(am^{2}+am)x+m}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm
="fricas")
```

output
```
-(a*m*x^2 - (m + 1)*x)*(e*x)^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2))/(m^2 - (a*m^2 + a*m)*x + m)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx = \text{Timed out}$$

input
```
integrate((c-c/a**2/x**2)**(1/2)*(e*x)**m*((a*x-1)/(a*x+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx = \frac{\left(a\sqrt{c}e^m mx-\sqrt{c}e^m(m+1)\right)(ax-1)x^m}{(m^2+m)a^2x-(m^2+m)a}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm
="maxima")
```

output
```
(a*sqrt(c)*e^m*m*x - sqrt(c)*e^m*(m + 1))*(a*x - 1)*x^m/((m^2 + m)*a^2*x -
 (m^2 + m)*a)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c/a^2/x^2)^(1/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx=\int \sqrt{c-\frac{c}{a^2\,x^2}}\,(e\,x)^m\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

input
```
int((c - c/(a^2*x^2))^(1/2)*(e*x)^m*((a*x - 1)/(a*x + 1))^(1/2),x)
```

output
```
int((c - c/(a^2*x^2))^(1/2)*(e*x)^m*((a*x - 1)/(a*x + 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.33

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}(ex)^m\,dx=\frac{x^m e^m\sqrt{c}\,(amx-m-1)}{am\,(m+1)}$$

input
```
int((c-c/a^2/x^2)^(1/2)*(e*x)^m*((a*x-1)/(a*x+1))^(1/2),x)
```

output
```
(x**m*e**m*sqrt(c)*(a*m*x - m - 1))/(a*m*(m + 1))
```

3.910 $\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}} dx$

Optimal result

Integrand size = 29, antiderivative size = 58

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}} dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}(ex)^{2+m}\text{Hypergeometric2F1}(1,2+m,3+m,-ax)}{e^2(2+m)\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
a*(1-1/a^2/x^2)^(1/2)*(e*x)^(2+m)*hypergeom([1, 2+m],[3+m],-a*x)/e^2/(2+m)
/(c-c/a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}} dx = \frac{a\sqrt{1-\frac{1}{a^2x^2}}x^2(ex)^m\text{Hypergeometric2F1}(1,2+m,3+m,-ax)}{(2+m)\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Integrate[(e*x)^m/(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*x^2*(e*x)^m*Hypergeometric2F1[1, 2 + m, 3 + m, -(
a*x)])/((2 + m)*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {6751, 6747, 8, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\sqrt{1-\frac{1}{a^2x^2}}}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \frac{x(ex)^m}{ax+1}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 8}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\int \frac{(ex)^{m+1}}{ax+1}dx}{e\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 74}$$

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}(ex)^{m+2}\text{Hypergeometric2F1}(1,m+2,m+3,-ax)}{e^2(m+2)\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[(e*x)^m/(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
```

output
```
(a*Sqrt[1 - 1/(a^2*x^2)]*(e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m,
-(a*x)])/(e^2*(2 + m)*Sqrt[c - c/(a^2*x^2)])
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [F]

$$\int \frac{(ex)^m \sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

input
```
int((e*x)^m*((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x)
```

output
```
int((e*x)^m*((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{(ex)^m\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate((e*x)^m*((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm
="fricas")
```

output
```
integral((e*x)^m*a^2*x^2*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 - c)/(a
^2*x^2))/(a^2*c*x^2 - c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\text{Timed out}$$

input
```
integrate((e*x)**m*((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{(ex)^m\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate((e*x)^m*((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((e*x)^m*sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a^2*x^2)), x)
```

Giac [F]

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{(ex)^m\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate((e*x)^m*((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm
="giac")
```

output
```
integrate((e*x)^m*sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a^2*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{(e\,x)^m\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\,dx$$

input
```
int(((e*x)^m*((a*x - 1)/(a*x + 1))^(1/2))/(c - c/(a^2*x^2))^(1/2),x)
```

output
```
int(((e*x)^m*((a*x - 1)/(a*x + 1))^(1/2))/(c - c/(a^2*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{e^m\sqrt{c}\left(x^m amx-x^m m-x^m+\left(\int \frac{x^m}{a\,x^2+x}dx\right)m^2+\left(\int \frac{x^m}{a\,x^2+x}dx\right)m\right)}{acm\,(m+1)}$$

input
```
int((e*x)^m*((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x)
```

output
```
(e**m*sqrt(c)*(x**m*a*m*x - x**m*m - x**m + int(x**m/(a*x**2 + x),x)*m**2
+ int(x**m/(a*x**2 + x),x)*m))/(a*c*m*(m + 1))
```

3.911 $\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 150

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = -\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}(ex)^{4+m}\,\text{Hypergeometric2F1}\left(2,\frac{4+m}{2},\frac{6+m}{2},a^2x^2\right)}{ce^4(4+m)\sqrt{c-\frac{c}{a^2x^2}}} + \frac{a^4\sqrt{1-\frac{1}{a^2x^2}}(ex)^{5+m}\,\text{Hypergeometric2F1}\left(2,\frac{5+m}{2},\frac{7+m}{2},a^2x^2\right)}{ce^5(5+m)\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
-a^3*(1-1/a^2/x^2)^(1/2)*(e*x)^(4+m)*hypergeom([2, 2+1/2*m],[3+1/2*m],a^2*
x^2)/c/e^4/(4+m)/(c-c/a^2/x^2)^(1/2)+a^4*(1-1/a^2/x^2)^(1/2)*(e*x)^(5+m)*h
ypergeom([2, 5/2+1/2*m],[7/2+1/2*m],a^2*x^2)/c/e^5/(5+m)/(c-c/a^2/x^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.75

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}x^4(ex)^m\left(-\left((5+m)\,\text{Hypergeometric2F1}\left(2,2+\frac{m}{2},3+\frac{m}{2},a^2x^2\right)\right)\right.}{c(4+m)(5+m)\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Integrate[(e*x)^m/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
```

output
```
(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^4*(e*x)^m*(-((5 + m)*Hypergeometric2F1[2, 2 +
 m/2, 3 + m/2, a^2*x^2]) + a*(4 + m)*x*Hypergeometric2F1[2, (5 + m)/2, (7
+ m)/2, a^2*x^2]))/(c*(4 + m)*(5 + m)*Sqrt[c - c/(a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {6751, 6747, 8, 25, 92, 82, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 6747}$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{x^3(ex)^m}{(1-ax)(ax+1)^2}dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{ 8}$$

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int -\frac{(ex)^{m+3}}{(1-ax)(ax+1)^2}dx}{ce^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\int \frac{(ex)^{m+3}}{(1-ax)(ax+1)^2}dx}{ce^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \; 92$$

$$-\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(\int \frac{(ex)^{m+3}}{(1-ax)^2(ax+1)^2}dx-\frac{a\int \frac{(ex)^{m+4}}{(1-ax)^2(ax+1)^2}dx}{e}\right)}{ce^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \; 82$$

$$-\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(\int \frac{(ex)^{m+3}}{(1-a^2x^2)^2}dx-\frac{a\int \frac{(ex)^{m+4}}{(1-a^2x^2)^2}dx}{e}\right)}{ce^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \; 278$$

$$-\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{(ex)^{m+4}\text{Hypergeometric2F1}\left(2,\frac{m+4}{2},\frac{m+6}{2},a^2x^2\right)}{e(m+4)}-\frac{a(ex)^{m+5}\text{Hypergeometric2F1}\left(2,\frac{m+5}{2},\frac{m+7}{2},a^2x^2\right)}{e^2(m+5)}\right)}{ce^3\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[(e*x)^m/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
```

output
```
-((a^3*Sqrt[1 - 1/(a^2*x^2)]*(((e*x)^(4 + m)*Hypergeometric2F1[2, (4 + m)/
2, (6 + m)/2, a^2*x^2])/(e*(4 + m)) - (a*(e*x)^(5 + m)*Hypergeometric2F1[2
, (5 + m)/2, (7 + m)/2, a^2*x^2])/(e^2*(5 + m))))/(c*e^3*Sqrt[c - c/(a^2*x
^2)]))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 92
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Simp[a   Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Simp[b/f   In
t[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, f, m,
 n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[p] &&
!IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6747
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Simp[c^p/a^(2*p)   Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
 + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Inte
gerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [F]

$$\int \frac{(ex)^m \sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m*((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x)
```

output
```
int((e*x)^m*((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \int \frac{(ex)^m \sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm
="fricas")
```

output
```
integral((e*x)^m*a^4*x^4*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 - c)/(a
^2*x^2))/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{(ex)^m\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm
="maxima")
```

output
```
integrate((e*x)^m*sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m*((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx=\int \frac{(e\,x)^m\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}}\,dx$$

input
```
int(((e*x)^m*((a*x - 1)/(a*x + 1))^(1/2))/(c - c/(a^2*x^2))^(3/2),x)
```

output
```
int(((e*x)^m*((a*x - 1)/(a*x + 1))^(1/2))/(c - c/(a^2*x^2))^(3/2), x)
```

Reduce [F]

$$\int \frac{e^{-\coth^{-1}(ax)}(ex)^m}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{e^m\sqrt{c}\left(x^m amx - x^m m - x^m - \left(\int \frac{x^m}{a^3x^4+a^2x^3-a\,x^2-x}dx\right)m^2 - \left(\int \frac{x^m}{a^3x^4+a^2x^3-a\,x^2-x}a\right.}{a\,c^2m\,(m+1}$$

input
```
int((e*x)^m*((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x)
```

output
```
(e**m*sqrt(c)*(x**m*a*m*x - x**m*m - x**m - int(x**m/(a**3*x**4 + a**2*x**
3 - a*x**2 - x),x)*m**2 - int(x**m/(a**3*x**4 + a**2*x**3 - a*x**2 - x),x)
*m + 2*int((x**m*x)/(a**3*x**3 + a**2*x**2 - a*x - 1),x)*a**2*m**2 + 2*int
((x**m*x)/(a**3*x**3 + a**2*x**2 - a*x - 1),x)*a**2*m))/(a*c**2*m*(m + 1))
```

3.912 $\int e^{n \coth^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 154

$$\int e^{n \coth^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right) \, dx = \frac{4c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}\left(2, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)} - \frac{2^{1+\frac{n}{2}} c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2}, -\frac{n}{2}, 2-\frac{n}{2}, \frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}$$

output
```
4*c*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2
*n],(a-1/x)/(a+1/x))/a/(2-n)-2^(1+1/2*n)*c*(1-1/a/x)^(1-1/2*n)*hypergeom([
-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/a/(2-n)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.80

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$$
$$=\frac{ce^{n\coth^{-1}(ax)}\left(2ax+anx+e^{2\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\,\text{Hyp}\right.}{a}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

output
```
(c*E^(n*ArcCoth[a*x])*(2*a*x + a*n*x + E^(2*ArcCoth[a*x])*n*Hypergeometric
2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*Hypergeometric2F1[1
, n/2, 1 + n/2, E^(2*ArcCoth[a*x])] + 4*E^(2*ArcCoth[a*x])*Hypergeometric2
F1[2, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]))/(a*(2 + n))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.68, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6748, 139, 27, 88, 79, 168, 25, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \ 6748$$

$$-c\int \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n+2}{2}}x^2d\frac{1}{x}$$

$$\Big\downarrow \ 139$$

$$-c\left(\frac{\int \frac{\left(3a+\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}}{a}d\frac{1}{x}}{a^2}+\int \frac{\left(a+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}x^2}{a}d\frac{1}{x}\right)$$

$$\Big\downarrow 27$$

$$-c\left(\frac{\int \left(3a+\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}d\frac{1}{x}}{a^3}+\frac{\int \left(a+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}x^2d\frac{1}{x}}{a}\right)$$

$$\Big\downarrow 88$$

$$-c\left(\frac{-\frac{an\int(1-\frac{1}{ax})^{1-\frac{n}{2}}(1+\frac{1}{ax})^{\frac{n-2}{2}}d\frac{1}{x}}{2-n}-\frac{2a^2(\frac{1}{ax}+1)^{\frac{n-2}{2}}(1-\frac{1}{ax})^{2-\frac{n}{2}}}{2-n}}{a^3}+\frac{\int \left(a+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}x^2d\frac{1}{x}}{a}\right)$$

$$\Big\downarrow 79$$

$$-c\left(\frac{\int \left(a+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}x^2d\frac{1}{x}}{a}+\frac{\frac{a^22^{n/2}n(1-\frac{1}{ax})^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},2-\frac{n}{2},3-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{(2-n)(4-n)}-\frac{2a^2(1-\cdots}{\cdots}}{a^3}\right.$$

$$\Big\downarrow 168$$

$$-c\left(\frac{-\int -n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}xd\frac{1}{x}-ax\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}}{a}+\frac{\frac{a^22^{n/2}n(1-\frac{1}{ax})^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2\cdots}{}\right.}{(2-n)(4-n)}}{}\right.$$

$$\Big\downarrow 25$$

$$-c\left(\frac{\int n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}xd\frac{1}{x}-ax\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{a}+\frac{\frac{a^22^{n/2}n(1-\frac{1}{ax})^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},2\cdots\right.}{(2-n)(4-n)}}{a\cdots}\right.$$

$$\Big\downarrow 27$$

$$-c\left(\frac{n\int \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}xd\frac{1}{x}-ax\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{a}+\frac{\frac{a^22^{n/2}n(1-\frac{1}{ax})^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},2\cdots\right.}{(2-n)(4-n)}}{a\cdots}\right.$$

$$\Big\downarrow 141$$

$$-c\left(\frac{\frac{a^2 2^{n/2} n\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\text{Hypergeometric2F1}\left(\frac{2-n}{2},2-\frac{n}{2},3-\frac{n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{(2-n)(4-n)}-\frac{2a^2\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{2-n}}{a^3}+\frac{2n\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\text{Hy}}{2}\right.$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

output
```
-(c*((-(a*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*x) + (2*n*(1
- 1/(a*x))^((2 - n)/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[1, (-2
 + n)/2, n/2, (a + x^(-1))/(a - x^(-1))])/(2 - n))/a + ((-2*a^2*(1 - 1/(a*
x))^(2 - n/2)*(1 + 1/(a*x))^((-2 + n)/2))/(2 - n) + (2^(n/2)*a^2*n*(1 - 1/
(a*x))^(2 - n/2)*Hypergeometric2F1[(2 - n)/2, 2 - n/2, 3 - n/2, (a - x^(-1
))/(2*a)])/((2 - n)*(4 - n)))/a^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 139
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[f^(p - 1)/d^p   Int[(a + b*x)^m*((d*e*p - c*f*(p - 1) +
 d*f*x)/(c + d*x)^(m + 1)), x], x] + Simp[f^(p - 1)   Int[(a + b*x)^m*((e +
 f*x)^p/(c + d*x)^(m + 1))*ExpandToSum[f^(-p + 1)*(c + d*x)^(-p + 1) - (d*e
*p - c*f*(p - 1) + d*f*x)/(d^p*(e + f*x)^p), x], x], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && EqQ[m + n + p, 0] && ILtQ[p, 0] && (LtQ[m, 0] || SumS
implerQ[m, 1] ||  !(LtQ[n, 0] || SumSimplerQ[n, 1]))
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [F]

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$$

input
```
int(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x)
```

output
```
int(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\int\left(c-\frac{c}{a^2x^2}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*x^2), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\frac{c\left(\int a^2e^{n\operatorname{acoth}(ax)}\,dx+\int\left(-\frac{e^{n\operatorname{acoth}(ax)}}{x^2}\right)\,dx\right)}{a^2}$$

input
```
integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2),x)
```

output
```
c*(Integral(a**2*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x))/x**2,
 x))/a**2
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\int\left(c-\frac{c}{a^2x^2}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
integrate((c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\int\left(c-\frac{c}{a^2x^2}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
integrate((c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a^2\,x^2}\right)\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - c/(a^2*x^2)),x)
```

output
```
int(exp(n*acoth(a*x))*(c - c/(a^2*x^2)), x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$$
$$=\frac{c\left(-e^{acoth(ax)n}a+\left(\int e^{acoth(ax)n}dx\right)a^2n+\left(\int\frac{e^{acoth(ax)n}}{a^2x^4-x^2}dx\right)n\right)}{a^2n}$$

input
```
int(exp(n*acoth(a*x))*(c-c/a^2/x^2),x)
```

output
```
(c*( - e**(acoth(a*x)*n)*a + int(e**(acoth(a*x)*n),x)*a**2*n + int(e**(aco
th(a*x)*n)/(a**2*x**4 - x**2),x)*n))/(a**2*n)
```

3.913 $\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=-\frac{(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{acn}+\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c}+\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{2+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{ac}$$

output
```
-(1+n)*(1+1/a/x)^(1/2*n)/a/c/n/((1-1/a/x)^(1/2*n))+(1+1/a/x)^(1/2*n)*x/c/(
(1-1/a/x)^(1/2*n))+2*(1+1/a/x)^(1/2*n)*hypergeom([1, 1/2*n],[1+1/2*n],(a+1
/x)/(a-1/x))/a/c/((1-1/a/x)^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.63

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$$
$$=\frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n^2\,\text{Hypergeometric2F1}\left(1,1+\frac{n}{2},2+\frac{n}{2},e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+anx+n\,\text{H}\right.\right.}{acn(2+n)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

output
```
(E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1, 1 + n/2,
2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + n*Hypergeometric2F1[1
, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6748, 144, 25, 27, 172, 25, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}x^2d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{144}$$

$$-\frac{x\left(-\left(\frac{1}{ax}+1\right)^{n/2}\right)\left(1-\frac{1}{ax}\right)^{-n/2}-\int -\frac{\left(an+\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}x}{a^2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\left(an+\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \left(an+\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}}{c}$$

$$\Big\downarrow \text{172}$$

$$-\frac{\frac{\frac{a(n+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}}{n}-\frac{a\int -n^2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}xd\frac{1}{x}}{n}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}}{c}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\frac{a\int n^2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}xd\frac{1}{x}}{n}+\frac{a(n+1)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}}{n}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}}{c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{an\int\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}xd\frac{1}{x}+\frac{a(n+1)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}}{n}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}}{c}$$

$$\Big\downarrow \text{141}$$

$$-\frac{\frac{\frac{a(n+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}}{n}-2a\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n+2}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{a^2}-x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}}{c}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

output
```
-((-(((1 + 1/(a*x))^(n/2)*x)/(1 - 1/(a*x))^(n/2)) + ((a*(1 + n)*(1 + 1/(a*
x))^(n/2))/(n*(1 - 1/(a*x))^(n/2)) - (2*a*(1 + 1/(a*x))^(n/2)*Hypergeometr
ic2F1[1, n/2, (2 + n)/2, (a + x^(-1))/(a - x^(-1))])/(1 - 1/(a*x))^(n/2))/
a^2)/c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 144
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[b*(a + b*x)^(m + 1)*(
c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x]
+ Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2))
- b*d*f*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && NeQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [F]

$$\int \frac{e^{n\ \mathrm{arccoth}(ax)}}{c-\frac{c}{a^2x^2}}dx$$

input
```
int(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x)
```

output
```
int(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{c-\frac{c}{a^2x^2}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="fricas")
```

output
```
integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \frac{a^2\int \frac{x^2e^{n\,\mathrm{acoth}\,(ax)}}{a^2x^2-1}\,dx}{c}$$

input
```
integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2),x)
```

output
```
a**2*Integral(x**2*exp(n*acoth(a*x))/(a**2*x**2 - 1), x)/c
```

Maxima [F]

$$\int \frac{e^{n \coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{c-\frac{c}{a^2x^2}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{n \coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Valu
e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{c-\frac{c}{a^2\,x^2}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - c/(a^2*x^2)),x)
```

output
```
int(exp(n*acoth(a*x))/(c - c/(a^2*x^2)), x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx=\frac{\left(\int \frac{e^{acoth(ax)n}x^2}{a^2x^2-1}dx\right)a^2}{c}$$

input
```
int(exp(n*acoth(a*x))/(c-c/a^2/x^2),x)
```

output
```
(int((e**(acoth(a*x)*n)*x**2)/(a**2*x**2 - 1),x)*a**2)/c
```

3.914 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 286

$$\begin{aligned}\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = &-\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} \\ &+\frac{(6+4n-n^2-n^3)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2n\,(4-n^2)} \\ &-\frac{(6+4n+n^2)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2n(2+n)} \\ &+\frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x}{c^2} \\ &+\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{2+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{ac^2}\end{aligned}$$

output
```
-(3+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(-1+1/2*n)/a/c^2/(2+n)+(-n^3-n^2+4*n
+6)*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)/a/c^2/n/(-n^2+4)-(n^2+4*n+6)*
(1+1/a/x)^(-1+1/2*n)/a/c^2/n/(2+n)/((1-1/a/x)^(1/2*n))+(1-1/a/x)^(-1-1/2*n
)*(1+1/a/x)^(-1+1/2*n)*x/c^2+2*(1+1/a/x)^(1/2*n)*hypergeom([1, 1/2*n],[1+1
/2*n],(a+1/x)/(a-1/x))/a/c^2/((1-1/a/x)^(1/2*n))
```

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.63

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{e^{n\coth^{-1}(ax)}\left(-6+n^2+6anx-an^3x+6a^2x^2-2a^2n^2x^2-4a^3nx^3+a^3n^3x^3+e^{2\coth^{-1}(ax)}(-2+n)n^2(-\right.}{a}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

output
```
(E^(n*ArcCoth[a*x])*(-6 + n^2 + 6*a*n*x - a*n^3*x + 6*a^2*x^2 - 2*a^2*n^2*
x^2 - 4*a^3*n*x^3 + a^3*n^3*x^3 + E^(2*ArcCoth[a*x])*(-2 + n)*n^2*(-1 + a^
2*x^2)*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + n*(-4
+ n^2)*(-1 + a^2*x^2)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x]
)]))/(a*c^2*(-2 + n)*n*(2 + n)*(-1 + a^2*x^2))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {6748, 144, 25, 27, 172, 25, 27, 172, 25, 27, 172, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}x^2d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{144}$$

$$-\frac{x\left(-\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}-\int -\frac{\left(an+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}x}{a^2}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{\left(an+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}x}{a^2}d\frac{1}{x}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{c^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\int \left(an+\frac{3}{x}\right)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}xd\frac{1}{x}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{c^2}$$

$$\Big\downarrow \ 172$$

$$-\frac{\frac{\frac{a(n+3)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{n+2}-\frac{a\int -\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}\left(an(n+2)+\frac{2(n+3)}{x}\right)x}{a}d\frac{1}{x}}{n+2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{c^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\frac{a\int \frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}\left(an(n+2)+\frac{2(n+3)}{x}\right)x}{a}d\frac{1}{x}}{n+2}+\frac{a(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{n+2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{c^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{\int \left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}\left(an(n+2)+\frac{2(n+3)}{x}\right)xd\frac{1}{x}}{n+2}+\frac{a(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{n+2}}{a^2}-x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{c^2}$$

$$\Big\downarrow \ 172$$

$$-\frac{\frac{\frac{\frac{a\left(n^2+4n+6\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{n}-\frac{a\int -\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}\left(a(n+2)n^2+\frac{n^2+4n+6}{x}\right)x}{a}d\frac{1}{x}}{n}}{n+2}+\frac{a(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{n+2}}{a^2}-x(}{c^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\frac{\frac{a\int \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}\left(a(n+2)n^2+\frac{n^2+4n+6}{x}\right)x}{a}d\frac{1}{x}}{n}+\frac{a\left(n^2+4n+6\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{n}}{n+2}+\frac{a(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{n+2}}{a^2}-x(1}{c^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{\frac{\int \left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-4}{2}}\left(a(n+2)n^2+\frac{n^2+4n+6}{x}\right)xd\frac{1}{x}}{n}+\frac{a\left(n^2+4n+6\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{n}}{n+2}+\frac{a(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{n+2}}{a^2} - x(1 -}{c^2}$$

$\downarrow$ 172

$$-\frac{\frac{\frac{\frac{a\int n^2\left(4-n^2\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}xd\frac{1}{x}}{2-n}-\frac{a\left(-n^3-n^2+4n+6\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{2-n}}{n}+\frac{a\left(n^2+4n+6\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{n}}{n+2}+\frac{a(n+3)\left(\frac{1}{a}\right.}{}}{a^2}}{c^2}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{\frac{an^2\left(4-n^2\right)\int\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{n-2}{2}}xd\frac{1}{x}}{2-n}-\frac{a\left(-n^3-n^2+4n+6\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{2-n}}{n}+\frac{a\left(n^2+4n+6\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}}{n}}{n+2}+\frac{a(n+3)\left(\frac{1}{a}\right.}{}}{a^2}}{c^2}$$

$\downarrow$ 141

$$-\frac{\frac{\frac{-\frac{2an\left(4-n^2\right)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}\text{Hypergeometric2F1}\left(1,\frac{n}{2},\frac{n+2}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{2-n}-\frac{a\left(-n^3-n^2+4n+6\right)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{2-n}}{n}+\frac{a\left(n^2+4n+6\right)\left(1-\frac{1}{ax}\right.}{n}}{n+2}}{a^2}}{c^2}$$

input
```
Int[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

output
```
-((-((1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*x) + ((a*(3 + n)*
(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2))/(2 + n) + ((a*(6 + 4*
n + n^2)*(1 + 1/(a*x))^((-2 + n)/2))/(n*(1 - 1/(a*x))^(n/2)) + (-((a*(6 +
4*n - n^2 - n^3)*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2))/(2 -
n)) - (2*a*n*(4 - n^2)*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, n/2, (2 +
n)/2, (a + x^(-1))/(a - x^(-1))])/((2 - n)*(1 - 1/(a*x))^(n/2)))/n)/(2 + n
))/a^2)/c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 144
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[b*(a + b*x)^(m + 1)*(
c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x]
+ Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2))
- b*d*f*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && NeQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\ \mathrm{arccoth}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}dx$$

input
```
int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x)
```

output
```
int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

output
```
integral(a^4*x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^4 - 2*a^2*c^2*x^
2 + c^2), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{x^4e^{n\,\mathrm{acoth}\,(ax)}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2)**2,x)
```

output
```
a**4*Integral(x**4*exp(n*acoth(a*x))/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**
2
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,2]%%%} Error: Bad Argument Valu
e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\left(c-\frac{c}{a^2\,x^2}\right)^2}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^2,x)
```

output
```
int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^2, x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{\left(\int \frac{e^{acoth(ax)n}x^4}{a^4x^4-2a^2x^2+1}dx\right)a^4}{c^2}$$

input
```
int(exp(n*acoth(a*x))/(c-c/a^2/x^2)^2,x)
```

output
```
(int((e**(acoth(a*x)*n)*x**4)/(a**4*x**4 - 2*a**2*x**2 + 1),x)*a**4)/c**2
```

3.915 $\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 295

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &+\frac{2n\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}}\\ &-\frac{2^{\frac{1+n}{2}}\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

output

```
(c-c/a^2/x^2)^(1/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(1/2+1/2*n)*x/(1-1/a^2
/x^2)^(1/2)+2*n*(c-c/a^2/x^2)^(1/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+
1/2*n)*hypergeom([1, 1/2-1/2*n],[3/2-1/2*n],(a-1/x)/(a+1/x))/a/(1-n)/(1-1/
a^2/x^2)^(1/2)-2^(1/2+1/2*n)*(c-c/a^2/x^2)^(1/2)*(1-1/a/x)^(1/2-1/2*n)*hyp
ergeom([1/2-1/2*n, 1/2-1/2*n],[3/2-1/2*n],1/2*(a-1/x)/a)/a/(1-n)/(1-1/a^2/
x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.49

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$
$$=\frac{ae^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{a^2x^2}}x^2\left(a(1+n)\sqrt{1-\frac{1}{a^2x^2}}x+2e^{\coth^{-1}(ax)}\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2}\right.\right.}{(1+n)\left(-1+a^2x^2\right)}$$

input
```
Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(a*E^(n*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a^2*x^2)]*x^2*(a*(
1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x + 2*E^ArcCoth[a*x]*Hypergeometric2F1[1, (1
+ n)/2, (3 + n)/2, -E^(2*ArcCoth[a*x])] + 2*E^ArcCoth[a*x]*n*Hypergeometri
c2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])]))/((1 + n)*(-1 + a^2*x^2
))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6751, 6748, 140, 79, 107, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-\frac{c}{a^2x^2}}e^{n\coth^{-1}(ax)}\,dx$$
$$\Big\downarrow \text{6751}$$
$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$\Big\downarrow \text{6748}$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n+1}{2}}x^2d\frac{1}{x}}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 140$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\int \left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}x^2d\frac{1}{x}-\frac{\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}d\frac{1}{x}}{a^2}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 79$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\int \left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}-1}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}x^2d\frac{1}{x}+\frac{2^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(1-n)}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 107$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{n\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}xd\frac{1}{x}}{a}+\frac{2^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2a}\right)}{a(1-n)}+x\left(-\left(\frac{1}{ax}+1\right.\right.}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$\Big\downarrow \; 141$$

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{2n\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)}+\frac{2^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-\right.}{a(1-n)}\right.}{\sqrt{1-\frac{1}{a^2x^2}}}$$

input

```
Int[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

output

```
-((Sqrt[c - c/(a^2*x^2)]*(-((1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((1 +
n)/2)*x) - (2*n*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*Hyper
geometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))])/(a*(1 -
n)) + (2^((1 + n)/2)*(1 - 1/(a*x))^((1 - n)/2)*Hypergeometric2F1[(1 - n)/2
, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(2*a)])/(a*(1 - n))))/Sqrt[1 - 1/(a^2
*x^2)])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [F]

$$\int e^{n\ \mathrm{arccoth}(ax)}\sqrt{c-\frac{c}{a^2x^2}}dx$$

input
```
int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x)
```

output
```
int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)), x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,\sqrt{c-\frac{c}{a^2\,x^2}}\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^(1/2),x)
```

output
```
int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^(1/2), x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c}\left(\int\frac{e^{acoth(ax)n}\sqrt{a^2x^2-1}}{x}dx\right)}{a}$$

input
```
int(exp(n*acoth(a*x))*(c-c/a^2/x^2)^(1/2),x)
```

output
```
(sqrt(c)*int((e**(acoth(a*x)*n)*sqrt(a**2*x**2 - 1))/x,x))/a
```

3.916 $\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}} dx$

Optimal result

Integrand size = 24, antiderivative size = 183

$$\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}} dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}} x}{\sqrt{c-\frac{c}{a^2x^2}}} + \frac{2n\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)} \text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{c-\frac{c}{a^2x^2}}}$$

output
```
(1-1/a^2/x^2)^(1/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(1/2+1/2*n)*x/(c-c/a^2
/x^2)^(1/2)+2*n*(1-1/a^2/x^2)^(1/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+
1/2*n)*hypergeom([1, 1/2-1/2*n],[3/2-1/2*n],(a-1/x)/(a+1/x))/a/(1-n)/(c-c/
a^2/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.61

$$\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}} dx = \frac{e^{n\coth^{-1}(ax)}(-1+a^2x^2)\left(a(1+n)\sqrt{1-\frac{1}{a^2x^2}}x+2e^{\coth^{-1}(ax)}n\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},e^{2\coth^{-1}(a
}{a^3(1+n)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{a^2x^2}}x^2}$$

input
```
Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
(E^(n*ArcCoth[a*x])*(-1 + a^2*x^2)*(a*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x + 2*
E^ArcCoth[a*x]*n*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a
*x])]))/(a^3*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a^2*x^2)]*x^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6751, 6748, 107, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}} dx$$

$$\Big\downarrow \text{6751}$$

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \text{6748}$$

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}x^2d\frac{1}{x}}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \ 107$$

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{n\int \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}xd\frac{1}{x}}{a}-x\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+1}{2}}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$\Big\downarrow \ 141$$

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(x\left(-\left(\frac{1}{ax}+1\right)^{\frac{n+1}{2}}\right)\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}-\frac{2n\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

input
```
Int[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

output
```
-((Sqrt[1 - 1/(a^2*x^2)]*(-((1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((1 +
n)/2)*x) - (2*n*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*Hyper
geometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))])/(a*(1 -
n))))/Sqrt[c - c/(a^2*x^2)])
```

Defintions of rubi rules used

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}dx$$

input
```
int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x)
```

output
```
int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x
^2))/(a^2*c*x^2 - c), x)
```

Sympy [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

input
```
integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2)**(1/2),x)
```

output
```
Integral(exp(n*acoth(a*x))/sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))), x)
```

Maxima [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a^2*x^2)), x)
```

Giac [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a^2*x^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\,dx$$

input
```
int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^(1/2),x)
```

output
```
int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx=\frac{\left(\int \frac{e^{acoth(ax)n}x}{\sqrt{a^2x^2-1}}dx\right)a}{\sqrt{c}}$$

input
```
int(exp(n*acoth(a*x))/(c-c/a^2/x^2)^(1/2),x)
```

output
```
(int((e**(acoth(a*x)*n)*x)/sqrt(a**2*x**2 - 1),x)*a)/sqrt(c)
```

3.917 $\int e^{n \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int e^{n \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^p \, dx = \frac{2^{1+\frac{n}{2}+p} \left(1-\frac{1}{a^2x^2}\right)^{-p} \left(c-\frac{c}{a^2x^2}\right)^p \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}+p} \text{AppellF1}\left(1-\frac{n}{2}+p,2,-\frac{n}{2}-p,2-\frac{n}{2}+p,1-\frac{1}{ax},\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n+2p)}$$

output
```
2^(1+1/2*n+p)*(c-c/a^2/x^2)^p*(1-1/a/x)^(1-1/2*n+p)*AppellF1(1-1/2*n+p,-1/
2*n-p,2,2-1/2*n+p,1/2*(a-1/x)/a,1-1/a/x)/a/(2-n+2*p)/((1-1/a^2/x^2)^p)
```

Mathematica [F]

$$\int e^{n \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^p \, dx = \int e^{n \coth^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^p \, dx$$

input
```
Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

output
```
Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6751, 6748, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^p e^{n\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6751}$$

$$\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p dx$$

$$\Big\downarrow \text{6748}$$

$$-\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\int \left(1-\frac{1}{ax}\right)^{p-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{n}{2}+p}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{153}$$

$$-\frac{2^{-\frac{n}{2}+p+1}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+p+1}\text{AppellF1}\left(\frac{n}{2}+p+1,\frac{1}{2}(n-2p),2,\frac{n}{2}+p+2,\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2p+2)}$$

input
```
Int[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

output
```
-((2^(1 - n/2 + p)*(c - c/(a^2*x^2))^p*(1 + 1/(a*x))^(1 + n/2 + p)*AppellF
1[1 + n/2 + p, (n - 2*p)/2, 2, 2 + n/2 + p, (a + x^(-1))/(2*a), 1 + 1/(a*x
)])/(a*(2 + n + 2*p)*(1 - 1/(a^2*x^2))^p))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [F]

$$\int e^{n \operatorname{arccoth}(ax)} \left(c - \frac{c}{a^2 x^2}\right)^p dx$$

input
```
int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

output
```
int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

Fricas [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^(1/2*n)*((a^2*c*x^2 - c)/(a^2*x^2))^p, x)
```

Sympy [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p e^{n\operatorname{acoth}(ax)}\,dx$$

input
```
integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2)**p,x)
```

output
```
Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(n*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="maxima")
```

output
```
integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Giac [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

input
```
integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="giac")
```

output
```
integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int e^{n\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a^2\,x^2}\right)^p\,dx$$

input
```
int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^p,x)
```

output
```
int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^p, x)
```

Reduce [F]

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\frac{\int\frac{e^{acoth(ax)n}\left(a^2c\,x^2-c\right)^p}{x^{2p}}dx}{a^{2p}}$$

input
```
int(exp(n*acoth(a*x))*(c-c/a^2/x^2)^p,x)
```

output
```
int((e**(acoth(a*x)*n)*(a**2*c*x**2 - c)**p)/x**(2*p),x)/a**(2*p)
```

3.918 $\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx = \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1-\frac{1}{ax}\right)^{1+2p}\text{Hypergeometric2F1}\left(2,1+2p,2(1+p),1-\frac{1}{ax}\right)}{a(1+2p)}$$

output
```
(c-c/a^2/x^2)^p*(1-1/a/x)^(1+2*p)*hypergeom([2, 1+2*p],[2*p+2],1-1/a/x)/a/
(1+2*p)/((1-1/a^2/x^2)^p)
```

Mathematica [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx = \int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$$

input
```
Integrate[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]),x]
```

output
```
Integrate[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]), x]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6751, 6748, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^p e^{-2p\coth^{-1}(ax)}\,dx$$

$$\Big\downarrow \ 6751$$

$$\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p dx$$

$$\Big\downarrow \ 6748$$

$$-\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\int \left(1-\frac{1}{ax}\right)^{2p}x^2d\frac{1}{x}$$

$$\Big\downarrow \ 75$$

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{2p+1}\left(c-\frac{c}{a^2x^2}\right)^p \text{Hypergeometric2F1}\left(2,2p+1,2(p+1),1-\frac{1}{ax}\right)}{a(2p+1)}$$

input
```
Int[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]),x]
```

output
```
((c - c/(a^2*x^2))^p*(1 - 1/(a*x))^(1 + 2*p)*Hypergeometric2F1[2, 1 + 2*p,
 2*(1 + p), 1 - 1/(a*x)])/(a*(1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [F]

$$\int \left(c-\frac{c}{a^2x^2}\right)^p \mathrm{e}^{-2p\ \mathrm{arccoth}(ax)}dx$$

input
```
int((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x)
```

output
```
int((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x)
```

Fricas [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int \frac{\left(c-\frac{c}{a^2x^2}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

input
```
integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")
```

output
```
integral(((a^2*c*x^2 - c)/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Sympy [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \int \left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p e^{-2p\,\mathrm{acoth}\,(ax)}\,dx$$

input
```
integrate((c-c/a**2/x**2)**p/exp(2*p*acoth(a*x)),x)
```

output
```
Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(-2*p*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \int \frac{\left(c-\frac{c}{a^2x^2}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

input
```
integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
```

output
```
integrate((c - c/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Giac [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \int \frac{\left(c-\frac{c}{a^2x^2}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

input
```
integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
```

output
```
integrate((c - c/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int e^{-2\,p\,\mathrm{acoth}(a\,x)}\left(c-\frac{c}{a^2\,x^2}\right)^p\,dx$$

input
```
int(exp(-2*p*acoth(a*x))*(c - c/(a^2*x^2))^p,x)
```

output
```
int(exp(-2*p*acoth(a*x))*(c - c/(a^2*x^2))^p, x)
```

Reduce [F]

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\frac{\int\frac{(a^2c\,x^2-c)^p}{x^{2p}e^{2acoth(ax)p}}dx}{a^{2p}}$$

input
```
int((c-c/a^2/x^2)^p/exp(2*p*acoth(a*x)),x)
```

output
```
int((a**2*c*x**2 - c)**p/(x**(2*p)*e**(2*acoth(a*x)*p)),x)/a**(2*p)
```

3.919 $\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx = -\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1+\frac{1}{ax}\right)^{1+2p}\text{Hypergeometric2F1}\left(2,1+2p,2(1+p),1+\frac{1}{ax}\right)}{a(1+2p)}$$

output
```
-(c-c/a^2/x^2)^p*(1+1/a/x)^(1+2*p)*hypergeom([2, 1+2*p],[2*p+2],1+1/a/x)/a
/(1+2*p)/((1-1/a^2/x^2)^p)
```

Mathematica [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx = \int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$$

input
```
Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

output
```
Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p, x]
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6751, 6748, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c-\frac{c}{a^2x^2}\right)^p e^{2p\coth^{-1}(ax)}dx$$

$$\Big\downarrow \text{6751}$$

$$\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p dx$$

$$\Big\downarrow \text{6748}$$

$$-\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\int \left(1+\frac{1}{ax}\right)^{2p}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{75}$$

$$-\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{ax}+1\right)^{2p+1}\left(c-\frac{c}{a^2x^2}\right)^p \text{Hypergeometric2F1}\left(2,2p+1,2(p+1),1+\frac{1}{ax}\right)}{a(2p+1)}$$

input
```
Int[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

output
```
-(((c - c/(a^2*x^2))^p*(1 + 1/(a*x))^(1 + 2*p)*Hypergeometric2F1[2, 1 + 2*
p, 2*(1 + p), 1 + 1/(a*x)])/(a*(1 + 2*p)*(1 - 1/(a^2*x^2))^p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 6748
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Simp[-c^p   Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x
, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[
n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

rule 6751
```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Simp[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p])   Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] ||
GtQ[c, 0])
```

Maple [F]

$$\int e^{2p \operatorname{arccoth}(ax)} \left(c - \frac{c}{a^2 x^2}\right)^p dx$$

input
```
int(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

output
```
int(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

Fricas [F]

$$\int e^{2p \coth^{-1}(ax)} \left(c - \frac{c}{a^2 x^2}\right)^p dx = \int \left(c - \frac{c}{a^2 x^2}\right)^p \left(\frac{ax+1}{ax-1}\right)^p dx$$

input
```
integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="fricas")
```

output
```
integral(((a*x + 1)/(a*x - 1))^p*((a^2*c*x^2 - c)/(a^2*x^2))^p, x)
```

Sympy [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx=\int\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p e^{2p\operatorname{acoth}(ax)}\,dx$$

input
```
integrate(exp(2*p*acoth(a*x))*(c-c/a**2/x**2)**p,x)
```

output
```
Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(2*p*acoth(a*x)), x)
```

Maxima [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx=\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^p dx$$

input
```
integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="maxima")
```

output
```
integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^p, x)
```

Giac [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx=\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^p dx$$

input
```
integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="giac")
```

output
```
integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\int e^{2\,p\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a^2\,x^2}\right)^p\,dx$$

input
```
int(exp(2*p*acoth(a*x))*(c - c/(a^2*x^2))^p,x)
```

output
```
int(exp(2*p*acoth(a*x))*(c - c/(a^2*x^2))^p, x)
```

Reduce [F]

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\frac{\int\frac{e^{2acoth(ax)p}\left(a^2c\,x^2-c\right)^p}{x^{2p}}dx}{a^{2p}}$$

input
```
int(exp(2*p*acoth(a*x))*(c-c/a^2/x^2)^p,x)
```

output
```
int((e**(2*acoth(a*x)*p)*(a**2*c*x**2 - c)**p)/x**(2*p),x)/a**(2*p)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*        GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```python
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
      return expnType(expn.operands()[0])   #expnType(expn.args[0])
   elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
      if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   else:
      return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
   m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
   m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
   return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
   return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
   m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
   return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
   m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
   return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
   m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
   return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
         #is checked before calling the grading function that is passed.
         #but kept it here.
   m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
   return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
   return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file